The Möbius–Hilbert Framework: A Possible Unitary Resolution to the Black Hole Information Paradox

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Abstract

The black hole information paradox is a fundamental tension between general relativity and quantum mechanics. This paper proposes the Möbius–Hilbert Framework (MHF), which is a possible topological resolution grounded in Hilbert space geometry rather than a spacetime structure. In this model, information is not destroyed but coherently transferred across a non-orientable Möbius-like manifold connecting the observable spacetime (O) to a hidden Hilbert domain ($\mathcal{H}_{\mathrm{H}}$). The underlying physical mechanism is proposed to be an intense hypermagnetic field generated by the interaction between accretion disk plasma and ergospheric frame-dragging, acting as the physical realization of a unitary Möbius inversion operator ($\mathcal{U}_{\mathrm{M}}$). To demonstrate theoretical viability, numerical simulations of a simplified tripartite quantum system (source, hidden domain, and radiation) were performed using a constant information migration rate. These simulations demonstrate that the framework can, in principle, preserve information unitarily, reproducing a Page-like entropy curve. We emphasize that this toy model employs idealized parameters to establish a proof of concept; deriving realistic, time-dependent migration rates from general relativistic magnetohydrodynamic (GRMHD) physics is the subject of subsequent work. The framework reframes apparent information loss as an epistemic illusion arising from observational limitations—a concept with precedent in the study of chaotic dynamical systems. This topological approach offers a novel perspective on the paradox, though empirical validation through analog black hole experiments or astrophysical observations remains necessary.

Keywords: black hole information paradox; quantum information; unitary evolution; Hilbert space; Möbius manifold; Page curve

1. Introduction

The reconciliation of quantum mechanics and general relativity is a daunting problem in the field of theoretical physics. The black hole information paradox illustrates this problem.

Although general relativity predicts the inevitable collapse of matter into a spacetime singularity where information is lost [1], the principles of quantum mechanics forbid the destruction of information and require that the evolution of a system remains unitary. This paradox emerges from the prediction that black holes emit thermal Hawking radiation [2], which appears uncorrelated with infalling matter, implying a fundamental violation of unitarity. Although numerous proposals, such as holography [3], firewalls [4], and the ER = EPR (Einstein-Rosen = Einstein-Podolsky-Rosen) conjecture [5, 6], have been advanced to resolve this conflict, no consensus has been reached.

This work proposes a possible resolution, grounded in the topology of the Hilbert space rather than in the spacetime geometry. To this end, the Möbius–Hilbert framework presents a unitary model that reinterprets the black hole as a Möbius-like topological bridge connecting observable (O) spacetime with a hidden information sector. Within this framework, the classical singularity is replaced by the hidden Hilbert domain ($\mathcal{H}_{\mathrm{H}}$), a finite, continuous quantum space in which information is preserved. Consequently, information loss is replaced by a reversible, rotational transfer of quantum coherence across entangled subspaces.

The physical mechanism driving this transfer is an intense hypermagnetic field that acts as a mediator of the information migration into the hidden Hilbert domain. Superheated plasma jets contribute to the formation of this field as extreme energy in the accretion disk strips electrons from the matter [7]. The synergistic interaction between this plasma and the ergospheric rotation of the black hole creates immense torque that serves as the physical driver for a unitary transformation, termed the Möbius inversion operator ($\mathcal{U}_{\mathrm{M}}$).

Entangled particle pairs split apart as they encounter a hypermagnetic environment. One particle emits as Hawking radiation (R), while the other migrates to the hidden domain (H), In this space, it persists as a phase-conjugated "film negative"—a complete but unobservable imprint of the original state. This mechanism also maintains the potential for informational reconstitution. Because the escaping and hidden components remain entangled as a globally pure state, the original information is, in principle, fully recoverable through the restoration of phase coherence.

To demonstrate the theoretical viability of this framework, we present a simplified proof of concept model using a tripartite quantum system with constant information migration parameters. We emphasize that this represents an idealized scenario; realistic black hole environments would require time-dependent migration rates derived from evolving GRMHD conditions and black hole geometry. The question of whether such realistic conditions can sustain sufficient information transfer throughout evaporation is addressed in subsequent work.

Ultimately, this framework reframes the complete evaporation of a black hole not as a physical termination, but as an observational limit akin to an ocean-going ship disappearing over the horizon. What appears as evaporation to the point of nonexistence can be understood as an epistemic illusion—a byproduct of our perspectival limitation—as its structure fully transitions into the timeless, unobservable Hilbert domain ($\mathcal{H}_{\mathrm{H}}$) where it continues to exist (Fig. 1).

2. Theoretical Framework

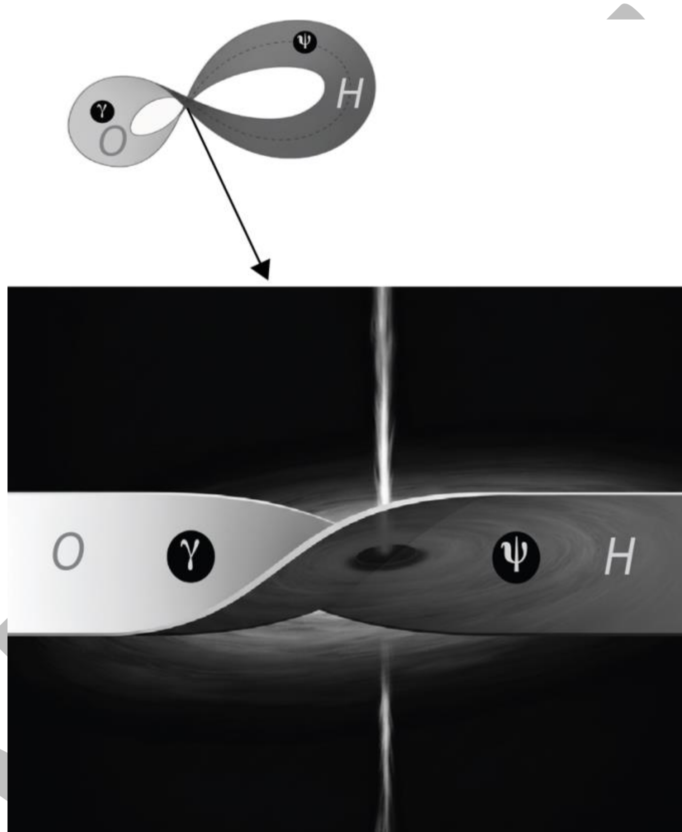

Fig. 1 Möbius–Hilbert manifold. A non-orientable manifold connects Observable Space (*O*) and the Hidden / Hilbert Space (*H*), with the twist representing the Event Horizon. An entangled pair is separated at the horizon: component γ is emitted as observable Hawking radiation (*R*), while its conjugate partner, ψ, is rotated into the hidden domain. This process, mediated by the unitary operator ($\mathcal{U}_{\mathrm{M}}$), preserves unitarity by reframing the singularity as a continuous topological transition.

2.1 Topological Reinterpretation of the Event Horizon

Instead of viewing the event horizon as an ending of information, the Möbius-Hilbert framework recognizes it as a region of topological non-orientability, geometrically analogous to a Möbius strip. The Möbius-Hilbert framework does not lead to a singularity. Instead, the model causes a topographical rotation of the local coordinate frame of the quantum state by means of a half-twist into the conjugate particle's orientation.

When an entangled particle pair approaches the horizon, its quantum state separates by this rotation: one component is emitted outward as Hawking radiation (R), while its conjugate counterpart migrates onto a hidden, complementary orientation of spacetime, which is defined as the hidden Hilbert domain ($\mathcal{H}_H$). Overall system coherence and unitarity are preserved. This preservation is due to the particles' continuing entanglement.

2.2 The Tripartite Hilbert Space

To formalize this process, we model the system using a tripartite Hilbert space decomposition:

$$\mathcal{H} = \mathcal{H}_s \otimes \mathcal{H}_H \otimes \mathcal{H}_R$$

where the subspaces represent:

- $\mathcal{H}_s$: The Hilbert space of the source (black hole's interior).
- $\mathcal{H}_H$: The Hilbert space of the hidden domain (phase-conjugated information).
- $\mathcal{H}_R$: The Hilbert space of the observable Hawking radiation (R) that escapes to infinity.

Within this framework, the total system state, ρ_{SHR}, remains pure, while its subsystems are mixed. The mutual information between any two subsystems is given by $I(A{:}B) = S(A) + S(B) - S(AB)$, where $S(X) = -\mathrm{Tr}(\rho_X \log \rho_X)$ is the von Neumann entropy. A central prediction of this model is that the radiation (R) will remain more strongly correlated with the hidden domain (H) than with the interior, such that:

$$I(R{:}H) > I(R{:}S)$$

2.3 The Möbius Inversion Operator and its Physical Driver

The Möbius inversion operator ($\mathcal{U}_{\mathrm{M}}$) describes the topological rotation and unitary transformation of information. The operator exchanges entanglement between the observable radiation subspace (R) and the hidden domain (H) coherently, while preserving the total quantum state.

The mathematical basis for this type of rotating and charged spacetime is given by the Kerr-Newman metric [8]. This metric is derived from a static solution using the Janis-Newman algorithm [9]. The hypermagnetic torque is proposed to be generated as the superheated plasma **in** the accretion disk interacts with the extreme frame-dragging within the ergosphere.

The complex dynamics of this region, including the formation of equatorial current sheets and magnetic reconnection, are supported [10] and confirmed by recent ab initio kinetic simulations [11]. This process twists and amplifies the magnetic fields that serve as the physical mediator of information transfer, providing a direct physical realization of the abstract ($\mathcal{U}_{\mathrm{M}}$) operator.

2.4 The Singularity as a "Film Negative" and Informational Reconstitution

This approach substitutes a classical singularity (the point of infinite density) with the hidden Hilbert domain ($\mathcal{H}_H$), a finite Hilbert curvature region in which information remains in conjugate phase. The domain operates in a "film negative" form: a timeless, phase-inverted repository that keeps all the structural data of the infalling matter in an unobservable encoding. In the mathematical description of this relationship, the operator $\mathcal{U}_{\mathrm{M}}$ (Möbius inversion) takes hold. If $|\psi R$ is the state of the escaping radiation (R), we have its hidden complement, being the phase-conjugate partner:

$$|\psi_H\rangle = \mathcal{U}_{\mathrm{M}}|\ \psi^*{}_R$$

This formalism guarantees, at least in principle, the preservation of the information. Since these two subsystems are still entangled as components of a globally pure state, the original information can be thoroughly recovered by reconstructing phase coherence over the whole non-orientable manifold. The total state (i.e., the globally pure state), $|\Psi_{total}\rangle$, reconstructs the original information in full fidelity:

$$|\psi_{total}\rangle = |\psi_R\rangle \otimes \mathcal{U}_{\mathrm{M}}/\psi^*{}_R\rangle$$

This demonstrates that the framework is mathematically consistent and preserves unitarity in principle. However, whether this topological structure describes actual black holes remains an open empirical question.

So, the paradox of information loss is resolved by a higher-level unity, in which what was separated by observation remains, in principle, coherent and recoverable**.**

3.1 Model Design

The simulation is based on a simplified system of three coupled qubits for the core subsystems:

- ***S* (Source):** Black hole's interior degrees of freedom.
- ***H* (Hidden):** Hidden-domain degrees of freedom that store the conjugate information.
- ***R* (Radiation):** Externally observable Hawking radiation.

The initial state is a tripartite entangled vector. Two main parameters can be changed to govern system dynamics:

- **Hypermagnetic Influence (μB):** The rotation angle corresponding to the strength of the hypermagnetic torque that drives information transfer between the hidden (H) and (R) domains.

- **Dephasing Strength (p):** Model's observational decoherence, representing the observer's limited access to the global quantum state, the result of radiation subsystem noise application.

We emphasize that this model employs a constant hypermagnetic influence parameter (μ_{B}) to establish proof of concept and isolate the core topological mechanism. In a physically realistic black hole, the information migration rate would be time-dependent, evolving with the black hole's changing mass, spin, and GRMHD environment during "evaporation." This simplified constant-parameter approach allows us to demonstrate that the proposed mechanism is mathematically consistent and capable of preserving unitarity in principle.

3.2 Simulation Process and Analysis

The simulation evolves the system's density matrix, $\rho = |\psi\rangle\langle\psi|$, through a series of sequential transformations corresponding to hypermagnetic rotation and dephasing. Dephasing is applied specifically to the radiation (R) qubit using Kraus operators to model environmental interaction without breaking unitarity.

To quantify the flow of information, the von Neumann entropy (S) and mutual information (I) are computed by taking partial traces across the subsystems. The mutual information between any two subsystems A and B is given by:

$$I(A{:}B) = S(A) + S(B) - S(AB).$$

The primary quantities tracked are the mutual information between the radiation (R) and hidden (H) domains, $I(R{:}H)$, the radiation (R) and source (S) domains, $I(R{:}S)$, and the entropy of the radiation subsystem, $S(R)$, over iterative cycles. We denote each emission step as k**,** representing the discrete time steps in the black hole's simulated "evaporation" process.

3.3 Computational Implementation

All numerical experiments were executed in Python. The NumPy library was used for stable matrix exponentiation to evolve the density matrices, while Matplotlib was used for data visualization.

4. Results

Numerical simulations of the tripartite quantum system (S, H, R) confirm that the Möbius-Hilbert framework preserves informational continuity and restores unitarity throughout the black hole's evolution (Fig. 2). The key findings are organized as follows.

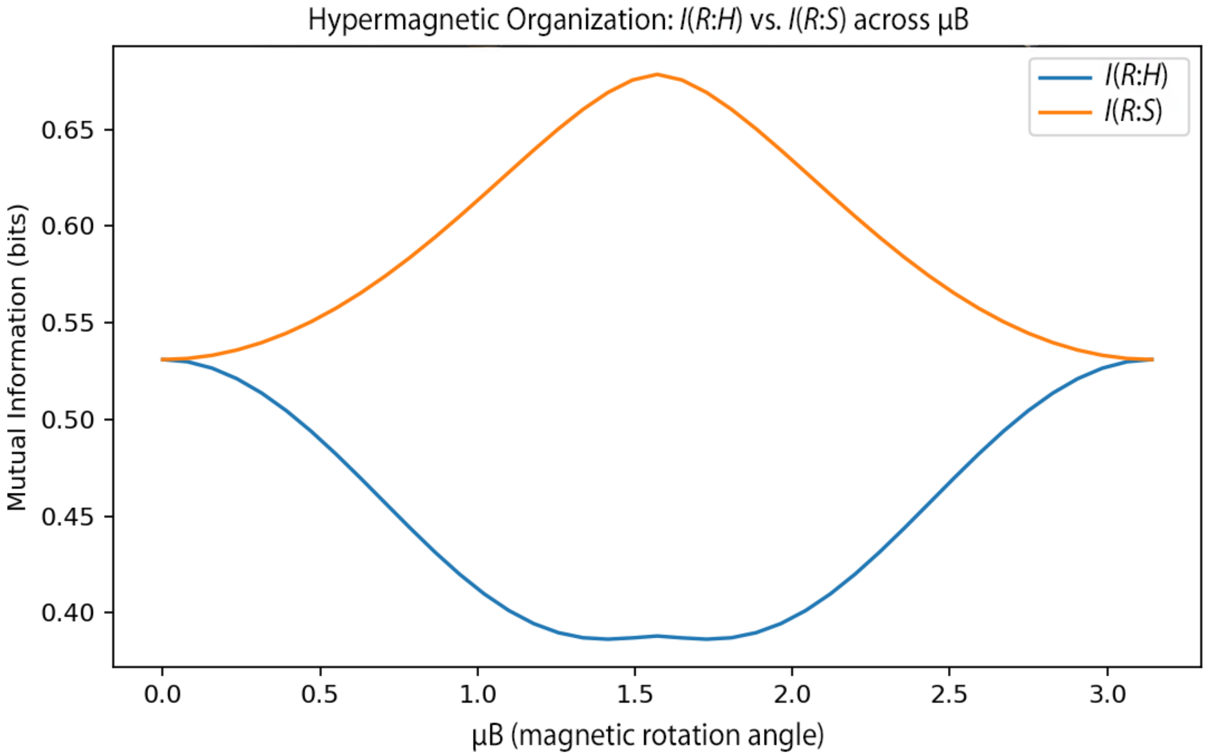

Fig. 2 Mutual information transfer driven by hypermagnetic rotation (μB). As rotation increases, the correlation with the Hidden domain (*I*(*R*:*H*), orange upper) surpasses the correlation with the Source (*I*(*R*:*S*), blue lower), demonstrating information migration.

4.1 Conservation and Transfer of Mutual Information

The primary result of the simulations is the demonstrated conservation of total mutual information. As the hypermagnetic rotation angle (μB) increases, mutual information is coherently transferred from the source to the hidden domain. The mutual information between the (R) and the hidden domain (*H*), $I(R\!:H)$, rises, while the mutual information between the radiation (R) and the source (*S*), $I(R\!:S)$, declines correspondingly. Throughout this process, $I(R\!:H)$, consistently exceeds $I(R\!:S)$, supporting the hypothesis that the outgoing radiation (R) consistently remains more strongly correlated with the unobservable hidden domain $(\mathcal{H}_H)$, than with the black hole's interior. This oscillatory conservation indicates a phase-coherent exchange between the hidden (*H*) and radiation (R) sectors, confirming that total correlation is preserved.

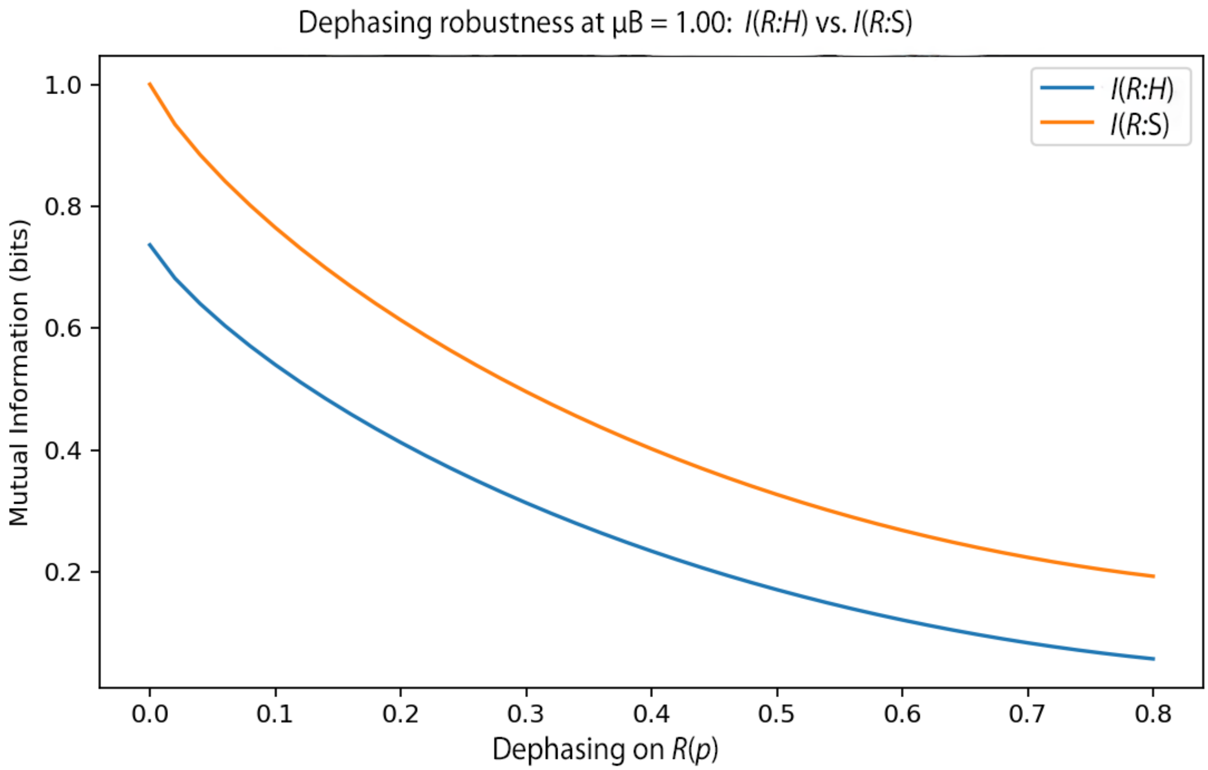

Fig. 3 Resilience to observational dephasing. The mutual information with the Hidden domain ($I(R{:}H)$, orange upper) remains dominant over the Source ($I(R{:}S)$, blue lower) as dephasing (p) increases, confirming the robustness of the information transfer.

4.2 Resilience to Observational Dephasing

The simulations demonstrated that the system's total correlation is resilient to moderate levels of observational dephasing. Applying dephasing noise to the radiation subsystem (R) resulted in coherence damping but did not cause total information loss until a critical threshold was reached. This suggests that the entanglement is redistributed across the total system rather than being destroyed by the observer's interaction.

4.3 Dynamic Reproduction of the Page Curve

To illustrate why an observer perceives a paradox, the following result (Fig. 4) confirms that this information is completely inaccessible when observing the radiation (R) in isolation.

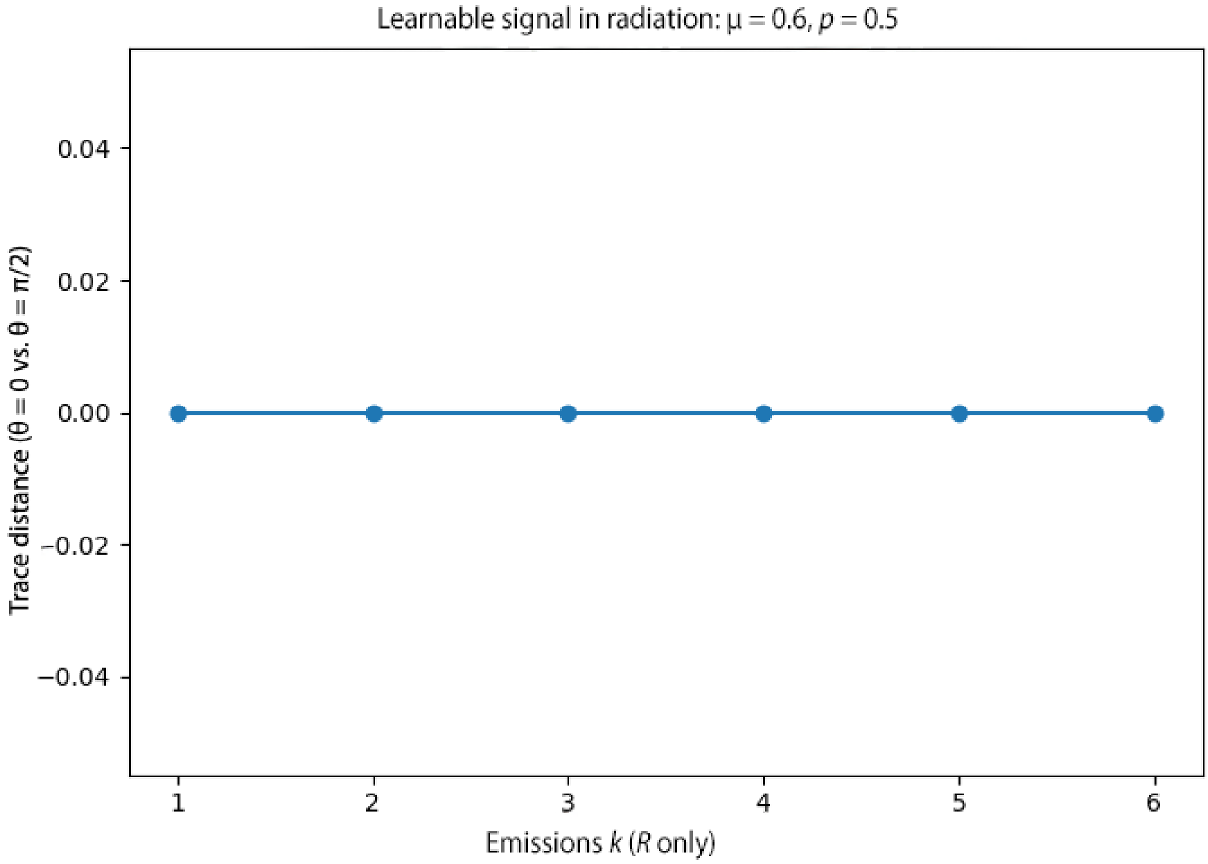

Fig. 4. Absence of learnable information in radiation alone. The trace distance between two encoded states of the radiation subsystem (R) is zero, showing that the information is not locally accessible to an observer.

When the Möbius inversion operator ($\mathcal{U}_{\mathrm{M}}$) was applied iteratively with constant μ_{B} to simulate the long-term "evaporation" of the black hole, the von Neumann entropy of the radiation subsystem ($S(R)$) was observed to follow a Page-like curve (Fig. 5) [12]. The entropy initially increases, consistent with thermal emission, and then decreases as information is dynamically recovered from the hidden domain. This result confirms that information recovery emerges directly from the unitary rotation of the Hilbert space.

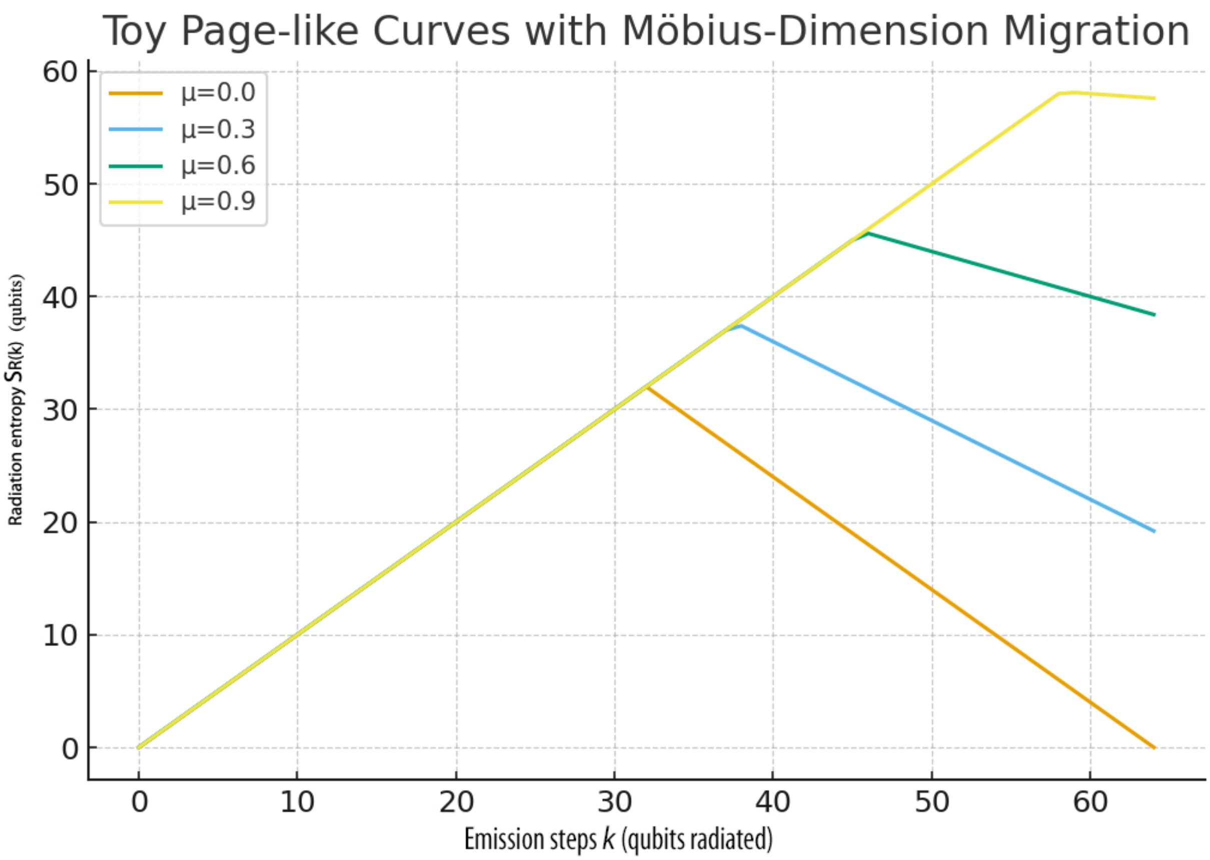

Fig. 5 Dynamical reproduction of the Page curve. The radiation entropy, $S_{R(k)}$, exhibits the characteristic rise and fall over emission steps (k), demonstrating the restoration of unitarity as information is recovered from the hidden domain. Different curves show the effect of varying constant migration rate (μ = 0.0, 0.3, 0.6, 0.9), with higher values producing more pronounced information recovery.

5. Discussion

Simulations are highly consistent with the Möbius-Hilbert framework as a possible unitary solution to the black hole information paradox. The observed conserved nature of mutual information and the dynamic repeatability of the Page curve under constant mitigation parameters indicate that information can, in principle, be transferred and preserved coherently in a hidden domain. This section interprets these results in relation to their physical and philosophical implications.

5.1 Reinterpreting the Singularity

This conceptual framework proposal drastically changes the nature of the black hole singularity. In contrast to a point of infinite density, where the laws of physics collapse [13], the singularity is reinterpreted as a transition point into the hidden Hilbert domain ($\mathcal{H}_H$). It is not a geometric point, but rather, a space with finite Hilbert curvature that is both continuous and unitary. This model proposes that the singularity acts as a "film negative"—a phase-inverted reflection of the external realm where information continues to persist in a conjugate, unobservable encoding. It appears not to be a domain of erasure, but one of encoded preservation.

This interpretation, while mathematically consistent within the MHF, remains a speculative proposal. The hidden Hilbert domain ($\mathcal{H}_H$) is not currently observable or directly testable with existing technology. The framework's value lies in providing an alternative conceptual structure that preserves unitarity mathematically. However, empirical validation of this topological structure for astrophysical black holes awaits future observational capabilities or analog experiments.

5.2 The Physical Mechanism of Information Transfer

A key strength of this model is its grounding in a physical mechanism. The extreme magnetic fields generated by the plasma jets, and amplified by the ergosphere's rotation, create a hypermagnetic torque that organizes the chaotic vacuum into an ordered flow of entanglement. This magnetic organization functions as the physical realization of the theoretical Möbius inversion operator ($\mathcal{U}_\mathrm{M}$), providing the force necessary to rotate one member of an entangled pair into the hidden domain. At the same time, its partner escapes as Hawking radiation (R). This process provides a constructive mathematical bridge between the spacetime curvature of general relativity and the quantum coherence of Hilbert space.

We emphasize that while the GRMHD environment (plasma jets, ergospheric rotation, magnetic field generation) is well-established astrophysics [7, 10, 11], the specific claim that these fields generate the unitary operator $\mathcal{U}_\mathrm{M}$ through the postulated equivalence $T_\mathrm{M} \approx \mathcal{U}_\mathrm{M}$ requires further theoretical development. The exact form of the interaction Hamiltonian and the quantum field theoretic derivation of how classical electromagnetic torques produce quantum unitary evolution represent important open questions for the framework. Investigation of whether realistic, time-dependent GRMHD scaling can provide sufficient information transfer throughout a black hole's "evaporation" is addressed in related work.

5.3 Relation to Existing Theories

The Möbius-Hilbert framework aligns with other theories while offering a novel perspective. The Möbius-like topological bridge that connects the observable (O) and hidden domains (H) serves a role analogous to that of the Einstein-Rosen bridge (wormhole) in the ER = EPR conjecture [5, 6], providing a topological connection that demonstrates preservation of quantum coherence. This framework is consistent, in part, with the holographic principle [3]. The distinction from the holographic perspective is that it reframes the mechanism as a geometric inversion into an unobservable domain (H) rather than an encoding on a lower-dimensional boundary surface. By proposing a smooth, unitary transfer of information across the event horizon, the framework provides a firewall-free resolution [4] to the paradox, making a destructive, high-energy barrier unnecessary.

5.4 Evaporation as an Epistemic Illusion

According to the Möbius-Hilbert framework, the information paradox is not the result of a physics failure, but rather of “directional blindness”—an observational limitation. What is referred to as information loss is a projection artifact, where an observer can see only one orientation of a globally entangled surface. Such observer measurement of Hawking radiation (R) essentially traces over the hidden domain, projecting a continuous, non-orientable geometry onto what appears to be a discontinuous local subspace.

This interpretation finds precedent in the behavior of perturbed chaotic dynamical systems. Recent work on hybrid Collatz-Lorenz systems [15] demonstrates that external perturbations can drive chaotic attractors through distinct regimes, including “Zones of Linearity” where complex, nonlinear dynamics become effectively linear and predictable from an observer’s limited perspective. Critically, the system’s internal memory (autocorrelation) determines whether its behavior appears simple or complex to an external observer, even though the underlying system remains fully deterministic and information-preserving. By analogy, what appears as “thermal” Hawking radiation—seemingly devoid of information—may represent an observer confined to a limited projection of a globally coherent, information-preserving state.

Thus, the complete evaporation of a black hole is not a physical termination but a perceptual limit. The apparent death of the black hole is shown to be an epistemic illusion. This reframing of a physical endpoint as a perspectival limit aligns closely with contemporary top-down cosmological models, which represent the universe’s fundamental properties within a framework that is observer-dependent and holographic [14] in a similar manner. Moreover, as the black hole has vanished from our observable manifold, its information and structure persist (on the hidden face) as a fully coherent entity that moved through this transition phase, from temporal to timeless existence. According to this idea, continuity is the single most fundamental invariant of existence: information can change shape or type, but it is not lost.

5.5 Future Work and Limitations

While demonstrating theoretical consistency through toy model simulations, empirical and computational limitations must guide future research.

The primary challenge lies in the model’s reliance on highly speculative, currently unverifiable entities. This paper does not offer empirical evidence for the key mechanisms of the proposed resolution: the theoretical Möbius inversion operator $(\mathcal{U}_\mathrm{M})$ or its physical realization, the hypermagnetic transfer mechanism. Perhaps the most daunting of these features is the hidden Hilbert domain $(\mathcal{H}_H)$, which modern physics cannot currently detect.

Notably, the simulation employs a constant information migration rate (μ_B). This is an idealization for demonstration purposes. A physically realistic black hole undergoes continuous evolution in mass $M(t)$ and spin $a(t)$ during its evaporation. The question of whether time-dependent GRMHD scaling can provide sufficient $\mu(t)$ to maintain unitary information recovery throughout the black hole’s lifetime is investigated in related work [16]. Preliminary results suggest that simple, local GRMHD scaling alone may be insufficient, indicating that the full MHF may require additional non-local mechanisms beyond the simplified equivalence postulate presented here.

The simulation utilizes a simplified toy model system of three coupled qubits for the source (S), hidden $\mathcal{H}$, and radiation (R) subsystems. An actual black hole would require a Hilbert space that is exponentially larger and more complex than the simplified model presented here. Therefore, it is currently unknown whether the conservation of mutual information and the recovery of unitarity observed in the simulation would hold at the full cosmological scale.

Future research should integrate this model with the concept of spacetime curvature. This integration can be accomplished by linking the information migration rate (μ) to surface gravity and the "evaporation" timeline of the black hole. Additionally, consideration of the influence of an intrinsic electric charge (Q) on a black hole is also important.

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Data Access

The Python simulation files, figures, and supplemental figures associated with this paper are available on GitHub (https://github.com/cgcaulkins/Mobius-Hilbert-Framework.git) and permanently archived on Zenodo (DOI: 10.5281/zenodo.17393450).

Conflicts of Interest

The author declares there are no conflicts of interest.

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References

1. Einstein, A.: Die grundlage der allgemeinen relativitätstheorie. Ann. Phys. 354, 769–822 (1916)
2. Hawking, S.W.: Particle creation by black holes. Commun. Math. Phys. 43, 199–220 (1975)
3. Susskind, L.: The world as a hologram. J. Math. Phys. 36, 6377–6396 (1995)
4. Almheiri, A., Marolf, D., Polchinski, J., Sully, J.: Black Holes: Complementarity or Firewalls? J. High Energy Phys. 2013, 62 (2013)
5. Maldacena, J., Susskind, L.: Cool horizons for entangled black holes. Fortschr. Phys. 61, 781–811 (2013)
6. Einstein, A., Podolsky, B., Rosen, N.: Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? Phys. Rev. 47, 777–780 (1935)
7. Frank, J., King, A., Raine, D.: Accretion Power in Astrophysics, 3rd edn. Cambridge University Press, Cambridge (2002)
8. Newman, E.T., Couch, R., Chinnapared, K., Exton, A., Prakash, A., Torrence, R.: Metric of a rotating, charged mass. J. Math. Phys. 6, 918–919 (1965)
9. Erbin, H.: Janis-Newman algorithm: simplifications and gauge field transformation. arXiv:1410.2602 (2015)
10. Manko, V.S., Mejía, I.M., Ruiz, E.: Metric of a rotating charged magnetized sphere. Phys. Lett. B 803, 135286 (2020)
11. Meringolo, C., Camilloni, F., Rezzolla, L.: Electromagnetic energy extraction from Kerr black holes: Ab Initio Calculations. Astrophys. J. Lett. 992, L8 (2025)
12. Page, D.N., Information in black hole radiation. Phys. Rev. Lett. 71, 3743–3746 (1993)
13. Penrose, R., Gravitational Collapse and Space-Time Singularities. Phys. Rev. Lett. 14, 57–59 (1965)
14. Hertog, T., On the Origin of Time: Stephen Hawking's Final Theory. Bantam Books, New York (2023)
15. Caulkins, C., Bounded chaos in perturbed Collatz-Lorenz sequences: Harnessing the power of complexity for applications in medicine, disaster mitigation, vehicle stabilization, and beyond. Chaos: An Interdisciplinary Journal of Nonlinear Science (submitted for publication, under review)

16. Caulkins, C., Geometric Unitary Evolution: Linking the Möbius Inversion Operator to Kerr Black Hole Dynamics (manuscript in preparation)