Theorem 2.5.

Given any scalar a and any three vectors P, Q, and R, the following properties hold.
(a) $P \cdot Q = Q \cdot P$
(b) $P \cdot P = ||P||^2$
(c) $|P \cdot Q| \leq ||P||||Q||$

$$\begin{aligned} ||P+Q||^2 &= (P+Q)\cdot(P+Q) \\ &= P^2 + Q^2 + 2P\cdot Q \\ &\leq P^2 + Q^2 + 2||P||||Q|| \\ &= (||P|| + ||Q||)^2 \end{aligned}$$

叉积定义

3D向量P,Q:

- $P \times Q = (P_yQ_z - P_zQ_y, P_zQ_x - P_xQ_z, P_xQ_y - P_yQ_x)$
- $||P \times Q|| = ||P||||Q||sin\alpha$
- $Q \times P = -P \times Q$
- $(aP) \times Q = a(P \times Q)$
- $P \times P = 0 = (0,0,0)$
- $P \times (Q \times P) = P \times Q \times P = P^2Q - (P \cdot Q)P$, 其中 $P^2 = P \cdot P = ||P||^2$
- $(P \times Q) \cdot R = (R \times P) \cdot Q = (Q \times R) \cdot P$

2D向量P,Q: $P \times Q = P_xQ_y - P_yQ_x$

二维叉积的几何意义 叉乘的几何意义是以两向量为邻边的平行四边形的有向面积。$P \times Q < 0$，则Q在P的顺时针方向; $P \times Q = 0$，则Q和P共线;$P \times Q > 0$，则Q在P的逆时针方向。

线性独立 **linearly independent**

Definition 2.11. A set of n vectors $e_1, e_2, \cdots, e_n$ is **linearly independent** if there do not exist real numbers $a_1, a_2, \cdots, a_n$, where at least one of the a_i is not
zero, such that
$a_1e_1 + a_2e_2 + \cdots + a_ne_n = 0$
Otherwise, the set $e_1, e_2, \cdots, e_n$ is called **linearly dependent(线性相关).**

> *Every basis of an n-dimensional vector space has exactly n vectors in it. For instance,it is impossible to find a set of four linearly independent vectors in $\mathbb{R}^3$, and a set of two linearly independent vectors is insufficient to generate the entire vector space.*

Given two nonzero vectorse_1 and e_2, if $e_1 \cdot e_2 = 0$, then e_1 and e_2 are linearly independent.

施密特正交化 **Gram-Schmidt Orthogonalization**

Given a set of n linearly independent vectors $B = \{e_1, e_2, \cdots, e_n\}$, this algorithm produces a set $B' = \{e'_1, e'_2, \cdots, e'_n\}$ such that