Emergent failures and cascades in power grids: a statistical physics perspective

Tommaso Nesti,[1] Alessandro Zocca,[2] and Bert Zwart[1]
[1] *CWI, Amsterdam 1098 XG, Netherlands*
[2] *California Institute of Technology, Pasadena, California 91125, USA*

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We model power grids transporting electricity generated by intermittent renewable sources as complex networks, where line failures can emerge indirectly by noisy power input at the nodes. By combining concepts from statistical physics and the physics of power flows, and taking weather correlations into account, we rank line failures according to their likelihood and establish the most likely way such failures occur and propagate. Our insights are mathematically rigorous in a small-noise limit and are validated with data from the German transmission grid.

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Understanding cascading failures in complex networks is of great importance and has received a lot of attention in recent years [1–17]. Despite proposing different mechanisms for their evolution, a common feature is that cascades are triggered by some *external* event. This initial attack is chosen either (i) deliberately, to target the most vulnerable or crucial network component or (ii) uniformly at random, to understand the average network reliability. This distinction led to the insight that complex networks are resilient to random attacks, but vulnerable to targeted attacks [7, 18, 19]. However, both lead to the *direct* failure of the attacked network component.

In this Letter, we focus on networks in which edge failures occur in a fundamentally different manner. Specifically, we consider networks where fluctuations of the node inputs can trigger edge failures. The realization (which we call configuration) of the noise at the nodes is not only

Previous works applying large-deviations techniques to problems in complex networks dynamics, such as epidemic extinction and biophysical networks, include [23, 24].

We model a transmission network by a connected graph G with n nodes representing the *buses* and m directed edges modeling *transmission lines*. The nominal values of net power injections at the nodes are given by $\boldsymbol{\mu} = \{\mu_i\}_{i=1,\dots,n}$. We model the stochastic fluctuation of the power injections around $\boldsymbol{\mu}$, due to variability in renewable generation, by means of the random vector $\mathbf{p} = \{p_i\}_{i=1,\dots,n}$, which is assumed to follow a multivariate Gaussian distribution with density

$$\varphi(\mathbf{x}) = \frac{\exp(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T(\varepsilon\boldsymbol{\Sigma}_p)^{-1}(\mathbf{x}-\boldsymbol{\mu}))}{(2\pi)^{\frac{n}{2}}\det(\varepsilon\boldsymbol{\Sigma}_p)^{\frac{1}{2}}}, \qquad (1)$$

with $\varepsilon\boldsymbol{\Sigma}_p \in \mathbb{R}^{n\times n}$ being the covariance matrix of $\mathbf{p}$. In