RADBOUD UNIVERSITY NIJMEGEN

FACULTY OF SCIENCE

Power Grid Failures & Stochastic Renewables

THESIS BSc MATHEMATICS

Supervisor:

dr. Henk DON

Author:

Fons VAN DER PLAS

Second reader:

prof. dr. Eric CATOR

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Notation

$\mathbf{Y}$ is a matrix or vector

$\mathrm{Y}_{i,j}$ is an element of a matrix or vecor

Y is a (real or complex) number

Y is a physical quantity (like 'admittance')

$[n] = \{1, \ldots, n\}$

Een geel doosje geeft aan dat een stuk tekst *nog geschreven moet worden.*

Een oranje doosje is meer *voor mezelf*, dingen waar ik later nog naar moet kijken.

Een groen doosje geeft *bronnen* aan die relevant (kunnen) zijn.

Contents

Part I

Theory of Electric Power Networks

Probability Theory

1.1   *Abstract Integration

Lebesgue-Stieltjes integral

See Rynne or Garling or Analyse de Hilbert et de Fourier course notes

1.2   *Probability Measures

Should cover: p. measure, cont. distr., stochastic vectors, E, moment gen?, CLT, multivar norm dist

See Pestman or MIT course notes or...

1.3   Large Deviation Theory

Should cover: application to multivariate normal dist

See Hugo or MIT course notes or ...

Chapter two

Graph theory

2.1 Directed graphs

When studying the flow of current in a network, it is useful to fix a direction for each line, relative to which the flow can be expressed. Semantically, a *positive* flow along an edge will mean a flow in the same direction as the edge, while a *negative* flow describes a 'positive' flow in the opposite direction. The choice of line direction could be seen as arbitrary.

Definition 2.1.1. A *directed graph* is a pair $(\mathcal{N}, \mathcal{L})$ such that the set of *nodes* $\mathcal{N}$ is finite, and the set of *lines* $\mathcal{L} \subseteq \mathcal{N} \times \mathcal{N}$ satisfies:

- If $(i, j) \in \mathcal{L}$, then $(j, i) \notin \mathcal{L}$. (i.e. there are no 'loops' between two nodes.)
- For each $i \in \mathcal{N}$: $(i, i) \notin \mathcal{L}$. (i.e. no node is directly connected to itself.)

Remark. Note that the second requirement follows from the first.

> undirected graph, subgraph, path, loop, euler path, connected, k-edge-connectivity, tree, minimum spanning subtree

> thm: every connected garph contains a spanning subtree

> thm: G is a tree iff m=n-1

> thm: removing a leaf from a tree results in a new tree

> thm: euleriaans pad in een boom is uniek

> If including maxwell derivation: graph topology, calculus on graphs

When flow is complex-valued, there is no pos/neg.

It turns out that this concept is called a 'Gain Graph' (wiki)

2.2 Flow

> intro

For the remainder of this section, assume that $G = (\mathcal{N}, \mathcal{L})$ is a directed graph, where the nodes are labelled $\mathcal{N} = \{1, 2, \ldots, n\} = [n]$ for some $n \in \mathbb{N}$.

The $m = \#\mathcal{L}$ lines of the network are labelled $\mathcal{L} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_m\}$.

Suppose $\mathbb{K}$ is a field.

Definition 2.2.1. An *injection* is an element of $\mathbb{K}^n$.

Definition 2.2.2. A *flow* is an element of $\mathbb{K}^m$.

semantics

When $\mathbb{K} = \mathbb{F}_2$, a flow can be seen as a subset of the collection of lines $\mathcal{L}$

Definition 2.2.3. An injection $\mathbf{p} \in \mathbb{K}^n$ is *induced* by a flow $\mathbf{f} \in \mathbb{K}^m$ if:

$$p_i = \sum_{\mathcal{L}_k = (i,j) \in \mathcal{L}} f_k - \sum_{\mathcal{L}_k = (j,i) \in \mathcal{L}} f_k \tag{2.1}$$

for each node $i \in \mathcal{N}$.

Note that $\mathbf{p}$ is uniquely defined for every choice of $\mathbf{f}$, under the condition that G has no *isolated nodes*: nodes with no lines connecting them. This allows us to define a function:

not necessary

Definition 2.2.4. Suppose G has no isolated nodes. We define the *Linear Power Flow* (LPF) as the function

$$\mathrm{LPF} : \mathbb{K}^m \to \mathbb{K}^n$$

which maps a flow $\mathbf{f} \in \mathbb{K}^m$ to the unique injection $\mathbf{p} \in \mathbb{K}^n$ that it induces.

From (2.1), it follows that LPF is a linear map. In fact, when written in matrix form, we find a familiar result: the transposed edge-vertex incidence matrix of G!

blub

$$\mathrm{LPF} = \mathbf{C}^T$$

The fact that LPF is a linear map raises an interesting question: how can the image and kernel of LPF be interpreted in the context of digraph flows? The reader is invited to revisit Example to visualise these sets.

examples

One can interpret the image of LPF as the set of injections that can be redistributed along the network. Because a flow only serves to redistribute injections from one node to another, nothing is lost or gained in the network. This imposes the condition that an injection vector must have *zero sum*. Moreover, when G is connected, *all* zero sum injections can be induced by a flow.

The kernel can be interpreted as the set of all flows that result in zero injections. Besides the trivial case of zero flow, any constant flow along a closed loop induces zero injection. We will find that these *loop flows* generate *all* flows in the kernel of LPF.

When G is a planar graph, loops around the faces of G actually form a *basis* for the kernel. In the more general case that G is connected, but not necessarily planar, such a basis can also be constructed by fixing a *spanning subtree* of G.

As an added bonus, applying the Rank-Nullity theorem to LPF when G is planar provides us with an alternative proof of Euler's Formula.

These statements will be made more precise in the next sections, when we study the image and kernel of LPF in more detail.

2.2.1 Image of LPF

Definition 2.2.5. The function $\sigma : \mathbb{K}^n \to \mathbb{K}$ defined by

$$\sigma : \mathbf{p} \mapsto \sum_{i=1}^{n} \mathrm{p}_i$$

is the linear map from an injection vector $\mathbf{p}$ to the *net injection* of $\mathbf{p}$.

The kernel of σ is the set of zero-sum injections:

$$\ker \sigma = \left\{ \mathbf{p} \in \mathbb{K}^n \mid \sum_{i=1}^{n} \mathrm{p}_i = 0 \right\}.$$

When G is connected, this is exactly the set of injections that can be induced by a flow on G:

Theorem 2.2.6. *Suppose that G is connected. Then*

$$\mathrm{Im\,LPF} = \left\{ \mathbf{p} \in \mathbb{K}^n \mid \sum_{i=1}^{n} \mathrm{p}_i = 0 \right\} \cong \mathbb{K}^{n-1} \tag{2.2}$$

We will prove this equality by considering the two inclusions $\subseteq$ and $\supseteq$ separately. The first inclusion follows from the observation that $\mathrm{LPF} = \mathbf{C}^T$:

Lemma 2.2.7. *Suppose that G is connected.*

$$\mathrm{Im\,LPF} \subseteq \ker \sigma \tag{2.3}$$

Proof. We write $\mathbf{e}^1, \dots, \mathbf{e}^m$ for the standard basis of $\mathbb{K}^m$.

Since LPF is linear, one only needs to verify that $\mathrm{LPF}(\mathbf{f}) \in \ker \sigma$ for each $\mathbf{f} = \mathbf{e}^k$ in the basis. The vectors $\mathrm{LPF}(\mathbf{e}^1), \dots, \mathrm{LPF}(\mathbf{e}^m)$ are exactly the columns of the matrix $\mathrm{LPF} = \mathbf{C}^T$, which all have zero sum. 🖉

To prove the inclusion $\supseteq$ implied by Theorem 2.2.6, we first consider the special case that G is a *tree*.

Lemma 2.2.8. *Suppose that G is a (connected) tree.*

$$\mathrm{Im\,LPF} \supseteq \ker \sigma \tag{2.4}$$

Proof. G is a tree, so $m = n - 1$. Because $\mathbb{K}^m$ and $\ker \sigma$ both have dimension $n - 1$, we only need to prove that LPF is injective: when LPF has nullity 0, it must have rank $n - 1$.

> This proof could benefit from an illustration.

If $n = 1$, then the digraph consist of a single node, and no lines. It then follows from (2.1) that the only injection that can be induced is $\mathbf{p} = (0) \in \mathbb{K}^1$, so LPF can only be injective.

> A single-node graph is not connected?

If $n > 1$, we will use the fact that the statement holds for any tree with fewer than n nodes. *(Proof by induction.)*

Suppose that $f \in \mathbb{K}^m$, such that $\mathbf{p} = \mathrm{LPF}(\mathbf{f}) = \mathbf{0}$. Because G is a tree, we can[1] pick a *leaf* $i \in \mathcal{N}$, which has a unique line $\mathcal{L}_k$ connecting i to some $j \in \mathcal{N}$. (We have either $\mathcal{L}_k = (i, j)$ or $\mathcal{L}_k = (j, i)$.)

Only one line is connected to i, so (2.1) gives: $\mathrm{p}_i = \pm \mathrm{f}_k$. (The sign depends on the orientation of $\mathcal{L}_k$.) We assumed $\mathrm{p}_i = 0$, so we must have $\mathrm{f}_k = 0$.

By removing node i and line $\mathcal{L}_k$, we obtain a smaller tree, for which the statement already holds. The Linear Power Flow of this subtree is essentially the restriction of LPF to the set $\{\mathbf{f} \in \mathbb{K}^m \mid \mathrm{f}_k = 0\}$. Because the restriction is injective, all other coefficients of $\mathbf{p}$ are also zero. This shows that LPF is injective, and the result follows. 🖊

Proof of Theorem 2.2.6. To prove $\mathrm{Im\,LPF} = \ker \sigma$, it remains to show that

$$\mathrm{Im\,LPF} \supseteq \ker \sigma \tag{2.5}$$

holds for *any* connected G, not just for trees.

Since G is connected, we can choose a minimum spanning subtree: choose $T \subseteq [m]$ with $\#T = n - 1$ such that $G_T = (\mathcal{N}, \{\mathcal{L}_k\}_{k \in T})$ is such a connected subdigraph.

Define $F_T = \mathrm{Span}\{\mathbf{e}^k\}_{k \in T} \subseteq \mathbb{K}^m$ as the subset of flows on G that are zero outside of G_T. Because F_T is a linear subspace of $\mathbb{K}^m$, we have

$$\mathrm{Im\,LPF} = \mathrm{LPF}(\mathbb{K}^m) \supseteq \mathrm{LPF}(F_T),$$

which reduces the problem to $\mathrm{LPF}(F_T) \supseteq \ker \sigma$, which follows from Lemma 2.2.8.

This shows that

$$\mathrm{Im\,LPF} = \ker \sigma.$$

Because $\sigma : \mathbb{K}^n \to \mathbb{K}$ is surjective, it has rank 1. It therefore has nullity $n - 1$, or equivalently, $\ker \sigma \cong \mathbb{K}^{n-1}$. 🖊

2.2.2 Kernel of LPF

Again, assume that G is connected. In Theorem 2.2.6, we derived an explicit formulation for the image of LPF, showing that $\mathrm{null\,LPF} = n - 1$.

Concerning the kernel of LPF, we already know that $\mathbf{0} \in \ker \mathrm{LPF}$, reflecting the fact that zero flow induces zero injection. In the special case that G is a tree, this is the only such flow. In general, however, the kernel of LPF is much bigger.

> introduce loops

Proposition 2.2.9. *Suppose that G is connected. The dimension of $\ker \mathrm{LPF}$ equals*

$$\mathrm{null\,LPF} = m - (n - 1).$$

Proof. This follows directly from the Rank-Nullity theorem, applied to Theorem 2.2.6.

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[1] Write $\mathrm{gr}(i)$ for the number of lines connected to i. G is connected, so $\mathrm{gr}(i) \geq 1$ for each $i \in \mathcal{N}$. If *no i exists with $\mathrm{gr}(i) = 1$*, then $\mathrm{gr}(i) \geq 2$ for each $i \in \mathcal{N}$, giving $\sum_{i=1}^{n} \mathrm{gr}(i) \geq 2n$. On the other hand, each of the $n - 1$ lines connects exactly two nodes, so $\sum_{i=1}^{n} \mathrm{gr}(i) = 2(n - 1) < 2n$, a contradiction.

> Make this mapping explicit? The fact that $\mathrm{p}_i = 0$ is required?

> Is this clear enough?

> state

Corollary 2.2.9.1. *If G is a tree, then* $\ker \mathrm{LPF} = \{\mathbf{0}\}$, *and* LPF *is a bijection between* $\mathbb{K}^m$ *and* $\ker \sigma$.

Proof. Applying Proposition 2.2.9 with $m = n - 1$, we find that $\mathrm{null\,LPF} = 0$, so that $\ker \mathrm{LPF} = \{\mathbf{0}\}$. Together with Theorem 2.2.6, we find the result. ✐

> talk about loops, use an example graph

Any flow along a *closed loop* results in zero power injection. When interpreting a loop as an element $\mathbf{f}$ of $\mathbb{K}^m$, we must be careful to *flip the sign of* f_k *if the line* $\mathcal{L}_k$ *is traversed in reverse.*

Theorem 2.2.10. *Suppose that G is connected and that $(i_1, i_2, \dots, i_p)$ is a closed loop. Then the* loop flow *$\mathbf{f} \in \mathbb{K}^m$ defined by:*

$$
\mathrm{f}_k = \begin{cases} 1 & \text{if } (i_s\phantom{_{+1}}, i_{s+1}) = \mathcal{L}_k \text{ for any } 1 \le s < p, \\ -1 & \text{if } (i_{s+1}, i_s\phantom{_{+1}}) = \mathcal{L}_k \text{ for any } 1 \le s < p, \\ 0 & \text{otherwise,} \end{cases} \tag{2.6}
$$

for each line $\mathcal{L}_k$, is an element of the kernel of LPF.

> this thm only works for Eulerian paths...

Proof. We will verify that $\mathbf{p} = \mathrm{LPF}(\mathbf{f})$ is zero. Choose any $i \in \mathcal{N}$.

Because the loop is closed, there is an *even* number (possibly zero) of lines with non-zero flow that connect to i. This means that the sums in (2.1) cancel each other (note the negative sign for reversed lines in (2.6)), resulting in $\mathrm{p}_i = 0$. ✐

Remark. Because LPF is linear, multiplying a loop flow with a scalar $\gamma \in \mathbb{K}$, or adding two loop flows, creates a new flow that induces zero injection. (The result of addition is a flow, but in general not a *loop* flow.)

Now that we know the dimension of $\ker \mathrm{LPF}$, a natural next step is to look for a *basis* that generates the kernel. Motivated by the previous theorem, we will look for a basis consisting of *loop flows.*

When looking at the previous examples, a logical choice for this basis would be the set of all flows in *loops surrounding the faces contained in the graph*. This approach, which only works for *planar graphs*, will be discussed in the next section.

For now, we would like to find a basis of loop flows for the more general case that G is connected, but not necessarily planar. We proceed as follows:

Definition 2.2.11.

Theorem 2.2.12. *The ???? constructed in Definition ??? is a basis for* $\ker \mathrm{LPF}$.

Proof. Because the number of fundamental loops equals the dimension of $\ker \mathrm{LPF}$, one only needs to prove that they are linearly indepent. Indeed, for each $k \in [m] \setminus T??????$, $??^k$ is the only fundamental loop for which the kth element is non-zero. Therefore, $???^k$ cannot be written as linear combination of the other fundamental loops. ✐

> th: (misschien eerst alleen voor K=F2) the fundamental group

2.2.3 Planar Graphs

th: in a planar embedding of a planar graph, the loops surrounding faces form a basis. proof: (in F2): they are linearly independent: these loops are all loops with the property that exactly one face is contained in the loop. ?? proof: (in F2): start with a fundamental basis. choose a loop f in the fundamental basis. this loop f contains a number of faces. f is equal to the sum of the loops around these faces. this shows that f can be written as linear combination of planar loops. QED

Corollary 2.2.12.1 (Euler's Formula). *$TODO$ In a planar, connected graph, we have:*

$$v + f - e = 1$$

where v is the number of vertices, e is the number of edges, and f is the number of faces enclosed by edges, excluding the 'exterior face'.

Grid connected iff admittance matrix has full rank?

Chapter three

Circuit Theory

3.1 *DC Circuits

A short summary of KCL, KVL, Ohm's Law, the Superposition principle, lumped circuits

See ? and ?

3.1.1 *Derivation of KVL& KCL from Maxwell's eq

applying Maxwell's equations to the discrete graph topology

3.2 AC Circuits

Impedance, real & reactive power

Chapter four

The Electric Grid

<div style="border:1px solid">

(100 W = 16-candela = 200lm = 2.2Watt bij ikea)

30%? of energy use is transported using the grid

list of synchronous grids

complexity

why interconnect? separate generation and load; economy of scale; security (distributed peak demand); stability;

</div>

This chapter is based on the books *Electric Power Systems* by **?**, which provides a clear summary of the working and operation of electric grids, and *Electric Power Principles: Sources, Conversion, Distribution and Use* by **?**, a more quantitative approach to the subject.

4.1 Overview

<div style="border:1px solid">

basics, scale, nomenclature

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4.2 Generators

Almost all sources of electric power in the electric grid use an *AC generator* to convert (rotational) kinetic energy to electric energy.[1] At a hydroelectric plant, the weight of water applies direct torque to the water pumps, mechanically connected to the generator shaft.[2] At most other types of plants (including coal-fired and nuclear plants), generators are powered by steam pressure.

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coils, 3 phase

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[1] A notable exception is the photovoltaic cell (PV cell) used in solar panels. PV cells produce a direct current, which is converted to AC power at nominal voltage using an *inverter*. Solar inverters are often complicated systems, capable of using battery storage, synchronising with the AC grid and varying PV operating voltage to maximise PV efficiency.

[2] Some hydroelectric plants are *reversible*: their generators can be used as electric motors, pumping water back up into a reservoir. This is a form of large-scale *energy storage*, which can increase the usability of intermittent renewable energy sources.

4.3   Loads

4.4   Transmission

4.4.1   DC transmission

4.4.2   3-phase transmission

The AC grid has 3 phases, but it is very common to model the grid as a single-line network (with a global, zero-impedance ground). Von Meier mentions material that justifies this change.

If you live in mainland Europe, North America, , your wall socket will have *two* connectors. Most modern sockets also have a third connector, the *ground wire*.

This makes sense, since most electrical devices only need a *potential difference* (between *two* wires) to function. In the early days of transmission systems, however, this caused a problem for factories using AC motors to power their production line. When a (single-phase) AC current is used to power a motor, the amount of *torque* [3] is not constant: when the potential difference is zero, which happens twice every $1/50^{\text{th}}$ of a second, no electric field is present in the armature windings, and no torque is applied to the rotor.

en waarom is dat een probleem?

To solve this problem, both the AC generator and AC motor are equipped with *three* identical sets of armature windings, each oriented 120° relative to the others. One end of each winding is connected to the common ground, and the remaining three connection points form the *three-phase connection* between generator and motor.

As the generator rotor is moving, there will always be one armature winding which is close [4] to being perpendicular to the rotor windings, and will therefore have a relatively high induced current.

In fact, when writing down the three currents in exact form (three sinusoids with the same frequency and magnitude, with phases 0°, 120° and 240°), we find that *the sum of their squares*, i.e. the total amount of transmitted power, *is constant*! Although the amount of power transmitted on just one of the phases (square of the sinusoid current) is not constant, the combination of the three is.

This means that the torque of a motor, which is the sum of torques applied by each

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[3]i.e. rotational force. In most systems, torque is proportional to *change in rotational velocity* (change in rpm).

[4]within 30°

armature winding, is constant.

When looking at AC transmission systems , you will always find three identical copies of the same component. For example, high-voltage AC transmission lines always have three, six or nine wires, which are identical in material, thickness and length.

There are a number of reasons to build transmission systems this way. One benefit of having three copies of the exact same network is that they will behave the same way, saving grid operators up to two thirds of potential headaches.

Single-line approximation

4.4.3   Line failures

When a transmission line stops transmitting power, we speak of a *line failure*. In most cases, a line failure is not caused by a broken or molten wire, but by a disconnection at one of its ends, as a safety measure. Some line failures are caused by a fault: a sudden abnormal peak in current, caused by anomalies like fallen trees or lightning strikes. A high current is detected by a *circuit breakers*, which then automatically breaks its connection. A household fuse is an example of a circuit breaker, although modern circuit breakers use a mechanical switch (which be reset, instead of requiring a replacement fuse). In high-voltage systems, circuit breakers can be quite advanced, sometimes using pressurised gas or oil to prevent arcing inside the switch.

Another type of line failure, the one most relevant to this report, is an *overload*. All transmission lines have a *thermal limit*: a maximum current that the line can sustain without risking overheating, causing sagging or even melting of transmission cables. To prevent these dangerous scenarios, transmission lines automatically disconnect when the current exceeds the predetermined *line capacity*.

It is important to note that grid operators cannot control the amount of current - and therefore - power that flows through a transmission line, grid operators can only connect or disconnect the line from the grid. The impedance of a wire does not act as a *bottleneck* (like a small-diameter pipe would do in an irrigation network). From Ohm's law ($ZI = V$), one could mistakenly conclude that the amount of current passing through a transmission line is inversely proportional to its resistance. However, the electric potential (V) in this equation is *not* the potential between the wire and the ground (e.g. 220 kV), but rather the potential difference between the two ends of the wire (which is not zero!). This difference (known as *line drop*) is quite low, compared to the potential between one of the ends and the ground (220 kV transmission lines usually have a 1% line drop). The amount of current carried by a wire is dominated by the current being 'generated' and 'used' at its ends.

4.5   Power Flow Analysis

4.6 Transmission grid operation

The transmission network is a crucial

The transmis

In societies that

Historically, failures of transmission network have had dramatic effects. The effects of such outages are even visible from space, since almost every light source indirectly depends on a functioning transmission network.

Luckily, TODO where a power-outage would be life-threatening, or has significant economic impact, often have backup generators in place.

In modern hospitals, medical equipment often has a built-in battery to sustain relatively short power cuts. (When unplugged by accident, for example.) Hospitals also use two sets of power lines with independent protection: one for essential equipment, and one for computers, lights, phone chargers, et cetera.

It should come as no surprise that the transmission network requires constant monitoring to ensure that it is working properly.

When the first power grids were built, grid security was ensured by *redundancy*. Trans-   `historic note`
mission lines were designed to carry currents much greater than predicted. One of the criteria for building a network was *N-1 contigency*: the network should remain operational after the failure of *any* line. In graph theory, this is equivalent to building a 2-edge-connected graph.

4.6.1 Power islanding

Because nodes often house both loads and generators, the disconnection of a single node, or of a subset of nodes, does not always lead to a power outage, since local power generation might be sufficient to serve the load. This situation, where a subsection of the transmission network still operates after being disconnected, is called a *power island*.

Since the power island and the remaining network have been operating independently of each other, it is likely that their AC phase angles no longer line up.[5] Unless this phase drift is corrected for (by temporarily increasing or decreasing the AC frequency on either side), the power island cannot be reconnected, since this phase drift likely exceeds the power angle limit of a new transmission line.   `adsf`

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[5]This misalignment likely occurs right after the disconnection, when the total generation in the power island is substantially higher or lower than the total load. Until this mismatch has settled, the AC frequency will be higher or lower, respectively, than the target frequency (say, 50 Hz).

4.6.2 The energy market

> intro. Very simplified world:

Every hour, all parties involved in the generation and transmission of power come together, to decide *what the configuration during the following hour will be*. For the sake of simplicity, we will assume that each power plant (renewable or non-renewable) is owned by its own company, and that the entire transmission network is owned by a single grid operator.

Phase one: offering prices

First, all generators offer to supply energy during the coming hour *for a certain price per kWh*. Generally, company A, which owns a wind farm, will offer their energy for a lower price than company B, which owns a coal plant. The price offered by company A is lower because a wind farm has zero *marginal cost* - the cost of generating an additional unit of electricity, once the turbine has been built. No fuel or labour is needed to generate power using a wind turbine, it simply needs to not be turned off.

Besides marginal cost, there are two more factors that influence the offered price:

- Company A needs to pay back their initial investment, which increases the offered price.
- Government subsidies, which promote the use of low-carbon energy, reduce the offered price.

In general, one can assume that the price offered for renewable energy is close to zero. In some cases, the subsidies are greater than all other costs, and energy is offered for a *negative price*. This has the counter-intuitive effect that a grid operator actually *earns money* by buying electricity from that company.

Energy from company B is generally more expensive, since it has a marginal cost (price of coal, labour wages, and so on), on top of paying back the investment.[6]

> Misschien is dit een goede plek om de *environmental cost* van elk type generator te geven

Phase two: grid operator

Now that all generators have offered a price, the grid operator needs to decide where to buy its energy from. Grid operators have a fairly good estimate for the load (energy demand) during the coming hour, based on the current load, historical load series, weather forecasts, sporting events, television programme schedules, and so on. Such a model is called a *load profile*.

load profile plotje

By design, the total generation capacity is much greater than the expected load, so

why? winter and incident load

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[6]In some cases, coal plants offer their electricity for a lower, or even negative, price, to avoid power off the plant. With the exception of open-cycle gas turbines, turning on non-renewable energy plants is expensive, and might take several hours. It might be cheaper for company B to offer their energy for free for one hour than to turn off the plant.

15 March 2019, 11:00-12:00		
generator	*offered price*	*requested output*
Sally's Solar Park	4 €/MWh	115 MW / *115 MW*
Chris's Coal Plant	122 €/MWh	0 MW / *240 MW*
Nellie's Nuclear Plant	45 €/MWh	1387 MW / *2500 MW*
Steven's Solar Park	-2 €/MWh	23 MW / *23 MW*
⋮	⋮	⋮

Table 4.1: A small section of a fictional configuration.

running every generator at full capacity would be problematic. As a grid operator, we need to tell each generator at what percentage of their capacity they should operate.

A complete set of generator instructions, called a *configuration*, might look something like Table 4.1. Note that generators could be told to operate at 0% of their capacity, which is equivalent to powering off the generator completely.

Some configurations are more expensive than others, and some will result in more greenhouse gas emissions than others. Perhaps surprisingly, most configurations are not even *admissible*: using them would cause some transmission lines to be overloaded, which could even result in a blackout.

More precisely, a configuration is called *ready to deploy* if it satisfies the following three conditions:

possible: For each generator, the power output dictated by a configuration must be a non-negative number, no greater than the maximum capacity of the generator.

balanced: When the dictated configuration is applied, total generation must equal total load, together with line losses.[7]

admissible: When the dictated configuration is applied, no line should be overloaded.

To determine whether a configuration is *admissible*, the grid operator can simply *calculate the line flows* that would be induced by that configuration, using a Power Flow (PF) algorithm, and check for any overloaded lines. Because power flow physics are quite well-understood, these results are accurate.

OPF

We can easily find a configuration that is both possible and balanced. First, set all generators to maximum capacity. We now have a *possible* configuration. By design, the

> stats: het percentage van totale capaciteit dat wordt gebruikt ligt tussen ?% en ?%.

> admissible is niet ja of nee

> accuracy

> geschiedenis van PF: modellen

---

[7]Of course, the total load is not perfectly constant for one hour. To accommodate for this, the grid operator designates a single *load following* generator, which will vary its output continually to make sure that the total demand is met exactly. (This is equivalent to maintaining a constant AC frequency!) The bus that this generator is connected to is called the *slack bus* of the transmission network.

total generation of this configuration is much greater than the total load. By scaling the output of all generators with the same factor $0 \leq \lambda \leq 1$, the configuration remains *possible*, and by the intermediate value theorem, a λ exists such that the configuration is *balanced*.

When trying out different *possible*, *balanced* configurations randomly, we will find that almost all of them are inadmissible; the subset of admissible configurations is actually quite small, and finding *any* ready-to-deploy configuration is a lot of work.

Once a ready-to-deploy configuration has been found, the grid operator now wants to minimise economic and environmental cost. Given this initial configuration, the associated cost can be calculated, since all generators have offered their price, and their environmental impacts are known. This ready-to-deploy configuration is not unique, and by varying the configuration only slightly, the configuration remains ready to deploy.[8] By doing so, we might be able to reduce the (environmental) cost! By repeating this process, a local minimum can be found.

The problem of

finding a ready-to-deploy configuration

that minimises the associated cost

is called the *Optimum Power Flow (OPF)* problem.

Even with modern-day computers, solving this problem is a difficult task indeed, and is often simplified by making a slight modification to our process. Instead of using the Power Flow (PF) algorithm to check a configuration's admissibility, we use the so-called *Linearised Power Flow (LPF)* algorithm to check linear admissibility. This linearised version can be computed much more quickly, with only a small loss in accuracy. We will derive the equations describing these two algorithms in Chapter 5, and compare their results.

> linear balance

With our slight modification, we restate our problem as:

minimise	*associated cost*
subject to	*possibility*
and	*balance*
and	*linear admissibility*

which is known as the *Linear Optimum Power Flow (LOPF)* problem. As suggested by the notation, this optimisation problem falls in the more general class of problems solvable using *linear programming*. First studied by Fourier in 1827, the field has seen many developments, most notably the *simplex* solution method developed by G. Dantzig in 1947.[9] Nowadays, several high-performance linear programming solvers have been created, including the open source `GNU Linear Programming Kit`, which was used in this project.

> Je kan de eisen van OPF naar beneden schalen om lineaire admissibility te krijgen (bv de contingecy factor!)

---

[8] As we will later learn, the function that maps a configuration to the vector of line flows that it induces is *continuous*. The set of vectors where each line flow is < 100% is an *open* box, and so its inverse image (the set of configurations that are ready to deploy) is also open.

[9] For the *simplex method*, see ? .

Chapter five

Modelling Transmission Networks

> intro

In practice, the grid structure remains unchanged during normal operation, while the grid state is continually changing over time. For example, an important aspect of grid operation is *load profiling*: examining and predicting the total load connected to a node, as a function of time. As a result of changing loads, the flow of power in a grid is constantly changing.

One example of a change in grid structure is a *line failure*, which can be modelled as the removal of an edge from the graph. In some cases, the removal of an edge from the graph result in an unconnected graph (i.e. there exist two nodes with no sequence of lines connecting them). This scenario is called *power islanding*. Most transmission networks are designed in such a way that no single (or double) line failure can cause power islanding or a blackout, by increasing the *edge-connectivity* of the graph.

In the case of a *line overload*, however, a line failure is caused by an exceptionally high power flow, as a result of high supply or demand.[1] The failure of an overloaded line will cause a redistribution of power flow, since the power flowing through the failed line now needs to 'find another path' between the nodes. In a highly stressed network, this redistribution can cause a second failure, which can then cause a third failure, and so on, eventually causing a blackout. We will study these *cascading failures* in Chapter ??.

5.1 Grid structure

A transmission grid is modelled as a directed graph $(\mathcal{N}, \mathcal{L})$. As vertices we take the *nodes* of the network, which are those points where transmission lines connect to a generator, load, or to each other. Nodes are electrically distinct, in the sense that there is some non-zero impedance between them, allowing them to sustain a potential difference. In a network of n nodes, they are represented by the natural numbers $1, \ldots, n$, in arbitrary order.

> We *model* the line protection mechanism as deterministic: overload causes instantanious failure.

> Require the graph to be connected? How about power islands?

---

[1] These high power injections are not necessarily located at the two endpoints of an overloaded line; it could also be a grid-wide pattern of power injections, all adding up to a high power flow on that line. We will study the *most likely power injection* in Chapter ??.

A pair of two nodes (i, j) is contained in the set of edges $\mathcal{L}$ iff there is a transmission line connecting the nodes, and $i < j$. (Note that by construction, this transmission line has non-zero impedance.) The choice of line orientation can be arbitrary, but we fix the orientation to $i < j$. In a network of m lines, lines are labeled $\mathcal{L}_1, \ldots, \mathcal{L}_m$, in arbitrary order.

Vertices will be called nodes, and edges will be called lines.

Given a directed graph with n nodes and m lines, we define the edge-vertex incidence matrix $\mathbf{C} \in \mathbb{R}^{m \times n}$ as:

$$
\mathbf{C}_{k,i} = \begin{cases} 1 & \text{if } \mathcal{L}_k = (i, j) \text{ for some } j \in [n], \\ -1 & \text{if } \mathcal{L}_k = (j, i) \text{ for some } j \in [n], \\ 0 & \text{otherwise.} \end{cases}
$$

The ordered set of lines is uniquely defined by the edge-vertex incidence matrix.

transmormer normalisation

We model transmission lines as time-invariant impedances, which are assumed to be constant under any electric potential and current, allowing us to apply Ohm's Law. Instead of the impedance Z of a line, we use its *admittance* $\mathsf{Y} = 1/\mathsf{Z} \in \mathbb{C}$, [2] and Ohm's Law becomes:

$$
\mathsf{I} = \mathsf{VY}
$$

This allows us to define the admittance between two unconnected nodes as 0. (i.e. no current is induced by a potential difference.) ~~We now construct the *admittance matrix* $\mathbf{Y} \in \mathbb{C}^{n \times n}$, where $\mathsf{Y}_{i,j}$ is the admittance between nodes i and j. Note that $\mathbf{Y}$ is a symmetric matrix, with only $2m$ non-zero entries.~~

We ~~also~~ define $\eta \in \mathbb{C}^m$ as the *admittance vector*, where η_l is the admittance of $\mathcal{L}_l$, for each $l \in [m]$. ~~Equivalently, one can write $\eta_l := Y_{\mathcal{L}_l}$ for each $l \in [m]$ since each $\mathcal{L}_l$ is a tuple of node indices.~~

Definition 5.1.1. To summarise, an (n, m)-*grid structure* is defined as a tuple $((\mathcal{N}, \mathcal{L}), \mathbf{C}, \eta)$ where:

- $n = \#\mathcal{N}$ is the number of nodes;

- $m = \#\mathcal{L}$ is the number of lines;

- $(\mathcal{N}, \mathcal{L})$ is a connected directed graph with $\mathcal{N} = [n]$;

- $\mathbf{C} \in \mathbb{R}^{m \times n}$ is the edge-vertex incidence matrix of $(\mathcal{N}, \mathcal{L})$;

- $\eta \in \mathbb{C}^m$ is the line admittance vector.

Example 5.1.2 (Two-node network). A very simple transmission network is one with just two nodes, and a single line connecting them. This *two-node network* is drawn in

---

[2]Written in Cartesian form, $\mathsf{Y} = \mathsf{G} + i\mathsf{B}$, where G is the conductance, and B the susceptance of a line. Both have unit siemens (S), or mho (the reverse of 'ohm'), defined as $1\,\mathrm{S} = 1\,\Omega^{-1}$.
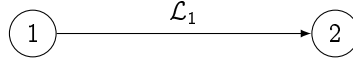
Figure 5.1: The simple *two-node* transmission network of two nodes and one line.

Figure 5.1. Although real networks are much bigger, this example is useful to illustrate some of the concepts introduced in this chapter.

We have:

$$\mathcal{N} = \{1, 2\} \text{ and}$$
$$\mathcal{L} = \{\mathcal{L}_1\} \text{ with } \mathcal{L}_1 = (1, 2).$$

Since $n = 2$ and $m = 1$, $\mathbf{C}$ is a 1×2 matrix:

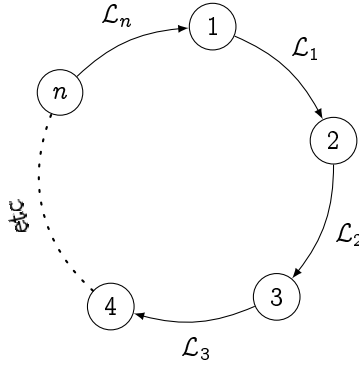$$\mathbf{C} = \begin{pmatrix} 1 & -1 \end{pmatrix}.$$

Figure 5.2: The *n-loop* transmission network with n nodes and n lines.

Example 5.1.3 (Loop network). A less trivial transmission network is the *n-loop network*, shown in Figure 5.2. It consists of n nodes, connected in a circular fashion (using n lines).

Note that this network is 2-edge connected, meaning that the network remains connected when any edge is removed. (Although removing *any* two lines will disconnect the network.)

We have:

$$\mathcal{N} = \{1, \ldots, n\} = [n] \text{ and}$$
$$\mathcal{L} = \{\mathcal{L}_1, \ldots, \mathcal{L}_n\} \text{ with } \mathcal{L}_i = (i, i+1) \text{ for } 1 \leq i < n \text{ and } \mathcal{L}_n = (n, 1).$$

Since $m = n$, $\mathbf{C}$ is an $n \times n$ matrix:

$$\mathbf{C} = \begin{pmatrix} 1 & -1 & & & & \\ & 1 & -1 & & & \\ & & 1 & -1 & & \\ & & & \ddots & \ddots & \\ & & & & 1 & -1 \\ -1 & & & & & 1 \end{pmatrix}.$$

(All unspecified elements are 0).

Note that rows of $\mathbf{C}$ correspond to lines of the network.

Example 5.1.4 (SciGRID Germany)**.** TODO

> Mostly for fun, and to emphasise that the dataset can be modelled in this form.
> The dataset will be treated in detail in Part II.
> Will also emphasise that in most real networks, $m > n$.

5.2   Power grid state

> TODO, I is directed along the graph edges

Definition 5.2.1. To summarise, a *grid state* of an (n, m)-grid structure is defined as a tuple $(\mathbf{S}, \mathbf{V}, \mathbf{I})$, where

- $\mathbf{S} \in \mathbb{C}^n$ is the *complex power injection vector*;
- $\mathbf{V} \in \mathbb{C}^n$ is the *bus voltage vector*;
- $\mathbf{I} \in \mathbb{C}^m$ is the *line current vector*.

> **I** could be left out of the grid state, since it is uniquely determined by **V** and the structure.

> U instead of V? Not common in lit

5.3   State validity

Given an (n, m)-grid structure $((\mathcal{N}, \mathcal{L}), \mathbf{C}, \eta)$, only some states are physically possible. A state that satisfies Kirchoff's Laws and Ohm's Law is called *valid*. Since we are studying a real-world system, we are mainly interested in states that are valid, or at least close to being valid (in the sense of *DC-valid*, see Section 5.5).

5.3.1   Electrical representation

An (n, m)-grid structure $((\mathcal{N}, \mathcal{L}), \mathbf{C}, \eta)$ represents an electrical circuit, consisting of impedances and AC sources. A *valid* grid state $(\mathbf{S}, \mathbf{V}, \mathbf{I})$ corresponds to a physical state of the circuit. Given the mathematical structure, we will construct the electric circuit in two layers, as follows:

In the first layer, each node i in $\mathcal{N}$ becomes a node in the circuit. The value of V_i is the electric potential of that node, relative to ground.

A line $\mathcal{L}_k = (i, j)$ is modelled as an impedance[3] (—▭—) with admittance η_k between the two nodes i and j. The value of I_k is the current flowing through the impedance from j to i.

See Figure 5.3 for the first constructed layer. This is not yet the final circuit: we have not yet added generators and loads to the circuit. Also, the transmission lines have no return wire. This means there is no closed loop between two nodes, and therefore no energy can be transmitted.

To construct the second layer, we add a new component to each node $i \in \mathcal{N}$, which can be seen as the collection of AC sources (generators) and impedances (loads), connected in parallel between the node and ground. Since an exact configuration[4] of loads and generators connected to the node is not important, we will simply state that this component:

- sustains a potential difference (which is V_i);[5]

- either supplies or draws a current, such that the amount of power generated or consumed by the component equals S_i (when S_i is positive or negative, respectively).

We will call this aggregation of generators and loads a *power injector* (—⌀—).

Each power injector is connected to a node on one end, and to a ground terminal on the other. In electric circuit theory, a ground terminal represents a direct connection to a universal ground: the zero reference of electric potential. One could say that between two different ground terminals, there exists a zero-resistance link connecting the two. We now have a simple closed circuit between any two nodes connected by a transmission line, consisting of a power injector for the first node, an impedance between the two nodes, a power injector for the second node and a ground link back to the first node.

The complete, two-layer model is shown in Figure 5.4.

Justify conductive ground wire in a 3-phase system

---

[3] complex-valued resistor

[4] This would be a circuit comprising the medium and low-voltage networks connected to the node, including every generator, fridge and phone charger that it serves. This is a common (and necessary) abstraction in power grid analysis.

[5] In physical systems, the *operating voltage* $|V_i|$ (remember that $\mathbf{V}$ is complex) can be controlled by power plant operators by adjusting excitation current of a generator. The *phase angle* $\theta_j = \mathrm{Arg}(V_j)$ can be controlled by adjusting the amount of energy (steam) supplied to a generator.

Both methods are an indirect form of control and the effects are very complex in nature, since the operating voltage and phase angle of one node is inherently linked to that of all other nodes in the network. In fact, *maintaining* a constant operating voltage and phase angle is a complicated task, requiring continuous adjustments to generator operation. See Section 4.3 of ? for a more detailed description.
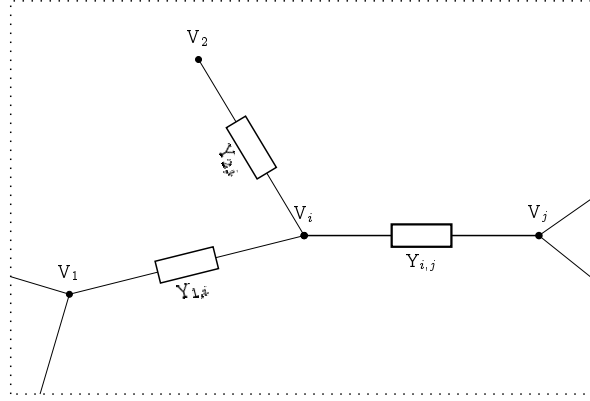
Figure 5.3: A section of a transmission network, showing the line (i, j). Two more nodes, both connected to i, are also shown. Note that this is only one layer of the electric circuit used in the model. In the full model (Figure 5.4), each transmission line forms a closed circuit, allowing current to flow.

5.3.2 Kirchoff's Voltage Law (KVL) & Ohm's Law

For each line $\mathcal{L}_k = (i, j)$, we can apply Kirchoff's Voltage Law to the loop "ground $\to i$ $\to j \to$ ground", as shown in Figure 5.4. This gives us:

$$\mathrm{V}_i + \Delta\mathrm{V}_k + -\mathrm{V}_j = 0,$$

where $\Delta\mathrm{V}_k$ is the electric potential of the line impedance. This potential relates to the line current according to Ohm's Law:

$$\mathrm{I}_k = \Delta\mathrm{V}_k \cdot \eta_k = (\mathrm{V}_j - \mathrm{V}_i) \cdot \eta_k.$$

Since the kth row of $\mathbf{C}$ (which corresponds to the line $\mathcal{L}_k = (i, j)$) has two non-zero entries: $\mathrm{C}_{k,i} = 1$ and $\mathrm{C}_{k,j} = -1$, we can write:

$$\mathrm{I}_k = (\mathrm{V}_j - \mathrm{V}_i) \cdot \eta_k = -(\mathbf{C}\mathbf{V})_k \eta_k.$$

This holds for every $k \in [m]$, and this system of equations can be written compactly as:

$$\mathbf{I} = i \operatorname{diag}(i\eta)\mathbf{C}\mathbf{V}$$

> Written in this form, we see that when $i\eta$, $\mathbf{C}$ and $\mathbf{V}$ are purely real, then $\mathbf{I}$ is purely imaginary. Physically, this means that line current is always 90° out of phase with node voltage. (Or it means that current is out of phase with voltage differences? Which are voltage angle differences?)

Writing node voltages in a circuit already implies that KVL holds.

5.3.3 Kirchoff's Current Law (KCL)

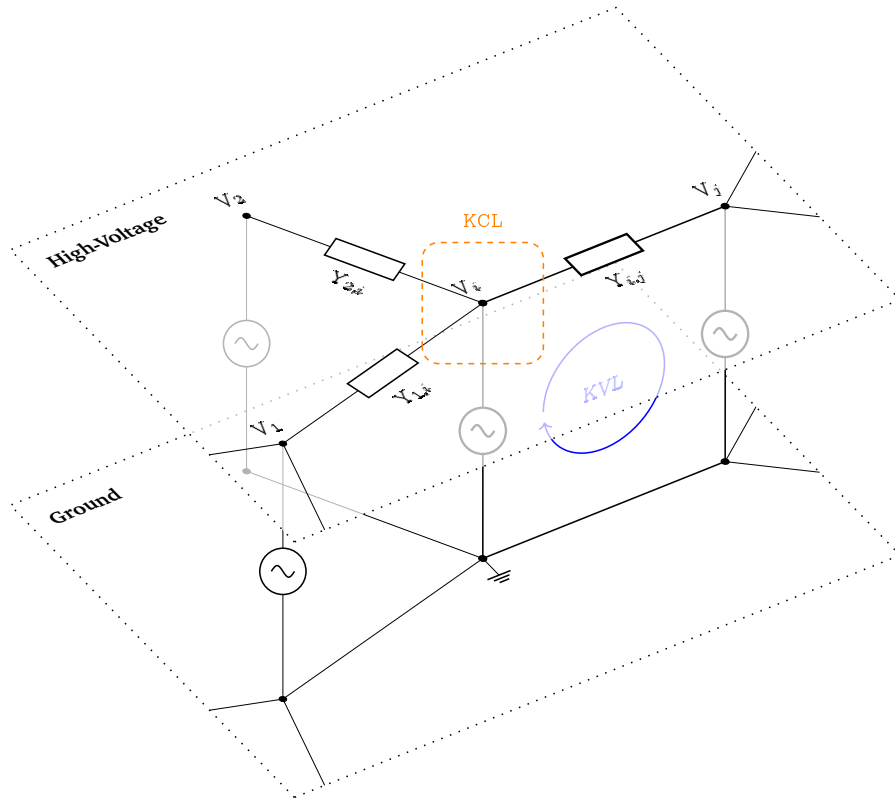Using KVL and Ohm's Law, we found a relation between line current and node voltages. We can use KCL to TODO

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Figure 5.4: The electric model of the transmission network, showing the line (i, j) and two other nodes. A node is represented by a single component (—Ⓢ—), which is the aggregation of all generators and loads connected to that node. A transmission line is modelled as an impedance (—▭—), a complex-valued resistor. The ground 'wires' have zero resistance.

KVL is applied to each **line**, by traversing the loop drawn in blue.

KCL is applied to each **node**, by summing all currents entering and leaving the orange area.

Let's apply KCL to every high-voltage node in the electric circuit, as shown in Figure 5.4. This gives us:

$$\mathbf{S} + (\mathbf{C}^T\mathbf{I})^* * \mathbf{V} = \mathbf{0}$$

which is the sum of line currents entering the node, minus the sum of line currents leaving the node.

> Show derivation of (S-KCL) from KCL in the last illustration

5.3.4 Validity conditions

> intro

Definition 5.3.1. Given an (n,m)-grid structure $((\mathcal{N}, \mathcal{L}), \mathbf{C}, \eta)$, a grid state $(\mathbf{S}, \mathbf{V}, \mathbf{I})$ is *valid* if it satisfies the *KVL-Ohm equality*:

$$\mathbf{I} = i \operatorname{diag}(i\eta)\mathbf{C}\mathbf{V} \tag{KVL-Ohm}$$

and the *KCL equality*:

$$\mathbf{S} + (\mathbf{C}^T\mathbf{I})^* * \mathbf{V} = \mathbf{0}. \tag{S-KCL}$$

Remark. In physical terms, the *KVL-Ohm equality* states:

> Each line $\mathcal{L}_k = (i,j)$ satisfies Ohm's Law $(\mathsf{I} = \mathsf{V}\mathsf{Y})$, where:
> $$\mathsf{I} = \mathsf{I}_k, \qquad \mathsf{V} = \mathsf{V}_i - \mathsf{V}_j, \qquad \mathsf{Y} = \eta_k$$

and the *KCL equality* states:

> At each node i, the sum of power injected at the node, S_i,
> and power injected from the grid, must equal 0.

Proposition 5.3.2. *Given a grid structure and state as in Definition 5.3.1, the following are equivalent:*

  i. *The grid state is valid.*

  ii. *The grid state satisfies (KVL-Ohm) and (S-KCL).*

  iii. *The grid state satisfies (KVL-Ohm), and each node i satisfies the* node flow equation*:*

$$\mathsf{S}_i = i \sum_{j=1}^{n} \mathsf{L}_{i,j}^* |\mathsf{V}_i||\mathsf{V}_j|e^{i(\theta_i - \theta_j)} \qquad \text{for each } i \in [n], \tag{5.1}$$

$$\text{where } \mathbf{L} = \mathbf{C}^T \operatorname{diag}(i\eta)\mathbf{C} \tag{5.2}$$

  and $\theta \in \mathbb{R}^n$ is the vector of voltage angles*, i.e. $\theta_j = \operatorname{Arg}(V_j)$ (the principle argument of V_j).*

Remark. Note that (5.1) does not depend on $\mathbf{I}$. This means that a valid state is uniquely determined by $\mathbf{S}$ and $\mathbf{V}$, since $\mathbf{I}$ can be computed from $\mathbf{V}$ using (KVL-Ohm).

Remark. Writing in real and imaginary components, we get the node flow equations for real and reactive power for each node $i \in [n]$:

$$P = \Re(S_i) = \sum_{j=1}^{n} |V_i||V_j|\Big[\Re(M_{i,j})\cos(\theta_i - \theta_j) + \Im(M_{i,j})\sin(\theta_i - \theta_j)\Big] \qquad (5.3)$$

$$Q = \Im(S_i) = \sum_{j=1}^{n} |V_i||V_j|\Big[\Re(M_{i,j})\sin(\theta_i - \theta_j) - \Im(M_{i,j})\cos(\theta_i - \theta_j)\Big] \qquad (5.4)$$

In literature on the subject, the node flow equation is often given in this form.

M should be $\pm i\mathbf{L}$

We will later study so called *DC-valid* grid structures, where the values of **M** are purely imaginary. If so, (5.3) simplifies to:

$$P = \Re(S_i) = \sum_{j=1}^{n} \Im(M_{i,j})|V_i||V_j|\sin(\theta_i - \theta_j),$$

which tells us that in a network of just two nodes and one line with purely imaginary admittance, the amount of real power transmitted is proportional to $\sin(\theta_2 - \theta_1)$.[6]

Proof. (i) $\iff$ (ii) is true by definition.

Suppose the grid state satisfies (KVL-Ohm). We have

$$S + (\mathbf{C}^T \mathbf{I})^* * V =$$
$$S + (i\mathbf{C}^T \operatorname{diag}(i\eta)\mathbf{C}V)^* * V =$$
$$S + (i\mathbf{L}V)^* * V = 0$$

iff for each $i \in [n]$

is this Newton's heat equation with M the discrete Laplacian?

$$S_i = -\left(\sum_{j=1}^{n} i L_{i,j} V_j\right)^* V_i$$

$$= -\left(\sum_{j=1}^{n} -i L_{i,j}^* V_j^*\right) V_i$$

$$= i \sum_{j=1}^{n} L_{i,j}^* V_i V_j^*$$

$$= i \sum_{j=1}^{n} L_{i,j}^* |V_i| e^{i\theta_i} |V_j| e^{-i\theta_j}$$

$$= i \sum_{j=1}^{n} L_{i,j}^* |V_i||V_j| e^{i(\theta_i - \theta_j)},$$

proving (ii) $\iff$ (iii).

---

[6]The quantity $\theta_2 - \theta_1$ is called the *power angle* of the transmission line, a common measure of the amount of power being transmitted. As discussed in Section ??, a power angle greater than 45° will cause the nodes to lose synchronicity, making power transmission between the two nodes impossible. For long lines (over 100km), this *stability limit* places an upper limit on the amount of power that a line can transmit. For shorter lines, the *thermal limit* dominates.

5.4 Power Flow

In the previous section, we derived a fundamental result: the *node flow equation* (5.1). Recall from Section **??** that the *power flow problem* entails the following:

Given an (n, m)-grid structure $((\mathcal{N}, \mathcal{L}), \mathbf{C}, \eta)$, and a power injection $\mathbf{S}$,
find $\mathbf{V}$ and $\mathbf{I}$ such that $(\mathbf{S}, \mathbf{V}, \mathbf{I})$ is a valid state.

Or put more simply:

Given the production or consumption at each node,
find the current flowing through each line.

It turns out that the easiest way to solve this problem is to solve the node flow equation to obtain $\mathbf{V}$. Once $\mathbf{V}$ is known, one can easily compute $\mathbf{I}$ using KVL-Ohm, giving a state $(\mathbf{S}, \mathbf{V}, \mathbf{I})$ that is valid by construction.

The node flow equation is a system of non-linear equations, and no closed-form solution is known to exist in general. Fortunately, solving the node flow equation is essentially a root-finding problem. This means that well-established techniques, such as the Newton-Raphson algorithm, can be used to find a numerical solution.

> Difficulty of finding initial values that convergence: flat start. Is DC approx used as initial value?

5.5 DC approximation

"DC approximation" is a name given to a collection of assumptions/approximations, described below. The name 'DC' refers to the approximation that the network is decoupled. Unlike the abbreviation might suggest, the power grid is still modelled as an AC network.

> intro

Scott1974 for the origin of the DC approximation

Compare the following with Definitions 5.1.1 and 5.2.1.

Definition 5.5.1. Suppose $((\mathcal{N}, \mathcal{L}), \mathbf{C}, \eta)$ is an (n, m)-grid structure, with a grid state $(\mathbf{S}, \mathbf{V}, \mathbf{I})$.

- If $\eta \in i\mathbb{R}^m \subseteq \mathbb{C}^m$ (i.e. η has purely imaginary values) then the grid structure *satisfies the DC approximation.* [7]

physics: conductance is neglected

- If $\mathbf{S} \in \mathbb{R}^n \subseteq \mathbb{C}^n$ then the grid state *satisfies the DC approximation.* (Note that $\mathbf{V}$ and $\mathbf{I}$ need not be real-valued.)

---

[7]In Nesti et al., we have $\beta = i\eta$. Transmission line impedance $(Z = R + iX)$ is dominated by inductance, which is positive reactance (X). Resistance (R) is always positive for passive components. Therefore, Z lies in the top-right quadrant of $\mathbb{C}$. Then the admittance, $Y = 1/Z = G + iB$, lies in the bottom-right quadrant of $\mathbb{C}$. In the DC approximation, line conductance (G) is neglected, so $(Y = iB$ with $B < 0)$. Therefore, $\beta_k = i\eta_k = iY = -B > 0$ is the *susceptance of line k*, **with reversed sign.**

- If $\mathbf{V} \in \mathbb{T}^n \cdot V_{op} \subseteq \mathbb{C}^n$ (i.e. $|V_i| = V_{op}$ for each node i) for some *operating voltage* $V_{op} \in \mathbb{R}_{\geq 0}$, then the grid state admits a *flat profile*.

  In the special case $V_{op} = 1$, the grid state admits a *normalised profile*.

By replacing the exp function in (5.1) by $z \mapsto 1 + z$ (the first two terms of the Maclaurin series of exp) we get the *approximate node flow equation*. These two conditions are different, but as we will later see, similar for networks with small phase shifts. Compare the following with Definition 5.3.1 and (5.1).

Definition 5.5.2. Given an (n, m)-grid structure $((\mathcal{N}, \mathcal{L}), \mathbf{C}, \eta)$, a grid state $(\mathbf{S}, \mathbf{V}, \mathbf{I})$ is *DC-valid* if it satisfies the *KVL-Ohm equality*:

$$\mathbf{I} = i \operatorname{diag}(i\eta)\mathbf{CV} \qquad \text{(KVL-Ohm)}$$

and each node i satisfies the *approximate node flow equation*:

$$S_i = i \sum_{j=1}^{n} \mathrm{L}_{i,j}^* |V_i||V_j|(1 + i(\theta_i - \theta_j)) \qquad \text{for each } i \in [n]. \qquad (5.5)$$

Theorem 5.5.3. *Suppose that a grid structure and state satisfy the DC approximation and that the state admits a flat, normalised profile. Then the approximated node flow equation is linear and real:*

$$\boxed{\mathbf{S} = \mathbf{L}\theta.} \qquad (5.6)$$

Proof. For each node i, we have:

$$S_i = i \sum_{j=1}^{n} \mathrm{L}_{i,j}^* |V_i||V_j|(1 + i(\theta_i - \theta_j))$$

$$= i \sum_{j=1}^{n} \mathrm{L}_{i,j} |V_i||V_j| - \sum_{j=1}^{n} \mathrm{L}_{i,j} |V_i||V_j|(\theta_i - \theta_j)$$

If we assume a DC state ($S_i \in \mathbb{R}$):

$$= - \sum_{j=1}^{n} \mathrm{L}_{i,j} |V_i||V_j|(\theta_i - \theta_j)$$

$$= - \sum_{j=1}^{n} \mathrm{L}_{i,j} |V_i||V_j|\theta_i + \sum_{j=1}^{n} \mathrm{L}_{i,j} |V_i||V_j|\theta_j$$

$$= -\theta_i |V_i| \sum_{j=1}^{n} \mathrm{L}_{i,j} |V_j| + |V_i| \sum_{j=1}^{n} \mathrm{L}_{i,j} |V_j|\theta_j$$

If we assume a flat profile ($|V_i| = |V_j| = V_{op}$ for every $i, j \in [n]$):

$$= -\theta_i V_{op}^2 \sum_{j=1}^{n} \mathrm{L}_{i,j} + V_{op}^2 \sum_{j=1}^{n} \mathrm{L}_{i,j}\theta_j$$

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The rows of L add up to 0:

$$= V_{op}^2 \sum_{j=1}^{n} \mathbf{L}_{i,j} \theta_j$$

$$= V_{op}^2 (\mathbf{L}\theta)_i$$

If we assume $\mathbf{V}$ to be normalised (if V is measured in p.u., instead of Volts):

$$= (\mathbf{L}\theta)_i$$

and we find:

$$\mathbf{S} = \mathbf{L}\theta$$

✎

Appendix: give a direct formula for $\mathbf{L}$

compute $\mathbf{L}$ for the example networks

give an alternative interpretation of $\mathbf{L}$ or $\mathbf{L}$: discrete laplacian

discuss the kernel of $\mathbf{L}$: it should follow from graph theoretical arguments that the kernel is spanned by $(1, 1, \ldots, 1)^T$, such that the nullity of $\mathbf{L}$ is 1. This corresponds to the fact that phase angle is a relative quantity. (The state remains (DC-)valid when increasing the phase angle at each node by the same amount. (If $\mathbf{S} = \mathbf{L}\theta$ and $\theta_0 \in \ker \mathbf{L}$, then $\mathbf{S} = \mathbf{L}(\theta + \theta_0)$.))
There are two paths to take after this point:
1: Fix the phase angle of the slack bus (to 0), which corresponds to taking the $n - 1 \times n - 1$ submatrix of $\mathbf{L}$, I think. **This is the approach of PyPSA, which might explain small differences in line flow.**
2: Leave $\mathbf{L}$ as-is, and consider the *Moore-Penrose pseudo-inverse* of $\mathbf{L}$. This is the approach of Nesti et al., which they attribute to 'distributive slack': not fixing a single slack bus.

5.5.1 Usefulness

Real transmission networks do not satisfy the DC approximation and, in general, a DC-valid state is not (physically) valid. Yet, using these two approximations has one important property: the approximated node flow equation becomes *linear*. This has a number of advantages:

- Solving the Power Flow problem (i.e. computing line flows induced by a power injection vector) becomes trivial. As we will see in Section 5.6, `ref`

    ugh

- A linear power flow is crucial for the study of stochastic power injections, since it can be used to transform a probability distribution (like a multivariate normal distribution) without losing its structure. `be specific`

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5.5.2 Accuracy

The DC approximation is a useful tool for understanding the complex nature of transmission networks. Therefore, it is crucial to verify that the DC approximation is, in fact, a good approximation, when real-world networks are studied. More precisely, one should ask:

1. How close is a DC-valid state to being valid? More precisely, when the approximate node flow equation (5.5) holds, what is the mismatch between the left hand and right hand side of the node flow equation (5.1)?

2. How close are real-world grid structures to satisfying the DC approximation?

The first question is answered in Proposition 5.5.5, where an upper bound is derived for the power mismatch. It shows that the mismatch is bounded by the *squares* of local differences in phase angles (i.e. $\theta_i - \theta_j$ for line (i, j)). **?** have shown that in the Belgian transmission network, all phase differences are below 7°, and 94% of lines have a phase difference below 2°.

The second question

> acc of DC approx struc vs acc of DC valid state

Lemma 5.5.4. *There exists $K \in \mathbb{R}_{>0}$ such that*

$$|\exp ix - (1 + ix)| = K|x|^2 \qquad \textit{for every } -2\pi \leq x \leq 2\pi.$$

Proof. **A proof using complex analysis.**[8] The complex (entire) function $z \mapsto \exp z$ is defined by the power series

$$z \mapsto \sum_{k=0}^{\infty} \frac{z^k}{k!} = 1 + z + z^2 \left(\frac{z^0}{2!} + \frac{z^1}{3!} + \dots \right)$$

which has infinite radius of convergence, and is continuous on $\mathbb{C}$.

The functions $z \mapsto \frac{1}{z^2}$, $z \mapsto \exp z$ and $z \mapsto 1 + z$ are all holomorphic on $\mathbb{C}^* = \mathbb{C} \setminus \{0\}$, and so the function

$$g : \mathbb{C}^* \to \mathbb{C} \qquad g : z \mapsto \frac{1}{z^2} (\exp z - (1 + z)) \tag{5.7}$$

is holomorphic on $\mathbb{C}^*$, with a removable singularity at 0. The (unique) extension of g to $\mathbb{C}$ is an entire function, and by construction:

$$\exp z = 1 + z + z^2 g(z) \qquad \text{for each } z \in \mathbb{C}.$$

Since g is entire, it is continuous on $\mathbb{C}$. $I = [-i2\pi, i2\pi]$ is a compact subset of $\mathbb{C}$, so g is bounded on I, proving the result. ✎

Proposition 5.5.5. *Suppose $((\mathcal{N}, \mathcal{L}), \mathbf{C}, \eta)$ is an (n, m)-grid structure, with a state $(\mathbf{S}, \mathbf{V}, \mathbf{I})$ that admits a flat profile with operating voltage V_{op}.*

---

[8]For an introduction, see ?.

Margin notes:
- europe
- SciGRID
- Purchala
- literature: ??
- Move to appendix?
- Find an upper bound for K, $K = 0.5$ works
- 0^0

If the state is DC-valid, then there exists $K \in \mathbb{R}_{\geq 0}$ such that for each node i:

$$\left| S_i - i \sum_{j=1}^{n} L_{i,j}^* |V_i||V_j|e^{i(\theta_i - \theta_j)} \right| \leq K V_{op}^2 \sum_{j=1}^{n} |L_{i,j}|(\theta_i - \theta_j)^2$$

$$\leq K V_{op}^2 \|\eta\|_\infty \|\mathbf{C}\theta\|_2^2 \,.$$

Remark. This result states that if the grid state is *DC-valid*, and the power angles (i.e. $\theta_i - \theta_j$ for line (i, j)) are low, then the state is *close* to also being *valid*.

Quantitatively, it tells us that for a given node i, the 'error' resulting from using the approximate node flow equation (5.5) is proportional to the *squares* of phase angles of lines that connect to i.

Proof. Suppose $i \in [n]$.

Choose a $K \in \mathbb{R}_{\geq 0}$ for which Lemma 5.5.4 holds. The state is DC-valid, so we can

> A more important result would be an upper limit for the error in *line flow*, instead of the error in *node flow*

> this follows from the proof, not the statement itself

substitute (5.5) for S_i:

$$\left| S_i - \sum_{j=1}^{n} M_{i,j}^* |V_i||V_j| e^{i(\theta_i - \theta_j)} \right|$$

$$= \left| i \sum_{j=1}^{n} L_{i,j}^* |V_i||V_j|(1 + i(\theta_i - \theta_j)) - i \sum_{j=1}^{n} L_{i,j}^* |V_i||V_j| e^{i(\theta_i - \theta_j)} \right|$$

$$= V_{op}^2 \left| \sum_{j=1}^{n} L_{i,j}^* \left(e^{i(\theta_i - \theta_j)} - (1 + i(\theta_i - \theta_j)) \right) \right|$$

$$\leq V_{op}^2 \sum_{j=1}^{n} |L_{i,j}| \left| e^{i(\theta_i - \theta_j)} - (1 + i(\theta_i - \theta_j)) \right| \qquad \text{(triangle inequality)}$$

$$\leq K V_{op}^2 \sum_{j=1}^{n} |L_{i,j}| (\theta_i - \theta_j)^2 \qquad \text{(Lemma 5.5.4)}$$

$$= K V_{op}^2 \left[\sum_{j=i} |L_{i,j}| (\theta_i - \theta_j)^2 + \sum_{\substack{j \neq i \\ i,j \text{ connected}}} |L_{i,j}| (\theta_i - \theta_j)^2 + \sum_{\substack{j \neq i \\ i,j \text{ not connected}}} |L_{i,j}| (\theta_i - \theta_j)^2 \right]$$

$$= K V_{op}^2 \left[|L_{i,i}| (\theta_i - \theta_i)^2 + \sum_{\substack{j \neq i \\ i,j \text{ connected}}} |L_{i,j}| (\theta_i - \theta_j)^2 + \sum_{\substack{j \neq i \\ i,j \text{ not connected}}} 0 \cdot (\theta_i - \theta_j)^2 \right]$$

$$= K V_{op}^2 \sum_{\substack{j \neq i \\ i,j \text{ connected}}} |L_{i,j}| (\theta_i - \theta_j)^2$$

$$\leq K V_{op}^2 \sum_{\mathcal{L}_k = (a,b) \in \mathcal{L}} |L_{a,b}| (\theta_a - \theta_b)^2$$

$$= K V_{op}^2 \sum_{\mathcal{L}_k = (a,b) \in \mathcal{L}} |\eta_k| (C\theta)_k^2$$

$$= K V_{op}^2 \|\eta * \mathbf{C}\theta * \mathbf{C}\theta\|_1 \qquad (\text{in } \ell^1)$$

$$\leq K V_{op}^2 \|\eta\|_\infty \|\mathbf{C}\theta * \mathbf{C}\theta\|_1 \qquad \text{(Hölder's inequality)}$$

$$= K V_{op}^2 \|\eta\|_\infty \|\mathbf{C}\theta\|_2^2$$

Discuss the PDEs used for computing the PF?

5.6 Linear Power Flow

> Physically, power does not actually flow. (Although *energy* does). A better term would be: *flow power*, since a high power flow from A to B is actually a *powerful flow of energy* from A to B.
>
> In the context of line overloads, we are only interested in line *current*, which can be seen as proportional to the amount of power being transmitted by the line. (Again, power is not a physical quantity that can be transmitted.) In a flat, normalised profile, power flow and line current are identical. To follow general convention, we will talk about power flow instead of line current. For example, line ratings are often given in Watts, not Amperes. (Even though the amount energy dissipated by a line, I^2R, depends only on line current. At a nearly constant operating voltage, however, these quantities are proportional to each other.)

DC approx, flat, normalised state and DC valid:

pseudoinverse:

$$\mathbf{p} = \mathbf{L}\theta$$

so there exists L^+ such that

$$\theta = \mathbf{L}^+\mathbf{p}$$

this is the moore-penrose pseudo-inverse,

> My DC approximated KVL-Ohm is incorrect, it should be: $\mathbf{I} = i\,\mathrm{diag}(i\eta)\mathbf{C}\theta V_{op}$.

$$\hat{\mathbf{f}} = -iV_{op}\mathbf{I} = -iV_{op}^2 i\,\mathrm{diag}(i\eta)\mathbf{C}\theta = \mathrm{diag}(i\eta)\mathbf{C}\mathbf{L}^+\mathbf{p}.$$

(We assumed a normalised profile, so $V_{op} = 1$.)

If the line capacities are $\mathbf{W} = (W_1, \ldots, W_m)^T \in \mathbb{R}^m$, then we can define the *normalised line flow* as:

$$\mathbf{f} = \mathrm{diag}(\mathbf{W})^{-1}\hat{\mathbf{f}}$$

and we find:

$$\mathbf{f} = \mathbf{F}\mathbf{p} \tag{5.8}$$

$$\text{where} \quad \mathbf{F} = \mathrm{diag}(\mathbf{W})^{-1}\,\mathrm{diag}(i\eta)\mathbf{C}\mathbf{L}^+ \tag{5.9}$$

This is a linear transformation from a power injection vector $\mathbf{p}$ *to the normalised line flow* $\mathbf{f}$ *that it induces.*

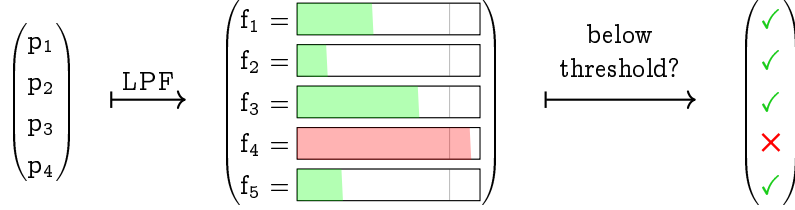## 5.7 Stochastic power injections

Using the linear transformation $\mathbf{F}$ that we just derived, we can compute the normalised line flow that a *zero-sum* power injection $\mathbf{p}$ induces. This is a useful result: for example, one could check whether a generator configuration is *admissible* by writing down the
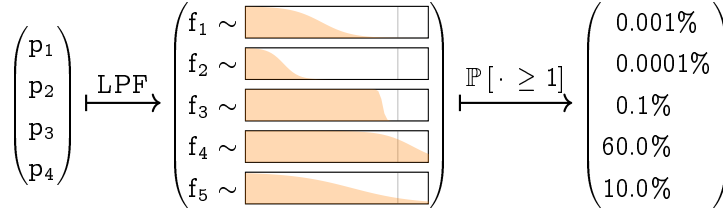
> C already used

> **F** is not the unique transformation with this property (otherwise **F** would be square and non-singular). Does **F** minimise $\|\mathbf{f}\|$?

> line drop: (real) power

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injection $\mathbf{p}$ associated to this configuration, and checking that no value of $\mathbf{Fp}$ is greater than 1. (Since we *normalised* the line flows, each line has unit 'threshold'.)

$$\begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix} \xmapsto{\text{LPF}} \begin{pmatrix} f_1 = \\ f_2 = \\ f_3 = \\ f_4 = \\ f_5 = \end{pmatrix} \xmapsto{\text{below threshold?}} \begin{pmatrix} \checkmark \\ \checkmark \\ \checkmark \\ \times \\ \checkmark \end{pmatrix}$$

Things get even more interesting when $\mathbf{p}$ is a *stochastic variable*. In this case, $\mathbf{f}$ is also a stochastic variable! In fact, given a probability distribution function for $\mathbf{p}$, we can use $\mathbf{F}$ to compute the *probability distribution function of* $\mathbf{f}$. Using this probability distribution, we can now answer questions such as: "What is the probability that line i overloads?" ($\mathbb{P}[f_i \geq 1]$) Or: "What is the probability that line i overloads, given that line j is operating at 95% capacity?" ($\mathbb{P}[f_i \geq 1 \mid f_j = 0.95]$).

$$\begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix} \xmapsto{\text{LPF}} \begin{pmatrix} f_1 \sim \\ f_2 \sim \\ f_3 \sim \\ f_4 \sim \\ f_5 \sim \end{pmatrix} \xmapsto{\mathbb{P}[\,\cdot\,\geq 1]} \begin{pmatrix} 0.001\% \\ 0.0001\% \\ 0.1\% \\ 60.0\% \\ 10.0\% \end{pmatrix}$$

5.7.1 Normally distributed power injection

Following **?**, we model $\mathbf{p}$ to be *multivariate normally distributed*:

$$\mathbf{p} \sim N(\mu_{\mathbf{p}}, \Sigma_{\mathbf{p}})$$

where $\mu_{\mathbf{p}}$ is the mean power injection, which is the sum of deterministic generation and expected stochastic generation, minus the load. $\Sigma_{\mathbf{p}}$ is the *power injection correlation matrix*. When power injection at different nodes is correlated (because of correlated weather), this matrix is non-diagonal.

Because $\mathbf{F}$ is a linear transformation, the vector of line flows $\mathbf{f}$ is also (multivariate) normally distributed, and its distribution is given by

$$\mathbf{f} \sim N(\mu_{\mathbf{f}}, \Sigma_{\mathbf{f}}), \quad \text{where} \quad \mu_{\mathbf{f}} = \mathbf{F}\mu_{\mathbf{f}} \quad \text{and} \quad \Sigma_{\mathbf{f}} = \mathbf{F}\Sigma_{\mathbf{p}}\mathbf{F}^T. \tag{5.10}$$

5.8 Applying the Large Devation Principle

We will study $\mathbb{P}(|f_l| \geq 1)$ for each line l using the LDP.

Part II

Predicting failures in Germany's grid

Chapter six

Methods

The goal of the model described in this thesis can be summarised as follows:

Given:

- A real-world *grid structure*

- Hourly *load series*

- Hourly *generation series*

Predict:

- *'Top 10'* of lines most likely to fail

- For each risky line: the most likely *cascading line failures*

The model, in particular the use of large deviation statistics to study cascading line failures, was proposed by **?**. Following their approach, the model is applied numerically to the SciGRID dataset (**?**). This dataset is an approximation of the transmission network in Germany, including grid structure, generation capacity, hourly load series and hourly renewable generation series.

As discussed in Section **??**, transmission lines can fail for a number of reasons, and this model only studies one specific type of failure: *short-term changes in renewable generation.*

Line overloads are, however, the most imminent type of failure in a network, in the sense that grid operators need to continually monitor the *configuration* (distribution among the nodes of power generation) to ensure that no line will overload. This strongly constrains the capability of the transmission network. At any point in time, there could be many configurations that are environmentally (or *economically*) better than the current configuration, but they might lead to a line overload.[1] Turning off functioning

> how common are these types of failures?

---

[1] It is for this reason that the Optimal Power Flow problem is computationally difficult solve: it often takes many iterations until a generation distribution is found that does not cause any transmission line to be overloaded.
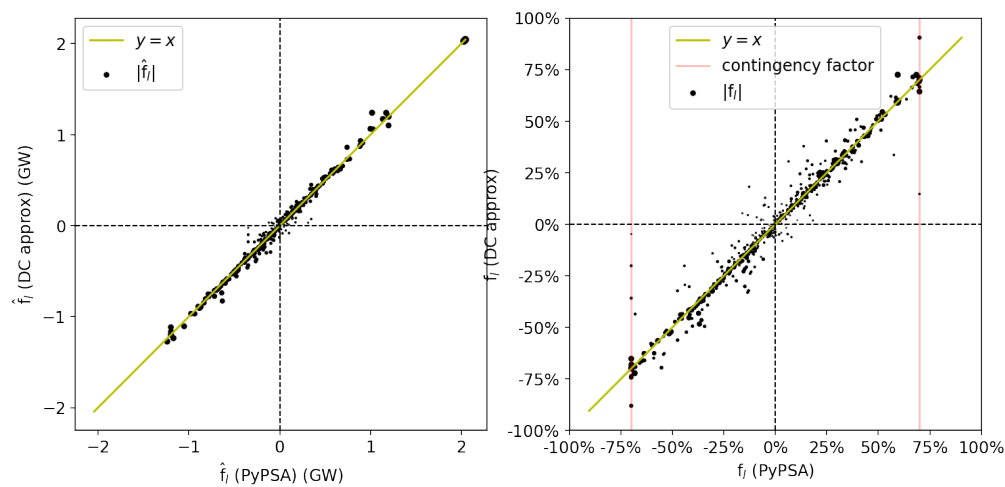
Figure 6.1: PLACEHOLDER: Line flow (left) and saturation (right) computed using LPF and (non-linear) PF.

wind turbines is known as *wind curtailment*, and switching off solar panels is called *solar curtailment*.[2]

In China, as of 2019 the world's largest producer of renewable energy *by a factor of two*, wind curtailment amounted to 16% between 2010 and 2016. (**?**)

ref, wiki

6.1 Constructing a complete dataset

6.1.1 PyPSA & SciGRID

Asdf

context,

some example graphs

context: PSAT for matlab **?**, references 110 and 111 of PSAT manual **?**

PyPSA: **?**

Maybe: PyPSA-EUR: **?**

6.2   LPF

6.2.1   Comparison with PyPSA results

6.3   Stochastic power injection

6.3.1   Explorative data analysis

6.3.2   ARMA model

Computing ARMA fits

Following **?**, the ARMA models were fitted to the first month of solar and wind generation series using the `arima` method in `R`. This produced the desired result for all wind series.

Unfortunately, the `arima` method failed to fit an ARMA model for 104 of the 489 buses, reflecting the hypothesis that these series are poorly modelled by an ARMA series. To recreate the results of **?**, it is crucial that a model can be fitted for *every* bus. Therefore, a modified method is required.

By the default, the `arima` method operates in two steps. First, a so-called Conditional-Sum-of-Squares (CSS) method is used as initial guess for the ARMA coefficients. Then, the coefficient *likelihood* function is maximised with the Nelder-Mead optimisation method, using the CSS result as initial value.

In our case, the CSS method often finds initial values that are *non-stationary*. A possible reason is that solar generation is exactly zero during nighttime, which lasts for up to 16 consecutive hours.[3] To resolve this issue, we use an optimisation method that works well without specifying an initial value: it should find the *global* maximum independently. The Simulated Annealing (SANN) algorithm included in `R` is such a method, and was chosen for this analysis.

With SANN as optimisation method, `arima` was able to fit solar models for 484 out of 489 buses. For the remaining 5 buses, a second technique is applied: to accommodate for the long periods of zero generation, a small amount of uniform noise is superimposed on the generation series. Initially, the noise magnitude is set to 1% of the solar capacity of the bus. This percentage is increased iteratively, until the modified method is able to fit a solar model. In fact, 1% noise proved sufficient for the remaining 5 buses in January, and no more than 2% noise was needed for any bus in the remaining months.

The modified method described in this section was used to fit solar and wind models to each bus of the network, for each month of 2011. Table 7.1 lists the number of buses that can be analysed using the default and modified methods, showing the poor performance of the default for solar series, especially during the months of summer.

> define?

> Belisle 1992, zie optim docs

---

[2]Wind turbines can indeed be turned off, by twisting the rotor blades towards 0° pitch and applying the *brakes*. In larger turbines, common types are drum brakes (similar to a back-pedalling brake for bikes) and disk brakes. Brakes are especially important during maintenance. Solar panels are switched off using a simple mechanical or electronic switch (a relay). (**?**)

[3]In reality, solar generation is never *exactly* zero, even after sunset, because solar panels also receive

? do not mention such problems (nor techniques to solve them). Because the same dataset, processing, programming language and method was used in this analysis, it is likely that they have also encountered and solved these problems, possibly in the same manner.

> discuss other possible techniques: warm start, intercept

Runtime

The default implementation has an estimated (single-core) runtime of 101 hours (4.2 days) when all 12 months are analysed, although only 78% of fits are successful using this method.

The modified version is able to fit all generation series successfully. The use of this more computationally expensive method increases the runtime to an estimated 224 hours (9.4 days). Because of the long runtime, fits were computed in parallel on a 40-core machine[4], provided by the IT department of the Radboud University.[5] This reduced the total runtime down to 7 hours. The results are publicly available at the GitHub repository.

6.3.3 Stochastic line flows

6.4 Line failures

6.5 Cascading line failures

---

ambient light. This is not included in the (artificial) dataset.

[4]Specifications: two *Intel Xeon E5 2630L v4* 10-core processors @2GHz with hyperthreading; 64GB RAM. CPU time was used as estimate for single-core runtime.

[5]Thank you!

Chapter seven

Results

asdf

l	$\mathbb{P}\left[|f_l| \geq 1\right]$	I_l
361	3.45 %	1.65
516	2.91 %	1.79
586	2.75 %	1.84
587	2.73 %	1.85
803	2.02 %	2.10
670	1.21 %	2.54
19	1.13 %	2.60
302	1.08 %	2.64
48	1.03 %	2.68
554	0.974%	2.73
488	0.971%	2.73
809	0.824%	2.88
28	0.748%	2.96
810	0.728%	2.98
29	0.396%	3.53
27	0.355%	3.62
389	0.247%	3.95
390	0.245%	3.96
486	0.235%	4.00
249	0.056%	5.30

Plotjes van kansen zoals in
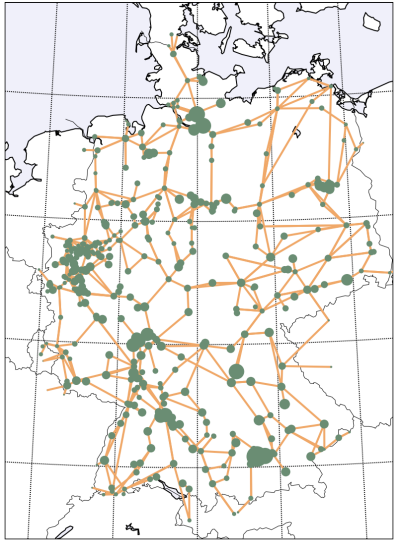
Figure 7.1: PLACEHOLDER: The load distribution of Germany at ???.
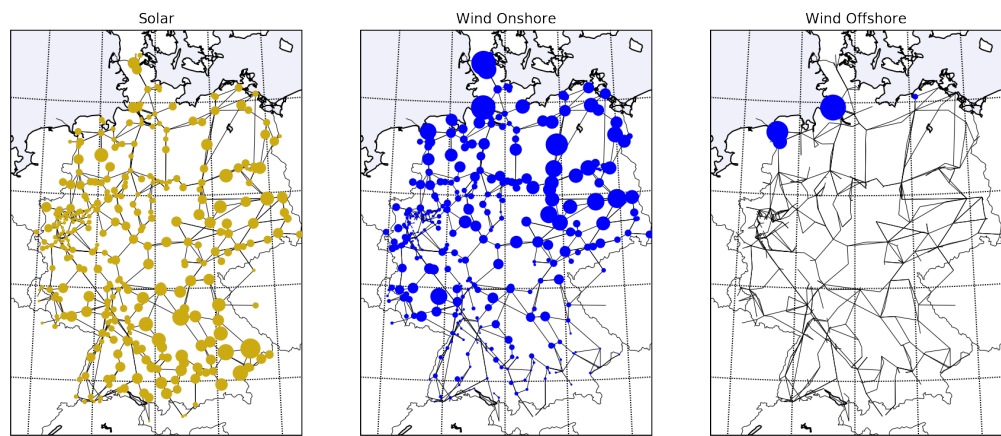

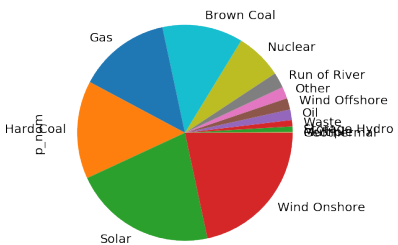**Figure 7.2:** PLACEHOLDER: Solar, wind onshore, wind offshore generation at ?? in Germany.

Figure 7.3: PLACEHOLDER: Generation capacity technology shares in Germany in 2011.

	SOLAR				WIND			
	default	*modified*	1%	2%	*default*	*modified*	1%	2%
Jan	385	**484**	**5**	**0**	**488**	488	**0**	**0**
Feb	364	**485**	**4**	**0**	**487**	487	**0**	**1**
Mar	334	**475**	**14**	**0**	**487**	487	**1**	**0**
Apr	211	**477**	**11**	**1**	**488**	488	**0**	**0**
May	189	**473**	**16**	**0**	**467**	467	**21**	**0**
Jun	255	**481**	**8**	**0**	**480**	480	**8**	**0**
Jul	292	**485**	**4**	**0**	**488**	488	**0**	**0**
Aug	179	**478**	**10**	**1**	**488**	488	**0**	**0**
Sep	259	**472**	**16**	**1**	**487**	487	**1**	**0**
Oct	263	**480**	**9**	**0**	**488**	488	**0**	**0**
Nov	290	**472**	**17**	**0**	**486**	486	**2**	**0**
Dec	317	**482**	**7**	**0**	**486**	486	**2**	**0**

Table 7.1: Number of buses (out of 489 solar, 488 wind) for which the ARMA model could be fitted using either the *default* implementation of arima in R, or the *modified* version (described in 6.3.2).

When even the modified method yielded no results, uniform noise was added to the series before fitting. Noise magnitude was set to 1% of generation capacity, or 2% when needed.

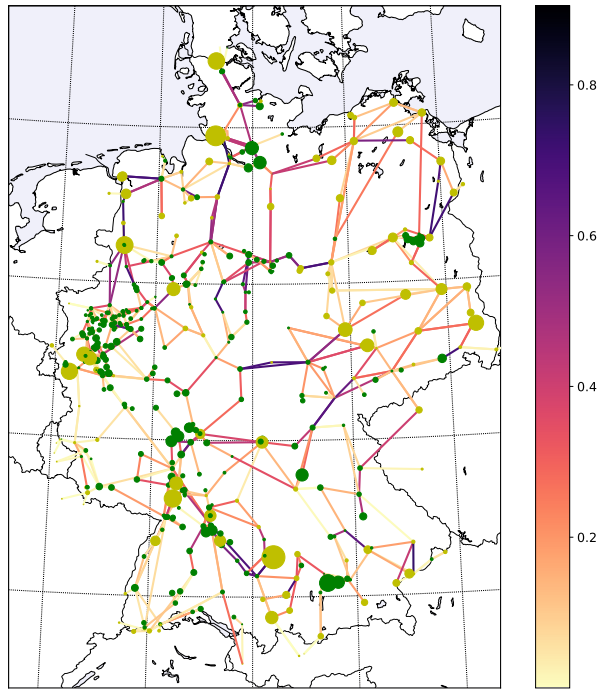Figures in **bold** indicate which results are used.

Figure 7.4: Nominal line flow during 11:00-12:00, as fraction of line capacity. Node size represents net power injection. When generation exceeds load, the injection is positive (yellow), otherwise negative (green). *Compare with Figure 1a of **?***.

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