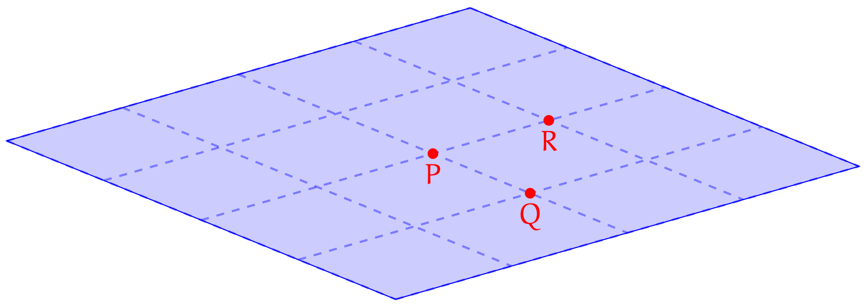
Example 1. Find a scalar equation for the plane that passes through the points $P(1, -2, 1)$, $Q(-4, 5, -8)$, and $R(7, -8, 7)$.



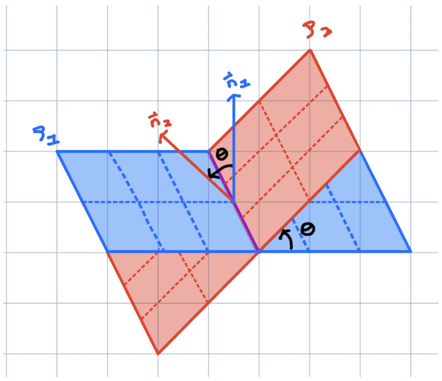
Example 2. Consider the planes:

$$\mathcal{P}_1 : 2x + y + 4z = 2$$

$$\mathcal{P}_2 : 2x - 4y + z = 2$$

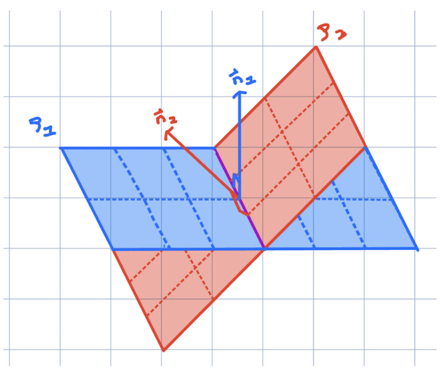
(a) Find the angle between the planes.

Typically we consider this angle (between the normal vectors) to be the angle between the planes, because it effectively captures how one is tilted with respect to the other. Therefore if you are asked for the angle between two planes, know that you are really being asked for the angle between their normal vectors.



(b) Parametrize the line of intersection of the two planes.

Hint: Note that $P(1, 0, 0)$ is on both planes.

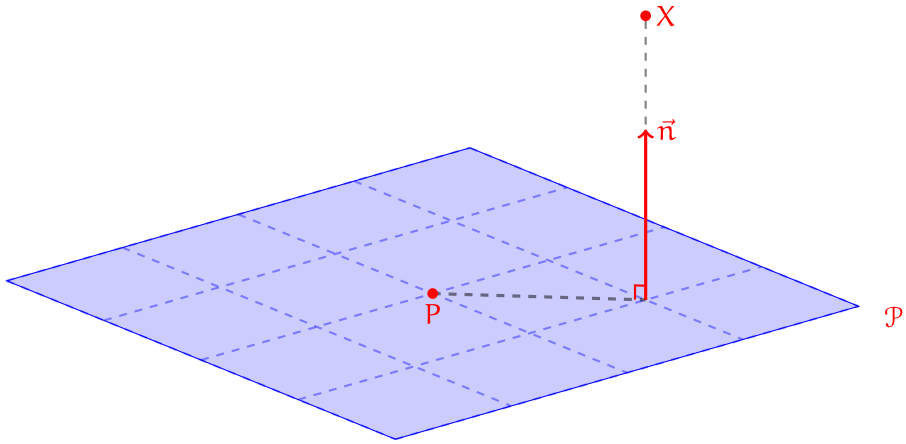


A. **Distances.** We can chat about distances between lines, planes, and points. Let us begin with the distance between a point X and a plane $\mathcal{P}$.

Precisely: the **distance** will always refer to the **shortest** distance between the objects.

Say we know a point P on the plane and a normal vector $\vec{n}$ to the plane.

We have to know something to do something.



The distance between point X and plane $\mathcal{P}$ is:

where P is a known point on the plane, and $\vec{n}$ is a known normal vector.

In this formula remember that P is a point on the plane and $\vec{n}$ is a normal vector to the plane.

Example 3. Consider the planes:

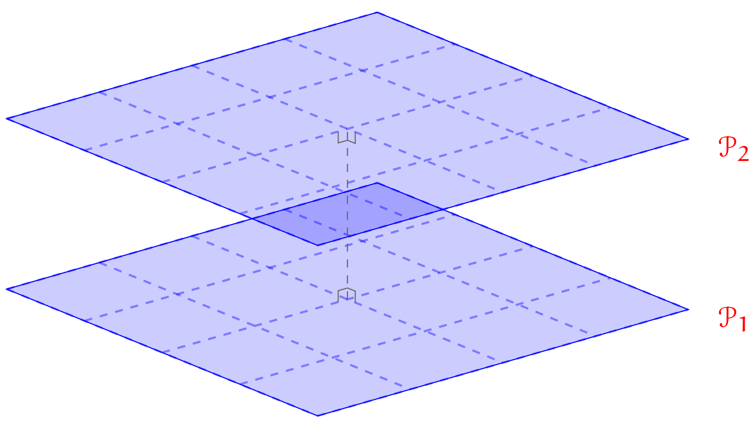
$$\mathcal{P}_1 : 10x + 2y - 2z = 6$$

$$\mathcal{P}_2 : 5x + y - z = 1$$

(a) Explain why these planes are parallel.

Planes are parallel if their normal vectors are parallel!

(b) Knowing that they are parallel, find the distance between them.



(c) Find the closest point on $\mathcal{P}_2$ to the point P_1 you used in part b.