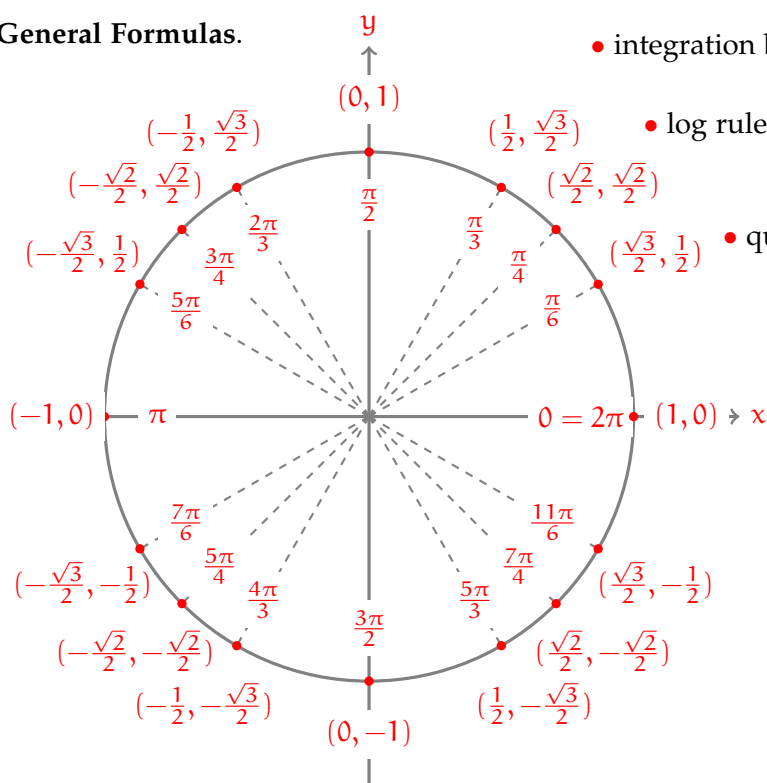


General Formulas.



• integration by parts: $\int u \, dv = uv - \int v \, du$

• log rules: $\ln(A) + \ln(B) = \ln(AB)$, $\ln(A) - \ln(B) = \ln\left(\frac{A}{B}\right)$,
 $c \ln(A) = \ln(A^c)$

• quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

• $\int e^{at} \cos bt \, dt = \frac{e^{at} (a \cos bt + b \sin bt)}{a^2 + b^2}$

• $\int e^{at} \sin bt \, dt = \frac{e^{at} (-b \cos bt + a \sin bt)}{a^2 + b^2}$

A2 Formulas.

- for $y' + a(x)y = f(x)$ we have:
 - integrating factor $I(x) = e^{\int a(x) \, dx}$
 - variation of parameters: $y = u y_h$
 - where $y_h = e^{-\int a(x) \, dx}$ and $u = \int \frac{f(x)}{y_h} \, dx$
- circuits
 - Kirchhoff's Voltage Law: $E = RI + LI' + Q/C$
 - derivative of charge is current: $I = Q'$
- Bernoulli equation: $y' + a(t)y = f(t)y^n$
 - substitute $u = y^{1-n}$
 - converts to $u' + (1-n)a(t)u = (1-n)f(t)$

A3 Formulas.

- spring with no external force: $my'' + \mu y' + ky = 0$
- Wronskian: $W(y_1, y_2) = y_1 y_2' - y_1' y_2$
- Abel's formula: for $y'' + p(t)y' + q(t)y = 0$
 - $W(y_1, y_2) = A e^{-\int p(t) \, dt}$ if y_1, y_2 are solutions
 - if y_1 is a solution, then Abel's formula with $A = 1$ yields:
 - so is $y_2 = u y_1$ where $u = \int \frac{e^{-\int p(t) \, dt}}{y_1^2} \, dt$

A4 Formulas.

forcing term	trial solution y_p
$[\deg m \text{ poly}]e^{at}$	$t^n [\deg m \text{ poly}]e^{at}$
$[\deg m \text{ poly}]e^{at} \cos(bt)$ or $[\deg m \text{ poly}]e^{at} \sin(bt)$	$t^n [\deg m \text{ poly}]e^{at} \cos(bt)$ + $t^n [\deg m \text{ poly}]e^{at} \sin(bt)$

note: n is the # of times $\lambda = a$ (or $\lambda = a + bi$) is a root of the characteristic equation.

A5 Formulas.

- variation of parameters for $y'' + p(t)y' + q(t)y = 0$:
 - $y = u_1 y_1 + u_2 y_2$ where:
 - y_1, y_2 are fundamental homogeneous solutions
 - $u_1 = - \int \frac{y_2 f}{W} \, dt$ and $u_2 = \int \frac{y_1 f}{W} \, dt$