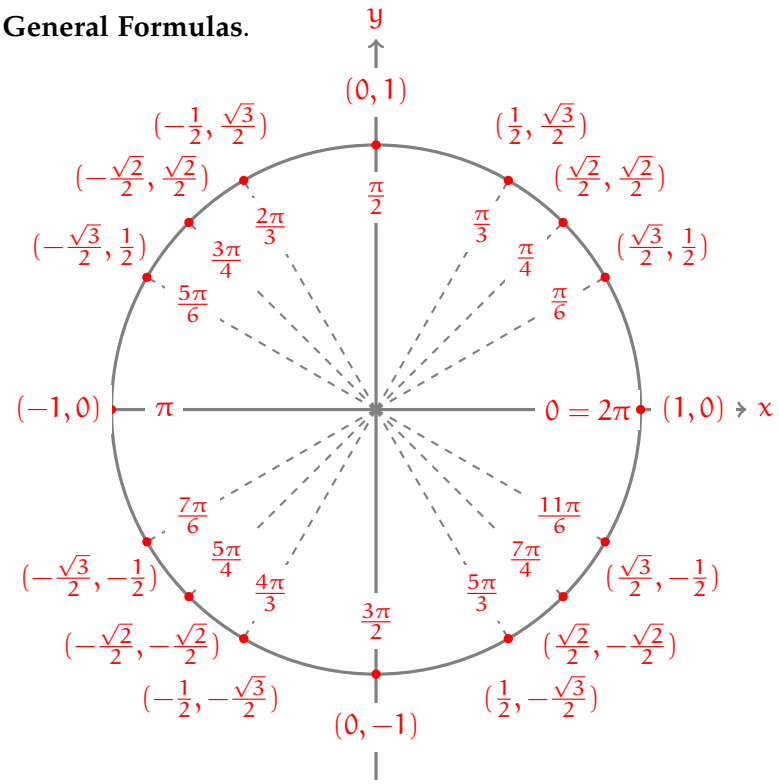
General Formulas.



- $\arctan(t) = \int \frac{1}{1+t^2}\,dt$
- $\arcsin(t) = \int \frac{1}{\sqrt{1-t^2}}\,dt$
- $\ln|t| = \int \frac{1}{t}\,dt$
- power reduction formulas:
  - $\cos^2\theta = \frac{1+\cos 2\theta}{2}$ and $\sin^2\theta = \frac{1-\cos 2\theta}{2}$
- double-angle formulas:
  - $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = \cos^2\theta - \sin^2\theta$

A1 Formulas.

- products and lengths and angles:
  - $\vec{v}\cdot\vec{w} = \|\vec{v}\|\|\vec{w}\|\cos\theta$
  - $\|\vec{v}\times\vec{w}\| = \|\vec{v}\|\|\vec{w}\|\sin\theta =$ [parallelogram area]
- projection and scalar component:
  - $\text{proj}_{\vec{v}}(\vec{w}) = \left(\frac{\vec{v}\cdot\vec{w}}{\vec{v}\cdot\vec{v}}\right)\vec{v}$ $\quad$ $\text{comp}_{\vec{v}}(\vec{w}) = \frac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|}$
- scalar triple product:
  - $\vec{v}\cdot(\vec{w}\times\vec{r}) = \vec{r}\cdot(\vec{v}\times\vec{w}) = \vec{w}\cdot(\vec{r}\times\vec{v})$

A2 Formulas.

- distance from point B to plane $\mathcal{P}$ with normal $\vec{n}$:
  - $\frac{|\mathbf{AB}\cdot\vec{n}|}{\|\vec{n}\|}$ where A is on $\mathcal{P}$
- distance from point B to line ℓ with direction vector $\vec{v}$:
  - $\frac{\|\mathbf{AB}\times\vec{v}\|}{\|\vec{v}\|}$ where A is on ℓ

A3 Formulas.

- standard form surfaces:
  - paraboloid: $\hat{z} = \hat{x}^2 + \hat{y}^2$
  - saddle: $\hat{z} = \hat{x}^2 - \hat{y}^2$
  - 1-sheeted hyperboloid: $\hat{x}^2 + \hat{y}^2 - \hat{z}^2 = 1$
  - 2-sheeted hyperboloid: $-\hat{x}^2 - \hat{y}^2 + \hat{z}^2 = 1$
  - ellipsoid: $\hat{x}^2 + \hat{y}^2 + \hat{z}^2 = 1$
  - double-cone: $\hat{z}^2 = \hat{x}^2 + \hat{y}^2$

A4 Formulas.

- tangent plane to $z = f(x,y)$ at $(a,b,f(a,b))$ is:
  - $z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

A5 Formulas.

- $D_{\vec{u}}f(P) = \nabla f(P)\cdot\vec{u}$ where $\vec{u}$ is a unit direction
  - max'ed in direction $\nabla f(P)$, with value $\|\nabla f(P)\|$
  - min'ed in direction $-\nabla f(P)$, with value $-\|\nabla f(P)\|$
  - equals 0 in directions $\perp$ to $\nabla f(P)$
- tangent plane to level set $F(x,y,z) = C$ at P is:
  - $\nabla F(P)\cdot(\vec{x}-\vec{p}) = 0$