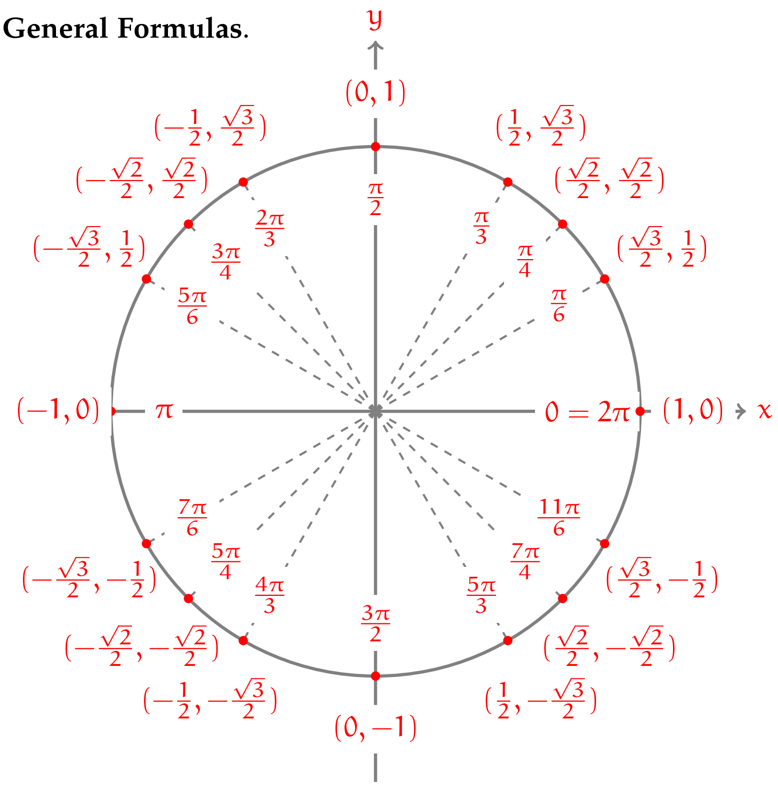
General Formulas.



- $\arctan(t) = \int \frac{1}{1+t^2}\, dt$
- $\arcsin(t) = \int \frac{1}{\sqrt{1-t^2}}\, dt$
- $\ln|t| = \int \frac{1}{t}\, dt$

A1 Formulas.

- products and lengths and angles:
  - $\mathbf{v} \cdot \mathbf{w} = \|\mathbf{v}\|\|\mathbf{w}\| \cos\theta$
  - $\|\mathbf{v} \times \mathbf{w}\| = \|\mathbf{v}\|\|\mathbf{w}\| \sin\theta =$ [parallelogram area]
- projection and scalar component:
  - $\text{proj}_{\mathbf{v}}(\mathbf{w}) = \left(\frac{\mathbf{v}\cdot\mathbf{w}}{\mathbf{v}\cdot\mathbf{v}}\right)\mathbf{v}$ ◦ $\text{comp}_{\mathbf{v}}(\mathbf{w}) = \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|}$
- scalar triple product $= \pm$ parallelepiped volume:
  - $\mathbf{v}\cdot(\mathbf{w}\times\mathbf{r}) = \mathbf{r}\cdot(\mathbf{v}\times\mathbf{w}) = \mathbf{w}\cdot(\mathbf{r}\times\mathbf{v})$

A2 Formulas.

- distance from point B to plane $\mathcal{P}$ with normal $\mathbf{n}$:
  - $\frac{|\mathbf{AB}\cdot\mathbf{n}|}{\|\mathbf{n}\|}$ where A is on $\mathcal{P}$
- distance from point B to line ℓ with direction vector $\mathbf{v}$:
  - $\frac{\|\mathbf{AB}\times\mathbf{v}\|}{\|\mathbf{v}\|}$ where A is on ℓ

A4 Formulas.

- tangent plane to $z = f(x,y)$ at $(a,b,f(a,b))$ is:
  - $z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

A3 Formulas.

- standard form surfaces:
  - paraboloid: $\hat{z} = \hat{x}^2 + \hat{y}^2$
  - saddle: $\hat{z} = \hat{x}^2 - \hat{y}^2$
  - 1-sheeted hyperboloid: $\hat{x}^2 + \hat{y}^2 - \hat{z}^2 = 1$
  - 2-sheeted hyperboloid: $-\hat{x}^2 - \hat{y}^2 + \hat{z}^2 = 1$
  - ellipsoid: $\hat{x}^2 + \hat{y}^2 + \hat{z}^2 = 1$
  - double-cone: $\hat{z}^2 = \hat{x}^2 + \hat{y}^2$

A5 Formulas.

- $D_{\mathbf{u}}f(P) = \nabla f(P)\cdot\mathbf{u}$ where $\mathbf{u}$ is a unit direction
  - max'ed in direction $\nabla f(P)$, with value $\|\nabla f(P)\|$
  - min'ed in direction $-\nabla f(P)$, with value $-\|\nabla f(P)\|$
  - equals 0 in directions $\perp$ to $\nabla f(P)$
- tangent plane to level set $F(x,y,z) = C$ at P is:
  - $\nabla F(P)\cdot(\mathbf{x}-\mathbf{p}) = 0$

B1 Formulas.

- If P is a critical point of $f(x, y)$ and:
  - $D = f_{xx}(P)f_{yy}(P) - (f_{xy}(P))^2$
  - $T = f_{xx}(P) + f_{yy}(P)$
- then:
  - $D < 0 \implies$ saddle
  - $D > 0$ and $T > 0 \implies$ local minimizer
  - $D > 0$ and $T < 0 \implies$ local maximizer

B2 Formulas.

- An extremizer P of f subject to $g = C$ satisfies:
  - $\nabla f(P) = \lambda \nabla g(P)$ or $\nabla g(P) = \mathbf{0}$