Sum Formulas.

- $1+2+\cdots+n = \frac{n(n+1)}{2}$
- $1^2+2^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$
- $1^3+2^3+\cdots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$
- if $|x|<1$, then:

$$\sum_{k=0}^{\infty} x^n = \frac{1}{1-x}$$
$$\sum_{k=1}^{\infty} nx^{n-1} = \frac{1}{(1-x)^2}$$

A1 Formulas.

- k–permutations: ${}_nP_k = \frac{n!}{(n-k)!}$
- k–combinations: ${}_nC_k = \binom{n}{k} = \frac{n!}{k!(n-k)!}$
- multiset permutations: $\binom{n}{n_1,n_2,\ldots,n_r} = \frac{n!}{n_1!n_2!\cdots n_r!}$
- binomial theorem: $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$
- multinomial theorem:
  $(x_1+\cdots+x_r)^n = \sum_{\substack{n_1+\cdots+n_r=n \\ n_1,\ldots,n_r\geqslant 0}} \binom{n}{n_1,\ldots,n_r} x_1^{n_1}\cdots x_r^{n_r}$
- the # of solutions $(x_1,\ldots,x_r)$ to $x_1+\cdots+x_r=n$ is:
  - $\binom{n-1}{r-1}$ if only positive integers are allowed
  - $\binom{n+r-1}{r-1}$ if only nonnegative integers are allowed

A2 Formulas.

- inclusion–exclusion:

$$\mathbb{P}(E_1\cup\cdots\cup E_n) = \sum_{r=1}^{n}(-1)^{r+1}\sum_{1\leqslant i_1<\cdots<i_r\leqslant n}\mathbb{P}(E_{i_1}\cdots E_{i_r})$$

A3 Formulas.

- conditional probability: $\mathbb{P}(E \mid F) = \frac{\mathbb{P}(EF)}{\mathbb{P}(F)}$
- independence: $E \perp F$ if and only if $\mathbb{P}(EF) = \mathbb{P}(E)\mathbb{P}(F)$
- law of total probability: if $F_1 \sqcup F_2 \sqcup \cdots \sqcup F_n = \Omega$, then:

$$\mathbb{P}(E) = \sum_{i=1}^{n} \mathbb{P}(E \mid F_i)\cdot\mathbb{P}(F_i)$$

- Bayes's formula: $\mathbb{P}(E \mid F) = \frac{\mathbb{P}(F|E)\mathbb{P}(E)}{\mathbb{P}(F)}$
- multiplication rule:
  $\mathbb{P}(E_1E_2\cdots E_n) = \mathbb{P}(E_1)\mathbb{P}(E_2 \mid E_1)\cdots\mathbb{P}(E_n \mid E_1E_2\cdots E_{n-1})$

Discrete Random Variables.

- uniform random variable: $X \sim \mathrm{Uniform}(\{1,\ldots,n\})$
  - $\mathbb{P}(X=k) = \frac{1}{n}$ if $k\in\{1,\ldots,n\}$
  - $\mathbb{E}[X] = \frac{n+1}{2}$ and $\mathrm{Var}(X) = \frac{n^2-1}{12}$
- Bernoulli random variable: $X \sim \mathrm{Bernoulli}(p)$
  - $\mathbb{P}(X=1) = p$ and $\mathbb{P}(X=0) = 1-p$
  - $\mathbb{E}[X] = p$ and $\mathrm{Var}(X) = p(1-p)$

Discrete Random Variables Continued.

- binomial random variable: $X \sim \mathrm{Binom}(n,p)$
  - $\mathbb{P}(X=k) = \binom{n}{k}p^k(1-p)^{n-k}$
  - $\mathbb{E}[X] = np$ and $\mathrm{Var}(X) = np(1-p)$
- geometric random variable: $X \sim \mathrm{Geom}(p)$
  - $\mathbb{P}(X=k) = (1-p)^{k-1}p$
  - $\mathbb{E}[X] = \frac{1}{p}$ and $\mathrm{Var}(X) = \frac{1}{p^2} - \frac{1}{p}$
- Poisson random variable: $X \sim \mathrm{Poisson}(\lambda)$
  - $\mathbb{P}(X=k) = e^{-\lambda}\frac{\lambda^k}{k!}$
  - $\mathbb{E}[X] = \lambda$ and $\mathrm{Var}(X) = \lambda$
- indicator function of an event E: $\mathbb{1}_E$
  - $\mathbb{E}[\mathbb{1}_E] = \mathbb{P}(E)$ and $\mathrm{Var}(\mathbb{1}_E) = \mathbb{P}(E)\mathbb{P}(E^c)$

A5 Formulas.

- if X is discrete:

$$\mathbb{E}[X] = \sum_x x\cdot\mathbb{P}(X=x)$$
$$\mathbb{E}[g(X)] = \sum_x g(x)\cdot\mathbb{P}(X=x)$$

- $\mathrm{Var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$
- $\mathrm{Cov}(X,Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$
- if $X \perp Y$ then:
  - $\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y]$
  - $\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$
  - $\mathrm{Cov}(X,Y) = 0$