**Integral Formulas.**

- $\int \frac{1}{1+t^2} dt = \arctan t + C$
- $\int \frac{1}{\sqrt{1-t^2}} dt = \arcsin t + C$
- $\int \frac{1}{t} dt = \ln |t| + C$

Trig Identities.

- power reduction: $\cos^2 \theta = \frac{1}{2} + \frac{1}{2} \cos 2\theta$
 $\sin^2 \theta = \frac{1}{2} - \frac{1}{2} \cos 2\theta$
- double angle: $\sin 2\theta = 2 \sin \theta \cos \theta$
 $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$







A1 Formulas.

- $\vec{v} \cdot \vec{w} = \|\vec{v}\| \|\vec{w}\| \cos \theta$
- $\|\vec{v} \times \vec{w}\| = \|\vec{v}\| \|\vec{w}\| \sin \theta = [\text{parallelogram area}]$
- $|\vec{v} \cdot (\vec{w} \times \vec{r})| = [\text{parallelepiped volume}]$
- $\text{proj}_{\vec{v}}(\vec{w}) = \left(\frac{\vec{v} \cdot \vec{w}}{\vec{v} \cdot \vec{v}} \right) \vec{v}$
- $\text{comp}_{\vec{v}}(\vec{w}) = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\|}$

A2 Formulas.

- distance from point **B** to plane \mathcal{P} with normal \vec{n} :
 $\frac{|\vec{AB} \cdot \vec{n}|}{\|\vec{n}\|}$ where **A** is on \mathcal{P}
- distance from point **B** to line ℓ with direction \vec{v} :
 $\frac{\|\vec{AB} \times \vec{v}\|}{\|\vec{v}\|}$ where **A** is on ℓ

A3 Formulas. Standard equations.

- paraboloid: $z = x^2 + y^2$ 
- saddle: $z = x^2 - y^2$ 
- 1-sheeted hyperboloid: $x^2 + y^2 - z^2 = 1$ 
- 2-sheeted hyperboloid: $-x^2 - y^2 + z^2 = 1$ 
- ellipsoid—sphere if uniformly scaled:
 $x^2 + y^2 + z^2 = 1$ 
- double-cone: $z^2 = x^2 + y^2$ 

A4 Formulas.

- tangent plane to $z = f(x, y)$ at $(a, b, f(a, b))$ is:
 $z = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$

A5 Formulas.

- given unit direction \vec{u} , the directional derivative:
 $D_{\vec{u}}f(P) = \vec{\nabla}f(P) \cdot \vec{u} = \|\vec{\nabla}f(P)\| \cos \theta$
- tangent plane to level set $F(x, y, z) = C$ at **P** is:
 $\vec{\nabla}F(P) \cdot \vec{x} = \vec{\nabla}F(P) \cdot \vec{p}$

B1 Formulas.

- If **P** is a critical point of $f(x, y)$ and:
 $D = f_{xx}(P)f_{yy}(P) - (f_{xy}(P))^2$
 $T = f_{xx}(P) + f_{yy}(P)$
- then:
 $D < 0 \implies$ saddle
 $D > 0$ and $T > 0 \implies$ local minimizer
 $D > 0$ and $T < 0 \implies$ local maximizer

B2 Formulas.

- An extremizer **P** of f subject to $g = C$ satisfies:
 $\vec{\nabla}f(P) = \lambda \vec{\nabla}g(P)$ or $\vec{\nabla}g(P) = \vec{0}$

B5 Formulas.

- polar and cylindrical
 - $r^2 = x^2 + y^2$ and $\tan \theta = \frac{y}{x}$
 - $x = r \cos \theta$ and $y = r \sin \theta$
 - $dA = r dr d\theta$ and $dV = r dz dr d\theta$