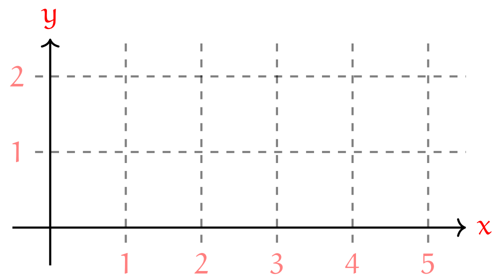
A. **Vectors.** A fundamental object in the study of multivariable calculus is the vector. It is intended to capture a "displacement" (a length and a direction).

We look at the following sketch to distinguish between a **point** $P = (2, 1)$ and its:

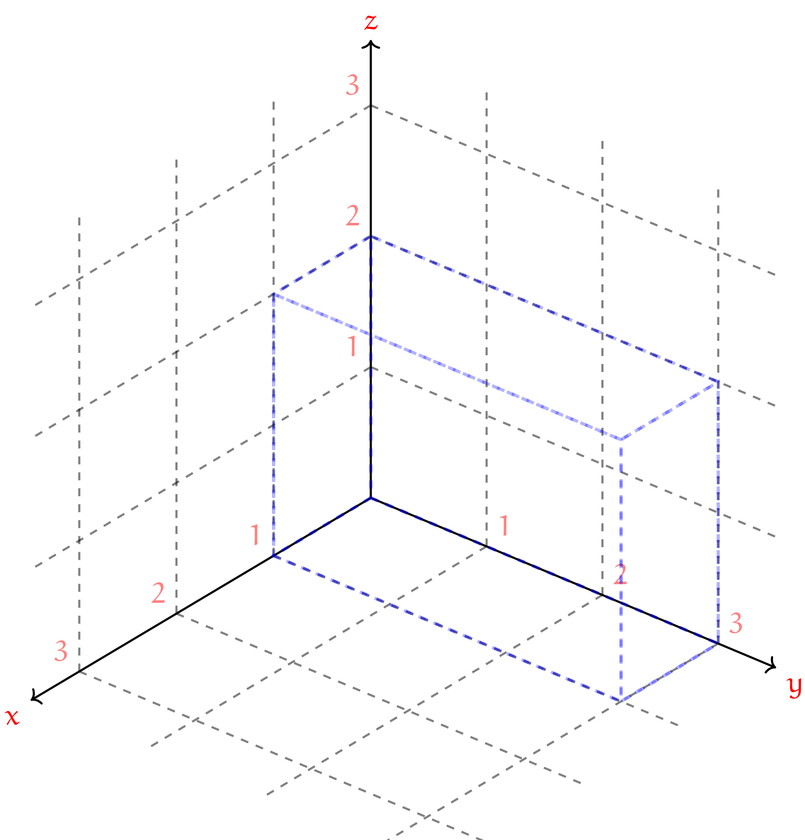
position vector $\mathbf{p} = \langle 2, 1 \rangle$ aka:



Unlike a point: a **vector** does not have a fixed location in space. It can be translated, so long as its length and direction are preserved. We call the entries of a vector its **components**.

We can do the same in three dimensions.

We use $P = (1, 3, 2)$ and its position vector $\mathbf{p} = \langle 1, 3, 2 \rangle$.



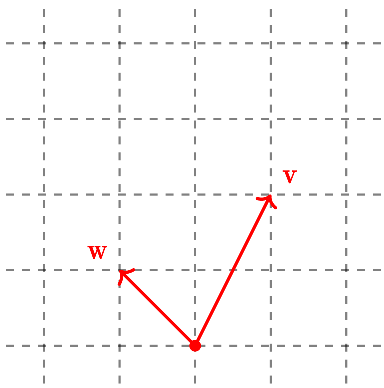
B. **Basic Vector Operations.** We can do some algebra with vectors.

Vector Addition: Vectors with the same number of components can be added:

$$\underbrace{\langle 1, 2\rangle}_{\mathbf{v}} + \underbrace{\langle -1, 1\rangle}_{\mathbf{w}} =$$

and this can be executed geometrically as follows.

- Translate the **tail** (start) of one vector to the **tip** (end) of the other.
- This produces a path made up of the two vectors.
- $\mathbf{v} + \mathbf{w}$ has its tail at the start of this path, and its tip at the end.

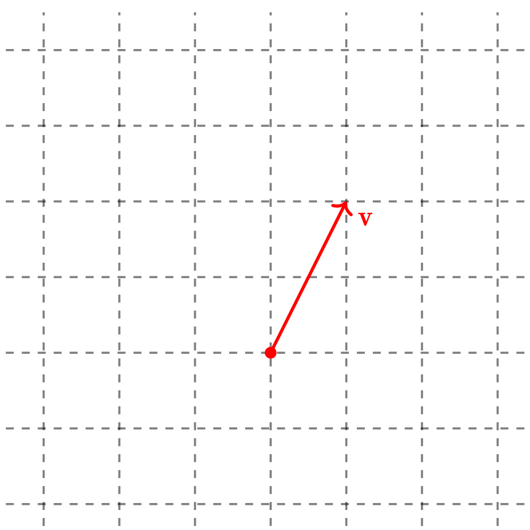


Scalar Multiplication: Vectors can also be **scaled** by **scalars** (real numbers):

$$2 \cdot \underbrace{\langle 1, 2\rangle}_{\mathbf{v}} =$$

$$-1 \cdot \underbrace{\langle 1, 2\rangle}_{\mathbf{v}} =$$

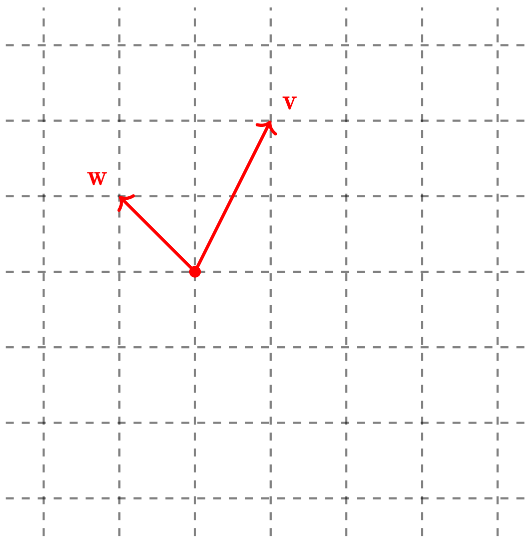
and this can be executed geometrically as depicted.



Example 1. Use the provided $\mathbf{v}$ and $\mathbf{w}$ to sketch:

$2(0.5\mathbf{v} - 1\mathbf{w}) =$

$0\mathbf{v} =$



C. **Dot Products and Length.** We have not talked about multiplying vectors yet. There are two useful ways to do so. We discuss the first of these: the **dot product**.

First we indicate what goes into a dot product and what comes out:

$$(\text{vector}) \cdot (\text{vector}) = (\text{scalar})$$

Now we break down how it works.

> If $\mathbf{v}$ and $\mathbf{w}$ have the same number of components then their **dot product** is:
>
> $$\underbrace{\langle v_1, \ldots, v_n \rangle}_{\mathbf{v}} \cdot \underbrace{\langle w_1, \ldots, w_n \rangle}_{\mathbf{w}} =$$

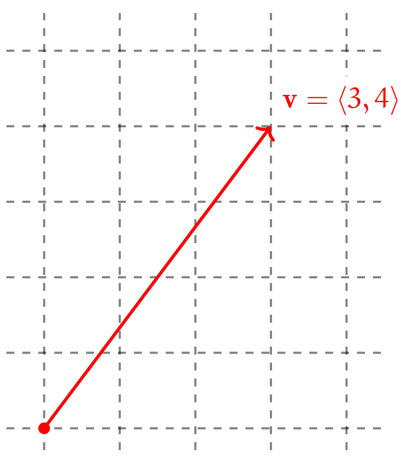
For example:

$$\langle 1, 2, 3 \rangle \cdot \langle 3, 2, 1 \rangle =$$

$$\langle 3, 4 \rangle \cdot \langle 3, 4 \rangle =$$

What is the meaning of the scalar we get out?

We first investigate the case of a vector being **dotted** with itself.



From this we see that:

> The **length** of a vector is:
>
> $$\|\mathbf{v}\| =$$