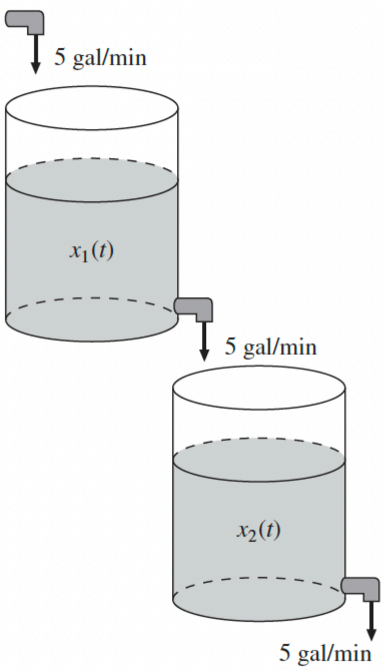
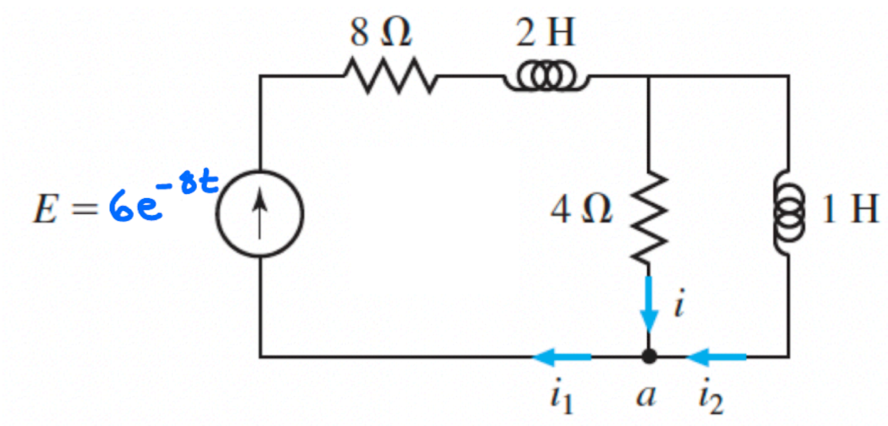
Example 1. Initially, the upper tank initially contains 200 gallons of salt solution with salt concentration of 0.2 lb/gal, and the lower tank also initially contains 200 gallon of salt solution, but with salt concentration 0.1 lb/gal. As time progresses, water flows between the tanks in the manner indicated, with **pure** water flowing into the upper tank. Find formulas for the salt contents $x_1(t)$ and $x_2(t)$ in the upper and lower tanks, in lbs.



Remember, if A is 2×2 with a repeated eigenvalue then we have the shortcut:

$e^{tA} = e^{ct}(I + tN)$ where $N = A - cI$

Example 2. Find the state–free solution—i.e. with initial values equal 0–for currents i_1 and i_2 in the circuit:



Some reminders about electrical circuits. The zigzags are resistors and the curlies are inductors.

Kirkhoff's Current Law:

- at any juncture, the current in equals the current out.

Kirkhoff's Voltage Law:

- directed sum of voltages around any closed loop equals 0

Impeding voltages across components:

- resistor: Ohm's Law $E_R = RI$
- capacitor: Capacitance Law $E_C = \frac{Q}{C}$
- inductor: Faraday's Law $E_L = L\frac{dI}{dt}$

Current is derivative of charge:

- $I = \frac{dQ}{dt}$

We apply Kirkhoff's voltage law to the left loop and the right loop.

Example 2 Continued.

You should find:

$$A = \begin{pmatrix} -6 & 2 \\ 4 & -4 \end{pmatrix} \text{ and } \mathbf{f}(t) = \begin{pmatrix} 3e^{-8t} \\ 0 \end{pmatrix}$$

in which case it will turn out that:

$$e^{tA} = \frac{1}{3}\begin{pmatrix} e^{-2t} + 2e^{-8t} & e^{-2t} - e^{-8t} \\ 2e^{-2t} - 2e^{-8t} & 2e^{-2t} + e^{-8t} \end{pmatrix}$$