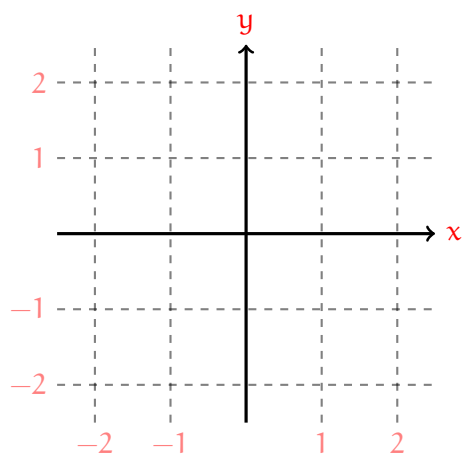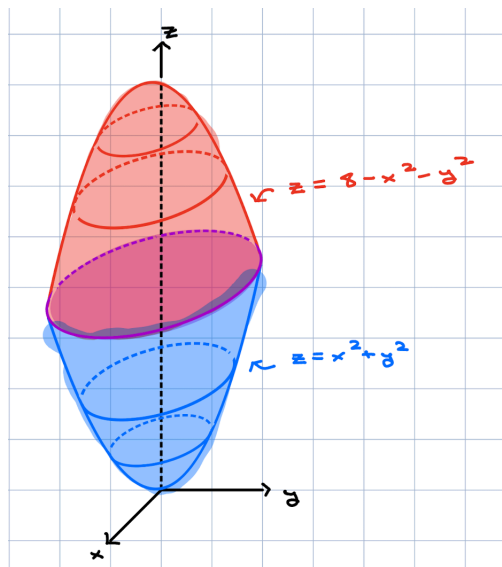Lecture 20. B4 – Triple Integrals.

Example 1. If E is the region bounded by the paraboloids $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$ then set up an iterated integral of an arbitrary function $f(x, y, z)$ over E in the in order $dz\,dy\,dx$.

In this case the region is the vertical space between the bottom paraboloid and the top paraboloid for all (x, y) inside the circle formed by their intersection.

Example 2. Sketch the region of integration for:

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{x^2+y^2}} f(x,y,z)\, dz\,dy\,dx$$
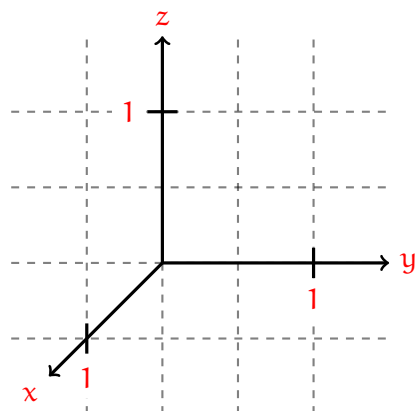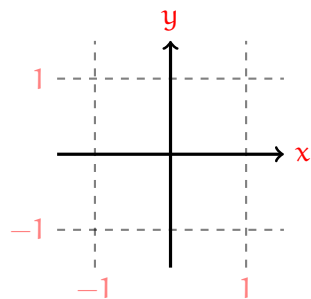
The equation $z = \sqrt{x^2 + y^2}$ describes the top–half of a vertical double cone, i.e. describes an upwards cone, while the equation $y = \sqrt{1 - x^2}$ describes the top–half of the unit circle in the xy-plane.