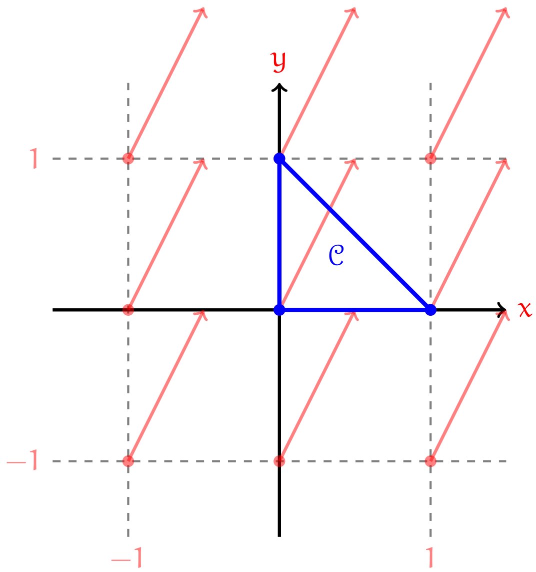
Example 1. Integrate:

$$\mathbf{F}(x, y) = \frac{1}{2}\langle 1, 2 \rangle$$

over the **clockwise** oriented triangle $\mathcal{C}$ depicted below.



Because the triangle is made up of three line segments, we need to compute three line integrals, one for each segment. We also need to select parametrizations that match with the orientation, which in this case means that as the parameter increases, the corresponding points on the curve move in the clockwise direction.

A. **Alternative Notation.** You may see notation for vector line integrals depicted a little differently.

$$\left\{ \begin{array}{l} \mathbf{F} = \langle P, Q, R \rangle \\ d\mathbf{r} = \langle dx, dy, dz \rangle \end{array} \right\} \implies \mathbf{F} \cdot d\mathbf{r} =$$

where:

$dx =$

$dy =$

$dz =$

Notation is a weird thing that, when understood correctly can significantly clarify complexities, but when understood poorly can only exacerbate them.

Example 2. Find:

$$\int_{\mathbf{r}} (y + z)\ dx + (x + z)\ dy + (x + y)\ dz$$

where $\mathbf{r}(t) = \left(t, t^2, t^3\right)$ and $0 \leqslant t \leqslant 1$.

B. **Conservative Vector Fields.** A special kind of vector field is a vector field that equals the **gradient** of some function.

A vector field $\mathbf{F}(\mathbf{x})$ is called **conservative** if it equals the gradient of some function $f(\mathbf{x})$, in other words:

$$\mathbf{F}(\mathbf{x}) =$$

We call f a **potential** for $\mathbf{F}$.

The word **conservative** comes from physics, with the notion of a conservative force. A conservative force is a force with the property that the work done by the force moving a particle between two points is independent of the path taken. We will explore this path independence later in this unit. Likewise the word **potential** also comes from physics, like graviational potential energy.

As we will see, there are plenty of vector fields that are **not** conservative.

Example 3. Show that the vector field below is conservative by finding all potential functions.

$\mathbf{F}(x, y, z) = (e^{yz} + z)\mathbf{i} + (xze^{yz} + 2yz)\mathbf{j} + (xye^{yz} + y^2 + x)\mathbf{k}$

The general strategy for finding all potential functions can be executed sequentially as: first find the general x–antiderivative of the first component of the vector field to find your candidate f, which will include an unkown function of y and z. Take a y–partial–derivative of your f, and use this to find more information about the unknown function of y and z, expresing that unknown function as a sum of a known function with an unknown function of z. Lastly take the z–partial–derivative of your f and compare to the third component of the vector field to determine the unknown function of z, up to a constant.