



### Integral Formulas.

- $\int \frac{1}{1+t^2} dt = \arctan t + C$
- $\int \frac{1}{\sqrt{1-t^2}} dt = \arcsin t + C$
- $\int \frac{1}{t} dt = \ln|t| + C$

### Trig Identities.

- power reduction:  $\cos^2 \theta = \frac{1}{2} + \frac{1}{2} \cos 2\theta$   
 $\sin^2 \theta = \frac{1}{2} - \frac{1}{2} \cos 2\theta$
- double angle:  $\sin 2\theta = 2 \sin \theta \cos \theta$   
 $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$

### A1 Formulas.

- $\vec{v} \cdot \vec{w} = \|\vec{v}\| \|\vec{w}\| \cos \theta$
- $\|\vec{v} \times \vec{w}\| = \|\vec{v}\| \|\vec{w}\| \sin \theta =$  [parallelogram area]
- $|\vec{v} \cdot (\vec{w} \times \vec{r})| =$  [parallelepiped volume]
- $\text{proj}_{\vec{v}}(\vec{w}) = \left( \frac{\vec{v} \cdot \vec{w}}{\vec{v} \cdot \vec{v}} \right) \vec{v}$
- $\text{comp}_{\vec{v}}(\vec{w}) = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\|}$

### A2 Formulas.

- distance from point  $B$  to plane  $P$  with normal  $\vec{n}$ :  
$$\frac{|\vec{AB} \cdot \vec{n}|}{\|\vec{n}\|}$$
 where  $A$  is on  $P$
- distance from point  $B$  to line  $\ell$  with direction  $\vec{v}$ :  
$$\frac{\|\vec{AB} \times \vec{v}\|}{\|\vec{v}\|}$$
 where  $A$  is on  $\ell$

### A3 Formulas. Standard equations.

- paraboloid:  $z = x^2 + y^2$
- saddle:  $z = x^2 - y^2$
- 1-sheeted hyperboloid:  $x^2 + y^2 - z^2 = 1$
- 2-sheeted hyperboloid:  $-x^2 - y^2 + z^2 = 1$
- ellipsoid—sphere if uniformly scaled:  
$$x^2 + y^2 + z^2 = 1$$
- double-cone:  $z^2 = x^2 + y^2$

### A4 Formulas.

- tangent plane to  $z = f(x, y)$  at  $(a, b, f(a, b))$  is:  
$$z = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

### A5 Formulas.

- given unit direction  $\vec{u}$ , the directional derivative:  
$$D_{\vec{u}} f(P) = \vec{\nabla} f(P) \cdot \vec{u} = \|\vec{\nabla} f(P)\| \cos \theta$$
- tangent plane to level set  $F(x, y, z) = C$  at  $P$  is:  
$$\vec{\nabla} F(P) \cdot \vec{x} = \vec{\nabla} F(P) \cdot \vec{p}$$

### B1 Formulas.

- If  $P$  is a critical point of  $f(x, y)$  and:  
$$D = f_{xx}(P)f_{yy}(P) - (f_{xy}(P))^2$$
  
$$T = f_{xx}(P) + f_{yy}(P)$$
- then:  
$$D < 0 \implies \text{saddle}$$
  
$$D > 0 \text{ and } T > 0 \implies \text{local minimizer}$$
  
$$D > 0 \text{ and } T < 0 \implies \text{local maximizer}$$

### B2 Formulas.

- An extremizer  $P$  of  $f$  subject to  $g = C$  satisfies:  
$$\vec{\nabla} f(P) = \lambda \vec{\nabla} g(P) \text{ or } \vec{\nabla} g(P) = \vec{0}$$

### B5 Formulas.

- polar and cylindrical
  - $r^2 = x^2 + y^2$  and  $\tan \theta = \frac{y}{x}$
  - $x = r \cos \theta$  and  $y = r \sin \theta$
  - $dA = r dr d\theta$  and  $dV = r dz dr d\theta$