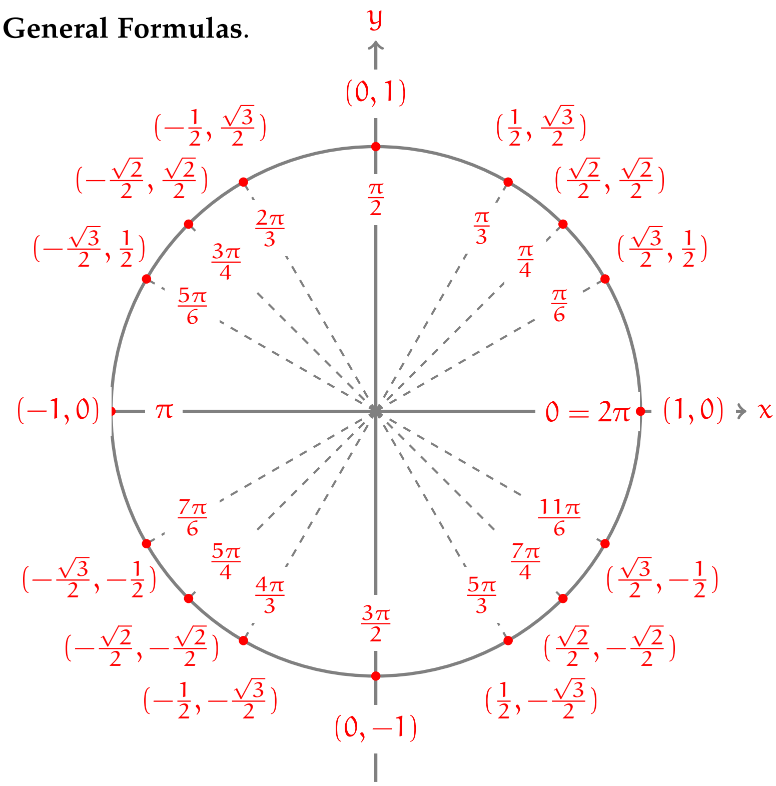
Math 226 – Formula Sheet. **Do not write on this sheet.**

General Formulas.



- $\arctan(t) = \int \frac{1}{1+t^2}\, dt$
- $\arcsin(t) = \int \frac{1}{\sqrt{1-t^2}}\, dt$
- $\ln|t| = \int \frac{1}{t}\, dt$
- power reduction formulas:
  - $\cos^2\theta = \frac{1+\cos 2\theta}{2}$ and $\sin^2\theta = \frac{1-\cos 2\theta}{2}$
- double-angle formulas:
  - $\sin 2t = 2\sin t\cos t$ and $\cos 2t = \cos^2 t - \sin^2 t$

A1 Formulas.

- products and lengths and angles:
  - $\mathbf{v}\cdot\mathbf{w} = \|\mathbf{v}\|\|\mathbf{w}\|\cos\theta$
  - $\|\mathbf{v}\times\mathbf{w}\| = \|\mathbf{v}\|\|\mathbf{w}\|\sin\theta =$ [parallelogram area]
- projection and scalar component:
  - $\mathrm{proj}_{\mathbf{v}}(\mathbf{w}) = \left(\frac{\mathbf{v}\cdot\mathbf{w}}{\mathbf{v}\cdot\mathbf{v}}\right)\mathbf{v}$
  - $\mathrm{comp}_{\mathbf{v}}(\mathbf{w}) = \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|}$
- scalar triple product $= \pm$ parallelepiped volume:
  - $\mathbf{v}\cdot(\mathbf{w}\times\mathbf{r}) = \mathbf{r}\cdot(\mathbf{v}\times\mathbf{w}) = \mathbf{w}\cdot(\mathbf{r}\times\mathbf{v})$

A2 Formulas.

- distance from point B to plane $\mathcal{P}$ with normal $\mathbf{n}$:
  - $\frac{|\mathbf{AB}\cdot\mathbf{n}|}{\|\mathbf{n}\|}$ where A is on $\mathcal{P}$
- distance from point B to line ℓ with direction vector $\mathbf{v}$:
  - $\frac{\|\mathbf{AB}\times\mathbf{v}\|}{\|\mathbf{v}\|}$ where A is on ℓ

A4 Formulas.

- tangent plane to $z = f(x,y)$ at $(a, b, f(a,b))$ is:
  - $z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

A3 Formulas.

- standard form surfaces:
  - paraboloid: $\hat{z} = \hat{x}^2 + \hat{y}^2$
  - saddle: $\hat{z} = \hat{x}^2 - \hat{y}^2$
  - 1-sheeted hyperboloid: $\hat{x}^2 + \hat{y}^2 - \hat{z}^2 = 1$
  - 2-sheeted hyperboloid: $-\hat{x}^2 - \hat{y}^2 + \hat{z}^2 = 1$
  - ellipsoid: $\hat{x}^2 + \hat{y}^2 + \hat{z}^2 = 1$
  - double-cone: $\hat{z}^2 = \hat{x}^2 + \hat{y}^2$

A5 Formulas.

- $D_{\mathbf{u}}f(P) = \nabla f(P)\cdot\mathbf{u}$ where $\mathbf{u}$ is a unit direction
  - max'ed in direction $\nabla f(P)$, with value $\|\nabla f(P)\|$
  - min'ed in direction $-\nabla f(P)$, with value $-\|\nabla f(P)\|$
  - equals 0 in directions $\perp$ to $\nabla f(P)$
- tangent plane to level set $F(x,y,z) = C$ at P is:
  - $\nabla F(P)\cdot(\mathbf{x}-\mathbf{p}) = 0$

B1 Formulas.

- If P is a critical point of $f(x,y)$ and:
  - $D = f_{xx}(P)f_{yy}(P) - (f_{xy}(P))^2$
  - $T = f_{xx}(P) + f_{yy}(P)$
- then:
  - $D < 0 \implies$ saddle
  - $D > 0$ and $T > 0 \implies$ local minimizer
  - $D > 0$ and $T < 0 \implies$ local maximizer

B2 Formulas.

- An extremizer P of f subject to $g = C$ satisfies:
  - $\nabla f(P) = \lambda \nabla g(P)$ or $\nabla g(P) = \mathbf{0}$

B5 Formulas.

- polar and cylindrical
  - $r^2 = x^2 + y^2$ and $\tan\theta = \frac{y}{x}$
  - $x = r\cos\theta$ and $y = r\sin\theta$
  - $dA = r\ drd\theta$ and $dV = r\ dzdrd\theta$

C1 Formulas.

- spherical
  - $\rho^2 = x^2 + y^2 + z^2$ and $\tan\phi = \frac{r}{z}$
  - $r = \rho\sin\theta$ and $z = \rho\cos\theta$
  - $dV = \rho^2 \sin\phi\ d\rho d\phi d\theta$

C2 Formulas.

- arclength $= \int_{\mathcal{C}} f\ ds = \int_a^b \|\mathbf{r}'(t)\|\ dt$
- scalar line integral:
  - $\int_{\mathcal{C}} f\ ds$ where $ds = \|\mathbf{r}'(t)\|\ dt$
- vector line integral:
  - $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ where $d\mathbf{r} = \mathbf{r}'(t)\ dt$
  - or $\int_{\mathcal{C}} P\,dx + Q\,dy + R\,dz$ where:
    - $\langle dx, dy, dz\rangle = \langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\rangle\ dt$

C3 Formulas.

- fundamental theorem of line integrals:
  - if $\mathbf{F} = \nabla f$, and given path $\mathbf{r}(t)$ with $a \leqslant t \leqslant b$:
    - $\int_{\mathbf{r}} \mathbf{F} \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$
- conservative vector fields are irrotational ($\operatorname{curl} = \mathbf{0}$)
- [irrotational + simply-connected domain] $\implies$ conservative
- conservative vec. fields have path-independent line integrals

C5 Formulas.

- Green's Theorem:
- $\mathcal{C} = \partial D$, oriented so D on left, then:

$$\oint_{\mathcal{C}} P\ dx + Q\ dy = \iint_D Q_x - P_y\ dA$$

- Stokes's Theorem:
- $\mathcal{C} = \partial S$, oriented by righthand rule for normals:

$$\oint_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r} = \iint_{\mathcal{S}} \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$$

C4 Formulas.

- graph $z = f(x,y)$ has:
  - $dS = \sqrt{f_x^2 + f_y^2 + 1}\,dxdy$
  - $d\mathbf{S} = \langle -f_x, -f_y, 1\rangle\ dxdy$ (upwards)
- graph $z = f(r)$ has:
  - $dS = r\sqrt{[f'(r)]^2 + 1}\,drd\theta$
  - $d\mathbf{S} = \langle -f'(r) r\cos\theta, -f'(r) r\sin\theta, r\rangle\ drd\theta$ (upwards, if $r > 0$)
- sphere $\rho = R$ has:
  - $dS = R^2 \sin\phi\ d\phi d\theta$
  - $d\mathbf{S} = R^2 \langle \cos\theta\sin\phi, \sin\theta\sin\phi, \cos\phi\rangle\ d\phi d\theta$ (outwards)
    $= (R\sin\phi)\ \langle x, y, z\rangle\ d\phi d\theta$ (outwards)