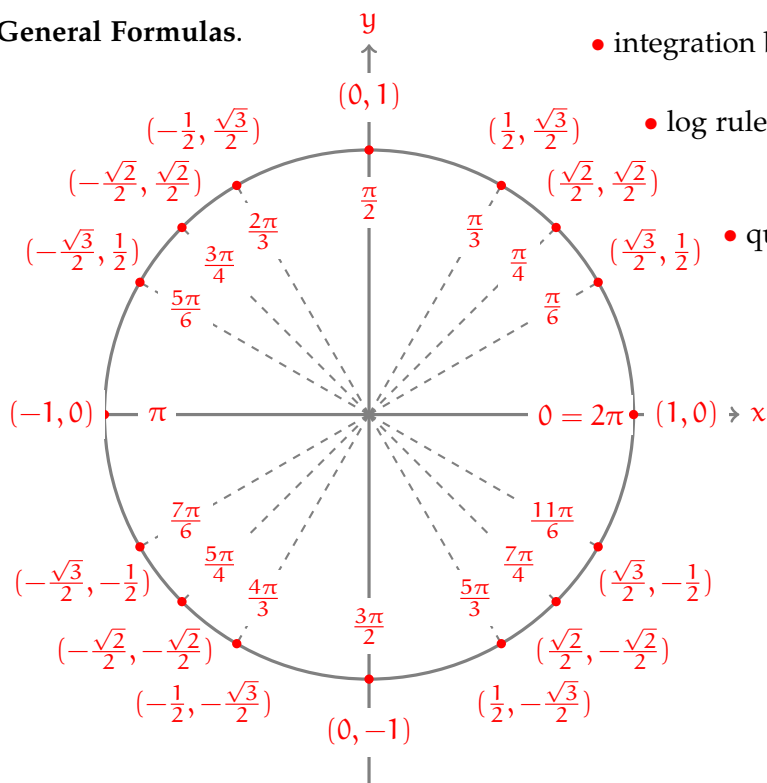


## General Formulas.



• integration by parts:  $\int u \, dv = uv - \int v \, du$

• log rules:  $\ln(A) + \ln(B) = \ln(AB)$ ,  $\ln(A) - \ln(B) = \ln\left(\frac{A}{B}\right)$ ,  
 $c \ln(A) = \ln(A^c)$

• quadratic formula:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

## A2 Formulas.

- for  $y' + a(x)y = f(x)$  we have:
  - integrating factor  $I(x) = e^{\int a(x) \, dx}$
  - variation of parameters:  $y = uy_h$ 
    - where  $y_h = e^{-\int a(x) \, dx}$  and  $u = \int \frac{f(x)}{y_h} \, dx$
- circuits
  - Kirchhoff's Voltage Law:  $E = RI + LI' + Q/C$
  - derivative of charge is current:  $I = Q'$
- Bernoulli equation:  $y' + a(t)y = f(t)y^n$ 
  - substitute  $u = y^{1-n}$
  - converts to  $u' + (1-n)a(t)u = (1-n)f(t)$

## A3 Formulas.

- spring with no external force:  $my'' + \mu y' + ky = 0$
- Wronskian:  $W(y_1, y_2) = y_1 y_2' - y_1' y_2$
- Abel's formula: for  $y'' + p(t)y' + q(t)y = 0$ 
  - $W(y_1, y_2) = Ae^{-\int p(t) \, dt}$  if  $y_1, y_2$  are solutions
  - if  $y_1$  is a solution, then Abel's formula with  $A = 1$  yields:
    - so is  $y_2 = uy_1$  where  $u = \int \frac{e^{-\int p(t) \, dt}}{y_1^2} \, dt$

## A4 Formulas.

forcing term	trial solution $y_p$
$[\deg m \text{ poly}]e^{at}$	$t^n[\deg m \text{ poly}]e^{at}$
$[\deg m \text{ poly}]e^{at} \cos(bt)$ or $[\deg m \text{ poly}]e^{at} \sin(bt)$	$t^n[\deg m \text{ poly}]e^{at} \cos(bt)$ + $t^n[\deg m \text{ poly}]e^{at} \sin(bt)$

note:  $n$  is the # of times  $\lambda = a$  (or  $\lambda = a + bi$ ) is a root of the characteristic equation.