

Example 1. Use Laplace transforms to solve the following initial value problem.

$$y'' - 3y' - 10y = 2 \text{ with } y(0) = 1 \text{ and } y'(0) = 2.$$

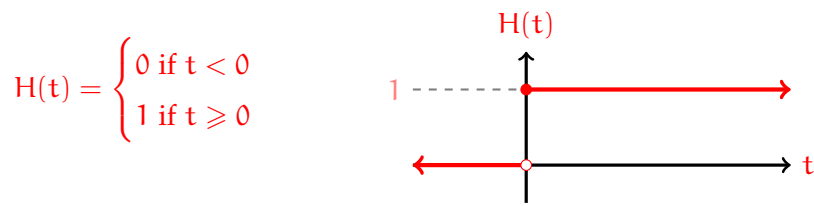
Recall the rule:

$$\mathcal{L}(y^{(n)}) = s^n Y - s^{n-1}y(0) - \dots - y^{(n-1)}(0)$$

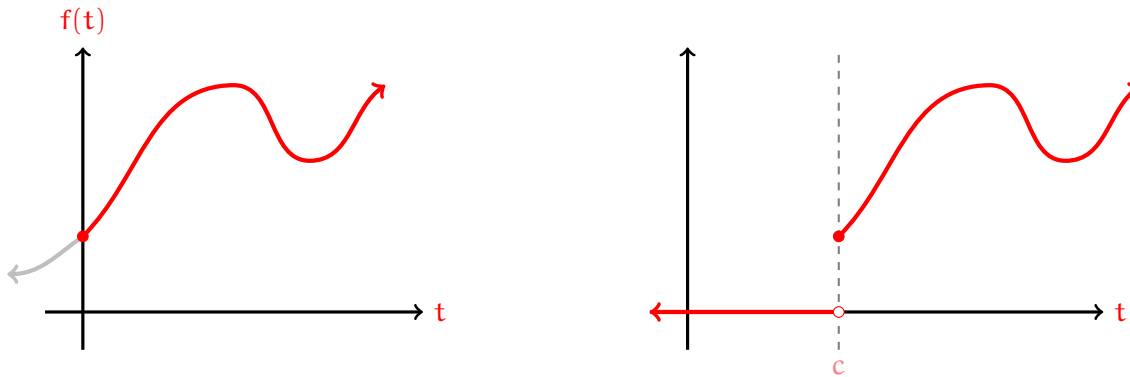
$$\mathcal{L}(1) = \frac{1}{s}$$

A. Heaviside Function.

The **Heaviside function** is:



The shift-right-by- c of the part of $f(t)$ with $t \geq 0$ is sketched below:



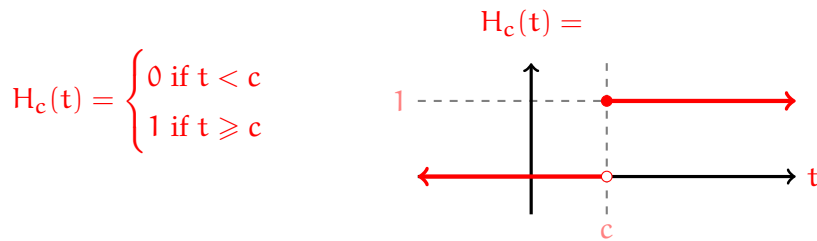
Find: $\mathcal{L} \{ H(t-c)f(t-c) \} (s) =$

Translation Formula.

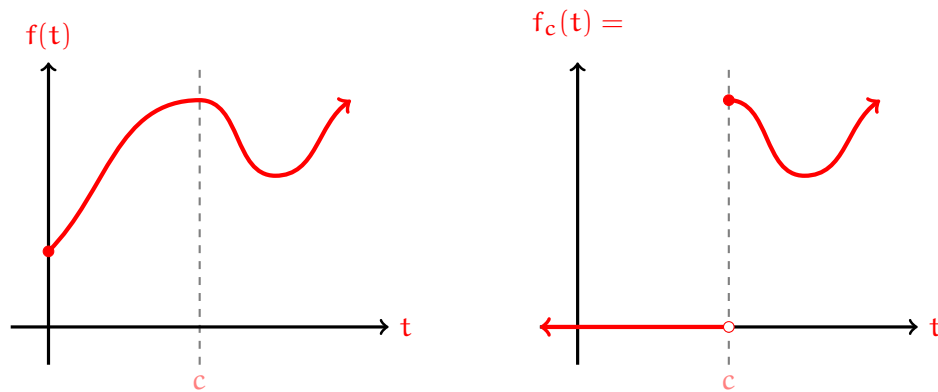
$$\mathcal{L} \{ H(t-c)f(t-c) \} (s) =$$

B. Truncation.

The **truncated Heaviside** for $t \geq c$ is sketched below:



The **truncation** of $f(t)$ for $t \geq c$ is sketched below:



Truncation Formula.

$$\mathcal{L}\{f_c(t)\} = \mathcal{L}\{H_c(t)f(t)\} = \mathcal{L}\{H(t-c)f(t)\} =$$

The idea is that we have shown:

$$\mathcal{L}\{H(t-c)f(t-c)\} = e^{-cs}\mathcal{L}\{f(t)\}$$

That is: multiplying by $H(t-c)$ in the t -domain is the same as adding c in the t -domain and multiplying by e^{cs} in the s -domain.

Example 2. Find:

$$\mathcal{L} \left\{ H \left(t - \frac{\pi}{4} \right) \sin(t) \right\} (s) =$$

Recall that:

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

Example 3. Find a formula for:

$$\mathcal{L} \left\{ H_c(t) \right\} (s) =$$

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