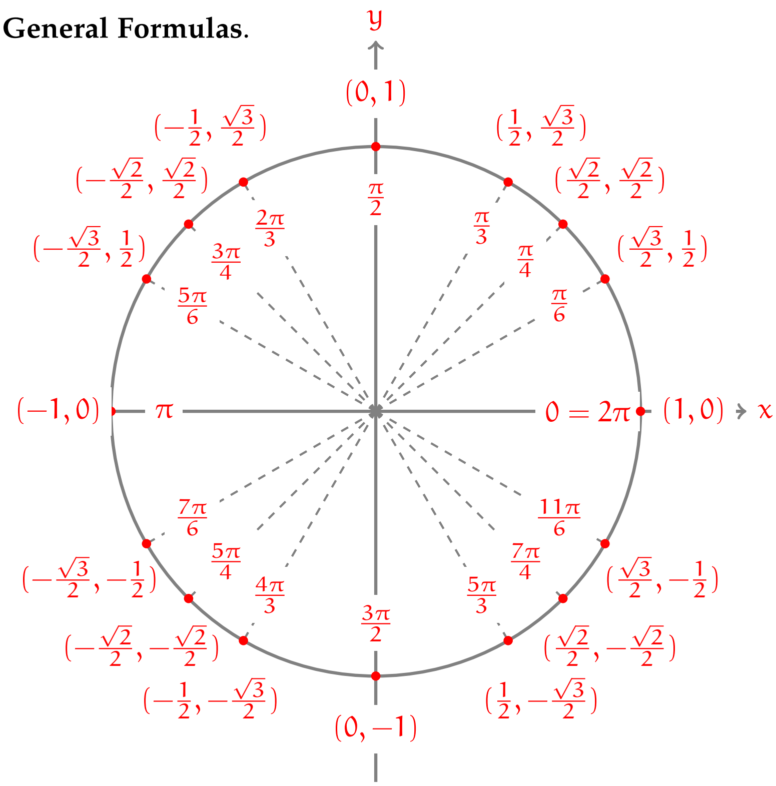
Math 226 – Formula Sheet. **Do not write on this sheet.**

General Formulas.



- $\arctan(t) = \int \frac{1}{1+t^2}\, dt$
- $\arcsin(t) = \int \frac{1}{\sqrt{1-t^2}}\, dt$
- $\ln|t| = \int \frac{1}{t}\, dt$

A1 Formulas.

- products and lengths and angles:
  - $\mathbf{v} \cdot \mathbf{w} = \|\mathbf{v}\|\|\mathbf{w}\| \cos\theta$
  - $\|\mathbf{v} \times \mathbf{w}\| = \|\mathbf{v}\|\|\mathbf{w}\| \sin\theta = [\text{parallelogram area}]$
- projection and scalar component:
  - $\text{proj}_{\mathbf{v}}(\mathbf{w}) = \left(\frac{\mathbf{v}\cdot\mathbf{w}}{\mathbf{v}\cdot\mathbf{v}}\right)\mathbf{v}$ ∘ $\text{comp}_{\mathbf{v}}(\mathbf{w}) = \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|}$
- scalar triple product $= \pm$ parallelepiped volume:
  - $\mathbf{v} \cdot (\mathbf{w} \times \mathbf{r}) = \mathbf{r} \cdot (\mathbf{v} \times \mathbf{w}) = \mathbf{w} \cdot (\mathbf{r} \times \mathbf{v})$

A2 Formulas.

- distance from point B to plane $\mathcal{P}$ with normal $\mathbf{n}$:
  - $\frac{|\mathbf{AB} \cdot \mathbf{n}|}{\|\mathbf{n}\|}$ where A is on $\mathcal{P}$
- distance from point B to line ℓ with direction vector $\mathbf{v}$:
  - $\frac{\|\mathbf{AB} \times \mathbf{v}\|}{\|\mathbf{v}\|}$ where A is on ℓ

A3 Formulas.

- standard form surfaces:
  - paraboloid: $\hat{z} = \hat{x}^2 + \hat{y}^2$
  - saddle: $\hat{z} = \hat{x}^2 - \hat{y}^2$
  - 1-sheeted hyperboloid: $\hat{x}^2 + \hat{y}^2 - \hat{z}^2 = 1$
  - 2-sheeted hyperboloid: $-\hat{x}^2 - \hat{y}^2 + \hat{z}^2 = 1$
  - ellipsoid: $\hat{x}^2 + \hat{y}^2 + \hat{z}^2 = 1$
  - double-cone: $\hat{z}^2 = \hat{x}^2 + \hat{y}^2$

A4 Formulas.

- tangent plane to $z = f(x, y)$ at $(a, b, f(a, b))$ is:
  - $z = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$

A5 Formulas.

- $D_{\mathbf{u}}f(P) = \nabla f(P) \cdot \mathbf{u}$ where $\mathbf{u}$ is a unit direction
  - max'ed in direction $\nabla f(P)$, with value $\|\nabla f(P)\|$
  - min'ed in direction $-\nabla f(P)$, with value $-\|\nabla f(P)\|$
  - equals 0 in directions $\perp$ to $\nabla f(P)$
- tangent plane to level set $F(x, y, z) = C$ at P is:
  - $\nabla F(P) \cdot (\mathbf{x} - \mathbf{p}) = 0$

B1 Formulas.

- If P is a critical point of $f(x, y)$ and:
  - $D = f_{xx}(P)f_{yy}(P) - (f_{xy}(P))^2$
  - $T = f_{xx}(P) + f_{yy}(P)$
- then:
  - $D < 0 \implies$ saddle
  - $D > 0$ and $T > 0 \implies$ local minimizer
  - $D > 0$ and $T < 0 \implies$ local maximizer

B2 Formulas.

- An extremizer P of f subject to $g = C$ satisfies:
  - $\nabla f(P) = \lambda \nabla g(P)$ or $\nabla g(P) = \mathbf{0}$

B5 Formulas.

- polar and cylindrical
  - $r^2 = x^2 + y^2$ and $\tan\theta = \frac{y}{x}$
  - $x = r\cos\theta$ and $y = r\sin\theta$
  - $dA = r\ drd\theta$ and $dV = r\ dzdrd\theta$