Waves

Waves are disturbances that travel through space with a finite velocity. Examples of disturbances include the collective motion of water molecules in ocean waves and of gas particles in sound waves. Waves can be characterized by a **wave equation**, a differential equation that describes the motion of the wave in space and time. **Harmonic waves** are waves with displacements that can be expressed as sine or cosine functions. These concepts are used in classical physics to describe the wave character of electromagnetic radiation, which is the focus of the following discussion.

A3.7 The electromagnetic field

In classical physics, electromagnetic radiation is understood in terms of the **electromagnetic field**, an oscillating electric and magnetic disturbance that spreads as a harmonic wave through empty space, the vacuum. The wave travels at a constant speed called the *speed of light*, c, which is about 3×10^8 m s^{-1}. As its name suggests, an electromagnetic field has two components, an **electric field** that acts on charged particles (whether stationary of moving) and a **magnetic field** that acts only on moving charged particles. The electromagnetic field is characterized by a **wavelength**, λ (lambda), the distance between the neighbouring peaks of the wave, and its **frequency**, ν (nu), the number of times per second at which its displacement at a fixed point returns to its original value (Fig. A3.5). The frequency is measured in *hertz*, where 1 Hz = 1 s^{-1}. The wavelength and frequency of an electromagnetic wave are related by

$$\lambda\nu = c \tag{A3.24}$$

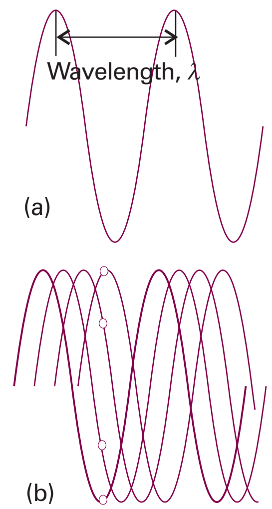

Fig. A3.5 (a) The wavelength, λ, of a wave is the peak-to-peak distance. (b) The wave is shown travelling to the right at a speed c. At a given location, the instantaneous amplitude of the wave changes through a complete cycle (the four dots show half a cycle) as it passes a given point. The frequency, ν, is the number of cycles per second that occur at a given point. Wavelength and frequency are related by $\lambda\nu = c$.

Therefore, the shorter the wavelength, the higher the frequency. The characteristics of a wave are also reported by giving the **wavenumber**, $\tilde{\nu}$ (nu tilde), of the radiation, where

$$\tilde{\nu} = \frac{\nu}{c} = \frac{1}{\lambda} \tag{A3.25}$$

A wavenumber can be interpreted as the number of complete wavelengths in a given length. Wavenumbers are normally reported in reciprocal centimetres (cm^{-1}), so a wavenumber of 5 cm^{-1} indicates that there are 5 complete wavelengths in 1 cm. The classification of the electromagnetic field according to its frequency and wavelength is summarized in Fig. A3.6.

A3.8 Features of electromagnetic radiation

Consider an electromagnetic disturbance travelling along the x direction with wavelength λ and frequency ν. The functions that describe the oscillating electric field, $\mathcal{E}(x,t)$, and magnetic field, $\mathcal{B}(x,t)$, may be written as

$$\mathcal{E}(x,t) = \mathcal{E}_0\cos\{2\pi\nu t - (2\pi/\lambda)x + \phi\} \tag{A3.26a}$$

$$\mathcal{B}(x,t) = \mathcal{B}_0\cos\{2\pi\nu t - (2\pi/\lambda)x + \phi\} \tag{A3.26b}$$

where $\mathcal{E}_0$ and $\mathcal{B}_0$ are the amplitudes of the electric and magnetic fields, respectively, and the parameter ϕ is the **phase** of the wave, which varies from $-\pi$ to π and gives the relative location of the peaks of two waves. If two waves, in the same region of space, with the same wavelength are shifted by $\phi = \pi$ or $-\pi$ (so the peaks of one wave coincide with the troughs of the other), then the resultant wave will have diminished

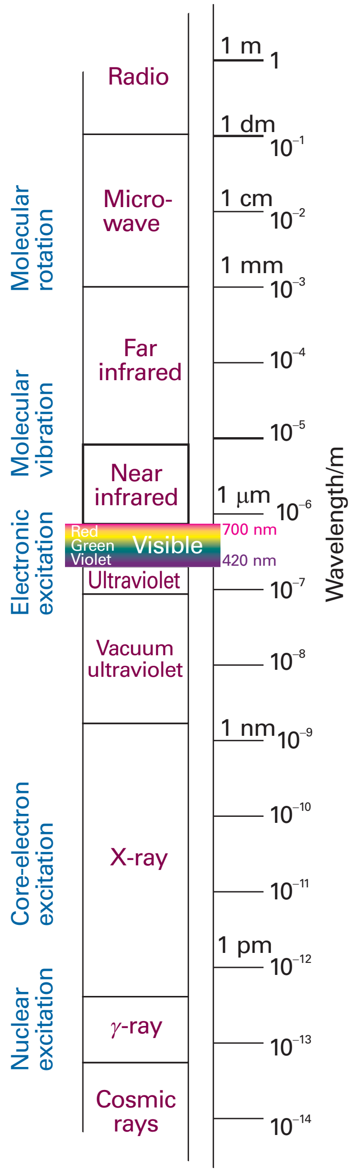

Fig. A3.6 The regions of the electromagnetic spectrum and the types of excitation that give rise to each region.

amplitudes. The waves are said to interfere destructively. A value of $\phi = 0$ (coincident peaks) corresponds to constructive interference, or the enhancement of the amplitudes.

Equations A3.26a and A3.26b represent electromagnetic radiation that is **plane-polarized**; it is so called because the electric and magnetic fields each oscillate in a single plane (in this case the *xy*-plane, Fig. A3.7). The plane of polarization may be orientated in any direction around the direction of propagation (the *x*-direction in Fig. A3.7), with the electric and magnetic fields perpendicular to that direction (and perpendicular to each other). An alternative mode of polarization is **circular polarization**, in which the electric and magnetic fields rotate around the direction of propagation in either a clockwise or a counterclockwise sense but remain perpendicular to it and each other.

It is easy to show by differentiation that eqns A3.26a and A3.26b satisfy the following equations:

$$\frac{\partial^2}{\partial x^2}\psi(x,t) = -\frac{4\pi^2}{\lambda^2}\psi(x,t) \qquad \frac{\partial^2}{\partial t^2}\psi(x,t) = -4\pi^2\nu^2\psi(x,t) \tag{A3.27}$$

where $\psi(x,t)$ is either $\mathcal{E}(x,t)$ or $\mathcal{B}(x,t)$.

According to classical electromagnetic theory, the intensity of electromagnetic radiation is proportional to the square of the amplitude of the wave. For example, the light detectors discussed in *Further information* 16.1 are based on the interaction between the electric field of the incident radiation and the detecting element, so light intensities are proportional to $\mathcal{E}_0^2$.

A3.9 Refraction

A beam of light changes direction ('bends') when it passes from one transparent medium to another. This effect, called **refraction**, depends on the **refractive index**, n_r, of the medium, the ratio of the speed of light in a vacuum, c, to its speed c' in the medium:

$$n_r = \frac{c}{c'} \qquad [A3.28]$$

It follows from the Maxwell equations (see *Further reading*), that the refractive index at a (visible or ultraviolet) specified frequency is related to the relative permittivity ε_r (discussed in Section 20.10) at that frequency by

$$n_r = \varepsilon_r^{1/2} \tag{A3.29}$$

Table A3.1 lists refractive indices of some materials.

Because the relative permittivity of a medium is related to its polarizability by eqn 20.10, the refractive index is related to the polarizability. To see why this is so, we need to realize that propagation of light through a medium induces an oscillating dipole moment, which then radiates light of the same frequency. The newly generated radiation is delayed slightly by this process, so it propagates more slowly through the medium than through a vacuum. Because photons of high-frequency light carry more energy than those of low-frequency light, they can distort the electronic distributions of the molecules in their path more effectively. Therefore, after allowing for the loss of contributions from low-frequency modes of motion, we can expect the electronic polarizabilities of molecules, and hence the refractive index, to increase as the incident frequency rises towards an absorption frequency. This dependence on frequency is the origin of the dispersion of white light by a prism: the refractive index is greater for blue light than for red, and therefore the blue rays are bent more than the red. The