The rates of chemical reactions

22

This chapter is the first of a sequence that explores the rates of chemical reactions. The chapter begins with a discussion of the definition of reaction rate and outlines the techniques for its measurement. The results of such measurements show that reaction rates depend on the concentration of reactants (and products) in characteristic ways that can be expressed in terms of differential equations known as rate laws. The solutions of these equations are used to predict the concentrations of species at any time after the start of the reaction. The form of the rate law also provides insight into the series of elementary steps by which a reaction takes place. The key task in this connection is the construction of a rate law from a proposed mechanism and its comparison with experiment. Simple elementary steps have simple rate laws, and these rate laws can be combined together by invoking one or more approximations. These approximations include the concept of the rate-determining stage of a reaction, the steady-state concentration of a reaction intermediate, and the existence of a pre-equilibrium.

This chapter introduces the principles of **chemical kinetics**, the study of reaction rates, by showing how the rates of reactions may be measured and interpreted. The remaining chapters of this part of the text then develop this material in more detail and apply it to more complicated or more specialized cases. The rate of a chemical reaction might depend on variables under our control, such as the pressure, the temperature, and the presence of a catalyst, and we may be able to optimize the rate by the appropriate choice of conditions. The study of reaction rates also leads to an understanding of the **mechanisms** of reactions, their analysis into a sequence of elementary steps.

Empirical chemical kinetics

The first steps in the kinetic analysis of reactions are to establish the stoichiometry of the reaction and identify any side reactions. The basic data of chemical kinetics are then the concentrations of the reactants and products at different times after a reaction has been initiated. The rates of most chemical reactions are sensitive to the temperature, so in conventional experiments the temperature of the reaction mixture must be held constant throughout the course of the reaction. This requirement puts severe demands on the design of an experiment. Gas-phase reactions, for instance, are often carried out in a vessel held in contact with a substantial block of metal. Liquid-phase reactions, including flow reactions, must be carried out in an efficient thermostat. Special efforts have to be made to study reactions at low temperatures, as in the study of the kinds of reactions that take place in interstellar clouds. Thus, supersonic expansion of the reaction gas can be used to attain temperatures as low as

Empirical chemical kinetics

22.1 Experimental techniques

22.2 The rates of reactions

22.3 Integrated rate laws

22.4 Reactions approaching equilibrium

22.5 The temperature dependence of reaction rates

Accounting for the rate laws

22.6 Elementary reactions

22.7 Consecutive elementary reactions

I22.1 Impact on biochemistry: The kinetics of the helix–coil transition in polypeptides

22.8 Unimolecular reactions

Checklist of key ideas

Further reading

Further information 22.1: The RRK model of unimolecular reactions

Discussion questions

Exercises

Problems

10 K. For work in the liquid phase and the solid phase, very low temperatures are often reached by flowing cold liquid or cold gas around the reaction vessel. Alternatively, the entire reaction vessel is immersed in a thermally insulated container filled with a cryogenic liquid, such as liquid helium (for work at around 4 K) or liquid nitrogen (for work at around 77 K). Non-isothermal conditions are sometimes employed. For instance, the shelf-life of an expensive pharmaceutical may be explored by slowly raising the temperature of a single sample.

22.1 Experimental techniques

The method used to monitor concentrations depends on the species involved and the rapidity with which their concentrations change. Many reactions reach equilibrium over periods of minutes or hours, and several techniques may then be used to follow the changing concentrations.

(a) Monitoring the progress of a reaction

A reaction in which at least one component is a gas might result in an overall change in pressure in a system of constant volume, so its progress may be followed by recording the variation of pressure with time.

Example 22.1 *Monitoring the variation in pressure*

Predict how the total pressure varies during the gas-phase decomposition $2\,N_2O_5(g) \rightarrow 4\,NO_2(g) + O_2(g)$ in a constant-volume container.

Method The total pressure (at constant volume and temperature and assuming perfect gas behaviour) is proportional to the number of gas-phase molecules. Therefore, because each mole of N_2O_5 gives rise to $\frac{5}{2}$ mol of gas molecules, we can expect the pressure to rise to $\frac{5}{2}$ times its initial value. To confirm this conclusion, express the progress of the reaction in terms of the fraction, α, of N_2O_5 molecules that have reacted.

Answer Let the initial pressure be p_0 and the initial amount of N_2O_5 molecules present be n. When a fraction α of the N_2O_5 molecules has decomposed, the amounts of the components in the reaction mixture are:

	N_2O_5	NO_2	O_2	Total
Amount:	$n(1-\alpha)$	$2\alpha n$	$\frac{1}{2}\alpha n$	$n(1+\frac{3}{2}\alpha)$

When $\alpha = 0$ the pressure is p_0, so at any stage the total pressure is

$$p = (1 + \tfrac{3}{2}\alpha)p_0$$

When the reaction is complete, the pressure will have risen to $\frac{5}{2}$ times its initial value.

Self-test 22.1 Repeat the calculation for $2\,NOBr(g) \rightarrow 2\,NO(g) + Br_2(g)$. $[p = (1 + \frac{1}{2}\alpha)p_0]$

Spectrophotometry, the measurement of absorption of radiation in a particular spectral region, is widely applicable, and is especially useful when one substance in the reaction mixture has a strong characteristic absorption in a conveniently accessible region of the electromagnetic spectrum. For example, the progress of the reaction

$$H_2(g) + Br_2(g) \rightarrow 2\,HBr(g)$$

can be followed by measuring the absorption of visible light by bromine. A reaction that changes the number or type of ions present in a solution may be followed by monitoring the electrical conductivity of the solution. The replacement of neutral molecules by ionic products can result in dramatic changes in the conductivity, as in the reaction

$$(CH_3)_3CCl(aq) + H_2O(l) \rightarrow (CH_3)_3COH(aq) + H^+(aq) + Cl^-(aq)$$

If hydrogen ions are produced or consumed, the reaction may be followed by monitoring the pH of the solution.

Other methods of determining composition include emission spectroscopy, mass spectrometry, gas chromatography, nuclear magnetic resonance, and electron paramagnetic resonance (for reactions involving radicals or paramagnetic *d*-metal ions).

(b) Application of the techniques

In a **real-time analysis** the composition of the system is analysed while the reaction is in progress. Either a small sample is withdrawn or the bulk solution is monitored. In the **flow method** the reactants are mixed as they flow together in a chamber (Fig. 22.1). The reaction continues as the thoroughly mixed solutions flow through the outlet tube, and observation of the composition at different positions along the tube is equivalent to the observation of the reaction mixture at different times after mixing. The disadvantage of conventional flow techniques is that a large volume of reactant solution is necessary. This makes the study of fast reactions particularly difficult because to spread the reaction over a length of tube the flow must be rapid. This disadvantage is avoided by the **stopped-flow technique**, in which the reagents are mixed very quickly in a small chamber fitted with a syringe instead of an outlet tube (Fig. 22.2). The flow ceases when the plunger of the syringe reaches a stop, and the reaction continues in the mixed solutions. Observations, commonly using spectroscopic techniques such as ultraviolet–visible absorption, circular dichroism, and fluorescence emission, are made on the sample as a function of time. The technique allows for the study of reactions that occur on the millisecond to second timescale. The suitability of the stopped-flow method to the study of small samples means that it is appropriate for many biochemical reactions, and it has been widely used to study the kinetics of protein folding and enzyme action (see *Impact* I22.1 later in the chapter).

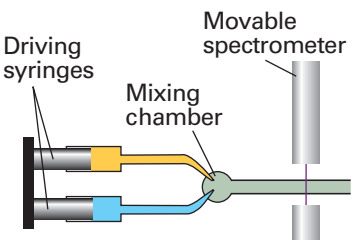

Fig. 22.1 The arrangement used in the flow technique for studying reaction rates. The reactants are injected into the mixing chamber at a steady rate. The location of the spectrometer corresponds to different times after initiation.

Very fast reactions can be studied by **flash photolysis**, in which the sample is exposed to a brief flash of light that initiates the reaction and then the contents of the reaction chamber are monitored. Most work is now done with lasers with photolysis pulse widths that range from femtoseconds to nanoseconds (Section 14.5). The apparatus used for flash photolysis studies is based on the experimental design for time-resolved spectroscopy (Section 14.6). Reactions occurring on a picosecond or femtosecond timescale may be monitored by using electronic absorption or emission, infrared absorption, or Raman scattering. The spectra are recorded at a series of times following laser excitation. The laser pulse can initiate the reaction by forming a reactive species, such as an excited electronic state of a molecule, a radical, or an ion. We discuss examples of excited state reactions in Chapter 23. An example of radical generation is the light-induced dissociation of $Cl_2(g)$ to yield Cl atoms that react with HBr to make HCl and Br according to the following sequence:

$$Cl_2 + h\nu \rightarrow Cl + Cl$$

$$Cl + HBr \rightarrow HCl^* + Br$$

$$HCl^* + M \rightarrow HCl + M$$

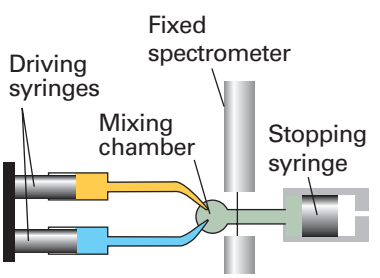

Fig. 22.2 In the stopped-flow technique the reagents are driven quickly into the mixing chamber by the driving syringes and then the time dependence of the concentrations is monitored.

Here HCl* denotes a vibrationally excited HCl molecule and M is a body (an unreactive molecule or the wall of the container) that removes the excess energy stored in HCl. A so-called 'third body' (M) is not always necessary for heteronuclear diatomic molecules because they can discard energy radiatively, but homonuclear diatomic molecules are vibrationally and rotationally inactive, and can discard energy only by collision.

In contrast to real-time analysis, **quenching methods** are based on stopping, or quenching, the reaction after it has been allowed to proceed for a certain time. In this way the composition is analysed at leisure and reaction intermediates may be trapped. These methods are suitable only for reactions that are slow enough for there to be little reaction during the time it takes to quench the mixture. In the **chemical quench flow method**, the reactants are mixed in much the same way as in the flow method but the reaction is quenched by another reagent, such as solution of acid or base, after the mixture has travelled along a fixed length of the outlet tube. Different reaction times can be selected by varying the flow rate along the outlet tube. An advantage of the chemical quench flow method over the stopped-flow method is that spectroscopic fingerprints are not needed in order to measure the concentration of reactants and products. Once the reaction has been quenched, the solution may be examined by 'slow' techniques, such as gel electrophoresis, mass spectrometry, and chromatography. In the **freeze quench method**, the reaction is quenched by cooling the mixture within milliseconds and the concentrations of reactants, intermediates, and products are measured spectroscopically.

22.2 The rates of reactions

Reaction rates depend on the composition and the temperature of the reaction mixture. The next few sections look at these observations in more detail.

(a) The definition of rate

Consider a reaction of the form A + 2 B → 3 C + D, in which at some instant the molar concentration of a participant J is [J] and the volume of the system is constant. The instantaneous **rate of consumption** of one of the reactants at a given time is $-\mathrm{d[R]}/\mathrm{d}t$, where R is A or B. This rate is a positive quantity (Fig. 22.3). The **rate of formation** of one of the products (C or D, which we denote P) is $\mathrm{d[P]}/\mathrm{d}t$ (note the difference in sign). This rate is also positive.

It follows from the stoichiometry for the reaction A + 2 B → 3 C + D that

$$\frac{\mathrm{d[D]}}{\mathrm{d}t}=\tfrac{1}{3}\frac{\mathrm{d[C]}}{\mathrm{d}t}=-\frac{\mathrm{d[A]}}{\mathrm{d}t}=-\tfrac{1}{2}\frac{\mathrm{d[B]}}{\mathrm{d}t}$$

so there are several rates connected with the reaction. The undesirability of having different rates to describe the same reaction is avoided by using the extent of reaction, ξ (xi, the quantity introduced in Section 7.1):

$$\xi=\frac{n_{\mathrm{J}}-n_{\mathrm{J},0}}{\nu_{\mathrm{J}}} \qquad (22.1)$$

where ν_{J} is the stoichiometric number of species J, and defining the unique **rate of reaction**, v, as the rate of change of the extent of reaction:

$$v=\frac{1}{V}\frac{\mathrm{d}\xi}{\mathrm{d}t} \qquad [22.2]$$

It follows that

$$v=\frac{1}{\nu_{\mathrm{J}}}\times\frac{1}{V}\frac{\mathrm{d}n_{\mathrm{J}}}{\mathrm{d}t} \qquad (22.3a)$$

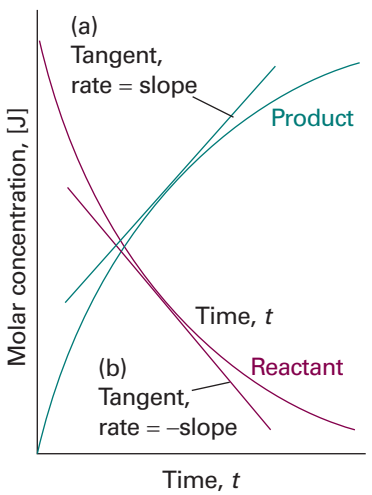

Fig. 22.3 The definition of (instantaneous) rate as the slope of the tangent drawn to the curve showing the variation of concentration with time. For negative slopes, the sign is changed when reporting the rate, so all reaction rates are positive.

(Remember that ν_J is negative for reactants and positive for products.) For a homogeneous reaction in a constant-volume system the volume V can be taken inside the differential and we use $[J] = n_J/V$ to write

$$v = \frac{1}{\nu_J}\frac{d[J]}{dt} \tag{22.3b}$$

For a heterogeneous reaction, we use the (constant) surface area, A, occupied by the species in place of V and use $\sigma_J = n_J/A$ to write

$$v = \frac{1}{\nu_J}\frac{d\sigma_J}{dt} \tag{22.3c}$$

In each case there is now a single rate for the entire reaction (for the chemical equation as written). With molar concentrations in moles per cubic decimetre and time in seconds, reaction rates of homogeneous reactions are reported in moles per cubic decimetre per second (mol dm^{-3} s^{-1}) or related units. For gas-phase reactions, such as those taking place in the atmosphere, concentrations are often expressed in molecules per cubic centimetre (molecules cm^{-3}) and rates in molecules per cubic centimetre per second (molecules cm^{-3} s^{-1}). For heterogeneous reactions, rates are expressed in moles per square metre per second (mol m^{-2} s^{-1}) or related units.

Illustration 22.1 *Rates of formation and consumption*

If the rate of formation of NO in the reaction $2\ NOBr(g) \rightarrow 2\ NO(g) + Br_2(g)$ is reported as 0.16 mmol dm^{-3} s^{-1}, we use $\nu_{NO} = +2$ to report that $v = 0.080$ mmol dm^{-3} s^{-1}. Because $\nu_{NOBr} = -2$ it follows that $d[NOBr]/dt = -0.16$ mmol dm^{-3} s^{-1}. The rate of consumption of NOBr is therefore 0.16 mmol dm^{-3} s^{-1}, or 9.6×10^{16} molecules cm^{-3} s^{-1}.

Self-test 22.2 The rate of change of molar concentration of CH_3 radicals in the reaction $2\ CH_3(g) \rightarrow CH_3CH_3(g)$ was reported as $d[CH_3]/dt = -1.2$ mol dm^{-3} s^{-1} under particular conditions. What is (a) the rate of reaction and (b) the rate of formation of CH_3CH_3? [(a) 0.60 mol dm^{-3} s^{-1}, (b) 0.60 mol dm^{-3} s^{-1}]

(b) Rate laws and rate constants

The rate of reaction is often found to be proportional to the concentrations of the reactants raised to a power. For example, the rate of a reaction may be proportional to the molar concentrations of two reactants A and B, so we write

$$v = k[A][B] \tag{22.4}$$

with each concentration raised to the first power. The coefficient k is called the **rate constant** for the reaction. The rate constant is independent of the concentrations but depends on the temperature. An experimentally determined equation of this kind is called the **rate law** of the reaction. More formally, a rate law is an equation that expresses the rate of reaction as a function of the concentrations of all the species present in the overall chemical equation for the reaction at some time:

$$v = f([A],[B],\dots) \tag{22.5a}$$

For homogeneous gas-phase reactions, it is often more convenient to express the rate law in terms of partial pressures, which are related to molar concentrations by $p_J = RT[J]$. In this case, we write

$$v = f(p_A, p_B, \dots) \qquad [22.5b]$$

The rate law of a reaction is determined experimentally, and cannot in general be inferred from the chemical equation for the reaction. The reaction of hydrogen and bromine, for example, has a very simple stoichiometry, $H_2(g) + Br_2(g) \rightarrow 2\,HBr(g)$, but its rate law is complicated:

$$v = \frac{k[H_2][Br_2]^{3/2}}{[Br_2] + k'[HBr]} \qquad (22.6)$$

In certain cases the rate law does reflect the stoichiometry of the reaction, but that is either a coincidence or reflects a feature of the underlying reaction mechanism (see later).

A practical application of a rate law is that, once we know the law and the value of the rate constant, we can predict the rate of reaction from the composition of the mixture. Moreover, as we shall see later, by knowing the rate law, we can go on to predict the composition of the reaction mixture at a later stage of the reaction. Moreover, a rate law is a guide to the mechanism of the reaction, for any proposed mechanism must be consistent with the observed rate law.

(c) Reaction order

Many reactions are found to have rate laws of the form

$$v = k[A]^a[B]^b \cdots \qquad (22.7)$$

The power to which the concentration of a species (a product or a reactant) is raised in a rate law of this kind is the **order** of the reaction with respect to that species. A reaction with the rate law in eqn 22.4 is **first-order** in A and first-order in B. The **overall order** of a reaction with a rate law like that in eqn 22.7 is the sum of the individual orders, $a + b + \cdots$. The rate law in eqn 22.4 is therefore second-order overall.

A reaction need not have an integral order, and many gas-phase reactions do not. For example, a reaction having the rate law

$$v = k[A]^{1/2}[B] \qquad (22.8)$$

is half-order in A, first-order in B, and three-halves-order overall. Some reactions obey a **zero-order rate law**, and therefore have a rate that is independent of the concentration of the reactant (so long as some is present). Thus, the catalytic decomposition of phosphine (PH_3) on hot tungsten at high pressures has the rate law

$$v = k \qquad (22.9)$$

The PH_3 decomposes at a constant rate until it has almost entirely disappeared. Zero-order reactions typically occur when there is a bottle-neck of some kind in the mechanism, as in heterogeneous reactions when the surface is saturated and the subsequent reaction slow and in a number of enzyme reactions when there is a large excess of substrate relative to the enzyme.

When a rate law is not of the form in eqn 22.7, the reaction does not have an overall order and may not even have definite orders with respect to each participant. Thus, although eqn 22.6 shows that the reaction of hydrogen and bromine is first-order in H_2, the reaction has an indefinite order with respect to both Br_2 and HBr and has no overall order.

These remarks point to three problems. First, we must see how to identify the rate law and obtain the rate constant from the experimental data. We concentrate on this aspect in this chapter. Second, we must see how to construct reaction mechanisms that are consistent with the rate law. We shall introduce the techniques of doing so in

this chapter and develop them further in Chapter 23. Third, we must account for the values of the rate constants and explain their temperature dependence. We shall see a little of what is involved in this chapter, but leave the details until Chapter 24.

(d) The determination of the rate law

The determination of a rate law is simplified by the **isolation method** in which the concentrations of all the reactants except one are in large excess. If B is in large excess, for example, then to a good approximation its concentration is constant throughout the reaction. Although the true rate law might be $v = k[A][B]$, we can approximate [B] by $[B]_0$, its initial value, and write

$$v = k'[A] \qquad k' = k[B]_0 \tag{22.10}$$

which has the form of a first-order rate law. Because the true rate law has been forced into first-order form by assuming that the concentration of B is constant, eqn 22.10 is called a **pseudofirst-order rate law**. The dependence of the rate on the concentration of each of the reactants may be found by isolating them in turn (by having all the other substances present in large excess), and so constructing a picture of the overall rate law.

In the **method of initial rates**, which is often used in conjunction with the isolation method, the rate is measured at the beginning of the reaction for several different initial concentrations of reactants. We shall suppose that the rate law for a reaction with A isolated is $v = k[A]^a$; then its initial rate, v_0, is given by the initial values of the concentration of A, and we write $v_0 = k[A]_0^a$. Taking logarithms gives:

$$\log v_0 = \log k + a \log [A]_0 \tag{22.11}$$

For a series of initial concentrations, a plot of the logarithms of the initial rates against the logarithms of the initial concentrations of A should be a straight line with slope a.

Example 22.2 *Using the method of initial rates*

The recombination of iodine atoms in the gas phase in the presence of argon was investigated and the order of the reaction was determined by the method of initial rates. The initial rates of reaction of $2\,I(g) + Ar(g) \rightarrow I_2(g) + Ar(g)$ were as follows:

$[I]_0/(10^{-5}$ mol dm$^{-3})$		1.0	2.0	4.0	6.0
$v_0/$(mol dm^{-3} s^{-1})	(a)	8.70×10^{-4}	3.48×10^{-3}	1.39×10^{-2}	3.13×10^{-2}
	(b)	4.35×10^{-3}	1.74×10^{-2}	6.96×10^{-2}	1.57×10^{-1}
	(c)	8.69×10^{-3}	3.47×10^{-2}	1.38×10^{-1}	3.13×10^{-1}

The Ar concentrations are (a) 1.0 mmol dm^{-3}, (b) 5.0 mmol dm^{-3}, and (c) 10.0 mmol dm^{-3}. Determine the orders of reaction with respect to the I and Ar atom concentrations and the rate constant.

Method Plot the logarithm of the initial rate, $\log v_0$, against $\log [I]_0$ for a given concentration of Ar, and, separately, against $\log [Ar]_0$ for a given concentration of I. The slopes of the two lines are the orders of reaction with respect to I and Ar, respectively. The intercepts with the vertical axis give $\log k$.

Answer The plots are shown in Fig. 22.4. The slopes are 2 and 1, respectively, so the (initial) rate law is

$$v_0 = k[I]_0^2[Ar]_0$$

This rate law signifies that the reaction is second-order in [I], first-order in [Ar], and third-order overall. The intercept corresponds to $k = 9 \times 10^9$ mol^{-2} dm^6 s^{-1}.

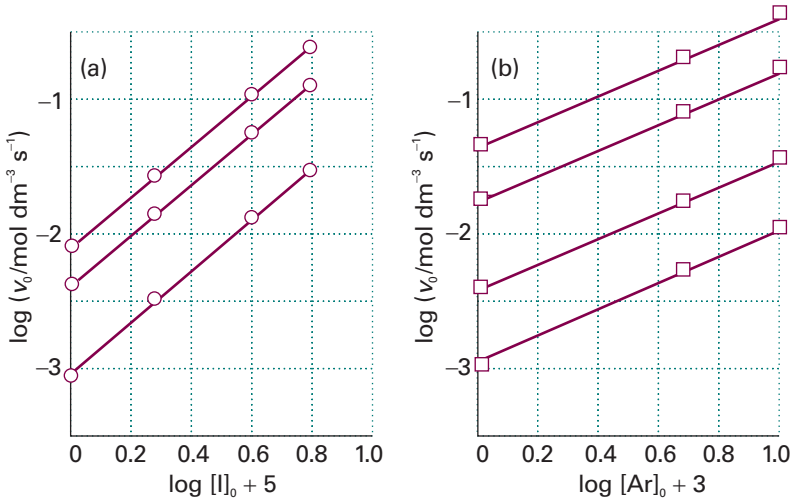

Fig. 22.4 The plot of log v_0 against (a) log $[I]_0$ for a given $[Ar]_0$, and (b) log $[Ar]_0$ for a given $[I]_0$.

A note on good practice The units of k come automatically from the calculation, and are always such as to convert the product of concentrations to a rate in concentration/time (for example, mol dm^{-3} s^{-1}).

Self-test 22.3 The initial rate of a reaction depended on concentration of a substance J as follows:

$[J]_0$/(mmol dm^{-3})	5.0	8.2	17	30
v_0/(10^{-7} mol dm^{-3} s^{-1})	3.6	9.6	41	130

Determine the order of the reaction with respect to J and calculate the rate constant.
[2, 1.4×10^{-2} dm^3 mol^{-1} s^{-1}]

The method of initial rates might not reveal the full rate law, for once the products have been generated they might participate in the reaction and affect its rate. For example, products participate in the synthesis of HBr, because eqn 22.6 shows that the full rate law depends on the concentration of HBr. To avoid this difficulty, the rate law should be fitted to the data throughout the reaction. The fitting may be done, in simple cases at least, by using a proposed rate law to predict the concentration of any component at any time, and comparing it with the data. A law should also be tested by observing whether the addition of products or, for gas-phase reactions, a change in the surface-to-volume ratio in the reaction chamber affects the rate.

22.3 Integrated rate laws

Because rate laws are differential equations, we must integrate them if we want to find the concentrations as a function of time. Even the most complex rate laws may be integrated numerically. However, in a number of simple cases analytical solutions, known as **integrated rate laws**, are easily obtained, and prove to be very useful. We examine a few of these simple cases here.

(a) First-order reactions

As shown in the *Justification* below, the integrated form of the first-order rate law

$$\frac{d[A]}{dt} = -k[A] \qquad (22.12a)$$

Synoptic table 22.1* Kinetic data for first-order reactions

Reaction	Phase	θ/°C	k/s^{-1}	$t_{1/2}$
$2\,N_2O_5 \rightarrow 4\,NO_2 + O_2$	g	25	3.38×10^{-5}	5.70 h
	Br_2(l)	25	4.27×10^{-5}	4.51 h
$C_2H_6 \rightarrow 2\,CH_3$	g	700	5.36×10^{-4}	21.6 min

* More values are given in the *Data section*.

Comment 22.1
The web site contains links to databases of rate constants of chemical reactions.

is

$$\ln\left(\frac{[A]}{[A]_0}\right) = -kt \qquad [A] = [A]_0 e^{-kt} \tag{22.12b}$$

where $[A]_0$ is the initial concentration of A (at $t = 0$).

Justification 22.1 *First-order integrated rate law*

First, we rearrange eqn 22.12a into

$$\frac{d[A]}{[A]} = -k\,dt$$

This expression can be integrated directly because k is a constant independent of t. Initially (at $t = 0$) the concentration of A is $[A]_0$, and at a later time t it is [A], so we make these values the limits of the integrals and write

$$\int_{[A]_0}^{[A]} \frac{d[A]}{[A]} = -k \int_0^t dt$$

Because the integral of $1/x$ is $\ln x$, eqn 22.12b is obtained immediately.

A note on good practice To set the limits of integration, identify the start time ($t = 0$) and the corresponding concentration of A ($[A]_0$), and write these quantities as the lower limits of their respective integrals. Then identify the time of interest (t) and the corresponding concentration ([A]), and write these quantities as the upper limits of their respective integrals.

Equation 22.12b shows that, if $\ln([A]/[A]_0)$ is plotted against t, then a first-order reaction will give a straight line of slope $-k$. Some rate constants determined in this way are given in Table 22.1. The second expression in eqn 22.12b shows that in a first-order reaction the reactant concentration decreases exponentially with time with a rate determined by k (Fig. 22.5).

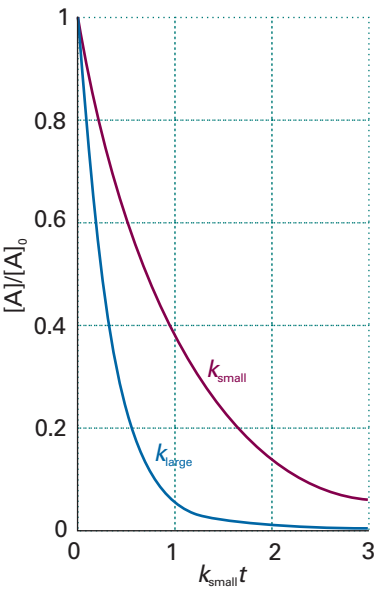

Fig. 22.5 The exponential decay of the reactant in a first-order reaction. The larger the rate constant, the more rapid the decay: here $k_{large} = 3k_{small}$.

Exploration For a first-order reaction of the form $A \rightarrow nB$ (with n possibly fractional), the concentration of the product varies with time as $[B] = n[B]_0(1 - e^{-kt})$. Plot the time dependence of [A] and [B] for the cases $n = 0.5$, 1, and 2.

Example 22.3 *Analysing a first-order reaction*

The variation in the partial pressure of azomethane with time was followed at 600 K, with the results given below. Confirm that the decomposition

$$CH_3N_2CH_3(g) \rightarrow CH_3CH_3(g) + N_2(g)$$

is first-order in azomethane, and find the rate constant at 600 K.

t/s	0	1000	2000	3000	4000
p/Pa	10.9	7.63	5.32	3.71	2.59

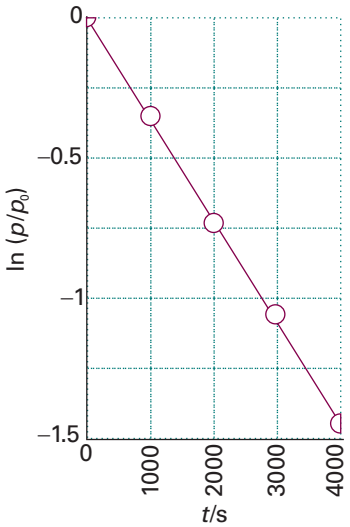

Fig. 22.6 The determination of the rate constant of a first-order reaction: a straight line is obtained when ln [A] (or, as here, ln p) is plotted against t; the slope gives k.

Method As indicated in the text, to confirm that a reaction is first-order, plot $\ln([A]/[A]_0)$ against time and expect a straight line. Because the partial pressure of a gas is proportional to its concentration, an equivalent procedure is to plot $\ln(p/p_0)$ against t. If a straight line is obtained, its slope can be identified with $-k$.

Answer We draw up the following table:

t/s	0	1000	2000	3000	4000
$\ln(p/p_0)$	1	−0.360	−0.720	−1.082	−1.441

Figure 22.6 shows the plot of $\ln(p/p_0)$ against t. The plot is straight, confirming a first-order reaction, and its slope is -3.6×10^{-4}. Therefore, $k = 3.6 \times 10^{-4}\ s^{-1}$.

A note on good practice Because the horizontal and vertical axes of graphs are labelled with pure numbers, the slope of a graph is always dimensionless. For a graph of the form $y = b + mx$ we can write

$$y = b + (m \text{ units})(x/\text{units})$$

where 'units' are the units of x, and identify the (dimensionless) slope with 'm units'. Then

$$m = \text{slope/units}$$

In the present case, because the graph shown here is a plot of $\ln(p/p_0)$ against t/s (with 'units' = s) and k is the negative value of the slope of $\ln(p/p_0)$ against t itself,

$$k = -\text{slope/s}$$

Self-test 22.4 In a particular experiment, it was found that the concentration of N_2O_5 in liquid bromine varied with time as follows:

t/s	0	200	400	600	1000
$[N_2O_5]/(\text{mol dm}^{-3})$	0.110	0.073	0.048	0.032	0.014

Confirm that the reaction is first-order in N_2O_5 and determine the rate constant.
$[k = 2.1 \times 10^{-3}\ s^{-1}]$

(b) Half-lives and time constants

A useful indication of the rate of a first-order chemical reaction is the **half-life**, $t_{1/2}$, of a substance, the time taken for the concentration of a reactant to fall to half its initial value. The time for [A] to decrease from $[A]_0$ to $\frac{1}{2}[A]_0$ in a first-order reaction is given by eqn 22.12b as

$$kt_{1/2} = -\ln\left(\frac{\frac{1}{2}[A]_0}{[A]_0}\right) = -\ln\tfrac{1}{2} = \ln 2$$

Hence

$$t_{1/2} = \frac{\ln 2}{k} \qquad (22.13)$$

($\ln 2 = 0.693$.) The main point to note about this result is that, for a first-order reaction, the half-life of a reactant is independent of its initial concentration. Therefore, if the concentration of A at some *arbitrary* stage of the reaction is [A], then it will have fallen to $\frac{1}{2}$[A] after a further interval of $(\ln 2)/k$. Some half-lives are given in Table 22.1.

Another indication of the rate of a first-order reaction is the **time constant**, τ (tau), the time required for the concentration of a reactant to fall to 1/e of its initial value. From eqn 22.12b it follows that

$$k\tau = -\ln\left(\frac{[A]_0/e}{[A]_0}\right) = -\ln\frac{1}{e} = 1$$

That is, the time constant of a first-order reaction is the reciprocal of the rate constant:

$$\tau = \frac{1}{k} \tag{22.14}$$

(c) Second-order reactions

We show in the *Justification* below that the integrated form of the second-order rate law

$$\frac{d[A]}{dt} = -k[A]^2 \tag{22.15a}$$

is either of the following two forms:

$$\frac{1}{[A]} - \frac{1}{[A]_0} = kt \tag{22.15b}$$

$$[A] = \frac{[A]_0}{1 + kt[A]_0} \tag{22.15c}$$

where $[A]_0$ is the initial concentration of A (at $t = 0$).

Justification 22.2 *Second-order integrated rate law*

To integrate eqn 22.15a we rearrange it into

$$\frac{d[A]}{[A]^2} = -k\,dt$$

The concentration of A is $[A_0]$ at $t = 0$ and $[A]$ at a general time t later. Therefore,

$$-\int_{[A]_0}^{[A]} \frac{d[A]}{[A]^2} = k\int_0^t dt$$

Because the integral of $1/x^2$ is $-1/x$, we obtain eqn 22.15b by substitution of the limits

$$\frac{1}{[A]}\Big|_{[A]_0}^{[A]} = \frac{1}{[A]} - \frac{1}{[A]_0} = kt$$

We can then rearrange this expression into eqn 22.15c.

Equation 22.15b shows that to test for a second-order reaction we should plot $1/[A]$ against t and expect a straight line. The slope of the graph is k. Some rate constants determined in this way are given in Table 22.2. The rearranged form, eqn 22.15c, lets us predict the concentration of A at any time after the start of the reaction. It shows that the concentration of A approaches zero more slowly than in a first-order reaction with the same initial rate (Fig. 22.7).

It follows from eqn 22.15b by substituting $t = t_{1/2}$ and $[A] = \frac{1}{2}[A]_0$ that the half-life of a species A that is consumed in a second-order reaction is

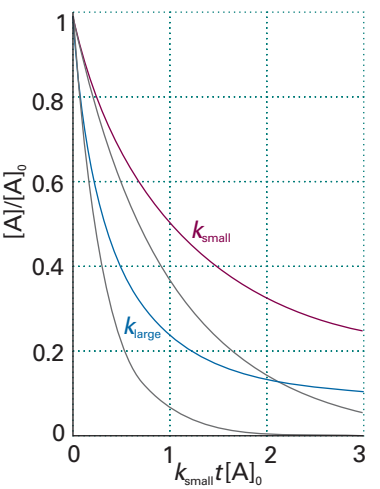

Fig. 22.7 The variation with time of the concentration of a reactant in a second-order reaction. The grey lines are the corresponding decays in a first-order reaction with the same initial rate. For this illustration, $k_{large} = 3k_{small}$.

Exploration For a second-order reaction of the form $A \rightarrow nB$ (with n possibly fractional), the concentration of the product varies with time as $[B] = nkt[A]_0^2/(1 + kt[A]_0)$. Plot the time dependence of [A] and [B] for the cases $n = 0.5$, 1, and 2.

Synoptic table 22.2* Kinetic data for second-order reactions

Reaction	Phase	θ/°C	k/(dm^3 mol^{-1} s^{-1})
$2\,NOBr \rightarrow 2\,NO + Br_2$	g	10	0.80
$2\,I \rightarrow I_2$	g	23	7×10^9
$CH_3Cl + CH_3O^-$	CH_3OH(l)	20	2.29×10^{-6}

* More values are given in the *Data section*.

$$t_{1/2} = \frac{1}{k[A]_0} \tag{22.16}$$

Therefore, unlike a first-order reaction, the half-life of a substance in a second-order reaction varies with the initial concentration. A practical consequence of this dependence is that species that decay by second-order reactions (which includes some environmentally harmful substances) may persist in low concentrations for long periods because their half-lives are long when their concentrations are low. In general, for an *n*th-order reaction of the form A → products, the half-life is related to the rate constant and the initial concentration of A by

$$t_{1/2} = \frac{1}{k[A]^{n-1}} \tag{22.17}$$

(See Exercise 12.12a.)

Another type of second-order reaction is one that is first-order in each of two reactants A and B:

$$\frac{d[A]}{dt} = -k[A][B] \tag{22.18}$$

Such a rate law cannot be integrated until we know how the concentration of B is related to that of A. For example, if the reaction is A + B → P, where P denotes products, and the initial concentrations are $[A]_0$ and $[B]_0$, then it is shown in the *Justification* below that, at a time t after the start of the reaction, the concentrations satisfy the relation

$$\ln\left(\frac{[B]/[B]_0}{[A]/[A]_0}\right) = ([B]_0 - [A]_0)kt \tag{22.19}$$

Therefore, a plot of the expression on the left against t should be a straight line from which k can be obtained.

Justification 22.3 *Overall second-order rate law*

It follows from the reaction stoichiometry that, when the concentration of A has fallen to $[A]_0 - x$, the concentration of B will have fallen to $[B]_0 - x$ (because each A that disappears entails the disappearance of one B). It follows that

$$\frac{d[A]}{dt} = -k([A]_0 - x)([B]_0 - x)$$

Because $[A] = [A]_0 - x$, it follows that $d[A]/dt = -dx/dt$ and the rate law may be written as

$$\frac{dx}{dt} = k([A]_0 - x)([B]_0 - x)$$

The initial condition is that $x = 0$ when $t = 0$; so the integration required is

$$\int_0^x \frac{dx}{([A]_0 - x)([B]_0 - x)} = k\int_0^t dt$$

The integral on the right is simply kt. The integral on the left is evaluated by using the method of partial fractions and by using $[A] = [A]_0$ and $[B] = [B]_0$ at $t = 0$ to give:

$$\int_0^x \frac{dx}{([A]_0 - x)([B]_0 - x)} = \frac{1}{[B]_0 - [A]_0}\left\{\ln\left(\frac{[A]_0}{[A]_0 - x}\right) - \ln\left(\frac{[B]_0}{[B]_0 - x}\right)\right\}$$

This expression can be simplified and rearranged into eqn 22.19 by combining the two logarithms by using $\ln y - \ln z = \ln(y/z)$ and noting that $[A] = [A]_0 - x$ and $[B] = [B]_0 - x$.

Similar calculations may be carried out to find the integrated rate laws for other orders, and some are listed in Table 22.3.

Comment 22.2

To use the method of partial fractions to evaluate an integral of the form $\int \frac{1}{(a-x)(b-x)}dx$, where a and b are constants, we write

$$\frac{1}{(a-x)(b-x)} = \frac{1}{b-a}\left(\frac{1}{a-x} - \frac{1}{b-x}\right)$$

and integrate the expression on the right. It follows that

$$\int \frac{dx}{(a-x)(b-x)} = \frac{1}{b-a}\left[\int \frac{dx}{a-x} - \int \frac{dx}{b-x}\right]$$

$$= \frac{1}{b-a}\left(\ln\frac{1}{a-x} - \ln\frac{1}{b-x}\right) + \text{constant}$$

Table 22.3 Integrated rate laws

Order	Reaction	Rate law*	$t_{1/2}$
0	$A \rightarrow P$	$v = k$ $kt = x$ for $0 \le x \le [A]_0$	$[A]_0/2k$
1	$A \rightarrow P$	$v = k[A]$ $kt = \ln\frac{[A]_0}{[A]_0 - x}$	$(\ln 2)/k$
2	$A \rightarrow P$	$v = k[A]^2$ $kt = \frac{x}{[A]_0([A]_0 - x)}$	$1/k[A]_0$
	$A + B \rightarrow P$	$v = k[A][B]$ $kt = \frac{1}{[B]_0 - [A]_0}\ln\frac{[A]_0([B]_0 - x)}{([A]_0 - x)[B]_0}$	
	$A + 2\,B \rightarrow P$	$v = k[A][B]$ $kt = \frac{1}{[B]_0 - 2[A]_0}\ln\frac{[A]_0([B]_0 - 2x)}{([A]_0 - x)[B]_0}$	
	$A \rightarrow P$ with autocatalysis	$v = k[A][P]$ $kt = \frac{1}{[A]_0 + [P]_0}\ln\frac{[A]_0([P]_0 + x)}{([A]_0 - x)[P]_0}$	
3	$A + 2\,B \rightarrow P$	$v = k[A][B]^2$ $kt = \frac{2x}{(2[A]_0 - [B]_0)([B]_0 - 2x)[B]_0} + \frac{1}{(2[A]_0 - [B]_0)^2}\ln\frac{[A]_0([B]_0 - 2x)}{([A]_0 - x)[B]_0}$	
$n \ge 2$	$A \rightarrow P$	$v = k[A]^n$ $kt = \frac{1}{n-1}\left\{\frac{1}{([A]_0 - x)^{n-1}} - \frac{1}{[A]_0^{n-1}}\right\}$	$\frac{2^{n-1} - 1}{(n-1)k[A]_0^{n-1}}$

* $x = [P]$ and $v = dx/dt$.

22.4 Reactions approaching equilibrium

Because all the laws considered so far disregard the possibility that the reverse reaction is important, none of them describes the overall rate when the reaction is close to equilibrium. At that stage the products may be so abundant that the reverse reaction must be taken into account. In practice, however, most kinetic studies are made on reactions that are far from equilibrium, and the reverse reactions are unimportant.

(a) First-order reactions close to equilibrium

We can explore the variation of the composition with time close to chemical equilibrium by considering the reaction in which A forms B and both forward and reverse reactions are first-order (as in some isomerizations). The scheme we consider is

$$\begin{aligned} &A \rightarrow B \qquad v = k[A] \\ &B \rightarrow A \qquad v = k'[B] \end{aligned} \tag{22.20}$$

The concentration of A is reduced by the forward reaction (at a rate $k[A]$) but it is increased by the reverse reaction (at a rate $k'[B]$). The net rate of change is therefore

$$\frac{d[A]}{dt} = -k[A] + k'[B] \tag{22.21}$$

If the initial concentration of A is $[A]_0$, and no B is present initially, then at all times $[A] + [B] = [A]_0$. Therefore,

$$\frac{d[A]}{dt} = -k[A] + k'([A]_0 - [A]) = -(k + k')[A] + k'[A]_0 \tag{22.22}$$

The solution of this first-order differential equation (as may be checked by differentiation) is

$$[A] = \frac{k' + ke^{-(k+k')t}}{k' + k}[A]_0 \tag{22.23}$$

Figure 22.8 shows the time dependence predicted by this equation.

As $t \rightarrow \infty$, the concentrations reach their equilibrium values, which are given by eqn 22.23 as:

$$[A]_{eq} = \frac{k'[A]_0}{k + k'} \qquad [B]_{eq} = [A]_0 - [A]_\infty = \frac{k[A]_0}{k + k'} \tag{22.24}$$

It follows that the equilibrium constant of the reaction is

$$K = \frac{[B]_{eq}}{[A]_{eq}} = \frac{k}{k'} \tag{22.25}$$

(This expression is only approximate because thermodynamic equilibrium constants are expressed in terms of activities, not concentrations.) Exactly the same conclusion can be reached—more simply, in fact—by noting that, at equilibrium, the forward and reverse rates must be the same, so

$$k[A]_{eq} = k'[B]_{eq} \tag{22.26}$$

This relation rearranges into eqn 22.25. The theoretical importance of eqn 22.25 is that it relates a thermodynamic quantity, the equilibrium constant, to quantities relating to rates. Its practical importance is that, if one of the rate constants can be measured, then the other may be obtained if the equilibrium constant is known.

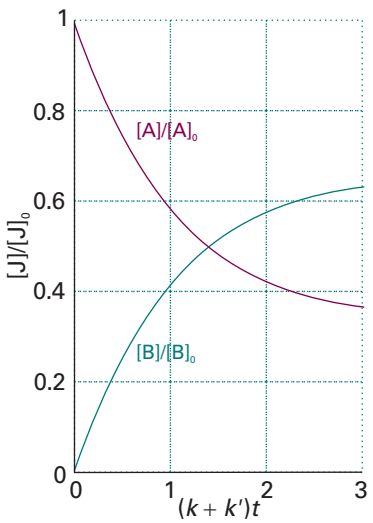

Fig. 22.8 The approach of concentrations to their equilibrium values as predicted by eqn 22.23 for a reaction A ⇌ B that is first-order in each direction, and for which $k = 2k'$.

Exploration Set up the rate equations and plot the corresponding graphs for the approach to and equilibrium of the form A ⇌ 2 B.

For a more general reaction, the overall equilibrium constant can be expressed in terms of the rate constants for all the intermediate stages of the reaction mechanism:

$$K = \frac{k_a}{k'_a} \times \frac{k_b}{k'_b} \times \cdots \qquad (22.27)$$

where the ks are the rate constants for the individual steps and the k's are those for the corresponding reverse steps.

(b) Relaxation methods

The term **relaxation** denotes the return of a system to equilibrium. It is used in chemical kinetics to indicate that an externally applied influence has shifted the equilibrium position of a reaction, normally suddenly, and that the reaction is adjusting to the equilibrium composition characteristic of the new conditions (Fig. 22.9). We shall consider the response of reaction rates to a **temperature jump**, a sudden change in temperature. We know from Section 7.4 that the equilibrium composition of a reaction depends on the temperature (provided $\Delta_r H^\ominus$ is nonzero), so a shift in temperature acts as a perturbation on the system. One way of achieving a temperature jump is to discharge a capacitor through a sample made conducting by the addition of ions, but laser or microwave discharges can also be used. Temperature jumps of between 5 and 10 K can be achieved in about 1 μs with electrical discharges. The high energy output of pulsed lasers (Section 14.5) is sufficient to generate temperature jumps of between 10 and 30 K within nanoseconds in aqueous samples. Some equilibria are also sensitive to pressure, and **pressure-jump techniques** may then also be used.

When a sudden temperature increase is applied to a simple A ⇌ B equilibrium that is first-order in each direction, we show in the *Justification* below that the composition relaxes exponentially to the new equilibrium composition:

$$x = x_0 e^{-t/\tau} \qquad \frac{1}{\tau} = k_a + k_b \qquad (22.28)$$

where x_0 is the departure from equilibrium immediately after the temperature jump and x is the departure from equilibrium at the new temperature after a time t.

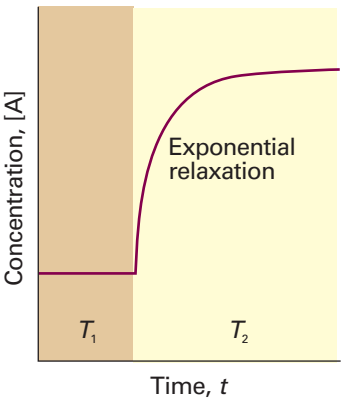

Fig. 22.9 The relaxation to the new equilibrium composition when a reaction initially at equilibrium at a temperature T_1 is subjected to a sudden change of temperature, which takes it to T_2.

Justification 22.4 *Relaxation to equilibrium*

When the temperature of a system at equilibrium is increased suddenly, the rate constants change from their earlier values to the new values k_a and k_b characteristic of that temperature, but the concentrations of A and B remain for an instant at their old equilibrium values. As the system is no longer at equilibrium, it readjusts to the new equilibrium concentrations, which are now given by

$$k_a[A]_{eq} = k_b[B]_{eq}$$

and it does so at a rate that depends on the new rate constants. We write the deviation of [A] from its new equilibrium value as x, so $[A] = x + [A]_{eq}$ and $[B] = [B]_{eq} - x$. The concentration of A then changes as follows:

$$\begin{aligned} \frac{d[A]}{dt} &= -k_a[A] + k_b[B] \\ &= -k_a([A]_{eq} + x) + k_b([B]_{eq} - x) \\ &= -(k_a + k_b)x \end{aligned}$$

because the two terms involving the equilibrium concentrations cancel. Because $d[A]/dt = dx/dt$, this equation is a first-order differential equation with the solution that resembles eqn 22.12b and is given in eqn 22.28.

Equation 22.28 shows that the concentrations of A and B relax into the new equilibrium at a rate determined by the sum of the two new rate constants. Because the equilibrium constant under the new conditions is $K \approx k_a/k_b$, its value may be combined with the relaxation time measurement to find the individual k_a and k_b.

Example 22.4 *Analysing a temperature-jump experiment*

The equilibrium constant for the autoprotolysis of water, $H_2O(l) \rightleftharpoons H^+(aq) + OH^-(aq)$, is $K_w = a(H^+)a(OH^-) = 1.008 \times 10^{-14}$ at 298 K. After a temperature-jump, the reaction returns to equilibrium with a relaxation time of 37 μs at 298 K and pH ≈ 7. Given that the forward reaction is first-order and the reverse is second-order overall, calculate the rate constants for the forward and reverse reactions.

Method We need to derive an expression for the relaxation time, τ (the time constant for return to equilibrium), in terms of k_1 (forward, first-order reaction) and k_2 (reverse, second-order reaction). We can proceed as above, but it will be necessary to make the assumption that the deviation from equilibrium (x) is so small that terms in x^2 can be neglected. Relate k_1 and k_2 through the equilibrium constant, but be careful with units because K_w is dimensionless.

Answer The forward rate at the final temperature is $k_1[H_2O]$ and the reverse rate is $k_2[H^+][OH^-]$. The net rate of deprotonation of H_2O is

$$\frac{d[H_2O]}{dt} = -k_1[H_2O] + k_2[H^+][OH^-]$$

We write $[H_2O] = [H_2O]_{eq} + x$, $[H^+] = [H^+]_{eq} - x$, and $[OH^-] = [OH^-]_{eq} - x$, and obtain

$$\frac{dx}{dt} = -\{k_1 + k_2([H^+]_{eq} + [OH^-]_{eq})\}x - k_1[H_2O]_{eq} + k_2[H^+]_{eq}[OH^-]_{eq} + k_2x^2$$
$$\approx -\{k_1 + k_2([H^+]_{eq} + [OH^-]_{eq})\}x$$

where we have neglected the term in x^2 and used the equilibrium condition

$$k_1[H_2O]_{eq} = k_2[H^+]_{eq}[OH^-]_{eq}$$

to eliminate the terms that are independent of x. It follows that

$$\frac{1}{\tau} = k_1 + k_2([H^+]_{eq} + [OH^-]_{eq})$$

At this point we note that

$$K_w = a(H^+)a(OH^-) \approx ([H^+]_{eq}/c^\ominus)([OH^-]_{eq}/c^\ominus) = [H^+]_{eq}[OH^-]_{eq}/c^{\ominus 2}$$

with $c^\ominus = 1$ mol dm^{-3}. For this electrically neutral system, $[H^+] = [OH^-]$, so the concentration of each type of ion is $K_w^{1/2}c^\ominus$, and hence

$$\frac{1}{\tau} = k_1 + k_2(K_w^{1/2}c^\ominus + K_w^{1/2}c^\ominus) = k_2\left\{\frac{k_1}{k_2} + 2K_w^{1/2}c^\ominus\right\}$$

At this point we note that

$$\frac{k_1}{k_2} = \frac{[H^+]_{eq}[OH^-]_{eq}}{[H_2O]_{eq}} = \frac{K_w c^{\ominus 2}}{[H_2O]_{eq}}$$

The molar concentration of pure water is 55.6 mol dm^{-3}, so $[H_2O]_{eq}/c^\ominus = 55.6$. If we write $K = K_w/55.6 = 1.81 \times 10^{-16}$, we obtain

$$\frac{1}{\tau} = k_2\{K + 2K_w^{1/2}\}c^\ominus$$

Hence,

$$k_2 = \frac{1}{\tau(K + 2K_w^{1/2})c^{\ominus}}$$

$$= \frac{1}{(3.7 \times 10^{-5}\ \text{s}) \times (2.0 \times 10^{-7}) \times (1\ \text{mol dm}^{-3})} = 1.4 \times 10^{11}\ \text{dm}^3\ \text{mol}^{-1}\ \text{s}^{-1}$$

It follows that

$$k_1 = k_2 K c^{\ominus} = 2.4 \times 10^{-5}\ \text{s}^{-1}$$

The reaction is faster in ice, where $k_2 = 8.6 \times 10^{12}\ \text{dm}^3\ \text{mol}^{-1}\ \text{s}^{-1}$.

A note on good practice Notice how we keep track of units through the use of $c^{\ominus}$: K and K_w are dimensionless; k_2 is expressed in $\text{dm}^3\ \text{mol}^{-1}\ \text{s}^{-1}$ and k_1 is expressed in s^{-1}.

Self-test 22.5 Derive an expression for the relaxation time of a concentration when the reaction A + B ⇌ C + D is second-order in both directions.

$[1/\tau = k([\text{A}] + [\text{B}])_{\text{eq}} + k'([\text{C}] + [\text{D}])_{\text{eq}}]$

22.5 The temperature dependence of reaction rates

The rate constants of most reactions increase as the temperature is raised. Many reactions in solution fall somewhere in the range spanned by the hydrolysis of methyl ethanoate (where the rate constant at 35°C is 1.82 times that at 25°C) and the hydrolysis of sucrose (where the factor is 4.13).

(a) The Arrhenius parameters

It is found experimentally for many reactions that a plot of ln k against $1/T$ gives a straight line. This behaviour is normally expressed mathematically by introducing two parameters, one representing the intercept and the other the slope of the straight line, and writing the **Arrhenius equation**

$$\ln k = \ln A - \frac{E_a}{RT} \qquad (22.29)$$

The parameter A, which corresponds to the intercept of the line at $1/T = 0$ (at infinite temperature, Fig. 22.10), is called the **pre-exponential factor** or the 'frequency factor'. The parameter E_a, which is obtained from the slope of the line $(-E_a/R)$, is called the **activation energy**. Collectively the two quantities are called the **Arrhenius parameters** (Table 22.4).

Synoptic table 22.4* Arrhenius parameters

(1) First-order reactions	A/s^{-1}	$E_a/(\text{kJ mol}^{-1})$
$CH_3NC \rightarrow CH_3CN$	3.98×10^{13}	160
$2\ N_2O_5 \rightarrow 4\ NO_2 + O_2$	4.94×10^{13}	103.4
(2) Second-order reactions	$A/(\text{dm}^3\ \text{mol}^{-1}\ \text{s}^{-1})$	$E_a/(\text{kJ mol}^{-1})$
$OH + H_2 \rightarrow H_2O + H$	8.0×10^{10}	42
$NaC_2H_5O + CH_3I$ in ethanol	2.42×10^{11}	81.6

* More values are given in the *Data section*.

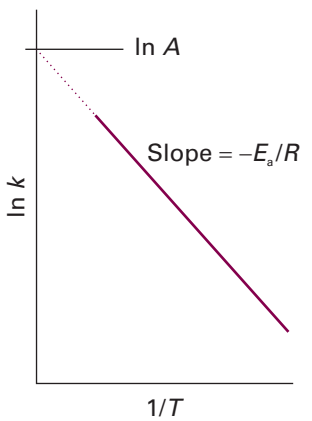

Fig. 22.10 A plot of ln k against $1/T$ is a straight line when the reaction follows the behaviour described by the Arrhenius equation (eqn 22.29). The slope gives $-E_a/R$ and the intercept at $1/T = 0$ gives ln A.

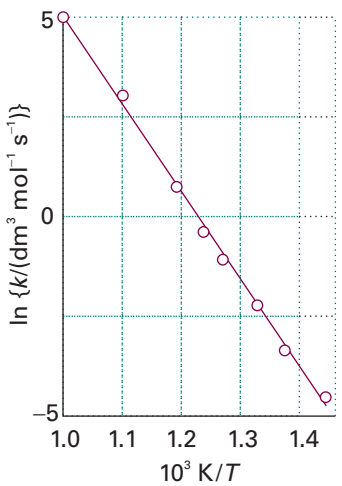

Fig. 22.11 The Arrhenius plot using the data in Example 22.5.

Example 22.5 *Determining the Arrhenius parameters*

The rate of the second-order decomposition of acetaldehyde (ethanal, CH_3CHO) was measured over the temperature range 700–1000 K, and the rate constants are reported below. Find E_a and A.

T/K	700	730	760	790	810	840	910	1000
$k/(\text{dm}^3\ \text{mol}^{-1}\ \text{s}^{-1})$	0.011	0.035	0.105	0.343	0.789	2.17	20.0	145

Method According to eqn 22.29, the data can be analysed by plotting $\ln(k/\text{dm}^3\ \text{mol}^{-1}\ \text{s}^{-1})$ against $1/(T/\text{K})$, or more conveniently $(10^3\ \text{K})/T$, and getting a straight line. As explained in Example 22.3, we obtain the activation energy from the dimensionless slope by writing $-E_a/R$ = slope/units, where in this case 'units' = $1/(10^3\ \text{K})$, so $E_a = -\text{slope} \times R \times 10^3\ \text{K}$. The intercept at $T = 0$ is $\ln(A/\text{dm}^3\ \text{mol}^{-1}\ \text{s}^{-1})$.

Answer We draw up the following table:

$(10^3\ \text{K})/T$	1.43	1.37	1.32	1.27	1.23	1.19	1.10	1.00
$\ln(k/\text{dm}^3\ \text{mol}^{-1}\ \text{s}^{-1})$	−4.51	−3.35	−2.25	−1.07	−0.24	0.77	3.00	4.98

Now plot ln k against $1/T$ (Fig. 22.11). The least-squares fit is to a line with slope −22.7 and intercept 27.7. Therefore,

$$E_a = 22.7 \times (8.3145\ \text{J K}^{-1}\ \text{mol}^{-1}) \times 10^3\ \text{K} = 189\ \text{kJ mol}^{-1}$$
$$A = e^{27.7}\ \text{dm}^3\ \text{mol}^{-1}\ \text{s}^{-1} = 1.1 \times 10^{12}\ \text{dm}^3\ \text{mol}^{-1}\ \text{s}^{-1}$$

A note on good practice Note that A has the same units as k. In practice, A is obtained from one of the mid-range data values rather than using a lengthy extrapolation.

Self-test 22.6 Determine A and E_a from the following data:

T/K	300	350	400	450	500
$k/(\text{dm}^3\ \text{mol}^{-1}\ \text{s}^{-1})$	7.9×10^6	3.0×10^7	7.9×10^7	1.7×10^8	3.2×10^8

[$8 \times 10^{10}\ \text{dm}^3\ \text{mol}^{-1}\ \text{s}^{-1}$, $23\ \text{kJ mol}^{-1}$]

The fact that E_a is given by the slope of the plot of ln k against $1/T$ means that, the higher the activation energy, the stronger the temperature dependence of the rate constant (that is, the steeper the slope). *A high activation energy signifies that the rate constant depends strongly on temperature.* If a reaction has zero activation energy, its rate is independent of temperature. In some cases the activation energy is negative, which indicates that the rate decreases as the temperature is raised. We shall see that such behaviour is a signal that the reaction has a complex mechanism.

The temperature dependence of some reactions is non-Arrhenius, in the sense that a straight line is not obtained when ln k is plotted against $1/T$. However, it is still possible to define an activation energy at any temperature as

$$E_a = RT^2\left(\frac{\mathrm{d}\ln k}{\mathrm{d}T}\right) \qquad [22.30]$$

This definition reduces to the earlier one (as the slope of a straight line) for a temperature-independent activation energy. However, the definition in eqn 22.30 is more general than eqn 22.29, because it allows E_a to be obtained from the slope (at the

temperature of interest) of a plot of ln k against $1/T$ even if the Arrhenius plot is not a straight line. Non-Arrhenius behaviour is sometimes a sign that quantum mechanical tunnelling is playing a significant role in the reaction (Section 22.7f).

(b) The interpretation of the parameters

For the present chapter we shall regard the Arrhenius parameters as purely empirical quantities that enable us to discuss the variation of rate constants with temperature; however, it is useful to have an interpretation in mind and write eqn 22.29 as

$$k = Ae^{-E_a/RT} \tag{22.31}$$

To interpret E_a we consider how the molecular potential energy changes in the course of a chemical reaction that begins with a collision between molecules of A and molecules of B (Fig. 22.12).

As the reaction event proceeds, A and B come into contact, distort, and begin to exchange or discard atoms. The **reaction coordinate** is the collection of motions, such as changes in interatomic distances and bond angles, that are directly involved in the formation of products from reactants. (The reaction coordinate is essentially a geometrical concept and quite distinct from the extent of reaction.) The potential energy rises to a maximum and the cluster of atoms that corresponds to the region close to the maximum is called the **activated complex**. After the maximum, the potential energy falls as the atoms rearrange in the cluster and reaches a value characteristic of the products. The climax of the reaction is at the peak of the potential energy, which corresponds to the activation energy E_a. Here two reactant molecules have come to such a degree of closeness and distortion that a small further distortion will send them in the direction of products. This crucial configuration is called the **transition state** of the reaction. Although some molecules entering the transition state might revert to reactants, if they pass through this configuration then it is inevitable that products will emerge from the encounter.

We also conclude from the preceding discussion that, for a reaction involving the collision of two molecules, *the activation energy is the minimum kinetic energy that reactants must have in order to form products*. For example, in a gas-phase reaction there are numerous collisions each second, but only a tiny proportion are sufficiently energetic to lead to reaction. The fraction of collisions with a kinetic energy in excess of an energy E_a is given by the Boltzmann distribution as $e^{-E_a/RT}$. Hence, we can interpret the exponential factor in eqn 22.31 as the fraction of collisions that have enough kinetic energy to lead to reaction.

The pre-exponential factor is a measure of the rate at which collisions occur irrespective of their energy. Hence, the product of A and the exponential factor, $e^{-E_a/RT}$, gives the rate of *successful* collisions. We shall develop these remarks in Chapter 24 and see that they have their analogues for reactions that take place in liquids.

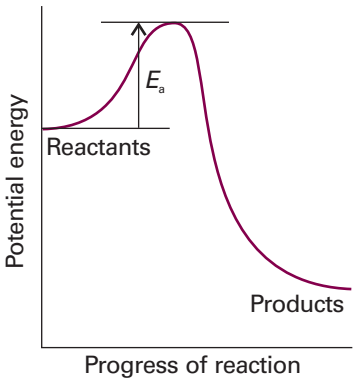

Fig. 22.12 A potential energy profile for an exothermic reaction. The height of the barrier between the reactants and products is the activation energy of the reaction.

Comment 22.3
The terms *activated complex* and *transition state* are often used as synonyms; however, we shall preserve a distinction.

Accounting for the rate laws

We now move on to the second stage of the analysis of kinetic data, their explanation in terms of a postulated reaction mechanism.

22.6 Elementary reactions

Most reactions occur in a sequence of steps called **elementary reactions**, each of which involves only a small number of molecules or ions. A typical elementary reaction is

$$H + Br_2 \rightarrow HBr + Br$$

Note that the phase of the species is not specified in the chemical equation for an elementary reaction, and the equation represents the specific process occurring to individual molecules. This equation, for instance, signifies that an H atom attacks a Br_2 molecule to produce an HBr molecule and a Br atom. The **molecularity** of an elementary reaction is the number of molecules coming together to react in an elementary reaction. In a **unimolecular reaction**, a single molecule shakes itself apart or its atoms into a new arrangement, as in the isomerization of cyclopropane to propene. In a **bimolecular reaction**, a pair of molecules collide and exchange energy, atoms, or groups of atoms, or undergo some other kind of change. It is most important to distinguish molecularity from order: reaction order is an empirical quantity, and obtained from the experimental rate law; molecularity refers to an elementary reaction proposed as an individual step in a mechanism.

The rate law of a unimolecular elementary reaction is first-order in the reactant:

$$A \rightarrow P \qquad \frac{d[A]}{dt} = -k[A] \tag{22.32}$$

where P denotes products (several different species may be formed). A unimolecular reaction is first-order because the number of A molecules that decay in a short interval is proportional to the number available to decay. (Ten times as many decay in the same interval when there are initially 1000 A molecules as when there are only 100 present.) Therefore, the rate of decomposition of A is proportional to its molar concentration.

An elementary bimolecular reaction has a second-order rate law:

$$A + B \rightarrow P \qquad \frac{d[A]}{dt} = -k[A][B] \tag{22.33}$$

A bimolecular reaction is second-order because its rate is proportional to the rate at which the reactant species meet, which in turn is proportional to their concentrations. Therefore, if we have evidence that a reaction is a single-step, bimolecular process, we can write down the rate law (and then go on to test it). Bimolecular elementary reactions are believed to account for many homogeneous reactions, such as the dimerizations of alkenes and dienes and reactions such as

$$CH_3I(alc) + CH_3CH_2O^-(alc) \rightarrow CH_3OCH_2CH_3(alc) + I^-(alc)$$

(where 'alc' signifies alcohol solution). There is evidence that the mechanism of this reaction is a single elementary step

$$CH_3I + CH_3CH_2O^- \rightarrow CH_3OCH_2CH_3 + I^-$$

This mechanism is consistent with the observed rate law

$$v = k[CH_3I][CH_3CH_2O^-] \tag{22.34}$$

We shall see below how to combine a series of simple steps together into a mechanism and how to arrive at the corresponding rate law. For the present we emphasize that, *if the reaction is an elementary bimolecular process, then it has second-order kinetics, but if the kinetics are second-order, then the reaction might be complex.* The postulated mechanism can be explored only by detailed detective work on the system, and by investigating whether side products or intermediates appear during the course of the reaction. Detailed analysis of this kind was one of the ways, for example, in which the reaction $H_2(g) + I_2(g) \rightarrow 2\,HI(g)$ was shown to proceed by a complex mechanism. For

many years the reaction had been accepted on good, but insufficiently meticulous evidence as a fine example of a simple bimolecular reaction, $H_2 + I_2 \rightarrow HI + HI$, in which atoms exchanged partners during a collision.

22.7 Consecutive elementary reactions

Some reactions proceed through the formation of an intermediate (I), as in the consecutive unimolecular reactions

$$A \xrightarrow{k_a} I \xrightarrow{k_b} P$$

An example is the decay of a radioactive family, such as

$$^{239}U \xrightarrow{23.5\ min} {}^{239}Np \xrightarrow{2.35\ day} {}^{239}Pu$$

(The times are half-lives.) We can discover the characteristics of this type of reaction by setting up the rate laws for the net rate of change of the concentration of each substance.

(a) The variation of concentrations with time

The rate of unimolecular decomposition of A is

$$\frac{d[A]}{dt} = -k_a[A] \tag{22.35}$$

and A is not replenished. The intermediate I is formed from A (at a rate $k_a[A]$) but decays to P (at a rate $k_b[I]$). The net rate of formation of I is therefore

$$\frac{d[I]}{dt} = k_a[A] - k_b[I] \tag{22.36}$$

The product P is formed by the unimolecular decay of I:

$$\frac{d[P]}{dt} = k_b[I] \tag{22.37}$$

We suppose that initially only A is present, and that its concentration is $[A]_0$.

The first of the rate laws, eqn 22.35, is an ordinary first-order decay, so we can write

$$[A] = [A]_0 e^{-k_a t} \tag{22.38}$$

When this equation is substituted into eqn 22.36, we obtain after rearrangement

$$\frac{d[I]}{dt} + k_b[I] = k_a[A]_0 e^{-k_a t} \tag{22.39}$$

This differential equation has a standard form and, after setting $[I]_0 = 0$, the solution is

$$[I] = \frac{k_a}{k_b - k_a}(e^{-k_a t} - e^{-k_b t})[A]_0 \tag{22.40}$$

At all times $[A] + [I] + [P] = [A]_0$, so it follows that

$$[P] = \left\{1 + \frac{k_a e^{-k_b t} - k_b e^{-k_a t}}{k_b - k_a}\right\}[A]_0 \tag{22.41}$$

The concentration of the intermediate I rises to a maximum and then falls to zero (Fig. 22.13). The concentration of the product P rises from zero towards $[A]_0$.

Comment 22.4

The solution of a first-order differential equation with the form

$$\frac{dy}{dx} + yf(x) = g(x)$$

is

$$e^{\int f(x)\,dx} y = \int e^{\int f(x)dx} g(x)dx + \text{constant}$$

Equation 22.39 is a special case of this standard form, with $f(x) =$ constant.

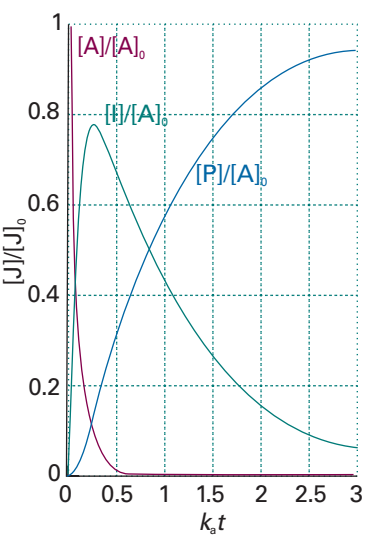

Fig. 22.13 The concentrations of A, I, and P in the consecutive reaction scheme A → I → P. The curves are plots of eqns 22.38, 22.40, and 21.41 with $k_a = 10k_b$. If the intermediate I is in fact the desired product, it is important to be able to predict when its concentration is greatest; see Example 22.6.

Exploration Use mathematical software, an electronic spreadsheet, or the applets found in the *Living graphs* section of the text's web site to investigate the effects on [A], [I], [P], and t_{max} of changing the ratio k_a/k_b from 10 (as in Fig. 22.13) to 0.01. Compare your results with those shown in Fig. 22.15.

Example 22.6 *Analysing consecutive reactions*

Suppose that in an industrial batch process a substance A produces the desired compound I which goes on to decay to a worthless product C, each step of the reaction being first-order. At what time will I be present in greatest concentration?

Method The time-dependence of the concentration of I is given by eqn 22.40. We can find the time at which [I] passes through a maximum, t_{max}, by calculating $d[I]/dt$ and setting the resulting rate equal to zero.

Answer It follows from eqn 22.40 that

$$\frac{d[I]}{dt} = -\frac{k_a[A]_0(k_a e^{-k_a t} - k_b e^{-k_b t})}{k_b - k_a}$$

This rate is equal to zero when

$$k_a e^{-k_a t} = k_b e^{-k_b t}$$

Therefore,

$$t_{max} = \frac{1}{k_a - k_b} \ln \frac{k_a}{k_b}$$

For a given value of k_a, as k_b increases both the time at which [I] is a maximum and the yield of I decrease.

Self-test 22.7 Calculate the maximum concentration of I and justify the last remark. $[[I]_{max}/[A]_0 = (k_a/k_b)^c, c = k_b/(k_b - k_a)]$

(b) The steady-state approximation

One feature of the calculation so far has probably not gone unnoticed: there is a considerable increase in mathematical complexity as soon as the reaction mechanism has more than a couple of steps. A reaction scheme involving many steps is nearly always unsolvable analytically, and alternative methods of solution are necessary. One approach is to integrate the rate laws numerically (see *Appendix* 2). An alternative approach, which continues to be widely used because it leads to convenient expressions and more readily digestible results, is to make an approximation.

The **steady-state approximation** (which is also widely called the **quasi-steady-state approximation**, QSSA, to distinguish it from a true steady state) assumes that, after an initial **induction period**, an interval during which the concentrations of intermediates, I, rise from zero, and during the major part of the reaction, the rates of change of concentrations of all reaction intermediates are negligibly small (Fig. 22.14):

$$\frac{d[I]}{dt} \approx 0 \qquad (22.42)$$

This approximation greatly simplifies the discussion of reaction schemes. For example, when we apply the approximation to the consecutive first-order mechanism, we set $d[I]/dt = 0$ in eqn 22.36, which then becomes

$$k_a[A] - k_b[I] \approx 0$$

Then

$$[I] \approx (k_a/k_b)[A] \qquad (22.43)$$

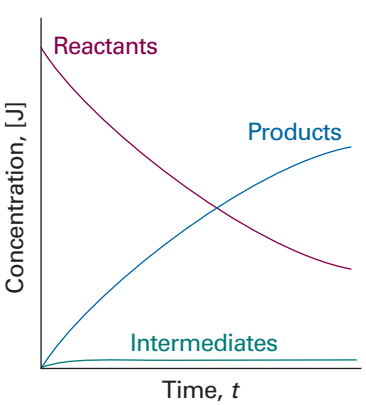

Fig. 22.14 The basis of the steady-state approximation. It is supposed that the concentrations of intermediates remain small and hardly change during most of the course of the reaction.

For this expression to be consistent with eqn 22.42, we require $k_a/k_b \ll 1$ (so that, even though [A] does depend on the time, the dependence of [I] on the time is negligible). On substituting this value of [I] into eqn 22.37, that equation becomes

$$\frac{d[P]}{dt} = k_b[I] \approx k_a[A] \tag{22.44}$$

and we see that P is formed by a first-order decay of A, with a rate constant k_a, the rate-constant of the slower, rate-determining, step. We can write down the solution of this equation at once by substituting the solution for [A], eqn 22.38, and integrating:

$$[P] = k_a[A]_0 \int_0^t e^{-k_a t} dt = (1 - e^{-k_a t})[A]_0 \tag{22.45}$$

This is the same (approximate) result as before, eqn 22.41, but much more quickly obtained. Figure 22.15 compares the approximate solutions found here with the exact solutions found earlier: k_b does not have to be very much bigger than k_a for the approach to be reasonably accurate.

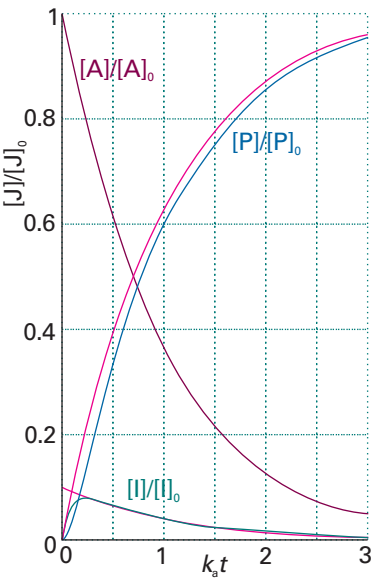

Fig. 22.15 A comparison of the exact result for the concentrations of a consecutive reaction and the concentrations obtained by using the steady-state approximation (red lines) for $k_b = 20k_a$. (The curve for [A] is unchanged.)

Example 22.7 *Using the steady-state approximation*

Devise the rate law for the decomposition of N_2O_5,

$$2\,N_2O_5(g) \rightarrow 4\,NO_2(g) + O_2(g)$$

on the basis of the following mechanism:

$N_2O_5 \rightarrow NO_2 + NO_3$	k_a
$NO_2 + NO_3 \rightarrow N_2O_5$	k_a'
$NO_2 + NO_3 \rightarrow NO_2 + O_2 + NO$	k_b
$NO + N_2O_5 \rightarrow NO_2 + NO_2 + NO_2$	k_c

A note on good practice Note that when writing the equation for an elementary reaction all the species are displayed individually; so we write A → B + B, for instance, not A → 2 B.

Method First identify the intermediates (species that occur in the reaction steps but do not appear in the overall reaction) and write expressions for their net rates of formation. Then, all net rates of change of the concentrations of intermediates are set equal to zero and the resulting equations are solved algebraically.

Answer The intermediates are NO and NO_3; the net rates of change of their concentrations are

$$\frac{d[NO]}{dt} = k_b[NO_2][NO_3] - k_c[NO][N_2O_5] \approx 0$$

$$\frac{d[NO_3]}{dt} = k_a[N_2O_5] - k_a'[NO_2][NO_3] - k_b[NO_2][NO_3] \approx 0$$

The net rate of change of concentration of N_2O_5 is

$$\frac{d[N_2O_5]}{dt} = -k_a[N_2O_5] + k_a'[NO_2][NO_3] - k_c[NO][N_2O_5]$$

and replacing the concentrations of the intermediates by using the equations above gives

$$\frac{d[N_2O_5]}{dt} = -\frac{2k_a k_b[N_2O_5]}{k'_a + k_b}$$

Self-test 22.8 Derive the rate law for the decomposition of ozone in the reaction $2\,O_3(g) \rightarrow 3\,O_2(g)$ on the basis of the (incomplete) mechanism

$$O_3 \rightarrow O_2 + O \qquad k_a$$
$$O_2 + O \rightarrow O_3 \qquad k'_a$$
$$O + O_3 \rightarrow O_2 + O_2 \qquad k_b$$

$[d[O_3]/dt = -k_a k_b[O_3]^2/(k'_a[O_2] + k_b[O_3])]$

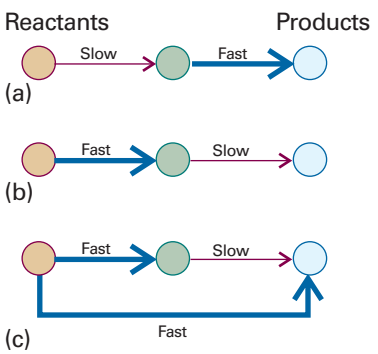

Fig. 22.16 In these diagrams of reaction schemes, heavy arrows represent fast steps and light arrows represent slow steps. (a) The first step is rate-determining; (b) the second step is rate-determining; (c) although one step is slow, it is not rate-determining because there is a fast route that circumvents it.

(c) The rate-determining step

Equation 22.45 shows that, when $k_b \gg k_a$, then the formation of the final product P depends on only the *smaller* of the two rate constants. That is, the rate of formation of P depends on the rate at which I is formed, not on the rate at which I changes into P. For this reason, the step $A \rightarrow I$ is called the 'rate-determining step' of the reaction. Its existence has been likened to building a six-lane highway up to a single-lane bridge: the traffic flow is governed by the rate of crossing the bridge. Similar remarks apply to more complicated reaction mechanisms, and in general the **rate-determining step** is the slowest step in a mechanism and controls the overall rate of the reaction. However, the rate-determining step is not just the slowest step: it must be slow *and* be a crucial gateway for the formation of products. If a faster reaction can also lead to products, then the slowest step is irrelevant because the slow reaction can then be sidestepped (Fig. 22.16).

The rate law of a reaction that has a rate-determining step can often be written down almost by inspection. If the first step in a mechanism is rate-determining, then the rate of the overall reaction is equal to the rate of the first step because all subsequent steps are so fast that once the first intermediate is formed it results immediately in the formation of products. Figure 22.17 shows the reaction profile for a mechanism of this kind in which the slowest step is the one with the highest activation energy. Once over the initial barrier, the intermediates cascade into products. However, a rate-determining step may also stem from the low concentration of a crucial reactant and need not correspond to the step with highest activation barrier.

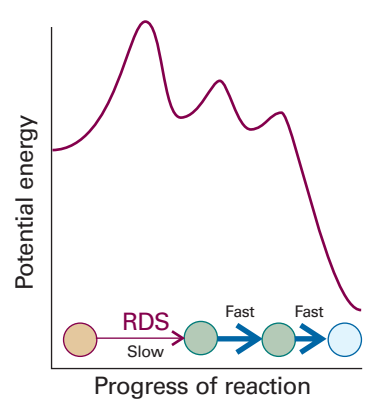

Fig. 22.17 The reaction profile for a mechanism in which the first step (RDS) is rate-determining.

(d) Kinetic and thermodynamic control of reactions

In some cases reactants can give rise to a variety of products, as in nitrations of mono-substituted benzene, when various proportions of the *ortho*-, *meta*-, and *para*-substituted products are obtained, depending on the directing power of the original substituent. Suppose two products, P_1 and P_2, are produced by the following competing reactions:

$$A + B \rightarrow P_1 \qquad \textit{Rate of formation of } P_1 = k_1[A][B]$$
$$A + B \rightarrow P_2 \qquad \textit{Rate of formation of } P_2 = k_2[A][B]$$

The relative proportion in which the two products have been produced at a given stage of the reaction (before it has reached equilibrium) is given by the ratio of the two rates, and therefore of the two rate constants:

$$\frac{[P_2]}{[P_1]} = \frac{k_2}{k_1} \tag{22.46}$$

This ratio represents the **kinetic control** over the proportions of products, and is a common feature of the reactions encountered in organic chemistry where reactants are chosen that facilitate pathways favouring the formation of a desired product. If a reaction is allowed to reach equilibrium, then the proportion of products is determined by thermodynamic rather than kinetic considerations, and the ratio of concentrations is controlled by considerations of the standard Gibbs energies of all the reactants and products.

(e) Pre-equilibria

From a simple sequence of consecutive reactions we now turn to a slightly more complicated mechanism in which an intermediate I reaches an equilibrium with the reactants A and B:

$$A + B \rightleftharpoons I \rightarrow P \tag{22.47}$$

The rate constants are k_a and k'_a for the forward and reverse reactions of the equilibrium and k_b for the final step. This scheme involves a **pre-equilibrium**, in which an intermediate is in equilibrium with the reactants. A pre-equilibrium can arise when the rate of decay of the intermediate back into reactants is much faster than the rate at which it forms products; thus, the condition is possible when $k'_a \gg k_b$ but not when $k_b \gg k'_a$. Because we assume that A, B, and I are in equilibrium, we can write

$$K = \frac{[I]}{[A][B]} \qquad K = \frac{k_a}{k'_a} \tag{22.48}$$

In writing these equations, we are presuming that the rate of reaction of I to form P is too slow to affect the maintenance of the pre-equilibrium (see the example below). The rate of formation of P may now be written:

$$\frac{d[P]}{dt} = k_b[I] = k_b K[A][B] \tag{22.49}$$

This rate law has the form of a second-order rate law with a composite rate constant:

$$\frac{d[P]}{dt} = k[A][B] \qquad k = k_b K = \frac{k_a k_b}{k'_a} \tag{22.50}$$

Example 22.8 *Analysing a pre-equilibrium*

Repeat the pre-equilibrium calculation but without ignoring the fact that I is slowly leaking away as it forms P.

Method Begin by writing the net rates of change of the concentrations of the substances and then invoke the steady-state approximation for the intermediate I. Use the resulting expression to obtain the rate of change of the concentration of P.

Answer The net rates of change of P and I are

$$\frac{d[P]}{dt} = k_b[I]$$

$$\frac{d[I]}{dt} = k_a[A][B] - k'_a[I] - k_b[I] \approx 0$$

The second equation solves to

$$[\mathrm{I}] \approx \frac{k_a[\mathrm{A}][\mathrm{B}]}{k'_a + k_b}$$

When we substitute this result into the expression for the rate of formation of P, we obtain

$$\frac{\mathrm{d}[\mathrm{P}]}{\mathrm{d}t} \approx k[\mathrm{A}][\mathrm{B}] \qquad k = \frac{k_a k_b}{k'_a + k_b}$$

This expression reduces to that in eqn 22.50 when the rate constant for the decay of I into products is much smaller than that for its decay into reactants, $k_b \ll k'_a$.

Self-test 22.9 Show that the pre-equilibrium mechanism in which 2 A $\rightleftharpoons$ I (K) followed by I + B $\rightarrow$ P (k_b) results in an overall third-order reaction.

$[\mathrm{d}[\mathrm{P}]/\mathrm{d}t = k_b K[\mathrm{A}]^2[\mathrm{B}]]$

(f) The kinetic isotope effect

The postulation of a plausible mechanism requires careful analysis of many experiments designed to determine the fate of atoms during the formation of products. Observation of the **kinetic isotope effect**, a decrease in the rate of a chemical reaction upon replacement of one atom in a reactant by a heavier isotope, facilitates the identification of bond-breaking events in the rate-determining step. A **primary kinetic isotope effect** is observed when the rate-determining step requires the scission of a bond involving the isotope. A **secondary kinetic isotope effect** is the reduction in reaction rate even though the bond involving the isotope is not broken to form product. In both cases, the effect arises from the change in activation energy that accompanies the replacement of an atom by a heavier isotope on account of changes in the zero-point vibrational energies (Section 13.9).

First, we consider the origin of the primary kinetic isotope effect in a reaction in which the rate-determining step is the scission of a C—H bond. The reaction coordinate corresponds to the stretching of the C—H bond and the potential energy profile is shown in Fig. 22.18. On deuteration, the dominant change is the reduction of the zero-point energy of the bond (because the deuterium atom is heavier). The whole reaction profile is not lowered, however, because the relevant vibration in the activated complex has a very low force constant, so there is little zero-point energy associated with the reaction coordinate in either isotopomeric form of the activated complex.

We assume that, to a good approximation, a change in the activation energy arises only from the change in zero-point energy of the stretching vibration, so

$$E_a(\mathrm{C{-}D}) - E_a(\mathrm{C{-}H}) = N_A\{\tfrac{1}{2}hc\tilde{\nu}(\mathrm{C{-}H}) - \tfrac{1}{2}hc\tilde{\nu}(\mathrm{C{-}D})\} \qquad (22.51)$$

where $\tilde{\nu}$ is the relevant vibrational wavenumber. From Section 13.9, we know that $\tilde{\nu}(\mathrm{C{-}D}) = (\mu_{CH}/\mu_{CD})^{1/2}\tilde{\nu}(\mathrm{C{-}H})$, where μ is the relevant effective mass. It follows that

$$E_a(\mathrm{C{-}D}) - E_a(\mathrm{C{-}H}) = \tfrac{1}{2}N_A hc\tilde{\nu}(\mathrm{C{-}H})\left\{1 - \left(\frac{\mu_{CH}}{\mu_{CD}}\right)^{1/2}\right\} \qquad (22.52)$$

If we assume further that the pre-exponential factor does not change upon deuteration, then the rate constants for the two species should be in the ratio

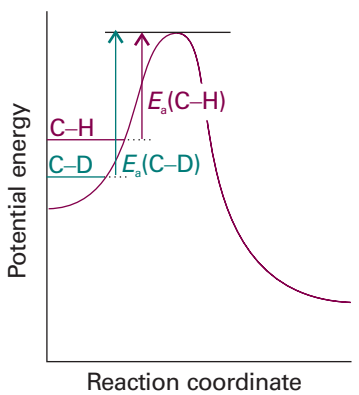

Fig. 22.18 Changes in the reaction profile when a C—H bond undergoing cleavage is deuterated. In this illustration, the C—H and C—D bonds are modelled as simple harmonic oscillators. The only significant change is in the zero-point energy of the reactants, which is lower for C—D than for C—H. As a result, the activation energy is greater for C—D cleavage than for C—H cleavage.

$$\frac{k(\text{C—D})}{k(\text{C—H})} = \mathrm{e}^{-\lambda} \qquad \text{with} \quad \lambda = \frac{hc\tilde{\nu}(\text{C—H})}{2kT}\left\{1 - \left(\frac{\mu_{\text{CH}}}{\mu_{\text{CD}}}\right)^{1/2}\right\} \tag{22.53}$$

Note that $\lambda > 0$ because $\mu_{\text{CD}} > \mu_{\text{CH}}$ and that $k(\text{C—D})/k(\text{C—H})$ decreases with decreasing temperature.

Illustration 22.2 *Assessing the primary kinetic isotope effect*

From infrared spectra, the fundamental vibrational wavenumber for stretching of a C—H bond is about 3000 cm^{-1}. From $\mu_{\text{CH}}/\mu_{\text{CD}} = 0.538$ and eqn 22.53, it follows that $k(\text{C—D})/k(\text{C—H}) = 0.145$ at 298 K. We predict that at room temperature C—H cleavage should be about seven times faster than C—D cleavage, other conditions being equal. However, experimental values of $k(\text{C—D})/k(\text{C—H})$ can differ significantly from those predicted by eqn 22.53 on account of the severity of the assumptions in the model.

In some cases, substitution of deuterium for hydrogen results in values of $k(\text{C—D})/k(\text{C—H})$ that are too low to be accounted for by eqn 22.53, even when more complete models are used to predict ratios of rate constants. Such abnormal kinetic isotope effects are evidence for a path in which quantum mechanical tunnelling of hydrogen atoms takes place through the activation barrier (Fig. 22.19). We saw in Section 9.3 that the probability of tunnelling through a barrier decreases as the mass of the particle increases, so deuterium tunnels less efficiently through a barrier than hydrogen and its reactions are correspondingly slower. Quantum mechanical tunnelling can be the dominant process in reactions involving hydrogen atom or proton transfer when the temperature is so low that very few reactant molecules can overcome the activation energy barrier. We shall see in Chapter 23 that, because m_{e} is so small, tunnelling is also a very important contributor to the rates of electron transfer reactions.

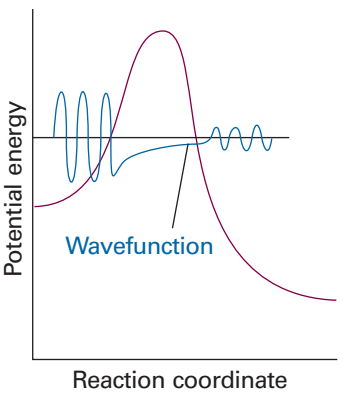

Fig. 22.19 A proton can tunnel through the activation energy barrier that separates reactants from products, so the effective height of the barrier is reduced and the rate of the proton transfer reaction increases. The effect is represented by drawing the wavefunction of the proton near the barrier. Proton tunnelling is important only at low temperatures, when most of the reactants are trapped on the left of the barrier.

Now consider the secondary isotope effect, which arises from differences in the zero-point energies between reactants and an activated complex with a significantly different structure. The activation energy of the undeuterated compound is

$$E_{\text{a}}(\text{H}) = E_{\text{a}} + E^{\ddagger}_{\text{vib},0}(\text{H}) - E_{\text{vib},0}(\text{H})$$

where E_{a} is the difference between the minima of the molecular potential energy curves of the activated complex and the ground state of the reactant and $E^{\ddagger}_{\text{vib},0}(\text{H})$ and $E_{\text{vib},0}(\text{H})$ are the zero-point vibrational energies of the two states (Fig. 22.20). For the deuterated compound

$$E_{\text{a}}(\text{D}) = E_{\text{a}} + E^{\ddagger}_{\text{vib},0}(\text{D}) - E_{\text{vib},0}(\text{D})$$

The difference in activation energies is therefore

$$E_{\text{a}}(\text{D}) - E_{\text{a}}(\text{H}) = \{E^{\ddagger}_{\text{vib},0}(\text{D}) - E_{\text{vib},0}(\text{D})\} - \{E^{\ddagger}_{\text{vib},0}(\text{H}) - E_{\text{vib},0}(\text{H})\}$$

We now suppose that the difference in zero-point energies is due solely to the vibration of a single C—H (or C—D) bond, and so write

$$\begin{aligned} E_{\text{a}}(\text{D}) - E_{\text{a}}(\text{H}) = {} & \tfrac{1}{2}N_{\text{A}}hc\{\tilde{\nu}^{\ddagger}(\text{C—D}) - \tilde{\nu}(\text{C—D})\} \\ & - \tfrac{1}{2}N_{\text{A}}hc\{\tilde{\nu}^{\ddagger}(\text{C—H}) - \tilde{\nu}(\text{C—H})\} \end{aligned} \tag{22.54}$$

where $\tilde{\nu}^{\ddagger}$ and $\tilde{\nu}$ denote vibrational wavenumbers in the activated complex and reactant, respectively. With $\tilde{\nu}^{\ddagger}(\text{C—D}) = (\mu_{\text{CH}}/\mu_{\text{CD}})^{1/2}\tilde{\nu}^{\ddagger}(\text{C—H})$ and $\tilde{\nu}(\text{C—D}) = (\mu_{\text{CH}}/\mu_{\text{CD}})^{1/2}\tilde{\nu}(\text{C—H})$, it follows that

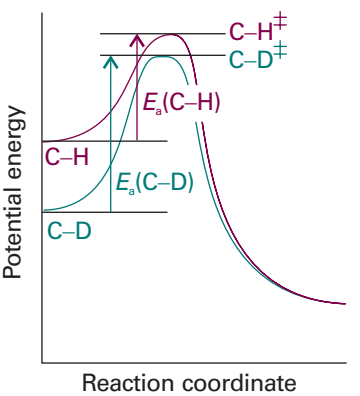

Fig. 22.20 The difference in zero-point vibrational energies used to explain the secondary isotope effect.

$$E_a(D) - E_a(H) = \tfrac{1}{2} N_A hc\{\tilde{\nu}^{\ddagger}(C{-}H) - \tilde{\nu}(C{-}H)\}\left\{\left(\frac{\mu_{CH}}{\mu_{CD}}\right)^{1/2} - 1\right\} \qquad (22.55)$$

and

$$\frac{k(D)}{k(H)} = e^{-\lambda} \quad \text{with} \quad \lambda = \frac{hc\{\tilde{\nu}^{\ddagger}(C{-}H) - \tilde{\nu}(C{-}H)\}}{2kT}\left\{\left(\frac{\mu_{CH}}{\mu_{CD}}\right)^{1/2} - 1\right\} \qquad (22.56)$$

Because $\mu_{CH}/\mu_{CD} < 1$, provided the vibrational wavenumber of the activated complex is less than that of the reactant, $\lambda > 1$ and the deuterated form reacts more slowly than the undeuterated compound.

Illustration 22.3 *Assessing the secondary kinetic isotope effect*

In the heterolytic dissociation $CHCl_3 \rightarrow CHCl_2^+ + Cl^-$ the activated complex resembles the product $CHCl_2^+$. From infrared spectra, the fundamental vibrational wavenumber for a bending motion involving the C—H group is about 1350 cm^{-1} in $CHCl_3$ and about 800 cm^{-1} in $CHCl_2^+$. Assuming that $\tilde{\nu}^{\ddagger}(C{-}H) = 800\ cm^{-1}$ on account of the structural similarity between $CHCl_2^+$ and the activated complex, it follows from $\mu_{CH}/\mu_{CD} = 0.538$ and eqn 22.56 that $k(D)/k(H) = 0.709$ at 298 K. We predict that at room temperature the dissociation of $CHCl_3$ should be about 40 per cent faster than dissociation of $CDCl_3$. Comparison with the result from *Illustration* 22.2 shows that the secondary kinetic isotope effect leads to higher values of $k(D)/k(H)$ than does the primary kinetic isotope effect. This conclusion is supported by a number of experimental observations.

IMPACT ON BIOCHEMISTRY

I22.1 Kinetics of the helix–coil transition in polypeptides

We saw in *Impact* I16.1 that a simple statistical model accounts for the thermodynamic aspects of the helix–coil transition in polypeptides. The unfolding of a helix begins somewhere in the middle of the chain with a nucleation step, which is less favourable than the remaining helix-to-coil conversions, and continues in a cooperative fashion, in which the polymer becomes increasingly more susceptible to structural changes as more conversions occur. Here we examine the kinetics of the helix–coil transition, focusing primarily on experimental strategies and some recent results.

Earlier work on folding and unfolding of small polypeptides and large proteins relied primarily on rapid mixing and stopped-flow techniques. In a typical stopped-flow experiment, a sample of the protein with a high concentration of a chemical denaturant, such as urea or guanidinium hydrochloride, is mixed with a solution containing a much lower concentration of the same denaturant. Upon entering the mixing chamber, the denaturant is diluted and the protein re-folds. Unfolding is observed by mixing a sample of folded protein with a solution containing a high concentration of denaturant. These experiments are ideal for sorting out events in the millisecond timescale, such as the formation of contacts between helical segments in a large protein. However, the available data also indicate that, in a number of proteins, a significant portion of the folding process occurs in less than 1 ms, a time range not accessible by the stopped-flow technique. More recent temperature-jump and flash photolysis experiments have uncovered faster events. For example, at ambient temperature the formation of a loop between helical or sheet segments may be as fast as 1 μs and the formation of tightly packed cores with significant tertiary structure occurs in the

10–100 μs range. Among the fastest events are the formation and denaturation of helices and sheets from fully unfolded peptide chains and we examine how the laser-induced temperature-jump technique has been used in the study of the helix–coil transition.

The laser-induced temperature-jump technique takes advantage of the fact that proteins unfold, or melt, at high temperatures and each protein has a characteristic melting temperature (Section 19.10). Proteins also lose their native structures at very low temperatures, a process known as *cold denaturation*, and re-fold when the temperature is increased but kept significantly below the melting temperature. Hence, a temperature-jump experiment can be configured to monitor either folding or unfolding of a polypeptide, depending on the initial and final temperatures of the sample. The challenge of using melting or cold denaturation as the basis of kinetic measurements lies in increasing the temperature of the sample very quickly so fast relaxation proccess can be monitored. A number of clever strategies have been employed. In one example, a pulsed laser excites dissolved dye molecules that decay largely by internal conversion, or heat transfer to the solution. Another variation makes use of direct excitation of overtones of the O—H or O—D stretching modes of H_2O or D_2O, respectively, with a pulsed infrared laser. The latter strategy leads to temperature jumps in a small irradiated volume of about 20 K in less than 100 ps. Relaxation of the sample can then be probed by a variety of spectroscopic techniques, including absorption, emission, or Raman scattering. For example, the infrared absorption spectrum of a polypeptide is sensitive to polypeptide conformation, as the N—H stretching vibrations in the range 1630–1670 cm^{-1} are significantly different in the helix and coil forms.

Much of the kinetic work on the helix–coil transition has been conducted in small synthetic polypeptides rich in alanine, an aminoacid that is known to stabilize helical structures. Both experimental results and statistical mechanical calculations suggest that the mechanism of unfolding consists of at least two steps: a very fast step in which aminoacids at either end of a helical segment undergo transitions to coil regions and a slower rate-determining step that corresponds to the cooperative melting of the rest of the chain and loss of helical content. Using *h* and *c* to denote an aminoacid residue belonging to a helical and coil region, respectively, the mechanism may be summarized as follows:

hhhh... → *chhh*...	very fast
chhh... → *cccc*...	rate-determining step

The rate-determining step is thought to account for the relaxation time of 160 ns measured with a laser-induced temperature jump between 282.5 K and 300.6 K in an alanine-rich polypeptide containing 21 amino acids. It is thought that the limitation on the rate of the helix–coil transition in this peptide arises from an activation energy barrier of 1.7 kJ mol^{-1} associated with nucleation events of the form ...*hhhh*... → ...*hhch*... in the middle of the chain. Therefore, nucleation is not only thermodynamically unfavourable but also kinetically slow. Models that use concepts of statistical thermodynamics also suggest that a *hhhh*... → *chhh*... transition at either end of a helical segment has a significantly lower activation energy on account of the converting aminoacid not being flanked by *h* regions.

The time constant for the helix–coil transition has also been measured in proteins. In apomyoglobin (myoglobin lacking the haem cofactor), the unfolding of the helices appears to have a relaxation time of about 50 ns, even shorter than in synthetic peptides. It is difficult to interpret these results because we do not yet know how the amino acid sequence or interactions between helices in a folded protein affect the helix–coil relaxation time.

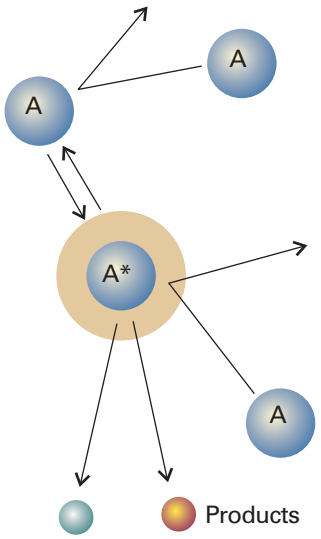

Fig. 22.21 A representation of the Lindemann–Hinshelwood mechanism of unimolecular reactions. The species A is excited by collision with A, and the excited A molecule (A*) may either be deactivated by a collision with A or go on to decay by a unimolecular process to form products.

22.8 Unimolecular reactions

A number of gas-phase reactions follow first-order kinetics, as in the isomerization of cyclopropane mentioned earlier:

$$cyclo\text{-}C_3H_6 \rightarrow CH_3CH{=}CH_2 \qquad v = k[cyclo\text{-}C_3H_6] \tag{22.57}$$

The problem with the interpretation of first-order rate laws is that presumably a molecule acquires enough energy to react as a result of its collisions with other molecules. However, collisions are simple bimolecular events, so how can they result in a first-order rate law? First-order gas-phase reactions are widely called 'unimolecular reactions' because they also involve an elementary unimolecular step in which the reactant molecule changes into the product. This term must be used with caution, though, because the overall mechanism has bimolecular as well as unimolecular steps.

(a) The Lindemann–Hinshelwood mechanism

The first successful explanation of unimolecular reactions was provided by Frederick Lindemann in 1921 and then elaborated by Cyril Hinshelwood. In the **Lindemann–Hinshelwood mechanism** it is supposed that a reactant molecule A becomes energetically excited by collision with another A molecule (Fig. 22.21):

$$A + A \rightarrow A^* + A \qquad \frac{d[A^*]}{dt} = k_a[A]^2 \tag{22.58}$$

The energized molecule (A*) might lose its excess energy by collision with another molecule:

$$A + A^* \rightarrow A + A \qquad \frac{d[A^*]}{dt} = -k'_a[A][A^*] \tag{22.59}$$

Alternatively, the excited molecule might shake itself apart and form products P. That is, it might undergo the unimolecular decay

$$A^* \rightarrow P \qquad \frac{d[A^*]}{dt} = -k_b[A^*] \tag{22.60}$$

If the unimolecular step is slow enough to be the rate-determining step, the overall reaction will have first-order kinetics, as observed. This conclusion can be demonstrated explicitly by applying the steady-state approximation to the net rate of formation of A*:

$$\frac{d[A^*]}{dt} = k_a[A]^2 - k'_a[A][A^*] - k_b[A^*] \approx 0 \tag{22.61}$$

This equation solves to

$$[A^*] = \frac{k_a[A]^2}{k_b + k'_a[A]} \tag{22.62}$$

so the rate law for the formation of P is

$$\frac{d[P]}{dt} = k_b[A^*] = \frac{k_a k_b[A]^2}{k_b + k'_a[A]} \tag{22.63}$$

At this stage the rate law is not first-order. However, if the rate of deactivation by (A*,A) collisions is much greater than the rate of unimolecular decay, in the sense that

$$k'_a[A^*][A] \gg k_b[A^*] \qquad \text{or} \qquad k'_a[A] \gg k_b$$

then we can neglect k_b in the denominator and obtain

$$\frac{d[P]}{dt} = k[A] \qquad k = \frac{k_a k_b}{k'_a} \tag{22.64}$$

Equation 22.64 is a first-order rate law, as we set out to show.

The Lindemann–Hinshelwood mechanism can be tested because it predicts that, as the concentration (and therefore the partial pressure) of A is reduced, the reaction should switch to overall second-order kinetics. Thus, when $k'_a[A] \ll k_b$, the rate law in eqn 22.63 is

$$\frac{d[P]}{dt} \approx k_a[A]^2 \tag{22.65}$$

The physical reason for the change of order is that at low pressures the rate-determining step is the bimolecular formation of A*. If we write the full rate law in eqn 22.63 as

$$\frac{d[P]}{dt} = k[A] \qquad k = \frac{k_a k_b[A]}{k_b + k'_a[A]} \tag{22.66}$$

then the expression for the effective rate constant, k, can be rearranged to

$$\frac{1}{k} = \frac{k'_a}{k_a k_b} + \frac{1}{k_a[A]} \tag{22.67}$$

Hence, a test of the theory is to plot $1/k$ against $1/[A]$, and to expect a straight line.

(b) The RRK model

Whereas the Lindemann–Hinshelwood mechanism agrees in general with the switch in order of unimolecular reactions, it does not agree in detail. Figure 22.22 shows a typical graph of $1/k$ against $1/[A]$. The graph has a pronounced curvature, corresponding to a larger value of k (a smaller value of $1/k$) at high pressures (low $1/[A]$) than would be expected by extrapolation of the reasonably linear low pressure (high $1/[A]$) data.

An improved model was proposed in 1926 by O.K. Rice and H.C. Ramsperger and almost simultaneously by L.S. Kassel, and is now known as the **Rice–Ramsperger–Kassel model** (RRK model). The model has been elaborated, largely by R.A. Marcus, into the RRKM model. Here we outline Kassel's original approach to the RRK model: the details are set out in *Further information* 22.1 at the end of the chapter. The essential feature of the model is that, although a molecule might have enough energy to react, that energy is distributed over all the modes of motion of the molecule, and reaction will occur only when enough of that energy has migrated into a particular location (such as a bond) in the molecule. Provided the rate constant is proportional to this probability, which we show in *Further information* 22.1 is

$$P = \left(1 - \frac{E^*}{E}\right)^{s-1} \tag{22.68a}$$

where s is the number of modes of motion over which the energy may be dissipated and E^* is the energy required for the bond of interest to break, we can write the **Kassel form** of the unimolecular rate constant for the decay of A* to products as

$$k_b(E) = \left(1 - \frac{E^*}{E}\right)^{s-1} k_b \qquad \text{for} \quad E \geq E^* \tag{22.68b}$$

where k_b is the rate constant used in the original Lindemann theory.

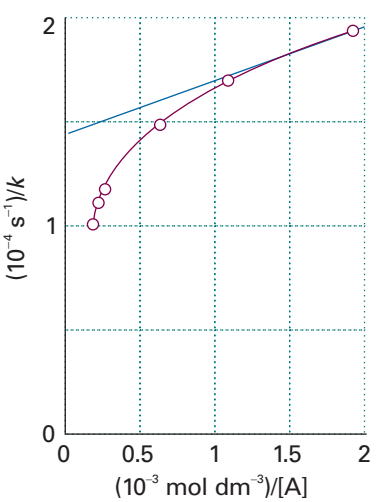

Fig. 22.22 The pressure dependence of the unimolecular isomerization of *trans*-CHD═CHD showing a pronounced departure from the straight line predicted by eqn 22.67 based on the Lindemann–Hinshelwood mechanism.

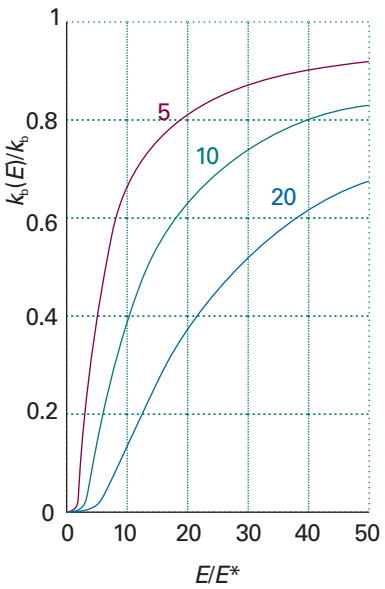

Fig. 22.23 The energy dependence of the rate constant given by eqn 22.68 for three values of *s*.

The energy dependence of the rate constant given by eqn 22.68 is shown in Fig. 22.23 for various values of *s*. We see that the rate constant is smaller at a given excitation energy if *s* is large, as it takes longer for the excitation energy to migrate through all the oscillators of a large molecule and accumulate in the critical mode. As E becomes very large, however, the term in parentheses approaches 1, and $k_b(E)$ becomes independent of the energy and the number of oscillators in the molecule, as there is now enough energy to accumulate immediately in the critical mode regardless of the size of the molecule.

(c) The activation energy of a composite reaction

Although the rate of each step of a complex mechanism might increase with temperature and show Arrhenius behaviour, is that true of a composite reaction? To answer this question, we consider the high-pressure limit of the Lindemann–Hinshelwood mechanism as expressed in eqn 22.64. If each of the rate constants has an Arrhenius-like temperature dependence, we can use eqn 22.31 for each of them, and write

$$k = \frac{k_a k_b}{k'_a} = \frac{(A_a e^{-E_a(a)/RT})(A_b e^{-E_a(b)/RT})}{(A'_a e^{-E'_a(a)/RT})} \tag{22.69}$$

$$= \frac{A_a A_b}{A'_a} e^{-\{E_a(a)+E_a(b)-E'_a(a)\}/RT}$$

That is, the composite rate constant k has an Arrhenius-like form with activation energy

$$E_a = E_a(a) + E_a(b) - E'_a(a) \tag{22.70}$$

Provided $E_a(a) + E_a(b) > E'_a(a)$, the activation energy is positive and the rate increases with temperature. However, it is conceivable that $E_a(a) + E_a(b) < E'_a(a)$ (Fig. 22.24), in which case the activation energy is negative and the rate will *decrease* as the temperature is raised. There is nothing remarkable about this behaviour: all it means is that the reverse reaction (corresponding to the deactivation of A*) is so sensitive to temperature that its rate increases sharply as the temperature is raised, and depletes the steady-state concentration of A*. The Lindemann–Hinshelwood mechanism is an unlikely candidate for this type of behaviour because the deactivation of A* has only a small activation energy, but there are reactions with analogous mechanisms in which a negative activation energy is observed.

When we examine the general rate law given in eqn 22.63, it is clear that the temperature dependence may be difficult to predict because each rate constant in the

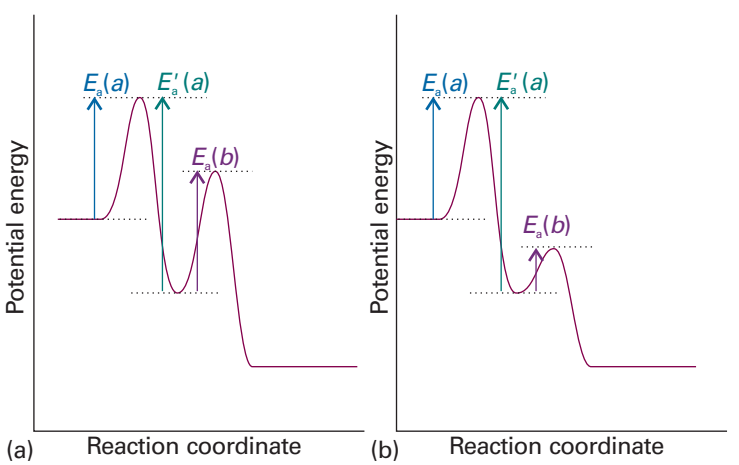

Fig. 22.24 For a reaction with a pre-equilibrium, there are three activation energies to take into account, two referring to the reversible steps of the pre-equilibrium and one for the final step. The relative magnitudes of the activation energies determine whether the overall activation energy is (a) positive or (b) negative.

expression for k increases with temperature, and the outcome depends on whether the terms in the numerator dominate those in the denominator, or vice versa. The fact that so many reactions do show Arrhenius-like behaviour with positive activation energies suggests that their rate laws are in a 'simple' regime, like eqn 22.65 rather than eqn 22.64, and that the temperature dependence is dominated by the activation energy of the rate-determining stage.

Checklist of key ideas

- ☐ 1. The rates of chemical reactions are measured by using techniques that monitor the concentrations of species present in the reaction mixture. Examples include real-time and quenching procedures, flow and stopped-flow techniques, and flash photolysis.
- ☐ 2. The instantaneous rate of a reaction is the slope of the tangent ot the graph of concentration against time (expressed as a positive quantity).
- ☐ 3. A rate law is an expression for the reaction rate in terms of the concentrations of the species that occur in the overall chemical reaction.
- ☐ 4. For a rate law of the form $v = k[A]^a[B]^b \ldots$, the rate constant is k, the order with respect to A is a, and the overall order is $a + b + \ldots$.
- ☐ 5. An integrated rate law is an expression for the concentration of a reactant or product as a function of time (Table 22.3).
- ☐ 6. The half-life $t_{1/2}$ of a reaction is the time it takes for the concentration of a species to fall to half its initial value. The time constant τ is the time required for the concentration of a reactant to fall to 1/e of its initial value. For a first-order reaction, $t_{1/2} = (\ln 2)/k$ and $\tau = 1/k$.
- ☐ 7. The equilibrium constant for a reaction is equal to the ratio of the forward and reverse rate constants, $K = k/k'$.
- ☐ 8. In relaxation methods of kinetic analysis, the equilibrium position of a reaction is first shifted suddenly and then allowed to readjust the equilibrium composition characteristic of the new conditions.
- ☐ 9. The temperature dependence of the rate constant of a reaction typically follows the Arrhenius equation, $\ln k = \ln A - E_a/RT$.
- ☐ 10. The activation energy, the parameter E_a in the Arrhenius equation, is the minimum kinetic energy for reaction during a molecular encounter. The larger the activation energy, the more sensitive the rate constant is to the temperature.
- ☐ 11. The mechanism of reaction is the sequence of elementary steps involved in a reaction.
- ☐ 12. The molecularity of an elementary reaction is the number of molecules coming together to react. An elementary unimolecular reaction has first-order kinetics; an elementary bimolecular reaction has second-order kinetics.
- ☐ 13. The rate-determining step is the slowest step in a reaction mechanism that controls the rate of the overall reaction.
- ☐ 14. In the steady-state approximation, it is assumed that the concentrations of all reaction intermediates remain constant and small throughout the reaction.
- ☐ 15. Provided a reaction has not reached equilibrium, the products of competing reactions are controlled by kinetics, with $[P_2]/[P_1] = k_2/k_1$.
- ☐ 16. Pre-equilibrium is a state in which an intermediate is in equilibrium with the reactants and which arises when the rates of formation of the intermediate and its decay back into reactants are much faster than its rate of formation of products.
- ☐ 17. The kinetic isotope effect is the decrease in the rate of a chemical reaction upon replacement of one atom in a reactant by a heavier isotope. A primary kinetic isotope effect is observed when the rate-determining step requires the scission of a bond involving the isotope. A secondary kinetic isotope effect is the reduction in reaction rate even though the bond involving the isotope is not broken to form product.
- ☐ 18. The Lindemann–Hinshelwood mechanism and the RRKM model of 'unimolecular' reactions account for the first-order kinetics of gas-phase reactions.

Further reading

Articles and texts

J. Andraos, A streamlined approach to solving simple and complex kinetic systems analytically. *J. Chem. Educ.* **76**, 1578 (1999).

C.H. Bamford, C.F. Tipper, and R.G. Compton (ed.), *Comprehensive chemical kinetics*. Vols. 1–38, Elsevier, Amsterdam (1969–2001).

M.N. Berberan-Santos and J.M.G. Martinho, Integration of kinetic rate equations by matrix methods. *J. Chem. Educ.* **67**, 375 (1990).

J.M. Goodman, How do approximations affect the solutions to kinetic equations? *J. Chem. Educ.* **76**, 275 (1999).

J.C. Lindon, G.E. Tranter, and J.L. Holmes (ed.), *Encyclopedia of spectroscopy and spectrometry*. Academic Press, San Diego (2000).

S.R. Logan, *Fundamentals of chemical kinetics*. Longman, Harlow (1996).

M.J. Pilling and P.W. Seakins, *Reaction kinetics*. Oxford University Press (1996).

J.I. Steinfeld, J.S. Francisco, and W.L. Hase, *Chemical kinetics and dynamics*. Prentice Hall, Englewood Cliffs (1998).

Sources of data and information

NDRL/NIST solution kinetics database, NIST standard reference database 40, National Institute of Standards and Technology, Gaithersburg (1994). For the URL, see the web site for this book.

NIST chemical kinetics database, NIST standard reference database 17, National Institute of Standards and Technology, Gaithersburg (1998). For the URL, see the web site for this book.

Further information

Further information 22.1 *The RRK model of unimolecular reactions*

To set up the RRK model, we suppose that a molecule consists of s identical harmonic oscillators, each of which has frequency ν. In practice, of course, the vibrational modes of a molecule have different frequencies, but assuming that they are all the same is a good first approximation. Next, we suppose that the vibrations are excited to a total energy $E = nh\nu$ and then set out to calculate the number of ways N in which the energy can be distributed over the oscillators.

We can represent the n quanta as follows:

□□□

However, because $s-1$ is small (in the sense $s-1 \ll n-n^*$), we can approximate this expression by

$$P \approx \frac{(n-n^*)(n-n^*)\ldots(n-n^*)_{s-1 \text{ factors}}}{(n)(n)\ldots(n)_{s-1 \text{ factors}}} = \left(\frac{n-n^*}{n}\right)^{s-1}$$

Because the energy of the excited molecule is $E = nh\nu$ and the critical energy is $E^* = n^*h\nu$, this expression may be written

$$P = \left(1 - \frac{E^*}{E}\right)^{s-1}$$

as in eqn 22.68a. The dispersal of the energy of the collision reduces the rate constant below its simple 'Lindemann' form, and to obtain the observed rate constant we should multiply the latter by the probability that the energy will in fact be localized in the bond of interest, which gives eqn 22.68b.

Discussion questions

22.1 Consult literature sources and list the observed timescales during which the following processes occur: radiative decay of excited electronic states, molecular rotational motion, molecular vibrational motion, proton transfer reactions, the initial event of vision, energy transfer in photosynthesis, the initial electron transfer events in photosynthesis, the helix-to-coil transition in polypeptides, and collisions in liquids.

22.2 Write a brief report on a recent research article in which at least one of the following techniques was used to study the kinetics of a chemical reaction: stopped-flow techniques, flash photolysis, chemical quench-flow methods, freeze quench methods, temperature-jump methods, or pressure-jump methods. Your report should be similar in content and size to one of the *Impact* sections found throughout this text.

22.3 Describe the main features, including advantages and disadvantages, of the following experimental methods for determining the rate law of a reaction: the isolation method, the method of initial rates, and fitting data to integrated rate law expressions.

22.4 Distinguish between reaction order and molecularity.

22.5 Assess the validity of the following statement: the rate-determining step is the slowest step in a reaction mechanism.

22.6 Distinguish between a pre-equilibrium approximation and a steady-state approximation.

22.7 Distinguish between kinetic and thermodynamic control of a reaction.

22.8 Define the terms in and limit the generality of the expression $\ln k = \ln A - E_a/RT$.

22.9 Distinguish between a primary and a secondary kinetic isotope effect. Discuss how kinetic isotope effects in general can provide insight into the mechanism of a reaction.

22.10 Discuss the limitations of the generality of the expression $k = k_a k_b[A]/(k_b + k'_a[A])$ for the effective rate constant of a unimolecular reaction $A \rightarrow P$ with the following mechanism: $A + A \rightleftharpoons A^* + A$ (k_a, k'_a), $A^* \rightarrow P$ (k_b). Suggest an experimental procedure that may either support or refute the mechanism.

Exercises

22.1a The rate of the reaction $A + 2\,B \rightarrow 3\,C + D$ was reported as 1.0 mol dm^{-3} s^{-1}. State the rates of formation and consumption of the participants.

22.1b The rate of the reaction $A + 3\,B \rightarrow C + 2\,D$ was reported as 1.0 mol dm^{-3} s^{-1}. State the rates of formation and consumption of the participants.

22.2a The rate of formation of C in the reaction $2\,A + B \rightarrow 2\,C + 3\,D$ is 1.0 mol dm^{-3} s^{-1}. State the reaction rate, and the rates of formation or consumption of A, B, and D.

22.2b The rate of consumption of B in the reaction $A + 3\,B \rightarrow C + 2\,D$ is 1.0 mol dm^{-3} s^{-1}. State the reaction rate, and the rates of formation or consumption of A, C, and D.

22.3a The rate law for the reaction in Exercise 22.1a was found to be $v = k[A][B]$. What are the units of k? Express the rate law in terms of the rates of formation and consumption of (a) A, (b) C.

22.3b The rate law for the reaction in Exercise 22.1b was found to be $v = k[A][B]^2$. What are the units of k? Express the rate law in terms of the rates of formation and consumption of (a) A, (b) C.

22.4a The rate law for the reaction in Exercise 22.2a was reported as $d[C]/dt = k[A][B][C]$. Express the rate law in terms of the reaction rate; what are the units for k in each case?

22.4b The rate law for the reaction in Exercise 22.2b was reported as $d[C]/dt = k[A][B][C]^{-1}$. Express the rate law in terms of the reaction rate; what are the units for k in each case?

22.5a At 518°C, the rate of decomposition of a sample of gaseous acetaldehyde, initially at a pressure of 363 Torr, was 1.07 Torr s^{-1} when 5.0 per cent had reacted and 0.76 Torr s^{-1} when 20.0 per cent had reacted. Determine the order of the reaction.

22.5b At 400 K, the rate of decomposition of a gaseous compound initially at a pressure of 12.6 kPa, was 9.71 Pa s^{-1} when 10.0 per cent had reacted and 7.67 Pa s^{-1} when 20.0 per cent had reacted. Determine the order of the reaction.

22.6a At 518°C, the half-life for the decomposition of a sample of gaseous acetaldehyde (ethanal) initially at 363 Torr was 410 s. When the pressure was 169 Torr, the half-life was 880 s. Determine the order of the reaction.

22.6b At 400 K, the half-life for the decomposition of a sample of a gaseous compound initially at 55.5 kPa was 340 s. When the pressure was 28.9 kPa, the half-life was 178 s. Determine the order of the reaction.

22.7a The rate constant for the first-order decomposition of N_2O_5 in the reaction $2\,N_2O_5(g) \rightarrow 4\,NO_2(g) + O_2(g)$ is $k = 3.38 \times 10^{-5}$ s^{-1} at 25°C. What is the half-life of N_2O_5? What will be the pressure, initially 500 Torr, at (a) 10 s, (b) 10 min after initiation of the reaction?

22.7b The rate constant for the first-order decomposition of a compound A in the reaction $2\,A \rightarrow P$ is $k = 2.78 \times 10^{-7}$ s^{-1} at 25°C. What is the half-life of A? What will be the pressure, initially 32.1 kPa, at (a) 10 s, (b) 10 min after initiation of the reaction?

22.8a A second-order reaction of the type A + B → P was carried out in a solution that was initially 0.050 mol dm^{-3} in A and 0.080 mol dm^{-3} in B. After 1.0 h the concentration of A had fallen to 0.020 mol dm^{-3}. (a) Calculate the rate constant. (b) What is the half-life of the reactants?

22.8b A second-order reaction of the type A + 2 B → P was carried out in a solution that was initially 0.075 mol dm^{-3} in A and 0.030 mol dm^{-3} in B. After 1.0 h the concentration of A had fallen to 0.045 mol dm^{-3}. (a) Calculate the rate constant. (b) What is the half-life of the reactants?

22.9a If the rate laws are expressed with (a) concentrations in moles per decimetre cubed, (b) pressures in kilopascals, what are the units of the second-order and third-order rate constants?

22.9b If the rate laws are expressed with (a) concentrations in molecules per metre cubed, (b) pressures in newtons per metre squared, what are the units of the second-order and third-order rate constants?

22.10a The second-order rate constant for the reaction

$$CH_3COOC_2H_5(aq) + OH^-(aq) \rightarrow CH_3CO_2^-(aq) + CH_3CH_2OH(aq)$$

is 0.11 dm^3 mol^{-1} s^{-1}. What is the concentration of ester after (a) 10 s, (b) 10 min when ethyl acetate is added to sodium hydroxide so that the initial concentrations are [NaOH] = 0.050 mol dm^{-3} and [$CH_3COOC_2H_5$] = 0.100 mol dm^{-3}?

22.10b The second-order rate constant for the reaction A + 2 B → C + D is 0.21 dm^3 mol^{-1} s^{-1}. What is the concentration of C after (a) 10 s, (b) 10 min when the reactants are mixed with initial concentrations of [A] = 0.025 mol dm^{-3} and [B] = 0.150 mol dm^{-3}?

22.11a A reaction 2 A → P has a second-order rate law with $k = 3.50 \times 10^{-4}$ dm^3 mol^{-1} s^{-1}. Calculate the time required for the concentration of A to change from 0.260 mol dm^{-3} to 0.011 mol dm^{-3}.

22.11b A reaction 2 A → P has a third-order rate law with $k = 3.50 \times 10^{-4}$ dm^6 mol^{-2} s^{-1}. Calculate the time required for the concentration of A to change from 0.077 mol dm^{-3} to 0.021 mol dm^{-3}.

22.12a Show that $t_{1/2} \propto 1/[A]^{n-1}$ for a reaction that is *n*th-order in A.

22.12b Deduce an expression for the time it takes for the concentration of a substance to fall to one-third its initial value in an *n*th-order reaction.

22.13a The pK_a of NH_4^+ is 9.25 at 25°C. The rate constant at 25°C for the reaction of NH_4^+ and OH^- to form aqueous NH_3 is 4.0×10^{10} dm^3 mol^{-1} s^{-1}. Calculate the rate constant for proton transfer to NH_3. What relaxation time would be observed if a temperature jump were applied to a solution of 0.15 mol dm^{-3} NH_3(aq) at 25°C?

22.13b The equilibrium A ⇌ B + C at 25°C is subjected to a temperature jump that slightly increases the concentrations of B and C. The measured relaxation time is 3.0 μs. The equilibrium constant for the system is 2.0×10^{-16} at 25°C, and the equilibrium concentrations of B and C at 25°C are both 2.0×10^{-4} mol dm^{-3}. Calculate the rate constants for steps (1) and (2).

22.14a The rate constant for the decomposition of a certain substance is 2.80×10^{-3} dm^3 mol^{-1} s^{-1} at 30°C and 1.38×10^{-2} dm^3 mol^{-1} s^{-1} at 50°C. Evaluate the Arrhenius parameters of the reaction.

22.14b The rate constant for the decomposition of a certain substance is 1.70×10^{-2} dm^3 mol^{-1} s^{-1} at 24°C and 2.01×10^{-2} dm^3 mol^{-1} s^{-1} at 37°C. Evaluate the Arrhenius parameters of the reaction.

22.15a The base-catalysed bromination of nitromethane-d_3 in water at room temperature (298 K) proceeds 4.3 times more slowly than the bromination of the undeuterated material. Account for this difference. Use k_f(C—H) = 450 N m^{-1}.

22.15b Predict the order of magnitude of the isotope effect on the relative rates of displacement of (a) ^{1}H and ^{3}H, (b) ^{16}O and ^{18}O. Will raising the temperature enhance the difference? Take k_f(C—H) = 450 N m^{-1}, k_f(C—O) = 1750 N m^{-1}.

22.16a The effective rate constant for a gaseous reaction that has a Lindemann–Hinshelwood mechanism is 2.50×10^{-4} s^{-1} at 1.30 kPa and 2.10×10^{-5} s^{-1} at 12 Pa. Calculate the rate constant for the activation step in the mechanism.

22.16b The effective rate constant for a gaseous reaction that has a Lindemann–Hinshelwood mechanism is 1.7×10^{-3} s^{-1} at 1.09 kPa and 2.2×10^{-4} s^{-1} at 25 Pa. Calculate the rate constant for the activation step in the mechanism.

Problems*

Numerical problems

22.1 The data below apply to the formation of urea from ammonium cyanate, $NH_4CNO \rightarrow NH_2CONH_2$. Initially 22.9 g of ammonium cyanate was dissolved in in enough water to prepare 1.00 dm^3 of solution. Determine the order of the reaction, the rate constant, and the mass of ammonium cyanate left after 300 min.

t/min	0	20.0	50.0	65.0	150
m(urea)/g	0	7.0	12.1	13.8	17.7

22.2 The data below apply to the reaction, $(CH_3)_3CBr + H_2O \rightarrow (CH_3)_3COH + HBr$. Determine the order of the reaction, the rate constant, and the molar concentration of $(CH_3)_3CBr$ after 43.8 h.

t/h	0	3.15	6.20	10.00	18.30	30.80
[$(CH_3)_3CBr$]/(10^{-2} mol dm^{-3})	10.39	8.96	7.76	6.39	3.53	2.07

22.3 The thermal decomposition of an organic nitrile produced the following data:

t/(10^3 s)	0	2.00	4.00	6.00	8.00	10.00	12.00	∞
[nitrile]/(mol dm^{-3})	1.10	0.86	0.67	0.52	0.41	0.32	0.25	0

Determine the order of the reaction and the rate constant.

22.4 The following data have been obtained for the decomposition of N_2O_5(g) at 67°C according to the reaction $2\ N_2O_5(g) \rightarrow 4\ NO_2(g) + O_2(g)$. Determine the order of the reaction, the rate constant, and the half-life. It is not necessary to obtain the result graphically, you may do a calculation using estimates of the rates of change of concentration.

t/min	0	1	2	3	4	5
[N_2O_5]/(mol dm^{-3})	1.000	0.705	0.497	0.349	0.246	0.173

22.5 A first-order decomposition reaction is observed to have the following rate constants at the indicated temperatures. Estimate the activation energy.

k/(10^{-3} s^{-1})	2.46	45.1	576
θ/°C	0	20.0	40.0

* Problems denoted with the symbol ‡ were supplied by Charles Trapp, Carmen Giunta, and Marshall Cady.

22.6 The gas-phase decomposition of acetic acid at 1189 K proceeds by way of two parallel reactions:

(1) $CH_3COOH \rightarrow CH_4 + CO_2$ $\quad k_1 = 3.74\ s^{-1}$

(2) $CH_3COOH \rightarrow H_2C{=}C{=}O + H_2O$ $\quad k_2 = 4.65\ s^{-1}$

What is the maximum percentage yield of the ketene CH_2CO obtainable at this temperature?

22.7 Sucrose is readily hydrolysed to glucose and fructose in acidic solution. The hydrolysis is often monitored by measuring the angle of rotation of plane-polarized light passing through the solution. From the angle of rotation the concentration of sucrose can be determined. An experiment on the hydrolysis of sucrose in 0.50 M HCl(aq) produced the following data:

t/min	0	14	39	60	80	110	140	170	210
[sucrose]/(mol dm^{-3})	0.316	0.300	0.274	0.256	0.238	0.211	0.190	0.170	0.146

Determine the rate constant of the reaction and the half-life of a sucrose molecule. What is the average liftime of a sucrose molecule?

22.8 The composition of a liquid-phase reaction 2 A → B was followed by a spectrophotometric method with the following results:

t/min	0	10	20	30	40	∞
[B]/(mol dm^{-3})	0	0.089	0.153	0.200	0.230	0.312

Determine the order of the reaction and its rate constant.

22.9 The ClO radical decays rapidly by way of the reaction, $2\ ClO \rightarrow Cl_2 + O_2$. The following data have been obtained:

t/(10^{-3} s)	0.12	0.62	0.96	1.60	3.20	4.00	5.75
[ClO]/(10^{-6} mol dm^{-3})	8.49	8.09	7.10	5.79	5.20	4.77	3.95

Determine the rate constant of the reaction and the half-life of a ClO radical.

22.10 Cyclopropane isomerizes into propene when heated to 500°C in the gas phase. The extent of conversion for various initial pressures has been followed by gas chromatography by allowing the reaction to proceed for a time with various initial pressures:

p_0/Torr	200	200	400	400	600	600
t/s	100	200	100	200	100	200
p/Torr	186	173	373	347	559	520

where p_0 is the initial pressure and p is the final pressure of cyclopropane. What are the order and rate constant for the reaction under these conditions?

22.11 The addition of hydrogen halides to alkenes has played a fundamental role in the investigation of organic reaction mechanisms. In one study (M.J. Haugh and D.R. Dalton, *J. Amer. Chem. Soc.* **97**, 5674 (1975)), high pressures of hydrogen chloride (up to 25 atm) and propene (up to 5 atm) were examined over a range of temperatures and the amount of 2-chloropropane formed was determined by NMR. Show that, if the reaction A + B → P proceeds for a short time δt, the concentration of product follows $[P]/[A] = k[A]^{m-1}[B]^n\delta t$ if the reaction is mth-order in A and nth-order in B. In a series of runs the ratio of [chloropropane] to [propene] was independent of [propene] but the ratio of [chloropropane] to [HCl] for constant amounts of propene depended on [HCl]. For $\delta t \approx 100$ h (which is short on the timescale of the reaction) the latter ratio rose from zero to 0.05, 0.03, 0.01 for p(HCl) = 10 atm, 7.5 atm, 5.0 atm, respectively. What are the orders of the reaction with respect to each reactant?

22.12 Use mathematical software or an electronic spreadsheet to examine the time dependence of [I] in the reaction mechanism A → I → P (k_1, k_2). You may either integrate eqn 22.39 numerically (see *Appendix* 2) or use eqn 22.40 directly. In all the following calculations, use $[A]_0 = 1$ mol dm^{-3} and a time range of 0 to 5 s. (a) Plot [I] against t for $k_1 = 10\ s^{-1}$ and $k_2 = 1\ s^{-1}$. (b) Increase the ratio k_2/k_1 steadily by decreasing the value of k_1 and examine the plot of [I] against t at each turn. What approximation about d[I]/dt becomes increasingly valid?

22.13 Show that the following mechanism can account for the rate law of the reaction in Problem 22.11:

$HCl + HCl \rightleftharpoons (HCl)_2$ $\quad K_1$

$HCl + CH_3CH{=}CH_2 \rightleftharpoons$ complex $\quad K_2$

$(HCl)_2 +$ complex $\rightarrow CH_3CHClCH_3 + 2\ HCl$ $\quad k$ (slow)

What further tests could you apply to verify this mechanism?

22.14 Consider the dimerization $2\ A \rightleftharpoons A_2$, with forward rate constant k_a and backward rate constant k_b. (a) Derive the following expression for the relaxation time in terms of the total concentration of protein, $[A]_{tot} = [A] + 2[A_2]$:

$$\frac{1}{\tau^2} = k_b^2 + 8k_ak_b[A]_{tot}$$

(b) Describe the computational procedures that lead to the determination of the rate constants k_a and k_b from measurements of τ for different values of $[A]_{tot}$. (c) Use the data provided below and the procedure you outlined in part (b) to calculate the rate constants k_a and k_b, and the equilibrium constant K for formation of hydrogen-bonded dimers of 2-pyridone:

$[A]_{tot}$/(mol dm^{-3})	0.500	0.352	0.251	0.151	0.101
τ/ns	2.3	2.7	3.3	4.0	5.3

22.15 In the experiments described in Problems 22.11 and 22.13 an inverse temperature dependence of the reaction rate was observed, the overall rate of reaction at 70°C being roughly one-third that at 19°C. Estimate the apparent activation energy and the activation energy of the rate-determining step given that the enthalpies of the two equilibria are both of the order of −14 kJ mol^{-1}.

22.16 The second-order rate constants for the reaction of oxygen atoms with aromatic hydrocarbons have been measured (R. Atkinson and J.N. Pitts, *J. Phys. Chem.* **79**, 295 (1975)). In the reaction with benzene the rate constants are 1.44×10^7 dm^3 mol^{-1} s^{-1} at 300.3 K, 3.03×10^7 dm^3 mol^{-1} s^{-1} at 341.2 K, and 6.9×10^7 dm^3 mol^{-1} s^{-1} at 392.2 K. Find the pre-exponential factor and activation energy of the reaction.

22.17 In Problem 22.10 the isomerization of cyclopropane over a limited pressure range was examined. If the Lindemann mechanism of first-order reactions is to be tested we also need data at low pressures. These have been obtained (H.O. Pritchard, R.G. Sowden, and A.F. Trotman-Dickenson, *Proc. R. Soc.* **A217**, 563 (1953)):

p/Torr	84.1	11.0	2.89	0.569	0.120	0.067
$10^4\ k_{eff}$/s^{-1}	2.98	2.23	1.54	0.857	0.392	0.303

Test the Lindemann theory with these data.

22.18‡ P.W. Seakins, M.J. Pilling, L.T. Niiranen, D. Gutman, and L.N. Krasnoperov (*J. Phys. Chem.* **96**, 9847 (1992)) measured the forward and reverse rate constants for the gas-phase reaction $C_2H_5(g) + HBr(g) \rightarrow C_2H_6(g) + Br(g)$ and used their findings to compute thermodynamic parameters for C_2H_5. The reaction is bimolecular in both directions with Arrhenius parameters $A = 1.0 \times 10^9$ dm^3 mol^{-1} s^{-1}, $E_a = -4.2$ kJ mol^{-1} for the forward reaction and $A' = 1.4 \times 10^{11}$ dm^3 mol^{-1} s^{-1}, $E_a = 53.3$ kJ mol^{-1} for the reverse reaction. Compute $\Delta_f H^\ominus$, $S_m^\ominus$, and $\Delta_f G^\ominus$ of C_2H_5 at 298 K.

22.19 Two products are formed in reactions in which there is kinetic control of the ratio of products. The activation energy for the reaction leading to Product 1 is greater than that leading to Product 2. Will the ratio of product concentrations $[P_1]/[P_2]$ increase or decrease if the temperature is raised?

Theoretical problems

22.20 The reaction mechanism

$A_2 \rightleftharpoons A + A$ $\quad$ (fast)

$A + B \rightarrow P$ $\quad$ (slow)

involves an intermediate A. Deduce the rate law for the reaction.

22.21 The equilibrium A ⇌ B is first-order in both directions. Derive an expression for the concentration of A as a function of time when the initial molar concentrations of A and B are $[A]_0$ and $[B]_0$. What is the final composition of the system?

22.22 Derive an integrated expression for a second-order rate law $v = k[A][B]$ for a reaction of stoichiometry 2 A + 3 B → P.

22.23 Derive the integrated form of a third-order rate law $v = k[A]^2[B]$ in which the stoichiometry is 2 A + B → P and the reactants are initially present in (a) their stoichiometric proportions, (b) with B present initially in twice the amount.

22.24 Set up the rate equations for the reaction mechanism:

$$A \underset{k'_a}{\overset{k_a}{\rightleftharpoons}} B \underset{k'_b}{\overset{k_b}{\rightleftharpoons}} C$$

Show that the mechanism is equivalent to

$$A \underset{k'_{eff}}{\overset{k_{eff}}{\rightleftharpoons}} C$$

under specified circumstances.

22.25 Show that the ratio $t_{1/2}/t_{3/4}$, where $t_{1/2}$ is the half-life and $t_{3/4}$ is the time for the concentration of A to decrease to $\frac{3}{4}$ of its initial value (implying that $t_{3/4} < t_{1/2}$) can be written as a function of n alone, and can therefore be used as a rapid assessment of the order of a reaction.

22.26 Derive an equation for the steady-state rate of the sequence of reactions A ⇌ B ⇌ C ⇌ D, with [A] maintained at a fixed value and the product D removed as soon as it is formed.

22.27‡ For a certain second-order reaction A + B → Products, the rate of reaction, v, may be written

$$v = \frac{dx}{dt} = k([A]_0 - x)([B]_0 + x)$$

where x is the decrease in concentration of A or B as a result of reaction. Find an expression for the maximum rate and the conditions under which it applies. Draw a graph of v against x, and noting that v and x cannot be negative, identify the portion of the curve that corresponds to reality.

22.28 Consider the dimerization A ⇔ A_2 with forward rate constant k_a and backward rate constant k_b. Show that the relaxation time is:

$$\tau = \frac{1}{k_b + 4k_a[A]_{eq}}$$

Applications: to archaeology, biochemistry, and environmental science

22.29 The half-life for the (first-order) radioactive decay of ^{14}C is 5730 y (it emits β rays with an energy of 0.16 MeV). An archaeological sample contained wood that had only 72 per cent of the ^{14}C found in living trees. What is its age?

22.30 One of the hazards of nuclear explosions is the generation of ^{90}Sr and its subsequent incorporation in place of calcium in bones. This nuclide emits β rays of energy 0.55 MeV, and has a half-life of 28.1 y. Suppose 1.00 μg was absorbed by a newly born child. How much will remain after (a) 18 y, (b) 70 y if none is lost metabolically?

22.31 Pharmacokinetics is the study of the rates of absorption and elimination of drugs by organisms. In most cases, elimination is slower than absorption and is a more important determinant of availability of a drug for binding to its target. A drug can be eliminated by many mechanisms, such as metabolism in the liver, intestine, or kidney followed by excretion of breakdown products through urine or faeces. As an example of pharmacokinetic analysis, consider the elimination of beta adrenergic blocking agents (beta blockers), drugs used in the treatment of hypertension. After intravenous administration of a beta blocker, the blood plasma of a patient was analysed for remaining drug and the data are shown below, where c is the drug concentration measured at a time t after the injection.

t/min	30	60	120	150	240	360	480
c/(ng cm^{-3})	699	622	413	292	152	60	24

(a) Is removal of the drug a first- or second-order process? (b) Calculate the rate constant and half-life of the process. *Comment.* An essential aspect of drug development is the optimization of the half-life of elimination, which needs to be long enough to allow the drug to find and act on its target organ but not so long that harmful side-effects become important.

22.32 The absorption and elimination of a drug in the body may be modelled with a mechanism consisting of two consecutive reactions:

A	→	B	→	C
drug at site of administration		drug dispersed in blood		eliminated drug

where the rate constants of absorption (A → B) and elimination are, respectively, k_1 and k_2. (a) Consider a case in which absorption is so fast that it may be regarded as instantaneous so that a dose of A at an initial concentration $[A]_0$ immediately leads to a drug concentration in blood of $[B]_0$. Also, assume that elimination follows first-order kinetics. Show that, after the administration of n equal doses separated by a time interval τ, the peak concentration of drug B in the blood, $[P]_n$, rises beyond the value of $[B]_0$ and eventually reaches a constant, maximum peak value given by

$$[P]_\infty = [B]_0(1 - e^{-k_2\tau})^{-1}$$

where $[P]_n$ is the (peak) concentration of B immediately after administration of the nth dose and $[P]_\infty$ is the value at very large n. Also, write a mathematical expression for the residual concentration of B, $[R]_n$, which we define to be the concentration of drug B immediately before the administration of the $(n + 1)$th dose. $[R]_n$ is always smaller than $[P]_n$ on account of drug elimination during the period τ between drug administrations. Show that $[P]_\infty - [R]_\infty = [B]_0$. (b) Consider a drug for which $k_2 = 0.0289\ h^{-1}$. (i) Calculate the value of τ required to achieve $[P]_\infty/[B]_0 = 10$. Prepare a graph that plots both $[P]_n/[B]_0$ and $[R]_n/[B]_0$ against n. (ii) How many doses must be administered to achieve a $[P]_n$ value that is 75 per cent of the maximum value? What time has passed during the administration of these doses? (iii) What actions can be taken to reduce the variation $[P]_\infty - [R]_\infty$ while maintaining the same value of $[P]_\infty$? (c) Now consider the administration of a single dose $[A]_0$ for which absorption follows first-order kinetics and elimination follows zero-order kinetics. Show that with the initial concentration $[B]_0 = 0$, the concentration of drug in the blood is given by

$$[B] = [A]_0(1 - e^{-k_1 t}) - k_2 t$$

Plot $[B]/[A]_0$ against t for the case $k_1 = 10\ h^{-1}$, $k_2 = 4.0 \times 10^{-3}$ mmol dm^{-3} h^{-1}, and $[A]_0 = 0.1$ mmol dm^{-3}. Comment on the shape of the curve. (d) Using the model from part (c), set $d[B]/dt = 0$ and show that the maximum value of [B] occurs at the time $t_{max} = \frac{1}{k_1}\ln\left(\frac{k_1[A]_0}{k_2}\right)$. Also, show that the maximum concentration of drug in blood is given by $[B]_{max} = [A]_0 - k_2/k_1 - k_2 t_{max}$.

22.33 Consider a mechanism for the helix–coil transition in which nucleation occurs in the middle of the chain:

hhhh . . . ⇌ *hchh* . . .

hchh . . . ⇌ *cccc* . . .

We saw in *Impact* I22.1 that this type of nucleation is relatively slow, so neither step may be rate-determining. (a) Set up the rate equations for this mechanism. (b) Apply the steady-state approximation and show that, under these circumstances, the mechanism is equivalent to *hhhh* . . . ⇔ *cccc* . . . .

(c) Use your knowledge of experimental techniques and your results from parts (a) and (b) to support or refute the following statement: It is very difficult to obtain experimental evidence for intermediates in protein folding by performing simple rate measurements and one must resort to special flow, relaxation, or trapping techniques to detect intermediates directly.

22.34 Propose a set of experiments in which analysis of the line-shapes of NMR transitions (Section 15.7) can be used to monitor fast events in protein folding and unfolding. What are the disadvantages and disadvantages of this NMR method over methods that use electronic and vibrational spectroscopy?

22.35 Consider the following mechanism for renaturation of a double helix from its strands A and B:

$$A + B \rightleftharpoons \text{unstable helix} \qquad \text{(fast)}$$

$$\text{Unstable helix} \rightarrow \text{stable double helix} \qquad \text{(slow)}$$

Derive the rate equation for the formation of the double helix and express the rate constant of the renaturation reaction in terms of the rate constants of the individual steps.

22.36‡ *Prebiotic reactions* are reactions that might have occurred under the conditions prevalent on the Earth before the first living creatures emerged and which can lead to analogues of molecules necessary for life as we now know it. To qualify, a reaction must proceed with favourable rates and equilibria. M.P. Robertson and S.I. Miller (*Science* **268**, 702(1995)) have studied the prebiotic synthesis of 5-substituted uracils, among them 5-hydroxymethyluracil (HMU). Amino acid analogues can be formed from HMU under prebiotic conditions by reaction with various nucleophiles, such as H_2S, HCN, indole, imidazole, etc. For the synthesis of HMU (the uracil analogue of serine) from uracil and formaldehyde (HCHO), the rate of addition is given by $\log\{k/(\text{dm}^3\ \text{mol}^{-1}\ \text{s}^{-1})\} = 11.75 - 5488/(T/\text{K})$ (at pH = 7), and $\log K = -1.36 + 1794/(T/\text{K})$. For this reaction, calculate the rates and equilibrium constants over a range of temperatures corresponding to possible prebiotic conditions, such as 0–50°C, and plot them against temperature. Also, calculate the activation energy and the standard reaction Gibbs energy and enthalpy at 25°C. Prebiotic conditions are not likely to be standard conditions. Speculate about how the actual values of the reaction Gibbs energy and enthalpy might differ from the standard values. Do you expect that the reaction would still be favourable?

22.37‡ Methane is a by-product of a number of natural processes (such as digestion of cellulose in ruminant animals, anaerobic decomposition of organic waste matter) and industrial processes (such as food production and fossil fuel use). Reaction with the hydroxyl radical OH is the main path by which CH_4 is removed from the lower atmosphere. T. Gierczak, R.K. Talukdar, S.C. Herndon, G.L. Vaghjiani, and A.R. Ravishankara (*J. Phys. Chem.* **A 101**, 3125 (1997)) measured the rate constants for the elementary bimolecular gas-phase reaction of methane with the hydroxyl radical over a range of temperatures of importance to atmospheric chemistry. Deduce the Arrhenius parameters A and E_a from the following measurements:

T/K	295	223	218	213	206	200	195
$k/(10^6\ \text{dm}^3\ \text{mol}^{-1}\ \text{s}^{-1})$	3.55	0.494	0.452	0.379	0.295	0.241	0.217

22.38‡ As we saw in Problem 22.37, reaction with the hydroxyl radical OH is the main path by which CH_4, a by-product of many natural and industrial processes, is removed from the lower atmosphere. T. Gierczak, R.K. Talukdar, S.C. Herndon, G.L. Vaghjiani, and A.R. Ravishankara (*J. Phys. Chem.* **A 101**, 3125 (1997)) measured the rate constants for the bimolecular gas-phase reaction $CH_4(g) + OH(g) \rightarrow CH_3(g) + H_2O(g)$ and found $A = 1.13 \times 10^9\ \text{dm}^3\ \text{mol}^{-1}\ \text{s}^{-1}$ and $E_a = 14.1\ \text{kJ mol}^{-1}$ for the Arrhenius parameters. (a) Estimate the rate of consumption of CH_4. Take the average OH concentration to be $1.5 \times 10^{-21}\ \text{mol dm}^{-3}$, that of CH_4 to be $4.0 \times 10^{-8}\ \text{mol dm}^{-3}$, and the temperature to be −10°C. (b) Estimate the global annual mass of CH_4 consumed by this reaction (which is slightly less than the amount introduced to the atmosphere) given an effective volume for the Earth's lower atmosphere of $4 \times 10^{21}\ \text{dm}^3$.

22.39‡ T. Gierczak, R.K. Talukdar, S.C. Herndon, G.L. Vaghjiani, and A.R. Ravishankara (*J. Phys. Chem.* **A 101**, 3125 (1997)) measured the rate constants for the bimolecular gas-phase reaction of methane with the hydroxyl radical in several isotopic variations. From their data, the following Arrhenius parameters can be obtained:

	$A/(\text{dm}^3\ \text{mol}^{-1}\ \text{s}^{-1})$	$E_a/(\text{kJ mol}^{-1})$
$CH_4 + OH \rightarrow CH_3 + H_2O$	1.13×10^9	14.1
$CD_4 + OH \rightarrow CD_3 + DOH$	6.0×10^8	17.5
$CH_4 + OD \rightarrow CH_3 + DOH$	1.01×10^9	13.6

Compute the rate constants at 298 K, and interpret the kinetic isotope effects.

22.40‡ The oxidation of HSO_3^- by O_2 in aqueous solution is a reaction of importance to the processes of acid rain formation and flue gas desulfurization. R.E. Connick, Y.-X. Zhang, S. Lee, R. Adamic, and P. Chieng (*Inorg. Chem.* **34**, 4543 (1995)) report that the reaction $2\ HSO_3^- + O_2 \rightarrow 2\ SO_4^{2-} + 2\ H^+$ follows the rate law $v = k[HSO_3^-]^2[H^+]^2$. Given a pH of 5.6 and an oxygen molar concentration of $2.4 \times 10^{-4}\ \text{mol dm}^{-3}$ (both presumed constant), an initial HSO_3^- molar concentration of $5 \times 10^{-5}\ \text{mol dm}^{-3}$, and a rate constant of $3.6 \times 10^6\ \text{dm}^9\ \text{mol}^{-3}\ \text{s}^{-1}$, what is the initial rate of reaction? How long would it take for HSO_3^- to reach half its initial concentration?