C H A P T E R 42

# Nuclear Physics

## 42.1 DISCOVERING THE NUCLEUS

### Learning Objectives

*After reading this module, you should be able to . . .*

42.1.1 Explain the general arrangement for Rutherford scattering and what was learned from it.

42.1.2 In a Rutherford scattering arrangement, apply the relationship between the projectile's initial kinetic energy and the distance of its closest approach to the target nucleus.

### Key Ideas

- The positive charge of an atom is concentrated in the central nucleus rather than being spread through the volume of the atom. This structure was proposed in 1910 by Ernest Rutherford of England after he conducted experiments with what we now call Rutherford scattering. Alpha particles (positively charged particles consisting of two protons and two neutrons) are directed through a thin metal foil to be scattered by the (positive) nuclei within the atoms.
- The total energy (kinetic energy plus electric potential energy) of the system of alpha particle and target nucleus is conserved as the alpha particle approaches the nucleus.

## What Is Physics?

We now turn to what lies at the center of an atom—the nucleus. For over 100 years, a principal goal of physics has been to work out the quantum physics of nuclei, and, for almost as long, a principal goal of some types of engineering has been to apply that quantum physics with applications ranging from radiation therapy in the war on cancer to detectors of radon gas in basements.

Before we get to such applications and the quantum physics of nuclei, let's first discuss how physicists discovered that an atom has a nucleus. As obvious as that fact is today, it initially came as an incredible surprise.

## Discovering the Nucleus

In the first years of the 20th century, not much was known about the structure of atoms beyond the fact that they contain electrons. The electron had been discovered (by J. J. Thomson) in 1897, and its mass was unknown in those early days. Thus, it was not possible even to say how many negatively charged electrons a given atom contained. Scientists reasoned that because atoms were electrically neutral, they must also contain some positive charge, but nobody knew what form this compensating positive charge took. One popular model was that the positive and negative charges were spread uniformly in a sphere.

In 1911 Ernest Rutherford proposed that the positive charge of the atom is densely concentrated at the center of the atom, forming its **nucleus,** and that, furthermore, the nucleus is responsible for most of the mass of the atom. Rutherford's proposal was no mere conjecture but was based firmly on the results of an experiment suggested by him and carried out by his collaborators, Hans Geiger (of Geiger counter fame) and Ernest Marsden, a 20-year-old student who had not yet earned his bachelor's degree.

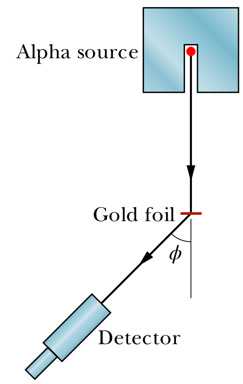


**Figure 42.1.1** An arrangement (top view) used in Rutherford's laboratory in 1911–1913 to study the scattering of $\alpha$ particles by thin metal foils. The detector can be rotated to various values of the scattering angle $\phi$. The alpha source was radon gas, a decay product of radium. With this simple "tabletop" apparatus, the atomic nucleus was discovered.

In Rutherford's day it was known that certain elements, called **radioactive,** transform into other elements spontaneously, emitting particles in the process. One such element is radon, which emits alpha ($\alpha$) particles that have an energy of about 5.5 MeV. We now know that these particles are helium nuclei.

Rutherford's idea was to direct energetic alpha particles at a thin target foil and measure the extent to which they were deflected as they passed through the foil. Alpha particles, which are about 7300 times more massive than electrons, have a charge of $+2e$.

Figure 42.1.1 shows the experimental arrangement of Geiger and Marsden. Their alpha source was a thin-walled glass tube of radon gas. The experiment involves counting the number of alpha particles that are deflected through various scattering angles $\phi$.

Figure 42.1.2 shows their results. Note especially that the vertical scale is logarithmic. We see that most of the particles are scattered through rather small angles, but—and this was the big surprise—a very small fraction of them are scattered through very large angles, approaching 180°. In Rutherford's words: "It was quite the most incredible event that ever happened to me in my life. It was almost as incredible as if you had fired a 15-inch shell at a piece of tissue paper and it [the shell] came back and hit you."

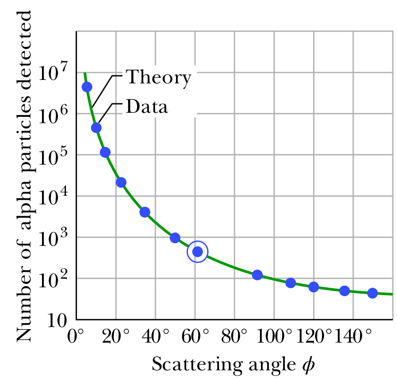


**Figure 42.1.2** The dots are alpha-particle scattering data for a gold foil, obtained by Geiger and Marsden using the apparatus of Fig. 42.1.1. The solid curve is the theoretical prediction, based on the assumption that the atom has a small, massive, positively charged nucleus. The data have been adjusted to fit the theoretical curve at the experimental point that is enclosed in a circle.

Why was Rutherford so surprised? At the time of these experiments, most physicists believed in the so-called plum pudding model of the atom, which had been advanced by J. J. Thomson. In this view the positive charge of the atom was thought to be spread out through the entire volume of the atom. The electrons (the "plums") were thought to vibrate about fixed points within this sphere of positive charge (the "pudding").

The maximum deflecting force that could act on an alpha particle as it passed through such a large positive sphere of charge would be far too small to deflect the alpha particle by even as much as 1°. (The expected deflection has been compared to what you would observe if you fired a bullet through a sack of snowballs.) The electrons in the atom would also have very little effect on the massive, energetic alpha particle. They would, in fact, be themselves strongly deflected, much as a swarm of gnats would be brushed aside by a stone thrown through them.

Rutherford saw that, to deflect the alpha particle backward, there must be a large force; this force could be provided if the positive charge, instead of being spread throughout the atom, were concentrated tightly at its center. Then the incoming alpha particle could get very close to the positive charge without penetrating it; such a close encounter would result in a large deflecting force.

Figure 42.1.3 shows possible paths taken by typical alpha particles as they pass through the atoms of the target foil. As we see, most are either undeflected or only slightly deflected, but a few (those whose incoming paths pass, by chance, very close to a nucleus) are deflected through large angles. From an analysis of the data, Rutherford concluded that the radius of the nucleus must be smaller than the radius of an atom by a factor of about $10^4$. In other words, the atom is mostly empty space.

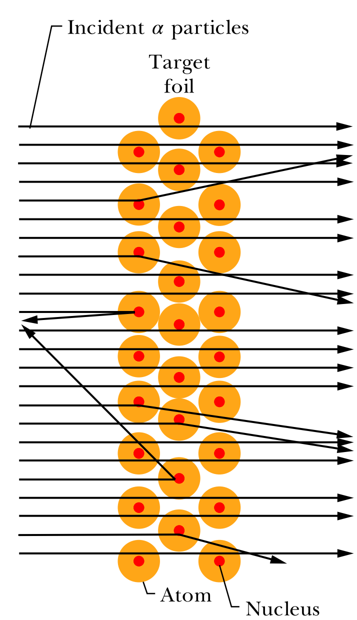


**Figure 42.1.3** The angle through which an incident alpha particle is scattered depends on how close the particle's path lies to an atomic nucleus. Large deflections result only from very close encounters.

## Sample Problem 42.1.1 Rutherford scattering of an alpha particle by a gold nucleus

An alpha particle with kinetic energy $K_i$ = 5.30 MeV happens, by chance, to be headed directly toward the nucleus of a neutral gold atom (Fig. 42.1.4*a*). What is its *distance of closest approach d* (least center-to-center separation) to the nucleus? Assume that the atom remains stationary.

### KEY IDEAS

(1) Throughout the motion, the total mechanical energy $E$ of the particle–atom system is conserved. (2) In addition to the kinetic energy, that total energy includes electric potential energy $U$ as given by Eq. 24.7.4 ($U = q_1q_2/4\pi\varepsilon_0 r$).

***Calculations:*** The alpha particle has a charge of $+2e$ because it contains two protons. The target nucleus has a charge of $q_{Au} = +79e$ because it contains 79 protons. However, that nuclear charge is surrounded by an electron "cloud" with a charge of $q_e = -79e$, and thus the alpha particle initially "sees" a neutral atom with a net charge of $q_{atom} = 0$. The electric force on the particle is zero and the initial electric potential energy of the particle–atom system is $U_i = 0$.

Once the alpha particle enters the atom, we say that it passes through the electron cloud surrounding the nucleus. That cloud then acts as a closed conducting spherical shell and, by Gauss' law, has no effect on the (now internal) charged alpha particle. Then the alpha particle "sees" only the nuclear charge $q_{Au}$. Because $q_\alpha$ and $q_{Au}$ are both positively charged, a repulsive electric force acts on the alpha particle, slowing it, and the particle–atom system has a potential energy

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{Au}}{r}$$

that depends on the center-to-center separation $r$ of the incoming particle and the target nucleus (Fig. 42.1.4*b*).

As the repulsive force slows the alpha particle, energy is transferred from kinetic energy to electric potential energy. The transfer is complete when the alpha particle momentarily stops at the distance of closest approach $d$ to the target nucleus (Fig. 42.1.4*c*). Just then the kinetic energy is $K_f = 0$ and the particle–atom system has the electric potential energy

$$U_f = \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{Au}}{d}.$$

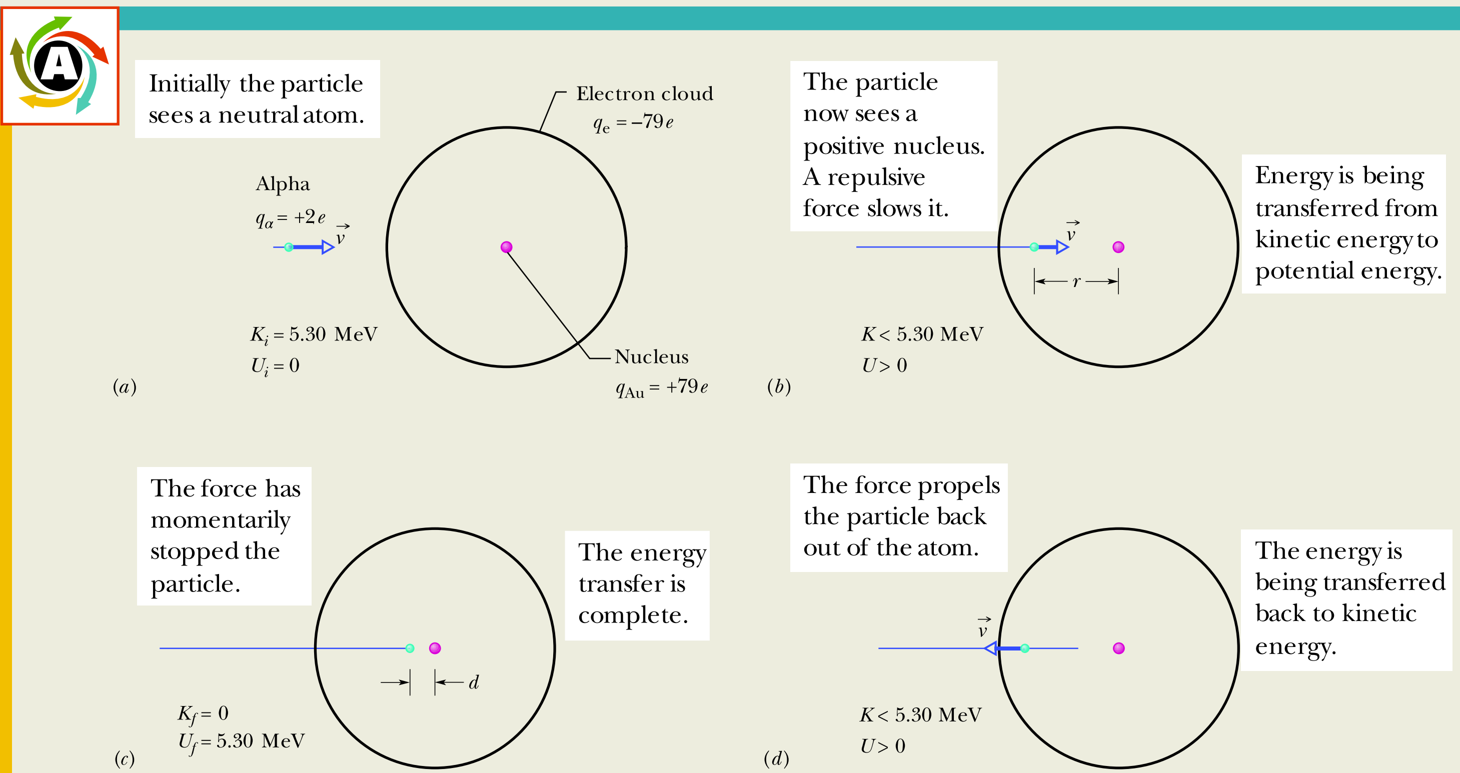


**Figure 42.1.4** An alpha particle (*a*) approaches and (*b*) then enters a gold atom, headed toward the nucleus. The alpha particle (*c*) comes to a stop at the point of closest approach and (*d*) is propelled back out of the atom.

To find $d$, we conserve the total mechanical energy between the initial state $i$ and this later state $f$, writing

$$K_i + U_i = K_f + U_f$$

and

$$K_i + 0 = 0 + \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{\mathrm{Au}}}{d}.$$

(We are assuming that the alpha particle is not affected by the force holding the nucleus together, which acts over only a short distance.) Solving for $d$ and then substituting for the charges and initial kinetic energy lead to

$$\begin{aligned} d &= \frac{(2e)(79e)}{4\pi\varepsilon_0 K_\alpha} \\ &= \frac{(2\times 79)(1.60\times 10^{-19}\ \mathrm{C})^2}{4\pi\varepsilon_0(5.30\ \mathrm{MeV})(1.60\times 10^{-13}\ \mathrm{J/MeV})} \\ &= 4.29\times 10^{-14}\ \mathrm{m}. \qquad \text{(Answer)} \end{aligned}$$

This distance is considerably larger than the sum of the radii of the gold nucleus and the alpha particle. Thus, this alpha particle reverses its motion (Fig. 42.1.4*d*) without ever actually "touching" the gold nucleus.

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# 42.2 SOME NUCLEAR PROPERTIES

## Learning Objectives

*After reading this module, you should be able to . . .*

**42.2.1** Identify nuclides, atomic number (or proton number), neutron number, mass number, nucleon, isotope, disintegration, neutron excess, isobar, zone of stable nuclei, and island of stability, and explain the symbols used for nuclei (such as $^{197}$Au).

**42.2.2** Sketch a graph of proton number versus neutron number and identify the approximate location of the stable nuclei, the proton-rich nuclei, and the neutron-rich nuclei.

**42.2.3** For spherical nuclei, apply the relationship between radius and mass number and calculate the nuclear density.

**42.2.4** Work with masses in atomic mass units, relate the mass number and the approximate nuclear mass, and convert between mass units and energy.

**42.2.5** Calculate mass excess.

**42.2.6** For a given nucleus, calculate the binding energy $\Delta E_{\mathrm{be}}$ and the binding energy per nucleon $\Delta E_{\mathrm{ben}}$, and explain the meaning of each term.

**42.2.7** Sketch a graph of the binding energy per nucleon versus mass number, indicating the nuclei that are the most tightly bound, those that can undergo fission with a release of energy, and those that can undergo fusion with a release of energy.

**42.2.8** Identify the force that holds nucleons together.

## Key Ideas

- Different types of nuclei are called nuclides. Each is characterized by an atomic number $Z$ (the number of protons), a neutron number $N$, and a mass number $A$ (the total number of nucleons—protons and neutrons). Thus, $A = Z + N$. A nuclide is represented with a symbol such as $^{197}$Au or $^{197}_{79}$Au, where the chemical symbol carries a superscript with the value of $A$ and (possibly) a subscript with the value of $Z$.
- Nuclides with the same atomic number but different neutron numbers are isotopes of one another.
- Nuclei have a mean radius $r$ given by

$$r = r_0 A^{1/3}$$

where $r_0 \approx 1.2$ fm.
- Atomic masses are often reported in terms of mass excess

$$\Delta = M - A,$$

where $M$ is the actual mass of an atom in atomic mass units and $A$ is the mass number for that atom's nucleus.
- The binding energy of a nucleus is the difference

$$\Delta E_{\mathrm{be}} = \sum(mc^2) - Mc^2,$$

where $\sum(mc^2)$ is the total mass energy of the individual protons and neutrons. The binding energy of a nucleus is the amount of energy needed to break the nucleus into its constituent parts (and is *not* an energy that resides in the nucleus).
- The binding energy per nucleon is

$$\Delta E_{\mathrm{ben}} = \frac{\Delta E_{\mathrm{be}}}{A}.$$

- The energy equivalent of one mass unit (u) is 931.494 013MeV.
- A plot of the binding energy per nucleon $\Delta E_{\mathrm{ben}}$ versus mass number $A$ shows that middle-mass nuclides are the most stable and that energy can be released both by fission of high-mass nuclei and by fusion of low-mass nuclei.

# Some Nuclear Properties

Table 42.2.1 shows some properties of a few atomic nuclei. When we are interested primarily in their properties as specific nuclear species (rather than as parts of atoms), we call these particles **nuclides.**

## Some Nuclear Terminology

Nuclei are made up of protons and neutrons. The number of protons in a nucleus (called the **atomic number** or **proton number** of the nucleus) is represented by the symbol $Z$; the number of neutrons (the **neutron number**) is represented by the symbol $N$. The total number of neutrons and protons in a nucleus is called its **mass number** $A$; thus

$$A = Z + N. \tag{42.2.1}$$

Neutrons and protons, when considered collectively as members of a nucleus, are called **nucleons.**

We represent nuclides with symbols such as those displayed in the first column of Table 42.2.1. Consider $^{197}$Au, for example. The superscript 197 is the mass number $A$. The chemical symbol Au tells us that this element is gold, whose atomic number is 79. Sometimes the atomic number is explicitly shown as a subscript, as in $^{197}_{79}$Au. From Eq. 42.2.1, the neutron number of this nuclide is the difference between the mass number and the atomic number, namely, 197 – 79, or 118.

Nuclides with the same atomic number $Z$ but different neutron numbers $N$ are called **isotopes** of one another. The element gold has 36 isotopes, ranging from $^{173}$Au to $^{204}$Au. Only one of them ($^{197}$Au) is stable; the remaining 35 are radioactive. Such **radionuclides** undergo **decay** (or **disintegration**) by emitting a particle and thereby transforming to a different nuclide.

## Organizing the Nuclides

The neutral atoms of all isotopes of an element (all with the same $Z$) have the same number of electrons and the same chemical properties, and they fit into the same box in the periodic table of the elements. The *nuclear* properties of the isotopes of a given element, however, are very different from one isotope to another. Thus, the periodic table is of limited use to the nuclear physicist, the nuclear chemist, or the nuclear engineer.

**Table 42.2.1** Some Properties of Selected Nuclides

| Nuclide | $Z$ | $N$ | $A$ | Stability[a] | Mass[b] (u) | Spin[c] | Binding Energy (MeV/nucleon) |
|---|---|---|---|---|---|---|---|
| $^{1}$H | 1 | 0 | 1 | 99.985% | 1.007 825 | $\frac{1}{2}$ | — |
| $^{7}$Li | 3 | 4 | 7 | 92.5% | 7.016 004 | $\frac{3}{2}$ | 5.60 |
| $^{31}$P | 15 | 16 | 31 | 100% | 30.973 762 | $\frac{1}{2}$ | 8.48 |
| $^{84}$Kr | 36 | 48 | 84 | 57.0% | 83.911 507 | 0 | 8.72 |
| $^{120}$Sn | 50 | 70 | 120 | 32.4% | 119.902 197 | 0 | 8.51 |
| $^{157}$Gd | 64 | 93 | 157 | 15.7% | 156.923 957 | $\frac{3}{2}$ | 8.21 |
| $^{197}$Au | 79 | 118 | 197 | 100% | 196.966 552 | $\frac{3}{2}$ | 7.91 |
| $^{227}$Ac | 89 | 138 | 227 | 21.8 y | 227.027 747 | $\frac{3}{2}$ | 7.65 |
| $^{239}$Pu | 94 | 145 | 239 | 24 100 y | 239.052 157 | $\frac{1}{2}$ | 7.56 |

[a] For stable nuclides, the **isotopic abundance** is given; this is the fraction of atoms of this type found in a typical sample of the element. For radioactive nuclides, the half-life is given.

[b] Following standard practice, the reported mass is that of the neutral atom, not that of the bare nucleus.

[c] Spin angular momentum in units of $\hbar$.

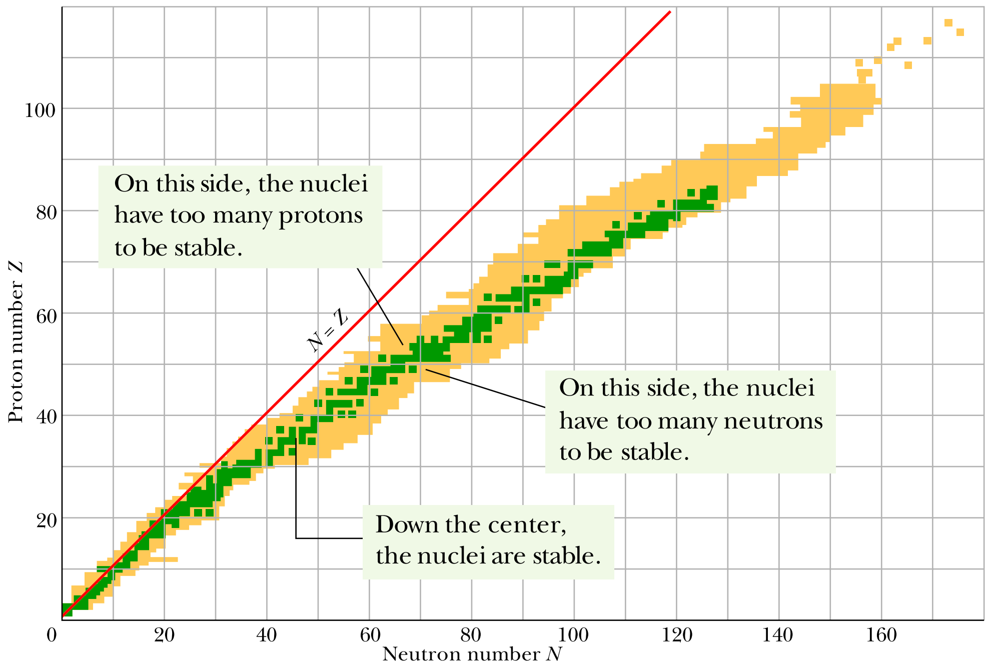


**Figure 42.2.1** A plot of the known nuclides. The green shading identifies the band of stable nuclides, the beige shading the radionuclides. Low-mass, stable nuclides have essentially equal numbers of neutrons and protons, but more massive nuclides have an increasing excess of neutrons. The figure shows that there are no stable nuclides with $Z > 83$ (bismuth).

We organize the nuclides on a **nuclidic chart** like that in Fig. 42.2.1, in which a nuclide is represented by plotting its proton number against its neutron number. The stable nuclides in this figure are represented by the green, the radionuclides by the beige. As you can see, the radionuclides tend to lie on either side of—and at the upper end of—a well-defined band of stable nuclides. Note too that light stable nuclides tend to lie close to the line $N = Z$, which means that they have about the same numbers of neutrons and protons. Heavier nuclides, however, tend to have many more neutrons than protons. As an example, we saw that $^{197}$Au has 118 neutrons and only 79 protons, a *neutron excess* of 39.

Nuclidic charts are available as wall charts, in which each small box on the chart is filled with data about the nuclide it represents. Figure 42.2.2 shows a section of such a chart, centered on $^{197}$Au. Relative abundances (usually, as found on Earth) are shown for stable nuclides, and half-lives (a measure of decay rate) are shown for radionuclides. The sloping line points out a line of **isobars**—nuclides of the same mass number, $A = 198$ in this case.

In recent years, nuclides with atomic numbers as high as $Z = 118$ ($A = 294$) have been found in laboratory experiments (no elements with $Z$ greater than 92 occur naturally). Although large nuclides generally should be highly unstable and last only a very brief time, certain supermassive nuclides are relatively stable, with fairly long lifetimes. These stable supermassive nuclides and other predicted ones form an *island of stability* at high values of $Z$ and $N$ on a nuclidic chart like Fig. 42.2.1.

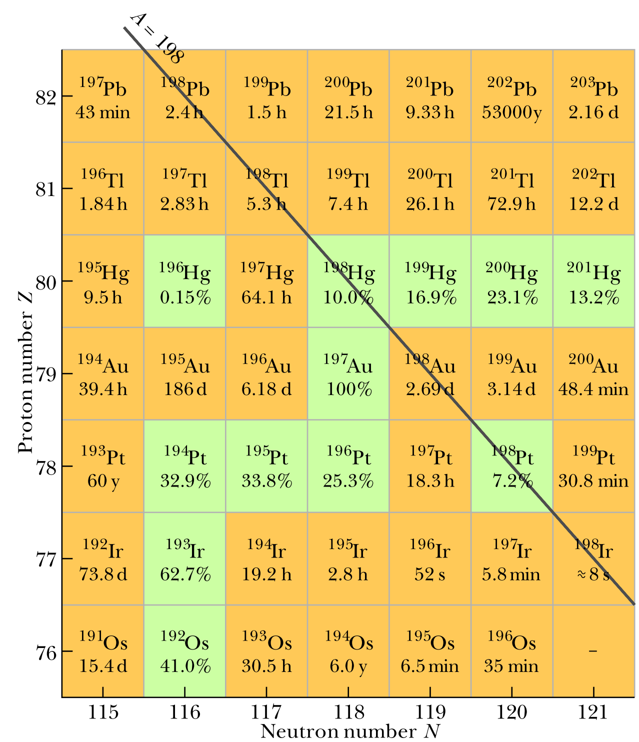


**Figure 42.2.2** An enlarged and detailed section of the nuclidic chart of Fig. 42.2.1, centered on $^{197}$Au. Green squares represent stable nuclides, for which relative isotopic abundances are given. Beige squares represent radionuclides, for which half-lives are given. Isobaric lines of constant mass number $A$ slope as shown by the example line for $A = 198$.

## Checkpoint 42.2.1

Based on Fig. 42.2.1, which of the following nuclides do you conclude are not likely to be detected: $^{52}$Fe ($Z = 26$), $^{90}$As ($Z = 33$), $^{158}$Nd ($Z = 60$), $^{175}$Lu ($Z = 71$), $^{208}$Pb ($Z = 82$)?

## Nuclear Radii

A convenient unit for measuring distances on the scale of nuclei is the *femtometer* ($= 10^{-15}$ m). This unit is often called the *fermi;* the two names share the same abbreviation. Thus,

$$1 \text{ femtometer} = 1 \text{ fermi} = 1 \text{ fm} = 10^{-15} \text{ m}. \tag{42.2.2}$$

We can learn about the size and structure of nuclei by bombarding them with a beam of high-energy electrons and observing how the nuclei deflect the incident electrons. The electrons must be energetic enough (at least 200 MeV) to have de Broglie wavelengths that are smaller than the nuclear structures they are to probe.

The nucleus, like the atom, is not a solid object with a well-defined surface. Furthermore, although most nuclides are spherical, some are notably ellipsoidal. Nevertheless, electron-scattering experiments (as well as experiments of other kinds) allow us to assign to each nuclide an effective radius given by

$$r = r_0 A^{1/3}, \tag{42.2.3}$$

in which $A$ is the mass number and $r_0 \approx 1.2$ fm. We see that the volume of a nucleus, which is proportional to $r^3$, is directly proportional to the mass number $A$ and is independent of the separate values of $Z$ and $N$. That is, we can treat most nuclei as being a sphere with a volume that depends on the number of nucleons, regardless of their type.

Equation 42.2.3 does not apply to *halo nuclides*, which are neutron-rich nuclides that were first produced in laboratories in the 1980s. These nuclides are larger than predicted by Eq. 42.2.3, because some of the neutrons form a *halo* around a spherical core of the protons and the rest of the neutrons. Lithium isotopes give an example. When a neutron is added to $^{8}$Li to form $^{9}$Li, neither of which are halo nuclides, the effective radius increases by about 4%. However, when two neutrons are added to $^{9}$Li to form the neutron-rich isotope $^{11}$Li (the largest of the lithium isotopes), they do not join that existing nucleus but instead form a halo around it, increasing the effective radius by about 30%. Apparently this halo configuration involves less energy than a core containing all 11 nucleons. (In this chapter we shall generally assume that Eq. 42.2.3 applies.)

## Atomic Masses

Atomic masses are now measured to great precision, but usually nuclear masses are not directly measurable because stripping off all the electrons from an atom is difficult. As we briefly discussed in Module 37.6, atomic masses are often reported in *atomic mass units,* a system in which the atomic mass of neutral $^{12}$C is defined to be exactly 12 u.

Precise atomic masses are available in tables on the web and are usually provided in homework problems. However, sometimes we need only an approximation of the mass of either a nucleus alone or a neutral atom. The mass number $A$ of a nuclide gives such an approximate mass in atomic mass units. For example, the approximate mass of both the nucleus and the neutral atom for $^{197}$Au is 197 u, which is close to the actual atomic mass of 196.966 552 u.

As we saw in Module 37.6,

$$1 \text{ u} = 1.660\,538\,86 \times 10^{-27} \text{ kg}. \tag{42.2.4}$$

We also saw that if the total mass of the participants in a nuclear reaction changes by an amount $\Delta m$, there is an energy release or absorption given by Eq. 37.6.11 ($Q = -\Delta m\, c^2$). As we shall now see, nuclear energies are often reported in multiples of 1 MeV. Thus, a convenient conversion between mass units and energy units is provided by Eq. 37.6.7:

$$c^2 = 931.494\,013 \text{ MeV/u}. \tag{42.2.5}$$

Scientists and engineers working with atomic masses often prefer to report the mass of an atom by means of the atom's *mass excess* $\Delta$, defined as

$$\Delta = M - A \quad \text{(mass excess)}, \tag{42.2.6}$$

where $M$ is the actual mass of the atom in atomic mass units and $A$ is the mass number for that atom's nucleus.

## Nuclear Binding Energies

The mass $M$ of a nucleus is *less* than the total mass $\Sigma m$ of its individual protons and neutrons. That means that the mass energy $Mc^2$ of a nucleus is *less* than the total mass energy $\Sigma(mc^2)$ of its individual protons and neutrons. The difference between these two energies is called the **binding energy** of the nucleus:

$$\Delta E_{be} = \Sigma(mc^2) - Mc^2 \quad \text{(binding energy)}. \tag{42.2.7}$$

*Caution:* Binding energy is not an energy that resides in the nucleus. Rather, it is a *difference* in mass energy between a nucleus and its individual nucleons: If we were able to separate a nucleus into its nucleons, we would have to transfer a total energy equal to $\Delta E_{be}$ to those particles during the separating process. Although we cannot actually tear apart a nucleus in this way, the nuclear binding energy is still a convenient measure of how well a nucleus is held together, in the sense that it measures how difficult the nucleus would be to take apart.

A better measure is the **binding energy per nucleon** $\Delta E_{ben}$, which is the ratio of the binding energy $\Delta E_{be}$ of a nucleus to the number $A$ of nucleons in that nucleus:

$$\Delta E_{ben} = \frac{\Delta E_{be}}{A} \quad \text{(binding energy per nucleon)}. \tag{42.2.8}$$

We can think of the binding energy per nucleon as the average energy needed to separate a nucleus into its individual nucleons. *A greater binding energy per nucleon means a more tightly bound nucleus.*

Figure 42.2.3 is a plot of the binding energy per nucleon $\Delta E_{ben}$ versus mass number $A$ for a large number of nuclei. Those high on the plot are very tightly

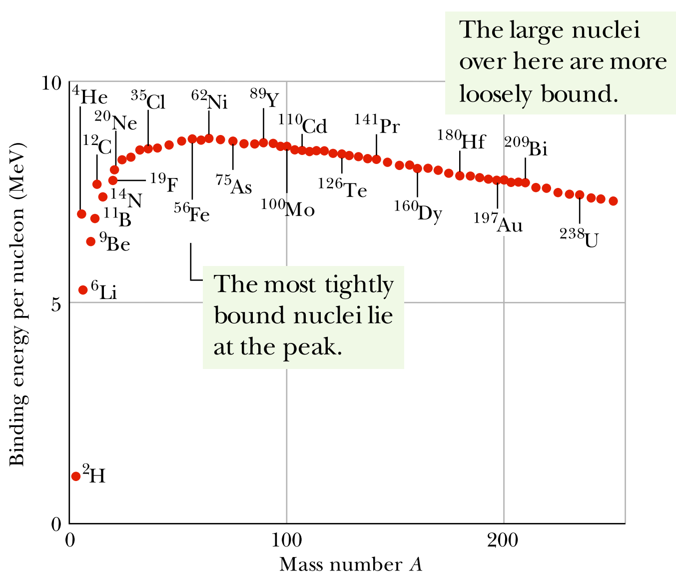


**Figure 42.2.3** The binding energy per nucleon for some representative nuclides. The nickel nuclide $^{62}$Ni has the highest binding energy per nucleon (about 8.794 60 MeV/nucleon) of any known stable nuclide. Note that the alpha particle ($^4$He) has a higher binding energy per nucleon than its neighbors in the periodic table and thus is also particularly stable.

bound; that is, we would have to supply a great amount of energy per nucleon to break apart one of those nuclei. The nuclei that are lower on the plot, at the left and right sides, are less tightly bound, and less energy per nucleon would be required to break them apart.

These simple statements about Fig. 42.2.3 have profound consequences. The nucleons in a nucleus on the right side of the plot would be more tightly bound if that nucleus were to split into two nuclei that lie near the top of the plot. Such a process, called **fission,** occurs naturally with large (high mass number $A$) nuclei such as uranium, which can undergo fission spontaneously (that is, without an external cause or source of energy). The process can also occur in nuclear weapons, in which many uranium or plutonium nuclei are made to fission all at once, to create an explosion.

The nucleons in any pair of nuclei on the left side of the plot would be more tightly bound if the pair were to combine to form a single nucleus that lies near the top of the plot. Such a process, called **fusion,** occurs naturally in stars. Were this not true, the Sun would not shine and thus life could not exist on Earth. As we shall discuss in the next chapter, fusion is also the basis of thermonuclear weapons (with an explosive release of energy) and anticipated power plants (with a sustained and controlled release of energy).

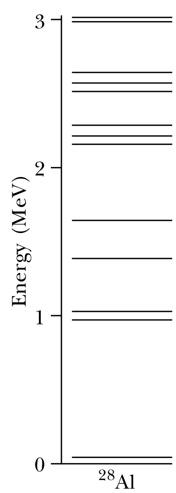


**Figure 42.2.4** Energy levels for the nuclide $^{28}$Al, deduced from nuclear reaction experiments.

## Nuclear Energy Levels

The energy of nuclei, like that of atoms, is quantized. That is, nuclei can exist only in discrete quantum states, each with a well-defined energy. Figure 42.2.4 shows some of these energy levels for $^{28}$Al, a typical low-mass nuclide. Note that the energy scale is in millions of electron-volts, rather than the electron-volts used for atoms. When a nucleus makes a transition from one level to a level of lower energy, the emitted photon is typically in the gamma-ray region of the electromagnetic spectrum.

## Nuclear Spin and Magnetism

Many nuclides have an intrinsic *nuclear angular momentum,* or spin, and an associated intrinsic *nuclear magnetic moment.* Although nuclear angular momenta are roughly of the same magnitude as the angular momenta of atomic electrons, nuclear magnetic moments are much smaller than typical atomic magnetic moments.

## The Nuclear Force

The force that controls the motions of atomic electrons is the familiar electromagnetic force. To bind the nucleus together, however, there must be a strong attractive nuclear force of a totally different kind, strong enough to overcome the repulsive force between the (positively charged) nuclear protons and to bind both protons and neutrons into the tiny nuclear volume. The nuclear force must also be of short range because its influence does not extend very far beyond the nuclear “surface.”

The present view is that the nuclear force that binds neutrons and protons in the nucleus is not a fundamental force of nature but is a secondary, or “spillover,” effect of the **strong force** that binds quarks together to form neutrons and protons. In much the same way, the attractive force between certain neutral molecules is a spillover effect of the Coulomb electric force that acts within each molecule to bind it together.

## Sample Problem 42.2.1 Binding energy per nucleon

What is the binding energy per nucleon for $^{120}$Sn?

### KEY IDEAS

1. We can find the binding energy per nucleon $\Delta E_{ben}$ if we first find the binding energy $\Delta E_{be}$ and then divide by the number of nucleons $A$ in the nucleus, according to Eq. 42.2.8 ($\Delta E_{ben} = \Delta E_{be}/A$).
2. We can find $\Delta E_{be}$ by finding the difference between the mass energy $Mc^2$ of the nucleus and the total mass energy $\Sigma(mc^2)$ of the individual nucleons that make up the nucleus, according to Eq. 42.2.7 ($\Delta E_{be} = \Sigma(mc^2) - Mc^2$).

***Calculations:*** From Table 42.2.1, we see that a $^{120}$Sn nucleus consists of 50 protons ($Z = 50$) and 70 neutrons ($N = A - Z = 120 - 50 = 70$). Thus, we need to imagine a $^{120}$Sn nucleus being separated into its 50 protons and 70 neutrons,

$$(^{120}\text{Sn nucleus}) \to 50\begin{pmatrix}\text{separate}\\\text{protons}\end{pmatrix} + 70\begin{pmatrix}\text{separate}\\\text{neutrons}\end{pmatrix}, \tag{42.2.9}$$

and then compute the resulting change in mass energy.

For that computation, we need the masses of a $^{120}$Sn nucleus, a proton, and a neutron. However, because the mass of a neutral atom (nucleus *plus* electrons) is much easier to measure than the mass of a bare nucleus, calculations of binding energies are traditionally done with atomic masses. Thus, let's modify Eq. 42.2.9 so that it has a neutral $^{120}$Sn atom on the left side. To do that, we include 50 electrons on the left side (to match the 50 protons in the $^{120}$Sn nucleus). We must also add 50 electrons on the right side to balance Eq. 42.2.9. Those 50 electrons can be combined with the 50 protons, to form 50 neutral hydrogen atoms. We then have

$$(^{120}\text{Sn atom}) \to 50\begin{pmatrix}\text{separate}\\\text{H atoms}\end{pmatrix} + 70\begin{pmatrix}\text{separate}\\\text{neutrons}\end{pmatrix}. \tag{42.2.10}$$

From the mass column of Table 42.2.1, the mass $M_{Sn}$ of a $^{120}$Sn atom is 119.902 197 u and the mass $m_H$ of a hydrogen atom is 1.007 825 u; the mass $m_n$ of a neutron is 1.008 665 u. Thus, Eq. 42.2.7 yields

$$\begin{aligned}\Delta E_{be} &= \sum(mc^2) - Mc^2\\ &= 50(m_H c^2) + 70(m_n c^2) - M_{Sn}c^2\\ &= 50(1.007\,825\text{ u})c^2 + 70(1.008\,665\text{ u})c^2\\ &\quad -(119.902\,197\text{u})c^2\\ &= (1.095\,603\text{ u})c^2\\ &= (1.095\,603\text{ u})(931.494\,013\text{ MeV/u})\\ &= 1020.5\text{ MeV},\end{aligned}$$

where Eq. 42.2.5 ($c^2 = 931.494013$ MeV/u) provides an easy unit conversion. Note that using atomic masses instead of nuclear masses does not affect the result because the mass of the 50 electrons in the $^{120}$Sn atom subtracts out from the mass of the electrons in the 50 hydrogen atoms.

Now Eq. 42.2.8 gives us the binding energy per nucleon as

$$\begin{aligned}\Delta E_{ben} &= \frac{\Delta E_{be}}{A} = \frac{1020.5\text{ MeV}}{120}\\ &= 8.50\text{ MeV/nucleon.}\end{aligned} \qquad \text{(Answer)}$$

## Sample Problem 42.2.2 Density of nuclear matter

We can think of all nuclides as made up of a neutron–proton mixture that we can call *nuclear matter.* What is the density of nuclear matter?

### KEY IDEA

We can find the (average) density $\rho$ of a nucleus by dividing its total mass by its volume.

***Calculations:*** Let $m$ represent the mass of a nucleon (either a proton or a neutron, because those particles have about the same mass). Then the mass of a nucleus containing $A$ nucleons is $Am$. Next, we assume the nucleus is spherical with radius $r$. Then its volume is $\frac{4}{3}\pi r^3$, and we can write the density of the nucleus as

$$\rho = \frac{Am}{\frac{4}{3}\pi r^3}.$$

The radius $r$ is given by Eq. 42.2.3 ($r = r_0 A^{1/3}$), where $r_0$ is 1.2 fm ($= 1.2 \times 10^{-15}$ m). Substituting for $r$ then leads to

$$\rho = \frac{Am}{\frac{4}{3}\pi r_0^3 A} = \frac{m}{\frac{4}{3}\pi r_0^3}.$$

Note that $A$ has canceled out; thus, this equation for density $\rho$ applies to any nucleus that can be treated as spherical with a radius given by Eq. 42.2.3. Using $1.67 \times 10^{-27}$ kg for the mass $m$ of a nucleon, we then have

$$\rho = \frac{1.67 \times 10^{-27}\text{ kg}}{\frac{4}{3}\pi(1.2 \times 10^{-15}\text{ m})^3} \approx 2 \times 10^{17}\text{ kg/m}^3. \qquad \text{(Answer)}$$

This is about $2 \times 10^{14}$ times the density of water and is the density of neutron stars, which contain primarily neutrons.

# 42.3 RADIOACTIVE DECAY

## Learning Objectives

*After reading this module, you should be able to . . .*

**42.3.1** Explain what is meant by radioactive decay and identify that it is a random process.

**42.3.2** Identify disintegration constant (or decay constant) $\lambda$.

**42.3.3** Identify that, at any given instant, the rate $dN/dt$ at which radioactive nuclei decay is proportional to the number $N$ of them still present then.

**42.3.4** Apply the relationship that gives the number $N$ of radioactive nuclei as a function of time.

**42.3.5** Apply the relationship that gives the decay rate $R$ of radioactive nuclei as a function of time.

**42.3.6** For any given time, apply the relationship between the decay rate $R$ and the remaining number $N$ of radioactive nuclei.

**42.3.7** Identify activity.

**42.3.8** Distinguish becquerel (Bq), curie (Ci), and counts per unit time.

**42.3.9** Distinguish half-life $T_{1/2}$ and mean life $\tau$.

**42.3.10** Apply the relationship between half-life $T_{1/2}$, mean life $\tau$, and disintegration constant $\lambda$.

**42.3.11** Identify that in any nuclear process, including radioactive decay, the charge and the number of nucleons are conserved.

## Key Ideas

- Most nuclides spontaneously decay at a rate $R = dN/dt$ that is proportional to the number $N$ of radioactive atoms present. The proportionality constant is the disintegration constant $\lambda$.
- The number of radioactive nuclei is given as a function of time by
$$N = N_0 e^{-\lambda t},$$
where $N_0$ is the number at time $t = 0$.
- The rate at which the nuclei decay is given as a function of time by
$$R = R_0 e^{-\lambda t},$$
where $R_0$ is the rate at time $t = 0$.
- The half-life $T_{1/2}$ and the mean life $\tau$ are measures of how quickly radioactive nuclei decay and are related by
$$T_{1/2} = \frac{\ln 2}{\lambda} = \tau \ln 2.$$

## Radioactive Decay

As Fig. 42.2.1 shows, most nuclides are radioactive. They each spontaneously (randomly) emit a particle and transform into a different nuclide. Thus these decays reveal that the laws for subatomic processes are statistical. For example, in a 1 mg sample of uranium metal, with $2.5 \times 10^{18}$ atoms of the very long-lived radionuclide $^{238}$U, only about 12 of the nuclei will decay in a given second by emitting an alpha particle and transforming into a nucleus of $^{234}$Th. However,

> There is absolutely no way to predict whether any given nucleus in a radioactive sample will be among the small number of nuclei that decay during any given second. All have the same chance.

Although we cannot predict which nuclei in a sample will decay, we can say that if a sample contains $N$ radioactive nuclei, then the rate ($= -dN/dt$) at which nuclei will decay is proportional to $N$:

$$-\frac{dN}{dt} = \lambda N, \tag{42.3.1}$$

in which $\lambda$, the **disintegration constant** (or **decay constant**) has a characteristic value for every radionuclide. Its SI unit is the inverse second ($s^{-1}$).

To find $N$ as a function of time $t$, we first rearrange Eq. 42.3.1 as

$$\frac{dN}{N} = -\lambda\, dt, \tag{42.3.2}$$

and then integrate both sides, obtaining

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_{t_0}^{t} dt,$$

or

$$\ln N - \ln N_0 = -\lambda(t - t_0). \tag{42.3.3}$$

Here $N_0$ is the number of radioactive nuclei in the sample at some arbitrary initial time $t_0$. Setting $t_0 = 0$ and rearranging Eq. 42.3.3 give us

$$\ln \frac{N}{N_0} = -\lambda t. \tag{42.3.4}$$

Taking the exponential of both sides (the exponential function is the antifunction of the natural logarithm) leads to

$$\frac{N}{N_0} = e^{-\lambda t},$$

or

$$N = N_0 e^{-\lambda t} \quad \text{(radioactive decay)}, \tag{42.3.5}$$

in which $N_0$ is the number of radioactive nuclei in the sample at $t = 0$ and $N$ is the number remaining at any subsequent time $t$. Note that lightbulbs (for one example) follow no such exponential decay law. If we life-test 1000 bulbs, we expect that they will all "decay" (that is, burn out) at more or less the same time. The decay of radionuclides follows quite a different law.

We are often more interested in the decay rate $R$ ($= -dN/dt$) than in $N$ itself. Differentiating Eq. 42.3.5, we find

$$R = -\frac{dN}{dt} = \lambda N_0 e^{-\lambda t},$$

or

$$R = R_0 e^{-\lambda t} \quad \text{(radioactive decay)}, \tag{42.3.6}$$

an alternative form of the law of radioactive decay (Eq. 42.3.5). Here $R_0$ is the decay rate at time $t = 0$ and $R$ is the rate at any subsequent time $t$. We can now rewrite Eq. 42.3.1 in terms of the decay rate $R$ of the sample as

$$R = \lambda N, \tag{42.3.7}$$

where $R$ and the number of radioactive nuclei $N$ that have not yet undergone decay must be evaluated at the same instant.

The total decay rate $R$ of a sample of one or more radionuclides is called the **activity** of that sample. The SI unit for activity is the **becquerel,** named for Henri Becquerel, the discoverer of radioactivity:

$$1 \text{ becquerel} = 1 \text{ Bq} = 1 \text{ decay per second}.$$

An older unit, the **curie,** is still in common use:

$$1 \text{ curie} = 1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq}.$$

Often a radioactive sample will be placed near a detector that does not record all the disintegrations that occur in the sample. The reading of the detector under these circumstances is proportional to (and smaller than) the true activity of the sample. Such proportional activity measurements are reported not in becquerel units but simply in counts per unit time.

***Lifetimes.*** There are two common time measures of how long any given type of radionuclides lasts. One measure is the **half-life** $T_{1/2}$ of a radionuclide, which is the time at which both $N$ and $R$ have been reduced to one-half their initial values. The other measure is the **mean** (or **average**) **life** $\tau$, which is the time at which both $N$ and $R$ have been reduced to $e^{-1}$ of their initial values.

To relate $T_{1/2}$ to the disintegration constant $\lambda$, we put $R = \frac{1}{2}R_0$ in Eq. 42.3.6 and substitute $T_{1/2}$ for $t$. We obtain

$$\tfrac{1}{2}R_0 = R_0 e^{-\lambda T_{1/2}}.$$

Taking the natural logarithm of both sides and solving for $T_{1/2}$, we find

$$T_{1/2} = \frac{\ln 2}{\lambda}.$$

Similarly, to relate $\tau$ to $\lambda$, we put $R = e^{-1}R_0$ in Eq. 42.3.6, substitute $\tau$ for $t$, and solve for $\tau$, finding

$$\tau = \frac{1}{\lambda}.$$

We summarize these results with the following:

$$T_{1/2} = \frac{\ln 2}{\lambda} = \tau \ln 2. \qquad (42.3.8)$$

### Checkpoint 42.3.1

The nuclide $^{131}I$ is radioactive, with a half-life of 8.04 days. At noon on January 1, the activity of a certain sample is 600 Bq. Using the concept of half-life, without written calculation, determine whether the activity at noon on January 24 will be a little less than 200 Bq, a little more than 200 Bq, a little less than 75 Bq, or a little more than 75 Bq.

## Sample Problem 42.3.1 Finding the disintegration constant and half-life from a graph

The table that follows shows some measurements of the decay rate of a sample of $^{128}I$, a radionuclide often used medically as a tracer to measure the rate at which iodine is absorbed by the thyroid gland.

| Time (min) | $R$ (counts/s) | Time (min) | $R$ (counts/s) |
|---|---|---|---|
| 4 | 392.2 | 132 | 10.9 |
| 36 | 161.4 | 164 | 4.56 |
| 68 | 65.5 | 196 | 1.86 |
| 100 | 26.8 | 218 | 1.00 |

Find the disintegration constant $\lambda$ and the half-life $T_{1/2}$ for this radionuclide.

### KEY IDEAS

The disintegration constant $\lambda$ determines the exponential rate at which the decay rate $R$ decreases with time $t$ (as indicated by Eq. 42.3.6, $R = R_0e^{-\lambda t}$). Therefore, we should be able to determine $\lambda$ by plotting the measurements of $R$ against the measurement times $t$. However, obtaining $\lambda$ from a plot of $R$ versus $t$ is difficult because $R$ decreases exponentially with $t$, according to Eq. 42.3.6. A neat solution is to transform Eq. 42.3.6 into a linear function of $t$, so that we can easily find $\lambda$. To do so, we take the natural logarithms of both sides of Eq. 42.3.6.

***Calculations:*** We obtain

$$\ln R = \ln(R_0e^{-\lambda t}) = \ln R_0 + \ln(e^{-\lambda t})$$
$$= \ln R_0 - \lambda t. \qquad (42.3.9)$$

Because Eq. 42.3.9 is of the form $y = b + mx$, with $b$ and $m$ constants, it is a linear equation giving the quantity $\ln R$ as a function of $t$. Thus, if we plot $\ln R$ (instead of $R$) versus $t$, we should get a straight line. Further, the slope of the line should be equal to $-\lambda$.

Figure 42.3.1 shows a plot of $\ln R$ versus time $t$ for the given measurements. The slope of the straight line that fits through the plotted points is

$$\text{slope} = \frac{0 - 6.2}{225\text{ min} - 0} = -0.0276\text{ min}^{-1}.$$

Thus, $\qquad -\lambda = -0.0276\text{ min}^{-1}$

or $\qquad \lambda = 0.0276\text{ min}^{-1} \approx 1.7\text{ h}^{-1}. \qquad$ (Answer)

The time for the decay rate $R$ to decrease by 1/2 is related to the disintegration constant $\lambda$ via Eq. 42.3.8 ($T_{1/2} = (\ln 2)/\lambda$). From that equation, we find

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{0.0276\text{ min}^{-1}} \approx 25\text{ min}. \qquad \text{(Answer)}$$

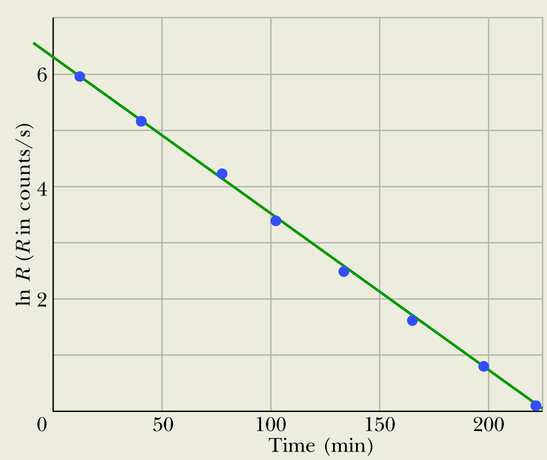


**Figure 42.3.1** A semilogarithmic plot of the decay of a sample of $^{128}I$, based on the data in the table.

### Sample Problem 42.3.2 Radioactivity of the potassium in a banana

Of the 600 mg of potassium in a large banana, 0.0117% is radioactive $^{40}$K, which has a half-life $T_{1/2}$ of $1.25 \times 10^9$ y. What is the activity of the banana?

**KEY IDEAS**

(1) We can relate the activity $R$ to the disintegration constant $\lambda$ with Eq. 42.3.7, but let's write it as $R = \lambda N_{40}$, where $N_{40}$ is the number of $^{40}$K nuclei (and thus atoms) in the banana. (2) We can relate the disintegration constant to the known half-life $T_{1/2}$ with Eq. 42.3.8 ($T_{1/2} = (\ln 2)/\lambda$).

***Calculations:*** Combining Eqs. 42.3.8 and 42.3.7 yields

$$R = \frac{N_{40} \ln 2}{T_{1/2}}. \tag{42.3.10}$$

We know that $N_{40}$ is 0.0117% of the total number $N$ of potassium atoms in the banana. We can find an expression for $N$ by combining two equations that give the number of moles $n$ of potassium in the banana. From Eq. 19.1.2, $n = N/N_A$, where $N_A$ is Avogadro's number ($6.02 \times 10^{23}$ mol$^{-1}$). From Eq. 19.1.3, $n = M_{sam}/M$, where $M_{sam}$ is the sample mass (here the given 600 mg of potassium) and $M$ is the molar mass of potassium. Combining those two equations to eliminate $n$, we can write

$$N_{40} = (1.17 \times 10^{-4}) \frac{M_{sam} N_A}{M}. \tag{42.3.11}$$

From Appendix F, we see that the molar mass of potassium is 39.102 g/mol. Equation 42.3.11 then yields

$$N_{40} = (1.17 \times 10^{-4}) \frac{(600 \times 10^{-3}\text{ g})(6.02 \times 10^{23}\text{ mol}^{-1})}{39.102\text{ g/mol}}$$
$$= 1.081 \times 10^{18}.$$

Substituting this value for $N_{40}$ and the given half-life of $1.25 \times 10^9$ y for $T_{1/2}$ into Eq. 42.3.10 leads to

$$R = \frac{(1.081 \times 10^{18})(\ln 2)}{(1.25 \times 10^9\text{ y})(3.16 \times 10^7\text{ s/y})}$$
$$= 18.96\text{ Bq} \approx 19.0\text{ Bq}. \quad \text{(Answer)}$$

This is about 0.51 nCi. Your body always has about 160 g of potassium. If you repeat our calculation here, you will find that the $^{40}$K component of that everyday amount has an activity of $5.06 \times 10^3$ Bq (or 0.14 $\mu$Ci). So, eating a banana adds less than 1% to the radiation your body receives daily from radioactive potassium.



# 42.4 ALPHA DECAY

## Learning Objectives

*After reading this module, you should be able to . . .*

**42.4.1** Identify alpha particle and alpha decay.
**42.4.2** For a given alpha decay, calculate the mass change and the $Q$ of the reaction.
**42.4.3** Determine the change in atomic number $Z$ and mass number $A$ of a nucleus undergoing alpha decay.
**42.4.4** In terms of the potential barrier, explain how an alpha particle can escape from a nucleus with less energy than the barrier height.

## Key Idea

- Some nuclides decay by emitting an alpha particle (a helium nucleus, $^4$He). Such decay is inhibited by a potential energy barrier that must be penetrated by tunneling.

## Alpha Decay

When a nucleus undergoes **alpha decay,** it transforms to a different nuclide by emitting an alpha particle (a helium nucleus, $^4$He). For example, when uranium $^{238}$U undergoes alpha decay, it transforms to thorium $^{234}$Th:

$$^{238}\text{U} \rightarrow {}^{234}\text{Th} + {}^4\text{He}. \tag{42.4.1}$$

This alpha decay of $^{238}$U can occur spontaneously (without an external source of energy) because the total mass of the decay products $^{234}$Th and $^4$He is less than the mass of the original $^{238}$U. Thus, the total mass energy of the decay products

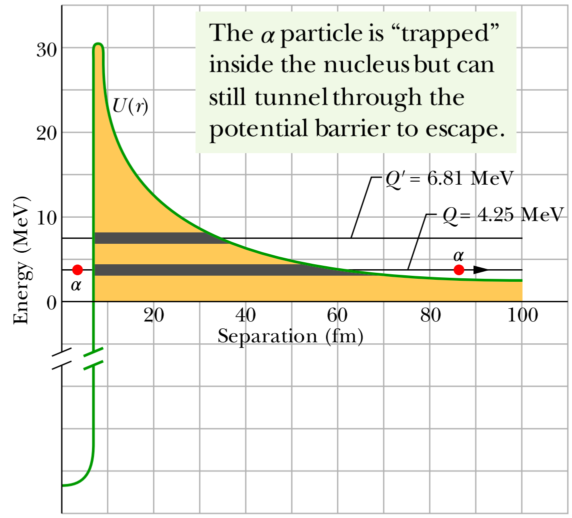


**Figure 42.4.1** A potential energy function for the emission of an alpha particle by $^{238}U$. The horizontal black line marked $Q = 4.25$ MeV shows the disintegration energy for the process. The thick gray portion of this line represents separations $r$ that are classically forbidden to the alpha particle. The alpha particle is represented by a dot, both inside this potential energy barrier (at the left) and outside it (at the right), after the particle has tunneled through. The horizontal black line marked $Q' = 6.81$ MeV shows the disintegration energy for the alpha decay of $^{228}U$. (Both isotopes have the same potential energy function because they have the same nuclear charge.)

is less than the mass energy of the original nuclide. As defined by Eq. 37.6.11 ($Q = -\Delta M\ c^2$), in such a process the difference between the initial mass energy and the total final mass energy is called the $Q$ of the process.

For a nuclear decay, we say that the difference in mass energy is the decay's *disintegration energy Q*. The $Q$ for the decay in Eq. 42.4.1 is 4.25 MeV—that amount of energy is said to be released by the alpha decay of $^{238}U$, with the energy transferred from mass energy to the kinetic energy of the two products.

The half-life of $^{238}U$ for this decay process is $4.5 \times 10^9$ y. Why so long? If $^{238}U$ can decay in this way, why doesn't every $^{238}U$ nuclide in a sample of $^{238}U$ atoms simply decay at once? To answer the questions, we must examine the process of alpha decay.

We choose a model in which the alpha particle is imagined to exist (already formed) inside the nucleus before it escapes from the nucleus. Figure 42.4.1 shows the approximate potential energy $U(r)$ of the system consisting of the alpha particle and the residual $^{234}Th$ nucleus, as a function of their separation $r$. This energy is a combination of (1) the potential energy associated with the (attractive) strong nuclear force that acts in the nuclear interior and (2) a Coulomb potential associated with the (repulsive) electric force that acts between the two particles before and after the decay has occurred.

The horizontal black line marked $Q = 4.25$ MeV shows the disintegration energy for the process. If we assume that this represents the total energy of the alpha particle during the decay process, then the part of the $U(r)$ curve above this line constitutes a potential energy barrier like that in Fig. 38.9.2. This barrier cannot be surmounted. If the alpha particle were able to be at some separation $r$ within the barrier, its potential energy $U$ would exceed its total energy $E$. This would mean, classically, that its kinetic energy $K$ (which equals $E - U$) would be negative, an impossible situation.

***Tunneling.*** We can see now why the alpha particle is not immediately emitted from the $^{238}U$ nucleus. That nucleus is surrounded by an impressive potential barrier, occupying—if you think of it in three dimensions—the volume lying between two spherical shells (of radii about 8 and 60 fm). This argument is so convincing that we now change our last question and ask: Since the particle seems permanently trapped inside the nucleus by the barrier, how can the $^{238}$U nucleus *ever* emit an alpha particle? The answer is that, as you learned in Module 38.9, there is a finite probability that a particle can tunnel through an energy barrier that is classically insurmountable. In fact, alpha decay occurs as a result of barrier tunneling.

**Table 42.4.1** Two Alpha Emitters Compared

| Radionuclide | $Q$ | Half-Life |
|---|---|---|
| $^{238}$U | 4.25 MeV | $4.5 \times 10^9$ y |
| $^{228}$U | 6.81 MeV | 9.1 min |

The very long half-life of $^{238}$U tells us that the barrier is apparently not very "leaky." If we imagine that an already-formed alpha particle is rattling back and forth inside the nucleus, it would arrive at the inner surface of the barrier about $10^{38}$ times before it would succeed in tunneling through the barrier. This is about $10^{21}$ times per second for about $4 \times 10^9$ years (the age of Earth)! We, of course, are waiting on the outside, able to count only the alpha particles that *do* manage to escape without being able to tell what's going on inside the nucleus.

We can test this explanation of alpha decay by examining other alpha emitters. For an extreme contrast, consider the alpha decay of another uranium isotope, $^{228}$U, which has a disintegration energy $Q'$ of 6.81 MeV, about 60% higher than that of $^{238}$U. (The value of $Q'$ is also shown as a horizontal black line in Fig. 42.4.1.) Recall from Module 38.9 that the transmission coefficient of a barrier is very sensitive to small changes in the total energy of the particle seeking to penetrate it. Thus, we expect alpha decay to occur more readily for this nuclide than for $^{238}$U. Indeed it does. As Table 42.4.1 shows, its half-life is only 9.1 min! An increase in $Q$ by a factor of only 1.6 produces a decrease in half-life (that is, in the effectiveness of the barrier) by a factor of $3 \times 10^{14}$. This is sensitivity indeed.

### Sample Problem 42.4.1 $Q$ value in an alpha decay, using masses

We are given the following atomic masses:

| | | | |
|---|---|---|---|
| $^{238}$U | 238.05079 u | $^{4}$He | 4.00260 u |
| $^{234}$Th | 234.04363 u | $^{1}$H | 1.00783 u |
| $^{237}$Pa | 237.05121 u | | |

Here Pa is the symbol for the element protactinium ($Z = 91$).

(a) Calculate the energy released during the alpha decay of $^{238}$U. The decay process is

$$^{238}\text{U} \rightarrow {}^{234}\text{Th} + {}^{4}\text{He}.$$

Note, incidentally, how nuclear charge is conserved in this equation: The atomic numbers of thorium (90) and helium (2) add up to the atomic number of uranium (92). The number of nucleons is also conserved: $238 = 234 + 4$.

**KEY IDEA**

The energy released in the decay is the disintegration energy $Q$, which we can calculate from the change in mass $\Delta M$ due to the $^{238}$U decay.

***Calculations:*** To do this, we use Eq. 37.6.11,

$$Q = M_i c^2 - M_f c^2, \qquad (42.4.2)$$

where the initial mass $M_i$ is that of $^{238}$U and the final mass $M_f$ is the sum of the $^{234}$Th and $^{4}$He masses. Using the atomic masses given in the problem statement, Eq. 42.4.2 becomes

$$\begin{aligned} Q &= (238.05079\ \text{u})c^2 - (234.04363\ \text{u} + 4.00260\ \text{u})c^2 \\ &= (0.00456\ \text{u})c^2 = (0.00456\ \text{u})(931.494013\ \text{MeV/u}) \\ &= 4.25\ \text{MeV}. \qquad \text{(Answer)} \end{aligned}$$

Note that using atomic masses instead of nuclear masses does not affect the result because the total mass of the electrons in the products subtracts out from the mass of the nucleons + electrons in the original $^{238}$U.

(b) Show that $^{238}$U cannot spontaneously emit a proton; that is, protons do not leak out of the nucleus in spite of the proton–proton repulsion within the nucleus.

***Solution:*** If this happened, the decay process would be

$$^{238}\text{U} \rightarrow {}^{237}\text{Pa} + {}^{1}\text{H}.$$

(You should verify that both nuclear charge and the number of nucleons are conserved in this process.) Using the same key ideas in part (a) and proceeding as we did there, we would find that the mass of the two decay products

$$237.05121\ \text{u} + 1.00783\ \text{u}$$

would *exceed* the mass of $^{238}$U by $\Delta m = 0.00825$ u, with disintegration energy

$$Q = -7.68\ \text{MeV}.$$

The minus sign indicates that we must *add* 7.68 MeV to a $^{238}$U nucleus before it will emit a proton; it will certainly not do so spontaneously.

# 42.5 BETA DECAY

## Learning Objectives

*After reading this module, you should be able to . . .*

42.5.1 Identify the two types of beta particles and the two types of beta decay.

42.5.2 Identify neutrino.

42.5.3 Explain why the beta particles in beta decays are emitted with a range of energies.

42.5.4 For a given beta decay, calculate the mass change and the $Q$ of the reaction.

42.5.5 Determine the change in the atomic number $Z$ of a nucleus undergoing a beta decay and identify that the mass number $A$ does not change.

## Key Ideas

- In beta decay, either an electron or a positron is emitted by a nucleus, along with a neutrino.
- The emitted particles share the available disintegration energy. Sometimes the neutrino gets most of the energy and sometimes the electron or positron gets most of it.

## Beta Decay

A nucleus that decays spontaneously by emitting an electron or a positron (a positively charged particle with the mass of an electron) is said to undergo **beta decay.** Like alpha decay, this is a spontaneous process, with a definite disintegration energy and half-life. Again like alpha decay, beta decay is a statistical process, governed by Eqs. 42.3.5 and 42.3.6. In *beta-minus* ($\beta^-$) decay, an electron is emitted by a nucleus, as in the decay

$$^{32}\text{P} \rightarrow {}^{32}\text{S} + \text{e}^- + \nu \quad (T_{1/2} = 14.3\ \text{d}). \tag{42.5.1}$$

In *beta-plus* ($\beta^+$) decay, a positron is emitted by a nucleus, as in the decay

$$^{64}\text{Cu} \rightarrow {}^{64}\text{Ni} + e^+ + \nu \quad (T_{1/2} = 12.7\ \text{h}). \tag{42.5.2}$$

The symbol $\nu$ represents a **neutrino,** a neutral particle that has a very small mass and that is emitted from the nucleus along with the electron or positron during the decay process. Neutrinos interact only very weakly with matter and—for that reason—are so extremely difficult to detect that their presence long went unnoticed.*

Both charge and nucleon number are conserved in the above two processes. In the decay of Eq. 42.5.1, for example, we can write for charge conservation

$$(+15e) = (+16e) + (-e) + (0),$$

because $^{32}$P has 15 protons, $^{32}$S has 16 protons, and the neutrino $\nu$ has zero charge. Similarly, for nucleon conservation, we can write

$$(32) = (32) + (0) + (0),$$

because $^{32}$P and $^{32}$S each have 32 nucleons and neither the electron nor the neutrino is a nucleon.

It may seem surprising that nuclei can emit electrons, positrons, and neutrinos, since we have said that nuclei are made up of neutrons and protons only. However, we saw earlier that atoms emit photons, and we certainly do not say that atoms "contain" photons. We say that the photons are created during the emission process.

*Beta decay also includes *electron capture*, in which a nucleus decays by absorbing one of its atomic electrons, emitting a neutrino in the process. We do not consider that process here. Also, the neutral particle emitted in the decay process of Eq. 42.5.1 is actually an *antineutrino*, a distinction we shall not make in this introductory treatment.

It is the same with the electrons, positrons, and neutrinos emitted from nuclei during beta decay. They are created during the emission process. For beta-minus decay, a neutron transforms into a proton within the nucleus according to

$$n \rightarrow p + e^- + \nu. \tag{42.5.3}$$

For beta-plus decay, a proton transforms into a neutron via

$$p \rightarrow n + e^+ + \nu. \tag{42.5.4}$$

These processes show why the mass number $A$ of a nuclide undergoing beta decay does not change; one of its constituent nucleons simply changes its character according to Eq. 42.5.3 or 42.5.4.

In both alpha decay and beta decay, the same amount of energy is released in every individual decay of a particular radionuclide. In the alpha decay of a particular radionuclide, every emitted alpha particle has the same sharply defined kinetic energy. However, in the beta-minus decay of Eq. 42.5.3 with electron emission, the disintegration energy $Q$ is shared—in varying proportions—between the emitted electron and neutrino. Sometimes the electron gets nearly all the energy, sometimes the neutrino does. In every case, however, the sum of the electron's energy and the neutrino's energy gives the same value $Q$. A similar sharing of energy, with a sum equal to $Q$, occurs in beta-plus decay (Eq. 42.5.4).

Thus, in beta decay the energy of the emitted electrons or positrons may range from near zero up to a certain maximum $K_{max}$. Figure 42.5.1 shows the distribution of positron energies for the beta decay of $^{64}$Cu (see Eq. 42.5.2). The maximum positron energy $K_{max}$ must equal the disintegration energy $Q$ because the neutrino has approximately zero energy when the positron has $K_{max}$:

$$Q = K_{max}. \tag{42.5.5}$$

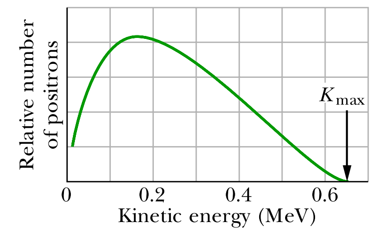


**Figure 42.5.1** The distribution of the kinetic energies of positrons emitted in the beta decay of $^{64}$Cu. The maximum kinetic energy of the distribution ($K_{max}$) is 0.653 MeV. In all $^{64}$Cu decay events, this energy is shared between the positron and the neutrino, in varying proportions. The *most probable* energy for an emitted positron is about 0.15 MeV.

## The Neutrino

Wolfgang Pauli first suggested the existence of neutrinos in 1930. His neutrino hypothesis not only permitted an understanding of the energy distribution of electrons or positrons in beta decay but also solved another early beta-decay puzzle involving "missing" angular momentum.

The neutrino is a truly elusive particle; the mean free path of an energetic neutrino in water has been calculated as no less than several thousand light-years. At the same time, neutrinos left over from the big bang that presumably marked the creation of the universe are the most abundant particles of physics. Billions of them pass through our bodies every second, leaving no trace.

In spite of their elusive character, neutrinos have been detected in the laboratory. This was first done in 1953 by F. Reines and C. L. Cowan, using neutrinos generated in a high-power nuclear reactor. (In 1995, Reines received a Nobel Prize for this work.) In spite of the difficulties of detection, experimental neutrino physics is now a well-developed branch of experimental physics, with avid practitioners at laboratories throughout the world.

The Sun emits neutrinos copiously from the nuclear furnace at its core, and at night these messengers from the center of the Sun come up at us from below, Earth being almost totally transparent to them. In February 1987, light from an exploding star in the Large Magellanic Cloud (a nearby galaxy) reached Earth after having traveled for 170 000 years. Enormous numbers of neutrinos were generated in this explosion, and about 10 of them were picked up by a sensitive neutrino detector in Japan; Fig. 42.5.2 shows a record of their passage.

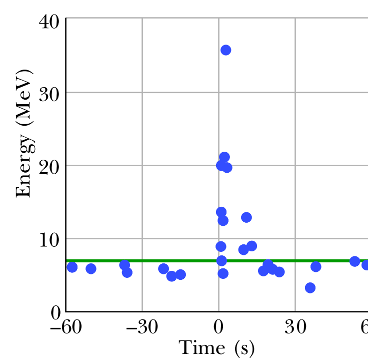


**Figure 42.5.2** A burst of neutrinos from the supernova SN 1987A, which occurred at (relative) time 0, stands out from the usual *background* of neutrinos. (For neutrinos, 10 is a "burst.") The particles were detected by an elaborate detector housed deep in a mine in Japan. The supernova was visible only in the Southern Hemisphere; so the neutrinos had to penetrate Earth (a trifling barrier for them) to reach the detector.

## Radioactivity and the Nuclidic Chart

We can increase the amount of information obtainable from the nuclidic chart of Fig. 42.2.1 by including a third axis showing the mass excess Δ expressed in

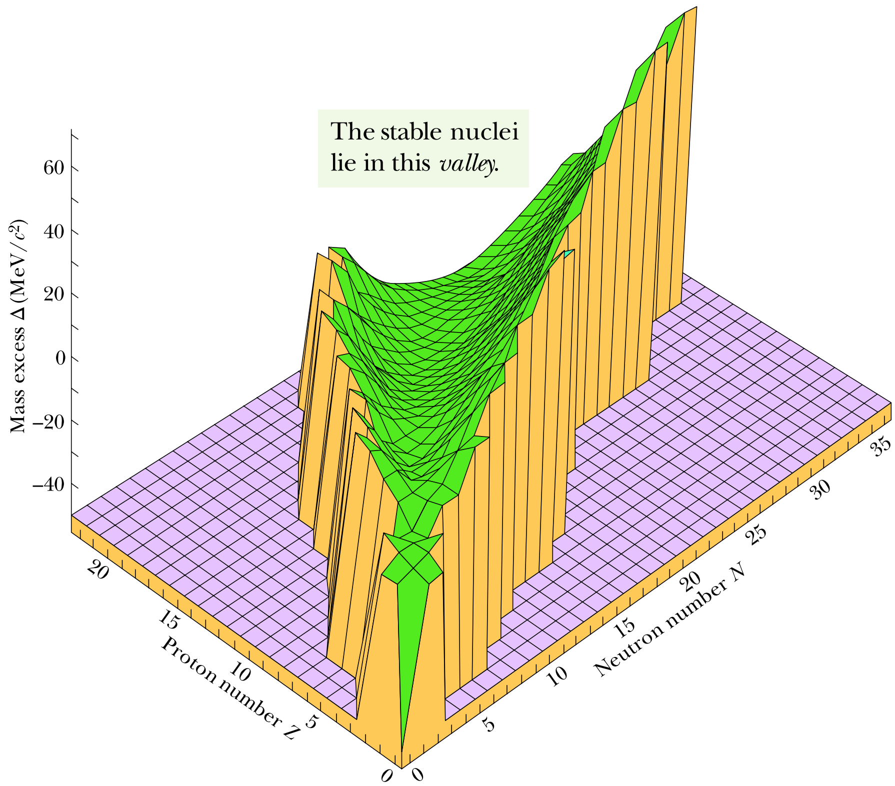


**Figure 42.5.3** A portion of the valley of the nuclides, showing only the nuclides of low mass. Deuterium, tritium, and helium lie at the near end of the plot, with helium at the high point. The valley stretches away from us, with the plot stopping at about $Z = 22$ and $N = 35$. Nuclides with large values of $A$, which would be plotted much beyond the valley, can decay into the valley by repeated alpha emissions and by fission (splitting of a nuclide).

the unit MeV/$c^2$. The inclusion of such an axis gives Fig. 42.5.3, which reveals the degree of nuclear stability of the nuclides. For the low-mass nuclides, we find a "valley of the nuclides," with the stability band of Fig. 42.2.1 running along its bottom. Nuclides on the proton-rich side of the valley decay into it by emitting positrons, and those on the neutron-rich side do so by emitting electrons.

### Checkpoint 42.5.1

$^{238}$U decays to $^{234}$Th by the emission of an alpha particle. There follows a chain of further radioactive decays, either by alpha decay or by beta decay. Eventually a stable nuclide is reached and, after that, no further radioactive decay is possible. Which of the following stable nuclides is the end product of the $^{238}$U radioactive decay chain: $^{206}$Pb, $^{207}$Pb, $^{208}$Pb, or $^{209}$Pb? (*Hint:* You can decide by considering the changes in mass number $A$ for the two types of decay.)

## Sample Problem 42.5.1 Q value in a beta decay, using masses

Calculate the disintegration energy $Q$ for the beta decay of $^{32}$P, as described by Eq. 42.5.1. The needed atomic masses are 31.97391 u for $^{32}$P and 31.97207 u for $^{32}$S.

### KEY IDEA

The disintegration energy $Q$ for the beta decay is the amount by which the mass energy is changed by the decay.

***Calculations:*** $Q$ is given by Eq. 37.6.11 ($Q = -\Delta M\ c^2$). However, we must be careful to distinguish between nuclear masses (which we do not know) and atomic masses (which we do know). Let the boldface symbols $\mathbf{m}_P$ and $\mathbf{m}_S$ represent the nuclear masses of $^{32}$P and $^{32}$S, and let the italic symbols $m_P$ and $m_S$ represent their atomic masses. Then we can write the change in mass for the decay of Eq. 42.5.1 as

$$\Delta m = (\mathbf{m}_S + m_e) - \mathbf{m}_P,$$

in which $m_e$ is the mass of the electron. If we add and subtract $15m_e$ on the right side of this equation, we obtain

$$\Delta m = (\mathbf{m}_S + 16m_e) - (\mathbf{m}_P + 15m_e).$$

The quantities in parentheses are the atomic masses of $^{32}$S and $^{32}$P, so

$$\Delta m = m_S - m_P.$$

We thus see that if we subtract only the atomic masses, the mass of the emitted electron is automatically taken into account. (This procedure will not work for positron emission.)

The disintegration energy for the $^{32}$P decay is then

$$Q = -\Delta m\, c^2$$
$$= -(31.97207 \text{ u} - 31.97391 \text{ u})(931.494013 \text{ MeV/u})$$
$$= 1.71 \text{ MeV}. \qquad \text{(Answer)}$$

Experimentally, this calculated quantity proves to be equal to $K_{max}$, the maximum energy the emitted electrons can have. Although 1.71 MeV is released every time a $^{32}$P nucleus decays, in essentially every case the electron carries away less energy than this. The neutrino gets all the rest, carrying it stealthily out of the laboratory.

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# 42.6 RADIOACTIVE DATING

### Learning Objectives

*After reading this module, you should be able to . . .*

**42.6.1** Apply the equations for radioactive decay to determine the age of rocks and archaeological materials.

**42.6.2** Explain how radiocarbon dating can be used to date the age of biological samples.

### Key Idea

- Naturally occurring radioactive nuclides provide a means for estimating the dates of historic and prehistoric events. For example, the ages of organic materials can often be found by measuring their $^{14}$C content, and rock samples can be dated using the radioactive $^{40}$K.

## Radioactive Dating

If you know the half-life of a given radionuclide, you can in principle use the decay of that radionuclide as a clock to measure time intervals. The decay of very long-lived nuclides, for example, can be used to measure the age of rocks—that is, the time that has elapsed since they were formed. Such measurements for rocks from Earth and the Moon, and for meteorites, yield a consistent maximum age of about $4.5 \times 10^9$ y for these bodies.

The radionuclide $^{40}$K, for example, decays to $^{40}$Ar, a stable isotope of the noble gas argon. The half-life for this decay is $1.25 \times 10^9$ y. A measurement of the ratio of $^{40}$K to $^{40}$Ar, as found in the rock in question, can be used to calculate the age of that rock. Other long-lived decays, such as that of $^{235}$U to $^{207}$Pb (involving a number of intermediate stages of unstable nuclei), can be used to verify this calculation.

For measuring shorter time intervals, in the range of historical interest, radiocarbon dating has proved invaluable. The radionuclide $^{14}$C (with $T_{1/2}$ = 5730 y) is produced at a constant rate in the upper atmosphere as atmospheric nitrogen is bombarded by cosmic rays. This radiocarbon mixes with the carbon that is normally present in the atmosphere (as $CO_2$) so that there is about one atom of $^{14}$C for every $10^{13}$ atoms of ordinary stable $^{12}$C. Through biological activity such as photosynthesis and breathing, the atoms of atmospheric carbon trade places randomly, one atom at a time, with the atoms of carbon in every living thing, including broccoli, mushrooms, penguins, and humans. Eventually an exchange equilibrium is reached at which the carbon atoms of every living thing contain a fixed small fraction of the radioactive nuclide $^{14}$C.

This equilibrium persists as long as the organism is alive. When the organism dies, the exchange with the atmosphere stops and the amount of radiocarbon trapped in the organism, since it is no longer being replenished, dwindles away with a half-life of 5730 y. By measuring the amount of radiocarbon per gram of organic matter, it is possible to measure the time that has elapsed since the organism died. Charcoal from ancient campfires, the Dead Sea scrolls (actually, the cloth used to plug the jars holding the scrolls), and many prehistoric artifacts have been dated in this way.



A fragment of the Dead Sea scrolls and the caves from which the scrolls were recovered.

### Sample Problem 42.6.1 Radioactive dating of a Moon rock

In a Moon rock sample, the ratio of the number of (stable) $^{40}$Ar atoms present to the number of (radioactive) $^{40}$K atoms is 10.3. Assume that all the argon atoms were produced by the decay of potassium atoms, with a half-life of $1.25 \times 10^9$ y. How old is the rock?

#### KEY IDEAS

(1) If $N_0$ potassium atoms were present at the time the rock was formed by solidification from a molten form, the number of potassium atoms now remaining at the time of analysis is

$$N_K = N_0 e^{-\lambda t}, \tag{42.6.1}$$

in which $t$ is the age of the rock. (2) For every potassium atom that decays, an argon atom is produced. Thus, the number of argon atoms present at the time of the analysis is

$$N_{Ar} = N_0 - N_K. \tag{42.6.2}$$

***Calculations:*** We cannot measure $N_0$; so let's eliminate it from Eqs. 42.6.1 and 42.6.2. We find, after some algebra, that

$$\lambda t = \ln\left(1 + \frac{N_{Ar}}{N_K}\right), \tag{42.6.3}$$

in which $N_{Ar}/N_K$ *can* be measured. Solving for $t$ and using Eq. 42.3.8 to replace $\lambda$ with $(\ln 2)/T_{1/2}$ yield

$$t = \frac{T_{1/2}\ln(1 + N_{Ar}/N_K)}{\ln 2}$$

$$= \frac{(1.25 \times 10^9 \text{ y})[\ln(1 + 10.3)]}{\ln 2}$$

$$= 4.37 \times 10^9 \text{ y}. \quad \text{(Answer)}$$

Lesser ages may be found for other lunar or terrestrial rock samples, but no substantially greater ones. Thus, the oldest rocks were formed soon after the Solar System formed, and the Solar System must be about 4 billion years old.

**WileyPLUS** Additional examples, video, and practice available at *WileyPLUS*

# 42.7 MEASURING RADIATION DOSAGE

## Learning Objectives

*After reading this module, you should be able to . . .*

**42.7.1** Identify absorbed dose, dose equivalent, and the associated units.

**42.7.2** Calculate absorbed dose and dose equivalent.

## Key Ideas

- The becquerel (1 Bq = 1 decay per second) measures the activity of a source.
- The amount of energy actually absorbed is measured in grays, with 1 Gy corresponding to 1 J/kg.
- The estimated biological effect of the absorbed energy is the dose equivalent and is measured in sieverts.

## Measuring Radiation Dosage

The effect of radiation such as gamma rays, electrons, and alpha particles on living tissue (particularly our own) is a matter of public interest. Such radiation is found in nature in cosmic rays (from astronomical sources) and in the emissions by radioactive elements in Earth's crust. Radiation associated with some human activities, such as using x rays and radionuclides in medicine and in industry, also contributes.

Our task here is not to explore the various sources of radiation but simply to describe the units in which the properties and effects of such radiations are expressed. We have already discussed the *activity* of a radioactive source. There are two remaining quantities of interest.

**1.** *Absorbed Dose.* This is a measure of the radiation dose (as energy per unit mass) actually absorbed by a specific object, such as a patient's hand or chest.

Its SI unit is the **gray** (Gy). An older unit, the **rad** (from **r**adiation **a**bsorbed **d**ose) is still in common use. The terms are defined and related as follows:

$$1 \text{ Gy} = 1 \text{ J/kg} = 100 \text{ rad}. \tag{42.7.1}$$

A typical dose-related statement is: "A whole-body, short-term gamma-ray dose of 3 Gy (= 300 rad) will cause death in 50% of the population exposed to it." Thankfully, our present average absorbed dose per year, from sources of both natural and human origin, is only about 2 mGy (= 0.2 rad).

**2.** *Dose Equivalent.* Although different types of radiation (gamma rays and neutrons, say) may deliver the same amount of energy to the body, they do not have the same biological effect. The dose equivalent allows us to express the biological effect by multiplying the absorbed dose (in grays or rads) by a numerical **RBE** factor (from **r**elative **b**iological **e**ffectiveness). For x rays and electrons, for example, RBE = 1; for slow neutrons, RBE = 5; for alpha particles, RBE = 10; and so on. Personnel-monitoring devices such as film badges register the dose equivalent.

The SI unit of dose equivalent is the **sievert** (Sv). An earlier unit, the **rem,** is still in common use. Their relationship is

$$1 \text{ Sv} = 100 \text{ rem}. \tag{42.7.2}$$

An example of the correct use of these terms is: "The recommendation of the National Council on Radiation Protection is that no individual who is (nonoccupationally) exposed to radiation should receive a dose equivalent greater than 5 mSv (= 0.5 rem) in any one year." This includes radiation of all kinds; of course the appropriate RBE factor must be used for each kind.

# 42.8 NUCLEAR MODELS

## Learning Objectives

*After reading this module, you should be able to . . .*

**42.8.1** Distinguish the collective model and the independent model, and explain the combined model.

**42.8.2** Identify compound nucleus.

**42.8.3** Identify magic numbers.

## Key Ideas

- The collective model of nuclear structure assumes that nucleons collide constantly with one another and that relatively long-lived compound nuclei are formed when a projectile is captured. The formation and eventual decay of a compound nucleus are totally independent events.
- The independent particle model of nuclear structure assumes that each nucleon moves, essentially without collision, in a quantized state within the nucleus. The model predicts nucleon levels and magic nucleon numbers associated with closed shells of nucleons.
- The combined model assumes that extra nucleons occupy quantized states outside a central core of closed shells.

## Nuclear Models

Nuclei are more complicated than atoms. For atoms, the basic force law (Coulomb's law) is simple in form and there is a natural force center, the nucleus. For nuclei, the force law is complicated and cannot, in fact, be written down explicitly in full detail. Furthermore, the nucleus—a jumble of protons and neutrons—has no natural force center to simplify the calculations.

In the absence of a comprehensive nuclear *theory,* we turn to the construction of nuclear *models.* A nuclear model is simply a way of looking at the nucleus that gives a physical insight into as wide a range of its properties as possible. The usefulness of a model is tested by its ability to provide predictions that can be verified experimentally in the laboratory.

Two models of the nucleus have proved useful. Although based on assumptions that seem flatly to exclude each other, each accounts very well for a selected group of nuclear properties. After describing them separately, we shall see how these two models may be combined to form a single coherent picture of the atomic nucleus.

## The Collective Model

In the *collective model,* formulated by Niels Bohr, the nucleons, moving around within the nucleus at random, are imagined to interact strongly with each other, like the molecules in a drop of liquid. A given nucleon collides frequently with other nucleons in the nuclear interior, its mean free path as it moves about being substantially less than the nuclear radius.

The collective model permits us to correlate many facts about nuclear masses and binding energies; it is useful (as you will see later) in explaining nuclear fission. It is also useful for understanding a large class of nuclear reactions.

Consider, for example, a generalized nuclear reaction of the form

$$X + a \rightarrow C \rightarrow Y + b. \tag{42.8.1}$$

We imagine that projectile $a$ enters target nucleus $X$, forming a **compound nucleus** $C$ and conveying to it a certain amount of excitation energy. The projectile, perhaps a neutron, is at once caught up by the random motions that characterize the nuclear interior. It quickly loses its identity—so to speak—and the excitation energy it carried into the nucleus is quickly shared with all the other nucleons in $C$.

The quasi-stable state represented by $C$ in Eq. 42.8.1 may have a mean life of $10^{-16}$ s before it decays to $Y$ and $b$. By nuclear standards, this is a very long time, being about one million times longer than the time required for a nucleon with a few million electron-volts of energy to travel across a nucleus.

The central feature of this compound-nucleus concept is that the formation of the compound nucleus and its eventual decay are totally independent events. At the time of its decay, the compound nucleus has "forgotten" how it was formed. Hence, its mode of decay is not influenced by its mode of formation. As an example, Fig. 42.8.1 shows three possible ways in which the compound nucleus $^{20}$Ne might be formed and three in which it might decay. Any of the three formation modes can lead to any of the three decay modes.

## The Independent Particle Model

In the collective model, we assume that the nucleons move around at random and bump into one another frequently. The *independent particle model,* however, is based on just the opposite assumption—namely, that each nucleon remains in a well-defined quantum state within the nucleus and makes hardly any collisions at all! The nucleus, unlike the atom, has no fixed center of charge; we assume in this model that each nucleon moves in a potential well that is determined by the smeared-out (time-averaged) motions of all the other nucleons.

A nucleon in a nucleus, like an electron in an atom, has a set of quantum numbers that defines its state of motion. Also, nucleons obey the Pauli exclusion principle, just as electrons do; that is, no two nucleons in a nucleus may occupy the same quantum state at the same time. In this regard, the neutrons and the protons are treated separately, each particle type with its own set of quantum states.

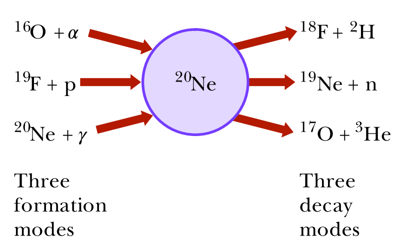


**Figure 42.8.1** The formation modes and the decay modes of the compound nucleus $^{20}$Ne.

The fact that nucleons obey the Pauli exclusion principle helps us to understand the relative stability of nucleon states. If two nucleons within the nucleus are to collide, the energy of each of them after the collision must correspond to the energy of an *unoccupied* state. If no such state is available, the collision simply cannot occur. Thus, any given nucleon experiencing repeated "frustrated collision opportunities" will maintain its state of motion long enough to give meaning to the statement that it exists in a quantum state with a well-defined energy.

In the atomic realm, the repetitions of physical and chemical properties that we find in the periodic table are associated with a property of atomic electrons—namely, they arrange themselves in shells that have a special stability when fully occupied. We can take the atomic numbers of the noble gases,

$$2,\ 10,\ 18,\ 36,\ 54,\ 86, \ldots,$$

as *magic electron numbers* that mark the completion (or closure) of such shells.

Nuclei also show such closed-shell effects, associated with certain **magic nucleon numbers:**

$$2,\ 8,\ 20,\ 28, 50, 82, 126, \ldots.$$

Any nuclide whose proton number $Z$ or neutron number $N$ has one of these values turns out to have a special stability that may be made apparent in a variety of ways.

Examples of "magic" nuclides are $^{18}$O ($Z = 8$), $^{40}$Ca ($Z = 20$, $N = 20$), $^{92}$Mo ($N = 50$), and $^{208}$Pb ($Z = 82$, $N = 126$). Both $^{40}$Ca and $^{208}$Pb are said to be "doubly magic" because they contain both filled shells of protons *and* filled shells of neutrons.

The magic number 2 shows up in the exceptional stability of the alpha particle ($^4$He), which, with $Z = N = 2$, is doubly magic. For example, on the binding energy curve of Fig. 42.2.3, the binding energy per nucleon for this nuclide stands well above those of its periodic-table neighbors hydrogen, lithium, and beryllium. The neutrons and protons making up the alpha particle are so tightly bound to one another, in fact, that it is impossible to add another proton or neutron to it; there is no stable nuclide with $A = 5$.

The central idea of a closed shell is that a single particle outside a closed shell can be relatively easily removed, but considerably more energy must be expended to remove a particle from the shell itself. The sodium atom, for example, has one (valence) electron outside a closed electron shell. Only about 5 eV is required to strip the valence electron away from a sodium atom; however, to remove a *second* electron (which must be plucked out of a closed shell) requires 22 eV. As a nuclear case, consider $^{121}$Sb ($Z = 51$), which contains a single proton outside a closed shell of 50 protons. To remove this lone proton requires 5.8 MeV; to remove a *second* proton, however, requires an energy of 11 MeV. There is much additional experimental evidence that the nucleons in a nucleus form closed shells and that these shells exhibit stable properties.

We have seen that quantum theory can account beautifully for the magic electron numbers—that is, for the populations of the subshells into which atomic electrons are grouped. It turns out that, under certain assumptions, quantum theory can account equally well for the magic nucleon numbers! The 1963 Nobel Prize in physics was, in fact, awarded to Maria Mayer and Hans Jensen "for their discoveries concerning nuclear shell structure."

## A Combined Model

Consider a nucleus in which a small number of neutrons (or protons) exist outside a core of closed shells that contains magic numbers of neutrons or protons. The outside nucleons occupy quantized states in a potential well established by the central core, thus preserving the central feature of the independent-particle model. These outside nucleons also interact with the core, deforming it and setting up "tidal wave" motions of rotation or vibration within it. These collective motions of the core preserve the central feature of the collective model. Such a model of nuclear structure thus succeeds in combining the seemingly irreconcilable points of view of the collective and independent-particle models. It has been remarkably successful in explaining observed nuclear properties.

### Sample Problem 42.8.1 Lifetime of a compound nucleus made by neutron capture

Consider the neutron capture reaction

$$^{109}\text{Ag} + \text{n} \rightarrow {}^{110}\text{Ag} \rightarrow {}^{110}\text{Ag} + \gamma, \tag{42.8.2}$$

in which a compound nucleus ($^{110}$Ag) is formed. Figure 42.8.2 shows the relative rate at which such events take place, plotted against the energy of the incoming neutron. Find the mean lifetime of this compound nucleus by using the uncertainty principle in the form

$$\Delta E \cdot \Delta t \approx \hbar. \tag{42.8.3}$$

Here $\Delta E$ is a measure of the uncertainty with which the energy of a state can be defined. The quantity $\Delta t$ is a measure of the time available to measure this energy. In fact, here $\Delta t$ is just $t_{avg}$, the average life of the compound nucleus before it decays to its ground state.

***Reasoning:*** We see that the relative reaction rate peaks sharply at a neutron energy of about 5.2 eV. This suggests that we are dealing with a single excited energy level of the compound nucleus $^{110}$Ag. When the available energy (of the incoming neutron) just matches the energy of this level above the $^{110}$Ag ground state, we have "resonance" and the reaction of Eq. 42.8.2 really "goes."

However, the resonance peak is not infinitely sharp but has an approximate half-width ($\Delta E$ in the figure) of about 0.20 eV. We can account for this resonance-peak width by saying that the excited level is not sharply defined in energy but has an energy uncertainty $\Delta E$ of about 0.20 eV.

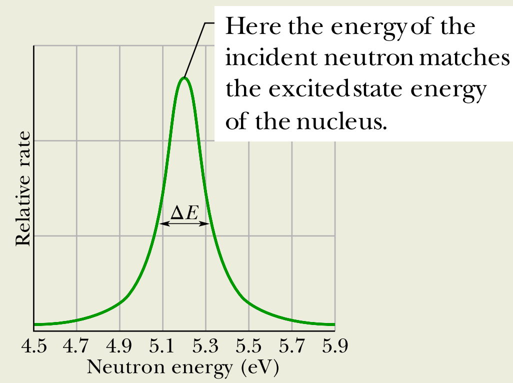


**Figure 42.8.2** A plot of the relative number of reaction events of the type described by Eq. 42.8.2 as a function of the energy of the incident neutron. The half-width $\Delta E$ of the resonance peak is about 0.20 eV.

***Calculation:*** Substituting that uncertainty of 0.20 eV into Eq. 42.8.3 gives us

$$\Delta t = t_{avg} \approx \frac{\hbar}{\Delta E} \approx \frac{(4.14 \times 10^{-15}\ \text{eV} \cdot \text{s})/2\pi}{0.20\ \text{eV}}$$

$$\approx 3 \times 10^{-15}\ \text{s}. \qquad \text{(Answer)}$$

This is several hundred times greater than the time a 5.2 eV neutron takes to cross the diameter of a $^{109}$Ag nucleus. Therefore, the neutron is spending this time of $3 \times 10^{-15}$ s *as part of* the nucleus.

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## Review & Summary

**The Nuclides** Approximately 2000 **nuclides** are known to exist. Each is characterized by an **atomic number** $Z$ (the number of protons), a **neutron number** $N$, and a **mass number** $A$ (the total number of **nucleons**—protons and neutrons). Thus, $A = Z + N$. Nuclides with the same atomic number but different neutron numbers are **isotopes** of one another. Nuclei have a mean radius $r$ given by

$$r = r_0 A^{1/3}, \tag{42.2.3}$$

where $r_0 \approx 1.2$ fm.

**Mass and Binding Energy** Atomic masses are often reported in terms of *mass excess*

$$\Delta = M - A \quad \text{(mass excess)}, \tag{42.2.6}$$

where $M$ is the actual mass of an atom in atomic mass units and $A$ is the mass number for that atom's nucleus. The **binding energy** of a nucleus is the difference

$$\Delta E_{be} = \Sigma(mc^2) - Mc^2 \quad \text{(binding energy)}, \tag{42.2.7}$$

where $\Sigma(mc^2)$ is the total mass energy of the *individual* protons and neutrons. The **binding energy per nucleon** is

$$\Delta E_{ben} = \frac{\Delta E_{be}}{A} \quad \text{(binding energy per nucleon)}. \tag{42.2.8}$$

**Mass–Energy Exchanges** The energy equivalent of one mass unit (u) is 931.494 013 MeV. The binding energy curve shows that middle-mass nuclides are the most stable and that energy can be released both by fission of high-mass nuclei and by fusion of low-mass nuclei.

**The Nuclear Force** Nuclei are held together by an attractive force acting among the nucleons, part of the **strong force** acting between the quarks that make up the nucleons.

**Radioactive Decay** Most known nuclides are radioactive; they spontaneously decay at a rate $R$ ($= -dN/dt$) that is proportional to the number $N$ of radioactive atoms present, the proportionality constant being the **disintegration constant** $\lambda$. This leads to the law of exponential decay:

$$N = N_0 e^{-\lambda t}, \qquad R = \lambda N = R_0 e^{-\lambda t} \quad \text{(radioactive decay)}. \qquad (42.3.5, 42.3.7, 42.3.6)$$

The **half-life** $T_{1/2} = (\ln 2)/\lambda$ of a radioactive nuclide is the time required for the decay rate $R$ (or the number $N$) in a sample to drop to half its initial value.

**Alpha Decay** Some nuclides decay by emitting an alpha particle (a helium nucleus, $^4$He). Such decay is inhibited by a potential energy barrier that cannot be penetrated according to classical physics but is subject to tunneling according to quantum physics. The barrier penetrability, and thus the half-life for alpha decay, is very sensitive to the energy of the emitted alpha particle.

**Beta Decay** In **beta decay** either an electron or a positron is emitted by a nucleus, along with a neutrino. The emitted particles share the available disintegration energy. The electrons and positrons emitted in beta decay have a continuous spectrum of energies from near zero up to a limit $K_{max}$ ($= Q = -\Delta m\, c^2$).

**Radioactive Dating** Naturally occurring radioactive nuclides provide a means for estimating the dates of historic and prehistoric events. For example, the ages of organic materials can often be found by measuring their $^{14}$C content; rock samples can be dated using the radioactive isotope $^{40}$K.

**Radiation Dosage** Three units are used to describe exposure to ionizing radiation. The **becquerel** (1 Bq = 1 decay per second) measures the **activity** of a source. The amount of energy actually absorbed is measured in **grays,** with 1 Gy corresponding to 1 J/kg. The estimated biological effect of the absorbed energy is measured in **sieverts;** a dose equivalent of 1 Sv causes the same biological effect regardless of the radiation type by which it was acquired.

**Nuclear Models** The **collective** model of nuclear structure assumes that nucleons collide constantly with one another and that relatively long-lived **compound nuclei** are formed when a projectile is captured. The formation and eventual decay of a compound nucleus are totally independent events.

The **independent particle** model of nuclear structure assumes that each nucleon moves, essentially without collisions, in a quantized state within the nucleus. The model predicts nucleon levels and **magic nucleon numbers** (2, 8, 20, 28, 50, 82, and 126) associated with closed shells of nucleons; nuclides with any of these numbers of neutrons or protons are particularly stable.

The **combined** model, in which extra nucleons occupy quantized states outside a central core of closed shells, is highly successful in predicting many nuclear properties.

# Questions

**1** The radionuclide $^{196}$Ir decays by emitting an electron. (a) Into which square in Fig. 42.2.2 is it transformed? (b) Do further decays then occur?

**2** Is the mass excess of an alpha particle (use a straightedge on Fig. 42.5.3) greater than or less than the particle's total binding energy (use the binding energy per nucleon from Fig. 42.2.3)?

**3** At $t = 0$, a sample of radionuclide $A$ has the same decay rate as a sample of radionuclide $B$ has at $t = 30$ min. The disintegration constants are $\lambda_A$ and $\lambda_B$, with $\lambda_A < \lambda_B$. Will the two samples ever have (simultaneously) the same decay rate? (*Hint:* Sketch a graph of their activities.)

**4** A certain nuclide is said to be particularly stable. Does its binding energy per nucleon lie slightly above or slightly below the binding energy curve of Fig. 42.2.3?

**5** Suppose the alpha particle in a Rutherford scattering experiment is replaced with a proton of the same initial kinetic energy and also headed directly toward the nucleus of the gold atom. (a) Will the distance from the center of the nucleus at which the proton stops be greater than, less than, or the same as that of the alpha particle? (b) If, instead, we switch the target to a nucleus with a larger value of $Z$, is the stopping distance of the alpha particle greater than, less than, or the same as with the gold target?

**6** Figure 42.1 gives the activities of three radioactive samples versus time. Rank the samples according to their (a) half-life and (b) disintegration constant, greatest first. (*Hint:* For (a), use a straightedge on the graph.)

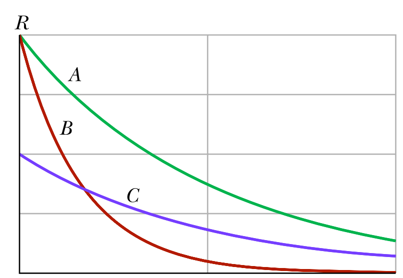


**Figure 42.1** Question 6.

**7** The nuclide $^{244}$Pu ($Z = 94$) is an alpha-emitter. Into which of the following nuclides does it decay: $^{240}$Np ($Z = 93$), $^{240}$U ($Z = 92$), $^{248}$Cm ($Z = 96$), or $^{244}$Am ($Z = 95$)?

**8** The radionuclide $^{49}$Sc has a half-life of 57.0 min. At $t = 0$, the counting rate of a sample of it is 6000 counts/min above the general background activity, which is 30 counts/min. Without computation, determine whether the counting rate of the sample will be about equal to the background rate in 3 h, 7 h, 10 h, or a time much longer than 10 h.

**9** At $t = 0$ we begin to observe two identical radioactive nuclei that have a half-life of 5 min. At $t = 1$ min, one of the nuclei decays. Does that event increase or decrease the chance that the second nucleus will decay in the next 4 min, or is there no effect on the second nucleus? (Are the events cause and effect, or random?)

**10** Figure 42.2 shows the curve for the binding energy per nucleon $\Delta E_{ben}$ versus mass number $A$. Three isotopes are indicated. Rank them according to the energy required to remove a nucleon from the isotope, greatest first.

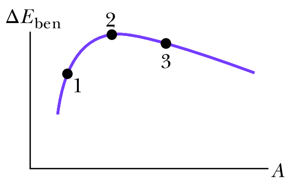


**Figure 42.2** Question 10.

**11** At $t = 0$, a sample of radionuclide $A$ has twice the decay rate as a sample of radionuclide $B$. The disintegration constants are $\lambda_A$ and $\lambda_B$, with $\lambda_A > \lambda_B$. Will the two samples ever have (simultaneously) the same decay rate?

**12** Figure 42.3 is a plot of mass number $A$ versus charge number $Z$. The location of a certain nucleus is represented by a dot. Which of the arrows extending from the dot would best represent the transition were the nucleus to undergo (a) a $\beta^-$ decay and (b) an $\alpha$ decay?

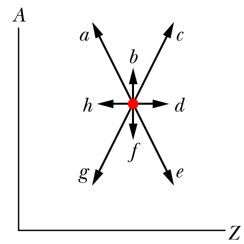


**Figure 42.3** Question 12.

**13** (a) Which of the following nuclides are magic: $^{122}$Sn, $^{132}$Sn, $^{98}$Cd, $^{198}$Au, $^{208}$Pb? (b) Which, if any, are doubly magic?

**14** If the mass of a radioactive sample is doubled, do (a) the activity of the sample and (b) the disintegration constant of the sample increase, decrease, or remain the same?

**15** The magic nucleon numbers for nuclei are given in Module 42.8 as 2, 8, 20, 28, 50, 82, and 126. Are nuclides magic (that is, especially stable) when (a) only the mass number $A$, (b) only the atomic number $Z$, (c) only the neutron number $N$, or (d) either $Z$ or $N$ (or both) is equal to one of these numbers? Pick all correct phrases.

# Problems

GO Tutoring problem available (at instructor's discretion) in *WileyPLUS*
SSM Worked-out solution available in Student Solutions Manual
E Easy M Medium H Hard
FCP Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com
CALC Requires calculus
BIO Biomedical application

## Module 42.1 Discovering the Nucleus

**1 E** A $^7$Li nucleus with a kinetic energy of 3.00 MeV is sent toward a $^{232}$Th nucleus. What is the least center-to-center separation between the two nuclei, assuming that the (more massive) $^{232}$Th nucleus does not move?

**2 E** Calculate the distance of closest approach for a head-on collision between a 5.30 MeV alpha particle and a copper nucleus.

**3 M** A 10.2 MeV Li nucleus is shot directly at the center of a Ds nucleus. At what center-to-center distance does the Li momentarily stop, assuming the Ds does not move?

**4 M GO** In a Rutherford scattering experiment, assume that an incident alpha particle (radius 1.80 fm) is headed directly toward a target gold nucleus (radius 6.23 fm). What energy must the alpha particle have to just barely "touch" the gold nucleus?

**5 M GO** When an alpha particle collides elastically with a nucleus, the nucleus recoils. Suppose a 5.00 MeV alpha particle has a head-on elastic collision with a gold nucleus that is initially at rest. What is the kinetic energy of (a) the recoiling nucleus and (b) the rebounding alpha particle?

## Module 42.2 Some Nuclear Properties

**6 E** The strong neutron excess (defined as $N - Z$) of high-mass nuclei is illustrated by noting that most high-mass nuclides could never fission into two stable nuclei without neutrons being left over. For example, consider the spontaneous fission of a $^{235}$U nucleus into two stable *daughter nuclei* with atomic numbers 39 and 53. From Appendix F, determine the name of the (a) first and (b) second daughter nucleus. From Fig. 42.2.1, approximately how many neutrons are in the (c) first and (d) second? (e) Approximately how many neutrons are left over?

**7 E** What is the nuclear mass density $\rho_m$ of (a) the fairly low-mass nuclide $^{55}$Mn and (b) the fairly high-mass nuclide $^{209}$Bi? (c) Compare the two answers, with an explanation. What is the nuclear charge density $\rho_q$ of (d) $^{55}$Mn and (e) $^{209}$Bi? (f) Compare the two answers, with an explanation.

**8 E** (a) Show that the mass $M$ of an atom is given approximately by $M_{app} = Am_p$, where $A$ is the mass number and $m_p$ is the proton mass. For (b) $^1$H, (c) $^{31}$P, (d) $^{120}$Sn, (e) $^{197}$Au, and (f) $^{239}$Pu, use Table 42.2.1 to find the percentage deviation between $M_{app}$ and $M$:

$$\text{Percentage deviation} = \frac{M_{app} - M}{M} 100.$$

(g) Is a value of $M_{app}$ accurate enough to be used in a calculation of a nuclear binding energy?

**9 E** The nuclide $^{14}$C contains (a) how many protons and (b) how many neutrons?

**10 E** What is the mass excess $\Delta_1$ of $^1$H (actual mass is 1.007 825 u) in (a) atomic mass units and (b) MeV/$c^2$? What is the mass excess $\Delta_n$ of a neutron (actual mass is 1.008 665 u) in (c) atomic mass units and (d) MeV/$c^2$? What is the mass excess $\Delta_{120}$ of $^{120}$Sn (actual mass is 119.902 197 u) in (e) atomic mass units and (f) MeV/$c^2$?

**11 E SSM** Nuclear radii may be measured by scattering high-energy (high-speed) electrons from nuclei. (a) What is the de Broglie wavelength for 200 MeV electrons? (b) Are these electrons suitable probes for this purpose?

**12** E The electric potential energy of a uniform sphere of charge $q$ and radius $r$ is given by

$$U = \frac{3q^2}{20\pi\varepsilon_0 r}.$$

(a) Does the energy represent a tendency for the sphere to bind together or blow apart? The nuclide $^{239}$Pu is spherical with radius 6.64 fm. For this nuclide, what are (b) the electric potential energy $U$ according to the equation, (c) the electric potential energy per proton, and (d) the electric potential energy per nucleon? The binding energy per nucleon is 7.56 MeV. (e) Why is the nuclide bound so well when the answers to (c) and (d) are large and positive?

**13** E A neutron star is a stellar object whose density is about that of nuclear matter, $2 \times 10^{17}$ kg/m$^3$. Suppose that the Sun were to collapse and become such a star without losing any of its present mass. What would be its radius?

**14** M GO What is the binding energy per nucleon of the americium isotope $^{244}_{95}$Am? Here are some atomic masses and the neutron mass.

| | | | |
|---|---|---|---|
| $^{244}_{95}$Am | 244.064 279 u | $^1$H | 1.007 825 u |
| n | 1.008 665 u | | |

**15** M (a) Show that the energy associated with the strong force between nucleons in a nucleus is proportional to $A$, the mass number of the nucleus in question. (b) Show that the energy associated with the Coulomb force between protons in a nucleus is proportional to $Z(Z - 1)$. (c) Show that, as we move to larger and larger nuclei (see Fig. 42.2.1), the importance of the Coulomb force increases more rapidly than does that of the strong force.

**16** M GO What is the binding energy per nucleon of the europium isotope $^{152}_{63}$Eu? Here are some atomic masses and the neutron mass.

| | | | |
|---|---|---|---|
| $^{152}_{63}$Eu | 151.921 742 u | $^1$H | 1.007 825 u |
| n | 1.008 665 u | | |

**17** M Because the neutron has no charge, its mass must be found in some way other than by using a mass spectrometer. When a neutron and a proton meet (assume both to be almost stationary), they combine and form a deuteron, emitting a gamma ray whose energy is 2.2233 MeV. The masses of the proton and the deuteron are 1.007 276 467 u and 2.013 553 212 u, respectively. Find the mass of the neutron from these data.

**18** M GO What is the binding energy per nucleon of the rutherfordium isotope $^{259}_{104}$Rf? Here are some atomic masses and the neutron mass.

| | | | |
|---|---|---|---|
| $^{259}_{104}$Rf | 259.105 63 u | $^1$H | 1.007 825 u |
| n | 1.008 665 u | | |

**19** M A periodic table might list the average atomic mass of magnesium as being 24.312 u, which is the result of *weighting* the atomic masses of the magnesium isotopes according to their natural abundances on Earth. The three isotopes and their masses are $^{24}$Mg (23.985 04 u), $^{25}$Mg (24.985 84 u), and $^{26}$Mg (25.982 59 u). The natural abundance of $^{24}$Mg is 78.99% by mass (that is, 78.99% of the mass of a naturally occurring sample of magnesium is due to the presence of $^{24}$Mg). What is the abundance of (a) $^{25}$Mg and (b) $^{26}$Mg?

**20** M What is the binding energy per nucleon of $^{262}$Bh? The mass of the atom is 262.1231 u.

**21** M SSM (a) Show that the total binding energy $E_{be}$ of a given nuclide is

$$E_{be} = Z\Delta_H + N\Delta_n - \Delta,$$

where $\Delta_H$ is the mass excess of $^1$H, $\Delta_n$ is the mass excess of a neutron, and $\Delta$ is the mass excess of the given nuclide. (b) Using this method, calculate the binding energy per nucleon for $^{197}$Au. Compare your result with the value listed in Table 42.2.1. The needed mass excesses, rounded to three significant figures, are $\Delta_H = +7.29$ MeV, $\Delta_n = +8.07$ MeV, and $\Delta_{197} = -31.2$ MeV. Note the economy of calculation that results when mass excesses are used in place of the actual masses.

**22** M GO An $\alpha$ particle ($^4$He nucleus) is to be taken apart in the following steps. Give the energy (work) required for each step: (a) remove a proton, (b) remove a neutron, and (c) separate the remaining proton and neutron. For an $\alpha$ particle, what are (d) the total binding energy and (e) the binding energy per nucleon? (f) Does either match an answer to (a), (b), or (c)? Here are some atomic masses and the neutron mass.

| | | | |
|---|---|---|---|
| $^4$He | 4.002 60 u | $^2$H | 2.014 10 u |
| $^3$H | 3.016 05 u | $^1$H | 1.007 83 u |
| n | 1.008 67 u | | |

**23** M SSM Verify the binding energy per nucleon given in Table 42.2.1 for the plutonium isotope $^{239}$Pu. The mass of the neutral atom is 239.052 16 u.

**24** M A penny has a mass of 3.0 g. Calculate the energy that would be required to separate all the neutrons and protons in this coin from one another. For simplicity, assume that the penny is made entirely of $^{63}$Cu atoms (of mass 62.929 60 u). The masses of the proton-plus-electron and the neutron are 1.007 83 u and 1.008 66 u, respectively.

## Module 42.3 Radioactive Decay

**25** E BIO Cancer cells are more vulnerable to x and gamma radiation than are healthy cells. In the past, the standard source for radiation therapy was radioactive $^{60}$Co, which decays, with a half-life of 5.27 y, into an excited nuclear state of $^{60}$Ni. That nickel isotope then immediately emits two gamma-ray photons, each with an approximate energy of 1.2 MeV. How many radioactive $^{60}$Co nuclei are present in a 6000 Ci source of the type used in hospitals? (Energetic particles from linear accelerators are now used in radiation therapy.)

**26** E The half-life of a radioactive isotope is 140 d. How many days would it take for the decay rate of a sample of this isotope to fall to one-fourth of its initial value?

**27** E A radioactive nuclide has a half-life of 30.0 y. What fraction of an initially pure sample of this nuclide will remain undecayed at the end of (a) 60.0 y and (b) 90.0 y?

**28** E The plutonium isotope $^{239}$Pu is produced as a by-product in nuclear reactors and hence is accumulating in our environment. It is radioactive, decaying with a half-life of $2.41 \times 10^4$ y. (a) How many nuclei of Pu constitute a chemically lethal dose of 2.00 mg? (b) What is the decay rate of this amount?

**29** E SSM A radioactive isotope of mercury, $^{197}$Hg, decays to gold, $^{197}$Au, with a disintegration constant of 0.0108 h$^{-1}$. (a) Calculate the half-life of the $^{197}$Hg. What fraction of a sample will remain at the end of (b) three half-lives and (c) 10.0 days?

**30** E The half-life of a particular radioactive isotope is 6.5 h. If there are initially $48 \times 10^{19}$ atoms of this isotope, how many remain at the end of 26 h?

**31** E Consider an initially pure 3.4 g sample of $^{67}$Ga, an isotope that has a half-life of 78 h. (a) What is its initial decay rate? (b) What is its decay rate 48 h later?

**32** E When aboveground nuclear tests were conducted, the explosions shot radioactive dust into the upper atmosphere. Global air circulations then spread the dust worldwide before it settled out on ground and water. One such test was conducted in October 1976. What fraction of the $^{90}$Sr produced by that explosion still existed in October 2006? The half-life of $^{90}$Sr is 29 y.

**33** M BIO The air in some caves includes a significant amount of radon gas, which can lead to lung cancer if breathed over a prolonged time. In British caves, the air in the cave with the greatest amount of the gas has an activity per volume of $1.55 \times 10^5$ Bq/m$^3$. Suppose that you spend two full days exploring (and sleeping in) that cave. Approximately how many $^{222}$Rn atoms would you take in and out of your lungs during your two-day stay? The radionuclide $^{222}$Rn in radon gas has a half-life of 3.82 days. You need to estimate your lung capacity and average breathing rate.

**34** M Calculate the mass of a sample of (initially pure) $^{40}$K that has an initial decay rate of $1.70 \times 10^5$ disintegrations/s. The isotope has a half-life of $1.28 \times 10^9$ y.

**35** M CALC SSM A certain radionuclide is being manufactured in a cyclotron at a constant rate $R$. It is also decaying with disintegration constant $\lambda$. Assume that the production process has been going on for a time that is much longer than the half-life of the radionuclide. (a) Show that the number of radioactive nuclei present after such time remains constant and is given by $N = R/\lambda$. (b) Now show that this result holds no matter how many radioactive nuclei were present initially. The nuclide is said to be in *secular equilibrium* with its source; in this state its decay rate is just equal to its production rate.

**36** M Plutonium isotope $^{239}$Pu decays by alpha decay with a half-life of 24 100 y. How many milligrams of helium are produced by an initially pure 12.0 g sample of $^{239}$Pu at the end of 20 000 y? (Consider only the helium produced directly by the plutonium and not by any by-products of the decay process.)

**37** M The radionuclide $^{64}$Cu has a half-life of 12.7 h. If a sample contains 5.50 g of initially pure $^{64}$Cu at $t = 0$, how much of it will decay between $t = 14.0$ h and $t = 16.0$ h?

**38** M BIO A dose of 8.60 $\mu$Ci of a radioactive isotope is injected into a patient. The isotope has a half-life of 3.0 h. How many of the isotope parents are injected?

**39** M The radionuclide $^{56}$Mn has a half-life of 2.58 h and is produced in a cyclotron by bombarding a manganese target with deuterons. The target contains only the stable manganese isotope $^{55}$Mn, and the manganese–deuteron reaction that produces $^{56}$Mn is

$$^{55}\text{Mn} + \text{d} \rightarrow {}^{56}\text{Mn} + \text{p}.$$

If the bombardment lasts much longer than the half-life of $^{56}$Mn, the activity of the $^{56}$Mn produced in the target reaches a final value of $8.88 \times 10^{10}$ Bq. (a) At what rate is $^{56}$Mn being produced? (b) How many $^{56}$Mn nuclei are then in the target? (c) What is their total mass?

**40** M A source contains two phosphorus radionuclides, $^{32}$P ($T_{1/2} = 14.3$ d) and $^{33}$P ($T_{1/2} = 25.3$ d). Initially, 10.0% of the decays come from $^{33}$P. How long must one wait until 90.0% do so?

**41** M A 1.00 g sample of samarium emits alpha particles at a rate of 120 particles/s. The responsible isotope is $^{147}$Sm, whose natural abundance in bulk samarium is 15.0%. Calculate the half-life.

**42** M What is the activity of a 20 ng sample of $^{92}$Kr, which has a half-life of 1.84 s?

**43** M BIO GO A radioactive sample intended for irradiation of a hospital patient is prepared at a nearby laboratory. The sample has a half-life of 83.61 h. What should its initial activity be if its activity is to be $7.4 \times 10^8$ Bq when it is used to irradiate the patient 24 h later?

**44** M GO Figure 42.4 shows the decay of parents in a radioactive sample. The axes are scaled by $N_s = 2.00 \times 10^6$ and $t_s = 10.0$ s. What is the activity of the sample at $t = 27.0$ s?

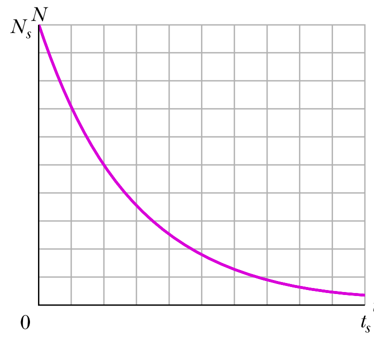


**Figure 42.4** Problem 44.

**45** M BIO In 1992, Swiss police arrested two men who were attempting to smuggle osmium out of Eastern Europe for a clandestine sale. However, by error, the smugglers had picked up $^{137}$Cs. Reportedly, each smuggler was carrying a 1.0 g sample of $^{137}$Cs *in a pocket*! In (a) bequerels and (b) curies, what was the activity of each sample? The isotope $^{137}$Cs has a half-life of 30.2 y. (The activities of radioisotopes commonly used in hospitals range up to a few millicuries.)

**46** M BIO The radioactive nuclide $^{99}$Tc can be injected into a patient's bloodstream in order to monitor the blood flow, measure the blood volume, or find a tumor, among other goals. The nuclide is produced in a hospital by a "cow" containing $^{99}$Mo, a radioactive nuclide that decays to $^{99}$Tc with a half-life of 67 h. Once a day, the cow is "milked" for its $^{99}$Tc, which is produced in an excited state by the $^{99}$Mo; the $^{99}$Tc de-excites to its lowest energy state by emitting a gamma-ray photon, which is recorded by detectors placed around the patient. The de-excitation has a half-life of 6.0 h. (a) By what process does $^{99}$Mo decay to $^{99}$Tc? (b) If a patient is injected with an $8.2 \times 10^7$ Bq sample of $^{99}$Tc, how many gamma-ray photons are initially produced within the patient each second? (c) If the emission rate of gamma-ray photons from a small tumor that has collected $^{99}$Tc is 38 per second at a certain time, how many excited-state $^{99}$Tc are located in the tumor at that time?

**47** M SSM After long effort, in 1902 Marie and Pierre Curie succeeded in separating from uranium ore the first substantial quantity of radium, one decigram of pure $RaCl_2$. The radium was the radioactive isotope $^{226}$Ra, which has a half-life of 1600 y. (a) How many radium nuclei had the Curies isolated? (b) What was the decay rate of their sample, in disintegrations per second?

## Module 42.4 Alpha Decay

**48** E How much energy is released when a $^{238}$U nucleus decays by emitting (a) an alpha particle and (b) a sequence of neutron, proton, neutron, proton? (c) Convince yourself both by reasoned argument and by direct calculation that the difference between these two numbers is just the total binding energy of the alpha particle. (d) Find that binding energy. Some needed atomic and particle masses are

| | | | |
|---|---|---|---|
| $^{238}$U | 238.050 79 u | $^{234}$Th | 234.043 63 u |
| $^{237}$U | 237.048 73 u | $^{4}$He | 4.002 60 u |
| $^{236}$Pa | 236.048 91 u | $^{1}$H | 1.007 83 u |
| $^{235}$Pa | 235.045 44 u | n | 1.008 66 u |

**49** E SSM Generally, more massive nuclides tend to be more unstable to alpha decay. For example, the most stable isotope of uranium, $^{238}$U, has an alpha decay half-life of $4.5 \times 10^9$ y. The most stable isotope of plutonium is $^{244}$Pu with an $8.0 \times 10^7$ y half-life, and for curium we have $^{248}$Cm and $3.4 \times 10^5$ y. When half of an original sample of $^{238}$U has decayed, what fraction of the original sample of (a) plutonium and (b) curium is left?

**50** M Large radionuclides emit an alpha particle rather than other combinations of nucleons because the alpha particle has such a stable, tightly bound structure. To confirm this statement, calculate the disintegration energies for these hypothetical decay processes and discuss the meaning of your findings:

(a) $^{235}\text{U} \to {}^{232}\text{Th} + {}^{3}\text{He}$, (b) $^{235}\text{U} \to {}^{231}\text{Th} + {}^{4}\text{He}$,
(c) $^{235}\text{U} \to {}^{230}\text{Th} + {}^{5}\text{He}$.

The needed atomic masses are

| | | | |
|---|---|---|---|
| $^{232}$Th | 232.0381 u | $^{3}$He | 3.0160 u |
| $^{231}$Th | 231.0363 u | $^{4}$He | 4.0026 u |
| $^{230}$Th | 230.0331 u | $^{5}$He | 5.0122 u |
| $^{235}$U | 235.0429 u | | |

**51** M A $^{238}$U nucleus emits a 4.196 MeV alpha particle. Calculate the disintegration energy $Q$ for this process, taking the recoil energy of the residual $^{234}$Th nucleus into account.

**52** M Under certain rare circumstances, a nucleus can decay by emitting a particle more massive than an alpha particle. Consider the decays

$$^{223}\text{Ra} \to {}^{209}\text{Pb} + {}^{14}\text{C} \quad \text{and} \quad {}^{223}\text{Ra} \to {}^{219}\text{Rn} + {}^{4}\text{He}.$$

Calculate the $Q$ value for the (a) first and (b) second decay and determine that both are energetically possible. (c) The Coulomb barrier height for alpha-particle emission is 30.0 MeV. What is the barrier height for $^{14}$C emission? (Be careful about the nuclear radii.) The needed atomic masses are

| | | | |
|---|---|---|---|
| $^{223}$Ra | 223.018 50 u | $^{14}$C | 14.003 24 u |
| $^{209}$Pb | 208.981 07 u | $^{4}$He | 4.002 60 u |
| $^{219}$Rn | 219.009 48 u | | |

## Module 42.5 Beta Decay

**53** E SSM The cesium isotope $^{137}$Cs is present in the fallout from aboveground detonations of nuclear bombs. Because it decays with a slow (30.2 y) half-life into $^{137}$Ba, releasing considerable energy in the process, it is of environmental concern. The atomic masses of the Cs and Ba are 136.9071 and 136.9058 u, respectively; calculate the total energy released in such a decay.

**54** E Some radionuclides decay by capturing one of their own atomic electrons, a $K$-shell electron, say. An example is

$$^{49}\text{V} + \text{e}^- \to {}^{49}\text{Ti} + \nu, \qquad T_{1/2} = 331 \text{ d}.$$

Show that the disintegration energy $Q$ for this process is given by

$$Q = (m_\text{V} - m_\text{Ti})c^2 - E_K,$$

where $m_\text{V}$ and $m_\text{Ti}$ are the atomic masses of $^{49}$V and $^{49}$Ti, respectively, and $E_K$ is the binding energy of the vanadium $K$-shell electron. (*Hint:* Put $\mathbf{m}_\text{V}$ and $\mathbf{m}_\text{Ti}$ as the corresponding nuclear masses and then add in enough electrons to use the atomic masses.)

**55** E A free neutron decays according to Eq. 42.5.3. If the neutron–hydrogen atom mass difference is 840 $\mu$u, what is the maximum kinetic energy $K_\text{max}$ possible for the electron produced in a neutron decay?

**56** E An electron is emitted from a middle-mass nuclide ($A$ = 150, say) with a kinetic energy of 1.0 MeV. (a) What is its de Broglie wavelength? (b) Calculate the radius of the emitting nucleus. (c) Can such an electron be confined as a standing wave in a “box” of such dimensions? (d) Can you use these numbers to disprove the (abandoned) argument that electrons actually exist in nuclei?

**57** M GO The radionuclide $^{11}$C decays according to

$$^{11}\text{C} \to {}^{11}\text{B} + \text{e}^+ + \nu, \qquad T_{1/2} = 20.3 \text{ min}.$$

The maximum energy of the emitted positrons is 0.960 MeV. (a) Show that the disintegration energy $Q$ for this process is given by

$$Q = (m_\text{C} - m_\text{B} - 2m_\text{e})c^2,$$

where $m_\text{C}$ and $m_\text{B}$ are the atomic masses of $^{11}$C and $^{11}$B, respectively, and $m_\text{e}$ is the mass of a positron. (b) Given the mass values $m_\text{C} = 11.011\,434$ u, $m_\text{B} = 11.009\,305$ u, and $m_\text{e} = 0.000\,548\,6$ u, calculate $Q$ and compare it with the maximum energy of the emitted positron given above. (*Hint:* Let $\mathbf{m}_\text{C}$ and $\mathbf{m}_\text{B}$ be the nuclear masses and then add in enough electrons to use the atomic masses.)

**58** M Two radioactive materials that alpha decay, $^{238}$U and $^{232}$Th, and one that beta decays, $^{40}$K, are sufficiently abundant in granite to contribute significantly to the heating of Earth through the decay energy produced. The alpha-decay isotopes give rise to decay chains that stop when stable lead isotopes are formed. The isotope $^{40}$K has a single beta decay. (Assume this is the only possible decay of that isotope.) Here is the information:

| Parent | Decay Mode | Half-Life (y) | Stable End Point | $Q$ (MeV) | $f$ (ppm) |
|---|---|---|---|---|---|
| $^{238}$U | $\alpha$ | $4.47 \times 10^9$ | $^{206}$Pb | 51.7 | 4 |
| $^{232}$Th | $\alpha$ | $1.41 \times 10^{10}$ | $^{208}$Pb | 42.7 | 13 |
| $^{40}$K | $\beta$ | $1.28 \times 10^9$ | $^{40}$Ca | 1.31 | 4 |

In the table $Q$ is the *total* energy released in the decay of one parent nucleus to the *final* stable end point and $f$ is the abundance of the isotope in kilograms per kilogram of granite; ppm means parts per million. (a) Show that these materials produce energy as heat at the rate of $1.0 \times 10^{-9}$ W for each kilogram of granite. (b) Assuming that there is $2.7 \times 10^{22}$ kg of granite in a 20-km-thick spherical shell at the surface of Earth, estimate the power of this decay process over all of Earth. Compare this power with the total solar power intercepted by Earth, $1.7 \times 10^{17}$ W.

**59** H SSM The radionuclide $^{32}$P decays to $^{32}$S as described by Eq. 42.5.1. In a particular decay event, a 1.71 MeV electron is emitted, the maximum possible value. What is the kinetic energy of the recoiling $^{32}S$ atom in this event? (*Hint:* For the electron it is necessary to use the relativistic expressions for kinetic energy and linear momentum. The $^{32}S$ atom is nonrelativistic.)

## Module 42.6 Radioactive Dating

**60** E A 5.00 g charcoal sample from an ancient fire pit has a $^{14}C$ activity of 63.0 disintegrations/min. A living tree has a $^{14}C$ activity of 15.3 disintegrations/min per 1.00 g. The half-life of $^{14}C$ is 5730 y. How old is the charcoal sample?

**61** E The isotope $^{238}U$ decays to $^{206}Pb$ with a half-life of $4.47 \times 10^9$ y. Although the decay occurs in many individual steps, the first step has by far the longest half-life; therefore, one can often consider the decay to go directly to lead. That is,

$$^{238}\mathrm{U} \rightarrow {}^{206}\mathrm{Pb} + \text{various decay products.}$$

A rock is found to contain 4.20 mg of $^{238}U$ and 2.135 mg of $^{206}Pb$. Assume that the rock contained no lead at formation, so all the lead now present arose from the decay of uranium. How many atoms of (a) $^{238}U$ and (b) $^{206}Pb$ does the rock now contain? (c) How many atoms of $^{238}U$ did the rock contain at formation? (d) What is the age of the rock?

**62** M A particular rock is thought to be 260 million years old. If it contains 3.70 mg of $^{238}U$, how much $^{206}Pb$ should it contain? See Problem 61.

**63** M GO A rock recovered from far underground is found to contain 0.86 mg of $^{238}U$, 0.15 mg of $^{206}Pb$, and 1.6 mg of $^{40}Ar$. How much $^{40}K$ will it likely contain? Assume that $^{40}K$ decays to only $^{40}Ar$ with a half-life of $1.25 \times 10^9$ y. Also assume that $^{238}U$ has a half-life of $4.47 \times 10^9$ y.

**64** H GO The isotope $^{40}K$ can decay to either $^{40}Ca$ or $^{40}Ar$; assume both decays have a half-life of $1.26 \times 10^9$ y. The ratio of the Ca produced to the Ar produced is 8.54/1 = 8.54. A sample originally had only $^{40}K$. It now has equal amounts of $^{40}K$ and $^{40}Ar$; that is, the ratio of K to Ar is 1/1 = 1. How old is the sample? (*Hint:* Work this like other radioactive-dating problems, except that this decay has two products.)

## Module 42.7 Measuring Radiation Dosage

**65** E BIO SSM The nuclide $^{198}Au$, with a half-life of 2.70 d, is used in cancer therapy. What mass of this nuclide is required to produce an activity of 250 Ci?

**66** E A radiation detector records 8700 counts in 1.00 min. Assuming that the detector records all decays, what is the activity of the radiation source in (a) becquerels and (b) curies?

**67** E BIO A 4.00 kg organic sample absorbs 2.00 mJ via slow neutron radiation (RBE = 5). What is the dose equivalent (mSv)?

**68** M BIO A 75 kg person receives a whole-body radiation dose of $2.4 \times 10^{-4}$ Gy, delivered by alpha particles for which the RBE factor is 12. Calculate (a) the absorbed energy in joules and the dose equivalent in (b) sieverts and (c) rem.

**69** M BIO An 85 kg worker at a breeder reactor plant accidentally ingests 2.5 mg of $^{239}Pu$ dust. This isotope has a half-life of 24 100 y, decaying by alpha decay. The energy of the emitted alpha particles is 5.2 MeV, with an RBE factor of 13. Assume that the plutonium resides in the worker's body for 12 h (it is eliminated naturally by the digestive system rather than being absorbed by any of the internal organs) and that 95% of the emitted alpha particles are stopped within the body. Calculate (a) the number of plutonium atoms ingested, (b) the number that decay during the 12 h, (c) the energy absorbed by the body, (d) the resulting physical dose in grays, and (e) the dose equivalent in sieverts.

## Module 42.8 Nuclear Models

**70** E A typical kinetic energy for a nucleon in a middle-mass nucleus may be taken as 5.00 MeV. To what effective nuclear temperature does this correspond, based on the assumptions of the collective model of nuclear structure?

**71** E A measurement of the energy $E$ of an intermediate nucleus must be made within the mean lifetime $\Delta t$ of the nucleus and necessarily carries an uncertainty $\Delta E$ according to the uncertainty principle

$$\Delta E \cdot \Delta t = \hbar.$$

(a) What is the uncertainty $\Delta E$ in the energy for an intermediate nucleus if the nucleus has a mean lifetime of $10^{-22}$ s? (b) Is the nucleus a compound nucleus?

**72** E In the following list of nuclides, identify (a) those with filled nucleon shells, (b) those with one nucleon outside a filled shell, and (c) those with one vacancy in an otherwise filled shell: $^{13}C$, $^{18}O$, $^{40}K$, $^{49}Ti$, $^{60}Ni$, $^{91}Zr$, $^{92}Mo$, $^{121}Sb$, $^{143}Nd$, $^{144}Sm$, $^{205}Tl$, and $^{207}Pb$.

**73** M SSM Consider the three formation processes shown for the compound nucleus $^{20}Ne$ in Fig. 42.8.1. Here are some of the atomic and particle masses:

| | | | |
|---|---|---|---|
| $^{20}Ne$ | 19.992 44 u | $\alpha$ | 4.002 60 u |
| $^{19}F$ | 18.998 40 u | p | 1.007 83 u |
| $^{16}O$ | 15.994 91 u | | |

What energy must (a) the alpha particle, (b) the proton, and (c) the $\gamma$-ray photon have to provide 25.0 MeV of excitation energy to the compound nucleus?

## Additional Problems

**74** GO In a certain rock, the ratio of lead atoms to uranium atoms is 0.300. Assume that uranium has a half-life of $4.47 \times 10^9$ y and that the rock had no lead atoms when it formed. How old is the rock?

**75** SSM A certain stable nuclide, after absorbing a neutron, emits an electron, and the new nuclide splits spontaneously into two alpha particles. Identify the nuclide.

**76** BIO A typical chest x-ray radiation dose is 250 $\mu$Sv, delivered by x rays with an RBE factor of 0.85. Assuming that the mass of the exposed tissue is one-half the patient's mass of 88 kg, calculate the energy absorbed in joules.

**77** SSM How many years are needed to reduce the activity of $^{14}C$ to 0.020 of its original activity? The half-life of $^{14}C$ is 5730 y.

**78** GO Radioactive element *AA* can decay to either element *BB* or element *CC*. The decay depends on chance, but the ratio of the resulting number of *BB* atoms to the resulting number of *CC* atoms is always 2/1. The decay has a half-life of 8.00 days. We start with a sample of pure *AA*. How long must we wait until the number of *CC* atoms is 1.50 times the number of *AA* atoms?

**79** GO BIO SSM One of the dangers of radioactive fallout from a nuclear bomb is its $^{90}Sr$, which decays with a 29-year half-life.

Because it has chemical properties much like those of calcium, the strontium, if ingested by a cow, becomes concentrated in the cow's milk. Some of the $^{90}$Sr ends up in the bones of whoever drinks the milk. The energetic electrons emitted in the beta decay of $^{90}$Sr damage the bone marrow and thus impair the production of red blood cells. A 1 megaton bomb produces approximately 400 g of $^{90}$Sr. If the fallout spreads uniformly over a 2000 km$^2$ area, what ground area would hold an amount of radioactivity equal to the "allowed" limit for one person, which is 74 000 counts/s?

**80** BIO Because of the 1986 explosion and fire in a reactor at the Chernobyl nuclear power plant in northern Ukraine, part of Ukraine is contaminated with $^{137}$Cs, which undergoes beta-minus decay with a half-life of 30.2 y. In 1996, the total activity of this contamination over an area of $2.6 \times 10^5$ km$^2$ was estimated to be $1 \times 10^{16}$ Bq. Assume that the $^{137}$Cs is uniformly spread over that area and that the beta-decay electrons travel either directly upward or directly downward. How many beta-decay electrons would you intercept were you to lie on the ground in that area for 1 h (a) in 1996 and (b) today? (You need to estimate your cross-sectional area that intercepts those electrons.)

**81** Figure 42.5 shows part of the decay scheme of $^{237}$Np on a plot of mass number $A$ versus proton number $Z$; five lines that represent either alpha decay or beta-minus decay connect dots that represent isotopes. What is the isotope at the end of the five decays (as marked with a question mark in Fig. 42.5)?

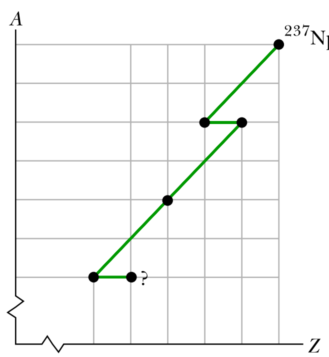


**Figure 42.5** Problem 81.

**82** After a brief neutron irradiation of silver, two isotopes are present: $^{108}$Ag ($T_{1/2} = 2.42$ min) with an initial decay rate of $3.1 \times 10^5$/s, and $^{110}$Ag ($T_{1/2} = 24.6$ s) with an initial decay rate of $4.1 \times 10^6$/s. Make a semilog plot similar to Fig. 42.3.1 showing the total combined decay rate of the two isotopes as a function of time from $t = 0$ until $t = 10$ min. We used Fig. 42.3.1 to illustrate the extraction of the half-life for simple (one isotope) decays. Given only your plot of total decay rate for the two-isotope system here, suggest a way to analyze it in order to find the half-lives of both isotopes.

**83** Because a nucleon is confined to a nucleus, we can take the uncertainty in its position to be the nuclear radius $r$. Use the uncertainty principle to determine the uncertainty $\Delta p$ in the linear momentum of the nucleon. Using the approximation $p \approx \Delta p$ and the fact that the nucleon is nonrelativistic, calculate the kinetic energy of the nucleon in a nucleus with $A = 100$.

**84** A radium source contains 1.00 mg of $^{226}$Ra, which decays with a half-life of 1600 y to produce $^{222}$Rn, a noble gas. This radon isotope in turn decays by alpha emission with a half-life of 3.82 d. If this process continues for a time much longer than the half-life of $^{222}$Rn, the $^{222}$Rn decay rate reaches a limiting value that matches the rate at which $^{222}$Rn is being produced, which is approximately constant because of the relatively long half-life of $^{226}$Ra. For the source under this limiting condition, what are (a) the activity of $^{226}$Ra, (b) the activity of $^{222}$Rn, and (c) the total mass of $^{222}$Rn?

**85** Make a nuclidic chart similar to Fig. 42.2.2 for the 25 nuclides $^{118-122}$Te, $^{117-121}$Sb, $^{116-120}$Sn, $^{115-119}$In, and $^{114-118}$Cd. Draw in and label (a) all isobaric (constant $A$) lines and (b) all lines of constant neutron excess, defined as $N - Z$.

**86** GO A projectile alpha particle is headed directly toward a target aluminum nucleus. Both objects are assumed to be spheres. What energy is required of the alpha particle if it is to momentarily stop just as its "surface" touches the "surface" of the aluminum nucleus? Assume that the target nucleus remains stationary.

**87** Consider a $^{238}$U nucleus to be made up of an alpha particle ($^4$He) and a residual nucleus ($^{234}$Th). Plot the electrostatic potential energy $U(r)$, where $r$ is the distance between these particles. Cover the approximate range 10 fm $< r <$ 100 fm and compare your plot with that of Fig. 42.4.1.

**88** Characteristic nuclear time is a useful but loosely defined quantity, taken to be the time required for a nucleon with a few million electron-volts of kinetic energy to travel a distance equal to the diameter of a middle-mass nuclide. What is the order of magnitude of this quantity? Consider 5 MeV neutrons traversing a nuclear diameter of $^{197}$Au; use Eq. 42.2.3.

**89** What is the likely mass number of a spherical nucleus with a radius of 3.6 fm as measured by electron-scattering methods?

**90** Using a nuclidic chart, write the symbols for (a) all stable isotopes with $Z = 60$, (b) all radioactive nuclides with $N = 60$, and (c) all nuclides with $A = 60$.

**91** If the unit for atomic mass were defined so that the mass of $^1$H were exactly 1.000 000 u, what would be the mass of (a) $^{12}$C (actual mass 12.000 000 u) and (b) $^{238}$U (actual mass 238.050 785 u)?

**92** High-mass radionuclides, which may be either alpha or beta emitters, belong to one of four decay chains, depending on whether their mass number $A$ is of the form $4n$, $4n + 1$, $4n + 2$, or $4n + 3$, where $n$ is a positive integer. (a) Justify this statement and show that if a nuclide belongs to one of these families, all its decay products belong to the same family. Classify the following nuclides as to family: (b) $^{235}$U, (c) $^{236}$U, (d) $^{238}$U, (e) $^{239}$Pu, (f) $^{240}$Pu, (g) $^{245}$Cm, (h) $^{246}$Cm, (i) $^{249}$Cf, and (j) $^{253}$Fm.

**93** Find the disintegration energy $Q$ for the decay of $^{49}$V by $K$-electron capture (see Problem 54). The needed data are $m_V =$ 48.948 52 u, $m_{Ti} = 48.947\,87$ u, and $E_K = 5.47$ keV.

**94** Locate the nuclides displayed in Table 42.2.1 on the nuclidic chart of Fig. 42.2.1. Verify that they lie in the stability zone.

**95** BIO The radionuclide $^{32}$P ($T_{1/2} = 14.28$ d) is often used as a tracer to follow the course of biochemical reactions involving phosphorus. (a) If the counting rate in a particular experimental setup is initially 3050 counts/s, how much time will the rate take to fall to 170 counts/s? (b) A solution containing $^{32}$P is fed to the root system of an experimental tomato plant, and the $^{32}$P activity in a leaf is measured 3.48 days later. By what factor must this reading be multiplied to correct for the decay that has occurred since the experiment began?

**96** CALC At the end of World War II, Dutch authorities arrested Dutch artist Hans van Meegeren for treason because, during the war, he had sold a masterpiece painting to the Nazi Hermann Goering. The painting, *Christ and His Disciples at Emmaus* by Dutch master Johannes Vermeer (1632–1675), had been discovered in 1937 by van Meegeren, after it had been lost for almost 300 years. Soon after the discovery, art experts proclaimed that *Emmaus* was possibly the best Vermeer ever seen. Selling such a Dutch national treasure to the enemy was unthinkable treason.

However, shortly after being imprisoned, van Meegeren suddenly announced that he, not Vermeer, had painted *Emmaus*. He explained that he had carefully mimicked Vermeer's style, using a 300-year-old canvas and Vermeer's choice of pigments; he had then signed Vermeer's name to the work and baked the painting to give it an authentically old look.

Was van Meegeren lying to avoid a conviction of treason, hoping to be convicted of only the lesser crime of fraud? To art experts, *Emmaus* certainly looked like a Vermeer but, at the time of van Meegeren's trial in 1947, there was no scientific way to answer the question. However, in 1968 Bernard Keisch of Carnegie-Mellon University was able to answer the question with newly developed techniques of radioactive analysis.

Specifically, he analyzed a small sample of white lead-bearing pigment removed from *Emmaus*. This pigment is refined from lead ore, in which the lead is produced by a long radioactive decay series that starts with unstable $^{238}$U and ends with stable $^{206}$Pb. To follow the spirit of Keisch's analysis, focus on the following abbreviated portion of that decay series, in which intermediate, relatively short-lived radionuclides have been omitted:

$$^{230}\text{Th} \xrightarrow[75.4\text{ ky}]{} {}^{226}\text{Ra} \xrightarrow[1.60\text{ ky}]{} {}^{210}\text{Pb} \xrightarrow[22.6\text{ ky}]{} {}^{206}\text{Pb}.$$

The longer and more important half-lives in this portion of the decay series are indicated.

(a) Show that in a sample of lead ore, the rate at which the number of $^{210}$Pb nuclei changes is given by

$$\frac{dN_{210}}{dt} = \lambda_{226}N_{226} - \lambda_{210}N_{210},$$

where $N_{210}$ and $N_{226}$ are the numbers of $^{210}$Pb nuclei and $^{226}$Ra nuclei in the sample and $\lambda_{210}$ and $\lambda_{226}$ are the corresponding disintegration constants.

Because the decay series has been active for billions of years and because the half-life of $^{210}$Pb is much less than that of $^{226}$Ra, the nuclides $^{226}$Ra and $^{210}$Pb are in *equilibrium*; that is, the numbers of these nuclides (and thus their concentrations) in the sample do not change. (b) What is the ratio $R_{226}/R_{210}$ of the activities of these nuclides in the sample of lead ore? (c) What is the ratio $N_{226}/N_{210}$ of their numbers?

When lead pigment is refined from the ore, most of the $^{226}$Ra is eliminated. Assume that only 1.00% remains. Just after the pigment is produced, what are the ratios (d) $R_{226}/R_{210}$ and (e) $N_{226}/N_{210}$?

Keisch realized that with time the ratio $R_{226}/R_{210}$ of the pigment would gradually change from the value in freshly refined pigment back to the value in the ore, as equilibrium between the $^{210}$Pb and the remaining $^{226}$Ra is established in the pigment. If *Emmaus* were painted by Vermeer and the sample of pigment taken from it were 300 years old when examined in 1968, the ratio would be close to the answer of (b). If *Emmaus* were painted by van Meegeren in the 1930s and the sample were only about 30 years old, the ratio would be close to the answer of (d). Keisch found a ratio of 0.09. (f) Is *Emmaus* a Vermeer?

**97** From data presented in the first few paragraphs of Module 42.3, find (a) the disintegration constant $\lambda$ and (b) the half-life of $^{238}$U.