CHAPTER 36

Diffraction

36.1 SINGLE-SLIT DIFFRACTION

Learning Objectives

After reading this module, you should be able to . . .

36.1.1 Describe the diffraction of light waves by a narrow opening and an edge, and also describe the resulting interference pattern.

36.1.2 Describe an experiment that demonstrates the Fresnel bright spot.

36.1.3 With a sketch, describe the arrangement for a single-slit diffraction experiment.

36.1.4 With a sketch, explain how splitting a slit width into equal zones leads to the equations giving the angles to the minima in the diffraction pattern.

36.1.5 Apply the relationships between width a of a thin, rectangular slit or object, the wavelength λ, the angle θ to any of the minima in the diffraction pattern, the distance to a viewing screen, and the distance between a minimum and the center of the pattern.

36.1.6 Sketch the diffraction pattern for monochromatic light, identifying what lies at the center and what the various bright and dark fringes are called (such as "first minimum").

36.1.7 Identify what happens to a diffraction pattern when the wavelength of the light or the width of the diffracting aperture or object is varied.

Key Ideas

- When waves encounter an edge, an obstacle, or an aperture the size of which is comparable to the wavelength of the waves, those waves spread out as they travel and, as a result, undergo interference. This type of interference is called diffraction.
- Waves passing through a long narrow slit of width a produce, on a viewing screen, a single-slit diffraction pattern that includes a central maximum (bright fringe) and other maxima. They are separated by minima that are located relative to the central axis by angles θ:

$$a \sin\theta = m\lambda, \quad \text{for } m = 1, 2, 3, \ldots \quad \text{(minima)}.$$

- The maxima are located approximately halfway between minima.

What Is Physics?

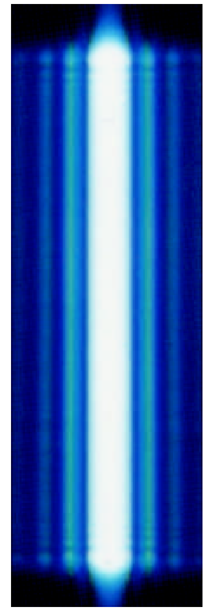
One focus of physics in the study of light is to understand and put to use the diffraction of light as it passes through a narrow slit or (as we shall discuss) past either a narrow obstacle or an edge. We touched on this phenomenon in Chapter 35 when we looked at how light flared—diffracted—through the slits in Young's experiment. Diffraction through a given slit is more complicated than simple flaring, however, because the light also interferes with itself and produces an interference pattern. It is because of such complications that light is rich with application opportunities. Even though the diffraction of light as it passes through a slit or past an obstacle seems awfully academic, countless engineers and scientists make their living using this physics, and the total worth of diffraction applications worldwide is probably incalculable.

Before we can discuss some of these applications, we first must discuss why diffraction is due to the wave nature of light.

Diffraction and the Wave Theory of Light

In Chapter 35 we defined diffraction rather loosely as the flaring of light as it emerges from a narrow slit. More than just flaring occurs, however, because the light produces an interference pattern called a **diffraction pattern.** For example, when monochromatic light from a distant source (or a laser) passes through a narrow slit and is then intercepted by a viewing screen, the light produces on the screen a diffraction pattern like that in Fig. 36.1.1. This pattern consists of a broad and intense (very bright) central maximum plus a number of narrower and less intense maxima (called **secondary** or **side** maxima) to both sides. In between the maxima are minima. Light flares into those dark regions, but the light waves cancel out one another.

Such a pattern would be totally unexpected in geometrical optics: If light traveled in straight lines as rays, then the slit would allow some of those rays through to form a sharp rendition of the slit on the viewing screen instead of a pattern of bright and dark bands as we see in Fig. 36.1.1. As in Chapter 35, we must conclude that geometrical optics is only an approximation.



Ken Kay/Fundamental Photographs

Figure 36.1.1 This diffraction pattern appeared on a viewing screen when light that had passed through a narrow vertical slit reached the screen. Diffraction caused the light to flare out perpendicular to the long sides of the slit. That flaring produced an interference pattern consisting of a broad central maximum plus less intense and narrower secondary (or side) maxima, with minima between them.

Edges. Diffraction is not limited to situations in which light passes through a narrow opening (such as a slit or pinhole). It also occurs when light passes an edge, such as the edges of the razor blade whose diffraction pattern is shown in Fig. 36.1.2. Note the lines of maxima and minima that run approximately parallel to the edges, at both the inside edges of the blade and the outside edges. As the light passes, say, the vertical edge at the left, it flares left and right and undergoes interference, producing the pattern along the left edge. The rightmost portion of that pattern actually lies behind the blade, within what would be the blade's shadow if geometrical optics prevailed.

Floaters. You encounter a common example of diffraction when you look at a clear blue sky and see tiny specks and hairlike structures floating in your view. These *floaters,* as they are called, are produced when light passes the edges of tiny deposits in the vitreous humor, the transparent material filling most of the eyeball. What you are seeing when a floater is in your field of vision is the diffraction pattern produced on the retina by one of these deposits. If you sight through a pinhole in a piece of cardboard so as to make the light entering your eye approximately a plane wave, you can distinguish individual maxima and minima in the patterns. FCP

Cheerleaders. Diffraction is a wave effect. That is, it occurs because light is a wave and it occurs with other types of waves as well. For example, you have probably seen diffraction in action at football games. When a cheerleader near the playing field yells up at several thousand noisy fans, the yell can hardly be heard because the sound waves diffract when they pass through the narrow opening of the cheerleader's mouth. This flaring leaves little of the waves traveling toward the fans in front of the cheerleader. To offset the diffraction, the cheerleader can yell through a megaphone. The sound waves then emerge from the much wider opening at the end of the megaphone. The flaring is thus reduced, and much more of the sound reaches the fans in front of the cheerleader. FCP

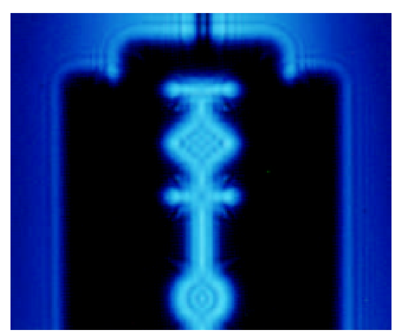


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Figure 36.1.2 The diffraction pattern produced by a razor blade in monochromatic light. Note the lines of alternating maximum and minimum intensity.

The Fresnel Bright Spot

Diffraction finds a ready explanation in the wave theory of light. However, this theory, originally advanced in the late 1600s by Huygens and used 123 years later by Young to explain double-slit interference, was very slow in being adopted, largely because it ran counter to Newton's theory that light was a stream of particles.

Newton's view was the prevailing view in French scientific circles of the early 19th century, when Augustin Fresnel was a young military engineer. Fresnel, who believed in the wave theory of light, submitted a paper to the French Academy of Sciences describing his experiments with light and his wave-theory explanations of them.

In 1819, the Academy, dominated by supporters of Newton and thinking to challenge the wave point of view, organized a prize competition for an essay on the subject of diffraction. Fresnel won. The Newtonians, however, were not swayed.

Courtesy of Jearl Walker

Figure 36.1.3 A photograph of the diffraction pattern of a disk. Note the concentric diffraction rings and the Fresnel bright spot at the center of the pattern. This experiment is essentially identical to that arranged by the committee testing Fresnel's theories, because both the sphere they used and the disk used here have a cross section with a circular edge.

One of them, S. D. Poisson, pointed out the "strange result" that if Fresnel's theories were correct, then light waves should flare into the shadow region of a sphere as they pass the edge of the sphere, producing a bright spot at the center of the shadow. The prize committee arranged a test of Poisson's prediction and discovered that the predicted *Fresnel bright spot,* as we call it today, was indeed there (Fig. 36.1.3). Nothing builds confidence in a theory so much as having one of its unexpected and counterintuitive predictions verified by experiment. FCP

Diffraction by a Single Slit: Locating the Minima

Let us now examine the diffraction pattern of plane waves of light of wavelength λ that are diffracted by a single long, narrow slit of width a in an otherwise opaque screen B, as shown in cross section in Fig. 36.1.4. (In that figure, the slit's length extends into and out of the page, and the incoming wavefronts are parallel to screen B.) When the diffracted light reaches viewing screen C, waves from different points within the slit undergo interference and produce a diffraction pattern of bright and dark fringes (interference maxima and minima) on the screen. To locate the fringes, we shall use a procedure somewhat similar to the one we used to locate the fringes in a two-slit interference pattern. However, diffraction is more mathematically challenging, and here we shall be able to find equations for only the dark fringes.

Before we do that, however, we can justify the central bright fringe seen in Fig. 36.1.1 by noting that the Huygens wavelets from all points in the slit travel about the same distance to reach the center of the pattern and thus are in phase there. As for the other bright fringes, we can say only that they are approximately halfway between adjacent dark fringes.

Pairings. To find the dark fringes, we shall use a clever (and simplifying) strategy that involves pairing up all the rays coming through the slit and then finding what conditions cause the wavelets of the rays in each pair to cancel each other. We apply this strategy in Fig. 36.1.4 to locate the first dark fringe, at point P_1. First, we mentally divide the slit into two *zones* of equal widths $a/2$. Then we extend to P_1 a light ray r_1 from the top point of the top zone and a light ray r_2 from the top point of the bottom zone. We want the wavelets along these two rays to cancel each other when they arrive at P_1. Then any similar pairing of rays from the two zones will give cancellation. A central axis is drawn from the center of the slit to screen C, and P_1 is located at an angle θ to that axis.

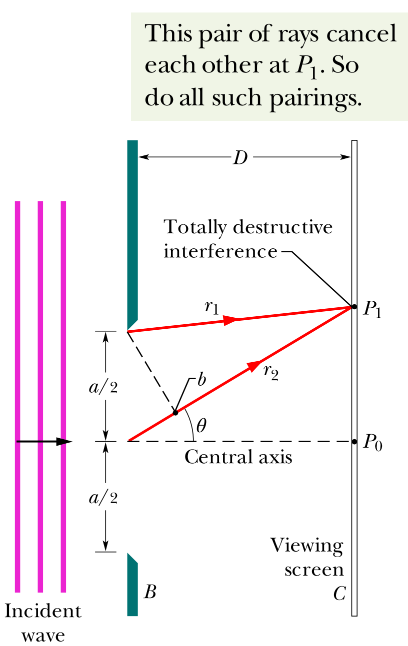

Figure 36.1.4 Waves from the top points of two zones of width $a/2$ undergo fully destructive interference at point P_1 on viewing screen C.

Path Length Difference. The wavelets of the pair of rays r_1 and r_2 are in phase within the slit because they originate from the same wavefront passing through the slit, along the width of the slit. However, to produce the first dark fringe they must be out of phase by $\lambda/2$ when they reach P_1; this phase difference is due to their path length difference, with the path traveled by the wavelet of r_2 to reach P_1 being longer than the path traveled by the wavelet of r_1. To display this path length difference, we find a point b on ray r_2 such that the path length from b to P_1 matches the path length of ray r_1. Then the path length difference between the two rays is the distance from the center of the slit to b.

When viewing screen C is near screen B, as in Fig. 36.1.4, the diffraction pattern on C is difficult to describe mathematically. However, we can simplify the mathematics considerably if we arrange for the screen separation D to be much larger than the slit width a. Then, as in Fig. 36.1.5, we can approximate rays r_1 and r_2 as being parallel, at angle θ to the central axis. We can also approximate the triangle formed by point b, the top point of the slit, and the center point of the slit as being a right triangle, and one of the angles inside that triangle as being θ. The path length difference between rays r_1 and r_2 (which is still the distance from the center of the slit to point b) is then equal to $(a/2)\sin\theta$.

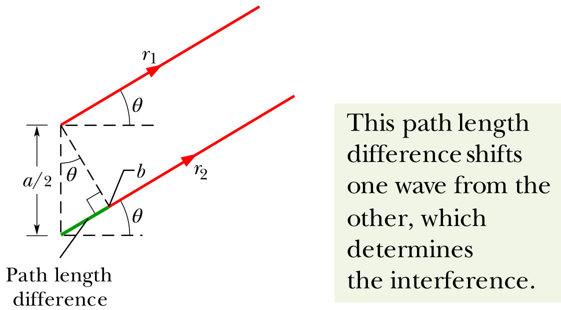

Figure 36.1.5 For $D \gg a$, we can approximate rays r_1 and r_2 as being parallel, at angle θ to the central axis.

First Minimum. We can repeat this analysis for any other pair of rays originating at corresponding points in the two zones (say, at the midpoints of the zones) and extending to point P_1. Each such pair of rays has the same path length difference $(a/2)\sin\theta$. Setting this common path length difference equal to $\lambda/2$ (our condition for the first dark fringe), we have

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2},$$

which gives us

$$a\sin\theta = \lambda \qquad \text{(first minimum).} \tag{36.1.1}$$

Given slit width a and wavelength λ, Eq. 36.1.1 tells us the angle θ of the first dark fringe above and (by symmetry) below the central axis.

Narrowing the Slit. Note that if we begin with $a > \lambda$ and then narrow the slit while holding the wavelength constant, we increase the angle at which the first dark fringes appear; that is, the extent of the diffraction (the extent of the flaring and the width of the pattern) is *greater* for a *narrower* slit. When we have reduced the slit width to the wavelength (that is, $a = \lambda$), the angle of the first dark fringes is 90°. Since the first dark fringes mark the two edges of the central bright fringe, that bright fringe must then cover the entire viewing screen.

Second Minimum. We find the second dark fringes above and below the central axis as we found the first dark fringes, except that we now divide the slit into *four* zones of equal widths $a/4$, as shown in Fig. 36.1.6*a*. We then extend rays r_1, r_2, r_3, and r_4 from the top points of the zones to point P_2, the location of the second dark fringe above the central axis. To produce that fringe, the path length difference between r_1 and r_2, that between r_2 and r_3, and that between r_3 and r_4 must all be equal to $\lambda/2$.

For $D \gg a$, we can approximate these four rays as being parallel, at angle θ to the central axis. To display their path length differences, we extend a perpendicular line through each adjacent pair of rays, as shown in Fig. 36.1.6*b*, to form a series of right triangles, each of which has a path length difference as one side. We see from the top triangle that the path length difference between r_1 and r_2 is $(a/4)\sin\theta$. Similarly, from the bottom triangle, the path length difference between r_3 and r_4 is also $(a/4)\sin\theta$. In fact, the path length difference for any two rays that originate at corresponding points in two adjacent zones is $(a/4)\sin\theta$. Since in each such case the path length difference is equal to $\lambda/2$, we have

$$\frac{a}{4}\sin\theta = \frac{\lambda}{2},$$

which gives us

$$a\sin\theta = 2\lambda \qquad \text{(second minimum).} \tag{36.1.2}$$

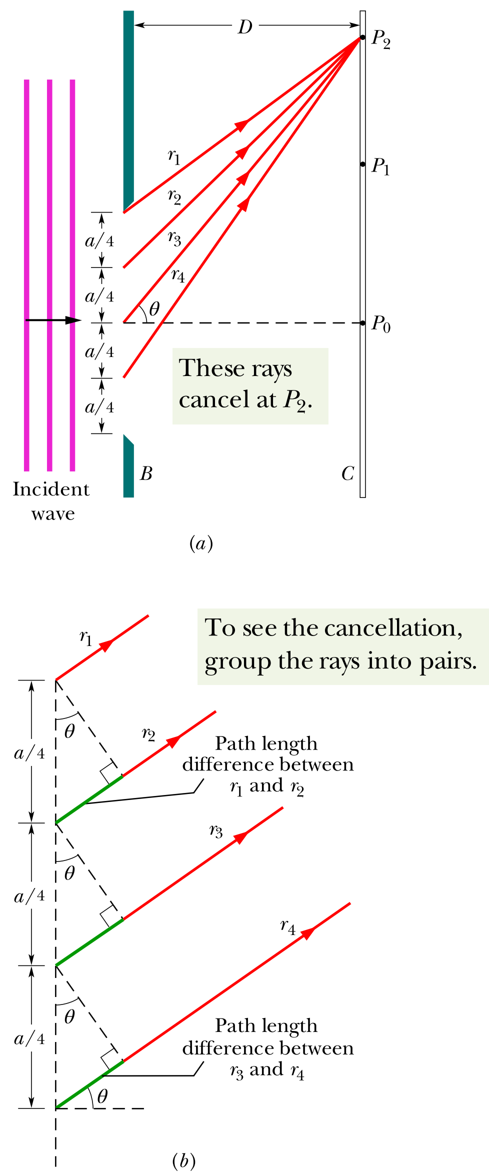

Figure 36.1.6 (a) Waves from the top points of four zones of width $a/4$ undergo fully destructive interference at point P_2. (b) For $D \gg a$, we can approximate rays r_1, r_2, r_3, and r_4 as being parallel, at angle θ to the central axis.

All Minima. We could now continue to locate dark fringes in the diffraction pattern by splitting up the slit into more zones of equal width. We would always choose an even number of zones so that the zones (and their waves) could be paired as we have been doing. We would find that the dark fringes above and below the central axis can be located with the general equation

$$a \sin\theta = m\lambda, \qquad \text{for } m = 1, 2, 3, \ldots \quad \text{(minima—dark fringes)}. \tag{36.1.3}$$

You can remember this result in the following way. Draw a triangle like the one in Fig. 36.1.5, but for the full slit width a, and note that the path length difference between the top and bottom rays equals $a \sin\theta$. Thus, Eq. 36.1.3 says:

> In a single-slit diffraction experiment, dark fringes are produced where the path length differences ($a \sin\theta$) between the top and bottom rays are equal to $\lambda, 2\lambda, 3\lambda, \ldots$.

This may seem to be wrong because the waves of those two particular rays will be exactly in phase with each other when their path length difference is an integer number of wavelengths. However, they each will still be part of a pair of waves that are exactly out of phase with each other; thus, *each* wave will be canceled by some other wave, resulting in darkness. (Two light waves that are exactly out of phase will always cancel each other, giving a net wave of zero, even if they happen to be exactly in phase with other light waves.)

Using a Lens. Equations 36.1.1, 36.1.2, and 36.1.3 are derived for the case of $D \gg a$. However, they also apply if we place a converging lens between the slit and the viewing screen and then move the screen in so that it coincides with the focal plane of the lens. The lens ensures that rays which now reach any point on the screen are *exactly* parallel (rather than approximately) back at the slit. They are like the initially parallel rays of Fig. 34.4.1*a* that are directed to the focal point by a converging lens.

Checkpoint 36.1.1

We produce a diffraction pattern on a viewing screen by means of a long narrow slit illuminated by blue light. Does the pattern expand away from the bright center (the maxima and minima shift away from the center) or contract toward it if we (a) switch to yellow light or (b) decrease the slit width?

Sample Problem 36.1.1 Single-slit diffraction pattern with white light

A slit of width a is illuminated by white light.

(a) For what value of a will the first minimum for red light of wavelength $\lambda = 650$ nm appear at $\theta = 15°$?

KEY IDEA

Diffraction occurs separately for each wavelength in the range of wavelengths passing through the slit, with the locations of the minima for each wavelength given by Eq. 36.1.3 ($a \sin\theta = m\lambda$).

Calculation: When we set $m = 1$ (for the first minimum) and substitute the given values of θ and λ, Eq. 36.1.3 yields

$$a = \frac{m\lambda}{\sin\theta} = \frac{(1)(650\text{ nm})}{\sin 15°}$$
$$= 2511\text{ nm} \approx 2.5\ \mu\text{m}. \qquad \text{(Answer)}$$

For the incident light to flare out that much ($\pm 15°$ to the first minima) the slit has to be very fine indeed—in this case, a mere four times the wavelength. For comparison, note that a fine human hair may be about 100 μm in diameter.

(b) What is the wavelength λ' of the light whose first side diffraction maximum is at 15°, thus coinciding with the first minimum for the red light?

KEY IDEA

The first side maximum for any wavelength is about halfway between the first and second minima for that wavelength.

Calculations: Those first and second minima can be located with Eq. 36.1.3 by setting $m = 1$ and $m = 2$, respectively. Thus, the first side maximum can be located *approximately* by setting $m = 1.5$. Then Eq. 36.1.3 becomes

$$a \sin \theta = 1.5\lambda'.$$

Solving for λ' and substituting known data yield

$$\lambda' = \frac{a \sin \theta}{1.5} = \frac{(2511 \text{ nm})(\sin 15°)}{1.5}$$
$$= 430 \text{ nm}. \qquad \text{(Answer)}$$

Light of this wavelength is violet (far blue, near the short-wavelength limit of the human range of visible light). From the two equations we used, can you see that the first side maximum for light of wavelength 430 nm will always coincide with the first minimum for light of wavelength 650 nm, no matter what the slit width is? However, the angle θ at which this overlap occurs does depend on slit width. If the slit is relatively narrow, the angle will be relatively large, and conversely.

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36.2 INTENSITY IN SINGLE-SLIT DIFFRACTION

Learning Objectives

After reading this module, you should be able to . . .

36.2.1 Divide a thin slit into multiple zones of equal width and write an expression for the phase difference of the wavelets from adjacent zones in terms of the angle θ to a point on the viewing screen.

36.2.2 For single-slit diffraction, draw phasor diagrams for the central maximum and several of the minima and maxima off to one side, indicating the phase difference between adjacent phasors, explaining how the net electric field is calculated, and identifying the corresponding part of the diffraction pattern.

36.2.3 Describe a diffraction pattern in terms of the net electric field at points in the pattern.

36.2.4 Evaluate α, the convenient connection between angle θ to a point in a diffraction pattern and the intensity I at that point.

36.2.5 For a given point in a diffraction pattern, at a given angle, calculate the intensity I in terms of the intensity I_m at the center of the pattern.

Key Idea

- The intensity of the diffraction pattern at any given angle θ is

$$I(\theta) = I_m\left(\frac{\sin \alpha}{\alpha}\right)^2,$$

where I_m is the intensity at the center of the pattern and

$$\alpha = \frac{\pi a}{\lambda} \sin \theta.$$

Intensity in Single-Slit Diffraction, Qualitatively

In Module 36.1 we saw how to find the positions of the minima and the maxima in a single-slit diffraction pattern. Now we turn to a more general problem: Find an expression for the intensity I of the pattern as a function of θ, the angular position of a point on a viewing screen.

To do this, we divide the slit of Fig. 36.1.4 into N zones of equal widths Δx small enough that we can assume each zone acts as a source of Huygens wavelets. We wish to superimpose the wavelets arriving at an arbitrary point P on the viewing screen, at angle θ to the central axis, so that we can determine the amplitude E_θ

of the electric component of the resultant wave at P. The intensity of the light at P is then proportional to the square of that amplitude.

To find E_θ, we need the phase relationships among the arriving wavelets. The point here is that in general they have different phases because they travel different distances to reach P. The phase difference between wavelets from adjacent zones is given by

$$\begin{pmatrix}\text{phase}\\ \text{difference}\end{pmatrix} = \left(\frac{2\pi}{\lambda}\right)\begin{pmatrix}\text{path length}\\ \text{difference}\end{pmatrix}.$$

For point P at angle θ, the path length difference between wavelets from adjacent zones is $\Delta x \sin\theta$. Thus, we can write the phase difference $\Delta\phi$ between wavelets from adjacent zones as

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x \sin\theta). \qquad (36.2.1)$$

We assume that the wavelets arriving at P all have the same amplitude ΔE. To find the amplitude E_θ of the resultant wave at P, we add the amplitudes ΔE via phasors. To do this, we construct a diagram of N phasors, one corresponding to the wavelet from each zone in the slit.

Central Maximum. For point P_0 at $\theta = 0$ on the central axis of Fig. 36.1.4, Eq. 36.2.1 tells us that the phase difference $\Delta\phi$ between the wavelets is zero; that is, the wavelets all arrive in phase. Figure 36.2.1*a* is the corresponding phasor diagram; adjacent phasors represent wavelets from adjacent zones and are arranged head to tail. Because there is zero phase difference between the wavelets, there is zero angle between each pair of adjacent phasors. The amplitude E_θ of the net wave at P_0 is the vector sum of these phasors. This arrangement of the phasors turns out to be the one that gives the greatest value for the amplitude E_θ. We call this value E_m; that is, E_m is the value of E_θ for $\theta = 0$.

We next consider a point P that is at a small angle θ to the central axis. Equation 36.2.1 now tells us that the phase difference $\Delta\phi$ between wavelets from adjacent zones is no longer zero. Figure 36.2.1*b* shows the corresponding phasor diagram; as before, the phasors are arranged head to tail, but now there is an angle $\Delta\phi$ between adjacent phasors. The amplitude E_θ at this new point is still the vector sum of the phasors, but it is smaller than that in Fig. 36.2.1*a*, which means that the intensity of the light is less at this new point P than at P_0.

First Minimum. If we continue to increase θ, the angle $\Delta\phi$ between adjacent phasors increases, and eventually the chain of phasors curls completely around so that the head of the last phasor just reaches the tail of the first phasor (Fig. 36.2.1*c*). The amplitude E_θ is now zero, which means that the intensity of the light is also zero. We have reached the first minimum, or dark fringe, in the diffraction pattern. The first and last phasors now have a phase difference of 2π rad, which means that the path length difference between the top and bottom rays through the slit equals one wavelength. Recall that this is the condition we determined for the first diffraction minimum.

First Side Maximum. As we continue to increase θ, the angle $\Delta\phi$ between adjacent phasors continues to increase, the chain of phasors begins to wrap back on itself, and the resulting coil begins to shrink. Amplitude E_θ now increases until it reaches a maximum value in the arrangement shown in Fig. 36.2.1*d*. This arrangement corresponds to the first side maximum in the diffraction pattern.

Second Minimum. If we increase θ a bit more, the resulting shrinkage of the coil decreases E_θ, which means that the intensity also decreases. When θ is increased enough, the head of the last phasor again meets the tail of the first phasor. We have then reached the second minimum.

We could continue this qualitative method of determining the maxima and minima of the diffraction pattern but, instead, we shall now turn to a quantitative method.

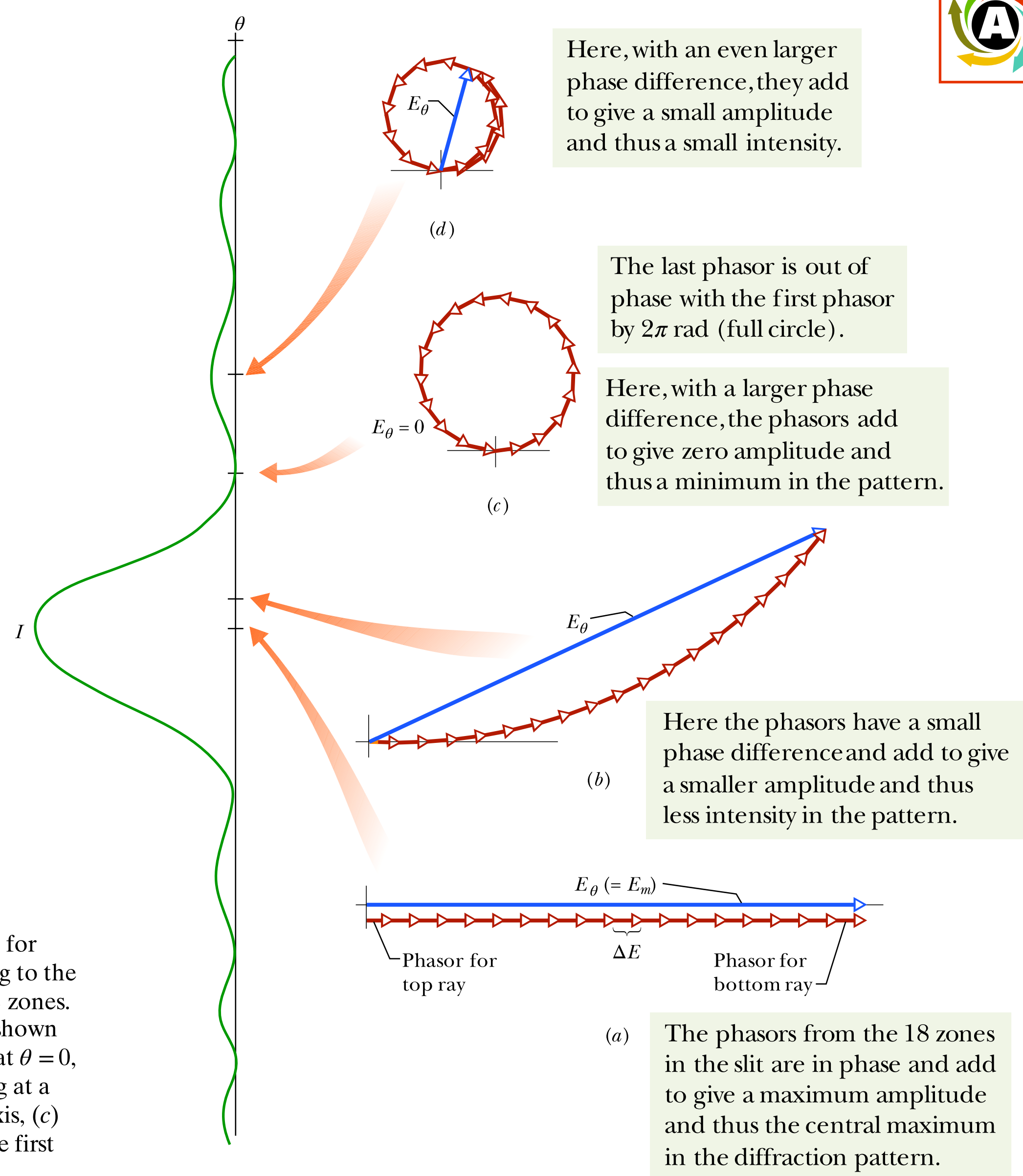
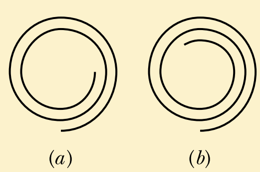


Figure 36.2.1 Phasor diagrams for $N = 18$ phasors, corresponding to the division of a single slit into 18 zones. Resultant amplitudes E_θ are shown for (*a*) the central maximum at $\theta = 0$, (*b*) a point on the screen lying at a small angle θ to the central axis, (*c*) the first minimum, and (*d*) the first side maximum.

Checkpoint 36.2.1

The figures represent, in smoother form (with more phasors) than Fig. 36.2.1, the phasor diagrams for two points of a diffraction pattern that are on opposite sides of a certain diffraction maximum. (a) Which maximum is it? (b) What is the approximate value of m (in Eq. 36.1.3) that corresponds to this maximum?



Intensity in Single-Slit Diffraction, Quantitatively

Equation 36.1.3 tells us how to locate the minima of the single-slit diffraction pattern on screen C of Fig. 36.1.4 as a function of the angle θ in that figure. Here we wish to derive an expression for the intensity $I(\theta)$ of the pattern as a function of θ. We state, and shall prove below, that the intensity is given by

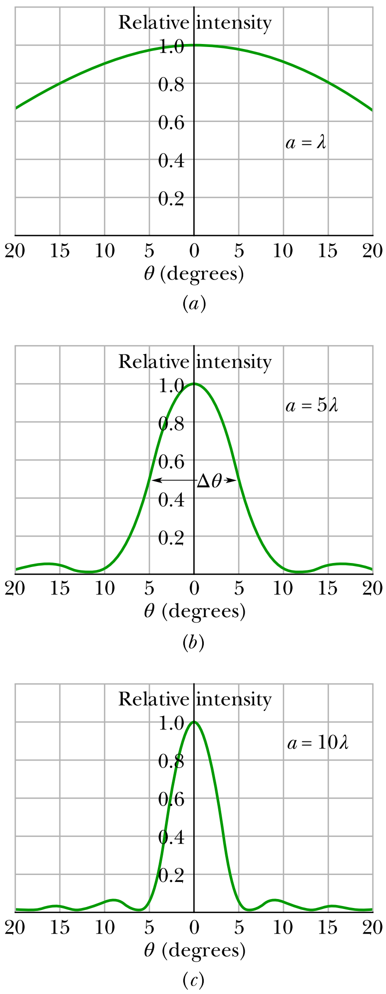


Figure 36.2.2 The relative intensity in single-slit diffraction for three values of the ratio a/λ. The wider the slit is, the narrower is the central diffraction maximum.

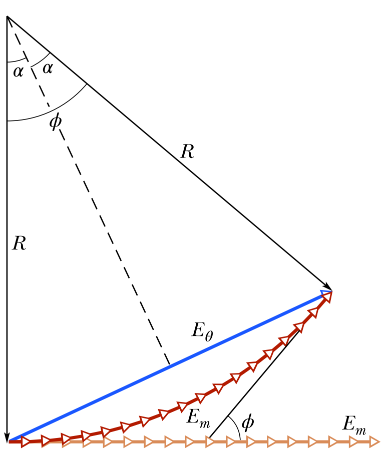


Figure 36.2.3 A construction used to calculate the intensity in single-slit diffraction. The situation shown corresponds to that of Fig. 36.2.1*b*.

$$I(\theta) = I_m \left(\frac{\sin \alpha}{\alpha}\right)^2, \tag{36.2.2}$$

where
$$\alpha = \tfrac{1}{2}\phi = \frac{\pi a}{\lambda} \sin \theta. \tag{36.2.3}$$

The symbol α is just a convenient connection between the angle θ that locates a point on the viewing screen and the light intensity $I(\theta)$ at that point. The intensity I_m is the greatest value of the intensities $I(\theta)$ in the pattern and occurs at the central maximum (where $\theta = 0$), and ϕ is the phase difference (in radians) between the top and bottom rays from the slit of width a.

Study of Eq. 36.2.2 shows that intensity minima will occur where

$$\alpha = m\pi, \qquad \text{for } m = 1, 2, 3, \ldots. \tag{36.2.4}$$

If we put this result into Eq. 36.2.3, we find

$$m\pi = \frac{\pi a}{\lambda} \sin \theta, \qquad \text{for } m = 1,2,3,\ldots,$$

or
$$a \sin \theta = m\lambda, \qquad \text{for } m = 1, 2, 3, \ldots \quad \text{(minima—dark fringes)}, \tag{36.2.5}$$

which is exactly Eq. 36.1.3, the expression that we derived earlier for the location of the minima.

Plots. Figure 36.2.2 shows plots of the intensity of a single-slit diffraction pattern, calculated with Eqs. 36.2.2 and 36.2.3 for three slit widths: $a = \lambda$, $a = 5\lambda$, and $a = 10\lambda$. Note that as the slit width increases (relative to the wavelength), the width of the *central diffraction maximum* (the central hill-like region of the graphs) decreases; that is, the light undergoes less flaring by the slit. The secondary maxima also decrease in width (and become weaker). In the limit of slit width a being much greater than wavelength λ, the secondary maxima due to the slit disappear; we then no longer have single-slit diffraction (but we still have diffraction due to the edges of the wide slit, like that produced by the edges of the razor blade in Fig. 36.1.2).

Proof of Eqs. 36.2.2 and 36.2.3

To find an expression for the intensity at a point in the diffraction pattern, we need to divide the slit into many zones and then add the phasors corresponding to those zones, as we did in Fig. 36.2.1. The arc of phasors in Fig. 36.2.3 represents the wavelets that reach an arbitrary point P on the viewing screen of Fig. 36.1.4, corresponding to a particular small angle θ. The amplitude E_θ of the resultant wave at P is the vector sum of these phasors. If we divide the slit of Fig. 36.1.4 into infinitesimal zones of width Δx, the arc of phasors in Fig. 36.2.3 approaches the arc of a circle; we call its radius R as indicated in that figure. The length of the arc must be E_m, the amplitude at the center of the diffraction pattern, because if we straightened out the arc we would have the phasor arrangement of Fig. 36.2.1*a* (shown lightly in Fig. 36.2.3).

The angle ϕ in the lower part of Fig. 36.2.3 is the difference in phase between the infinitesimal vectors at the left and right ends of arc E_m. From the geometry, ϕ is also the angle between the two radii marked R in Fig. 36.2.3. The dashed line in that figure, which bisects ϕ, then forms two congruent right triangles. From either triangle we can write

$$\sin \tfrac{1}{2}\phi = \frac{E_\theta}{2R}. \tag{36.2.6}$$

In radian measure, ϕ is (with E_m considered to be a circular arc)

$$\phi = \frac{E_m}{R}.$$

Solving this equation for R and substituting in Eq. 36.2.6 lead to

$$E_\theta = \frac{E_m}{\frac{1}{2}\phi}\sin\tfrac{1}{2}\phi. \tag{36.2.7}$$

Intensity. In Module 33.2 we saw that the intensity of an electromagnetic wave is proportional to the square of the amplitude of its electric field. Here, this means that the maximum intensity I_m (at the center of the pattern) is proportional to E_m^2 and the intensity $I(\theta)$ at angle θ is proportional to E_θ^2. Thus,

$$\frac{I(\theta)}{I_m} = \frac{E_\theta^2}{E_m^2}. \tag{36.2.8}$$

Substituting for E_θ with Eq. 36.2.7 and then substituting $\alpha = \frac{1}{2}\phi$, we are led to Eq. 36.2.2 for the intensity as a function of θ:

$$I(\theta) = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2.$$

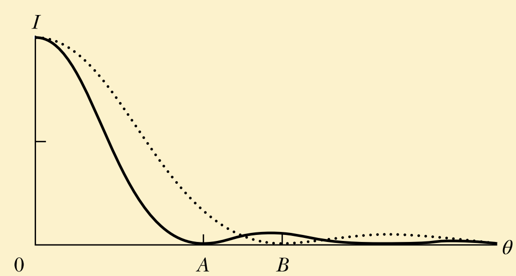
The second equation we wish to prove relates α to θ. The phase difference ϕ between the rays from the top and bottom of the entire slit may be related to a path length difference with Eq. 36.2.1; it tells us that

$$\phi = \left(\frac{2\pi}{\lambda}\right)(a\sin\theta),$$

where a is the sum of the widths Δx of the infinitesimal zones. However, $\phi = 2\alpha$, so this equation reduces to Eq. 36.2.3.

Checkpoint 36.2.2

Two wavelengths, 650 and 430 nm, are used separately in a single-slit diffraction experiment. The figure shows the results as graphs of intensity I versus angle θ for the two diffraction patterns. If both wavelengths are then used simultaneously, what color will be seen in the combined diffraction pattern at (a) angle A and (b) angle B?



Sample Problem 36.2.1 Intensities of the maxima in a single-slit interference pattern

Find the intensities of the first three secondary maxima (side maxima) in the single-slit diffraction pattern of Fig. 36.1.1, measured as a percentage of the intensity of the central maximum.

KEY IDEAS

The secondary maxima lie approximately halfway between the minima, whose angular locations are given by Eq. 36.2.4 ($\alpha = m\pi$). The locations of the secondary maxima are then given (approximately) by

$$a = \left(m + \tfrac{1}{2}\right)\pi, \qquad \text{for } m = 1, 2, 3, \ldots,$$

with α in radian measure. We can relate the intensity I at any point in the diffraction pattern to the intensity I_m of the central maximum via Eq. 36.2.2.

Calculations: Substituting the approximate values of α for the secondary maxima into Eq. 36.2.2 to obtain the relative intensities at those maxima, we get

$$\frac{I}{I_m} = \left(\frac{\sin\alpha}{\alpha}\right)^2 = \left(\frac{\sin\left(m+\frac{1}{2}\right)\pi}{\left(m+\frac{1}{2}\right)\pi}\right)^2, \quad \text{for } m = 1, 2, 3, \ldots.$$

The first of the secondary maxima occurs for $m = 1$, and its relative intensity is

$$\frac{I_1}{I_m} = \left(\frac{\sin(1 + \frac{1}{2})\pi}{(1 + \frac{1}{2})\pi}\right)^2 = \left(\frac{\sin 1.5\pi}{1.5\pi}\right)^2$$

$$= 4.50 \times 10^{-2} \approx 4.5\%. \qquad \text{(Answer)}$$

For $m = 2$ and $m = 3$ we find that

$$\frac{I_2}{I_m} = 1.6\% \quad \text{and} \quad \frac{I_3}{I_m} = 0.83\%. \qquad \text{(Answer)}$$

As you can see from these results, successive secondary maxima decrease rapidly in intensity. Figure 36.1.1 was deliberately overexposed to reveal them.

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36.3 DIFFRACTION BY A CIRCULAR APERTURE

Learning Objectives

After reading this module, you should be able to . . .

36.3.1 Describe and sketch the diffraction pattern from a small circular aperture or obstacle.

36.3.2 For diffraction by a small circular aperture or obstacle, apply the relationships between the angle θ to the first minimum, the wavelength λ of the light, the diameter d of the aperture, the distance D to a viewing screen, and the distance y between the minimum and the center of the diffraction pattern.

36.3.3 By discussing the diffraction patterns of point objects, explain how diffraction limits visual resolution of objects.

36.3.4 Identify that Rayleigh's criterion for resolvability gives the (approximate) angle at which two point objects are just barely resolvable.

36.3.5 Apply the relationships between the angle θ_R in Rayleigh's criterion, the wavelength λ of the light, the diameter d of the aperture (for example, the diameter of the pupil of an eye), the angle θ subtended by two distant point objects, and the distance L to those objects.

Key Ideas

- Diffraction by a circular aperture or a lens with diameter d produces a central maximum and concentric maxima and minima, with the first minimum at an angle θ given by

$$\sin\theta = 1.22\,\frac{\lambda}{d} \quad \text{(first minimum—circular aperture)}.$$

- Rayleigh's criterion suggests that two objects are on the verge of resolvability if the central diffraction maximum of one is at the first minimum of the other. Their angular separation can then be no less than

$$\theta_R = 1.22\,\frac{\lambda}{d} \quad \text{(Rayleigh's criterion)},$$

in which d is the diameter of the aperture through which the light passes.

Diffraction by a Circular Aperture

Here we consider diffraction by a circular aperture—that is, a circular opening, such as a circular lens, through which light can pass. Figure 36.3.1 shows the image formed by light from a laser that was directed onto a circular aperture with a very small diameter. This image is not a point, as geometrical optics would suggest, but a circular disk surrounded by several progressively fainter secondary rings. Comparison with Fig. 36.1.1 leaves little doubt that we are dealing with a diffraction phenomenon. Here, however, the aperture is a circle of diameter d rather than a rectangular slit.

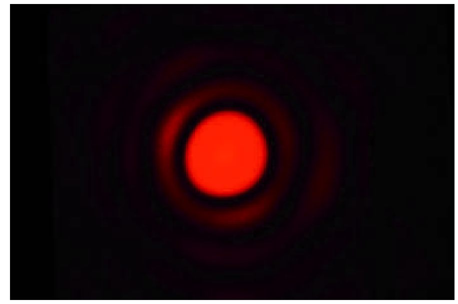
The (complex) analysis of such patterns shows that the first minimum for the diffraction pattern of a circular aperture of diameter d is located by

$$\sin\theta = 1.22\,\frac{\lambda}{d} \quad \text{(first minimum—circular aperture).} \tag{36.3.1}$$

The angle θ here is the angle from the central axis to any point on that (circular) minimum. Compare this with Eq. 36.1.1,

$$\sin\theta = \frac{\lambda}{a} \quad \text{(first minimum—single slit),} \tag{36.3.2}$$

which locates the first minimum for a long narrow slit of width a. The main difference is the factor 1.22, which enters because of the circular shape of the aperture.



Courtesy of Jearl Walker

Figure 36.3.1 The diffraction pattern of a circular aperture. Note the central maximum and the circular secondary maxima. The figure has been overexposed to bring out these secondary maxima, which are much less intense than the central maximum.

Resolvability

The fact that lens images are diffraction patterns is important when we wish to *resolve* (distinguish) two distant point objects whose angular separation is small. Figure 36.3.2 shows, in three different cases, the visual appearance and corresponding intensity pattern for two distant point objects (stars, say) with small angular separation. In Figure 36.3.2*a*, the objects are not resolved because of diffraction; that is, their diffraction patterns (mainly their central maxima) overlap so much that the two objects cannot be distinguished from a single point object. In Fig. 36.3.2*b* the objects are barely resolved, and in Fig. 36.3.2*c* they are fully resolved.

In Fig. 36.3.2*b* the angular separation of the two point sources is such that the central maximum of the diffraction pattern of one source is centered on the first minimum of the diffraction pattern of the other, a condition called **Rayleigh's criterion** for resolvability. From Eq. 36.3.3, two objects that are barely resolvable by this criterion must have an angular separation θ_R of

$$\theta_R = \sin^{-1}\frac{1.22\lambda}{d}.$$

Since the angles are small, we can replace $\sin\theta_R$ with θ_R expressed in radians:

$$\theta_R = 1.22\,\frac{\lambda}{d} \quad \text{(Rayleigh's criterion).} \tag{36.3.3}$$

Human Vision. Applying Rayleigh's criterion for resolvability to human vision is only an approximation because visual resolvability depends on many factors, such as the relative brightness of the sources and their surroundings, turbulence in the air between the sources and the observer, and the functioning of the observer's visual system. Experimental results show that the least angular separation that can

Courtesy of Jearl Walker

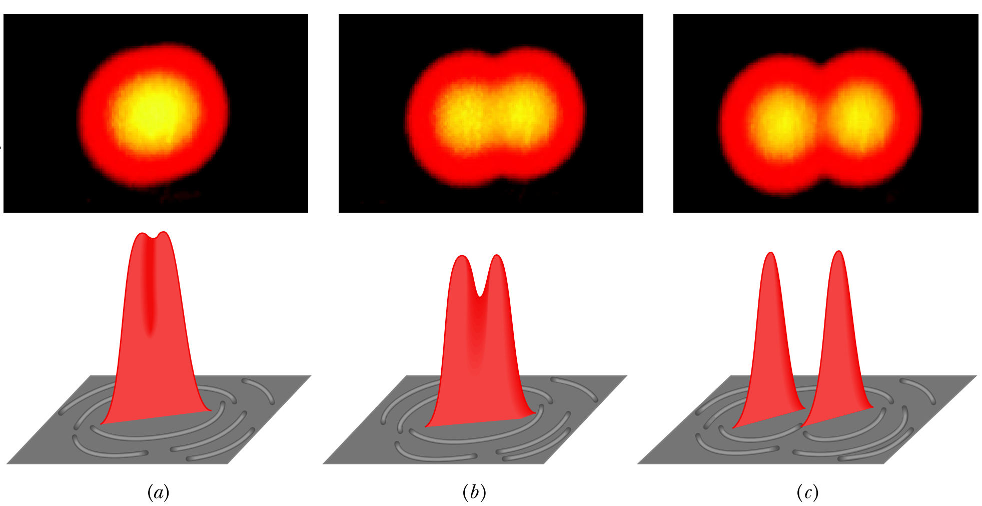
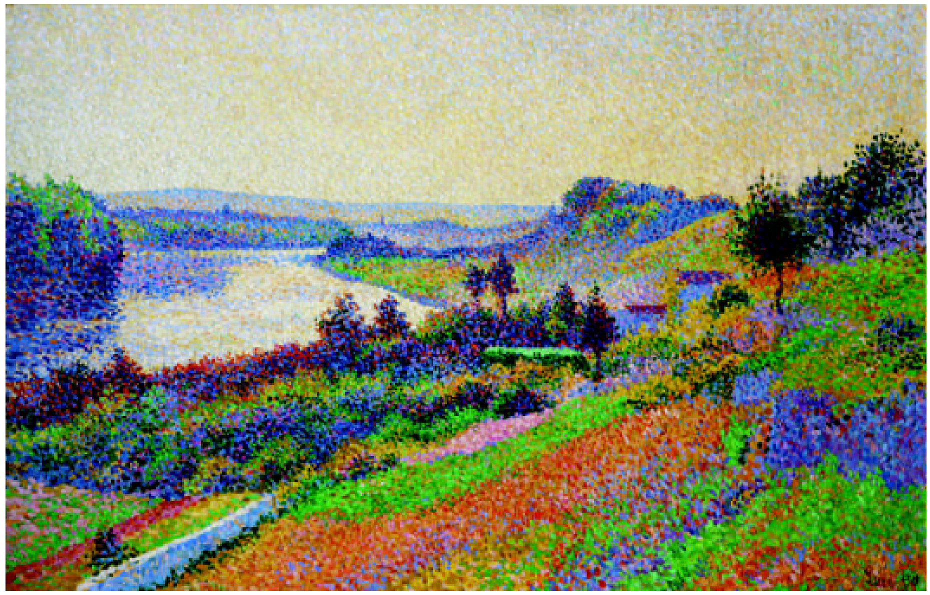


Figure 36.3.2 At the top, the images of two point sources formed by a converging lens. At the bottom, representations of the image intensities. In (*a*) the angular separation of the sources is too small for them to be distinguished, in (*b*) they can be marginally distinguished, and in (*c*) they are clearly distinguished. Rayleigh's criterion is satisfied in (*b*), with the central maximum of one diffraction pattern coinciding with the first minimum of the other.

Figure 36.3.3 The pointillistic painting *The Seine at Herblay* by Maximilien Luce consists of thousands of colored dots. With the viewer very close to the canvas, the dots and their true colors are visible. At normal viewing distances, the dots are irresolvable and thus blend.



Maximilien Luce, *The Seine at Herblay*, 1890.Musée d'Orsay, Paris, France. Photo by Erich Lessing/Art Resource

actually be resolved by a person is generally somewhat greater than the value given by Eq. 36.3.3. However, for calculations here, we shall take Eq. 36.3.3 as being a precise criterion: If the angular separation θ between the sources is greater than θ_R, we can visually resolve the sources; if it is less, we cannot.

When we wish to use a lens instead of our visual system to resolve objects of small angular separation, it is desirable to make the diffraction pattern as small as possible. According to Eq. 36.3.3, this can be done either by increasing the lens diameter or by using light of a shorter wavelength. For this reason ultraviolet light is often used with microscopes because its wavelength is shorter than a visible light wavelength.

Pointillism. Rayleigh's criterion can explain the arresting illusions of color in the style of painting known as pointillism (Fig. 36.3.3). In this style, a painting is made not with brushstrokes in the usual sense but rather with a myriad of small colored dots. One fascinating aspect of a pointillistic painting is that when you change your distance from it, the colors shift in subtle, almost subconscious ways. This color shifting has to do with whether you can resolve the colored dots. When you stand close enough to the painting, the angular separations θ of adjacent dots are greater than θ_R and thus the dots can be seen individually. Their colors are the true colors of the paints used. However, when you stand far enough from the painting, the angular separations θ are less than θ_R and the dots cannot be seen individually. The resulting blend of colors coming into your eye from any group of dots can then cause your brain to "make up" a color for that group—a color that may not actually exist in the group. In this way, a pointillistic painter uses your visual system to create the colors of the art. FCP

Checkpoint 36.3.1

Suppose that you can barely resolve two red dots because of diffraction by the pupil of your eye. If we increase the general illumination around you so that the pupil decreases in diameter, does the resolvability of the dots improve or diminish? Consider only diffraction. (You might experiment to check your answer.)

Sample Problem 36.3.1 Pointillistic paintings use the diffraction of your eye

Figure 36.3.4*a* is a representation of the colored dots on a pointillistic painting. Assume that the average center-to-center separation of the dots is $D = 2.0$ mm. Also assume that the diameter of the pupil of your eye is $d = 1.5$ mm and that the least angular separation between dots you can resolve is set only by Rayleigh's criterion. What is the least viewing distance from which you cannot distinguish any dots on the painting? FCP

KEY IDEA

Consider any two adjacent dots that you can distinguish when you are close to the painting. As you move away, you continue to distinguish the dots until their angular separation θ (in your view) has decreased to the angle given by Rayleigh's criterion:

$$\theta_R = 1.22\frac{\lambda}{d}. \tag{36.3.4}$$

Calculations: Figure 36.3.4*b* shows, from the side, the angular separation θ of the dots, their center-to-center separation D, and your distance L from them. Because D/L is small, angle θ is also small and we can make the approximation

$$\theta = \frac{D}{L}. \tag{36.3.5}$$

Setting θ of Eq. 36.3.5 equal to θ_R of Eq. 36.3.4 and solving for L, we then have

$$L = \frac{Dd}{1.22\lambda}. \tag{36.3.6}$$

Equation 36.3.6 tells us that L is larger for smaller λ. Thus, as you move away from the painting, adjacent red dots (long wavelengths) become indistinguishable before adjacent blue dots do. To find the least distance L at which *no* colored dots are distinguishable, we substitute $\lambda = 400$ nm (blue or violet light) into Eq. 36.3.6:

$$L = \frac{(2.0\times 10^{-3}\text{ m})(1.5\times 10^{-3}\text{ m})}{(1.22)(400\times 10^{-9}\text{ m})} = 6.1\text{ m}. \quad \text{(Answer)}$$

At this or a greater distance, the color you perceive at any given spot on the painting is a blended color that may not actually exist there.

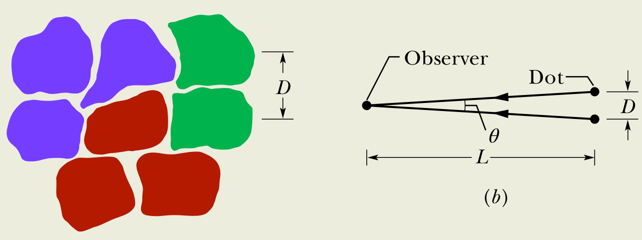


Figure 36.3.4 (*a*) Representation of some dots on a pointillistic painting, showing an average center-to-center separation D. (*b*) The arrangement of separation D between two dots, their angular separation θ, and the viewing distance L.

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Sample Problem 36.3.2 Rayleigh's criterion for resolving two distant objects

A circular converging lens, with diameter $d = 32$ mm and focal length $f = 24$ cm, forms images of distant point objects in the focal plane of the lens. The wavelength is $\lambda = 550$ nm.

(a) Considering diffraction by the lens, what angular separation must two distant point objects have to satisfy Rayleigh's criterion?

KEY IDEA

Figure 36.3.5 shows two distant point objects P_1 and P_2, the lens, and a viewing screen in the focal plane of the lens. It also shows, on the right, plots of light intensity I versus position on the screen for the central maxima of the images formed by the lens. Note that the angular

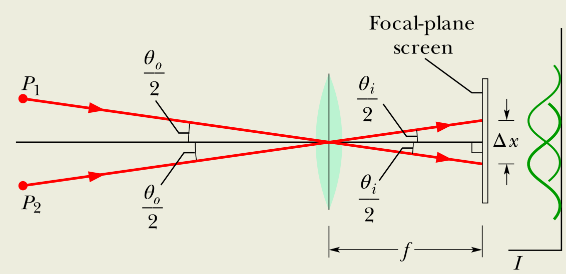

Figure 36.3.5 Light from two distant point objects P_1 and P_2 passes through a converging lens and forms images on a viewing screen in the focal plane of the lens. Only one representative ray from each object is shown. The images are not points but diffraction patterns, with intensities approximately as plotted at the right.

separation θ_o of the objects equals the angular separation θ_i of the images. Thus, if the images are to satisfy Rayleigh's criterion, these separations must be given by Eq. 36.3.3 (for small angles).

Calculations: From Eq. 36.3.3, we obtain

$$\theta_o = \theta_i = \theta_R = 1.22\frac{\lambda}{d}$$
$$= \frac{(1.22)(550 \times 10^{-9}\text{ m})}{32 \times 10^{-3}\text{ m}} = 2.1 \times 10^{-5}\text{ rad.} \quad \text{(Answer)}$$

Each central maximum in the two intensity curves of Fig. 36.3.5 is centered on the first minimum of the other curve.

(b) What is the separation Δx of the centers of the *images* in the focal plane? (That is, what is the separation of the *central* peaks in the two intensity-versus-position curves?)

Calculations: From either triangle between the lens and the screen in Fig. 36.3.5, we see that $\tan \theta_i/2 = \Delta x/2f$. Rearranging this equation and making the approximation $\tan\theta \approx \theta$, we find

$$\Delta x = f\theta_i, \tag{36.3.7}$$

where θ_i is in radian measure. We then find

$$\Delta x = (0.24\text{ m})(2.1 \times 10^{-5}\text{ rad}) = 5.0\ \mu\text{m}. \quad \text{(Answer)}$$

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36.4 DIFFRACTION BY A DOUBLE SLIT

Learning Objectives

After reading this module, you should be able to . . .

36.4.1 In a sketch of a double-slit experiment, explain how the diffraction through each slit modifies the two-slit interference pattern, and identify the diffraction envelope, the central peak, and the side peaks of that envelope.

36.4.2 For a given point in a double-slit diffraction pattern, calculate the intensity I in terms of the intensity I_m at the center of the pattern.

36.4.3 In the intensity equation for a double-slit diffraction pattern, identify what part corresponds to the interference between the two slits and what part corresponds to the diffraction by each slit.

36.4.4 For double-slit diffraction, apply the relationship between the ratio d/a and the locations of the diffraction minima in the single-slit diffraction pattern, and then count the number of two-slit maxima that are contained in the central peak and in the side peaks of the diffraction envelope.

Key Ideas

- Waves passing through two slits produce a combination of double-slit interference and diffraction by each slit.
- For identical slits with width a and center-to-center separation d, the intensity in the pattern varies with the angle θ from the central axis as

$$I(\theta) = I_m(\cos^2\beta)\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \text{(double slit)},$$

where I_m is the intensity at the center of the pattern,

$$\beta = \left(\frac{\pi d}{\lambda}\right)\sin\theta,$$

and

$$\alpha = \left(\frac{\pi a}{\lambda}\right)\sin\theta.$$

Diffraction by a Double Slit

In the double-slit experiments of Chapter 35, we implicitly assumed that the slits were much narrower than the wavelength of the light illuminating them; that is, $a \ll \lambda$. For such narrow slits, the central maximum of the diffraction pattern of either slit covers the entire viewing screen. Moreover, the interference of light from the two slits produces bright fringes with approximately the same intensity (Fig. 35.3.1).

In practice with visible light, however, the condition $a \ll \lambda$ is often not met. For relatively wide slits, the interference of light from two slits produces bright fringes that do not all have the same intensity. That is, the intensities of the fringes produced by double-slit interference (as discussed in Chapter 35) are modified by diffraction of the light passing through each slit (as discussed in this chapter).

Plots. As an example, the intensity plot of Fig. 36.4.1*a* suggests the double-slit interference pattern that would occur if the slits were infinitely narrow (and thus $a \ll \lambda$); all the bright interference fringes would have the same intensity. The intensity plot of Fig. 36.4.1*b* is that for diffraction by a single actual slit; the diffraction pattern has a broad central maximum and weaker secondary maxima at $\pm 17°$. The plot of Fig. 36.4.1*c* suggests the interference pattern for two actual slits. That plot was constructed by using the curve of Fig. 36.4.1*b* as an *envelope* on the intensity plot in Fig. 36.4.1*a*. The positions of the fringes are not changed; only the intensities are affected.

Photos. Figure 36.4.2*a* shows an actual pattern in which both double-slit interference and diffraction are evident. If one slit is covered, the single-slit diffraction pattern of Fig. 36.4.2*b* results. Note the correspondence between Figs. 36.4.2*a* and 36.4.1*c*, and between Figs. 36.4.2*b* and 36.4.1*b*. In comparing these figures, bear in mind that Fig. 36.4.2 has been deliberately overexposed to bring out the faint secondary maxima and that several secondary maxima (rather than one) are shown.

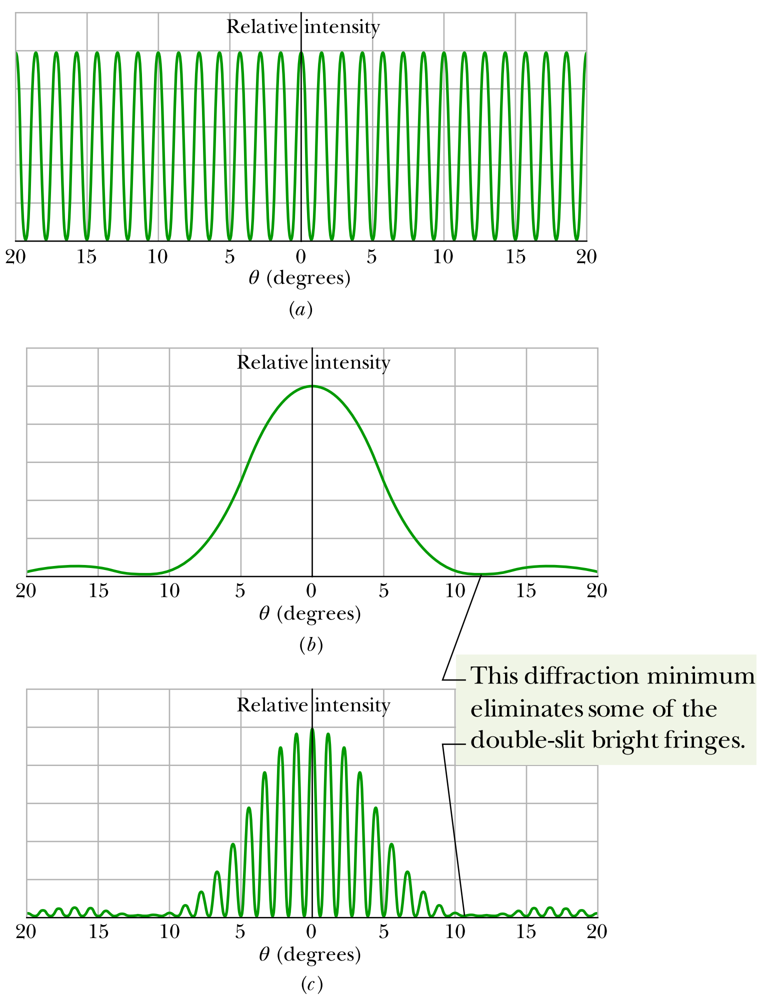
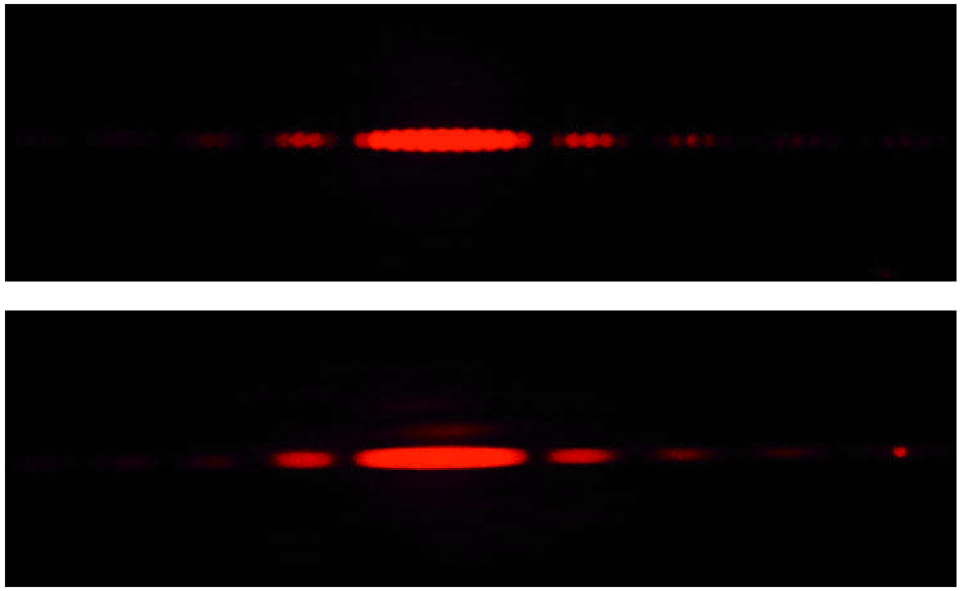


Figure 36.4.1 (*a*) The intensity plot to be expected in a double-slit interference experiment with vanishingly narrow slits. (*b*) The intensity plot for diffraction by a typical slit of width *a* (not vanishingly narrow). (*c*) The intensity plot to be expected for two slits of width *a*. The curve of (*b*) acts as an envelope, limiting the intensity of the double-slit fringes in (*a*). Note that the first minima of the diffraction pattern of (*b*) eliminate the double-slit fringes that would occur near 12° in (*c*).

(*a*)

(*b*)



Courtesy of Jearl Walker

Figure 36.4.2 (*a*) Interference fringes for an actual double-slit system; compare with Fig. 36.4.1*c*. (*b*) The diffraction pattern of a single slit; compare with Fig. 36.4.1*b*.

Intensity. With diffraction effects taken into account, the intensity of a double-slit interference pattern is given by

$$I(\theta) = I_m(\cos^2 \beta)\left(\frac{\sin \alpha}{\alpha}\right)^2 \quad \text{(double slit)}, \tag{36.4.1}$$

in which

$$\beta = \frac{\pi d}{\lambda} \sin \theta \tag{36.4.2}$$

and

$$\alpha = \frac{\pi a}{\lambda} \sin \theta. \tag{36.4.3}$$

Here d is the distance between the centers of the slits and a is the slit width. Note carefully that the right side of Eq. 36.4.1 is the product of I_m and two factors. (1) The *interference factor* $\cos^2 \beta$ is due to the interference between two slits with slit separation d (as given by Eqs. 35.3.3 and 35.3.4). (2) The *diffraction factor* $[(\sin \alpha)/\alpha]^2$ is due to diffraction by a single slit of width a (as given by Eqs. 36.2.2 and 36.2.3).

Let us check these factors. If we let $a \to 0$ in Eq. 36.4.3, for example, then $\alpha \to 0$ and $(\sin \alpha)/\alpha \to 1$. Equation 36.4.1 then reduces, as it must, to an equation describing the interference pattern for a pair of vanishingly narrow slits with slit separation d. Similarly, putting $d = 0$ in Eq. 36.4.2 is equivalent physically to causing the two slits to merge into a single slit of width a. Then Eq. 36.4.2 yields $\beta = 0$ and $\cos^2 \beta = 1$. In this case Eq. 36.4.1 reduces, as it must, to an equation describing the diffraction pattern for a single slit of width a.

Language. The double-slit pattern described by Eq. 36.4.1 and displayed in Fig. 36.4.2*a* combines interference and diffraction in an intimate way. Both are superposition effects, in that they result from the combining of waves with different phases at a given point. If the combining waves originate from a small number of elementary coherent sources—as in a double-slit experiment with $a \ll \lambda$—we call the process *interference*. If the combining waves originate in a single wavefront—as in a single-slit experiment—we call the process *diffraction*. This distinction between interference and diffraction (which is somewhat arbitrary and not always adhered to) is a convenient one, but we should not forget that both are superposition effects and usually both are present simultaneously (as in Fig. 36.4.2*a*).

Checkpoint 36.4.1

The first diffraction minima on the two sides of a double-slit diffraction pattern happen to coincide with the fourth side bright fringes at a certain angle θ. (a) How many bright fringes are in the central diffraction envelope? (b) To shift the coincidence to the fifth side bright fringe, how should the slit separation be changed? (c) To make the shift by changing the slit widths instead of the slit separation, how should the widths be changed?

Sample Problem 36.4.1 Double-slit experiment with diffraction of each slit included

In a double-slit experiment, the wavelength λ of the light source is 405 nm, the slit separation d is 19.44 μm, and the slit width a is 4.050 μm. Consider the interference of the light from the two slits and also the diffraction of the light through each slit.

(a) How many bright interference fringes are within the central peak of the diffraction envelope?

KEY IDEAS

We first analyze the two basic mechanisms responsible for the optical pattern produced in the experiment:

1. *Single-slit diffraction:* The limits of the central peak are set by the first minima in the diffraction pattern due to either slit individually. (See Fig. 36.4.1.) The angular locations of those minima are given by Eq. 36.1.3 ($a \sin \theta = m\lambda$). Here let us rewrite this equation as $a \sin \theta = m_1\lambda$, with the subscript 1 referring to the one-slit diffraction. For the first minima in the diffraction pattern, we substitute $m_1 = 1$, obtaining
$$a \sin \theta = \lambda. \tag{36.4.4}$$
2. *Double-slit interference:* The angular locations of the bright fringes of the double-slit interference pattern are given by Eq. 35.2.3, which we can write as
$$d \sin \theta = m_2\lambda, \quad \text{for } m_2 = 0, 1, 2, \ldots. \tag{36.4.5}$$
Here the subscript 2 refers to the double-slit interference.

Calculations: We can locate the first diffraction minimum within the double-slit fringe pattern by dividing Eq. 36.4.5 by Eq. 36.4.4 and solving for m_2. By doing so and then substituting the given data, we obtain

$$m_2 = \frac{d}{a} = \frac{19.44\,\mu\text{m}}{4.050\,\mu\text{m}} = 4.8.$$

This tells us that the bright interference fringe for $m_2 = 4$ fits into the central peak of the one-slit diffraction pattern, but the fringe for $m_2 = 5$ does not fit. Within the central diffraction peak we have the central bright fringe ($m_2 = 0$), and four bright fringes (up to $m_2 = 4$) on each side of it. Thus, a total of nine bright fringes of the double-slit interference pattern are within the central peak of the diffraction envelope. The bright fringes to one side of the central bright fringe are shown in Fig. 36.4.3.

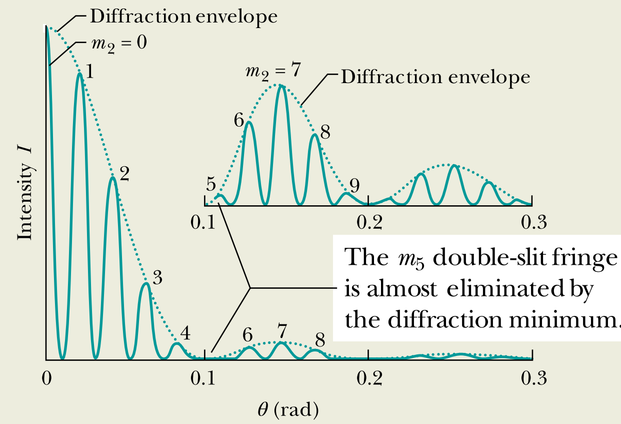


Figure 36.4.3 One side of the intensity plot for a two-slit interference experiment. The inset shows (vertically expanded) the plot within the first and second side peaks of the diffraction envelope.

(b) How many bright fringes are within either of the first side peaks of the diffraction envelope?

KEY IDEA

The outer limits of the first side diffraction peaks are the second diffraction minima, each of which is at the angle θ given by $a \sin \theta = m_1\lambda$ with $m_1 = 2$:

$$a \sin \theta = 2\lambda. \tag{36.4.6}$$

Calculation: Dividing Eq. 36.4.5 by Eq. 36.4.6, we find

$$m_2 = \frac{2d}{a} = \frac{(2)(19.44\,\mu\text{m})}{4.050\,\mu\text{m}} = 9.6.$$

This tells us that the second diffraction minimum occurs just before the bright interference fringe for $m_2 = 10$ in Eq. 36.4.5. Within either first side diffraction peak we have the fringes from $m_2 = 5$ to $m_2 = 9$, for a total of five bright fringes of the double-slit interference pattern (shown in the inset of Fig. 36.4.3). However, if the $m_2 = 5$ bright fringe, which is almost eliminated by the first diffraction minimum, is considered too dim to count, then only four bright fringes are in the first side diffraction peak.

WileyPLUS Additional examples, video, and practice available at *WileyPLUS*

36.5 DIFFRACTION GRATINGS

Learning Objectives

After reading this module, you should be able to . . .

36.5.1 Describe a diffraction grating and sketch the interference pattern it produces in monochromatic light.

36.5.2 Distinguish the interference patterns of a diffraction grating and a double-slit arrangement.

36.5.3 Identify the terms line and order number.

36.5.4 For a diffraction grating, relate order number m to the path length difference of rays that give a bright fringe.

36.5.5 For a diffraction grating, relate the slit separation d, the angle θ to a bright fringe in the pattern, the order number m of that fringe, and the wavelength λ of the light.

36.5.6 Identify the reason why there is a maximum order number for a given diffraction grating.

36.5.7 Explain the derivation of the equation for a line's half-width in a diffraction-grating pattern.

36.5.8 Calculate the half-width of a line at a given angle in a diffraction-grating pattern.

36.5.9 Explain the advantage of increasing the number of slits in a diffraction grating.

36.5.10 Explain how a grating spectroscope works.

Key Ideas

- A diffraction grating is a series of "slits" used to separate an incident wave into its component wavelengths by separating and displaying their diffraction maxima. Diffraction by N (multiple) slits results in maxima (lines) at angles θ such that

$$d \sin \theta = m\lambda, \quad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima)}.$$

- A line's half-width is the angle from its center to the point where it disappears into the darkness and is given by

$$\Delta\theta_{\text{hw}} = \frac{\lambda}{Nd \cos \theta} \text{ (half-width)}.$$

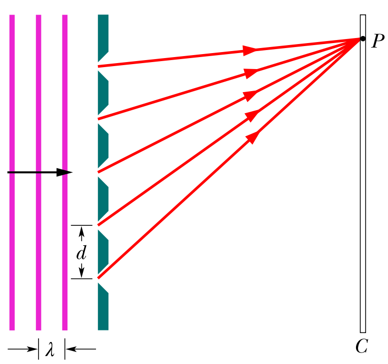


Figure 36.5.1 An idealized diffraction grating, consisting of only five rulings, that produces an interference pattern on a distant viewing screen C.

Diffraction Gratings

One of the most useful tools in the study of light and of objects that emit and absorb light is the **diffraction grating.** This device is somewhat like the double-slit arrangement of Fig. 36.3.1 but has a much greater number N of slits, often called *rulings,* perhaps as many as several thousand per millimeter. An idealized grating consisting of only five slits is represented in Fig. 36.5.1. When monochromatic light is sent through the slits, it forms narrow interference fringes that can be analyzed to determine the wavelength of the light. (Diffraction gratings can also be opaque surfaces with narrow parallel grooves arranged like the slits in Fig. 36.5.1. Light then scatters back from the grooves to form interference fringes rather than being transmitted through open slits.)

Pattern. With monochromatic light incident on a diffraction grating, if we gradually increase the number of slits from two to a large number N, the intensity plot changes from the typical double-slit plot of Fig. 36.4.1*c* to a much more complicated one and then eventually to a simple graph like that shown in Fig. 36.5.2*a*.

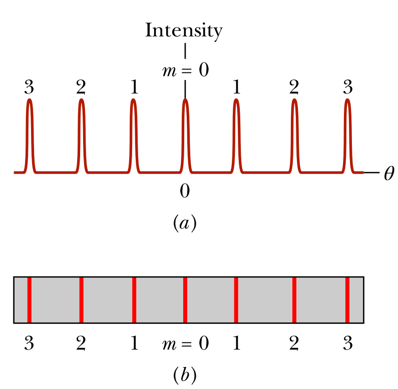


Figure 36.5.2 (*a*) The intensity plot produced by a diffraction grating with a great many rulings consists of narrow peaks, here labeled with their order numbers m. (*b*) The corresponding bright fringes seen on the screen are called lines and are here also labeled with order numbers m.

The pattern you would see on a viewing screen using monochromatic red light from, say, a helium–neon laser is shown in Fig. 36.5.2*b*. The maxima are now very narrow (and so are called *lines*); they are separated by relatively wide dark regions.

Equation. We use a familiar procedure to find the locations of the bright lines on the viewing screen. We first assume that the screen is far enough from the grating so that the rays reaching a particular point P on the screen are approximately parallel when they leave the grating (Fig. 36.5.3). Then we apply to each pair of adjacent rulings the same reasoning we used for double-slit interference. The separation d between rulings is called the *grating spacing*. (If N rulings occupy a total width w, then $d = w/N$.) The path length difference between adjacent rays is again $d \sin\theta$ (Fig. 36.5.3), where θ is the angle from the central axis of the grating (and of the diffraction pattern) to point P. A line will be located at P if the path length difference between adjacent rays is an integer number of wavelengths:

$$d \sin\theta = m\lambda, \quad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima—lines)}, \qquad (36.5.1)$$

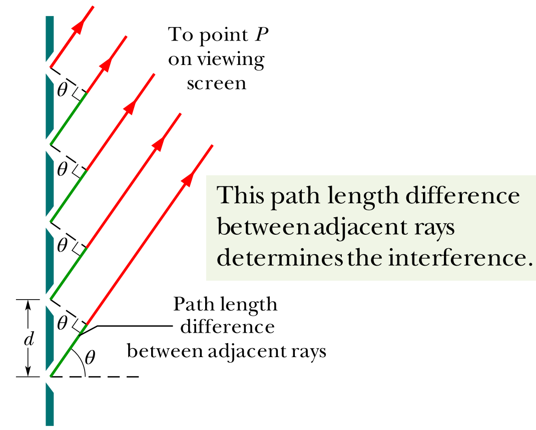


Figure 36.5.3 The rays from the rulings in a diffraction grating to a distant point P are approximately parallel. The path length difference between each two adjacent rays is $d \sin\theta$, where θ is measured as shown. (The rulings extend into and out of the page.)

where λ is the wavelength of the light. Each integer m represents a different line; hence these integers can be used to label the lines, as in Fig. 36.5.2. The integers are then called the *order numbers,* and the lines are called the zeroth-order line (the central line, with $m = 0$), the first-order line ($m = 1$), the second-order line ($m = 2$), and so on.

Determining Wavelength. If we rewrite Eq. 36.5.1 as $\theta = \sin^{-1}(m\lambda/d)$, we see that, for a given diffraction grating, the angle from the central axis to any line (say, the third-order line) depends on the wavelength of the light being used. Thus, when light of an unknown wavelength is sent through a diffraction grating, measurements of the angles to the higher-order lines can be used in Eq. 36.5.1 to determine the wavelength. Even light of several unknown wavelengths can be distinguished and identified in this way. We cannot do that with the double-slit arrangement of Module 35.2, even though the same equation and wavelength dependence apply there. In double-slit interference, the bright fringes due to different wavelengths overlap too much to be distinguished.

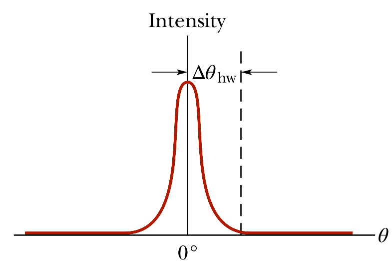


Figure 36.5.4 The half-width $\Delta\theta_{hw}$ of the central line is measured from the center of that line to the adjacent minimum on a plot of I versus θ like Fig. 36.5.2*a*.

Width of the Lines

A grating's ability to resolve (separate) lines of different wavelengths depends on the width of the lines. We shall here derive an expression for the *half-width* of the central line (the line for which $m = 0$) and then state an expression for the half-widths of the higher-order lines. We define the **half-width** of the central line as being the angle $\Delta\theta_{hw}$ from the center of the line at $\theta = 0$ outward to where the line effectively ends and darkness effectively begins with the first minimum (Fig. 36.5.4). At such a minimum, the N rays from the N slits of the grating cancel one another. (The actual width of the central line is, of course, $2(\Delta\theta_{hw})$, but line widths are usually compared via half-widths.)

In Module 36.1 we were also concerned with the cancellation of a great many rays, there due to diffraction through a single slit. We obtained Eq. 36.1.3, which, because of the similarity of the two situations, we can use to find the first minimum here. It tells us that the first minimum occurs where the path length difference between the top and bottom rays equals λ. For single-slit diffraction, this difference is $a \sin\theta$. For a grating of N rulings, each separated from the next by distance d, the distance between the top and bottom rulings is Nd (Fig. 36.5.5), and so the path length difference between the top and bottom rays here is $Nd \sin\Delta\theta_{hw}$. Thus, the first minimum occurs where

$$Nd \sin\Delta\theta_{hw} = \lambda. \qquad (36.5.2)$$

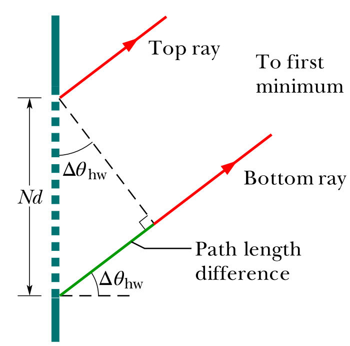


Figure 36.5.5 The top and bottom rulings of a diffraction grating of N rulings are separated by Nd. The top and bottom rays passing through these rulings have a path length difference of $Nd \sin\Delta\theta_{hw}$, where $\Delta\theta_{hw}$ is the angle to the first minimum. (The angle is here greatly exaggerated for clarity.)

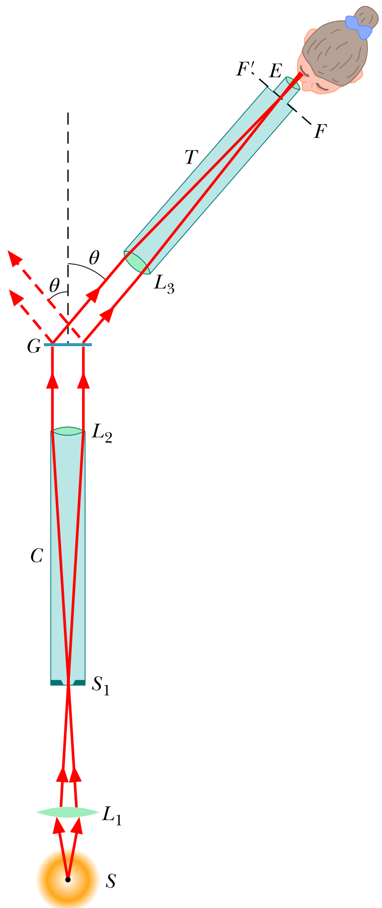


Figure 36.5.6 A simple type of grating spectroscope used to analyze the wavelengths of light emitted by source S.

Because $\Delta\theta_{hw}$ is small, $\sin \Delta\theta_{hw} = \Delta\theta_{hw}$ (in radian measure). Substituting this in Eq. 36.5.2 gives the half-width of the central line as

$$\Delta\theta_{hw} = \frac{\lambda}{Nd} \quad \text{(half-width of central line)}. \tag{36.5.3}$$

We state without proof that the half-width of any other line depends on its location relative to the central axis and is

$$\Delta\theta_{hw} = \frac{\lambda}{Nd\cos\theta} \quad \text{(half-width of line at } \theta\text{)}. \tag{36.5.4}$$

Note that for light of a given wavelength λ and a given ruling separation d, the widths of the lines decrease with an increase in the number N of rulings. Thus, of two diffraction gratings, the grating with the larger value of N is better able to distinguish between wavelengths because its diffraction lines are narrower and so produce less overlap.

Grating Spectroscope

Diffraction gratings are widely used to determine the wavelengths that are emitted by sources of light ranging from lamps to stars. Figure 36.5.6 shows a simple *grating spectroscope* in which a grating is used for this purpose. Light from source S is focused by lens L_1 on a vertical slit S_1 placed in the focal plane of lens L_2. The light emerging from tube C (called a *collimator*) is a plane wave and is incident perpendicularly on grating G, where it is diffracted into a diffraction pattern, with the $m = 0$ order diffracted at angle $\theta = 0$ along the central axis of the grating.

We can view the diffraction pattern that would appear on a viewing screen at any angle θ simply by orienting telescope T in Fig. 36.5.6 to that angle. Lens L_3 of the telescope then focuses the light diffracted at angle θ (and at slightly smaller and larger angles) onto a focal plane FF' within the telescope. When we look through eyepiece E, we see a magnified view of this focused image.

By changing the angle θ of the telescope, we can examine the entire diffraction pattern. For any order number other than $m = 0$, the original light is spread out according to wavelength (or color) so that we can determine, with Eq. 36.5.1, just what wavelengths are being emitted by the source. If the source emits discrete wavelengths, what we see as we rotate the telescope horizontally through the angles corresponding to an order m is a vertical line of color for each wavelength, with the shorter-wavelength line at a smaller angle θ than the longer-wavelength line.

Hydrogen. For example, the light emitted by a hydrogen lamp, which contains hydrogen gas, has four discrete wavelengths in the visible range. If our eyes intercept this light directly, it appears to be white. If, instead, we view it through a grating spectroscope, we can distinguish, in several orders, the lines of the four colors corresponding to these visible wavelengths. (Such lines are called *emission lines*.) Four orders are represented in Fig. 36.5.7. In the central order ($m = 0$), the lines corresponding to all four wavelengths are superimposed, giving a single white line at $\theta = 0$. The colors are separated in the higher orders.

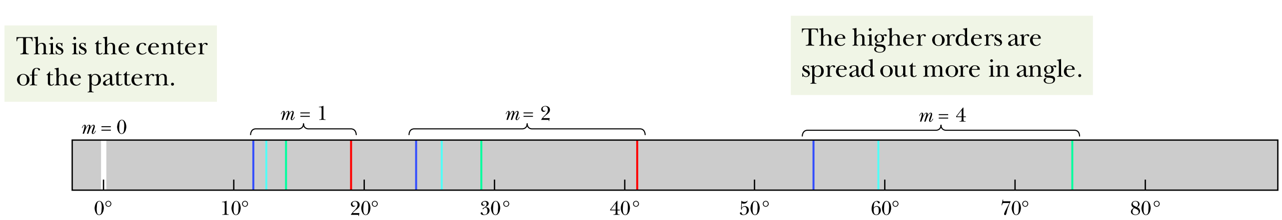
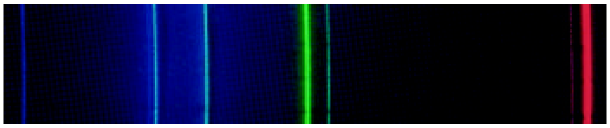


Figure 36.5.7 The zeroth, first, second, and fourth orders of the visible emission lines from hydrogen. Note that the lines are farther apart at greater angles. (They are also dimmer and wider, although that is not shown here.)



Department of Physics, Imperial College/Science Photo Library/ Science Source

Figure 36.5.8 The visible emission lines of cadmium, as seen through a grating spectroscope.

The third order is not shown in Fig. 36.5.7 for the sake of clarity; it actually overlaps the second and fourth orders. The fourth-order red line is missing because it is not formed by the grating used here. That is, when we attempt to solve Eq. 36.5.1 for the angle θ for the red wavelength when $m = 4$, we find that $\sin\theta$ is greater than unity, which is not possible. The fourth order is then said to be *incomplete* for this grating; it might not be incomplete for a grating with greater spacing d, which will spread the lines less than in Fig. 36.5.7. Figure 36.5.8 is a photograph of the visible emission lines produced by cadmium.

Optically Variable Graphics

Holograms are made by having laser light scatter from an object onto an emulsion. Once the hologram is developed, an image of the object can be created by illuminating the hologram with the same type of laser light. The image is arresting because, unlike common photographs, it has depth, and you can change your perspective of the object by changing the angle at which you view the hologram.

Holograms were thought to be an ideal anticounterfeiting measure for credit cards and other types of personal cards. However, they have several disadvantages. (1) A holographic image can be sharp when viewed in laser light, which is coherent and incident from a single direction. However, the image is murky ("milky") when viewed in the normal light of a store. (Such *diffuse light* is incoherent and incident from many directions.) Thus, a store clerk is unlikely to examine a display on a credit card closely enough to see whether it is a legitimate hologram. (2) A hologram can be easily counterfeited because it is a photograph of an actual object. A counterfeiter merely makes a model of that object, then makes a hologram of it, and then attaches the hologram to a counterfeit credit card.

Most credit cards and many identification cards now carry *optically variable graphics* (OVG), which produce an image via the diffraction of diffuse light by gratings embedded in the device (Fig. 36.5.9). The gratings send out hundreds or even thousands of different orders. Someone viewing the card intercepts some of these orders, and the combined light creates a virtual image that is part of, say, a credit-card logo. For example, in Fig. 36.5.10*a*, gratings at point *a* produce a certain image when the viewer is at orientation *A*, and in Fig. 36.5.10*b*, gratings

stu49/Alamy Stock Photo

Figure 36.5.9 A card with OVG.

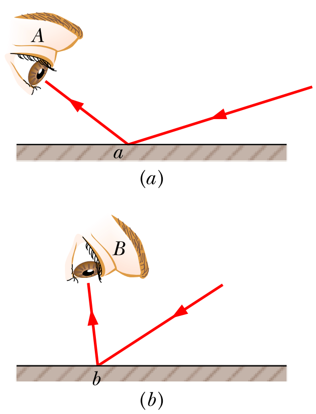
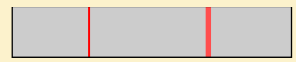


Figure 36.5.10 (*a*) Gratings at point *a* on the surface of an OVG device send light to a viewer at orientation *A*, creating a certain virtual image. (*b*) Gratings at point *b* send light to the viewer at orientation *B*, creating a different virtual image.

at point b produce a different image when the viewer is at orientation B. These images are bright and sharp because the gratings have been designed to be viewed in diffuse light.

An OVG is very difficult to design because optical engineers must work backwards from a graphic, such as a given logo. The engineers must determine the grating properties across the OVG if a certain image is to be seen from one set of viewing angles and a different image is to be seen from a different set of viewing angles. Such work requires sophisticated programming on computers. Once designed, the OVG structure is so complicated that counterfeiting it is extremely difficult.

Checkpoint 36.5.1



The figure shows lines of different orders produced by a diffraction grating in monochromatic red light.
(a) Is the center of the pattern to the left or right?
(b) In monochromatic green light, are the half-widths of the lines produced in the same orders greater than, less than, or the same as the half-widths of the lines shown?

36.6 GRATINGS: DISPERSION AND RESOLVING POWER

Learning Objectives

After reading this module, you should be able to . . .

36.6.1 Identify dispersion as the spreading apart of the diffraction lines associated with different wavelengths.

36.6.2 Apply the relationships between dispersion D, wavelength difference $\Delta\lambda$, angular separation $\Delta\theta$, slit separation d, order number m, and the angle θ corresponding to the order number.

36.6.3 Identify the effect on the dispersion of a diffraction grating if the slit separation is varied.

36.6.4 Identify that for us to resolve lines, a diffraction grating must make them distinguishable.

36.6.5 Apply the relationship between resolving power R, wavelength difference $\Delta\lambda$, average wavelength λ_{avg}, number of rulings N, and order number m.

36.6.6 Identify the effect on the resolving power R if the number of slits N is increased.

Key Ideas

- The dispersion D of a diffraction grating is a measure of the angular separation $\Delta\theta$ of the lines it produces for two wavelengths differing by $\Delta\lambda$. For order number m, at angle θ, the dispersion is given by

$$D = \frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta} \quad \text{(dispersion)}.$$

- The resolving power R of a diffraction grating is a measure of its ability to make the emission lines of two close wavelengths distinguishable. For two wavelengths differing by $\Delta\lambda$ and with an average value of λ_{avg}, the resolving power is given by

$$R = \frac{\lambda_{avg}}{\Delta\lambda} = Nm \quad \text{(resolving power)}.$$

Gratings: Dispersion and Resolving Power

Dispersion

To be useful in distinguishing wavelengths that are close to each other (as in a grating spectroscope), a grating must spread apart the diffraction lines associated with the various wavelengths. This spreading, called **dispersion,** is defined as

$$D = \frac{\Delta\theta}{\Delta\lambda} \quad \text{(dispersion defined)}. \tag{36.6.1}$$

Here $\Delta\theta$ is the angular separation of two lines whose wavelengths differ by $\Delta\lambda$. The greater D is, the greater is the distance between two emission lines whose wavelengths differ by $\Delta\lambda$. We show below that the dispersion of a grating at angle θ is given by

$$D = \frac{m}{d \cos \theta} \quad \text{(dispersion of a grating).} \tag{36.6.2}$$

Thus, to achieve higher dispersion we must use a grating of smaller grating spacing d and work in a higher-order m. Note that the dispersion does not depend on the number of rulings N in the grating. The SI unit for D is the degree per meter or the radian per meter.

Kristen Brochmann/Fundamental Photographs

The fine rulings, each 0.5 μm wide, on a compact disc function as a diffraction grating. When a small source of white light illuminates a disc, the diffracted light forms colored "lanes" that are the composite of the diffraction patterns from the rulings. FCP

Resolving Power

To *resolve* lines whose wavelengths are close together (that is, to make the lines distinguishable), the line should also be as narrow as possible. Expressed otherwise, the grating should have a high **resolving power** R, defined as

$$R = \frac{\lambda_{avg}}{\Delta\lambda} \quad \text{(resolving power defined).} \tag{36.6.3}$$

Here λ_{avg} is the mean wavelength of two emission lines that can barely be recognized as separate, and $\Delta\lambda$ is the wavelength difference between them. The greater R is, the closer two emission lines can be and still be resolved. We shall show below that the resolving power of a grating is given by the simple expression

$$R = Nm \quad \text{(resolving power of a grating).} \tag{36.6.4}$$

To achieve high resolving power, we must use many rulings (large N).

Proof of Eq. 36.6.2

Let us start with Eq. 36.5.1, the expression for the locations of the lines in the diffraction pattern of a grating:

$$d \sin \theta = m\lambda.$$

Let us regard θ and λ as variables and take differentials of this equation. We find

$$d(\cos \theta)\, d\theta = m\, d\lambda.$$

For small enough angles, we can write these differentials as small differences, obtaining

$$d(\cos \theta)\, \Delta\theta = m\, \Delta\lambda \tag{36.6.5}$$

or

$$\frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d \cos \theta}.$$

The ratio on the left is simply D (see Eq. 36.6.1), and so we have indeed derived Eq. 36.6.2.

Proof of Eq. 36.6.4

We start with Eq. 36.6.5, which was derived from Eq. 36.5.1, the expression for the locations of the lines in the diffraction pattern formed by a grating. Here $\Delta\lambda$ is the small wavelength difference between two waves that are diffracted by the grating, and $\Delta\theta$ is the angular separation between them in the diffraction pattern. If $\Delta\theta$ is to be the smallest angle that will permit the two lines to be resolved, it must (by Rayleigh's criterion) be equal to the half-width of each line, which is given by Eq. 36.5.4:

$$\Delta\theta_{hw} = \frac{\lambda}{Nd \cos \theta}.$$

Table 36.6.1 Three Gratings[a]

Grating	N	d (nm)	θ	D (°/μm)	R
A	10 000	2540	13.4°	23.2	10 000
B	20 000	2540	13.4°	23.2	20 000
C	10 000	1360	25.5°	46.3	10 000

[a]Data are for $\lambda = 589$ nm and $m = 1$.

If we substitute $\Delta\theta_{hw}$ as given here for $\Delta\theta$ in Eq. 36.6.5, we find that

$$\frac{\lambda}{N} = m\ \Delta\lambda,$$

from which it readily follows that

$$R = \frac{\lambda}{\Delta\lambda} = Nm.$$

This is Eq. 36.6.4, which we set out to derive.

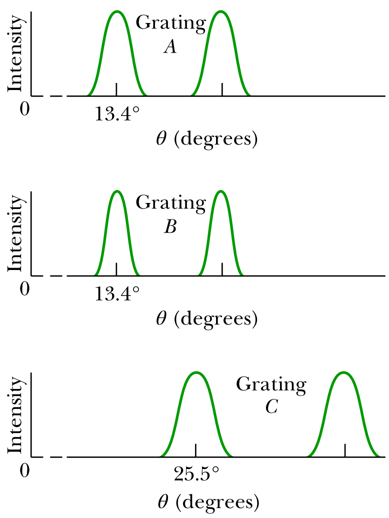


Figure 36.6.1 The intensity patterns for light of two wavelengths sent through the gratings of Table 36.6.1. Grating B has the highest resolving power, and grating C the highest dispersion.

Dispersion and Resolving Power Compared

The resolving power of a grating must not be confused with its dispersion. Table 36.6.1 shows the characteristics of three gratings, all illuminated with light of wavelength $\lambda = 589$ nm, whose diffracted light is viewed in the first order ($m = 1$ in Eq. 36.5.1). You should verify that the values of D and R as given in the table can be calculated with Eqs. 36.6.2 and 36.6.4, respectively. (In the calculations for D, you will need to convert radians per meter to degrees per micrometer.)

For the conditions noted in Table 36.6.1, gratings A and B have the same *dispersion* D and A and C have the same *resolving power* R.

Figure 36.6.1 shows the intensity patterns (also called *line shapes*) that would be produced by these gratings for two lines of wavelengths λ_1 and λ_2, in the vicinity of $\lambda = 589$ nm. Grating B, with the higher resolving power, produces narrower lines and thus is capable of distinguishing lines that are much closer together in wavelength than those in the figure. Grating C, with the higher dispersion, produces the greater angular separation between the lines.

Checkpoint 36.6.1

If we cover half a grating with opaque tape, what happens to resolving power R of the grating?

Sample Problem 36.6.1 Dispersion and resolving power of a diffraction grating

A diffraction grating has 1.26×10^4 rulings uniformly spaced over width $w = 25.4$ mm. It is illuminated at normal incidence by yellow light from a sodium vapor lamp. This light contains two closely spaced emission lines (known as the sodium doublet) of wavelengths 589.00 nm and 589.59 nm.

(a) At what angle does the first-order maximum occur (on either side of the center of the diffraction pattern) for the wavelength of 589.00 nm?

KEY IDEA

The maxima produced by the diffraction grating can be determined with Eq. 36.5.1 ($d \sin\theta = m\lambda$).

Calculations: The grating spacing d is

$$d = \frac{w}{N} = \frac{25.4 \times 10^{-3}\ \text{m}}{1.26 \times 10^4}$$
$$= 2.016 \times 10^{-6}\ \text{m} = 2016\ \text{nm}.$$

The first-order maximum corresponds to $m = 1$. Substituting these values for d and m into Eq. 36.5.1 leads to

$$\theta = \sin^{-1}\frac{m\lambda}{d} = \sin^{-1}\frac{(1)(589.00\ \text{nm})}{2016\ \text{nm}}$$
$$= 16.99° \approx 17.0°. \qquad \text{(Answer)}$$

(b) Using the dispersion of the grating, calculate the angular separation between the two lines in the first order.

KEY IDEAS

(1) The angular separation $\Delta\theta$ between the two lines in the first order depends on their wavelength difference $\Delta\lambda$ and the dispersion D of the grating, according to Eq. 36.6.1 ($D = \Delta\theta/\Delta\lambda$). (2) The dispersion D depends on the angle θ at which it is to be evaluated.

Calculations: We can assume that, in the first order, the two sodium lines occur close enough to each other for us to evaluate D at the angle $\theta = 16.99°$ we found in part (a) for one of those lines. Then Eq. 36.6.2 gives the dispersion as

$$D = \frac{m}{d\cos\theta} = \frac{1}{(2016\text{ nm})(\cos 16.99°)}$$
$$= 5.187 \times 10^{-4}\text{ rad/nm}.$$

From Eq. 36.6.1 and with $\Delta\lambda$ in nanometers, we then have

$$\Delta\theta = D\,\Delta\lambda = (5.187 \times 10^{-4}\text{ rad/nm})(589.59 - 589.00)$$
$$= 3.06 \times 10^{-4}\text{ rad} = 0.0175°. \qquad \text{(Answer)}$$

You can show that this result depends on the grating spacing d but not on the number of rulings there are in the grating.

(c) What is the least number of rulings a grating can have and still be able to resolve the sodium doublet in the first order?

KEY IDEAS

(1) The resolving power of a grating in any order m is physically set by the number of rulings N in the grating according to Eq. 36.6.4 ($R = Nm$). (2) The smallest wavelength difference $\Delta\lambda$ that can be resolved depends on the average wavelength involved and on the resolving power R of the grating, according to Eq. 36.6.3 ($R = \lambda_{avg}/\Delta\lambda$).

Calculation: For the sodium doublet to be barely resolved, $\Delta\lambda$ must be their wavelength separation of 0.59 nm, and λ_{avg} must be their average wavelength of 589.30 nm. Thus, we find that the smallest number of rulings for a grating to resolve the sodium doublet is

$$N = \frac{R}{m} = \frac{\lambda_{avg}}{m\,\Delta\lambda}$$
$$= \frac{589.30\text{ nm}}{(1)(0.59\text{ nm})} = 999\text{ rulings}. \qquad \text{(Answer)}$$

36.7 X-RAY DIFFRACTION

Learning Objectives

After reading this module, you should be able to . . .

36.7.1 Identify approximately where x rays are located in the electromagnetic spectrum.
36.7.2 Define a unit cell.
36.7.3 Define reflecting planes (or crystal planes) and interplanar spacing.
36.7.4 Sketch two rays that scatter from adjacent planes, showing the angle that is used in calculations.
36.7.5 For the intensity maxima in x-ray scattering by a crystal, apply the relationship between the interplanar spacing d, the angle θ of scattering, the order number m, and the wavelength λ of the x rays.
36.7.6 Given a drawing of a unit cell, demonstrate how an interplanar spacing can be determined.

Key Ideas

- If x rays are directed toward a crystal structure, they undergo Bragg scattering, which is easiest to visualize if the crystal atoms are considered to be in parallel planes.
- For x rays of wavelength λ scattering from crystal planes with separation d, the angles θ at which the scattered intensity is maximum are given by

$$2d\sin\theta = m\lambda, \qquad \text{for } m = 1, 2, 3, \ldots \quad \text{(Bragg's law)}.$$

X-Ray Diffraction

X rays are electromagnetic radiation whose wavelengths are of the order of 1 Å ($= 10^{-10}$ m). Compare this with a wavelength of 550 nm ($= 5.5 \times 10^{-7}$ m) at the

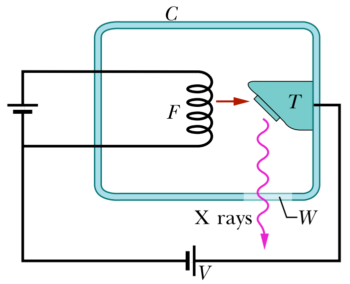


Figure 36.7.1 X rays are generated when electrons leaving heated filament F are accelerated through a potential difference V and strike a metal target T. The "window" W in the evacuated chamber C is transparent to x rays.

center of the visible spectrum. Figure 36.7.1 shows that x rays are produced when electrons escaping from a heated filament F are accelerated by a potential difference V and strike a metal target T.

A standard optical diffraction grating cannot be used to discriminate between different wavelengths in the x-ray wavelength range. For $\lambda = 1$ Å $(= 0.1$ nm$)$ and $d = 3000$ nm, for example, Eq. 36.5.1 shows that the first-order maximum occurs at

$$\theta = \sin^{-1}\frac{m\lambda}{d} = \sin^{-1}\frac{(1)(0.1\text{ nm})}{3000\text{ nm}} = 0.0019°.$$

This is too close to the central maximum to be practical. A grating with $d \approx \lambda$ is desirable, but, because x-ray wavelengths are about equal to atomic diameters, such gratings cannot be constructed mechanically.

In 1912, it occurred to German physicist Max von Laue that a crystalline solid, which consists of a regular array of atoms, might form a natural three-dimensional "diffraction grating" for x rays. The idea is that, in a crystal such as sodium chloride (NaCl), a basic unit of atoms (called the *unit cell*) repeats itself throughout the array. Figure 36.7.2*a* represents a section through a crystal of NaCl and identifies this basic unit. The unit cell is a cube measuring a_0 on each side.

When an x-ray beam enters a crystal such as NaCl, x rays are *scattered*—that is, redirected—in all directions by the crystal structure. In some directions the scattered waves undergo destructive interference, resulting in intensity minima; in other directions the interference is constructive, resulting in intensity maxima. This process of scattering and interference is a form of diffraction.

Fictional Planes. Although the process of diffraction of x rays by a crystal is complicated, the maxima turn out to be in directions *as if* the x rays were

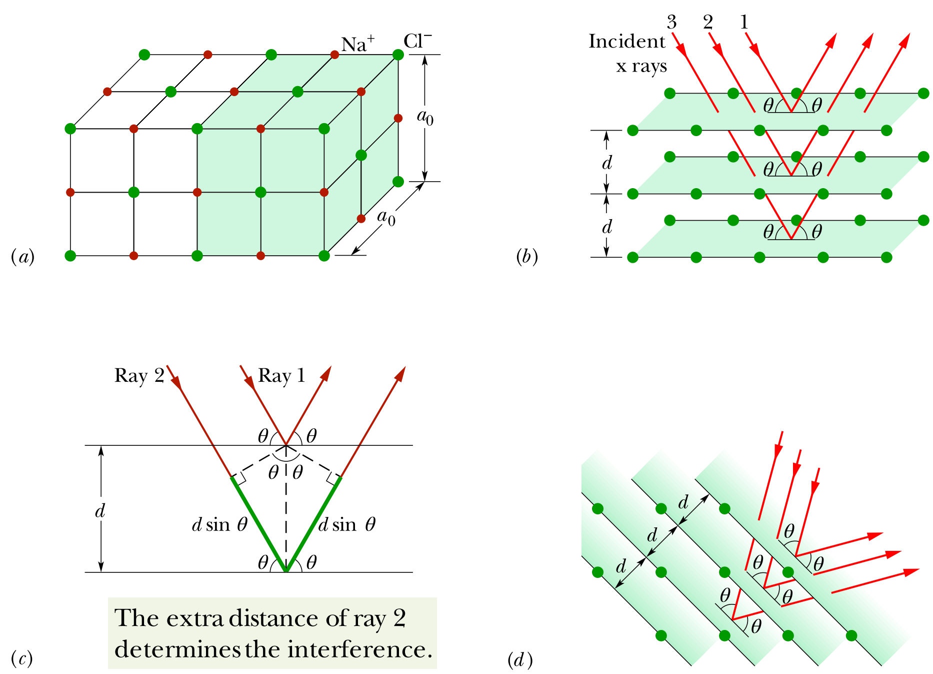


Figure 36.7.2 (*a*) The cubic structure of NaCl, showing the sodium and chlorine ions and a unit cell (shaded). (*b*) Incident x rays undergo diffraction by the structure of (*a*). The x rays are diffracted as if they were reflected by a family of parallel planes, with angles measured relative to the planes (not relative to a normal as in optics). (*c*) The path length difference between waves effectively reflected by two adjacent planes is $2d \sin \theta$. (*d*) A different orientation of the incident x rays relative to the structure. A different family of parallel planes now effectively reflects the x rays.

reflected by a family of parallel *reflecting planes* (or *crystal planes*) that extend through the atoms within the crystal and that contain regular arrays of the atoms. (The x rays are not actually reflected; we use these fictional planes only to simplify the analysis of the actual diffraction process.)

Figure 36.7.2*b* shows three reflecting planes (part of a family containing many parallel planes) with *interplanar spacing d*, from which the incident rays shown are said to reflect. Rays 1, 2, and 3 reflect from the first, second, and third planes, respectively. At each reflection the angle of incidence and the angle of reflection are represented with θ. Contrary to the custom in optics, these angles are defined relative to the *surface* of the reflecting plane rather than a normal to that surface. For the situation of Fig. 36.7.2*b*, the interplanar spacing happens to be equal to the unit cell dimension a_0.

Figure 36.7.2*c* shows an edge-on view of reflection from an adjacent pair of planes. The waves of rays 1 and 2 arrive at the crystal in phase. After they are reflected, they must again be in phase because the reflections and the reflecting planes have been defined solely to explain the intensity maxima in the diffraction of x rays by a crystal. Unlike light rays, the x rays do not refract upon entering the crystal; moreover, we do not define an index of refraction for this situation. Thus, the relative phase between the waves of rays 1 and 2 as they leave the crystal is set solely by their path length difference. For these rays to be in phase, the path length difference must be equal to an integer multiple of the wavelength λ of the x rays.

Diffraction Equation. By drawing the dashed perpendiculars in Fig. 36.7.2*c*, we find that the path length difference is $2d \sin\theta$. In fact, this is true for any pair of adjacent planes in the family of planes represented in Fig. 36.7.2*b*. Thus, we have, as the criterion for intensity maxima for x-ray diffraction,

$$2d \sin\theta = m\lambda, \qquad \text{for } m = 1, 2, 3, \ldots \quad \text{(Bragg's law)}, \qquad (36.7.1)$$

where m is the order number of an intensity maximum. Equation 36.7.1 is called **Bragg's law** after British physicist W. L. Bragg, who first derived it. (He and his father shared the 1915 Nobel Prize in physics for their use of x rays to study the structures of crystals.) The angle of incidence and reflection in Eq. 36.7.1 is called a *Bragg angle.*

Regardless of the angle at which x rays enter a crystal, there is always a family of planes from which they can be said to reflect so that we can apply Bragg's law. In Fig. 36.7.2*d*, notice that the crystal structure has the same orientation as it does in Fig. 36.7.2*a*, but the angle at which the beam enters the structure differs from that shown in Fig. 36.7.2*b*. This new angle requires a new family of reflecting planes, with a different interplanar spacing d and different Bragg angle θ, in order to explain the x-ray diffraction via Bragg's law.

Determining a Unit Cell. Figure 36.7.3 shows how the interplanar spacing d can be related to the unit cell dimension a_0. For the particular family of planes shown there, the Pythagorean theorem gives

$$5d = \sqrt{\tfrac{5}{4}a_0^2},$$

or

$$d = \frac{a_0}{\sqrt{20}} = 0.2236a_0. \qquad (36.7.2)$$

Figure 36.7.3 suggests how the dimensions of the unit cell can be found once the interplanar spacing has been measured by means of x-ray diffraction.

X-ray diffraction is a powerful tool for studying both x-ray spectra and the arrangement of atoms in crystals. To study spectra, a particular set of crystal planes, having a known spacing d, is chosen. These planes effectively reflect different wavelengths at different angles. A detector that can discriminate one angle from another can then be used to determine the wavelength of radiation

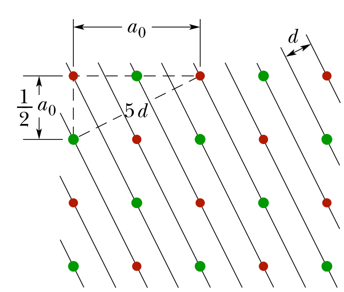
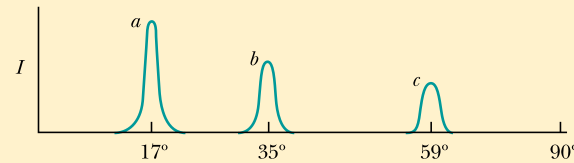


Figure 36.7.3 A family of planes through the structure of Fig. 36.7.2*a*, and a way to relate the edge length a_0 of a unit cell to the interplanar spacing d.

reaching it. The crystal itself can be studied with a monochromatic x-ray beam, to determine not only the spacing of various crystal planes but also the structure of the unit cell.

Checkpoint 36.7.1

The figure gives the intensity versus diffraction angle for the diffraction of a monochromatic x-ray beam by a particular family of reflecting planes in a crystal. Rank the three intensity peaks according to the associated path length differences of the x rays, greatest first.



Review & Summary

Diffraction When waves encounter an edge, an obstacle, or an aperture the size of which is comparable to the wavelength of the waves, those waves spread out as they travel and, as a result, undergo interference. This is called **diffraction.**

Single-Slit Diffraction Waves passing through a long narrow slit of width a produce, on a viewing screen, a **single-slit diffraction pattern** that includes a central maximum and other maxima, separated by minima located at angles θ to the central axis that satisfy

$$a \sin\theta = m\lambda, \quad \text{for } m = 1, 2, 3, \ldots \quad \text{(minima)}. \qquad (36.1.3)$$

The intensity of the diffraction pattern at any given angle θ is

$$I(\theta) = I_m \left(\frac{\sin\alpha}{\alpha}\right)^2, \quad \text{where } \alpha = \frac{\pi a}{\lambda}\sin\theta \qquad (36.2.2, 36.2.3)$$

and I_m is the intensity at the center of the pattern.

Circular-Aperture Diffraction Diffraction by a circular aperture or a lens with diameter d produces a central maximum and concentric maxima and minima, with the first minimum at an angle θ given by

$$\sin\theta = 1.22\frac{\lambda}{d} \quad \text{(first minimum—circular aperture)}. \qquad (36.3.1)$$

Rayleigh's Criterion *Rayleigh's criterion* suggests that two objects are on the verge of resolvability if the central diffraction maximum of one is at the first minimum of the other. Their angular separation can then be no less than

$$\theta_R = 1.22\frac{\lambda}{d} \quad \text{(Rayleigh's criterion)}, \qquad (36.3.3)$$

in which d is the diameter of the aperture through which the light passes.

Double-Slit Diffraction Waves passing through two slits, each of width a, whose centers are a distance d apart, display diffraction patterns whose intensity I at angle θ is

$$I(\theta) = I_m(\cos^2\beta)\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \text{(double slit)}, \qquad (36.4.1)$$

with $\beta = (\pi d/\lambda)\sin\theta$ and α as for single-slit diffraction.

Diffraction Gratings A *diffraction grating* is a series of "slits" used to separate an incident wave into its component wavelengths by separating and displaying their diffraction maxima. Diffraction by N (multiple) slits results in maxima (lines) at angles θ such that

$$d\sin\theta = m\lambda, \quad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima)}, \qquad (36.5.1)$$

with the **half-widths** of the lines given by

$$\Delta\theta_{hw} = \frac{\lambda}{Nd\cos\theta} \quad \text{(half-widths)}. \qquad (36.5.4)$$

The dispersion D and resolving power R are given by

$$D = \frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta} \qquad (36.6.1, 36.6.2)$$

and

$$R = \frac{\lambda_{avg}}{\Delta\lambda} = Nm. \qquad (36.6.3, 36.6.4)$$

X-Ray Diffraction The regular array of atoms in a crystal is a three-dimensional diffraction grating for short-wavelength waves such as x rays. For analysis purposes, the atoms can be visualized as being arranged in planes with characteristic interplanar spacing d. Diffraction maxima (due to constructive interference) occur if the incident direction of the wave, measured from the surfaces of these planes, and the wavelength λ of the radiation satisfy **Bragg's law:**

$$2d\sin\theta = m\lambda, \quad \text{for } m = 1, 2, 3, \ldots \quad \text{(Bragg's law)}. \qquad (36.7.1)$$

Questions

1 You are conducting a single-slit diffraction experiment with light of wavelength λ. What appears, on a distant viewing screen, at a point at which the top and bottom rays through the slit have a path length difference equal to (a) 5λ and (b) 4.5λ?

2 In a single-slit diffraction experiment, the top and bottom rays through the slit arrive at a certain point on the viewing screen with a path length difference of 4.0 wavelengths. In a phasor representation like those in Fig. 36.2.1, how many overlapping circles does the chain of phasors make?

3 For three experiments, Fig. 36.1 gives the parameter β of Eq. 36.4.2 versus angle θ for two-slit interference using light of wavelength 500 nm. The slit separations in the three experiments differ. Rank the experiments according to (a) the slit separations and (b) the total number of two-slit interference maxima in the pattern, greatest first.

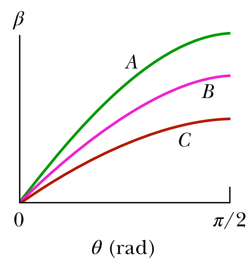


Figure 36.1 Question 3.

4 For three experiments, Fig. 36.2 gives α versus angle θ in one-slit diffraction using light of wavelength 500 nm. Rank the experiments according to (a) the slit widths and (b) the total number of diffraction minima in the pattern, greatest first.

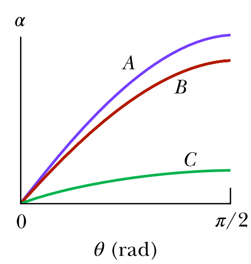


Figure 36.2 Question 4.

5 Figure 36.3 shows four choices for the rectangular opening of a source of either sound waves or light waves. The sides have lengths of either L or $2L$, with L being 3.0 times the wavelength of the waves. Rank the openings according to the extent of (a) left–right spreading and (b) up–down spreading of the waves due to diffraction, greatest first.

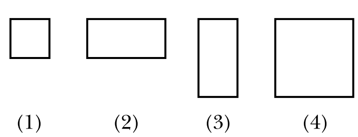


Figure 36.3 Question 5.

6 Light of frequency f illuminating a long narrow slit produces a diffraction pattern. (a) If we switch to light of frequency $1.3f$, does the pattern expand away from the center or contract toward the center? (b) Does the pattern expand or contract if, instead, we submerge the equipment in clear corn syrup?

7 At night many people see rings (called *entoptic halos*) surrounding bright outdoor lamps in otherwise dark surroundings. The rings are the first of the side maxima in diffraction patterns produced by structures that are thought to be within the cornea (or possibly the lens) of the observer's eye. (The central maxima of such patterns overlap the lamp.) (a) Would a particular ring become smaller or larger if the lamp were switched from blue to red light? (b) If a lamp emits white light, is blue or red on the outside edge of the ring?

8 (a) For a given diffraction grating, does the smallest difference $\Delta\lambda$ in two wavelengths that can be resolved increase, decrease, or remain the same as the wavelength increases? (b) For a given wavelength region (say, around 500 nm), is $\Delta\lambda$ greater in the first order or in the third order?

9 Figure 36.4 shows a red line and a green line of the same order in the pattern produced by a diffraction grating. If we increased the number of rulings in the grating—say, by removing tape that had covered the outer half of the rulings—would (a) the half-widths of the lines and (b) the separation of the lines increase, decrease, or remain the same? (c) Would the lines shift to the right, shift to the left, or remain in place?

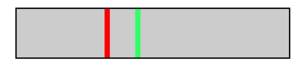


Figure 36.4 Questions 9 and 10.

10 For the situation of Question 9 and Fig. 36.4, if instead we increased the grating spacing, would (a) the half-widths of the lines and (b) the separation of the lines increase, decrease, or remain the same? (c) Would the lines shift to the right, shift to the left, or remain in place?

11 (a) Figure 36.5*a* shows the lines produced by diffraction gratings A and B using light of the same wavelength; the lines are of the same order and appear at the same angles θ. Which grating has the greater number of rulings? (b) Figure 36.5*b* shows lines of two orders produced by a single diffraction grating using light of two wavelengths, both in the red region of the spectrum. Which lines, the left pair or right pair, are in the order with greater m? Is the center of the diffraction pattern located to the left or to the right in (c) Fig. 36.5*a* and (d) Fig. 36.5*b*?

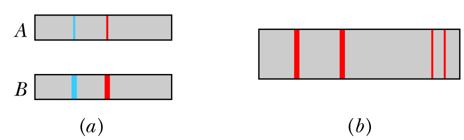


Figure 36.5 Question 11.

12 Figure 36.6 shows the bright fringes that lie within the central diffraction envelope in two double-slit diffraction experiments using the same wavelength of light. Are (a) the slit width a, (b) the slit separation d, and (c) the ratio d/a in experiment B greater than, less than, or the same as those quantities in experiment A?

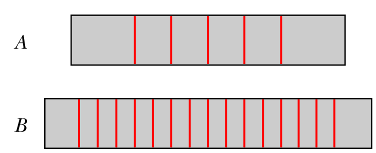


Figure 36.6 Question 12.

13 In three arrangements you view two closely spaced small objects that are the same large distance from you. The angles that the objects occupy in your field of view and their distances from you are the following: (1) 2ϕ and R; (2) 2ϕ and $2R$; (3) $\phi/2$ and $R/2$. (a) Rank the arrangements according to the separation between the objects, greatest first. If you can just barely resolve the two objects in arrangement 2, can you resolve them in (b) arrangement 1 and (c) arrangement 3?

14 For a certain diffraction grating, the ratio λ/a of wavelength to ruling spacing is 1/3.5. Without written calculation or use of a calculator, determine which of the orders beyond the zeroth order appear in the diffraction pattern.

Problems

GO Tutoring problem available (at instructor's discretion) in *WileyPLUS*
SSM Worked-out solution available in Student Solutions Manual
E Easy M Medium H Hard
FCP Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com
CALC Requires calculus
BIO Biomedical application

Module 36.1 Single-Slit Diffraction

1 E GO The distance between the first and fifth minima of a single-slit diffraction pattern is 0.35 mm with the screen 40 cm away from the slit, when light of wavelength 550 nm is used. (a) Find the slit width. (b) Calculate the angle θ of the first diffraction minimum.

2 E What must be the ratio of the slit width to the wavelength for a single slit to have the first diffraction minimum at $\theta = 45.0°$?

3 E A plane wave of wavelength 590 nm is incident on a slit with a width of $a = 0.40$ mm. A thin converging lens of focal length +70 cm is placed between the slit and a viewing screen and focuses the light on the screen. (a) How far is the screen from the lens? (b) What is the distance on the screen from the center of the diffraction pattern to the first minimum?

4 E In conventional television, signals are broadcast from towers to home receivers. Even when a receiver is not in direct view of a tower because of a hill or building, it can still intercept a signal if the signal diffracts enough around the obstacle, into the obstacle's "shadow region." Previously, television signals had a wavelength of about 50 cm, but digital television signals that are transmitted from towers have a wavelength of about 10 mm. (a) Did this change in wavelength increase or decrease the diffraction of the signals into the shadow regions of obstacles? Assume that a signal passes through an opening of 5.0 m width between two adjacent buildings. What is the angular spread of the central diffraction maximum (out to the first minima) for wavelengths of (b) 50 cm and (c) 10 mm?

5 E A single slit is illuminated by light of wavelengths λ_a and λ_b, chosen so that the first diffraction minimum of the λ_a component coincides with the second minimum of the λ_b component. (a) If $\lambda_b = 350$ nm, what is λ_a? For what order number m_b (if any) does a minimum of the λ_b component coincide with the minimum of the λ_a component in the order number (b) $m_a = 2$ and (c) $m_a = 3$?

6 E Monochromatic light of wavelength 441 nm is incident on a narrow slit. On a screen 2.00 m away, the distance between the second diffraction minimum and the central maximum is 1.50 cm. (a) Calculate the angle of diffraction θ of the second minimum. (b) Find the width of the slit.

7 E Light of wavelength 633 nm is incident on a narrow slit. The angle between the first diffraction minimum on one side of the central maximum and the first minimum on the other side is 1.20°. What is the width of the slit?

8 M Sound waves with frequency 3000 Hz and speed 343 m/s diffract through the rectangular opening of a speaker cabinet and into a large auditorium of length $d = 100$ m. The opening, which has a horizontal width of 30.0 cm, faces a wall 100 m away (Fig. 36.7). Along that wall, how far from the central axis will a listener be at the first diffraction minimum and thus have difficulty hearing the sound? (Neglect reflections.)

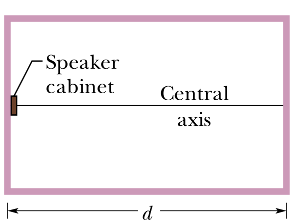


Figure 36.7 Problem 8.

9 M SSM A slit 1.00 mm wide is illuminated by light of wavelength 589 nm. We see a diffraction pattern on a screen 3.00 m away. What is the distance between the first two diffraction minima on the same side of the central diffraction maximum?

10 M GO Manufacturers of wire (and other objects of small dimension) sometimes use a laser to continually monitor the thickness of the product. The wire intercepts the laser beam, producing a diffraction pattern like that of a single slit of the same width as the wire diameter (Fig. 36.8). Suppose a helium–neon laser, of wavelength 632.8 nm, illuminates a wire, and the diffraction pattern appears on a screen at distance $L = 2.60$ m. If the desired wire diameter is 1.37 mm, what is the observed distance between the two tenth-order minima (one on each side of the central maximum)?

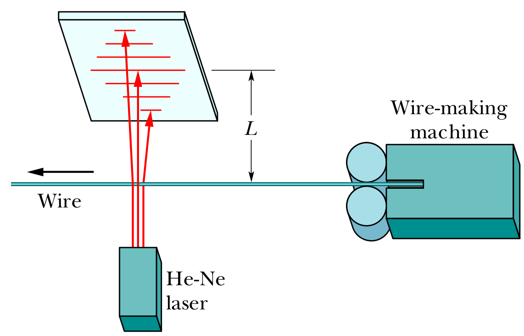


Figure 36.8 Problem 10.

Module 36.2 Intensity in Single-Slit Diffraction

11 E A 0.10-mm-wide slit is illuminated by light of wavelength 589 nm. Consider a point P on a viewing screen on which the diffraction pattern of the slit is viewed; the point is at 30° from the central axis of the slit. What is the phase difference between the Huygens wavelets arriving at point P from the top and midpoint of the slit? (*Hint:* See Eq. 36.2.1.)

12 E Figure 36.9 gives α versus the sine of the angle θ in a single-slit diffraction experiment using light of wavelength 610 nm. The vertical axis scale is set by $\alpha_s = 12$ rad. What are (a) the slit width, (b) the total number of diffraction minima in the pattern (count them on both sides of the center of the diffraction pattern), (c) the least angle for a minimum, and (d) the greatest angle for a minimum?

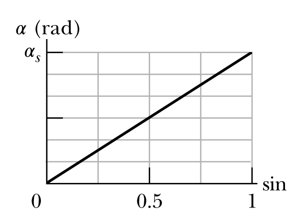


Figure 36.9 Problem 12.

13 E Monochromatic light with wavelength 538 nm is incident on a slit with width 0.025 mm. The distance from the slit to a screen is 3.5 m. Consider a point on the screen 1.1 cm from the central maximum. Calculate (a) θ for that point, (b) α, and (c) the ratio of the intensity at that point to the intensity at the central maximum.

14 E In the single-slit diffraction experiment of Fig. 36.1.4, let the wavelength of the light be 500 nm, the slit width be 6.00 μm, and the viewing screen be at distance $D = 3.00$ m. Let a y axis extend upward along the viewing screen, with its origin at the center of the diffraction pattern. Also let I_P represent the intensity of the diffracted light at point P at $y = 15.0$ cm. (a) What is the ratio of I_P to the intensity I_m at the center of the pattern? (b) Determine where point P is in the diffraction pattern by giving the maximum and minimum between which it lies, or the two minima between which it lies.

15 M SSM The full width at half-maximum (FWHM) of a central diffraction maximum is defined as the angle between the two points in the pattern where the intensity is one-half that at the center of the pattern. (See Fig. 36.2.2*b*.) (a) Show that the intensity drops to one-half the maximum value when $\sin^2 \alpha = \alpha^2/2$. (b) Verify that $\alpha = 1.39$ rad (about 80°) is a solution to the transcendental equation of (a). (c) Show that the FWHM is $\Delta\theta = 2 \sin^{-1}(0.442\lambda/a)$, where a is the slit width. Calculate the FWHM of the central maximum for slit width (d) 1.00λ, (e) 5.00λ, and (f) 10.0λ.

16 M *Babinet's principle.* A monochromatic beam of parallel light is incident on a "collimating" hole of diameter $x \gg \lambda$. Point P lies in the geometrical shadow region on a *distant* screen (Fig. 36.10*a*). Two diffracting objects, shown in Fig. 36.10*b*, are placed in turn over the collimating hole. Object A is an opaque circle with a hole in it, and B is the "photographic negative" of A. Using superposition concepts, show that the intensity at P is identical for the two diffracting objects A and B.

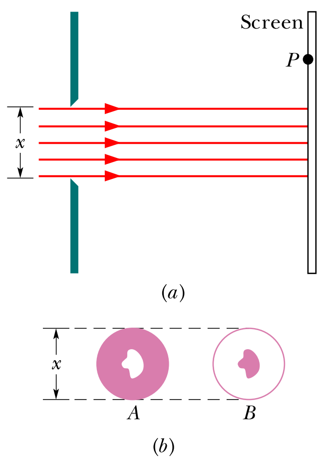


Figure 36.10 Problem 16.

17 M (a) Show that the values of α at which intensity maxima for single-slit diffraction occur can be found exactly by differentiating Eq. 36.2.2 with respect to α and equating the result to zero, obtaining the condition $\tan \alpha = \alpha$. To find values of α satisfying this relation, plot the curve $y = \tan \alpha$ and the straight line $y = \alpha$ and then find their intersections, or use a calculator to find an appropriate value of α by trial and error. Next, from $\alpha = (m + \frac{1}{2})\pi$, determine the values of m associated with the maxima in the single-slit pattern. (These m values are *not* integers because secondary maxima do not lie exactly halfway between minima.) What are the (b) smallest α and (c) associated m, the (d) second smallest α and (e) associated m, and the (f) third smallest α and (g) associated m?

Module 36.3 Diffraction by a Circular Aperture

18 E BIO The wall of a large room is covered with acoustic tile in which small holes are drilled 5.0 mm from center to center. How far can a person be from such a tile and still distinguish the individual holes, assuming ideal conditions, the pupil diameter of the observer's eye to be 4.0 mm, and the wavelength of the room light to be 550 nm?

19 E BIO (a) How far from grains of red sand must you be to position yourself just at the limit of resolving the grains if your pupil diameter is 1.5 mm, the grains are spherical with radius 50 μm, and the light from the grains has wavelength 650 nm? (b) If the grains were blue and the light from them had wavelength 400 nm, would the answer to (a) be larger or smaller?

20 E The radar system of a navy cruiser transmits at a wavelength of 1.6 cm, from a circular antenna with a diameter of 2.3 m. At a range of 6.2 km, what is the smallest distance that two speedboats can be from each other and still be resolved as two separate objects by the radar system?

21 E BIO SSM Estimate the linear separation of two objects on Mars that can just be resolved under ideal conditions by an observer on Earth (a) using the naked eye and (b) using the 200 in. (= 5.1 m) Mount Palomar telescope. Use the following data: distance to Mars $= 8.0 \times 10^7$ km, diameter of pupil $= 5.0$ mm, wavelength of light $= 550$ nm.

22 E BIO Assume that Rayleigh's criterion gives the limit of resolution of an astronaut's eye looking down on Earth's surface from a typical space shuttle altitude of 400 km. (a) Under that idealized assumption, estimate the smallest linear width on Earth's surface that the astronaut can resolve. Take the astronaut's pupil diameter to be 5 mm and the wavelength of visible light to be 550 nm. (b) Can the astronaut resolve the Great Wall of China (Fig. 36.11), which is more than 3000 km long, 5 to 10 m thick at its base, 4 m thick at its top, and 8 m in height? (c) Would the astronaut be able to resolve any unmistakable sign of intelligent life on Earth's surface?

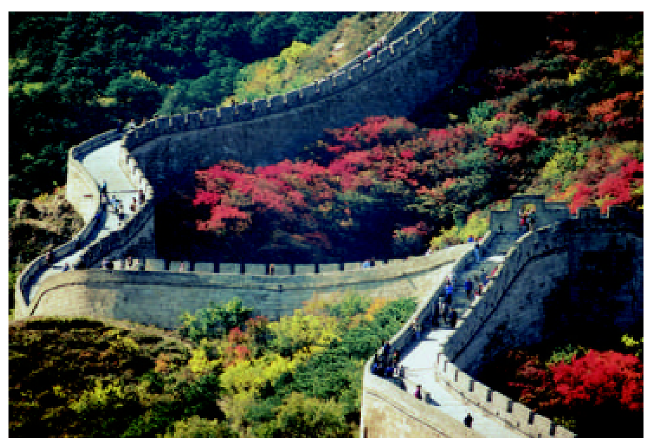


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Figure 36.11 Problem 22. The Great Wall of China.

23 E BIO SSM The two headlights of an approaching automobile are 1.4 m apart. At what (a) angular separation and (b) maximum distance will the eye resolve them? Assume that the pupil diameter is 5.0 mm, and use a wavelength of 550 nm for the light. Also assume that diffraction effects alone limit the resolution so that Rayleigh's criterion can be applied.

24 E BIO FCP *Entoptic halos.* If someone looks at a bright outdoor lamp in otherwise dark surroundings, the lamp appears to be surrounded by bright and dark rings (hence *halos*) that are actually a circular diffraction pattern as in Fig. 36.3.1, with the central maximum overlapping the direct light from the lamp. The diffraction is produced by structures within the cornea or lens of the eye (hence *entoptic*). If the lamp is monochromatic at wavelength 550 nm and the first dark ring subtends angular diameter 2.5° in the observer's view, what is the (linear) diameter of the structure producing the diffraction?

25 E Find the separation of two points on the Moon's surface that can just be resolved by the 200 in. (= 5.1 m) telescope at Mount Palomar, assuming that this separation is determined by diffraction effects. The distance from Earth to the Moon is 3.8×10^5 km. Assume a wavelength of 550 nm for the light.

26 E The telescopes on some commercial surveillance satellites can resolve objects on the ground as small as 85 cm across (see Google Earth), and the telescopes on military surveillance satellites reportedly can resolve objects as small as 10 cm across. Assume first that object resolution is determined entirely by Rayleigh's criterion and is not degraded by turbulence in the atmosphere. Also assume that the satellites are at a typical altitude of 400 km and that the wavelength of visible light is 550 nm. What would be the required diameter of the telescope aperture for (a) 85 cm resolution and (b) 10 cm resolution? (c) Now, considering that turbulence is certain to degrade resolution and that the aperture diameter of the Hubble Space Telescope is 2.4 m, what can you say about the answer to (b) and about how the military surveillance resolutions are accomplished?

27 E BIO If Superman really had x-ray vision at 0.10 nm wavelength and a 4.0 mm pupil diameter, at what maximum altitude could he distinguish villains from heroes, assuming that he needs to resolve points separated by 5.0 cm to do this?

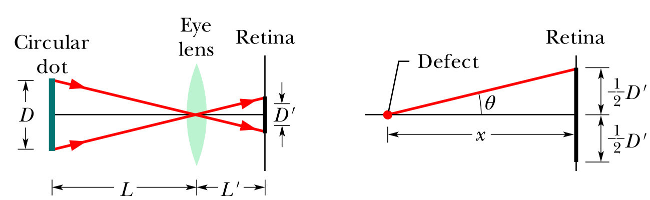
28 M BIO GO FCP The wings of tiger beetles (Fig. 36.12) are colored by interference due to thin cuticle-like layers. In addition, these layers are arranged in patches that are 60 μm across and produce different colors. The color you see is a pointillistic mixture of thin-film interference colors that varies with perspective. Approximately what viewing distance from a wing puts you at the limit of resolving the different colored patches according to Rayleigh's criterion? Use 550 nm as the wavelength of light and 3.00 mm as the diameter of your pupil.

Kjell B. Sandved/Bruce Coleman, Inc./Photoshot Holdings Ltd.

Figure 36.12 Problem 28. Tiger beetles are colored by pointillistic mixtures of thin-film interference colors.

29 M (a) What is the angular separation of two stars if their images are barely resolved by the Thaw refracting telescope at the Allegheny Observatory in Pittsburgh? The lens diameter is 76 cm and its focal length is 14 m. Assume $\lambda = 550$ nm. (b) Find the distance between these barely resolved stars if each of them is 10 light-years distant from Earth. (c) For the image of a single star in this telescope, find the diameter of the first dark ring in the diffraction pattern, as measured on a photographic plate placed at the focal plane of the telescope lens. Assume that the structure of the image is associated entirely with diffraction at the lens aperture and not with lens "errors."

30 M BIO GO FCP *Floaters.* The floaters you see when viewing a bright, featureless background are diffraction patterns of defects in the vitreous humor that fills most of your eye. Sighting through a pinhole sharpens the diffraction pattern. If you also view a small circular dot, you can approximate the defect's size. Assume that the defect diffracts light as a circular aperture does. Adjust the dot's distance L from your eye (or eye lens) until the dot and the circle of the first minimum in the diffraction pattern appear to have the same size in your view. That is, until they have the same diameter D' on the retina at distance $L' = 2.0$ cm from the front of the eye, as suggested in Fig. 36.13*a*, where the angles on the two sides of the eye lens are equal. Assume that the wavelength of visible light is $\lambda = 550$ nm. If the dot has diameter $D = 2.0$ mm and is distance $L = 45.0$ cm from the eye and the defect is $x = 6.0$ mm in front of the retina (Fig. 36.13*b*), what is the diameter of the defect?



(*a*) (*b*)

Figure 36.13 Problem 30.

31 M SSM Millimeter-wave radar generates a narrower beam than conventional microwave radar, making it less vulnerable to antiradar missiles than conventional radar. (a) Calculate the angular width 2θ of the central maximum, from first minimum to first minimum, produced by a 220 GHz radar beam emitted by a 55.0-cm-diameter circular antenna. (The frequency is chosen to coincide with a low-absorption atmospheric "window.") (b) What is 2θ for a more conventional circular antenna that has a diameter of 2.3 m and emits at wavelength 1.6 cm?

32 M (a) A circular diaphragm 60 cm in diameter oscillates at a frequency of 25 kHz as an underwater source of sound used for submarine detection. Far from the source, the sound intensity is distributed as the diffraction pattern of a circular hole whose diameter equals that of the diaphragm. Take the speed of sound in water to be 1450 m/s and find the angle between the normal to the diaphragm and a line from the diaphragm to the first minimum. (b) Is there such a minimum for a source having an (audible) frequency of 1.0 kHz?

33 M GO Nuclear-pumped x-ray lasers are seen as a possible weapon to destroy ICBM booster rockets at ranges up to 2000 km. One limitation on such a device is the spreading of the beam due to diffraction, with resulting dilution of beam intensity. Consider such a laser operating at a wavelength of 1.40 nm. The element that emits light is the end of a wire with diameter 0.200 mm. (a) Calculate the diameter of the central beam at a target 2000 km away from the beam source. (b) What is the ratio of the beam intensity at the target to that at the end of the wire? (The laser is fired from space, so neglect any atmospheric absorption.)

34 H GO FCP A circular obstacle produces the same diffraction pattern as a circular hole of the same diameter (except very near $\theta = 0$). Airborne water drops are examples of such obstacles. When you see the Moon through suspended water drops, such as in a fog, you intercept the diffraction pattern from many drops. The composite of the central diffraction maxima of those drops forms a white region that surrounds the Moon and may obscure it. Figure 36.14 is a photograph in which the Moon is obscured. There are two faint, colored rings around the Moon (the larger one may be too faint to be seen in your copy of the photograph). The smaller ring is on the outer edge of the central maxima from the drops; the somewhat larger ring is on the outer edge of the smallest of the secondary maxima from the drops (see Fig. 36.3.1). The color is visible because the rings are adjacent to the diffraction minima (dark rings) in the patterns. (Colors in other parts of the pattern overlap too much to be visible.)

(a) What is the color of these rings on the outer edges of the diffraction maxima? (b) The colored ring around the central maxima in Fig. 36.14 has an angular diameter that is 1.35 times the angular diameter of the Moon, which is 0.50°. Assume that the drops all have about the same diameter. Approximately what is that diameter?

Pekka Parvianen/ScienceSource

Figure 36.14 Problem 34. The corona around the Moon is a composite of the diffraction patterns of airborne water drops.

Module 36.4 Diffraction by a Double Slit

35 E Suppose that the central diffraction envelope of a double-slit diffraction pattern contains 11 bright fringes and the first diffraction minima eliminate (are coincident with) bright fringes. How many bright fringes lie between the first and second minima of the diffraction envelope?

36 E A beam of light of a single wavelength is incident perpendicularly on a double-slit arrangement, as in Fig. 35.2.5. The slit widths are each 46 μm and the slit separation is 0.30 mm. How many complete bright fringes appear between the two first-order minima of the diffraction pattern?

37 E In a double-slit experiment, the slit separation d is 2.00 times the slit width w. How many bright interference fringes are in the central diffraction envelope?

38 E In a certain two-slit interference pattern, 10 bright fringes lie within the second side peak of the diffraction envelope and diffraction minima coincide with two-slit interference maxima. What is the ratio of the slit separation to the slit width?

39 M Light of wavelength 440 nm passes through a double slit, yielding a diffraction pattern whose graph of intensity I versus angular position θ is shown in Fig. 36.15. Calculate (a) the slit width and (b) the slit separation. (c) Verify the displayed intensities of the $m = 1$ and $m = 2$ interference fringes.

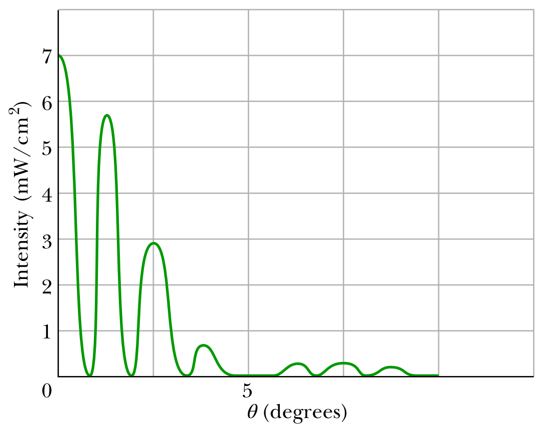


Figure 36.15 Problem 39.

40 M GO Figure 36.16 gives the parameter β of Eq. 36.4.2 versus the sine of the angle θ in a two-slit interference experiment using light of wavelength 435 nm. The vertical axis scale is set by $\beta_s = 80.0$ rad. What are (a) the slit separation, (b) the total number of interference maxima (count them on both sides of the pattern's center), (c) the smallest angle for a maxima, and (d) the greatest angle for a minimum? Assume that none of the interference maxima are completely eliminated by a diffraction minimum.

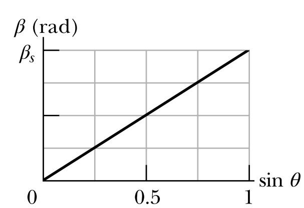


Figure 36.16 Problem 40.

41 M GO In the two-slit interference experiment of Fig. 35.2.5, the slit widths are each 12.0 μm, their separation is 24.0 μm, the wavelength is 600 nm, and the viewing screen is at a distance of 4.00 m. Let I_P represent the intensity at point P on the screen, at height $y = 70.0$ cm. (a) What is the ratio of I_P to the intensity I_m at the center of the pattern? (b) Determine where P is in the two-slit interference pattern by giving the maximum or minimum on which it lies or the maximum and minimum between which it lies. (c) In the same way, for the diffraction that occurs, determine where point P is in the diffraction pattern.

42 M GO (a) In a double-slit experiment, what largest ratio of d to a causes diffraction to eliminate the fourth bright side fringe? (b) What other bright fringes are also eliminated? (c) How many other ratios of d to a cause the diffraction to (exactly) eliminate that bright fringe?

43 M SSM (a) How many bright fringes appear between the first diffraction-envelope minima to either side of the central maximum in a double-slit pattern if $\lambda = 550$ nm, $d = 0.150$ mm, and $a = 30.0\,\mu$m? (b) What is the ratio of the intensity of the third bright fringe to the intensity of the central fringe?

Module 36.5 Diffraction Gratings

44 E BIO FCP Perhaps to confuse a predator, some tropical gyrinid beetles (whirligig beetles) are colored by optical interference that is due to scales whose alignment forms a diffraction grating (which scatters light instead of transmitting it). When the incident light rays are perpendicular to the grating, the angle between the first-order maxima (on opposite sides of the zeroth-order maximum) is about 26° in light with a wavelength of 550 nm. What is the grating spacing of the beetle?

45 E A diffraction grating 20.0 mm wide has 6000 rulings. Light of wavelength 589 nm is incident perpendicularly on the grating. What are the (a) largest, (b) second largest, and (c) third largest values of θ at which maxima appear on a distant viewing screen?

46 E Visible light is incident perpendicularly on a grating with 315 rulings/mm. What is the longest wavelength that can be seen in the fifth-order diffraction?

47 E SSM A grating has 400 lines/mm. How many orders of the entire visible spectrum (400–700 nm) can it produce in a diffraction experiment, in addition to the $m = 0$ order?

48 M A diffraction grating is made up of slits of width 300 nm with separation 900 nm. The grating is illuminated by monochromatic plane waves of wavelength $\lambda = 600$ nm at normal incidence. (a) How many maxima are there in the full diffraction pattern? (b) What is the angular width of a spectral line observed in the first order if the grating has 1000 slits?

49 M SSM Light of wavelength 600 nm is incident normally on a diffraction grating. Two adjacent maxima occur at angles given by $\sin\theta = 0.2$ and $\sin\theta = 0.3$. The fourth-order maxima are missing. (a) What is the separation between adjacent slits? (b) What is the smallest slit width this grating can have? For that slit width, what are the (c) largest, (d) second largest, and (e) third largest values of the order number m of the maxima produced by the grating?

50 M With light from a gaseous discharge tube incident normally on a grating with slit separation 1.73 μm, sharp maxima of green light are experimentally found at angles $\theta = \pm 17.6°, 37.3°, -37.1°, 65.2°$, and $-65.0°$. Compute the wavelength of the green light that best fits these data.

51 M GO A diffraction grating having 180 lines/mm is illuminated with a light signal containing only two wavelengths, $\lambda_1 = 400$ nm and $\lambda_2 = 500$ nm. The signal is incident perpendicularly on the grating. (a) What is the angular separation between the second-order maxima of these two wavelengths? (b) What is the smallest angle at which two of the resulting maxima are superimposed? (c) What is the highest order for which maxima for both wavelengths are present in the diffraction pattern?

52 M GO A beam of light consisting of wavelengths from 460.0 nm to 640.0 nm is directed perpendicularly onto a diffraction grating with 160 lines/mm. (a) What is the lowest order that is overlapped by another order? (b) What is the highest order for which the complete wavelength range of the beam is present? In that highest order, at what angle does the light at wavelength (c) 460.0 nm and (d) 640.0 nm appear? (e) What is the greatest angle at which the light at wavelength 460.0 nm appears?

53 M GO A grating has 350 rulings/mm and is illuminated at normal incidence by white light. A spectrum is formed on a screen 30.0 cm from the grating. If a hole 10.0 mm square is cut in the screen, its inner edge being 50.0 mm from the central maximum and parallel to it, what are the (a) shortest and (b) longest wavelengths of the light that passes through the hole?

54 M Derive this expression for the intensity pattern for a three-slit "grating":

$$I = \tfrac{1}{9} I_m (1 + 4\cos\phi + 4\cos^2\phi),$$

where $\phi = (2\pi d \sin\theta)/\lambda$ and $a \ll \lambda$.

Module 36.6 Gratings: Dispersion and Resolving Power

55 E SSM A source containing a mixture of hydrogen and deuterium atoms emits red light at two wavelengths whose mean is 656.3 nm and whose separation is 0.180 nm. Find the minimum number of lines needed in a diffraction grating that can resolve these lines in the first order.

56 E (a) How many rulings must a 4.00-cm-wide diffraction grating have to resolve the wavelengths 415.496 and 415.487 nm in the second order? (b) At what angle are the second-order maxima found?

57 E Light at wavelength 589 nm from a sodium lamp is incident perpendicularly on a grating with 40 000 rulings over width 76 mm. What are the first-order (a) dispersion D and (b) resolving power R, the second-order (c) D and (d) R, and the third-order (e) D and (f) R?

58 E A grating has 600 rulings/mm and is 5.0 mm wide. (a) What is the smallest wavelength interval it can resolve in the third order at $\lambda = 500$ nm? (b) How many higher orders of maxima can be seen?

59 E A diffraction grating with a width of 2.0 cm contains 1000 lines/cm across that width. For an incident wavelength of 600 nm, what is the smallest wavelength difference this grating can resolve in the second order?

60 E The D line in the spectrum of sodium is a doublet with wavelengths 589.0 and 589.6 nm. Calculate the minimum number of lines needed in a grating that will resolve this doublet in the second-order spectrum.

61 E With a particular grating the sodium doublet (589.00 nm and 589.59 nm) is viewed in the third order at 10° to the normal and is barely resolved. Find (a) the grating spacing and (b) the total width of the rulings.

62 M A diffraction grating illuminated by monochromatic light normal to the grating produces a certain line at angle θ. (a) What is the product of that line's half-width and the grating's resolving power? (b) Evaluate that product for the first order of a grating of slit separation 900 nm in light of wavelength 600 nm.

63 **M** Assume that the limits of the visible spectrum are arbitrarily chosen as 430 and 680 nm. Calculate the number of rulings per millimeter of a grating that will spread the first-order spectrum through an angle of 20.0°.

Module 36.7 X-Ray Diffraction

64 **E** What is the smallest Bragg angle for x rays of wavelength 30 pm to reflect from reflecting planes spaced 0.30 nm apart in a calcite crystal?

65 **E** An x-ray beam of wavelength A undergoes first-order reflection (Bragg law diffraction) from a crystal when its angle of incidence to a crystal face is 23°, and an x-ray beam of wavelength 97 pm undergoes third-order reflection when its angle of incidence to that face is 60°. Assuming that the two beams reflect from the same family of reflecting planes, find (a) the interplanar spacing and (b) the wavelength A.

66 **E** An x-ray beam of a certain wavelength is incident on an NaCl crystal, at 30.0° to a certain family of reflecting planes of spacing 39.8 pm. If the reflection from those planes is of the first order, what is the wavelength of the x rays?

67 **E** Figure 36.17 is a graph of intensity versus angular position θ for the diffraction of an x-ray beam by a crystal. The horizontal scale is set by $\theta_s = 2.00°$. The beam consists of two wavelengths, and the spacing between the reflecting planes is 0.94 nm. What are the (a) shorter and (b) longer wavelengths in the beam?

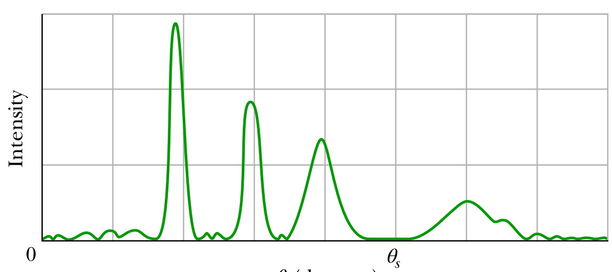


Figure 36.17 Problem 67.

68 **E** If first-order reflection occurs in a crystal at Bragg angle 3.4°, at what Bragg angle does second-order reflection occur from the same family of reflecting planes?

69 **E** X rays of wavelength 0.12 nm are found to undergo second-order reflection at a Bragg angle of 28° from a lithium fluoride crystal. What is the interplanar spacing of the reflecting planes in the crystal?

70 **M** GO In Fig. 36.18, first-order reflection from the reflection planes shown occurs when an x-ray beam of wavelength 0.260 nm makes an angle $\theta = 63.8°$ with the top face of the crystal. What is the unit cell size a_0?

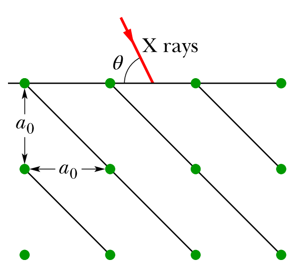


Figure 36.18 Problem 70.

71 **M** In Fig. 36.19, let a beam of x rays of wavelength 0.125 nm be incident on an NaCl crystal at angle $\theta = 45.0°$ to the top face of the crystal and a family of reflecting planes. Let the reflecting planes have separation $d = 0.252$ nm. The crystal is turned through angle ϕ around an axis perpendicular to the plane of the page until these reflecting planes give diffraction maxima. What are the (a) smaller and (b) larger value of ϕ if the crystal is turned clockwise and the (c) smaller and (d) larger value of ϕ if it is turned counterclockwise?

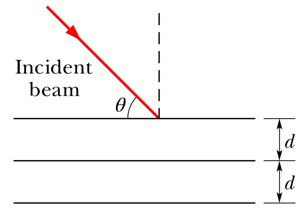


Figure 36.19 Problems 71 and 72.

72 **M** In Fig. 36.19, an x-ray beam of wavelengths from 95.0 to 140 pm is incident at $\theta = 45.0°$ to a family of reflecting planes with spacing $d = 275$ pm. What are the (a) longest wavelength λ and (b) associated order number m and the (c) shortest λ and (d) associated m of the intensity maxima in the diffraction of the beam?

73 **M** Consider a two-dimensional square crystal structure, such as one side of the structure shown in Fig. 36.7.2*a*. The largest interplanar spacing of reflecting planes is the unit cell size a_0. Calculate and sketch the (a) second largest, (b) third largest, (c) fourth largest, (d) fifth largest, and (e) sixth largest interplanar spacing. (f) Show that your results in (a) through (e) are consistent with the general formula

$$d = \frac{a_0}{\sqrt{h^2 + k^2}},$$

where h and k are relatively prime integers (they have no common factor other than unity).

Additional Problems

74 **BIO** An astronaut in a space shuttle claims she can just barely resolve two point sources on Earth's surface, 160 km below. Calculate their (a) angular and (b) linear separation, assuming ideal conditions. Take $\lambda = 540$ nm and the pupil diameter of the astronaut's eye to be 5.0 mm.

75 **SSM** Visible light is incident perpendicularly on a diffraction grating of 200 rulings/mm. What are the (a) longest, (b) second longest, and (c) third longest wavelengths that can be associated with an intensity maximum at $\theta = 30.0°$?

76 A beam of light consists of two wavelengths, 590.159 nm and 590.220 nm, that are to be resolved with a diffraction grating. If the grating has lines across a width of 3.80 cm, what is the minimum number of lines required for the two wavelengths to be resolved in the second order?

77 **SSM** In a single-slit diffraction experiment, there is a minimum of intensity for orange light ($\lambda = 600$ nm) and a minimum of intensity for blue-green light ($\lambda = 500$ nm) at the same angle of 1.00 mrad. For what minimum slit width is this possible?

78 GO A double-slit system with individual slit widths of 0.030 mm and a slit separation of 0.18 mm is illuminated with 500 nm light directed perpendicular to the plane of the slits. What is the total number of complete bright fringes appearing between the two first-order minima of the diffraction pattern? (Do not count the fringes that coincide with the minima of the diffraction pattern.)

79 **SSM** Consider a diffraction grating that has resolving power $R = \lambda_{avg}/\Delta\lambda = Nm$. (a) Show that the corresponding frequency range Δf that can just be resolved is given by $\Delta f = c/Nm\lambda$. (b) From Fig. 36.5.5, show that the times required for light to travel along the ray at the bottom of the figure and the ray at the top differ by $\Delta t = (Nd/c) \sin\theta$. (c) Show that $(\Delta f)(\Delta t) = 1$, this relation being independent of the various grating parameters. Assume $N \gg 1$.

80 BIO The pupil of a person's eye has a diameter of 5.00 mm. According to Rayleigh's criterion, what distance apart must two small objects be if their images are just barely resolved when they are 250 mm from the eye? Assume they are illuminated with light of wavelength 500 nm.

81 Light is incident on a grating at an angle ψ as shown in Fig. 36.20. Show that bright fringes occur at angles θ that satisfy the equation

$$d(\sin\psi + \sin\theta) = m\lambda, \quad \text{for } m = 0, 1, 2, \ldots.$$

(Compare this equation with Eq. 36.5.1.) Only the special case $\psi = 0$ has been treated in this chapter.

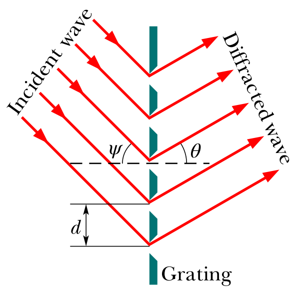


Figure 36.20 Problem 81.

82 A grating with $d = 1.50\,\mu$m is illuminated at various angles of incidence by light of wavelength 600 nm. Plot, as a function of the angle of incidence (0 to 90°), the angular deviation of the first-order maximum from the incident direction. (See Problem 81.)

83 SSM In two-slit interference, if the slit separation is 14 μm and the slit widths are each 2.0 μm, (a) how many two-slit maxima are in the central peak of the diffraction envelope and (b) how many are in either of the first side peak of the diffraction envelope?

84 GO In a two-slit interference pattern, what is the ratio of slit separation to slit width if there are 17 bright fringes within the central diffraction envelope and the diffraction minima coincide with two-slit interference maxima?

85 A beam of light with a narrow wavelength range centered on 450 nm is incident perpendicularly on a diffraction grating with a width of 1.80 cm and a line density of 1400 lines/cm across that width. For this light, what is the smallest wavelength difference this grating can resolve in the third order?

86 BIO If you look at something 40 m from you, what is the smallest length (perpendicular to your line of sight) that you can resolve, according to Rayleigh's criterion? Assume the pupil of your eye has a diameter of 4.00 mm, and use 500 nm as the wavelength of the light reaching you.

87 BIO Two yellow flowers are separated by 60 cm along a line perpendicular to your line of sight to the flowers. How far are you from the flowers when they are at the limit of resolution according to the Rayleigh criterion? Assume the light from the flowers has a single wavelength of 550 nm and that your pupil has a diameter of 5.5 mm.

88 In a single-slit diffraction experiment, what must be the ratio of the slit width to the wavelength if the second diffraction minima are to occur at an angle of 37.0° from the center of the diffraction pattern on a viewing screen?

89 A diffraction grating 3.00 cm wide produces the second order at 33.0° with light of wavelength 600 nm. What is the total number of lines on the grating?

90 A single-slit diffraction experiment is set up with light of wavelength 420 nm, incident perpendicularly on a slit of width 5.10 μm. The viewing screen is 3.20 m distant. On the screen, what is the distance between the center of the diffraction pattern and the second diffraction minimum?

91 A diffraction grating has 8900 slits across 1.20 cm. If light with a wavelength of 500 nm is sent through it, how many orders (maxima) lie to one side of the central maximum?

92 In an experiment to monitor the Moon's surface with a light beam, pulsed radiation from a ruby laser ($\lambda = 0.69\,\mu$m) was directed to the Moon through a reflecting telescope with a mirror radius of 1.3 m. A reflector on the Moon behaved like a circular flat mirror with radius 10 cm, reflecting the light directly back toward the telescope on Earth. The reflected light was then detected after being brought to a focus by this telescope. Approximately what fraction of the original light energy was picked up by the detector? Assume that for each direction of travel all the energy is in the central diffraction peak.

93 In June 1985, a laser beam was sent out from the Air Force Optical Station on Maui, Hawaii, and reflected back from the shuttle *Discovery* as it sped by 354 km overhead. The diameter of the central maximum of the beam at the shuttle position was said to be 9.1 m, and the beam wavelength was 500 nm. What is the effective diameter of the laser aperture at the Maui ground station? (*Hint:* A laser beam spreads only because of diffraction; assume a circular exit aperture.)

94 A diffraction grating 1.00 cm wide has 10 000 parallel slits. Monochromatic light that is incident normally is diffracted through 30° in the first order. What is the wavelength of the light?

95 SSM If you double the width of a single slit, the intensity of the central maximum of the diffraction pattern increases by a factor of 4, even though the energy passing through the slit only doubles. Explain this quantitatively.

96 When monochromatic light is incident on a slit 22.0 μm wide, the first diffraction minimum lies at 1.80° from the direction of the incident light. What is the wavelength?

97 A spy satellite orbiting at 160 km above Earth's surface has a lens with a focal length of 3.6 m and can resolve objects on the ground as small as 30 cm. For example, it can easily measure the size of an aircraft's air intake port. What is the effective diameter of the lens as determined by diffraction consideration alone? Assume $\lambda = 550$ nm.

98 *Epidural with fiber Bragg grating.* A fiber Bragg grating is an optical fiber that has had its core treated with ultraviolet light so that it has a periodic variation in its index of refraction, with a certain spacing d. Along a few millimeters, there are "lines" with a greater index than the rest of the core (Fig. 36.21*a*). When light over a broad wavelength range is sent into the fiber, one wavelength, called the Bragg wavelength λ_B, is reflected and the rest is transmitted. The value of λ_B depends on d. If a force F decreases the length of the grating, decreasing d, then λ_B decreases. Thus, the grating acts as a *strain gauge*. Figure 36.21*b* gives the change $\Delta\lambda_B$ in the Bragg wavelength versus applied force F.

Recent research suggests that a fiber Bragg grating could be used in robotic assisted surgery in an epidural procedure in which a needle is inserted into the epidural space of the spinal column to release an anesthetic fluid. The surgeon first inserts the needle into the back and then manually monitors the force magnitude required to advance the needle. This tricky procedure requires much practice so that the surgeon knows when the needle has reached the epidural space and not overshot it, an error that could result in serious complications. Figure 36.21*c* is a graph of the force magnitude F versus displacement x of the needle tip in a typical epidural procedure. (The line segments have been straightened somewhat from the original data.) (1) As x increases from 0, the skin resists the needle, but at x = 8.0 mm the force is finally great enough to pierce the skin, and then the required force decreases. (2) Next, the needle finally pierces the interspinous ligament at x = 18 mm and (3) the relatively tough ligamentum flavum at x = 30 mm. (4) As the needle then enters the epidural space, the force drops sharply. A new surgeon must learn this pattern of force magnitude versus displacement to recognize when to stop pushing on the needle.

If a fiber Bragg grating could be incorporated into an epidural needle, an automated system could monitor $\Delta\lambda_B$ to determine when the needle is properly placed. For the plot of Fig. 36.21*b*, what is $\Delta\lambda_B$ for the peak force at (a) x = 8.0 mm, (b) x = 18 mm, and (c) x = 30 mm?

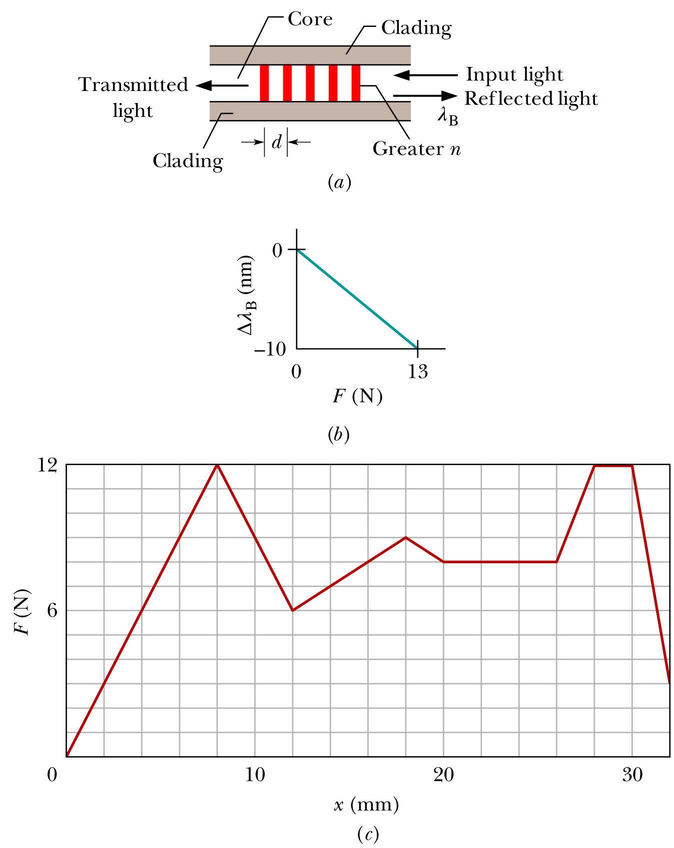


Figure 36.21 Problem 98.