CHAPTER 34

Images

34.1 IMAGES AND PLANE MIRRORS

Learning Objectives

After reading this module, you should be able to . . .

34.1.1 Distinguish virtual images from real images.

34.1.2 Explain the common roadway mirage.

34.1.3 Sketch a ray diagram for the reflection of a point source of light by a plane mirror, indicating the object distance and image distance.

34.1.4 Using the proper algebraic sign, relate the object distance p to the image distance i.

34.1.5 Give an example of the apparent hallway that you can see in a mirror maze based on equilateral triangles.

Key Ideas

- An image is a reproduction of an object via light. If the image can form on a surface, it is a real image and can exist even if no observer is present. If the image requires the visual system of an observer, it is a virtual image.
- A plane (flat) mirror can form a virtual image of a light source (said to be the object) by redirecting light rays emerging from the source. The image can be seen where backward extensions of reflected rays pass through one another. The objectâ€“distance p from the mirror is related to the (apparent) image distance i from the mirror by

$$i = -p \quad \text{(plane mirror)}.$$

Object distance p is a positive quantity. Image distance i for a virtual image is a negative quantity.

What Is Physics?

One goal of physics is to discover the basic laws governing light, such as the law of refraction. A broader goal is to put those laws to use, and perhaps the most important use is the production of images. The first photographic images, made in 1824, were only novelties, but our world now thrives on images. Huge industries are based on the production of images on television, computer, and theater screens. Images from satellites guide military strategists during times of conflict and environmental strategists during times of blight. Camera surveillance can make a subway system more secure, but it can also invade the privacy of unsuspecting citizens. Physiologists and medical engineers are still puzzled by how images are produced by the human eye and the visual cortex of the brain, but they have managed to create mental images in some sightless people by electrical stimulation of the visual cortex.

Our first step in this chapter is to define and classify images. Then we examine several basic ways in which they can be produced.

Two Types of Image

For you to see, say, a penguin, your eye must intercept some of the light rays spreading from the penguin and then redirect them onto the retina at the rear of the eye. Your visual system, starting with the retina and ending with the visual cortex at the rear of your brain, automatically and subconsciously processes the information provided by the light. That system identifies edges, orientations, textures, shapes, and colors and then rapidly brings to your consciousness an **image** (a reproduction derived from light) of the penguin; you perceive and recognize the penguin as being in the direction from which the light rays came and at the proper distance.

Your visual system goes through this processing and recognition even if the light rays do not come directly from the penguin, but instead reflect toward you from a mirror or refract through the lenses in a pair of binoculars. However, you now see the penguin in the direction from which the light rays came after they reflected or refracted, and the distance you perceive may be quite different from the penguinâ€™se distance.

For example, if the light rays have been reflected toward you from a standard flat mirror, the penguin appears to be behind the mirror because the rays you intercept come from that direction. Of course, the penguin is not back there. This type of image, which is called a **virtual image,** truly exists only within the brain but nevertheless is *said* to exist at the perceived location.

A **real image** differs in that it can be formed on a surface, such as a card or a movie screen. You can see a real image (otherwise movie theaters would be empty), but the existence of the image does not depend on your seeing it and it is present even if you are not. Before we discuss real and virtual images in detail, letâ€™sxamine a natural virtual image.

A Common Mirage

A common example of a virtual image is a pool of water that appears to lie on the road some distance ahead of you on a sunny day, but that you can never reach. The pool is a *mirage* (a type of illusion), formed by light rays coming from the low section of the sky in front of you (Fig. 34.1.1*a*). As the rays approach the road, they travel through progressively warmer air that has been heated by the road, which is usually relatively warm. With an increase in air temperature, the density of the airâ€”and hence the index of refraction of the airâ€”decreases slightly. Thus, as the rays descend, encountering progressively smaller indexes of refraction, they continuously bend toward the horizontal (Fig. 34.1.1*b*).

Once a ray is horizontal, somewhat above the roadâ€™surface, it still bends because the lower portion of each associated wavefront is in slightly warmer air and is moving slightly faster than the upper portion of the wavefront (Fig. 34.1.1*c*). This nonuniform motion of the wavefronts bends the ray upward. As the ray then ascends, it continues to bend upward through progressively greater indexes of refraction (Fig. 34.1.1*d*).

If you intercept some of this light, your visual system automatically infers that it originated along a backward extension of the rays you have intercepted and, to make sense of the light, assumes that it came from the road surface. If the light happens to be bluish from blue sky, the mirage appears bluish, like water. Because the air is probably turbulent due to the heating, the mirage shimmies, as if water waves were present. The bluish coloring and the shimmy enhance the illusion of a pool of water, but you are actually seeing a virtual image of a low section of the sky. As you travel toward the illusionary pool, you no longer intercept the shallow refracted rays and the illusion disappears. FCP

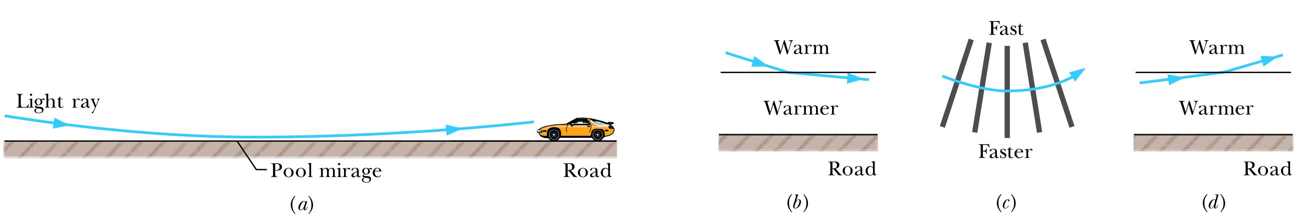


Figure 34.1.1 (*a*) A ray from a low section of the sky refracts through air that is heated by a road (without reaching the road). An observer who intercepts the light perceives it to be from a pool of water on the road. (*b*) Bending (exaggerated) of a light ray descending across an imaginary boundary from warm air to warmer air. (*c*) Shifting of wavefronts and associated bending of a ray, which occur because the lower ends of wavefronts move faster in warmer air. (*d*) Bending of a ray ascending across an imaginary boundary to warm air from warmer air.

In a plane mirror the light seems to come from an object on the other side.

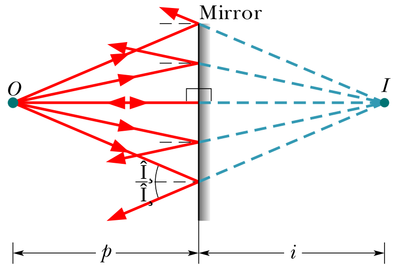


Figure 34.1.2 A point source of light O, called the *object*, is a perpendicular distance p in front of a plane mirror. Light rays reaching the mirror from O reflect from the mirror. If your eye intercepts some of the reflected rays, you perceive a point source of light I to be behind the mirror, at a perpendicular distance i. The perceived source I is a virtual image of object O.

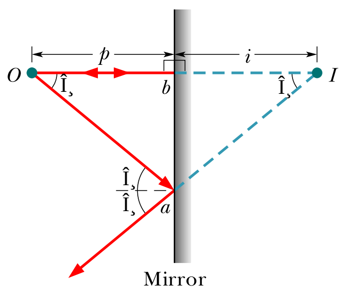


Figure 34.1.3 Two rays from Fig. 34.1.2. Ray Oa makes an arbitrary angle θ with the normal to the mirror surface. Ray Ob is perpendicular to the mirror.

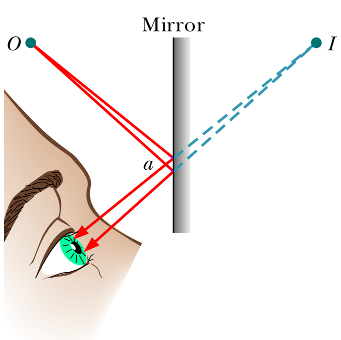


Figure 34.1.4 A "pencil" of rays from O enters the eye after reflection at the mirror. Only a small portion of the mirror near a is involved in this reflection. The light appears to originate at point I behind the mirror.

Plane Mirrors

A **mirror** is a surface that can reflect a beam of light in one direction instead of either scattering it widely in many directions or absorbing it. A shiny metal surface acts as a mirror; a concrete wall does not. In this module we examine the images that a **plane mirror** (a flat reflecting surface) can produce.

Figure 34.1.2 shows a point source of light O, which we shall call the *object,* at a perpendicular distance p in front of a plane mirror. The light that is incident on the mirror is represented with rays spreading from O. The reflection of that light is represented with reflected rays spreading from the mirror. If we extend the reflected rays backward (behind the mirror), we find that the extensions intersect at a point that is a perpendicular distance i behind the mirror.

If you look into the mirror of Fig. 34.1.2, your eyes intercept some of the reflected light. To make sense of what you see, you perceive a point source of light located at the point of intersection of the extensions. This point source is the image I of object O. It is called a *point image* because it is a point, and it is a virtual image because the rays do not actually pass through it. (As you will see, rays *do* pass through a point of intersection for a real image.)

Ray Tracing. Figure 34.1.3 shows two rays selected from the many rays in Fig. 34.1.2. One reaches the mirror at point b, perpendicularly. The other reaches it at an arbitrary point a, with an angle of incidence θ. The extensions of the two reflected rays are also shown. The right triangles $aOba$ and $aIba$ have a common side and three equal angles and are thus congruent (equal in size), so their horizontal sides have the same length. That is,

$$Ib = Ob, \tag{34.1.1}$$

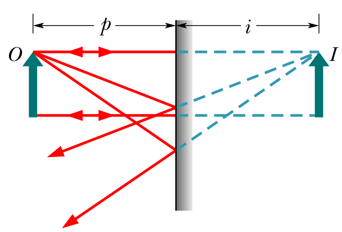
where Ib and Ob are the distances from the mirror to the image and the object, respectively. Equation 34.1.1 tells us that the image is as far behind the mirror as the object is in front of it. By convention (that is, to get our equations to work out), *object distances p* are taken to be positive quantities and *image distances i* for virtual images (as here) are taken to be negative quantities. Thus, Eq. 34.1.1 can be written as $|i| = p$ or as

$$i = -p \quad \text{(plane mirror)}. \tag{34.1.2}$$

Only rays that are fairly close together can enter the eye after reflection at a mirror. For the eye position shown in Fig. 34.1.4, only a small portion of the mirror near point a (a portion smaller than the pupil of the eye) is useful in forming the image. To find this portion, close one eye and look at the mirror image of a small object such as the tip of a pencil. Then move your fingertip over the mirror surface until you cannot see the image. Only that small portion of the mirror under your fingertip produced the image. FCP

Extended Objects

In Fig. 34.1.5, an extended object O, represented by an upright arrow, is at perpendicular distance p in front of a plane mirror. Each small portion of the



In a plane mirror the image is just as far from the mirror as the object.

Figure 34.1.5 An extended object O and its virtual image I in a plane mirror.

Courtesy of Adrian Fisher, www.mazemaker.com

Figure 34.1.6 A maze of mirrors.

object that faces the mirror acts like the point source O of Figs. 34.1.2 and 34.1.3. If you intercept the light reflected by the mirror, you perceive a virtual image I that is a composite of the virtual point images of all those portions of the object. This virtual image seems to be at (negative) distance i behind the mirror, with i and p related by Eq. 34.1.2.

We can also locate the image of an extended object as we did for a point object in Fig. 34.1.2: We draw some of the rays that reach the mirror from the top of the object, draw the corresponding reflected rays, and then extend those reflected rays behind the mirror until they intersect to form an image of the top of the object. We then do the same for rays from the bottom of the object. As shown in Fig. 34.1.5, we find that virtual image I has the same orientation and *height* (measured parallel to the mirror) as object O.

Mirror Maze

In a mirror maze (Fig. 34.1.6), each wall is covered, floor to ceiling, with a mirror. Walk through such a maze and what you see in most directions is a confusing montage of reflections. In some directions, however, you see a hallway that seems to offer a path through the maze. Take these hallways, though, and you soon learn, after smacking into mirror after mirror, that the hallways are largely an illusion.

Figure 34.1.7*a* is an overhead view of a simple mirror maze in which differently painted floor sections form equilateral triangles (60Âângles) and walls are covered with vertical mirrors. You look into the maze while standing at point O at the middle of the maze entrance. In most directions, you see a confusing jumble of images. However, you see something curious in the direction of the ray shown in Fig. 34.1.7*a*. That ray leaves the middle of mirror B and reflects to you at the middle of mirror A. (The reflection obeys the law of reflection, with the angle of incidence and the angle of reflection both equal to 30Â°.)

To make sense of the origin of the ray reaching you, your brain automatically extends the ray backward. It appears to originate at a point lying *behind* mirror A. That is, you perceive a virtual image of B behind A, at a distance equal to the actual distance between A and B (Fig. 34.1.7*b*). Thus, when you face into the maze in this direction, you see B along an apparent straight hallway consisting of four triangular floor sections.

This story is incomplete, however, because the ray reaching you does not *originate* at mirror Bâ€"itonly reflects there. To find the origin, we continue to apply the law of reflection as we work backwards, reflection by reflection on the mirrors (Fig. 34.1.7*c*). We finally come to the origin of the ray: you! What you see when you look along the apparent hallway is a virtual image of yourself, at a distance of nine triangular floor sections from you (Fig. 34.1.7*d*). FCP

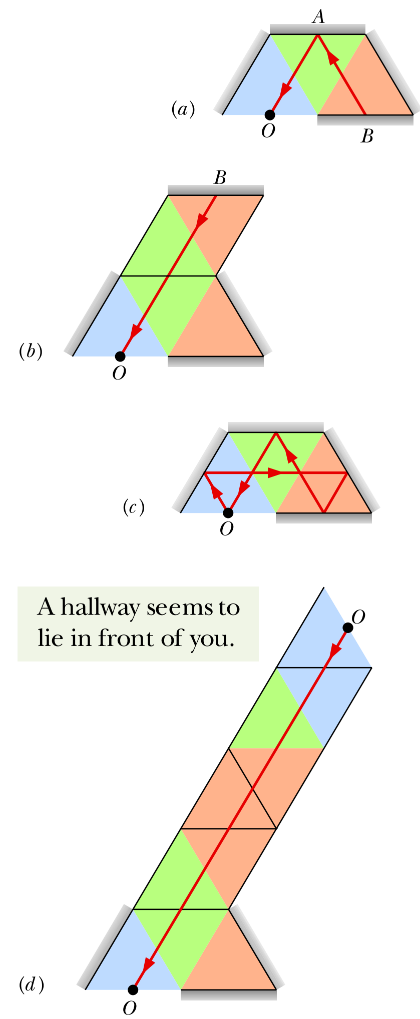
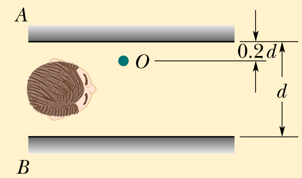


Figure 34.1.7 (*a*) Overhead view of a mirror maze. A ray from mirror B reaches you at O by reflecting from mirror A. (*b*) Mirror B appears to be behind A. (*c*) The ray reaching you comes from you. (*d*) You see a virtual image of yourself at the end of an apparent hallway. (Can you find a second apparent hallway extending away from point O?)

Checkpoint 34.1.1



In the figure you are in a system of two vertical parallel mirrors A and B separated by distance d. A grinning gargoyle is perched at point O, a distance $0.2d$ from mirror A. Each mirror produces a *first* (least deep) image of the gargoyle. Then each mirror produces a *second* image with the object being the first image in the opposite mirror. Then each mirror produces a *third* image with the object being the second image in the opposite mirror, and so onâ€"you might see hundreds of grinning gargoyle images. How deep behind mirror A are the first, second, and third images in mirror A? FCP

34.2 SPHERICAL MIRRORS

Learning Objectives

After reading this module, you should be able to . . .

34.2.1 Distinguish a concave spherical mirror from a convex spherical mirror.

34.2.2 For concave and convex mirrors, sketch a ray diagram for the reflection of light rays that are initially parallel to the central axis, indicating how they form the focal points, and identifying which is real and which is virtual.

34.2.3 Distinguish a real focal point from a virtual focal point, identify which corresponds to which type of mirror, and identify the algebraic sign associated with each focal length.

34.2.4 Relate a focal length of a spherical mirror to the radius.

34.2.5 Identify the terms â€œinsideâ€ the focal pointâ€ and â€œoutsideâ€ the focal point.â€

34.2.6 For an object (a) inside and (b) outside the focal point of a concave mirror, sketch the reflections of at least two rays to find the image and identify the type and orientation of the image.

34.2.7 For a concave mirror, distinguish the locations and orientations of a real image and a virtual image.

34.2.8 For an object in front of a convex mirror, sketch the reflections of at least two rays to find the image and identify the type and orientation of the image.

34.2.9 Identify which type of mirror can produce both real and virtual images and which type can produce only virtual images.

34.2.10 Identify the algebraic signs of the image distance i for real images and virtual images.

34.2.11 For convex, concave, and plane mirrors, apply the relationship between the focal length f, object distance p, and image distance i.

34.2.12 Apply the relationships between lateral magnification m, image height hâ€², object height h, image distance i, and object distance p.

Key Ideas

â— A spherical mirror is in the shape of a small section of a spherical surface and can be concave (the radius of curvature r is a positive quantity), convex (r is a negative quantity), or plane (flat, r is infinite).

â— If parallel rays are sent into a (spherical) concave mirror parallel to the central axis, the reflected rays pass through a common point (a real focus F) at a distance f (a positive quantity) from the mirror. If they are sent toward a (spherical) convex mirror, backward extensions of the reflected rays pass through a common point (a virtual focus F) at a distance f (a negative quantity) from the mirror.

â— A concave mirror can form a real image (if the object is outside the focal point) or a virtual image (if the object is inside the focal point).

â— A convex mirror can form only a virtual image.

â— The mirror equation relates an object distance p, the mirrorâ€™s focal length f and radius of curvature r, and the image distance i:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} = \frac{2}{r}.$$

â— The magnitude of the lateral magnification m of an object is the ratio of the image height hâ€² to object height h,

$$|m| = \frac{h'}{h},$$

and is related to the object distance p and image distance i by

$$m = -\frac{i}{p}.$$

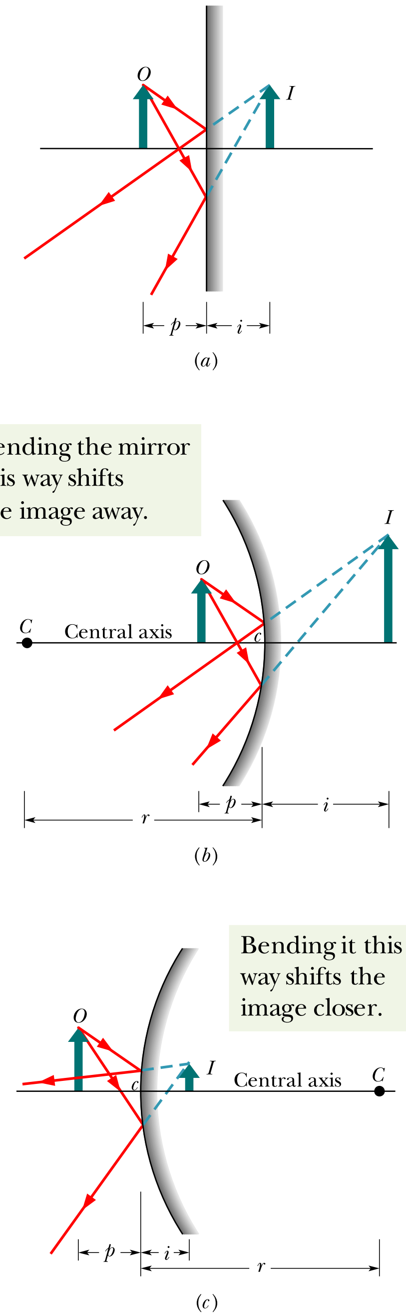


Figure 34.2.1 (*a*) An object *O* forms a virtual image *I* in a plane mirror. (*b*) If the mirror is bent so that it becomes *concave*, the image moves farther away and becomes larger. (*c*) If the plane mirror is bent so that it becomes *convex*, the image moves closer and becomes smaller.

Spherical Mirrors

We turn now from images produced by plane mirrors to images produced by mirrors with curved surfaces. In particular, we consider spherical mirrors, which are simply mirrors in the shape of a small section of the surface of a sphere. A plane mirror is in fact a spherical mirror with an infinitely large *radius of curvature* and thus an approximately flat surface.

Making a Spherical Mirror

We start with the plane mirror of Fig. 34.2.1*a*, which faces leftward toward an object *O* that is shown and an observer that is not shown. We make a **concave mirror** by curving the mirrorâ€™s surface so it is *concave* (â€œcaved inâ€) as in Fig. 34.2.1*b*. Curving the surface in this way changes several characteristics of the mirror and the image it produces of the object:

1. The *center of curvature C* (the center of the sphere of which the mirrorâ€™s surface is part) was infinitely far from the plane mirror; it is now closer but still in front of the concave mirror.
2. The *field of view*â€”the extent of the scene that is reflected to the observerâ€” was wide; it is now smaller.
3. The image of the object was as far behind the plane mirror as the object was in front; the image is farther behind the concave mirror; that is, $|i|$ is greater.
4. The height of the image was equal to the height of the object; the height of the image is now greater. This feature is why many makeup mirrors and shaving mirrors are concaveâ€”they produce a larger image of a face.

We can make a **convex mirror** by curving a plane mirror so its surface is *convex* (â€œflexed outâ€) in Fig. 34.2.1*c*. Curving the surface in this way (1) moves the center of curvature *C* to *behind* the mirror and (2) *increases* the field of view. It also (3) moves the image of the object *closer* to the mirror and (4) *shrinks* it. Store surveillance mirrors are usually convex to take advantage of the increase in the field of viewâ€”more of the store can then be seen with a single mirror. In some states, a law requires that convex mirrors be mounted at the front of a bus or large truck so that the driver can see a pedestrian in a crosswalk directly in front of the vehicle (Fig. 34.2.2).

Focal Points of Spherical Mirrors

For a plane mirror, the magnitude of the image distance *i* is always equal to the object distance *p*. Before we can determine how these two distances are related for a spherical mirror, we must consider the reflection of light from an object *O* located an effectively infinite distance in front of a spherical mirror, on the mirrorâ€™s *central axis*. That axis extends through the center of curvature *C* and the center *c* of the mirror. Because of the great distance between the object and the mirror, the light waves spreading from the object are plane waves when they reach the mirror along the central axis. This means that the rays representing the light waves are all parallel to the central axis when they reach the mirror.

Forming a Focus. When these parallel rays reach a concave mirror like that of Fig. 34.2.3*a*, those near the central axis are reflected through a common point *F*; two of these reflected rays are shown in the figure. If we placed a (small) card at *F*, a point image of the infinitely distant object *O* would appear on the card. (This would occur for any infinitely distant object.) Point *F* is called the **focal**

sangfoto/iStock/GettyImages

Figure 34.2.2 A *crossover mirror* allows a truck or bus driver to see a pedestrian in the blind spot in front of the vehicle.

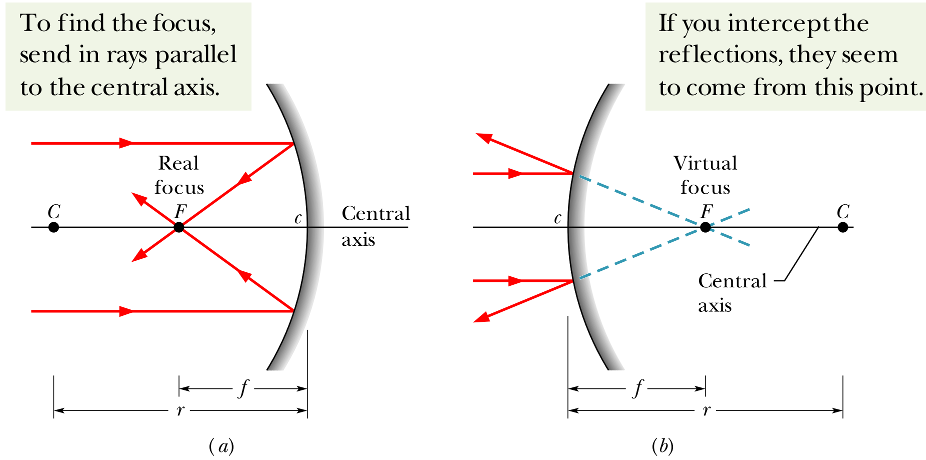


Figure 34.2.3 (*a*) In a concave mirror, incident parallel light rays are brought to a real focus at F, on the same side of the mirror as the incident light rays. (*b*) In a convex mirror, incident parallel light rays seem to diverge from a virtual focus at F, on the side of the mirror opposite the light rays.

point (or **focus**) of the mirror, and its distance from the center of the mirror c is the **focal length** f of the mirror.

If we now substitute a convex mirror for the concave mirror, we find that the parallel rays are no longer reflected through a common point. Instead, they diverge as shown in Fig. 34.2.3*b*. However, if your eye intercepts some of the reflected light, you perceive the light as originating from a point source behind the mirror. This perceived source is located where extensions of the reflected rays pass through a common point (F in Fig. 34.2.3*b*). That point is the focal point (or focus) F of the convex mirror, and its distance from the mirror surface is the focal length f of the mirror. If we placed a card at this focal point, an image of object O would *not* appear on the card, so this focal point is not like that of a concave mirror.

Two Types. To distinguish the actual focal point of a concave mirror from the perceived focal point of a convex mirror, the former is said to be a *real focal point* and the latter is said to be a *virtual focal point.* Moreover, the focal length f of a concave mirror is taken to be a positive quantity, and that of a convex mirror a negative quantity. For mirrors of both types, the focal length f is related to the radius of curvature r of the mirror by

$$f = \tfrac{1}{2}r \quad \text{(spherical mirror)}, \tag{34.2.1}$$

where r is positive for a concave mirror and negative for a convex mirror.

Images from Spherical Mirrors

Inside. With the focal point of a spherical mirror defined, we can find the relation between image distance i and object distance p for concave and convex spherical mirrors. We begin by placing the object O *inside the focal point* of the concave mirrorâ€"that is, between the mirror and its focal point F (Fig. 34.2.4*a*). An observer can then see a virtual image of O in the mirror: The image appears to be behind the mirror, and it has the same orientation as the object.

If we now move the object away from the mirror until it is at the focal point, the image moves farther and farther back from the mirror until, when the object is at the focal point, the image is at infinity (Fig. 34.2.4*b*). The image is then

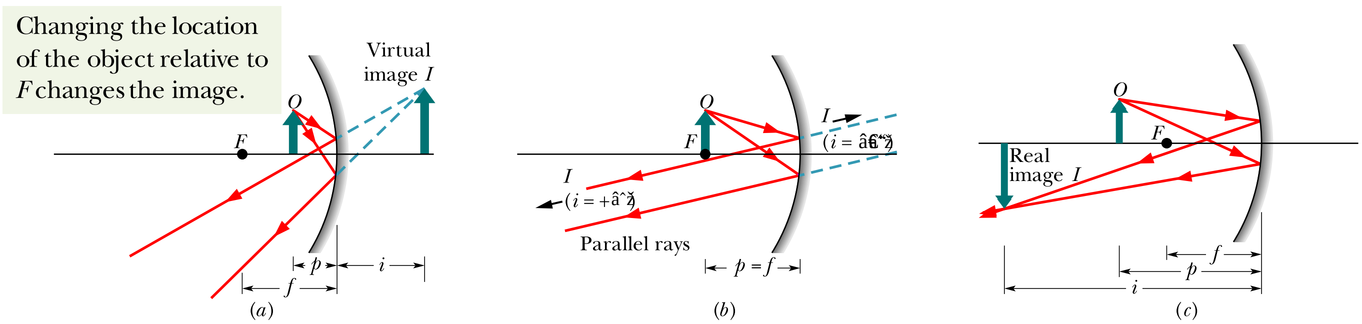


Figure 34.2.4 (*a*) An object O inside the focal point of a concave mirror, and its virtual image I. (*b*) The object at the focal point F. (*c*) The object outside the focal point, and its real image I.

ambiguous and imperceptible because neither the rays reflected by the mirror nor the ray extensions behind the mirror cross to form an image of O.

Outside. If we next move the object *outside the focal point*â€”that is, farther away from the mirror than the focal pointâ€”the rays reflected by the mirror converge to form an *inverted* image of object O (Fig. 34.2.4*c*) in front of the mirror. That image moves in from infinity as we move the object farther outside F. If you were to hold a card at the position of the image, the image would show up on the cardâ€”the image is said to be *focused* on the card by the mirror. (The verb â€œfocus,â€ which in this context means to produce an image, differs from the noun â€œfocus,â€ which is another name for the focal point.) Because this image can actually appear on a surface, it is a real imageâ€”the rays actually intersect to create the image, regardless of whether an observer is present. The image distance i of a real image is a positive quantity, in contrast to that for a virtual image. We can now generalize about the location of images from spherical mirrors:

> Real images form on the side of a mirror where the object is, and virtual images form on the opposite side.

Main Equation. As we shall prove in Module 34.6, when light rays from an object make only small angles with the central axis of a spherical mirror, a simple equation relates the object distance p, the image distance i, and the focal length f:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \quad \text{(spherical mirror)}. \tag{34.2.2}$$

We assume such small angles in figures such as Fig. 34.2.4, but for clarity the rays are drawn with exaggerated angles. With that assumption, Eq. 34.2.2 applies to any concave, convex, or plane mirror. For a convex or plane mirror, only a virtual image can be formed, regardless of the objectâ€™s location on the central axis. As shown in the example of a convex mirror in Fig. 34.2.1*c*, the image is always on the opposite side of the mirror from the object and has the same orientation as the object.

Magnification. The size of an object or image, as measured *perpendicular* to the mirrorâ€™s central axis, is called the object or image *height*. Let h represent the height of the object, and hâ€² the height of the image. Then the ratio hâ€²/h is called the **lateral magnification** m produced by the mirror. However, by convention, the lateral magnification always includes a plus sign when the image orientation is that of the object and a minus sign when the image orientation is opposite that of the object. For this reason, we write the formula for m as

$$|m| = \frac{h'}{h} \quad \text{(lateral magnification).} \tag{34.2.3}$$

We shall soon prove that the lateral magnification can also be written as

$$m = -\frac{i}{p} \quad \text{(lateral magnification).} \tag{34.2.4}$$

For a plane mirror, for which $i = -p$, we have $m = +1$. The magnification of 1 means that the image is the same size as the object. The plus sign means that the image and the object have the same orientation. For the concave mirror of Fig. 34.2.4*c*, $m \approx -1.5$.

Organizing Table. Equations 34.2.1 through 34.2.4 hold for all plane mirrors, concave spherical mirrors, and convex spherical mirrors. In addition to those equations, you have been asked to absorb a lot of information about these mirrors, and you should organize it for yourself by filling in Table 34.2.1. Under Image Location, note whether the image is on the *same* side of the mirror as the object or on the *opposite* side. Under Image Type, note whether the image is *real* or *virtual*. Under Image Orientation, note whether the image has the *same* orientation as the object or is *inverted*. Under Sign, give the sign of the quantity or fill in ± if the sign is ambiguous. You will need this organization to tackle homework or a test.

Locating Images by Drawing Rays

Figures 34.2.5*a* and *b* show an object O in front of a concave mirror. We can graphically locate the image of any off-axis point of the object by drawing a *ray diagram* with any two of four special rays through the point:

1. A ray that is initially parallel to the central axis reflects through the focal point F (ray 1 in Fig. 34.2.5*a*).
2. A ray that reflects from the mirror after passing through the focal point emerges parallel to the central axis (ray 2 in Fig. 34.2.5*a*).
3. A ray that reflects from the mirror after passing through the center of curvature C returns along itself (ray 3 in Fig. 34.2.5*b*).
4. A ray that reflects from the mirror at point c is reflected symmetrically about that axis (ray 4 in Fig. 34.2.5*b*).

The image of the point is at the intersection of the two special rays you choose. The image of the object can then be found by locating the images of two or more

Table 34.2.1 Your Organizing Table for Mirrors

Mirror Type	Object Location	Image Location	Image Type	Image Orientation	Sign of f	Sign of r	Sign of i	Sign of m
Plane	Anywhere							
Concave	Inside F							
	Outside F							
Convex	Anywhere							

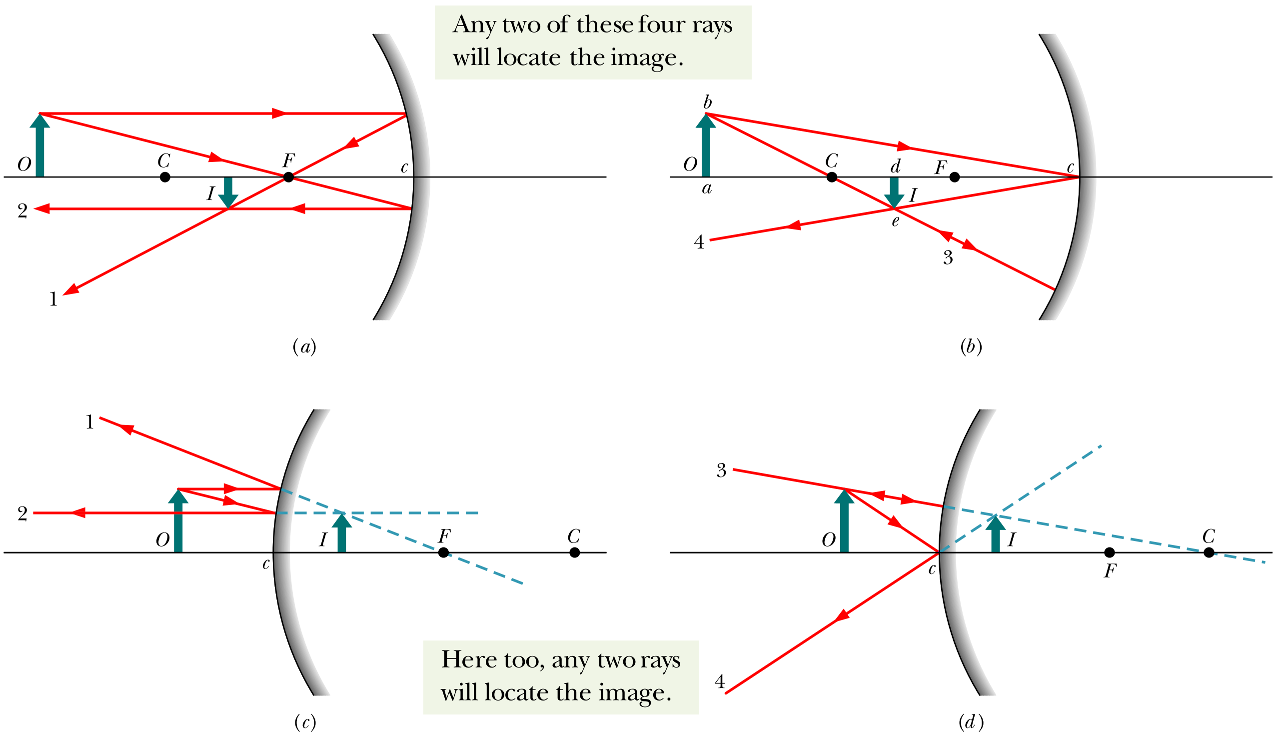


Figure 34.2.5 (a, b) Four rays that may be drawn to find the image formed by a concave mirror. For the object position shown, the image is real, inverted, and smaller than the object. (c, d) Four similar rays for the case of a convex mirror. For a convex mirror, the image is always virtual, oriented like the object, and smaller than the object. [In (c), ray 2 is initially directed toward focal point F. In (d), ray 3 is initially directed toward center of curvature C.]

of its off-axis points (say, the point most off axis) and then sketching in the rest of the image. You need to modify the descriptions of the rays slightly to apply them to convex mirrors, as in Figs. 34.2.5c and d.

A concave mirror does not require a continuous curved surface. Instead, it can consist of an array of flat reflecting surfaces arranged on a larger concave surface. For example, Fig. 34.2.6 shows one side of Londonâ€™s skyscraper 20 Fenchurch Street, dubbed the Walkie Talkie for its shape like an early communication device. The side facing the Sun is concave with flat windows that partially reflect and focus the sunlight. Soon after the building was completed, the city discovered that this giant concave mirror could focus light down onto the street so intensely that pedestrians had to guard their eyes, a parked car and other objects were melted, and (in a demonstration) an egg was cooked in a skillet placed on the sidewalk. To eliminate the glare, the windows were eventually fitted with sunshades.

amer ghazzal/AlamyStock Photo

Figure 34.2.6 Blinding light reflections from the Walkie Talkie building in London.

Glaucoma and Air-Puff Tonometry

Glaucoma, a leading cause of blindness, occurs when the optic nerve is damaged by abnormally high pressure on it from the fluid inside the eye, the *interocular*

PJF Military Collection/Alamy Stock Photo

Figure 34.2.7 Air-puff tonometer being used to measure interocular pressure in testing for glaucoma.

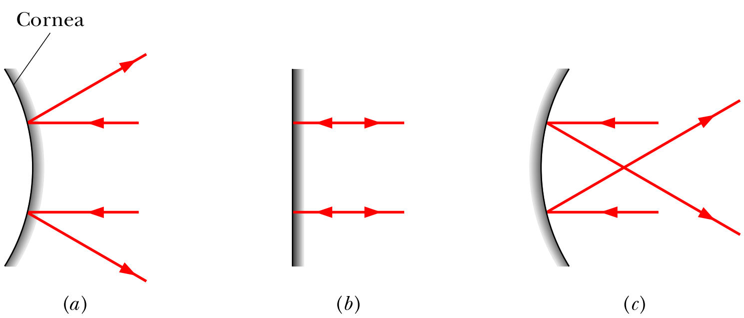


Figure 34.2.8 (*a*) The reflections of light back to the light meter in the tonometer are spread out. (*b*) The reflections are brightest when the cornea is momentarily flat. (*c*) The reflections are again spread out as the cornea becomes concave.

pressure IOP. That nerve transfers image information from the retina to the vision center of the brain. The damage might be so gradual that a person might be unaware of the resulting loss of sight. One way to monitor the IOP without contact with the eye is with an *air-puff tonometer* (Fig. 34.2.7). The patient sits so that the instrument is near the eyeâ€™s cornea, the transparent convex structure through which light passes into the eye to reach the retina. The instrument shoots a brief puff of air against the cornea with enough pressure to flatten it and then push it slightly inward so that it is momentarily concave. To monitor this change in shape, the instrument directs parallel rays of light onto the cornea and measures the intensity of the light reflected to it by the cornea. Initially some of the light reflects to a light meter in the instrument (Fig 34.2.8*a*). As the cornea flattens, the intensity increases and reaches a maximum when the cornea is flat (Fig 34.2.8*b*), and then it decreases as the cornea becomes a concave surface (Fig 34.2.8*c*). By measuring the time required to reach maximum intensity, the instrument can compute the IOP.

Proof of Equation 34.2.4

We are now in a position to derive Eq. 34.2.4 ($m =$ â€“i/p), the equation for the lateral magnification of an object reflected in a mirror. Consider ray 4 in Fig. 34.2.5*b*. It is reflected at point *c* so that the incident and reflected rays make equal angles with the axis of the mirror at that point.

The two right triangles *abc* and *dec* in the figure are similar (have the same set of angles), so we can write

$$\frac{de}{ab} = \frac{cd}{ca}.$$

The quantity on the left (apart from the question of sign) is the lateral magnification *m* produced by the mirror. Because we indicate an inverted image as a *negative* magnification, we symbolize this as â€“m. However, $cd = i$ and $ca = p$, so we have

$$m = \text{â€“}\frac{i}{p} \quad \text{(magniï¬ cation)}, \tag{34.2.5}$$

which is the relation we set out to prove.

Checkpoint 34.2.1

A Central American vampire bat, dozing on the central axis of a spherical mirror, is magnified by $m =$ â€“4. Is its image (a) real or virtual, (b) inverted or of the same orientation as the bat, and (c) on the same side of the mirror as the bat or on the opposite side?

Sample Problem 34.2.1 Image produced by a spherical mirror

A tarantula of height h sits cautiously before a spherical mirror whose focal length has absolute value $|f| = 40$ cm. The image of the tarantula produced by the mirror has the same orientation as the tarantula and has height $h' = 0.20h$.

(a) Is the image real or virtual, and is it on the same side of the mirror as the tarantula or the opposite side?

Reasoning: Because the image has the same orientation as the tarantula (the object), it must be virtual and on the opposite side of the mirror. (You can easily see this result if you have filled out Table 34.2.1.)

(b) Is the mirror concave or convex, and what is its focal length f, sign included?

KEY IDEA

We *cannot* tell the type of mirror from the type of image because both types of mirror can produce virtual images. Similarly, we cannot tell the type of mirror from the sign of the focal length f, as obtained from Eq. 34.2.1 or Eq. 34.2.2, because we lack enough information to use either equation. However, we can make use of the magnification information.

Calculations: From the given information, we know that the ratio of image height h' to object height h is 0.20. Thus, from Eq. 34.2.3 we have

$$|m| = \frac{h'}{h} = 0.20.$$

Because the object and image have the same orientation, we know that m must be positive: $m = +0.20$. Substituting this into Eq. 34.2.4 and solving for, say, i gives us

$$i = -0.20p,$$

which does not appear to be of help in finding f. However, it is helpful if we substitute it into Eq. 34.2.2. That equation gives us

$$\frac{1}{f} = \frac{1}{i} + \frac{1}{p} = \frac{1}{-0.20p} + \frac{1}{p} = \frac{1}{p}(-5 + 1),$$

from which we find

$$f = -p/4.$$

Now we have it: Because p is positive, f must be negative, which means that the mirror is convex with

$$f = -40 \text{ cm}. \qquad \text{(Answer)}$$

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34.3 SPHERICAL REFRACTING SURFACES

Learning Objectives

After reading this module, you should be able to . . .

34.3.1 Identify that the refraction of rays by a spherical surface can produce real images and virtual images of an object, depending on the indexes of refraction on the two sides, the surface's radius of curvature r, and whether the object faces a concave or convex surface.

34.3.2 For a point object on the central axis of a spherical refracting surface, sketch the refraction of a ray in the six general arrangements and identify whether the image is real or virtual.

34.3.3 For a spherical refracting surface, identify what type of image appears on the same side as the object and what type appears on the opposite side.

34.3.4 For a spherical refracting surface, apply the relationship between the two indexes of refraction, the object distance p, the image distance i, and the radius of curvature r.

34.3.5 Identify the algebraic signs of the radius r for an object facing a concave refracting surface and a convex refracting surface.

Key Ideas

- A single spherical surface that refracts light can form an image.
- The object distance p, the image distance i, and the radius of curvature r of the surface are related by

$$\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{r},$$

where n_1 is the index of refraction of the material where the object is located and n_2 is the index of refraction on the other side of the surface.

- If the surface faced by the object is convex, r is positive, and if it is concave, r is negative.
- Images on the object's side of the surface are virtual, and images on the opposite side are real.

Dr. Paul A. Zahl/Science Source

This insect has been entombed in amber for about 25 million years. Because we view the insect through a curved refracting surface, the location of the image we see does not coincide with the location of the insect (see Fig. 34.3.1*d*).

Spherical Refracting Surfaces

We now turn from images formed by reflection to images formed by refraction through surfaces of transparent materials, such as glass. We shall consider only spherical surfaces, with radius of curvature r and center of curvature C. The light will be emitted by a point object O in a medium with index of refraction n_1; it will refract through a spherical surface into a medium of index of refraction n_2.

Our concern is whether the light rays, after refracting through the surface, form a real image (no observer necessary) or a virtual image (assuming that an observer intercepts the rays). The answer depends on the relative values of n_1 and n_2 and on the geometry of the situation.

Six possible results are shown in Fig. 34.3.1. In each part of the figure, the medium with the greater index of refraction is shaded, and object O is always in the medium with index of refraction n_1, to the left of the refracting surface. In each part, a representative ray is shown refracting through the surface. (That ray and a ray along the central axis suffice to determine the position of the image in each case.)

At the point of refraction of each ray, the normal to the refracting surface is a radial line through the center of curvature C. Because of the refraction, the ray bends toward the normal if it is entering a medium of greater index of refraction and away from the normal if it is entering a medium of lesser index of refraction. If the bending sends the ray toward the central axis, that ray and others (undrawn) form a real image on that axis. If the bending sends the ray away from the central axis, the ray cannot form a real image; however, backward extensions of it and other refracted rays can form a virtual image, provided (as with mirrors) some of those rays are intercepted by an observer.

Real images I are formed (at image distance i) in parts *a* and *b* of Fig. 34.3.1, where the refraction directs the ray *toward* the central axis. Virtual images are formed in parts *c* and *d*, where the refraction directs the ray *away* from the central axis. Note, in these four parts, that real images are formed when the object is relatively far from the refracting surface and virtual images are formed when the object is nearer the refracting surface. In the final situations (Figs. 34.3.1*e* and *f*), refraction always directs the ray away from the central axis and virtual images are always formed, regardless of the object distance.

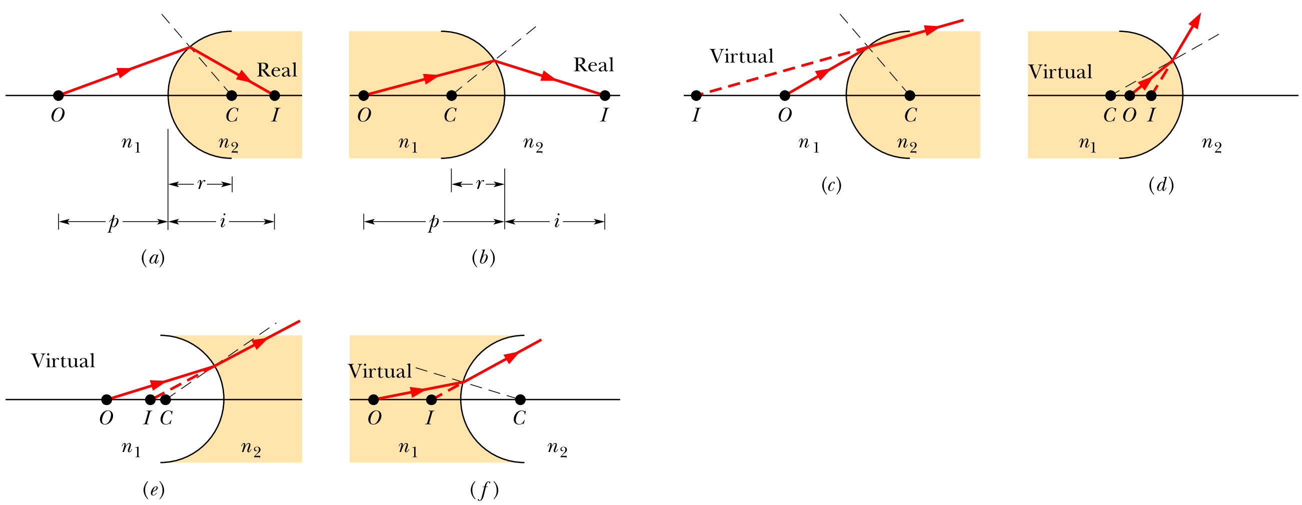

Figure 34.3.1 Six possible ways in which an image can be formed by refraction through a spherical surface of radius r and center of curvature C. The surface separates a medium with index of refraction n_1 from a medium with index of refraction n_2. The point object O is always in the medium with n_1, to the left of the surface. The material with the lesser index of refraction is unshaded (think of it as being air, and the other material as being glass). Real images are formed in (*a*) and (*b*); virtual images are formed in the other four situations.

Note the following major difference from reflected images:

> Real images form on the side of a refracting surface that is opposite the object, and virtual images form on the same side as the object.

In Module 34.6, we shall show that (for light rays making only small angles with the central axis)

$$\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{r}. \tag{34.3.1}$$

Just as with mirrors, the object distance p is positive, and the image distance i is positive for a real image and negative for a virtual image. However, to keep all the signs correct in Eq. 34.3.1, we must use the following rule for the sign of the radius of curvature r:

> When the object faces a convex refracting surface, the radius of curvature r is positive. When it faces a concave surface, r is negative.

Be careful: This is just the reverse of the sign convention we have for mirrors, which can be a slippery point in the heat of an exam.

Checkpoint 34.3.1

A bee is hovering in front of the concave spherical refracting surface of a glass sculpture. (a) Which part of Fig. 34.3.1 is like this situation? (b) Is the image produced by the surface real or virtual, and (c) is it on the same side as the bee or the opposite side?

Sample Problem 34.3.1 Images from a half-submerged eye

Underwater vision is usually difficult even if you have perfect vision above water. The reason has to do with how water affects the refraction of light entering the eye. The refraction may be appropriate in the air but quite wrong in the water. However, the peculiar fish *Anableps anableps* (Fig. 34.3.2) swims with its eyes partially extending above the water surface so that it can see simultaneously above and below water. Figure 34.3.3 shows a vertical cross section through the eye of the fish, with a pigment band separating the two halves at the water surface. The front of the eye (the cornea) is a spherically convex refracting surface of radius $r = 1.95$ mm and index of refraction $n_2 = 1.335$. The refraction at the cornea is the first step in the eye's focusing of a *real* image onto the back of the retina at the back of the eye, where visual processing begins. If the cornea faces an insect (lunch) at object distance $p = 0.200$ m, what is the image distance i of that refraction for the cornea in air ($n_1 = 1.000$) and water ($n = 1.333$)?

KEY IDEAS

(1) Because the object and its image are on opposite sides of the refracting surface with its convex side facing the object, the situation is like either Fig. 34.3.1*a* (for a real image) or Fig. 34.3.1*c* (for a virtual image). (2) Image distance and object distance are related by Eq. 34.3.1.

Calculations: Solving Eq. 34.3.1 for i gives us

$$i = \frac{n_2}{\dfrac{n_2 - n_1}{r} - \dfrac{n_1}{p}}.$$

Substituting the given values and using the index of refraction $n_1 = 1.000$ for air, we find

$$i = \frac{1.335}{\dfrac{1.335 - 1.000}{0.00195} - \dfrac{1.000}{0.200}}$$

$$= 8.00 \text{ mm.} \qquad \text{(Answer)}$$

Repeating the calculation but using the index of refraction $n_1 = 1.333$ for water, we find

$$i = \frac{1.335}{\dfrac{1.335 - 1.333}{0.00195} - \dfrac{1.333}{0.200}}$$

$$= -0.237 \text{ m.} \qquad \text{(Answer)}$$

After light is refracted by the cornea, it is further refracted by a lens to give a final real image on the retina. (The function of a lens is discussed in the next module.) Our first answer is positive (indicating a real image, as needed) and about twice the diameter of the eye. Thus, the role of the lens in focusing the light onto the retina is moderate because so much refraction occurs at the cornea. Our second answer is quite different: It is negative (indicating a virtual image) and much larger. So, more focusing is required by the lens to put a real image on the retina. To provide moderate focusing of light from the air and much stronger focusing of light from the water, the eye lens in *Anableps anableps* is egg shaped, with much greater curvature in the bottom half than in the top.

Figure 34.3.2 The fish that can see both above and below water.

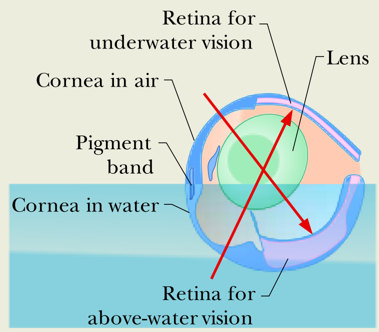

Figure 34.3.3 A cross section of the eye that can see both above and below water.

34.4 THIN LENSES

Learning Objectives

After reading this module, you should be able to . . .

34.4.1 Distinguish converging lenses from diverging lenses.

34.4.2 For converging and diverging lenses, sketch a ray diagram for rays initially parallel to the central axis, indicating how they form focal points, and identifying which is real and which is virtual.

34.4.3 Distinguish a real focal point from a virtual focal point, identify which corresponds to which type of lens and under which circumstances, and identify the algebraic sign associated with each focal length.

34.4.4 For an object (a) inside and (b) outside the focal point of a converging lens, sketch at least two rays to find the image and identify the type and orientation of the image.

34.4.5 For a converging lens, distinguish the locations and orientations of a real image and a virtual image.

34.4.6 For an object in front of a diverging lens, sketch at least two rays to find the image and identify the type and orientation of the image.

34.4.7 Identify which type of lens can produce both real and virtual images and which type can produce only virtual images.

34.4.8 Identify the algebraic sign of the image distance i for a real image and for a virtual image.

34.4.9 For converging and diverging lenses, apply the relationship between the focal length f, object distance p, and image distance i.

34.4.10 Apply the relationships between lateral magnification m, image height hâ€²object height h, image distance i, and object distance p.

34.4.11 Apply the lens makerâ€™s equation to relate a focal length to the index of refraction of a lens (assumed to be in air) and the radii of curvature of the two sides of the lens.

34.4.12 For a multiple-lens system with the object in front of lens 1, find the image produced by lens 1 and then use it as the object for lens 2, and so on.

34.4.13 For a multiple-lens system, determine the overall magnification (of the final image) from the magnifications produced by each lens.

Key Ideas

- This module primarily considers thin lenses with symmetric, spherical surfaces.
- If parallel rays are sent through a converging lens parallel to the central axis, the refracted rays pass through a common point (a real focus F) at a focal distance f (a positive quantity) from the lens. If they are sent through a diverging lens, backward extensions of the refracted rays pass through a common point (a virtual focus F) at a focal distance f (a negative quantity) from the lens.
- A converging lens can form a real image (if the object is outside the focal point) or a virtual image (if the object is inside the focal point).
- A diverging lens can form only a virtual image.
- For an object in front of a lens, object distance p and image distance i are related to the lensâ€™s focal length f, index of refraction n, and radii of curvature r_1 and r_2 by

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right).$$

- The magnitude of the lateral magnification m of an object is the ratio of the image height h' to object height h,

$$|m|=\frac{h'}{h},$$

and is related to the object distance p and image distance i by

$$m=-\frac{i}{p}.$$

- For a system of lenses with a common central axis, the image produced by the first lens acts as the object for the second lens, and so on, and the overall magnification is the product of the individual magnifications.

Thin Lenses

A **lens** is a transparent object with two refracting surfaces whose central axes coincide. The common central axis is the central axis of the lens. When a lens is surrounded by air, light refracts from the air into the lens, crosses through the lens, and then refracts back into the air. Each refraction can change the direction of travel of the light.

A lens that causes light rays initially parallel to the central axis to converge is (reasonably) called a **converging lens.** If, instead, it causes such rays to diverge, the lens is a **diverging lens.** When an object is placed in front of a lens of either type, light rays from the object that refract into and out of the lens can produce an image of the object.

Lens Equations. We shall consider only the special case of a **thin lens**â€”that is, a lens in which the thickest part is thin relative to the object distance p, the image distance i, and the radii of curvature r_1 and r_2 of the two surfaces of the lens. We shall also consider only light rays that make small angles with the central axis (they are exaggerated in the figures here). In Module 34.6 we shall prove that for such rays, a thin lens has a focal length f. Moreover, i and p are related to each other by

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{i} \quad \text{(thin lens)}, \tag{34.4.1}$$

which is the same as we had for mirrors. We shall also prove that when a thin lens with index of refraction n is surrounded by air, this focal length f is given by

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \quad \text{(thin lens in air)}, \tag{34.4.2}$$

Courtesy of Matthew G. Wheeler

A fire is being started by focusing sunlight onto newspaper by means of a converging lens made of clear ice. The lens was made by melting both sides of a flat piece of ice into a convex shape in the shallow vessel (which has a curved bottom). FCP which is often called the *lens makerâ€™s equation.* Here r_1 is the radius of curvature of the lens surface nearer the object and r_2 is that of the other surface. The signs of these radii are found with the rules in Module 34.3 for the radii of spherical refracting surfaces. If the lens is surrounded by some medium other than air (say, corn oil) with index of refraction n_{medium}, we replace n in Eq. 34.4.2 with n/n_{medium}. Keep in mind the basis of Eqs. 34.4.1 and 34.4.2:

> A lens can produce an image of an object only because the lens can bend light rays, but it can bend light rays only if its index of refraction differs from that of the surrounding medium.

Forming a Focus. Figure 34.4.1*a* shows a thin lens with convex refracting surfaces, or *sides*. When rays that are parallel to the central axis of the lens are sent through the lens, they refract twice, as is shown enlarged in Fig. 34.4.1*b*. This double refraction causes the rays to converge and pass through a common point F_2 at a distance f from the center of the lens. Hence, this lens is a converging lens; further, a *real* focal point (or focus) exists at F_2 (because the rays really do pass through it), and the associated focal length is f. When rays parallel to the central axis are sent in the opposite direction through the lens, we find another real focal point at F_1 on the other side of the lens. For a thin lens, these two focal points are equidistant from the lens.

Signs, Signs, Signs. Because the focal points of a converging lens are real, we take the associated focal lengths f to be positive, just as we do with a real focus of a concave mirror. However, signs in optics can be tricky, so we had better check this in Eq. 34.4.2. The left side of that equation is positive if f is positive; how about the right side? We examine it term by term. Because the index of refraction n of glass or any other material is greater than 1, the term (n â€‘) must be positive. Because the source of the light (which is the object) is at the left and

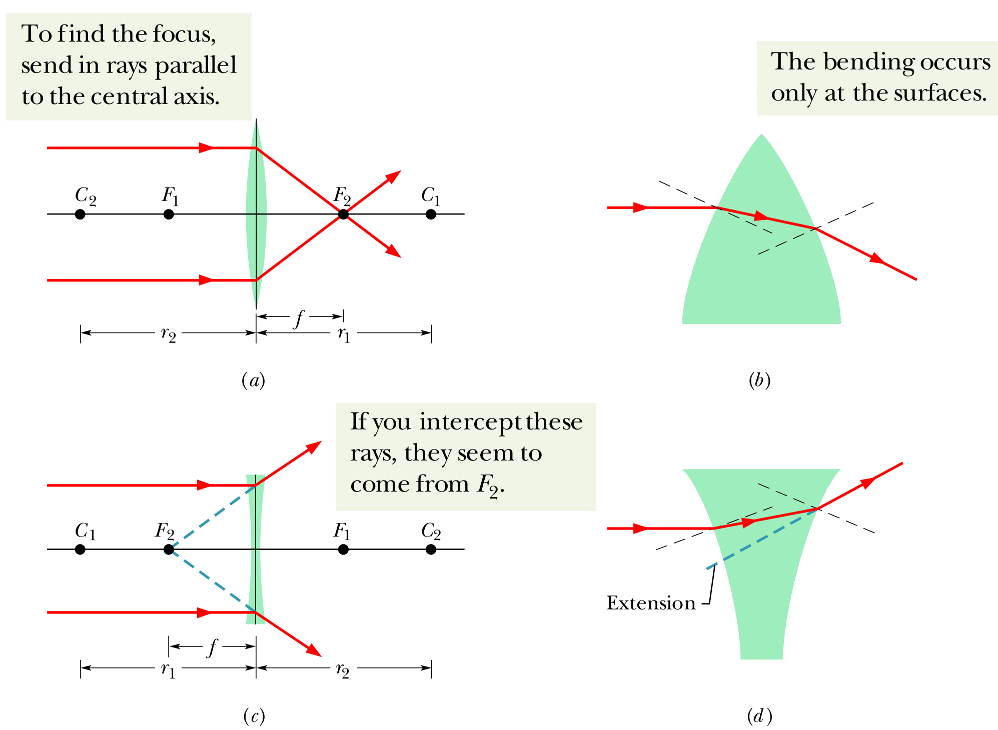

Figure 34.4.1 (*a*) Rays initially parallel to the central axis of a converging lens are made to converge to a real focal point F_2 by the lens. The lens is thinner than drawn, with a width like that of the vertical line through it. We shall consider all the bending of rays as occurring at this central line. (*b*) An enlargement of the top part of the lens of (*a*); normals to the surfaces are shown dashed. Note that both refractions bend the ray downward, toward the central axis. (*c*) The same initially parallel rays are made to diverge by a diverging lens. Extensions of the diverging rays pass through a virtual focal point F_2. (*d*) An enlargement of the top part of the lens of (*c*). Note that both refractions bend the ray upward, away from the central axis.

faces the convex left side of the lens, the radius of curvature r_1 of that side must be positive according to the sign rule for refracting surfaces. Similarly, because the object faces a concave right side of the lens, the radius of curvature r_2 of that side must be negative according to that rule. Thus, the term ($1/r_1$ â€‘$/r_2$) is positive, the whole right side of Eq. 34.4.2 is positive, and all the signs are consistent.

Figure 34.4.1*c* shows a thin lens with concave sides. When rays that are parallel to the central axis of the lens are sent through this lens, they refract twice, as is shown enlarged in Fig. 34.4.1*d*; these rays *diverge,* never passing through any common point, and so this lens is a diverging lens. However, extensions of the rays do pass through a common point F_2 at a distance f from the center of the lens. Hence, the lens has a *virtual* focal point at F_2. (If your eye intercepts some of the diverging rays, you perceive a bright spot to be at F_2, as if it is the source of the light.) Another virtual focus exists on the opposite side of the lens at F_1, symmetrically placed if the lens is thin. Because the focal points of a diverging lens are virtual, we take the focal length f to be negative.

Images from Thin Lenses

We now consider the types of image formed by converging and diverging lenses. Figure 34.4.2*a* shows an object O outside the focal point F_1 of a converging lens. The two rays drawn in the figure show that the lens forms a real, inverted image I of the object on the side of the lens opposite the object.

When the object is placed inside the focal point F_1, as in Fig. 34.4.2*b*, the lens forms a virtual image I on the same side of the lens as the object and with the same orientation. Hence, a converging lens can form either a real image or a virtual image, depending on whether the object is outside or inside the focal point, respectively.

Figure 34.4.2*c* shows an object O in front of a diverging lens. Regardless of the object distance (regardless of whether O is inside or outside the virtual focal point), this lens produces a virtual image that is on the same side of the lens as the object and has the same orientation.

As with mirrors, we take the image distance i to be positive when the image is real and negative when the image is virtual. However, the locations of real and virtual images from lenses are the reverse of those from mirrors:

> Real images form on the side of a lens that is opposite the object, and virtual images form on the side where the object is.

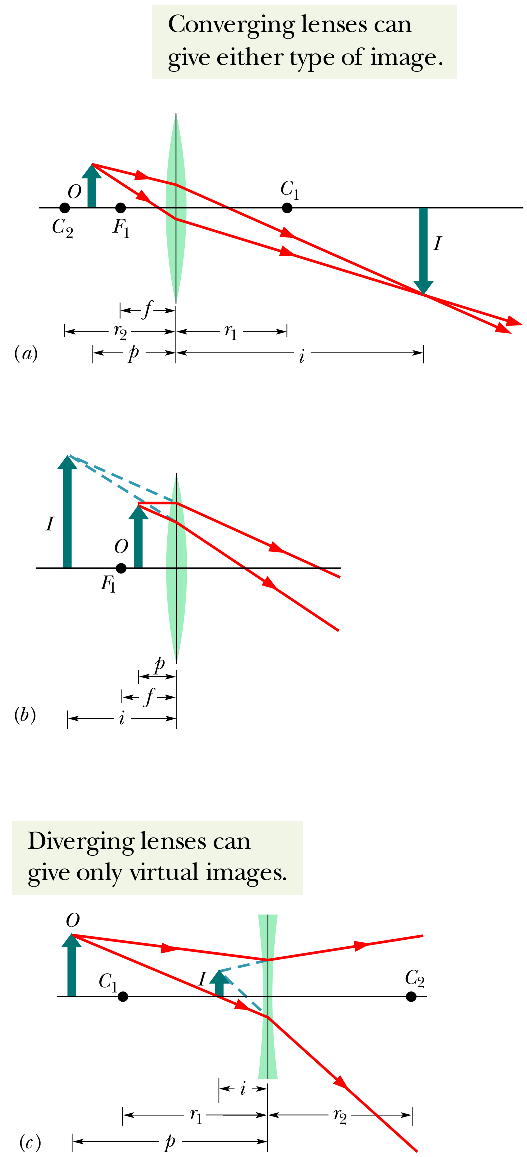

Figure 34.4.2 (*a*) A real, inverted image I is formed by a converging lens when the object O is outside the focal point F_1. (*b*) The image I is virtual and has the same orientation as O when O is inside the focal point. (*c*) A diverging lens forms a virtual image I, with the same orientation as the object O, whether O is inside or outside the focal point of the lens.

The lateral magnification m produced by converging and diverging lenses is given by Eqs. 34.2.3 and 34.2.4, the same as for mirrors.

You have been asked to absorb a lot of information in this module, and you should organize it for yourself by filling in Table 34.4.1 for thin *symmetric lenses* (both sides are convex or both sides are concave). Under Image Location note whether the image is on the *same* side of the lens as the object or on the *opposite* side. Under Image Type note whether the image is *real* or *virtual.* Under Image Orientation note whether the image has the *same* orientation as the object or is *inverted.*

Table 34.4.1 Your Organizing Table for Thin Lenses

		Image			Sign		
Lens Type	Object Location	Location	Type	Orientation	of f	of i	of m
Converging	Inside F						
	Outside F						
Diverging	Anywhere						

Locating Images of Extended Objects by Drawing Rays

Figure 34.4.3*a* shows an object O outside focal point F_1 of a converging lens. We can graphically locate the image of any off-axis point on such an object (such as the tip of the arrow in Fig. 34.4.3*a*) by drawing a ray diagram with any two of three special rays through the point. These special rays, chosen from all those that pass through the lens to form the image, are the following:

1. A ray that is initially parallel to the central axis of the lens will pass through focal point F_2 (ray 1 in Fig. 34.4.3*a*).
2. A ray that initially passes through focal point F_1 will emerge from the lens parallel to the central axis (ray 2 in Fig. 34.4.3*a*).
3. A ray that is initially directed toward the center of the lens will emerge from the lens with no change in its direction (ray 3 in Fig. 34.4.3*a*) because the ray encounters the two sides of the lens where they are almost parallel.

The image of the point is located where the rays intersect on the far side of the lens. The image of the object is found by locating the images of two or more of its points.

Figure 34.4.3*b* shows how the extensions of the three special rays can be used to locate the image of an object placed inside focal point F_1 of a converging lens. Note that the description of ray 2 requires modification (it is now a ray whose backward extension passes through F_1).

You need to modify the descriptions of rays 1 and 2 to use them to locate an image placed (anywhere) in front of a diverging lens. In Fig. 34.4.3*c*, for example, we find the point where ray 3 intersects the backward extensions of rays 1 and 2.

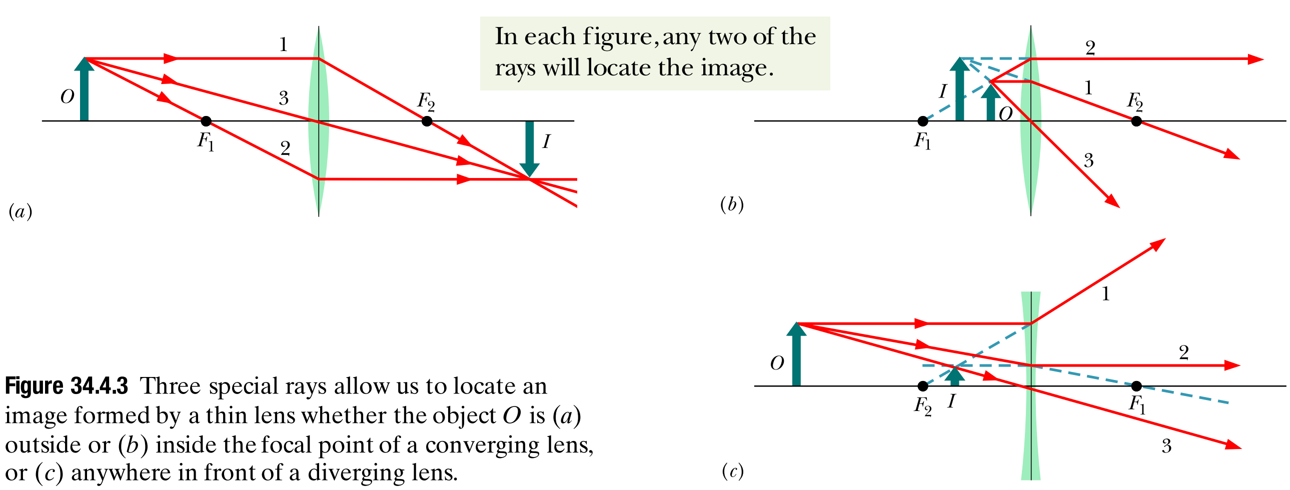

Figure 34.4.3 Three special rays allow us to locate an image formed by a thin lens whether the object O is (*a*) outside or (*b*) inside the focal point of a converging lens, or (*c*) anywhere in front of a diverging lens.

Two-Lens Systems

Here we consider an object sitting in front of a system of two lenses whose central axes coincide. Some of the possible two-lens systems are sketched in Fig. 34.4.4, but the figures are not drawn to scale. In each, the object sits to the left of lens 1 but can be inside or outside the focal point of the lens. Although tracing the light rays through any such two-lens system can be challenging, we can use the following simple two-step solution:

Step 1 Neglecting lens 2, use Eq. 34.4.1 to locate the image I_1 produced by lens 1. Determine whether the image is on the left or right side of the lens, whether it is real or virtual, and whether it has the same orientation as the object. Roughly sketch I_1. The top part of Fig. 34.4.4*a* gives an example.

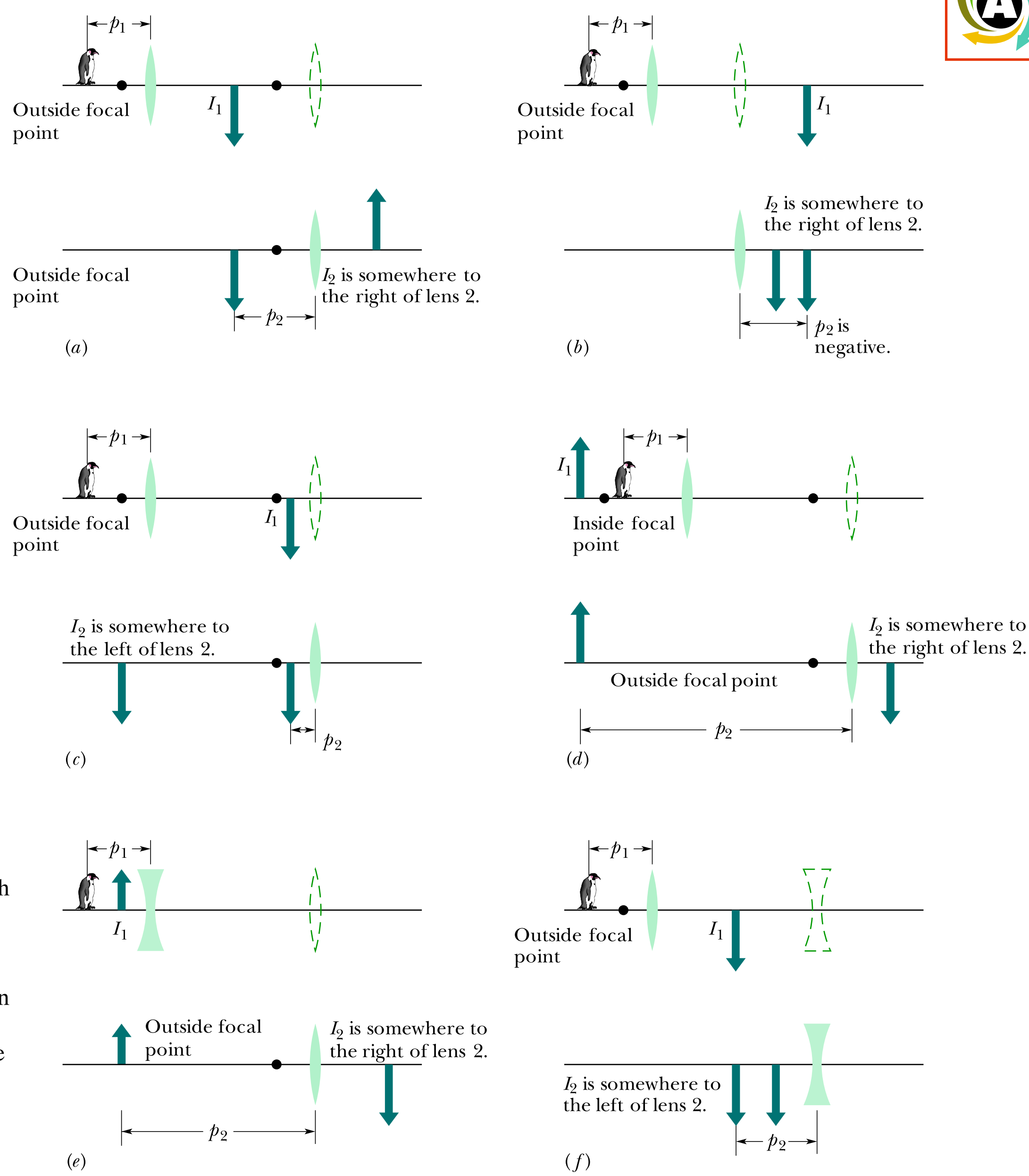

Figure 34.4.4 Several sketches (not to scale) of a two-lens system in which an object sits to the left of lens 1. In step 1 of the solution, we consider lens 1 and ignore lens 2 (shown in dashes). In step 2, we consider lens 2 and ignore lens 1 (no longer shown). We want to find the final image, that is, the image produced by lens 2.

Step 2 Neglecting lens 1, treat I_1 as though it is the *object* for lens 2. Use Eq. 34.4.1 to locate the image I_2 produced by lens 2. This is the final image of the system. Determine whether the image is on the left or right side of the lens, whether it is real or virtual, and whether it has the same orientation as the object for lens 2. Roughly sketch I_2. The bottom part of Fig. 34.4.4*a* gives an example.

Thus we treat the two-lens system with two single-lens calculations, using the normal decisions and rules for a single lens. The only exception to the procedure occurs if I_1 lies to the right of lens 2 (past lens 2). We still treat it as the object for lens 2, but we take the object distance p_2 as a *negative* number when we use Eq. 34.4.1 to find I_2. Then, as in our other examples, if the image distance i_2 is positive, the image is real and on the right side of the lens. An example is sketched in Fig. 34.4.4*b*.

This same step-by-step analysis can be applied for any number of lenses. It can also be applied if a mirror is substituted for lens 2. The *overall* (or *net*) lateral magnification M of a system of lenses (or lenses and a mirror) is the product of the individual lateral magnifications as given by Eq. 34.2.5 ($m =$ â€“i/p). Thus, for a two-lens system, we have

$$M = m_1 m_2. \tag{34.4.3}$$

If M is positive, the final image has the same orientation as the object (the one in front of lens 1). If M is negative, the final image is inverted from the object. In the situation where p_2 is negative, such as in Fig. 34.4.4*b*, determining the orientation of the final image is probably easiest by examining the sign of M.

Checkpoint 34.4.1

A thin symmetric lens provides an image of a fingerprint with a magnification of +0.2 when the fingerprint is 1.0 cm farther from the lens than the focal point of the lens. What are the (a) type and (b) orientation of the image, and (c) what is the type of lens?

Sample Problem 34.4.1 Image produced by a thin symmetric lens

A praying mantis preys along the central axis of a thin symmetric lens, 20 cm from the lens. The lateral magnification of the mantis provided by the lens is $m =$ â€“0.25, and the index of refraction of the lens material is 1.65.

(a) Determine the type of image produced by the lens, the type of lens, whether the object (mantis) is inside or outside the focal point, on which side of the lens the image appears, and whether the image is inverted.

Reasoning: We can tell a lot about the lens and the image from the given value of m. From it and Eq. 34.2.4 ($m =$ â€“i/p), we see that

$$i = \text{â€“}mp = 0.25p.$$

Even without finishing the calculation, we can answer the questions. Because p is positive, i here must be positive. That means we have a real image, which means we have a converging lens (the only lens that can produce a real image). The object must be outside the focal point (the only way a real image can be produced). Also, the image is inverted and on the side of the lens opposite the object. (That is how a converging lens makes a real image.)

(b) What are the two radii of curvature of the lens?

KEY IDEAS

1. Because the lens is symmetric, r_1 (for the surface nearer the object) and r_2 have the same magnitude r.
2. Because the lens is a converging lens, the object faces a convex surface on the nearer side and so $r_1 = +r$. Similarly, it faces a concave surface on the farther side, so $r_2 =$ â€“r.
3. We can relate these radii of curvature to the focal length f via the lens makerâ€™s equation, Eq. 34.4.2 (our only equation involving the radii of curvature of a lens).
4. We can relate f to the object distance p and image distance i via Eq. 34.4.1.

Calculations: We know p, but we do not know i. Thus, our starting point is to finish the calculation for i in part (a); we obtain

$$i = (0.25)(20\text{cm}) = 5.0\text{ cm}.$$

Now Eq. 34.4.1 gives us

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i} = \frac{1}{20\text{ cm}} + \frac{1}{5.0\text{ cm}},$$

from which we find $f = 4.0$ cm.

Equation 34.4.2 then gives us

$$\frac{1}{f} = (n\ \text{â^'}1)\left(\frac{1}{r_1}\ \text{â^'}\frac{1}{r_2}\right) = (n\ \text{â^'}1)\left(\frac{1}{+r}\ \text{â^'}\frac{1}{\text{â^'}r}\right)$$

or, with known values inserted,

$$\frac{1}{4.0\text{ cm}} = (1.65\ \text{â^'}1)\,\frac{2}{r},$$

which yields

$$r = (0.65)(2)(4.0\text{cm}) = 5.2\text{ cm}. \qquad \text{(Answer)}$$

Sample Problem 34.4.2 Image produced by a system of two thin lenses

Figure 34.4.5*a* shows a jalapeÃ±seed O_1 that is placed in front of two thin symmetrical coaxial lenses 1 and 2, with focal lengths $f_1 = +24$ cm and $f_2 = +9.0$ cm, respectively, and with lens separation $L = 10$ cm. The seed is 6.0 cm from lens 1. Where does the system of two lenses produce an image of the seed?

KEY IDEA

We could locate the image produced by the system of lenses by tracing light rays from the seed through the two lenses. However, we can, instead, calculate the location of that image by working through the system in steps, lens by lens. We begin with the lens closer to the seed. The image we seek is the final oneâ€"that is, image I_2 produced by lens 2.

Lens 1: Ignoring lens 2, we locate the image I_1 produced by lens 1 by applying Eq. 34.4.1 to lens 1 alone:

$$\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1}.$$

The object O_1 for lens 1 is the seed, which is 6.0 cm from the lens; thus, we substitute $p_1 = +6.0$ cm. Also substituting the given value of f_1, we then have

$$\frac{1}{+6.0\text{ cm}} + \frac{1}{i_1} = \frac{1}{+24\text{ cm}},$$

which yields $i_1 =$ â8.0 cm.

This tells us that image I_1 is 8.0 cm from lens 1 and virtual. (We could have guessed that it is virtual by noting that the seed is inside the focal point of lens 1, that is, between the lens and its focal point.) Because I_1 is virtual, it is on the same side of the lens as object O_1 and has the same orientation as the seed, as shown in Fig. 34.4.5*b*.

Lens 2: In the second step of our solution, we treat image I_1 as an object O_2 for the second lens and now ignore lens 1. We first note that this object O_2 is outside the focal point of lens 2. So the image I_2 produced by lens 2 must

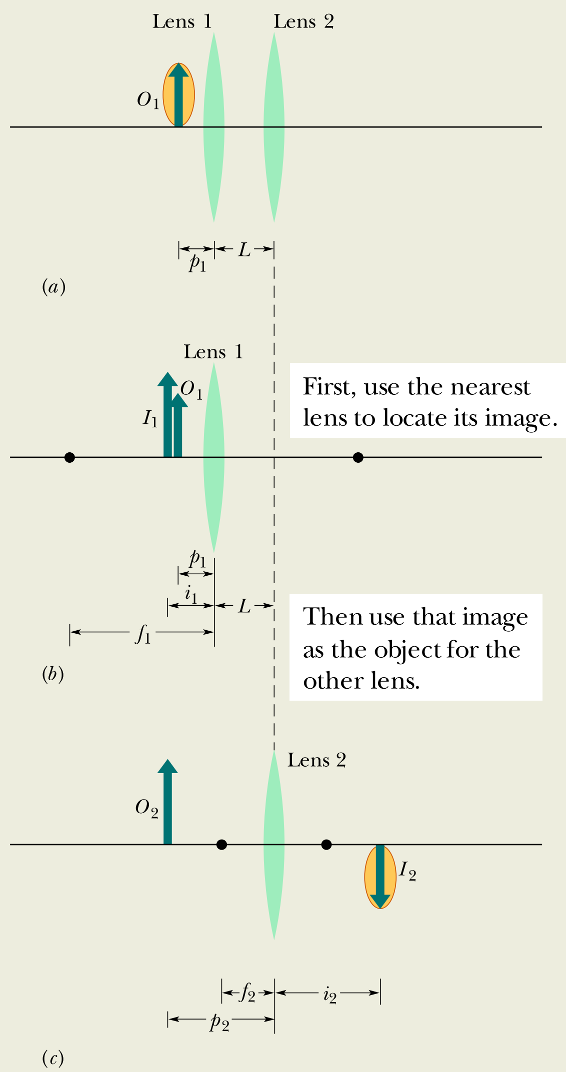

Figure 34.4.5 (*a*) Seed O_1 is distance p_1 from a two-lens system with lens separation L. We use the arrow to orient the seed. (*b*) The image I_1 produced by lens 1 alone. (*c*) Image I_1 acts as object O_2 for lens 2 alone, which produces the final image I_2.

be real, inverted, and on the side of the lens opposite O_2. Let us see.

The distance p_2 between this object O_2 and lens 2 is, from Fig. 34.4.5*c*,

$$p_2 = L + |i_1| = 10\text{ cm} + 8.0\text{ cm} = 18\text{ cm}.$$

Then Eq. 34.4.1, now written for lens 2, yields

$$\frac{1}{+18\text{ cm}} + \frac{1}{i_2} = \frac{1}{+9.0\text{ cm}}.$$

Hence, $$i_2 = +18\text{ cm}. \qquad \text{(Answer)}$$

The plus sign confirms our guess: Image I_2 produced by lens 2 is real, inverted, and on the side of lens 2 opposite O_2, as shown in Fig. 34.4.5*c*. Thus, the image would appear on a card placed at its location.

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34.5 OPTICAL INSTRUMENTS

Learning Objectives

After reading this module, you should be able to . . .

34.5.1 Identify the near point in vision.
34.5.2 With sketches, explain the function of a simple magnifying lens.
34.5.3 Identify angular magnification.
34.5.4 Determine the angular magnification for an object at the focal point of a simple magnifying lens.
34.5.5 With a sketch, explain a compound microscope.
34.5.6 Identify that the overall magnification of a compound microscope is due to the lateral magnification by the objective and the angular magnification by the eyepiece.
34.5.7 Calculate the overall magnification of a compound microscope.
34.5.8 With a sketch, explain a refracting telescope.
34.5.9 Calculate the angular magnification of a refracting telescope.

Key Ideas

- The angular magnification of a simple magnifying lens is

$$m_\theta = \frac{25\text{ cm}}{f},$$

where f is the focal length of the lens and 25 cm is a reference value for the near point value.

- The overall magnification of a compound microscope is

$$M = m m_\theta = -\frac{s}{f_{ob}}\frac{25\text{ cm}}{f_{ey}},$$

where m is the lateral magnification of the objective, m_θ is the angular magnification of the eyepiece, s is the tube length, f_{ob} is the focal length of the objective, and f_{ey} is the focal length of the eyepiece.

- The angular magnification of a refracting telescope is

$$m_\theta = -\frac{f_{ob}}{f_{ey}}.$$

Optical Instruments

The human eye is a remarkably effective organ, but its range can be extended in many ways by optical instruments such as eyeglasses, microscopes, and telescopes. Many such devices extend the scope of our vision beyond the visible range; satellite-borne infrared cameras and x-ray microscopes are just two examples.

The mirror and thin-lens formulas can be applied only as approximations to most sophisticated optical instruments. The lenses in typical laboratory microscopes are by no means “thin.” In most optical instruments the lenses are compound lenses; that is, they are made of several components, the interfaces rarely being exactly spherical. Now we discuss three optical instruments, assuming, for simplicity, that the thin-lens formulas apply.

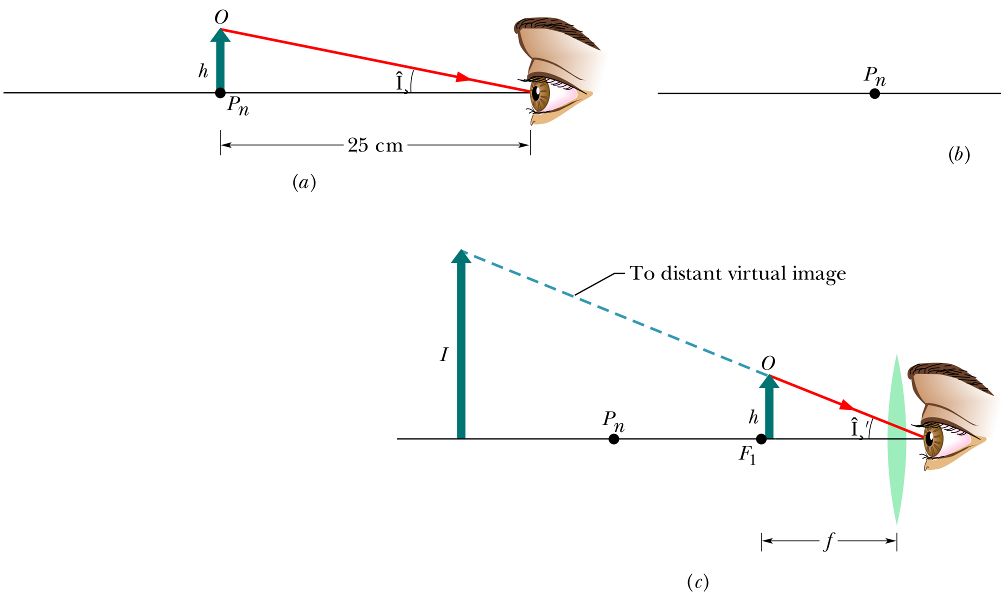


Figure 34.5.1 (*a*) An object O of height h placed at the near point of a human eye occupies angle θ in the eye's view. (*b*) The object is moved closer to increase the angle, but now the observer cannot bring the object into focus. (*c*) A converging lens is placed between the object and the eye, with the object just inside the focal point F_1 of the lens. The image produced by the lens is then far enough away to be focused by the eye, and the image occupies a larger angle θ' than object O does in (*a*).

Simple Magnifying Lens

The normal human eye can focus a sharp image of an object on the retina (at the rear of the eye) if the object is located anywhere from infinity to a certain point called the *near point* P_n. If you move the object closer to the eye than the near point, the perceived retinal image becomes fuzzy. The location of the near point normally varies with age, generally moving away from the person. To find your own near point, remove your glasses or contacts if you wear any, close one eye, and then bring this page closer to your open eye until it becomes indistinct. In what follows, we take the near point to be 25 cm from the eye, a bit more than the typical value for 20-year-olds.

Figure 34.5.1*a* shows an object O placed at the near point P_n of an eye. The size of the image of the object produced on the retina depends on the angle θ that the object occupies in the field of view from that eye. By moving the object closer to the eye, as in Fig. 34.5.1*b*, you can increase the angle and, hence, the possibility of distinguishing details of the object. However, because the object is then closer than the near point, it is no longer *in focus*; that is, the image is no longer clear.

You can restore the clarity by looking at O through a converging lens, placed so that O is just inside the focal point F_1 of the lens, which is at focal length f (Fig. 34.5.1*c*). What you then see is the virtual image of O produced by the lens. That image is farther away than the near point; thus, the eye can see it clearly.

Moreover, the angle θ' occupied by the virtual image is larger than the largest angle θ that the object alone can occupy and still be seen clearly. The *angular magnification* m_θ (not to be confused with lateral magnification m) of what is seen is

$$m_\theta = \theta'/\theta.$$

In words, the angular magnification of a simple magnifying lens is a comparison of the angle occupied by the image the lens produces with the angle occupied by the object when the object is moved to the near point of the viewer.

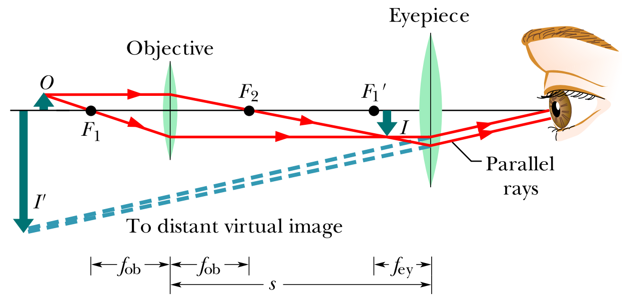


Figure 34.5.2 A thin-lens representation of a compound microscope (not to scale). The objective produces a real image I of object O just inside the focal point F_1â€²of the eyepiece. Image I then acts as an object for the eyepiece, which produces a virtual final image Iâ€²that is seen by the observer. The objective has focal length f_{ob}; the eyepiece has focal length f_{ey}; and s is the tube length.

From Fig. 34.5.1, assuming that O is at the focal point of the lens, and approximating tan Î¸ as Î¸ and tan Î¸â€² as Î¸â€² for small angles, we have

$$\text{Î¸ â‰ˆ } h/25\text{ cm} \quad \text{and} \quad \text{Î¸â€² â‰ˆ } h/f.$$

We then find that

$$m_{\text{Î¸}} \text{ â‰ˆ } \frac{25\text{ cm}}{f} \quad \text{(simple magnifier).} \tag{34.5.1}$$

Compound Microscope

Figure 34.5.2 shows a thin-lens version of a compound microscope. The instrument consists of an *objective* (the front lens) of focal length f_{ob} and an *eyepiece* (the lens near the eye) of focal length f_{ey}. It is used for viewing small objects that are very close to the objective.

The object O to be viewed is placed just outside the first focal point F_1 of the objective, close enough to F_1 that we can approximate its distance p from the lens as being f_{ob}. The separation between the lenses is then adjusted so that the enlarged, inverted, real image I produced by the objective is located just inside the first focal point F_1â€²of the eyepiece. The *tube length* s shown in Fig. 34.5.2 is actually large relative to f_{ob}, and therefore we can approximate the distance i between the objective and the image I as being length s.

From Eq. 34.2.4, and using our approximations for p and i, we can write the lateral magnification produced by the objective as

$$m = \text{âˆ’}\frac{i}{p} = \text{âˆ’}\frac{s}{f_{ob}}. \tag{34.5.2}$$

Because the image I is located just inside the focal point F_1â€²of the eyepiece, the eyepiece acts as a simple magnifying lens, and an observer sees a final (virtual, inverted) image Iâ€²through it. The overall magnification of the instrument is the product of the lateral magnification m produced by the objective, given by Eq. 34.5.2, and the angular magnification $m_{\text{Î¸}}$ produced by the eyepiece, given by Eq. 34.5.1; that is,

$$M = m\, m_{\text{Î¸}} = \text{âˆ’}\frac{s}{f_{ob}}\frac{25\text{ cm}}{f_{ey}} \quad \text{(microscope).} \tag{34.5.3}$$

Refracting Telescope

Telescopes come in a variety of forms. The form we describe here is the simple refracting telescope that consists of an objective and an eyepiece; both are represented in Fig. 34.5.3 with simple lenses, although in practice, as is also true for most microscopes, each lens is actually a compound lens system.

The lens arrangements for telescopes and for microscopes are similar, but telescopes are designed to view large objects, such as galaxies, stars, and planets, at large distances, whereas microscopes are designed for just the opposite purpose. This difference requires that in the telescope of Fig. 34.5.3 the second focal point of the objective F_2 coincide with the first focal point of the eyepiece F_1â€² whereas in the microscope of Fig. 34.5.2 these points are separated by length *s*.

In Fig. 34.5.3*a*, parallel rays from a distant object strike the objective, making an angle Î¸$_{ob}$ with the telescope axis and forming a real, inverted image *I* at the common focal point F_2, F_1â€² This image *I* acts as an object for the eyepiece, through which an observer sees a distant (still inverted) virtual image *I*â€² The rays defining the image make an angle Î¸$_{ey}$ with the telescope axis.

The angular magnification $m_{Î¸}$ of the telescope is Î¸$_{ey}$/Î¸$_{ob}$. From Fig. 34.5.3, for rays close to the central axis, we can write Î¸$_{ob}$ = *h*â€²/f_{ob} and Î¸$_{ey}$ â‰ˆ *h*â€²/f_{ey}, which gives us

$$m_{Î¸} = â\frac{f_{ob}}{f_{ey}} \quad \text{(telescope)}, \qquad (34.5.4)$$

where the minus sign indicates that *I*â€² is inverted. In words, the angular magnification of a telescope is a comparison of the angle occupied by the image the telescope produces with the angle occupied by the distant object as seen without the telescope.

Magnification is only one of the design factors for an astronomical telescope and is indeed easily achieved. A good telescope needs *light-gathering power,* which determines how bright the image is. This is important for viewing faint objects such as distant galaxies and is accomplished by making the objective diameter as large as possible. A telescope also needs *resolving power,* which is the ability to distinguish between two distant objects (stars, say) whose angular separation is small. *Field of view* is another important design parameter. A telescope designed to look at galaxies (which occupy a tiny field of view) is much different from one designed to track meteors (which move over a wide field of view).

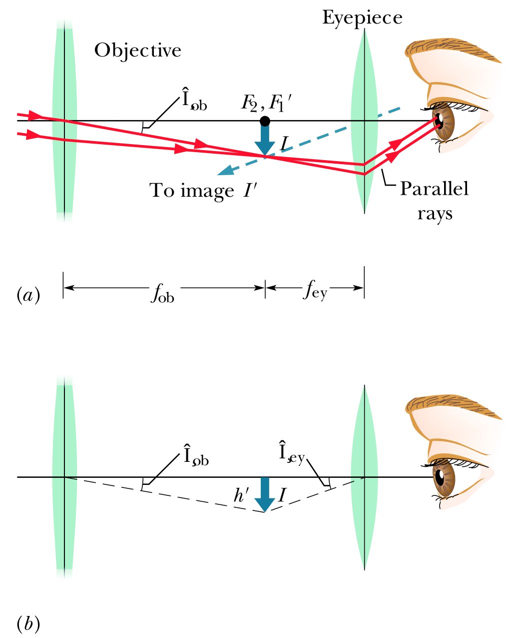


Figure 34.5.3 (*a*) A thin-lens representation of a refracting telescope. From rays that are approximately parallel when they reach the objective, the objective produces a real image *I* of a distant source of light (the object). (One end of the object is assumed to lie on the central axis.) Image *I*, formed at the common focal points F_2 and F_1â€² acts as an object for the eyepiece, which produces a virtual final image *I*â€² at a great distance from the observer. The objective has focal length f_{ob}; the eyepiece has focal length f_{ey}. (*b*) Image *I* has height *h*â€² and takes up angle Î¸$_{ob}$ measured from the objective and angle Î¸$_{ey}$ measured from the eyepiece.

The telescope designer must also take into account the difference between real lenses and the ideal thin lenses we have discussed. A real lens with spherical surfaces does not form sharp images, a flaw called *spherical aberration.* Also, because refraction by the two surfaces of a real lens depends on wavelength, a real lens does not focus light of different wavelengths to the same point, a flaw called *chromatic aberration.*

This brief discussion by no means exhausts the design parameters of astronomical telescopesâ€”many others are involved. We could make a similar listing for any other high-performance optical instrument.

Optical Neuroimaging

In *functional near infrared spectroscopy* (fNIRS), a person wears a close-fitting cap with LEDs emitting in the near infrared range with wavelengths of 650 to 950 nm (Fig. 34.5.4). The light can penetrate the scalp, the skull, and the outer layer (1 to 2 cm) of the brain, where it is either absorbed or scattered by hemoglobin, the protein in the blood that can carry oxygen from the lungs to the rest of the body. The absorption and scattering differ for the hemoglobin (Hb) without oxygen and the hemoglobin (HbO) with oxygen. If the person switches from seeing a

Bloomberg/Getty Images

Figure 34.5.4 Headgear for optical imaging of the brain.

gray featureless image to a patterned image, the flow of oxygenated hemoglobin (HbO) increases into the part of the brain responsible for producing images. That change in HbO is indicated by the amount of scattered light. Researchers are now using fNIRS to map which parts of the brain are activated by which activities. The advantages of fNIRS over other ways of "looking inside" the brain are that it is noninvasive, inexpensive, and relatively portable, so the equipment can be used anywhere from a baseball field to an airplane cockpit.

Checkpoint 34.5.1

Consider the compound microscope and the refracting telescope of this module. Which type of image is produced for an observer with (a) a microscope and (b) a telescope? (c) Which instrument involves a separation of the objective's focal point and the eyepiece focal point?

34.6 THREE PROOFS

The Spherical Mirror Formula (Eq. 34.2.2)

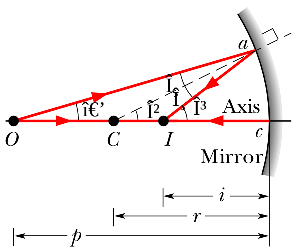


Figure 34.6.1 A concave spherical mirror forms a real point image I by reflecting light rays from a point object O.

Figure 34.6.1 shows a point object O placed on the central axis of a concave spherical mirror, outside its center of curvature C. A ray from O that makes an angle α with the axis intersects the axis at I after reflection from the mirror at a. A ray that leaves O along the axis is reflected back along itself at c and also passes through I. Thus, because both rays pass through that common point, I is the image of O; it is a *real* image because light actually passes through it. Let us find the image distance i.

A trigonometry theorem that is useful here tells us that an exterior angle of a triangle is equal to the sum of the two opposite interior angles. Applying this to triangles OaC and OaI in Fig. 34.6.1 yields

$$\beta = \alpha + \theta \quad \text{and} \quad \gamma = \alpha + 2\theta.$$

If we eliminate θ between these two equations, we find

$$\alpha + \gamma = 2\beta. \tag{34.6.1}$$

We can write angles α, β, and γ, in radian measure, as

$$\alpha \approx \frac{\overset{\frown}{ac}}{cO} = \frac{\overset{\frown}{ac}}{p}, \quad \beta = \frac{\overset{\frown}{ac}}{cC} = \frac{\overset{\frown}{ac}}{r},$$

and

$$\gamma \approx \frac{\overset{\frown}{ac}}{cI} = \frac{\overset{\frown}{ac}}{i}, \tag{34.6.2}$$

where the overhead symbol means "arc." Only the equation for β is exact, because the center of curvature of $\overset{\frown}{ac}$ is at C. However, the equations for α and γ are approximately correct if these angles are small enough (that is, for rays close to the central axis). Substituting Eqs. 34.6.2 into Eq. 34.6.1, using Eq. 34.2.1 to replace r with $2f$, and canceling $\overset{\frown}{ac}$ lead exactly to Eq. 34.2.2, the relation that we set out to prove.

The Refracting Surface Formula (Eq. 34.3.1)

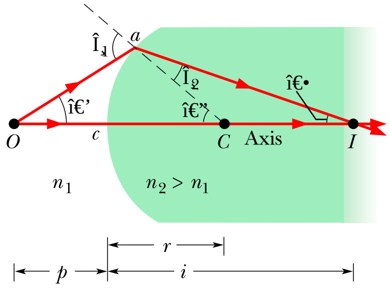


Figure 34.6.2 A real point image I of a point object O is formed by refraction at a spherical convex surface between two media.

The incident ray from point object O in Fig. 34.6.2 that falls on point a of a spherical refracting surface is refracted there according to Eq. 33.5.2,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2.$$

If α is small, θ_1 and θ_2 will also be small and we can replace the sines of these angles with the angles themselves. Thus, the equation above becomes

$$n_1\theta_1 \approx n_2\theta_2. \tag{34.6.3}$$

We again use the fact that an exterior angle of a triangle is equal to the sum of the two opposite interior angles. Applying this to triangles COa and ICa yields

$$\theta_1 = \alpha + \beta \quad \text{and} \quad \beta = \theta_2 + \gamma. \tag{34.6.4}$$

If we use Eqs. 34.6.4 to eliminate θ_1 and θ_2 from Eq. 34.6.3, we find

$$n_1\alpha + n_2\gamma = (n_2 - n_1)\beta. \tag{34.6.5}$$

In radian measure the angles α, β, and γ are

$$\alpha \approx \frac{\overset{\frown}{ac}}{p}; \quad \beta = \frac{\overset{\frown}{ac}}{r}; \quad \gamma \approx \frac{\overset{\frown}{ac}}{i}. \tag{34.6.6}$$

Only the second of these equations is exact. The other two are approximate because I and O are not the centers of circles of which $\overset{\frown}{ac}$ is a part. However, for α small enough (for rays close to the axis), the inaccuracies in Eqs. 34.6.6 are small. Substituting Eqs. 34.6.6 into Eq. 34.6.5 leads directly to Eq. 34.3.1, as we wanted.

The Thin-Lens Formulas (Eqs. 34.4.1 and 34.4.2)

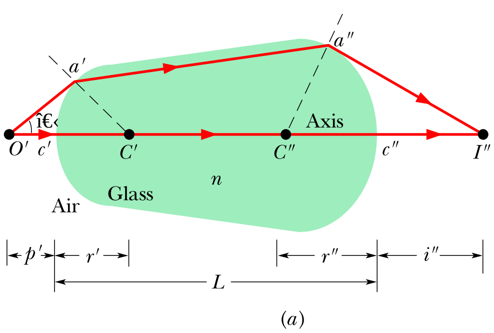
Our plan is to consider each lens surface as a separate refracting surface, and to use the image formed by the first surface as the object for the second.

We start with the thick glass "lens" of length L in Fig. 34.6.3a whose left and right refracting surfaces are ground to radii r' and r''. A point object O' is placed near the left surface as shown. A ray leaving O' along the central axis is not deflected on entering or leaving the lens.

A second ray leaving O' at an angle α with the central axis intersects the left surface at point a', is refracted, and intersects the second (right) surface at point a''. The ray is again refracted and crosses the axis at I'', which, being the intersection of two rays from O', is the image of point O', formed after refraction at two surfaces.

Figure 34.6.3b shows that the first (left) surface also forms a virtual image of O' at I'. To locate I', we use Eq. 34.3.1,

$$\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{r}.$$



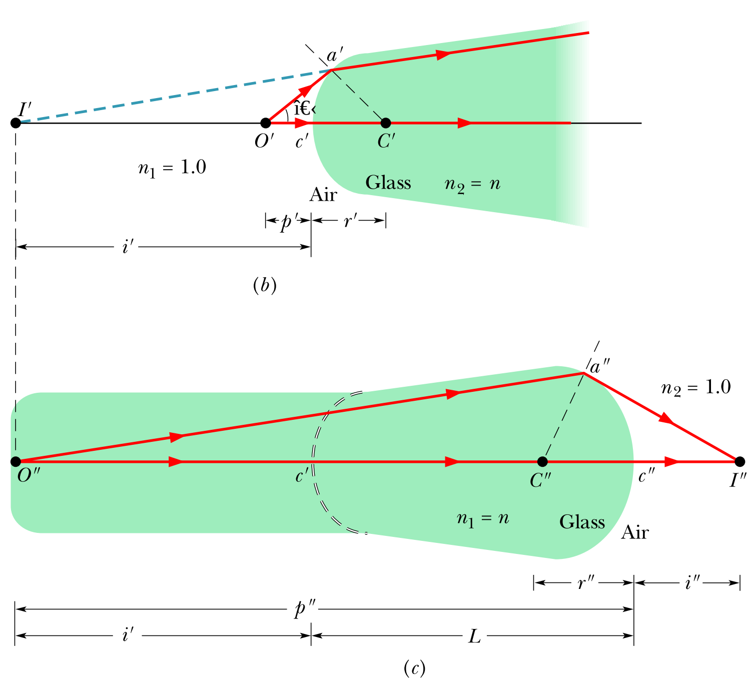


Figure 34.6.3 (a) Two rays from point object O' form a real image I'' after refracting through two spherical surfaces of a lens. The object faces a convex surface at the left side of the lens and a concave surface at the right side. The ray traveling through points a' and a'' is actually close to the central axis through the lens. (b) The left side and (c) the right side of the lens in (a), shown separately.

Putting $n_1 = 1$ for air and $n_2 = n$ for lens glass and bearing in mind that the (virtual) image distance is negative (that is, $i = -i'$ in Fig. 34.6.3*b*), we obtain

$$\frac{1}{p'} - \frac{n}{i'} = \frac{n-1}{r'}. \tag{34.6.7}$$

(Because the minus sign is explicit, i' will be a positive number.)

Figure 34.6.3*c* shows the second surface again. Unless an observer at point a' were aware of the existence of the first surface, the observer would think that the light striking that point originated at point I' in Fig. 34.6.3*b* and that the region to the left of the surface was filled with glass as indicated. Thus, the (virtual) image I' formed by the first surface serves as a real object O'' for the second surface. The distance of this object from the second surface is

$$p'' = i' + L. \tag{34.6.8}$$

To apply Eq. 34.3.1 to the second surface, we must insert $n_1 = n$ and $n_2 = 1$ because the object now is effectively imbedded in glass. If we substitute with Eq. 34.6.8, then Eq. 34.3.1 becomes

$$\frac{n}{i' + L} + \frac{1}{i''} = \frac{1-n}{r''}. \tag{34.6.9}$$

Let us now assume that the thickness L of the "lens" in Fig. 34.6.3*a* is so small that we can neglect it in comparison with our other linear quantities (such as p', i', p'', i'', r', and r''). In all that follows we make this *thin-lens approximation.* Putting $L = 0$ in Eq. 34.6.9 and rearranging the right side lead to

$$\frac{n}{i'} + \frac{1}{i''} = -\frac{n-1}{r''}. \tag{34.6.10}$$

Adding Eqs. 34.6.7 and 34.6.10 leads to

$$\frac{1}{p'} + \frac{1}{i''} = (n-1)\left(\frac{1}{r'} - \frac{1}{r''}\right).$$

Finally, calling the original object distance simply p and the final image distance simply i leads to

$$\frac{1}{p} + \frac{1}{i} = (n-1)\left(\frac{1}{r'} - \frac{1}{r''}\right), \tag{34.6.11}$$

which, with a small change in notation, is Eqs. 34.4.1 and 34.4.2.

Review & Summary

Real and Virtual Images An *image* is a reproduction of an object via light. If the image can form on a surface, it is a *real image* and can exist even if no observer is present. If the image requires the visual system of an observer, it is a *virtual image.*

Image Formation *Spherical mirrors, spherical refracting surfaces,* and *thin lenses* can form images of a source of light—the object—by redirecting rays emerging from the source. The image occurs where the redirected rays cross (forming a real image) or where backward extensions of those rays cross (forming a virtual image). If the rays are sufficiently close to the *central axis* through the spherical mirror, refracting surface, or thin lens, we have the following relations between the *object distance p* (which is positive) and the *image distance i* (which is positive for real images and negative for virtual images):

1. Spherical Mirror:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} = \frac{2}{r}, \tag{34.2.2, 34.2.1}$$

where f is the mirror's focal length and r is its radius of curvature. A *plane mirror* is a special case for which $r \rightarrow \infty$ so that $p = -i$. Real images form on the side of a mirror where the object is located, and virtual images form on the opposite side.

2. Spherical Refracting Surface:

$$\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{r} \quad \text{(single surface)}, \tag{34.3.1}$$

where n_1 is the index of refraction of the material where the object is located, n_2 is the index of refraction of the material on the other side of the refracting surface, and r is the radius of curvature of the surface. When the object faces a convex refracting surface, the radius r is positive. When it faces a concave surface, r is negative. Real images form on the side of a refracting surface that is opposite the object, and virtual images form on the same side as the object.

3. Thin Lens:

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f}=(n\ \text{â}^{\text{`}}1)\left(\frac{1}{r_1}\ \text{â}^{\text{`}}\frac{1}{r_2}\right), \qquad (34.4.1, 34.4.2)$$

where f is the lensâ€™s focal length, n is the index of refraction of the lens material, and r_1 and r_2 are the radii of curvature of the two sides of the lens, which are spherical surfaces. A convex lens surface that faces the object has a positive radius of curvature; a concave lens surface that faces the object has a negative radius of curvature. Real images form on the side of a lens that is opposite the object, and virtual images form on the same side as the object.

Lateral Magnification The *lateral magnification* m produced by a spherical mirror or a thin lens is

$$m=\text{â}^{\text{`}}\frac{i}{p}. \qquad (34.2.4)$$

The magnitude of m is given by

$$|m|=\frac{h\text{â€²}}{h}, \qquad (34.2.3)$$

where h and hâ€²are the heights (measured perpendicular to the central axis) of the object and image, respectively.

Optical Instruments Three optical instruments that extend human vision are:

1. The *simple magnifying lens,* which produces an *angular magnification* $m_{\text{Î¸}}$ given by

$$m_{\text{Î¸}}=\frac{25\text{ cm}}{f}, \qquad (34.5.1)$$

where f is the focal length of the magnifying lens. The distance of 25 cm is a traditionally chosen value that is a bit more than the typical near point for someone 20 years old.

2. The *compound microscope,* which produces an *overall magnification* M given by

$$M=mm_{\text{Î¸}}=\text{â}^{\text{`}}\frac{s}{f_{ob}}\frac{25\text{ cm}}{f_{ey}}, \qquad (34.5.3)$$

where m is the lateral magnification produced by the objective, $m_{\text{Î¸}}$ is the angular magnification produced by the eyepiece, s is the tube length, and f_{ob} and f_{ey} are the focal lengths of the objective and eyepiece, respectively.

3. The *refracting telescope,* which produces an *angular magnification* $m_{\text{Î¸}}$ given by

$$m_{\text{Î¸}}=\text{â}^{\text{`}}\frac{f_{ob}}{f_{ey}}. \qquad (34.5.4)$$

Questions

1 Figure 34.1 shows a fish and a fish stalker in water. (a) Does the stalker see the fish in the general region of point a or point b? (b) Does the fish see the (wild) eyes of the stalker in the general region of point c or point d?

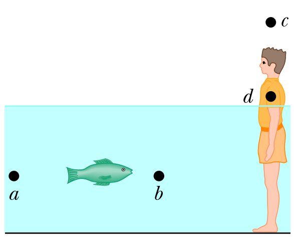


Figure 34.1 Question 1.

2 In Fig. 34.2, stick figure O stands in front of a spherical mirror that is mounted within the boxed region; the central axis through the mirror is shown. The four stick figures I_1 to I_4 suggest general locations and orientations for the images that might be produced by the mirror. (The figures are only sketched in; neither their heights nor their distances from the mirror are drawn to scale.) (a) Which of the stick figures could not possibly represent images? Of the possible images, (b) which would be due to a concave mirror, (c) which would be due to a convex mirror, (d) which would be virtual, and (e) which would involve negative magnification?

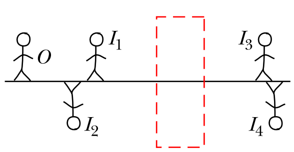


Figure 34.2 Questions 2 and 10.

3 Figure 34.3 is an overhead view of a mirror maze based on floor sections that are equilateral triangles. Every wall within the maze is mirrored. If you stand at entrance x, (a) which of the maze monsters a, b, and c hiding in the maze can you see along the virtual hallways extending from entrance x; (b) how many times does each visible monster appear in a hallway; and (c) what is at the far end of a hallway?

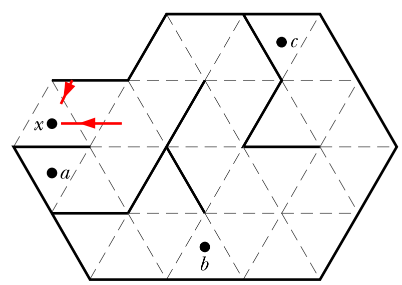


Figure 34.3 Question 3.

4 A penguin waddles along the central axis of a concave mirror, from the focal point to an effectively infinite distance. (a) How does its image move? (b) Does the height of its image increase continuously, decrease continuously, or change in some more complicated manner?

5 When a *T. rex* pursues a jeep in the movie *Jurassic Park,* we see a reflected image of the *T. rex* via a side-view mirror, on which is printed the (then darkly humorous) warning: â€œObjects in mirror are closer than they appear.â€ Is the mirror flat, convex, or concave?

6 An object is placed against the center of a concave mirror and then moved along the central axis until it is 5.0 m from the mirror. During the motion, the distance $|i|$ between the mirror and the image it produces is measured. The procedure is then repeated with a convex mirror and a plane mirror. Figure 34.4 gives the results versus object distance p. Which curve corresponds to which mirror? (Curve 1 has two segments.)

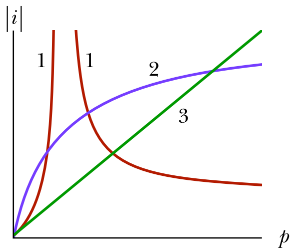


Figure 34.4 Questions 6 and 8.

7 The table details six variations of the basic arrangement of two thin lenses represented in Fig. 34.5. (The points labeled F_1 and F_2 are the focal points of lenses 1 and 2.) An object is distance p_1 to the left of lens 1, as in Fig. 34.4.5. (a) For which variations can we tell, *without calculation,* whether the final image (that due to lens 2) is to the left or right of lens 2 and whether it has the same orientation as the object? (b) For those â€œeasyâ€ variations, give the image location as â€œleftâ€ or â€œrightâ€ and the orientation as â€œsameâ€ or â€œinverted.â€

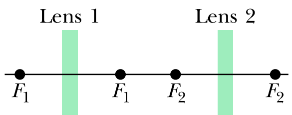


Figure 34.5 Question 7.

Variation	Lens 1	Lens 2	
1	Converging	Converging	$p_1 < \|f_1\|$
2	Converging	Converging	$p_1 > \|f_1\|$
3	Diverging	Converging	$p_1 < \|f_1\|$
4	Diverging	Converging	$p_1 > \|f_1\|$
5	Diverging	Diverging	$p_1 < \|f_1\|$
6	Diverging	Diverging	$p_1 > \|f_1\|$

8 An object is placed against the center of a converging lens and then moved along the central axis until it is 5.0 m from the lens. During the motion, the distance $|i|$ between the lens and the image it produces is measured. The procedure is then repeated with a diverging lens. Which of the curves in Fig. 34.4 best gives $|i|$ versus the object distance p for these lenses? (Curve 1 consists of two segments. Curve 3 is straight.)

9 Figure 34.6 shows four thin lenses, all of the same material, with sides that either are flat or have a radius of curvature of magnitude 10 cm. Without written calculation, rank the lenses according to the magnitude of the focal length, greatest first.

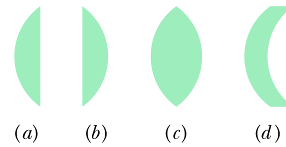


Figure 34.6 Question 9.

10 In Fig. 34.2, stick figure O stands in front of a thin, symmetric lens that is mounted within the boxed region; the central axis through the lens is shown. The four stick figures I_1 to I_4 suggest general locations and orientations for the images that might be produced by the lens. (The figures are only sketched in; neither their height nor their distance from the lens is drawn to scale.) (a) Which of the stick figures could not possibly represent images? Of the possible images, (b) which would be due to a converging lens, (c) which would be due to a diverging lens, (d) which would be virtual, and (e) which would involve negative magnification?

11 Figure 34.7 shows a coordinate system in front of a flat mirror, with the x axis perpendicular to the mirror. Draw the image of the system in the mirror. (a) Which axis is reversed by the reflection? (b) If you face a mirror, is your image inverted (top for bottom)? (c) Is it reversed left and right (as commonly believed)? (d) What then is reversed?

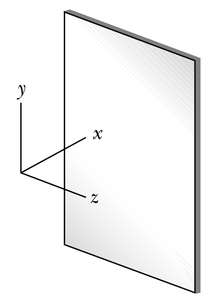


Figure 34.7 Question 11.

Problems

GO Tutoring problem available (at instructorâ€™s discretion) in *WileyPLUS*
SSM Worked-out solution available in Student Solutions Manual
E Easy M Medium H Hard
FCP Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com
CALC Requires calculus
BIO Biomedical application

Module 34.1 Images and Plane Mirrors

1 E You look through a camera toward an image of a hummingbird in a plane mirror. The camera is 4.30 m in front of the mirror. The bird is at camera level, 5.00 m to your right and 3.30 m from the mirror. What is the distance between the camera and the apparent position of the birdâ€™s image in the mirror?

2 E A moth at about eye level is 10 cm in front of a plane mirror; you are behind the moth, 30 cm from the mirror. What is the distance between your eyes and the apparent position of the mothâ€™s image in the mirror?

3 M In Fig. 34.8, an isotropic point source of light S is positioned at distance d from a viewing screen A and the light intensity I_P at point P (level with S) is measured. Then a plane mirror M is placed behind S at distance d. By how much is I_P multiplied by the presence of the mirror?

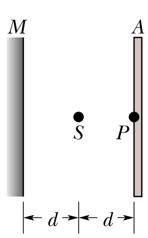


Figure 34.8 Problem 3.

4 M Figure 34.9 shows an overhead view of a corridor with a plane mirror M mounted at one end. A burglar B sneaks along the corridor directly toward the center of the mirror. If $d = 3.0$ m, how far from the mirror will she be when the security guard S can first see her in the mirror?

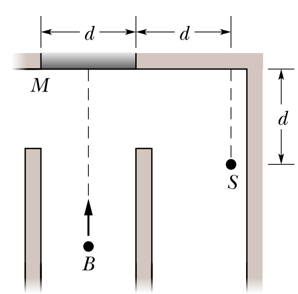


Figure 34.9 Problem 4.

5 H SSM Figure 34.10 shows a small lightbulb suspended at distance $d_1 = 250$ cm above the surface of the water in a swimming pool where the water depth is $d_2 = 200$ cm. The bottom of the pool is a large mirror. How far below the mirror surface is the image of the bulb? (*Hint:* Assume that the rays are close to a vertical axis through the bulb, and use the small-angle approximation in which sin Î¸â‰ˆtan Î¸â‰ˆÎ¸.)

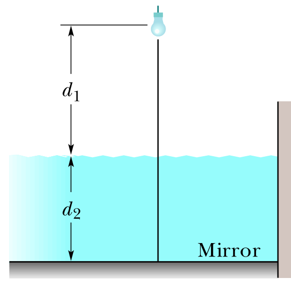


Figure 34.10 Problem 5.

Module 34.2 Spherical Mirrors

6 E An object is moved along the central axis of a spherical mirror while the lateral magnification m of it is measured. Figure 34.11 gives m versus object distance p for the range $p_a = 2.0$ cm to $p_b = 8.0$ cm. What is m for $p = 14.0$ cm?

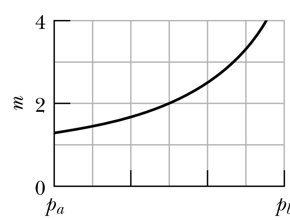


Figure 34.11 Problem 6.

7 E A concave shaving mirror has a radius of curvature of 35.0 cm. It is positioned so that the (upright) image of a manâ€™s face is 2.50 times the size of the face. How far is the mirror from the face?

8 E An object is placed against the center of a spherical mirror and then moved 70 cm from it along the central axis as the image distance i is measured. Figure 34.12 gives i versus object distance p out to $p_s = 40$ cm. What is i for $p = 70$ cm?

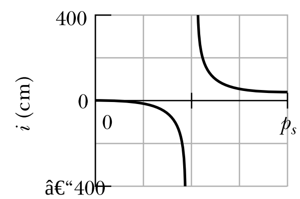


Figure 34.12 Problem 8.

9 through 16 M GO 12 SSM 9, 11, 13 *Spherical mirrors.* Object O stands on the central axis of a spherical mirror. For this situation, each problem in Table 34.1 gives object distance p_s (centimeters), the type of mirror, and then the distance (centimeters, without proper sign) between the focal point and the mirror. Find (a) the radius of curvature r (including sign), (b) the image distance i, and (c) the lateral magnification m. Also, determine whether the image is (d) real (R) or virtual (V), (e) inverted (I) from object O or noninverted (NI), and (f) on the *same* side of the mirror as O or on the *opposite* side.

17 through 29 M GO 22 SSM 23, 29 *More mirrors.* Object O stands on the central axis of a spherical or plane mirror. For this situation, each problem in Table 34.2 refers to (a) the type of mirror, (b) the focal distance f, (c) the radius of curvature r, (d) the object distance p, (e) the image distance i, and (f) the lateral magnification m. (All distances are in centimeters.) It also refers to whether (g) the image is real (R) or virtual (V), (h) inverted (I) or noninverted (NI) from O, and (i) on the *same* side of the mirror as object O or on the *opposite* side. Fill in the missing information. Where only a sign is missing, answer with the sign.

30 M GO Figure 34.13 gives the lateral magnification m of an object versus the object distance p from a spherical mirror as the object is moved along the mirrorâ€™s central axis through a range of values for p. The horizontal scale is set by $p_s = 10.0$ cm. What is the magnification of the object when the object is 21 cm from the mirror?

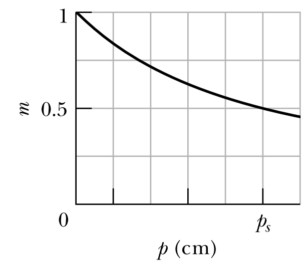


Figure 34.13 Problem 30.

31 M CALC (a) A luminous point is moving at speed v_O toward a spherical mirror with radius of curvature r, along the central axis of the mirror. Show that the image of this point is moving at speed

$$v_I = \text{â}ˆ\left(\frac{r}{2p\ \text{â}ˆ r}\right)^2 v_O,$$

where p is the distance of the luminous point from the mirror at any given time. Now assume the mirror is concave, with $r = 15$ cm, and let $v_O = 5.0$ cm/s. Find v_I when (b) $p = 30$ cm (far outside the focal point), (c) $p = 8.0$ cm (just outside the focal point), and (d) $p = 10$ mm (very near the mirror).

Module 34.3 Spherical Refracting Surfaces

32 through 38 M GO 37, 38 SSM 33, 35 *Spherical refracting surfaces.* An object O stands on the central axis of a spherical refracting surface. For this situation, each problem in Table 34.3 refers to the index of refraction n_1 where the object is located, (a) the index of refraction n_2 on the other side of the

Table 34.1 Problems 9 through 16: Spherical Mirrors. See the setup for these problems.

	p	Mirror	(a) r	(b) i	(c) m	(d) R/V	(e) I/NI	(f) Side
9	+18	Concave, 12						
10	+15	Concave, 10						
11	+8.0	Convex, 10						
12	+24	Concave, 36						
13	+12	Concave, 18						
14	+22	Convex, 35						
15	+10	Convex, 8.0						
16	+17	Convex, 14						

Table 34.2 Problems 17 through 29: More Mirrors. See the setup for these problems.

	(a) Type	(b) f	(c) r	(d) p	(e) i	(f) m	(g) R/V	(h) I/NI	(i) Side
17	Concave	20		+10					
18				+24		0.50		I	
19			−40		−10				
20				+40		−0.70			
21		+20		+30					
22		20				+0.10			
23		30				+0.20			
24				+60		−0.50			
25				+30		0.40		I	
26		20		+60					Same
27		−30			−15				
28				+10		+1.0			
29	Convex		40		4.0				

refracting surface, (b) the object distance p, (c) the radius of curvature r of the surface, and (d) the image distance i. (All distances are in centimeters.) Fill in the missing information, including whether the image is (e) real (R) or virtual (V) and (f) on the *same* side of the surface as object O or on the *opposite* side.

39 M In Fig. 34.14, a beam of parallel light rays from a laser is incident on a solid transparent sphere of index of refraction n. (a) If a point image is produced at the back of the sphere, what is the index of refraction of the sphere? (b) What index of refraction, if any, will produce a point image at the center of the sphere?

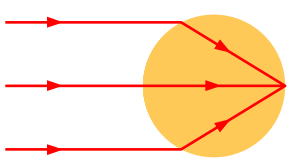


Figure 34.14 Problem 39.

40 M A glass sphere has radius $R = 5.0$ cm and index of refraction 1.6. A paperweight is constructed by slicing through the sphere along a plane that is 2.0 cm from the center of the sphere, leaving height $h = 3.0$ cm. The paperweight is placed on a table and viewed from directly above by an observer who is distance $d = 8.0$ cm from the tabletop (Fig. 34.15). When viewed through the paperweight, how far away does the tabletop appear to be to the observer?

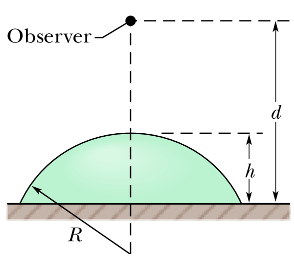


Figure 34.15 Problem 40.

Module 34.4 Thin Lenses

41 E A lens is made of glass having an index of refraction of 1.5. One side of the lens is flat, and the other is convex with a radius of curvature of 20 cm. (a) Find the focal length of the lens. (b) If an object is placed 40 cm in front of the lens, where is the image?

42 E Figure 34.16 gives the lateral magnification m of an object versus the object distance p from a lens as the object is moved along the central axis of the lens through a range of values for p out to $p_s = 20.0$ cm. What is the magnification of the object when the object is 35 cm from the lens?

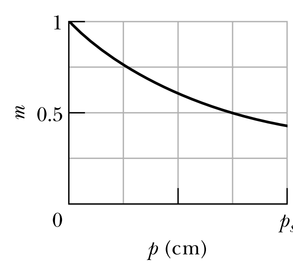


Figure 34.16 Problem 42.

43 E A movie camera with a (single) lens of focal length 75 mm takes a picture of a person standing 27 m away. If the person is 180 cm tall, what is the height of the image on the film?

Table 34.3 Problems 32 through 38: Spherical Refracting Surfaces. See the setup for these problems.

	n_1	(a) n_2	(b) p	(c) r	(d) i	(e) R/V	(f) Side
32	1.0	1.5	+10	+30			
33	1.0	1.5	+10		−13		
34	1.5		+100	−30	+600		
35	1.5	1.0	+70	+30			
36	1.5	1.0		−30	−7.5		
37	1.5	1.0	+10		−6.0		
38	1.0	1.5		+30	+600		

44 **E** An object is placed against the center of a thin lens and then moved away from it along the central axis as the image distance i is measured. Figure 34.17 gives i versus object distance p out to $p_s = 60$ cm. What is the image distance when $p = 100$ cm?

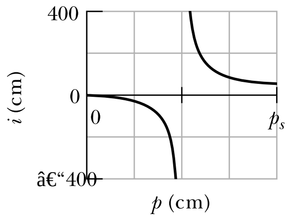


Figure 34.17 Problem 44.

45 **E** You produce an image of the Sun on a screen, using a thin lens whose focal length is 20.0 cm. What is the diameter of the image? (See Appendix C for needed data on the Sun.)

46 **E** An object is placed against the center of a thin lens and then moved 70 cm from it along the central axis as the image distance i is measured. Figure 34.18 gives i versus object distance p out to $p_s = 40$ cm. What is the image distance when $p = 70$ cm?

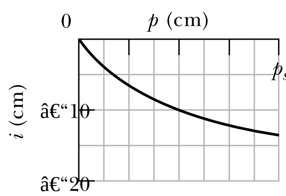


Figure 34.18 Problem 46.

47 **E** **SSM** A double-convex lens is to be made of glass with an index of refraction of 1.5. One surface is to have twice the radius of curvature of the other and the focal length is to be 60 mm. What is the (a) smaller and (b) larger radius?

48 **E** An object is moved along the central axis of a thin lens while the lateral magnification m is measured. Figure 34.19 gives m versus object distance p out to $p_s = 8.0$ cm. What is the magnification of the object when the object is 14.0 cm from the lens?

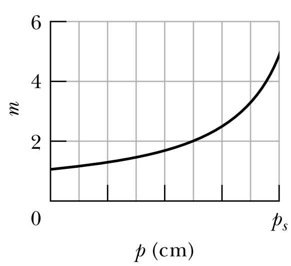


Figure 34.19 Problem 48.

49 **E** **SSM** An illuminated slide is held 44 cm from a screen. How far from the slide must a lens of focal length 11 cm be placed (between the slide and the screen) to form an image of the slideâ€™s picture on the screen?

50 through 57 **M** **GO** 55, 57 **SSM** 53 *Thin lenses.* Object O stands on the central axis of a thin symmetric lens. For this situation, each problem in Table 34.4 gives object distance p (centimeters), the type of lens (C stands for converging and D for diverging), and then the distance (centimeters, without proper sign) between a focal point and the lens. Find (a) the image distance i and (b) the lateral magnification m of the object, including signs. Also, determine whether the image is (c) real (R) or virtual (V), (d) inverted (I) from object O or noninverted (NI), and (e) on the *same* side of the lens as object O or on the *opposite* side.

58 through 67 **M** **GO** 61 **SSM** 59 *Lenses with given radii.* Object O stands in front of a thin lens, on the central axis. For this situation, each problem in Table 34.5 gives object distance p, index of refraction n of the lens, radius r_1 of the nearer lens surface, and radius r_2 of the farther lens surface. (All distances are in centimeters.) Find (a) the image distance i and (b) the lateral magnification m of the object, including signs. Also, determine whether the image is (c) real (R) or virtual (V), (d) inverted (I) from object O or noninverted (NI), and (e) on the *same* side of the lens as object O or on the *opposite* side.

68 **M** In Fig. 34.20, a real inverted image I of an object O is formed by a particular lens (not shown); the objectâ€“image separation is $d = 40.0$ cm, measured along the central axis of the lens. The image is just half the size of the object. (a) What kind of lens must be used to produce this image? (b) How far from the object must the lens be placed? (c) What is the focal length of the lens?

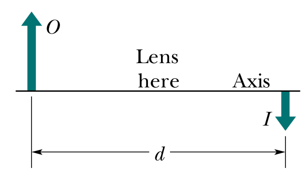


Figure 34.20 Problem 68.

69 through 79 **M** **GO** 76, 78 **SSM** 75, 77 *More lenses.* Object O stands on the central axis of a thin symmetric lens. For this situation, each problem in Table 34.6 refers to (a) the lens type, converging (C) or diverging (D), (b) the focal distance f, (c) the object distance p, (d) the image distance i, and (e) the lateral magnification m. (All distances are in centimeters.) It also refers to whether (f) the image is real (R) or virtual (V), (g) inverted (I) or noninverted (NI) from O, and (h) on the *same* side of the lens as O or on the *opposite* side. Fill in the missing information, including the value of m when only an inequality is given. Where only a sign is missing, answer with the sign.

Table 34.4 **Problems 50 through 57: Thin Lenses. See the setup for these problems.**

	p	Lens	(a) i	(b) m	(c) R/V	(d) I/NI	(e) Side
50	+16	C, 4.0					
51	+12	C, 16					
52	+25	C, 35					
53	+8.0	D, 12					
54	+10	D, 6.0					
55	+22	D, 14					
56	+12	D, 31					
57	+45	C, 20					

Table 34.5 **Problems 58 through 67: Lenses with Given Radii. See the setup for these problems.**

	p	n	r_1	r_2	(a) i	(b) m	(c) R/V	(d) I/NI	(e) Side
58	+29	1.65	+35	â^ž					
59	+75	1.55	+30	â€“42					
60	+6.0	1.70	+10	â€“12					
61	+24	1.50	â€“15	â€“25					
62	+10	1.50	+30	â€“30					
63	+35	1.70	+42	+33					
64	+10	1.50	â€“30	â€“60					
65	+10	1.50	â€“30	+30					
66	+18	1.60	â€“27	+24					
67	+60	1.50	+35	â€“35					

Table 34.6 **Problems 69 through 79: More Lenses. See the setup for these problems.**

	(a) Type	(b) f	(c) p	(d) i	(e) m	(f) R/V	(g) I/NI	(h) Side
69		+10	+5.0					
70		20	+8.0		<1.0		NI	
71			+16		+0.25			
72			+16		â€“0.25			
73			+10		â€“0.50			
74	C	10	+20					
75		10	+5.0		<1.0			Same
76		10	+5.0		>1.0			
77			+16		+1.25			
78			+10		0.50		NI	
79		20	+8.0		>1.0			

80 through 87 M GO 80, 87 SSM 83 *Two-lens systems.* In Fig. 34.21, stick figure O (the object) stands on the common central axis of two thin, symmetric lenses, which are mounted in the boxed regions. Lens 1 is mounted within the boxed region closer to O, which is at object distance p_1. Lens 2 is mounted within the farther boxed region, at distance d. Each problem in Table 34.7 refers to a different combination of lenses and different values for distances, which are given in centimeters. The type of lens is indicated by C for converging and D for diverging; the number after C or D is the distance between a lens and either of its focal points (the proper sign of the focal distance is not indicated).

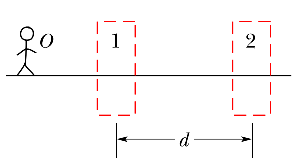


Figure 34.21 Problems 80 through 87.

Find (a) the image distance i_2 for the image produced by lens 2 (the final image produced by the system) and (b) the overall lateral magnification M for the system, including signs. Also, determine whether the final image is (c) real (R) or virtual (V), (d) inverted (I) from object O or noninverted (NI), and (e) on the *same* side of lens 2 as object O or on the *opposite* side.

Module 34.5 Optical Instruments

88 E If the angular magnification of an astronomical telescope is 36 and the diameter of the objective is 75 mm, what is the minimum diameter of the eyepiece required to collect all the light entering the objective from a distant point source on the telescope axis?

89 E SSM In a microscope of the type shown in Fig. 34.5.2, the focal length of the objective is 4.00 cm, and that of the eyepiece is 8.00 cm. The distance between the lenses is 25.0 cm. (a) What is the tube length s? (b) If image I in Fig. 34.5.2 is to be

Table 34.7 Problems 80 through 87: Two-Lens Systems. See the setup for these problems.

	p_1	Lens 1	d	Lens 2	(a) i_2	(b) M	(c) R/V	(d) I/NI	(e) Side
80	+10	C, 15	10	C, 8.0					
81	+12	C, 8.0	32	C, 6.0					
82	+8.0	D, 6.0	12	C, 6.0					
83	+20	C, 9.0	8.0	C, 5.0					
84	+15	C, 12	67	C, 10					
85	+4.0	C, 6.0	8.0	D, 6.0					
86	+12	C, 8.0	30	D, 8.0					
87	+20	D, 12	10	D, 8.0					

just inside focal point F_1â€?how far from the objective should the object be? What then are (c) the lateral magnification m of the objective, (d) the angular magnification m_{θ} of the eyepiece, and (e) the overall magnification M of the microscope?

90 M Figure 34.22*a* shows the basic structure of an old film camera. A lens can be moved forward or back to produce an image on film at the back of the camera. For a certain camera, with the distance i between the lens and the film set at $f = 5.0$ cm, parallel light rays from a very distant object O converge to a point image on the film, as shown. The object is now brought closer, to a distance of $p = 100$ cm, and the lensâ€“film distance is adjusted so that an inverted real image forms on the film (Fig. 34.22*b*). (a) What is the lensâ€“film distance i now? (b) By how much was distance i changed?

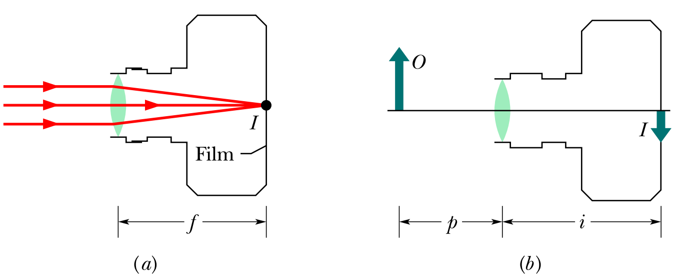


Figure 34.22 Problem 90.

91 M BIO SSM Figure 34.23*a* shows the basic structure of a human eye. Light refracts into the eye through the cornea and is then further redirected by a lens whose shape (and thus ability to focus the light) is controlled by muscles. We can treat the cornea and eye lens as a single effective thin lens (Fig. 34.23*b*). A â€œnormalâ€ eye can focus parallel light rays from a distant object O to a point on the retina at the back of the eye, where processing of the visual information begins. As an object is brought close to the eye, however, the muscles must change the shape of the lens so that rays form an inverted real image on the retina (Fig. 34.23*c*). (a) Suppose that for the parallel rays of Figs. 34.23*a* and *b*, the focal length f of the effective thin lens of the eye is 2.50 cm. For an object at distance $p = 40.0$ cm, what focal length f' of the effective lens is required for the object to be seen clearly? (b) Must the eye muscles increase or decrease the radii of curvature of the eye lens to give focal length f'?

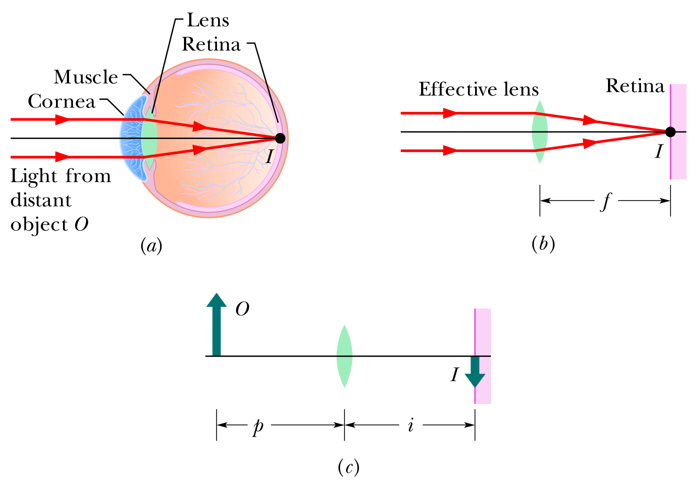


Figure 34.23 Problem 91.

92 M An object is 10.0 mm from the objective of a certain compound microscope. The lenses are 300 mm apart, and the intermediate image is 50.0 mm from the eyepiece. What overall magnification is produced by the instrument?

93 M BIO Someone with a near point P_n of 25 cm views a thimble through a simple magnifying lens of focal length 10 cm by placing the lens near his eye. What is the angular magnification of the thimble if it is positioned so that its image appears at (a) P_n and (b) infinity?

Additional Problems

94 An object is placed against the center of a spherical mirror and then moved 70 cm from it along the central axis as the image distance i is measured. Figure 34.24 gives i versus object distance p out to $p_s = 40$ cm. What is the image distance when the object is 70 cm from the mirror?

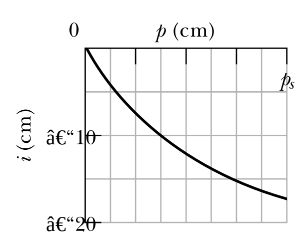


Figure 34.24 Problem 94.

95 through 100 GO 95, 96, 99 *Three-lens systems.* In Fig. 34.25, stick figure O (the object) stands on the common central axis of three thin, symmetric lenses, which are mounted in the boxed regions. Lens 1 is mounted within the boxed region closest to O, which is at object distance p_1. Lens 2 is mounted within the middle boxed region, at distance d_{12} from lens 1. Lens 3 is mounted in the farthest boxed region, at distance d_{23} from lens 2. Each problem in Table 34.8 refers to a different combination of lenses and different values for distances, which are given in centimeters. The type of lens is indicated by C for converging and D for diverging; the number after C or D is the distance between a lens and either of the focal points (the proper sign of the focal distance is not indicated).

Find (a) the image distance i_3 for the (final) image produced by lens 3 (the final image produced by the system) and (b) the overall lateral magnification M for the system, including signs. Also, determine whether the final image is (c) real (R) or virtual (V), (d) inverted (I) from object O or noninverted (NI), and (e) on the same side of lens 3 as object O or on the opposite side.

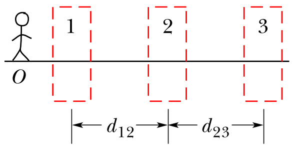


Figure 34.25 Problems 95 through 100.

101 SSM The formula $1/p + 1/i = 1/f$ is called the *Gaussian* form of the thin-lens formula. Another form of this formula, the *Newtonian* form, is obtained by considering the distance x from the object to the first focal point and the distance xâ€² from the second focal point to the image. Show that xxâ€² = f^2 is the Newtonian form of the thin-lens formula.

102 Figure 34.26*a* is an overhead view of two vertical plane mirrors with an object O placed between them. If you look into the mirrors, you see multiple images of O. You can find them by drawing the reflection in each mirror of the angular region between the mirrors, as is done in Fig. 34.26*b* for the left-hand mirror. Then draw the reflection of the reflection. Continue this on the left and on the right until the reflections meet or overlap at the rear of the mirrors. Then you can count the number of images of O. How many images of O would you see if Î¸ is (a) 90Â°, (b) 45Â°, and (c) 60Â°? If Î¸ = 120Â°, determine the (d) smallest and (e) largest number of images that can be seen, depending on your perspective and the location of O. (f) In each situation, draw the image locations and orientations as in Fig. 34.26*b*.

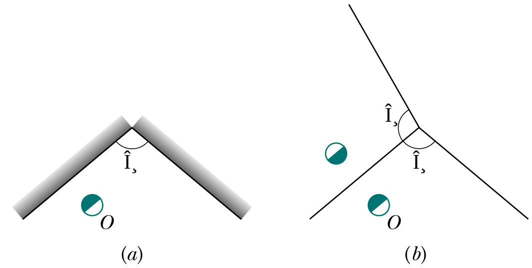


Figure 34.26 Problem 102.

103 SSM Two thin lenses of focal lengths f_1 and f_2 are in contact and share the same central axis. Show that, in image formation, they are equivalent to a single thin lens for which the focal length is $f = f_1f_2/(f_1 + f_2)$.

104 Two plane mirrors are placed parallel to each other and 40 cm apart. An object is placed 10 cm from one mirror. Determine the (a) smallest, (b) second smallest, (c) third smallest (occurs twice), and (d) fourth smallest distance between the object and image of the object.

105 In Fig. 34.27, a box is somewhere at the left, on the central axis of the thin converging lens. The image I_m of the box produced by the plane mirror is 4.00 cm â€œinsideâ€ the mirror. The lensâ€“mirror separation is 10.0 cm, and the focal length of the lens is 2.00 cm. (a) What is the distance between the box and the lens? Light reflected by the mirror travels back through the lens, which produces a final image of the box. (b) What is the distance between the lens and that final image?

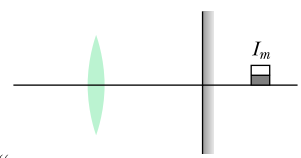


Figure 34.27 Problem 105.

106 In Fig. 34.28, an object is placed in front of a converging lens at a distance equal to twice the focal length f_1 of the lens. On the other side of the lens is a concave mirror of focal length f_2 separated from the lens by a distance $2(f_1 + f_2)$. Light from the object passes rightward through the lens, reflects from

Table 34.8 **Problems 95 through 100: Three-Lens Systems. See the setup for these problems.**

	p_1	Lens 1	d_{12}	Lens 2	d_{23}	Lens 3	(a) i_3	(b) M	(c) R/V	(d) I/NI	(e) Side
95	+12	C, 8.0	28	C, 6.0	8.0	C, 6.0					
96	+4.0	D, 6.0	9.6	C, 6.0	14	C, 4.0					
97	+18	C, 6.0	15	C, 3.0	11	C, 3.0					
98	+2.0	C, 6.0	15	C, 6.0	19	C, 5.0					
99	+8.0	D, 8.0	8.0	D, 16	5.1	C, 8.0					
100	+4.0	C, 6.0	8.0	D, 4.0	5.7	D, 12					

the mirror, passes leftward through the lens, and forms a final image of the object. What are (a) the distance between the lens and that final image and (b) the overall lateral magnification M of the object? Is the image (c) real or virtual (if it is virtual, it requires someone looking through the lens toward the mirror), (d) to the left or right of the lens, and (e) inverted or noninverted relative to the object?

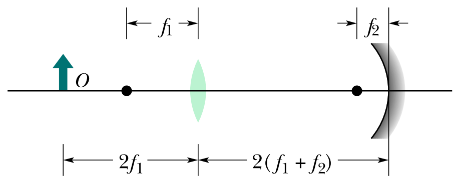


Figure 34.28 Problem 106.

107 SSM A fruit fly of height H sits in front of lens 1 on the central axis through the lens. The lens forms an image of the fly at a distance $d = 20$ cm from the fly; the image has the flyâ€™s orientation and height $H_I = 2.0H$. What are (a) the focal length f_1 of the lens and (b) the object distance p_1 of the fly? The fly then leaves lens 1 and sits in front of lens 2, which also forms an image at $d =$ 20 cm that has the same orientation as the fly, but now $H_I =$ $0.50H$. What are (c) f_2 and (d) p_2?

108 You grind the lenses shown in Fig. 34.29 from flat glass disks ($n = 1.5$) using a machine that can grind a radius of curvature of either 40 cm or 60 cm. In a lens where either radius is appropriate, you select the 40 cm radius. Then you hold each lens in sunshine to form an image of the Sun. What are the (a) focal length f and (b) image type (real or virtual) for (*bi-convex*) lens 1, (c) f and (d) image type for (*plane-convex*) lens 2, (e) f and (f) image type for (*meniscus convex*) lens 3, (g) f and (h) image type for (*bi-concave*) lens 4, (i) f and (j) image type for (*plane-concave*) lens 5, and (k) f and (l) image type for (*meniscus concave*) lens 6?

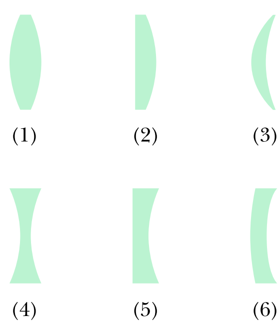


Figure 34.29 Problem 108.

109 In Fig. 34.30, a fish watcher at point P watches a fish through a glass wall of a fish tank. The watcher is level with the fish; the index of refraction of the glass is 8/5, and that of the water is 4/3. The distances are $d_1 = 8.0$ cm, $d_2 = 3.0$ cm, and $d_3 = 6.8$ cm. (a) To the fish, how far away does the watcher appear to be? (*Hint:* The watcher is the object. Light from that object passes through the wallâ€™s outside surface, which acts as a refracting surface. Find the image produced by that surface. Then treat that image as an object whose light passes through the wallâ€™s inside surface, which acts as another refracting surface.) (b) To the watcher, how far away does the fish appear to be?

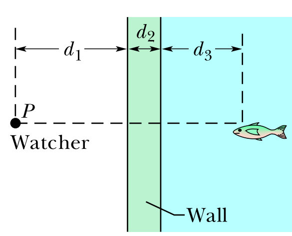


Figure 34.30 Problem 109.

110 A goldfish in a spherical fish bowl of radius R is at the level of the center C of the bowl and at distance $R/2$ from the glass (Fig. 34.31). What magnification of the fish is produced by the water in the bowl for a viewer looking along a line that includes the fish and the center, with the fish on the near side of the center? The index of refraction of the water is 1.33. Neglect the glass wall of the bowl. Assume the viewer looks with one eye. (*Hint:* Equation 34.2.3 holds, but Eq. 34.2.4 does not. You need to work with a ray diagram of the situation and assume that the rays are close to the observerâ€™s line of sightâ€”that is, they deviate from that line by only small angles.)

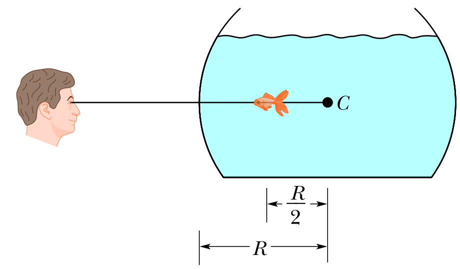


Figure 34.31 Problem 110.

111 Figure 34.32 shows a *beam expander* made with two coaxial converging lenses of focal lengths f_1 and f_2 and separation $d = f_1 + f_2$. The device can expand a laser beam while keeping the light rays in the beam parallel to the central axis through the lenses. Suppose a uniform laser beam of width $W_i = 2.5$ mm and intensity $I_i = 9.0$ kW/m^2 enters a beam expander for which $f_1 = 12.5$ cm and $f_2 = 30.0$ cm. What are (a) W_f and (b) I_f of the beam leaving the expander? (c) What value of d is needed for the beam expander if lens 1 is replaced with a diverging lens of focal length $f_1 =$ â€26.0 cm?

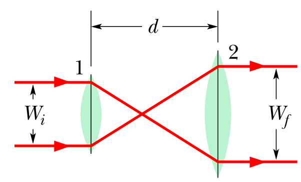


Figure 34.32 Problem 111.

112 You look down at a coin that lies at the bottom of a pool of liquid of depth d and index of refraction n (Fig. 34.33). Because you view with two eyes, which intercept different rays of light from the coin, you perceive the coin to be where extensions of the intercepted rays cross, at depth d_a instead of d. Assuming that the intercepted rays in Fig. 34.33 are close to a vertical axis through the coin, show that $d_a = d/n$. (*Hint:* Use the small-angle approximation sin Î¸ â‰ˆ tan Î¸ â‰ˆ Î¸.)

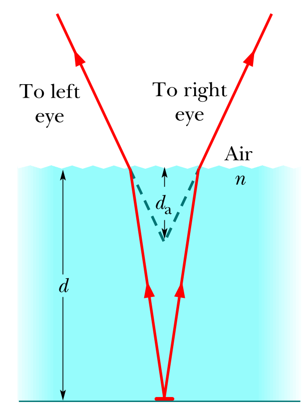


Figure 34.33 Problem 112.

113 A pinhole camera has the hole a distance 12 cm from the film plane, which is a rectangle of height 8.0 cm and width 6.0 cm. How far from a painting of dimensions 50 cm by 50 cm should the camera be placed so as to get the largest complete image possible on the film plane?

114 Light travels from point A to point B via reflection at point O on the surface of a mirror. Without using calculus, show that length AOB is a minimum when the angle of incidence θ is equal to the angle of reflection ϕ. (*Hint:* Consider the image of A in the mirror.)

115 A point object is 10 cm away from a plane mirror, and the eye of an observer (with pupil diameter 5.0 mm) is 20 cm away. Assuming the eye and the object to be on the same line perpendicular to the mirror surface, find the area of the mirror used in observing the reflection of the point. (*Hint:* Adapt Fig. 34.1.4.)

116 Show that the distance between an object and its real image formed by a thin converging lens is always greater than or equal to four times the focal length of the lens.

117 **CALC** A luminous object and a screen are a fixed distance D apart. (a) Show that a converging lens of focal length f, placed between object and screen, will form a real image on the screen for two lens positions that are separated by a distance $d = \sqrt{D(D - 4f)}$. (b) Show that

$$\left(\frac{D - d}{D + d}\right)^2$$

gives the ratio of the two image sizes for these two positions of the lens.

118 An eraser of height 1.0 cm is placed 10.0 cm in front of a two-lens system. Lens 1 (nearer the eraser) has focal length $f_1 = -5$ cm, lens 2 has $f_2 = 12$ cm, and the lens separation is $d = 12$ cm. For the image produced by lens 2, what are (a) the image distance i_2 (including sign), (b) the image height, (c) the image type (real or virtual), and (d) the image orientation (inverted relative to the eraser or not inverted)?

119 A peanut is placed 40 cm in front of a two-lens system: lens 1 (nearer the peanut) has focal length $f_1 = +20$ cm, lens 2 has $f_2 = -5$ cm, and the lens separation is $d = 10$ cm. For the image produced by lens 2, what are (a) the image distance i_2 (including sign), (b) the image orientation (inverted relative to the peanut or not inverted), and (c) the image type (real or virtual)? (d) What is the net lateral magnification?

120 A coin is placed 20 cm in front of a two-lens system. Lens 1 (nearer the coin) has focal length $f_1 = +10$ cm, lens 2 has $f_2 = +12.5$ cm, and the lens separation is $d = 30$ cm. For the image produced by lens 2, what are (a) the image distance i_2 (including sign), (b) the overall lateral magnification, (c) the image type (real or virtual), and (d) the image orientation (inverted relative to the coin or not inverted)?

121 An object is 20 cm to the left of a thin diverging lens that has a 30 cm focal length. (a) What is the image distance i? (b) Draw a ray diagram showing the image position.

122 *Mirror length.* If a basketball player is 206 cm tall, how tall must the mirror be if the player is to see that entire length?