Coulomb's Law

21.1 COULOMB'S LAW

Learning Objectives

After reading this module, you should be able to . . .

21.1.1 Distinguish between being electrically neutral, negatively charged, and positively charged and identify excess charge.

21.1.2 Distinguish between conductors, nonconductors (insulators), semiconductors, and superconductors.

21.1.3 Describe the electrical properties of the particles inside an atom.

21.1.4 Identify conduction electrons and explain their role in making a conducting object negatively or positively charged.

21.1.5 Identify what is meant by "electrically isolated" and by "grounding."

21.1.6 Explain how a charged object can set up induced charge in a second object.

21.1.7 Identify that charges with the same electrical sign repel each other and those with opposite electrical signs attract each other.

21.1.8 For either of the particles in a pair of charged particles, draw a free-body diagram, showing the electrostatic force (Coulomb force) on it and anchoring the tail of the force vector on that particle.

21.1.9 For either of the particles in a pair of charged particles, apply Coulomb's law to relate the magnitude of the electrostatic force, the charge magnitudes of the particles, and the separation between the particles.

21.1.10 Identify that Coulomb's law applies only to (point-like) particles and objects that can be treated as particles.

21.1.11 If more than one force acts on a particle, find the net force by adding all the forces as vectors, not scalars.

21.1.12 Identify that a shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated as a particle at the shell's center.

21.1.13 Identify that if a charged particle is located inside a shell of uniform charge, there is no net electrostatic force on the particle from the shell.

21.1.14 Identify that if excess charge is put on a spherical conductor, it spreads out uniformly over the external surface area.

21.1.15 Identify that if two identical spherical conductors touch or are connected by conducting wire, any excess charge will be shared equally.

21.1.16 Identify that a nonconducting object can have any given distribution of charge, including charge at interior points.

21.1.17 Identify current as the rate at which charge moves through a point.

21.1.18 For current through a point, apply the relationship between the current, a time interval, and the amount of charge that moves through the point in that time interval.

Key Ideas

● The strength of a particle's electrical interaction with objects around it depends on its electric charge (usually represented as q), which can be either positive or negative. Particles with the same sign of charge repel each other, and particles with opposite signs of charge attract each other.

● An object with equal amounts of the two kinds of charge is electrically neutral, whereas one with an imbalance is electrically charged and has an excess charge.

● Conductors are materials in which a significant number of electrons are free to move. The charged particles in nonconductors (insulators) are not free to move.

● Electric current i is the rate dq/dt at which charge passes a point:

$$i = \frac{dq}{dt}.$$

● Coulomb's law describes the electrostatic force (or electric force) between two charged particles. If the

particles have charges q_1 and q_2, are separated by distance r, and are at rest (or moving only slowly) relative to each other, then the magnitude of the force acting on each due to the other is given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2} \quad \text{(Coulomb's law)},$$

where $\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$ is the permittivity constant. The ratio $1/4\pi\varepsilon_0$ is often replaced with the electrostatic constant (or Coulomb constant) $k = 8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$.

● The electrostatic force vector acting on a charged particle due to a second charged particle is either directly toward the second particle (opposite signs of charge) or directly away from it (same sign of charge).

● If multiple electrostatic forces act on a particle, the net force is the vector sum (not scalar sum) of the individual forces.

● Shell theorem 1: A charged particle outside a shell with charge uniformly distributed on its surface is attracted or repelled as if the shell's charge were concentrated as a particle at its center.

● Shell theorem 2: A charged particle inside a shell with charge uniformly distributed on its surface has no net force acting on it due to the shell.

● Charge on a conducting spherical shell spreads uniformly over the (external) surface.

What Is Physics?

You are surrounded by devices that depend on the physics of electromagnetism, which is the combination of electric and magnetic phenomena. This physics is at the root of computers, television, radio, telecommunications, household lighting, and even the ability of food wrap to cling to a container. This physics is also the basis of the natural world. Not only does it hold together all the atoms and molecules in the world, it also produces lightning, auroras, and rainbows.

The physics of electromagnetism was first studied by the early Greek philosophers, who discovered that if a piece of amber is rubbed and then brought near bits of straw, the straw will jump to the amber. We now know that the attraction between amber and straw is due to an electric force. The Greek philosophers also discovered that if a certain type of stone (a naturally occurring magnet) is brought near bits of iron, the iron will jump to the stone. We now know that the attraction between magnet and iron is due to a magnetic force.

From these modest origins with the Greek philosophers, the sciences of electricity and magnetism developed separately for centuries—until 1820, in fact, when Hans Christian Oersted found a connection between them: An electric current in a wire can deflect a magnetic compass needle. Interestingly enough, Oersted made this discovery, a big surprise, while preparing a lecture demonstration for his physics students.

The new science of electromagnetism was developed further by workers in many countries. One of the best was Michael Faraday, a truly gifted experimenter with a talent for physical intuition and visualization. That talent is attested to by the fact that his collected laboratory notebooks do not contain a single equation. In the mid-nineteenth century, James Clerk Maxwell put Faraday's ideas into mathematical form, introduced many new ideas of his own, and put electromagnetism on a sound theoretical basis.

Our discussion of electromagnetism is spread through the next 16 chapters. We begin with electrical phenomena, and our first step is to discuss the nature of electric charge and electric force.
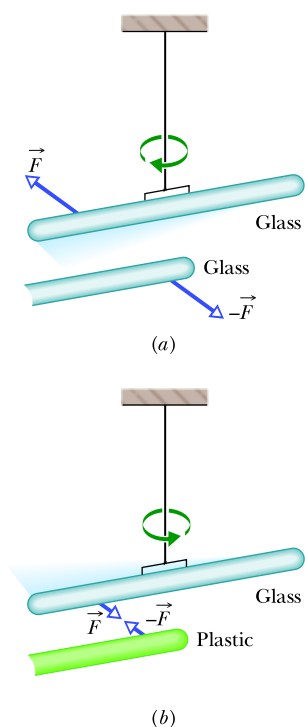
Figure 21.1.1 (*a*) The two glass rods were each rubbed with a silk cloth and one was suspended by thread. When they are close to each other, they repel each other. (*b*) The plastic rod was rubbed with fur. When brought close to the glass rod, the rods attract each other.

Electric Charge

Here are two demonstrations that seem to be magic, but our job here is to make sense of them. After rubbing a glass rod with a silk cloth (on a day when the humidity is low), we hang the rod by means of a thread tied around its center (Fig. 21.1.1*a*). Then we rub a second glass rod with the silk cloth and bring it near

the hanging rod. The hanging rod magically moves away. We can see that a force repels it from the second rod, but how? There is no contact with that rod, no breeze to push on it, and no sound wave to disturb it.

In the second demonstration we replace the second rod with a plastic rod that has been rubbed with fur. This time, the hanging rod moves toward the nearby rod (Fig. 21.1.1*b*). Like the repulsion, this attraction occurs without any contact or obvious communication between the rods.

In the next chapter we shall discuss how the hanging rod knows of the presence of the other rods, but in this chapter let's focus on just the forces that are involved. In the first demonstration, the force on the hanging rod was *repulsive*, and in the second, *attractive*. After a great many investigations, scientists figured out that the forces in these types of demonstrations are due to the *electric charge* that we set up on the rods when they are in contact with silk or fur. Electric charge is an intrinsic property of the fundamental particles that make up objects such as the rods, silk, and fur. That is, charge is a property that comes automatically with those particles wherever they exist.

Two Types. There are two types of electric charge, named by the American scientist and statesman Benjamin Franklin as positive charge and negative charge. He could have called them anything (such as cherry and walnut), but using algebraic signs as names comes in handy when we add up charges to find the net charge. In most everyday objects, such as a mug, there are about equal numbers of negatively charged particles and positively charged particles, and so the net charge is zero, the charge is said to be *balanced*, and the object is said to be *electrically neutral* (or just *neutral* for short).

Excess Charge. Normally you are approximately neutral. However, if you live in regions where the humidity is low, you know that the charge on your body can become slightly unbalanced when you walk across certain carpets. Either you gain negative charge from the carpet (at the points of contact between your shoes with the carpet) and become negatively charged, or you lose negative charge and become positively charged. Either way, the extra charge is said to be an *excess charge*. You probably don't notice it until you reach for a door handle or another person. Then, if your excess charge is enough, a spark leaps between you and the other object, eliminating your excess charge. Such sparking can be annoying and even somewhat painful. Such *charging* and *discharging* do not happen in humid conditions because the water in the air *neutralizes* your excess charge about as fast as you acquire it.

Two of the grand mysteries in physics are (1) *why* does the universe have particles with electric charge (what is it, really?) and (2) *why* does electric charge come in two types (and not, say, one type or three types). We are still working on the answers. Nevertheless, with lots of experiments similar to our two demonstrations scientists discovered that

> Particles with the same sign of electrical charge repel each other, and particles with opposite signs attract each other.

In a moment we shall put this rule into quantitative form as Coulomb's law of *electrostatic force* (or *electric force*) between charged particles. The term *electrostatic* is used to emphasize that, relative to each other, the charges are either stationary or moving only very slowly.

Demos. Now let's get back to the demonstrations to understand the motions of the rod as being something other than just magic. When we rub the glass rod with a silk cloth, a small amount of negative charge moves from the rod to the silk (a transfer like that between you and a carpet), leaving the rod with a small amount of excess positive charge. (Which way the negative charge moves is not obvious and requires a lot of experimentation.) We *rub* the silk over the rod to increase the number of contact points and thus the amount, still tiny, of transferred charge. We hang the rod from the thread so as to *electrically isolate* it from

(a)

(b)
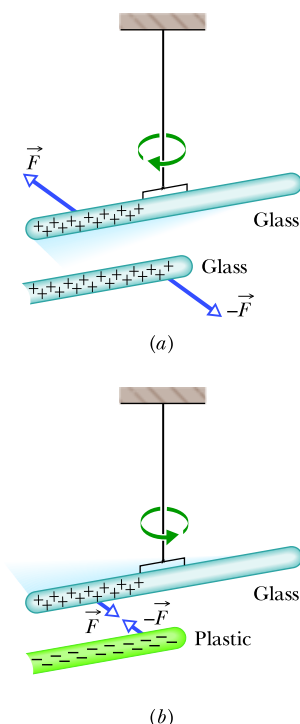
Figure 21.1.2 (a) Two charged rods of the same sign repel each other. (b) Two charged rods of opposite signs attract each other. Plus signs indicate a positive net charge, and minus signs indicate a negative net charge.
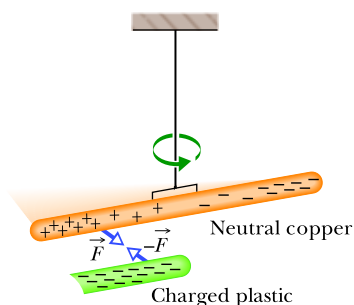
Figure 21.1.3 A neutral copper rod is electrically isolated from its surroundings by being suspended on a nonconducting thread. Either end of the copper rod will be attracted by a charged rod. Here, conduction electrons in the copper rod are repelled to the far end of that rod by the negative charge on the plastic rod. Then that negative charge attracts the remaining positive charge on the near end of the copper rod, rotating the copper rod to bring that near end closer to the plastic rod.

its surroundings (so that the surroundings cannot neutralize the rod by giving it enough negative charge to rebalance its charge). When we rub the second rod with the silk cloth, it too becomes positively charged. So when we bring it near the first rod, the two rods repel each other (Fig. 21.1.2a).

Next, when we rub the plastic rod with fur, it gains excess negative charge from the fur. (Again, the transfer direction is learned through many experiments.) When we bring the plastic rod (with negative charge) near the hanging glass rod (with positive charge), the rods are attracted to each other (Fig. 21.1.2b). All this is subtle. You cannot see the charge or its transfer, only the results.

Conductors and Insulators

We can classify materials generally according to the ability of charge to move through them. **Conductors** are materials through which charge can move rather freely; examples include metals (such as copper in common lamp wire), the human body, and tap water. **Nonconductors**—also called **insulators**—are materials through which charge cannot move freely; examples include rubber (such as the insulation on common lamp wire), plastic, glass, and chemically pure water. **Semiconductors** are materials that are intermediate between conductors and insulators; examples include silicon and germanium in computer chips. **Superconductors** are materials that are *perfect* conductors, allowing charge to move without *any* hindrance. In these chapters we discuss only conductors and insulators.

Conducting Path. Here is an example of how conduction can eliminate excess charge on an object. If you rub a copper rod with wool, charge is transferred from the wool to the rod. However, if you are holding the rod while also touching a faucet connected to metal piping, you cannot charge the rod in spite of the transfer. The reason is that you, the rod, and the faucet are all conductors connected, via the plumbing, to Earth's surface, which is a huge conductor. Because the excess charges put on the rod by the wool repel one another, they move away from one another by moving first through the rod, then through you, and then through the faucet and plumbing to reach Earth's surface, where they can spread out. The process leaves the rod electrically neutral.

In thus setting up a pathway of conductors between an object and Earth's surface, we are said to *ground* the object, and in neutralizing the object (by eliminating an unbalanced positive or negative charge), we are said to *discharge* the object. If instead of holding the copper rod in your hand, you hold it by an insulating handle, you eliminate the conducting path to Earth, and the rod can then be charged by rubbing (the charge remains on the rod), as long as you do not touch it directly with your hand.

Charged Particles. The properties of conductors and insulators are due to the structure and electrical nature of atoms. Atoms consist of positively charged *protons*, negatively charged *electrons*, and electrically neutral *neutrons*. The protons and neutrons are packed tightly together in a central *nucleus*.

The charge of a single electron and that of a single proton have the same magnitude but are opposite in sign. Hence, an electrically neutral atom contains equal numbers of electrons and protons. Electrons are held near the nucleus because they have the electrical sign opposite that of the protons in the nucleus and thus are attracted to the nucleus. Were this not true, there would be no atoms and thus no you.

When atoms of a conductor like copper come together to form the solid, some of their outermost (and so most loosely held) electrons become free to wander about within the solid, leaving behind positively charged atoms (*positive ions*). We call the mobile electrons *conduction electrons*. There are few (if any) free electrons in a nonconductor.

Induced Charge. The experiment of Fig. 21.1.3 demonstrates the mobility of charge in a conductor. A negatively charged plastic rod will attract either end

of an isolated neutral copper rod. What happens is that many of the conduction electrons in the closer end of the copper rod are repelled by the negative charge on the plastic rod. Some of the conduction electrons move to the far end of the copper rod, leaving the near end depleted in electrons and thus with an unbalanced positive charge. This positive charge is attracted to the negative charge in the plastic rod. Although the copper rod is still neutral, it is said to have an *induced charge*, which means that some of its positive and negative charges have been separated due to the presence of a nearby charge.

Similarly, if a positively charged glass rod is brought near one end of a neutral copper rod, induced charge is again set up in the neutral copper rod but now the near end gains conduction electrons, becomes negatively charged, and is attracted to the glass rod, while the far end is positively charged.

Note that only conduction electrons, with their negative charges, can move; positive ions are fixed in place. Thus, an object becomes positively charged only through the *removal of negative charges*.

Blue Flashes from a Wintergreen Candy

Indirect evidence for the attraction of charges with opposite signs can be seen with a wintergreen LifeSaver (the candy shaped in the form of a marine life-saver). If you adapt your eyes to darkness for about 15 minutes and then have a friend chomp on a piece of the candy in the darkness, you will see a faint blue flash from your friend's mouth with each chomp. Whenever a chomp breaks a sugar crystal into pieces, each piece will probably end up with a different number of electrons. Suppose a crystal breaks into pieces A and B, with A ending up with more electrons on its surface than B (Fig. 21.1.4). This means that B has positive ions (atoms that lost electrons to A) on its surface. Because the electrons on A are strongly attracted to the positive ions on B, some of those electrons jump across the gap between the pieces.
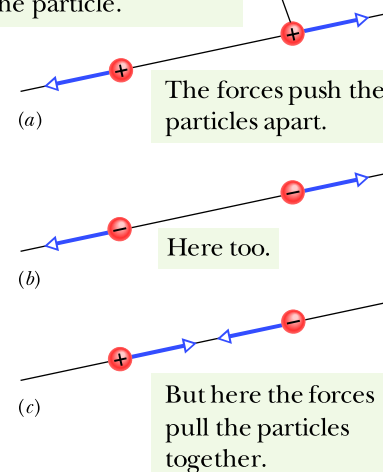
As A and B move away from each other, air (primarily nitrogen, N_2) flows into the gap, and many of the jumping electrons collide with nitrogen molecules in the air, causing the molecules to emit ultraviolet light. You cannot see this type of light. However, the wintergreen molecules on the surfaces of the candy pieces absorb the ultraviolet light and then emit blue light, which you *can* see—it is the blue light coming from your friend's mouth. FCP

Coulomb's Law

Now we come to the equation for Coulomb's law, but first a caution. This equation works for only charged particles (and a few other things that can be treated as particles). For extended objects, with charge located in many different places, we need more powerful techniques. So, here we consider just charged particles and not, say, two charged cats.

If two charged particles are brought near each other, they each exert an **electrostatic force** on the other. The direction of the force vectors depends on the signs of the charges. If the particles have the same sign of charge, they repel each other. That means that the force vector on each is directly away from the other particle (Figs. 21.1.5a and b). If we release the particles, they accelerate away from each other. If, instead, the particles have opposite signs of charge, they attract each other. That means that the force vector on each is directly toward the other particle (Fig. 21.1.5c). If we release the particles, they accelerate toward each other.

The equation for the electrostatic forces acting on the particles is called **Coulomb's law** after Charles-Augustin de Coulomb, whose experiments in 1785 led him to it. Let's write the equation in vector form and in terms of the particles shown in Fig. 21.1.6, where particle 1 has charge q_1 and particle 2 has charge q_2.

Figure 21.1.4 Two pieces of a wintergreen candy as they fall away from each other. Electrons jumping from the negative surface of piece A to the positive surface of piece B collide with nitrogen (N_2) molecules in the air.

Always draw the force vector with the tail on the particle.

(a) The forces push the particles apart.

(b) Here too.

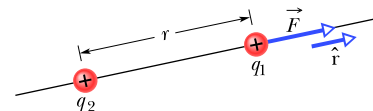(c) But here the forces pull the particles together.

Figure 21.1.5 Two charged particles repel each other if they have the same sign of charge, either (a) both positive or (b) both negative. (c) They attract each other if they have opposite signs of charge.

Figure 21.1.6 The electrostatic force on particle 1 can be described in terms of a unit vector $\hat{r}$ along an axis through the two particles, radially away from particle 2.

(These symbols can represent either positive or negative charge.) Let's also focus on particle 1 and write the force acting on it in terms of a unit vector $\hat{r}$ that points along a radial axis extending through the two particles, radially away from particle 2. (As with other unit vectors, $\hat{r}$ has a magnitude of exactly 1 and no unit; its purpose is to point, like a direction arrow on a street sign.) With these decisions, we write the electrostatic force as

$$\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r} \quad \text{(Coulomb's law)}, \tag{21.1.1}$$

where r is the separation between the particles and k is a positive constant called the *electrostatic constant* or the *Coulomb constant*. (We'll discuss k below.)

Let's first check the direction of the force on particle 1 as given by Eq. 21.1.1. If q_1 and q_2 have the same sign, then the product $q_1 q_2$ gives us a positive result. So, Eq. 21.1.1 tells us that the force on particle 1 is in the direction of $\hat{r}$. That checks, because particle 1 is being repelled from particle 2. Next, if q_1 and q_2 have opposite signs, the product $q_1 q_2$ gives us a negative result. So, now Eq. 21.1.1 tells us that the force on particle 1 is in the direction opposite $\hat{r}$. That checks because particle 1 is being attracted toward particle 2.

Checkpoint 21.1.1

The figure shows five pairs of plates: A, B, and D are charged plastic plates and C is an electrically neutral copper plate. The electrostatic forces between the pairs of plates are shown for three of the pairs. For the remaining two pairs, do the plates repel or attract each other?
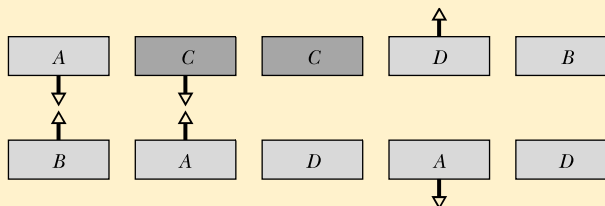
An Aside. Here is something that is very curious. The form of Eq. 21.1.1 is the same as that of Newton's equation (Eq. 13.1.3) for the gravitational force between two particles with masses m_1 and m_2 and separation r:

$$\vec{F} = G\frac{m_1 m_2}{r^2}\hat{r} \quad \text{(Newton's law)}, \tag{21.1.2}$$

where G is the gravitational constant. Although the two types of forces are wildly different, both equations describe inverse square laws (the $1/r^2$ dependences) that involve a product of a property of the interacting particles—the charge in one case and the mass in the other. However, the laws differ in that gravitational forces are always attractive but electrostatic forces may be either attractive or repulsive, depending on the signs of the charges. This difference arises from the fact that there is only one type of mass but two types of charge.

Unit. The SI unit of charge is the **coulomb**. For practical reasons having to do with the accuracy of measurements, the coulomb unit is derived from the SI unit *ampere* for electric current i. We shall discuss current in detail in Chapter 26, but here let's just note that current i is the rate dq/dt at which charge moves past a point or through a region:

$$i = \frac{dq}{dt} \quad \text{(electric current)}. \tag{21.1.3}$$

Rearranging Eq. 21.1.3 and replacing the symbols with their units (coulombs C, amperes A, and seconds s) we see that

$$1\,\text{C} = (1\,\text{A})(1\,\text{s}).$$

Force Magnitude. For historical reasons (and because doing so simplifies many other formulas), the electrostatic constant k in Eq. 21.1.1 is often written as $1/4\pi\varepsilon_0$. Then the magnitude of the electrostatic force in Coulomb's law becomes

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2} \quad \text{(Coulomb's law)}. \qquad (21.1.4)$$

The constants in Eqs. 21.1.1 and 21.1.4 have the value

$$k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 \,\text{N} \cdot \text{m}^2/\text{C}^2. \qquad (21.1.5)$$

The quantity ε_0, called the **permittivity constant,** sometimes appears separately in equations and is

$$\varepsilon_0 = 8.85 \times 10^{-12} \,\text{C}^2/\text{N} \cdot \text{m}^2. \qquad (21.1.6)$$

Working a Problem. Note that the charge magnitudes appear in Eq. 21.1.4, which gives us the force magnitude. So, in working problems in this chapter, we use Eq. 21.1.4 to find the magnitude of a force on a chosen particle due to a second particle and we separately determine the direction of the force by considering the charge signs of the two particles.

Multiple Forces. As with all forces in this book, the electrostatic force obeys the principle of superposition. Suppose we have n charged particles near a chosen particle called particle 1; then the net force on particle 1 is given by the vector sum

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + \ldots + \vec{F}_{1n}, \qquad (21.1.7)$$

in which, for example, $\vec{F}_{14}$ is the force on particle 1 due to the presence of particle 4.

This equation is the key to many of the homework problems, so let's state it in words. If you want to know the net force acting on a chosen charged particle that is surrounded by other charged particles, first clearly identify that chosen particle and then find the force on it due to each of the other particles. Draw those force vectors in a free-body diagram of the chosen particle, with the tails anchored on the particle. (That may sound trivial, but failing to do so easily leads to errors.) Then add all those forces *as vectors* according to the rules of Chapter 3, not as scalars. (You cannot just willy-nilly add up their magnitudes.) The result is the net force (or resultant force) acting on the particle.

Although the vector nature of the forces makes the homework problems harder than if we simply had scalars, be thankful that Eq. 21.1.7 works. If two force vectors did not simply add but for some reason amplified each other, the world would be very difficult to understand and manage.

Shell Theories. Analogous to the shell theories for the gravitational force (Module 13.1), we have two shell theories for the electrostatic force:

Shell theory 1. A charged particle outside a shell with charge uniformly distributed on its surface is attracted or repelled as if the shell's charge were concentrated as a particle at its center.

Shell theory 2. A charged particle inside a shell with charge uniformly distributed on its surface has no net force acting on it due to the shell.

(In the first theory, we assume that the charge on the shell is much greater than the particle's charge. Thus the presence of the particle has negligible effect on the distribution of charge on the shell.)

Spherical Conductors

If excess charge is placed on a spherical shell that is made of conducting material, the excess charge spreads uniformly over the (external) surface. For example, if we place excess electrons on a spherical metal shell, those electrons repel one another and tend to move apart, spreading over the available surface until they are uniformly distributed. That arrangement maximizes the distances between all pairs of the excess electrons. According to the first shell theorem, the shell then will attract or repel an external charge as if all the excess charge on the shell were concentrated at its center.

If we remove negative charge from a spherical metal shell, the resulting positive charge of the shell is also spread uniformly over the surface of the shell. For example, if we remove n electrons, there are then n sites of positive charge (sites missing an electron) that are spread uniformly over the shell. According to the first shell theorem, the shell will again attract or repel an external charge as if all the shell's excess charge were concentrated at its center.

Checkpoint 21.1.2

The figure shows two protons (symbol p) and one electron (symbol e) on an axis. On the central proton, what is the direction of (a) the force due to the electron, (b) the force due to the other proton, and (c) the net force?
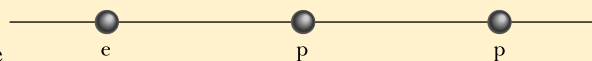
Sample Problem 21.1.1   Finding the net force due to two other particles

This sample problem actually contains three examples, to build from basic stuff to harder stuff. In each we have the same charged particle 1. First there is a single force acting on it (easy stuff). Then there are two forces, but they are just in opposite directions (not too bad). Then there are again two forces but they are in very different directions (ah, now we have to get serious about the fact that they are vectors). The key to all three examples is to draw the forces correctly *before* you reach for a calculator, otherwise you may be calculating nonsense on the calculator. (Figure 21.1.7 is available in *WileyPLUS* as an animation with voiceover.)
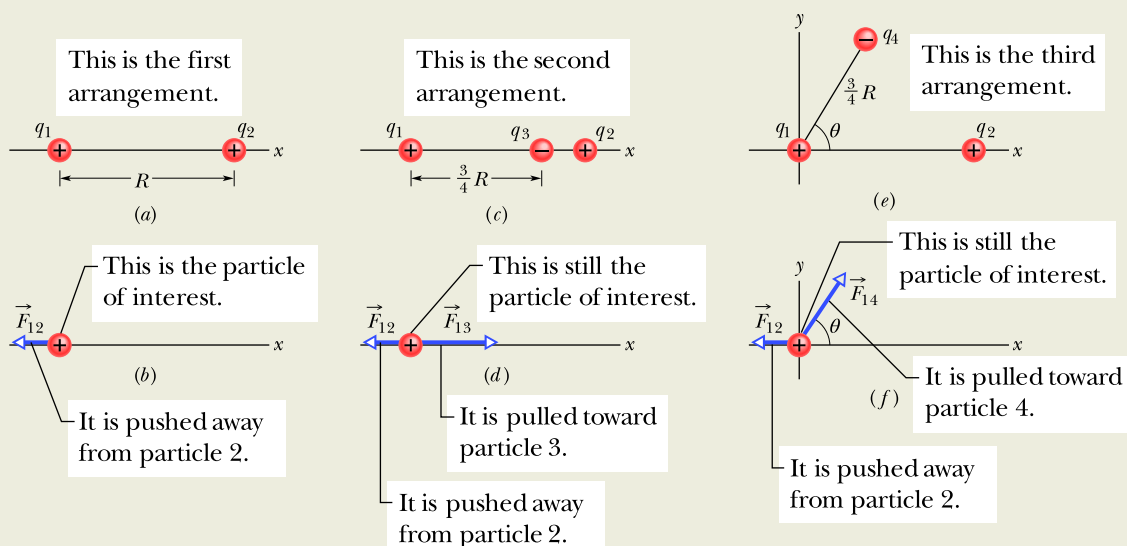
Figure 21.1.7 (*a*) Two charged particles of charges q_1 and q_2 are fixed in place on an *x* axis. (*b*) The free-body diagram for particle 1, showing the electrostatic force on it from particle 2. (*c*) Particle 3 included. (*d*) Free-body diagram for particle 1. (*e*) Particle 4 included. (*f*) Free-body diagram for particle 1.

(a) Figure 21.1.7a shows two positively charged particles fixed in place on an x axis. The charges are $q_1 = 1.60 \times 10^{-19}$ C and $q_2 = 3.20 \times 10^{-19}$ C, and the particle separation is $R = 0.0200$ m. What are the magnitude and direction of the electrostatic force $\vec{F}_{12}$ on particle 1 from particle 2?

KEY IDEAS

Because both particles are positively charged, particle 1 is repelled by particle 2, with a force magnitude given by Eq. 21.1.4. Thus, the direction of force $\vec{F}_{12}$ on particle 1 is *away from* particle 2, in the negative direction of the x axis, as indicated in the free-body diagram of Fig. 21.1.7b.

Two particles: Using Eq. 21.1.4 with separation R substituted for r, we can write the magnitude F_{12} of this force as

$$F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{R^2}$$
$$= (8.99 \times 10^9 \text{ N·m}^2/\text{C}^2)$$
$$\times \frac{(1.60 \times 10^{-19}\text{ C})(3.20 \times 10^{-19}\text{ C})}{(0.0200\text{ m})^2}$$
$$= 1.15 \times 10^{-24} \text{ N}.$$

Thus, force $\vec{F}_{12}$ has the following magnitude and direction (relative to the positive direction of the x axis):

$$1.15 \times 10^{-24} \text{ N} \quad \text{and} \quad 180°. \qquad \text{(Answer)}$$

We can also write $\vec{F}_{12}$ in unit-vector notation as

$$\vec{F}_{12} = -(1.15 \times 10^{-24} \text{ N})\hat{i}. \qquad \text{(Answer)}$$

(b) Figure 21.1.7c is identical to Fig. 21.1.7a except that particle 3 now lies on the x axis between particles 1 and 2. Particle 3 has charge $q_3 = -3.20 \times 10^{-19}$ C and is at a distance $\frac{3}{4}R$ from particle 1. What is the net electrostatic force $\vec{F}_{1,\text{net}}$ on particle 1 due to particles 2 and 3?

KEY IDEA

The presence of particle 3 does not alter the electrostatic force on particle 1 from particle 2. Thus, force $\vec{F}_{12}$ still acts on particle 1. Similarly, the force $\vec{F}_{13}$ that acts on particle 1 due to particle 3 is not affected by the presence of particle 2. Because particles 1 and 3 have charge of opposite signs, particle 1 is attracted to particle 3. Thus, force $\vec{F}_{13}$ is directed *toward* particle 3, as indicated in the free-body diagram of Fig. 21.1.7d.

Three particles: To find the magnitude of $\vec{F}_{13}$, we can rewrite Eq. 21.1.4 as

$$F_{13} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_3|}{(\frac{3}{4}R)^2}$$
$$= (8.99 \times 10^9 \text{ N·m}^2/\text{C}^2)$$
$$\times \frac{(1.60 \times 10^{-19}\text{ C})(3.20 \times 10^{-19}\text{ C})}{(\frac{3}{4})^2(0.0200\text{ m})^2}$$
$$= 2.05 \times 10^{-24} \text{ N}.$$

We can also write $\vec{F}_{13}$ in unit-vector notation:

$$\vec{F}_{13} = (2.05 \times 10^{-24} \text{ N})\hat{i}.$$

The net force $\vec{F}_{1,\text{net}}$ on particle 1 is the vector sum of $\vec{F}_{12}$ and $\vec{F}_{13}$; that is, from Eq. 21.1.7, we can write the net force $\vec{F}_{1,\text{net}}$ on particle 1 in unit-vector notation as

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13}$$
$$= -(1.15 \times 10^{-24} \text{ N})\hat{i} + (2.05 \times 10^{-24} \text{ N})\hat{i}$$
$$= (9.00 \times 10^{-25} \text{ N})\hat{i}. \qquad \text{(Answer)}$$

Thus, $\vec{F}_{1,\text{net}}$ has the following magnitude and direction (relative to the positive direction of the x axis):

$$9.00 \times 10^{-25} \text{ N} \quad \text{and} \quad 0°. \qquad \text{(Answer)}$$

(c) Figure 21.1.7e is identical to Fig. 21.1.7a except that particle 4 is now included. It has charge $q_4 = -3.20 \times 10^{-19}$ C, is at a distance $\frac{3}{4}R$ from particle 1, and lies on a line that makes an angle $\theta = 60°$ with the x axis. What is the net electrostatic force $\vec{F}_{1,\text{net}}$ on particle 1 due to particles 2 and 4?

KEY IDEA

The net force $\vec{F}_{1,\text{net}}$ is the vector sum of $\vec{F}_{12}$ and a new force $\vec{F}_{14}$ acting on particle 1 due to particle 4. Because particles 1 and 4 have charge of opposite signs, particle 1 is attracted to particle 4. Thus, force $\vec{F}_{14}$ on particle 1 is directed *toward* particle 4, at angle $\theta = 60°$, as indicated in the free-body diagram of Fig. 21.1.7f.

Four particles: We can rewrite Eq. 21.1.4 as

$$F_{14} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_4|}{(\frac{3}{4}R)^2}$$
$$= (8.99 \times 10^9 \text{ N·m}^2/\text{C}^2)$$
$$\times \frac{(1.60 \times 10^{-19}\text{ C})(3.20 \times 10^{-19}\text{ C})}{(\frac{3}{4})^2(0.0200\text{ m})^2}$$
$$= 2.05 \times 10^{-24} \text{ N}.$$

Then from Eq. 21.1.7, we can write the net force $\vec{F}_{1,\text{net}}$ on particle 1 as

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{14}.$$

Because the forces $\vec{F}_{12}$ and $\vec{F}_{14}$ are not directed along the same axis, we *cannot* sum simply by combining their magnitudes. Instead, we must add them as vectors, using one of the following methods.

Method 1. *Summing directly on a vector-capable calculator*. For $\vec{F}_{12}$, we enter the magnitude 1.15×10^{-24} and the angle $180°$. For $\vec{F}_{14}$, we enter the magnitude 2.05×10^{-24} and the angle $60°$. Then we add the vectors.

Method 2. *Summing in unit-vector notation*. First we rewrite $\vec{F}_{14}$ as

$$\vec{F}_{14} = (F_{14} \cos\theta)\hat{i} + (F_{14} \sin\theta)\hat{j}.$$

Substituting 2.05×10^{-24} N for F_{14} and $60°$ for θ, this becomes

$$\vec{F}_{14} = (1.025 \times 10^{-24}\,\text{N})\hat{i} + (1.775 \times 10^{-24}\,\text{N})\hat{j}.$$

Then we sum:

$$\vec{F}_{1,\,net} = \vec{F}_{12} + \vec{F}_{14}$$
$$= -(1.15 \times 10^{-24}\,\text{N})\hat{i}$$
$$+ (1.025 \times 10^{-24}\,\text{N})\hat{i} + (1.775 \times 10^{-24}\,\text{N})\hat{j}$$
$$\approx (-1.25 \times 10^{-25}\,\text{N})\hat{i} + (1.78 \times 10^{-24}\,\text{N})\hat{j}.$$

(Answer)

Method 3. *Summing components axis by axis*. The sum of the x components gives us

$$F_{1,netx} = F_{12x} + F_{14x} = F_{12} + F_{14} \cos 60°$$
$$= -1.15 \times 10^{-24}\,\text{N} + (2.05 \times 10^{-24}\,\text{N})(\cos 60°)$$
$$= -1.25 \times 10^{-25}\,\text{N}.$$

The sum of the y components gives us

$$F_{1,nety} = F_{12,y} + F_{14,y} = 0 + F_{14} \sin 60°$$
$$= (2.05 \times 10^{-24}\,\text{N})(\sin 60°)$$
$$= 1.78 \times 10^{-24}\,\text{N}.$$

The net force $\vec{F}_{1,net}$ has the magnitude

$$F_{1,net} = \sqrt{F_{1,netx}^2 + F_{1,nety}^2} = 1.78 \times 10^{-24}\,\text{N}. \quad \text{(Answer)}$$

To find the direction of $\vec{F}_{1,net}$ we take

$$\theta = \tan^{-1} \frac{F_{1,nety}}{F_{1,netx}} = -86.0°.$$

However, this is an unreasonable result because $\vec{F}_{1,net}$ must have a direction between the directions of $\vec{F}_{12}$ and $\vec{F}_{14}$. To correct θ, we add $180°$, obtaining

$$-86.0° + 180° = 94.0°. \quad \text{(Answer)}$$

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Checkpoint 21.1.3

The figure here shows three arrangements of an electron e and two protons p. (a) Rank the arrangements according to the magnitude of the net electrostatic force on the electron due to the protons, largest first. (b) In situation c, is the angle between the net force on the electron and the line labeled d less than or more than $45°$?

(a) (b) (c)
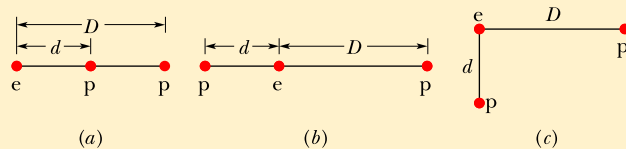
Sample Problem 21.1.2   Equilibrium of two forces on a particle

Figure 21.1.8a shows two particles fixed in place: a particle of charge $q_1 = +8q$ at the origin and a particle of charge $q_2 = -2q$ at $x = L$. At what point (other than infinitely far away) can a proton be placed so that it is in *equilibrium* (the net force on it is zero)? Is that equilibrium *stable* or *unstable*? (That is, if the proton is displaced, do the forces drive it back to the point of equilibrium or drive it farther away?)

KEY IDEA

If $\vec{F}_1$ is the force on the proton due to charge q_1 and $\vec{F}_2$ is the force on the proton due to charge q_2, then the point we seek is where $\vec{F}_1 + \vec{F}_2 = 0$. Thus,

$$\vec{F}_1 = -\vec{F}_2. \quad (21.1.8)$$

This tells us that at the point we seek, the forces acting on the proton due to the other two particles must be of equal magnitudes,

$$F_1 = F_2, \quad (21.1.9)$$

and that the forces must have opposite directions.

Reasoning: Because a proton has a positive charge, the proton and the particle of charge q_1 are of the same sign, and force $\vec{F}_1$ on the proton must point away from q_1. Also, the proton and the particle of charge q_2 are of opposite signs, so force $\vec{F}_2$ on the proton must point toward q_2. "Away from q_1" and "toward q_2" can be in opposite directions only if the proton is located on the x axis.

If the proton is on the x axis at any point between q_1 and q_2, such as point P in Fig. 21.1.8b, then $\vec{F}_1$ and $\vec{F}_2$

are in the same direction and not in opposite directions as required. If the proton is at any point on the x axis to the left of q_1, such as point S in Fig. 21.1.8c, then $\vec{F}_1$ and $\vec{F}_2$ are in opposite directions. However, Eq. 21.1.4 tells us that $\vec{F}_1$ and $\vec{F}_2$ cannot have equal magnitudes there: F_1 must be greater than F_2, because F_1 is produced by a closer charge (with lesser r) of greater magnitude ($8q$ versus $2q$).

Finally, if the proton is at any point on the x axis to the right of q_2, such as point R in Fig. 21.1.8d, then $\vec{F}_1$ and $\vec{F}_2$ are again in opposite directions. However, because now the charge of greater magnitude (q_1) is *farther* away from the proton than the charge of lesser magnitude, there is a point at which F_1 is equal to F_2. Let x be the coordinate of this point, and let q_p be the charge of the proton.

Calculations: With Eq. 21.1.4, we can now rewrite Eq. 21.1.9:

$$\frac{1}{4\pi\varepsilon_0}\frac{8qq_p}{x^2}=\frac{1}{4\pi\varepsilon_0}\frac{2qq_p}{(x-L)^2}. \qquad (21.1.10)$$

(Note that only the charge magnitudes appear in Eq. 21.1.10. We already decided about the directions of the forces in drawing Fig. 21.1.8d and do not want to include any positive or negative signs here.) Rearranging Eq. 21.1.10 gives us

$$\left(\frac{x-L}{x}\right)^2=\frac{1}{4}.$$

After taking the square roots of both sides, we find

$$\frac{x-L}{x}=\frac{1}{2}$$

and $\qquad\qquad\qquad x=2L.$ (Answer)
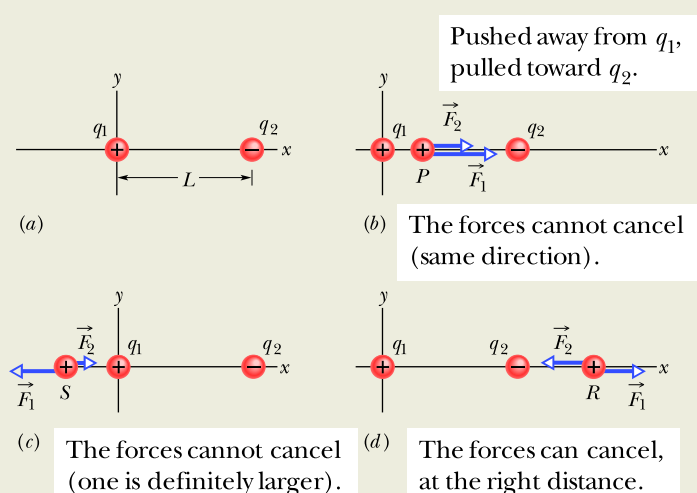
Figure 21.1.8 (*a*) Two particles of charges q_1 and q_2 are fixed in place on an x axis, with separation L. (*b*)–(*d*) Three possible locations P, S, and R for a proton. At each location, $\vec{F}_1$ is the force on the proton from particle 1 and $\vec{F}_2$ is the force on the proton from particle 2.

The equilibrium at $x = 2L$ is unstable; that is, if the proton is displaced leftward from point R, then F_1 and F_2 both increase but F_2 increases more (because q_2 is closer than q_1), and a net force will drive the proton farther leftward. If the proton is displaced rightward, both F_1 and F_2 decrease but F_2 decreases more, and a net force will then drive the proton farther rightward. In a stable equilibrium, if the proton is displaced slightly, it returns to the equilibrium position.

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Sample Problem 21.1.3   Charge sharing by two identical conducting spheres

In Fig. 21.1.9a, two identical, electrically isolated conducting spheres A and B are separated by a (center-to-center) distance a that is large compared to the spheres. Sphere A has a positive charge of $+Q$, and sphere B is electrically neutral. Initially, there is no electrostatic force between the spheres. (The large separation means there is no induced charge.)

(a) Suppose the spheres are connected for a moment by a conducting wire. The wire is thin enough so that any net charge on it is negligible. What is the electrostatic force between the spheres after the wire is removed?

KEY IDEAS

(1) Because the spheres are identical, connecting them means that they end up with identical charges (same sign
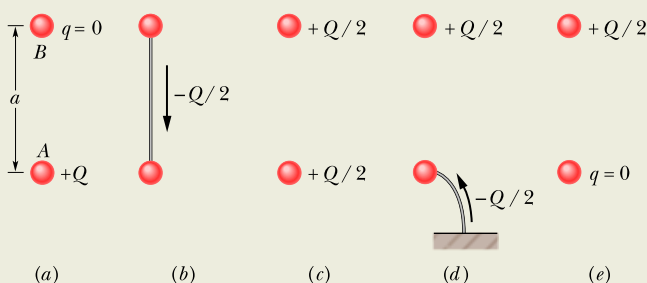
Figure 21.1.9 Two small conducting spheres A and B. (*a*) To start, sphere A is charged positively. (*b*) Negative charge is transferred from B to A through a connecting wire. (*c*) Both spheres are then charged positively. (*d*) Negative charge is transferred through a grounding wire to sphere A. (*e*) Sphere A is then neutral.

and same amount). (2) The initial sum of the charges (including the signs of the charges) must equal the final sum of the charges.

Reasoning: When the spheres are wired together, the (negative) conduction electrons on B, which repel one another, have a way to move away from one another (along the wire to positively charged A, which attracts them—Fig. 21.1.9b). As B loses negative charge, it becomes positively charged, and as A gains negative charge, it becomes *less* positively charged. The transfer of charge stops when the charge on B has increased to $+Q/2$ and the charge on A has decreased to $+Q/2$, which occurs when $-Q/2$ has shifted from B to A.

    After the wire has been removed (Fig. 21.1.9c), we can assume that the charge on either sphere does not disturb the uniformity of the charge distribution on the other sphere, because the spheres are small relative to their separation. Thus, we can apply the first shell theorem to each sphere. By Eq. 21.1.4 with $q_1 = q_2 = Q/2$ and $r = a$,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{(Q/2)(Q/2)}{a^2} = \frac{1}{16\pi\varepsilon_0}\left(\frac{Q}{a}\right)^2. \qquad \text{(Answer)}$$

The spheres, now positively charged, repel each other.

(b) Next, suppose sphere A is grounded momentarily, and then the ground connection is removed. What now is the electrostatic force between the spheres?

Reasoning: When we provide a conducting path between a charged object and the ground (which is a huge conductor), we neutralize the object. Were sphere A negatively charged, the mutual repulsion between the excess electrons would cause them to move from the sphere to the ground. However, because sphere A is positively charged, electrons with a total charge of $-Q/2$ move *from* the ground up onto the sphere (Fig. 21.1.9d), leaving the sphere with a charge of 0 (Fig. 21.1.9e). Thus, the electrostatic force is again zero.

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21.2 CHARGE IS QUANTIZED

Learning Objectives

After reading this module, you should be able to . . .

21.2.1 Identify the elementary charge.

21.2.2 Identify that the charge of a particle or object must be a positive or negative integer times the elementary charge.

Key Ideas

● Electric charge is quantized (restricted to certain values).

● The charge of a particle can be written as ne, where n is a positive or negative integer and e is the

elementary charge, which is the magnitude of the charge of the electron and proton ($\approx 1.602 \times 10^{-19}$ C).

Charge Is Quantized

In Benjamin Franklin's day, electric charge was thought to be a continuous fluid—an idea that was useful for many purposes. However, we now know that fluids themselves, such as air and water, are not continuous but are made up of atoms and molecules; matter is discrete. Experiment shows that "electrical fluid" is also not continuous but is made up of multiples of a certain elementary charge. Any positive or negative charge q that can be detected can be written as

$$q = ne, \qquad n = \pm 1, \pm 2, \pm 3, \ldots, \qquad (21.2.1)$$

in which e, the **elementary charge,** has the approximate value

$$e = 1.602 \times 10^{-19} \text{ C}. \qquad (21.2.2)$$

The electron has a charge of $-e$ and the proton has a charge of $+e$ (Table 21.2.1). The neutron is electrically neutral with no charge. Those three particles are the

Table 21.2.1 The Charges of Three Particles and Their Antiparticles

Particle	Symbol	Charge	Antiparticle	Symbol	Charge
Electron	e or e⁻	$-e$	Positron	e⁺	$+e$
Proton	p	$+e$	Antiproton	$\overline{\text{p}}$	$-e$
Neutron	n	0	Antineutron	$\overline{\text{n}}$	0

Table 21.2.2 The Charges of Two Quarks and Their Antiparticles

Quark	Symbol	Charge	Antiparticle	Symbol	Charge
Up	u	$+\frac{2}{3}e$	Antiup	$\overline{\text{u}}$	$-\frac{2}{3}e$
Down	d	$-\frac{1}{3}e$	Antidown	$\overline{\text{d}}$	$+\frac{1}{3}e$

only particles in your body and any common material. The electron does not consist of internal particles, but the proton and neutron consist of three quarks (Table 21.2.2). Uncommon particles consist of a quark and an antiquark or either three quarks or three antiquarks. The quarks and antiquarks have fractional charges of $\pm e/3$ or $\pm 2e/3$. However, because quarks cannot be detected individually and for historical reasons, we do not take their charge to be the elementary charge.

You often see phrases—such as "the charge on a sphere," "the amount of charge transferred," and "the charge carried by the electron"—that suggest that charge is a substance. (Indeed, such statements have already appeared in this chapter.) You should, however, keep in mind what is intended: *Particles* are the substance and charge happens to be one of their properties, just as mass is.

When a physical quantity such as charge can have only discrete values rather than any value, we say that the quantity is **quantized**. It is possible, for example, to find a particle that has no charge at all or a charge of $+10e$ or $-6e$, but not a particle with a charge of, say, $3.57e$.

The quantum of charge is small. In an ordinary 100 W lightbulb, for example, about 10^{19} elementary charges enter the bulb every second and just as many leave. However, the graininess of electricity does not show up in such large-scale phenomena (the bulb does not flicker with each electron).

Checkpoint 21.2.1

Initially, sphere A has a charge of $-50e$ and sphere B has a charge of $+20e$. The spheres are made of conducting material and are identical in size. If the spheres then touch, what is the resulting charge on sphere A?

Sample Problem 21.2.1 Mutual electric repulsion in a nucleus

The nucleus in an iron atom has a radius of about 4.0×10^{-15} m and contains 26 protons.

(a) What is the magnitude of the repulsive electrostatic force between two of the protons that are separated by 4.0×10^{-15} m?

KEY IDEA

The protons can be treated as charged particles, so the magnitude of the electrostatic force on one from the other is given by Coulomb's law.

Calculation: Table 21.2.1 tells us that the charge of a proton is $+e$. Thus, Eq. 21.1.4 gives us

$$F = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

$$= \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.602 \times 10^{-19} \text{ C})^2}{(4.0 \times 10^{-15} \text{ m})^2}$$

$$= 14 \text{ N.} \qquad \text{(Answer)}$$

No explosion: This is a small force to be acting on a macroscopic object like a cantaloupe, but an enormous force

to be acting on a proton. Such forces should explode the nucleus of any element but hydrogen (which has only one proton in its nucleus). However, they don't, not even in nuclei with a great many protons. Therefore, there must be some enormous attractive force to counter this enormous repulsive electrostatic force.

(b) What is the magnitude of the gravitational force between those same two protons?

KEY IDEA

Because the protons are particles, the magnitude of the gravitational force on one from the other is given by Newton's equation for the gravitational force (Eq. 21.1.2).

Calculation: With m_p (= 1.67×10^{-27} kg) representing the mass of a proton, Eq. 21.1.2 gives us

$$F = G \frac{m_p^2}{r^2}$$

$$= \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.67 \times 10^{-27} \text{ kg})^2}{(4.0 \times 10^{-15} \text{ m})^2}$$

$$= 1.2 \times 10^{-35} \text{ N}. \qquad \text{(Answer)}$$

Weak versus strong: This result tells us that the (attractive) gravitational force is far too weak to counter the repulsive electrostatic forces between protons in a nucleus. Instead, the protons are bound together by an enormous force called (aptly) the *strong nuclear force* — a force that acts between protons (and neutrons) when they are close together, as in a nucleus.

Although the gravitational force is many times weaker than the electrostatic force, it is more important in large-scale situations because it is always attractive. This means that it can collect many small bodies into huge bodies with huge masses, such as planets and stars, that then exert large gravitational forces. The electrostatic force, on the other hand, is repulsive for charges of the same sign, so it is unable to collect either positive charge or negative charge into large concentrations that would then exert large electrostatic forces.

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21.3 CHARGE IS CONSERVED

Learning Objectives

After reading this module, you should be able to . . .

21.3.1 Identify that in any isolated physical process, the net charge cannot change (the net charge is always conserved).

21.3.2 Identify an annihilation process of particles and a pair production of particles.

21.3.3 Identify mass number and atomic number in terms of the number of protons, neutrons, and electrons.

Key Ideas

● The net electric charge of any isolated system is always conserved.

● If two charged particles undergo an annihilation process, they have opposite signs of charge.

● If two charged particles appear as a result of a pair production process, they have opposite signs of charge.

Charge Is Conserved

If you rub a glass rod with silk, a positive charge appears on the rod. Measurement shows that a negative charge of equal magnitude appears on the silk. This suggests that rubbing does not create charge but only transfers it from one body to another, upsetting the electrical neutrality of each body during the process. This hypothesis of **conservation of charge,** first put forward by Benjamin Franklin, has stood up under close examination, both for large-scale charged bodies and for atoms, nuclei, and elementary particles. No exceptions have ever been found.

Thus, we add electric charge to our list of quantities—including energy and both linear momentum and angular momentum—that obey a conservation law.

Important examples of the conservation of charge occur in the *radioactive decay* of nuclei, said to be *radionuclides*. In the process, a nucleus transforms into (becomes) a different type of nucleus. For example, a uranium-238 nucleus ($^{238}_{92}$U) transforms into a thorium-234 nucleus ($^{234}_{90}$Th) by emitting an alpha particle. That particle can be symbolized with α, but because it has the same makeup as a helium-4 nucleus, it can also be symbolized with $^{4}_{2}$He. In these symbols, we use the chemical notation for the element. The superscript is the *mass number A* that gives the total number of protons and neutrons in the nucleus (collectively called the *nucleons*), and the subscript is the *atomic number* or *charge number Z* that gives the number of protons. Appendix E gives the chemical symbols and the values of Z for all elements.

We can write the alpha decay of uranium-238 as

$$^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}. \qquad (21.3.1)$$

The initial uranium nucleus is said to be the *parent nucleus*, and the resulting thorium nucleus is said to be the *daughter nucleus*. Note the conservation of charge: On the left the parent nucleus has 92 protons (and thus a charge of $+92e$), and on the right the two nuclei together have 92 protons (and thus the same charge). The mass number A is also conserved, but that is not our focus here.

Another type of radioactive decay is *electron capture* in which a proton in a parent nucleus "captures" one of the inner electrons of the atom to form a neutron (which remains in the daughter nucleus) and to release a *neutrino ν* (which has no charge):

$$\text{p} + \text{e}^- \rightarrow \text{n} + \nu. \qquad (21.3.2)$$

The process reduces the atomic number Z by one, but the net charge is conserved: The net charge on the left is $+e + (-e) = 0$ and the net charge on the right is $0 + 0 = 0$. After the capture, an outer electron can drop into the inner vacancy left by the captured electron. That transition can release an x ray. Alternatively, it can provide the energy for another of the outer electrons to escape the atom, a process first discovered by Lise Meitner in 1922 and then independently by Pierre Auger in 1923. Today the process is used in cancer therapy: A radionuclide that undergoes electron capture is encapsulated and placed next to the cancer cells so that the *Auger–Meitner electrons* can lethally damage the cells and thus reduce the cancer.

Another example of charge conservation occurs when an electron e$^-$ (charge $-e$) and its antiparticle, the positron e$^+$ (charge $+e$), undergo an *annihilation process,* transforming into two *gamma rays* (high-energy light):

$$\text{e}^- + \text{e}^+ \rightarrow \gamma + \gamma \qquad \text{(annihilation)}. \qquad (21.3.3)$$
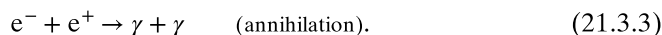
In applying the conservation-of-charge principle, we must add the charges algebraically, with due regard for their signs. In the annihilation process of Eq. 21.3.3 then, the net charge of the system is zero both before and after the event. Charge is conserved.

In *pair production*, the converse of annihilation, charge is also conserved. In this process a gamma ray transforms into an electron and a positron:

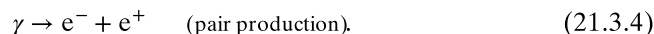$$\gamma \rightarrow \text{e}^- + \text{e}^+ \qquad \text{(pair production)}. \qquad (21.3.4)$$

Figure 21.3.1 shows such a pair-production event that occurred in a bubble chamber. (This is a device in which a liquid is suddenly made hotter than its boiling point. If a charged particle passes through it, tiny vapor bubbles form along the particle's trail.) A gamma ray entered the chamber from the bottom and at one point transformed into an electron and a positron. Because those new particles
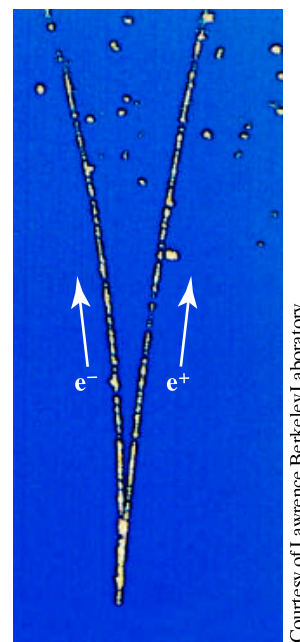
Figure 21.3.1 A photograph of trails of bubbles left in a bubble chamber by an electron and a positron. The pair of particles was produced by a gamma ray that entered the chamber directly from the bottom. Being electrically neutral, the gamma ray did not generate a telltale trail of bubbles along its path, as the electron and positron did.

were charged and moving, each left a trail of bubbles. (The trails were curved because a magnetic field had been set up in the chamber.) The gamma ray, being electrically neutral, left no trail. Still, you can tell exactly where it underwent pair production—at the tip of the curved V, which is where the trails of the electron and positron begin.

PET Scans

A widely used method of obtaining images inside a human body is *positron emission tomography* (PET). A *beta-plus (positron) emitter* is injected into a patient, where it will tend to collect in a tumor. When one of the nuclei emits a positron as a proton transforms to a neutron, that positron undergoes particle annihilation with an electron in the surrounding tissue within a micron of the nucleus (Eq. 21.3.3). We can approximate the total momentum of the positron and electron as being zero. From the conservation of momentum, the two gamma rays that are produced must also have a total momentum of zero, which requires that they travel away from their production site in opposite directions.

A PET apparatus consists of gamma-ray detectors that are commonly arranged in a ring around the production site in the patient (Fig. 21.3.2a). When two detectors are triggered on opposite sides of the ring (Fig. 21.3.2b) within a narrow time interval, the system backtracks the gamma-ray paths to determine the location of the production site and thus the site of the tumor. An image of the site is built up as this determination is done repeatedly with gamma-ray pairs emitted in various directions into the ring.

U.S. Navy/ZUMA Press/Newscom

(a)   (b)
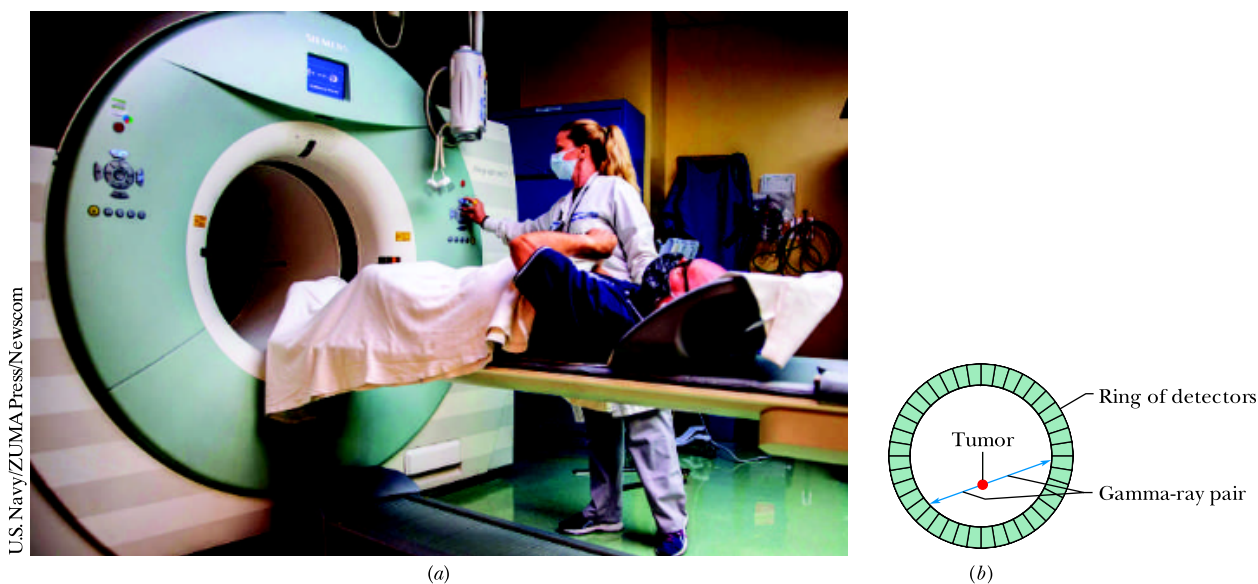
Figure 21.3.2 (*a*) Patient in a PET scan apparatus. (*b*) Annihilation of a positron and electron sends gamma rays in opposite directions to the ring of detectors.

Review & Summary

Electric Charge   The strength of a particle's electrical interaction with objects around it depends on its **electric charge** (usually represented as q), which can be either positive or negative. Particles with the same sign of charge repel each other, and particles with opposite signs of charge attract each other. An object with equal amounts of the two kinds of charge is electrically neutral, whereas one with an imbalance is electrically charged and has an excess charge.

Conductors are materials in which a significant number of electrons are free to move. The charged particles in **nonconductors (insulators)** are not free to move.

Electric current i is the rate dq/dt at which charge passes a point:

$$i = \frac{dq}{dt} \quad \text{(electric current)}. \tag{21.1.3}$$

Coulomb's Law   Coulomb's law describes the electrostatic force (or electric force) between two charged particles. If the particles have charges q_1 and q_2, are separated by distance r, and are at rest (or moving only slowly) relative to each other, then the magnitude of the force acting on each due to the other is given by

$$F = \frac{1}{4\pi\varepsilon_0} \frac{|q_1||q_2|}{r^2} \quad \text{(Coulomb's law)}, \tag{21.1.4}$$

where $\varepsilon_0 = 8.85 \times 10^{-12} \ \mathrm{C^2/N \cdot m^2}$ is the **permittivity constant**. The ratio $1/4\pi\varepsilon_0$ is often replaced with the **electrostatic constant** (or **Coulomb constant**) $k = 8.99 \times 10^9 \ \mathrm{N \cdot m^2/C^2}$.

The electrostatic force vector acting on a charged particle due to a second charged particle is either directly toward the second particle (opposite signs of charge) or directly away from it (same sign of charge). As with other types of forces, if multiple electrostatic forces act on a particle, the net force is the vector sum (not scalar sum) of the individual forces.

The two shell theories for electrostatics are

Shell theorem 1: A charged particle outside a shell with charge uniformly distributed on its surface is attracted or repelled as if the shell's charge were concentrated as a particle at its center.

Shell theorem 2: A charged particle inside a shell with charge uniformly distributed on its surface has no net force acting on it due to the shell.

Charge on a conducting spherical shell spreads uniformly over the (external) surface.

The Elementary Charge   Electric charge is quantized (restricted to certain values). The charge of a particle can be written as ne, where n is a positive or negative integer and e is the elementary charge, which is the magnitude of the charge of the electron and proton ($\approx 1.602 \times 10^{-19}$ C).

Conservation of Charge   The net electric charge of any isolated system is always conserved.

Questions

1   Figure 21.1 shows four situations in which five charged particles are evenly spaced along an axis. The charge values are indicated except for the central particle, which has the same charge in all four situations. Rank the situations according to the magnitude of the net electrostatic force on the central particle, greatest first.
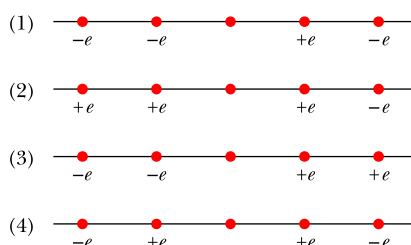
Figure 21.1 Question 1.

2   Figure 21.2 shows three pairs of identical spheres that are to be touched together and then separated. The initial charges on them are indicated. Rank the pairs according to (a) the magnitude of the charge transferred during touching and (b) the charge left on the positively charged sphere, greatest first.
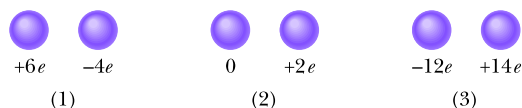
Figure 21.2 Question 2.

3   Figure 21.3 shows four situations in which charged particles are fixed in place on an axis. In which situations is there
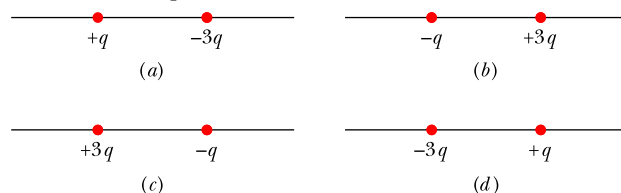

Figure 21.3 Question 3.

a point to the left of the particles where an electron will be in equilibrium?

4   Figure 21.4 shows two charged particles on an axis. The charges are free to move. However, a third charged particle can be placed at a certain point such that all three particles are then in equilibrium. (a) Is that point to the left of the first two particles, to their right, or between them? (b) Should the third particle be positively or negatively charged? (c) Is the equilibrium stable or unstable?
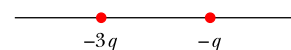

Figure 21.4 Question 4.

5   In Fig. 21.5, a central particle of charge $-q$ is surrounded by two circular rings of charged particles. What are the magnitude and direction of the net electrostatic force on the central particle due to the other particles? (*Hint*: Consider symmetry.)
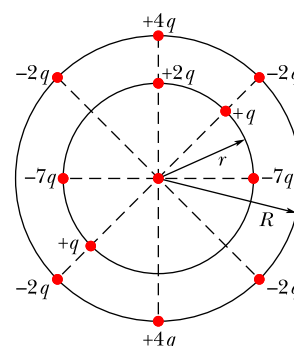

Figure 21.5 Question 5.

6   A positively charged ball is brought close to an electrically neutral isolated conductor. The conductor is then grounded while the ball is kept close. Is the conductor charged positively, charged negatively, or neutral if (a) the ball is first taken away and then the ground connection is removed and (b) the ground connection is first removed and then the ball is taken away?

7   Figure 21.6 shows three situations involving a charged particle and a uniformly charged spherical shell. The charges are given, and the radii of the shells are indicated. Rank the

situations according to the magnitude of the force on the particle due to the presence of the shell, greatest first.
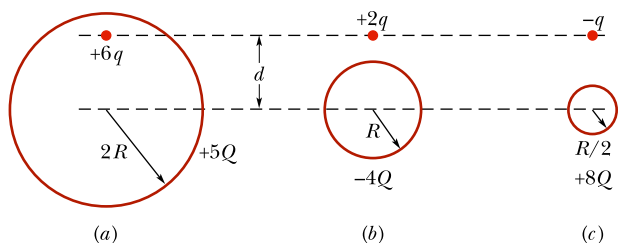

Figure 21.6 Question 7.

8 Figure 21.7 shows four arrangements of charged particles. Rank the arrangements according to the magnitude of the net electrostatic force on the particle with charge $+Q$, greatest first.
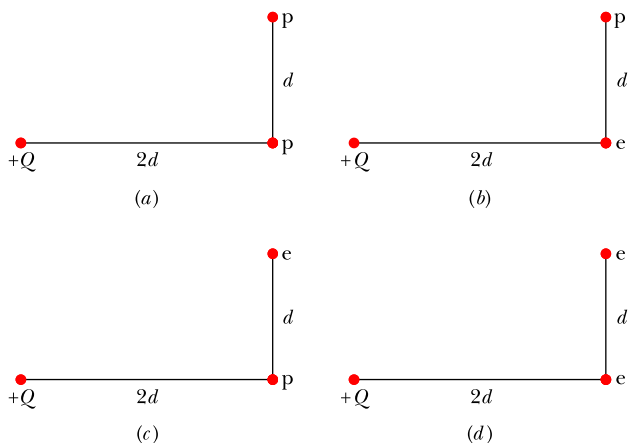

Figure 21.7 Question 8.

9 Figure 21.8 shows four situations in which particles of charge $+q$ or $-q$ are fixed in place. In each situation, the particles on the x axis are equidistant from the y axis. First, consider the middle particle in situation 1; the middle particle experiences an electrostatic force from each of the other two particles. (a) Are the magnitudes F of those forces the same or different? (b) Is the magnitude of the net force on the middle particle equal to, greater than, or less than $2F$? (c) Do the x components of the two forces add or cancel? (d) Do their y components add or cancel? (e) Is the direction of the net force on the middle particle
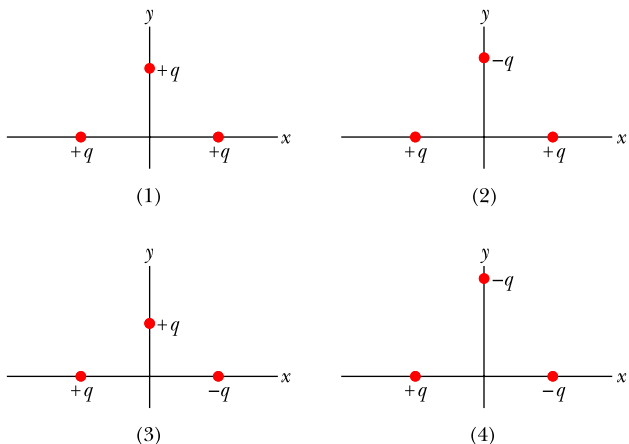
that of the canceling components or the adding components? (f) What is the direction of that net force? Now consider the remaining situations: What is the direction of the net force on the middle particle in (g) situation 2, (h) situation 3, and (i) situation 4? (In each situation, consider the symmetry of the charge distribution and determine the canceling components and the adding components.)

10 In Fig. 21.9, a central particle of charge $-2q$ is surrounded by a square array of charged particles, separated by either distance d or $d/2$ along the perimeter of the square. What are the magnitude and direction of the net electrostatic force on the central particle due to the other particles? (*Hint:* Consideration of symmetry can greatly reduce the amount of work required here.)
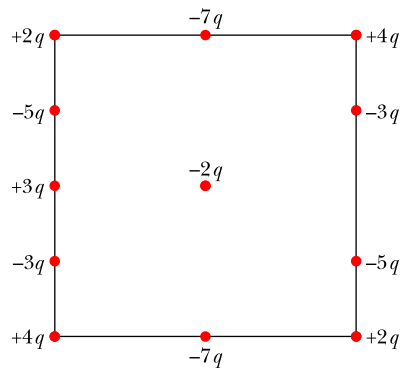
Figure 21.9 Question 10.

11 Figure 21.10 shows three identical conducting bubbles A, B, and C floating in a conducting container that is grounded by a wire. The bubbles initially have the same charge. Bubble A bumps into the container's ceiling and then into bubble B. Then bubble B bumps into bubble C, which then drifts to the container's floor. When bubble C reaches the floor, a charge of $-3e$ is transferred upward through the wire, from the ground to the container, as indicated. (a) What was the initial charge of each bubble? When (b) bubble A and (c) bubble B reach the floor, what is the charge transfer through the wire? (d) During this whole process, what is the total charge transfer through the wire?
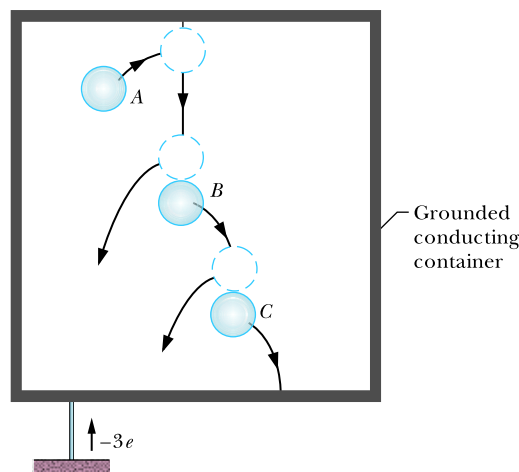

Figure 21.10 Question 11.

12 Figure 21.11 shows four situations in which a central proton is partially surrounded by protons or electrons fixed in place along a half-circle. The angles θ are identical; the angles ϕ are also. (a) In each situation, what is the direction of the net force on the central proton due to the other particles? (b) Rank the four situations according to the magnitude of that net force on the central proton, greatest first.
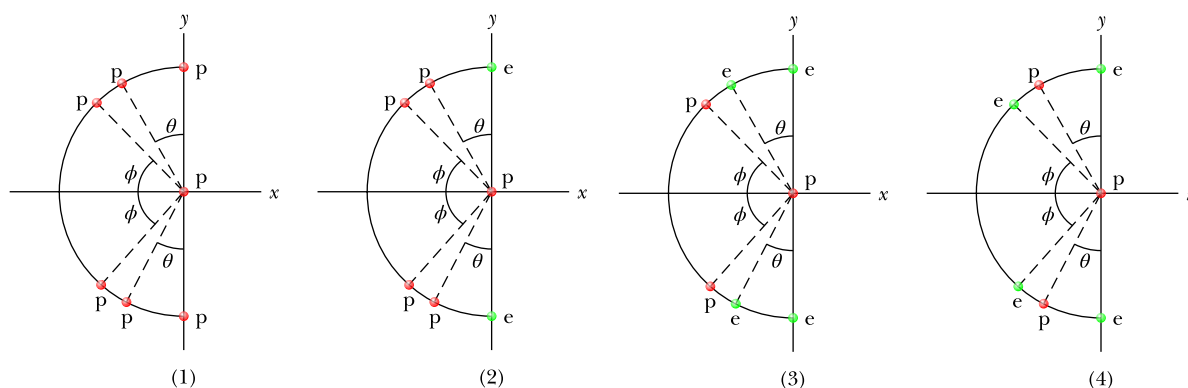

Figure 21.8 Question 9.

Figure 21.11 Question 12.

Problems

GO Tutoring problem available (at instructor's discretion) in *WileyPLUS*	
SSM Worked-out solution available in Student Solutions Manual	**CALC** Requires calculus
E Easy **M** Medium **H** Hard	**BIO** Biomedical application
FCP Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com	

Module 21.1   Coulomb's Law

1 **E** **SSM** Of the charge Q initially on a tiny sphere, a portion q is to be transferred to a second, nearby sphere. Both spheres can be treated as particles and are fixed with a certain separation. For what value of q/Q will the electrostatic force between the two spheres be maximized?

2 **E** Identical isolated conducting spheres 1 and 2 have equal charges and are separated by a distance that is large compared with their diameters (Fig. 21.12a). The electrostatic force acting on sphere 2 due to sphere 1 is $\vec{F}$. Suppose now that a third identical sphere 3, having an insulating handle and initially neutral, is touched first to sphere 1 (Fig. 21.12b), then to sphere 2 (Fig. 21.12c), and finally removed (Fig. 21.12d). The electrostatic force that now acts on sphere 2 has magnitude F'. What is the ratio F'/F?
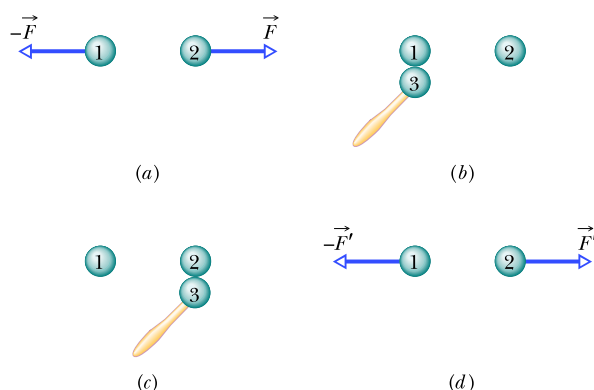
Figure 21.12 Problem 2.

3 **E** **SSM** What must be the distance between point charge $q_1 = 26.0\,\mu\text{C}$ and point charge $q_2 = -47.0\,\mu\text{C}$ for the electrostatic force between them to have a magnitude of 5.70 N?

4 **E** **FCP** In the return stroke of a typical lightning bolt, a current of 2.5×10^4 A exists for 20 μs. How much charge is transferred in this event?

5 **E** A particle of charge $+3.00 \times 10^{-6}$ C is 12.0 cm distant from a second particle of charge -1.50×10^{-6} C. Calculate the magnitude of the electrostatic force between the particles.

6 **E** Two equally charged particles are held 3.2×10^{-3} m apart and then released from rest. The initial acceleration of the first particle is observed to be 7.0 m/s^2 and that of the second to be 9.0 m/s^2. If the mass of the first particle is 6.3×10^{-7} kg, what are (a) the mass of the second particle and (b) the magnitude of the charge of each particle?

7 **M** In Fig. 21.13, three charged particles lie on an x axis. Particles 1 and 2 are fixed in place. Particle 3 is free

Figure 21.13 Problems 7 and 40.

to move, but the net electrostatic force on it from particles 1 and 2 happens to be zero. If $L_{23} = L_{12}$, what is the ratio q_1/q_2?
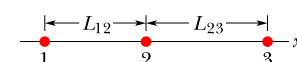
8 **M** In Fig. 21.14, three identical conducting spheres initially have the following charges: sphere A, $4Q$; sphere B, $-6Q$; and sphere C, 0. Spheres A and B are fixed in place, with a center-to-center separation that is much larger than the spheres. Two experiments are conducted. In experiment 1, sphere C is touched to sphere A

Figure 21.14 Problem 8.

and then (separately) to sphere B, and then it is removed. In experiment 2, starting with the same initial states, the procedure is reversed: Sphere C is touched to sphere B and then (separately) to sphere A, and then it is removed. What is the ratio of the electrostatic force between A and B at the end of experiment 2 to that at the end of experiment 1?
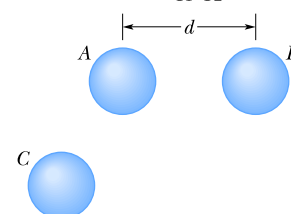
9 **M** **SSM** Two identical conducting spheres, fixed in place, attract each other with an electrostatic force of 0.108 N when their center-to-center separation is 50.0 cm. The spheres are then connected by a thin conducting wire. When the wire is

removed, the spheres repel each other with an electrostatic force of 0.0360 N. Of the initial charges on the spheres, with a positive net charge, what was (a) the negative charge on one of them and (b) the positive charge on the other?

10 M GO In Fig. 21.15, four particles form a square. The charges are $q_1 = q_4 = Q$ and $q_2 = q_3 = q$. (a) What is Q/q if the net electrostatic force on particles 1 and 4 is zero? (b) Is there any value of q that makes the net electrostatic force on each of the four particles zero? Explain.
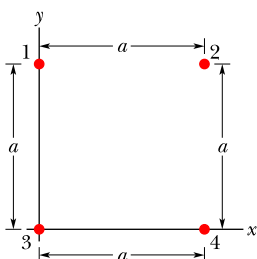
Figure 21.15
Problems 10 and 11.

11 M In Fig. 21.15, the particles have charges $q_1 = -q_2 = 100$ nC and $q_3 = -q_4 = 200$ nC, and distance $a = 5.0$ cm. What are the (a) x and (b) y components of the net electrostatic force on particle 3?

12 M Two particles are fixed on an x axis. Particle 1 of charge $40\,\mu$C is located at $x = -2.0$ cm; particle 2 of charge Q is located at $x = 3.0$ cm. Particle 3 of charge magnitude $20\,\mu$C is released from rest on the y axis at $y = 2.0$ cm. What is the value of Q if the initial acceleration of particle 3 is in the positive direction of (a) the x axis and (b) the y axis?

13 M GO In Fig. 21.16, particle 1 of charge $+1.0\,\mu$C and particle 2 of charge $-3.0\,\mu$C are held at separation $L = 10.0$ cm on an x axis. If particle 3 of unknown charge q_3 is to be located such that the net electrostatic force on it from particles 1 and 2 is zero, what must be the (a) x and (b) y coordinates of particle 3?
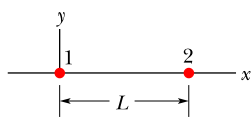
Figure 21.16 Problems 13, 19, 30, and 58.

14 M Three particles are fixed on an x axis. Particle 1 of charge q_1 is at $x = -a$, and particle 2 of charge q_2 is at $x = +a$. If their net electrostatic force on particle 3 of charge $+Q$ is to be zero, what must be the ratio q_1/q_2 when particle 3 is at (a) $x = +0.500a$ and (b) $x = +1.50a$?

15 M GO The charges and coordinates of two charged particles held fixed in an xy plane are $q_1 = +3.0\,\mu$C, $x_1 = 3.5$ cm, $y_1 = 0.50$ cm, and $q_2 = -4.0\,\mu$C, $x_2 = -2.0$ cm, $y_2 = 1.5$ cm. Find the (a) magnitude and (b) direction of the electrostatic force on particle 2 due to particle 1. At what (c) x and (d) y coordinates should a third particle of charge $q_3 = +4.0\,\mu$C be placed such that the net electrostatic force on particle 2 due to particles 1 and 3 is zero?

16 M GO In Fig. 21.17a, particle 1 (of charge q_1) and particle 2 (of charge q_2) are fixed in place on an x axis, 8.00 cm apart. Particle 3 (of charge $q_3 = +8.00 \times 10^{-19}$ C) is to be placed on the line between particles 1 and 2 so that they produce a net electrostatic force $\vec{F}_{3,\,net}$ on it. Figure 21.17b gives the x component of that force versus the coordinate x at which particle 3 is placed.
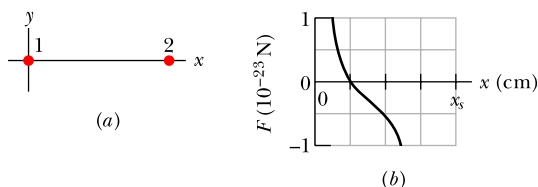

Figure 21.17 Problem 16.

The scale of the x axis is set by $x_s = 8.0$ cm. What are (a) the sign of charge q_1 and (b) the ratio q_2/q_1?

17 M In Fig. 21.18a, particles 1 and 2 have charge $20.0\,\mu$C each and are held at separation distance $d = 1.50$ m. (a) What is the magnitude of the electrostatic force on particle 1 due to particle 2? In Fig. 21.18b, particle 3 of charge $20.0\,\mu$C is positioned so as to complete an equilateral triangle. (b) What is the magnitude of the net electrostatic force on particle 1 due to particles 2 and 3?
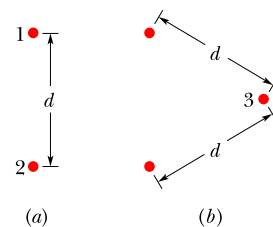

Figure 21.18 Problem 17.

18 M In Fig. 21.19a, three positively charged particles are fixed on an x axis. Particles B and C are so close to each other that they can be considered to be at the same distance from particle A. The net force on particle A due to particles B and C is 2.014×10^{-23} N in the negative direction of the x axis. In Fig. 21.19b, particle B has been moved to the opposite side of A but is still at the same distance from it. The net force on A is now 2.877×10^{-24} N in the negative direction of the x axis. What is the ratio q_C/q_B?
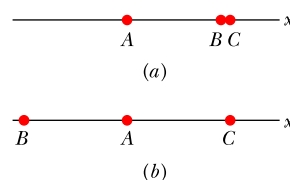

Figure 21.19 Problem 18.

19 M SSM In Fig. 21.16, particle 1 of charge $+q$ and particle 2 of charge $+4.00q$ are held at separation $L = 9.00$ cm on an x axis. If particle 3 of charge q_3 is to be located such that the three particles remain in place when released, what must be the (a) x and (b) y coordinates of particle 3, and (c) the ratio q_3/q?

20 H GO Figure 21.20a shows an arrangement of three charged particles separated by distance d. Particles A and C are fixed on the x axis, but particle B can be moved along a circle centered on particle A. During the movement, a radial line between A and B makes an angle θ relative to the positive direction of the x axis (Fig. 21.20b). The curves in Fig. 21.20c give, for two situations, the magnitude F_{net} of the net electrostatic force on particle A due to the other particles. That net force is given as a function of angle θ and as a multiple of a basic amount F_0. For example on curve 1, at $\theta = 180°$, we see that $F_{net} = 2F_0$. (a) For the situation corresponding to curve 1, what is the ratio of the charge of particle C to that of particle B (including sign)? (b) For the situation corresponding to curve 2, what is that ratio?
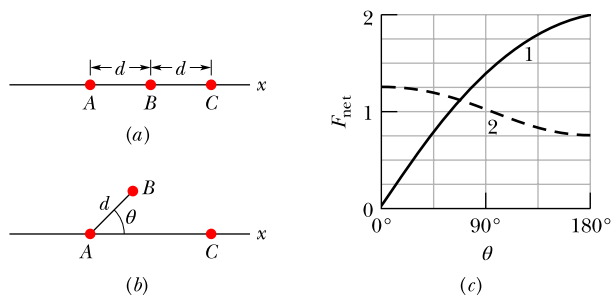

Figure 21.20 Problem 20.

21 H CALC GO A nonconducting spherical shell, with an inner radius of 4.0 cm and an outer radius of 6.0 cm, has charge spread nonuniformly through its volume between its inner and outer

surfaces. The *volume charge density* ρ is the charge per unit volume, with the unit coulomb per cubic meter. For this shell $\rho = b/r$, where r is the distance in meters from the center of the shell and $b = 3.0\,\mu C/m^2$. What is the net charge in the shell?

22 H GO Figure 21.21 shows an arrangement of four charged particles, with angle $\theta = 30.0°$ and distance $d = 2.00$ cm. Particle 2 has charge $q_2 = +8.00 \times 10^{-19}$ C; particles 3 and 4 have charges $q_3 = q_4 = -1.60 \times 10^{-19}$ C. (a) What is distance D between the origin and particle 2 if the net electrostatic force on particle 1 due to the other particles is zero? (b) If particles 3 and 4 were moved closer to the x axis but maintained their symmetry about that axis, would the required value of D be greater than, less than, or the same as in part (a)?
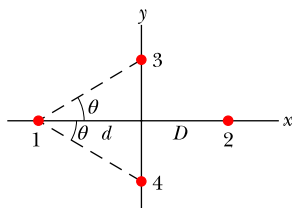
Figure 21.21 Problem 22.

23 H GO In Fig. 21.22, particles 1 and 2 of charge $q_1 = q_2 = +3.20 \times 10^{-19}$ C are on a y axis at distance $d = 17.0$ cm from the origin. Particle 3 of charge $q_3 = +6.40 \times 10^{-19}$ C is moved gradually along the x axis from $x = 0$ to $x = +5.0$ m. At what values of x will the magnitude of the electrostatic force on the third particle from the other two particles be (a) minimum and (b) maximum? What are the (c) minimum and (d) maximum magnitudes?
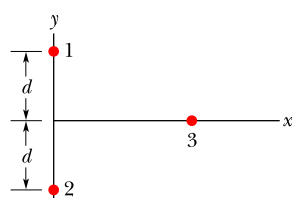

Figure 21.22 Problem 23.

Module 21.2 Charge Is Quantized

24 E Two tiny, spherical water drops, with identical charges of -1.00×10^{-16} C, have a center-to-center separation of 1.00 cm. (a) What is the magnitude of the electrostatic force acting between them? (b) How many excess electrons are on each drop, giving it its charge imbalance?

25 E How many electrons would have to be removed from a coin to leave it with a charge of $+1.0 \times 10^{-7}$ C?

26 E What is the magnitude of the electrostatic force between a singly charged sodium ion (Na^+, of charge $+e$) and an adjacent singly charged chlorine ion (Cl^-, of charge $-e$) in a salt crystal if their separation is 2.82×10^{-10} m?

27 E SSM The magnitude of the electrostatic force between two identical ions that are separated by a distance of 5.0×10^{-10} m is 3.7×10^{-9} N. (a) What is the charge of each ion? (b) How many electrons are "missing" from each ion (thus giving the ion its charge imbalance)?

28 E BIO FCP A current of 0.300 A through your chest can send your heart into fibrillation, ruining the normal rhythm of heartbeat and disrupting the flow of blood (and thus oxygen) to your brain. If that current persists for 2.00 min, how many conduction electrons pass through your chest?
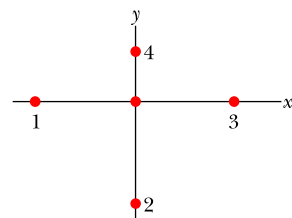
29 M GO In Fig. 21.23, particles 2 and 4, of charge $-e$, are fixed in place on a y axis, at $y_2 = -10.0$ cm

Figure 21.23 Problem 29.

and $y_4 = 5.00$ cm. Particles 1 and 3, of charge $-e$, can be moved along the x axis. Particle 5, of charge $+e$, is fixed at the origin. Initially particle 1 is at $x_1 = -10.0$ cm and particle 3 is at $x_3 = 10.0$ cm. (a) To what x value must particle 1 be moved to rotate the direction of the net electric force $\vec{F}_{net}$ on particle 5 by 30° counterclockwise? (b) With particle 1 fixed at its new position, to what x value must you move particle 3 to rotate $\vec{F}_{net}$ back to its original direction?

30 M In Fig. 21.16, particles 1 and 2 are fixed in place on an x axis, at a separation of $L = 8.00$ cm. Their charges are $q_1 = +e$ and $q_2 = -27e$. Particle 3 with charge $q_3 = +4e$ is to be placed on the line between particles 1 and 2, so that they produce a net electrostatic force $\vec{F}_{3,net}$ on it. (a) At what coordinate should particle 3 be placed to minimize the magnitude of that force? (b) What is that minimum magnitude?

31 M Earth's atmosphere is constantly bombarded by *cosmic ray protons* that originate somewhere in space. If the protons all passed through the atmosphere, each square meter of Earth's surface would intercept protons at the average rate of 1500 protons per second. What would be the electric current intercepted by the total surface area of the planet?

32 M GO Figure 21.24*a* shows charged particles 1 and 2 that are fixed in place on an x axis. Particle 1 has a charge with a magnitude of $|q_1| = 8.00e$. Particle 3 of charge $q_3 = +8.00e$ is initially on the x axis near particle 2. Then particle 3 is gradually moved in the positive direction of the x axis. As a result, the magnitude of the net electrostatic force $\vec{F}_{2,net}$ on particle 2 due to particles 1 and 3 changes. Figure 21.24*b* gives the x component of that net force as a function of the position x of particle 3. The scale of the x axis is set by $x_s = 0.80$ m. The plot has an asymptote of $F_{2,net} = 1.5 \times 10^{-25}$ N as $x \to \infty$. As a multiple of e and including the sign, what is the charge q_2 of particle 2?
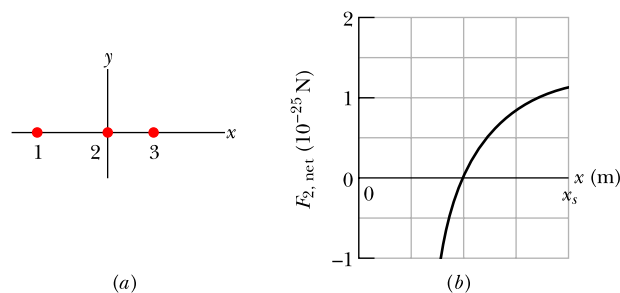

Figure 21.24 Problem 32.

33 M Calculate the number of coulombs of positive charge in 250 cm^3 of (neutral) water. (*Hint:* A hydrogen atom contains one proton; an oxygen atom contains eight protons.)

34 H GO Figure 21.25 shows electrons 1 and 2 on an x axis and charged ions 3 and 4 of identical charge $-q$ and at identical angles θ. Electron 2 is free to move; the other three particles are fixed in place at horizontal distances R from electron 2 and are intended to hold electron 2 in place. For physically possible values of $q \leq 5e$, what are the (a) smallest, (b) second smallest, and (c) third smallest values of θ for which electron 2 is held in place?
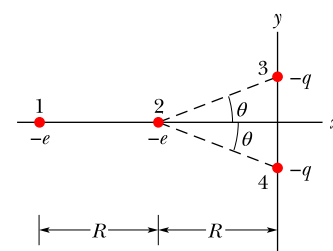

Figure 21.25 Problem 34.

35 H SSM In crystals of the salt cesium chloride, cesium ions Cs^+ form the eight corners of a cube and a chlorine ion Cl^- is at the cube's center (Fig. 21.26). The edge length of the cube is 0.40 nm. The Cs^+ ions are each deficient by one electron (and thus each has a charge of $+e$), and the Cl^- ion has one excess electron (and thus has a charge of $-e$). (a) What is the magnitude of the net electrostatic force exerted on the Cl^- ion by the eight Cs^+ ions at the corners of the cube? (b) If one of the Cs^+ ions is missing, the crystal is said to have a *defect*; what is the magnitude of the net electrostatic force exerted on the Cl^- ion by the seven remaining Cs^+ ions?
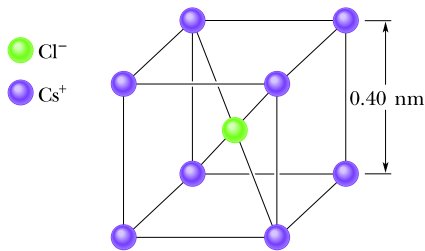
Figure 21.26 Problem 35.

Module 21.3 Charge Is Conserved

36 E Electrons and positrons are produced by the nuclear transformations of protons and neutrons known as *beta decay*. (a) If a proton transforms into a neutron, is an electron or a positron produced? (b) If a neutron transforms into a proton, is an electron or a positron produced?

37 E SSM Identify X in the following nuclear reactions: (a) $^1H + ^9Be \rightarrow X + n$; (b) $^{12}C + ^1H \rightarrow X$; (c) $^{15}N + ^1H \rightarrow ^4He + X$. Appendix F will help.

Additional Problems

38 GO Figure 21.27 shows four identical conducting spheres that are actually well separated from one another. Sphere W (with an initial charge of zero) is touched to sphere A and then they are separated. Next, sphere W is touched to sphere B (with an initial charge of $-32e$) and then they are separated. Finally, sphere W is touched to sphere C (with an initial charge of $+48e$), and then they are separated. The final charge on sphere W is $+18e$. What was the initial charge on sphere A?
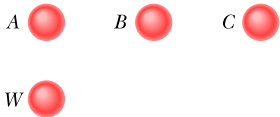

Figure 21.27 Problem 38.

39 SSM In Fig. 21.28, particle 1 of charge $+4e$ is above a floor by distance $d_1 = 2.00$ mm and particle 2 of charge $+6e$ is on the floor, at distance $d_2 = 6.00$ mm horizontally from particle 1. What is the x component of the electrostatic force on particle 2 due to particle 1?
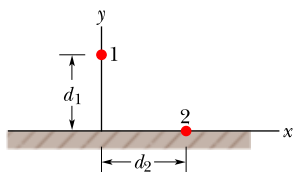

Figure 21.28 Problem 39.

40 In Fig. 21.13, particles 1 and 2 are fixed in place, but particle 3 is free to move. If the net electrostatic force on particle 3 due to particles 1 and 2 is zero and $L_{23} = 2.00L_{12}$, what is the ratio q_1/q_2?

41 (a) What equal positive charges would have to be placed on Earth and on the Moon to neutralize their gravitational attraction? (b) Why don't you need to know the lunar distance to solve this problem? (c) How many kilograms of hydrogen ions (that is, protons) would be needed to provide the positive charge calculated in (a)?

42 In Fig. 21.29, two tiny conducting balls of identical mass m and identical charge q hang from nonconducting threads of length L. Assume that θ is so small that $\tan \theta$ can be replaced by its approximate equal, $\sin \theta$. (a) Show that

$$x = \left(\frac{q^2 L}{2\pi \varepsilon_0 mg} \right)^{1/3}$$

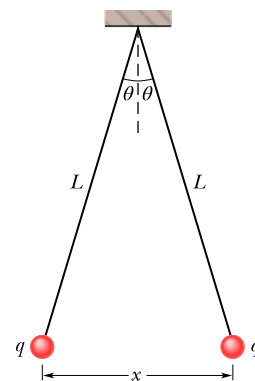gives the equilibrium separation x of the balls. (b) If $L = 120$ cm, $m = 10$ g, and $x = 5.0$ cm, what is $|q|$?

43 (a) Explain what happens to the balls of Problem 42 if one of them is discharged (loses its charge q to, say, the ground). (b) Find the new equilibrium separation x, using the given values of L and m and the computed value of $|q|$.

Figure 21.29 Problems 42 and 43.

44 SSM How far apart must two protons be if the magnitude of the electrostatic force acting on either one due to the other is equal to the magnitude of the gravitational force on a proton at Earth's surface?

45 How many megacoulombs of positive charge are in 1.00 mol of neutral molecular-hydrogen gas (H_2)?

46 In Fig. 21.30, four particles are fixed along an x axis, separated by distances $d = 2.00$ cm. The charges are $q_1 = +2e$, $q_2 = -e$, $q_3 = +e$, and $q_4 = +4e$, with $e = 1.60 \times 10^{-19}$ C. In unit-vector notation, what is the net electrostatic force on (a) particle 1 and (b) particle 2 due to the other particles?
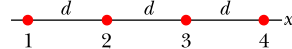

Figure 21.30 Problem 46.

47 GO Point charges of $+6.0\,\mu C$ and $-4.0\,\mu C$ are placed on an x axis, at $x = 8.0$ m and $x = 16$ m, respectively. What charge must be placed at $x = 24$ m so that any charge placed at the origin would experience no electrostatic force?

48 In Fig. 21.31, three identical conducting spheres form an equilateral triangle of side length $d = 20.0$ cm. The sphere radii are much smaller than d, and the sphere charges are $q_A = -2.00$ nC, $q_B = -4.00$ nC, and $q_C = +8.00$ nC. (a) What is the magnitude of the electrostatic force between spheres A and C? The following steps are then taken: A and B are connected by a thin wire and then disconnected; B is grounded by the wire, and the wire is then removed; B and C are connected by the wire and then disconnected. What now are the magnitudes of the electrostatic force (b) between spheres A and C and (c) between spheres B and C?
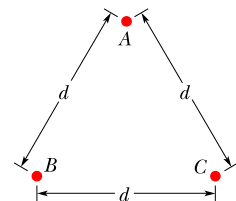

Figure 21.31 Problem 48.

49 A neutron consists of one "up" quark of charge $+2e/3$ and two "down" quarks each having charge $-e/3$. If we assume that the down quarks are 2.6×10^{-15} m apart inside the neutron, what is the magnitude of the electrostatic force between them?

50 Figure 21.32 shows a long, nonconducting, massless rod of length L, pivoted at its center and balanced with a block of weight W at a distance x from the left end. At the left and right ends of the rod are attached small conducting spheres with positive charges q and $2q$, respectively. A distance h directly beneath each of these spheres is a fixed sphere with positive charge Q. (a) Find the distance x when the rod is horizontal and balanced. (b) What value should h have so that the rod exerts no vertical force on the bearing when the rod is horizontal and balanced?
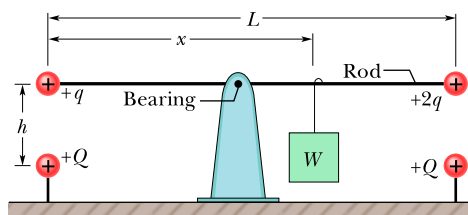

Figure 21.32 Problem 50.

51 CALC A charged nonconducting rod, with a length of 2.00 m and a cross-sectional area of $4.00\ cm^2$, lies along the positive side of an x axis with one end at the origin. The *volume charge density* ρ is charge per unit volume in coulombs per cubic meter. How many excess electrons are on the rod if ρ is (a) uniform, with a value of $-4.00\ \mu C/m^3$, and (b) nonuniform, with a value given by $\rho = bx^2$, where $b = -2.00\ \mu C/m^5$?

52 A particle of charge Q is fixed at the origin of an xy coordinate system. At $t = 0$ a particle ($m = 0.800$ g, $q = 4.00\ \mu C$) is located on the x axis at $x = 20.0$ cm, moving with a speed of 50.0 m/s in the positive y direction. For what value of Q will the moving particle execute circular motion? (Neglect the gravitational force on the particle.)

53 What would be the magnitude of the electrostatic force between two 1.00 C point charges separated by a distance of (a) 1.00 m and (b) 1.00 km if such point charges existed (they do not) and this configuration could be set up?

54 A charge of $6.0\ \mu C$ is to be split into two parts that are then separated by 3.0 mm. What is the maximum possible magnitude of the electrostatic force between those two parts?

55 Of the charge Q on a tiny sphere, a fraction α is to be transferred to a second, nearby sphere. The spheres can be treated as particles. (a) What value of α maximizes the magnitude F of the electrostatic force between the two spheres? What are the (b) smaller and (c) larger values of α that put F at half the maximum magnitude?

56 BIO FCP If a cat repeatedly rubs against your cotton slacks on a dry day, the charge transfer between the cat hair and the cotton can leave you with an excess charge of $-2.00\ \mu C$. (a) How many electrons are transferred between you and the cat?

You will gradually discharge via the floor, but if instead of waiting, you immediately reach toward a faucet, a painful spark can suddenly appear as your fingers near the faucet. (b) In that spark, do electrons flow from you to the faucet or vice versa? (c) Just before the spark appears, do you induce positive or negative charge in the faucet? (d) If, instead, the cat reaches a paw toward the faucet, which way do electrons flow in the resulting spark? (e) If you stroke a cat with a bare hand on a dry day, you should take care not to bring your fingers near the cat's nose or you will hurt it with a spark. Considering that cat hair is an insulator, explain how the spark can appear.

57 We know that the negative charge on the electron and the positive charge on the proton are equal. Suppose, however, that these magnitudes differ from each other by 0.00010%. With what force would two copper coins, placed 1.0 m apart, repel each other? Assume that each coin contains 3×10^{22} copper atoms. (*Hint:* A neutral copper atom contains 29 protons and 29 electrons.) What do you conclude?

58 In Fig. 21.16, particle 1 of charge $-80.0\ \mu C$ and particle 2 of charge $+40.0\ \mu C$ are held at separation $L = 20.0$ cm on an x axis. In unit-vector notation, what is the net electrostatic force on particle 3, of charge $q_3 = 20.0\ \mu C$, if particle 3 is placed at (a) $x = 40.0$ cm and (b) $x = 80.0$ cm? What should be the (c) x and (d) y coordinates of particle 3 if the net electrostatic force on it due to particles 1 and 2 is zero?

59 What is the total charge in coulombs of 75.0 kg of electrons?

60 GO In Fig. 21.33, six charged particles surround particle 7 at radial distances of either $d = 1.0$ cm or $2d$, as drawn. The charges are $q_1 = +2e$, $q_2 = +4e$, $q_3 = +e$, $q_4 = +4e$, $q_5 = +2e$, $q_6 = +8e$, $q_7 = +6e$, with $e = 1.60 \times 10^{-19}$ C. What is the magnitude of the net electrostatic force on particle 7?
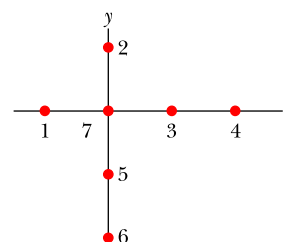

Figure 21.33 Problem 60.

61 Three charged particles form a triangle: Particle 1 with charge $Q_1 = 80.0$ nC is at xy coordinates (0, 3.00 mm), particle 2 with charge Q_2 is at (0, -3.00 mm), and particle 3 with charge $q = 18.0$ nC is at (4.00 mm, 0). In unit-vector notation, what is the electrostatic force on particle 3 due to the other two particles if Q_2 is equal to (a) 80.0 nC and (b) -80.0 nC?

62 SSM In Fig. 21.34, what are the (a) magnitude and (b) direction of the net electrostatic force on particle 4 due to the other three particles? All four particles are fixed in the xy plane, and $q_1 = -3.20 \times 10^{-19}$ C, $q_2 = +3.20 \times 10^{-19}$ C, $q_3 = +6.40 \times 10^{-19}$ C, $q_4 = +3.20 \times 10^{-19}$ C, $\theta_1 = 35.0°$, $d_1 = 3.00$ cm, and $d_2 = d_3 = 2.00$ cm.
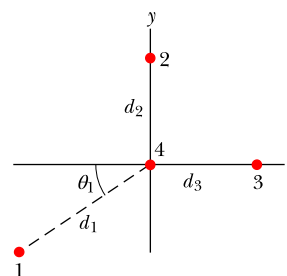

Figure 21.34 Problem 62.

63 *Charge of a penny.* A U.S. penny has mass $m = 3.11$ g and contains equal amounts of positive and negative charge. Assume the coin is made entirely of copper (molar mass $M = 63.5$ g/mol, atomic number $Z = 29$). (a) What is the magnitude q of these charges? (b) If the charges could be concentrated into two separate bundles held 100 m apart, what would be the attractive force between the bundles?

64 *Quarks.* What is the quark composition of (a) a proton, (b) a neutron, and (c) an antiproton? (The symbols can be in any order.) (d) When a nucleon undergoes beta-minus emission (see problem 67), what is the change in the quark composition?

65 *Electrostatic and gravitational forces.* The average distance r between the electron and the central proton in the hydrogen atom is 5.3×10^{-11} m. What are the magnitudes of (a) the electrostatic force and (b) the gravitational force acting between the particles? (c) Can the latter account for holding the atom together?

66 BIO *Electron capture in cancer therapy.* Certain radionuclides can decay by electron capture,

$$p + e^- \rightarrow n + \nu,$$
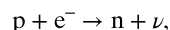
and then the daughter nuclei can release Auger–Meitner electrons. If the radionuclides are placed next to cancer cells, those released electrons can lethally damage the cells. What is the daughter atom if the parent atom is (a) iodine $^{123}_{53}$I, (b) iodine $^{125}_{53}$I, and (c) gallium $^{67}_{31}$Ga?

67 BIO *Beta-minus decay in cancer therapy.* Certain radionuclides undergo beta-minus decay, in which a neutron transforms into a proton (which remains in the nucleus) and releases an electron and a neutrino:

$$n \rightarrow p + e^- + \nu.$$
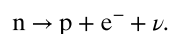
If the radionuclides are placed next to a tumor, the released electrons can lethally damage the tumor. What is the daughter atom if the parent atom is (a) iodine $^{131}_{53}$I, (b) copper $^{67}_{29}$Cu, and (c) yttrium $^{90}_{39}$Y? The first two are used for smaller tumors, the third for larger tumors.

68 *Smoke detectors.* Many household smoke detectors (Fig. 21.35) contain radioactive americium-241 $^{241}_{95}$Am, which is an alpha emitter. The alpha particles ionize the air (they strip electrons from the air molecules) between two charged plates. The freed electrons then flow to the positively charged plate. Thus, there is an electrical current between the plates. If smoke particles enter the air, they reduce that current, which triggers an alarm. What is the daughter nucleus produced by the alpha decay?


Figure 21.35 Problem 68.

69 BIO *Alpha decay in cancer therapy.* Certain radionuclides decay by emitting an alpha particle. To treat bone cancer, an alpha emitter such as radium $^{223}_{88}$Ra is attached to a carrier molecule that is then taken up by bone as though it were calcium. What is the daughter atom if the parent atom is (a) radium $^{223}_{88}$Ra, (b) radium $^{226}_{88}$Ra, and (c) actinium $^{225}_{89}$Ac?

70 BIO *Competitive decays in cancer diagnosis.* Some radionuclides can undergo either beta-plus decay or electron capture. What is the resulting daughter when carbon $^{11}_{6}$C undergoes (a) beta-plus decay and (b) electron capture? What is the daughter when $^{18}_{9}$F undergoes (c) beta-plus decay and (d) electron capture?

71 *Uranium fission.* If a slow neutron is captured by a uranium-235 nucleus (a large nucleus), the nucleus can fission (split) into two intermediate-size nuclei and release two or three neutrons. Here is one possibility:

$$^{235}_{92}U + n \rightarrow {}^{144}_{56}Ba + {}^{(a)}_{(b)}(c) + 3n.$$
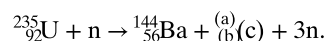
What numbers go into superscript (a) and subscript (b), and what chemical symbol goes into position (c)? (d) The intermediate-size nuclei have too many neutrons to be stable and so they decay by beta-minus decays, in which a neutron becomes a proton and the nucleus emits an electron and a neutrino (which has no charge). See Problem 67. What daughter nucleus results from the decay of $^{144}_{56}$Ba?

72 BIO *Gamma cameras.* To image organs in a patient's body, the patient is injected with a substance containing radioactive molybdenum $^{99}_{42}$Mo, which decays to technetium $^{99}_{43}$Tc, and then slid into a gamma camera (Fig. 21.36). The technetium is produced in a higher energy state but while in the gamma camera it reduces its energy by emitting a gamma ray. The patient is partially surrounded by an array of gamma-ray detectors that function much like a conventional camera capturing an image with visible light. The detection system produces an image of the patient showing the emission sites of the gamma rays. (Radionuclides that emit electrons, positrons, or alpha particles are not useful in deep imaging because those particles travel only a short distance through the body whereas the gamma rays can escape and reach the detectors.) In the decay of $^{99}_{42}$Mo to $^{99}_{43}$Tc, what particle is emitted in addition to a neutrino?

Figure 21.36 Problem 72.