

# Waves—II

## 17.1 SPEED OF SOUND

### Learning Objectives

After reading this module, you should be able to . . .

**17.1.1** Distinguish between a longitudinal wave and a transverse wave.

**17.1.2** Explain wavefronts and rays.

**17.1.3** Apply the relationship between the speed of sound through a material, the material's bulk modulus, and the material's density.

**17.1.4** Apply the relationship between the speed of sound, the distance traveled by a sound wave, and the time required to travel that distance.

### Key Idea

- Sound waves are longitudinal mechanical waves that can travel through solids, liquids, or gases. The speed  $v$  of a sound wave in a medium having bulk modulus  $B$  and density  $\rho$  is

$$v = \sqrt{\frac{B}{\rho}} \quad (\text{speed of sound}).$$

In air at 20°C, the speed of sound is 343 m/s.

## What Is Physics?

The physics of sound waves is the basis of countless studies in the research journals of many fields. Here are just a few examples. Some physiologists are concerned with how speech is produced, how speech impairment might be corrected, how hearing loss can be alleviated, and even how snoring is produced. Some acoustic engineers are concerned with improving the acoustics of cathedrals and concert halls, with reducing noise near freeways and road construction, and with reproducing music by speaker systems. Some aviation engineers are concerned with the shock waves produced by supersonic aircraft and the aircraft noise produced in communities near an airport. Some medical researchers are concerned with how noises produced by the heart and lungs can signal a medical problem in a patient. Some paleontologists are concerned with how a dinosaur's fossil might reveal the dinosaur's vocalizations. Some military engineers are concerned with how the sounds of sniper fire might allow a soldier to pinpoint the sniper's location, and, on the gentler side, some biologists are concerned with how a cat purrs. **FCP**

To begin our discussion of the physics of sound, we must first answer the question “What are sound waves?”

## Sound Waves

As we saw in Chapter 16, mechanical waves are waves that require a material medium to exist. There are two types of mechanical waves: *Transverse waves* involve oscillations perpendicular to the direction in which the wave travels; *longitudinal waves* involve oscillations parallel to the direction of wave travel.



**Figure 17.1.1** A loggerhead turtle is being checked with ultrasound (which has a frequency above your hearing range); an image of its interior is being produced on a monitor off to the right.

In this book, a **sound wave** is defined roughly as any longitudinal wave. Seismic prospecting teams use such waves to probe Earth's crust for oil. Ships carry sound-ranging gear (sonar) to detect underwater obstacles. Submarines use sound waves to stalk other submarines, largely by listening for the characteristic noises produced by the propulsion system. Figure 17.1.1 suggests how sound waves can be used to explore the soft tissues of an animal or human body. In this chapter we shall focus on sound waves that travel through the air and that are audible to people.

Figure 17.1.2 illustrates several ideas that we shall use in our discussions. Point *S* represents a tiny sound source, called a *point source*, that emits sound waves in all directions. The *wavefronts* and *rays* indicate the direction of travel and the spread of the sound waves. **Wavefronts** are surfaces over which the oscillations due to the sound wave have the same value; such surfaces are represented by whole or partial circles in a two-dimensional drawing for a point source. **Rays** are directed lines perpendicular to the wavefronts that indicate the direction of travel of the wavefronts. The short double arrows superimposed on the rays of Fig. 17.1.2 indicate that the longitudinal oscillations of the air are parallel to the rays.

Near a point source like that of Fig. 17.1.2, the wavefronts are spherical and spread out in three dimensions, and there the waves are said to be *spherical*. As the wavefronts move outward and their radii become larger, their curvature decreases. Far from the source, we approximate the wavefronts as planes (or lines on two-dimensional drawings), and the waves are said to be *planar*.

## The Speed of Sound

The speed of any mechanical wave, transverse or longitudinal, depends on both an inertial property of the medium (to store kinetic energy) and an elastic property of the medium (to store potential energy). Thus, we can generalize Eq. 16.2.5, which gives the speed of a transverse wave along a stretched string, by writing

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}}, \quad (17.1.1)$$

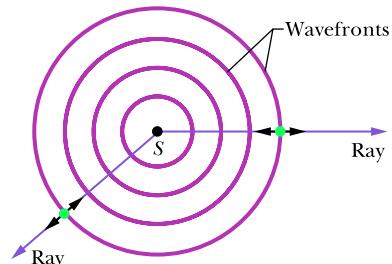
where (for transverse waves)  $\tau$  is the tension in the string and  $\mu$  is the string's linear density. If the medium is air and the wave is longitudinal, we can guess that the inertial property, corresponding to  $\mu$ , is the volume density  $\rho$  of air. What shall we put for the elastic property?

In a stretched string, potential energy is associated with the periodic stretching of the string elements as the wave passes through them. As a sound wave passes through air, potential energy is associated with periodic compressions and expansions of small volume elements of the air. The property that determines the extent to which an element of a medium changes in volume when the pressure (force per unit area) on it changes is the **bulk modulus**  $B$ , defined (from Eq. 12.3.4) as

$$B = -\frac{\Delta p}{\Delta V/V} \quad (\text{definition of bulk modulus}). \quad (17.1.2)$$

Here  $\Delta V/V$  is the fractional change in volume produced by a change in pressure  $\Delta p$ . As explained in Module 14.1, the SI unit for pressure is the newton per square meter, which is given a special name, the *pascal* (Pa). From Eq. 17.1.2 we see that the unit for  $B$  is also the pascal. The signs of  $\Delta p$  and  $\Delta V$  are always opposite: When we increase the pressure on an element ( $\Delta p$  is positive), its volume decreases ( $\Delta V$  is negative). We include a minus sign in Eq. 17.1.2 so that  $B$  is always a positive quantity. Now substituting  $B$  for  $\tau$  and  $\rho$  for  $\mu$  in Eq. 17.1.1 yields

$$v = \sqrt{\frac{B}{\rho}} \quad (\text{speed of sound}) \quad (17.1.3)$$



**Figure 17.1.2** A sound wave travels from a point source *S* through a three-dimensional medium. The wavefronts form spheres centered on *S*; the rays are radial to *S*. The short, double-headed arrows indicate that elements of the medium oscillate parallel to the rays.

as the speed of sound in a medium with bulk modulus  $B$  and density  $\rho$ . Table 17.1.1 lists the speed of sound in various media.

The density of water is almost 1000 times greater than the density of air. If this were the only relevant factor, we would expect from Eq. 17.1.3 that the speed of sound in water would be considerably less than the speed of sound in air. However, Table 17.1.1 shows us that the reverse is true. We conclude (again from Eq. 17.1.3) that the bulk modulus of water must be more than 1000 times greater than that of air. This is indeed the case. Water is much more incompressible than air, which (see Eq. 17.1.2) is another way of saying that its bulk modulus is much greater.

### Formal Derivation of Eq. 17.1.3

We now derive Eq. 17.1.3 by direct application of Newton's laws. Let a single pulse in which air is compressed travel (from right to left) with speed  $v$  through the air in a long tube, like that in Fig. 16.1.2. Let us run along with the pulse at that speed, so that the pulse appears to stand still in our reference frame. Figure 17.1.3a shows the situation as it is viewed from that frame. The pulse is standing still, and air is moving at speed  $v$  through it from left to right.

Let the pressure of the undisturbed air be  $p$  and the pressure inside the pulse be  $p + \Delta p$ , where  $\Delta p$  is positive due to the compression. Consider an element of air of thickness  $\Delta x$  and face area  $A$ , moving toward the pulse at speed  $v$ . As this element enters the pulse, the leading face of the element encounters a region of higher pressure, which slows the element to speed  $v + \Delta v$ , in which  $\Delta v$  is negative. This slowing is complete when the rear face of the element reaches the pulse, which requires time interval

$$\Delta t = \frac{\Delta x}{v}. \quad (17.1.4)$$

Let us apply Newton's second law to the element. During  $\Delta t$ , the average force on the element's trailing face is  $pA$  toward the right, and the average force on the leading face is  $(p + \Delta p)A$  toward the left (Fig. 17.1.3b). Therefore, the average net force on the element during  $\Delta t$  is

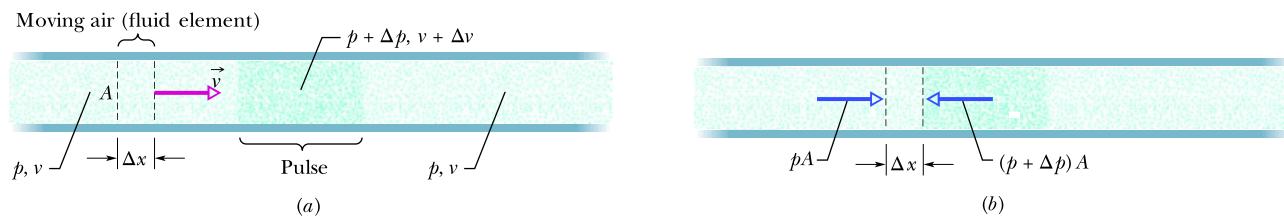
$$\begin{aligned} F &= pA - (p + \Delta p)A \\ &= -\Delta p A \quad (\text{net force}). \end{aligned} \quad (17.1.5)$$

The minus sign indicates that the net force on the air element is directed to the left in Fig. 17.1.3b. The volume of the element is  $A \Delta x$ , so with the aid of Eq. 17.1.4, we can write its mass as

$$\Delta m = \rho \Delta V = \rho A \Delta x = \rho A v \Delta t \quad (\text{mass}). \quad (17.1.6)$$

The average acceleration of the element during  $\Delta t$  is

$$a = \frac{\Delta v}{\Delta t} \quad (\text{acceleration}). \quad (17.1.7)$$



**Figure 17.1.3** A compression pulse is sent from right to left down a long air-filled tube. The reference frame of the figure is chosen so that the pulse is at rest and the air moves from left to right. (a) An element of air of width  $\Delta x$  moves toward the pulse with speed  $v$ . (b) The leading face of the element enters the pulse. The forces acting on the leading and trailing faces (due to air pressure) are shown.

**Table 17.1.1** The Speed of Sound<sup>a</sup>

Medium	Speed (m/s)
<i>Gases</i>	
Air (0°C)	331
Air (20°C)	343
Helium	965
Hydrogen	1284
<i>Liquids</i>	
Water (0°C)	1402
Water (20°C)	1482
Seawater <sup>b</sup>	1522
<i>Solids</i>	
Aluminum	6420
Steel	5941
Granite	6000

<sup>a</sup>At 0°C and 1 atm pressure, except where noted.

<sup>b</sup>At 20°C and 3.5% salinity.

Thus, from Newton's second law ( $F = ma$ ), we have, from Eqs. 17.1.5, 17.1.6, and 17.1.7,

$$-\Delta p A = (\rho A v \Delta t) \frac{\Delta v}{\Delta t}, \quad (17.1.8)$$

which we can write as

$$\rho v^2 = -\frac{\Delta p}{\Delta v/v}. \quad (17.1.9)$$

The air that occupies a volume  $V (= A v \Delta t)$  outside the pulse is compressed by an amount  $\Delta V (= A \Delta v \Delta t)$  as it enters the pulse. Thus,

$$\frac{\Delta V}{V} = \frac{A \Delta v \Delta t}{A v \Delta t} = \frac{\Delta v}{v}. \quad (17.1.10)$$

Substituting Eq. 17.1.10 and then Eq. 17.1.2 into Eq. 17.1.9 leads to

$$\rho v^2 = -\frac{\Delta p}{\Delta v/v} = -\frac{\Delta p}{\Delta V/V} = B. \quad (17.1.11)$$

Solving for  $v$  yields Eq. 17.1.3 for the speed of the air toward the right in Fig. 17.1.3, and thus for the actual speed of the pulse toward the left.

### Checkpoint 17.1.1

The same change  $\Delta p$  in pressure is applied to two materials with the same initial volume: Material A has a greater bulk modulus than material B. Which material undergoes the greater change in volume?

## 17.2 TRAVELING SOUND WAVES

### Learning Objectives

*After reading this module, you should be able to . . .*

- 17.2.1 For any particular time and position, calculate the displacement  $s(x, t)$  of an element of air as a sound wave travels through its location.
- 17.2.2 Given a displacement function  $s(x, t)$  for a sound wave, calculate the time between two given displacements.
- 17.2.3 Apply the relationships between wave speed  $v$ , angular frequency  $\omega$ , angular wave number  $k$ , wavelength  $\lambda$ , period  $T$ , and frequency  $f$ .
- 17.2.4 Sketch a graph of the displacement  $s(x)$  of an element of air as a function of position, and identify the amplitude  $s_m$  and wavelength  $\lambda$ .
- 17.2.5 For any particular time and position, calculate the pressure variation  $\Delta p$  (variation from atmospheric

pressure) of an element of air as a sound wave travels through its location.

- 17.2.6 Sketch a graph of the pressure variation  $\Delta p(x)$  of an element as a function of position, and identify the amplitude  $\Delta p_m$  and wavelength  $\lambda$ .
- 17.2.7 Apply the relationship between pressure-variation amplitude  $\Delta p_m$  and displacement amplitude  $s_m$ .
- 17.2.8 Given a graph of position  $s$  versus time for a sound wave, determine the amplitude  $s_m$  and the period  $T$ .
- 17.2.9 Given a graph of pressure variation  $\Delta p$  versus time for a sound wave, determine the amplitude  $\Delta p_m$  and the period  $T$ .

### Key Ideas

- A sound wave causes a longitudinal displacement  $s$  of a mass element in a medium as given by

$$s = s_m \cos(kx - \omega t),$$

where  $s_m$  is the displacement amplitude (maximum displacement) from equilibrium,  $k = 2\pi/\lambda$ , and  $\omega = 2\pi f$ ,  $\lambda$  and  $f$  being the wavelength and frequency, respectively, of the sound wave.

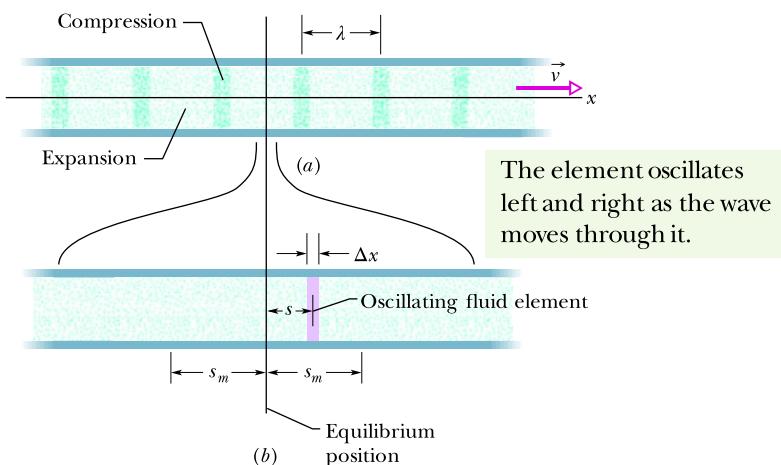
- The sound wave also causes a pressure change  $\Delta p$  of the medium from the equilibrium pressure:

$$\Delta p = \Delta p_m \sin(kx - \omega t),$$

where the pressure amplitude is

$$\Delta p_m = (v\rho\omega)s_m.$$

**Figure 17.2.1** (a) A sound wave, traveling through a long air-filled tube with speed  $v$ , consists of a moving, periodic pattern of expansions and compressions of the air. The wave is shown at an arbitrary instant. (b) A horizontally expanded view of a short piece of the tube. As the wave passes, an air element of thickness  $\Delta x$  oscillates left and right in simple harmonic motion about its equilibrium position. At the instant shown in (b), the element happens to be displaced a distance  $s$  to the right of its equilibrium position. Its maximum displacement, either right or left, is  $s_m$ .



## Traveling Sound Waves

Here we examine the displacements and pressure variations associated with a sinusoidal sound wave traveling through air. Figure 17.2.1a displays such a wave traveling rightward through a long air-filled tube. Recall from Chapter 16 that we can produce such a wave by sinusoidally moving a piston at the left end of the tube (as in Fig. 16.1.2). The piston's rightward motion moves the element of air next to the piston face and compresses that air; the piston's leftward motion allows the element of air to move back to the left and the pressure to decrease. As each element of air pushes on the next element in turn, the right-left motion of the air and the change in its pressure travel along the tube as a sound wave.

Consider the thin element of air of thickness  $\Delta x$  shown in Fig. 17.2.1b. As the wave travels through this portion of the tube, the element of air oscillates left and right in simple harmonic motion about its equilibrium position. Thus, the oscillations of each air element due to the traveling sound wave are like those of a string element due to a transverse wave, except that the air element oscillates *longitudinally* rather than *transversely*. Because string elements oscillate parallel to the  $y$  axis, we write their displacements in the form  $y(x, t)$ . Similarly, because air elements oscillate parallel to the  $x$  axis, we could write their displacements in the confusing form  $x(x, t)$ , but we shall use  $s(x, t)$  instead.

**Displacement.** To show that the displacements  $s(x, t)$  are sinusoidal functions of  $x$  and  $t$ , we can use either a sine function or a cosine function. In this chapter we use a cosine function, writing

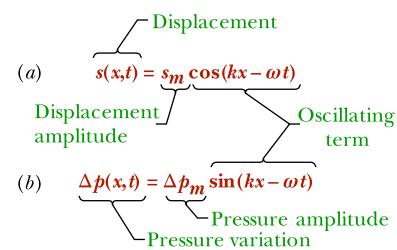
$$s(x, t) = s_m \cos(kx - \omega t). \quad (17.2.1)$$

Figure 17.2.2a labels the various parts of this equation. In it,  $s_m$  is the **displacement amplitude**—that is, the maximum displacement of the air element to either side of its equilibrium position (see Fig. 17.2.1b). The angular wave number  $k$ , angular frequency  $\omega$ , frequency  $f$ , wavelength  $\lambda$ , speed  $v$ , and period  $T$  for a sound (longitudinal) wave are defined and interrelated exactly as for a transverse wave, except that  $\lambda$  is now the distance (again along the direction of travel) in which the pattern of compression and expansion due to the wave begins to repeat itself (see Fig. 17.2.1a). (We assume  $s_m$  is much less than  $\lambda$ .)

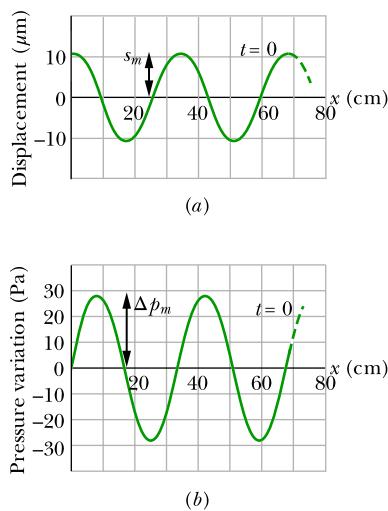
**Pressure.** As the wave moves, the air pressure at any position  $x$  in Fig. 17.2.1a varies sinusoidally, as we prove next. To describe this variation we write

$$\Delta p(x, t) = \Delta p_m \sin(kx - \omega t). \quad (17.2.2)$$

Figure 17.2.2b labels the various parts of this equation. A negative value of  $\Delta p$  in Eq. 17.2.2 corresponds to an expansion of the air, and a positive value to a



**Figure 17.2.2** (a) The displacement function and (b) the pressure-variation function of a traveling sound wave consist of an amplitude and an oscillating term.



**Figure 17.2.3** (a) A plot of the displacement function (Eq. 17.2.1) for  $t = 0$ . (b) A similar plot of the pressure-variation function (Eq. 17.2.2). Both plots are for a 1000 Hz sound wave whose pressure amplitude is at the threshold of pain.

compression. Here  $\Delta p_m$  is the **pressure amplitude**, which is the maximum increase or decrease in pressure due to the wave;  $\Delta p_m$  is normally very much less than the pressure  $p$  present when there is no wave. As we shall prove, the pressure amplitude  $\Delta p_m$  is related to the displacement amplitude  $s_m$  in Eq. 17.2.1 by

$$\Delta p_m = (v\rho\omega)s_m. \quad (17.2.3)$$

Figure 17.2.3 shows plots of Eqs. 17.2.1 and 17.2.2 at  $t = 0$ ; with time, the two curves would move rightward along the horizontal axes. Note that the displacement and pressure variation are  $\pi/2$  rad (or  $90^\circ$ ) out of phase. Thus, for example, the pressure variation  $\Delta p$  at any point along the wave is zero when the displacement there is a maximum.

### Checkpoint 17.2.1

When the oscillating air element in Fig. 17.2.1b is moving rightward through the point of zero displacement, is the pressure in the element at its equilibrium value, just beginning to increase, or just beginning to decrease?

### Derivation of Eqs. 17.2.2 and 17.2.3

Figure 17.2.1b shows an oscillating element of air of cross-sectional area  $A$  and thickness  $\Delta x$ , with its center displaced from its equilibrium position by distance  $s$ . From Eq. 17.1.2 we can write, for the pressure variation in the displaced element,

$$\Delta p = -B \frac{\Delta V}{V}. \quad (17.2.4)$$

The quantity  $V$  in Eq. 17.2.4 is the volume of the element, given by

$$V = A \Delta x. \quad (17.2.5)$$

The quantity  $\Delta V$  in Eq. 17.2.4 is the change in volume that occurs when the element is displaced. This volume change comes about because the displacements of the two faces of the element are not quite the same, differing by some amount  $\Delta s$ . Thus, we can write the change in volume as

$$\Delta V = A \Delta s. \quad (17.2.6)$$

Substituting Eqs. 17.2.5 and 17.2.6 into Eq. 17.2.4 and passing to the differential limit yield

$$\Delta p = -B \frac{\Delta s}{\Delta x} = -B \frac{\partial s}{\partial x}. \quad (17.2.7)$$

The symbols  $\partial$  indicate that the derivative in Eq. 17.2.7 is a *partial derivative*, which tells us how  $s$  changes with  $x$  when the time  $t$  is fixed. From Eq. 17.2.1 we then have, treating  $t$  as a constant,

$$\frac{\partial s}{\partial x} = \frac{\partial}{\partial x} [s_m \cos(kx - \omega t)] = -k s_m \sin(kx - \omega t).$$

Substituting this quantity for the partial derivative in Eq. 17.2.7 yields

$$\Delta p = B k s_m \sin(kx - \omega t).$$

This tells us that the pressure varies as a sinusoidal function of time and that the amplitude of the variation is equal to the terms in front of the sine function. Setting  $\Delta p_m = B k s_m$ , this yields Eq. 17.2.2, which we set out to prove.

Using Eq. 17.1.3, we can now write

$$\Delta p_m = (Bk)s_m = (v^2\rho k)s_m.$$

Equation 17.2.3, which we also wanted to prove, follows at once if we substitute  $\omega/v$  for  $k$  from Eq. 16.1.12.

### Sample Problem 17.2.1 Pressure amplitude, displacement amplitude

The maximum pressure amplitude  $\Delta p_m$  that the human ear can tolerate in loud sounds is about 28 Pa (which is very much less than the normal air pressure of about  $10^5$  Pa). What is the displacement amplitude  $s_m$  for such a sound in air of density  $\rho = 1.21 \text{ kg/m}^3$ , at a frequency of 1000 Hz and a speed of 343 m/s?

#### KEY IDEA

The displacement amplitude  $s_m$  of a sound wave is related to the pressure amplitude  $\Delta p_m$  of the wave according to Eq. 17.2.3.

**Calculations:** Solving that equation for  $s_m$  yields

$$s_m = \frac{\Delta p_m}{v\rho\omega} = \frac{\Delta p_m}{v\rho(2\pi f)}.$$

Substituting known data then gives us

$$s_m = \frac{28 \text{ Pa}}{(343 \text{ m/s})(1.21 \text{ kg/m}^3)(2\pi)(1000 \text{ Hz})} \\ = 1.1 \times 10^{-5} \text{ m} = 11 \mu\text{m}. \quad (\text{Answer})$$

That is only about one-seventh the thickness of a book page. Obviously, the displacement amplitude of even the loudest sound that the ear can tolerate is very small. Temporary exposure to such loud sound produces temporary hearing loss, probably due to a decrease in blood supply to the inner ear. Prolonged exposure produces permanent damage.

The pressure amplitude  $\Delta p_m$  for the *faintest* detectable sound at 1000 Hz is  $2.8 \times 10^{-5}$  Pa. Proceeding as above leads to  $s_m = 1.1 \times 10^{-11} \text{ m}$  or 11 pm, which is about one-tenth the radius of a typical atom. The ear is indeed a sensitive detector of sound waves.

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## 17.3 INTERFERENCE

### Learning Objectives

After reading this module, you should be able to . . .

**17.3.1** If two waves with the same wavelength begin in phase but reach a common point by traveling along different paths, calculate their phase difference  $\phi$  at that point by relating the path length difference  $\Delta L$  to the wavelength  $\lambda$ .

**17.3.2** Given the phase difference between two sound waves with the same amplitude, wavelength, and

travel direction, determine the type of interference between the waves (fully destructive interference, fully constructive interference, or indeterminate interference).

**17.3.3** Convert a phase difference between radians, degrees, and number of wavelengths.

### Key Ideas

- The interference of two sound waves with identical wavelengths passing through a common point depends on their phase difference  $\phi$  there. If the sound waves were emitted in phase and are traveling in approximately the same direction,  $\phi$  is given by

$$\phi = \frac{\Delta L}{\lambda} 2\pi,$$

where  $\Delta L$  is their path length difference.

- Fully constructive interference occurs when  $\phi$  is an integer multiple of  $2\pi$ ,

$$\phi = m(2\pi), \quad \text{for } m = 0, 1, 2, \dots,$$

and, equivalently, when  $\Delta L$  is related to wavelength  $\lambda$  by

$$\frac{\Delta L}{\lambda} = 0, 1, 2, \dots$$

- Fully destructive interference occurs when  $\phi$  is an odd multiple of  $\pi$ ,

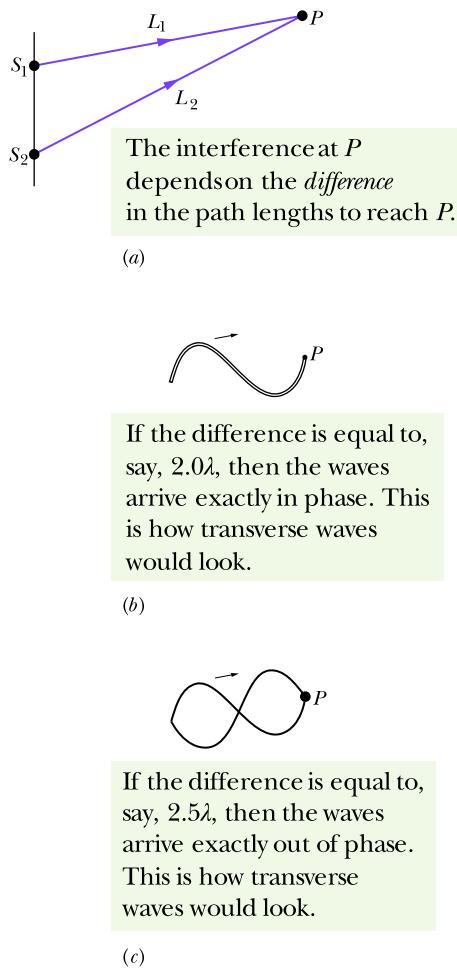
$$\phi = (2m + 1)\pi, \quad \text{for } m = 0, 1, 2, \dots,$$

and

$$\frac{\Delta L}{\lambda} = 0.5, 1.5, 2.5, \dots$$

## Interference

Like transverse waves, sound waves can undergo interference. In fact, we can write equations for the interference as we did in Module 16.5 for transverse



**Figure 17.3.1** (a) Two point sources  $S_1$  and  $S_2$  emit spherical sound waves in phase. The rays indicate that the waves pass through a common point  $P$ . The waves (represented with *transverse* waves) arrive at  $P$  (b) exactly in phase and (c) exactly out of phase.

waves. Suppose two sound waves with the same amplitude and wavelength are traveling in the positive direction of an  $x$  axis with a phase difference of  $\phi$ . We can express the waves in the form of Eqs. 16.5.2 and 16.5.3 but, to be consistent with Eq. 17.2.1, we use cosine functions instead of sine functions:

$$s_1(x, t) = s_m \cos(kx - \omega t)$$

and

$$s_2(x, t) = s_m \cos(kx - \omega t + \phi).$$

These waves overlap and interfere. From Eq. 16.5.6, we can write the resultant wave as

$$s' = [2s_m \cos \frac{1}{2}\phi] \cos(kx - \omega t + \frac{1}{2}\phi).$$

As we saw with transverse waves, the resultant wave is itself a traveling wave. Its amplitude is the magnitude

$$s'_m = |2s_m \cos \frac{1}{2}\phi|. \quad (17.3.1)$$

As with transverse waves, the value of  $\phi$  determines what type of interference the individual waves undergo.

One way to control  $\phi$  is to send the waves along paths with different lengths. Figure 17.3.1a shows how we can set up such a situation: Two point sources  $S_1$  and  $S_2$  emit sound waves that are in phase and of identical wavelength  $\lambda$ . Thus, the *sources* themselves are said to be in phase; that is, as the waves emerge from the sources, their displacements are always identical. We are interested in the waves that then travel through point  $P$  in Fig. 17.3.1a. We assume that the distance to  $P$  is much greater than the distance between the sources so that we can approximate the waves as traveling in the same direction at  $P$ .

If the waves traveled along paths with identical lengths to reach point  $P$ , they would be in phase there. As with transverse waves, this means that they would undergo fully constructive interference there. However, in Fig. 17.3.1a, path  $L_2$  traveled by the wave from  $S_2$  is longer than path  $L_1$  traveled by the wave from  $S_1$ . The difference in path lengths means that the waves may not be in phase at point  $P$ . In other words, their phase difference  $\phi$  at  $P$  depends on their **path length difference**  $\Delta L = |L_2 - L_1|$ .

To relate phase difference  $\phi$  to path length difference  $\Delta L$ , we recall (from Module 16.1) that a phase difference of  $2\pi$  rad corresponds to one wavelength. Thus, we can write the proportion

$$\frac{\phi}{2\pi} = \frac{\Delta L}{\lambda}, \quad (17.3.2)$$

from which

$$\phi = \frac{\Delta L}{\lambda} 2\pi. \quad (17.3.3)$$

Fully constructive interference occurs when  $\phi$  is zero,  $2\pi$ , or any integer multiple of  $2\pi$ . We can write this condition as

$$\phi = m(2\pi), \quad \text{for } m = 0, 1, 2, \dots \quad (\text{fully constructive interference}). \quad (17.3.4)$$

From Eq. 17.3.3, this occurs when the ratio  $\Delta L/\lambda$  is

$$\frac{\Delta L}{\lambda} = 0, 1, 2, \dots \quad (\text{fully constructive interference}). \quad (17.3.5)$$

For example, if the path length difference  $\Delta L = |L_2 - L_1|$  in Fig. 17.3.1a is equal to  $2\lambda$ , then  $\Delta L/\lambda = 2$  and the waves undergo fully constructive interference at

point  $P$  (Fig. 17.3.1b). The interference is fully constructive because the wave from  $S_2$  is phase-shifted relative to the wave from  $S_1$  by  $2\lambda$ , putting the two waves *exactly in phase* at  $P$ .

Fully destructive interference occurs when  $\phi$  is an odd multiple of  $\pi$ :

$$\phi = (2m + 1)\pi, \quad \text{for } m = 0, 1, 2, \dots \quad (\text{fully destructive interference}). \quad (17.3.6)$$

From Eq. 17.3.3, this occurs when the ratio  $\Delta L/\lambda$  is

$$\frac{\Delta L}{\lambda} = 0.5, 1.5, 2.5, \dots \quad (\text{fully destructive interference}). \quad (17.3.7)$$

For example, if the path length difference  $\Delta L = |L_2 - L_1|$  in Fig. 17.3.1a is equal to  $2.5\lambda$ , then  $\Delta L/\lambda = 2.5$  and the waves undergo fully destructive interference at point  $P$  (Fig. 17.3.1c). The interference is fully destructive because the wave from  $S_2$  is phase-shifted relative to the wave from  $S_1$  by 2.5 wavelengths, which puts the two waves *exactly out of phase* at  $P$ .

Of course, two waves could produce intermediate interference as, say, when  $\Delta L/\lambda = 1.2$ . This would be closer to fully constructive interference ( $\Delta L/\lambda = 1.0$ ) than to fully destructive interference ( $\Delta L/\lambda = 1.5$ ).

### Checkpoint 17.3.1

Here are three pairs of sound waves. The waves in each pair are sent along the same axis so that they undergo interference. Rank the pairs according to the amplitude of the resultant wave, greatest amplitude.

Pair A:

$$\begin{aligned}s_1(x, t) &= s_m \cos(kx - \omega t) \\ s_2(x, t) &= s_m \cos(kx - \omega t + 0.90\pi)\end{aligned}$$

Pair B:

$$\begin{aligned}s_1(x, t) &= s_m \cos(kx - \omega t) \\ s_2(x, t) &= s_m \cos(kx - \omega t + 1.10\pi)\end{aligned}$$

Pair C:

$$\begin{aligned}s_1(x, t) &= s_m \cos(kx - \omega t) \\ s_2(x, t) &= s_m \cos(kx - \omega t + 0.20\pi)\end{aligned}$$

### Sample Problem 17.3.1 Interference points along a big circle

In Fig. 17.3.2a, two point sources  $S_1$  and  $S_2$ , which are in phase and separated by distance  $D = 1.5\lambda$ , emit identical sound waves of wavelength  $\lambda$ .

(a) What is the path length difference of the waves from  $S_1$  and  $S_2$  at point  $P_1$ , which lies on the perpendicular bisector of distance  $D$ , at a distance greater than  $D$  from the sources (Fig. 17.3.2b)? (That is, what is the difference in the distance from source  $S_1$  to point  $P_1$  and the distance from source  $S_2$  to  $P_1$ ?) What type of interference occurs at  $P_1$ ?

**Reasoning:** Because the waves travel identical distances to reach  $P_1$ , their path length difference is

$$\Delta L = 0. \quad (\text{Answer})$$

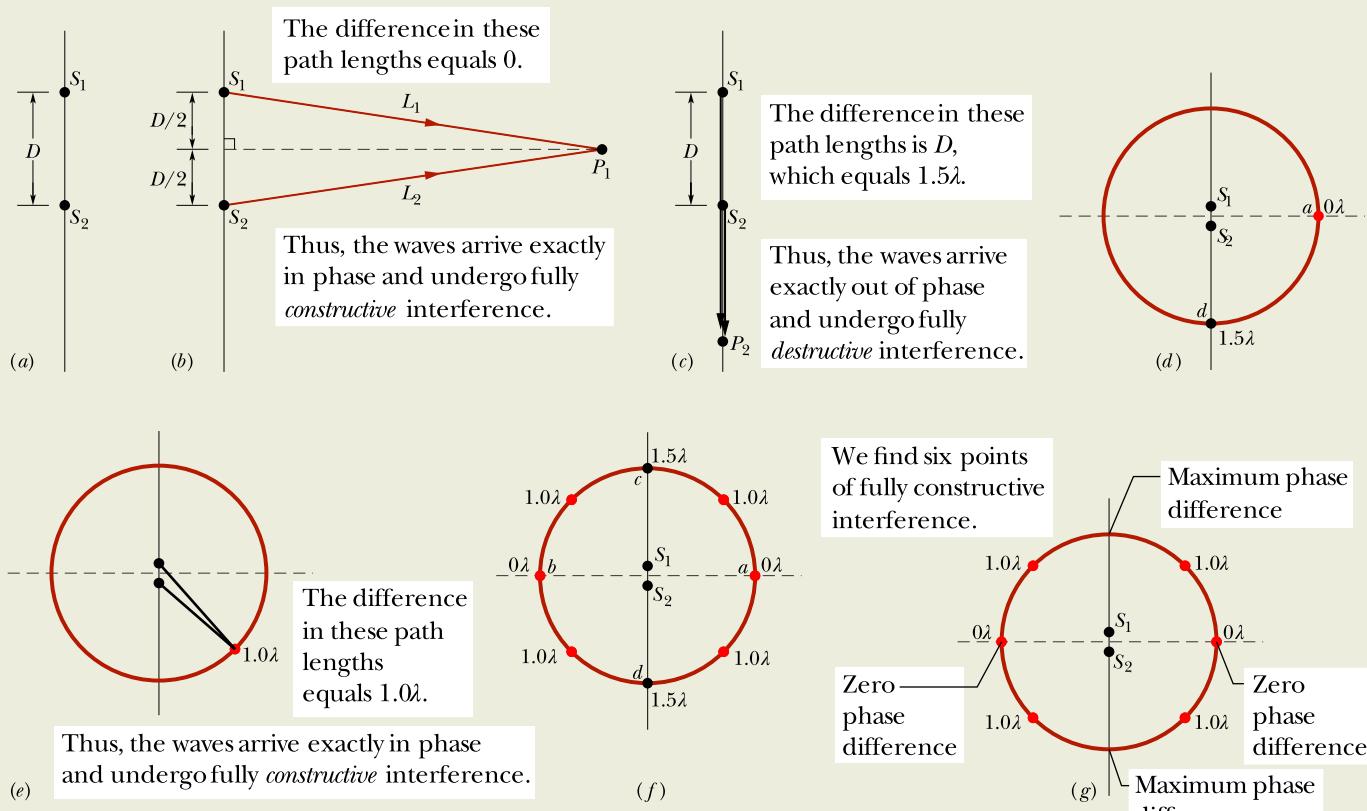
From Eq. 17.3.5, this means that the waves undergo fully constructive interference at  $P_1$  because they start in phase at the sources and reach  $P_1$  in phase.

(b) What are the path length difference and type of interference at point  $P_2$  in Fig. 17.3.2c?

**Reasoning:** The wave from  $S_1$  travels the extra distance  $D$  ( $= 1.5\lambda$ ) to reach  $P_2$ . Thus, the path length difference is

$$\Delta L = 1.5\lambda. \quad (\text{Answer})$$

From Eq. 17.3.7, this means that the waves are exactly out of phase at  $P_2$  and undergo fully destructive interference there.



**Figure 17.3.2** (a) Two point sources  $S_1$  and  $S_2$ , separated by distance  $D$ , emit spherical sound waves in phase. (b) The waves travel equal distances to reach point  $P_1$ . (c) Point  $P_2$  is on the line extending through  $S_1$  and  $S_2$ . (d) We move around a large circle. (e) Another point of fully constructive interference. (f) Using symmetry to determine other points. (g) The six points of fully constructive interference.

(c) Figure 17.3.2d shows a circle with a radius much greater than  $D$ , centered on the midpoint between sources  $S_1$  and  $S_2$ . What is the number of points  $N$  around this circle at which the interference is fully constructive? (That is, at how many points do the waves arrive exactly in phase?)

**Reasoning:** Starting at point  $a$ , let's move clockwise along the circle to point  $d$ . As we move, path length difference  $\Delta L$  increases and so the type of interference changes. From (a), we know that  $\Delta L = 0\lambda$  at point  $a$ . From (b), we know that  $\Delta L = 1.5\lambda$  at point  $d$ . Thus, there must be one point between  $a$  and  $d$  at which

$\Delta L = \lambda$  (Fig. 17.3.2e). From Eq. 17.3.5, fully constructive interference occurs at that point. Also, there can be no other point along the way from point  $a$  to point  $d$  at which fully constructive interference occurs, because there is no other integer than 1 between 0 at point  $a$  and 1.5 at point  $d$ .

We can now use symmetry to locate other points of fully constructive or destructive interference (Fig. 17.3.2f). Symmetry about line  $cd$  gives us point  $b$ , at which  $\Delta L = 0\lambda$ . Also, there are three more points at which  $\Delta L = \lambda$ . In all (Fig. 17.3.2g) we have

$$N = 6.$$

(Answer)

# 17.4 INTENSITY AND SOUND LEVEL

## Learning Objectives

After reading this module, you should be able to . . .

- 17.4.1 Calculate the sound intensity  $I$  at a surface as the ratio of the power  $P$  to the surface area  $A$ .
- 17.4.2 Apply the relationship between the sound intensity  $I$  and the displacement amplitude  $s_m$  of the sound wave.
- 17.4.3 Identify an isotropic point source of sound.
- 17.4.4 For an isotropic point source, apply the relationship involving the emitting power  $P_s$ , the distance

$r$  to a detector, and the sound intensity  $I$  at the detector.

- 17.4.5 Apply the relationship between the sound level  $\beta$ , the sound intensity  $I$ , and the standard reference intensity  $I_0$ .
- 17.4.6 Evaluate a logarithm function (log) and an anti-logarithm function ( $\log^{-1}$ ).
- 17.4.7 Relate the change in a sound level to the change in sound intensity.

## Key Ideas

- The intensity  $I$  of a sound wave at a surface is the average rate per unit area at which energy is transferred by the wave through or onto the surface:

$$I = \frac{P}{A},$$

where  $P$  is the time rate of energy transfer (power) of the sound wave and  $A$  is the area of the surface intercepting the sound. The intensity  $I$  is related to the displacement amplitude  $s_m$  of the sound wave by

$$I = \frac{1}{2} \rho v \omega^2 s_m^2.$$

- The intensity at a distance  $r$  from a point source that emits sound waves of power  $P_s$  equally in all directions (isotropically) is

$$I = \frac{P_s}{4\pi r^2}.$$

- The sound level  $\beta$  in decibels (dB) is defined as

$$\beta = (10 \text{ dB}) \log \frac{I}{I_0},$$

where  $I_0 (= 10^{-12} \text{ W/m}^2)$  is a reference intensity level to which all intensities are compared. For every factor-of-10 increase in intensity, 10 dB is added to the sound level.

## Intensity and Sound Level

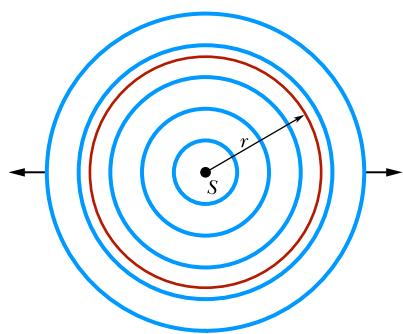
If you have ever tried to sleep while someone played loud music nearby, you are well aware that there is more to sound than frequency, wavelength, and speed. There is also intensity. The **intensity**  $I$  of a sound wave at a surface is the average rate per unit area at which energy is transferred by the wave through or onto the surface. We can write this as

$$I = \frac{P}{A}, \quad (17.4.1)$$

where  $P$  is the time rate of energy transfer (the power) of the sound wave and  $A$  is the area of the surface intercepting the sound. As we shall derive shortly, the intensity  $I$  is related to the displacement amplitude  $s_m$  of the sound wave by

$$I = \frac{1}{2} \rho v \omega^2 s_m^2. \quad (17.4.2)$$

Intensity can be measured on a detector. *Loudness* is a perception, something that you sense. The two can differ because your perception depends on factors such as the sensitivity of your hearing mechanism to various frequencies.



**Figure 17.4.1** A point source  $S$  emits sound waves uniformly in all directions. The waves pass through an imaginary sphere of radius  $r$  that is centered on  $S$ .

### Variation of Intensity with Distance

How intensity varies with distance from a real sound source is often complex. Some real sources (like loudspeakers) may transmit sound only in particular directions, and the environment usually produces echoes (reflected sound waves) that overlap the direct sound waves. In some situations, however, we can ignore echoes and assume that the sound source is a point source that emits the sound *isotropically*—that is, with equal intensity in all directions. The wavefronts spreading from such an isotropic point source  $S$  at a particular instant are shown in Fig. 17.4.1.

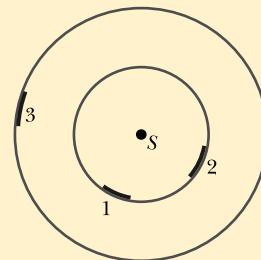
Let us assume that the mechanical energy of the sound waves is conserved as they spread from this source. Let us also center an imaginary sphere of radius  $r$  on the source, as shown in Fig. 17.4.1. All the energy emitted by the source must pass through the surface of the sphere. Thus, the time rate at which energy is transferred through the surface by the sound waves must equal the time rate at which energy is emitted by the source (that is, the power  $P_s$  of the source). From Eq. 17.4.1, the intensity  $I$  at the sphere must then be

$$I = \frac{P_s}{4\pi r^2}, \quad (17.4.3)$$

where  $4\pi r^2$  is the area of the sphere. Equation 17.4.3 tells us that the intensity of sound from an isotropic point source decreases with the square of the distance  $r$  from the source.

### Checkpoint 17.4.1

The figure indicates three small patches 1, 2, and 3 that lie on the surfaces of two imaginary spheres; the spheres are centered on an isotropic point source  $S$  of sound. The rates at which energy is transmitted through the three patches by the sound waves are equal. Rank the patches according to (a) the intensity of the sound on them and (b) their area, greatest first.



### The Decibel Scale

The displacement amplitude at the human ear ranges from about  $10^{-5}$  m for the loudest tolerable sound to about  $10^{-11}$  m for the faintest detectable sound, a ratio of  $10^6$ . From Eq. 17.4.2 we see that the intensity of a sound varies as the *square* of its amplitude, so the ratio of intensities at these two limits of the human auditory system is  $10^{12}$ . Humans can hear over an enormous range of intensities.

We deal with such an enormous range of values by using logarithms. Consider the relation

$$y = \log x,$$

in which  $x$  and  $y$  are variables. It is a property of this equation that if we multiply  $x$  by 10, then  $y$  increases by 1. To see this, we write

$$y' = \log(10x) = \log 10 + \log x = 1 + y.$$

Similarly, if we multiply  $x$  by  $10^{12}$ ,  $y$  increases by only 12.



© Ben Rose

Sound can cause the wall of a drinking glass to oscillate. If the sound produces a standing wave of oscillations and if the intensity of the sound is large enough, the glass will shatter.

Thus, instead of speaking of the intensity  $I$  of a sound wave, it is much more convenient to speak of its **sound level**  $\beta$ , defined as

$$\beta = (10 \text{ dB}) \log \frac{I}{I_0}. \quad (17.4.4)$$

Here dB is the abbreviation for **decibel**, the unit of sound level, a name that was chosen to recognize the work of Alexander Graham Bell.  $I_0$  in Eq. 17.4.4 is a standard reference intensity ( $= 10^{-12} \text{ W/m}^2$ ), chosen because it is near the lower limit of the human range of hearing. For  $I = I_0$ , Eq. 17.4.4 gives  $\beta = 10 \log 1 = 0$ , so our standard reference level corresponds to zero decibels. Then  $\beta$  increases by 10 dB every time the sound intensity increases by an order of magnitude (a factor of 10). Thus,  $\beta = 40$  corresponds to an intensity that is  $10^4$  times the standard reference level. Table 17.4.1 lists the sound levels for a variety of environments.

### Derivation of Eq. 17.4.2

Consider, in Fig. 17.2.1a, a thin slice of air of thickness  $dx$ , area  $A$ , and mass  $dm$ , oscillating back and forth as the sound wave of Eq. 17.2.1 passes through it. The kinetic energy  $dK$  of the slice of air is

$$dK = \frac{1}{2} dm v_s^2. \quad (17.4.5)$$

Here  $v_s$  is not the speed of the wave but the speed of the oscillating element of air, obtained from Eq. 17.2.1 as

$$v_s = \frac{ds}{dt} = -\omega s_m \sin(kx - \omega t).$$

Using this relation and putting  $dm = \rho A dx$  allow us to rewrite Eq. 17.4.5 as

$$dK = \frac{1}{2} (\rho A dx) (-\omega s_m)^2 \sin^2(kx - \omega t). \quad (17.4.6)$$

Dividing Eq. 17.4.6 by  $dt$  gives the rate at which kinetic energy moves along with the wave. As we saw in Chapter 16 for transverse waves,  $dx/dt$  is the wave speed  $v$ , so we have

$$\frac{dK}{dt} = \frac{1}{2} \rho A v \omega^2 s_m^2 \sin^2(kx - \omega t). \quad (17.4.7)$$

The *average* rate at which kinetic energy is transported is

$$\begin{aligned} \left( \frac{dK}{dt} \right)_{\text{avg}} &= \frac{1}{2} \rho A v \omega^2 s_m^2 [\sin^2(kx - \omega t)]_{\text{avg}} \\ &= \frac{1}{4} \rho A v \omega^2 s_m^2. \end{aligned} \quad (17.4.8)$$

To obtain this equation, we have used the fact that the average value of the square of a sine (or a cosine) function over one full oscillation is  $\frac{1}{2}$ .

We assume that *potential* energy is carried along with the wave at this same average rate. The wave intensity  $I$ , which is the average rate per unit area at which energy of both kinds is transmitted by the wave, is then, from Eq. 17.4.8,

$$I = \frac{2(dK/dt)_{\text{avg}}}{A} = \frac{1}{2} \rho v \omega^2 s_m^2,$$

which is Eq. 17.4.2, the equation we set out to derive.

**Table 17.4.1 Some Sound Levels (dB)**

Hearing threshold	0
Rustle of leaves	10
Conversation	60
Rock concert	110
Pain threshold	120
Jet engine	130

### Sample Problem 17.4.1 Led Zeppelin

During a 1969 outdoor concert by Led Zeppelin (Fig. 17.4.2), the maximum displacement amplitude  $s_m$  of the sound waves during the song “Heartbreaker” was  $70.0\text{ }\mu\text{m}$ , which is the thickness of a common book page. What was the pressure amplitude and intensity? Use an air density of  $1.21\text{ kg/m}^3$ , a sound speed of  $343\text{ m/s}$ , and a frequency of  $1000\text{ Hz}$ .



Chris Walter/WireImage/Getty Images

Figure 17.4.2 Led Zeppelin

### KEY IDEA

The displacement amplitude  $s_m$  of a sound wave is related to the pressure amplitude  $\Delta p_m$  of the wave according to Eq. 17.2.3.

**Calculations:** Substituting data into that equation yields

$$\begin{aligned}\Delta p_m &= \nu\rho\omega s_m = \nu\rho(2\pi f)s_m \\ &= (343\text{ m/s})(1.21\text{ kg/m}^3)(2\pi)(1000\text{ Hz})(70 \times 10^{-6}\text{ m}) \\ &= 182.53\text{ Pa} \approx 183\text{ Pa.}\end{aligned}\quad (\text{Answer})$$

The maximum pressure amplitude that the human ear can tolerate is about  $28\text{ Pa}$ . So, standing directly in front of the speaker systems during the Zeppelin concert was impossible without wearing hearing protection, even if you were a fanatic Zeppelin fan. From Eq. 17.4.2 (with the units suppressed), we find that the intensity was

$$\begin{aligned}I &= \frac{1}{2}\nu\rho(2\pi f)^2 s_m^2 \\ &= \frac{1}{2}(343)(1.21)(2\pi)^2(1000)^2(70.0 \times 10^{-6})^2 \\ &= 40.1\text{ W/m}^2.\end{aligned}\quad (\text{Answer})$$

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## 17.5 SOURCES OF MUSICAL SOUND

### Learning Objectives

After reading this module, you should be able to . . .

- 17.5.1 Using standing wave patterns for string waves, sketch the standing wave patterns for the first several acoustical harmonics of a pipe with only one open end and with two open ends.
- 17.5.2 For a standing wave of sound, relate the distance between nodes and the wavelength.

### Key Ideas

- Standing sound wave patterns can be set up in pipes (that is, resonance can be set up) if sound of the proper wavelength is introduced in the pipe.
- A pipe open at both ends will resonate at frequencies

$$f = \frac{v}{\lambda} = \frac{nv}{2L}, \quad n = 1, 2, 3, \dots,$$

- 17.5.3 Identify which type of pipe has even harmonics.

- 17.5.4 For any given harmonic and for a pipe with only one open end or with two open ends, apply the relationships between the pipe length  $L$ , the speed of sound  $v$ , the wavelength  $\lambda$ , the harmonic frequency  $f$ , and the harmonic number  $n$ .

where  $v$  is the speed of sound in the air in the pipe.

- For a pipe closed at one end and open at the other, the resonant frequencies are

$$f = \frac{v}{\lambda} = \frac{nv}{4L}, \quad n = 1, 3, 5, \dots$$

### Sources of Musical Sound

Musical sounds can be set up by oscillating strings (guitar, piano, violin), membranes (kettledrum, snare drum), air columns (flute, oboe, pipe organ, and the didgeridoo of Fig. 17.5.1), wooden blocks or steel bars (marimba, xylophone),

and many other oscillating bodies. Most common instruments involve more than a single oscillating part.

FCP

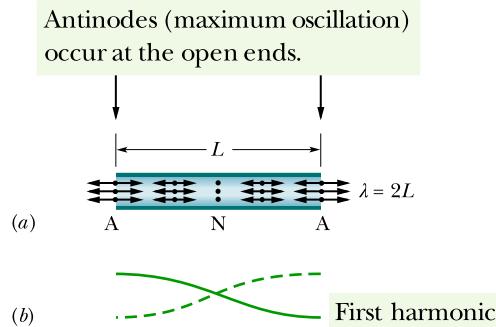
Recall from Chapter 16 that standing waves can be set up on a stretched string that is fixed at both ends. They arise because waves traveling along the string are reflected back onto the string at each end. If the wavelength of the waves is suitably matched to the length of the string, the superposition of waves traveling in opposite directions produces a standing wave pattern (or oscillation mode). The wavelength required of the waves for such a match is one that corresponds to a *resonant frequency* of the string. The advantage of setting up standing waves is that the string then oscillates with a large, sustained amplitude, pushing back and forth against the surrounding air and thus generating a noticeable sound wave with the same frequency as the oscillations of the string. This production of sound is of obvious importance to, say, a guitarist.

**Sound Waves.** We can set up standing waves of sound in an air-filled pipe in a similar way. As sound waves travel through the air in the pipe, they are reflected at each end and travel back through the pipe. (The reflection occurs even if an end is open, but the reflection is not as complete as when the end is closed.) If the wavelength of the sound waves is suitably matched to the length of the pipe, the superposition of waves traveling in opposite directions through the pipe sets up a standing wave pattern. The wavelength required of the sound waves for such a match is one that corresponds to a resonant frequency of the pipe. The advantage of such a standing wave is that the air in the pipe oscillates with a large, sustained amplitude, emitting at any open end a sound wave that has the same frequency as the oscillations in the pipe. This emission of sound is of obvious importance to, say, an organist.

Many other aspects of standing sound wave patterns are similar to those of string waves: The closed end of a pipe is like the fixed end of a string in that there must be a node (zero displacement) there, and the open end of a pipe is like the end of a string attached to a freely moving ring, as in Fig. 16.7.3b, in that there must be an antinode there. (Actually, the antinode for the open end of a pipe is located slightly beyond the end, but we shall not dwell on that detail.)

**Two Open Ends.** The simplest standing wave pattern that can be set up in a pipe with two open ends is shown in Fig. 17.5.2a. There is an antinode across each open end, as required. There is also a node across the middle of the pipe. An easier way of representing this standing longitudinal sound wave is shown in Fig. 17.5.2b—by drawing it as a standing transverse string wave.

The standing wave pattern of Fig. 17.5.2a is called the *fundamental mode* or *first harmonic*. For it to be set up, the sound waves in a pipe of length  $L$  must have a wavelength given by  $L = \lambda/2$ , so that  $\lambda = 2L$ . Several more standing sound wave patterns for a pipe with two open ends are shown in Fig. 17.5.3a

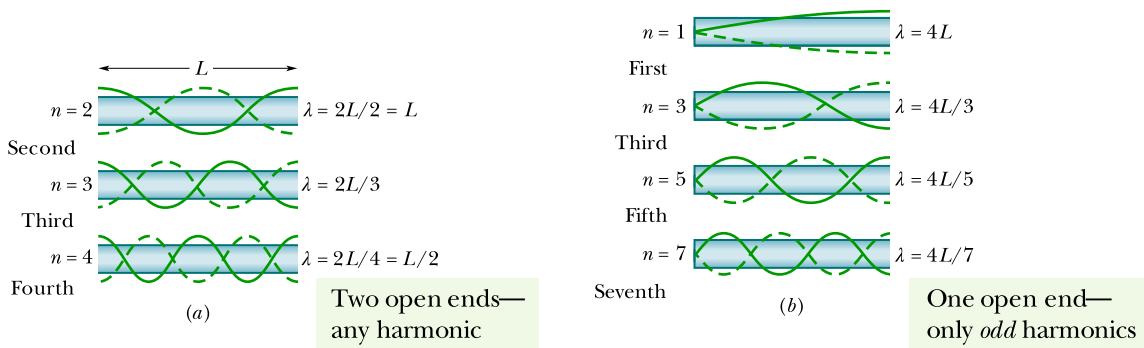


**Figure 17.5.2** (a) The simplest standing wave pattern of displacement for (longitudinal) sound waves in a pipe with both ends open has an antinode (A) across each end and a node (N) across the middle. (The longitudinal displacements represented by the double arrows are greatly exaggerated.) (b) The corresponding standing wave pattern for (transverse) string waves.



WorldFoto/Alamy Stock Photo

**Figure 17.5.1** The air column within a didgeridoo (“a pipe”) oscillates when the instrument is played.



**Figure 17.5.3** Standing wave patterns for string waves superimposed on pipes to represent standing sound wave patterns in the pipes. (a) With *both* ends of the pipe open, any harmonic can be set up in the pipe. (b) With only *one* end open, only odd harmonics can be set up.

using string wave representations. The *second harmonic* requires sound waves of wavelength  $\lambda = L$ , the *third harmonic* requires wavelength  $\lambda = 2L/3$ , and so on.

More generally, the resonant frequencies for a pipe of length  $L$  with two open ends correspond to the wavelengths

$$\lambda = \frac{2L}{n}, \quad \text{for } n = 1, 2, 3, \dots, \quad (17.5.1)$$

where  $n$  is called the *harmonic number*. Letting  $v$  be the speed of sound, we write the resonant frequencies for a pipe with two open ends as

$$f = \frac{v}{\lambda} = \frac{nv}{2L}, \quad \text{for } n = 1, 2, 3, \dots \quad (\text{pipe, two open ends}). \quad (17.5.2)$$

**One Open End.** Figure 17.5.3b shows (using string wave representations) some of the standing sound wave patterns that can be set up in a pipe with only one open end. As required, across the open end there is an antinode and across the closed end there is a node. The simplest pattern requires sound waves having a wavelength given by  $L = \lambda/4$ , so that  $\lambda = 4L$ . The next simplest pattern requires a wavelength given by  $L = 3\lambda/4$ , so that  $\lambda = 4L/3$ , and so on.

More generally, the resonant frequencies for a pipe of length  $L$  with only one open end correspond to the wavelengths

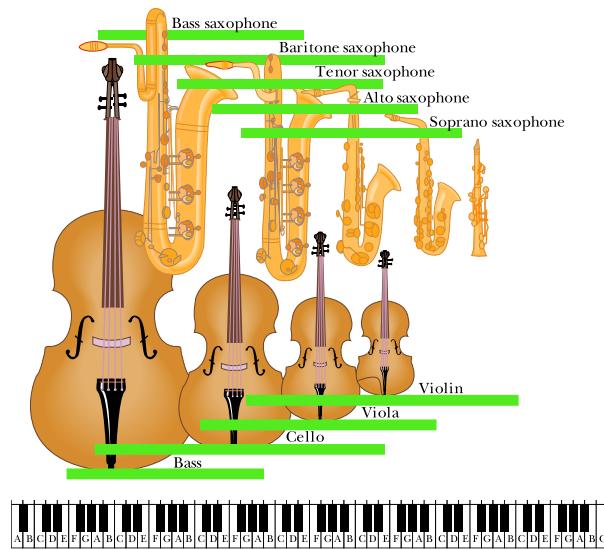
$$\lambda = \frac{4L}{n}, \quad \text{for } n = 1, 3, 5, \dots, \quad (17.5.3)$$

in which the harmonic number  $n$  must be an odd number. The resonant frequencies are then given by

$$f = \frac{v}{\lambda} = \frac{nv}{4L}, \quad \text{for } n = 1, 3, 5, \dots \quad (\text{pipe, one open end}). \quad (17.5.4)$$

Note again that only odd harmonics can exist in a pipe with one open end. For example, the second harmonic, with  $n = 2$ , cannot be set up in such a pipe. Note also that for such a pipe the adjective in a phrase such as “the third harmonic” still refers to the harmonic number  $n$  (and not to, say, the third possible harmonic). Finally note that Eqs. 17.5.1 and 17.5.2 for two open ends contain the number 2 and any integer value of  $n$ , but Eqs. 17.5.3 and 17.5.4 for one open end contain the number 4 and only odd values of  $n$ .

**Length.** The length of a musical instrument reflects the range of frequencies over which the instrument is designed to function, and smaller length implies higher frequencies, as we can tell from Eq. 16.7.9 for string instruments and Eqs. 17.5.2 and 17.5.4 for instruments with air columns. Figure 17.5.4, for example, shows the saxophone and violin families, with their frequency ranges suggested



**Figure 17.5.4** The saxophone and violin families, showing the relations between instrument length and frequency range. The frequency range of each instrument is indicated by a horizontal bar along a frequency scale suggested by the keyboard at the bottom; the frequency increases toward the right.

by the piano keyboard. Note that, for every instrument, there is overlap with its higher- and lower-frequency neighbors.

**Net Wave.** In any oscillating system that gives rise to a musical sound, whether it is a violin string or the air in an organ pipe, the fundamental and one or more of the higher harmonics are usually generated simultaneously. Thus, you hear them together—that is, superimposed as a net wave. When different instruments are played at the same note, they produce the same fundamental frequency but different intensities for the higher harmonics. For example, the fourth harmonic of middle C might be relatively loud on one instrument and relatively quiet or even missing on another. Thus, because different instruments produce different net waves, they sound different to you even when they are played at the same note. That would be the case for the two net waves shown in Fig. 17.5.5, which were produced at the same note by different instruments. If you heard only the fundamentals, the music would not be musical.

### Checkpoint 17.5.1

Pipe *A*, with length *L*, and pipe *B*, with length *2L*, both have two open ends. Which harmonic of pipe *B* has the same frequency as the fundamental of pipe *A*?

### Sample Problem 17.5.1 Resonance between pipes of different lengths

Pipe *A* is open at both ends and has length  $L_A = 0.343\text{ m}$ . We want to place it near three other pipes in which standing waves have been set up, so that the sound can set up a standing wave in pipe *A*. Those other three pipes are each closed at one end and have lengths  $L_B = 0.500L_A$ ,  $L_C = 0.250L_A$ , and  $L_D = 2.00L_A$ . For each of these three pipes, which of their harmonics can excite a harmonic in pipe *A*?

#### KEY IDEAS

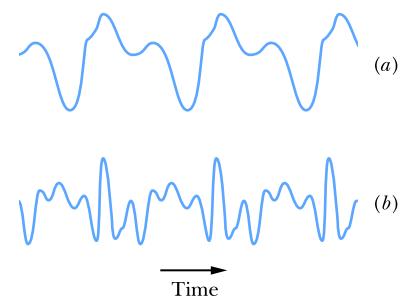
- (1) The sound from one pipe can set up a standing wave in another pipe only if the harmonic frequencies match.
- (2) Equation 17.5.2 gives the harmonic frequencies in a pipe with two open ends (a symmetric pipe) as  $f = nv/2L$ , for

$n = 1, 2, 3, \dots$ , that is, for any positive integer. (3) Equation 17.5.4 gives the harmonic frequencies in a pipe with only one open end (an asymmetric pipe) as  $f = nv/4L$ , for  $n = 1, 3, 5, \dots$ , that is, for only odd positive integers.

**Pipe A:** Let's first find the resonant frequencies of symmetric pipe *A* (with two open ends) by evaluating Eq. 17.5.2:

$$\begin{aligned} f_A &= \frac{n_A v}{2L_A} = \frac{n_A(343 \text{ m/s})}{2(0.343 \text{ m})} \\ &= n_A(500 \text{ Hz}) = n_A(0.50 \text{ kHz}), \quad \text{for } n_A = 1, 2, 3, \dots \end{aligned}$$

The first six harmonic frequencies are shown in the top plot in Fig. 17.5.6.



**Figure 17.5.5** The wave forms produced by (a) a flute and (b) an oboe when played at the same note, with the same first harmonic frequency.

**Pipe B:** Next let's find the resonant frequencies of asymmetric pipe *B* (with only one open end) by evaluating Eq. 17.5.4, being careful to use only odd integers for the harmonic numbers:

$$f_B = \frac{n_B v}{4L_B} = \frac{n_B v}{4(0.500L_A)} = \frac{n_B(343 \text{ m/s})}{2(0.343 \text{ m})}$$

$$= n_B(500 \text{ Hz}) = n_B(0.500 \text{ kHz}), \quad \text{for } n_B = 1, 3, 5, \dots$$

Comparing our two results, we see that we get a match for each choice of  $n_B$ :

$$f_A = f_B \quad \text{for } n_A = n_B \quad \text{with } n_B = 1, 3, 5, \dots \quad (\text{Answer})$$

For example, as shown in Fig. 17.5.6, if we set up the fifth harmonic in pipe *B* and bring the pipe close to pipe *A*, the fifth harmonic will then be set up in pipe *A*. However, no harmonic in *B* can set up an even harmonic in *A*.

**Pipe C:** Let's continue with pipe *C* (with only one end) by writing Eq. 17.5.4 as

$$f_C = \frac{n_C v}{4L_C} = \frac{n_C v}{4(0.250L_A)} = \frac{n_C(343 \text{ m/s})}{0.343 \text{ m/s}}$$

$$= n_C(1000 \text{ Hz}) = n_C(1.00 \text{ kHz}), \quad \text{for } n_C = 1, 3, 5, \dots$$

From this we see that *C* can excite some of the harmonics of *A* but only those with harmonic numbers  $n_A$  that are twice an odd integer:

$$f_A = f_C \quad \text{for } n_A = 2n_C, \quad \text{with } n_C = 1, 3, 5, \dots \quad (\text{Answer})$$

**Pipe D:** Finally, let's check *D* with our same procedure:

$$f_D = \frac{n_D v}{4L_D} = \frac{n_D v}{4(2L_A)} = \frac{n_D(343 \text{ m/s})}{8(0.343 \text{ m/s})}$$

$$= n_D(125 \text{ Hz}) = n_D(0.125 \text{ kHz}), \quad \text{for } n_D = 1, 3, 5, \dots$$

As shown in Fig. 17.5.6, none of these frequencies match a harmonic frequency of *A*. (Can you see that we would get a match if  $n_D = 4n_A$ ? But that is impossible because  $4n_A$  cannot yield an odd integer, as required of  $n_D$ .) Thus *D* cannot set up a standing wave in *A*.

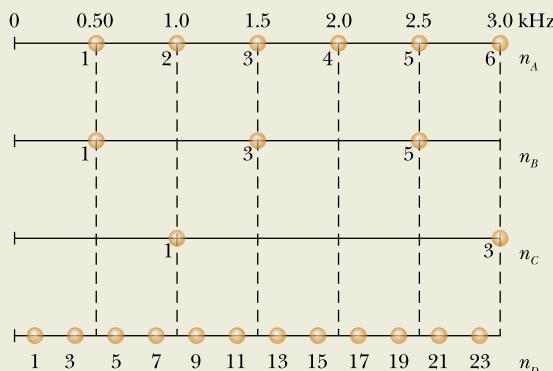


Figure 17.5.6 Harmonic frequencies of four pipes.

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## 17.6 BEATS

### Learning Objectives

After reading this module, you should be able to . . .

**17.6.1** Explain how beats are produced.

**17.6.2** Add the displacement equations for two sound waves of the same amplitude and slightly different angular frequencies to find the displacement equation of the resultant wave and identify the time-varying amplitude.

### Key Idea

- Beats arise when two waves having slightly different frequencies,  $f_1$  and  $f_2$ , are detected together. The beat frequency is

$$f_{\text{beat}} = f_1 - f_2.$$

### Beats

If we listen, a few minutes apart, to two sounds whose frequencies are, say, 552 and 564 Hz, most of us cannot tell one from the other because the frequencies are so close to each other. However, if the sounds reach our ears simultaneously,

**17.6.3** Apply the relationship between the beat frequency and the frequencies of two sound waves that have the same amplitude when the frequencies (or, equivalently, the angular frequencies) differ by a small amount.

what we hear is a sound whose frequency is 558 Hz, the *average* of the two combining frequencies. We also hear a striking variation in the intensity of this sound—it increases and decreases in slow, wavering **beats** that repeat at a frequency of 12 Hz, the *difference* between the two combining frequencies. Figure 17.6.1 shows this beat phenomenon.

Let the time-dependent variations of the displacements due to two sound waves of equal amplitude  $s_m$  be

$$s_1 = s_m \cos \omega_1 t \quad \text{and} \quad s_2 = s_m \cos \omega_2 t, \quad (17.6.1)$$

where  $\omega_1 > \omega_2$ . From the superposition principle, the resultant displacement is the sum of the individual displacements:

$$s = s_1 + s_2 = s_m(\cos \omega_1 t + \cos \omega_2 t).$$

Using the trigonometric identity (see Appendix E)

$$\cos \alpha + \cos \beta = 2 \cos\left[\frac{1}{2}(\alpha - \beta)\right] \cos\left[\frac{1}{2}(\alpha + \beta)\right]$$

allows us to write the resultant displacement as

$$s = 2s_m \cos\left[\frac{1}{2}(\omega_1 - \omega_2)t\right] \cos\left[\frac{1}{2}(\omega_1 + \omega_2)t\right]. \quad (17.6.2)$$

If we write

$$\omega' = \frac{1}{2}(\omega_1 - \omega_2) \quad \text{and} \quad \omega = \frac{1}{2}(\omega_1 + \omega_2), \quad (17.6.3)$$

we can then write Eq. 17.6.2 as

$$s(t) = [2s_m \cos \omega' t] \cos \omega t. \quad (17.6.4)$$

We now assume that the angular frequencies  $\omega_1$  and  $\omega_2$  of the combining waves are almost equal, which means that  $\omega \gg \omega'$  in Eq. 17.6.3. We can then regard Eq. 17.6.4 as a cosine function whose angular frequency is  $\omega$  and whose amplitude (which is not constant but varies with angular frequency  $\omega'$ ) is the absolute value of the quantity in the brackets.

A maximum amplitude will occur whenever  $\cos \omega' t$  in Eq. 17.6.4 has the value +1 or -1, which happens twice in each repetition of the cosine function. Because  $\cos \omega' t$  has angular frequency  $\omega'$ , the angular frequency  $\omega_{\text{beat}}$  at which beats occur is  $\omega_{\text{beat}} = 2\omega'$ . Then, with the aid of Eq. 17.6.3, we can write the beat angular frequency as

$$\omega_{\text{beat}} = 2\omega' = (2)\left(\frac{1}{2}\right)(\omega_1 - \omega_2) = \omega_1 - \omega_2.$$

Because  $\omega = 2\pi f$ , we can recast this as

$$f_{\text{beat}} = f_1 - f_2 \quad (\text{beat frequency}). \quad (17.6.5)$$

Musicians use the beat phenomenon in tuning instruments. If an instrument is sounded against a standard frequency (for example, the note called “concert A” played on an orchestra’s first oboe) and tuned until the beat disappears, the instrument is in tune with that standard. In musical Vienna, concert A (440 Hz) is available as a convenient telephone service for the city’s many musicians.

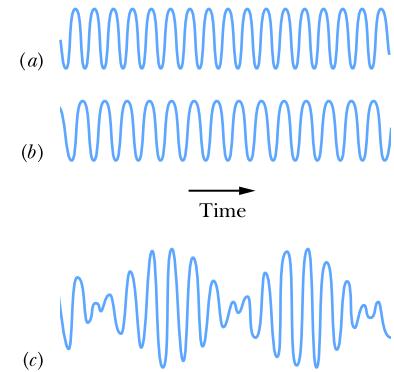
### Checkpoint 17.6.1

Here are three pairs of sound frequencies. (a) Rank them according to their beat frequency, greatest first. (b) Next, rank them according to the frequency of the sound that would be perceived, greatest first.

Pair A: 486 Hz and 490 Hz

Pair B: 501 Hz and 504 Hz

Pair C: 760 Hz and 762 Hz



**Figure 17.6.1** (a, b) The pressure variations  $\Delta p$  of two sound waves as they would be detected separately. The frequencies of the waves are nearly equal. (c) The resultant pressure variation if the two waves are detected simultaneously.

### Sample Problem 17.6.1 Beat frequencies and penguins finding one another

When an emperor penguin returns from a search for food, how can it find its mate among the thousands of penguins huddled together for warmth in the harsh Antarctic weather? It is not by sight, because penguins all look alike, even to a penguin.

The answer lies in the way penguins vocalize. Most birds vocalize by using only one side of their two-sided vocal organ, called the *syrinx*. Emperor penguins, however, vocalize by using both sides simultaneously. Each side sets up acoustic standing waves in the bird's throat and mouth, much like in a pipe with two open ends. Suppose that the frequency of the first harmonic produced by side *A* is  $f_{A1} = 432 \text{ Hz}$  and the frequency of the first harmonic produced by side *B* is  $f_{B1} = 371 \text{ Hz}$ . What is the beat frequency between those two first-harmonic frequencies and between the two second-harmonic frequencies?

FCP

#### KEY IDEA

The beat frequency between two frequencies is their difference, as given by Eq. 17.6.5 ( $f_{\text{beat}} = f_1 - f_2$ ).

**Calculations:** For the two first-harmonic frequencies  $f_{A1}$  and  $f_{B1}$ , the beat frequency is

$$\begin{aligned} f_{\text{beat},1} &= f_{A1} - f_{B1} = 432 \text{ Hz} - 371 \text{ Hz} \\ &= 61 \text{ Hz.} \end{aligned} \quad (\text{Answer})$$

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## 17.7 THE DOPPLER EFFECT

#### Learning Objectives

After reading this module, you should be able to . . .

**17.7.1** Identify that the Doppler effect is the shift in the detected frequency from the frequency emitted by a sound source due to the relative motion between the source and the detector.

**17.7.2** Identify that in calculating the Doppler shift in sound, the speeds are measured relative to the medium (such as air or water), which may be moving.

**17.7.3** Calculate the shift in sound frequency for (a) a source moving either directly toward or away from

Because the standing waves in the penguin are effectively in a pipe with two open ends, the resonant frequencies are given by Eq. 17.5.2 ( $f = nv/2L$ ), in which  $L$  is the (unknown) length of the effective pipe. The first-harmonic frequency is  $f_1 = v/2L$ , and the second-harmonic frequency is  $f_2 = 2v/2L$ . Comparing these two frequencies, we see that, in general,

$$f_2 = 2f_1.$$

For the penguin, the second harmonic of side *A* has frequency  $f_{A2} = 2f_{A1}$  and the second harmonic of side *B* has frequency  $f_{B2} = 2f_{B1}$ . Using Eq. 17.6.5 with frequencies  $f_{A2}$  and  $f_{B2}$ , we find that the corresponding beat frequency associated with the second harmonics is

$$\begin{aligned} f_{\text{beat},2} &= f_{A2} - f_{B2} = 2f_{A1} - 2f_{B1} \\ &= 2(432 \text{ Hz}) - 2(371 \text{ Hz}) \\ &= 122 \text{ Hz.} \end{aligned} \quad (\text{Answer})$$

Experiments indicate that penguins can perceive such large beat frequencies. (Humans cannot hear a beat frequency any higher than about 12 Hz—we perceive the two separate frequencies.) Thus, a penguin's cry can be rich with different harmonics and different beat frequencies, allowing the voice to be recognized even among the voices of thousands of other, closely huddled penguins.

#### Key Ideas

• The Doppler effect is a change in the observed frequency of a wave when the source or the detector moves relative to the transmitting medium (such as air). For sound the observed frequency  $f'$  is given in terms of the source frequency  $f$  by

$$f' = f \frac{v \pm v_D}{v \pm v_S} \quad (\text{general Doppler effect}),$$

a stationary detector, (b) a detector moving either directly toward or away from a stationary source, and (c) both source and detector moving either directly toward each other or directly away from each other.

**17.7.4** Identify that for relative motion between a sound source and a sound detector, motion toward tends to shift the frequency up and motion away tends to shift it down.

where  $v_D$  is the speed of the detector relative to the medium,  $v_S$  is that of the source, and  $v$  is the speed of sound in the medium.

• The signs are chosen such that  $f'$  tends to be greater for relative motion toward (one of the objects moves toward the other) and less for motion away.

## The Doppler Effect

A police car is parked by the side of the highway, sounding its 1000 Hz siren. If you are also parked by the highway, you will hear that same frequency. However, if there is relative motion between you and the police car, either toward or away from each other, you will hear a different frequency. For example, if you are driving *toward* the police car at 120 km/h (about 75 mi/h), you will hear a *higher* frequency (1096 Hz, an *increase* of 96 Hz). If you are driving *away from* the police car at that same speed, you will hear a *lower* frequency (904 Hz, a *decrease* of 96 Hz).

FCP

These motion-related frequency changes are examples of the **Doppler effect**. The effect was proposed (although not fully worked out) in 1842 by Austrian physicist Johann Christian Doppler. It was tested experimentally in 1845 by Buys Ballot in Holland, “using a locomotive drawing an open car with several trumpeters.”

The Doppler effect holds not only for sound waves but also for electromagnetic waves, including microwaves, radio waves, and visible light. Here, however, we shall consider only sound waves, and we shall take as a reference frame the body of air through which these waves travel. This means that we shall measure the speeds of a source  $S$  of sound waves and a detector  $D$  of those waves *relative to that body of air*. (Unless otherwise stated, the body of air is stationary relative to the ground, so the speeds can also be measured relative to the ground.) We shall assume that  $S$  and  $D$  move either directly toward or directly away from each other, at speeds less than the speed of sound.

**General Equation.** If either the detector or the source is moving, or both are moving, the emitted frequency  $f$  and the detected frequency  $f'$  are related by

$$f' = f \frac{v \pm v_D}{v \pm v_S} \quad (\text{general Doppler effect}), \quad (17.7.1)$$

where  $v$  is the speed of sound through the air,  $v_D$  is the detector's speed relative to the air, and  $v_S$  is the source's speed relative to the air. The choice of plus or minus signs is set by this rule:



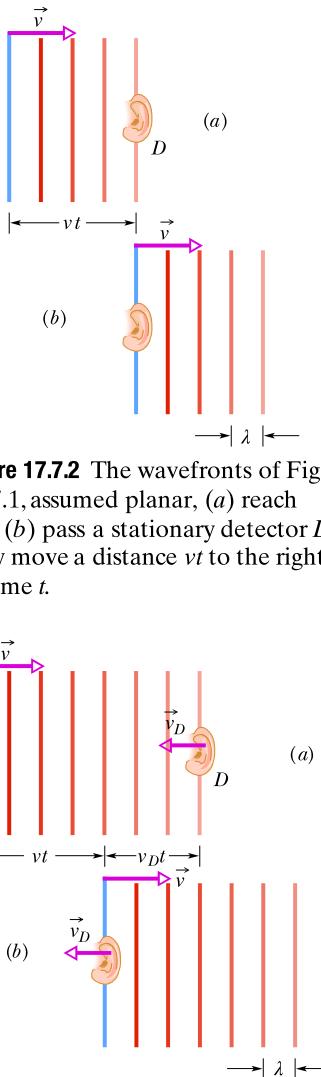
When the motion of detector or source is toward the other, the sign on its speed must give an upward shift in frequency. When the motion of detector or source is away from the other, the sign on its speed must give a downward shift in frequency.

In short, *toward* means *shift up*, and *away* means *shift down*.

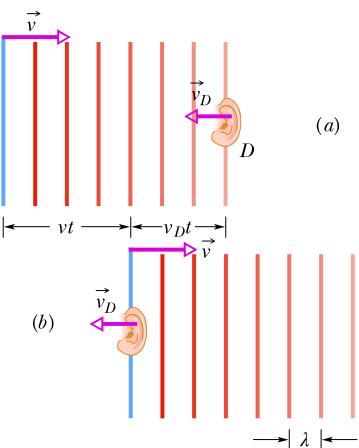
Here are some examples of the rule. If the detector moves toward the source, use the plus sign in the numerator of Eq. 17.7.1 to get a shift up in the frequency. If it moves away, use the minus sign in the numerator to get a shift down. If it is stationary, substitute 0 for  $v_D$ . If the source moves toward the detector, use the minus sign in the denominator of Eq. 17.7.1 to get a shift up in the frequency. If it moves away, use the plus sign in the denominator to get a shift down. If the source is stationary, substitute 0 for  $v_S$ .

Next, we derive equations for the Doppler effect for the following two specific situations and then derive Eq. 17.7.1 for the general situation.

1. When the detector moves relative to the air and the source is stationary relative to the air, the motion changes the frequency at which the detector intercepts wavefronts and thus changes the detected frequency of the sound wave.
2. When the source moves relative to the air and the detector is stationary relative to the air, the motion changes the wavelength of the sound wave and thus changes the detected frequency (recall that frequency is related to wavelength).

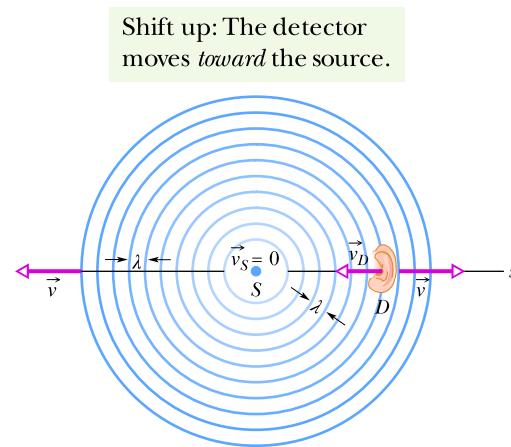


**Figure 17.7.2** The wavefronts of Fig. 17.7.1, assumed planar, (a) reach and (b) pass a stationary detector  $D$ ; they move a distance  $vt$  to the right in time  $t$ .



**Figure 17.7.3** Wavefronts traveling to the right (a) reach and (b) pass detector  $D$ , which moves in the opposite direction. In time  $t$ , the wavefronts move a distance  $vt$  to the right and  $D$  moves a distance  $v_D t$  to the left.

**Figure 17.7.1** A stationary source of sound  $S$  emits spherical wavefronts, shown one wavelength apart, that expand outward at speed  $v$ . A sound detector  $D$ , represented by an ear, moves with velocity  $\vec{v}_D$  toward the source. The detector senses a higher frequency because of its motion.



### Detector Moving, Source Stationary

In Fig. 17.7.1, a detector  $D$  (represented by an ear) is moving at speed  $v_D$  toward a stationary source  $S$  that emits spherical wavefronts, of wavelength  $\lambda$  and frequency  $f$ , moving at the speed  $v$  of sound in air. The wavefronts are drawn one wavelength apart. The frequency detected by detector  $D$  is the rate at which  $D$  intercepts wavefronts (or individual wavelengths). If  $D$  were stationary, that rate would be  $f$ , but since  $D$  is moving into the wavefronts, the rate of interception is greater, and thus the detected frequency  $f'$  is greater than  $f$ .

Let us for the moment consider the situation in which  $D$  is stationary (Fig. 17.7.2). In time  $t$ , the wavefronts move to the right a distance  $vt$ . The number of wavelengths in that distance  $vt$  is the number of wavelengths intercepted by  $D$  in time  $t$ , and that number is  $vt/\lambda$ . The rate at which  $D$  intercepts wavelengths, which is the frequency  $f$  detected by  $D$ , is

$$f = \frac{vt/\lambda}{t} = \frac{v}{\lambda}. \quad (17.7.2)$$

In this situation, with  $D$  stationary, there is no Doppler effect—the frequency detected by  $D$  is the frequency emitted by  $S$ .

Now let us again consider the situation in which  $D$  moves in the direction opposite the wavefront velocity (Fig. 17.7.3). In time  $t$ , the wavefronts move to the right a distance  $vt$  as previously, but now  $D$  moves to the left a distance  $v_D t$ . Thus, in this time  $t$ , the distance moved by the wavefronts relative to  $D$  is  $vt + v_D t$ . The number of wavelengths in this relative distance  $vt + v_D t$  is the number of wavelengths intercepted by  $D$  in time  $t$  and is  $(vt + v_D t)/\lambda$ . The rate at which  $D$  intercepts wavelengths in this situation is the frequency  $f'$ , given by

$$f' = \frac{(vt + v_D t)/\lambda}{t} = \frac{v + v_D}{\lambda}. \quad (17.7.3)$$

From Eq. 17.7.2, we have  $\lambda = v/f$ . Then Eq. 17.7.3 becomes

$$f' = \frac{v + v_D}{v/f} = f \frac{v + v_D}{v}. \quad (17.7.4)$$

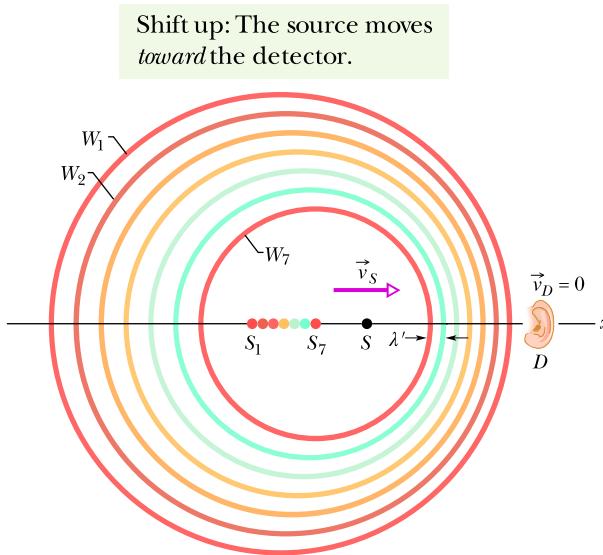
Note that in Eq. 17.7.4,  $f' > f$  unless  $v_D = 0$  (the detector is stationary).

Similarly, we can find the frequency detected by  $D$  if  $D$  moves away from the source. In this situation, the wavefronts move a distance  $vt - v_D t$  relative to  $D$  in time  $t$ , and  $f'$  is given by

$$f' = f \frac{v - v_D}{v}. \quad (17.7.5)$$

In Eq. 17.7.5,  $f' < f$  unless  $v_D = 0$ . We can summarize Eqs. 17.7.4 and 17.7.5 with

$$f' = f \frac{v \pm v_D}{v} \quad (\text{detector moving, source stationary}). \quad (17.7.6)$$



**Figure 17.7.4** A detector  $D$  is stationary, and a source  $S$  is moving toward it at speed  $v_S$ . Wavefront  $W_1$  was emitted when the source was at  $S_1$ , wavefront  $W_7$  when it was at  $S_7$ . At the moment depicted, the source is at  $S$ . The detector senses a higher frequency because the moving source, chasing its own wavefronts, emits a reduced wavelength  $\lambda'$  in the direction of its motion.

### Source Moving, Detector Stationary

Let detector  $D$  be stationary with respect to the body of air, and let source  $S$  move toward  $D$  at speed  $v_S$  (Fig. 17.7.4). The motion of  $S$  changes the wavelength of the sound waves it emits and thus the frequency detected by  $D$ .

To see this change, let  $T$  ( $= 1/f$ ) be the time between the emission of any pair of successive wavefronts  $W_1$  and  $W_2$ . During  $T$ , wavefront  $W_1$  moves a distance  $vT$  and the source moves a distance  $v_S T$ . At the end of  $T$ , wavefront  $W_2$  is emitted. In the direction in which  $S$  moves, the distance between  $W_1$  and  $W_2$ , which is the wavelength  $\lambda'$  of the waves moving in that direction, is  $vT - v_S T$ . If  $D$  detects those waves, it detects frequency  $f'$  given by

$$\begin{aligned} f' &= \frac{v}{\lambda'} = \frac{v}{vT - v_S T} = \frac{v}{v/f - v_S/f} \\ &= f \frac{v}{v - v_S}. \end{aligned} \quad (17.7.7)$$

Note that  $f'$  must be greater than  $f$  unless  $v_S = 0$ .

In the direction opposite that taken by  $S$ , the wavelength  $\lambda'$  of the waves is again the distance between successive waves but now that distance is  $vT + v_S T$ . If  $D$  detects those waves, it detects frequency  $f'$  given by

$$f' = f \frac{v}{v + v_S}. \quad (17.7.8)$$

Now  $f'$  must be less than  $f$  unless  $v_S = 0$ .

We can summarize Eqs. 17.7.7 and 17.7.8 with

$$f' = f \frac{v}{v \pm v_S} \quad (\text{source moving, detector stationary}). \quad (17.7.9)$$

### General Doppler Effect Equation

We can now derive the general Doppler effect equation by replacing  $f$  in Eq. 17.7.9 (the source frequency) with  $f'$  of Eq. 17.7.6 (the frequency associated with motion of the detector). That simple replacement gives us Eq. 17.7.1 for the general Doppler effect. That general equation holds not only when both detector and source are moving but also in the two specific situations we just discussed. For the situation in which the detector is moving and the source is stationary, substitution of  $v_S = 0$  into Eq. 17.7.1 gives us Eq. 17.7.6, which we previously found. For the situation in which the source is moving and the detector is stationary, substitution of  $v_D = 0$  into Eq. 17.7.1 gives us Eq. 17.7.9, which we previously found. Thus, Eq. 17.7.1 is the equation to remember.

### Checkpoint 17.7.1

The figure indicates the directions of motion of a sound source and a detector for six situations in stationary air. For each situation, is the detected frequency greater than or less than the emitted frequency, or can't we tell without more information about the actual speeds?

Source	Detector	Source	Detector
(a)	• 0 speed	(d)	
(b)	• 0 speed	(e)	
(c)		(f)	

### Sample Problem 17.7.1 Double Doppler shift in the echoes used by bats

Bats navigate and search out prey by emitting, and then detecting reflections of, ultrasonic waves, which are sound waves with frequencies greater than can be heard by a human. Suppose a bat emits ultrasound at frequency  $f_{be} = 82.52 \text{ kHz}$  while flying with velocity  $\vec{v}_b = (9.00 \text{ m/s})\hat{i}$  as it chases a moth that flies with velocity  $\vec{v}_m = (8.00 \text{ m/s})\hat{i}$ . What frequency  $f_{md}$  does the moth detect? What frequency  $f_{bd}$  does the bat detect in the returning echo from the moth? FCP

#### KEY IDEAS

The frequency is shifted by the relative motion of the bat and moth. Because they move along a single axis, the shifted frequency is given by Eq. 17.7.1. Motion *toward* tends to shift the frequency *up*, and motion *away* tends to shift it *down*.

**Detection by moth:** The general Doppler equation is

$$f' = f \frac{v \pm v_D}{v \pm v_S} \quad (17.7.10)$$

Here, the detected frequency  $f'$  that we want to find is the frequency  $f_{md}$  detected by the moth. On the right side, the emitted frequency  $f$  is the bat's emission frequency  $f_{be} = 82.52 \text{ kHz}$ , the speed of sound is  $v = 343 \text{ m/s}$ , the speed  $v_D$  of the detector is the moth's speed  $v_m = 8.00 \text{ m/s}$ , and the speed  $v_S$  of the source is the bat's speed  $v_b = 9.00 \text{ m/s}$ .

The decisions about the plus and minus signs can be tricky. Think in terms of *toward* and *away*. We have the speed of the moth (the detector) in the numerator of Eq. 17.7.10. The moth moves *away* from the bat, which

tends to lower the detected frequency. Because the speed is in the numerator, we choose the minus sign to meet that tendency (the numerator becomes smaller). These reasoning steps are shown in Table 17.7.1.

We have the speed of the bat in the denominator of Eq. 17.7.10. The bat moves *toward* the moth, which tends to increase the detected frequency. Because the speed is in the denominator, we choose the minus sign to meet that tendency (the denominator becomes smaller).

With these substitutions and decisions, we have

$$\begin{aligned} f_{md} &= f_{be} \frac{v - v_m}{v - v_b} \\ &= (82.52 \text{ kHz}) \frac{343 \text{ m/s} - 8.00 \text{ m/s}}{343 \text{ m/s} - 9.00 \text{ m/s}} \\ &= 82.767 \text{ kHz} \approx 82.8 \text{ kHz} \quad (\text{Answer}) \end{aligned}$$

**Detection of echo by bat:** In the echo back to the bat, the moth acts as a source of sound, emitting at the frequency  $f_{md}$  we just calculated. So now the moth is the source (moving *away*) and the bat is the detector (moving *toward*). The reasoning steps are shown in Table 17.7.1. To find the frequency  $f_{bd}$  detected by the bat, we write Eq. 17.7.10as

$$\begin{aligned} f_{bd} &= f_{md} \frac{v + v_b}{v + v_m} \\ &= (82.767 \text{ kHz}) \frac{343 \text{ m/s} + 9.00 \text{ m/s}}{343 \text{ m/s} + 8.00 \text{ m/s}} \\ &= 83.00 \text{ kHz} \approx 83.0 \text{ kHz} \quad (\text{Answer}) \end{aligned}$$

Some moths evade bats by “jamming” the detection system with ultrasonic clicks.

Table 17.7.1

Bat to Moth		Echo Back to Bat	
Detector	Source	Detector	Source
moth	bat	bat	moth
speed $v_D = v_m$	speed $v_S = v_b$	speed $v_D = v_b$	speed $v_S = v_m$
away	toward	toward	away
shift down	shift up	shift up	shift down
numerator	denominator	denominator	denominator
minus	minus	plus	plus

# 17.8 SUPERSONIC SPEEDS, SHOCK WAVES

## Learning Objectives

After reading this module, you should be able to . . .

**17.8.1** Sketch the bunching of wavefronts for a sound source traveling at the speed of sound or faster.

**17.8.2** Calculate the Mach number for a sound source exceeding the speed of sound.

**17.8.3** For a sound source exceeding the speed of sound, apply the relationship between the Mach cone angle, the speed of sound, and the speed of the source.

## Key Idea

- If the speed of a source relative to the medium exceeds the speed of sound in the medium, the Doppler equation no longer applies. In such a case, shock waves result.

The half-angle  $\theta$  of the Mach cone is given by

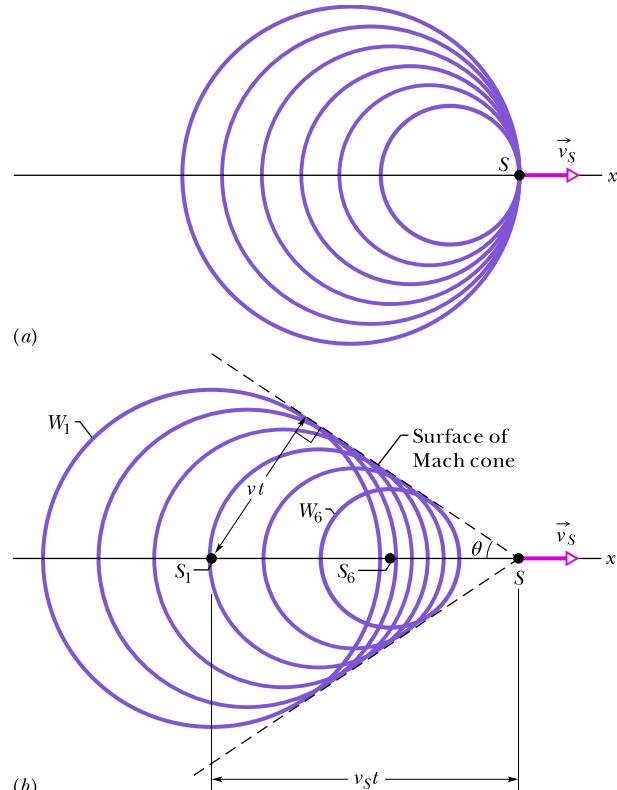
$$\sin \theta = \frac{v}{v_S} \quad (\text{Mach cone angle}).$$

## Supersonic Speeds, Shock Waves

If a source is moving toward a stationary detector at a speed  $v_S$  equal to the speed of sound  $v$ , Eqs. 17.7.1 and 17.7.9 predict that the detected frequency  $f'$  will be infinitely great. This means that the source is moving so fast that it keeps pace with its own spherical wavefronts (Fig. 17.8.1a). What happens when  $v_S > v$ ? For such *supersonic* speeds, Eqs. 17.7.1 and 17.7.9 no longer apply. Figure 17.8.1b depicts the spherical wavefronts that originated at various positions of the source. The radius of any wavefront is  $vt$ , where  $t$  is the time that has elapsed since the source emitted that wavefront. Note that all the wavefronts bunch along a V-shaped envelope in this two-dimensional drawing. The wavefronts actually extend in three dimensions, and the bunching actually forms a cone called the *Mach cone*. A *shock wave* exists along the surface of this cone, because the bunching of wavefronts causes an abrupt rise and fall of air pressure as the surface passes through any point. From Fig. 17.8.1b, we see that the half-angle  $\theta$  of the cone (the *Mach cone angle*) is given by

$$\sin \theta = \frac{v}{v_S t} = \frac{v}{v_S} \quad (\text{Mach cone angle}). \quad (17.8.1)$$

The ratio  $v_S/v$  is the *Mach number*. If a plane flies at Mach 2.3, its speed is 2.3 times the speed of sound in the air through which the plane is flying. The shock wave generated by a supersonic aircraft (Fig. 17.8.2) or projectile produces a burst of sound,



**Figure 17.8.1** (a) A source of sound  $S$  moves at speed  $v_S$  equal to the speed of sound and thus as fast as the wavefronts it generates. (b) A source  $S$  moves at speed  $v_S$  faster than the speed of sound and thus faster than the wavefronts. When the source was at position  $S_1$  it generated wavefront  $W_1$ , and at position  $S_6$  it generated  $W_6$ . All the spherical wavefronts expand at the speed of sound  $v$  and bunch along the surface of a cone called the *Mach cone*, forming a shock wave. The surface of the cone has half-angle  $\theta$  and is tangent to all the wavefronts.

Courtesy of U.S. Navy photo by Ensign John Gay



**Figure 17.8.2** Shock waves produced by the wings of a Navy FA 18 jet. The shock waves are visible because the sudden decrease in air pressure in them caused water molecules in the air to condense, forming a fog.

called a *sonic boom*, in which the air pressure first suddenly increases and then suddenly decreases below normal before returning to normal. Part of the sound that is heard when a rifle is fired is the sonic boom produced by the bullet. When a long bull whip is snapped, its tip is moving faster than sound and produces a small sonic boom—the *crack* of the whip.

FCP

### Checkpoint 17.8.1

The speed of sound varies with altitude in Earth's atmosphere. If a supersonic airplane changes altitude such that the speed of sound in the air is then slower, does the Mach angle increase or decrease?

## Review & Summary

**Sound Waves** Sound waves are longitudinal mechanical waves that can travel through solids, liquids, or gases. The speed  $v$  of a sound wave in a medium having **bulk modulus**  $B$  and density  $\rho$  is

$$v = \sqrt{\frac{B}{\rho}} \quad (\text{speed of sound}) \quad (17.1.3)$$

In air at 20°C, the speed of sound is 343 m/s.

A sound wave causes a longitudinal displacement  $s$  of a mass element in a medium as given by

$$s = s_m \cos(kx - \omega t), \quad (17.2.1)$$

where  $s_m$  is the **displacement amplitude** (maximum displacement) from equilibrium,  $k = 2\pi/\lambda$ , and  $\omega = 2\pi f$ ,  $\lambda$  and  $f$  being the wavelength and frequency of the sound wave. The wave also causes a pressure change  $\Delta p$  from the equilibrium pressure:

$$\Delta p = \Delta p_m \sin(kx - \omega t), \quad (17.2.2)$$

where the **pressure amplitude** is

$$\Delta p_m = (v\rho\omega)s_m. \quad (17.2.3)$$

**Interference** The interference of two sound waves with identical wavelengths passing through a common point depends on their phase difference  $\phi$  there. If the sound waves were emitted in phase and are traveling in approximately the same direction,  $\phi$  is given by

$$\phi = \frac{\Delta L}{\lambda} 2\pi, \quad (17.3.3)$$

where  $\Delta L$  is their **path length difference** (the difference in the distances traveled by the waves to reach the common point). Fully constructive interference occurs when  $\phi$  is an integer multiple of  $2\pi$ ,

$$\phi = m(2\pi), \quad \text{for } m = 0, 1, 2, \dots, \quad (17.3.4)$$

and, equivalently, when  $\Delta L$  is related to wavelength  $\lambda$  by

$$\frac{\Delta L}{\lambda} = 0, 1, 2, \dots. \quad (17.3.5)$$

Fully destructive interference occurs when  $\phi$  is an odd multiple of  $\pi$ ,

$$\phi = (2m + 1)\pi, \quad \text{for } m = 0, 1, 2, \dots, \quad (17.3.6)$$

and, equivalently, when  $\Delta L$  is related to  $\lambda$  by

$$\frac{\Delta L}{\lambda} = 0.5, 1.5, 2.5, \dots \quad (17.3.7)$$

**Sound Intensity** The **intensity**  $I$  of a sound wave at a surface is the average rate per unit area at which energy is transferred by the wave through or onto the surface:

$$I = \frac{P}{A}, \quad (17.4.1)$$

where  $P$  is the time rate of energy transfer (power) of the sound wave and  $A$  is the area of the surface intercepting the sound. The intensity  $I$  is related to the displacement amplitude  $s_m$  of the sound wave by

$$I = \frac{1}{2} \rho v \omega^2 s_m^2. \quad (17.4.2)$$

The intensity at a distance  $r$  from a point source that emits sound waves of power  $P_s$  is

$$I = \frac{P_s}{4\pi r^2}. \quad (17.4.3)$$

**Sound Level in Decibels** The *sound level*  $\beta$  in *decibels* (dB) is defined as

$$\beta = (10 \text{ dB}) \log \frac{I}{I_0}, \quad (17.4.4)$$

where  $I_0 (= 10^{-12} \text{ W/m}^2)$  is a reference intensity level to which all intensities are compared. For every factor-of-10 increase in intensity, 10 dB is added to the sound level.

**Standing Wave Patterns in Pipes** Standing sound wave patterns can be set up in pipes. A pipe open at both ends will resonate at frequencies

$$f = \frac{v}{\lambda} = \frac{nv}{2L}, \quad n = 1, 2, 3, \dots, \quad (17.5.2)$$

where  $v$  is the speed of sound in the air in the pipe. For a pipe closed at one end and open at the other, the resonant frequencies are

$$f = \frac{v}{\lambda} = \frac{nv}{4L}, \quad n = 1, 3, 5, \dots. \quad (17.5.4)$$

**Beats** *Beats* arise when two waves having slightly different frequencies,  $f_1$  and  $f_2$ , are detected together. The beat frequency is

$$f_{\text{beat}} = f_1 - f_2. \quad (17.6.5)$$

**The Doppler Effect** The *Doppler effect* is a change in the observed frequency of a wave when the source or the detector moves relative to the transmitting medium (such as air). For sound the observed frequency  $f'$  is given in terms of the source frequency  $f$  by

$$f' = f \frac{v \pm v_D}{v \mp v_S} \quad (\text{general Doppler effect}), \quad (17.7.1)$$

where  $v_D$  is the speed of the detector relative to the medium,  $v_S$  is that of the source, and  $v$  is the speed of sound in the medium.

The signs are chosen such that  $f'$  tends to be *greater* for motion toward and *less* for motion away.

**Shock Wave** If the speed of a source relative to the medium exceeds the speed of sound in the medium, the Doppler equation no longer applies. In such a case, shock waves result. The half-angle  $\theta$  of the Mach cone is given by

$$\sin \theta = \frac{v}{v_S} \quad (\text{Mach cone angle}). \quad (17.8.1)$$

## Questions

**1** In a first experiment, a sinusoidal sound wave is sent through a long tube of air, transporting energy at the average rate of  $P_{\text{avg},1}$ . In a second experiment, two other sound waves, identical to the first one, are to be sent simultaneously through the tube with a phase difference  $\phi$  of either 0, 0.2 wavelength, or 0.5 wavelength between the waves. (a) With only mental calculation, rank those choices of  $\phi$  according to the average rate at which the waves will transport energy, greatest first. (b) For the first choice of  $\phi$ , what is the average rate in terms of  $P_{\text{avg},1}$ ?

**2** In Fig. 17.1, two point sources  $S_1$  and  $S_2$ , which are in phase, emit identical sound waves of wavelength 2.0 m. In terms of wavelengths, what is the phase difference between the waves arriving at point  $P$  if (a)  $L_1 = 38$  m and  $L_2 = 34$  m, and (b)  $L_1 = 39$  m and  $L_2 = 36$  m? (c) Assuming that the source separation is much smaller than  $L_1$  and  $L_2$ , what type of interference occurs at  $P$  in situations (a) and (b)?

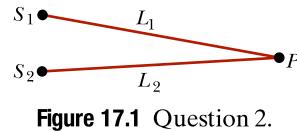


Figure 17.1 Question 2.

**3** In Fig. 17.2, three long tubes ( $A$ ,  $B$ , and  $C$ ) are filled with different gases under different pressures. The ratio of the bulk modulus to the density is indicated for each gas in terms of a basic value  $B_0/\rho_0$ . Each tube has a piston at its left end that can send a sound pulse through the tube (as in Fig. 16.1.2). The three pulses are sent simultaneously. Rank the tubes according to the time of arrival of the pulses at the open right ends of the tubes, earliest first.

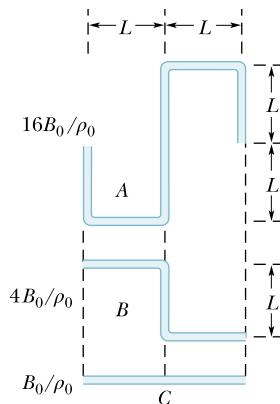


Figure 17.2 Question 3.

**4** The sixth harmonic is set up in a pipe. (a) How many open ends does the pipe have (it has at least one)? (b) Is there a node, antinode, or some intermediate state at the midpoint?

**5** In Fig. 17.3, pipe  $A$  is made to oscillate in its third harmonic by a small internal sound source. Sound emitted at the right end happens to resonate four nearby pipes, each with only one open end (they are *not* drawn to scale). Pipe  $B$  oscillates in its lowest harmonic, pipe  $C$  in its second lowest harmonic, pipe  $D$  in its

third lowest harmonic, and pipe  $E$  in its fourth lowest harmonic. Without computation, rank all five pipes according to their length, greatest first. (*Hint:* Draw the standing waves to scale and then draw the pipes to scale.)

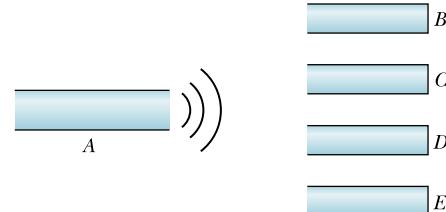


Figure 17.3 Question 5.

**6** Pipe  $A$  has length  $L$  and one open end. Pipe  $B$  has length  $2L$  and two open ends. Which harmonics of pipe  $B$  have a frequency that matches a resonant frequency of pipe  $A$ ?

**7** Figure 17.4 shows a moving sound source  $S$  that emits at a certain frequency, and four stationary sound detectors. Rank the detectors according to the frequency of the sound they detect from the source, greatest first.

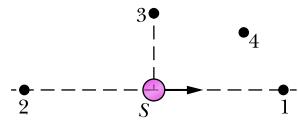


Figure 17.4 Question 7.

**8** A friend rides, in turn, the rims of three fast merry-go-rounds while holding a sound source that emits isotropically at a certain frequency. You stand far from each merry-go-round. The frequency you hear for each of your friend's three rides varies as the merry-go-round rotates. The variations in frequency for the three rides are given by the three curves in Fig. 17.5. Rank the curves according to (a) the linear speed  $v$  of the sound source, (b) the angular speeds  $\omega$  of the merry-go-rounds, and (c) the radii  $r$  of the merry-go-rounds, greatest first.

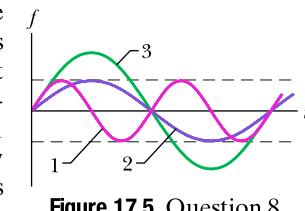


Figure 17.5 Question 8.

**9** For a particular tube, here are four of the six harmonic frequencies below 1000 Hz: 300, 600, 750, and 900 Hz. What two frequencies are missing from the list?

**10** Figure 17.6 shows a stretched string of length  $L$  and pipes  $a$ ,  $b$ ,  $c$ , and  $d$  of lengths  $L$ ,  $2L$ ,  $L/2$ , and  $L/2$ , respectively. The string's tension is adjusted until the speed of waves on the string equals the speed of sound waves in the air. The fundamental

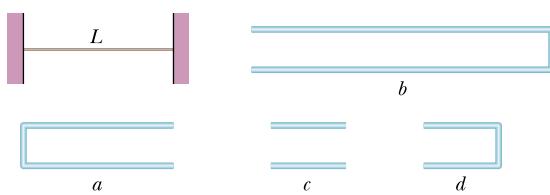


Figure 17.6 Question 10.

## Problems

**GO** Tutoring problem available (at instructor's discretion) in WileyPLUS

**SSM** Worked-out solution available in Student Solutions Manual

**E** Easy **M** Medium **H** Hard

**FCP** Additional information available in *The Flying Circus of Physics* and at [flyingcircusofphysics.com](http://flyingcircusofphysics.com)

**CALC** Requires calculus

**BIO** Biomedical application

Where needed in the problems, use

$$\text{speed of sound in air} = 343 \text{ m/s}$$

and

$$\text{density of air} = 1.21 \text{ kg/m}^3$$

unless otherwise specified.

### Module 17.1 Speed of Sound

**1 E** Two spectators at a soccer game see, and a moment later hear, the ball being kicked on the playing field. The time delay for spectator *A* is 0.23 s, and for spectator *B* it is 0.12 s. Sight lines from the two spectators to the player kicking the ball meet at an angle of 90°. How far are (a) spectator *A* and (b) spectator *B* from the player? (c) How far are the spectators from each other?

**2 E** What is the bulk modulus of oxygen if 32.0 g of oxygen occupies 22.4 L and the speed of sound in the oxygen is 317 m/s?

**3 E FCP** When the door of the Chapel of the Mausoleum in Hamilton, Scotland, is slammed shut, the last echo heard by someone standing just inside the door reportedly comes 15 s later. (a) If that echo were due to a single reflection off a wall opposite the door, how far from the door is the wall? (b) If, instead, the wall is 25.7 m away, how many reflections (back and forth) occur?

**4 E** A column of soldiers, marching at 120 paces per minute, keep in step with the beat of a drummer at the head of the column. The soldiers in the rear end of the column are striding forward with the left foot when the drummer is advancing with the right foot. What is the approximate length of the column?

**5 M SSM** Earthquakes generate sound waves inside Earth. Unlike a gas, Earth can experience both transverse (S) and longitudinal (P) sound waves. Typically, the speed of S waves is about 4.5 km/s, and that of P waves 8.0 km/s. A seismograph records P and S waves from an earthquake. The first P waves arrive 3.0 min before the first S waves. If the waves travel in a straight line, how far away did the earthquake occur?

**6 M** A man strikes one end of a thin rod with a hammer. The speed of sound in the rod is 15 times the speed of sound in air. A woman, at the other end with her ear close to the rod, hears the sound of the blow twice with a 0.12 s interval between; one sound comes through the rod and the other comes

mode of oscillation is then set up on the string. In which pipe will the sound produced by the string cause resonance, and what oscillation mode will that sound set up?

**11** You are given four tuning forks. The fork with the lowest frequency oscillates at 500 Hz. By striking two tuning forks at a time, you can produce the following beat frequencies, 1, 2, 3, 5, 7, and 8 Hz. What are the possible frequencies of the other three forks? (There are two sets of answers.)

through the air alongside the rod. If the speed of sound in air is 343 m/s, what is the length of the rod?

**7 M SSM** A stone is dropped into a well. The splash is heard 3.00 s later. What is the depth of the well?

**8 M CALC GO FCP** *Hot chocolate effect.* Tap a metal spoon inside a mug of water and note the frequency  $f_i$  you hear. Then add a spoonful of powder (say, chocolate mix or instant coffee) and tap again as you stir the powder. The frequency you hear has a lower value  $f_s$  because the tiny air bubbles released by the powder change the water's bulk modulus. As the bubbles reach the water surface and disappear, the frequency gradually shifts back to its initial value. During the effect, the bubbles don't appreciably change the water's density or volume or the sound's wavelength. Rather, they change the value of  $dV/dp$ —that is, the differential change in volume due to the differential change in the pressure caused by the sound wave in the water. If  $f_s/f_i = 0.333$ , what is the ratio  $(dV/dp)_s/(dV/dp)_i$ ?

### Module 17.2 Traveling Sound Waves

**9 E** If the form of a sound wave traveling through air is

$$s(x, t) = (6.0 \text{ nm}) \cos(kx + (3000 \text{ rad/s})t + \phi),$$

how much time does any given air molecule along the path take to move between displacements  $s = +2.0 \text{ nm}$  and  $s = -2.0 \text{ nm}$ ?

**10 E BIO FCP** *Underwater illusion.* One clue used by your brain to determine the direction of a source of sound is the time delay  $\Delta t$  between the arrival of the sound at the ear closer to the source and the arrival at the farther ear. Assume that the source is distant so that a wavefront from it is approximately planar when it reaches you, and let  $D$  represent the separation between your ears. (a) If the source is located at angle  $\theta$  in front of you (Fig. 17.7), what is  $\Delta t$  in terms of  $D$  and the speed of sound  $v$  in air? (b) If you are submerged in water and the sound source is directly to your right, what is  $\Delta t$  in terms of  $D$  and the speed of sound  $v_w$  in water? (c) Based on the time-delay clue, your brain interprets the submerged sound to arrive at an angle  $\theta$  from the forward direction. Evaluate  $\theta$  for fresh water at 20°C.

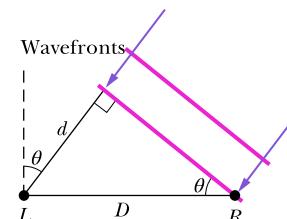


Figure 17.7 Problem 10.

**11 E BIO SSM** Diagnostic ultrasound of frequency 4.50 MHz is used to examine tumors in soft tissue. (a) What is the wavelength in air of such a sound wave? (b) If the speed of sound in tissue is 1500 m/s, what is the wavelength of this wave in tissue?

**12 E** The pressure in a traveling sound wave is given by the equation

$$\Delta p = (1.50 \text{ Pa}) \sin \pi[(0.900 \text{ m}^{-1})x - (315 \text{ s}^{-1})t].$$

Find the (a) pressure amplitude, (b) frequency, (c) wavelength, and (d) speed of the wave.

**13 M** A sound wave of the form  $s = s_m \cos(kx - \omega t + \phi)$  travels at 343 m/s through air in a long horizontal tube. At one instant, air molecule *A* at  $x = 2.000 \text{ m}$  is at its maximum positive displacement of 6.00 nm and air molecule *B* at  $x = 2.070 \text{ m}$  is at a positive displacement of 2.00 nm. All the molecules between *A* and *B* are at intermediate displacements. What is the frequency of the wave?

**14 M** Figure 17.8 shows the output from a pressure monitor mounted at a point along the path taken by a sound wave of a single frequency traveling at 343 m/s through air with a uniform density of  $1.21 \text{ kg/m}^3$ . The vertical axis scale is set by  $\Delta p_s = 4.0 \text{ mPa}$ . If the displacement function of the wave is  $s(x, t) = s_m \cos(kx - \omega t)$ , what are (a)  $s_m$ , (b)  $k$ , and (c)  $\omega$ ?

The air is then cooled so that its density is  $1.35 \text{ kg/m}^3$  and the speed of a sound wave through it is 320 m/s. The sound source again emits the sound wave at the same frequency and same pressure amplitude. What now are (d)  $s_m$ , (e)  $k$ , and (f)  $\omega$ ?

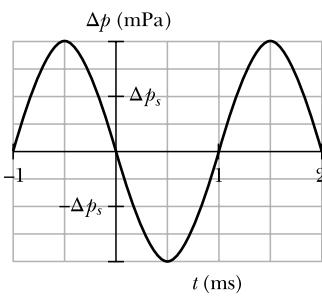


Figure 17.8 Problem 14.

**15 M GO FCP** A handclap on stage in an amphitheater sends out sound waves that scatter from terraces of width  $w = 0.75 \text{ m}$  (Fig. 17.9). The sound returns to the stage as a periodic series of pulses, one from each terrace; the parade of pulses sounds like a played note. (a) Assuming that all the rays in Fig. 17.9 are horizontal, find the frequency at which the pulses return (that is, the frequency of the perceived note). (b) If the width  $w$  of the terraces were smaller, would the frequency be higher or lower?

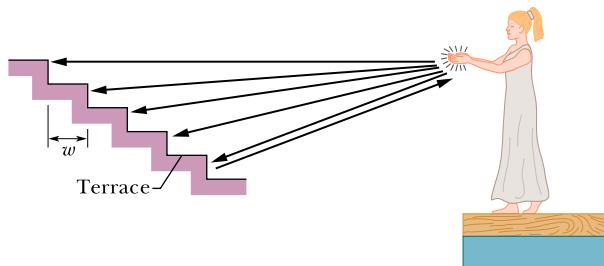


Figure 17.9 Problem 15.

### Module 17.3 Interference

**16 E** Two sound waves, from two different sources with the same frequency, 540 Hz, travel in the same direction at 330 m/s. The sources are in phase. What is the phase difference of the waves at a point that is 4.40 m from one source and 4.00 m from the other?

**17 M FCP** Two loudspeakers are located 3.35 m apart on an outdoor stage. A listener is 18.3 m from one and 19.5 m from the other. During the sound check, a signal generator drives the two speakers in phase with the same amplitude and frequency. The transmitted frequency is swept through the audible range (20 Hz to 20 kHz). (a) What is the lowest frequency  $f_{\min,1}$  that gives minimum signal (destructive interference) at the listener's location? By what number must  $f_{\min,1}$  be multiplied to get (b) the second lowest frequency  $f_{\min,2}$  that gives minimum signal and (c) the third lowest frequency  $f_{\min,3}$  that gives minimum signal? (d) What is the lowest frequency  $f_{\max,1}$  that gives maximum signal (constructive interference) at the listener's location? By what number must  $f_{\max,1}$  be multiplied to get (e) the second lowest frequency  $f_{\max,2}$  that gives maximum signal and (f) the third lowest frequency  $f_{\max,3}$  that gives maximum signal?

**18 M GO** In Fig. 17.10, sound waves *A* and *B*, both of wavelength  $\lambda$ , are initially in phase and traveling rightward, as indicated by the two rays. Wave *A* is reflected from four surfaces but ends up traveling in its original direction. Wave *B* ends in that direction after reflecting from two surfaces. Let distance  $L$  in the figure be expressed as a multiple  $q$  of  $\lambda$ :  $L = q\lambda$ . What are the (a) smallest and (b) second smallest values of  $q$  that put *A* and *B* exactly out of phase with each other after the reflections?

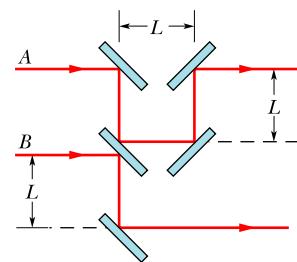


Figure 17.10 Problem 18.

**19 M GO** Figure 17.11 shows two isotropic point sources of sound,  $S_1$  and  $S_2$ . The sources emit waves in phase at wavelength  $0.50 \text{ m}$ ; they are separated by  $D = 1.75 \text{ m}$ . If we move a sound detector along a large circle centered at the midpoint between the sources, at how many points do waves arrive at the detector (a) exactly in phase and (b) exactly out of phase?

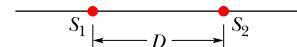


Figure 17.11 Problem 19.

**20 M** Figure 17.12 shows four isotropic point sources of sound that are uniformly spaced on an  $x$  axis. The sources emit sound at the same wavelength  $\lambda$  and same amplitude  $s_m$ , and they emit in phase. A point  $P$  is shown on the  $x$  axis. Assume that as the sound waves travel to  $P$ , the decrease in their amplitude is negligible. What multiple of  $s_m$  is the amplitude of the net wave at  $P$  if distance  $d$  in the figure is (a)  $\lambda/4$ , (b)  $\lambda/2$ , and (c)  $\lambda$ ?

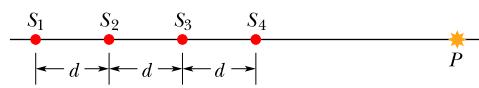


Figure 17.12 Problem 20.

**21 M SSM** In Fig. 17.13, two speakers separated by distance  $d_1 = 2.00 \text{ m}$  are in phase. Assume the amplitudes of the sound waves from the speakers are approximately the same at the listener's ear at distance  $d_2 = 3.75 \text{ m}$  directly in front of one speaker. Consider the full audible range for normal

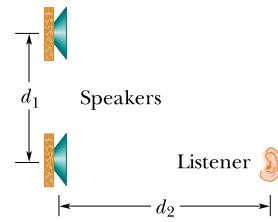


Figure 17.13 Problem 21.

hearing, 20 Hz to 20 kHz. (a) What is the lowest frequency  $f_{\min,1}$  that gives minimum signal (destructive interference) at the listener's ear? By what number must  $f_{\min,1}$  be multiplied to get (b) the second lowest frequency  $f_{\min,2}$  that gives minimum signal and (c) the third lowest frequency  $f_{\min,3}$  that gives minimum signal? (d) What is the lowest frequency  $f_{\max,1}$  that gives maximum signal (constructive interference) at the listener's ear? By what number must  $f_{\max,1}$  be multiplied to get (e) the second lowest frequency  $f_{\max,2}$  that gives maximum signal and (f) the third lowest frequency  $f_{\max,3}$  that gives maximum signal?

**22 M** In Fig. 17.14, sound with a 40.0 cm wavelength travels rightward from a source and through a tube that consists of a straight portion and a half-circle. Part of the sound wave travels through the half-circle and then rejoins the rest of the wave, which goes directly through the straight portion. This rejoining results in interference. What is the smallest radius  $r$  that results in an intensity minimum at the detector?

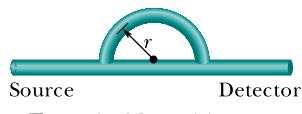


Figure 17.14 Problem 22.

**23 H GO** Figure 17.15 shows two point sources  $S_1$  and  $S_2$  that emit sound of wavelength  $\lambda = 2.00 \text{ m}$ . The emissions are isotropic and in phase, and the separation between the sources is  $d = 16.0 \text{ m}$ . At any point  $P$  on the  $x$  axis, the wave from  $S_1$  and the wave from  $S_2$  interfere. When  $P$  is very far away ( $x \approx \infty$ ), what are (a) the phase difference between the arriving waves from  $S_1$  and  $S_2$  and (b) the type of interference they produce? Now move point  $P$  along the  $x$  axis toward  $S_1$ . (c) Does the phase difference between the waves increase or decrease? At what distance  $x$  do the waves have a phase difference of (d)  $0.50\lambda$ , (e)  $1.00\lambda$ , and (f)  $1.50\lambda$ ?

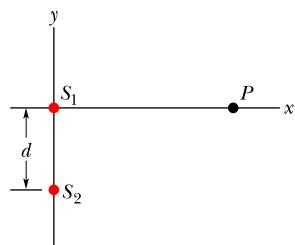


Figure 17.15 Problem 23.

#### Module 17.4 Intensity and Sound Level

**24 E BIO** Suppose that the sound level of a conversation is initially at an angry 70 dB and then drops to a soothing 50 dB. Assuming that the frequency of the sound is 500 Hz, determine the (a) initial and (b) final sound intensities and the (c) initial and (d) final sound wave amplitudes.

**25 E** A sound wave of frequency 300 Hz has an intensity of  $1.00 \mu\text{W}/\text{m}^2$ . What is the amplitude of the air oscillations caused by this wave?

**26 E** A 1.0 W point source emits sound waves isotropically. Assuming that the energy of the waves is conserved, find the intensity (a) 1.0 m from the source and (b) 2.5 m from the source.

**27 E SSM** A certain sound source is increased in sound level by 30.0 dB. By what multiple is (a) its intensity increased and (b) its pressure amplitude increased?

**28 E** Two sounds differ in sound level by 1.00 dB. What is the ratio of the greater intensity to the smaller intensity?

**29 E SSM** A point source emits sound waves isotropically. The intensity of the waves 2.50 m from the source is  $1.91 \times 10^{-4} \text{ W/m}^2$ . Assuming that the energy of the waves is conserved, find the power of the source.

**30 E** The source of a sound wave has a power of  $1.00 \mu\text{W}$ . If it is a point source, (a) what is the intensity 3.00 m away and (b) what is the sound level in decibels at that distance?

**31 E BIO FCP GO** When you "crack" a knuckle, you suddenly widen the knuckle cavity, allowing more volume for the synovial fluid inside it and causing a gas bubble suddenly to appear in the fluid. The sudden production of the bubble, called "cavitation," produces a sound pulse—the cracking sound. Assume that the sound is transmitted uniformly in all directions and that it fully passes from the knuckle interior to the outside. If the pulse has a sound level of 62 dB at your ear, estimate the rate at which energy is produced by the cavitation.

**32 E BIO FCP** Approximately a third of people with normal hearing have ears that continuously emit a low-intensity sound outward through the ear canal. A person with such *spontaneous otoacoustic emission* is rarely aware of the sound, except perhaps in a noise-free environment, but occasionally the emission is loud enough to be heard by someone else nearby. In one observation, the sound wave had a frequency of 1665 Hz and a pressure amplitude of  $1.13 \times 10^{-3} \text{ Pa}$ . What were (a) the displacement amplitude and (b) the intensity of the wave emitted by the ear?

**33 E BIO FCP** Male *Rana catesbeiana* bullfrogs are known for their loud mating call. The call is emitted not by the frog's mouth but by its eardrums, which lie on the surface of the head. And, surprisingly, the sound has nothing to do with the frog's inflated throat. If the emitted sound has a frequency of 260 Hz and a sound level of 85 dB (near the eardrum), what is the amplitude of the eardrum's oscillation? The air density is  $1.21 \text{ kg/m}^3$ .

**34 M GO** Two atmospheric sound sources  $A$  and  $B$  emit isotropically at constant power. The sound levels  $\beta$  of their emissions are plotted in Fig. 17.16 versus the radial distance  $r$  from the sources. The vertical axis scale is set by  $\beta_1 = 85.0 \text{ dB}$  and  $\beta_2 = 65.0 \text{ dB}$ . What are (a) the ratio of the larger power to the smaller power and (b) the sound level difference at  $r = 10 \text{ m}$ ?

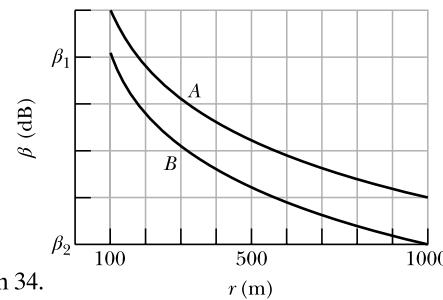


Figure 17.16 Problem 34.

**35 M** A point source emits 30.0 W of sound isotropically. A small microphone intercepts the sound in an area of  $0.750 \text{ cm}^2$ , 200 m from the source. Calculate (a) the sound intensity there and (b) the power intercepted by the microphone.

**36 M BIO FCP** *Party hearing.* As the number of people at a party increases, you must raise your voice for a listener to hear you against the *background noise* of the other partygoers. However, once you reach the level of yelling, the only way you can be heard is if you move closer to your listener, into the listener's "personal space." Model the situation by replacing you with an isotropic point source of fixed power  $P$  and replacing your listener with a point that absorbs part of your sound waves. These points are initially separated by  $r_i = 1.20 \text{ m}$ . If the background

noise increases by  $\Delta\beta = 5$  dB, the sound level at your listener must also increase. What separation  $r_f$  is then required?

- 37 H GO** A sound source sends a sinusoidal sound wave of angular frequency 3000 rad/s and amplitude 12.0 nm through a tube of air. The internal radius of the tube is 2.00 cm. (a) What is the average rate at which energy (the sum of the kinetic and potential energies) is transported to the opposite end of the tube? (b) If, simultaneously, an identical wave travels along an adjacent, identical tube, what is the total average rate at which energy is transported to the opposite ends of the two tubes by the waves? If, instead, those two waves are sent along the same tube simultaneously, what is the total average rate at which they transport energy when their phase difference is (c) 0, (d)  $0.40\pi$  rad, and (e)  $\pi$  rad?

### Module 17.5 Sources of Musical Sound

**38 E** The water level in a vertical glass tube 1.00 m long can be adjusted to any position in the tube. A tuning fork vibrating at 686 Hz is held just over the open top end of the tube, to set up a standing wave of sound in the air-filled top portion of the tube. (That air-filled top portion acts as a tube with one end closed and the other end open.) (a) For how many different positions of the water level will sound from the fork set up resonance in the tube's air-filled portion? What are the (b) least and (c) second least water heights in the tube for resonance to occur?

**39 E SSM** (a) Find the speed of waves on a violin string of mass 800 mg and length 22.0 cm if the fundamental frequency is 920 Hz. (b) What is the tension in the string? For the fundamental, what is the wavelength of (c) the waves on the string and (d) the sound waves emitted by the string?

**40 E** Organ pipe *A*, with both ends open, has a fundamental frequency of 300 Hz. The third harmonic of organ pipe *B*, with one end open, has the same frequency as the second harmonic of pipe *A*. How long are (a) pipe *A* and (b) pipe *B*?

**41 E** A violin string 15.0 cm long and fixed at both ends oscillates in its  $n = 1$  mode. The speed of waves on the string is 250 m/s, and the speed of sound in air is 348 m/s. What are the (a) frequency and (b) wavelength of the emitted sound wave?

**42 E** A sound wave in a fluid medium is reflected at a barrier so that a standing wave is formed. The distance between nodes is 3.8 cm, and the speed of propagation is 1500 m/s. Find the frequency of the sound wave.

**43 E SSM** In Fig. 17.17, *S* is a small loudspeaker driven by an audio oscillator with a frequency that is varied from 1000 Hz to 2000 Hz, and *D* is a cylindrical pipe with two open ends and a length of 45.7 cm. The speed of sound in the air-filled pipe is 344 m/s. (a) At how many frequencies does the sound from the loudspeaker set up resonance in the pipe? What are the (b) lowest and (c) second lowest frequencies at which resonance occurs?

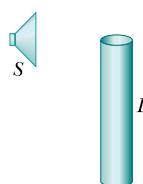


Figure 17.17  
Problem 43.

**44 E BIO FCP** The crest of a *Parasaurolophus* dinosaur skull is shaped somewhat like a trombone and contains a nasal passage in the form of a long, bent tube open at both ends. The dinosaur may have used the passage to produce sound by setting up the fundamental mode in it. (a) If the nasal passage in a certain *Parasaurolophus* fossil is 2.0 m long, what frequency would have

been produced? (b) If that dinosaur could be recreated (as in *Jurassic Park*), would a person with a hearing range of 60 Hz to 20 kHz be able to hear that fundamental mode and, if so, would the sound be high or low frequency? Fossil skulls that contain shorter nasal passages are thought to be those of the female *Parasaurolophus*. (c) Would that make the female's fundamental frequency higher or lower than the male's?

**45 E** In pipe *A*, the ratio of a particular harmonic frequency to the next lower harmonic frequency is 1.2. In pipe *B*, the ratio of a particular harmonic frequency to the next lower harmonic frequency is 1.4. How many open ends are in (a) pipe *A* and (b) pipe *B*?

**46 M GO** Pipe *A*, which is 1.20 m long and open at both ends, oscillates at its third lowest harmonic frequency. It is filled with air for which the speed of sound is 343 m/s. Pipe *B*, which is closed at one end, oscillates at its second lowest harmonic frequency. This frequency of *B* happens to match the frequency of *A*. An *x* axis extends along the interior of *B*, with *x* = 0 at the closed end. (a) How many nodes are along that axis? What are the (b) smallest and (c) second smallest value of *x* locating those nodes? (d) What is the fundamental frequency of *B*?

**47 M** A well with vertical sides and water at the bottom resonates at 7.00 Hz and at no lower frequency. The air-filled portion of the well acts as a tube with one closed end (at the bottom) and one open end (at the top). The air in the well has a density of  $1.10 \text{ kg/m}^3$  and a bulk modulus of  $1.33 \times 10^5 \text{ Pa}$ . How far down in the well is the water surface?

**48 M** One of the harmonic frequencies of tube *A* with two open ends is 325 Hz. The next-highest harmonic frequency is 390 Hz. (a) What harmonic frequency is next highest after the harmonic frequency 195 Hz? (b) What is the number of this next-highest harmonic? One of the harmonic frequencies of tube *B* with only one open end is 1080 Hz. The next-highest harmonic frequency is 1320 Hz. (c) What harmonic frequency is next highest after the harmonic frequency 600 Hz? (d) What is the number of this next-highest harmonic?

**49 M SSM** A violin string 30.0 cm long with linear density 0.650 g/m is placed near a loudspeaker that is fed by an audio oscillator of variable frequency. It is found that the string is set into oscillation only at the frequencies 880 and 1320 Hz as the frequency of the oscillator is varied over the range 500–1500 Hz. What is the tension in the string?

**50 M GO** A tube 1.20 m long is closed at one end. A stretched wire is placed near the open end. The wire is 0.330 m long and has a mass of 9.60 g. It is fixed at both ends and oscillates in its fundamental mode. By resonance, it sets the air column in the tube into oscillation at that column's fundamental frequency. Find (a) that frequency and (b) the tension in the wire.

### Module 17.6 Beats

**51 E** The A string of a violin is a little too tightly stretched. Beats at 4.00 per second are heard when the string is sounded together with a tuning fork that is oscillating accurately at concert A (440 Hz). What is the period of the violin string oscillation?

**52 E** A tuning fork of unknown frequency makes 3.00 beats per second with a standard fork of frequency 384 Hz. The beat frequency decreases when a small piece of wax is put on a prong of the first fork. What is the frequency of this fork?

**53 M SSM** Two identical piano wires have a fundamental frequency of 600 Hz when kept under the same tension. What fractional increase in the tension of one wire will lead to the occurrence of 6.0 beats/s when both wires oscillate simultaneously?

**54 M** You have five tuning forks that oscillate at close but different resonant frequencies. What are the (a) maximum and (b) minimum number of different beat frequencies you can produce by sounding the forks two at a time, depending on how the resonant frequencies differ?

### Module 17.7 The Doppler Effect

**55 E** A whistle of frequency 540 Hz moves in a circle of radius 60.0 cm at an angular speed of 15.0 rad/s. What are the (a) lowest and (b) highest frequencies heard by a listener a long distance away, at rest with respect to the center of the circle?

**56 E** An ambulance with a siren emitting a whine at 1600 Hz overtakes and passes a cyclist pedaling a bike at 2.44 m/s. After being passed, the cyclist hears a frequency of 1590 Hz. How fast is the ambulance moving?

**57 E** A state trooper chases a speeder along a straight road; both vehicles move at 160 km/h. The siren on the trooper's vehicle produces sound at a frequency of 500 Hz. What is the Doppler shift in the frequency heard by the speeder?

**58 M** A sound source *A* and a reflecting surface *B* move directly toward each other. Relative to the air, the speed of source *A* is 29.9 m/s, the speed of surface *B* is 65.8 m/s, and the speed of sound is 329 m/s. The source emits waves at frequency 1200 Hz as measured in the source frame. In the reflector frame, what are the (a) frequency and (b) wavelength of the arriving sound waves? In the source frame, what are the (c) frequency and (d) wavelength of the sound waves reflected back to the source?

**59 M GO** In Fig. 17.18, a French submarine and a U.S. submarine move toward each other during maneuvers in motionless water in the North Atlantic. The French sub moves at speed  $v_F = 50.00 \text{ km/h}$ , and the U.S. sub at  $v_{US} = 70.00 \text{ km/h}$ . The French sub sends out a sonar signal (sound wave in water) at  $1.000 \times 10^3 \text{ Hz}$ . Sonar waves travel at 5470 km/h. (a) What is the signal's frequency as detected by the U.S. sub? (b) What frequency is detected by the French sub in the signal reflected back to it by the U.S. sub?

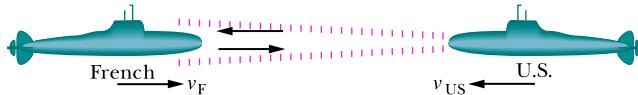


Figure 17.18 Problem 59.

**60 M** A stationary motion detector sends sound waves of frequency 0.150 MHz toward a truck approaching at a speed of 45.0 m/s. What is the frequency of the waves reflected back to the detector?

**61 M BIO GO FCP** A bat is flitting about in a cave, navigating via ultrasonic beeps. Assume that the sound emission frequency of the bat is 39 000 Hz. During one fast swoop directly toward a flat wall surface, the bat is moving at 0.025 times the speed of sound in air. What frequency does the bat hear reflected off the wall?

**62 M** Figure 17.19 shows four tubes with lengths 1.0 m or 2.0 m, with one or two open ends as drawn. The third harmonic is set

up in each tube, and some of the sound that escapes from them is detected by detector *D*, which moves directly away from the tubes. In terms of the speed of sound *v*, what speed must the detector have such that the detected frequency of the sound from (a) tube 1, (b) tube 2, (c) tube 3, and (d) tube 4 is equal to the tube's fundamental frequency?

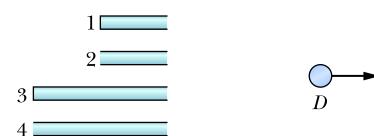


Figure 17.19 Problem 62.

**63 M** An acoustic burglar alarm consists of a source emitting waves of frequency 28.0 kHz. What is the beat frequency between the source waves and the waves reflected from an intruder walking at an average speed of 0.950 m/s directly away from the alarm?

**64 M** A stationary detector measures the frequency of a sound source that first moves at constant velocity directly toward the detector and then (after passing the detector) directly away from it. The emitted frequency is *f*. During the approach the detected frequency is  $f'_{\text{app}}$  and during the recession it is  $f'_{\text{rec}}$ . If  $(f'_{\text{app}} - f'_{\text{rec}})/f = 0.500$ , what is the ratio  $v_s/v$  of the speed of the source to the speed of sound?

**65 H GO** A 2000 Hz siren and a civil defense official are both at rest with respect to the ground. What frequency does the official hear if the wind is blowing at 12 m/s (a) from source to official and (b) from official to source?

**66 H GO** Two trains are traveling toward each other at 30.5 m/s relative to the ground. One train is blowing a whistle at 500 Hz. (a) What frequency is heard on the other train in still air? (b) What frequency is heard on the other train if the wind is blowing at 30.5 m/s toward the whistle and away from the listener? (c) What frequency is heard if the wind direction is reversed?

**67 H SSM** A girl is sitting near the open window of a train that is moving at a velocity of 10.00 m/s to the east. The girl's uncle stands near the tracks and watches the train move away. The locomotive whistle emits sound at frequency 500.0 Hz. The air is still. (a) What frequency does the uncle hear? (b) What frequency does the girl hear? A wind begins to blow from the east at 10.00 m/s. (c) What frequency does the uncle now hear? (d) What frequency does the girl now hear?

### Module 17.8 Supersonic Speeds, Shock Waves

**68 E** The shock wave off the cockpit of the FA 18 in Fig. 17.8.2 has an angle of about 60°. The airplane was traveling at about 1350 km/h when the photograph was taken. Approximately what was the speed of sound at the airplane's altitude?

**69 M SSM FCP** A jet plane passes over you at a height of 5000 m and a speed of Mach 1.5. (a) Find the Mach cone angle (the sound speed is 331 m/s). (b) How long after the jet passes directly overhead does the shock wave reach you?

**70 M** A plane flies at 1.25 times the speed of sound. Its sonic boom reaches a man on the ground 1.00 min after the plane passes directly overhead. What is the altitude of the plane? Assume the speed of sound to be 330 m/s.

### Additional Problems

**71** At a distance of 10 km, a 100 Hz horn, assumed to be an isotropic point source, is barely audible. At what distance would it begin to cause pain?

**72** A bullet is fired with a speed of 685 m/s. Find the angle made by the shock cone with the line of motion of the bullet.

**73 BIO FCP** A sperm whale (Fig. 17.20a) vocalizes by producing a series of clicks. Actually, the whale makes only a single sound near the front of its head to start the series. Part of that sound then emerges from the head into the water to become the first click of the series. The rest of the sound travels backward through the spermaceti sac (a body of fat), reflects from the frontal sac (an air layer), and then travels forward through the spermaceti sac. When it reaches the distal sac (another air layer) at the front of the head, some of the sound escapes into the water to form the second click, and the rest is sent back through the spermaceti sac (and ends up forming later clicks).

Figure 17.20b shows a strip-chart recording of a series of clicks. A unit time interval of 1.0 ms is indicated on the chart. Assuming that the speed of sound in the spermaceti sac is 1372 m/s, find the length of the spermaceti sac. From such a calculation, marine scientists estimate the length of a whale from its click series.

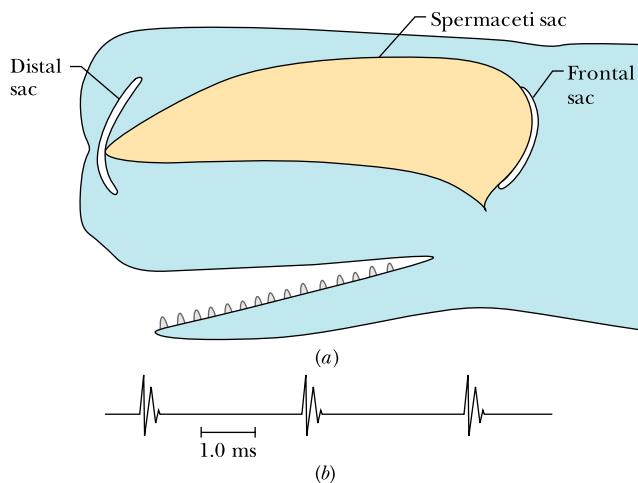


Figure 17.20 Problem 73.

**74** The average density of Earth's crust 10 km beneath the continents is  $2.7 \text{ g/cm}^3$ . The speed of longitudinal seismic waves at that depth, found by timing their arrival from distant earthquakes, is 5.4 km/s. Find the bulk modulus of Earth's crust at that depth. For comparison, the bulk modulus of steel is about  $16 \times 10^{10} \text{ Pa}$ .

**75** A certain loudspeaker system emits sound isotropically with a frequency of 2000 Hz and an intensity of  $0.960 \text{ mW/m}^2$  at a distance of 6.10 m. Assume that there are no reflections. (a) What is the intensity at 30.0 m? At 6.10 m, what are (b) the displacement amplitude and (c) the pressure amplitude?

**76** Find the ratios (greater to smaller) of the (a) intensities, (b) pressure amplitudes, and (c) particle displacement amplitudes for two sounds whose sound levels differ by 37 dB.

**77** In Fig. 17.21, sound waves *A* and *B*, both of wavelength  $\lambda$ , are initially in phase and traveling rightward, as indicated by the two rays. Wave *A* is reflected from four surfaces but ends up

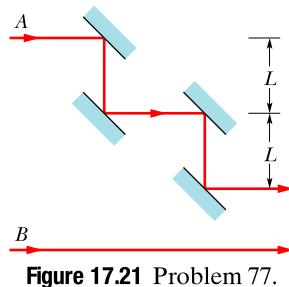


Figure 17.21 Problem 77.

traveling in its original direction. What multiple of wavelength  $\lambda$  is the smallest value of distance  $L$  in the figure that puts *A* and *B* exactly out of phase with each other after the reflections?

**78** A trumpet player on a moving railroad flatcar moves toward a second trumpet player standing alongside the track while both play a 440 Hz note. The sound waves heard by a stationary observer between the two players have a beat frequency of 4.0 beats/s. What is the flatcar's speed?

**79 GO** In Fig. 17.22, sound of wavelength 0.850 m is emitted isotropically by point source *S*. Sound ray 1 extends directly to detector *D*, at distance  $L = 10.0 \text{ m}$ . Sound ray 2 extends to *D* via a reflection (effectively, a “bouncing”) of the sound at a flat surface. That reflection occurs on a perpendicular bisector to the *SD* line, at distance  $d$  from the line. Assume that the reflection shifts the sound wave by  $0.500\lambda$ . For what least value of  $d$  (other than zero) do the direct sound and the reflected sound arrive at *D* (a) exactly out of phase and (b) exactly in phase?

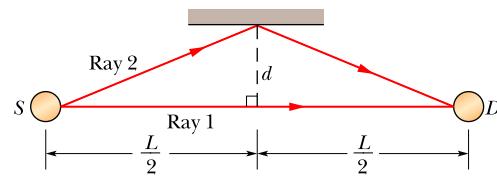


Figure 17.22 Problem 79.

**80 GO** A detector initially moves at constant velocity directly toward a stationary sound source and then (after passing it) directly from it. The emitted frequency is  $f$ . During the approach the detected frequency is  $f'_{\text{app}}$  and during the recession it is  $f'_{\text{rec}}$ . If the frequencies are related by  $(f'_{\text{app}} - f'_{\text{rec}})/f = 0.500$ , what is the ratio  $v_D/v$  of the speed of the detector to the speed of sound?

**81 SSM** (a) If two sound waves, one in air and one in (fresh) water, are equal in intensity and angular frequency, what is the ratio of the pressure amplitude of the wave in water to that of the wave in air? Assume the water and the air are at  $20^\circ\text{C}$ . (See Table 14.1.1.) (b) If the pressure amplitudes are equal instead, what is the ratio of the intensities of the waves?

**82** A continuous sinusoidal longitudinal wave is sent along a very long coiled spring from an attached oscillating source. The wave travels in the negative direction of an *x* axis; the source frequency is 25 Hz; at any instant the distance between successive points of maximum expansion in the spring is 24 cm; the maximum longitudinal displacement of a spring particle is 0.30 cm; and the particle at  $x = 0$  has zero displacement at time  $t = 0$ . If the wave is written in the form  $s(x, t) = s_m \cos(kx \pm \omega t)$ , what are (a)  $s_m$ , (b)  $k$ , (c)  $\omega$ , (d) the wave speed, and (e) the correct choice of sign in front of  $\omega$ ?

**83 BIO SSM** Ultrasound, which consists of sound waves with frequencies above the human audible range, can be used to produce an image of the interior of a human body. Moreover, ultrasound can be used to measure the speed of the blood in the body; it does so by comparing the frequency of the ultrasound sent into the body with the frequency of the ultrasound reflected back

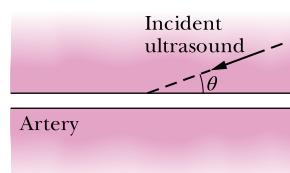


Figure 17.23 Problem 83.

to the body's surface by the blood. As the blood pulses, this detected frequency varies.

Suppose that an ultrasound image of the arm of a patient shows an artery that is angled at  $\theta = 20^\circ$  to the ultrasound's line of travel (Fig. 17.23). Suppose also that the frequency of the ultrasound reflected by the blood in the artery is increased by a maximum of 5495 Hz from the original ultrasound frequency of 5.000 000 MHz. (a) In Fig. 17.23, is the direction of the blood flow rightward or leftward? (b) The speed of sound in the human arm is 1540 m/s. What is the maximum speed of the blood? (*Hint:* The Doppler effect is caused by the component of the blood's velocity along the ultrasound's direction of travel.) (c) If angle  $\theta$  were greater, would the reflected frequency be greater or less?

**84** The speed of sound in a certain metal is  $v_m$ . One end of a long pipe of that metal of length  $L$  is struck a hard blow. A listener at the other end hears two sounds, one from the wave that travels along the pipe's metal wall and the other from the wave that travels through the air inside the pipe. (a) If  $v$  is the speed of sound in air, what is the time interval  $\Delta t$  between the arrivals of the two sounds at the listener's ear? (b) If  $\Delta t = 1.00\text{ s}$  and the metal is steel, what is the length  $L$ ?

**85 FCP** An avalanche of sand along some rare desert sand dunes can produce a booming that is loud enough to be heard 10 km away. The booming apparently results from a periodic oscillation of the sliding layer of sand—the layer's thickness expands and contracts. If the emitted frequency is 90 Hz, what are (a) the period of the thickness oscillation and (b) the wavelength of the sound?

**86** A sound source moves along an  $x$  axis, between detectors  $A$  and  $B$ . The wavelength of the sound detected at  $A$  is 0.500 that of the sound detected at  $B$ . What is the ratio  $v_s/v$  of the speed of the source to the speed of sound?

**87 SSM** A siren emitting a sound of frequency 1000 Hz moves away from you toward the face of a cliff at a speed of 10 m/s. Take the speed of sound in air as 330 m/s. (a) What is the frequency of the sound you hear coming directly from the siren? (b) What is the frequency of the sound you hear reflected off the cliff? (c) What is the beat frequency between the two sounds? Is it perceptible (less than 20 Hz)?

**88** At a certain point, two waves produce pressure variations given by  $\Delta p_1 = \Delta p_m \sin \omega t$  and  $\Delta p_2 = \Delta p_m \sin(\omega t - \phi)$ . At this point, what is the ratio  $\Delta p_r/\Delta p_m$ , where  $\Delta p_r$  is the pressure amplitude of the resultant wave, if  $\phi$  is (a) 0, (b)  $\pi/2$ , (c)  $\pi/3$ , and (d)  $\pi/4$ ?

**89** Two sound waves with an amplitude of 12 nm and a wavelength of 35 cm travel in the same direction through a long tube, with a phase difference of  $\pi/3$  rad. What are the (a) amplitude and (b) wavelength of the net sound wave produced by their interference? If, instead, the sound waves travel through the tube in opposite directions, what are the (c) amplitude and (d) wavelength of the net wave?

**90** A sinusoidal sound wave moves at 343 m/s through air in the positive direction of an  $x$  axis. At one instant during the oscillations, air molecule  $A$  is at its maximum displacement in the negative direction of the axis while air molecule  $B$  is at its equilibrium position. The separation between those molecules is 15.0 cm, and the molecules between  $A$  and  $B$  have intermediate

displacements in the negative direction of the axis. (a) What is the frequency of the sound wave?

In a similar arrangement but for a different sinusoidal sound wave, at one instant air molecule  $C$  is at its maximum displacement in the positive direction while molecule  $D$  is at its maximum displacement in the negative direction. The separation between the molecules is again 15.0 cm, and the molecules between  $C$  and  $D$  have intermediate displacements. (b) What is the frequency of the sound wave?

**91** Two identical tuning forks can oscillate at 440 Hz. A person is located somewhere on the line between them. Calculate the beat frequency as measured by this individual if (a) she is standing still and the tuning forks move in the same direction along the line at 3.00 m/s, and (b) the tuning forks are stationary and the listener moves along the line at 3.00 m/s.

**92** You can estimate your distance from a lightning stroke by counting the seconds between the flash you see and the thunder you later hear. By what integer should you divide the number of seconds to get the distance in kilometers?

**93 SSM** Figure 17.24 shows an air-filled, acoustic interferometer, used to demonstrate the interference of sound waves. Soundsource  $S$  is an oscillating diaphragm;  $D$  is a sound detector, such as the ear or a microphone. Path  $SBD$  can be varied in length, but path  $SAD$  is fixed. At  $D$ , the sound wave coming along path  $SBD$  interferes with that coming along path  $SAD$ . In one demonstration, the sound intensity at  $D$  has a minimum value of 100 units at one position of the movable arm and continuously climbs to a maximum value of 900 units when that arm is shifted by 1.65 cm. Find (a) the frequency of the sound emitted by the source and (b) the ratio of the amplitude at  $D$  of the  $SAD$  wave to that of the  $SBD$  wave. (c) How can it happen that these waves have different amplitudes, considering that they originate at the same source?

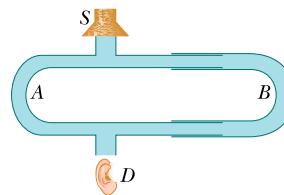


Figure 17.24 Problem 93.

**94** On July 10, 1996, a granite block broke away from a wall in Yosemite Valley and, as it began to slide down the wall, was launched into projectile motion. Seismic waves produced by its impact with the ground triggered seismographs as far away as 200 km. Later measurements indicated that the block had a mass between  $7.3 \times 10^7\text{ kg}$  and  $1.7 \times 10^8\text{ kg}$  and that it landed 500 m vertically below the launch point and 30 m horizontally from it. (The launch angle is not known.) (a) Estimate the block's kinetic energy just before it landed.

Consider two types of seismic waves that spread from the impact point—a hemispherical *body wave* traveled through the ground in an expanding hemisphere and a cylindrical *surface wave* traveled along the ground in an expanding shallow vertical cylinder (Fig. 17.25). Assume that the impact lasted 0.50 s, the vertical cylinder had a depth  $d$  of 5.0 m, and each wave type received 20% of the energy the block had just before impact. Neglecting any mechanical energy loss the waves experienced as they traveled, determine the intensities of (b) the body wave and (c) the surface wave when they reached a seismograph 200 km away. (d) On the basis of these results, which wave is more easily detected on a distant seismograph?

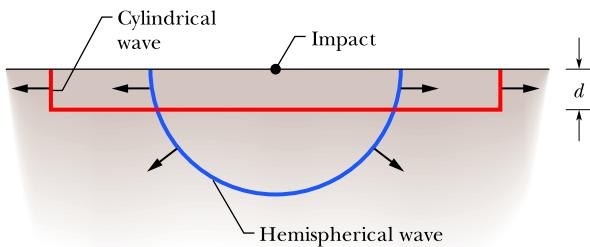


Figure 17.25 Problem 94.

**95 SSM** The sound intensity is  $0.0080 \text{ W/m}^2$  at a distance of 10 m from an isotropic point source of sound. (a) What is the power of the source? (b) What is the sound intensity 5.0 m from the source? (c) What is the sound level 10 m from the source?

**96** Four sound waves are to be sent through the same tube of air, in the same direction:

$$\begin{aligned}s_1(x, t) &= (9.00 \text{ nm}) \cos(2\pi x - 700\pi t), \\ s_2(x, t) &= (9.00 \text{ nm}) \cos(2\pi x - 700\pi t + 0.7\pi), \\ s_3(x, t) &= (9.00 \text{ nm}) \cos(2\pi x - 700\pi t + \pi), \\ s_4(x, t) &= (9.00 \text{ nm}) \cos(2\pi x - 700\pi t + 1.7\pi).\end{aligned}$$

What is the amplitude of the resultant wave? (*Hint:* Use a phasor diagram to simplify the problem.)

**97** Straight line  $AB$  connects two point sources that are 5.00 m apart, emit 300 Hz sound waves of the same amplitude, and emit exactly out of phase. (a) What is the shortest distance between the midpoint of  $AB$  and a point on  $AB$  where the interfering waves cause maximum oscillation of the air molecules? What are the (b) second and (c) third shortest distances?

**98** A point source that is stationary on an  $x$  axis emits a sinusoidal sound wave at a frequency of 686 Hz and speed 343 m/s. The wave travels radially outward from the source, causing air molecules to oscillate radially inward and outward. Let us define a wavefront as a line that connects points where the air molecules have the maximum, radially outward displacement. At any given instant, the wavefronts are concentric circles that are centered on the source. (a) Along  $x$ , what is the adjacent wavefront separation? Next, the source moves along  $x$  at a speed of 110 m/s. Along  $x$ , what are the wavefront separations (b) in front of and (c) behind the source?

**99** You are standing at a distance  $D$  from an isotropic point source of sound. You walk 50.0 m toward the source and observe that the intensity of the sound has doubled. Calculate the distance  $D$ .

**100** Pipe  $A$  has only one open end; pipe  $B$  is four times as long and has two open ends. Of the lowest 10 harmonic numbers  $n_B$  of pipe  $B$ , what are the (a) smallest, (b) second smallest, and (c) third smallest values at which a harmonic frequency of  $B$  matches one of the harmonic frequencies of  $A$ ?

**101** A toy rocket moves at a speed of 242 m/s directly toward a stationary pole (through stationary air) while emitting sound waves at frequency  $f = 1250 \text{ Hz}$ . (a) What frequency  $f'$  is sensed by a detector that is attached to the pole? (b) Some of the sound reaching the pole reflects to the rocket, which has an onboard detector. What frequency  $f''$  does it detect?

**102 SHM Doppler shift.** A microscopic structure is in simple harmonic motion in air along an  $x$  axis with angular frequency

$\omega = 6.80 \times 10^6 \text{ rad/s}$  (Fig. 17.26).

From a stationary source, a beam of ultrasound of frequency  $f_0$  is directed toward the structure along that axis. The echo returned to a stationary detector at the ultrasound source varies in frequency during the oscillation from a lowest value  $f_L$  to a highest value  $f_H$ . The ratio  $f_L/f_0$  is 0.800. (a) Where in the oscillation is the structure when  $f_L$  is emitted? What are (b) the amplitude  $x_m$  of the oscillation and (c) the ratio  $f_H/f_0$ ?



Figure 17.26 Problem 102.

### 103 Beats with Doppler shifts.

Figure 17.27 shows two isotropic point sources of sound: Sources  $S_1$  and  $S_2$  are moving in the positive direction of an  $x$  axis toward detector  $D$ , and both emit sound with frequency  $f = 500 \text{ Hz}$ .  $S_1$  moves at speed  $v_{S1} = 0.180v$ , where  $v$  is the speed of sound.  $S_2$  moves at  $v_{S2} = 0.185v$ . What is the beat frequency of the sound at the detector?

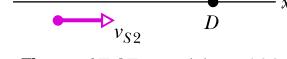


Figure 17.27 Problem 103.

### 104 Off-axis Doppler shift.

In Fig. 17.28, a bat flies at speed  $v_b = 9.00 \text{ m/s}$  along an  $x$  axis while emitting sound at frequency  $f = 80.0 \text{ kHz}$ . The sound is detected by  $D$ , located at distance  $d = 20.0 \text{ m}$  off the axis. Assume that the bat is an isotropic sound source. (a) What frequency  $f'$  is detected by  $D$  when the bat emits the sound at  $x_b = -113 \text{ m}$ . (*Hint:* What is the velocity component toward  $D$  just then?) (b) What is the bat's coordinate at the moment of detection? (c) As the bat continues toward the origin of the coordinate system, does  $f'$  increase, decrease, or stay the same, and what is the detected frequency of the sound that is emitted at the origin?

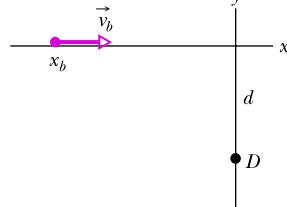


Figure 17.28 Problem 104.

**105 Sound speed in aluminum.** An experimenter wishes to measure the speed of sound in an aluminum rod 10 cm long by measuring the time a sound pulse takes to travel the length of the rod. If results good to four significant figures are desired, (a) how precisely must the length of the rod be known, and (b) how closely must the experimenter be able to resolve time intervals?

**106 The loudest.** One way to measure “loudness” is with the sound pressure level SPL (units dB SPL), which is defined as

$$\text{SPL} = 20 \log\left(\frac{\Delta p_m}{p_0}\right),$$

where  $\Delta p_m$  is the measured pressure amplitude of the sound wave and  $p_0$  is the reference pressure of  $20 \mu\text{Pa}$  ( $= 20 \mu\text{N/m}^2$ ). For a sustained sinusoidal sound wave, the upper limit for the pressure amplitude is  $\Delta p_m = 1 \text{ atm}$ . In that case, what are (a) the maximum pressure and (b) the minimum pressure in the wave? (c) What is the SPL? (This is the “loudest” that a sustained sound wave can be in Earth’s atmosphere.)

**107 Longest string-can telephone.** To make a string-can telephone (Fig. 17.29), punch a hole in the bottom of two empty food cans (or paper cups). Then, from the exterior of each can, run a string through the hole and tie a knot at the end to prevent the end from slipping back through the hole. Give one can to someone with the instruction to walk away and then pull on that

can so that the string is under tension. You can then talk to the other person by speaking into your can. Your sound causes the can bottom to oscillate, which periodically pulls and releases the string, sending pulses along the string. When those pulses reach the bottom of the other can, that bottom oscillates, producing sound waves in the can's air. The other person thus hears your message. A better design is with steel wire (instead of ordinary string) that is welded to the can bottoms to form rigid connections. The Guinness World Record for the longest string-can telephone is 242.62 m, set in Chosei, Chiba, Japan, in 2019. Assume that steel wire was used. What is the time difference  $\Delta t$  between a pulse sent along the wire and a pulse sent through the air?



**Figure 17.29** Problem 107.

**108 Chalk squealing on a chalkboard.** To make a chalk stick squeal when drawn across a ceramic (clay) chalkboard (Fig. 17.30), first rub it several times over the area to be used. Then hold the chalk stick about  $30^\circ$  from a perpendicular to the board and pull it. The chalk can undergo repeated *stick and slip*, producing the irritating squeal. The oscillations responsible for the sound occur in a *shear layer* that is a 0.3-mm-thick layer between the board and chalk stick whenever the chalk slips after sticking. Waves from that oscillating layer travel into the board, which radiates the sound much like a drumhead. If



**Figure 17.30** Problem 108.

the frequency of the squealing is 2050 Hz, what is the wavelength of the sound that reaches you? (If the chalk stick is short, your grip on it damps out the oscillations, eliminating the squeal.)

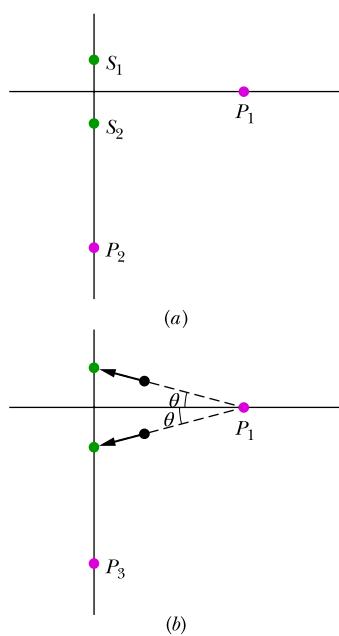
**109 Wave interference.** Figure 17.31a shows two point sound sources  $S_1$  and  $S_2$  located on a line. The sources emit sound isotropically, in phase, and at the same wavelength  $\lambda$  and same amplitude. Detection point  $P_1$  is on a perpendicular bisector to the line between the sources; waves arrive there with zero phase difference. Detection point  $P_2$  is on the line through the sources; waves arrive there with a phase difference of 5.0 wavelengths. (a) In terms of wavelengths, what is the distance between the sources? (b) What type of interference occurs at point  $P_1$ ?

The sources are now both moved directly away from point  $P_1$  by a distance of  $\lambda/2$ , at an angle of  $\theta = 30^\circ$  (Fig. 17.31b). (c) In terms of wavelengths, what is the phase difference between the waves arriving at point  $P_3$ , which is on the new line through the sources? (d) What type of interference occurs at point  $P_3$ ?

**110 BIO Firing range.** The most common handgun carried by United States police officers is the Glock 22 (0.40 caliber). The officers must qualify and train with the weapon on firing ranges where hearing protection is required to avoid hearing loss, possibly permanent loss. In one investigation of the protection, a Glock 22 was fired 1.0 m from a mannequin. The noise was detected by two microphones. One was under protective gear on the mannequin's ear and one was adjacent to the mannequin and unprotected. The noise level was measured in terms of the sound pressure level SPL (units dB SPL), which is defined as

$$SPL = 20 \log\left(\frac{p}{p_0}\right),$$

where  $p$  is the measured sound pressure and  $p_0$  is the reference pressure of  $20 \mu\text{Pa} (= 20 \mu\text{N/m}^2)$ . What were the pressures for (a) 158 dB SPL for the unprotected microphone, (b) 123 dB SPL for the mannequin with the protective earmuffs, and (c) 103 dB SPL for the mannequin with both earmuffs and earplugs? The recommended allowable exposure set by the National Institute for Occupational Safety and Health is 140 dB SPL, which would be exceeded by an officer firing a Glock 22 without protection, as is sometimes required in the line of duty.



**Figure 17.31** Problem 109.