CHAPTER 26

# Current and Resistance

## 26.1 ELECTRIC CURRENT

### Learning Objectives

*After reading this module, you should be able to . . .*

**26.1.1** Apply the definition of current as the rate at which charge moves through a point, including solving for the amount of charge that passes the point in a given time interval.

**26.1.2** Identify that current is normally due to the motion of conduction electrons that are driven by electric fields (such as those set up in a wire by a battery).

**26.1.3** Identify a junction in a circuit and apply the fact that (due to conservation of charge) the total current into a junction must equal the total current out of the junction.

**26.1.4** Explain how current arrows are drawn in a schematic diagram of a circuit, and identify that the arrows are not vectors.

### Key Ideas

- An electric current $i$ in a conductor is defined by

$$i = \frac{dq}{dt},$$

where $dq$ is the amount of positive charge that passes in time $dt$.

- By convention, the direction of electric current is taken as the direction in which positive charge carriers would move even though (normally) only conduction electrons can move.

### What Is Physics?

In the last five chapters we discussed electrostatics—the physics of stationary charges. In this and the next chapter, we discuss the physics of **electric currents**—that is, charges in motion.

Examples of electric currents abound and involve many professions. Meteorologists are concerned with lightning and with the less dramatic slow flow of charge through the atmosphere. Biologists, physiologists, and engineers working in medical technology are concerned with the nerve currents that control muscles and especially with how those currents can be reestablished after spinal cord injuries. Electrical engineers are concerned with countless electrical systems, such as power systems, lightning protection systems, information storage systems, and music systems. Space engineers monitor and study the flow of charged particles from our Sun because that flow can wipe out telecommunication systems in orbit and even power transmission systems on the ground. In addition to such scholarly work, almost every aspect of daily life now depends on information carried by electric currents, from stock trades to ATM transfers and from video entertainment to social networking.

In this chapter we discuss the basic physics of electric currents and why they can be established in some materials but not in others. We begin with the meaning of electric current.

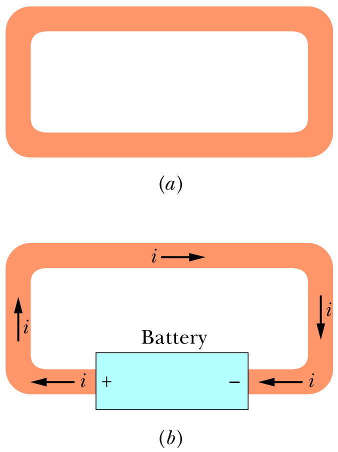



**Figure 26.1.1** (*a*) A loop of copper in electrostatic equilibrium. The entire loop is at a single potential, and the electric field is zero at all points inside the copper. (*b*) Adding a battery imposes an electric potential difference between the ends of the loop that are connected to the terminals of the battery. The battery thus produces an electric field within the loop, from terminal to terminal, and the field causes charges to move around the loop. This movement of charges is a current $i$.

## Electric Current

Although an electric current is a stream of moving charges, not all moving charges constitute an electric current. If there is to be an electric current through a given surface, there must be a net flow of charge through that surface. Two examples clarify our meaning.

1. The free electrons (conduction electrons) in an isolated length of copper wire are in random motion at speeds of the order of $10^6$ m/s. If you pass a hypothetical plane through such a wire, conduction electrons pass through it *in both directions* at the rate of many billions per second—but there is *no net transport* of charge and thus *no current* through the wire. However, if you connect the ends of the wire to a battery, you slightly bias the flow in one direction, with the result that there now is a net transport of charge and thus an electric current through the wire.
2. The flow of water through a garden hose represents the directed flow of positive charge (the protons in the water molecules) at a rate of perhaps several million coulombs per second. There is no net transport of charge, however, because there is a parallel flow of negative charge (the electrons in the water molecules) of exactly the same amount moving in exactly the same direction.

In this chapter we restrict ourselves largely to the study—within the framework of classical physics—of *steady* currents of *conduction electrons* moving through *metallic conductors* such as copper wires.

As Fig. 26.1.1*a* reminds us, any isolated conducting loop—regardless of whether it has an excess charge—is all at the same potential. No electric field can exist within it or along its surface. Although conduction electrons are available, no net electric force acts on them and thus there is no current.

If, as in Fig. 26.1.1*b*, we insert a battery in the loop, the conducting loop is no longer at a single potential. Electric fields act inside the material making up the loop, exerting forces on the conduction electrons, causing them to move and thus establishing a current. After a very short time, the electron flow reaches a constant value and the current is in its *steady state* (it does not vary with time).

Figure 26.1.2 shows a section of a conductor, part of a conducting loop in which current has been established. If charge $dq$ passes through a hypothetical plane (such as $aa'$) in time $dt$, then the current $i$ through that plane is defined as

$$i = \frac{dq}{dt} \quad \text{(definition of current).} \tag{26.1.1}$$

We can find the charge that passes through the plane in a time interval extending from 0 to $t$ by integration:

$$q = \int dq = \int_0^t i\,dt, \tag{26.1.2}$$

in which the current $i$ may vary with time.

Under steady-state conditions, the current is the same for planes $aa'$, $bb'$, and $cc'$ and indeed for all planes that pass completely through the conductor, no matter what their location or orientation. This follows from the fact that charge is conserved. Under the steady-state conditions assumed here, an electron must pass through plane $aa'$ for every electron that passes through plane $cc'$. In the same way, if we have a steady flow of water through a garden hose, a drop of water must leave the nozzle for every drop that enters the hose at the other end. The amount of water in the hose is a conserved quantity.

The SI unit for current is the coulomb per second, or the ampere (A), which is an SI base unit:

$$1 \text{ ampere} = 1 \text{ A} = 1 \text{ coulomb per second} = 1 \text{ C/s}.$$

The formal definition of the ampere is discussed in Chapter 29.

The current is the same in any cross section.

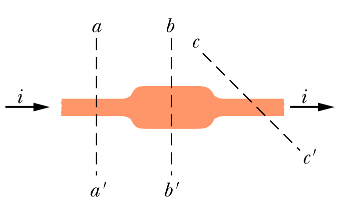



**Figure 26.1.2** The current $i$ through the conductor has the same value at planes $aa'$, $bb'$, and $cc'$.

Current, as defined by Eq. 26.1.1, is a scalar because both charge and time in that equation are scalars. Yet, as in Fig. 26.1.1*b*, we often represent a current with an arrow to indicate that charge is moving. Such arrows are not vectors, however, and they do not require vector addition. Figure 26.1.3*a* shows a conductor with current $i_0$ splitting at a junction into two branches. Because charge is conserved, the magnitudes of the currents in the branches must add to yield the magnitude of the current in the original conductor, so that

$$i_0 = i_1 + i_2. \tag{26.1.3}$$

As Fig. 26.1.3*b* suggests, bending or reorienting the wires in space does not change the validity of Eq. 26.1.3. Current arrows show only a direction (or sense) of flow along a conductor, not a direction in space.

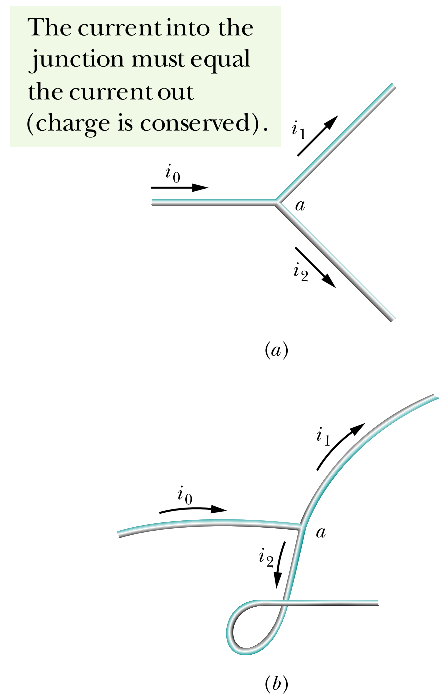



**Figure 26.1.3** The relation $i_0 = i_1 + i_2$ is true at junction $a$ no matter what the orientation in space of the three wires. Currents are scalars, not vectors.

## The Directions of Currents

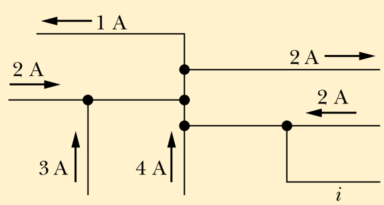
In Fig. 26.1.1*b* we drew the current arrows in the direction in which positively charged particles would be forced to move through the loop by the electric field. Such positive *charge carriers,* as they are often called, would move away from the positive battery terminal and toward the negative terminal. Actually, the charge carriers in the copper loop of Fig. 26.1.1*b* are electrons and thus are negatively charged. The electric field forces them to move in the direction opposite the current arrows, from the negative terminal to the positive terminal. For historical reasons, however, we use the following convention:

> A current arrow is drawn in the direction in which positive charge carriers would move, even if the actual charge carriers are negative and move in the opposite direction.

We can use this convention because in *most* situations, the assumed motion of positive charge carriers in one direction has the same effect as the actual motion of negative charge carriers in the opposite direction. (When the effect is not the same, we shall drop the convention and describe the actual motion.)

### Checkpoint 26.1.1

The figure here shows a portion of a circuit. What are the magnitude and direction of the current $i$ in the lower right-hand wire?




### Sample Problem 26.1.1 Current is the rate at which charge passes a point

Water flows through a garden hose at a volume flow rate $dV/dt$ of 450 cm$^3$/s. What is the current of negative charge?

**KEY IDEAS**

The current $i$ of negative charge is due to the electrons in the water molecules moving through the hose. The current is the rate at which that negative charge passes through any plane that cuts completely across the hose.

***Calculations:*** We can write the current in terms of the number of molecules that pass through such a plane per second as

$$i = \begin{pmatrix}\text{charge}\\ \text{per}\\ \text{electron}\end{pmatrix}\begin{pmatrix}\text{electrons}\\ \text{per}\\ \text{molecule}\end{pmatrix}\begin{pmatrix}\text{molecules}\\ \text{per}\\ \text{second}\end{pmatrix}$$

or

$$i = (e)(10)\frac{dN}{dt}.$$

We substitute 10 electrons per molecule because a water ($H_2O$) molecule contains 8 electrons in the single oxygen atom and 1 electron in each of the two hydrogen atoms.

We can express the rate $dN/dt$ in terms of the given volume flow rate $dV/dt$ by first writing

$$\begin{pmatrix}\text{molecules}\\ \text{per}\\ \text{second}\end{pmatrix} = \begin{pmatrix}\text{molecules}\\ \text{per}\\ \text{mole}\end{pmatrix}\begin{pmatrix}\text{moles}\\ \text{per unit}\\ \text{mass}\end{pmatrix} \times \begin{pmatrix}\text{mass}\\ \text{per unit}\\ \text{volume}\end{pmatrix}\begin{pmatrix}\text{volume}\\ \text{per}\\ \text{second}\end{pmatrix}.$$

"Molecules per mole" is Avogadro's number $N_A$. "Moles per unit mass" is the inverse of the mass per mole, which is the molar mass $M$ of water. "Mass per unit volume" is the (mass) density $\rho_{mass}$ of water. The volume per second is the volume flow rate $dV/dt$. Thus, we have

$$\frac{dN}{dt} = N_A\left(\frac{1}{M}\right)\rho_{mass}\left(\frac{dV}{dt}\right) = \frac{N_A\rho_{mass}}{M}\frac{dV}{dt}.$$

Substituting this into the equation for $i$, we find

$$i = 10eN_A M^{-1}\rho_{mass}\frac{dV}{dt}.$$

We know that Avogadro's number $N_A$ is $6.02 \times 10^{23}$ molecules/mol, or $6.02 \times 10^{23}\ \text{mol}^{-1}$, and from Table 14.1.1 we know that the density of water $\rho_{mass}$ under normal conditions is $1000\ \text{kg/m}^3$. We can get the molar mass of water from the molar masses listed in Appendix F (in grams per mole): We add the molar mass of oxygen (16 g/mol) to twice the molar mass of hydrogen (1 g/mol), obtaining 18 g/mol = 0.018 kg/mol. So, the current of negative charge due to the electrons in the water is

$$\begin{aligned} i &= (10)(1.6 \times 10^{-19}\ \text{C})(6.02 \times 10^{23}\ \text{mol}^{-1}) \\ &\quad \times (0.018\ \text{kg/mol})^{-1}(1000\ \text{kg/m}^3)(450 \times 10^{-6}\ \text{m}^3/\text{s}) \\ &= 2.41 \times 10^7\ \text{C/s} = 2.41 \times 10^7\ \text{A} \\ &= 24.1\ \text{MA}. \end{aligned} \quad \text{(Answer)}$$

This current of negative charge is exactly compensated by a current of positive charge associated with the nuclei of the three atoms that make up the water molecule. Thus, there is no net flow of charge through the hose.

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# 26.2 CURRENT DENSITY

## Learning Objectives

*After reading this module, you should be able to . . .*

**26.2.1** Identify a current density and a current density vector.

**26.2.2** For current through an area element on a cross section through a conductor (such as a wire), identify the element's area vector $d\vec{A}$.

**26.2.3** Find the current through a cross section of a conductor by integrating the dot product of the current density vector $\vec{J}$ and the element area vector $d\vec{A}$ over the full cross section.

**26.2.4** For the case where current is uniformly spread over a cross section in a conductor, apply the relationship between the current $i$, the current density magnitude $J$, and the area $A$.

**26.2.5** Identify streamlines.

**26.2.6** Explain the motion of conduction electrons in terms of their drift speed.

**26.2.7** Distinguish the drift speeds of conduction electrons from their random-motion speeds, including relative magnitudes.

**26.2.8** Identify charge carrier density $n$.

**26.2.9** Apply the relationship between current density $J$, charge carrier density $n$, and charge carrier drift speed $v_d$.

## Key Ideas

- Current $i$ (a scalar quantity) is related to current density $\vec{J}$ (a vector quantity) by

$$i = \int \vec{J} \cdot d\vec{A},$$

where $d\vec{A}$ is a vector perpendicular to a surface element of area $dA$ and the integral is taken over any surface cutting across the conductor. The current density $\vec{J}$ has the same direction as the velocity of the moving charges if they are positive and the opposite direction if they are negative.

- When an electric field $\vec{E}$ is established in a conductor, the charge carriers (assumed positive) acquire a drift speed $v_d$ in the direction of $\vec{E}$.
- The drift velocity $\vec{v}_d$ is related to the current density by

$$\vec{J} = (ne)\vec{v}_d,$$

where $ne$ is the carrier charge density.

## Current Density

Sometimes we are interested in the current $i$ in a particular conductor. At other times we take a localized view and study the flow of charge through a cross section of the conductor at a particular point. To describe this flow, we can use the **current density** $\vec{J}$, which has the same direction as the velocity of the moving charges if they are positive and the opposite direction if they are negative. For each element of the cross section, the magnitude $J$ is equal to the current per unit area through that element. We can write the amount of current through the element as $\vec{J} \cdot d\vec{A}$, where $d\vec{A}$ is the area vector of the element, perpendicular to the element. The total current through the surface is then

$$i = \int \vec{J} \cdot d\vec{A}. \tag{26.2.1}$$

If the current is uniform across the surface and parallel to $d\vec{A}$, then $\vec{J}$ is also uniform and parallel to $d\vec{A}$. Then Eq. 26.2.1 becomes

$$i = \int J\, dA = J \int dA = JA,$$

so

$$J = \frac{i}{A}, \tag{26.2.2}$$

where $A$ is the total area of the surface. From Eq. 26.2.1 or 26.2.2 we see that the SI unit for current density is the ampere per square meter ($A/m^2$).

In Chapter 22 we saw that we can represent an electric field with electric field lines. Figure 26.2.1 shows how current density can be represented with a similar set of lines, which we can call *streamlines*. The current, which is toward the right in Fig. 26.2.1, makes a transition from the wider conductor at the left to the narrower conductor at the right. Because charge is conserved during the transition, the amount of charge and thus the amount of current cannot change. However, the current density does change—it is greater in the narrower conductor. The spacing of the streamlines suggests this increase in current density; streamlines that are closer together imply greater current density.

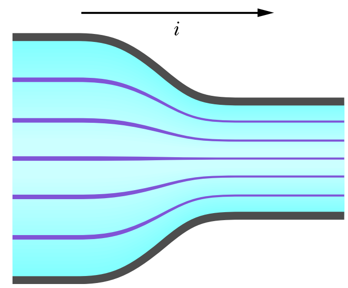



**Figure 26.2.1** Streamlines representing current density in the flow of charge through a constricted conductor.

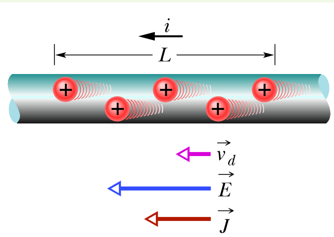
### Drift Speed

When a conductor does not have a current through it, its conduction electrons move randomly, with no net motion in any direction. When the conductor does have a current through it, these electrons actually still move randomly, but now they tend to *drift* with a **drift speed** $v_d$ in the direction opposite that of the applied electric field that causes the current. The drift speed is tiny compared with the speeds in the random motion. For example, in the copper conductors of household wiring, electron drift speeds are perhaps $10^{-5}$ or $10^{-4}$ m/s, whereas the random-motion speeds are around $10^6$ m/s.

We can use Fig. 26.2.2 to relate the drift speed $v_d$ of the conduction electrons in a current through a wire to the magnitude $J$ of the current density in the wire.

**Figure 26.2.2** Positive charge carriers drift at speed $v_d$ in the direction of the applied electric field $\vec{E}$. By convention, the direction of the current density $\vec{J}$ and the sense of the current arrow are drawn in that same direction.

Current is said to be due to positive charges that are propelled by the electric field.



For convenience, Fig. 26.2.2 shows the equivalent drift of *positive* charge carriers in the direction of the applied electric field $\vec{E}$. Let us assume that these charge carriers all move with the same drift speed $v_d$ and that the current density $J$ is uniform across the wire's cross-sectional area $A$. The number of charge carriers in a length $L$ of the wire is $nAL$, where $n$ is the number of carriers per unit volume. The total charge of the carriers in the length $L$, each with charge $e$, is then

$$q = (nAL)e.$$

Because the carriers all move along the wire with speed $v_d$, this total charge moves through any cross section of the wire in the time interval

$$t = \frac{L}{v_d}.$$

Equation 26.1.1 tells us that the current $i$ is the time rate of transfer of charge across a cross section, so here we have

$$i = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d. \tag{26.2.3}$$

Solving for $v_d$ and recalling Eq. 26.2.2 ($J = i/A$), we obtain

$$v_d = \frac{i}{nAe} = \frac{J}{ne}$$

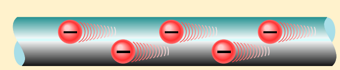
or, extended to vector form,

$$\vec{J} = (ne)\vec{v}_d. \tag{26.2.4}$$

Here the product $ne$, whose SI unit is the coulomb per cubic meter (C/m$^3$), is the *carrier charge density*. For positive carriers, $ne$ is positive and Eq. 26.2.4 predicts that $\vec{J}$ and $\vec{v}_d$ have the same direction. For negative carriers, $ne$ is negative and $\vec{J}$ and $\vec{v}_d$ have opposite directions.

**Checkpoint 26.2.1**

The figure shows conduction electrons moving leftward in a wire. Are the following leftward or rightward: (a) the current $i$, (b) the current density $\vec{J}$, (c) the electric field $\vec{E}$ in the wire?



### Sample Problem 26.2.1 Current density, uniform and nonuniform

(a) The current density in a cylindrical wire of radius $R = 2.0$ mm is uniform across a cross section of the wire and is $J = 2.0 \times 10^5$ A/m$^2$. What is the current through the outer portion of the wire between radial distances $R/2$ and $R$ (Fig. 26.2.3*a*)?

**KEY IDEA**

Because the current density is uniform across the cross section, the current density $J$, the current $i$, and the cross-sectional area $A$ are related by Eq. 26.2.2 ($J = i/A$).

***Calculations:*** We want only the current through a reduced cross-sectional area $A'$ of the wire (rather than the entire area), where

$$A' = \pi R^2 - \pi\left(\frac{R}{2}\right)^2 = \pi\left(\frac{3R^2}{4}\right)$$

$$= \frac{3\pi}{4}(0.0020 \text{ m})^2 = 9.424 \times 10^{-6} \text{ m}^2.$$

So, we rewrite Eq. 26.2.2 as

$$i = JA'$$

and then substitute the data to find

$$i = (2.0 \times 10^5 \text{ A/m}^2)(9.424 \times 10^{-6} \text{ m}^2)$$
$$= 1.9 \text{ A}. \quad \text{(Answer)}$$

(b) Suppose, instead, that the current density through a cross section varies with radial distance $r$ as $J = ar^2$, in which $a = 3.0 \times 10^{11}$ A/m$^4$ and $r$ is in meters. What now is the current through the same outer portion of the wire?

## KEY IDEA

Because the current density is not uniform across a cross section of the wire, we must resort to Eq. 26.2.1 ($i = \int \vec{J} \cdot d\vec{A}$) and integrate the current density over the portion of the wire from $r = R/2$ to $r = R$.

***Calculations:*** The current density vector $\vec{J}$ (along the wire's length) and the differential area vector $d\vec{A}$ (perpendicular to a cross section of the wire) have the same direction. Thus,

$$\vec{J} \cdot d\vec{A} = J\,dA \cos 0 = J\,dA.$$

We need to replace the differential area $dA$ with something we can actually integrate between the limits $r = R/2$ and $r = R$. The simplest replacement (because $J$ is given as a function of $r$) is the area $2\pi r\, dr$ of a thin ring of circumference $2\pi r$ and width $dr$ (Fig. 26.2.3$b$). We can then integrate with $r$ as the variable of integration. Equation 26.2.1 then gives us

$$\begin{aligned} i &= \int \vec{J} \cdot d\vec{A} = \int J\,dA \\ &= \int_{R/2}^{R} ar^2\, 2\pi r\, dr = 2\pi a \int_{R/2}^{R} r^3\, dr \\ &= 2\pi a \left[\frac{r^4}{4}\right]_{R/2}^{R} = \frac{\pi a}{2}\left[R^4 - \frac{R^4}{16}\right] = \frac{15}{32}\pi a R^4 \\ &= \frac{15}{32}\pi(3.0 \times 10^{11}\ \mathrm{A/m^4})(0.0020\ \mathrm{m})^4 = 7.1\ \mathrm{A}. \end{aligned}$$

(Answer)

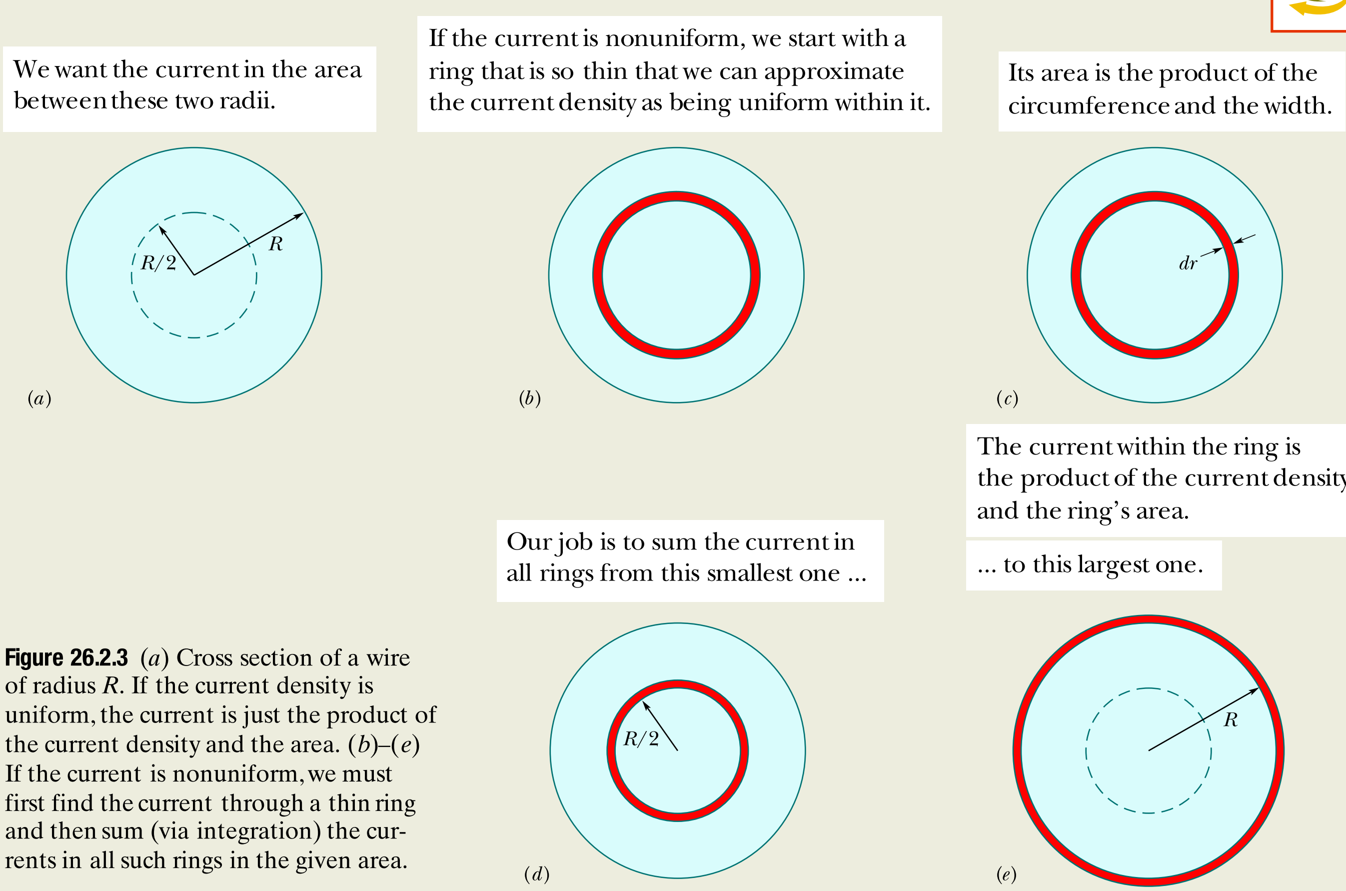



**Figure 26.2.3** ($a$) Cross section of a wire of radius $R$. If the current density is uniform, the current is just the product of the current density and the area. ($b$)–($e$) If the current is nonuniform, we must first find the current through a thin ring and then sum (via integration) the currents in all such rings in the given area.

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### Sample Problem 26.2.2 In a current, the conduction electrons move very slowly

What is the drift speed of the conduction electrons in a copper wire with radius $r = 900\ \mu\text{m}$ when it has a uniform current $i = 17$ mA? Assume that each copper atom contributes one conduction electron to the current and that the current density is uniform across the wire's cross section.

#### KEY IDEAS

1. The drift speed $v_d$ is related to the current density $\vec{J}$ and the number $n$ of conduction electrons per unit volume according to Eq. 26.2.4, which we can write as $J = nev_d$.
2. Because the current density is uniform, its magnitude $J$ is related to the given current $i$ and wire size by Eq. 26.2.2 ($J = i/A$, where $A$ is the cross-sectional area of the wire).
3. Because we assume one conduction electron per atom, the number $n$ of conduction electrons per unit volume is the same as the number of atoms per unit volume.

***Calculations:*** Let us start with the third idea by writing

$$n = \begin{pmatrix}\text{atoms}\\ \text{per unit}\\ \text{volume}\end{pmatrix} = \begin{pmatrix}\text{atoms}\\ \text{per}\\ \text{mole}\end{pmatrix}\begin{pmatrix}\text{moles}\\ \text{per unit}\\ \text{mass}\end{pmatrix}\begin{pmatrix}\text{mass}\\ \text{per unit}\\ \text{volume}\end{pmatrix}.$$

The number of atoms per mole is just Avogadro's number $N_A$ $(= 6.02 \times 10^{23}\ \text{mol}^{-1})$. Moles per unit mass is the inverse of the mass per mole, which here is the molar mass $M$ of copper. The mass per unit volume is the (mass) density $\rho_{\text{mass}}$ of copper. Thus,

$$n = N_A\left(\frac{1}{M}\right)\rho_{\text{mass}} = \frac{N_A\rho_{\text{mass}}}{M}.$$

Taking copper's molar mass $M$ and density $\rho_{\text{mass}}$ from Appendix F, we then have (with some conversions of units)

$$n = \frac{(6.02 \times 10^{23}\ \text{mol}^{-1})(8.96 \times 10^3\ \text{kg/m}^3)}{63.54 \times 10^{-3}\ \text{kg/mol}}$$
$$= 8.49 \times 10^{28}\ \text{electrons/m}^3$$

or

$$n = 8.49 \times 10^{28}\ \text{m}^{-3}.$$

Next let us combine the first two key ideas by writing

$$\frac{i}{A} = nev_d.$$

Substituting for $A$ with $\pi r^2$ $(= 2.54 \times 10^{-6}\ \text{m}^2)$ and solving for $v_d$, we then find

$$v_d = \frac{i}{ne(\pi r^2)}$$
$$= \frac{17 \times 10^{-3}\ \text{A}}{(8.49 \times 10^{28}\ \text{m}^{-3})(1.6 \times 10^{-19}\ \text{C})(2.54 \times 10^{-6}\ \text{m}^2)}$$
$$= 4.9 \times 10^{-7}\ \text{m/s}, \qquad \text{(Answer)}$$

which is only 1.8 mm/h, slower than a sluggish snail.

***Lights are fast:*** You may well ask: "If the electrons drift so slowly, why do the room lights turn on so quickly when I throw the switch?" Confusion on this point results from not distinguishing between the drift speed of the electrons and the speed at which *changes* in the electric field configuration travel along wires. This latter speed is nearly that of light; electrons everywhere in the wire begin drifting almost at once, including into the lightbulbs. Similarly, when you open the valve on your garden hose with the hose full of water, a pressure wave travels along the hose at the speed of sound in water. The speed at which the water itself moves through the hose—measured perhaps with a dye marker—is much slower.

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# 26.3 RESISTANCE AND RESISTIVITY

## Learning Objectives

*After reading this module, you should be able to . . .*

**26.3.1** Apply the relationship between the potential difference $V$ applied across an object, the object's resistance $R$, and the resulting current $i$ through the object, between the application points.

**26.3.2** Identify a resistor.

**26.3.3** Apply the relationship between the electric field magnitude $E$ set up at a point in a given material, the material's resistivity $\rho$, and the resulting current density magnitude $J$ at that point.

**26.3.4** For a uniform electric field set up in a wire, apply the relationship between the electric field magnitude $E$, the potential difference $V$ between the two ends, and the wire's length $L$.

**26.3.5** Apply the relationship between resistivity $\rho$ and conductivity $\sigma$.

**26.3.6** Apply the relationship between an object's resistance $R$, the resistivity of its material $\rho$, its length $L$, and its cross-sectional area $A$.

**26.3.7** Apply the equation that approximately gives a conductor's resistivity $\rho$ as a function of temperature $T$.

**26.3.8** Sketch a graph of resistivity $\rho$ versus temperature $T$ for a metal.

## Key Ideas

- The resistance $R$ of a conductor is defined as

$$R = \frac{V}{i},$$

where $V$ is the potential difference across the conductor and $i$ is the current.

- The resistivity $\rho$ and conductivity $\sigma$ of a material are related by

$$\rho = \frac{1}{\sigma} = \frac{E}{J},$$

where $E$ is the magnitude of the applied electric field and $J$ is the magnitude of the current density.

- The electric field and current density are related to the resistivity by

$$\vec{E} = \rho\vec{J}.$$

- The resistance $R$ of a conducting wire of length $L$ and uniform cross section is

$$R = \rho\frac{L}{A},$$

where $A$ is the cross-sectional area.

- The resistivity $\rho$ for most materials changes with temperature. For many materials, including metals, the relation between $\rho$ and temperature $T$ is approximated by the equation

$$\rho - \rho_0 = \rho_0\alpha(T - T_0).$$

Here $T_0$ is a reference temperature, $\rho_0$ is the resistivity at $T_0$, and $\alpha$ is the temperature coefficient of resistivity for the material.

## Resistance and Resistivity

If we apply the same potential difference between the ends of geometrically similar rods of copper and of glass, very different currents result. The characteristic of the conductor that enters here is its electrical **resistance.** We determine the resistance between any two points of a conductor by applying a potential difference $V$ between those points and measuring the current $i$ that results. The resistance $R$ is then

$$R = \frac{V}{i} \quad \text{(definition of } R\text{).} \tag{26.3.1}$$

The SI unit for resistance that follows from Eq. 26.3.1 is the volt per ampere. This combination occurs so often that we give it a special name, the **ohm** (symbol Ω); that is,

$$\begin{aligned} 1 \text{ ohm} &= 1\ \Omega = 1 \text{ volt per ampere} \\ &= 1 \text{ V/A}. \end{aligned} \tag{26.3.2}$$

A conductor whose function in a circuit is to provide a specified resistance is called a **resistor** (see Fig. 26.3.1). In a circuit diagram, we represent a resistor and a resistance with the symbol -⁄\/\/\/-. If we write Eq. 26.3.1 as

$$i = \frac{V}{R},$$

we see that, for a given $V$, the greater the resistance, the smaller the current.

The resistance of a conductor depends on the manner in which the potential difference is applied to it. Figure 26.3.2, for example, shows a given potential difference applied in two different ways to the same conductor. As the current density streamlines suggest, the currents in the two cases—hence the measured resistances—will be different. Unless otherwise stated, we shall assume that any given potential difference is applied as in Fig. 26.3.2*b*.

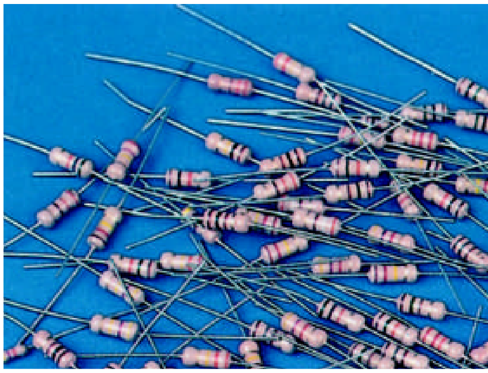

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**Figure 26.3.1** An assortment of resistors. The circular bands are color-coding marks that identify the value of the resistance.


**Figure 26.3.2** Two ways of applying a potential difference to a conducting rod. The gray connectors are assumed to have negligible resistance. When they are arranged as in (*a*) in a small region at each rod end, the measured resistance is larger than when they are arranged as in (*b*) to cover the entire rod end.

**Table 26.3.1** Resistivities of Some Materials at Room Temperature (20°C)

| Material | Resistivity, $\rho$ ($\Omega\cdot$m) | Temperature Coefficient of Resistivity, $\alpha$ (K$^{-1}$) |
|---|---|---|
| *Typical Metals* | | |
| Silver | $1.62\times10^{-8}$ | $4.1\times10^{-3}$ |
| Copper | $1.69\times10^{-8}$ | $4.3\times10^{-3}$ |
| Gold | $2.35\times10^{-8}$ | $4.0\times10^{-3}$ |
| Aluminum | $2.75\times10^{-8}$ | $4.4\times10^{-3}$ |
| Manganin[a] | $4.82\times10^{-8}$ | $0.002\times10^{-3}$ |
| Tungsten | $5.25\times10^{-8}$ | $4.5\times10^{-3}$ |
| Iron | $9.68\times10^{-8}$ | $6.5\times10^{-3}$ |
| Platinum | $10.6\times10^{-8}$ | $3.9\times10^{-3}$ |
| *Typical Semiconductors* | | |
| Silicon, pure | $2.5\times10^{3}$ | $-70\times10^{-3}$ |
| Silicon, $n$-type[b] | $8.7\times10^{-4}$ | |
| Silicon, $p$-type[c] | $2.8\times10^{-3}$ | |
| *Typical Insulators* | | |
| Glass | $10^{10}$–$10^{14}$ | |
| Fused quartz | $\sim10^{16}$ | |

[a]An alloy specifically designed to have a small value of $\alpha$.
[b]Pure silicon doped with phosphorus impurities to a charge carrier density of $10^{23}$ m$^{-3}$.
[c]Pure silicon doped with aluminum impurities to a charge carrier density of $10^{23}$ m$^{-3}$.

As we have done several times in other connections, we often wish to take a general view and deal not with particular objects but with materials. Here we do so by focusing not on the potential difference $V$ across a particular resistor but on the electric field $\vec{E}$ at a point in a resistive material. Instead of dealing with the current $i$ through the resistor, we deal with the current density $\vec{J}$ at the point in question. Instead of the resistance $R$ of an object, we deal with the **resistivity** $\rho$ of the *material:*

$$\rho = \frac{E}{J} \quad \text{(definition of } \rho\text{)}. \tag{26.3.3}$$

(Compare this equation with Eq. 26.3.1.)

If we combine the SI units of $E$ and $J$ according to Eq. 26.3.3, we get, for the unit of $\rho$, the ohm-meter ($\Omega\cdot$m):

$$\frac{\text{unit }(E)}{\text{unit }(J)} = \frac{\text{V/m}}{\text{A/m}^2} = \frac{\text{V}}{\text{A}}\,\text{m} = \Omega\cdot\text{m}.$$

(Do not confuse the *ohm-meter,* the unit of resistivity, with the *ohmmeter,* which is an instrument that measures resistance.) Table 26.3.1 lists the resistivities of some materials.

We can write Eq. 26.3.3 in vector form as

$$\vec{E} = \rho\vec{J}. \tag{26.3.4}$$

Equations 26.3.3 and 26.3.4 hold only for *isotropic* materials—materials whose electrical properties are the same in all directions.

We often speak of the **conductivity** $\sigma$ of a material. This is simply the reciprocal of its resistivity, so

$$\sigma = \frac{1}{\rho} \quad \text{(definition of } \sigma\text{)}. \tag{26.3.5}$$

The SI unit of conductivity is the reciprocal ohm-meter, $(\Omega\cdot\text{m})^{-1}$. The unit name mhos per meter is sometimes used (mho is ohm backwards). The definition of $\sigma$ allows us to write Eq. 26.3.4 in the alternative form

$$\vec{J} = \sigma\vec{E}. \tag{26.3.6}$$

## Calculating Resistance from Resistivity

We have just made an important distinction:

Resistance is a property of an object. Resistivity is a property of a material.

If we know the resistivity of a substance such as copper, we can calculate the resistance of a length of wire made of that substance. Let $A$ be the cross-sectional area of the wire, let $L$ be its length, and let a potential difference $V$ exist between its ends (Fig. 26.3.3). If the streamlines representing the current density are uniform throughout the wire, the electric field and the current density will be constant for all points within the wire and, from Eqs. 24.6.5 and 26.2.2, will have the values

$$E = V/L \quad \text{and} \quad J = i/A. \tag{26.3.7}$$

We can then combine Eqs. 26.3.3 and 26.3.7 to write

$$\rho = \frac{E}{J} = \frac{V/L}{i/A}. \tag{26.3.8}$$

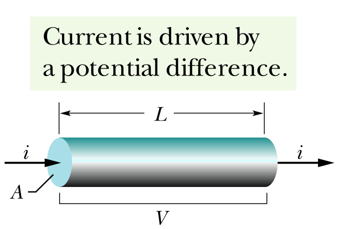



**Figure 26.3.3** A potential difference $V$ is applied between the ends of a wire of length $L$ and cross section $A$, establishing a current $i$.

However, $V/i$ is the resistance $R$, which allows us to recast Eq. 26.3.8 as

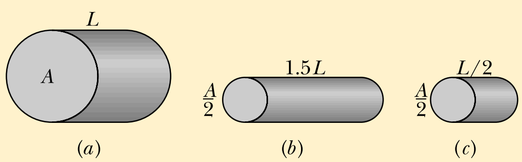
$$R = \rho \frac{L}{A}. \tag{26.3.9}$$

Equation 26.3.9 can be applied only to a homogeneous isotropic conductor of uniform cross section, with the potential difference applied as in Fig. 26.3.2*b*.

The macroscopic quantities $V$, $i$, and $R$ are of greatest interest when we are making electrical measurements on specific conductors. They are the quantities that we read directly on meters. We turn to the microscopic quantities $E$, $J$, and $\rho$ when we are interested in the fundamental electrical properties of materials.

### Checkpoint 26.3.1

The figure here shows three cylindrical copper conductors along with their face areas and lengths. Rank them according to the current through them, greatest first, when the same potential difference $V$ is placed across their lengths.




## Variation with Temperature

The values of most physical properties vary with temperature, and resistivity is no exception. Figure 26.3.4, for example, shows the variation of this property for copper over a wide temperature range. The relation between temperature and resistivity for copper—and for metals in general—is fairly linear over a rather broad temperature range. For such linear relations we can write an empirical approximation that is good enough for most engineering purposes:

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0). \tag{26.3.10}$$

Here $T_0$ is a selected reference temperature and $\rho_0$ is the resistivity at that temperature. Usually $T_0 = 293$ K (room temperature), for which $\rho_0 = 1.69 \times 10^{-8}\ \Omega \cdot \text{m}$ for copper.

Because temperature enters Eq. 26.3.10 only as a difference, it does not matter whether you use the Celsius or Kelvin scale in that equation because the sizes of degrees on these scales are identical. The quantity $\alpha$ in Eq. 26.3.10, called the *temperature coefficient of resistivity,* is chosen so that the equation gives good agreement with experiment for temperatures in the chosen range. Some values of $\alpha$ for metals are listed in Table 26.3.1.

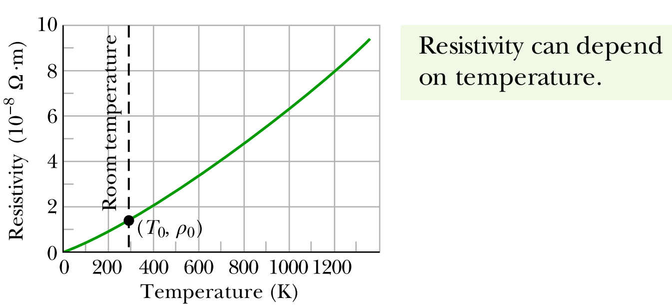



**Figure 26.3.4** The resistivity of copper as a function of temperature. The dot on the curve marks a convenient reference point at temperature $T_0 = 293$ K and resistivity $\rho_0 = 1.69 \times 10^{-8}\ \Omega \cdot \text{m}$.

## Sample Problem 26.3.1 Danger of ground current in a lightning strike

Figure 26.3.5*a* shows a person and a cow, each a radial distance $D = 60.0$ m from the point where lightning of current $I = 100$ kA strikes the ground. The current spreads through the ground uniformly over a hemisphere centered on the strike point. The person's feet are separated by radial distance $\Delta r_{per} = 0.50$ m; the cow's front and rear hooves are separated by radial distance $\Delta r_{cow} = 1.50$ m. The resistivity of the ground is $\rho_{gr} = 100\ \Omega\cdot\text{m}$. The resistance both across the person, between left and right feet, and across the cow, between front and rear hooves, is $R = 4.00$ k$\Omega$.

(a) What is the current $i_p$ through the person?

### KEY IDEAS

(1) The lightning strike sets up an electric field and an electric potential in the surrounding ground. (2) Because one foot is closer to the strike point than the other foot, a potential difference $\Delta V$ is set up across the person. (3) That $\Delta V$ drives a current $i_p$ through the person.

***Potential difference:*** Because the lightning's current $I$ spreads uniformly over a hemisphere in the ground, the current density at any given radius $r$ from the strike point is, from Eq. 26.2.2 ($J = i/A$),

$$J = \frac{I}{2\pi r^2},$$

where $2\pi r^2$ is the area of the curved surface of a hemisphere. From Eq. 26.3.4 ($\rho = E/J$), the magnitude of the electric field is then

$$E = \rho_{gr} J = \frac{\rho_{gr} I}{2\pi r^2}.$$

From Eq. 24.2.4 ($\Delta V = -\int \vec{E}\cdot d\vec{s}$), the potential difference $\Delta V$ between a point at radial distance $D$ and a point at radial distance $D + \Delta r$ is

$$\Delta V = -\int_D^{D+\Delta r} E\, dr.$$

Substituting our expression for $E$ and then integrating give us the potential difference:

$$\Delta V = -\int_D^{D+\Delta r} \frac{\rho_{gr} I}{2\pi r^2} dr = -\frac{\rho_{gr} I}{2\pi}\left[-\frac{1}{r}\right]_D^{D+\Delta r}$$

$$= \frac{\rho_{gr} I}{2\pi}\left(\frac{1}{D+\Delta r} - \frac{1}{D}\right)$$

$$= -\frac{\rho_{gr} I}{2\pi}\frac{\Delta r}{D(D+\Delta r)}.$$

***Current:*** If one of the person's feet is at radial distance $D$ from the strike point and the other foot is at radial distance $D + \Delta r$, the potential difference between the feet is given by our result for $\Delta V$. That potential difference drives a current $i_p$ through the person. To find that current, we use Eq. 26.3.1 ($R = V/i$), in which $V$ represents the magnitude of $\Delta V$. We can then write

$$i = \frac{V}{R} = \frac{\rho_{gr} I}{2\pi}\frac{\Delta r}{D(D+\Delta r)}\frac{1}{R}.$$

Substituting known values, including the foot-to-foot separation $\Delta r_{per} = 0.50$ m, gives the current through the person:

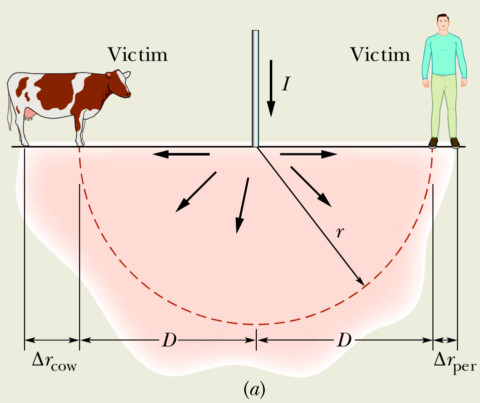




**Figure 26.3.5** (*a*) Current from a lightning strike spreads hemispherically through the ground and reaches a cow and a person, each located distance $D$ from the strike point. The danger to them depends on the separation $\Delta r$. (*b*) Marks left on turf by ground currents from a lightning strike.

$$i_p = \frac{(100\ \Omega \cdot \text{m})(100\ \text{kA})}{2\pi} \times \frac{0.50\ \text{m}}{(60.0\ \text{m})(60.0\ \text{m} + 0.50\ \text{m})} \frac{1}{4.00\ \text{k}\Omega}$$

$$= 0.0548\ \text{A} = 54.8\ \text{mA}. \qquad \text{(Answer)}$$

This amount of current causes involuntary muscle contraction; the person will collapse but probably soon recover. Note that the person could reduce the current an order of magnitude by standing with feet together so that $\Delta r$ is only a few centimeters.

(b) What is the current $i_c$ through the cow?

**Calculation:** We again use our result for current $i$ but now $\Delta r$ is $\Delta r_{cow} = 1.50$ m. We now find that the current through the cow is

$$i_c = 0.162\ \text{A} = 162\ \text{mA}, \qquad \text{(Answer)}$$

which is fatal. The cow is in more danger from the ground current because of its greater value of $\Delta r$. The cow is, of course, unable to reduce its danger by standing with its hooves together (which would be a bizarre sight).

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# 26.4 OHM'S LAW

## Learning Objectives

*After reading this module, you should be able to . . .*

26.4.1 Distinguish between an *object* that obeys Ohm's law and one that does not.

26.4.2 Distinguish between a *material* that obeys Ohm's law and one that does not.

26.4.3 Describe the general motion of a conduction electron in a current.

26.4.4 For the conduction electrons in a conductor, explain the relationship between the mean free time $\tau$, the effective speed, and the thermal (random) motion.

26.4.5 Apply the relationship between resistivity $\rho$, number density $n$ of conduction electrons, and the mean free time $\tau$ of the electrons.

## Key Ideas

- A given device (conductor, resistor, or any other electrical device) obeys Ohm's law if its resistance $R$ $(= V/i)$ is independent of the applied potential difference $V$.
- A given material obeys Ohm's law if its resistivity $\rho$ $(= E/J)$ is independent of the magnitude and direction of the applied electric field $\vec{E}$.
- The assumption that the conduction electrons in a metal are free to move like the molecules in a gas leads to an expression for the resistivity of a metal:

$$\rho = \frac{m}{e^2 n \tau}.$$

Here $n$ is the number of free electrons per unit volume and $\tau$ is the mean time between the collisions of an electron with the atoms of the metal.

- Metals obey Ohm's law because the mean free time $\tau$ is approximately independent of the magnitude $E$ of any electric field applied to a metal.

## Ohm's Law

As we just discussed, a resistor is a conductor with a specified resistance. It has that same resistance no matter what the magnitude and direction (*polarity*) of the applied potential difference are. Other conducting devices, however, might have resistances that change with the applied potential difference.

Figure 26.4.1*a* shows how to distinguish such devices. A potential difference $V$ is applied across the device being tested, and the resulting current $i$ through the device is measured as $V$ is varied in both magnitude and polarity. The polarity of $V$ is arbitrarily taken to be positive when the left terminal of the device is

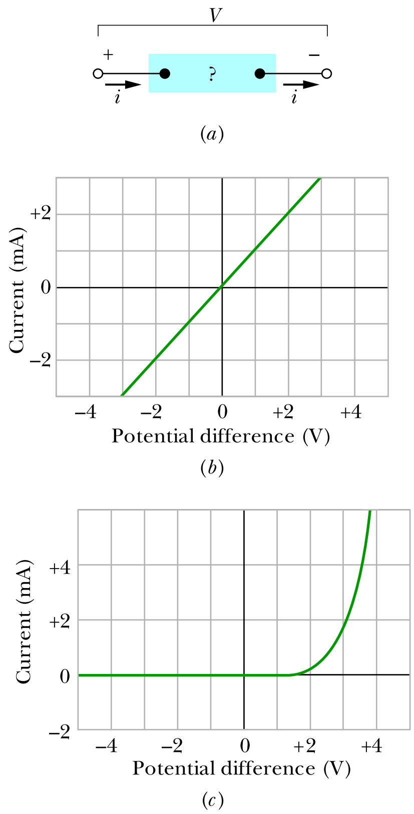


**Figure 26.4.1** (*a*) A potential difference $V$ is applied to the terminals of a device, establishing a current $i$. (*b*) A plot of current $i$ versus applied potential difference $V$ when the device is a 1000 Ω resistor. (*c*) A plot when the device is a semiconducting *pn* junction diode.

at a higher potential than the right terminal. The direction of the resulting current (from left to right) is arbitrarily assigned a plus sign. The reverse polarity of $V$ (with the right terminal at a higher potential) is then negative; the current it causes is assigned a minus sign.

Figure 26.4.1*b* is a plot of $i$ versus $V$ for one device. This plot is a straight line passing through the origin, so the ratio $i/V$ (which is the slope of the straight line) is the same for all values of $V$. This means that the resistance $R = V/i$ of the device is independent of the magnitude and polarity of the applied potential difference $V$.

Figure 26.4.1*c* is a plot for another conducting device. Current can exist in this device only when the polarity of $V$ is positive and the applied potential difference is more than about 1.5 V. When current does exist, the relation between $i$ and $V$ is not linear; it depends on the value of the applied potential difference $V$.

We distinguish between the two types of device by saying that one obeys Ohm's law and the other does not.

**Ohm's law** is an assertion that the current through a device is *always* directly proportional to the potential difference applied to the device.

(This assertion is correct only in certain situations; still, for historical reasons, the term "law" is used.) The device of Fig. 26.4.1*b*—which turns out to be a 1000 Ω resistor—obeys Ohm's law. The device of Fig. 26.4.1*c*—which is called a *pn* junction diode—does not.

A conducting device obeys Ohm's law when the resistance of the device is independent of the magnitude and polarity of the applied potential difference.

It is often contended that $V = iR$ is a statement of Ohm's law. That is not true! This equation is the defining equation for resistance, and it applies to all conducting devices, whether they obey Ohm's law or not. If we measure the potential difference $V$ across, and the current $i$ through, any device, even a *pn* junction diode, we can find its resistance *at that value of* $V$ as $R = V/i$. The essence of Ohm's law, however, is that a plot of $i$ versus $V$ is linear; that is, $R$ is independent of $V$. We can generalize this for conducting materials by using Eq. 26.3.4 ($\vec{E} = \rho\vec{J}$):

A conducting material obeys Ohm's law when the resistivity of the material is independent of the magnitude and direction of the applied electric field.

All homogeneous materials, whether they are conductors like copper or semiconductors like pure silicon or silicon containing special impurities, obey Ohm's law within some range of values of the electric field. If the field is too strong, however, there are departures from Ohm's law in all cases.

### Checkpoint 26.4.1

The following table gives the current $i$ (in amperes) through two devices for several values of potential difference $V$ (in volts). From these data, determine which device does not obey Ohm's law.

| Device 1 | | Device 2 | |
|---|---|---|---|
| $V$ | $i$ | $V$ | $i$ |
| 2.00 | 4.50 | 2.00 | 1.50 |
| 3.00 | 6.75 | 3.00 | 2.20 |
| 4.00 | 9.00 | 4.00 | 2.80 |

## A Microscopic View of Ohm's Law

To find out *why* particular materials obey Ohm's law, we must look into the details of the conduction process at the atomic level. Here we consider only conduction in metals, such as copper. We base our analysis on the *free-electron model,* in which we assume that the conduction electrons in the metal are free to move throughout the volume of a sample, like the molecules of a gas in a closed container. We also assume that the electrons collide not with one another but only with atoms of the metal.

According to classical physics, the electrons should have a Maxwellian speed distribution somewhat like that of the molecules in a gas (Module 19.6), and thus the average electron speed should depend on the temperature. The motions of electrons are, however, governed not by the laws of classical physics but by those of quantum physics. As it turns out, an assumption that is much closer to the quantum reality is that conduction electrons in a metal move with a single effective speed $v_{\text{eff}}$, and this speed is essentially independent of the temperature. For copper, $v_{\text{eff}} \approx 1.6 \times 10^6$ m/s.

When we apply an electric field to a metal sample, the electrons modify their random motions slightly and drift very slowly—in a direction opposite that of the field—with an average drift speed $v_d$. The drift speed in a typical metallic conductor is about $5 \times 10^{-7}$ m/s, less than the effective speed ($1.6 \times 10^6$ m/s) by many orders of magnitude. Figure 26.4.2 suggests the relation between these two speeds. The gray lines show a possible random path for an electron in the absence of an applied field; the electron proceeds from $A$ to $B$, making six collisions along the way. The green lines show how the same events *might* occur when an electric field $\vec{E}$ is applied. We see that the electron drifts steadily to the right, ending at $B'$ rather than at $B$. Figure 26.4.2 was drawn with the assumption that $v_d \approx 0.02 v_{\text{eff}}$. However, because the actual value is more like $v_d \approx (10^{-13}) v_{\text{eff}}$, the drift displayed in the figure is greatly exaggerated.

The motion of conduction electrons in an electric field $\vec{E}$ is thus a combination of the motion due to random collisions and that due to $\vec{E}$. When we consider all the free electrons, their random motions average to zero and make no contribution to the drift speed. Thus, the drift speed is due only to the effect of the electric field on the electrons.

If an electron of mass $m$ is placed in an electric field of magnitude $E$, the electron will experience an acceleration given by Newton's second law:

$$a = \frac{F}{m} = \frac{eE}{m}. \tag{26.4.1}$$

After a typical collision, each electron will—so to speak—completely lose its memory of its previous drift velocity, starting fresh and moving off in a random direction. In the average time $\tau$ between collisions, the average electron will acquire a drift speed of $v_d = a\tau$. Moreover, if we measure the drift speeds of all the electrons at any instant, we will find that their average drift speed is also $a\tau$. Thus, at any instant, on average, the electrons will have drift speed $v_d = a\tau$. Then Eq. 26.4.1 gives us

$$v_d = a\tau = \frac{eE\tau}{m}. \tag{26.4.2}$$

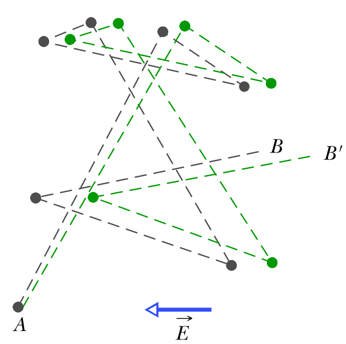


**Figure 26.4.2** The gray lines show an electron moving from $A$ to $B$, making six collisions en route. The green lines show what the electron's path might be in the presence of an applied electric field $\vec{E}$. Note the steady drift in the direction of $-\vec{E}$. (Actually, the green lines should be slightly curved, to represent the parabolic paths followed by the electrons between collisions, under the influence of an electric field.)

Combining this result with Eq. 26.2.4 ($\vec{J} = ne\vec{v}_d$), in magnitude form, yields

$$v_d = \frac{J}{ne} = \frac{eE\tau}{m}, \tag{26.4.3}$$

which we can write as

$$E = \left(\frac{m}{e^2 n\tau}\right)J. \tag{26.4.4}$$

Comparing this with Eq. 26.3.4 ($\vec{E} = \rho\vec{J}$), in magnitude form, leads to

$$\rho = \frac{m}{e^2 n\tau}. \tag{26.4.5}$$

Equation 26.4.5 may be taken as a statement that metals obey Ohm's law if we can show that, for metals, their resistivity $\rho$ is a constant, independent of the strength of the applied electric field $\vec{E}$. Let's consider the quantities in Eq. 26.4.5. We can reasonably assume that $n$, the number of conduction electrons per volume, is independent of the field, and $m$ and $e$ are constants. Thus, we only need to convince ourselves that $\tau$, the average time (or *mean free time*) between collisions, is a constant, independent of the strength of the applied electric field. Indeed, $\tau$ can be considered to be a constant because the drift speed $v_d$ caused by the field is so much smaller than the effective speed $v_{\text{eff}}$ that the electron speed—and thus $\tau$—is hardly affected by the field. Thus, because the right side of Eq. 26.4.5 is independent of the field magnitude, metals obey Ohm's law.

## Sample Problem 26.4.1 Mean free time and mean free distance

(a) What is the mean free time $\tau$ between collisions for the conduction electrons in copper?

### KEY IDEAS

The mean free time $\tau$ of copper is approximately constant, and in particular does not depend on any electric field that might be applied to a sample of the copper. Thus, we need not consider any particular value of applied electric field. However, because the resistivity $\rho$ displayed by copper under an electric field depends on $\tau$, we can find the mean free time $\tau$ from Eq. 26.4.5 ($\rho = m/e^2 n\tau$).

***Calculations:*** That equation gives us

$$\tau = \frac{m}{ne^2\rho}. \tag{26.4.6}$$

The number of conduction electrons per unit volume in copper is $8.49 \times 10^{28}\ \text{m}^{-3}$. We take the value of $\rho$ from Table 26.3.1. The denominator then becomes

$$(8.49 \times 10^{28}\ \text{m}^{-3})(1.6 \times 10^{-19}\ \text{C})^2(1.69 \times 10^{-8}\ \Omega \cdot \text{m})$$
$$= 3.67 \times 10^{-17}\ \text{C}^2 \cdot \Omega/\text{m}^2 = 3.67 \times 10^{-17}\ \text{kg/s},$$

where we converted units as

$$\frac{\text{C}^2 \cdot \Omega}{\text{m}^2} = \frac{\text{C}^2 \cdot \text{V}}{\text{m}^2 \cdot \text{A}} = \frac{\text{C}^2 \cdot \text{J/C}}{\text{m}^2 \cdot \text{C/s}} = \frac{\text{kg} \cdot \text{m}^2/\text{s}^2}{\text{m}^2/\text{s}} = \frac{\text{kg}}{\text{s}}.$$

Using these results and substituting for the electron mass $m$, we then have

$$\tau = \frac{9.1 \times 10^{-31}\ \text{kg}}{3.67 \times 10^{-17}\ \text{kg/s}} = 2.5 \times 10^{-14}\ \text{s}. \quad \text{(Answer)}$$

(b) The mean free path $\lambda$ of the conduction electrons in a conductor is the average distance traveled by an electron between collisions. (This definition parallels that in Module 19.5 for the mean free path of molecules in a gas.) What is $\lambda$ for the conduction electrons in copper, assuming that their effective speed $v_{\text{eff}}$ is $1.6 \times 10^6$ m/s?

### KEY IDEA

The distance $d$ any particle travels in a certain time $t$ at a constant speed $v$ is $d = vt$.

***Calculation:*** For the electrons in copper, this gives us

$$\lambda = v_{\text{eff}}\tau \tag{26.4.7}$$
$$= (1.6 \times 10^6\ \text{m/s})(2.5 \times 10^{-14}\ \text{s})$$
$$= 4.0 \times 10^{-8}\ \text{m} = 40\ \text{nm}. \quad \text{(Answer)}$$

This is about 150 times the distance between nearest-neighbor atoms in a copper lattice. Thus, on the average, each conduction electron passes many copper atoms before finally hitting one.

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# 26.5 POWER, SEMICONDUCTORS, SUPERCONDUCTORS

## Learning Objectives

*After reading this module, you should be able to . . .*

**26.5.1** Explain how conduction electrons in a circuit lose energy in a resistive device.

**26.5.2** Identify that power is the rate at which energy is transferred from one type to another.

**26.5.3** For a resistive device, apply the relationships between power $P$, current $i$, voltage $V$, and resistance $R$.

**26.5.4** For a battery, apply the relationship between power $P$, current $i$, and potential difference $V$.

**26.5.5** Apply the conservation of energy to a circuit with a battery and a resistive device to relate the energy transfers in the circuit.

**26.5.6** Distinguish conductors, semiconductors, and superconductors.

## Key Ideas

- The power $P$, or rate of energy transfer, in an electrical device across which a potential difference $V$ is maintained is

$$P = iV.$$

- If the device is a resistor, the power can also be written as

$$P = i^2R = \frac{V^2}{R}.$$

- In a resistor, electric potential energy is converted to internal thermal energy via collisions between charge carriers and atoms.
- Semiconductors are materials that have few conduction electrons but can become conductors when they are doped with other atoms that contribute charge carriers.
- Superconductors are materials that lose all electrical resistance. Most such materials require very low temperatures, but some become superconducting at temperatures as high as room temperature.

## Power in Electric Circuits

Figure 26.5.1 shows a circuit consisting of a battery B that is connected by wires, which we assume have negligible resistance, to an unspecified conducting device. The device might be a resistor, a storage battery (a rechargeable battery), a motor, or some other electrical device. The battery maintains a potential difference of magnitude $V$ across its own terminals and thus (because of the wires) across the terminals of the unspecified device, with a greater potential at terminal $a$ of the device than at terminal $b$.

Because there is an external conducting path between the two terminals of the battery, and because the potential differences set up by the battery are maintained, a steady current $i$ is produced in the circuit, directed from terminal $a$ to terminal $b$. The amount of charge $dq$ that moves between those terminals in time interval $dt$ is equal to $i\,dt$. This charge $dq$ moves through a decrease in potential of magnitude $V$, and thus its electric potential energy decreases in magnitude by the amount

$$dU = dq\ V = i\ dt\ V. \tag{26.5.1}$$

The principle of conservation of energy tells us that the decrease in electric potential energy from $a$ to $b$ is accompanied by a transfer of energy to some other form. The power $P$ associated with that transfer is the rate of transfer $dU/dt$, which is given by Eq. 26.5.1 as

$$P = iV \quad \text{(rate of electrical energy transfer).} \tag{26.5.2}$$

Moreover, this power $P$ is also the rate at which energy is transferred from the battery to the unspecified device. If that device is a motor connected to a mechanical load, the energy is transferred as work done on the load. If the device is a storage battery that is being charged, the energy is transferred to stored chemical energy in the storage battery. If the device is a resistor, the energy is transferred to internal thermal energy, tending to increase the resistor's temperature.

The battery at the left supplies energy to the conduction electrons that form the current.

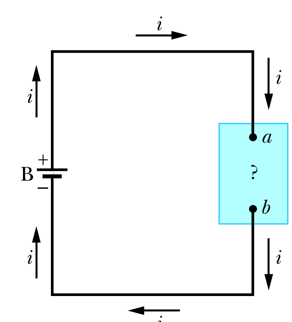



**Figure 26.5.1** A battery B sets up a current $i$ in a circuit containing an unspecified conducting device.

The unit of power that follows from Eq. 26.5.2 is the volt-ampere (V · A). We can write it as

$$1\ \text{V}\cdot\text{A} = \left(1\,\frac{\text{J}}{\text{C}}\right)\left(1\,\frac{\text{C}}{\text{s}}\right) = 1\,\frac{\text{J}}{\text{s}} = 1\ \text{W}.$$

As an electron moves through a resistor at constant drift speed, its average kinetic energy remains constant and its lost electric potential energy appears as thermal energy in the resistor and the surroundings. On a microscopic scale this energy transfer is due to collisions between the electron and the molecules of the resistor, which leads to an increase in the temperature of the resistor lattice. The mechanical energy thus transferred to thermal energy is *dissipated* (lost) because the transfer cannot be reversed.

For a resistor or some other device with resistance $R$, we can combine Eqs. 26.3.1 ($R = V/i$) and 26.5.2 to obtain, for the rate of electrical energy dissipation due to a resistance, either

$$P = i^2R \quad \text{(resistive dissipation)} \tag{26.5.3}$$

or

$$P = \frac{V^2}{R} \quad \text{(resistive dissipation).} \tag{26.5.4}$$

*Caution:* We must be careful to distinguish these two equations from Eq. 26.5.2: $P = iV$ applies to electrical energy transfers of all kinds; $P = i^2R$ and $P = V^2/R$ apply only to the transfer of electric potential energy to thermal energy in a device with resistance.

### Checkpoint 26.5.1

A potential difference $V$ is connected across a device with resistance $R$, causing current $i$ through the device. Rank the following variations according to the change in the rate at which electrical energy is converted to thermal energy due to the resistance, greatest change first: (a) $V$ is doubled with $R$ unchanged, (b) $i$ is doubled with $R$ unchanged, (c) $R$ is doubled with $V$ unchanged, (d) $R$ is doubled with $i$ unchanged.

## Sample Problem 26.5.1 Rate of energy dissipation in a wire carrying current

You are given a length of uniform heating wire made of a nickel–chromium–iron alloy called Nichrome; it has a resistance $R$ of 72 Ω. At what rate is energy dissipated in each of the following situations? (1) A potential difference of 120 V is applied across the full length of the wire. (2) The wire is cut in half, and a potential difference of 120 V is applied across the length of each half.

### KEY IDEA

Current in a resistive material produces a transfer of mechanical energy to thermal energy; the rate of transfer (dissipation) is given by Eqs. 26.5.2 to 26.5.4.

***Calculations:*** Because we know the potential $V$ and resistance $R$, we use Eq. 26.5.4, which yields, for situation 1,

$$P = \frac{V^2}{R} = \frac{(120\ \text{V})^2}{72\ \Omega} = 200\ \text{W}. \qquad \text{(Answer)}$$

In situation 2, the resistance of each half of the wire is (72 Ω)/2, or 36 Ω. Thus, the dissipation rate for each half is

$$P' = \frac{(120\ \text{V})^2}{36\ \Omega} = 400\ \text{W},$$

and that for the two halves is

$$P = 2P' = 800\ \text{W}. \qquad \text{(Answer)}$$

This is four times the dissipation rate of the full length of wire. Thus, you might conclude that you could buy a heating coil, cut it in half, and reconnect it to obtain four times the heat output. Why is this unwise? (What would happen to the amount of current in the coil?)

## Semiconductors

Semiconducting devices are at the heart of the microelectronic revolution that ushered in the information age. Table 26.5.1 compares the properties of silicon—a typical semiconductor—and copper—a typical metallic conductor. We see that silicon has many fewer charge carriers, a much higher resistivity, and a temperature coefficient of resistivity that is both large and negative. Thus, although the resistivity of copper increases with increasing temperature, that of pure silicon decreases.

Pure silicon has such a high resistivity that it is effectively an insulator and thus not of much direct use in microelectronic circuits. However, its resistivity can be greatly reduced in a controlled way by adding minute amounts of specific "impurity" atoms in a process called *doping*. Table 26.3.1 gives typical values of resistivity for silicon before and after doping with two different impurities.

We can roughly explain the differences in resistivity (and thus in conductivity) between semiconductors, insulators, and metallic conductors in terms of the energies of their electrons. (We need quantum physics to explain in more detail.) In a metallic conductor such as copper wire, most of the electrons are firmly locked in place within the atoms; much energy would be required to free them so they could move and participate in an electric current. However, there are also some electrons that, roughly speaking, are only loosely held in place and that require only little energy to become free. Thermal energy can supply that energy, as can an electric field applied across the conductor. The field would not only free these loosely held electrons but would also propel them along the wire; thus, the field would drive a current through the conductor.

In an insulator, significantly greater energy is required to free electrons so they can move through the material. Thermal energy cannot supply enough energy, and neither can any reasonable electric field applied to the insulator. Thus, no electrons are available to move through the insulator, and hence no current occurs even with an applied electric field.

A semiconductor is like an insulator *except* that the energy required to free some electrons is not quite so great. More important, doping can supply electrons or positive charge carriers that are very loosely held within the material and thus are easy to get moving. Moreover, by controlling the doping of a semiconductor, we can control the density of charge carriers that can participate in a current and thereby can control some of its electrical properties. Most semiconducting devices, such as transistors and junction diodes, are fabricated by the selective doping of different regions of the silicon with impurity atoms of different kinds.

Let us now look again at Eq. 26.4.5 for the resistivity of a conductor:

$$\rho = \frac{m}{e^2 n\tau}, \tag{26.5.5}$$

where $n$ is the number of charge carriers per unit volume and $\tau$ is the mean time between collisions of the charge carriers. The equation also applies to semiconductors. Let's consider how $n$ and $\tau$ change as the temperature is increased.

In a conductor, $n$ is large but very nearly constant with any change in temperature. The increase of resistivity with temperature for metals (Fig. 26.3.4) is due to an increase in the collision rate of the charge carriers, which shows up in Eq. 26.5.5 as a decrease in $\tau$, the mean time between collisions.

**Table 26.5.1** Some Electrical Properties of Copper and Silicon

| Property | Copper | Silicon |
|---|---|---|
| Type of material | Metal | Semiconductor |
| Charge carrier density, $m^{-3}$ | $8.49 \times 10^{28}$ | $1 \times 10^{16}$ |
| Resistivity, $\Omega \cdot m$ | $1.69 \times 10^{-8}$ | $2.5 \times 10^{3}$ |
| Temperature coefficient of resistivity, $K^{-1}$ | $+4.3 \times 10^{-3}$ | $-70 \times 10^{-3}$ |

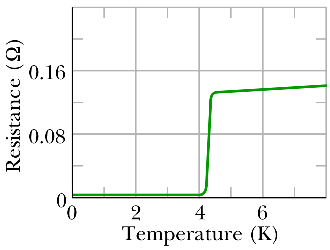



**Figure 26.5.2** The resistance of mercury drops to zero at a temperature of about 4 K.

In a semiconductor, $n$ is small but increases very rapidly with temperature as the increased thermal agitation makes more charge carriers available. This causes a *decrease* of resistivity with increasing temperature, as indicated by the negative temperature coefficient of resistivity for silicon in Table 26.5.1. The same increase in collision rate that we noted for metals also occurs for semiconductors, but its effect is swamped by the rapid increase in the number of charge carriers.

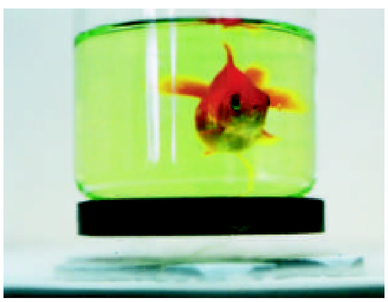
## Superconductors

In 1911, Dutch physicist Kamerlingh Onnes discovered that the resistivity of mercury absolutely disappears at temperatures below about 4 K (Fig. 26.5.2). This phenomenon of **superconductivity** is of vast potential importance in technology because it means that charge can flow through a superconducting conductor without losing its energy to thermal energy. Currents created in a superconducting ring, for example, have persisted for several years without loss; the electrons making up the current require a force and a source of energy at start-up time but not thereafter.

Prior to 1986, the technological development of superconductivity was throttled by the cost of producing the extremely low temperatures required to achieve the effect. In 1986, however, new ceramic materials were discovered that become superconducting at considerably higher (and thus cheaper to produce) temperatures. Practical application of superconducting devices at room temperature may eventually become commonplace.

Superconductivity is a phenomenon much different from conductivity. In fact, the best of the normal conductors, such as silver and copper, cannot become superconducting at any temperature, and the new ceramic superconductors are actually good insulators when they are not at low enough temperatures to be in a superconducting state.

One explanation for superconductivity is that the electrons that make up the current move in coordinated pairs. One of the electrons in a pair may electrically distort the molecular structure of the superconducting material as it moves through, creating nearby a short-lived concentration of positive charge. The other electron in the pair may then be attracted toward this positive charge. According to the theory, such coordination between electrons would prevent them from colliding with the molecules of the material and thus would eliminate electrical resistance. The theory worked well to explain the pre-1986, lower temperature superconductors, but new theories appear to be needed for the newer, higher temperature superconductors.





A disk-shaped magnet is levitated above a superconducting material that has been cooled by liquid nitrogen. The goldfish is along for the ride.

# Review & Summary

**Current** An **electric current** $i$ in a conductor is defined by

$$i = \frac{dq}{dt}. \tag{26.1.1}$$

Here $dq$ is the amount of (positive) charge that passes in time $dt$ through a hypothetical surface that cuts across the conductor. By convention, the direction of electric current is taken as the direction in which positive charge carriers would move. The SI unit of electric current is the **ampere** (A): 1 A = 1 C/s.

**Current Density** Current (a scalar) is related to **current density** $\vec{J}$ (a vector) by

$$i = \int \vec{J} \cdot d\vec{A}, \tag{26.2.1}$$

where $d\vec{A}$ is a vector perpendicular to a surface element of area $dA$ and the integral is taken over any surface cutting across the conductor. $\vec{J}$ has the same direction as the velocity of the moving charges if they are positive and the opposite direction if they are negative.

**Drift Speed of the Charge Carriers** When an electric field $\vec{E}$ is established in a conductor, the charge carriers (assumed positive) acquire a **drift speed** $v_d$ in the direction of $\vec{E}$; the velocity $\vec{v}_d$ is related to the current density by

$$\vec{J} = (ne)\vec{v}_d, \tag{26.2.4}$$

where $ne$ is the *carrier charge density*.

**Resistance of a Conductor** The **resistance** $R$ of a conductor is defined as

$$R = \frac{V}{i} \quad \text{(definition of } R\text{)}, \tag{26.3.1}$$

where $V$ is the potential difference across the conductor and $i$ is the current. The SI unit of resistance is the **ohm** ($\Omega$): $1\ \Omega = 1\ \text{V/A}$. Similar equations define the **resistivity** $\rho$ and **conductivity** $\sigma$ of a material:

$$\rho = \frac{1}{\sigma} = \frac{E}{J} \quad \text{(definitions of } \rho \text{ and } \sigma\text{)}, \tag{26.3.5, 26.3.3}$$

where $E$ is the magnitude of the applied electric field. The SI unit of resistivity is the ohm-meter ($\Omega \cdot \text{m}$). Equation 26.3.3 corresponds to the vector equation

$$\vec{E} = \rho \vec{J}. \tag{26.3.4}$$

The resistance $R$ of a conducting wire of length $L$ and uniform cross section is

$$R = \rho \frac{L}{A}, \tag{26.3.9}$$

where $A$ is the cross-sectional area.

**Change of $\rho$ with Temperature** The resistivity $\rho$ for most materials changes with temperature. For many materials, including metals, the relation between $\rho$ and temperature $T$ is approximated by the equation

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0). \tag{26.3.10}$$

Here $T_0$ is a reference temperature, $\rho_0$ is the resistivity at $T_0$, and $\alpha$ is the temperature coefficient of resistivity for the material.

**Ohm's Law** A given device (conductor, resistor, or any other electrical device) obeys *Ohm's law* if its resistance $R$, defined by Eq. 26.3.1 as $V/i$, is independent of the applied potential difference $V$. A given *material* obeys Ohm's law if its resistivity, defined by Eq. 26.3.3, is independent of the magnitude and direction of the applied electric field $\vec{E}$.

**Resistivity of a Metal** By assuming that the conduction electrons in a metal are free to move like the molecules of a gas, it is possible to derive an expression for the resistivity of a metal:

$$\rho = \frac{m}{e^2 n \tau}. \tag{26.4.5}$$

Here $n$ is the number of free electrons per unit volume and $\tau$ is the mean time between the collisions of an electron with the atoms of the metal. We can explain why metals obey Ohm's law by pointing out that $\tau$ is essentially independent of the magnitude $E$ of any electric field applied to a metal.

**Power** The power $P$, or rate of energy transfer, in an electrical device across which a potential difference $V$ is maintained is

$$P = iV \quad \text{(rate of electrical energy transfer)}. \tag{26.5.2}$$

**Resistive Dissipation** If the device is a resistor, we can write Eq. 26.5.2 as

$$P = i^2 R = \frac{V^2}{R} \quad \text{(resistive dissipation)}. \tag{26.5.3, 26.5.4}$$

In a resistor, electric potential energy is converted to internal thermal energy via collisions between charge carriers and atoms.

**Semiconductors** *Semiconductors* are materials that have few conduction electrons but can become conductors when they are *doped* with other atoms that contribute charge carriers.

**Superconductors** *Superconductors* are materials that lose all electrical resistance at low temperatures. Some materials are superconducting at surprisingly high temperatures.

# Questions

**1** Figure 26.1 shows cross sections through three long conductors of the same length and material, with square cross sections of edge lengths as shown. Conductor $B$ fits snugly within conductor $A$, and conductor $C$ fits snugly within conductor $B$. Rank the following according to their end-to-end resistances, greatest first: the individual conductors and the combinations of $A + B$ ($B$ inside $A$), $B + C$ ($C$ inside $B$), and $A + B + C$ ($B$ inside $A$ inside $C$).

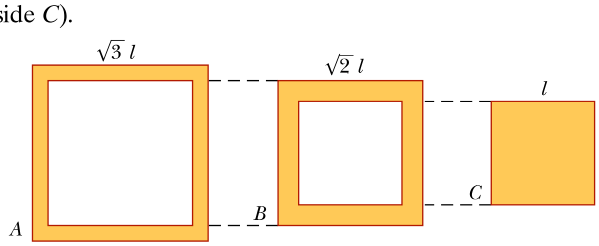



**Figure 26.1** Question 1.

**2** Figure 26.2 shows cross sections through three wires of identical length and material; the sides are given in millimeters. Rank the wires according to their resistance (measured end to end along each wire's length), greatest first.

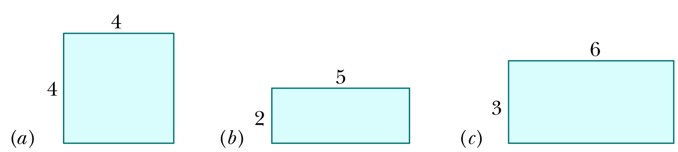



**Figure 26.2** Question 2.

**3** Figure 26.3 shows a rectangular solid conductor of edge lengths $L$, $2L$, and $3L$. A potential difference $V$ is to be applied uniformly between pairs of opposite faces of the conductor as in Fig. 26.3.2*b*. (The potential difference is applied between the entire face on one side and the entire face on the other side.) First $V$ is applied between the left–right faces, then between the top–bottom faces, and then between the front–back faces. Rank those pairs, greatest first, according to the following (within the conductor): (a) the magnitude of the electric field, (b) the current density, (c) the current, and (d) the drift speed of the electrons.

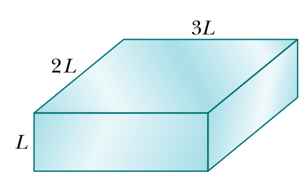



**Figure 26.3** Question 3.

**4** Figure 26.4 shows plots of the current $i$ through a certain cross section of a wire over four different time periods. Rank the periods according to the net charge that passes through the cross section during the period, greatest first.

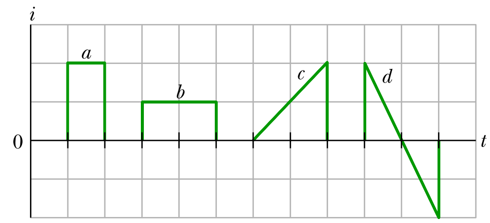



**Figure 26.4** Question 4.

**5** Figure 26.5 shows four situations in which positive and negative charges move horizontally and gives the rate at which each charge moves. Rank the situations according to the effective current through the regions, greatest first.

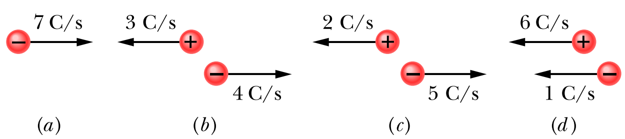



**Figure 26.5** Question 5.

**6** In Fig. 26.6, a wire that carries a current consists of three sections with different radii. Rank the sections according to the following quantities, greatest first: (a) current, (b) magnitude of current density, and (c) magnitude of electric field.

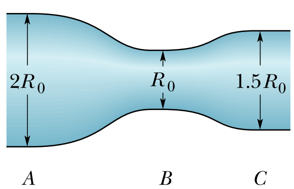



**Figure 26.6** Question 6.

**7** Figure 26.7 gives the electric potential $V(x)$ versus position $x$ along a copper wire carrying current. The wire consists of three sections that differ in radius. Rank the three sections according to the magnitude of the (a) electric field and (b) current density, greatest first.

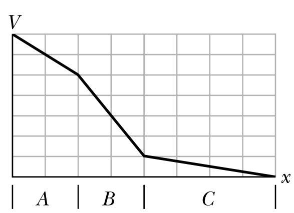



**Figure 26.7** Question 7.

**8** The following table gives the lengths of three copper rods, their diameters, and the potential differences between their ends. Rank the rods according to (a) the magnitude of the electric field within them, (b) the current density within them, and (c) the drift speed of electrons through them, greatest first.

| Rod | Length | Diameter | Potential Difference |
|---|---|---|---|
| 1 | $L$ | $3d$ | $V$ |
| 2 | $2L$ | $d$ | $2V$ |
| 3 | $3L$ | $2d$ | $2V$ |

**9** Figure 26.8 gives the drift speed $v_d$ of conduction electrons in a copper wire versus position $x$ along the wire. The wire consists of three sections that differ in radius. Rank the three sections according to the following quantities, greatest first: (a) radius, (b) number of conduction electrons per cubic meter, (c) magnitude of electric field, (d) conductivity.

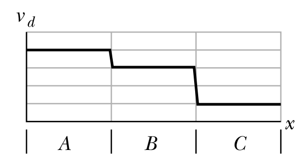



**Figure 26.8** Question 9.

**10** Three wires, of the same diameter, are connected in turn between two points maintained at a constant potential difference. Their resistivities and lengths are $\rho$ and $L$ (wire $A$), $1.2\rho$ and $1.2L$ (wire $B$), and $0.9\rho$ and $L$ (wire $C$). Rank the wires according to the rate at which energy is transferred to thermal energy within them, greatest first.

**11** Figure 26.9 gives, for three wires of radius $R$, the current density $J(r)$ versus radius $r$, as measured from the center of a circular cross section through the wire. The wires are all made from the same material. Rank the wires according to the magnitude of the electric field (a) at the center, (b) halfway to the surface, and (c) at the surface, greatest first.

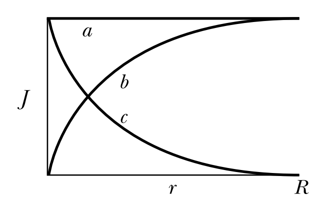



**Figure 26.9** Question 11.

# Problems

GO Tutoring problem available (at instructor's discretion) in *WileyPLUS*
SSM Worked-out solution available in Student Solutions Manual
**E** Easy **M** Medium **H** Hard
FCP Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com
CALC Requires calculus
BIO Biomedical application

## Module 26.1 Electric Current

**1** **E** During the 4.0 min a 5.0 A current is set up in a wire, how many (a) coulombs and (b) electrons pass through any cross section across the wire's width?

**2** **M** An isolated conducting sphere has a 10 cm radius. One wire carries a current of 1.000 002 0 A into it. Another wire carries a current of 1.000 000 0 A out of it. How long would it take for the sphere to increase in potential by 1000 V?

**3** **M** A charged belt, 50 cm wide, travels at 30 m/s between a source of charge and a sphere. The belt carries charge into the sphere at a rate corresponding to 100 $\mu$A. Compute the surface charge density on the belt.

## Module 26.2 Current Density

**4** **E** The (United States) National Electric Code, which sets maximum safe currents for insulated copper wires of various diameters, is given (in part) in the table. Plot the safe current density as a function of diameter. Which wire gauge has the maximum safe current density? ("Gauge" is a way of identifying wire diameters, and 1 mil = $10^{-3}$ in.)

| Gauge | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
|---|---|---|---|---|---|---|---|---|
| Diameter, mils | 204 | 162 | 129 | 102 | 81 | 64 | 51 | 40 |
| Safe current, A | 70 | 50 | 35 | 25 | 20 | 15 | 6 | 3 |

**5** E SSM A beam contains $2.0 \times 10^8$ doubly charged positive ions per cubic centimeter, all of which are moving north with a speed of $1.0 \times 10^5$ m/s. What are the (a) magnitude and (b) direction of the current density $\vec{J}$? (c) What additional quantity do you need to calculate the total current $i$ in this ion beam?

**6** E A certain cylindrical wire carries current. We draw a circle of radius $r$ around its central axis in Fig. 26.10$a$ to determine the current $i$ within the circle. Figure 26.10$b$ shows current $i$ as a function of $r^2$. The vertical scale is set by $i_s = 4.0$ mA, and the horizontal scale is set by $r_s^2 = 4.0\ \text{mm}^2$. (a) Is the current density uniform? (b) If so, what is its magnitude?

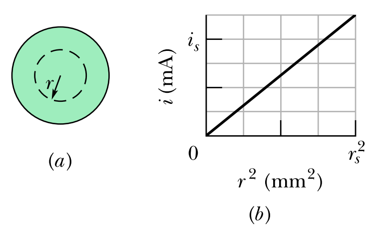



**Figure 26.10** Problem 6.

**7** E A fuse in an electric circuit is a wire that is designed to melt, and thereby open the circuit, if the current exceeds a predetermined value. Suppose that the material to be used in a fuse melts when the current density rises to 440 A/cm$^2$. What diameter of cylindrical wire should be used to make a fuse that will limit the current to 0.50 A?

**8** E A small but measurable current of $1.2 \times 10^{-10}$ A exists in a copper wire whose diameter is 2.5 mm. The number of charge carriers per unit volume is $8.49 \times 10^{28}\ \text{m}^{-3}$. Assuming the current is uniform, calculate the (a) current density and (b) electron drift speed.

**9** M CALC The magnitude $J(r)$ of the current density in a certain cylindrical wire is given as a function of radial distance from the center of the wire's cross section as $J(r) = Br$, where $r$ is in meters, $J$ is in amperes per square meter, and $B = 2.00 \times 10^5\ \text{A/m}^3$. This function applies out to the wire's radius of 2.00 mm. How much current is contained within the width of a thin ring concentric with the wire if the ring has a radial width of 10.0 $\mu$m and is at a radial distance of 1.20 mm?

**10** M CALC The magnitude $J$ of the current density in a certain lab wire with a circular cross section of radius $R = 2.00$ mm is given by $J = (3.00 \times 10^8)r^2$, with $J$ in amperes per square meter and radial distance $r$ in meters. What is the current through the outer section bounded by $r = 0.900R$ and $r = R$?

**11** M CALC What is the current in a wire of radius $R = 3.40$ mm if the magnitude of the current density is given by (a) $J_a = J_0 r/R$ and (b) $J_b = J_0(1 - r/R)$, in which $r$ is the radial distance and $J_0 = 5.50 \times 10^4\ \text{A/m}^2$? (c) Which function maximizes the current density near the wire's surface?

**12** M Near Earth, the density of protons in the solar wind (a stream of particles from the Sun) is 8.70 cm$^{-3}$, and their speed is 470 km/s. (a) Find the current density of these protons. (b) If Earth's magnetic field did not deflect the protons, what total current would Earth receive?

**13** M GO How long does it take electrons to get from a car battery to the starting motor? Assume the current is 300 A and the electrons travel through a copper wire with cross-sectional area 0.21 cm$^2$ and length 0.85 m. The number of charge carriers per unit volume is $8.49 \times 10^{28}\ \text{m}^{-3}$.

## Module 26.3 Resistance and Resistivity

**14** E BIO FCP A human being can be electrocuted if a current as small as 50 mA passes near the heart. An electrician working with sweaty hands makes good contact with the two conductors he is holding, one in each hand. If his resistance is 2000 Ω, what might the fatal voltage be?

**15** E SSM A coil is formed by winding 250 turns of insulated 16-gauge copper wire (diameter = 1.3 mm) in a single layer on a cylindrical form of radius 12 cm. What is the resistance of the coil? Neglect the thickness of the insulation. (Use Table 26.3.1.)

**16** E Copper and aluminum are being considered for a high-voltage transmission line that must carry a current of 60.0 A. The resistance per unit length is to be 0.150 Ω/km. The densities of copper and aluminum are 8960 and 2600 kg/m$^3$, respectively. Compute (a) the magnitude $J$ of the current density and (b) the mass per unit length $\lambda$ for a copper cable and (c) $J$ and (d) $\lambda$ for an aluminum cable.

**17** E A wire of Nichrome (a nickel–chromium–iron alloy commonly used in heating elements) is 1.0 m long and 1.0 mm$^2$ in cross-sectional area. It carries a current of 4.0 A when a 2.0 V potential difference is applied between its ends. Calculate the conductivity $\sigma$ of Nichrome.

**18** E A wire 4.00 m long and 6.00 mm in diameter has a resistance of 15.0 mΩ. A potential difference of 23.0 V is applied between the ends. (a) What is the current in the wire? (b) What is the magnitude of the current density? (c) Calculate the resistivity of the wire material. (d) Using Table 26.3.1, identify the material.

**19** E SSM What is the resistivity of a wire of 1.0 mm diameter, 2.0 m length, and 50 mΩ resistance?

**20** E A certain wire has a resistance $R$. What is the resistance of a second wire, made of the same material, that is half as long and has half the diameter?

**21** M A common flashlight bulb is rated at 0.30 A and 2.9 V (the values of the current and voltage under operating conditions). If the resistance of the tungsten bulb filament at room temperature (20°C) is 1.1 Ω, what is the temperature of the filament when the bulb is on?

**22** M BIO FCP *Kiting during a storm.* The legend that Benjamin Franklin flew a kite as a storm approached is only a legend—he was neither stupid nor suicidal. Suppose a kite string of radius 2.00 mm extends directly upward by 0.800 km and is coated with a 0.500 mm layer of water having resistivity 150 Ω·m. If the potential difference between the two ends of the string is 160 MV, what is the current through the water layer? The danger is not this current but the chance that the string draws a lightning strike, which can have a current as large as 500 000 A (way beyond just being lethal).

**23** M When 115 V is applied across a wire that is 10 m long and has a 0.30 mm radius, the magnitude of the current density is $1.4 \times 10^8$ A/m$^2$. Find the resistivity of the wire.

**24** M GO Figure 26.11*a* gives the magnitude $E(x)$ of the electric fields that have been set up by a battery along a resistive rod of length 9.00 mm (Fig. 26.11*b*). The vertical scale is set by $E_s = 4.00 \times 10^3$ V/m. The rod consists of three sections of the same material but with different radii. (The schematic diagram of Fig. 26.11*b* does not indicate the different radii.) The radius of section 3 is 2.00 mm. What is the radius of (a) section 1 and (b) section 2?

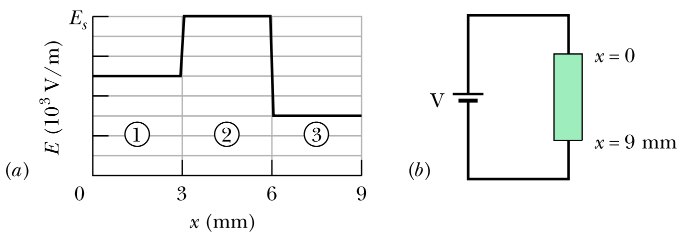



**Figure 26.11** Problem 24.

**25** M SSM A wire with a resistance of 6.0 Ω is drawn out through a die so that its new length is three times its original length. Find the resistance of the longer wire, assuming that the resistivity and density of the material are unchanged.

**26** M In Fig. 26.12*a*, a 9.00 V battery is connected to a resistive strip that consists of three sections with the same cross-sectional areas but different conductivities. Figure 26.12*b* gives the electric potential $V(x)$ versus position $x$ along the strip. The horizontal scale is set by $x_s = 8.00$ mm. Section 3 has conductivity $3.00 \times 10^7$ $(\Omega \cdot \text{m})^{-1}$. What is the conductivity of section (a) 1 and (b) 2?

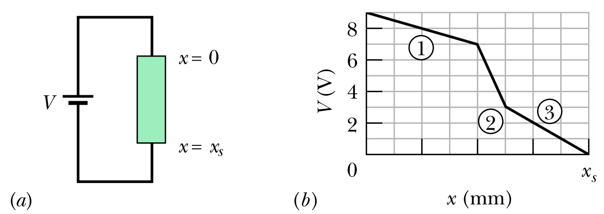



**Figure 26.12** Problem 26.

**27** M SSM Two conductors are made of the same material and have the same length. Conductor *A* is a solid wire of diameter 1.0 mm. Conductor *B* is a hollow tube of outside diameter 2.0 mm and inside diameter 1.0 mm. What is the resistance ratio $R_A/R_B$, measured between their ends?

**28** M GO Figure 26.13 gives the electric potential $V(x)$ along a copper wire carrying uniform current, from a point of higher potential $V_s = 12.0$ μV at $x = 0$ to a point of zero potential at $x_s = 3.00$ m. The wire has a radius of 2.00 mm. What is the current in the wire?

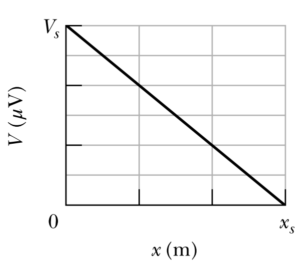



**Figure 26.13** Problem 28.

**29** M A potential difference of 3.00 nV is set up across a 2.00 cm length of copper wire that has a radius of 2.00 mm. How much charge drifts through a cross section in 3.00 ms?

**30** M If the gauge number of a wire is increased by 6, the diameter is halved; if a gauge number is increased by 1, the diameter decreases by the factor $2^{1/6}$ (see the table in Problem 4). Knowing this, and knowing that 1000 ft of 10-gauge copper wire has a resistance of approximately 1.00 Ω, estimate the resistance of 25 ft of 22-gauge copper wire.

**31** M An electrical cable consists of 125 strands of fine wire, each having 2.65 μΩ resistance. The same potential difference is applied between the ends of all the strands and results in a total current of 0.750 A. (a) What is the current in each strand? (b) What is the applied potential difference? (c) What is the resistance of the cable?

**32** M Earth's lower atmosphere contains negative and positive ions that are produced by radioactive elements in the soil and cosmic rays from space. In a certain region, the atmospheric electric field strength is 120 V/m and the field is directed vertically down. This field causes singly charged positive ions, at a density of 620 cm$^{-3}$, to drift downward and singly charged negative ions, at a density of 550 cm$^{-3}$, to drift upward (Fig. 26.14). The measured conductivity of the air in that region is $2.70 \times 10^{-14}$ $(\Omega \cdot \text{m})^{-1}$. Calculate (a) the magnitude of the current density and (b) the ion drift speed, assumed to be the same for positive and negative ions.

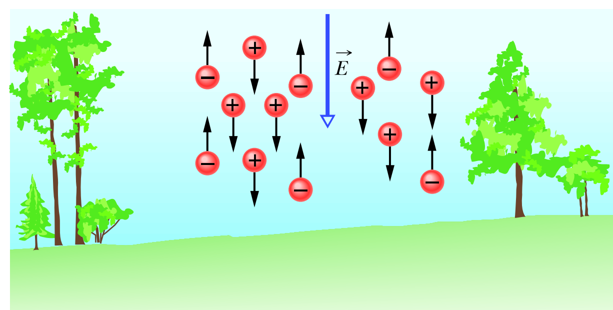



**Figure 26.14** Problem 32.

**33** M A block in the shape of a rectangular solid has a cross-sectional area of 3.50 cm$^2$ across its width, a front-to-rear length of 15.8 cm, and a resistance of 935 Ω. The block's material contains $5.33 \times 10^{22}$ conduction electrons/m$^3$. A potential difference of 35.8 V is maintained between its front and rear faces. (a) What is the current in the block? (b) If the current density is uniform, what is its magnitude? What are (c) the drift velocity of the conduction electrons and (d) the magnitude of the electric field in the block?

**34** H GO Figure 26.15 shows wire section 1 of diameter $D_1 = 4.00R$ and wire section 2 of diameter $D_2 = 2.00R$, connected by a tapered section. The wire is copper and carries a current. Assume that the current is uniformly distributed across any cross-sectional area through the wire's width. The electric potential change $V$ along the length $L = 2.00$ m shown in section 2 is 10.0 μV. The number of charge carriers per unit volume is $8.49 \times 10^{28}$ m$^{-3}$. What is the drift speed of the conduction electrons in section 1?

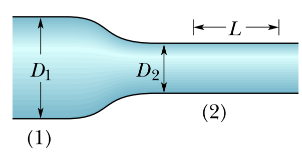



**Figure 26.15** Problem 34.

**35** H CALC GO In Fig. 26.16, current is set up through a truncated right circular cone of resistivity 731 Ω·m, left radius

$a = 2.00$ mm, right radius $b = 2.30$ mm, and length $L = 1.94$ cm. Assume that the current density is uniform across any cross section taken perpendicular to the length. What is the resistance of the cone?

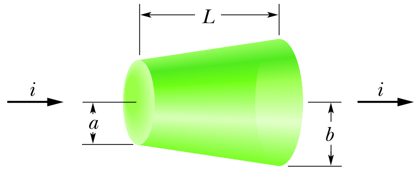



**Figure 26.16** Problem 35.

**36** H CALC BIO GO FCP *Swimming during a storm.* Figure 26.17 shows a swimmer at distance $D = 35.0$ m from a lightning strike to the water, with current $I = 78$ kA. The water has resistivity 30 Ω·m, the width of the swimmer along a radial line from the strike is 0.70 m, and his resistance across that width is 4.00 kΩ. Assume that the current spreads through the water over a hemisphere centered on the strike point. What is the current through the swimmer?

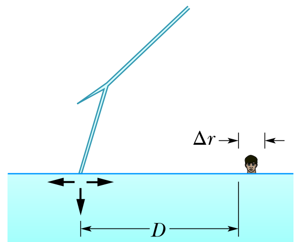



**Figure 26.17** Problem 36.

## Module 26.4 Ohm's Law

**37** M Show that, according to the free-electron model of electrical conduction in metals and classical physics, the resistivity of metals should be proportional to $\sqrt{T}$, where $T$ is the temperature in kelvins. (See Eq. 19.6.5.)

## Module 26.5 Power, Semiconductors, Superconductors

**38** E In Fig. 26.18*a*, a 20 Ω resistor is connected to a battery. Figure 26.18*b* shows the increase of thermal energy $E_{th}$ in the resistor as a function of time $t$. The vertical scale is set by $E_{th,s} = 2.50$ mJ, and the horizontal scale is set by $t_s = 4.0$ s. What is the electric potential across the battery?

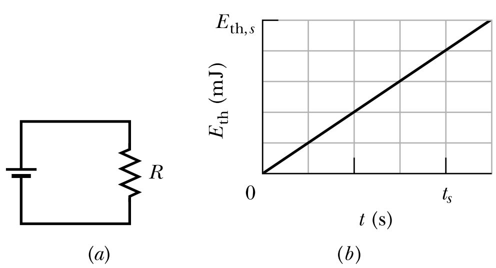



**Figure 26.18** Problem 38.

**39** E A certain brand of hot-dog cooker works by applying a potential difference of 120 V across opposite ends of a hot dog and allowing it to cook by means of the thermal energy produced. The current is 10.0 A, and the energy required to cook one hot dog is 60.0 kJ. If the rate at which energy is supplied is unchanged, how long will it take to cook three hot dogs simultaneously?

**40** E Thermal energy is produced in a resistor at a rate of 100 W when the current is 3.00 A. What is the resistance?

**41** E SSM A 120 V potential difference is applied to a space heater whose resistance is 14 Ω when hot. (a) At what rate is electrical energy transferred to thermal energy? (b) What is the cost for 5.0 h at US\$0.05/kW·h?

**42** E In Fig. 26.19, a battery of potential difference $V = 12$ V is connected to a resistive strip of resistance $R = 6.0$ Ω. When an electron moves through the strip from one end to the other, (a) in which direction in the figure does the electron move, (b) how much work is done on the electron by the electric field in the strip, and (c) how much energy is transferred to the thermal energy of the strip by the electron?

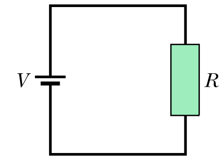



**Figure 26.19** Problem 42.

**43** E An unknown resistor is connected between the terminals of a 3.00 V battery. Energy is dissipated in the resistor at the rate of 0.540 W. The same resistor is then connected between the terminals of a 1.50 V battery. At what rate is energy now dissipated?

**44** E A student kept his 9.0 V, 7.0 W radio turned on at full volume from 9:00 P.M. until 2:00 A.M. How much charge went through it?

**45** E SSM A 1250 W radiant heater is constructed to operate at 115 V. (a) What is the current in the heater when the unit is operating? (b) What is the resistance of the heating coil? (c) How much thermal energy is produced in 1.0 h?

**46** M GO A copper wire of cross-sectional area $2.00 \times 10^{-6}$ m$^2$ and length 4.00 m has a current of 2.00 A uniformly distributed across that area. (a) What is the magnitude of the electric field along the wire? (b) How much electrical energy is transferred to thermal energy in 30 min?

**47** M A heating element is made by maintaining a potential difference of 75.0 V across the length of a Nichrome wire that has a $2.60 \times 10^{-6}$ m$^2$ cross section. Nichrome has a resistivity of $5.00 \times 10^{-7}$ Ω·m. (a) If the element dissipates 5000 W, what is its length? (b) If 100 V is used to obtain the same dissipation rate, what should the length be?

**48** M BIO FCP *Exploding shoes.* The rain-soaked shoes of a person may explode if ground current from nearby lightning vaporizes the water. The sudden conversion of water to water vapor causes a dramatic expansion that can rip apart shoes. Water has density 1000 kg/m$^3$ and requires 2256 kJ/kg to be vaporized. If horizontal current lasts 2.00 ms and encounters water with resistivity 150 Ω·m, length 12.0 cm, and vertical cross-sectional area $15 \times 10^{-5}$ m$^2$, what average current is required to vaporize the water?

**49** M A 100 W lightbulb is plugged into a standard 120 V outlet. (a) How much does it cost per 31-day month to leave the light turned on continuously? Assume electrical energy costs US\$0.06/kW·h. (b) What is the resistance of the bulb? (c) What is the current in the bulb?

**50** M GO The current through the battery and resistors 1 and 2 in Fig. 26.20*a* is 2.00 A. Energy is transferred from the current to thermal energy $E_{th}$ in both resistors. Curves 1 and 2 in Fig. 26.20*b* give that thermal energy $E_{th}$ for resistors 1 and 2, respectively, as a function of time $t$. The vertical scale is set by $E_{th,s} = 40.0$ mJ, and the horizontal scale is set by $t_s = 5.00$ s. What is the power of the battery?

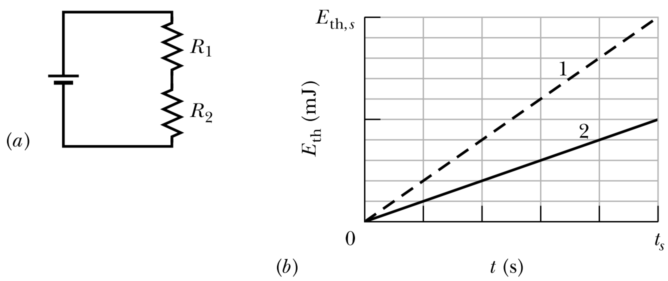



**Figure 26.20** Problem 50.

**51** M GO SSM Wire $C$ and wire $D$ are made from different materials and have length $L_C = L_D = 1.0$ m. The resistivity and diameter of wire $C$ are $2.0 \times 10^{-6}\ \Omega \cdot$m and 1.00 mm, and those of wire $D$ are $1.0 \times 10^{-6}\ \Omega \cdot$m and 0.50 mm. The wires are joined as shown in Fig. 26.21, and a current of 2.0 A is set up in them. What is the electric potential difference between (a) points 1 and 2 and (b) points 2 and 3? What is the rate at which energy is dissipated between (c) points 1 and 2 and (d) points 2 and 3?

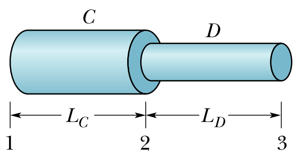



**Figure 26.21** Problem 51.

**52** M CALC GO The current-density magnitude in a certain circular wire is $J = (2.75 \times 10^{10}\ \text{A/m}^4)r^2$, where $r$ is the radial distance out to the wire's radius of 3.00 mm. The potential applied to the wire (end to end) is 60.0 V. How much energy is converted to thermal energy in 1.00 h?

**53** M A 120 V potential difference is applied to a space heater that dissipates 500 W during operation. (a) What is its resistance during operation? (b) At what rate do electrons flow through any cross section of the heater element?

**54** H CALC GO Figure 26.22*a* shows a rod of resistive material. The resistance per unit length of the rod increases in the positive direction of the $x$ axis. At any position $x$ along the rod, the resistance $dR$ of a narrow (differential) section of width $dx$ is given by $dR = 5.00x\, dx$, where $dR$ is in ohms and $x$ is in meters. Figure 26.22*b* shows such a narrow section. You are to slice off a length of the rod between $x = 0$ and some position $x = L$ and then connect that length to a battery with potential difference $V = 5.0$ V (Fig. 26.22*c*). You want the current in the length to transfer energy to thermal energy at the rate of 200 W. At what position $x = L$ should you cut the rod?

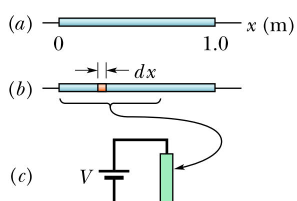



**Figure 26.22** Problem 54.

## Additional Problems

**55** SSM A Nichrome heater dissipates 500 W when the applied potential difference is 110 V and the wire temperature is 800°C. What would be the dissipation rate if the wire temperature were held at 200°C by immersing the wire in a bath of cooling oil? The applied potential difference remains the same, and $\alpha$ for Nichrome at 800°C is $4.0 \times 10^{-4}\ \text{K}^{-1}$.

**56** A potential difference of 1.20 V will be applied to a 33.0 m length of 18-gauge copper wire (diameter = 0.0400 in.). Calculate (a) the current, (b) the magnitude of the current density, (c) the magnitude of the electric field within the wire, and (d) the rate at which thermal energy will appear in the wire.

**57** An 18.0 W device has 9.00 V across it. How much charge goes through the device in 4.00 h?

**58** An aluminum rod with a square cross section is 1.3 m long and 5.2 mm on edge. (a) What is the resistance between its ends? (b) What must be the diameter of a cylindrical copper rod of length 1.3 m if its resistance is to be the same as that of the aluminum rod?

**59** A cylindrical metal rod is 1.60 m long and 5.50 mm in diameter. The resistance between its two ends (at 20°C) is $1.09 \times 10^{-3}\ \Omega$. (a) What is the material? (b) A round disk, 2.00 cm in diameter and 1.00 mm thick, is formed of the same material. What is the resistance between the round faces, assuming that each face is an equipotential surface?

**60** FCP *The chocolate crumb mystery.* This story begins with Problem 60 in Chapter 23 and continues through Chapters 24 and 25. The chocolate crumb powder moved to the silo through a pipe of radius $R$ with uniform speed $v$ and uniform charge density $\rho$. (a) Find an expression for the current $i$ (the rate at which charge on the powder moved) through a perpendicular cross section of the pipe. (b) Evaluate $i$ for the conditions at the factory: pipe radius $R = 5.0$ cm, speed $v = 2.0$ m/s, and charge density $\rho = 1.1 \times 10^{-3}\ \text{C/m}^3$.

If the powder were to flow through a change $V$ in electric potential, its energy could be transferred to a spark at the rate $P = iV$. (c) Could there be such a transfer within the pipe due to the radial potential difference discussed in Problem 70 of Chapter 24?

As the powder flowed from the pipe into the silo, the electric potential of the powder changed. The magnitude of that change was at least equal to the radial potential difference within the pipe (as evaluated in Problem 70 of Chapter 24). (d) Assuming that value for the potential difference and using the current found in (b) above, find the rate at which energy could have been transferred from the powder to a spark as the powder exited the pipe. (e) If a spark did occur at the exit and lasted for 0.20 s (a reasonable expectation), how much energy would have been transferred to the spark? Recall from Problem 60 in Chapter 23 that a minimum energy transfer of 150 mJ is needed to cause an explosion. (f) Where did the powder explosion most likely occur: in the powder cloud at the unloading bin (Problem 60 of Chapter 25), within the pipe, or at the exit of the pipe into the silo?

**61** SSM A steady beam of alpha particles ($q = +2e$) traveling with constant kinetic energy 20 MeV carries a current of 0.25 $\mu$A. (a) If the beam is directed perpendicular to a flat surface, how many alpha particles strike the surface in 3.0 s? (b) At any instant, how many alpha particles are there in a given 20 cm length of the beam? (c) Through what potential difference is it necessary to accelerate each alpha particle from rest to bring it to an energy of 20 MeV?

**62** A resistor with a potential difference of 200 V across it transfers electrical energy to thermal energy at the rate of 3000 W. What is the resistance of the resistor?

**63** A 2.0 kW heater element from a dryer has a length of 80 cm. If a 10 cm section is removed, what power is used by the now shortened element at 120 V?

**64** A cylindrical resistor of radius 5.0 mm and length 2.0 cm is made of material that has a resistivity of $3.5 \times 10^{-5}\ \Omega \cdot m$. What are (a) the magnitude of the current density and (b) the potential difference when the energy dissipation rate in the resistor is 1.0 W?

**65** A potential difference $V$ is applied to a wire of cross-sectional area $A$, length $L$, and resistivity $\rho$. You want to change the applied potential difference and stretch the wire so that the energy dissipation rate is multiplied by 30.0 and the current is multiplied by 4.00. Assuming the wire's density does not change, what are (a) the ratio of the new length to $L$ and (b) the ratio of the new cross-sectional area to $A$?

**66** The headlights of a moving car require about 10 A from the 12 V alternator, which is driven by the engine. Assume the alternator is 80% efficient (its output electrical power is 80% of its input mechanical power), and calculate the horsepower the engine must supply to run the lights.

**67** A 500 W heating unit is designed to operate with an applied potential difference of 115 V. (a) By what percentage will its heat output drop if the applied potential difference drops to 110 V? Assume no change in resistance. (b) If you took the variation of resistance with temperature into account, would the actual drop in heat output be larger or smaller than that calculated in (a)?

**68** The copper windings of a motor have a resistance of 50 Ω at 20°C when the motor is idle. After the motor has run for several hours, the resistance rises to 58 Ω. What is the temperature of the windings now? Ignore changes in the dimensions of the windings. (Use Table 26.3.1.)

**69** How much electrical energy is transferred to thermal energy in 2.00 h by an electrical resistance of 400 Ω when the potential applied across it is 90.0 V?

**70** A caterpillar of length 4.0 cm crawls in the direction of electron drift along a 5.2-mm-diameter bare copper wire that carries a uniform current of 12 A. (a) What is the potential difference between the two ends of the caterpillar? (b) Is its tail positive or negative relative to its head? (c) How much time does the caterpillar take to crawl 1.0 cm if it crawls at the drift speed of the electrons in the wire? (The number of charge carriers per unit volume is $8.49 \times 10^{28}\ m^{-3}$.)

**71** SSM (a) At what temperature would the resistance of a copper conductor be double its resistance at 20.0°C? (Use 20.0°C as the reference point in Eq. 26.3.10; compare your answer with Fig. 26.3.4.)(b) Does this same "doubling temperature" hold for all copper conductors, regardless of shape or size?

**72** A steel trolley-car rail has a cross-sectional area of 56.0 cm². What is the resistance of 10.0 km of rail? The resistivity of the steel is $3.00 \times 10^{-7}\ \Omega \cdot m$.

**73** A coil of current-carrying Nichrome wire is immersed in a liquid. (Nichrome is a nickel–chromium–iron alloy commonly used in heating elements.) When the potential difference across the coil is 12 V and the current through the coil is 5.2 A, the liquid evaporates at the steady rate of 21 mg/s. Calculate the heat of vaporization of the liquid (see Module 18.4).

**74** GO The current density in a wire is uniform and has magnitude $2.0 \times 10^{6}\ A/m^2$, the wire's length is 5.0 m, and the density of conduction electrons is $8.49 \times 10^{28}\ m^{-3}$. How long does an electron take (on the average) to travel the length of the wire?

**75** A certain x-ray tube operates at a current of 7.00 mA and a potential difference of 80.0 kV. What is its power in watts?

**76** A current is established in a gas discharge tube when a sufficiently high potential difference is applied across the two electrodes in the tube. The gas ionizes; electrons move toward the positive terminal and singly charged positive ions toward the negative terminal. (a) What is the current in a hydrogen discharge tube in which $3.1 \times 10^{18}$ electrons and $1.1 \times 10^{18}$ protons move past a cross-sectional area of the tube each second? (b) Is the direction of the current density $\vec{J}$ toward or away from the negative terminal?

**77** *Two drift speeds.* One end of an aluminum wire with diameter 2.5 mm is welded to one end of a copper wire with diameter 1.8 mm. The composite carries a steady current $i$ of 1.3 A. For points that are not next to the junction, what is the current density in (a) the aluminum wire and (b) the copper wire? (c) What is the magnitude of the electric field in the copper?

**78** *Semiconductor.* A strip of silicon has width $w = 3.2$ mm and thickness $t = 250\ \mu m$ and carries current $i = 5.2$ mA. The silicon is said to be an *n-type semiconductor* because it has been "doped" with a controlled phosphorus impurity. The process greatly increased $n$, the number of charge carriers per unit volume, to a value of $1.5 \times 10^{23}\ m^{-3}$. What are (a) the current density, (b) the drift speed, and (c) the electric field magnitude in the strip?