

Measurement

1.1 MEASURING THINGS, INCLUDING LENGTHS

Learning Objectives

After reading this module, you should be able to . . .

1.1.1 Identify the base quantities in the SI system.

1.1.2 Name the most frequently used prefixes for SI units.

1.1.3 Change units (here for length, area, and volume) by using chain-link conversions.

1.1.4 Explain that the meter is defined in terms of the speed of light in a vacuum.

Key Ideas

- Physics is based on measurement of physical quantities. Certain physical quantities have been chosen as base quantities (such as length, time, and mass); each has been defined in terms of a standard and given a unit of measure (such as meter, second, and kilogram). Other physical quantities are defined in terms of the base quantities and their standards and units.

- The unit system emphasized in this book is the International System of Units (SI). The three physical quantities displayed in Table 1.1.1 are used in the early chapters. Standards, which must be both accessible and invariable, have been established for these base

quantities by international agreement. These standards are used in all physical measurement, for both the base quantities and the quantities derived from them. Scientific notation and the prefixes of Table 1.1.2 are used to simplify measurement notation.

- Conversion of units may be performed by using chain-link conversions in which the original data are multiplied successively by conversion factors written as unity and the units are manipulated like algebraic quantities until only the desired units remain.

- The meter is defined as the distance traveled by light during a precisely specified time interval.

What Is Physics?

Science and engineering are based on measurements and comparisons. Thus, we need rules about how things are measured and compared, and we need experiments to establish the units for those measurements and comparisons. One purpose of physics (and engineering) is to design and conduct those experiments.

For example, physicists strive to develop clocks of extreme accuracy so that any time or time interval can be precisely determined and compared. You may wonder whether such accuracy is actually needed or worth the effort. Here is one example of the worth: Without clocks of extreme accuracy, the Global Positioning System (GPS) that is now vital to worldwide navigation would be useless.

Measuring Things

We discover physics by learning how to measure the quantities involved in physics. Among these quantities are length, time, mass, temperature, pressure, and electric current.

We measure each physical quantity in its own units, by comparison with a **standard**. The **unit** is a unique name we assign to measures of that quantity—for example, meter (m) for the quantity length. The standard corresponds to exactly 1.0 unit of the quantity. As you will see, the standard for length, which corresponds to exactly 1.0 m, is the distance traveled by light in a vacuum during a certain fraction of a second. We can define a unit and its standard in any way we care to. However, the important thing is to do so in such a way that scientists around the world will agree that our definitions are both sensible and practical.

Once we have set up a standard—say, for length—we must work out procedures by which any length whatever, be it the radius of a hydrogen atom, the wheelbase of a skateboard, or the distance to a star, can be expressed in terms of the standard. Rulers, which approximate our length standard, give us one such procedure for measuring length. However, many of our comparisons must be indirect. You cannot use a ruler, for example, to measure the radius of an atom or the distance to a star.

Base Quantities. There are so many physical quantities that it is a problem to organize them. Fortunately, they are not all independent; for example, speed is the ratio of a length to a time. Thus, what we do is pick out—by international agreement—a small number of physical quantities, such as length and time, and assign standards to them alone. We then define all other physical quantities in terms of these *base quantities* and their standards (called *base standards*). Speed, for example, is defined in terms of the base quantities length and time and their base standards.

Base standards must be both accessible and invariable. If we define the length standard as the distance between one's nose and the index finger on an outstretched arm, we certainly have an accessible standard—but it will, of course, vary from person to person. The demand for precision in science and engineering pushes us to aim first for invariability. We then exert great effort to make duplicates of the base standards that are accessible to those who need them.

The International System of Units

In 1971, the 14th General Conference on Weights and Measures picked seven quantities as base quantities, thereby forming the basis of the International System of Units, abbreviated SI from its French name and popularly known as the *metric system*. Table 1.1.1 shows the units for the three base quantities—length, mass, and time—that we use in the early chapters of this book. These units were defined to be on a “human scale.”

Many SI *derived units* are defined in terms of these base units. For example, the SI unit for power, called the **watt** (W), is defined in terms of the base units for mass, length, and time. Thus, as you will see in Chapter 7,

$$1 \text{ watt} = 1 \text{ W} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^3, \quad (1.1.1)$$

where the last collection of unit symbols is read as kilogram-meter squared per second cubed.

To express the very large and very small quantities we often run into in physics, we use *scientific notation*, which employs powers of 10. In this notation,

$$3\,560\,000\,000 \text{ m} = 3.56 \times 10^9 \text{ m} \quad (1.1.2)$$

$$\text{and} \quad 0.000\,000\,492 \text{ s} = 4.92 \times 10^{-7} \text{ s.} \quad (1.1.3)$$

Scientific notation on computers sometimes takes on an even briefer look, as in 3.56 E9 and 4.92 E-7, where E stands for “exponent of ten.” It is briefer still on some calculators, where E is replaced with an empty space.

Table 1.1.1 Units for Three SI Base Quantities

Quantity	Unit Name	Unit Symbol
Length	meter	m
Time	second	s
Mass	kilogram	kg

As a further convenience when dealing with very large or very small measurements, we use the prefixes listed in Table 1.1.2. As you can see, each prefix represents a certain power of 10, to be used as a multiplication factor. Attaching a prefix to an SI unit has the effect of multiplying by the associated factor. Thus, we can express a particular electric power as

$$1.27 \times 10^9 \text{ watts} = 1.27 \text{ gigawatts} = 1.27 \text{ GW} \quad (1.1.4)$$

or a particular time interval as

$$2.35 \times 10^{-9} \text{ s} = 2.35 \text{ nanoseconds} = 2.35 \text{ ns.} \quad (1.1.5)$$

Some prefixes, as used in milliliter, centimeter, kilogram, and megabyte, are probably familiar to you.

Changing Units

We often need to change the units in which a physical quantity is expressed. We do so by a method called *chain-link conversion*. In this method, we multiply the original measurement by a **conversion factor** (a ratio of units that is equal to unity). For example, because 1 min and 60 s are identical time intervals, we have

$$\frac{1 \text{ min}}{60 \text{ s}} = 1 \quad \text{and} \quad \frac{60 \text{ s}}{1 \text{ min}} = 1.$$

Thus, the ratios $(1 \text{ min})/(60 \text{ s})$ and $(60 \text{ s})/(1 \text{ min})$ can be used as conversion factors. This is *not* the same as writing $\frac{1}{60} = 1$ or $60 = 1$; each *number* and its *unit* must be treated together.

Because multiplying any quantity by unity leaves the quantity unchanged, we can introduce conversion factors wherever we find them useful. In chain-link conversion, we use the factors to cancel unwanted units. For example, to convert 2 min to seconds, we have

$$2 \text{ min} = (2 \text{ min})(1) = (2 \text{ min})\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 120 \text{ s.} \quad (1.1.6)$$

If you introduce a conversion factor in such a way that unwanted units do *not* cancel, invert the factor and try again. In conversions, the units obey the same algebraic rules as variables and numbers.

Appendix D gives conversion factors between SI and other systems of units, including non-SI units still used in the United States. However, the conversion factors are written in the style of “1 min = 60 s” rather than as a ratio. So, you need to decide on the numerator and denominator in any needed ratio.

Length

In 1792, the newborn Republic of France established a new system of weights and measures. Its cornerstone was the meter, defined to be one ten-millionth of the distance from the north pole to the equator. Later, for practical reasons, this Earth standard was abandoned and the meter came to be defined as the distance between two fine lines engraved near the ends of a platinum–iridium bar, the **standard meter bar**, which was kept at the International Bureau of Weights and Measures near Paris. Accurate copies of the bar were sent to standardizing laboratories throughout the world. These **secondary standards** were used to produce other, still more accessible standards, so that ultimately every

Table 1.1.2 Prefixes for SI Units

Factor	Prefix ^a	Symbol
10^{24}	yotta-	Y
10^{21}	zetta-	Z
10^{18}	exa-	E
10^{15}	peta-	P
10^{12}	tera-	T
10^9	giga-	G
10^6	mega-	M
10^3	kilo-	k
10^2	hecto-	h
10^1	deka-	da
10^{-1}	deci-	d
10^{-2}	centi-	c
10^{-3}	milli-	m
10^{-6}	micro-	μ
10^{-9}	nano-	n
10^{-12}	pico-	p
10^{-15}	femto-	f
10^{-18}	atto-	a
10^{-21}	zepto-	z
10^{-24}	yocto-	y

^aThe most frequently used prefixes are shown in bold type.

measuring device derived its authority from the standard meter bar through a complicated chain of comparisons.

Eventually, a standard more precise than the distance between two fine scratches on a metal bar was required. In 1960, a new standard for the meter, based on the wavelength of light, was adopted. Specifically, the standard for the meter was redefined to be 1 650 763.73 wavelengths of a particular orange-red light emitted by atoms of krypton-86 (a particular isotope, or type, of krypton) in a gas discharge tube that can be set up anywhere in the world. This awkward number of wavelengths was chosen so that the new standard would be close to the old meter-bar standard.

By 1983, however, the demand for higher precision had reached such a point that even the krypton-86 standard could not meet it, and in that year a bold step was taken. The meter was redefined as the distance traveled by light in a specified time interval. In the words of the 17th General Conference on Weights and Measures:



The meter is the length of the path traveled by light in a vacuum during a time interval of $1/299\,792\,458$ of a second.

This time interval was chosen so that the speed of light c is exactly

$$c = 299\,792\,458 \text{ m/s.}$$

Measurements of the speed of light had become extremely precise, so it made sense to adopt the speed of light as a defined quantity and to use it to redefine the meter.

Table 1.1.3 shows a wide range of lengths, from that of the universe (top line) to those of some very small objects.

Table 1.1.3 Some Approximate Lengths

Measurement	Length in Meters
Distance to the first galaxies formed	2×10^{26}
Distance to the Andromeda galaxy	2×10^{22}
Distance to the nearby star Proxima Centauri	4×10^{16}
Distance to Pluto	6×10^{12}
Radius of Earth	6×10^6
Height of Mt. Everest	9×10^3
Thickness of this page	1×10^{-4}
Length of a typical virus	1×10^{-8}
Radius of a hydrogen atom	5×10^{-11}
Radius of a proton	1×10^{-15}

Significant Figures and Decimal Places

Suppose that you work out a problem in which each value consists of two digits. Those digits are called **significant figures** and they set the number of digits that you can use in reporting your final answer. With data given in two significant figures, your final answer should have only two significant figures. However, depending on the mode setting of your calculator, many more digits might be displayed. Those extra digits are meaningless.

In this book, final results of calculations are often rounded to match the least number of significant figures in the given data. (However, sometimes an extra significant figure is kept.) When the leftmost of the digits to be discarded is 5 or more, the last remaining digit is rounded up; otherwise it is retained as is. For example, 11.3516 is rounded to three significant figures as 11.4 and 11.3279 is rounded to three significant figures as 11.3. (The answers to sample problems in this book are usually presented with the symbol = instead of \approx even if rounding is involved.)

When a number such as 3.15 or 3.15×10^3 is provided in a problem, the number of significant figures is apparent, but how about the number 3000? Is it known to only one significant figure (3×10^3)? Or is it known to as many as four significant figures (3.000×10^3)? In this book, we assume that all the zeros in such given numbers as 3000 are significant, but you had better not make that assumption elsewhere.

Don't confuse *significant figures* with *decimal places*. Consider the lengths 35.6 mm, 3.56 m, and 0.00356 m. They all have three significant figures but they have one, two, and five decimal places, respectively.

Sample Problem 1.1.1 Estimating order of magnitude, ball of string

The world's largest ball of string is about 2 m in radius. To the nearest order of magnitude, what is the total length L of the string in the ball?

KEY IDEA

We could, of course, take the ball apart and measure the total length L , but that would take great effort and make the ball's builder most unhappy. Instead, because we want only the nearest order of magnitude, we can estimate any quantities required in the calculation.

Calculations: Let us assume the ball is spherical with radius $R = 2$ m. The string in the ball is not closely packed (there are uncountable gaps between adjacent sections of string). To allow for these gaps, let us somewhat overestimate the cross-sectional area of the string by assuming the cross section is square, with an edge length $d = 4$ mm.

Then, with a cross-sectional area of d^2 and a length L , the string occupies a total volume of

$$V = (\text{cross-sectional area})(\text{length}) = d^2 L.$$

This is approximately equal to the volume of the ball, given by $\frac{4}{3}\pi R^3$, which is about $4R^3$ because π is about 3. Thus, we have the following

$$\begin{aligned} d^2 L &= 4R^3, \\ \text{or } L &= \frac{4R^3}{d^2} = \frac{4(2 \text{ m})^3}{(4 \times 10^{-3} \text{ m})^2} \\ &= 2 \times 10^6 \text{ m} \approx 10^6 \text{ m} = 10^3 \text{ km}. \end{aligned} \quad (\text{Answer})$$

(Note that you do not need a calculator for such a simplified calculation.) To the nearest order of magnitude, the ball contains about 1000 km of string!

WileyPLUS Additional examples, video, and practice available at *WileyPLUS*

1.2 TIME

Learning Objectives

After reading this module, you should be able to . . .

1.2.1 Change units for time by using chain-link conversions.

1.2.2 Use various measures of time, such as for motion or as determined on different clocks.

Key Idea

- The second is defined in terms of the oscillations of light emitted by an atomic (cesium-133) source. Accurate

time signals are sent worldwide by radio signals keyed to atomic clocks in standardizing laboratories.

Time

Time has two aspects. For civil and some scientific purposes, we want to know the time of day so that we can order events in sequence. In much scientific work, we want to know how long an event lasts. Thus, any time standard must be able to answer two questions: “*When* did it happen?” and “*What* is its *duration*?”. Table 1.2.1 shows some time intervals.

Any phenomenon that repeats itself is a possible time standard. Earth's rotation, which determines the length of the day, has been used in this way for centuries; Fig. 1.2.1 shows one novel example of a watch based on that rotation. A quartz clock, in which a quartz ring is made to vibrate continuously, can be calibrated against Earth's rotation via astronomical observations and used to measure time intervals in the laboratory. However, the calibration cannot be carried out with the accuracy called for by modern scientific and engineering technology.



Figure 1.2.1 When the metric system was proposed in 1792, the hour was redefined to provide a 10-hour day. The idea did not catch on. The maker of this 10-hour watch wisely provided a small dial that kept conventional 12-hour time. Do the two dials indicate the same time?

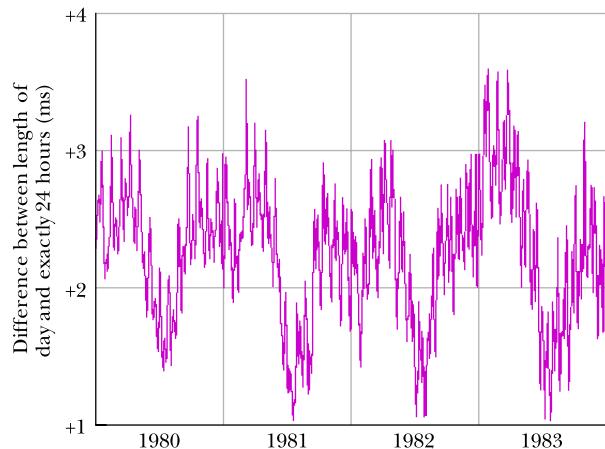


Figure 1.2.2 Variations in the length of the day over a 4-year period. Note that the entire vertical scale amounts to only 3 ms ($= 0.003\text{ s}$).

Table 1.2.1 Some Approximate Time Intervals

Measurement	Time Interval in Seconds	Measurement	Time Interval in Seconds
Lifetime of the proton (predicted)	3×10^{40}	Time between human heartbeats	8×10^{-1}
Age of the universe	5×10^{17}	Lifetime of the muon	2×10^{-6}
Age of the pyramid of Cheops	1×10^{11}	Shortest lab light pulse	1×10^{-16}
Human life expectancy	2×10^9	Lifetime of the most unstable particle	1×10^{-23}
Length of a day	9×10^4	The Planck time ^a	1×10^{-43}

^aThis is the earliest time after the big bang at which the laws of physics as we know them can be applied.

To meet the need for a better time standard, atomic clocks have been developed. An atomic clock at the National Institute of Standards and Technology (NIST) in Boulder, Colorado, is the standard for Coordinated Universal Time (UTC) in the United States. Its time signals are available by shortwave radio (stations WWV and WWVH) and by telephone (303-499-7111). Time signals (and related information) are also available from the United States Naval Observatory at website <https://www.usno.navy.mil/USNO/time>. (To set a clock extremely accurately at your particular location, you would have to account for the travel time required for these signals to reach you.)

Figure 1.2.2 shows variations in the length of one day on Earth over a 4-year period, as determined by comparison with a cesium (atomic) clock. Because the variation displayed by Fig. 1.2.2 is seasonal and repetitious, we suspect the rotating Earth when there is a difference between Earth and atom as timekeepers. The variation is due to tidal effects caused by the Moon and to large-scale winds.

The 13th General Conference on Weights and Measures in 1967 adopted a standard second based on the cesium clock:



One second is the time taken by 9 192 631 770 oscillations of the light (of a specified wavelength) emitted by a cesium-133 atom.

Atomic clocks are so consistent that, in principle, two cesium clocks would have to run for 6000 years before their readings would differ by more than 1 s. Even such accuracy pales in comparison with that of clocks currently being developed; their precision may be 1 part in 10^{18} —that is, 1 s in $1 \times 10^{18}\text{ s}$ (which is about $3 \times 10^{10}\text{ y}$).

1.3 MASS

Learning Objectives

After reading this module, you should be able to . . .

1.3.1 Change units for mass by using chain-link conversions.

1.3.2 Relate density to mass and volume when the mass is uniformly distributed.

Key Ideas

The kilogram is defined in terms of a platinum–iridium standard mass kept near Paris. For measurements on an atomic scale, the atomic mass unit, defined in terms of the atom carbon-12, is usually used.

The density ρ of a material is the mass per unit volume:

$$\rho = \frac{m}{V}.$$

Mass

The Standard Kilogram

The SI standard of mass is a cylinder of platinum and iridium (Fig. 1.3.1) that is kept at the International Bureau of Weights and Measures near Paris and assigned, by international agreement, a mass of 1 kilogram. Accurate copies have been sent to standardizing laboratories in other countries, and the masses of other bodies can be determined by balancing them against a copy. Table 1.3.1 shows some masses expressed in kilograms, ranging over about 83 orders of magnitude.

The U.S. copy of the standard kilogram is housed in a vault at NIST. It is removed, no more than once a year, for the purpose of checking duplicate copies that are used elsewhere. Since 1889, it has been taken to France twice for recomparison with the primary standard.

Kibble Balance

A far more accurate way of measuring mass is now being adopted. In a Kibble balance (named after its inventor Brian Kibble), a standard mass can be measured when the downward pull on it by gravity is balanced by an upward force from a magnetic field due to an electrical current. The precision of this technique comes from the fact that the electric and magnetic properties can be determined in terms of quantum mechanical quantities that have been precisely defined or measured. Once a standard mass is measured, it can be sent to other labs where the masses of other bodies can be determined from it.

A Second Mass Standard

The masses of atoms can be compared with one another more precisely than they can be compared with the standard kilogram. For this reason, we have a second mass standard. It is the carbon-12 atom, which, by international agreement, has been assigned a mass of 12 **atomic mass units** (u). The relation between the two units is

$$1 \text{ u} = 1.660\,538\,86 \times 10^{-27} \text{ kg}, \quad (1.3.1)$$

with an uncertainty of ± 10 in the last two decimal places. Scientists can, with reasonable precision, experimentally determine the masses of other atoms relative to the mass of carbon-12. What we presently lack is a reliable means of extending that precision to more common units of mass, such as a kilogram.

Density

As we shall discuss further in Chapter 14, **density** ρ (lowercase Greek letter rho) is the mass per unit volume:

$$\rho = \frac{m}{V}. \quad (1.3.2)$$

Densities are typically listed in kilograms per cubic meter or grams per cubic centimeter. The density of water (1.00 gram per cubic centimeter) is often used as a comparison. Fresh snow has about 10% of that density; platinum has a density that is about 21 times that of water.



Courtesy of Bureau International des Poids et Mesures. Reproduced with permission of the BIPM.

Figure 1.3.1 The international 1 kg standard of mass, a platinum–iridium cylinder 3.9 cm in height and in diameter.

Table 1.3.1 Some Approximate Masses

Object	Mass in Kilograms
Known universe	1×10^{53}
Our galaxy	2×10^{41}
Sun	2×10^{30}
Moon	7×10^{22}
Asteroid Eros	5×10^{15}
Small mountain	1×10^{12}
Ocean liner	7×10^7
Elephant	5×10^3
Grape	3×10^{-3}
Speck of dust	7×10^{-10}
Penicillin molecule	5×10^{-17}
Uranium atom	4×10^{-25}
Proton	2×10^{-27}
Electron	9×10^{-31}

Review & Summary

Measurement in Physics Physics is based on measurement of physical quantities. Certain physical quantities have been chosen as **base quantities** (such as length, time, and mass); each has been defined in terms of a **standard** and given a **unit** of measure (such as meter, second, and kilogram). Other physical quantities are defined in terms of the base quantities and their standards and units.

SI Units The unit system emphasized in this book is the International System of Units (SI). The three physical quantities displayed in Table 1.1.1 are used in the early chapters. Standards, which must be both accessible and invariable, have been established for these base quantities by international agreement. These standards are used in all physical measurement, for both the base quantities and the quantities derived from them. Scientific notation and the prefixes of Table 1.1.2 are used to simplify measurement notation.

Changing Units Conversion of units may be performed by using *chain-link conversions* in which the original data are

multiplied successively by conversion factors written as unity and the units are manipulated like algebraic quantities until only the desired units remain.

Length The meter is defined as the distance traveled by light during a precisely specified time interval.

Time The second is defined in terms of the oscillations of light emitted by an atomic (cesium-133) source. Accurate time signals are sent worldwide by radio signals keyed to atomic clocks in standardizing laboratories.

Mass The kilogram is defined in terms of a platinum-iridium standard mass kept near Paris. For measurements on an atomic scale, the atomic mass unit, defined in terms of the atom carbon-12, is usually used.

Density The density ρ of a material is the mass per unit volume:

$$\rho = \frac{m}{V} \quad (1.3.2)$$

Problems

Tutoring problem available (at instructor's discretion) in WileyPLUS

Worked-out solution available in Student Solutions Manual

Easy Medium Hard

Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

Requires calculus

Biomedical application

Module 1.1 Measuring Things, Including Lengths

1 E SSM Earth is approximately a sphere of radius 6.37×10^6 m. What are (a) its circumference in kilometers, (b) its surface area in square kilometers, and (c) its volume in cubic kilometers?

2 E A *gry* is an old English measure for length, defined as 1/10 of a line, where *line* is another old English measure for length, defined as 1/12 inch. A common measure for length in the publishing business is a *point*, defined as 1/72 inch. What is an area of 0.50 gry² in points squared (points²)?

3 E The micrometer ($1 \mu\text{m}$) is often called the *micron*. (a) How many microns make up 1.0 km? (b) What fraction of a centimeter equals $1.0 \mu\text{m}$? (c) How many microns are in 1.0 yd?

4 E Spacing in this book was generally done in units of points and picas: 12 points = 1 pica, and 6 picas = 1 inch. If a figure was misplaced in the page proofs by 0.80 cm, what was the misplacement in (a) picas and (b) points?

5 E SSM Horses are to race over a certain English meadow for a distance of 4.0 furlongs. What is the race distance in (a) rods and (b) chains? (1 furlong = 201.168 m, 1 rod = 5.0292 m, and 1 chain = 20.117 m.)

6 M You can easily convert common units and measures electronically, but you still should be able to use a conversion table, such as those in Appendix D. Table 1.1 is part of a conversion table for a system of volume measures once common in Spain; a volume of 1 fanega is equivalent to 55.501 dm³ (cubic decimeters). To complete the table, what numbers (to three significant

Table 1.1 Problem 6

	cahiz	fanega	cuartilla	almude	medio
1 cahiz =	1	12	48	144	288
1 fanega =		1	4	12	24
1 cuartilla =			1	3	6
1 almude =				1	2
1 medio =					1

figures) should be entered in (a) the cahiz column, (b) the fanega column, (c) the cuartilla column, and (d) the almude column, starting with the top blank? Express 7.00 almudes in (e) medios, (f) cahizes, and (g) cubic centimeters (cm³).

7 M Hydraulic engineers in the United States often use, as a unit of volume of water, the *acre-foot*, defined as the volume of water that will cover 1 acre of land to a depth of 1 ft. A severe thunderstorm dumped 2.0 in. of rain in 30 min on a town of area 26 km². What volume of water, in acre-feet, fell on the town?

8 M GO Harvard Bridge, which connects MIT with its fraternities across the Charles River, has a length of 364.4 Smoots plus one ear. The unit of one Smoot is based on the length of Oliver Reed Smoot, Jr., class of 1962, who was carried or dragged length by length across the bridge so that other pledge members of the Lambda Chi Alpha fraternity could mark off (with paint) 1-Smoot lengths along the bridge. The marks have

been repainted biannually by fraternity pledges since the initial measurement, usually during times of traffic congestion so that the police cannot easily interfere. (Presumably, the police were originally upset because the Smoot is not an SI base unit, but these days they seem to have accepted the unit.) Figure 1.1 shows three parallel paths, measured in Smoots (S), Willies (W), and Zeldas (Z). What is the length of 50.0 Smoots in (a) Willies and (b) Zeldas?



Figure 1.1 Problem 8.

9 M Antarctica is roughly semicircular, with a radius of 2000 km (Fig. 1.2). The average thickness of its ice cover is 3000 m. How many cubic centimeters of ice does Antarctica contain? (Ignore the curvature of Earth.)

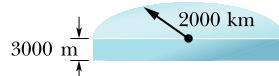


Figure 1.2 Problem 9.

Module 1.2 Time

10 E Until 1883, every city and town in the United States kept its own local time. Today, travelers reset their watches only when the time change equals 1.0 h. How far, on the average, must you travel in degrees of longitude between the time-zone boundaries at which your watch must be reset by 1.0 h? (*Hint:* Earth rotates 360° in about 24 h.)

11 E For about 10 years after the French Revolution, the French government attempted to base measures of time on multiples of ten: One week consisted of 10 days, one day consisted of 10 hours, one hour consisted of 100 minutes, and one minute consisted of 100 seconds. What are the ratios of (a) the French decimal week to the standard week and (b) the French decimal second to the standard second?

12 E The fastest growing plant on record is a *Hesperoyucca whipplei* that grew 3.7 m in 14 days. What was its growth rate in micrometers per second?

13 E Three digital clocks A, B, and C run at different rates and do not have simultaneous readings of zero. Figure 1.3 shows simultaneous readings on pairs of the clocks for four occasions. (At the earliest occasion, for example, B reads 25.0 s and C reads 92.0 s.) If two events are 600 s apart on clock A, how far apart are they on (a) clock B and (b) clock C? (c) When clock A reads 400 s, what does clock B read? (d) When clock C reads 15.0 s, what does clock B read? (Assume negative readings for prezero times.)

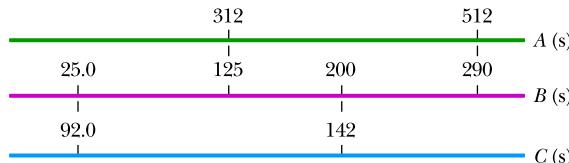


Figure 1.3 Problem 13.

14 E A lecture period (50 min) is close to 1 microcentury. (a) How long is a microcentury in minutes? (b) Using

$$\text{percentage difference} = \left(\frac{\text{actual} - \text{approximation}}{\text{actual}} \right) 100,$$

find the percentage difference from the approximation.

15 E A fortnight is a charming English measure of time equal to 2.0 weeks (the word is a contraction of “fourteen nights”). That is a nice amount of time in pleasant company but perhaps a painful string of microseconds in unpleasant company. How many microseconds are in a fortnight?

16 E Time standards are now based on atomic clocks. A promising second standard is based on *pulsars*, which are rotating neutron stars (highly compact stars consisting only of neutrons). Some rotate at a rate that is highly stable, sending out a radio beacon that sweeps briefly across Earth once with each rotation, like a lighthouse beacon. Pulsar PSR 1937 + 21 is an example; it rotates once every $1.557\ 806\ 448\ 872\ 75 \pm 3$ ms, where the trailing ± 3 indicates the uncertainty in the last decimal place (it does *not* mean ± 3 ms). (a) How many rotations does PSR 1937 + 21 make in 7.00 days? (b) How much time does the pulsar take to rotate exactly one million times and (c) what is the associated uncertainty?

17 E Five clocks are being tested in a laboratory. Exactly at noon, as determined by the WWV time signal, on successive days of a week the clocks read as in the following table. Rank the five clocks according to their relative value as good time-keepers, best to worst. Justify your choice.

Clock	Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
A	12:36:40	12:36:56	12:37:12	12:37:27	12:37:44	12:37:59	12:38:14
B	11:59:59	12:00:02	11:59:57	12:00:07	12:00:02	11:59:56	12:00:03
C	15:50:45	15:51:43	15:52:41	15:53:39	15:54:37	15:55:35	15:56:33
D	12:03:59	12:02:52	12:01:45	12:00:38	11:59:31	11:58:24	11:57:17
E	12:03:59	12:02:49	12:01:54	12:01:52	12:01:32	12:01:22	12:01:12

18 M Because Earth’s rotation is gradually slowing, the length of each day increases: The day at the end of 1.0 century is 1.0 ms longer than the day at the start of the century. In 20 centuries, what is the total of the daily increases in time?

19 H Suppose that, while lying on a beach near the equator watching the Sun set over a calm ocean, you start a stopwatch just as the top of the Sun disappears. You then stand, elevating your eyes by a height $H = 1.70$ m, and stop the watch when the top of the Sun again disappears. If the elapsed time is $t = 11.1$ s, what is the radius r of Earth?

Module 1.3 Mass

20 E The record for the largest glass bottle was set in 1992 by a team in Millville, New Jersey—they blew a bottle with a volume of 193 U.S. fluid gallons. (a) How much short of 1.0 million cubic centimeters is that? (b) If the bottle were filled with water at the leisurely rate of 1.8 g/min, how long would the filling take? Water has a density of $1000\ \text{kg/m}^3$.

21 E Earth has a mass of 5.98×10^{24} kg. The average mass of the atoms that make up Earth is 40 u. How many atoms are there in Earth?

22 E Gold, which has a density of 19.32 g/cm^3 , is the most ductile metal and can be pressed into a thin leaf or drawn out into a long fiber. (a) If a sample of gold, with a mass of 27.63 g , is pressed into a leaf of $1.000 \mu\text{m}$ thickness, what is the area of the leaf? (b) If, instead, the gold is drawn out into a cylindrical fiber of radius $2.500 \mu\text{m}$, what is the length of the fiber?

23 E SSM (a) Assuming that water has a density of exactly 1 g/cm^3 , find the mass of one cubic meter of water in kilograms. (b) Suppose that it takes 10.0 h to drain a container of 5700 m^3 of water. What is the “mass flow rate,” in kilograms per second, of water from the container?

24 M GO Grains of fine California beach sand are approximately spheres with an average radius of $50 \mu\text{m}$ and are made of silicon dioxide, which has a density of 2600 kg/m^3 . What mass of sand grains would have a total surface area (the total area of all the individual spheres) equal to the surface area of a cube 1.00 m on an edge?

25 M FCP During heavy rain, a section of a mountainside measuring 2.5 km horizontally, 0.80 km up along the slope, and 2.0 m deep slips into a valley in a mud slide. Assume that the mud ends up uniformly distributed over a surface area of the valley measuring $0.40 \text{ km} \times 0.40 \text{ km}$ and that mud has a density of 1900 kg/m^3 . What is the mass of the mud sitting above a 4.0 m^2 area of the valley floor?

26 M One cubic centimeter of a typical cumulus cloud contains 50 to 500 water drops, which have a typical radius of $10 \mu\text{m}$. For that range, give the lower value and the higher value, respectively, for the following. (a) How many cubic meters of water are in a cylindrical cumulus cloud of height 3.0 km and radius 1.0 km ? (b) How many 1-liter pop bottles would that water fill? (c) Water has a density of 1000 kg/m^3 . How much mass does the water in the cloud have?

27 M Iron has a density of 7.87 g/cm^3 , and the mass of an iron atom is $9.27 \times 10^{-26} \text{ kg}$. If the atoms are spherical and tightly packed, (a) what is the volume of an iron atom and (b) what is the distance between the centers of adjacent atoms?

28 M A mole of atoms is 6.02×10^{23} atoms. To the nearest order of magnitude, how many moles of atoms are in a large domestic cat? The masses of a hydrogen atom, an oxygen atom, and a carbon atom are 1.0 u , 16 u , and 12 u , respectively. (*Hint:* Cats are sometimes known to kill a mole.)

29 M On a spending spree in Malaysia, you buy an ox with a weight of 28.9 piculs in the local unit of weights: 1 picul = 100 gins, 1 gin = 16 tahils, 1 tahil = 10 chees, and 1 chee = 10 hoons. The weight of 1 hoon corresponds to a mass of 0.3779 g . When you arrange to ship the ox home to your astonished family, how much mass in kilograms must you declare on the shipping manifest? (*Hint:* Set up multiple chain-link conversions.)

30 M CALC GO Water is poured into a container that has a small leak. The mass m of the water is given as a function of time t by $m = 5.00t^{0.8} - 3.00t + 20.00$, with $t \geq 0$, m in grams, and t in seconds. (a) At what time is the water mass greatest, and (b) what is that greatest mass? In kilograms per minute, what is the rate of mass change at (c) $t = 2.00 \text{ s}$ and (d) $t = 5.00 \text{ s}$?

31 H CALC GO A vertical container with base area measuring $14.0 \text{ cm} \times 17.0 \text{ cm}$ is being filled with identical pieces of candy, each with a volume of 50.0 mm^3 and a mass of 0.0200 g . Assume that the volume of the empty spaces between the candies is

negligible. If the height of the candies in the container increases at the rate of 0.250 cm/s , at what rate (kilograms per minute) does the mass of the candies in the container increase?

Additional Problems

32 In the United States, a doll house has the scale of 1:12 of a real house (that is, each length of the doll house is $\frac{1}{12}$ that of the real house) and a miniature house (a doll house to fit within a doll house) has the scale of 1:144 of a real house. Suppose a real house (Fig. 1.4) has a front length of 20 m , a depth of 12 m , a height of 6.0 m , and a standard sloped roof (vertical triangular faces on the ends) of height 3.0 m . In cubic meters, what are the volumes of the corresponding (a) doll house and (b) miniature house?

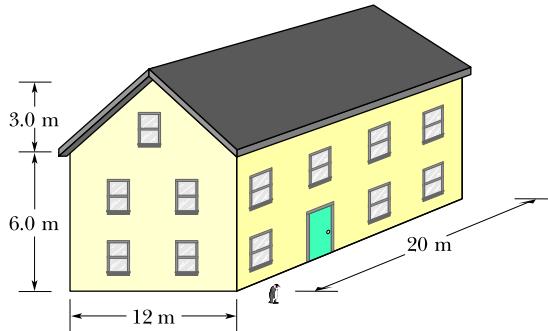


Figure 1.4 Problem 32.

33 SSM A ton is a measure of volume frequently used in shipping, but that use requires some care because there are at least three types of tons: A *displacement ton* is equal to 7 barrels bulk, a *freight ton* is equal to 8 barrels bulk, and a *register ton* is equal to 20 barrels bulk. A *barrel bulk* is another measure of volume: 1 barrel bulk = 0.1415 m^3 . Suppose you spot a shipping order for “73 tons” of M&M candies, and you are certain that the client who sent the order intended “ton” to refer to volume (instead of weight or mass, as discussed in Chapter 5). If the client actually meant displacement tons, how many extra U.S. bushels of the candies will you erroneously ship if you interpret the order as (a) 73 freight tons and (b) 73 register tons? ($1 \text{ m}^3 = 28.378 \text{ U.S. bushels}$)

34 Two types of *barrel* units were in use in the 1920s in the United States. The apple barrel had a legally set volume of 7056 cubic inches; the cranberry barrel, 5826 cubic inches. If a merchant sells 20 cranberry barrels of goods to a customer who thinks he is receiving apple barrels, what is the discrepancy in the shipment volume in liters?

35 An old English children’s rhyme states, “Little Miss Muffet sat on a tuffet, eating her curds and whey, when along came a spider who sat down beside her. . . .” The spider sat down not because of the curds and whey but because Miss Muffet had a stash of 11 tuffets of dried flies. The volume measure of a tuffet is given by 1 tuffet = 2 pecks = 0.50 Imperial bushel, where 1 Imperial bushel = 36.3687 liters (L). What was Miss Muffet’s stash in (a) pecks, (b) Imperial bushels, and (c) liters?

36 Table 1.2 shows some old measures of liquid volume. To complete the table, what numbers (to three significant figures) should be entered in (a) the wey column, (b) the chaldron column, (c) the bag column, (d) the pottle column, and (e) the gill column, starting from the top down? (f) The volume of 1 bag is equal to 0.1091 m^3 . If an old story has a witch cooking up some

vile liquid in a cauldron of volume 1.5 chaldrons, what is the volume in cubic meters?

Table 1.2 Problem 36

	wey	chaldrón	bag	pottle	gill
1 wey =	1	10/9	40/3	640	120 240
1 chaldrón =					
1 bag =					
1 pottle =					
1 gill =					

37 A typical sugar cube has an edge length of 1 cm. If you had a cubical box that contained a mole of sugar cubes, what would its edge length be? (One mole = 6.02×10^{23} units.)

38 An old manuscript reveals that a landowner in the time of King Arthur held 3.00 acres of plowed land plus a livestock area of 25.0 perches by 4.00 perches. What was the total area in (a) the old unit of roods and (b) the more modern unit of square meters? Here, 1 acre is an area of 40 perches by 4 perches, 1 rood is an area of 40 perches by 1 perch, and 1 perch is the length 16.5 ft.

39 SSM A tourist purchases a car in England and ships it home to the United States. The car sticker advertised that the car's fuel consumption was at the rate of 40 miles per gallon on the open road. The tourist does not realize that the U.K. gallon differs from the U.S. gallon:

$$1 \text{ U.K. gallon} = 4.546\,090\,0 \text{ liters}$$

$$1 \text{ U.S. gallon} = 3.785\,411\,8 \text{ liters.}$$

For a trip of 750 miles (in the United States), how many gallons of fuel does (a) the mistaken tourist believe she needs and (b) the car actually require?

40 Using conversions and data in the chapter, determine the number of hydrogen atoms required to obtain 1.0 kg of hydrogen. A hydrogen atom has a mass of 1.0 u.

41 SSM A cord is a volume of cut wood equal to a stack 8 ft long, 4 ft wide, and 4 ft high. How many cords are in 1.0 m³?

42 One molecule of water (H₂O) contains two atoms of hydrogen and one atom of oxygen. A hydrogen atom has a mass of 1.0 u and an atom of oxygen has a mass of 16 u, approximately. (a) What is the mass in kilograms of one molecule of water? (b) How many molecules of water are in the world's oceans, which have an estimated total mass of 1.4×10^{21} kg?

43 A person on a diet might lose 2.3 kg per week. Express the mass loss rate in milligrams per second, as if the dieter could sense the second-by-second loss.

44 What mass of water fell on the town in Problem 7? Water has a density of $1.0 \times 10^3 \text{ kg/m}^3$.

45 (a) A unit of time sometimes used in microscopic physics is the *shake*. One shake equals 10^{-8} s. Are there more shakes in a second than there are seconds in a year? (b) Humans have existed for about 10^6 years, whereas the universe is about 10^{10} years old. If the age of the universe is defined as 1 "universe day," where a universe day consists of "universe seconds" as a normal day consists of normal seconds, how many universe seconds have humans existed?

46 A unit of area often used in measuring land areas is the *hectare*, defined as 10^4 m^2 . An open-pit coal mine consumes 75 hectares of land, down to a depth of 26 m, each year. What volume of earth, in cubic kilometers, is removed in this time?

47 SSM An astronomical unit (AU) is the average distance between Earth and the Sun, approximately 1.50×10^8 km. The speed of light is about $3.0 \times 10^8 \text{ m/s}$. Express the speed of light in astronomical units per minute.

48 The common Eastern mole, a mammal, typically has a mass of 75 g, which corresponds to about 7.5 moles of atoms. (A mole of atoms is 6.02×10^{23} atoms.) In atomic mass units (u), what is the average mass of the atoms in the common Eastern mole?

49 A traditional unit of length in Japan is the ken (1 ken = 1.97 m). What are the ratios of (a) square kens to square meters and (b) cubic kens to cubic meters? What is the volume of a cylindrical water tank of height 5.50 kens and radius 3.00 kens in (c) cubic kens and (d) cubic meters?

50 You receive orders to sail due east for 24.5 mi to put your salvage ship directly over a sunken pirate ship. However, when your divers probe the ocean floor at that location and find no evidence of a ship, you radio back to your source of information, only to discover that the sailing distance was supposed to be 24.5 *nautical miles*, not regular miles. Use the Length table in Appendix D to calculate how far horizontally you are from the pirate ship in kilometers.

51 Density and liquefaction. A heavy object can sink into the ground during an earthquake if the shaking causes the ground to undergo *liquefaction*, in which the soil grains experience little friction as they slide over one another. The ground is then effectively quicksand. The possibility of liquefaction in sandy ground can be predicted in terms of the *void ratio* e for a sample of the ground: $e = V_{\text{voids}}/V_{\text{grains}}$. Here, V_{grains} is the total volume of the sand grains in the sample and V_{voids} is the total volume between the grains (in the voids). If e exceeds a critical value of 0.80, liquefaction can occur during an earthquake. What is the corresponding sand density ρ_{sand} ? Solid silicon dioxide (the primary component of sand) has a density of $\rho_{\text{SiO}_2} = 2.600 \times 10^3 \text{ kg/m}^3$.

52 Billion and trillion. Until 1974, the U.S. and the U.K. used the same names to mean different large numbers. Here are two examples: In American English a billion means a number with 9 zeros after the 1 and in British English it formerly meant a number with 12 zeros after the 1. In American English a trillion means a number with 12 zeros after the 1 and in British English it formerly meant a number with 18 zeros after the 1. In scientific notation with the prefixes in Table 1.1.2, what is 4.0 billion meters in (a) the American use and (b) the former British use? What is 5.0 trillion meters in (c) the American use and (d) the former British use?

53 Townships. In the United States, real estate can be measured in terms of *townships*: 1 township = 36 mi^2 , $1 \text{ mi}^2 = 640 \text{ acres}$, $1 \text{ acre} = 4840 \text{ yd}^2$, $1 \text{ yd}^2 = 9 \text{ ft}^2$. If you own 3.0 townships, how many square feet of real estate do you own?

54 Measures of a man. Leonardo da Vinci, renowned for his understanding of human anatomy, valued the measures of a man stated by Vitruvius Pollio, a Roman architect and engineer of the first century BC: four fingers make one palm, four palms make one foot, six palms make one cubit, and four cubits make a man's height. If we take a finger width to be 0.75 in., what then

are (a) the length of a man's foot and (b) the height of a man, both in centimeters?

55 Dog years. Dog owners like to convert the age of a dog (dubbed *dog years*) to the usual meaning of years to account for the more rapid aging of dogs. One measure of the aging process in both dogs and humans is the rate at which the DNA changes in a process called methylation. Research on that process shows that after the first year, the equivalent age of a dog is given by

$$\text{equivalent age} = 16 \ln(\text{dog years}) + 31,$$

where \ln is the natural logarithm. What then is the equivalent age of a dog on its 13th birthday?

56 Galactic years. The time the Solar System takes to circle around the center of the Milky Way galaxy, a galactic year, is about 230 My. In galactic years, how long ago did (a) the *Tyrannosaurus rex* dinosaurs live (67 My ago), (b) the first major ice age occur (2.2 Gy ago), and (c) Earth form (4.54 Gy ago)?

57 Planck time. The smallest time interval defined in physics is the Planck time $t_P = 5.39 \times 10^{-44}$ s, which is the time required for light to travel across a certain length in a vacuum. The universe began with the big bang 13.772 billion years ago. What is the number of Planck times since that beginning?

58 20,000 Leagues Under the Sea. In Jules Verne's classic science fiction story (published as a serial from 1869 to 1870), Captain Nemo travels in his underwater ship *Nautilus* through the seas of the world for a distance of 20,000 leagues, where a (metric) league is equal to 4.000 km. Assume Earth is spherical with a radius of 6378 km. How many times could Nemo have traveled around Earth?

59 Sea mile. A sea mile is a commonly used measure of distance in navigation but, unlike the *nautical mile*, it does not have a fixed value because it depends on the latitude at which it is measured. It is the distance measured along any given longitude that subtends 1 arc minute, as measured from Earth's center (Fig. 1.5). That distance depends on the radius r of Earth at that point, but because Earth is not a perfect sphere but is wider at the equator and has slightly flattened polar regions, the radius depends on the latitude. At the equator, the radius is 6378 km; at the pole it is 6356 km. What is the difference in a sea mile measured at the equator and at the pole?

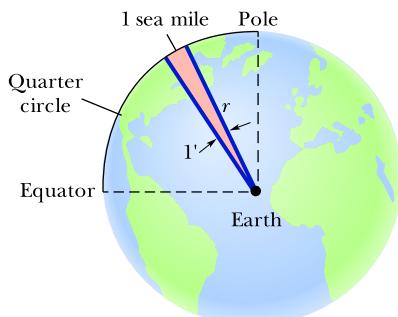


Figure 1.5 Problem 59.

60 Noctilucent clouds. Soon after the huge 1883 volcanic explosion of Krakatoa Island (near Java in the southeast Pacific), silvery, blue clouds began to appear nightly in the Northern Hemisphere early at night. The explosion was so violent that it hurled dust to the *mesosphere*, a cool portion of the atmosphere located well above the stratosphere. There water collected and froze on the dust to form the particles that made the first of these clouds. Termed *noctilucent clouds* ("night shining"), these clouds are now appearing frequently (Fig. 1.6a), signaling a major change in Earth's atmosphere, not because of volcanic explosions, but because of the increased production of methane by industries, rice paddies, landfills, and livestock flatulence.

The clouds are visible after sunset because they are in the upper portion of the atmosphere that is still illuminated by sunlight. Figure 1.6b shows the situation for an observer at point A who sees the clouds overhead 38 min after sunset. The two lines of light are tangent to Earth's surface at A and B , at radius r from Earth's center. Earth rotates through angle θ between the two lines of light. What is the height H of the clouds?



(a)

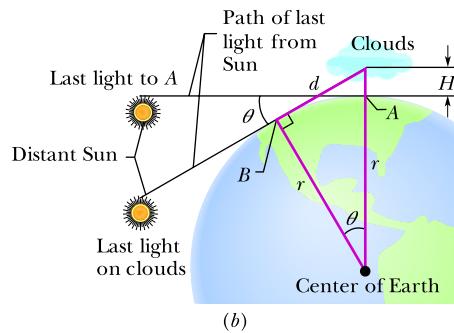


Figure 1.6 Problem 60. (a) Noctilucent clouds. (b) Sunlight reaching the observer and the clouds.

61 Class time, the long of it. For a common four-year undergraduate program, what are the total number of (a) hours and (b) seconds spent in class? Enter your answer in scientific notation.

Noctilucent clouds over the Baltic Sea as viewed from Laboe, Germany, 2019. Source: Matthias Süßen.
Licensed under CC BY-SA 4.0