Conduction of Electricity in Solids

41.1 THE ELECTRICAL PROPERTIES OF METALS

Learning Objectives

After reading this module, you should be able to . . .

41.1.1 Identify the three basic properties of crystalline solids and sketch unit cells for them.

41.1.2 Distinguish insulators, metals, and semiconductors.

41.1.3 With sketches, explain the transition of an energy-level diagram for a single atom to an energy-band diagram for many atoms.

41.1.4 Draw a band–gap diagram for an insulator, indicating the filled and empty bands and explaining what prevents the electrons from participating in a current.

41.1.5 Draw a band–gap diagram for a metal, and explain what feature, in contrast to an insulator, allows electrons to participate in a current.

41.1.6 Identify the Fermi level, Fermi energy, and Fermi speed.

41.1.7 Distinguish monovalent atoms, bivalent atoms, and trivalent atoms.

41.1.8 For a conducting material, apply the relationships between the number density n of conduction electrons and the material's density, volume V, and molar mass M.

41.1.9 Identify that in a metal's partially filled band, thermal agitation can jump some of the conduction electrons to higher energy levels.

41.1.10 For a given energy level in a band, calculate the density of states $N(E)$ and identify that it is actually a double density (per volume and per energy).

41.1.11 Find the number of states per unit volume in a range ΔE at height E in a band by integrating $N(E)$ over that range or, if ΔE is small relative to E, by evaluating the product $N(E)\,\Delta E$.

41.1.12 For a given energy level, calculate the probability $P(E)$ that the level is occupied by electrons.

41.1.13 Identify that probability $P(E)$ is 0.5 at the Fermi level.

41.1.14 At a given energy level, calculate the density $N_o(E)$ of occupied states.

41.1.15 For a given range in energy levels, calculate the number of states and the number of occupied states.

41.1.16 Sketch graphs of the density of states $N(E)$, occupancy probability $P(E)$, and the density of occupied states $N_o(E)$, all versus height in a band.

41.1.17 Apply the relationship between the Fermi energy E_F and the number density of conduction electrons n.

Key Ideas

● Crystalline solids can be broadly divided into insulators, metals, and semiconductors.

● The quantized energy levels for a crystalline solid form bands that are separated by gaps.

● In a metal, the highest band that contains any electrons is only partially filled, and the highest filled level at a temperature of 0 K is called the Fermi level E_F.

● The electrons in the partially filled band are the conduction electrons, and their number density (number per unit volume) is

$$n = \frac{\text{material's density}}{M/N_A},$$

where M is the material's molar mass and N_A is Avogadro's number.

● The number density of states of the allowed energy levels per unit volume and per unit energy interval is

$$N(E) = \frac{8\sqrt{2}\,\pi m^{3/2}}{h^3}E^{1/2},$$

where m is the electron mass and E is the energy *in joules* at which $N(E)$ is to be evaluated.

● The occupancy probability $P(E)$ is the probability that a given available state will be occupied by an electron:
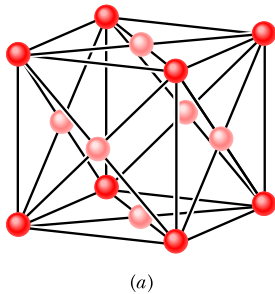
$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1}.$$

● The density of occupied states $N_o(E)$ is given by the product of the density of states function and the occupancy probability function:
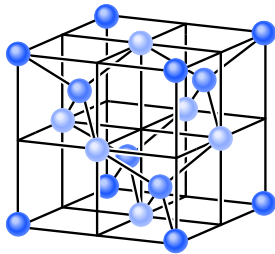
$$N_o(E) = N(E)\, P(E).$$

● The Fermi energy E_F for a metal can be found by integrating $N_o(E)$ for temperature $T = 0$ K (absolute zero) from $E = 0$ to $E = E_F$. The result is

$$E_F = \left(\frac{3}{16\sqrt{2}\pi}\right)^{2/3} \frac{h^2}{m} n^{2/3} = \frac{0.121 h^2}{m} n^{2/3}.$$

(a)

(b)

Figure 41.1.1 (a) The unit cell for copper is a cube. There is one copper atom (darker) at each corner of the cube and one copper atom (lighter) at the center of each face of the cube. The arrangement is called *face-centered cubic.* (b) The unit cell for either silicon or the carbon atoms in diamond is also a cube, the atoms being arranged in what is called a *diamond lattice.* There is one atom (darkest) at each corner of the cube and one atom (lightest) at the center of each cube face; in addition, four atoms (medium color) lie within the cube. Every atom is bonded to its four nearest neighbors by a two-electron covalent bond (only the four atoms within the cube show all four nearest neighbors).

What Is Physics?

A major question in physics, which underlies *solid-state* electronic devices, is this: What are the mechanisms by which a material conducts, or does not conduct, electricity? The answers are complex and poorly understood, largely because they involve the application of quantum physics to a tremendous number of particles and atoms grouped together and interacting. Let's start by characterizing conducting and nonconducting materials.

The Electrical Properties of Solids

We shall examine only **crystalline solids**—that is, solids whose atoms are arranged in a repetitive three-dimensional structure called a **lattice.** We shall not consider such solids as wood, plastic, glass, and rubber, whose atoms are not arranged in such repetitive patterns. Figure 41.1.1 shows the basic repetitive units (the **unit cells**) of the lattice structures of copper, our prototype of a metal, and silicon and diamond (carbon), our prototypes of a semiconductor and an insulator, respectively.

We can classify solids electrically according to three basic properties:

1. Their **resistivity** ρ at room temperature, with the SI unit ohm-meter ($\Omega \cdot$m); resistivity is defined in Module 26.3.
2. Their **temperature coefficient of resistivity** α, defined as $\alpha = (1/\rho)(d\rho/dT)$ in Eq. 26.3.10 and having the SI unit inverse kelvin (K^{-1}). We can evaluate α for any solid by measuring ρ over a range of temperatures.
3. Their **number density of charge carriers** n. This quantity, the number of charge carriers per unit volume, can be found from measurements of the Hall effect, as discussed in Module 28.3, and has the SI unit inverse cubic meter (m^{-3}).

From measurements of resistivity, we find that there are some materials, **insulators**, that do not conduct electricity at all. These are materials with very high resistivity. Diamond, an excellent example, has a resistivity greater than that of copper by the enormous factor of about 10^{24}.

We can then use measurements of ρ, α, and n to divide most noninsulators, at least at low temperatures, into two categories: **metals** and **semiconductors.**

Semiconductors have a considerably greater resistivity ρ than metals.

Semiconductors have a temperature coefficient of resistivity α that is both high and negative. That is, the resistivity of a semiconductor *decreases* with temperature, whereas that of a metal *increases.*

Semiconductors have a considerably lower number density of charge carriers n than metals.

Table 41.1.1 shows values of these quantities for copper, our prototype metal, and silicon, our prototype semiconductor.

Now let's consider our central question: *What features make diamond an insulator, copper a metal, and silicon a semiconductor?*

Table 41.1.1 Some Electrical Properties of Two Materials[a]

Property	Unit	Material	
		Copper	Silicon
Type of conductor		Metal	Semiconductor
Resistivity, ρ	$\Omega \cdot m$	2×10^{-8}	3×10^{3}
Temperature coefficient of resistivity, α	K^{-1}	$+4 \times 10^{-3}$	-70×10^{-3}
Number density of charge carriers, n	m^{-3}	9×10^{28}	1×10^{16}

[a]All values are for room temperature.

(a)

(b)
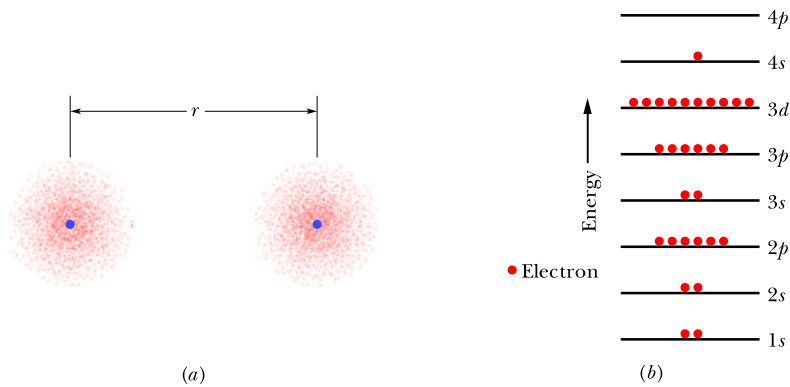
Figure 41.1.2 (*a*) Two copper atoms separated by a large distance; their electron distributions are represented by dot plots. (*b*) Each copper atom has 29 electrons distributed among a set of subshells. In the neutral atom in its ground state, all subshells up through the 3*d* level are filled, the 4*s* subshell contains one electron (it can hold two), and higher subshells are empty. For simplicity, the subshells are shown as being evenly spaced in energy.

Energy Levels in a Crystalline Solid

The distance between adjacent copper atoms in solid copper is 260 pm. Figure 41.1.2*a* shows two isolated copper atoms separated by a distance *r* that is much greater than that. As Fig. 41.1.2*b* shows, each of these isolated neutral atoms stacks up its 29 electrons in an array of discrete subshells as follows:

$$1s^2 \ 2s^2 \ 2p^6 \ 3s^2 \ 3p^6 \ 3d^{10} \ 4s^1.$$

Here we use the shorthand notation of Module 40.5 to identify the subshells. Recall, for example, that the subshell with principal quantum number $n = 3$ and orbital quantum number $\ell = 1$ is called the 3*p* subshell; it can hold up to $2(2\ell + 1) = 6$ electrons; the number it actually contains is indicated by a numerical superscript. We see above that the first six subshells in copper are filled, but the (outermost) 4*s* subshell, which can hold two electrons, holds only one.

If we bring the atoms of Fig. 41.1.2*a* closer, their wave functions begin to overlap, starting with those of the outer electrons. We then have a single two-atom system with 58 electrons, not two independent atoms. The Pauli exclusion principle requires that each of these electrons occupy a different quantum state. In fact, 58 quantum states are available because each energy level of the isolated atom splits into *two* levels for the two-atom system.

If we bring up more atoms, we gradually assemble a lattice of solid copper. For *N* atoms, each level of an isolated copper atom must split into *N* levels in the solid. Thus, the individual energy levels of the solid form energy **bands,** adjacent bands being separated by an energy **gap,** with the gap representing a range of energies that no electron can possess. A typical band ranges over only a few electron-volts. Since *N* may be of the order of 10^{24}, the individual levels within a band are very close together indeed, and there are a vast number of levels.

Figure 41.1.3 suggests the band–gap structure of the energy levels in a generalized crystalline solid. Note that bands of lower energy are narrower than those of higher energy. This occurs because electrons that occupy the lower energy
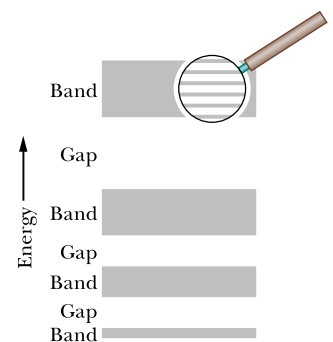
Figure 41.1.3 The band–gap pattern of energy levels for an idealized crystalline solid. As the magnified view suggests, each band consists of a very large number of very closely spaced energy levels. (In many solids, adjacent bands may overlap; for clarity, this condition is not shown.)

In an insulator, electrons need a big energy jump.
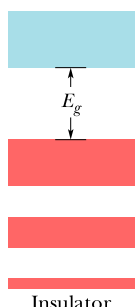
Figure 41.1.4 The band–gap pattern for an insulator; filled levels are shown in red and empty levels in blue.

bands spend most of their time deep within the atom's electron cloud. The wave functions of these core electrons do not overlap as much as the wave functions of the outer electrons do. Hence the splitting of the lower energy levels (core electrons) is less than that of the higher energy levels (outer electrons).

Insulators

A solid is said to be an electrical insulator if no current exists within it when we apply a potential difference across it. For a current to exist, the kinetic energy of the average electron must increase. In other words, some electrons in the solid must move to a higher energy level. However, as Fig. 41.1.4 shows, in an insulator the highest band containing any electrons is fully occupied. Because the Pauli exclusion principle keeps electrons from moving to occupied levels, no electrons in the solid are allowed to move. Thus, the electrons in the filled band of an insulator have no place to go; they are in gridlock, like a child on a ladder filled with children.

There are plenty of unoccupied levels (or *vacant levels*) in the band above the filled band in Fig. 41.1.4. However, if an electron is to occupy one of those levels, it must acquire enough energy to jump across the substantial energy gap E_g that separates the two bands. In diamond, this gap is so wide (the energy needed to cross it is 5.5 eV, about 140 times the average thermal energy of a free particle at room temperature) that essentially no electron can jump across it. Diamond is thus an electrical insulator, and a very good one.

Sample Problem 41.1.1 Probability of electron excitation in an insulator

Approximately what is the probability that, at room temperature (300 K), an electron at the top of the highest filled band in diamond (an insulator) will jump the energy gap E_g in Fig. 41.1.4? For diamond, E_g is 5.5 eV.

KEY IDEA

In Chapter 40 we used Eq. 40.7.2,

$$\frac{N_x}{N_0} = e^{-(E_x - E_0)/kT}, \tag{41.1.1}$$

to relate the population N_x of atoms at energy level E_x to the population N_0 at energy level E_0, where the atoms are part of a system at temperature T (measured in kelvins); k is the Boltzmann constant (8.62×10^{-5} eV/K). In this chapter we can use Eq. 41.1.1 to *approximate* the probability P that an electron in an insulator will jump the energy gap E_g in Fig. 41.1.4.

Calculations: We first set the energy difference $E_x - E_0$ to E_g. Then the probability P of the jump is approximately equal to the ratio N_x/N_0 of the number of electrons just above the energy gap to the number of electrons just below the gap.

For diamond, the exponent in Eq. 41.1.1 is

$$-\frac{E_g}{kT} = -\frac{5.5 \text{ eV}}{(8.62 \times 10^{-5} \text{ eV/K})(300 \text{ K})} = -213.$$

The required probability is then

$$P = \frac{N_x}{N_0} = e^{-(E_g/kT)} = e^{-213} \approx 3 \times 10^{-93}. \quad \text{(Answer)}$$

This result tells us that approximately 3 electrons out of 10^{93} electrons would jump across the energy gap. Because any diamond stone has fewer than 10^{23} electrons, we see that the probability of the jump is vanishingly small. No wonder diamond is such a good insulator.

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Metals

The feature that defines a metal is that, as Fig. 41.1.5 shows, the highest occupied energy level falls somewhere near the middle of an energy band. If we apply a potential difference across a metal, a current can exist because there are plenty of

vacant levels at nearby higher energies into which electrons (the charge carriers in a metal) can jump. Thus, a metal can conduct electricity because electrons in its highest occupied band can easily move into higher energy levels.

In Module 26.4 we discussed the **free-electron model** of a metal, in which the **conduction electrons** are free to move throughout the volume of the sample like the molecules of a gas in a closed container. We used this model to derive an expression for the resistivity of a metal. Here we use the model to explain the behavior of the conduction electrons in the partially filled band of Fig. 41.1.5. However, we now assume the energies of these electrons to be quantized and the Pauli exclusion principle to hold.

Assuming that the electric potential energy U of a conduction electron is uniform throughout the lattice, let's set $U = 0$ so that the mechanical energy E is entirely kinetic. Then the level at the bottom of the partially filled band of Fig. 41.1.5 corresponds to $E = 0$. The highest occupied level in this band at absolute zero ($T = 0$ K) is called the **Fermi level,** and the energy corresponding to it is called the **Fermi energy** E_F; for copper, $E_F = 7.0$ eV.

The electron speed corresponding to the Fermi energy is called the **Fermi speed** v_F. For copper the Fermi speed is 1.6×10^6 m/s. Thus, all motion does *not* cease at absolute zero; at that temperature—and solely because of the Pauli exclusion principle—the conduction electrons are stacked up in the partially filled band of Fig. 41.1.5 with energies that range from zero to the Fermi energy.

How Many Conduction Electrons Are There?

If we could bring individual atoms together to form a sample of a metal, we would find that the conduction electrons in the metal are the *valence electrons* of the atoms (the electrons in the outermost occupied shells of the atoms). A *monovalent* atom contributes one such electron to the conduction electrons in a metal; a *bivalent* atom contributes two such electrons. Thus, the total number of conduction electrons is

$$\begin{pmatrix} \text{number of conduction} \\ \text{electrons in sample} \end{pmatrix} = \begin{pmatrix} \text{number of atoms} \\ \text{in sample} \end{pmatrix} \begin{pmatrix} \text{number of valence} \\ \text{electrons per atom} \end{pmatrix}. \quad (41.1.2)$$

(In this chapter, we shall write several equations largely in words because the symbols we have previously used for the quantities in them now represent other quantities.) The *number density n* of conduction electrons in a sample is the number of conduction electrons per unit volume:

$$n = \frac{\text{number of conduction electrons in sample}}{\text{sample volume } V}. \quad (41.1.3)$$

We can relate the number of atoms in a sample to various other properties of the sample and to the material making up the sample with the following:

$$\begin{pmatrix} \text{number of atoms} \\ \text{in sample} \end{pmatrix} = \frac{\text{sample mass } M_{sam}}{\text{atomic mass}} = \frac{\text{sample mass } M_{sam}}{(\text{molar mass } M)/N_A}$$
$$= \frac{(\text{material's density})(\text{sample volume } V)}{(\text{molar mass } M)/N_A}, \quad (41.1.4)$$

where the molar mass M is the mass of one mole of the material in the sample and N_A is Avogadro's number (6.02×10^{23} mol^{-1}).

In a conductor, electrons need only a small energy jump.
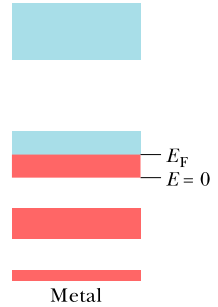
Figure 41.1.5 The band–gap pattern for a metal. The highest filled level, called the Fermi level, lies near the middle of a band. Since vacant levels are available within that band, electrons in the band can easily change levels, and conduction can take place.

Sample Problem 41.1.2   Number of conduction electrons in a metal

How many conduction electrons are in a cube of magnesium of volume $2.00 \times 10^{-6}\,\mathrm{m}^3$? Magnesium atoms are bivalent.

KEY IDEAS

1. Because magnesium atoms are bivalent, each magnesium atom contributes two conduction electrons.
2. The cube's number of conduction electrons is related to its number of magnesium atoms by Eq. 41.1.2.
3. We can find the number of atoms with Eq. 41.1.4 and known data about the cube's volume and magnesium's properties.

Calculations: We can write Eq. 41.1.4 as

$$\begin{pmatrix} \text{number} \\ \text{of atoms} \\ \text{in sample} \end{pmatrix} = \frac{(\text{density})(\text{sample volume } V)N_{\mathrm{A}}}{\text{molar mass } M}.$$

Magnesium has density $1.738\,\mathrm{g/cm}^3$ ($= 1.738 \times 10^3\,\mathrm{kg/m}^3$) and molar mass $24.312\,\mathrm{g/mol}$ ($= 24.312 \times 10^{-3}\,\mathrm{kg/mol}$) (see Appendix F). The numerator gives us

$$(1.738 \times 10^3\,\mathrm{kg/m}^3)(2.00 \times 10^{-6}\,\mathrm{m}^3)$$
$$\times\,(6.02 \times 10^{23}\,\mathrm{atoms/mol}) = 2.0926 \times 10^{21}\,\mathrm{kg/mol}.$$

Thus, $$\begin{pmatrix} \text{number of atoms} \\ \text{in sample} \end{pmatrix} = \frac{2.0926 \times 10^{21}\,\mathrm{kg/mol}}{24.312 \times 10^{-3}\,\mathrm{kg/mol}}$$
$$= 8.61 \times 10^{22}.$$

Using this result and the fact that magnesium atoms are bivalent, we find that Eq. 41.1.2 yields

$$\begin{pmatrix} \text{number of} \\ \text{conduction electrons} \\ \text{in sample} \end{pmatrix}$$
$$= (8.61 \times 10^{22}\,\mathrm{atoms})\left(2\,\frac{\text{electrons}}{\text{atom}}\right)$$
$$= 1.72 \times 10^{23}\,\text{electrons.} \qquad \text{(Answer)}$$

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Conductivity Above Absolute Zero

Our practical interest in the conduction of electricity in metals is at temperatures above absolute zero. What happens to the electron distribution of Fig. 41.1.5 at such higher temperatures? As we shall see, surprisingly little. Of the electrons in the partially filled band of Fig. 41.1.5, only those that are close to the Fermi energy find unoccupied levels above them, and only those electrons are free to be boosted to these higher levels by thermal agitation. Even at $T = 1000\,\mathrm{K}$ (the copper would glow brightly in a dark room), the electron distribution among the available levels does not differ much from the distribution at $T = 0$ K.

Let us see why. The quantity kT, where k is the Boltzmann constant, is a convenient measure of the energy that may be given to a conduction electron by the random thermal motions of the lattice. At $T = 1000\,\mathrm{K}$, we have $kT = 0.086\,\mathrm{eV}$. No electron can hope to have its energy changed by more than a few times this relatively small amount by thermal agitation alone; so at best only those few conduction electrons whose energies are close to the Fermi energy are likely to jump to higher energy levels due to thermal agitation. Poetically stated, thermal agitation normally causes only ripples on the surface of the Fermi sea of electrons; the vast depths of that sea lie undisturbed.

How Many Quantum States Are There?

The ability of a metal to conduct electricity depends on how many quantum states are available to its electrons and what the energies of those states are. Thus, a question arises: What are the energies of the individual states in the partially filled band of Fig. 41.1.5? This question is too difficult to answer because we cannot possibly list the energies of so many states individually. We ask instead: How many states in a unit volume of a sample have energies in the energy range E to $E + dE$? We write this number as $N(E)\,dE$, where $N(E)$ is called the **density**

of states at energy E. The conventional unit for $N(E)\,dE$ is states per cubic meter (states/m^3, or simply m^{-3}), and the conventional unit for $N(E)$ is states per cubic meter per electron-volt (m^{-3} eV^{-1}).

We can find an expression for the density of states by counting the number of standing electron matter waves that can fit into a box the size of the metal sample we are considering. This is analogous to counting the number of standing waves of sound that can exist in a closed organ pipe. Here the problem is three-dimensional (not one-dimensional) and the waves are matter waves (not sound waves). Such counting is covered in more advanced treatments of solid state physics; the result is

$$N(E) = \frac{8\sqrt{2}\,\pi m^{3/2}}{h^3}E^{1/2} \quad \text{(density of states, m}^{-3}\text{ J}^{-1}), \qquad (41.1.5)$$

where m ($= 9.109 \times 10^{-31}$ kg) is the electron mass, h ($= 6.626 \times 10^{-34}$ J·s) is the Planck constant, E is the energy in joules at which $N(E)$ is to be evaluated, and $N(E)$ is in states per cubic meter per joule (m^{-3} J^{-1}). To modify this equation so that the value of E is in electron-volts and the value of $N(E)$ is in states per cubic meter per electron-volt (m^{-3} eV^{-1}), multiply the right side of the equation by $e^{3/2}$, where e is the fundamental charge, 1.602×10^{-19} C. Figure 41.1.6 is a plot of such a modified version of Eq. 41.1.5. Note that nothing in Eq. 41.1.5 or Fig. 41.1.6 involves the shape, temperature, or composition of the sample.

The density of energy levels increases upward in a band.
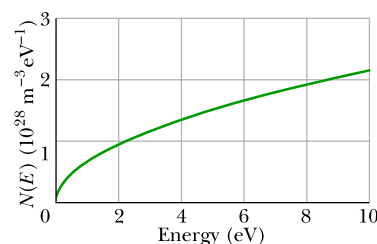
Figure 41.1.6 The density of states $N(E)$—that is, the number of electron energy levels per unit energy interval per unit volume—plotted as a function of electron energy. The density of states function simply counts the available states; it says nothing about whether these states are occupied by electrons.

Checkpoint 41.1.1

Is the spacing between adjacent energy levels at $E = 4$ eV in copper larger than, the same as, or smaller than the spacing at $E = 6$ eV?

Sample Problem 41.1.3 Number of states per electron-volt in a metal

(a) Using the data of Fig. 41.1.6, determine the number of states per electron-volt at 7 eV in a metal sample with a volume V of 2×10^{-9} m^3.

KEY IDEA

We can obtain the number of states per electron-volt at a given energy by using the density of states $N(E)$ at that energy and the sample's volume V.

Calculations: At an energy of 7 eV, we write

$$\begin{pmatrix} \text{number of states} \\ \text{per eV at 7 eV} \end{pmatrix} = \begin{pmatrix} \text{density of states} \\ N(E) \text{ at 7 eV} \end{pmatrix}\begin{pmatrix} \text{volume } V \\ \text{of sample} \end{pmatrix}.$$

From Fig. 41.1.6, we see that at an energy E of 7 eV, the density of states is about 1.8×10^{28} m^{-3} eV^{-1}. Thus,

$$\begin{pmatrix} \text{number of states} \\ \text{per eV at 7 eV} \end{pmatrix} = (1.8 \times 10^{28}\text{ m}^{-3}\text{ eV}^{-1})(2 \times 10^{-9}\text{ m}^3)$$

$$= 3.6 \times 10^{19}\text{ eV}^{-1}$$

$$\approx 4 \times 10^{19}\text{ eV}^{-1}. \qquad \text{(Answer)}$$

(b) Next, determine the number of states N in the sample within a *small* energy range ΔE of 0.003 eV centered at 7 eV (the range is small relative to the energy level in the band).

Calculation: From Eq. 41.1.5 and Fig. 41.1.6, we know that the density of states is a function of energy E. However, for an energy range ΔE that is small relative to E, we can approximate the density of states (and thus the number of states per electron-volt) to be constant. Thus, at an energy of 7 eV, we find the number of states N in the energy range ΔE of 0.003 eV as

$$\begin{pmatrix} \text{number of states } N \\ \text{in range } \Delta E \text{ at 7 eV} \end{pmatrix} = \begin{pmatrix} \text{number of states} \\ \text{per eV at 7 eV} \end{pmatrix}\begin{pmatrix} \text{energy} \\ \text{range } \Delta E \end{pmatrix}$$

or

$$N = (3.6 \times 10^{19}\text{ eV}^{-1})(0.003\text{ eV})$$

$$= 1.1 \times 10^{17} \approx 1 \times 10^{17}. \qquad \text{(Answer)}$$

(When you are asked for the number of states in a certain energy range, first see if that range is small enough to allow this type of approximation.)

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Figure 41.1.7 The occupancy probability $P(E)$ is the probability that an energy level will be occupied by an electron. (a) At $T = 0$ K, $P(E)$ is unity for levels with energies E up to the Fermi energy E_F and zero for levels with higher energies. (b) At $T = 1000$ K, a few electrons whose energies were slightly less than the Fermi energy at $T = 0$ K move up to states with energies slightly greater than the Fermi energy. The dot on the curve shows that, for $E = E_F$, $P(E) = 0.5$.

The occupancy probability is high below the Fermi level.

(a)

(b)
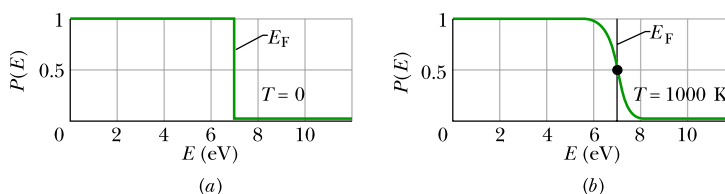
The Occupancy Probability $P(E)$

If an energy level is available at energy E, what is the probability $P(E)$ that it is actually occupied by an electron? At $T = 0$ K, we know that all levels with energies below the Fermi energy are certainly occupied ($P(E) = 1$) and all higher levels are certainly not occupied ($P(E) = 0$). Figure 41.1.7a illustrates this situation. To find $P(E)$ at temperatures above absolute zero, we must use a set of quantum counting rules called **Fermi–Dirac statistics,** named for the physicists who introduced them. With these rules, the **occupancy probability** $P(E)$ is

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1} \quad \text{(occupancy probability)}, \qquad (41.1.6)$$

in which E_F is the Fermi energy. Note that $P(E)$ depends not on the energy E of the level but only on the difference $E - E_F$, which may be positive or negative.

To see whether Eq. 41.1.6 describes Fig. 41.1.7a, we substitute $T = 0$ K in it. Then,

For $E < E_F$, the exponential term in Eq. 41.1.6 is $e^{-\infty}$, or zero; so $P(E) = 1$, in agreement with Fig. 41.1.7a.

For $E > E_F$, the exponential term is $e^{+\infty}$; so $P(E) = 0$, again in agreement with Fig. 41.1.7a.

Figure 41.1.7b is a plot of $P(E)$ for $T = 1000$ K. Compared with Fig. 41.1.7a, it shows that, as stated above, changes in the distribution of electrons among the available states involve only states whose energies are near the Fermi energy E_F. Note that if $E = E_F$ (no matter what the temperature T), the exponential term in Eq. 41.1.6 is $e^0 = 1$ and $P(E) = 0.5$. This leads us to a more useful definition of the Fermi energy:

> The Fermi energy of a given material is the energy of a quantum state that has the probability 0.5 of being occupied by an electron.

Figures 41.1.7a and b are plotted for copper, which has a Fermi energy of 7.0 eV. Thus, for copper both at $T = 0$ K and at $T = 1000$ K, a state at energy $E = 7.0$ eV has a probability of 0.5 of being occupied.

Sample Problem 41.1.4 Probability of occupancy of an energy state in a metal

(a) What is the probability that a quantum state whose energy is 0.10 eV above the Fermi energy will be occupied? Assume a sample temperature of 800 K.

KEY IDEA

The occupancy probability of any state in a metal can be found from Fermi–Dirac statistics according to Eq. 41.1.6.

Calculations: Let's start with the exponent in Eq. 41.1.6:

$$\frac{E - E_F}{kT} = \frac{0.10 \text{ eV}}{(8.62 \times 10^{-5} \text{ eV/K})(800 \text{ K})} = 1.45.$$

Inserting this exponent into Eq. 41.1.6 yields

$$P(E) = \frac{1}{e^{1.45} + 1} = 0.19 \text{ or } 19\%. \quad \text{(Answer)}$$

(b) What is the probability of occupancy for a state that is 0.10 eV *below* the Fermi energy?

Calculation: The key idea of part (a) applies here also except that now the state has an energy *below* the Fermi energy. Thus, the exponent in Eq. 41.1.6 has the same

magnitude we found in part (a) but is negative, and that makes the denominator smaller. Equation 41.1.6 now yields

$$P(E) = \frac{1}{e^{-1.45} + 1} = 0.81 \text{ or } 81\%. \quad \text{(Answer)}$$

For states below the Fermi energy, we are often more interested in the probability that the state is *not* occupied. This probability is just $1 - P(E)$, or 19%. Note that it is the same as the probability of occupancy in (a).

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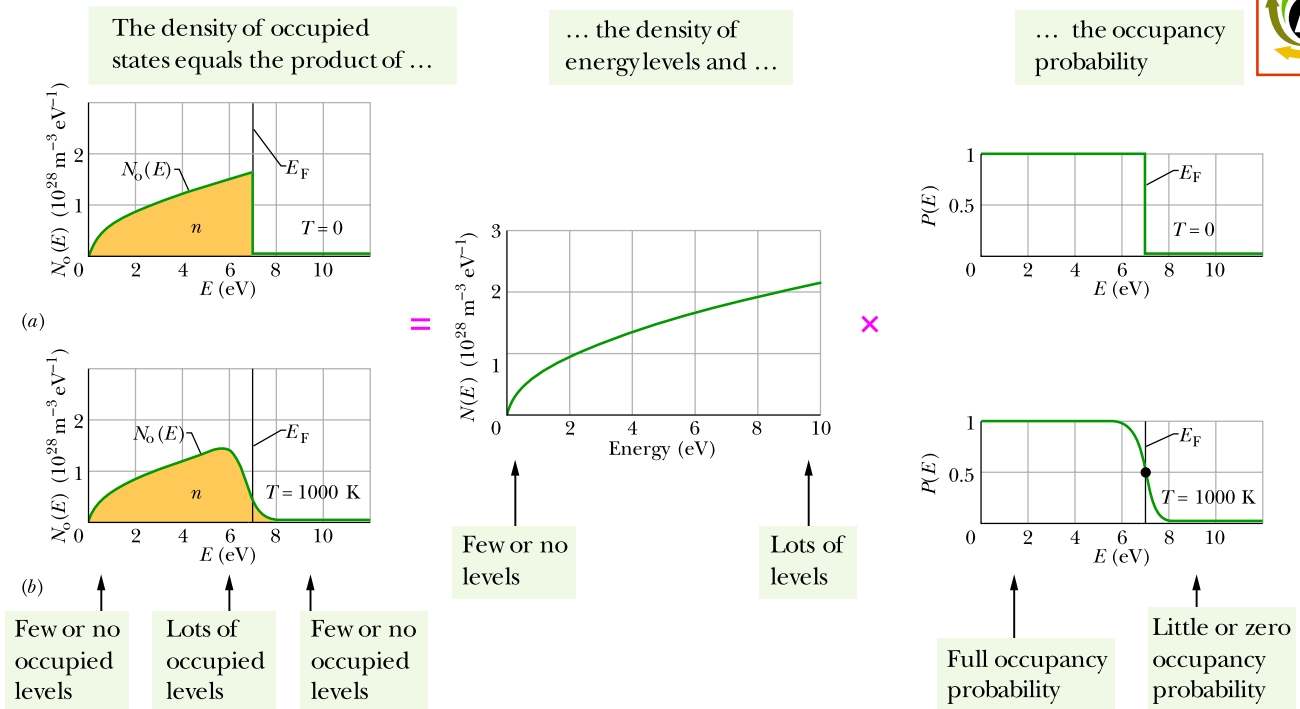
How Many Occupied States Are There?

Equation 41.1.5 and Fig. 41.1.6 tell us how the available states are distributed in energy. The occupancy probability of Eq. 41.1.6 gives us the probability that any given state will actually be occupied by an electron. To find $N_o(E)$, the **density of occupied states,** we must multiply each available state by the corresponding value of the occupancy probability; that is,

$$\begin{pmatrix} \text{density of occupied states} \\ N_o(E) \text{ at energy } E \end{pmatrix} = \begin{pmatrix} \text{density of states} \\ N(E) \text{ at energy } E \end{pmatrix} \begin{pmatrix} \text{occupancy probability} \\ P(E) \text{ at energy } E \end{pmatrix},$$

or

$$N_o(E) = N(E)\,P(E) \quad \text{(density of occupied states).} \quad (41.1.7)$$

For copper at $T = 0$ K, Eq. 41.1.7 tells us to multiply, at each energy, the value of the density of states function (Eq. 41.1.6) by the value of the occupancy probability for absolute zero (Fig. 41.1.7a). The result is Fig. 41.1.8a. Figure 41.1.8b shows the density of occupied states at $T = 1000$ K.

The density of occupied states equals the product of …

… the density of energy levels and …

… the occupancy probability

(a)

(b)

Few or no occupied levels

Lots of occupied levels

Few or no occupied levels

Few or no levels

Lots of levels

Full occupancy probability

Little or zero occupancy probability

Figure 41.1.8 (*a*) The density of occupied states $N_o(E)$ for copper at absolute zero. The area under the curve is the number density of electrons n. Note that all states with energies up to the Fermi energy $E_F = 7$ eV are occupied, and all those with energies above the Fermi energy are vacant. (*b*) The same for copper at $T = 1000$ K. Note that only electrons with energies near the Fermi energy have been affected and redistributed.

Sample Problem 41.1.5   Number of occupied states in an energy range in a metal

A lump of copper (Fermi energy = 7.0 eV) has volume 2×10^{-9} m³. How many occupied states per eV lie in a narrow energy range around 7.0 eV?

KEY IDEAS

(1) First we want the density of occupied states $N_o(E)$ as given by Eq. 41.1.7 ($N_o(E) = N(E)\,P(E)$). (2) Because we want to evaluate quantities for a narrow energy range around 7.0 eV (the Fermi energy for copper), the occupancy probability $P(E)$ is 0.50.

Calculations: From Fig. 41.1.6, we see that the density of states at 7 eV is about 1.8×10^{28} m⁻³ eV⁻¹. Thus, Eq. 41.1.7 tells us that the density of occupied states is

$$N_o(E) = N(E)\,P(E) = (1.8 \times 10^{28}\ \text{m}^{-3}\ \text{eV}^{-1})(0.50)$$
$$= 0.9 \times 10^{28}\ \text{m}^{-3}\ \text{eV}^{-1}.$$

Next, we write

$$\begin{pmatrix} \text{number of } occupied \\ \text{states per eV at 7 eV} \end{pmatrix} = \begin{pmatrix} \text{density of } occupied \\ \text{states } N_o(E) \text{ at 7 eV} \end{pmatrix}$$
$$\times \begin{pmatrix} \text{volume } V \\ \text{of sample} \end{pmatrix}.$$

Substituting for $N_o(E)$ and V gives us

$$\begin{pmatrix} \text{number of occupied} \\ \text{states per eV} \\ \text{at 7 eV} \end{pmatrix} = (0.9 \times 10^{28}\,\text{m}^{-3}\,\text{eV}^{-1})(2 \times 10^{-9}\,\text{m}^3)$$
$$= 1.8 \times 10^{19}\ \text{eV}^{-1}$$
$$\approx 2 \times 10^{19}\ \text{eV}^{-1}. \qquad \text{(Answer)}$$

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Calculating the Fermi Energy

Suppose we add up (via integration) the number of occupied states per unit volume in Fig. 41.1.8*a* (for $T = 0$ K) at all energies between $E = 0$ and $E = E_F$. The result must equal n, the number of conduction electrons per unit volume for the metal, because at that temperature none of the energy states above the Fermi level are occupied. In equation form, we have

$$n = \int_0^{E_F} N_o(E)\, dE. \qquad (41.1.8)$$

(Graphically, the integral here represents the area under the distribution curve of Fig. 41.1.8*a*.) Because $P(E) = 1$ for all energies below the Fermi energy when $T = 0$ K, Eq. 41.1.7 tells us we can replace $N_o(E)$ in Eq. 41.1.8 with $N(E)$ and then use Eq. 41.1.8 to find the Fermi energy E_F. If we substitute Eq. 41.1.5 into Eq. 41.1.8, we find that

$$n = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} \int_0^{E_F} E^{1/2}\, dE = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} \frac{2E_F^{3/2}}{3},$$

in which m is the electron mass. Solving for E_F now leads to

$$E_F = \left(\frac{3}{16\sqrt{2}\pi}\right)^{2/3} \frac{h^2}{m} n^{2/3} = \frac{0.121 h^2}{m} n^{2/3}. \qquad (41.1.9)$$

Thus, when we know n, the number of conduction electrons per unit volume for a metal, we can find the Fermi energy for that metal.

41.2   SEMICONDUCTORS AND DOPING

Learning Objectives

After reading this module, you should be able to . . .

41.2.1 Sketch a band–gap diagram for a semiconductor, identifying the conduction and valence bands, conduction electrons, holes, and the energy gap.

41.2.2 Compare the energy gap of a semiconductor with that of an insulator.

41.2.3 Apply the relationship between a semiconductor's energy gap and the wavelength of light associated with a transition across the gap.

41.2.4 Sketch the lattice structure for pure silicon and doped silicon.

41.2.5 Identify holes, how they are produced, and how they move in an applied electric field.

41.2.6 For metals and semiconductors, compare the resistivity ρ and the temperature coefficient of resistivity α, and explain how the resistivity changes with temperature.

41.2.7 Explain the procedure for producing n-type semiconductors and p-type semiconductors.

41.2.8 Apply the relationship between the number of conduction electrons in a pure material and the number in the doped material.

41.2.9 Identify donors and acceptors and indicate where their energy levels lie in an energy-level diagram.

41.2.10 Identify majority carriers and minority carriers.

41.2.11 Explain the advantage of doping a semiconductor.

Key Ideas

● The band structure of a semiconductor is like that of an insulator except it has a much smaller gap width E_g, which can be jumped by thermally excited electrons.

● In silicon at room temperature, thermal agitation raises a few electrons to the conduction band, leaving an equal number of holes in the valence band. When the silicon is put under a potential difference, both electrons and holes serve as charge carriers.

● The number of electrons in the conduction band of silicon can be increased greatly by doping with small amounts of phosphorus, thus forming n-type material. The phosphorus atoms are said to be donor atoms.

● The number of holes in the valence band of silicon can be greatly increased by doping with small amounts of aluminum, thus forming p-type material. The aluminum atoms are said to be acceptor atoms.

Semiconductors

If you compare Fig. 41.2.1a with Fig. 41.1.4, you can see that the band structure of a semiconductor is like that of an insulator. The main difference is that the semiconductor has a much smaller energy gap E_g between the top of the highest filled band (called the **valence band**) and the bottom of the vacant band just above it (called the **conduction band**). Thus, there is no doubt that silicon ($E_g = 1.1$ eV) is a semiconductor and diamond ($E_g = 5.5$ eV) is an insulator. In silicon—but not in diamond—there is a real possibility that thermal agitation at room temperature will cause electrons to jump the gap from valence to conduction band.

In Table 41.1.1 we compared three basic electrical properties of copper, our prototype metallic conductor, and silicon, our prototype semiconductor. Let us look again at that table, one row at a time, to see how a semiconductor differs from a metal.
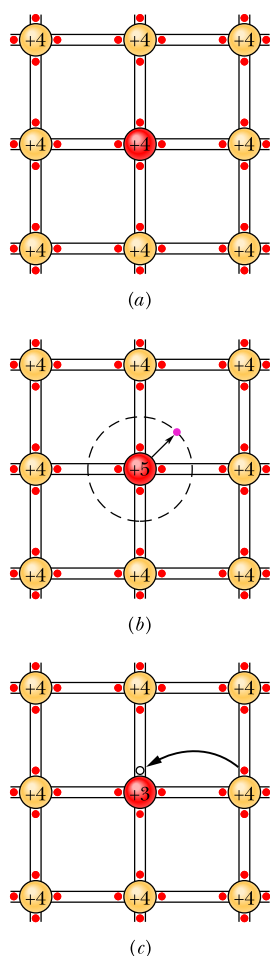
Number Density of Charge Carriers n

The bottom row of Table 41.1.1 shows that copper has far more charge carriers per unit volume than silicon, by a factor of about 10^{13}. For copper, each atom contributes one electron, its single valence electron, to the conduction process. Charge carriers in silicon arise only because, at thermal equilibrium, thermal agitation causes a certain (very small) number of valence-band electrons to jump the energy gap into the conduction band, leaving an equal number of unoccupied energy states, called **holes,** in the valence band. Figure 41.2.1b shows the situation.

Both the electrons in the conduction band and the holes in the valence band serve as charge carriers. The holes do so by permitting a certain freedom of movement to the electrons remaining in the valence band, electrons that, in the absence of holes, would be gridlocked. If an electric field $\vec{E}$ is set up in a semiconductor, the electrons in the valence band, being negatively charged, tend to drift in the direction opposite $\vec{E}$. This causes the positions of the holes to drift in the direction of $\vec{E}$. In effect, the holes behave like moving particles of charge $+e$.

It may help to think of a row of cars parked bumper to bumper, with the lead car at one car's length from a barrier and the empty one-car-length distance being an available parking space. If the leading car moves forward to the barrier, it opens up a parking space behind it. The second car can then move up to fill that space, allowing the third car to move up, and so on. The motions of the many cars toward the barrier are most simply analyzed by focusing attention on the drift of the single "hole" (parking space) away from the barrier.
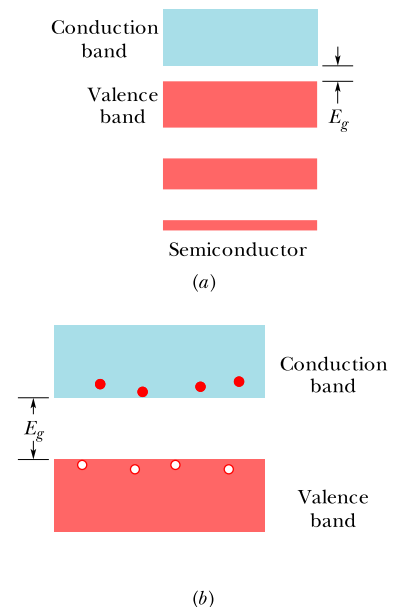
Figure 41.2.1 (a) The band–gap pattern for a semiconductor. It resembles that of an insulator (see Fig. 41.1.4) except that here the energy gap E_g is much smaller; thus electrons, because of their thermal agitation, have some reasonable probability of being able to jump the gap. (b) Thermal agitation has caused a few electrons to jump the gap from the valence band to the conduction band, leaving an equal number of holes in the valence band.

In semiconductors, conduction by holes is just as important as conduction by electrons. In thinking about hole conduction, we can assume that all unoccupied states in the valence band are occupied by particles of charge $+e$ and that all electrons in the valence band have been removed, so that these positive charge carriers can move freely throughout the band.

Resistivity ρ

Recall from Chapter 26 that the resistivity ρ of a material is $m/e^2n\tau$, where m is the electron mass, e is the fundamental charge, n is the number of charge carriers per unit volume, and τ is the mean time between collisions of the charge carriers. Table 41.1.1 shows that, at room temperature, the resistivity of silicon is higher than that of copper by a factor of about 10^{11}. This vast difference can be accounted for by the vast difference in n. Other factors enter, but their effect on the resistivity is swamped by the enormous difference in n.

Temperature Coefficient of Resistivity α

Recall that α (see Eq. 26.3.10) is the fractional change in resistivity per unit change in temperature:

$$\alpha = \frac{1}{\rho}\frac{d\rho}{dT}. \qquad (41.2.1)$$

The resistivity of copper *increases* with temperature (that is, $d\rho/dT > 0$) because collisions of copper's charge carriers occur more frequently at higher temperatures. Thus, α is *positive* for copper.

The collision frequency also increases with temperature for silicon. However, the resistivity of silicon actually *decreases* with temperature ($d\rho/dT < 0$) because the number of charge carriers n (electrons in the conduction band and holes in the valence band) increases so rapidly with temperature. (More electrons jump the gap from the valence band to the conduction band.) Thus, the fractional change α is *negative* for silicon.

Doped Semiconductors

The usefulness of semiconductors in technology can be greatly improved by introducing a small number of suitable replacement atoms (called impurities) into the semiconductor lattice—a process called **doping.** Typically, only about 1 silicon atom in 10^7 is replaced by a dopant atom in the doped semiconductor. Essentially all modern semiconducting devices are based on doped material. Such materials are of two types, called **n-type** and **p-type;** we discuss each in turn.

n-Type Semiconductors

The electrons in an isolated silicon atom are arranged in subshells according to the scheme

$$1s^2 \ 2s^2 \ 2p^6 \ 3s^2 \ 3p^2,$$

in which, as usual, the superscripts (which add to 14, the atomic number of silicon) represent the numbers of electrons in the specified subshells.

Figure 41.2.2*a* is a flattened-out representation of a portion of the lattice of pure silicon in which the portion has been projected onto a plane; compare the figure with Fig. 41.1.1*b*, which represents the unit cell of the lattice in three dimensions. Each silicon atom contributes its pair of 3*s* electrons and its pair of 3*p* electrons to form a rigid two-electron covalent bond with each of its four nearest neighbors. (A covalent bond is a link between two atoms in which the atoms share a pair of electrons.) The four atoms that lie within the unit cell in Fig. 41.1.1*b* show these four bonds.

Figure 41.2.2 (*a*) A flattened-out representation of the lattice structure of pure silicon. Each silicon ion is coupled to its four nearest neighbors by a two-electron covalent bond (represented by a pair of red dots between two parallel black lines). The electrons belong to the bond—not to the individual atoms—and form the valence band of the sample. (*b*) One silicon atom is replaced by a phosphorus atom (valence = 5). The "extra" electron is only loosely bound to its ion core and may easily be elevated to the conduction band, where it is free to wander through the volume of the lattice. (*c*) One silicon atom is replaced by an aluminum atom (valence = 3). There is now a hole in one of the covalent bonds and thus in the valence band of the sample. The hole can easily migrate through the lattice as electrons from neighboring bonds move in to fill it. Here the hole migrates rightward.

The electrons that form the silicon–silicon bonds constitute the valence band of the silicon sample. If an electron is torn from one of these bonds so that it becomes free to wander throughout the lattice, we say that the electron has been raised from the valence band to the conduction band. The minimum energy required to do this is the gap energy E_g.

Because four of its electrons are involved in bonds, each silicon "atom" is actually an ion consisting of an inert neon-like electron cloud (containing 10 electrons) surrounding a nucleus whose charge is $+14e$, where 14 is the atomic number of silicon. The net charge of each of these ions is thus $+4e$, and the ions are said to have a *valence number* of 4.

In Fig. 41.2.2*b* the central silicon ion has been replaced by an atom of phosphorus (valence = 5). Four of the valence electrons of the phosphorus form bonds with the four surrounding silicon ions. The fifth ("extra") electron is only loosely bound to the phosphorus ion core. On an energy-band diagram, we usually say that such an electron occupies a localized energy state that lies within the energy gap, at an average energy interval E_d below the bottom of the conduction band; this is indicated in Fig. 41.2.3*a*. Because $E_d \ll E_g$, the energy required to excite electrons from *these* levels into the conduction band is much less than that required to excite silicon valence electrons into the conduction band.

The phosphorus atom is called a **donor** atom because it readily *donates* an electron to the conduction band. In fact, at room temperature virtually *all* the electrons contributed by the donor atoms are in the conduction band. By adding donor atoms, it is possible to increase greatly the number of electrons in the conduction band, by a factor very much larger than Fig. 41.2.3*a* suggests.
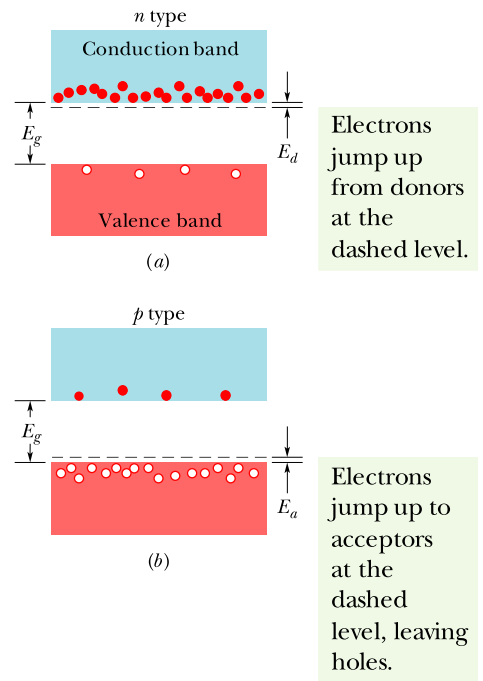
Semiconductors doped with donor atoms are called **n-type semiconductors;** the *n* stands for *negative,* to imply that the negative charge carriers introduced into the conduction band greatly outnumber the positive charge carriers, which are the holes in the valence band. In *n*-type semiconductors, the electrons are called the **majority carriers** and the holes are called the **minority carriers.**

p-Type Semiconductors

Now consider Fig. 41.2.2*c*, in which one of the silicon atoms (valence = 4) has been replaced by an atom of aluminum (valence = 3). The aluminum atom can bond covalently with only three silicon atoms, and so there is now a "missing" electron (a hole) in one aluminum–silicon bond. With a small expenditure of energy, an electron can be torn from a neighboring silicon–silicon bond to fill this hole, thereby creating a hole in *that* bond. Similarly, an electron from some other bond can be moved to fill the newly created hole. In this way, the hole can migrate through the lattice.

The aluminum atom is called an **acceptor** atom because it readily *accepts* an electron from a neighboring bond—that is, from the valence band of silicon. As Fig. 41.2.3*b* suggests, this electron occupies a localized energy state that lies within the energy gap, at an average energy interval E_a above the top of the valence band. Because this energy interval E_a is small, valence electrons are easily bumped up to the acceptor level, leaving holes in the valence band. Thus, by adding acceptor atoms, it is possible to greatly increase the number of holes in the valence band, by a factor much larger than Fig. 41.2.3*b* suggests. In silicon at room temperature, virtually *all* the acceptor levels are occupied by electrons.

Semiconductors doped with acceptor atoms are called **p-type semiconductors;** the *p* stands for *positive* to imply that the holes introduced into the valence band, which behave like positive charge carriers, greatly outnumber the electrons in the conduction band. In *p*-type semiconductors, holes are the majority carriers and electrons are the minority carriers.

Electrons jump up from donors at the dashed level.

Electrons jump up to acceptors at the dashed level, leaving holes.

Figure 41.2.3 (*a*) In a doped *n*-type semiconductor, the energy levels of donor electrons lie a small interval E_d below the bottom of the conduction band. Because donor electrons can be easily excited to the conduction band, there are now many more electrons in that band. The valence band contains the same small number of holes as before the dopant was added. (*b*) In a doped *p*-type semiconductor, the acceptor levels lie a small energy interval E_a above the top of the valence band. There are now many more holes in the valence band. The conduction band contains the same small number of electrons as before the dopant was added. The ratio of majority carriers to minority carriers in both (*a*) and (*b*) is very much greater than is suggested by these diagrams.

Table 41.2.1 Properties of Two Doped Semiconductors

	Type of Semiconductor	
Property	n	p
Matrix material	Silicon	Silicon
Matrix nuclear charge	$+14e$	$+14e$
Matrix energy gap	1.2 eV	1.2 eV
Dopant	Phosphorus	Aluminum
Type of dopant	Donor	Acceptor
Majority carriers	Electrons	Holes
Minority carriers	Holes	Electrons
Dopant energy gap	$E_d = 0.045$ eV	$E_a = 0.067$ eV
Dopant valence	5	3
Dopant nuclear charge	$+15e$	$+13e$
Dopant net ion charge	$+e$	$-e$

Table 41.2.1 summarizes the properties of a typical n-type and a typical p-type semiconductor. Note particularly that the donor and acceptor ion cores, although they are charged, are not charge *carriers* because they are fixed in place.

Sample Problem 41.2.1   Doping silicon with phosphorus

The number density n_0 of conduction electrons in pure silicon at room temperature is about 10^{16} m^{-3}. Assume that, by doping the silicon lattice with phosphorus, we want to increase this number by a factor of a million (10^6). What fraction of silicon atoms must we replace with phosphorus atoms? (Recall that at room temperature, thermal agitation is so effective that essentially every phosphorus atom donates its "extra" electron to the conduction band.)

Number of phosphorus atoms: Because each phosphorus atom contributes one conduction electron and because we want the total number density of conduction electrons to be $10^6 n_0$, the number density of phosphorus atoms n_P must be given by

$$10^6 n_0 = n_0 + n_P.$$

Then $n_P = 10^6 n_0 - n_0 \approx 10^6 n_0$
$$= (10^6)(10^{16} \text{ m}^{-3}) = 10^{22} \text{ m}^{-3}.$$

This tells us that we must add 10^{22} atoms of phosphorus per cubic meter of silicon.

Fraction of silicon atoms: We can find the number density n_{Si} of silicon atoms in pure silicon (before the doping) from Eq. 41.1.4, which we can write as

$$\begin{pmatrix} \text{number of atoms} \\ \text{in sample} \end{pmatrix}$$
$$= \frac{(\text{silicon density})(\text{sample volume } V)}{(\text{silicon molar mass } M_{Si})/N_A}.$$

Dividing both sides by the sample volume V to get the number density of silicon atoms n_{Si} on the left, we then have

$$n_{Si} = \frac{(\text{silicon density})N_A}{M_{Si}}.$$

Appendix F tells us that the density of silicon is 2.33 g/cm^3 ($= 2330$ kg/m^3) and the molar mass of silicon is 28.1 g/mol ($= 0.0281$ kg/mol). Thus, we have

$$n_{Si} = \frac{(2330 \text{ kg/m}^3)(6.02 \times 10^{23} \text{ atoms/mol})}{0.0281 \text{ kg/mol}}$$
$$= 5 \times 10^{28} \text{ atoms/m}^3 = 5 \times 10^{28} \text{ m}^{-3}.$$

The fraction we seek is approximately

$$\frac{n_P}{n_{Si}} = \frac{10^{22} \text{ m}^{-3}}{5 \times 10^{28} \text{ m}^{-3}} = \frac{1}{5 \times 10^6}. \qquad \text{(Answer)}$$

If we replace only *one silicon atom in five million* with a phosphorus atom, the number of electrons in the conduction band will be increased by a factor of a million.

How can such a tiny admixture of phosphorus have what seems to be such a big effect? The answer is that, although the effect is very significant, it is not "big." The number density of conduction electrons was 10^{16} m^{-3} before doping and 10^{22} m^{-3} after doping. For copper, however, the conduction-electron number density (given in Table 41.1.1) is about 10^{29} m^{-3}. Thus, even after doping, the number density of conduction electrons in silicon remains much less than that of a typical metal, such as copper, by a factor of about 10^7.

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41.3 THE *p-n* JUNCTION AND THE TRANSISTOR

Learning Objectives

After reading this module, you should be able to . . .

41.3.1 Describe a *p-n* junction and outline how it works.

41.3.2 Identify diffusion current, space charge, depletion zone, contact potential difference, and drift current.

41.3.3 Describe the functioning of a junction rectifier.

41.3.4 Distinguish forward bias and back bias.

41.3.5 Explain the general properties of a light-emitting diode, a photodiode, a junction laser, and a MOSFET.

Key Ideas

● A *p-n* junction is a single semiconducting crystal with one end doped to form *p*-type material and the other end doped to form *n*-type material. The two types meet at a junction plane.

● At thermal equilibrium, the following occur at the junction plane: (1) Majority carriers diffuse across the plane, producing a diffusion current I_{diff}. (2) Minority carriers are swept across the plane, forming a drift current I_{drift}. (3) A depletion zone forms at the plane. (4) A contact potential V_0 develops across the depletion zone.

● A *p-n* junction conducts electricity better for one direction of an applied potential difference (forward biased) than for the opposite direction (back biased), and thus the device can serve as a junction rectifier.

● A *p-n* junction made with certain materials can emit light when forward biased and thus can serve as a light-emitting diode (LED).

● A light-emitting *p-n* junction can also be made to emit stimulated emission and thus can serve as a laser.

The *p-n* Junction

A **p-n junction** (Fig. 41.3.1*a*), essential to most semiconductor devices, is a single semiconductor crystal that has been selectively doped so that one region is *n*-type material and the adjacent region is *p*-type material. Let's assume that the junction has been formed mechanically by jamming together a bar of *n*-type semiconductor and a bar of *p*-type semiconductor. Thus, the transition from one region to the other is perfectly sharp, occurring at a single **junction plane.**

Let us discuss the motions of electrons and holes just after the *n*-type bar and the *p*-type bar, both electrically neutral, have been jammed together to form the junction. We first examine the majority carriers, which are electrons in the *n*-type material and holes in the *p*-type material.

Motions of the Majority Carriers

If you burst a helium-filled balloon, helium atoms will diffuse (spread) outward into the surrounding air. This happens because there are very few helium atoms in normal air. In more formal language, there is a helium *density gradient* at the balloon–air interface (the number density of helium atoms varies across the interface); the helium atoms move so as to reduce the gradient.

In the same way, electrons on the *n* side of Fig. 41.3.1*a* that are close to the junction plane tend to diffuse across it (from right to left in the figure) and into the *p* side, where there are very few free electrons. Similarly, holes on the *p* side that are close to the junction plane tend to diffuse across that plane (from left to right) and into the *n* side, where there are very few holes. The motions of both the electrons and the holes contribute to a **diffusion current** I_{diff}, conventionally directed from left to right as indicated in Fig. 41.3.1*d*.

Recall that the *n*-side is studded throughout with positively charged donor ions, fixed firmly in their lattice sites. Normally, the excess positive charge of each of these ions is compensated electrically by one of the conduction-band electrons. When an *n*-side electron diffuses across the junction plane, however, the diffusion "uncovers" one of these donor ions, thus introducing a fixed

Figure 41.3.1 (*a*) A *p-n* junction. (*b*) Motions of the majority charge carriers across the junction plane uncover a space charge associated with uncompensated donor ions (to the right of the plane) and acceptor ions (to the left). (*c*) Associated with the space charge is a contact potential difference V_0 across d_0. (*d*) The diffusion of majority carriers (both electrons and holes) across the junction plane produces a diffusion current I_{diff}. (In a real *p-n* junction, the boundaries of the depletion zone would not be sharp, as shown here, and the contact potential curve (*c*) would be smooth, with no sharp corners.)
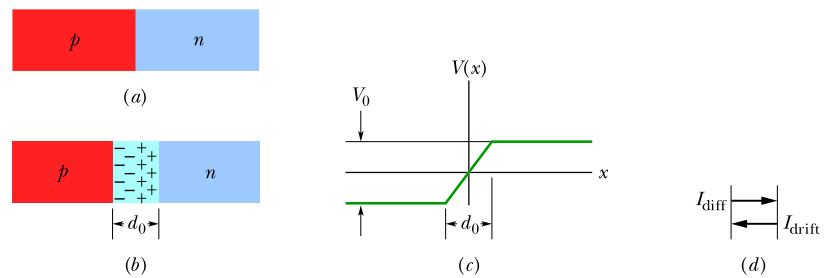
positive charge near the junction plane on the *n* side. When the diffusing electron arrives on the *p* side, it quickly combines with an acceptor ion (which lacks one electron), thus introducing a fixed negative charge near the junction plane on the *p* side.

In this way electrons diffusing through the junction plane from right to left in Fig. 41.3.1*a* result in a buildup of **space charge** on each side of the junction plane, with positive charge on the *n* side and negative charge on the *p* side, as shown in Fig. 41.3.1*b*. Holes diffusing through the junction plane from left to right have exactly the same effect. (Take the time now to convince yourself of that.) The motions of both majority carriers—electrons and holes—contribute to the buildup of these two space charge regions, one positive and one negative. These two regions form a **depletion zone,** so named because it is relatively free of *mobile* charge carriers; its width is shown as d_0 in Fig. 41.3.1*b*.

The buildup of space charge generates an associated **contact potential difference** V_0 across the depletion zone, as Fig. 41.3.1*c* shows. This potential difference limits further diffusion of electrons and holes across the junction plane. Negative charges tend to avoid regions of low potential. Thus, an electron approaching the junction plane from the right in Fig. 41.3.1*b* is moving toward a region of low potential and would tend to turn back into the *n* side. Similarly, a positive charge (a hole) approaching the junction plane from the left is moving toward a region of high potential and would tend to turn back into the *p* side.

Motions of the Minority Carriers

As Fig. 41.2.3*a* shows, although the majority carriers in *n*-type material are electrons, there are a few holes. Likewise in *p*-type material (Fig. 41.2.3*b*), although the majority carriers are holes, there are also a few electrons. These few holes and electrons are the minority carriers in the corresponding materials.

Although the potential difference V_0 in Fig. 41.3.1*c* acts as a barrier for the majority carriers, it is a downhill trip for the minority carriers, be they electrons on the *p* side or holes on the *n* side. Positive charges (holes) tend to seek regions of low potential; negative charges (electrons) tend to seek regions of high potential. Thus, both types of minority carriers are *swept across* the junction plane by the contact potential difference and together constitute a **drift current** I_{drift} across the junction plane from right to left, as Fig. 41.3.1*d* indicates.

Thus, an isolated *p-n* junction is in an equilibrium state in which a contact potential difference V_0 exists between its ends. At equilibrium, the average diffusion current I_{diff} that moves through the junction plane from the *p* side to the *n* side is just balanced by an average drift current I_{drift} that moves in the opposite direction. These two currents cancel because the net current through the junction plane must be zero; otherwise charge would be transferred without limit from one end of the junction to the other.

The Junction Rectifier

Look now at Fig. 41.3.2. It shows that, if we place a potential difference across a *p-n* junction in one direction (here labeled + and "Forward bias"), there will be a current through the junction. However, if we reverse the direction of the potential difference, there will be approximately zero current through the junction.

One application of this property is the **junction rectifier,** whose symbol is shown in Fig. 41.3.3*b*; the arrowhead corresponds to the *p*-type end of the device and points in the allowed direction of conventional current. A sine wave input potential to the device (Fig. 41.3.3*a*) is transformed to a half-wave output potential (Fig. 41.3.3*c*) by the junction rectifier; that is, the rectifier acts as essentially a closed switch (zero resistance) for one polarity of the input potential and as essentially an open switch (infinite resistance) for the other. The average input voltage is zero, but the average output voltage is not. Thus, a junction rectifier can be used as part of an apparatus to convert an alternating potential difference into a constant potential difference, as for a power supply.

Figure 41.3.4 shows why a *p-n* junction operates as a junction rectifier. In Fig. 41.3.4*a*, a battery is connected across the junction with its positive terminal connected at the *p* side. In this **forward-bias connection,** the *p* side becomes more positive and the *n* side becomes more negative, thus *decreasing* the height of the potential barrier V_0 of Fig. 41.3.1*c*. More of the majority carriers can now surmount this smaller barrier; hence, the diffusion current I_{diff} increases markedly.

The minority carriers that form the drift current, however, sense no barrier; so the drift current I_{drift} is not affected by the external battery. The nice current balance that existed at zero bias (see Fig. 41.3.1*d*) is thus upset, and, as shown in Fig. 41.3.4*a*, a large net forward current I_F appears in the circuit.

Another effect of forward bias is to narrow the depletion zone, as a comparison of Fig. 41.3.1*b* and Fig. 41.3.4*a* shows. The depletion zone narrows because the reduced potential barrier associated with forward bias must be associated with a smaller space charge. Because the ions producing the space charge are fixed in their lattice sites, a reduction in their number can come about only through a reduction in the width of the depletion zone.

Because the depletion zone normally contains very few charge carriers, it is normally a region of high resistivity. However, when its width is substantially reduced by a forward bias, its resistance is also reduced substantially, as is consistent with the large forward current.
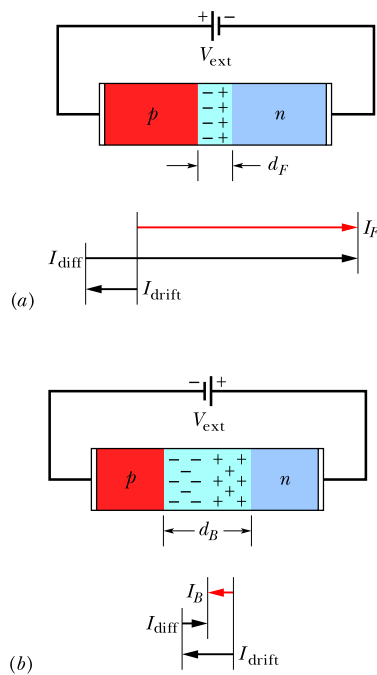
Figure 41.3.4*b* shows the **back-bias** connection, in which the negative terminal of the battery is connected at the *p*-type end of the *p-n* junction. Now the applied emf *increases* the contact potential difference, the diffusion current *decreases* substantially while the drift current remains unchanged, and a relatively *small* back current I_B results. The depletion zone *widens,* its *high* resistance being consistent with the *small* back current I_B.
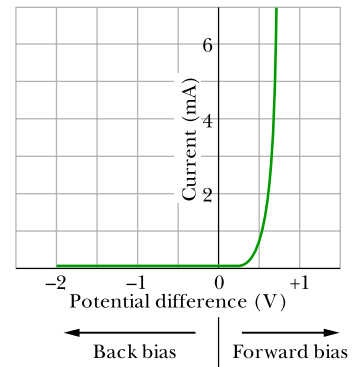

Figure 41.3.2 A current–voltage plot for a *p-n* junction, showing that the junction is highly conducting when forward-biased and essentially nonconducting when back-biased.
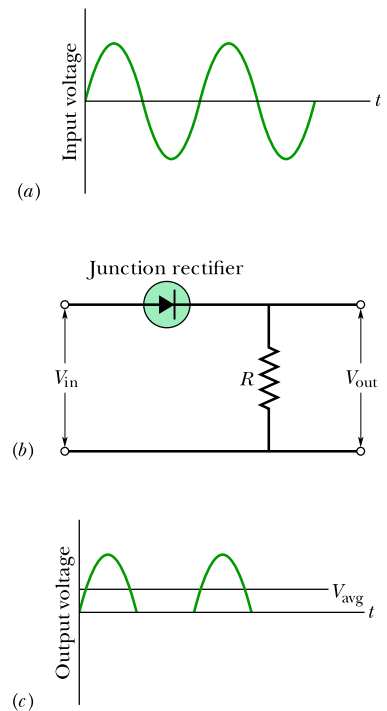
Figure 41.3.3 A *p-n* junction connected as a junction rectifier. The action of the circuit in (*b*) is to pass the positive half of the input wave form in (*a*) but to suppress the negative half. The average potential of the input wave form is zero; that of the output wave form in (*c*) has a positive value V_{avg}.

Figure 41.3.4 (*a*) The forward-bias connection of a *p-n* junction, showing the narrowed depletion zone and the large forward current I_F. (*b*) The back-bias connection, showing the widened depletion zone and the small back current I_B.
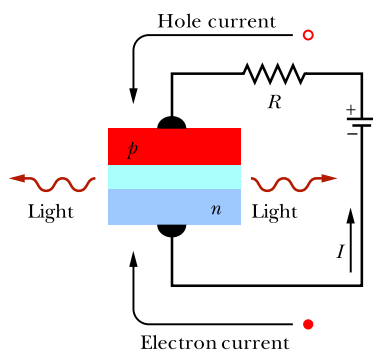
Figure 41.3.5 A forward-biased *p-n* junction, showing electrons being injected into the *n*-type material and holes into the *p*-type material. (Holes move in the conventional direction of the current *I*, equivalent to electrons moving in the opposite direction.) Light is emitted from the narrow depletion zone each time an electron and a hole combine across that zone.

The Light-Emitting Diode (LED)

Nowadays, we can hardly avoid the brightly colored "electronic" numbers that glow at us from cash registers and gasoline pumps, microwave ovens and alarm clocks, and we cannot seem to do without the invisible infrared beams that control elevator doors and operate television sets via remote control. In nearly all cases this light is emitted from a *p-n* junction operating as a **light-emitting diode** (LED). How can a *p-n* junction generate light?

Consider first a simple semiconductor. When an electron from the bottom of the conduction band falls into a hole at the top of the valence band, an energy E_g equal to the gap width is released. In silicon, germanium, and many other semiconductors, this energy is largely transformed into thermal energy of the vibrating lattice, and as a result, no light is emitted.

In some semiconductors, however, including gallium arsenide, the energy can be emitted as a photon of energy hf at wavelength

$$\lambda = \frac{c}{f} = \frac{c}{E_g/h} = \frac{hc}{E_g}. \tag{41.3.1}$$

To emit enough light to be useful as an LED, the material must have a suitably large number of electron–hole transitions. This condition is not satisfied by a pure semiconductor because, at room temperature, there are simply not enough electron–hole pairs. As Fig. 41.2.3 suggests, doping will not help. In doped *n*-type material the number of conduction electrons is greatly increased, but there are not enough holes for them to combine with; in doped *p*-type material there are plenty of holes but not enough electrons to combine with them. Thus, neither a pure semiconductor nor a doped semiconductor can provide enough electron–hole transitions to serve as a practical LED.

What we need is a semiconductor material with a very large number of electrons in the conduction band *and* a correspondingly large number of holes in the valence band. A device with this property can be fabricated by placing a strong forward bias on a heavily doped *p-n* junction, as in Fig. 41.3.5. In such an arrangement the current *I* through the device serves to inject electrons into the *n*-type material and to inject holes into the *p*-type material. If the doping is heavy enough and the current is great enough, the depletion zone can become very narrow, perhaps only a few micrometers wide. The result is a great number density of electrons in the *n*-type material facing a correspondingly great number density of holes in the *p*-type material, across the narrow depletion zone. With such great number densities so near each other, many electron–hole combinations occur, causing light to be emitted from that zone. Figure 41.3.6 shows the construction of an actual LED.

Commercial LEDs designed for the visible region are commonly based on gallium suitably doped with arsenic and phosphorus atoms. An arrangement in which 60% of the nongallium sites are occupied by arsenic ions and 40% by phosphorus ions results in a gap width E_g of about 1.8 eV, corresponding to red light. Other doping and transition-level arrangements make it possible to construct LEDs that emit light in essentially any desired region of the visible and near-visible spectra.

The Photodiode

Passing a current through a suitably arranged *p-n* junction can generate light. The reverse is also true; that is, shining light on a suitably arranged *p-n* junction can produce a current in a circuit that includes the junction. This is the basis for the **photodiode.**

When you click your television remote control, an LED in the device sends out a coded sequence of pulses of infrared light. The receiving device in your television set is an elaboration of the simple (two-terminal) photodiode that not only detects the infrared signals but also amplifies them and transforms them into electrical signals that change the channel or adjust the volume, among other tasks.

Figure 41.3.6 Cross section of an LED (the device has rotational symmetry about the central axis). The *p*-type material, which is thin enough to transmit light, is in the form of a circular disk. A connection is made to the *p*-type material through a circular metal ring that touches the disk at its periphery. The depletion zone between the *n*-type material and the *p*-type material is not shown.

The Junction Laser

In the arrangement of Fig. 41.3.5, there are many electrons in the conduction band of the *n*-type material and many holes in the valence band of the *p*-type material. Thus, there is a **population inversion** for the electrons; that is, there are more electrons in higher energy levels than in lower energy levels. As we discussed in Module 40.7, this can lead to lasing.

When a single electron moves from the conduction band to the valence band, it can release its energy as a photon. This photon can stimulate a second electron to fall into the valence band, producing a second photon by stimulated emission. In this way, if the current through the junction is great enough, a chain reaction of stimulated emission events can occur and laser light can be generated. To bring this about, opposite faces of the *p-n* junction crystal must be flat and parallel, so that light can be reflected back and forth within the crystal. (Recall that in the helium–neon laser of Fig. 40.7.4, a pair of mirrors served this purpose.) Thus, a *p-n* junction can act as a **junction laser,** its light output being highly coherent and much more sharply defined in wavelength than light from an LED.

Junction lasers are built into CD and DVD players, where, by detecting reflections from the rotating disc, they are used to translate microscopic pits in the disc into sound. They are also much used in optical communication systems based on optical fibers. Figure 41.3.7 suggests their tiny scale. Junction lasers are usually designed to operate in the infrared region of the electromagnetic spectrum because optical fibers have two "windows" in that region (at $\lambda = 1.31$ and $1.55\,\mu$m) for which the energy absorption per unit length of the fiber is a minimum.
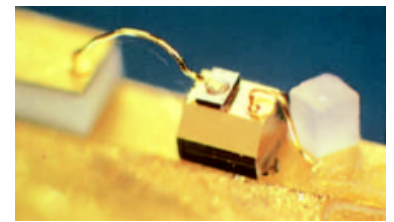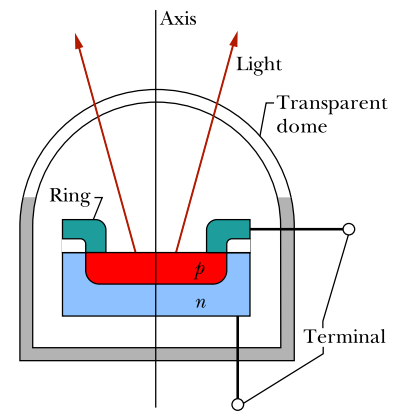
Courtesy of AT&T Archives and History Center, Warren, NJ

Figure 41.3.7 A junction laser developed at the AT&T Bell Laboratories. The cube at the right is a grain of salt.

Sample Problem 41.3.1 **Light-emitting diode (LED)**

An LED is constructed from a *p-n* junction based on a certain Ga-As-P semiconducting material whose energy gap is 1.9 eV. What is the wavelength of the emitted light?

Calculation: For jumps from the bottom of the conduction band to the top of the valence band, Eq. 41.3.1 tells us

$$\lambda = \frac{hc}{E_g} = \frac{(6.63 \times 10^{-34}\,\text{J} \cdot \text{s})(3.00 \times 10^8\,\text{m/s})}{(1.9\,\text{eV})(1.60 \times 10^{-19}\,\text{J/eV})}$$

$$= 6.5 \times 10^{-7}\,\text{m} = 650\,\text{nm}. \qquad \text{(Answer)}$$

Light of this wavelength is red.

WileyPLUS Additional examples, video, and practice available at *WileyPLUS*
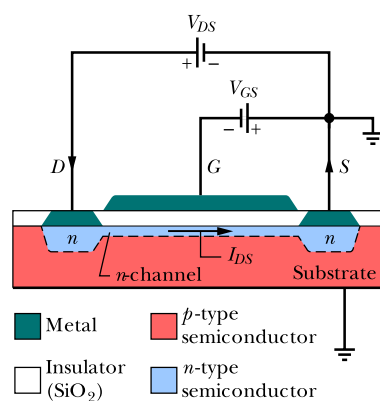

The Transistor

A **transistor** is a three-terminal semiconducting device that can be used to amplify input signals. Figure 41.3.8 shows a generalized **f**ield-**e**ffect **t**ransistor (FET); in it, the flow of electrons from terminal *S* (the *source*) leftward through the shaded region to terminal *D* (the *drain*) can be controlled by an electric field (hence field effect) set up within the device by a suitable electric potential applied to terminal *G* (the *gate*). Transistors are available in many types; we shall discuss only a particular FET called a MOSFET, or **m**etal-**o**xide-**s**emiconductor-**f**ield-**e**ffect **t**ransistor. The MOSFET has been described as the workhorse of the modern electronics industry.

For many applications the MOSFET is operated in only two states: with the drain-to-source current I_{DS} ON (gate open) or with it OFF (gate closed). The first of these can represent a 1 and the other a 0 in the binary arithmetic
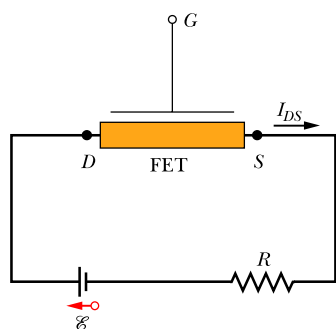
Figure 41.3.8 A circuit containing a generalized field-effect transistor through which electrons flow from the source terminal S to the drain terminal D. (The conventional current I_{DS} is in the opposite direction.) The magnitude of I_{DS} is controlled by the electric field set up within the FET by a potential applied to G, the gate terminal.

Figure 41.3.9 A particular type of field-effect transistor known as a MOSFET. The magnitude of the drain-to-source conventional current I_{DS} through the n channel is controlled by the potential difference V_{GS} applied between the source S and the gate G. A depletion zone that exists between the n-type material and the p-type substrate is not shown.

on which digital logic is based, and therefore MOSFETs can be used in digital logic circuits. Switching between the ON and OFF states can occur at high speed, so that binary logic data can be moved through MOSFET-based circuits very rapidly. MOSFETs about 500 nm in length—about the same as the wavelength of yellow light—are routinely fabricated for use in electronic devices of all kinds.

Figure 41.3.9 shows the basic structure of a MOSFET. A single crystal of silicon or other semiconductor is lightly doped to form p-type material that serves as the *substrate*. Embedded in this substrate, by heavily "overdoping" with n-type dopants, are two "islands" of n-type material, forming the drain D and the source S. The drain and source are connected by a thin channel of n-type material, called the **n channel.** A thin insulating layer of silicon dioxide (hence the O in MOSFET) is deposited on the crystal and penetrated by two metallic terminals (hence the M) at D and S, so that electrical contact can be made with the drain and the source. A thin metallic layer—the gate G—is deposited facing the n channel. Note that the gate makes no electrical contact with the transistor proper, being separated from it by the insulating oxide layer.

Consider first that the source and p-type substrate are grounded (at zero potential) and the gate is "floating"; that is, the gate is not connected to an external source of emf. Let a potential V_{DS} be applied between the drain and the source, such that the drain is positive. Electrons will then flow through the n channel from source to drain, and the conventional current I_{DS}, as shown in Fig. 41.3.9, will be from drain to source through the n channel.

Now let a potential V_{GS} be applied to the gate, making it negative with respect to the source. The negative gate sets up within the device an electric field (hence the "field effect") that tends to repel electrons from the n channel down into the substrate. This electron movement widens the (naturally occurring) depletion zone between the n channel and the substrate, at the expense of the n channel. The reduced width of the n channel, coupled with a reduction in the number of charge carriers in that channel, increases the resistance of that channel and thus decreases the current I_{DS}. With the proper value of V_{GS}, this current can be shut off completely; hence, by controlling V_{GS}, the MOSFET can be switched between its ON and OFF modes.

Charge carriers do not flow through the substrate because it (1) is lightly doped, (2) is not a good conductor, and (3) is separated from the n channel and the two n-type islands by an insulating depletion zone, not specifically shown in Fig. 41.3.9. Such a depletion zone always exists at a boundary between n-type material and p-type material, as Fig. 41.3.1b shows.

Computers and other electronic devices employ thousands (if not millions) of transistors and other electronic components, such as capacitors and resistors. These are not assembled as separate units but are crafted into a single semiconducting **chip,** forming an **integrated circuit** with millions of transistors and many other electronic components.

Review & Summary

Metals, Semiconductors, and Insulators Three electrical properties that can be used to distinguish among crystalline solids are **resistivity** ρ, **temperature coefficient of resistivity** α, and **number density of charge carriers** n. Solids can be broadly divided into **insulators** (very high ρ), **metals** (low ρ, positive and low α, large n), and **semiconductors** (high ρ, negative and high α, small n).

Energy Levels and Gaps in a Crystalline Solid An isolated atom can exist in only a discrete set of energy levels. As atoms come together to form a solid, the levels of the individual atoms merge to form the discrete energy **bands** of the solid. These energy bands are separated by energy **gaps,** each of which corresponds to a range of energies that no electron may possess.

Any energy band is made up of an enormous number of very closely spaced levels. The Pauli exclusion principle asserts that only one electron may occupy each of these levels.

Insulators   In an insulator, the highest band containing electrons is completely filled and is separated from the vacant band above it by an energy gap so large that electrons can essentially never become thermally agitated enough to jump across the gap.

Metals   In a metal, the highest band that contains any electrons is only partially filled. The energy of the highest filled level at a temperature of 0 K is called the **Fermi energy** E_F for the metal.

The electrons in the partially filled band are the **conduction electrons** and their number is

$$\begin{pmatrix} \text{number of conduction} \\ \text{electrons in sample} \end{pmatrix} = \begin{pmatrix} \text{number of atoms} \\ \text{in sample} \end{pmatrix}$$
$$\times \begin{pmatrix} \text{number of valence} \\ \text{electrons per atom} \end{pmatrix}. \quad (41.1.2)$$

The number of atoms in a sample is given by

$$\begin{pmatrix} \text{number of atoms} \\ \text{in sample} \end{pmatrix} = \frac{\text{sample mass } M_{\text{sam}}}{\text{atomic mass}}$$
$$= \frac{\text{sample mass } M_{\text{sam}}}{(\text{molar mass } M)/N_A}$$
$$= \frac{\begin{pmatrix} \text{material's} \\ \text{density} \end{pmatrix} \begin{pmatrix} \text{sample} \\ \text{volume } V \end{pmatrix}}{(\text{molar mass } M)/N_A}. \quad (41.1.4)$$

The number density n of the conduction electrons is

$$n = \frac{\text{number of conduction electrons in sample}}{\text{sample volume } V}. \quad (41.1.3)$$

The **density of states** function $N(E)$ is the number of available energy levels per unit volume of the sample and per unit energy interval and is given by

$$N(E) = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} E^{1/2} \quad \text{(density of states, m}^{-3}\text{ J}^{-1}), \quad (41.1.5)$$

where m ($= 9.109 \times 10^{-31}$ kg) is the electron mass, h ($= 6.626 \times 10^{-34}$ J·s) is the Planck constant, and E is the energy in joules at which $N(E)$ is to be evaluated. To modify the equation so that the value of E is in eV and the value of $N(E)$ is in m^{-3} eV^{-1}, multiply the right side by $e^{3/2}$ (where $e = 1.602 \times 10^{-19}$ C).

The **occupancy probability** $P(E)$, the probability that a given available state will be occupied by an electron, is

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1} \quad \text{(occupancy probability)}. \quad (41.1.6)$$

The **density of occupied states** $N_o(E)$ is given by the product of the two quantities in Eqs. 41.1.5 and 41.1.6:

$$N_o(E) = N(E)\, P(E) \quad \text{(density of occupied states)}. \quad (41.1.7)$$

The Fermi energy for a metal can be found by integrating $N_o(E)$ for $T = 0$ from $E = 0$ to $E = E_F$. The result is

$$E_F = \left(\frac{3}{16\sqrt{2}\pi}\right)^{2/3} \frac{h^2}{m} n^{2/3} = \frac{0.121 h^2}{m} n^{2/3}. \quad (41.1.9)$$

Semiconductors   The band structure of a semiconductor is like that of an insulator except that the gap width E_g is much smaller in the semiconductor. For silicon (a semiconductor) at room temperature, thermal agitation raises a few electrons to the **conduction band,** leaving an equal number of **holes** in the **valence band.** Both electrons and holes serve as charge carriers. The number of electrons in the conduction band of silicon can be increased greatly by doping with small amounts of phosphorus, thus forming **n-type material.** The number of holes in the valence band can be greatly increased by doping with aluminum, thus forming **p-type material.**

The p-n Junction   A **p-n junction** is a single semiconducting crystal with one end doped to form p-type material and the other end doped to form n-type material, the two types meeting at a **junction plane.** At thermal equilibrium, the following occurs at that plane:

The **majority carriers** (electrons on the n side and holes on the p side) diffuse across the junction plane, producing a **diffusion current** I_{diff}.

The **minority carriers** (holes on the n side and electrons on the p side) are swept across the junction plane, forming a **drift current** I_{drift}. These two currents are equal in magnitude, making the net current zero.

A **depletion zone,** consisting largely of charged donor and acceptor ions, forms across the junction plane.

A **contact potential difference** V_0 develops across the depletion zone.

Applications of the p-n Junction   When a potential difference is applied across a *p-n* junction, the device conducts electricity more readily for one polarity of the applied potential difference than for the other. Thus, a *p-n* junction can serve as a **junction rectifier.**

When a *p-n* junction is forward biased, it can emit light, hence can serve as a **light-emitting diode** (LED). The wavelength of the emitted light is given by

$$\lambda = \frac{c}{f} = \frac{hc}{E_g}. \quad (41.3.1)$$

A strongly forward-biased *p-n* junction with parallel end faces can operate as a **junction laser,** emitting light of a sharply defined wavelength.

Questions

1   On which of the following does the interval between adjacent energy levels in the highest occupied band of a metal depend: (a) the material of which the sample is made, (b) the size of the sample, (c) the position of the level in the band, (d) the temperature of the sample, (e) the Fermi energy of the metal?

2   Figure 41.1.1*a* shows 14 atoms that represent the unit cell of copper. However, because each of these atoms is shared with one or more adjoining unit cells, only a fraction of each atom belongs to the unit cell shown. What is the number of atoms per unit cell for copper? (To answer, count up the fractional atoms belonging to a single unit cell.)

3 Figure 41.1.1*b* shows 18 atoms that represent the unit cell of silicon. Fourteen of these atoms, however, are shared with one or more adjoining unit cells. What is the number of atoms per unit cell for silicon? (See Question 2.)

4 Figure 41.1 shows three labeled levels in a band and also the Fermi level for the material. The temperature is 0 K. Rank the three levels according to the probability of occupation, greatest first if the temperature is (a) 0 K and (b) 1000 K. (c) At the latter temperature, rank the levels according to the density of states $N(E)$ there, greatest first.
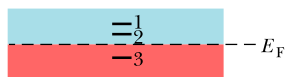
Figure 41.1 Question 4.

5 The occupancy probability at a certain energy E_1 in the valence band of a metal is 0.60 when the temperature is 300 K. Is E_1 above or below the Fermi energy?

6 An isolated atom of germanium has 32 electrons, arranged in subshells according to this scheme:

$$1s^2 \ 2s^2 \ 2p^6 \ 3s^2 \ 3p^6 \ 3d^{10} \ 4s^2 \ 4p^2.$$

This element has the same crystal structure as silicon and, like silicon, is a semiconductor. Which of these electrons form the valence band of crystalline germanium?

7 If the temperature of a piece of a metal is increased, does the probability of occupancy 0.1 eV above the Fermi level increase, decrease, or remain the same?

8 In the biased *p-n* junctions shown in Fig. 41.3.4, there is an electric field $\vec{E}$ in each of the two depletion zones, associated with the potential difference that exists across that zone. (a) Is the electric field vector directed from left to right in the figure or from right to left? (b) Is the magnitude of the field greater for forward bias or for back bias?

9 Consider a copper wire that is carrying, say, a few amperes of current. Is the drift speed v_d of the conduction electrons that form that current about equal to, much greater than, or much less than the Fermi speed v_F for copper (the speed associated with the Fermi energy for copper)?

10 In a silicon lattice, where should you look if you want to find (a) a conduction electron, (b) a valence electron, and (c) an electron associated with the 2*p* subshell of the isolated silicon atom?

11 The energy gaps E_g for the semiconductors silicon and germanium are, respectively, 1.12 and 0.67 eV. Which of the following statements, if any, are true? (a) Both substances have the same number density of charge carriers at room temperature. (b) At room temperature, germanium has a greater number density of charge carriers than silicon. (c) Both substances have a greater number density of conduction electrons than holes. (d) For each substance, the number density of electrons equals that of holes.

Problems

Module 41.1 The Electrical Properties of Metals

1 E Show that Eq. 41.1.9 can be written as $E_F = An^{2/3}$, where the constant A has the value 3.65×10^{-19} m² · eV.

2 E Calculate the density of states $N(E)$ for a metal at energy $E = 8.0$ eV and show that your result is consistent with the curve of Fig. 41.1.6.

3 E Copper, a monovalent metal, has molar mass 63.54 g/mol and density 8.96 g/cm³. What is the number density n of conduction electrons in copper?

4 E A state 63 meV above the Fermi level has a probability of occupancy of 0.090. What is the probability of occupancy for a state 63 meV *below* the Fermi level?

5 E (a) Show that Eq. 41.1.5 can be written as $N(E) = CE^{1/2}$. (b) Evaluate C in terms of meters and electron-volts. (c) Calculate $N(E)$ for $E = 5.00$ eV.

6 E Use Eq. 41.1.9 to verify 7.0 eV as copper's Fermi energy.

7 E SSM What is the probability that a state 0.0620 eV above the Fermi energy will be occupied at (a) $T = 0$ K and (b) $T = 320$ K?

8 E What is the number density of conduction electrons in gold, which is a monovalent metal? Use the molar mass and density provided in Appendix F.

9 M SSM Silver is a monovalent metal. Calculate (a) the number density of conduction electrons, (b) the Fermi energy, (c) the Fermi speed, and (d) the de Broglie wavelength corresponding to this electron speed. See Appendix F for the needed data on silver.

10 M Show that the probability $P(E)$ that an energy level having energy E is not occupied is

$$P(E) = \frac{1}{e^{-\Delta E/kT} + 1},$$

where $\Delta E = E - E_F$.

11 M Calculate $N_o(E)$, the density of occupied states, for copper at $T = 1000$ K for an energy E of (a) 4.00 eV, (b) 6.75 eV, (c) 7.00 eV, (d) 7.25 eV, and (e) 9.00 eV. Compare your results with the graph of Fig. 41.1.8*b*. The Fermi energy for copper is 7.00 eV.

12 M What is the probability that, at a temperature of $T = 300$ K, an electron will jump across the energy gap E_g ($= 5.5$ eV) in a diamond that has a mass equal to the mass of Earth? Use the molar mass of carbon in Appendix F; assume that in diamond there is one valence electron per carbon atom.

13 M GO The Fermi energy for copper is 7.00 eV. For copper at 1000 K, (a) find the energy of the energy level whose probability of being occupied by an electron is 0.900. For this energy,

evaluate (b) the density of states $N(E)$ and (c) the density of occupied states $N_o(E)$.

14 M Assume that the total volume of a metal sample is the sum of the volume occupied by the metal ions making up the lattice and the (separate) volume occupied by the conduction electrons. The density and molar mass of sodium (a metal) are 971 kg/m³ and 23.0 g/mol, respectively; assume the radius of the Na⁺ ion is 98.0 pm. (a) What percent of the volume of a sample of metallic sodium is occupied by its conduction electrons? (b) Carry out the same calculation for copper, which has density, molar mass, and ionic radius of 8960 kg/m³, 63.5 g/mol, and 135 pm, respectively. (c) For which of these metals do you think the conduction electrons behave more like a free-electron gas?

15 M SSM In Eq. 41.1.6 let $E - E_F = \Delta E = 1.00$ eV. (a) At what temperature does the result of using this equation differ by 1.0% from the result of using the classical Boltzmann equation $P(E) = e^{-\Delta E/kT}$ (which is Eq. 41.1.1 with two changes in notation)? (b) At what temperature do the results from these two equations differ by 10%?

16 M Calculate the number density (number per unit volume) for (a) molecules of oxygen gas at 0.0°C and 1.0 atm pressure and (b) conduction electrons in copper. (c) What is the ratio of the latter to the former? What is the average distance between (d) the oxygen molecules and (e) the conduction electrons, assuming this distance is the edge length of a cube with a volume equal to the available volume per particle (molecule or electron)?

17 M The Fermi energy of aluminum is 11.6 eV; its density and molar mass are 2.70 g/cm³ and 27.0 g/mol, respectively. From these data, determine the number of conduction electrons per atom.

18 M GO A sample of a certain metal has a volume of 4.0×10^{-5} m³. The metal has a density of 9.0 g/cm³ and a molar mass of 60 g/mol. The atoms are bivalent. How many conduction electrons (or valence electrons) are in the sample?

19 M The Fermi energy for silver is 5.5 eV. At $T = 0°C$, what are the probabilities that states with the following energies are occupied: (a) 4.4 eV, (b) 5.4 eV, (c) 5.5 eV, (d) 5.6 eV, and (e) 6.4 eV? (f) At what temperature is the probability 0.16 that a state with energy $E = 5.6$ eV is occupied?

20 M GO What is the number of occupied states in the energy range of 0.0300 eV that is centered at a height of 6.10 eV in the valence band if the sample volume is 5.00×10^{-8} m³, the Fermi level is 5.00 eV, and the temperature is 1500 K?

21 M CALC At 1000 K, the fraction of the conduction electrons in a metal that have energies greater than the Fermi energy is equal to the area under the curve of Fig. 41.1.8b beyond E_F divided by the area under the entire curve. It is difficult to find these areas by direct integration. However, an approximation to this fraction at any temperature T is

$$frac = \frac{3kT}{2E_F}.$$

Note that $frac = 0$ for $T = 0$ K, just as we would expect. What is this fraction for copper at (a) 300 K and (b) 1000 K? For copper, $E_F = 7.0$ eV. (c) Check your answers by numerical integration using Eq. 41.1.7.

22 M At what temperature do 1.30% of the conduction electrons in lithium (a metal) have energies greater than the Fermi energy E_F, which is 4.70 eV? (See Problem 21.)

23 M CALC Show that, at $T = 0$ K, the average energy E_{avg} of the conduction electrons in a metal is equal to $\frac{3}{5}E_F$. (*Hint:* By definition of average, $E_{avg} = (1/n) \int E N_o(E) \, dE$, where n is the number density of charge carriers.)

24 M GO A certain material has a molar mass of 20.0 g/mol, a Fermi energy of 5.00 eV, and 2 valence electrons per atom. What is the density (g/cm³)?

25 M (a) Using the result of Problem 23 and 7.00 eV for copper's Fermi energy, determine how much energy would be released by the conduction electrons in a copper coin with mass 3.10 g if we could suddenly turn off the Pauli exclusion principle. (b) For how long would this amount of energy light a 100 W lamp? (*Note:* There is no way to turn off the Pauli principle!)

26 M At $T = 300$ K, how far above the Fermi energy is a state for which the probability of occupation by a conduction electron is 0.10?

27 M Zinc is a bivalent metal. Calculate (a) the number density of conduction electrons, (b) the Fermi energy, (c) the Fermi speed, and (d) the de Broglie wavelength corresponding to this electron speed. See Appendix F for the needed data on zinc.

28 M GO What is the Fermi energy of gold (a monovalent metal with molar mass 197 g/mol and density 19.3 g/cm³)?

29 M Use the result of Problem 23 to calculate the total translational kinetic energy of the conduction electrons in 1.00 cm³ of copper at $T = 0$ K.

30 H GO A certain metal has 1.70×10^{28} conduction electrons per cubic meter. A sample of that metal has a volume of 6.00×10^{-6} m³ and a temperature of 200 K. How many occupied states are in the energy range of 3.20×10^{-20} J that is centered on the energy 4.00×10^{-19} J? (*Caution:* Avoid round-off in the exponential.)

Module 41.2 Semiconductors and Doping

31 E SSM (a) What maximum light wavelength will excite an electron in the valence band of diamond to the conduction band? The energy gap is 5.50 eV. (b) In what part of the electromagnetic spectrum does this wavelength lie?

32 M The compound gallium arsenide is a commonly used semiconductor, having an energy gap E_g of 1.43 eV. Its crystal structure is like that of silicon, except that half the silicon atoms are replaced by gallium atoms and half by arsenic atoms. Draw a flattened-out sketch of the gallium arsenide lattice, following the pattern of Fig. 41.2.2a. What is the net charge of the (a) gallium and (b) arsenic ion core? (c) How many electrons per bond are there? (*Hint:* Consult the periodic table in Appendix G.)

33 M The occupancy probability function (Eq. 41.1.6) can be applied to semiconductors as well as to metals. In semiconductors the Fermi energy is close to the midpoint of the gap between the valence band and the conduction band. For germanium, the gap width is 0.67 eV. What is the probability that (a) a state at the bottom of the conduction band is occupied and (b) a state at the top of the valence band is not occupied? Assume that $T = 290$ K. (*Note:* In a pure semiconductor, the Fermi energy lies symmetrically between the population of conduction electrons and the population of holes and thus is at the center of the gap. There need not be an available state at the location of the Fermi energy.)

34 [M] In a simplified model of an undoped semiconductor, the actual distribution of energy states may be replaced by one in which there are N_v states in the valence band, all these states having the same energy E_v, and N_c states in the conduction band, all these states having the same energy E_c. The number of electrons in the conduction band equals the number of holes in the valence band. (a) Show that this last condition implies that

$$\frac{N_c}{\exp(\Delta E_c/kT)+1} = \frac{N_v}{\exp(\Delta E_v/kT)+1},$$

in which

$$\Delta E_c = E_c - E_F \quad \text{and} \quad \Delta E_v = -(E_v - E_F).$$

(b) If the Fermi level is in the gap between the two bands and its distance from each band is large relative to kT, then the exponentials dominate in the denominators. Under these conditions, show that

$$E_F = \frac{(E_c + E_v)}{2} + \frac{kT\ln(N_v/N_c)}{2}$$

and that, if $N_v \approx N_c$, the Fermi level for the undoped semiconductor is close to the gap's center.

35 [M] [SSM] What mass of phosphorus is needed to dope 1.0 g of silicon so that the number density of conduction electrons in the silicon is increased by a multiply factor of 10^6 from the 10^{16} m^{-3} in pure silicon?

36 [M] [GO] A silicon sample is doped with atoms having donor states 0.110 eV below the bottom of the conduction band. (The energy gap in silicon is 1.11 eV.) If each of these donor states is occupied with a probability of 5.00×10^{-5} at $T = 300$ K, (a) is the Fermi level above or below the top of the silicon valence band and (b) how far above or below? (c) What then is the probability that a state at the bottom of the silicon conduction band is occupied?

37 [M] [GO] Doping changes the Fermi energy of a semiconductor. Consider silicon, with a gap of 1.11 eV between the top of the valence band and the bottom of the conduction band. At 300 K the Fermi level of the pure material is nearly at the midpoint of the gap. Suppose that silicon is doped with donor atoms, each of which has a state 0.15 eV below the bottom of the silicon conduction band, and suppose further that doping raises the Fermi level to 0.11 eV below the bottom of that band (Fig. 41.2). For (a) pure and (b) doped silicon, calculate the probability that a state at the bottom of the silicon conduction band is occupied. (c) Calculate the probability that a state in the doped material (at the donor level) is occupied.
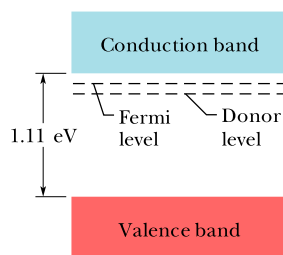
Figure 41.2 Problem 37.

38 [M] Pure silicon at room temperature has an electron number density in the conduction band of about 5×10^{15} m^{-3} and an equal density of holes in the valence band. Suppose that one of every 10^7 silicon atoms is replaced by a phosphorus atom. (a) Which type will the doped semiconductor be, n or p? (b) What charge carrier number density will the phosphorus add? (c) What is the ratio of the charge carrier number density (electrons in the conduction band and holes in the valence band) in the doped silicon to that in pure silicon?

Module 41.3 The *p-n* Junction and the Transistor
39 [E] [SSM] When a photon enters the depletion zone of a *p-n* junction, the photon can scatter from the valence electrons there, transferring part of its energy to each electron, which then jumps to the conduction band. Thus, the photon creates electron–hole pairs. For this reason, the junctions are often used as light detectors, especially in the x-ray and gamma-ray regions of the electromagnetic spectrum. Suppose a single 662 keV gamma-ray photon transfers its energy to electrons in multiple scattering events inside a semiconductor with an energy gap of 1.1 eV, until all the energy is transferred. Assuming that each electron jumps the gap from the top of the valence band to the bottom of the conduction band, find the number of electron–hole pairs created by the process.

40 [E] For an ideal *p-n* junction rectifier with a sharp boundary between its two semiconducting sides, the current I is related to the potential difference V across the rectifier by

$$I = I_0(e^{eV/kT} - 1),$$

where I_0, which depends on the materials but not on I or V, is called the *reverse saturation current*. The potential difference V is positive if the rectifier is forward-biased and negative if it is back-biased. (a) Verify that this expression predicts the behavior of a junction rectifier by graphing I versus V from -0.12 V to $+0.12$ V. Take $T = 300$ K and $I_0 = 5.0$ nA. (b) For the same temperature, calculate the ratio of the current for a 0.50 V forward bias to the current for a 0.50 V back bias.

41 [E] In a particular crystal, the highest occupied band is full. The crystal is transparent to light of wavelengths longer than 295 nm but opaque at shorter wavelengths. Calculate, in electron-volts, the gap between the highest occupied band and the next higher (empty) band for this material.

42 [E] A potassium chloride crystal has an energy band gap of 7.6 eV above the topmost occupied band, which is full. Is this crystal opaque or transparent to light of wavelength 140 nm?

43 [E] A certain computer chip that is about the size of a postage stamp (2.54 cm × 2.22 cm) contains about 3.5 million transistors. If the transistors are square, what must be their *maximum* dimension? (*Note:* Devices other than transistors are also on the chip, and there must be room for the interconnections among the circuit elements. Transistors smaller than 0.7 μm are now commonly and inexpensively fabricated.)

44 [E] A silicon-based MOSFET has a square gate 0.50 μm on edge. The insulating silicon oxide layer that separates the gate from the *p*-type substrate is 0.20 μm thick and has a dielectric constant of 4.5. (a) What is the equivalent gate–substrate capacitance (treating the gate as one plate and the substrate as the other plate)? (b) Approximately how many elementary charges *e* appear in the gate when there is a gate–source potential difference of 1.0 V?

Additional Problems
45 [CALC] [SSM] (a) Show that the slope dP/dE of Eq. 41.1.6 evaluated at $E = E_F$ is $-1/4kT$. (b) Show that the tangent line to the curve of Fig. 41.1.7b evaluated at $E = E_F$ intercepts the horizontal axis at $E = E_F + 2kT$.

46 [CALC] Calculate $d\rho/dT$ at room temperature for (a) copper and (b) silicon, using data from Table 41.1.1.

47 (a) Find the angle θ between adjacent nearest-neighbor bonds in the silicon lattice. Recall that each silicon atom is bonded to four of its nearest neighbors. The four neighbors form a regular tetrahedron—a pyramid whose sides and base are equilateral triangles. (b) Find the bond length, given that the atoms at the corners of the tetrahedron are 388 pm apart.

48 Show that $P(E)$, the occupancy probability in Eq. 41.1.6, is symmetrical about the value of the Fermi energy; that is, show that

$$P(E_F + \Delta E) + P(E_F - \Delta E) = 1.$$

49 (a) Show that the density of states at the Fermi energy is given by

$$N(E_F) = \frac{(4)(3^{1/3})(\pi^{2/3})mn^{1/3}}{h^2}$$
$$= (4.11 \times 10^{18} \text{ m}^{-2} \text{ eV}^{-1})n^{1/3},$$

in which n is the number density of conduction electrons. (b) Calculate $N(E_F)$ for copper, which is a monovalent metal with molar mass 63.54 g/mol and density 8.96 g/cm³. (c) Verify your calculation with the curve of Fig. 41.1.6, recalling that $E_F = 7.0$ eV for copper.

50 Silver melts at 961°C. At the melting point, what fraction of the conduction electrons are in states with energies greater than the Fermi energy of 5.5 eV? (See Problem 21.)

51 The Fermi energy of copper is 7.0 eV. Verify that the corresponding Fermi speed is 1600 km/s.

52 Verify the numerical factor 0.121 in Eq. 41.1.9.

53 At what pressure, in atmospheres, would the number of molecules per unit volume in an ideal gas be equal to the number density of the conduction electrons in copper, with both gas and copper at temperature $T = 300$ K?