

Force and Motion—II

6.1 FRICTION

Learning Objectives

After reading this module, you should be able to . . .

- 6.1.1** Distinguish between friction in a static situation and in a kinetic situation.
- 6.1.2** Determine direction and magnitude of a frictional force.

Key Ideas

● When a force \vec{F} tends to slide a body along a surface, a frictional force from the surface acts on the body. The frictional force is parallel to the surface and directed so as to oppose the sliding. It is due to bonding between the body and the surface.

If the body does not slide, the frictional force is a static frictional force \vec{f}_s . If there is sliding, the frictional force is a kinetic frictional force \vec{f}_k .

● If a body does not move, the static frictional force \vec{f}_s and the component of \vec{F} parallel to the surface are equal in magnitude, and \vec{f}_s is directed opposite that component. If the component increases, f_s also increases.

- 6.1.3** For objects on horizontal, vertical, or inclined planes in situations involving friction, draw free-body diagrams and apply Newton's second law.

● The magnitude of \vec{f}_s has a maximum value $\vec{f}_{s,\max}$ given by

$$f_{s,\max} = \mu_s F_N,$$

where μ_s is the coefficient of static friction and F_N is the magnitude of the normal force. If the component of \vec{F} parallel to the surface exceeds $f_{s,\max}$, the body slides on the surface.

● If the body begins to slide on the surface, the magnitude of the frictional force rapidly decreases to a constant value \vec{f}_k given by

$$f_k = \mu_k F_N,$$

where μ_k is the coefficient of kinetic friction.

What Is Physics?

In this chapter we focus on the physics of three common types of force: frictional force, drag force, and centripetal force. An engineer preparing a car for the Indianapolis 500 must consider all three types. Frictional forces acting on the tires are crucial to the car's acceleration out of the pit and out of a curve (if the car hits an oil slick, the friction is lost and so is the car). Drag forces acting on the car from the passing air must be minimized or else the car will consume too much fuel and have to pit too early (even one 14 s pit stop can cost a driver the race). Centripetal forces are crucial in the turns (if there is insufficient centripetal force, the car slides into the wall). We start our discussion with frictional forces.

Friction

Frictional forces are unavoidable in our daily lives. If we were not able to counteract them, they would stop every moving object and bring to a halt every rotating shaft. About 20% of the gasoline used in an automobile is needed to counteract friction in the engine and in the drive train. On the other hand, if friction were totally absent, we could not get an automobile to go anywhere, and we could not walk or ride a

bicycle. We could not hold a pencil, and, if we could, it would not write. Nails and screws would be useless, woven cloth would fall apart, and knots would untie.

Three Experiments. Here we deal with the frictional forces that exist between dry solid surfaces, either stationary relative to each other or moving across each other at slow speeds. Consider three simple thought experiments:

1. Send a book sliding across a long horizontal counter. As expected, the book slows and then stops. This means the book must have an acceleration parallel to the counter surface, in the direction opposite the book's velocity. From Newton's second law, then, a force must act on the book parallel to the counter surface, in the direction opposite its velocity. That force is a frictional force.
2. Push horizontally on the book to make it travel at constant velocity along the counter. Can the force from you be the only horizontal force on the book? No, because then the book would accelerate. From Newton's second law, there must be a second force, directed opposite your force but with the same magnitude, so that the two forces balance. That second force is a frictional force, directed parallel to the counter.
3. Push horizontally on a heavy crate. The crate does not move. From Newton's second law, a second force must also be acting on the crate to counteract your force. Moreover, this second force must be directed opposite your force and have the same magnitude as your force, so that the two forces balance. That second force is a frictional force. Push even harder. The crate still does not move. Apparently the frictional force can change in magnitude so that the two forces still balance. Now push with all your strength. The crate begins to slide. Evidently, there is a maximum magnitude of the frictional force. When you exceed that maximum magnitude, the crate slides.

Two Types of Friction. Figure 6.1.1 shows a similar situation. In Fig. 6.1.1a, a block rests on a tabletop, with the gravitational force \vec{F}_g balanced by a normal force \vec{F}_N . In Fig. 6.1.1b, you exert a force \vec{F} on the block, attempting to pull it to the left. In response, a frictional force \vec{f}_s is directed to the right, exactly

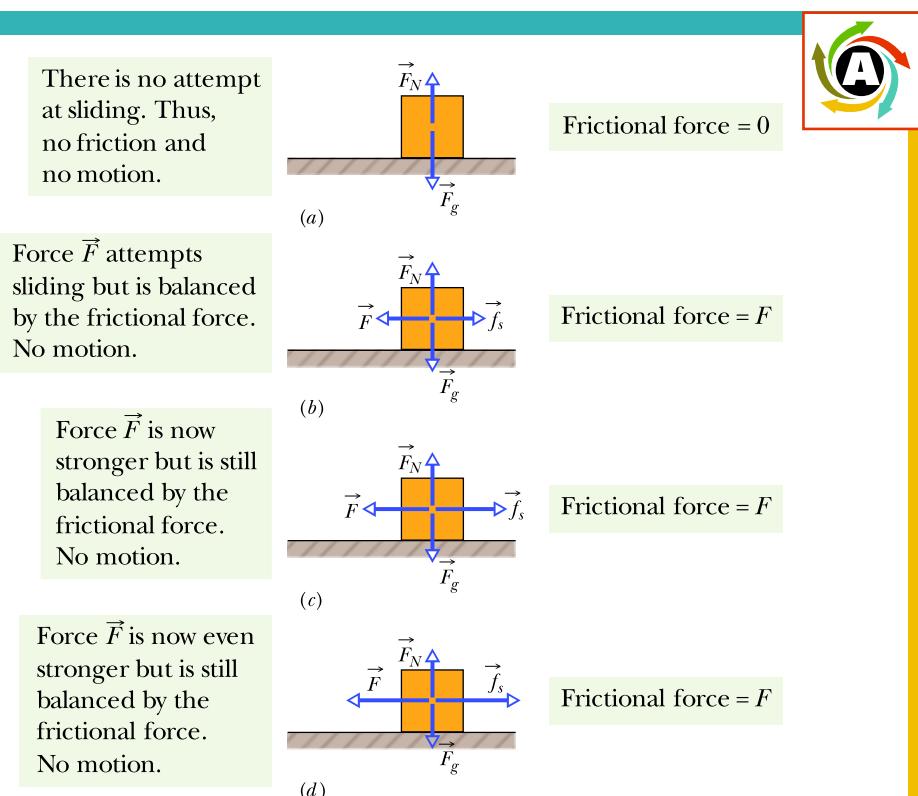
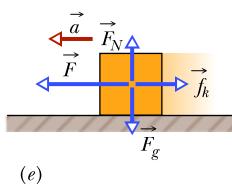


Figure 6.1.1 (a) The forces on a stationary block. (b–d) An external force \vec{F} , applied to the block, is balanced by a static frictional force \vec{f}_s . As F is increased, f_s also increases, until f_s reaches a certain maximum value. (*Figure continues*)

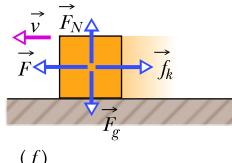
Figure 6.1.1 (Continued) (e) Once f_s reaches its maximum value, the block “breaks away,” accelerating suddenly in the direction of \vec{F} . (f) If the block is now to move with constant velocity, F must be reduced from the maximum value it had just before the block broke away. (g) Some experimental results for the sequence (a) through (f). In WileyPLUS, this figure is available as an animation with voiceover.

Finally, the applied force has overwhelmed the static frictional force. Block slides and accelerates.



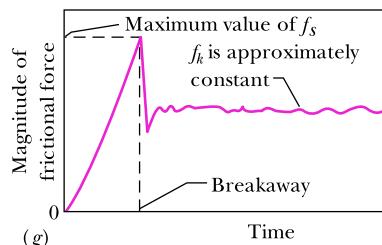
Weak kinetic frictional force

To maintain the speed, weaken force \vec{F} to match the weak frictional force.



Same weak kinetic frictional force

Static frictional force can only match growing applied force.



Kinetic frictional force has only one value (no matching).

balancing your force. The force \vec{f}_s is called the **static frictional force**. The block does not move.

Figures 6.1.1c and 6.1.1d show that as you increase the magnitude of your applied force, the magnitude of the static frictional force \vec{f}_s also increases and the block remains at rest. When the applied force reaches a certain magnitude, however, the block “breaks away” from its intimate contact with the tabletop and accelerates leftward (Fig. 6.1.1e). The frictional force that then opposes the motion is called the **kinetic frictional force** \vec{f}_k .

Usually, the magnitude of the kinetic frictional force, which acts when there is motion, is less than the maximum magnitude of the static frictional force, which acts when there is no motion. Thus, if you wish the block to move across the surface with a constant speed, you must usually decrease the magnitude of the applied force once the block begins to move, as in Fig. 6.1.1f. As an example, Fig. 6.1.1g shows the results of an experiment in which the force on a block was slowly increased until breakaway occurred. Note the reduced force needed to keep the block moving at constant speed after breakaway.

Microscopic View. A frictional force is, in essence, the vector sum of many forces acting between the surface atoms of one body and those of another body. If two highly polished and carefully cleaned metal surfaces are brought together in a very good vacuum (to keep them clean), they cannot be made to slide over each other. Because the surfaces are so smooth, many atoms of one surface contact many atoms of the other surface, and the surfaces *cold-weld* together instantly, forming a single piece of metal. If a machinist’s specially polished gage blocks are brought together in air, there is less atom-to-atom contact, but the blocks stick firmly to each other and can be separated only by means of a wrenching motion. Usually, however, this much atom-to-atom contact is not possible. Even a highly polished metal surface is far from being flat on the atomic scale. Moreover, the surfaces of everyday objects have layers of oxides and other contaminants that reduce cold-welding.

When two ordinary surfaces are placed together, only the high points touch each other. (It is like having the Alps of Switzerland turned over and placed down on the Alps of Austria.) The actual *microscopic* area of contact is much less than the apparent *macroscopic* contact area, perhaps by a factor of 10^4 . Nonetheless,

many contact points do cold-weld together. These welds produce static friction when an applied force attempts to slide the surfaces relative to each other.

If the applied force is great enough to pull one surface across the other, there is first a tearing of welds (at breakaway) and then a continuous re-forming and tearing of welds as movement occurs and chance contacts are made (Fig. 6.1.2). The kinetic frictional force \vec{f}_k that opposes the motion is the vector sum of the forces at those many chance contacts.

If the two surfaces are pressed together harder, many more points cold-weld. Now getting the surfaces to slide relative to each other requires a greater applied force: The static frictional force \vec{f}_s has a greater maximum value. Once the surfaces are sliding, there are many more points of momentary cold-welding, so the kinetic frictional force \vec{f}_k also has a greater magnitude.

Often, the sliding motion of one surface over another is “jerky” because the two surfaces alternately stick together and then slip. Such repetitive *stick-and-slip* can produce squeaking or squealing, as when tires skid on dry pavement, fingernails scratch along a chalkboard, or a rusty hinge is opened. It can also produce beautiful and captivating sounds, as in music when a bow is drawn properly across a violin string. FCP

Properties of Friction

Experiment shows that when a dry and unlubricated body presses against a surface in the same condition and a force \vec{F} attempts to slide the body along the surface, the resulting frictional force has three properties:

Property 1. If the body does not move, then the static frictional force \vec{f}_s and the component of \vec{F} that is parallel to the surface balance each other. They are equal in magnitude, and \vec{f}_s is directed opposite that component of \vec{F} .

Property 2. The magnitude of \vec{f}_s has a maximum value $f_{s,\max}$ that is given by

$$f_{s,\max} = \mu_s F_N, \quad (6.1.1)$$

where μ_s is the **coefficient of static friction** and F_N is the magnitude of the normal force on the body from the surface. If the magnitude of the component of \vec{F} that is parallel to the surface exceeds $f_{s,\max}$, then the body begins to slide along the surface.

Property 3. If the body begins to slide along the surface, the magnitude of the frictional force rapidly decreases to a value f_k given by

$$f_k = \mu_k F_N, \quad (6.1.2)$$

where μ_k is the **coefficient of kinetic friction**. Thereafter, during the sliding, a kinetic frictional force \vec{f}_k with magnitude given by Eq. 6.1.2 opposes the motion.

The magnitude F_N of the normal force appears in properties 2 and 3 as a measure of how firmly the body presses against the surface. If the body presses harder, then, by Newton's third law, F_N is greater. Properties 1 and 2 are worded in terms of a single applied force \vec{F} , but they also hold for the net force of several applied forces acting on the body. Equations 6.1.1 and 6.1.2 are *not* vector equations; the direction of \vec{f}_s or \vec{f}_k is always parallel to the surface and opposed to the attempted sliding, and the normal force \vec{F}_N is perpendicular to the surface.

The coefficients μ_s and μ_k are dimensionless and must be determined experimentally. Their values depend on certain properties of both the body and the surface; hence, they are usually referred to with the preposition “between,” as in “the value of μ_s between an egg and a Teflon-coated skillet is 0.04, but that between rock-climbing shoes and rock is as much as 1.2.” We assume that the value of μ_k does not depend on the speed at which the body slides along the surface.

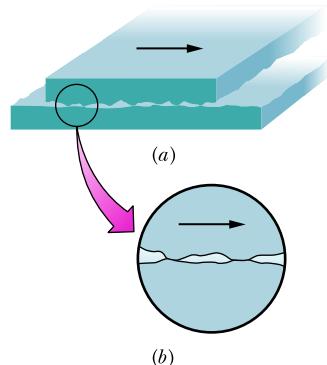


Figure 6.1.2 The mechanism of sliding friction. (a) The upper surface is sliding to the right over the lower surface in this enlarged view. (b) A detail, showing two spots where cold-welding has occurred. Force is required to break the welds and maintain the motion.

Checkpoint 6.1.1

A block lies on a floor. (a) What is the magnitude of the frictional force on it from the floor? (b) If a horizontal force of 5 N is now applied to the block, but the block does not move, what is the magnitude of the frictional force on it? (c) If the maximum value $f_{s,\max}$ of the static frictional force on the block is 10 N, will the block move if the magnitude of the horizontally applied force is 8 N? (d) If it is 12 N? (e) What is the magnitude of the frictional force in part (c)?

Sample Problem 6.1.1 Angled force applied to an initially stationary block

This sample problem involves a tilted applied force, which requires that we work with components to find a frictional force. The main challenge is to sort out all the components. Figure 6.1.3a shows a force of magnitude $F = 12.0\text{N}$ applied to an 8.00 kg block at a downward angle of $\theta = 30.0^\circ$. The coefficient of static friction between block and floor is $\mu_s = 0.700$; the coefficient of kinetic friction is $\mu_k = 0.400$. Does the block begin to slide or does it remain stationary? What is the magnitude of the frictional force on the block?

KEY IDEAS

- (1) When the object is stationary on a surface, the static frictional force balances the force component that is attempting to slide the object along the surface.
- (2) The maximum possible magnitude of that force is given by Eq. 6.1.1 ($f_{s,\max} = \mu_s F_N$).
- (3) If the component of the applied force along the surface exceeds this limit on the static friction, the block begins to slide.
- (4) If the object slides, the kinetic frictional force is given by Eq. 6.1.2 ($f_k = \mu_k F_N$).

Calculations: To see if the block slides (and thus to calculate the magnitude of the frictional force), we must compare the applied force component F_x with the maximum magnitude $f_{s,\max}$ that the static friction can have. From the triangle of components and full force shown in Fig. 6.1.3b, we see that

$$\begin{aligned} F_x &= F \cos \theta \\ &= (12.0\text{ N}) \cos 30^\circ = 10.39\text{ N}. \end{aligned} \quad (6.1.3)$$

From Eq. 6.1.1, we know that $f_{s,\max} = \mu_s F_N$, but we need the magnitude F_N of the normal force to evaluate $f_{s,\max}$. Because the normal force is vertical, we need to write Newton's second law ($F_{\text{net},y} = ma_y$) for the vertical force components acting on the block, as displayed in Fig. 6.1.3c. The gravitational force with magnitude mg acts downward. The applied force has a downward component $F_y = F \sin \theta$. And the vertical acceleration a_y is just zero. Thus, we can write Newton's second law as

$$F_N - mg - F \sin \theta = m(0), \quad (6.1.4)$$

which gives us

$$F_N = mg + F \sin \theta. \quad (6.1.5)$$

Now we can evaluate $f_{s,\max} = \mu_s F_N$:

$$\begin{aligned} f_{s,\max} &= \mu_s(mg + F \sin \theta) \\ &= (0.700)((8.00\text{kg})(9.8\text{ m/s}^2) + (12.0\text{N})(\sin 30^\circ)) \\ &= 59.08\text{ N}. \end{aligned} \quad (6.1.6)$$

Because the magnitude $F_x (= 10.39\text{N})$ of the force component attempting to slide the block is less than $f_{s,\max} (= 59.08\text{ N})$, the block remains stationary. That means that the magnitude f_s of the frictional force matches F_x . From Fig. 6.1.3d, we can write Newton's second law for x components as

$$F_x - f_s = m(0), \quad (6.1.7)$$

and thus $f_s = F_x = 10.39\text{ N} \approx 10.4\text{ N}$. (Answer)

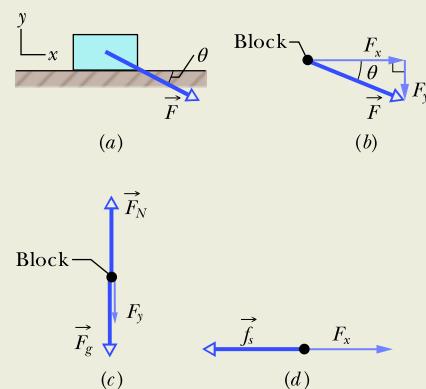


Figure 6.1.3 (a) A force is applied to an initially stationary block. (b) The components of the applied force. (c) The vertical force components. (d) The horizontal force components.

Sample Problem 6.1.2 Snowboarding

Most snowboarders (Fig. 6.1.4a) realize that a snowboard will easily slide down a snowy slope because the friction between the snow and the moving board warms the snow, producing a micron-thick layer of meltwater with a low coefficient of kinetic friction. However, few snowboarders realize that the normal force supporting them is mainly due to air pressure, not the snow itself. Here we examine the forces on a 70 kg snowboarder sliding directly down the *fall line* of an 18° slope, which is a *blue square slope* in the rating system of North America. The coefficient of kinetic friction is 0.040. We will use an x axis that is directed down the slope (Fig. 6.1.4b). (a) What is the acceleration down the slope?

KEY IDEAS

- (1) The snowboarder accelerates (the speed increases) down the slope due to a net force $F_{\text{net},x}$, which is the vector sum of the frictional force \vec{f}_k up the slope and the component $F_{g,x}$ of the gravitational force down the slope.
- (2) The frictional force is a kinetic frictional force with a magnitude given by Eq. 6.1.2 ($f_k = \mu_k F_N$), in which F_N is the magnitude of the normal force on the snowboarder (perpendicular to the slope).
- (3) We can relate the acceleration of the snowboarder to the net force along the slope by writing Newton's second law ($F_{\text{net},x} = ma_x$) for motion along the slope.

Calculations: Figure 6.1.4b shows the component $mg \sin \theta$ of the gravitational force down the slope and the component $mg \cos \theta$ perpendicular to it. The normal force F_N matches that perpendicular component, so

$$F_N = mg \cos \theta$$

and thus the magnitude of the frictional force up the slope is

$$f_k = \mu_k F_N = \mu_k mg \cos \theta.$$

We then find the acceleration a_x along the x axis from Newton's second law:

$$\begin{aligned} F_{\text{net},x} &= ma_x \\ -f_k + mg \sin \theta &= ma_x \\ -\mu_k mg \cos \theta + mg \sin \theta &= ma_x \\ g(-\mu_k \cos \theta + \sin \theta) &= a_x \\ a_x &= (9.8 \text{ m/s}^2)(-0.040 \cos 18^\circ + \sin 18^\circ) \\ &= -2.7 \text{ m/s}^2. \end{aligned} \quad (\text{Answer})$$

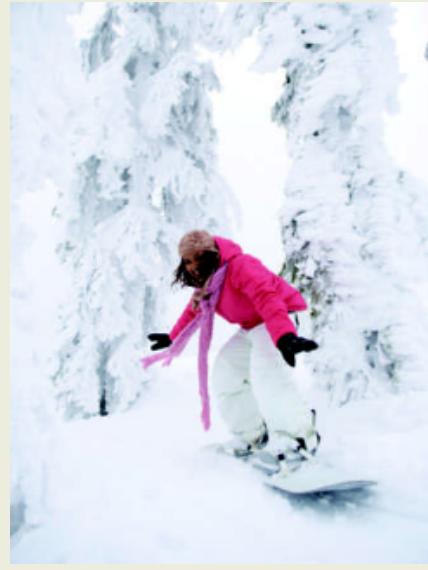
(b) If the board's speed is less than 10 m/s, the air between the snow particles beneath the board flows off to the sides and the normal force is provided directly by the particles. The result is the same for fresh snow for even faster speeds because the snow is porous so that the air can be squeezed out. However, for those faster speeds over

wind-packed snow, the board's passage is too brief for the air to be squeezed out and is momentarily trapped. For a board with a length of 1.5 m and moving at 15 m/s, how long is it over any given part of the snow?

$$t = \frac{L}{v} = \frac{1.5 \text{ m}}{15 \text{ m/s}} = 0.10 \text{ s.} \quad (\text{Answer})$$

(c) For such a brief interval, the compression by the snowboarder increases the pressure of the trapped air, which then contributes $2/3$ of the normal force. What is the contribution F_{air} for the 70 kg snowboarder?

$$\begin{aligned} F_{\text{air}} &= \frac{2}{3} mg \cos \theta \\ &= \frac{2}{3} (70 \text{ kg})(9.8 \text{ m/s}^2)(\cos 18^\circ) \\ &= 435 \text{ N.} \end{aligned} \quad (\text{Answer})$$



Darryl Leniu/DigitalVision/GettyImages

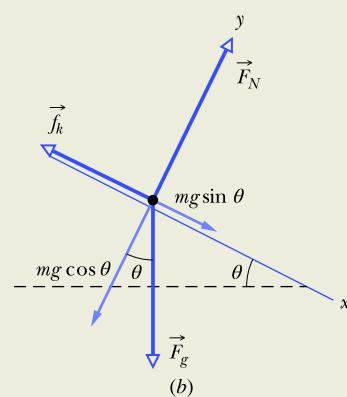


Figure 6.1.4 (a) Snowboarding. (b) A free-body diagram for the snowboarder.

6.2 THE DRAG FORCE AND TERMINAL SPEED

Learning Objectives

After reading this module, you should be able to . . .

6.2.1 Apply the relationship between the drag force on an object moving through air and the speed of the object.

Key Ideas

- When there is relative motion between air (or some other fluid) and a body, the body experiences a drag force \vec{D} that opposes the relative motion and points in the direction in which the fluid flows relative to the body. The magnitude of \vec{D} is related to the relative speed v by an experimentally determined drag coefficient C according to

$$D = \frac{1}{2} C \rho A v^2,$$

where ρ is the fluid density (mass per unit volume) and A is the effective cross-sectional area of the body (the

6.2.2 Determine the terminal speed of an object falling through air.

area of a cross section taken perpendicular to the relative velocity \vec{v}).

- When a blunt object has fallen far enough through air, the magnitudes of the drag force \vec{D} and the gravitational force \vec{F}_g on the body become equal. The body then falls at a constant terminal speed v_t given by

$$v_t = \sqrt{\frac{2F_g}{C\rho A}}.$$

The Drag Force and Terminal Speed

A **fluid** is anything that can flow—generally either a gas or a liquid. When there is a relative velocity between a fluid and a body (either because the body moves through the fluid or because the fluid moves past the body), the body experiences a **drag force** \vec{D} that opposes the relative motion and points in the direction in which the fluid flows relative to the body.

Here we examine only cases in which air is the fluid, the body is blunt (like a baseball) rather than slender (like a javelin), and the relative motion is fast enough so that the air becomes turbulent (breaks up into swirls) behind the body. In such cases, the magnitude of the drag force \vec{D} is related to the relative speed v by an experimentally determined **drag coefficient** C according to

$$D = \frac{1}{2} C \rho A v^2, \quad (6.2.1)$$

where ρ is the air density (mass per volume) and A is the **effective cross-sectional area** of the body (the area of a cross section taken perpendicular to the velocity \vec{v}). The drag coefficient C (typical values range from 0.4 to 1.0) is not truly a constant for a given body because if v varies significantly, the value of C can vary as well. Here, we ignore such complications.

Downhill speed skiers know well that drag depends on A and v^2 . To reach high speeds a skier must reduce D as much as possible by, for example, riding the skis in the “egg position” (Fig. 6.2.1) to minimize A .

Falling. When a blunt body falls from rest through air, the drag force \vec{D} is directed upward; its magnitude gradually increases from zero as the speed of the body increases. This upward force \vec{D} opposes the downward gravitational force \vec{F}_g on the body. We can relate these forces to the body’s acceleration by writing Newton’s second law for a vertical y axis ($F_{\text{net},y} = ma_y$) as

$$D - F_g = ma, \quad (6.2.2)$$

where m is the mass of the body. As suggested in Fig. 6.2.2, if the body falls long enough, D eventually equals F_g . From Eq. 6.2.2, this means that $a = 0$, and so the body’s speed no longer increases. The body then falls at a constant speed, called the **terminal speed** v_t .



technoir/E+/GettyImages

Figure 6.2.1 This skier crouches in an “egg position” so as to minimize her effective cross-sectional area and thus minimize the air drag acting on her.

Table 6.2.1 Some Terminal Speeds in Air

Object	Terminal Speed (m/s)	95% Distance ^a (m)
Shot (from shot put)	145	2500
Sky diver (typical)	60	430
Baseball	42	210
Tennis ball	31	115
Basketball	20	47
Ping-Pong ball	9	10
Raindrop (radius = 1.5 mm)	7	6
Parachutist (typical)	5	3

^aThis is the distance through which the body must fall from rest to reach 95% of its terminal speed.

Based on Peter J. Brancazio, *Sport Science*, 1984, Simon & Schuster, New York.

To find v_t , we set $a = 0$ in Eq. 6.2.2 and substitute for D from Eq. 6.2.1, obtaining

$$\frac{1}{2}C\rho A v_t^2 - F_g = 0,$$

which gives

$$v_t = \sqrt{\frac{2F_g}{C\rho A}}. \quad (6.2.3)$$

Table 6.2.1 gives values of v_t for some common objects.

According to calculations* based on Eq. 6.2.1, a cat must fall about six floors to reach terminal speed. Until it does so, $F_g > D$ and the cat accelerates downward because of the net downward force. Recall from Chapter 2 that your body is an accelerometer, not a speedometer. Because the cat also senses the acceleration, it is frightened and keeps its feet underneath its body, its head tucked in, and its spine bent upward, making A small, v_t large, and injury likely.

However, if the cat does reach v_t during a longer fall, the acceleration vanishes and the cat relaxes somewhat, stretching its legs and neck horizontally outward and straightening its spine (it then resembles a flying squirrel). These actions increase area A and thus also, by Eq. 6.2.1, the drag D . The cat begins to slow because now $D > F_g$ (the net force is upward), until a new, smaller v_t is reached. The decrease in v_t reduces the possibility of serious injury on landing. Just before the end of the fall, when it sees it is nearing the ground, the cat pulls its legs back beneath its body to prepare for the landing. FCP

Humans often fall from great heights for the fun of skydiving. However, in April 1987, during a jump, sky diver Gregory Robertson noticed that fellow sky diver Debbie Williams had been knocked unconscious in a collision with a third sky diver and was unable to open her parachute. Robertson, who was well above Williams at the time and who had not yet opened his parachute for the 4 km plunge, reoriented his body head-down so as to minimize A and maximize his downward speed. Reaching an estimated v_t of 320 km/h, he caught up with Williams and then went into a horizontal “spread eagle” (as in Fig. 6.2.3) to increase D so that he could grab her. He opened her parachute and then, after releasing her, his own, a scant 10 s before impact. Williams received extensive internal injuries due to her lack of control on landing but survived. FCP

As the cat's speed increases, the upward drag force increases until it balances the gravitational force.

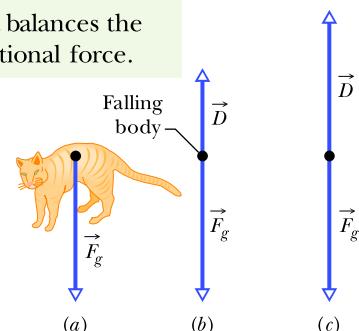


Figure 6.2.2 The forces that act on a body falling through air: (a) the body when it has just begun to fall and (b) the free-body diagram a little later, after a drag force has developed. (c) The drag force has increased until it balances the gravitational force on the body. The body now falls at its constant terminal speed.



Figure 6.2.3 Sky divers in a horizontal “spread eagle” maximize air drag.

*W. O. Whitney and C. J. Mehlhaff, “High-Rise Syndrome in Cats.” *The Journal of the American Veterinary Medical Association*, 1987.

Sample Problem 6.2.1 Terminal speed of falling raindrop

A raindrop with radius $R = 1.5 \text{ mm}$ falls from a cloud that is at height $h = 1200 \text{ m}$ above the ground. The drag coefficient C for the drop is 0.60. Assume that the drop is spherical throughout its fall. The density of water ρ_w is 1000 kg/m^3 , and the density of air ρ_a is 1.2 kg/m^3 .

(a) As Table 6.1.1 indicates, the raindrop reaches terminal speed after falling just a few meters. What is the terminal speed?

KEY IDEA

The drop reaches a terminal speed v_t when the gravitational force on it is balanced by the air drag force on it, so its acceleration is zero. We could then apply Newton's second law and the drag force equation to find v_t , but Eq. 6.2.3 does all that for us.

Calculations: To use Eq. 6.2.3, we need the drop's effective cross-sectional area A and the magnitude F_g of the gravitational force. Because the drop is spherical, A is the area of a circle (πR^2) that has the same radius as the sphere. To find F_g , we use three facts: (1) $F_g = mg$, where m is the drop's mass; (2) the (spherical) drop's volume is $V = \frac{4}{3}\pi R^3$; and (3) the density of the water in the drop is the mass per volume, or $\rho_w = m/V$. Thus, we find

$$F_g = V\rho_w g = \frac{4}{3}\pi R^3 \rho_w g.$$

We next substitute this, the expression for A , and the given data into Eq. 6.2.3. Being careful to distinguish between the air density ρ_a and the water density ρ_w , we obtain

$$\begin{aligned} v_t &= \sqrt{\frac{2F_g}{C\rho_a A}} = \sqrt{\frac{8\pi R^3 \rho_w g}{3C\rho_a \pi R^2}} = \sqrt{\frac{8R\rho_w g}{3C\rho_a}} \\ &= \sqrt{\frac{(8)(1.5 \times 10^{-3} \text{ m})(1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)}{(3)(0.60)(1.2 \text{ kg/m}^3)}} \\ &= 7.4 \text{ m/s} \approx 27 \text{ km/h}. \end{aligned} \quad (\text{Answer})$$

Note that the height of the cloud does not enter into the calculation.

(b) What would be the drop's speed just before impact if there were no drag force?

KEY IDEA

With no drag force to reduce the drop's speed during the fall, the drop would fall with the constant free-fall acceleration g , so the constant-acceleration equations of Table 2.4.1 apply.

Calculation: Because we know the acceleration is g , the initial velocity v_0 is 0, and the displacement $x - x_0$ is $-h$, we use Eq. 2.4.6 to find v :

$$\begin{aligned} v &= \sqrt{2gh} = \sqrt{(2)(9.8 \text{ m/s}^2)(1200 \text{ m})} \\ &= 153 \text{ m/s} \approx 550 \text{ km/h}. \end{aligned} \quad (\text{Answer})$$

Had he known this, Shakespeare would scarcely have written, "it droppeth as the gentle rain from heaven, upon the place beneath." In fact, the speed is close to that of a bullet from a large-caliber handgun!

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Checkpoint 6.2.1

Is the terminal speed for a large raindrop greater than, less than, or the same as that for a small raindrop, assuming that both drops are spherical?

6.3 UNIFORM CIRCULAR MOTION

Learning Objectives

After reading this module, you should be able to . . .

6.3.1 Sketch the path taken in uniform circular motion and explain the velocity, acceleration, and force vectors (magnitudes and directions) during the motion.

6.3.2 Identify that unless there is a radially inward net force (a centripetal force), an object cannot move in circular motion.

6.3.3 For a particle in uniform circular motion, apply the relationship between the radius of the path, the particle's speed and mass, and the net force acting on the particle.

Key Ideas

If a particle moves in a circle or a circular arc of radius R at constant speed v , the particle is said to be in uniform circular motion. It then has a centripetal acceleration \vec{a} with magnitude given by

$$a = \frac{v^2}{R}.$$

- This acceleration is due to a net centripetal force on the particle, with magnitude given by

$$F = \frac{mv^2}{R},$$

where m is the particle's mass. The vector quantities \vec{a} and \vec{F} are directed toward the center of curvature of the particle's path.

Uniform Circular Motion

From Module 4.5, recall that when a body moves in a circle (or a circular arc) at constant speed v , it is said to be in uniform circular motion. Also recall that the body has a centripetal acceleration (directed toward the center of the circle) of constant magnitude given by

$$a = \frac{v^2}{R} \quad (\text{centripetal acceleration}), \quad (6.3.1)$$

where R is the radius of the circle. Here are two examples:

- Rounding a curve in a car.* You are sitting in the center of the rear seat of a car moving at a constant high speed along a flat road. When the driver suddenly turns left, rounding a corner in a circular arc, you slide across the seat toward the right and then jam against the car wall for the rest of the turn. What is going on?

While the car moves in the circular arc, it is in uniform circular motion; that is, it has an acceleration that is directed toward the center of the circle. By Newton's second law, a force must cause this acceleration. Moreover, the force must also be directed toward the center of the circle. Thus, it is a **centripetal force**, where the adjective indicates the direction. In this example, the centripetal force is a frictional force on the tires from the road; it makes the turn possible.

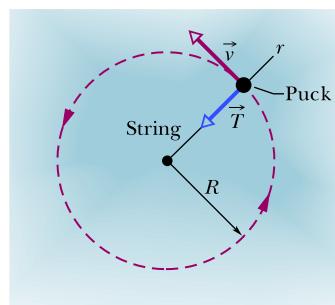
If you are to move in uniform circular motion along with the car, there must also be a centripetal force on you. However, apparently the frictional force on you from the seat was not great enough to make you go in a circle with the car. Thus, the seat slid beneath you, until the right wall of the car jammed into you. Then its push on you provided the needed centripetal force on you, and you joined the car's uniform circular motion.

- Orbiting Earth.* This time you are a passenger in the space shuttle *Atlantis*. As it and you orbit Earth, you float through your cabin. What is going on?

Both you and the shuttle are in uniform circular motion and have accelerations directed toward the center of the circle. Again by Newton's second law, centripetal forces must cause these accelerations. This time the centripetal forces are gravitational pulls (the pull on you and the pull on the shuttle) exerted by Earth and directed radially inward, toward the center of Earth.

In both car and shuttle you are in uniform circular motion, acted on by a centripetal force—yet your sensations in the two situations are quite different. In the car, jammed up against the wall, you are aware of being compressed by the wall. In the orbiting shuttle, however, you are floating around with no sensation of any force acting on you. Why this difference?

The difference is due to the nature of the two centripetal forces. In the car, the centripetal force is the push on the part of your body touching the car wall. You can sense the compression on that part of your body. In the shuttle, the centripetal force is Earth's gravitational pull on every atom of your body.



The puck moves in uniform circular motion only because of a toward-the-center force.

Figure 6.3.1 An overhead view of a hockey puck moving with constant speed v in a circular path of radius R on a horizontal frictionless surface. The centripetal force on the puck is \vec{T} , the pull from the string, directed inward along the radial axis r extending through the puck.

Thus, there is no compression (or pull) on any one part of your body and no sensation of a force acting on you. (The sensation is said to be one of “weightlessness,” but that description is tricky. The pull on you by Earth has certainly not disappeared and, in fact, is only a little less than it would be with you on the ground.)

Another example of a centripetal force is shown in Fig. 6.3.1. There a hockey puck moves around in a circle at constant speed v while tied to a string looped around a central peg. This time the centripetal force is the radially inward pull on the puck from the string. Without that force, the puck would slide off in a straight line instead of moving in a circle.

Note again that a centripetal force is not a new kind of force. The name merely indicates the direction of the force. It can, in fact, be a frictional force, a gravitational force, the force from a car wall or a string, or any other force. For any situation:



A centripetal force accelerates a body by changing the direction of the body's velocity without changing the body's speed.

From Newton's second law and Eq. 6.3.1 ($a = v^2/R$), we can write the magnitude F of a centripetal force (or a net centripetal force) as

$$F = m \frac{v^2}{R} \quad (\text{magnitude of centripetal force}). \quad (6.3.2)$$

Because the speed v here is constant, the magnitudes of the acceleration and the force are also constant.

However, the directions of the centripetal acceleration and force are not constant; they vary continuously so as to always point toward the center of the circle. For this reason, the force and acceleration vectors are sometimes drawn along a radial axis r that moves with the body and always extends from the center of the circle to the body, as in Fig. 6.3.1. The positive direction of the axis is radially outward, but the acceleration and force vectors point radially inward.

Checkpoint 6.3.1

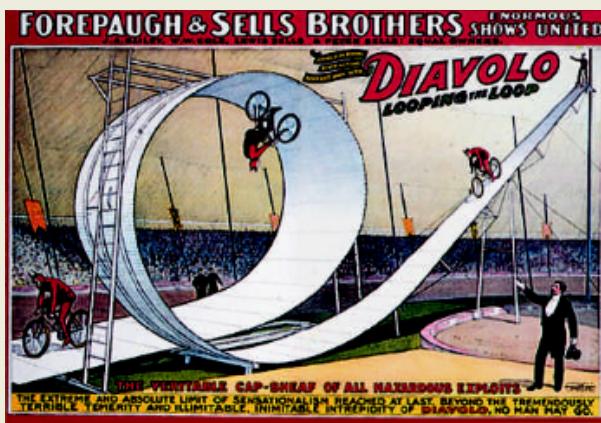
As every amusement park fan knows, a Ferris wheel is a ride consisting of seats mounted on a tall ring that rotates around a horizontal axis. When you ride in a Ferris wheel at constant speed, what are the directions of your acceleration \vec{a} and the normal force \vec{F}_N on you (from the always upright seat) as you pass through (a) the highest point and (b) the lowest point of the ride? (c) How does the magnitude of the acceleration at the highest point compare with that at the lowest point? (d) How do the magnitudes of the normal force compare at those two points?

Sample Problem 6.3.1 Vertical circular loop, Diavolo

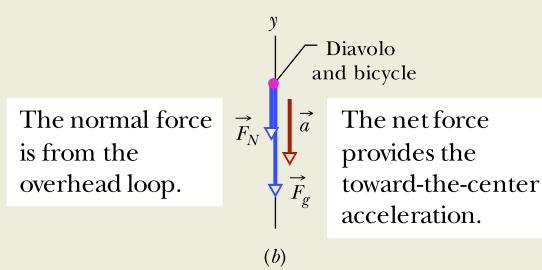
Largely because of riding in cars, you are used to horizontal circular motion. Vertical circular motion would be a novelty. In this sample problem, such motion seems to defy the gravitational force.

In a 1901 circus performance, Allo “Dare Devil” Diavolo introduced the stunt of riding a bicycle in a loop-the-loop (Fig. 6.3.2a). Assuming that the loop is a circle with radius $R = 2.7\text{ m}$, what is the least speed v that Diavolo and his bicycle could have at the top of the loop to remain in contact with it there? FCP

Photograph reproduced with permission of Circus World Museum



(a)



(b)

Figure 6.3.2 (a) Contemporary advertisement for Diavolo and (b) free-body diagram for the performer at the top of the loop.

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Sample Problem 6.3.2 Driving in flat turns and upside down

Upside-down racing: A modern race car is designed so that the passing air pushes down on it, allowing the car to travel much faster through a flat turn in a Grand Prix without friction failing. This downward push is called *negative lift*. Can a race car have so much negative lift that it could be driven upside down on a long ceiling, as done fictionally by a sedan in the first *Men in Black* movie?

Figure 6.3.3a represents a Grand Prix race car of mass $m = 600\text{ kg}$ as it travels on a flat track in a circular arc of radius $R = 100\text{ m}$. Because of the shape of the car and the wings on it, the passing air exerts a negative lift \vec{F}_L downward on the car. The coefficient of static friction between

KEY IDEA

We can assume that Diavolo and his bicycle travel through the top of the loop as a single particle in uniform circular motion. Thus, at the top, the acceleration \vec{a} of this particle must have the magnitude $a = v^2/R$ given by Eq. 6.3.1 and be directed downward, toward the center of the circular loop.

Calculations: The forces on the particle when it is at the top of the loop are shown in the free-body diagram of Fig 6.3.2b. The gravitational force \vec{F}_g is downward along a y axis; so is the normal force \vec{F}_N on the particle from the loop (the loop can push down, not pull up); so also is the centripetal acceleration of the particle. Thus, Newton’s second law for y components ($F_{\text{net},y} = ma_y$) gives us

$$-F_N - F_g = m(-a)$$

$$\text{and } -F_N - mg = m\left(-\frac{v^2}{R}\right). \quad (6.3.3)$$

If the particle has the *least speed* v needed to remain in contact, then it is on the *verge of losing contact* with the loop (falling away from the loop), which means that $F_N = 0$ at the top of the loop (the particle and loop touch but without any normal force). Substituting 0 for F_N in Eq. 6.3.3, solving for v , and then substituting known values give us

$$v = \sqrt{gR} = \sqrt{(9.8\text{ m/s}^2)(2.7\text{ m})} \\ = 5.1\text{ m/s.} \quad (\text{Answer})$$

Comments: Diavolo made certain that his speed at the top of the loop was greater than 5.1 m/s so that he did not lose contact with the loop and fall away from it. Note that this speed requirement is independent of the mass of Diavolo and his bicycle. Had he feasted on, say, pierogies before his performance, he still would have had to exceed only 5.1 m/s to maintain contact as he passed through the top of the loop.

the tires and the track is 0.75. (Assume that the forces on the four tires are identical.) FCP

- (a) If the car is on the verge of sliding out of the turn when its speed is 28.6 m/s, what is the magnitude of the negative lift \vec{F}_L acting downward on the car?

KEY IDEAS

1. A centripetal force must act on the car because the car is moving around a circular arc; that force must be directed toward the center of curvature of the arc (here, that is horizontally).

2. The only horizontal force acting on the car is a frictional force on the tires from the road. So the required centripetal force is a frictional force.
3. Because the car is not sliding, the frictional force must be a *static* frictional force \vec{f}_s (Fig. 6.3.3a).
4. Because the car is on the verge of sliding, the magnitude f_s is equal to the maximum value $f_{s,\max} = \mu_s F_N$, where F_N is the magnitude of the normal force \vec{F}_N acting on the car from the track.

Radial calculations: The frictional force \vec{f}_s is shown in the free-body diagram of Fig. 6.3.3b. It is in the negative direction of a radial axis r that always extends from the center of curvature through the car as the car moves. The force produces a centripetal acceleration of magnitude v^2/R . We can relate the force and acceleration by writing Newton's second law for components along the r axis ($F_{\text{net},r} = ma_r$) as

$$-f_s = m\left(-\frac{v^2}{R}\right). \quad (6.3.4)$$

Substituting $f_{s,\max} = \mu_s F_N$ for f_s leads us to

$$\mu_s F_N = m\left(\frac{v^2}{R}\right). \quad (6.3.5)$$

Vertical calculations: Next, let's consider the vertical forces on the car. The normal force \vec{F}_N is directed up, in the positive direction of the y axis in Fig. 6.3.3b. The gravitational force $\vec{F}_g = m\vec{g}$ and the negative lift \vec{F}_L are directed down. The acceleration of the car along the y axis is zero. Thus we can write Newton's second law for components along the y axis ($F_{\text{net},y} = ma_y$) as

$$F_N - mg - F_L = 0, \quad \text{or} \quad F_N = mg + F_L. \quad (6.3.6)$$

Combining results: Now we can combine our results along the two axes by substituting Eq. 6.3.6 for F_N in Eq. 6.3.5. Doing so and then solving for F_L lead to

$$\begin{aligned} F_L &= m\left(\frac{v^2}{\mu_s R} - g\right) \\ &= (600 \text{ kg})\left(\frac{(28.6 \text{ m/s})^2}{(0.75)(100 \text{ m})} - 9.8 \text{ m/s}^2\right) \\ &= 663.7 \text{ N} \approx 660 \text{ N}. \end{aligned} \quad (\text{Answer})$$

(b) The magnitude F_L of the negative lift on a car depends on the square of the car's speed v^2 , just as the drag force does (Eq. 6.2.1). Thus, the negative lift on the car here is greater when the car travels faster, as it does on a straight section of track. What is the magnitude of the negative lift for a speed of 90 m/s?

KEY IDEA

F_L is proportional to v^2 .

Calculations: Thus we can write a ratio of the negative lift $F_{L,90}$ at $v = 90 \text{ m/s}$ to our result for the negative lift F_L at $v = 28.6 \text{ m/s}$ as

$$\frac{F_{L,90}}{F_L} = \frac{(90 \text{ m/s})^2}{(28.6 \text{ m/s})^2}.$$

Substituting our known negative lift of $F_L = 663.7 \text{ N}$ and solving for $F_{L,90}$ give us

$$F_{L,90} = 6572 \text{ N} \approx 6600 \text{ N}. \quad (\text{Answer})$$

Upside-down racing: The gravitational force is, of course, the force to beat if there is a chance of racing upside down:

$$\begin{aligned} F_g &= mg = (600 \text{ kg})(9.8 \text{ m/s}^2) \\ &= 5880 \text{ N}. \end{aligned}$$

With the car upside down, the negative lift is an *upward* force of 6600 N, which exceeds the downward 5880 N. Thus, the car could run on a long ceiling *provided* that it moves at about 90 m/s ($= 324 \text{ km/h} = 201 \text{ mi/h}$). However, moving that fast while right side up on a horizontal track is dangerous enough, so you are not likely to see upside-down racing except in the movies.

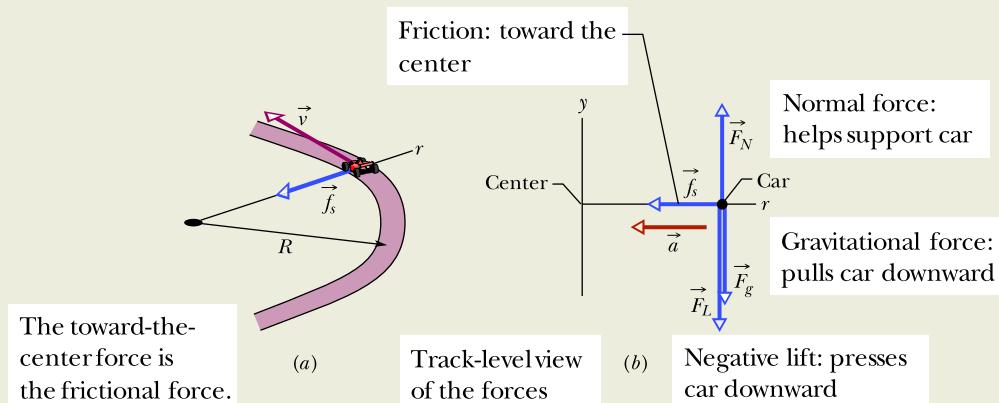


Figure 6.3.3 (a) A race car moves around a flat curved track at constant speed v . The frictional force \vec{f}_s provides the necessary centripetal force along a radial axis r . (b) A free-body diagram (not to scale) for the car, in the vertical plane containing r .

Review & Summary

Friction When a force \vec{F} tends to slide a body along a surface, a **frictional force** from the surface acts on the body. The frictional force is parallel to the surface and directed so as to oppose the sliding. It is due to bonding between the atoms on the body and the atoms on the surface, an effect called cold-welding.

If the body does not slide, the frictional force is a **static frictional force** f_s . If there is sliding, the frictional force is a **kinetic frictional force** f_k .

1. If a body does not move, the static frictional force f_s and the component of \vec{F} parallel to the surface are equal in magnitude, and f_s is directed opposite that component. If the component increases, f_s also increases.

2. The magnitude of f_s has a maximum value $f_{s,\max}$ given by

$$f_{s,\max} = \mu_s F_N, \quad (6.1.1)$$

where μ_s is the **coefficient of static friction** and F_N is the magnitude of the normal force. If the component of \vec{F} parallel to the surface exceeds $f_{s,\max}$, the static friction is overwhelmed and the body slides on the surface.

3. If the body begins to slide on the surface, the magnitude of the frictional force rapidly decreases to a constant value f_k given by

$$f_k = \mu_k F_N, \quad (6.1.2)$$

where μ_k is the **coefficient of kinetic friction**.

Drag Force When there is relative motion between air (or some other fluid) and a body, the body experiences a **drag force** \vec{D} that opposes the relative motion and points in the direction in which the fluid flows relative to the body. The magnitude of \vec{D} is

related to the relative speed v by an experimentally determined **drag coefficient** C according to

$$D = \frac{1}{2} C \rho A v^2, \quad (6.2.1)$$

where ρ is the fluid density (mass per unit volume) and A is the **effective cross-sectional area** of the body (the area of a cross section taken perpendicular to the relative velocity \vec{v}).

Terminal Speed When a blunt object has fallen far enough through air, the magnitudes of the drag force \vec{D} and the gravitational force \vec{F}_g on the body become equal. The body then falls at a constant **terminal speed** v_t given by

$$v_t = \sqrt{\frac{2F_g}{C\rho A}}. \quad (6.2.3)$$

Uniform Circular Motion If a particle moves in a circle or a circular arc of radius R at constant speed v , the particle is said to be in **uniform circular motion**. It then has a **centripetal acceleration** \vec{a} with magnitude given by

$$a = \frac{v^2}{R}. \quad (6.3.1)$$

This acceleration is due to a net **centripetal force** on the particle, with magnitude given by

$$F = \frac{mv^2}{R}, \quad (6.3.2)$$

where m is the particle's mass. The vector quantities \vec{a} and \vec{F} are directed toward the center of curvature of the particle's path. A particle can move in circular motion only if a net centripetal force acts on it.

Questions

- 1 In Fig. 6.1, if the box is stationary and the angle θ between the horizontal and force \vec{F} is increased somewhat, do the following quantities increase, decrease, or remain the same: (a) F_x ; (b) f_s ; (c) F_N ; (d) $f_{s,\max}$? (e) If, instead, the box is sliding and θ is increased, does the magnitude of the frictional force on the box increase, decrease, or remain the same?

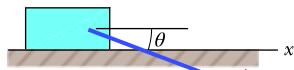


Figure 6.1 Question 1.

- 2 Repeat Question 1 for force \vec{F} angled upward instead of downward as drawn.
- 3 In Fig. 6.2, horizontal force \vec{F}_1 of magnitude 10 N is applied to a box on a floor, but the box does not slide. Then, as the magnitude of vertical force \vec{F}_2 is increased from zero, do the following quantities increase, decrease, or stay the same: (a) the magnitude of the frictional force f_s on the box; (b) the magnitude of the normal force F_N on the box from the floor; (c) the maximum value $f_{s,\max}$ of the magnitude of the static frictional force on the box? (d) Does the box eventually slide?

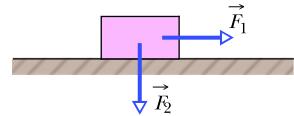


Figure 6.2 Question 3.

- 4 In three experiments, three different horizontal forces are applied to the same block lying on the same countertop. The force magnitudes are $F_1 = 12$ N, $F_2 = 8$ N, and $F_3 = 4$ N. In each experiment, the block remains stationary in spite of the applied force. Rank the forces according to (a) the magnitude f_s of the static frictional force on the block from the countertop and (b) the maximum value $f_{s,\max}$ of that force, greatest first.

- 5 If you press an apple crate against a wall so hard that the crate cannot slide down the wall, what is the direction of (a) the static frictional force \vec{f}_s on the crate from the wall and (b) the normal force \vec{F}_N on the crate from the wall? If you increase your push, what happens to (c) f_s , (d) F_N , and (e) $f_{s,\max}$?

- 6 In Fig. 6.3, a block of mass m is held stationary on a ramp by the frictional force on it from the ramp. A force \vec{F} , directed up the ramp, is then applied to the block and gradually increased in magnitude from zero. During the increase, what happens to the direction and magnitude of the frictional force on the block?

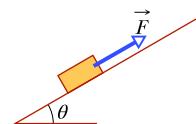


Figure 6.3
Question 6.

7 Reconsider Question 6 but with the force \vec{F} now directed down the ramp. As the magnitude of \vec{F} is increased from zero, what happens to the direction and magnitude of the frictional force on the block?

8 In Fig. 6.4, a horizontal force of 100 N is to be applied to a 10 kg slab that is initially stationary on a frictionless floor, to accelerate the slab. A 10 kg block lies on top of the slab; the coefficient of friction μ between the block and the slab is not known, and the block might slip. In fact, the contact between the block and the slab might even be frictionless. (a) Considering that possibility, what is the possible range of values for the magnitude of the slab's acceleration a_{slab} ? (*Hint:* You don't need written calculations; just consider extreme values for μ). (b) What is the possible range for the magnitude a_{block} of the block's acceleration?

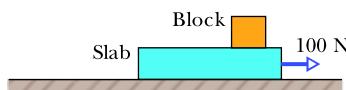


Figure 6.4 Question 8.

9 Figure 6.5 shows the overhead view of the path of an amusement-park ride that travels at constant speed through five circular arcs of radii R_0 , $2R_0$, and $3R_0$. Rank the arcs according to the magnitude of the centripetal force on a rider traveling in the arcs, greatest first.

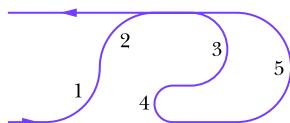


Figure 6.5 Question 9.

10 FCP In 1987, as a Halloween stunt, two sky divers passed a pumpkin back and forth between them while they were in free

fall just west of Chicago. The stunt was great fun until the last sky diver with the pumpkin opened his parachute. The pumpkin broke free from his grip, plummeted about 0.5 km, ripped through the roof of a house, slammed into the kitchen floor, and splattered all over the newly remodeled kitchen. From the sky diver's viewpoint and from the pumpkin's viewpoint, why did the sky diver lose control of the pumpkin?

11 A person riding a Ferris wheel moves through positions at (1) the top, (2) the bottom, and (3) midheight. If the wheel rotates at a constant rate, rank these three positions according to (a) the magnitude of the person's centripetal acceleration, (b) the magnitude of the net centripetal force on the person, and (c) the magnitude of the normal force on the person, greatest first.

12 During a routine flight in 1956, test pilot Tom Attridge put his jet fighter into a 20° dive for a test of the aircraft's 20 mm machine cannons. While traveling faster than sound at 4000 m altitude, he shot a burst of rounds. Then, after allowing the cannons to cool, he shot another burst at 2000 m; his speed was then 344 m/s, the speed of the rounds relative to him was 730 m/s, and he was still in a dive.

Almost immediately the canopy around him was shredded and his right air intake was damaged. With little flying capability left, the jet crashed into a wooded area, but Attridge managed to escape the resulting explosion. Explain what apparently happened just after the second burst of cannon rounds. (Attridge has been the only pilot who has managed to shoot himself down.)

13 A box is on a ramp that is at angle θ to the horizontal. As θ is increased from zero, and before the box slips, do the following increase, decrease, or remain the same: (a) the component of the gravitational force on the box, along the ramp, (b) the magnitude of the static frictional force on the box from the ramp, (c) the component of the gravitational force on the box, perpendicular to the ramp, (d) the magnitude of the normal force on the box from the ramp, and (e) the maximum value $f_{s,\text{max}}$ of the static frictional force?

Problems

GO Tutoring problem available (at instructor's discretion) in WileyPLUS

SSM Worked-out solution available in Student Solutions Manual

E Easy **M** Medium **H** Hard

FCP Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

CALC Requires calculus

BIO Biomedical application

Module 6.1 Friction

1 E The floor of a railroad flatcar is loaded with loose crates having a coefficient of static friction of 0.25 with the floor. If the train is initially moving at a speed of 48 km/h, in how short a distance can the train be stopped at constant acceleration without causing the crates to slide over the floor?

2 E In a pickup game of dorm shuffleboard, students crazed by final exams use a broom to propel a calculus book along the dorm hallway. If the 3.5 kg book is pushed from rest through a distance of 0.90 m by the horizontal 25 N force from the broom and then has a speed of 1.60 m/s, what is the coefficient of kinetic friction between the book and floor?

3 E SSM A bedroom bureau with a mass of 45 kg, including drawers and clothing, rests on the floor. (a) If the coefficient

of static friction between the bureau and the floor is 0.45, what is the magnitude of the minimum horizontal force that a person must apply to start the bureau moving? (b) If the drawers and clothing, with 17 kg mass, are removed before the bureau is pushed, what is the new minimum magnitude?

4 E A slide-loving pig slides down a certain 35° slide in twice the time it would take to slide down a frictionless 35° slide. What is the coefficient of kinetic friction between the pig and the slide?

5 E GO A 2.5 kg block is initially at rest on a horizontal surface. A horizontal force \vec{F} of magnitude 6.0 N and a vertical force \vec{P} are then applied to the block (Fig. 6.6). The coefficients of friction for the block and surface are $\mu_s = 0.40$ and $\mu_k = 0.25$. Determine the magnitude of the frictional force acting on the block if the magnitude of \vec{P} is (a) 8.0 N, (b) 10 N, and (c) 12 N.

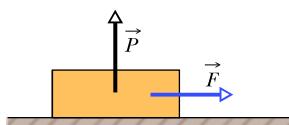


Figure 6.6 Problem 5.

6 E A baseball player with mass $m = 79 \text{ kg}$, sliding into second base, is retarded by a frictional force of magnitude 470 N . What is the coefficient of kinetic friction μ_k between the player and the ground?

7 E SSM A person pushes horizontally with a force of 220 N on a 55 kg crate to move it across a level floor. The coefficient of kinetic friction between the crate and the floor is 0.35 . What is the magnitude of (a) the frictional force and (b) the acceleration of the crate?

8 E FCP *The mysterious sliding stones.* Along the remote Racetrack Playa in Death Valley, California, stones sometimes gouge out prominent trails in the desert floor, as if the stones had been migrating (Fig. 6.7). For years curiosity mounted about why the stones moved. One explanation was that strong winds during occasional rainstorms would drag the rough stones over ground softened by rain. When the desert dried out, the trails behind the stones were hard-baked in place. According to measurements, the coefficient of kinetic friction between the stones and the wet playa ground is about 0.80 . What horizontal force must act on a 20 kg stone (a typical mass) to maintain the stone's motion once a gust has started it moving? (Story continues with Problem 37.)



Jerry Schad/Science Source

Figure 6.7 Problem 8. What moved the stone?

9 E GO A 3.5 kg block is pushed along a horizontal floor by a force \vec{F} of magnitude 15 N at an angle $\theta = 40^\circ$ with the horizontal (Fig. 6.8). The coefficient of kinetic friction between the block and the floor is 0.25 . Calculate the magnitudes of (a) the frictional force on the block from the floor and (b) the block's acceleration.

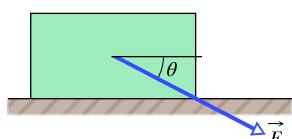
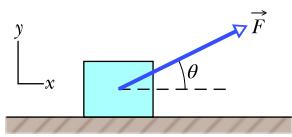
Figure 6.8
Problems 9 and 32.

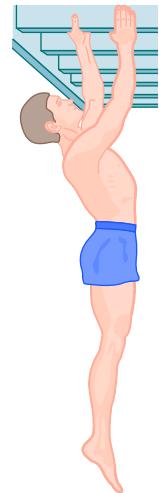
Figure 6.9 Problem 10.

10 E Figure 6.9 shows an initially stationary block of mass m on a floor. A force of magnitude

$0.500mg$ is then applied at upward angle $\theta = 20^\circ$. What is the magnitude of the acceleration of the block across the floor if the friction coefficients are (a) $\mu_s = 0.600$ and $\mu_k = 0.500$ and (b) $\mu_s = 0.400$ and $\mu_k = 0.300$?

11 E SSM A 68 kg crate is dragged across a floor by pulling on a rope attached to the crate and inclined 15° above the horizontal. (a) If the coefficient of static friction is 0.50 , what minimum force magnitude is required from the rope to start the crate moving? (b) If $\mu_k = 0.35$, what is the magnitude of the initial acceleration of the crate?

12 E BIO In about 1915, Henry Sincosky of Philadelphia suspended himself from a rafter by gripping the rafter with the thumb of each hand on one side and the fingers on the opposite side (Fig. 6.10). Sincosky's mass was 79 kg . If the coefficient of static friction between hand and rafter was 0.70 , what was the least magnitude of the normal force on the rafter from each thumb or opposite fingers? (After suspending himself, Sincosky chinned himself on the rafter and then moved hand-over-hand along the rafter. If you do not think Sincosky's grip was remarkable, try to repeat his stunt.)

Figure 6.10
Problem 12.

13 E A worker pushes horizontally on a 35 kg crate with a force of magnitude 110 N . The coefficient of static friction between the crate and the floor is 0.37 . (a) What is the value of $f_{s,\max}$ under the circumstances? (b) Does the crate move? (c) What is the frictional force on the crate from the floor? (d) Suppose, next, that a second worker pulls directly upward on the crate to help out. What is the least vertical pull that will allow the first worker's 110 N push to move the crate? (e) If, instead, the second worker pulls horizontally to help out, what is the least pull that will get the crate moving?

14 E Figure 6.11 shows the cross section of a road cut into the side of a mountain. The solid line AA' represents a weak bedding plane along which sliding is possible. Block B directly above the highway is separated from uphill rock by a large crack (called a *joint*), so that only friction between the block and the bedding plane prevents sliding. The mass of the block is $1.8 \times 10^7 \text{ kg}$, the *dip angle* θ of the bedding plane is 24° , and the coefficient of static friction between block and plane is 0.63 . (a) Show that the block will not slide under these circumstances. (b) Next, water seeps into the joint and expands upon freezing, exerting on the block a force \vec{F} parallel to AA' . What minimum value of force magnitude F will trigger a slide down the plane?

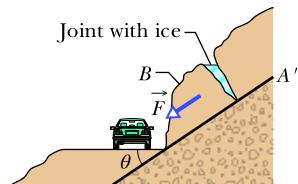


Figure 6.11 Problem 14.

15 E The coefficient of static friction between Teflon and scrambled eggs is about 0.04 . What is the smallest angle from the horizontal that will cause the eggs to slide across the bottom of a Teflon-coated skillet?

16 M A loaded penguin sled weighing 80 N rests on a plane inclined at angle $\theta = 20^\circ$ to the horizontal (Fig. 6.12). Between the sled and the plane, the coefficient of static friction is 0.25 , and the coefficient of kinetic friction is 0.15 . (a) What is the

least magnitude of the force \vec{F} , parallel to the plane, that will prevent the sled from slipping down the plane? (b) What is the minimum magnitude F that will start the sled moving up the plane? (c) What value of F is required to move the sled up the plane at constant velocity?

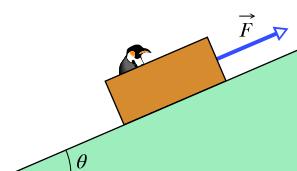


Figure 6.12
Problems 16 and 22.

17 M In Fig. 6.13, a force \vec{P} acts on a block weighing 45 N. The block is initially at rest on a plane inclined at angle $\theta = 15^\circ$ to the horizontal. The positive direction of the x axis is up the plane. Between block and plane, the coefficient of static friction is $\mu_s = 0.50$ and the coefficient of kinetic friction is $\mu_k = 0.34$. In unit-vector notation, what is the frictional force on the block from the plane when \vec{P} is (a) $(-5.0 \text{ N})\hat{i}$, (b) $(-8.0 \text{ N})\hat{i}$, and (c) $(-15 \text{ N})\hat{i}$?

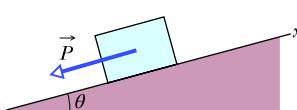


Figure 6.13 Problem 17.

18 M GO You testify as an *expert witness* in a case involving an accident in which car A slid into the rear of car B , which was stopped at a red light along a road headed down a hill (Fig. 6.14). You find that the slope of the hill is $\theta = 12.0^\circ$, that the cars were separated by distance $d = 24.0 \text{ m}$ when the driver of car A put the car into a slide (it lacked any automatic anti-brake-lock system), and that the speed of car A at the onset of braking was $v_0 = 18.0 \text{ m/s}$. With what speed did car A hit car B if the coefficient of kinetic friction was (a) 0.60 (dry road surface) and (b) 0.10 (road surface covered with wet leaves)?

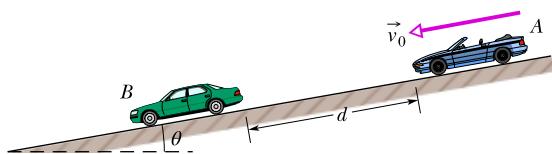


Figure 6.14 Problem 18.

19 M A 12 N horizontal force \vec{F} pushes a block weighing 5.0 N against a vertical wall (Fig. 6.15). The coefficient of static friction between the wall and the block is 0.60, and the coefficient of kinetic friction is 0.40. Assume that the block is not moving initially. (a) Will the block move? (b) In unit-vector notation, what is the force on the block from the wall?

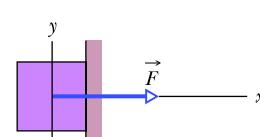


Figure 6.15 Problem 19.

20 M GO In Fig. 6.16, a box of Cheerios (mass $m_C = 1.0 \text{ kg}$) and a box of Wheaties (mass $m_W = 3.0 \text{ kg}$) are accelerated across a horizontal surface by a horizontal force \vec{F} applied to the Cheerios box. The magnitude of the frictional force on the Cheerios box is 2.0 N, and the magnitude of the frictional force on the Wheaties box is 4.0 N. If the magnitude of \vec{F} is 12 N, what is the magnitude of the force on the Wheaties box from the Cheerios box?

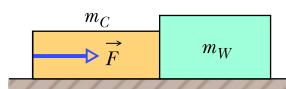


Figure 6.16 Problem 20.

21 M CALC An initially stationary box of sand is to be pulled across a floor by means of a cable in which the tension should not exceed 1100 N. The coefficient of static friction between the box and the floor is 0.35. (a) What should be the angle between the cable and the horizontal in order to pull the greatest possible amount of sand, and (b) what is the weight of the sand and box in that situation?

22 M GO In Fig. 6.12, a sled is held on an inclined plane by a cord pulling directly up the plane. The sled is to be on the verge of moving up the plane. In Fig. 6.17, the magnitude F required of the cord's force on the sled is plotted versus a range of values for the coefficient of static friction μ_s between sled and plane: $F_1 = 2.0 \text{ N}$, $F_2 = 5.0 \text{ N}$, and $\mu_2 = 0.50$. At what angle θ is the plane inclined?

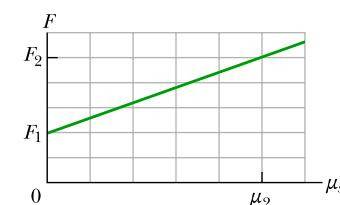


Figure 6.17 Problem 22.

23 M When the three blocks in Fig. 6.18 are released from rest, they accelerate with a magnitude of 0.500 m/s^2 . Block 1 has mass M , block 2 has $2M$, and block 3 has $2M$. What is the coefficient of kinetic friction between block 2 and the table?

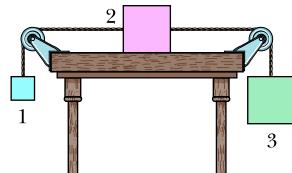


Figure 6.18 Problem 23.

24 M A 4.10 kg block is pushed along a floor by a constant applied force that is horizontal and has a magnitude of 40.0 N . Figure 6.19 gives the block's speed v versus time t as the block moves along an x axis on the floor. The scale of the figure's vertical axis is set by $v_s = 5.0 \text{ m/s}$. What is the coefficient of kinetic friction between the block and the floor?

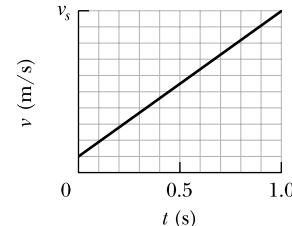


Figure 6.19 Problem 24.

25 M SSM Block B in Fig. 6.20 weighs 711 N . The coefficient of static friction between block and table is 0.25; angle θ is 30° ; assume that the cord between B and the knot is horizontal. Find the maximum weight of block A for which the system will be stationary.

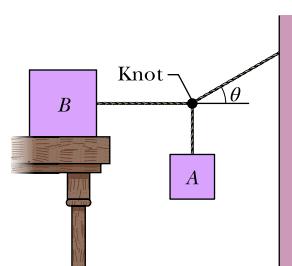


Figure 6.20 Problem 25.

26 M GO Figure 6.21 shows three crates being pushed over a concrete floor by a horizontal force \vec{F} of magnitude 440 N . The masses of the crates are $m_1 = 30.0 \text{ kg}$, $m_2 = 10.0 \text{ kg}$, and $m_3 = 20.0 \text{ kg}$. The coefficient of kinetic friction between the floor and each of the crates is 0.700. (a) What is the magnitude F_{32} of the force on crate 3 from crate 2? (b) If the crates then slide onto a polished floor, where the coefficient of kinetic friction is less than 0.700, is

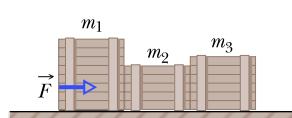


Figure 6.21 Problem 26.

magnitude F_{32} more than, less than, or the same as it was when the coefficient was 0.700?

- 27 M GO** Body *A* in Fig. 6.22 weighs 102 N, and body *B* weighs 32 N. The coefficients of friction between *A* and the incline are $\mu_s = 0.56$ and $\mu_k = 0.25$. Angle θ is 40° . Let the positive direction of an *x* axis be up the incline. In unit-vector notation, what is the acceleration of *A* if *A* is initially (a) at rest, (b) moving up the incline, and (c) moving down the incline?

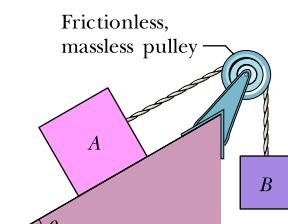


Figure 6.22
Problems 27 and 28.

- 28 M** In Fig. 6.22, two blocks are connected over a pulley. The mass of block *A* is 10 kg, and the coefficient of kinetic friction between *A* and the incline is 0.20. Angle θ of the incline is 30° . Block *A* slides down the incline at constant speed. What is the mass of block *B*? Assume the connecting rope has negligible mass. (The pulley's function is only to redirect the rope.)

- 29 M GO** In Fig. 6.23, blocks *A* and *B* have weights of 44 N and 22 N, respectively. (a) Determine the minimum weight of block *C* to keep *A* from sliding if μ_s between *A* and the table is 0.20. (b) Block *C* suddenly is lifted off *A*. What is the acceleration of block *A* if μ_k between *A* and the table is 0.15?

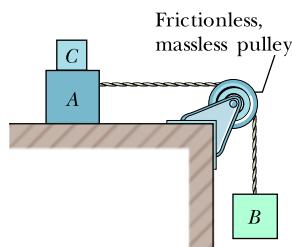


Figure 6.23 Problem 29.

- 30 M CALC** A toy chest and its contents have a combined weight of 180 N. The coefficient of static friction between toy chest and floor is 0.42. The child in Fig. 6.24 attempts to move the chest across the floor by pulling on an attached rope. (a) If θ is 42° , what is the magnitude of the force \vec{F} that the child must exert on the rope to put the chest on the verge of moving? (b) Write an expression for the magnitude F required to put the chest on the verge of moving as a function of the angle θ . Determine (c) the value of θ for which F is a minimum and (d) that minimum magnitude.

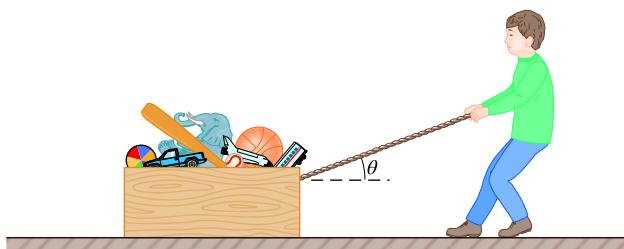


Figure 6.24 Problem 30.

- 31 M SSM** Two blocks, of weights 3.6 N and 7.2 N, are connected by a massless string and slide down a 30° inclined plane. The coefficient of kinetic friction between the lighter block and the plane is 0.10, and the coefficient between the heavier block and the plane is 0.20. Assuming that the lighter block leads, find (a) the magnitude of the acceleration of the blocks and (b) the tension in the taut string.

- 32 M GO** A block is pushed across a floor by a constant force that is applied at downward angle θ (Fig. 6.8). Figure 6.25 gives the acceleration magnitude a versus a range of values for the coefficient of kinetic friction μ_k between block and floor: $a_1 = 3.0 \text{ m/s}^2$, $\mu_{k2} = 0.20$, and $\mu_{k3} = 0.40$. What is the value of θ ?

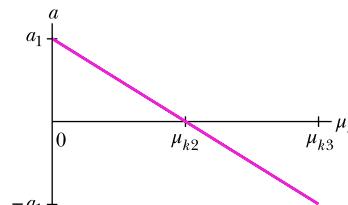


Figure 6.25 Problem 32.

- 33 H SSM** A 1000 kg boat is traveling at 90 km/h when its engine is shut off. The magnitude of the frictional force \vec{f}_k between boat and water is proportional to the speed v of the boat: $f_k = 70v$, where v is in meters per second and f_k is in newtons. Find the time required for the boat to slow to 45 km/h.

- 34 H GO** In Fig. 6.26, a slab of mass $m_1 = 40 \text{ kg}$ rests on a frictionless floor, and a block of mass $m_2 = 10 \text{ kg}$ rests on top of the slab. Between block and slab, the coefficient of static friction is 0.60, and the coefficient of kinetic friction is 0.40. A horizontal force \vec{F} of magnitude 100 N begins to pull directly on the block, as shown. In unit-vector notation, what are the resulting accelerations of (a) the block and (b) the slab?

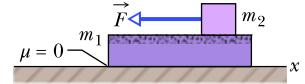


Figure 6.26 Problem 34.

- 35 H** The two blocks ($m = 16 \text{ kg}$ and $M = 88 \text{ kg}$) in Fig. 6.27 are not attached to each other. The coefficient of static friction between the blocks is $\mu_s = 0.38$, but the surface beneath the larger block is frictionless. What is the minimum magnitude of the horizontal force \vec{F} required to keep the smaller block from slipping down the larger block?

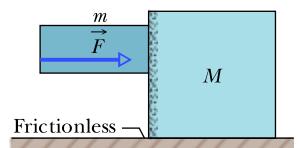


Figure 6.27 Problem 35.

Module 6.2 The Drag Force and Terminal Speed

- 36 E** The terminal speed of a sky diver is 160 km/h in the spread-eagle position and 310 km/h in the nosedive position. Assuming that the diver's drag coefficient C does not change from one position to the other, find the ratio of the effective cross-sectional area A in the slower position to that in the faster position.

- 37 M FCP** Continuation of Problem 8. Now assume that Eq. 6.2.1 gives the magnitude of the air drag force on the typical 20 kg stone, which presents to the wind a vertical cross-sectional area of 0.040 m^2 and has a drag coefficient C of 0.80. Take

the air density to be 1.21 kg/m^3 , and the coefficient of kinetic friction to be 0.80. (a) In kilometers per hour, what wind speed V along the ground is needed to maintain the stone's motion once it has started moving? Because winds along the ground are retarded by the ground, the wind speeds reported for storms are often measured at a height of 10 m. Assume wind speeds are 2.00 times those along the ground. (b) For your answer to (a), what wind speed would be reported for the storm? (c) Is that value reasonable for a high-speed wind in a storm? (Story continues with Problem 65.)

38 M Assume Eq. 6.2.1 gives the drag force on a pilot plus ejection seat just after they are ejected from a plane traveling horizontally at 1300 km/h. Assume also that the mass of the seat is equal to the mass of the pilot and that the drag coefficient is that of a sky diver. Making a reasonable guess of the pilot's mass and using the appropriate v_t value from Table 6.2.1, estimate the magnitudes of (a) the drag force on the *pilot + seat* and (b) their horizontal deceleration (in terms of g), both just after ejection. (The result of (a) should indicate an engineering requirement: The seat must include a protective barrier to deflect the initial wind blast away from the pilot's head.)

39 M Calculate the ratio of the drag force on a jet flying at 1000 km/h at an altitude of 10 km to the drag force on a prop-driven transport flying at half that speed and altitude. The density of air is 0.38 kg/m^3 at 10 km and 0.67 kg/m^3 at 5.0 km. Assume that the airplanes have the same effective cross-sectional area and drag coefficient C .

40 M FCP In downhill speed skiing a skier is retarded by both the air drag force on the body and the kinetic frictional force on the skis. (a) Suppose the slope angle is $\theta = 40.0^\circ$, the snow is dry snow with a coefficient of kinetic friction $\mu_k = 0.0400$, the mass of the skier and equipment is $m = 85.0 \text{ kg}$, the cross-sectional area of the (tucked) skier is $A = 1.30 \text{ m}^2$, the drag coefficient is $C = 0.150$, and the air density is 1.20 kg/m^3 . (a) What is the terminal speed? (b) If a skier can vary C by a slight amount dC by adjusting, say, the hand positions, what is the corresponding variation in the terminal speed?

Module 6.3 Uniform Circular Motion

41 E A cat dozes on a stationary merry-go-round in an amusement park, at a radius of 5.4 m from the center of the ride. Then the operator turns on the ride and brings it up to its proper turning rate of one complete rotation every 6.0 s. What is the least coefficient of static friction between the cat and the merry-go-round that will allow the cat to stay in place, without sliding (or the cat clinging with its claws)?

42 E Suppose the coefficient of static friction between the road and the tires on a car is 0.60 and the car has no negative lift. What speed will put the car on the verge of sliding as it rounds a level curve of 30.5 m radius?

43 E What is the smallest radius of an unbanked (flat) track around which a bicyclist can travel if her speed is 29 km/h and the μ_s between tires and track is 0.32?

44 E During an Olympic bobsled run, the Jamaican team makes a turn of radius 7.6 m at a speed of 96.6 km/h. What is their acceleration in terms of g ?

45 M SSM FCP A student of weight 667 N rides a steadily rotating Ferris wheel (the student sits upright). At the highest point, the magnitude of the normal force F_N on the student from the seat is 556 N. (a) Does the student feel "light" or "heavy" there? (b) What is the magnitude of F_N at the lowest point?

If the wheel's speed is doubled, what is the magnitude F_N at the (c) highest and (d) lowest point?

46 M A police officer in hot pursuit drives her car through a circular turn of radius 300 m with a constant speed of 80.0 km/h. Her mass is 55.0 kg. What are (a) the magnitude and (b) the angle (relative to vertical) of the net force of the officer on the car seat? (*Hint:* Consider both horizontal and vertical forces.)

47 M FCP A circular-motion addict of mass 80 kg rides a Ferris wheel around in a vertical circle of radius 10 m at a constant speed of 6.1 m/s. (a) What is the period of the motion? What is the magnitude of the normal force on the addict from the seat when both go through (b) the highest point of the circular path and (c) the lowest point?

48 M FCP A roller-coaster car at an amusement park has a mass of 1200 kg when fully loaded with passengers. As the car passes over the top of a circular hill of radius 18 m, assume that its speed is not changing. At the top of the hill, what are the (a) magnitude F_N and (b) direction (up or down) of the normal force on the car from the track if the car's speed is $v = 11 \text{ m/s}$? What are (c) F_N and (d) the direction if $v = 14 \text{ m/s}$?

49 M GO In Fig. 6.28, a car is driven at constant speed over a circular hill and then into a circular valley with the same radius. At the top of the hill, the normal force on the driver from the car seat is 0. The driver's mass is 70.0 kg. What is the magnitude of the normal force on the driver from the seat when the car passes through the bottom of the valley?

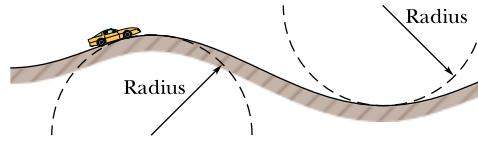


Figure 6.28 Problem 49.

50 M CALC An 85.0 kg passenger is made to move along a circular path of radius $r = 3.50 \text{ m}$ in uniform circular motion. (a) Figure 6.29a is a plot of the required magnitude F of the net centripetal force for a range of possible values of the passenger's speed v . What is the plot's slope at $v = 8.30 \text{ m/s}$? (b) Figure 6.29b is a plot of F for a range of possible values of T , the period of the motion. What is the plot's slope at $T = 2.50 \text{ s}$?

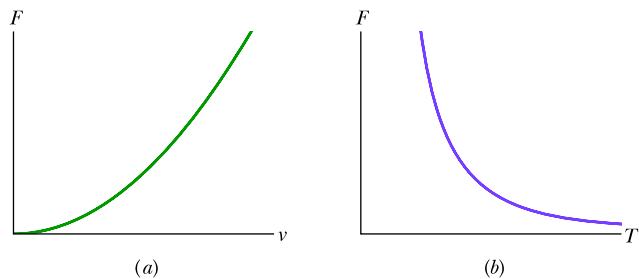


Figure 6.29 Problem 50.

51 M SSM An airplane is flying in a horizontal circle at a speed of 480 km/h (Fig. 6.30). If its wings are tilted at angle $\theta = 40^\circ$ to the horizontal, what is the radius of the circle in which the plane is flying? Assume that the required force is provided entirely by an "aerodynamic lift" that is perpendicular to the wing surface.

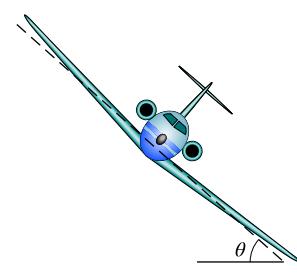


Figure 6.30 Problem 51.

52 M FCP An amusement park ride consists of a car moving in a vertical circle on the end of a rigid boom of negligible mass. The combined weight of the car and riders is 5.0 kN, and the circle's radius is 10 m. At the top of the circle, what are the (a) magnitude F_B and (b) direction (up or down) of the force on the car from the boom if the car's speed is $v = 5.0 \text{ m/s}$? What are (c) F_B and (d) the direction if $v = 12 \text{ m/s}$?

53 M An old streetcar rounds a flat corner of radius 9.1 m, at 16 km/h. What angle with the vertical will be made by the loosely hanging hand straps?

54 M CALC FCP In designing circular rides for amusement parks, mechanical engineers must consider how small variations in certain parameters can alter the net force on a passenger. Consider a passenger of mass m riding around a horizontal circle of radius r at speed v . What is the variation dF in the net force magnitude for (a) a variation dr in the radius with v held constant, (b) a variation dv in the speed with r held constant, and (c) a variation dT in the period with r held constant?

55 M A bolt is threaded onto one end of a thin horizontal rod, and the rod is then rotated horizontally about its other end. An engineer monitors the motion by flashing a strobe lamp onto the rod and bolt, adjusting the strobe rate until the bolt appears to be in the same eight places during each full rotation of the rod (Fig. 6.31). The strobe rate is 2000 flashes per second; the bolt has mass 30 g and is at radius 3.5 cm. What is the magnitude of the force on the bolt from the rod?

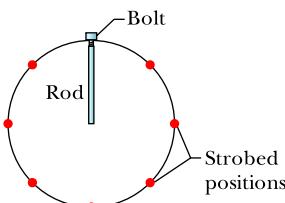


Figure 6.31 Problem 55.

56 M GO A banked circular highway curve is designed for traffic moving at 60 km/h. The radius of the curve is 200 m. Traffic is moving along the highway at 40 km/h on a rainy day. What is the minimum coefficient of friction between tires and road that will allow cars to take the turn without sliding off the road? (Assume the cars do not have negative lift.)

57 M GO A puck of mass $m = 1.50 \text{ kg}$ slides in a circle of radius $r = 20.0 \text{ cm}$ on a frictionless table while attached to a hanging cylinder of mass $M = 2.50 \text{ kg}$ by means of a cord that extends through a hole in the table (Fig. 6.32). What speed keeps the cylinder at rest?

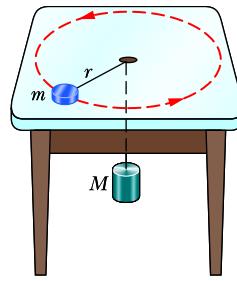


Figure 6.32 Problem 57.

58 M FCP *Brake or turn?* Figure 6.33 depicts an overhead view of a car's path as the car travels toward a wall. Assume that the driver begins to brake the car when the distance to the wall is $d = 107 \text{ m}$, and take the car's mass as $m = 1400 \text{ kg}$, its initial speed as $v_0 = 35 \text{ m/s}$, and the coefficient of static friction as $\mu_s = 0.50$. Assume that the car's weight is distributed evenly on the four wheels, even during braking. (a) What magnitude of static friction is needed (between tires and road) to stop the car just as it reaches the wall? (b) What is the maximum possible static friction $f_{s,\max}$? (c) If the coefficient of kinetic friction between the (sliding) tires and the road is $\mu_k = 0.40$, at what speed will

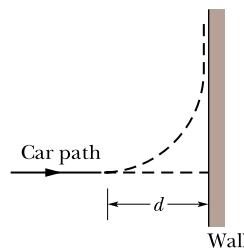


Figure 6.33
Problem 58.

the car hit the wall? To avoid the crash, a driver could elect to turn the car so that it just barely misses the wall, as shown in the figure. (d) What magnitude of frictional force would be required to keep the car in a circular path of radius d and at the given speed v_0 , so that the car moves in a quarter circle and then parallel to the wall? (e) Is the required force less than $f_{s,\max}$ so that a circular path is possible?

59 H SSM In Fig. 6.34, a 1.34 kg ball is connected by means of two massless strings, each of length $L = 1.70 \text{ m}$, to a vertical, rotating rod. The strings are tied to the rod with separation $d = 1.70 \text{ m}$ and are taut. The tension in the upper string is 35 N. What are the (a) tension in the lower string, (b) magnitude of the net force \vec{F}_{net} on the ball, and (c) speed of the ball? (d) What is the direction of $\vec{F}_{\text{net}}?$

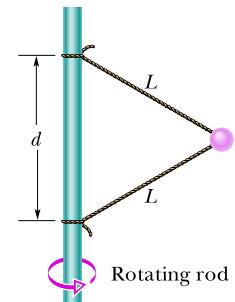


Figure 6.34
Problem 59.

Additional Problems

60 GO In Fig. 6.35, a box of ant aunts (total mass $m_1 = 1.65 \text{ kg}$) and a box of ant uncles (total mass $m_2 = 3.30 \text{ kg}$) slide down an inclined plane while attached by a massless rod parallel to the plane. The angle of incline is $\theta = 30.0^\circ$. The coefficient of kinetic friction between the aunt box and the incline is $\mu_1 = 0.226$; that between the uncle box and the incline is $\mu_2 = 0.113$. Compute (a) the tension in the rod and (b) the magnitude of the common acceleration of the two boxes. (c) How would the answers to (a) and (b) change if the uncles trailed the aunts?

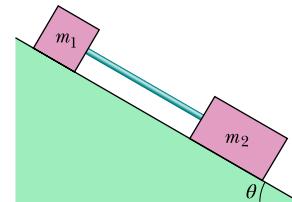


Figure 6.35 Problem 60.

61 SSM A block of mass $m_t = 4.0 \text{ kg}$ is put on top of a block of mass $m_b = 5.0 \text{ kg}$. To cause the top block to slip on the bottom one while the bottom one is held fixed, a horizontal force of at least 12 N must be applied to the top block. The assembly of blocks is now placed on a horizontal, frictionless table (Fig. 6.36). Find the magnitudes of (a) the maximum horizontal force \vec{F} that can be applied to the lower block so that the blocks will move together and (b) the resulting acceleration of the blocks.

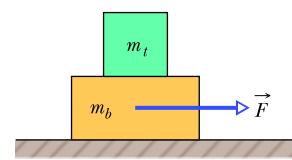


Figure 6.36 Problem 61.

62 A 5.00 kg stone is rubbed across the horizontal ceiling of a cave passageway (Fig. 6.37). If the coefficient of kinetic friction is 0.65 and the force applied to the stone is angled at $\theta = 70.0^\circ$, what must the magnitude of the force be for the stone to move at constant velocity?

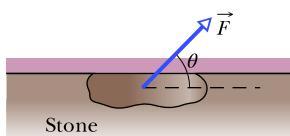


Figure 6.37 Problem 62.

- 63 BIO FCP** In Fig. 6.38, a 49 kg rock climber is climbing a “chimney.” The coefficient of static friction between her shoes and the rock is 1.2; between her back and the rock is 0.80. She has reduced her push against the rock until her back and her shoes are on the verge of slipping. (a) Draw a free-body diagram of her. (b) What is the magnitude of her push against the rock? (c) What fraction of her weight is supported by the frictional force on her shoes?

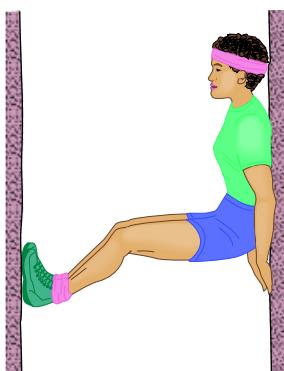


Figure 6.38 Problem 63.

- 64** A high-speed railway car goes around a flat, horizontal circle of radius 470 m at a constant speed. The magnitudes of the horizontal and vertical components of the force of the car on a 51.0 kg passenger are 210 N and 500 N, respectively. (a) What is the magnitude of the net force (of *all* the forces) on the passenger? (b) What is the speed of the car?

65 FCP Continuation of Problems 8 and 37. Another explanation is that the stones move only when the water dumped on the playa during a storm freezes into a large, thin sheet of ice. The stones are trapped in place in the ice. Then, as air flows across the ice during a wind, the air-drag forces on the ice and stones move them both, with the stones gouging out the trails. The magnitude of the air-drag force on this horizontal “ice sail” is given by $D_{\text{ice}} = 4C_{\text{ice}}\rho A_{\text{ice}}v^2$, where C_{ice} is the drag coefficient (2.0×10^{-3}), ρ is the air density (1.21 kg/m^3), A_{ice} is the horizontal area of the ice, and v is the wind speed along the ice.

Assume the following: The ice sheet measures 400 m by 500 m by 4.0 mm and has a coefficient of kinetic friction of 0.10 with the ground and a density of 917 kg/m^3 . Also assume that 100 stones identical to the one in Problem 8 are trapped in the ice. To maintain the motion of the sheet, what are the required wind speeds (a) near the sheet and (b) at a height of 10 m? (c) Are these reasonable values for high-speed winds in a storm?

- 66 GO** In Fig. 6.39, block 1 of mass $m_1 = 2.0 \text{ kg}$ and block 2 of mass $m_2 = 3.0 \text{ kg}$ are connected by a string of negligible mass and are initially held in place. Block 2 is on a frictionless surface tilted at $\theta = 30^\circ$. The coefficient of kinetic friction between block 1 and the horizontal surface is 0.25. The pulley has negligible mass and friction. Once they are released, the blocks move. What then is the tension in the string?

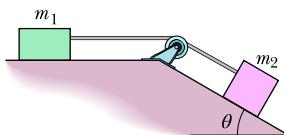


Figure 6.39 Problem 66.

- 67** In Fig. 6.40, a crate slides down an inclined right-angled trough. The coefficient of kinetic friction between the crate and the trough is μ_k . What is the acceleration of the crate in terms of μ_k , θ , and g ?

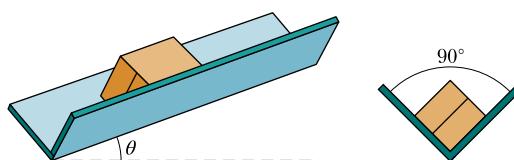


Figure 6.40 Problem 67.

- 68 Engineering a highway curve.** If a car goes through a curve too fast, the car tends to slide out of the curve. For a banked curve with friction, a frictional force acts on a fast car to oppose the tendency to slide out of the curve; the force is directed down the bank (in the direction water would drain). Consider a circular curve of radius $R = 200 \text{ m}$ and bank angle θ , where the coefficient of static friction between tires and pavement is μ_s . A car (without negative lift) is driven around the curve. (a) Find an expression for the car speed v_{\max} that puts the car on the verge of sliding out. (b) On the same graph, plot v_{\max} versus angle θ for the range 0° to 50° , first for $\mu_s = 0.60$ (dry pavement) and then for $\mu_s = 0.050$ (wet or icy pavement). In kilometers per hour, evaluate v_{\max} for a bank angle of $\theta = 10^\circ$ and for (c) $\mu_s = 0.60$ and (d) $\mu_s = 0.050$. (Now you can see why accidents occur in highway curves when icy conditions are not obvious to drivers, who tend to drive at normal speeds.)

- 69** A student, crazed by final exams, uses a force \vec{P} of magnitude 80 N and angle $\theta = 70^\circ$ to push a 5.0 kg block across the ceiling of his room (Fig. 6.41). If the coefficient of kinetic friction between the block and the ceiling is 0.40, what is the magnitude of the block’s acceleration?

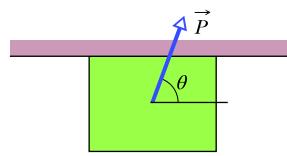


Figure 6.41 Problem 69.

- 70 GO** Figure 6.42 shows a *conical pendulum*, in which the bob (the small object at the lower end of the cord) moves in a horizontal circle at constant speed. (The cord sweeps out a cone as the bob rotates.) The bob has a mass of 0.040 kg , the string has length $L = 0.90 \text{ m}$ and negligible mass, and the bob follows a circular path of circumference 0.94 m . What are (a) the tension in the string and (b) the period of the motion?

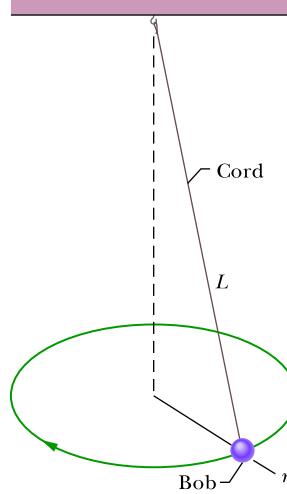


Figure 6.42 Problem 70.

71 An 8.00 kg block of steel is at rest on a horizontal table. The coefficient of static friction between the block and the table is 0.450. A force is to be applied to the block. To three significant figures, what is the magnitude of that applied force if it puts the block on the verge of sliding when the force is directed (a) horizontally, (b) upward at 60.0° from the horizontal, and (c) downward at 60.0° from the horizontal?

72 A box of canned goods slides down a ramp from street level into the basement of a grocery store with acceleration 0.75 m/s^2 directed down the ramp. The ramp makes an angle of 40° with the horizontal. What is the coefficient of kinetic friction between the box and the ramp?

73 In Fig. 6.43, the coefficient of kinetic friction between the block and inclined plane is 0.20, and angle θ is 60° . What are the (a) magnitude a and (b) direction (up or down the plane) of the block's acceleration if the block is sliding down the plane? What are (c) a and (d) the direction if the block is sent sliding up the plane?

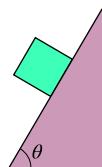


Figure 6.43
Problem 73.

74 A 110 g hockey puck sent sliding over ice is stopped in 15 m by the frictional force on it from the ice. (a) If its initial speed is 6.0 m/s , what is the magnitude of the frictional force? (b) What is the coefficient of friction between the puck and the ice?

75 A locomotive accelerates a 25-car train along a level track. Every car has a mass of $5.0 \times 10^4 \text{ kg}$ and is subject to a frictional force $f = 250v$, where the speed v is in meters per second and the force f is in newtons. At the instant when the speed of the train is 30 km/h , the magnitude of its acceleration is 0.20 m/s^2 . (a) What is the tension in the coupling between the first car and the locomotive? (b) If this tension is equal to the maximum force the locomotive can exert on the train, what is the steepest grade up which the locomotive can pull the train at 30 km/h ?

76 A house is built on the top of a hill with a nearby slope at angle $\theta = 45^\circ$ (Fig. 6.44). An engineering study indicates that the slope angle should be reduced because the top layers of soil along the slope might slip past the lower layers. If the coefficient of static friction between two such layers is 0.5, what is the least angle ϕ through which the present slope should be reduced to prevent slippage?

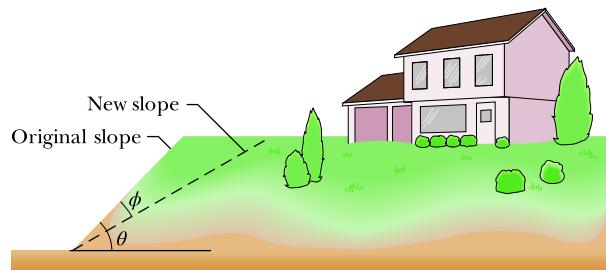


Figure 6.44 Problem 76.

77 What is the terminal speed of a 6.00 kg spherical ball that has a radius of 3.00 cm and a drag coefficient of 1.60? The density of the air through which the ball falls is 1.20 kg/m^3 .

78 A student wants to determine the coefficients of static friction and kinetic friction between a box and a plank. She places the box on the plank and gradually raises one end of

the plank. When the angle of inclination with the horizontal reaches 30° , the box starts to slip, and it then slides 2.5 m down the plank in 4.0 s at constant acceleration. What are (a) the coefficient of static friction and (b) the coefficient of kinetic friction between the box and the plank?

79 SSM Block A in Fig. 6.45 has mass $m_A = 4.0 \text{ kg}$, and block B has mass $m_B = 2.0 \text{ kg}$. The coefficient of kinetic friction between block B and the horizontal plane is $\mu_k = 0.50$. The inclined plane is frictionless and at angle $\theta = 30^\circ$. The pulley serves only to change the direction of the cord connecting the blocks. The cord has negligible mass. Find (a) the tension in the cord and (b) the magnitude of the acceleration of the blocks.

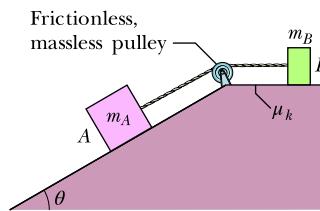


Figure 6.45 Problem 79.

80 Calculate the magnitude of the drag force on a missile 53 cm in diameter cruising at 250 m/s at low altitude, where the density of air is 1.2 kg/m^3 . Assume $C = 0.75$.

81 SSM A bicyclist travels in a circle of radius 25.0 m at a constant speed of 9.00 m/s . The bicycle-rider mass is 85.0 kg . Calculate the magnitudes of (a) the force of friction on the bicycle from the road and (b) the net force on the bicycle from the road.

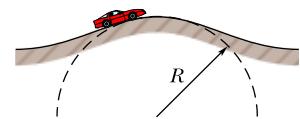


Figure 6.46 Problem 82.

82 In Fig. 6.46, a stuntman drives a car (without negative lift) over the top of a hill, the cross section of which can be approximated by a circle of radius $R = 250 \text{ m}$. What is the greatest speed at which he can drive without the car leaving the road at the top of the hill?

83 You must push a crate across a floor to a docking bay. The crate weighs 165 N. The coefficient of static friction between crate and floor is 0.510, and the coefficient of kinetic friction is 0.32. Your force on the crate is directed horizontally. (a) What magnitude of your push puts the crate on the verge of sliding? (b) With what magnitude must you then push to keep the crate moving at a constant velocity? (c) If, instead, you then push with the same magnitude as the answer to (a), what is the magnitude of the crate's acceleration?

84 In Fig. 6.47, force \vec{F} is applied to a crate of mass m on a floor where the coefficient of static friction between crate and floor is μ_s . Angle θ is initially 0° but is gradually increased so that the force vector rotates clockwise in the figure. During the rotation, the magnitude F of the force is continuously adjusted so that the crate is always on the verge of sliding. For $\mu_s = 0.70$, (a) plot the ratio F/mg versus θ and (b) determine the angle θ_{inf} at which the ratio approaches an infinite value. (c) Does lubricating the floor increase or decrease θ_{inf} , or is the value unchanged? (d) What is θ_{inf} for $\mu_s = 0.60$?

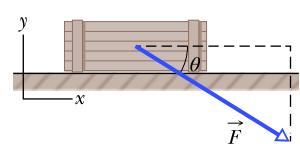


Figure 6.47 Problem 84.

85 In the early afternoon, a car is parked on a street that runs down a steep hill, at an angle of 35.0° relative to the horizontal. Just then the coefficient of static friction between the tires and the street surface is 0.725. Later, after nightfall, a sleet storm hits the area, and the coefficient decreases due to both the ice and a chemical change in the road surface because of the temperature decrease. By what percentage must the coefficient decrease if the car is to be in danger of sliding down the street?

86 FCP A sling-thrower puts a stone (0.250 kg) in the sling's pouch (0.010 kg) and then begins to make the stone and pouch move in a vertical circle of radius 0.650 m. The cord between the pouch and the person's hand has negligible mass and will break when the tension in the cord is 33.0 N or more. Suppose the sling-thrower can gradually increase the speed of the stone. (a) Will the breaking occur at the lowest point of the circle or at the highest point? (b) At what speed of the stone will that breaking occur?

87 SSM A car weighing 10.7 kN and traveling at 13.4 m/s without negative lift attempts to round an unbanked curve with a radius of 61.0 m. (a) What magnitude of the frictional force on the tires is required to keep the car on its circular path? (b) If the coefficient of static friction between the tires and the road is 0.350, is the attempt at taking the curve successful?

88 In Fig. 6.48, block 1 of mass $m_1 = 2.0 \text{ kg}$ and block 2 of mass $m_2 = 1.0 \text{ kg}$ are connected by a string of negligible mass. Block 2 is pushed by force \vec{F} of magnitude 20 N and angle $\theta = 35^\circ$. The coefficient of kinetic friction between each block and the horizontal surface is 0.20. What is the tension in the string?

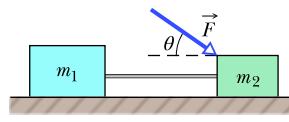


Figure 6.48 Problem 88.

89 SSM A filing cabinet weighing 556 N rests on the floor. The coefficient of static friction between it and the floor is 0.68, and the coefficient of kinetic friction is 0.56. In four different attempts to move it, it is pushed with horizontal forces of magnitudes (a) 222 N, (b) 334 N, (c) 445 N, and (d) 556 N. For each attempt, calculate the magnitude of the frictional force on it from the floor. (The cabinet is initially at rest.) (e) In which of the attempts does the cabinet move?

90 In Fig. 6.49, a block weighing 22 N is held at rest against a vertical wall by a horizontal force \vec{F} of magnitude 60 N. The coefficient of static friction between the wall and the block is 0.55, and the coefficient of kinetic friction between them is 0.38. In six experiments, a second force \vec{P} is applied to the block and directed parallel to the wall with these magnitudes and directions: (a) 34 N, up, (b) 12 N, up, (c) 48 N, up, (d) 62 N, up, (e) 10 N, down, and (f) 18 N, down.

In each experiment, what is the magnitude of the frictional force on the block? In which does the block move (g) up the wall and (h) down the wall? (i) In which is the frictional force directed down the wall?

91 Down and up. (a) Let an x axis extend up a plane inclined at angle $\theta = 60^\circ$ with the horizontal. What is the acceleration of a block sliding down the plane if the coefficient of kinetic friction between the block and plane is 0.20? (b) What is the

acceleration if the block is given an upward shove and is still sliding up the slope?

92 Airport luggage. In an airport, luggage is transported from one location to another by a conveyor belt. At a certain location, the belt moves down an incline that makes an angle of 2.5° with the horizontal. Assume that with such a slight angle there is no slipping of the luggage and that an x axis extends up the incline. Determine the frictional force from the belt on a box with weight 69 N when the box is on that incline for the following situations: (a) The belt is stationary. (b) The belt has a constant speed of 0.65 m/s. (c) The belt has a speed of 0.65 m/s that is increasing at a rate of 0.20 m/s^2 . (d) The belt has a speed of 0.65 m/s that is decreasing at a rate of 0.20 m/s^2 . (e) The belt has a speed of 0.65 m/s that is increasing at a rate of 0.57 m/s^2 .

93 How far? A block slides down an inclined plane of slope angle θ with a constant velocity. It is then projected up the plane with an initial speed v_0 . (a) How far up the plane will it move before coming to rest? (b) Will it slide down again?

94 Turntable candy. An M&M candy piece is placed on a stationary turntable, at a distance of 6.6 cm from the center. Then the turntable is turned on and its turning rate adjusted so that it makes 2.00 full turns in 2.45 s. The candy rides the turntable without slipping. (a) What is the magnitude of its acceleration? (b) If the turntable rate is gradually increased to 2.00 full turns every 1.80 s, the candy piece then begins to slide. What is the coefficient of static friction between it and the turntable?

95 Circling the Galaxy. The Solar System is moving along an approximately circular path of radius $2.5 \times 10^4 \text{ ly}$ (light-years) around the center of the Milky Way Galaxy with a speed of 205 km/s. (a) In years, what is the period of the circling? (b) The mass of the Solar System is approximately equal to the mass of the Sun ($1.99 \times 10^{30} \text{ kg}$). What is the magnitude of the centripetal force on the Solar System in the circling?

96 BIO Icy curb ramps. Slipping and falling while walking on an icy surface is a very common occurrence in winter weather. The danger comes when a person steps forward because then the full weight is over only the rear foot while that foot pushes backward. The static coefficient of friction of a common shoe (flat shoe or pump) on ice is $\mu_s = 0.050$. Let the person's mass be $m = 70 \text{ kg}$. (a) On a level surface, what magnitude F of the horizontal backward push puts the person on the verge of slipping? (b) Most urban communities require curb ramps (Fig. 6.50) so that wheelchairs can travel between sidewalk and street. The common allowed slope is for a rise of 1 distance unit to a (horizontal) length of 12 distance units, which gives a slope of 8.33%. If the person attempted to stand on such a ramp that is covered with ice, what would be the magnitude of the component of the gravitational force on the person down the ramp? (c) What would be the maximum magnitude of the static frictional force up the ramp? (d) Can the person stand like that without slipping (or walk up the ramp)? (e) What is the maximum angle θ_{verge} of a ramp such that the person is on the verge of slipping? (f) Some brands of winter footwear allow a person to walk up a ramp that is at the maximum allowed angle of 7.0° without slipping. If the backward force magnitude that puts the person on the verge of slipping is 34 N, what is the coefficient of static friction between the footwear and the ice?

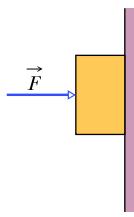


Figure 6.49
Problem 90.

Sidney de Almeida/Shutterstock.com

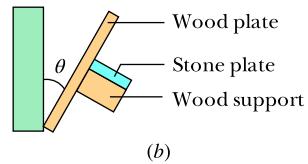
**Figure 6.50** Problem 96.

97 Braking distance in snow. You are driving at 60 mi/h (=26.8 m/s). Assume that you then brake at a constant acceleration with the maximum static friction on the tires from the roadway (no sliding). What is your minimum braking distance if the coefficient of static friction μ_s is (a) 0.70 for dry asphalt, (b) 0.30 for a snow layer of depth 2.5 mm, and (c) 0.15 for a snow layer of a depth 3.20 cm? (The lesson here is simple: Slow down when driving in snow.)

98 BIO Rock climbing chalk. Many rock climbers, whether outdoors or in a gym, periodically dip their fingers into a chalk bag hanging from the back of their belt to coat their fingers (Fig. 6.51a). (The chalk is magnesium carbonate, not classroom chalk.) Climbers claim that the chalk dries their fingers from sweat, allowing better grips on the holds but other climbers doubt that effect. In a research experiment, experienced climbers supported their weight on a *hang bar*, something commonly used in training. However, this hang bar could be tilted (Fig. 6.51b). On its wood support, researchers attached a stone plate from which a climber would hang by one hand, and then the researchers increased the tilt angle θ until the climber slipped off. From the slip angle θ_{slip} we can calculate the coefficient of static friction μ_s between the fingers and the stone plate. What is μ_s for limestone when θ_{slip} is (a) 32.6° for unchalked fingers and (b) 37.2° for chalked fingers? What is μ_s for sandstone when θ_{slip} is (a) 36.5° for unchalked fingers and (b) 41.9° for chalked fingers?



aluxum/GettyImages

**Figure 6.51** Problem 98. (a) Chalk used in rock climbing. (b) Hang bar arrangement.

99 Sliding down a ski slope on your back. Suppose you fall off your skis while skiing down a uniform slope at a moderate speed of 12 m/s. If you are wearing common ski overalls and land on your back, the coefficient of kinetic friction between the overalls and the snow is 0.25. Assume an *x* axis extends up the slope. What is your velocity along the axis when you hit a tree after sliding for 7.0 s if you are on (a) a *blue slope* (for beginners) at slope angle 12°, (b) a *red slope* at angle 18°, and (c) a *black slope* at angle 25°? (d) If you avoid all obstacles, on which slope will you slide to a stop and how far will that take?

100 Car on an icy hill—destined for YouTube video. Some of the funniest videos on YouTube show cars in uncontrolled slides on icy roads, especially icy hills. Here is an example of such a slide. A car with an initial speed $v_0 = 10.0$ m/s slides down a long icy road with an inclination of $\theta = 5.00^\circ$ (a mild incline, nothing like the hills of San Francisco). The coefficient of kinetic friction between the tires and the ice is $\mu_k = 0.10$. How far does the car take to slide to a stop if we assume that it does not hit anything along the way?

101 Controlled platoon of cars. A platoon of cars moves along a road under computer control with uniform spacings and equal speeds that are set by a dynamic system that monitors the weather road conditions. It determines the speed for a given spacing such that, in an emergency, each car would stop just as it reached the car in front of it. The coefficient of static friction μ_s is 0.700 for dry asphalt and 0.300 for rain-wetted asphalt. The cars each have length $L = 4.50$ m. What should the controlled speed be (meters per second, miles per hour, and kilometers per hour) if the bumper-to-bumper spacing is $2L$ for (a) dry asphalt and (b) wet asphalt? What should it be if the spacing is $5L$ for (a) dry asphalt and (b) wet asphalt?