C H A P T E R 30

Induction and Inductance

30.1 FARADAY'S LAW AND LENZ'S LAW

Learning Objectives

After reading this module, you should be able to . . .

30.1.1 Identify that the amount of magnetic field piercing a surface (not skimming along the surface) is the magnetic flux Φ_B through the surface.

30.1.2 Identify that an area vector for a flat surface is a vector that is perpendicular to the surface and that has a magnitude equal to the area of the surface.

30.1.3 Identify that any surface can be divided into area elements (patch elements) that are each small enough and flat enough for an area vector $d\vec{A}$ to be assigned to it, with the vector perpendicular to the element and having a magnitude equal to the area of the element.

30.1.4 Calculate the magnetic flux Φ_B through a surface by integrating the dot product of the magnetic field vector $\vec{B}$ and the area vector $d\vec{A}$ (for patch elements) over the surface, in magnitude-angle notation and unit-vector notation.

30.1.5 Identify that a current is induced in a conducting loop while the number of magnetic field lines intercepted by the loop is changing.

30.1.6 Identify that an induced current in a conducting loop is driven by an induced emf.

30.1.7 Apply Faraday's law, which is the relationship between an induced emf in a conducting loop and the rate at which magnetic flux through the loop changes.

30.1.8 Extend Faraday's law from a loop to a coil with multiple loops.

30.1.9 Identify the three general ways in which the magnetic flux through a coil can change.

30.1.10 Use a right-hand rule for Lenz's law to determine the direction of induced emf and induced current in a conducting loop.

30.1.11 Identify that when a magnetic flux through a loop changes, the induced current in the loop sets up a magnetic field to oppose that change.

30.1.12 If an emf is induced in a conducting loop containing a battery, determine the net emf and calculate the corresponding current in the loop.

Key Ideas

- The magnetic flux Φ_B through an area A in a magnetic field $\vec{B}$ is defined as

$$\Phi_B = \int \vec{B} \cdot d\vec{A},$$

where the integral is taken over the area. The SI unit of magnetic flux is the weber, where $1\ \mathrm{Wb} = 1\ \mathrm{T \cdot m^2}$.

- If $\vec{B}$ is perpendicular to the area and uniform over it, the flux is

$$\Phi_B = BA \quad (\vec{B} \perp A, \vec{B} \text{ uniform}).$$

- If the magnetic flux Φ_B through an area bounded by a closed conducting loop changes with time, a current and an emf are produced in the loop; this process is called induction. The induced emf is

$$\mathscr{E} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's law)}.$$

- If the loop is replaced by a closely packed coil of N turns, the induced emf is

$$\mathscr{E} = -N\frac{d\Phi_B}{dt}.$$

- An induced current has a direction such that the magnetic field *due to the current* opposes the change in the magnetic flux that induces the current. The induced emf has the same direction as the induced current.

What Is Physics?

In Chapter 29 we discussed the fact that a current produces a magnetic field. That fact came as a surprise to the scientists who discovered the effect. Perhaps even more surprising was the discovery of the reverse effect: A magnetic field can produce an electric field that can drive a current. This link between a magnetic field and the electric field it produces (*induces*) is now called *Faraday's law of induction.*

The observations by Michael Faraday and other scientists that led to this law were at first just basic science. Today, however, applications of that basic science are almost everywhere. For example, induction is the basis of the electric guitars that revolutionized early rock and still drive heavy metal and punk today. It is also the basis of the electric generators that power cities and transportation lines and of the huge induction furnaces that are commonplace in foundries where large amounts of metal must be melted rapidly.

Before we get to applications like the electric guitar, we must examine two simple experiments about Faraday's law of induction.

Two Experiments

Let us examine two simple experiments to prepare for our discussion of Faraday's law of induction.

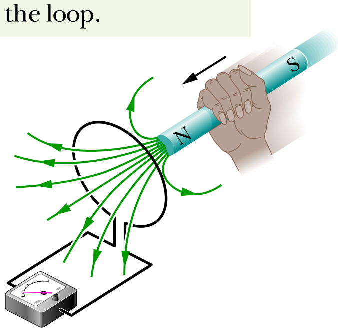


Figure 30.1.1 An ammeter registers a current in the wire loop when the magnet is moving with respect to the loop.

First Experiment. Figure 30.1.1 shows a conducting loop connected to a sensitive ammeter. Because there is no battery or other source of emf included, there is no current in the circuit. However, if we move a bar magnet toward the loop, a current suddenly appears in the circuit. The current disappears when the magnet stops. If we then move the magnet away, a current again suddenly appears, but now in the opposite direction. If we experimented for a while, we would discover the following:

1. A current appears only if there is relative motion between the loop and the magnet (one must move relative to the other); the current disappears when the relative motion between them ceases.
2. Faster motion produces a greater current.
3. If moving the magnet's north pole toward the loop causes, say, clockwise current, then moving the north pole away causes counterclockwise current. Moving the south pole toward or away from the loop also causes currents, but in the reversed directions.

The current produced in the loop is called an **induced current;** the work done per unit charge to produce that current (to move the conduction electrons that constitute the current) is called an **induced emf;** and the process of producing the current and emf is called **induction.**

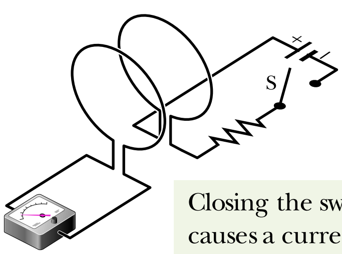


Figure 30.1.2 An ammeter registers a current in the left-hand wire loop just as switch S is closed (to turn on the current in the right-hand wire loop) or opened (to turn off the current in the right-hand loop). No motion of the coils is involved.

Second Experiment. For this experiment we use the apparatus of Fig. 30.1.2, with the two conducting loops close to each other but not touching. If we close switch S, to turn on a current in the right-hand loop, the meter suddenly and briefly registers a current—an induced current—in the left-hand loop. If we then open the switch, another sudden and brief induced current appears in the left-hand loop, but in the opposite direction. We get an induced current (and thus an induced emf) only when the current in the right-hand loop is changing (either turning on or turning off) and not when it is constant (even if it is large).

The induced emf and induced current in these experiments are apparently caused when something changes—but what is that "something"? Faraday knew.

Faraday's Law of Induction

Faraday realized that an emf and a current can be induced in a loop, as in our two experiments, by changing the *amount of magnetic field* passing through the loop. He further realized that the "amount of magnetic field" can be visualized in terms of the magnetic field lines passing through the loop. **Faraday's law of induction,** stated in terms of our experiments, is this:

> An emf is induced in the loop at the left in Figs. 30.1.1 and 30.1.2 when the number of magnetic field lines that pass through the loop is changing.

The actual number of field lines passing through the loop does not matter; the values of the induced emf and induced current are determined by the *rate* at which that number changes.

In our first experiment (Fig. 30.1.1), the magnetic field lines spread out from the north pole of the magnet. Thus, as we move the north pole closer to the loop, the number of field lines passing through the loop increases. That increase apparently causes conduction electrons in the loop to move (the induced current) and provides energy (the induced emf) for their motion. When the magnet stops moving, the number of field lines through the loop no longer changes and the induced current and induced emf disappear.

In our second experiment (Fig. 30.1.2), when the switch is open (no current), there are no field lines. However, when we turn on the current in the right-hand loop, the increasing current builds up a magnetic field around that loop and at the left-hand loop. While the field builds, the number of magnetic field lines through the left-hand loop increases. As in the first experiment, the increase in field lines through that loop apparently induces a current and an emf there. When the current in the right-hand loop reaches a final, steady value, the number of field lines through the left-hand loop no longer changes, and the induced current and induced emf disappear.

A Quantitative Treatment

To put Faraday's law to work, we need a way to calculate the *amount of magnetic field* that passes through a loop. In Chapter 23, in a similar situation, we needed to calculate the amount of electric field that passes through a surface. There we defined an electric flux $\Phi_E = \int \vec{E} \cdot d\vec{A}$. Here we define a *magnetic flux*: Suppose a loop enclosing an area A is placed in a magnetic field $\vec{B}$. Then the **magnetic flux** through the loop is

$$\Phi_B = \int \vec{B} \cdot d\vec{A} \quad \text{(magnetic flux through area } A). \tag{30.1.1}$$

As in Chapter 23, $d\vec{A}$ is a vector of magnitude dA that is perpendicular to a differential area dA. As with electric flux, we want the component of the field that *pierces* the surface (not skims along it). The dot product of the field and the area vector automatically gives us that piercing component.

Special Case. As a special case of Eq. 30.1.1, suppose that the loop lies in a plane and that the magnetic field is perpendicular to the plane of the loop. Then we can write the dot product in Eq. 30.1.1 as $B\ dA \cos 0° = B\ dA$. If the magnetic field is also uniform, then B can be brought out in front of the integral sign. The remaining $\int dA$ then gives just the area A of the loop. Thus, Eq. 30.1.1 reduces to

$$\Phi_B = BA \quad (\vec{B} \perp \text{area } A, \vec{B} \text{ uniform}). \tag{30.1.2}$$

Unit. From Eqs. 30.1.1 and 30.1.2, we see that the SI unit for magnetic flux is the tesla-square meter, which is called the *weber* (abbreviated Wb):

$$1 \text{ weber} = 1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2. \tag{30.1.3}$$

Faraday's Law. With the notion of magnetic flux, we can state Faraday's law in a more quantitative and useful way:

> The magnitude of the emf $\mathscr{E}$ induced in a conducting loop is equal to the rate at which the magnetic flux Φ_B through that loop changes with time.

As you will see below, the induced emf $\mathscr{E}$ tends to oppose the flux change, so Faraday's law is formally written as

$$\mathscr{E} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's law)}, \tag{30.1.4}$$

with the minus sign indicating that opposition. We often neglect the minus sign in Eq. 30.1.4, seeking only the magnitude of the induced emf.

If we change the magnetic flux through a coil of N turns, an induced emf appears in every turn and the total emf induced in the coil is the sum of these individual induced emfs. If the coil is tightly wound (*closely packed*), so that the same magnetic flux Φ_B passes through all the turns, the total emf induced in the coil is

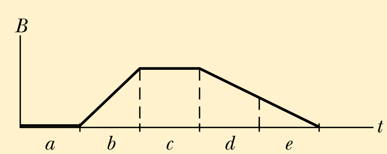
$$\mathscr{E} = -N\frac{d\Phi_B}{dt} \quad \text{(coil of } N \text{ turns)}. \tag{30.1.5}$$

Here are the general means by which we can change the magnetic flux through a coil:

1. Change the magnitude B of the magnetic field within the coil.
2. Change either the total area of the coil or the portion of that area that lies within the magnetic field (for example, by expanding the coil or sliding it into or out of the field).
3. Change the angle between the direction of the magnetic field $\vec{B}$ and the plane of the coil (for example, by rotating the coil so that field $\vec{B}$ is first perpendicular to the plane of the coil and then is along that plane).

Checkpoint 30.1.1

The graph gives the magnitude $B(t)$ of a uniform magnetic field that exists throughout a conducting loop, with the direction of the field perpendicular to the plane of the loop. Rank the five regions of the graph according to the magnitude of the emf induced in the loop, greatest first.



Sample Problem 30.1.1 Induced emf in coil due to a solenoid

The long solenoid S shown (in cross section) in Fig. 30.1.3 has 220 turns/cm and carries a current $i = 1.5$ A; its diameter D is 3.2 cm. At its center we place a 130-turn closely packed coil C of diameter $d = 2.1$ cm. The current in the solenoid is reduced to zero at a steady rate in 25 ms. What is the magnitude of the emf that is induced in coil C while the current in the solenoid is changing?

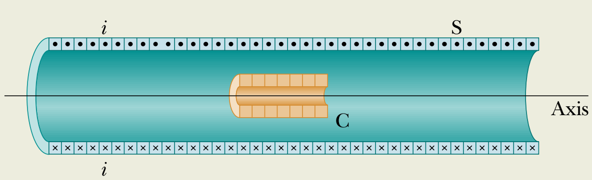


Figure 30.1.3 A coil C is located inside a solenoid S, which carries current i.

KEY IDEAS

1. Because it is located in the interior of the solenoid, coil C lies within the magnetic field produced by current i in the solenoid; thus, there is a magnetic flux Φ_B through coil C.
2. Because current i decreases, flux Φ_B also decreases.
3. As Φ_B decreases, emf $\mathscr{E}$ is induced in coil C.
4. The flux through each turn of coil C depends on the area A and orientation of that turn in the solenoid's magnetic field $\vec{B}$. Because $\vec{B}$ is uniform and directed perpendicular to area A, the flux is given by Eq. 30.1.2 ($\Phi_B = BA$).
5. The magnitude B of the magnetic field in the interior of a solenoid depends on the solenoid's current i and its number n of turns per unit length, according to Eq. 29.4.3 ($B = \mu_0 in$).

Calculations: Because coil C consists of more than one turn, we apply Faraday's law in the form of Eq. 30.1.5 ($\mathscr{E} = -N\,d\Phi_B/dt$), where the number of turns N is 130 and $d\Phi_B/dt$ is the rate at which the flux changes.

Because the current in the solenoid decreases at a steady rate, flux Φ_B also decreases at a steady rate, and so we can write $d\Phi_B/dt$ as $\Delta\Phi_B/\Delta t$. Then, to evaluate $\Delta\Phi_B$, we need the final and initial flux values. The final flux $\Phi_{B,f}$ is zero because the final current in the solenoid is zero. To find the initial flux $\Phi_{B,i}$, we note that area A is $\frac{1}{4}\pi d^2$ ($= 3.464 \times 10^{-4}$ m^2) and the number n is 220 turns/cm, or 22 000 turns/m. Substituting Eq. 29.4.3 into Eq. 30.1.2 then leads to

$$\begin{aligned}\Phi_{B,i} &= BA = (\mu_0 in)A\\ &= (4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(1.5\ \text{A})(22\,000\ \text{turns/m})\\ &\quad \times (3.464 \times 10^{-4}\ \text{m}^2)\\ &= 1.44 \times 10^{-5}\ \text{Wb}.\end{aligned}$$

Now we can write

$$\begin{aligned}\frac{d\Phi_B}{dt} &= \frac{\Delta\Phi_B}{\Delta t} = \frac{\Phi_{B,f} - \Phi_{B,i}}{\Delta t}\\ &= \frac{(0 - 1.44 \times 10^{-5}\ \text{Wb})}{25 \times 10^{-3}\ \text{s}}\\ &= -5.76 \times 10^{-4}\ \text{Wb/s}\\ &= -5.76 \times 10^{-4}\ \text{V}.\end{aligned}$$

We are interested only in magnitudes; so we ignore the minus signs here and in Eq. 30.1.5, writing

$$\begin{aligned}\mathscr{E} &= N\frac{d\Phi_B}{dt} = (130\ \text{turns})(5.76 \times 10^{-4}\ \text{V})\\ &= 7.5 \times 10^{-2}\ \text{V}\\ &= 75\ \text{mV}. \qquad \text{(Answer)}\end{aligned}$$

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Lenz's Law

Soon after Faraday proposed his law of induction, Heinrich Friedrich Lenz devised a rule for determining the direction of an induced current in a loop:

> An induced current has a direction such that the magnetic field due to *the current* opposes the change in the magnetic flux that induces the current.

Furthermore, the direction of an induced emf is that of the induced current. The key word in Lenz's law is "opposition." Let's apply the law to the motion of the north pole toward the conducting loop in Fig. 30.1.4.

1. ***Opposition to Pole Movement.*** The approach of the magnet's north pole in Fig. 30.1.4 increases the magnetic flux through the loop and thereby induces a current in the loop. From Fig. 29.5.1, we know that the loop then acts as a magnetic dipole with a south pole and a north pole, and that its magnetic dipole moment $\vec{\mu}$ is directed from south to north. To *oppose* the magnetic flux increase being caused by the approaching magnet, the loop's north pole (and thus $\vec{\mu}$) must face *toward* the approaching north pole so as to repel it (Fig. 30.1.4). Then the curled–straight right-hand rule for $\vec{\mu}$ (Fig. 29.5.1) tells us that the current induced in the loop must be counterclockwise in Fig. 30.1.4.

   If we next pull the magnet away from the loop, a current will again be induced in the loop. Now, however, the loop will have a south pole facing the retreating north pole of the magnet, so as to oppose the retreat. Thus, the induced current will be clockwise.

2. ***Opposition to Flux Change.*** In Fig. 30.1.4, with the magnet initially distant, no magnetic flux passes through the loop. As the north pole of the magnet then nears the loop with its magnetic field $\vec{B}$ directed *downward*, the flux through the loop increases. To oppose this increase in flux, the induced current i must set up its own field $\vec{B}_{\text{ind}}$ directed *upward* inside the loop, as shown in Fig. 30.1.5*a*; then the upward flux of field $\vec{B}_{\text{ind}}$ opposes the increasing downward flux of field $\vec{B}$. The curled–straight right-hand rule of Fig. 29.5.1 then tells us that i must be counterclockwise in Fig. 30.1.5*a*.

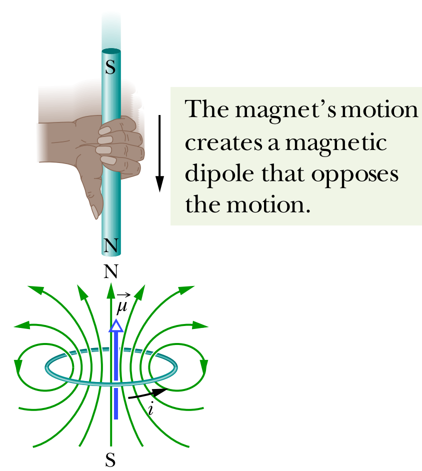


Figure 30.1.4 Lenz's law at work. As the magnet is moved toward the loop, a current is induced in the loop. The current produces its own magnetic field, with magnetic dipole moment $\vec{\mu}$ oriented so as to oppose the motion of the magnet. Thus, the induced current must be counterclockwise as shown.

Heads Up. The flux of $\vec{B}_{ind}$ always opposes the *change* in the flux of $\vec{B}$, but $\vec{B}_{ind}$ is not always opposite $\vec{B}$. For example, if we next pull the magnet away from the loop in Fig. 30.1.4, the magnet's flux Φ_B is still downward through the loop, but it is now decreasing. The flux of $\vec{B}_{ind}$ must now be downward inside the loop, to oppose that *decrease* (Fig. 30.1.5*b*). Thus, $\vec{B}_{ind}$ and $\vec{B}$ are now in the same direction. In Figs. 30.1.5*c* and *d*, the south pole of the magnet approaches and retreats from the loop, again with opposition to change.

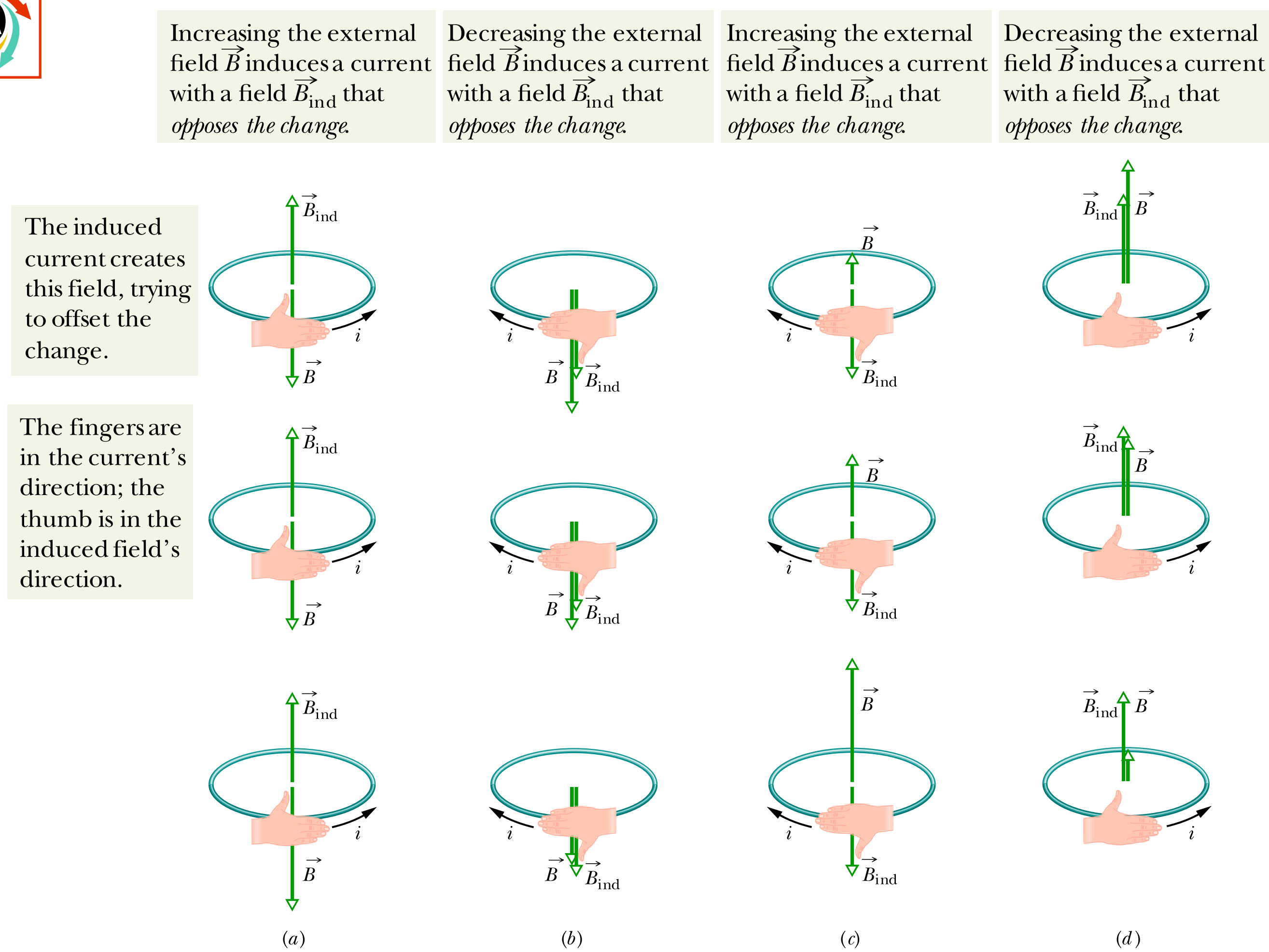


Figure 30.1.5 The direction of the current i induced in a loop is such that the current's magnetic field $\vec{B}_{ind}$ opposes the *change* in the magnetic field $\vec{B}$ inducing i. The field $\vec{B}_{ind}$ is always directed opposite an increasing field $\vec{B}$ (*a*, *c*) and in the same direction as a decreasing field $\vec{B}$ (*b*, *d*). The curled–straight right-hand rule gives the direction of the induced current based on the direction of the induced field.

Electric Guitars

Figure 30.1.6shows a Fender® Stratocaster®, one type of electric guitar. Whereas an acoustic guitar depends for its sound on the acoustic resonance produced in the hollow body of the instrument by the oscillations of the strings, an electric guitar is a solid instrument, so there is no body resonance. Instead, the oscillations of the metal strings are sensed by electric "pickups" that send signals to an amplifier and a set of speakers.

The basic construction of pickup is shown in Fig. 30.1.7. Wire connecting the instrument to the amplifier is coiled around a small magnet. The magnetic field of the magnet produces a north and south pole in the section of the metal string just above the magnet. That section of string then has its own magnetic field. When the string is plucked and thus made to oscillate, its motion relative to the coil changes the flux of its magnetic field through the coil, inducing a current in the coil. As the string oscillates toward and away from the coil, the induced current changes direction at the same frequency as the string's oscillations, thus relaying the frequency of oscillation to the amplifier and speaker.

On a Stratocaster, there are three groups of pickups, placed at the near end of the strings (on the wide part of the body). The group closest to the near end better detects the high-frequency oscillations of the strings; the group farthest from the near end better detects the low-frequency oscillations. By throwing a toggle switch on the guitar, the musician can select which group or which pair of groups will send signals to the amplifier and speakers.

To gain further control over his music, the legendary Jimi Hendrix sometimes rewrapped the wire in the pickup coils of his guitar to change the number of turns. In this way, he altered the amount of emf induced in the coils and thus their relative sensitivity to string oscillations. Even without this additional measure, you can see that the electric guitar offers far more control over the sound that is produced than can be obtained with an acoustic guitar.

Figure 30.1.6 A Fender® Stratocaster® guitar.

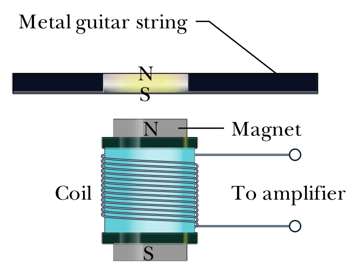
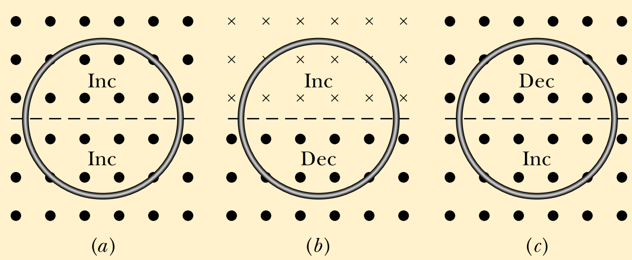


Figure 30.1.7 A side view of an electric guitar pickup. When the metal string (which acts like a magnet) is made to oscillate, it causes a variation in magnetic flux that induces a current in the coil.

Checkpoint 30.1.2

The figure shows three situations in which identical circular conducting loops are in uniform magnetic fields that are either increasing (Inc) or decreasing (Dec) in magnitude at identical rates. In each, the dashed line coincides with a diameter. Rank the situations according to the magnitude of the current induced in the loops, greatest first.



Sample Problem 30.1.2 Induced emf and current due to a changing uniform *B* field

Figure 30.1.8 shows a conducting loop consisting of a half-circle of radius $r = 0.20$ m and three straight sections. The half-circle lies in a uniform magnetic field $\vec{B}$ that is directed out of the page; the field magnitude is given by $B = 4.0t^2 + 2.0t + 3.0$, with B in teslas and t in seconds. An ideal battery with emf $\mathscr{E}_{bat} = 2.0$ V is connected to the loop. The resistance of the loop is 2.0 Ω.

(a) What are the magnitude and direction of the emf $\mathscr{E}_{ind}$ induced around the loop by field $\vec{B}$ at $t = 10$ s?

KEY IDEAS

1. According to Faraday's law, the magnitude of $\mathscr{E}_{ind}$ is equal to the rate $d\Phi_B/dt$ at which the magnetic flux through the loop changes.
2. The flux through the loop depends on how much of the loop's area lies within the flux and how the area is oriented in the magnetic field $\vec{B}$.
3. Because $\vec{B}$ is uniform and is perpendicular to the plane of the loop, the flux is given by Eq. 30.1.2 ($\Phi_B = BA$). (We don't need to integrate B over the area to get the flux.)
4. The induced field B_{ind} (due to the induced current) must always oppose the *change* in the magnetic flux.

Magnitude: Using Eq. 30.1.2 and realizing that only the field magnitude B changes in time (not the area A), we rewrite Faraday's law, Eq. 30.1.4, as

$$\mathscr{E}_{ind} = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A\frac{dB}{dt}.$$

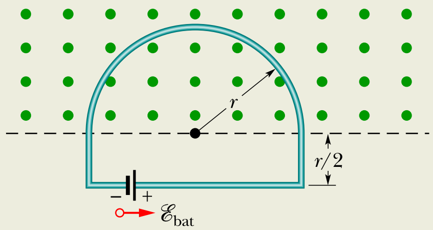


Figure 30.1.8 A battery is connected to a conducting loop that includes a half-circle of radius r lying in a uniform magnetic field. The field is directed out of the page; its magnitude is changing.

Because the flux penetrates the loop only within the half-circle, the area A in this equation is $\frac{1}{2}\pi r^2$. Substituting this and the given expression for B yields

$$\mathscr{E}_{ind} = A\frac{dB}{dt} = \frac{\pi r^2}{2}\frac{d}{dt}(4.0t^2 + 2.0t + 3.0)$$
$$= \frac{\pi r^2}{2}(8.0t + 2.0).$$

At $t = 10$ s, then,

$$\mathscr{E}_{ind} = \frac{\pi(0.20\text{ m})^2}{2}[8.0(10) + 2.0]$$
$$= 5.152\text{ V} \approx 5.2\text{ V}. \qquad \text{(Answer)}$$

Direction: To find the direction of $\mathscr{E}_{ind}$, we first note that in Fig. 30.1.8 the flux through the loop is out of the page and increasing. Because the induced field B_{ind} (due to the induced current) must oppose that increase, it must be *into* the page. Using the curled–straight right-hand rule (Fig. 30.1.5*c*), we find that the induced current is clockwise around the loop, and thus so is the induced emf $\mathscr{E}_{ind}$.

(b) What is the current in the loop at $t = 10$ s?

KEY IDEA

The point here is that *two* emfs tend to move charges around the loop.

Calculation: The induced emf $\mathscr{E}_{ind}$ tends to drive a current clockwise around the loop; the battery's emf $\mathscr{E}_{bat}$ tends to drive a current counterclockwise. Because $\mathscr{E}_{ind}$ is greater than $\mathscr{E}_{bat}$, the net emf $\mathscr{E}_{net}$ is clockwise, and thus so is the current. To find the current at $t = 10$ s, we use Eq. 27.1.2 ($i = \mathscr{E}/R$):

$$i = \frac{\mathscr{E}_{net}}{R} = \frac{\mathscr{E}_{ind} - \mathscr{E}_{bat}}{R}$$
$$= \frac{5.152\text{ V} - 2.0\text{ V}}{2.0\,\Omega} = 1.58\text{ A} \approx 1.6\text{ A}. \qquad \text{(Answer)}$$

Sample Problem 30.1.3 Induced emf due to a changing nonuniform *B* field

Figure 30.1.9 shows a rectangular loop of wire immersed in a nonuniform and varying magnetic field $\vec{B}$ that is perpendicular to and directed into the page. The field's magnitude is given by $B = 4t^2x^2$, with B in teslas, t in seconds, and x in meters. (Note that the function depends on *both* time and position.) The loop has width $W = 3.0$ m and height $H = 2.0$ m. What are the magnitude and direction of the induced emf $\mathscr{E}$ around the loop at $t = 0.10$ s?

KEY IDEAS

1. Because the magnitude of the magnetic field $\vec{B}$ is changing with time, the magnetic flux Φ_B through the loop is also changing.
2. The changing flux induces an emf $\mathscr{E}$ in the loop according to Faraday's law, which we can write as $\mathscr{E} = d\Phi_B/dt$.
3. To use that law, we need an expression for the flux Φ_B at any time t. However, because B is *not* uniform over the area enclosed by the loop, we *cannot* use Eq. 30.1.2 ($\Phi_B = BA$) to find that expression; instead we must use Eq. 30.1.1 ($\Phi_B = \int \vec{B} \cdot d\vec{A}$).

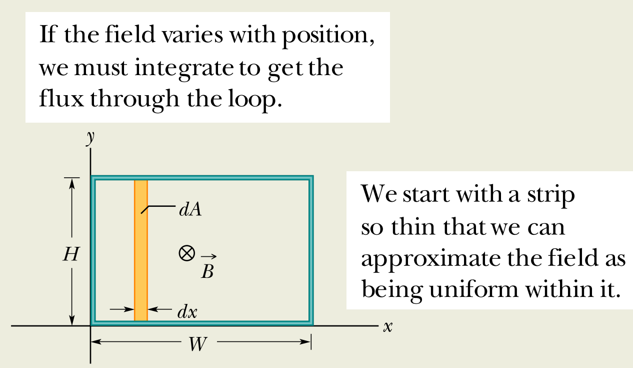


Figure 30.1.9 A closed conducting loop, of width W and height H, lies in a nonuniform, varying magnetic field that points directly into the page. To apply Faraday's law, we use the vertical strip of height H, width dx, and area dA.

Calculations: In Fig. 30.1.9, $\vec{B}$ is perpendicular to the plane of the loop (and hence parallel to the differential area vector $d\vec{A}$); so the dot product in Eq. 30.1.1 gives $B\,dA$. Because the magnetic field varies with the coordinate x but not with the coordinate y, we can take the differential area dA to be the area of a vertical strip of height H and width dx (as shown in Fig. 30.1.9). Then $dA = H\,dx$, and the flux through the loop is

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B\,dA = \int BH\,dx = \int 4t^2x^2H\,dx.$$

Treating t as a constant for this integration and inserting the integration limits $x = 0$ and $x = 3.0$ m, we obtain

$$\Phi_B = 4t^2 H \int_0^{3.0} x^2\,dx = 4t^2 H\left[\frac{x^3}{3}\right]_0^{3.0} = 72t^2,$$

where we have substituted $H = 2.0$ m and Φ_B is in webers. Now we can use Faraday's law to find the magnitude of $\mathscr{E}$ at any time t:

$$\mathscr{E} = \frac{d\Phi_B}{dt} = \frac{d(72t^2)}{dt} = 144t,$$

in which $\mathscr{E}$ is in volts. At $t = 0.10$ s,

$$\mathscr{E} = (144\text{ V/s})(0.10\text{ s}) \approx 14\text{ V}. \qquad \text{(Answer)}$$

The flux of $\vec{B}$ through the loop is into the page in Fig. 30.1.9 and is increasing in magnitude because B is increasing in magnitude with time. By Lenz's law, the field B_{ind} of the induced current opposes this increase and so is directed out of the page. The curled–straight right-hand rule in Fig. 30.1.5*a* then tells us that the induced current is counterclockwise around the loop, and thus so is the induced emf $\mathscr{E}$.

30.2 INDUCTION AND ENERGY TRANSFERS

Learning Objectives

After reading this module, you should be able to . . .

30.2.1 For a conducting loop pulled into or out of a magnetic field, calculate the rate at which energy is transferred to thermal energy.

30.2.2 Apply the relationship between an induced current and the rate at which it produces thermal energy.

30.2.3 Describe eddy currents.

Key Idea

- The induction of a current by a changing flux means that energy is being transferred to that current. The energy can then be transferred to other forms, such as thermal energy.

Induction and Energy Transfers

By Lenz's law, whether you move the magnet toward or away from the loop in Fig. 30.1.1, a magnetic force resists the motion, requiring your applied force to do positive work. At the same time, thermal energy is produced in the material of the loop because of the material's electrical resistance to the current that is induced by the motion. The energy you transfer to the closed *loop + magnet* system via your applied force ends up in this thermal energy. (For now, we neglect energy that is radiated away from the loop as electromagnetic waves during the induction.) The faster you move the magnet, the more rapidly your applied force does work and the greater the rate at which your energy is transferred to thermal energy in the loop; that is, the power of the transfer is greater.

Regardless of how current is induced in a loop, energy is always transferred to thermal energy during the process because of the electrical resistance of the loop (unless the loop is superconducting). For example, in Fig. 30.1.2, when switch S is closed and a current is briefly induced in the left-hand loop, energy is transferred from the battery to thermal energy in that loop.

Figure 30.2.1 shows another situation involving induced current. A rectangular loop of wire of width L has one end in a uniform external magnetic field that is directed perpendicularly into the plane of the loop. This field may be produced, for example, by a large electromagnet. The dashed lines in Fig. 30.2.1

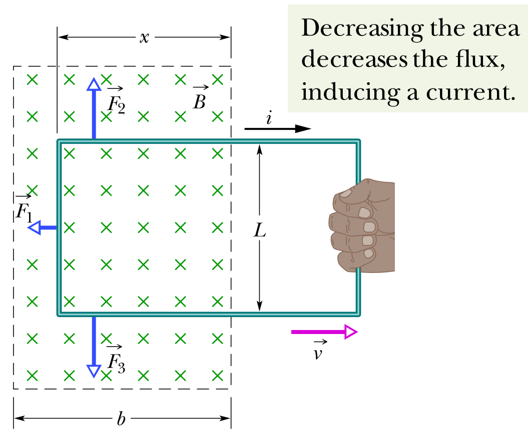


Figure 30.2.1 You pull a closed conducting loop out of a magnetic field at constant velocity $\vec{v}$. While the loop is moving, a clockwise current i is induced in the loop, and the loop segments still within the magnetic field experience forces $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$.

show the assumed limits of the magnetic field; the fringing of the field at its edges is neglected. You are to pull this loop to the right at a constant velocity $\vec{v}$.

Flux Change. The situation of Fig. 30.2.1 does not differ in any essential way from that of Fig. 30.1.1. In each case a magnetic field and a conducting loop are in relative motion; in each case the flux of the field through the loop is changing with time. It is true that in Fig. 30.1.1 the flux is changing because $\vec{B}$ is changing and in Fig. 30.2.1 the flux is changing because the area of the loop still in the magnetic field is changing, but that difference is not important. The important difference between the two arrangements is that the arrangement of Fig. 30.2.1 makes calculations easier. Let us now calculate the rate at which you do mechanical work as you pull steadily on the loop in Fig. 30.2.1.

Rate of Work. As you will see, to pull the loop at a constant velocity $\vec{v}$, you must apply a constant force $\vec{F}$ to the loop because a magnetic force of equal magnitude but opposite direction acts on the loop to oppose you. From Eq. 7.6.7, the rate at which you do work—that is, the power—is then

$$P = Fv, \tag{30.2.1}$$

where F is the magnitude of your force. We wish to find an expression for P in terms of the magnitude B of the magnetic field and the characteristics of the loop—namely, its resistance R to current and its dimension L.

As you move the loop to the right in Fig. 30.2.1, the portion of its area within the magnetic field decreases. Thus, the flux through the loop also decreases and, according to Faraday's law, a current is produced in the loop. It is the presence of this current that causes the force that opposes your pull.

Induced Emf. To find the current, we first apply Faraday's law. When x is the length of the loop still in the magnetic field, the area of the loop still in the field is Lx. Then from Eq. 30.1.2, the magnitude of the flux through the loop is

$$\Phi_B = BA = BLx. \tag{30.2.2}$$

As x decreases, the flux decreases. Faraday's law tells us that with this flux decrease, an emf is induced in the loop. Dropping the minus sign in Eq. 30.1.4 and using Eq. 30.2.2, we can write the magnitude of this emf as

$$\mathscr{E} = \frac{d\Phi_B}{dt} = \frac{d}{dt} BLx = BL\frac{dx}{dt} = BLv, \tag{30.2.3}$$

in which we have replaced dx/dt with v, the speed at which the loop moves.

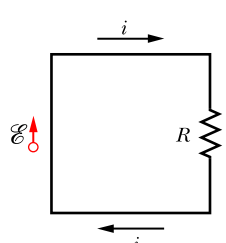


Figure 30.2.2 A circuit diagram for the loop of Fig. 30.2.1 while the loop is moving.

Figure 30.2.2 shows the loop as a circuit: Induced emf $\mathscr{E}$ is represented on the left, and the collective resistance R of the loop is represented on the right. The direction of the induced current i is obtained with a right-hand rule as in Fig. 30.1.5*b* for decreasing flux; applying the rule tells us that the current must be clockwise, and $\mathscr{E}$ must have the same direction.

Induced Current. To find the magnitude of the induced current, we cannot apply the loop rule for potential differences in a circuit because, as you will see in Module 30.3, we cannot define a potential difference for an induced emf. However, we can apply the equation $i = \mathscr{E}/R$. With Eq. 30.2.3, this becomes

$$i = \frac{BLv}{R}. \tag{30.2.4}$$

Because three segments of the loop in Fig. 30.2.1 carry this current through the magnetic field, sideways deflecting forces act on those segments. From Eq. 28.6.2 we know that such a deflecting force is, in general notation,

$$\vec{F}_d = i\vec{L} \times \vec{B}. \tag{30.2.5}$$

In Fig. 30.2.1, the deflecting forces acting on the three segments of the loop are marked $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$. Note, however, that from the symmetry, forces $\vec{F}_2$ and $\vec{F}_3$ are equal in magnitude and cancel. This leaves only force $\vec{F}_1$, which is directed opposite your force $\vec{F}$ on the loop and thus is the force opposing you. So, $\vec{F} = -\vec{F}_1$.

Using Eq. 30.2.5 to obtain the magnitude of $\vec{F}_1$ and noting that the angle between $\vec{B}$ and the length vector $\vec{L}$ for the left segment is 90°, we write

$$F = F_1 = iLB \sin 90° = iLB. \tag{30.2.6}$$

Substituting Eq. 30.2.4 for i in Eq. 30.2.6 then gives us

$$F = \frac{B^2L^2v}{R}. \tag{30.2.7}$$

Because B, L, and R are constants, the speed v at which you move the loop is constant if the magnitude F of the force you apply to the loop is also constant.

Rate of Work. By substituting Eq. 30.2.7 into Eq. 30.2.1, we find the rate at which you do work on the loop as you pull it from the magnetic field:

$$P = Fv = \frac{B^2L^2v^2}{R} \quad \text{(rate of doing work).} \tag{30.2.8}$$

Thermal Energy. To complete our analysis, let us find the rate at which thermal energy appears in the loop as you pull it along at constant speed. We calculate it from Eq. 26.5.3,

$$P = i^2R. \tag{30.2.9}$$

Substituting for i from Eq. 30.2.4, we find

$$P = \left(\frac{BLv}{R}\right)^2 R = \frac{B^2L^2v^2}{R} \quad \text{(thermal energy rate),} \tag{30.2.10}$$

which is exactly equal to the rate at which you are doing work on the loop (Eq. 30.2.8). Thus, the work that you do in pulling the loop through the magnetic field appears as thermal energy in the loop.

Burns During MRI Scans

A patient undergoing an MRI scan (Fig. 30.2.3) lies in an apparatus containing two magnetic fields: a large constant field $\vec{B}_{con}$ and a small sinusoidally varying field $\vec{B}(t)$. Normally the scan requires the patient to lie motionless for a long time. Any patient unable to lie motionless, such as a child, say, is sedated. Because sedation, especially a general anesthetic, can be dangerous, a sedated patient must be carefully monitored, usually with a *pulse oximeter*, a device that measures the oxygen level in the patient's blood. This device includes a probe attached to one of the patient's fingers and a cable running from the probe to a monitor located outside the MRI apparatus.

MRI scans should be perfectly harmless to a patient. In a few cases, however, disregard of Faraday's law of induction led to a sedated patient receiving severe burns. In those cases, the oximeter cable was allowed to touch the patient's arm (Fig. 30.2.4). The cable and the lower part of the arm then formed a closed loop

Figure 30.2.3 A patient about to enter an MRI apparatus.

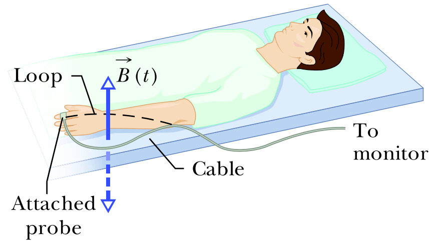


Figure 30.2.4 A probe attached to a finger of a patient undergoing an MRI scan, in which a vertical magnetic field $\vec{B}(t)$ varies sinusoidally. The probe cable touches the patient's skin along the arm, and the cable and the lower part of the arm form a closed loop.

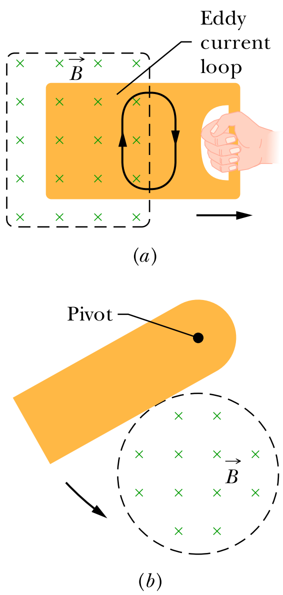


Figure 30.2.5 (*a*) As you pull a solid conducting plate out of a magnetic field, *eddy currents* are induced in the plate. A typical loop of eddy current is shown. (*b*) A conducting plate is allowed to swing like a pendulum about a pivot and into a region of magnetic field. As it enters and leaves the field, eddy currents are induced in the plate.

through which the varying magnetic field $\vec{B}(t)$ produced a varying flux. This flux variation induced an emf around the loop. Although the cable insulation and the skin had high electrical resistance, the induced emf was large enough to drive a significant current around the loop. As with any other circuit in which there is resistance, the current transferred energy to thermal energy at the points of resistance. In this way, the finger and the skin where the cable touched the lower arm were burned. MRI staff are now trained to keep any monitor cable from touching a patient at more than one point.

Eddy Currents

Suppose we replace the conducting loop of Fig. 30.2.1 with a solid conducting plate. If we then move the plate out of the magnetic field as we did the loop (Fig. 30.2.5*a*), the relative motion of the field and the conductor again induces a current in the conductor. Thus, we again encounter an opposing force and must do work because of the induced current. With the plate, however, the conduction electrons making up the induced current do not follow one path as they do with the loop. Instead, the electrons swirl about within the plate as if they were caught in an eddy (whirlpool) of water. Such a current is called an *eddy current* and can be represented, as it is in Fig. 30.2.5*a*, *as if* it followed a single path.

As with the conducting loop of Fig. 30.2.1, the current induced in the plate results in mechanical energy being dissipated as thermal energy. The dissipation is more apparent in the arrangement of Fig. 30.2.5*b*; a conducting plate, free to rotate about a pivot, is allowed to swing down through a magnetic field like a pendulum. Each time the plate enters and leaves the field, a portion of its mechanical energy is transferred to its thermal energy. After several swings, no mechanical energy remains and the warmed-up plate just hangs from its pivot.

Induction Furnaces

Traditionally, foundries used flame-heated furnaces to melt metals. However, many modern foundries avoid the resulting air pollution by using an induction furnace (Fig. 30.2.6) in which the metal is heated by the current in insulated wires wrapped around the crucible that holds the metal. However, the wires themselves do not get hot enough to melt the metal (or they would also melt). Indeed, they are kept cool by a water bath.

Figure 30.2.7 shows the basic design. Metal is held within a crucible around which the insulated wires are wrapped. The current in the wires alternates in direction and magnitude. Thus, the magnetic field due to the current continuously varies in direction and magnitude. This changing field $\vec{B}(t)$ creates eddy

Monty Rakusen/Cultura/GettyImages

Figure 30.2.6 Molten metal pouring from a tilted induction furnace.

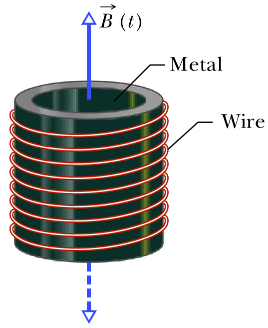
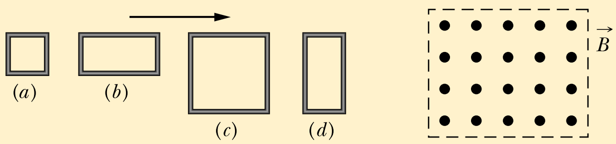


Figure 30.2.7 Basic design of an induction furnace.

currents within the metal, and electrical energy is dissipated as thermal energy at the rate given by Eq. 30.2.9 ($P = i^2R$). The dissipation increases the temperature of the metal to the melting point, and then the molten metal can be poured by tilting the furnace.

Checkpoint 30.2.1

The figure shows four wire loops, with edge lengths of either L or $2L$. All four loops will move through a region of uniform magnetic field $\vec{B}$ (directed out of the page) at the same constant velocity. Rank the four loops according to the maximum magnitude of the emf induced as they move through the field, greatest first.



30.3 INDUCED ELECTRIC FIELDS

Learning Objectives

After reading this module, you should be able to . . .

30.3.1 Identify that a changing magnetic field induces an electric field, regardless of whether there is a conducting loop.

30.3.2 Apply Faraday's law to relate the electric field $\vec{E}$ induced along a closed path (whether it has conducting material or not) to the rate of change $d\Phi/dt$ of the magnetic flux encircled by the path.

30.3.3 Identify that an electric potential cannot be associated with an induced electric field.

Key Ideas

- An emf is induced by a changing magnetic flux even if the loop through which the flux is changing is not a physical conductor but an imaginary line. The changing magnetic field induces an electric field $\vec{E}$ at every point of such a loop; the induced emf is related to $\vec{E}$ by

$$\mathscr{E} = \oint \vec{E} \cdot d\vec{s}.$$

- Using the induced electric field, we can write Faraday's law in its most general form as

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's law)}.$$

A changing magnetic field induces an electric field $\vec{E}$.

Induced Electric Fields

Let us place a copper ring of radius r in a uniform external magnetic field, as in Fig. 30.3.1*a*. The field—neglecting fringing—fills a cylindrical volume of radius R. Suppose that we increase the strength of this field at a steady rate, perhaps by increasing—in an appropriate way—the current in the windings of the electromagnet that produces the field. The magnetic flux through the ring will then change at a steady rate and—by Faraday's law—an induced emf and thus an induced current will appear in the ring. From Lenz's law we can deduce that the direction of the induced current is counterclockwise in Fig. 30.3.1*a*.

If there is a current in the copper ring, an electric field must be present along the ring because an electric field is needed to do the work of moving the conduction electrons. Moreover, the electric field must have been produced by the changing magnetic flux. This **induced electric field** $\vec{E}$ is just as real as an electric field produced by static charges; either field will exert a force $q_0\vec{E}$ on a particle of charge q_0.

By this line of reasoning, we are led to a useful and informative restatement of Faraday's law of induction:

A changing magnetic field produces an electric field.

The striking feature of this statement is that the electric field is induced even if there is no copper ring. Thus, the electric field would appear even if the changing magnetic field were in a vacuum.

To fix these ideas, consider Fig. 30.3.1*b*, which is just like Fig. 30.3.1*a* except the copper ring has been replaced by a hypothetical circular path of radius r. We assume, as previously, that the magnetic field $\vec{B}$ is increasing in magnitude at a constant rate dB/dt. The electric field induced at various points around the

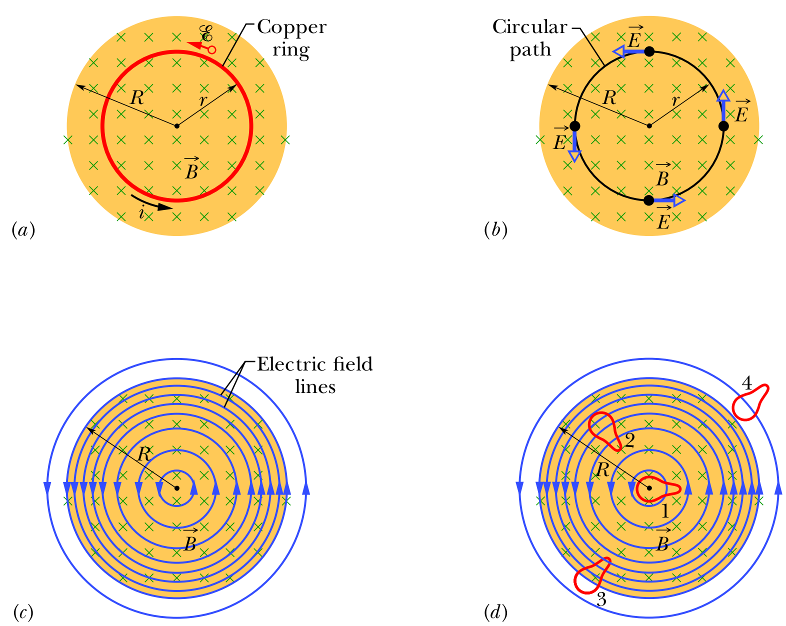


Figure 30.3.1 (*a*) If the magnetic field increases at a steady rate, a constant induced current appears, as shown, in the copper ring of radius r. (*b*) An induced electric field exists even when the ring is removed; the electric field is shown at four points. (*c*) The complete picture of the induced electric field, displayed as field lines. (*d*) Four similar closed paths that enclose identical areas. Equal emfs are induced around paths 1 and 2, which lie entirely within the region of changing magnetic field. A smaller emf is induced around path 3, which only partially lies in that region. No net emf is induced around path 4, which lies entirely outside the magnetic field.

circular path must—from the symmetry—be tangent to the circle, as Fig. 30.3.1*b* shows.* Hence, the circular path is an electric field line. There is nothing special about the circle of radius r, so the electric field lines produced by the changing magnetic field must be a set of concentric circles, as in Fig. 30.3.1*c*.

As long as the magnetic field is *increasing* with time, the electric field represented by the circular field lines in Fig. 30.3.1*c* will be present. If the magnetic field remains *constant* with time, there will be no induced electric field and thus no electric field lines. If the magnetic field is *decreasing* with time (at a constant rate), the electric field lines will still be concentric circles as in Fig. 30.3.1*c*, but they will now have the opposite direction. All this is what we have in mind when we say "A changing magnetic field produces an electric field."

A Reformulation of Faraday's Law

Consider a particle of charge q_0 moving around the circular path of Fig. 30.3.1*b*. The work W done on it in one revolution by the induced electric field is $W = \mathscr{E}q_0$, where $\mathscr{E}$ is the induced emf—that is, the work done per unit charge in moving the test charge around the path. From another point of view, the work is

$$W = \int \vec{F} \cdot d\vec{s} = (q_0 E)(2\pi r), \tag{30.3.1}$$

where q_0E is the magnitude of the force acting on the test charge and $2\pi r$ is the distance over which that force acts. Setting these two expressions for W equal to each other and canceling q_0, we find that

$$\mathscr{E} = 2\pi r E. \tag{30.3.2}$$

Next we rewrite Eq. 30.3.1 to give a more general expression for the work done on a particle of charge q_0 moving along any closed path:

$$W = \oint \vec{F} \cdot d\vec{s} = q_0 \oint \vec{E} \cdot d\vec{s}. \tag{30.3.3}$$

(The loop on each integral sign indicates that the integral is to be taken around the closed path.) Substituting $\mathscr{E}q_0$ for W, we find that

$$\mathscr{E} = \oint \vec{E} \cdot d\vec{s}. \tag{30.3.4}$$

This integral reduces at once to Eq. 30.3.2 if we evaluate it for the special case of Fig. 30.3.1*b*.

Meaning of Emf. With Eq. 30.3.4, we can expand the meaning of induced emf. Up to this point, induced emf has meant the work per unit charge done in maintaining current due to a changing magnetic flux, or it has meant the work done per unit charge on a charged particle that moves around a closed path in a changing magnetic flux. However, with Fig. 30.3.1*b* and Eq. 30.3.4, an induced emf can exist without the need of a current or particle: An induced emf is the sum—via integration—of quantities $\vec{E} \cdot d\vec{s}$ around a closed path, where $\vec{E}$ is the electric field induced by a changing magnetic flux and $d\vec{s}$ is a differential length vector along the path.

If we combine Eq. 30.3.4 with Faraday's law in Eq. 30.1.4 ($\mathscr{E} = -d\Phi_B/dt$), we can rewrite Faraday's law as

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's law).} \tag{30.3.5}$$

*Arguments of symmetry would also permit the lines of $\vec{E}$ around the circular path to be *radial*, rather than tangential. However, such radial lines would imply that there are free charges, distributed symmetrically about the axis of symmetry, on which the electric field lines could begin or end; there are no such charges.

This equation says simply that a changing magnetic field induces an electric field. The changing magnetic field appears on the right side of this equation, the electric field on the left.

Faraday's law in the form of Eq. 30.3.5 can be applied to *any* closed path that can be drawn in a changing magnetic field. Figure 30.3.1*d*, for example, shows four such paths, all having the same shape and area but located in different positions in the changing field. The induced emfs $\mathscr{E}(=\oint \vec{E}\cdot d\vec{s})$ for paths 1 and 2 are equal because these paths lie entirely in the magnetic field and thus have the same value of $d\Phi_B/dt$. This is true even though the electric field vectors at points along these paths are different, as indicated by the patterns of electric field lines in the figure. For path 3 the induced emf is smaller because the enclosed flux Φ_B (hence $d\Phi_B/dt$) is smaller, and for path 4 the induced emf is zero even though the electric field is not zero at any point on the path.

A New Look at Electric Potential

Induced electric fields are produced not by static charges but by a changing magnetic flux. Although electric fields produced in either way exert forces on charged particles, there is an important difference between them. The simplest evidence of this difference is that the field lines of induced electric fields form closed loops, as in Fig. 30.3.1*c*. Field lines produced by static charges never do so but must start on positive charges and end on negative charges. Thus, a field line from a charge can never loop around and back onto itself as we see for each of the field lines in Fig. 30.3.1*c*.

In a more formal sense, we can state the difference between electric fields produced by induction and those produced by static charges in these words:

> Electric potential has meaning only for electric fields that are produced by static charges; it has no meaning for electric fields that are produced by induction.

You can understand this statement qualitatively by considering what happens to a charged particle that makes a single journey around the circular path in Fig. 30.3.1*b*. It starts at a certain point and, on its return to that same point, has experienced an emf $\mathscr{E}$ of, let us say, 5 V; that is, work of 5 J/C has been done on the particle by the electric field, and thus the particle should then be at a point that is 5 V greater in potential. However, that is impossible because the particle is back at the same point, which cannot have two different values of potential. Thus, potential has no meaning for electric fields that are set up by changing magnetic fields.

We can take a more formal look by recalling Eq. 24.2.4, which defines the potential difference between two points i and f in an electric field $\vec{E}$ in terms of an integration between those points:

$$V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s}. \qquad (30.3.6)$$

In Chapter 24 we had not yet encountered Faraday's law of induction; so the electric fields involved in the derivation of Eq. 24.2.4 were those due to static charges. If i and f in Eq. 30.3.6 are the same point, the path connecting them is a closed loop, V_i and V_f are identical, and Eq. 30.3.6 reduces to

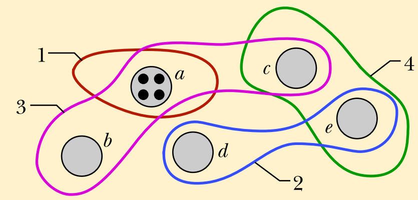
$$\oint \vec{E}\cdot d\vec{s} = 0. \qquad (30.3.7)$$

However, when a changing magnetic flux is present, this integral is *not* zero but is $-d\Phi_B/dt$, as Eq. 30.3.5 asserts. Thus, assigning electric potential to an induced electric field leads us to a contradiction. We must conclude that electric potential has no meaning for electric fields associated with induction.

Checkpoint 30.3.1

The figure shows five lettered regions in which a uniform magnetic field extends either directly out of the page or into the page, with the direction indicated only for region *a*. The field is increasing in magnitude at the same steady rate in all five regions; the regions are identical in area. Also shown are four numbered paths along which $\oint \vec{E} \cdot d\vec{s}$ has the magnitudes given below in terms of a quantity "mag." Determine whether the magnetic field is directed into or out of the page for regions *b* through *e*.

Path	1	2	3	4
$\oint \vec{E} \cdot d\vec{s}$	mag	2(mag)	3(mag)	0



Sample Problem 30.3.1 Induced electric field due to changing *B* field, inside and outside

In Fig. 30.3.1*b*, take $R = 8.5$ cm and $dB/dt = 0.13$ T/s.

(a) Find an expression for the magnitude *E* of the induced electric field at points within the magnetic field, at radius *r* from the center of the magnetic field. Evaluate the expression for $r = 5.2$ cm.

KEY IDEA

An electric field is induced by the changing magnetic field, according to Faraday's law.

Calculations: To calculate the field magnitude *E*, we apply Faraday's law in the form of Eq. 30.3.5. We use a circular path of integration with radius $r \le R$ because we want *E* for points within the magnetic field. We assume from the symmetry that $\vec{E}$ in Fig. 30.3.1*b* is tangent to the circular path at all points. The path vector $d\vec{s}$ is also always tangent to the circular path; so the dot product $\vec{E} \cdot d\vec{s}$ in Eq. 30.3.5 must have the magnitude $E\,ds$ at all points on the path. We can also assume from the symmetry that *E* has the same value at all points along the circular path. Then the left side of Eq. 30.3.5 becomes

$$\oint \vec{E} \cdot d\vec{s} = \oint E\,ds = E \oint ds = E(2\pi r). \tag{30.3.8}$$

(The integral $\oint ds$ is the circumference $2\pi r$ of the circular path.)

Next, we need to evaluate the right side of Eq. 30.3.5. Because $\vec{B}$ is uniform over the area *A* encircled by the path of integration and is directed perpendicular to that area, the magnetic flux is given by Eq. 30.1.2:

$$\Phi_B = BA = B(\pi r^2). \tag{30.3.9}$$

Substituting this and Eq. 30.3.8 into Eq. 30.3.5 and dropping the minus sign, we find that

$$E(2\pi r) = (\pi r^2)\frac{dB}{dt}$$

or

$$E = \frac{r}{2}\frac{dB}{dt}. \qquad \text{(Answer)} \tag{30.3.10}$$

Equation 30.3.10 gives the magnitude of the electric field at any point for which $r \le R$ (that is, within the magnetic field). Substituting given values yields, for the magnitude of $\vec{E}$ at $r = 5.2$ cm,

$$E = \frac{(5.2 \times 10^{-2}\text{ m})}{2}(0.13\text{ T/s})$$
$$= 0.0034\text{ V/m} = 3.4\text{ mV/m}. \qquad \text{(Answer)}$$

(b) Find an expression for the magnitude *E* of the induced electric field at points that are outside the magnetic field, at radius *r* from the center of the magnetic field. Evaluate the expression for $r = 12.5$ cm.

KEY IDEAS

Here again an electric field is induced by the changing magnetic field, according to Faraday's law, except that now we use a circular path of integration with radius $r \ge R$ because we want to evaluate *E* for points outside the magnetic field. Proceeding as in (a), we again obtain Eq. 30.3.8. However, we do not then obtain Eq. 30.3.9 because the new path of integration is now outside the magnetic field, and so the magnetic flux encircled by the new path is only that in the area πR^2 of the magnetic field region.

Calculations: We can now write

$$\Phi_B = BA = B(\pi R^2). \tag{30.3.11}$$

Substituting this and Eq. 30.3.8 into Eq. 30.3.5 (without the minus sign) and solving for E yield

$$E = \frac{R^2}{2r}\frac{dB}{dt}. \qquad \text{(Answer)} \qquad (30.3.12)$$

Because E is not zero here, we know that an electric field is induced even at points that are outside the changing magnetic field, an important result that (as you will see in Module 31.6) makes transformers possible.

With the given data, Eq. 30.3.12 yields the magnitude of $\vec{E}$ at $r = 12.5$ cm:

$$E = \frac{(8.5 \times 10^{-2}\ \text{m})^2}{(2)(12.5 \times 10^{-2}\ \text{m})}(0.13\ \text{T/s})$$

$$= 3.8 \times 10^{-3}\ \text{V/m} = 3.8\ \text{mV/m}. \qquad \text{(Answer)}$$

Equations 30.3.10 and 30.3.12 give the same result for $r = R$. Figure 30.3.2 shows a plot of $E(r)$. Note that the inside and outside plots meet at $r = R$.

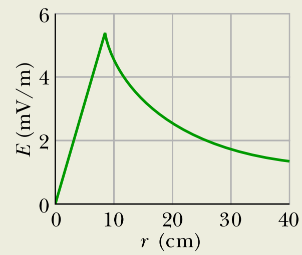


Figure 30.3.2 A plot of the induced electric field $E(r)$.

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30.4 INDUCTORS AND INDUCTANCE

Learning Objectives

After reading this module, you should be able to . . .

30.4.1 Identify an inductor.

30.4.2 For an inductor, apply the relationship between inductance L, total flux $N\Phi$, and current i.

30.4.3 For a solenoid, apply the relationship between the inductance per unit length L/l, the area A of each turn, and the number of turns per unit length n.

Key Ideas

- An inductor is a device that can be used to produce a known magnetic field in a specified region. If a current i is established through each of the N windings of an inductor, a magnetic flux Φ_B links those windings. The inductance L of the inductor is

$$L = \frac{N\Phi_B}{i} \quad \text{(inductance defined)}.$$

- The SI unit of inductance is the henry (H), where $1\ \text{henry} = 1\ \text{H} = 1\ \text{T} \cdot \text{m}^2/\text{A}$.
- The inductance per unit length near the middle of a long solenoid of cross-sectional area A and n turns per unit length is

$$\frac{L}{l} = \mu_0 n^2 A \quad \text{(solenoid)}.$$

Inductors and Inductance

We found in Chapter 25 that a capacitor can be used to produce a desired electric field. We considered the parallel-plate arrangement as a basic type of capacitor. Similarly, an **inductor** (symbol ꝏꝏ) can be used to produce a desired magnetic field. We shall consider a long solenoid (more specifically, a short length near the middle of a long solenoid, to avoid any fringing effects) as our basic type of inductor.

If we establish a current i in the windings (turns) of the solenoid we are taking as our inductor, the current produces a magnetic flux Φ_B through the central region of the inductor. The **inductance** of the inductor is then defined in terms of that flux as

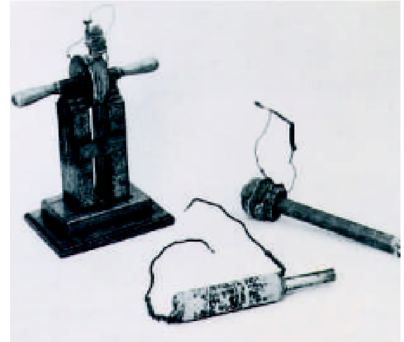
$$L = \frac{N\Phi_B}{i} \quad \text{(inductance defined)}, \qquad (30.4.1)$$

in which N is the number of turns. The windings of the inductor are said to be *linked* by the shared flux, and the product $N\Phi_B$ is called the *magnetic flux linkage*. The inductance L is thus a measure of the flux linkage produced by the inductor per unit of current.

Because the SI unit of magnetic flux is the tesla-square meter, the SI unit of inductance is the tesla-square meter per ampere ($T \cdot m^2/A$). We call this the **henry** (H), after American physicist Joseph Henry, the codiscoverer of the law of induction and a contemporary of Faraday. Thus,

$$1 \text{ henry} = 1 \text{ H} = 1 \text{ T} \cdot \text{m}^2/\text{A}. \tag{30.4.2}$$

Through the rest of this chapter we assume that all inductors, no matter what their geometric arrangement, have no magnetic materials such as iron in their vicinity. Such materials would distort the magnetic field of an inductor.



The Royal Institution/Bridgeman Art Library/NY

The crude inductors with which Michael Faraday discovered the law of induction. In those days amenities such as insulated wire were not commercially available. It is said that Faraday insulated his wires by wrapping them with strips cut from one of his wife's petticoats.

Inductance of a Solenoid

Consider a long solenoid of cross-sectional area A. What is the inductance per unit length near its middle? To use the defining equation for inductance (Eq. 30.4.1), we must calculate the flux linkage set up by a given current in the solenoid windings. Consider a length l near the middle of this solenoid. The flux linkage there is

$$N\Phi_B = (nl)(BA),$$

in which n is the number of turns per unit length of the solenoid and B is the magnitude of the magnetic field within the solenoid.

The magnitude B is given by Eq. 29.4.3,

$$B = \mu_0 in,$$

and so from Eq. 30.4.1,

$$\begin{aligned} L &= \frac{N\Phi_B}{i} = \frac{(nl)(BA)}{i} = \frac{(nl)(\mu_0 in)(A)}{i} \\ &= \mu_0 n^2 lA. \end{aligned} \tag{30.4.3}$$

Thus, the inductance per unit length near the center of a long solenoid is

$$\frac{L}{l} = \mu_0 n^2 A \quad \text{(solenoid)}. \tag{30.4.4}$$

Inductance—like capacitance—depends only on the geometry of the device. The dependence on the square of the number of turns per unit length is to be expected. If you, say, triple n, you not only triple the number of turns (N) but you also triple the flux ($\Phi_B = BA = \mu_0 inA$) through each turn, multiplying the flux linkage $N\Phi_B$ and thus the inductance L by a factor of 9.

If the solenoid is very much longer than its radius, then Eq. 30.4.3 gives its inductance to a good approximation. This approximation neglects the spreading of the magnetic field lines near the ends of the solenoid, just as the parallel-plate capacitor formula ($C = \varepsilon_0 A/d$) neglects the fringing of the electric field lines near the edges of the capacitor plates.

From Eq. 30.4.3, and recalling that n is a number per unit length, we can see that an inductance can be written as a product of the permeability constant μ_0 and a quantity with the dimensions of a length. This means that μ_0 can be expressed in the unit henry per meter:

$$\begin{aligned} \mu_0 &= 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \\ &= 4\pi \times 10^{-7} \text{ H/m}. \end{aligned} \tag{30.4.5}$$

The latter is the more common unit for the permeability constant.

Checkpoint 30.4.1

We have three inductors in different circuits with the same length and the following number of turns n, area A, and current i, each given as a multiple of a basic amount. Rank the inductors according to their inductance, greatest first.

Inductor	n	A	i
a	$2n_0$	$4A_0$	$16i_0$
b	n_0	$13A_0$	$20i_0$
c	$3n_0$	A_0	$25i_0$

30.5 SELF-INDUCTION

Learning Objectives

After reading this module, you should be able to . . .

30.5.1 Identify that an induced emf appears in a coil when the current through the coil is changing.

30.5.2 Apply the relationship between the induced emf in a coil, the coil's inductance L, and the rate di/dt at which the current is changing.

30.5.3 When an emf is induced in a coil because the current in the coil is changing, determine the direction of the emf by using Lenz's law to show that the emf always opposes the change in the current, attempting to maintain the initial current.

Key Ideas

- If a current i in a coil changes with time, an emf is induced in the coil. This self-induced emf is

$$\mathscr{E}_L = -L\frac{di}{dt}.$$

- The direction of $\mathscr{E}_L$ is found from Lenz's law: The self-induced emf acts to oppose the change that produces it.

Self-Induction

If two coils—which we can now call inductors—are near each other, a current i in one coil produces a magnetic flux Φ_B through the second coil. We have seen that if we change this flux by changing the current, an induced emf appears in the second coil according to Faraday's law. An induced emf appears in the first coil as well.

An induced emf $\mathscr{E}_L$ appears in any coil in which the current is changing.

This process (see Fig. 30.5.1) is called **self-induction,** and the emf that appears is called a **self-induced emf.** It obeys Faraday's law of induction just as other induced emfs do.

For any inductor, Eq. 30.4.1 tells us that

$$N\Phi_B = Li. \tag{30.5.1}$$

Faraday's law tells us that

$$\mathscr{E}_L = -\frac{d(N\Phi_B)}{dt}. \tag{30.5.2}$$

By combining Eqs. 30.5.1 and 30.5.2 we can write

$$\mathscr{E}_L = -L\frac{di}{dt} \quad \text{(self-induced emf)}. \tag{30.5.3}$$

Thus, in any inductor (such as a coil, a solenoid, or a toroid) a self-induced emf appears whenever the current changes with time. The magnitude of the current has no influence on the magnitude of the induced emf; only the rate of change of the current counts.

Direction. You can find the *direction* of a self-induced emf from Lenz's law. The minus sign in Eq. 30.5.3 indicates that—as the law states—the self-induced emf $\mathscr{E}_L$ has the orientation such that it opposes the change in current i. We can drop the minus sign when we want only the magnitude of $\mathscr{E}_L$.

Suppose that you set up a current i in a coil and arrange to have the current increase with time at a rate di/dt. In the language of Lenz's law, this increase in the current in the coil is the "change" that the self-induction must oppose. Thus, a self-induced emf must appear in the coil, pointing so as to oppose the increase in the current, trying (but failing) to maintain the initial condition, as shown in Fig. 30.5.2*a*. If, instead, the current decreases with time, the self-induced emf

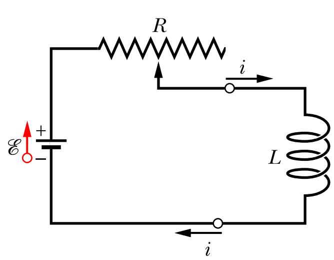


Figure 30.5.1 If the current in a coil is changed by varying the contact position on a variable resistor, a self-induced emf $\mathscr{E}_L$ will appear in the coil *while the current is changing.*

must point in a direction that tends to oppose the decrease (Fig. 30.5.2*b*), again trying to maintain the initial condition.

Electric Potential. In Module 30.3 we saw that we cannot define an electric potential for an electric field (and thus for an emf) that is induced by a changing magnetic flux. This means that when a self-induced emf is produced in the inductor of Fig. 30.5.1, we cannot define an electric potential within the inductor itself, where the flux is changing. However, potentials can still be defined at points of the circuit that are not within the inductor—points where the electric fields are due to charge distributions and their associated electric potentials.

Moreover, we can define a self-induced potential difference V_L *across an inductor* (between its terminals, which we assume to be outside the region of changing flux). For an *ideal inductor* (its wire has negligible resistance), the magnitude of V_L is equal to the magnitude of the self-induced emf $\mathscr{E}_L$.

If, instead, the wire in the inductor has resistance r, we mentally separate the inductor into a resistance r (which we take to be outside the region of changing flux) and an ideal inductor of self-induced emf $\mathscr{E}_L$. As with a real battery of emf $\mathscr{E}$ and internal resistance r, the potential difference across the terminals of a real inductor then differs from the emf. Unless otherwise indicated, we assume here that inductors are ideal.

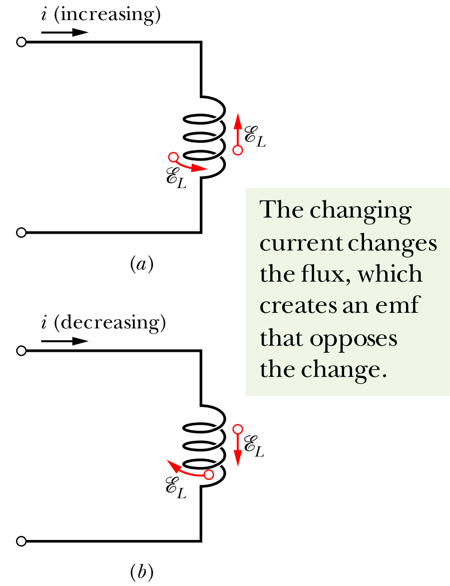
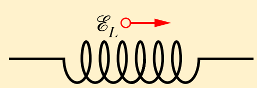


Figure 30.5.2 (*a*) The current i is increasing, and the self-induced emf $\mathscr{E}_L$ appears along the coil in a direction such that it opposes the increase. The arrow representing $\mathscr{E}_L$ can be drawn along a turn of the coil or alongside the coil. Both are shown. (*b*) The current i is decreasing, and the self-induced emf appears in a direction such that it opposes the decrease.

Checkpoint 30.5.1

The figure shows an emf $\mathscr{E}_L$ induced in a coil. Which of the following can describe the current through the coil: (a) constant and rightward, (b) constant and leftward, (c) increasing and rightward, (d) decreasing and rightward, (e) increasing and leftward, (f) decreasing and leftward?



30.6 *RL* CIRCUITS

Learning Objectives

After reading this module, you should be able to . . .

30.6.1 Sketch a schematic diagram of an RL circuit in which the current is rising.

30.6.2 Write a loop equation (a differential equation) for an RL circuit in which the current is rising.

30.6.3 For an RL circuit in which the current is rising, apply the equation $i(t)$ for the current as a function of time.

30.6.4 For an RL circuit in which the current is rising, find equations for the potential difference V across the resistor, the rate di/dt at which the current changes, and the emf of the inductor, as functions of time.

30.6.5 Calculate an inductive time constant τ_L.

30.6.6 Sketch a schematic diagram of an RL circuit in which the current is decaying.

30.6.7 Write a loop equation (a differential equation) for an RL circuit in which the current is decaying.

30.6.8 For an RL circuit in which the current is decaying, apply the equation $i(t)$ for the current as a function of time.

30.6.9 From an equation for decaying current in an RL circuit, find equations for the potential difference V across the resistor, the rate di/dt at which current is changing, and the emf of the inductor, as functions of time.

30.6.10 For an RL circuit, identify the current through the inductor and the emf across it just as current in the circuit begins to change (the initial condition) and a long time later when equilibrium is reached (the final condition).

Key Ideas

- If a constant emf $\mathscr{E}$ is introduced into a single-loop circuit containing a resistance R and an inductance L, the current rises to an equilibrium value of $\mathscr{E}/R$ according to

$$i = \frac{\mathscr{E}}{R}(1 - e^{-t/\tau_L}) \quad \text{(rise of current).}$$

Here τ_L $(= L/R)$ governs the rate of rise of the current and is called the inductive time constant of the circuit.

- When the source of constant emf is removed, the current decays from a value i_0 according to

$$i = i_0 e^{-t/\tau_L} \quad \text{(decay of current).}$$

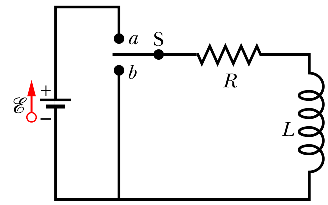


Figure 30.6.1 An RL circuit. When switch S is closed on a, the current rises and approaches a limiting value $\mathscr{E}/R$.

RL Circuits

In Module 27.4 we saw that if we suddenly introduce an emf $\mathscr{E}$ into a single-loop circuit containing a resistor R and a capacitor C, the charge on the capacitor does not build up immediately to its final equilibrium value $C\mathscr{E}$ but approaches it in an exponential fashion:

$$q = C\mathscr{E}(1 - e^{-t/\tau_C}). \tag{30.6.1}$$

The rate at which the charge builds up is determined by the capacitive time constant τ_C, defined in Eq. 27.4.7 as

$$\tau_C = RC. \tag{30.6.2}$$

If we suddenly remove the emf from this same circuit, the charge does not immediately fall to zero but approaches zero in an exponential fashion:

$$q = q_0 e^{-t/\tau_C}. \tag{30.6.3}$$

The time constant τ_C describes the fall of the charge as well as its rise.

An analogous slowing of the rise (or fall) of the current occurs if we introduce an emf $\mathscr{E}$ into (or remove it from) a single-loop circuit containing a resistor R and an inductor L. When the switch S in Fig. 30.6.1 is closed on a, for example, the current in the resistor starts to rise. If the inductor were not present, the current would rise rapidly to a steady value $\mathscr{E}/R$. Because of the inductor, however, a self-induced emf $\mathscr{E}_L$ appears in the circuit; from Lenz's law, this emf opposes the rise of the current, which means that it opposes the battery emf $\mathscr{E}$ in polarity. Thus, the current in the resistor responds to the difference between two emfs, a constant $\mathscr{E}$ due to the battery and a variable $\mathscr{E}_L$ $(= -L\, di/dt)$ due to self-induction. As long as this $\mathscr{E}_L$ is present, the current will be less than $\mathscr{E}/R$.

As time goes on, the rate at which the current increases becomes less rapid and the magnitude of the self-induced emf, which is proportional to di/dt, becomes smaller. Thus, the current in the circuit approaches $\mathscr{E}/R$ asymptotically.

We can generalize these results as follows:

> Initially, an inductor acts to oppose changes in the current through it. A long time later, it acts like ordinary connecting wire.

Now let us analyze the situation quantitatively. With the switch S in Fig. 30.6.1 thrown to a, the circuit is equivalent to that of Fig. 30.6.2. Let us apply the loop rule, starting at point x in this figure and moving clockwise around the loop along with current i.

1. *Resistor.* Because we move through the resistor in the direction of current i, the electric potential decreases by iR. Thus, as we move from point x to point y, we encounter a potential change of $-iR$.
2. *Inductor.* Because current i is changing, there is a self-induced emf $\mathscr{E}_L$ in the inductor. The magnitude of $\mathscr{E}_L$ is given by Eq. 30.5.3 as $L\, di/dt$. The direction of $\mathscr{E}_L$ is upward in Fig. 30.6.2 because current i is downward through the inductor *and* increasing. Thus, as we move from point y to point z, opposite the direction of $\mathscr{E}_L$, we encounter a potential change of $-L\, di/dt$.
3. *Battery.* As we move from point z back to starting point x, we encounter a potential change of $+\mathscr{E}$ due to the battery's emf.

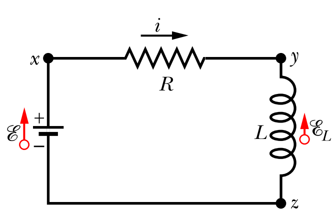


Figure 30.6.2 The circuit of Fig. 30.6.1 with the switch closed on a. We apply the loop rule for the circuit clockwise, starting at x.

Thus, the loop rule gives us

$$-iR - L\frac{di}{dt} + \mathscr{E} = 0,$$

or

$$L\frac{di}{dt} + Ri = \mathscr{E} \quad (RL \text{ circuit}). \tag{30.6.4}$$

Equation 30.6.4 is a differential equation involving the variable i and its first derivative di/dt. To solve it, we seek the function $i(t)$ such that when $i(t)$ and its first derivative are substituted in Eq. 30.6.4, the equation is satisfied and the initial condition $i(0) = 0$ is satisfied.

Equation 30.6.4 and its initial condition are of exactly the form of Eq. 27.4.3 for an RC circuit, with i replacing q, L replacing R, and R replacing $1/C$. The solution of Eq. 30.6.4 must then be of exactly the form of Eq. 27.4.4 with the same replacements. That solution is

$$i = \frac{\mathscr{E}}{R}(1 - e^{-Rt/L}), \tag{30.6.5}$$

which we can rewrite as

$$i = \frac{\mathscr{E}}{R}(1 - e^{-t/\tau_L}) \quad (\text{rise of current}). \tag{30.6.6}$$

Here τ_L, the **inductive time constant,** is given by

$$\tau_L = \frac{L}{R} \quad (\text{time constant}). \tag{30.6.7}$$

Let's examine Eq. 30.6.6 for just after the switch is closed (at time $t = 0$) and for a time long after the switch is closed ($t \to \infty$). If we substitute $t = 0$ into Eq. 30.6.6, the exponential becomes $e^{-0} = 1$. Thus, Eq. 30.6.6 tells us that the current is initially $i = 0$, as we expected. Next, if we let t go to ∞, then the exponential goes to $e^{-\infty} = 0$. Thus, Eq. 30.6.6 tells us that the current goes to its equilibrium value of $\mathscr{E}/R$.

We can also examine the potential differences in the circuit. For example, Fig. 30.6.3 shows how the potential differences V_R ($= iR$) across the resistor and V_L ($= L\, di/dt$) across the inductor vary with time for particular values of $\mathscr{E}$, L, and R. Compare this figure carefully with the corresponding figure for an RC circuit (Fig. 27.1.16).

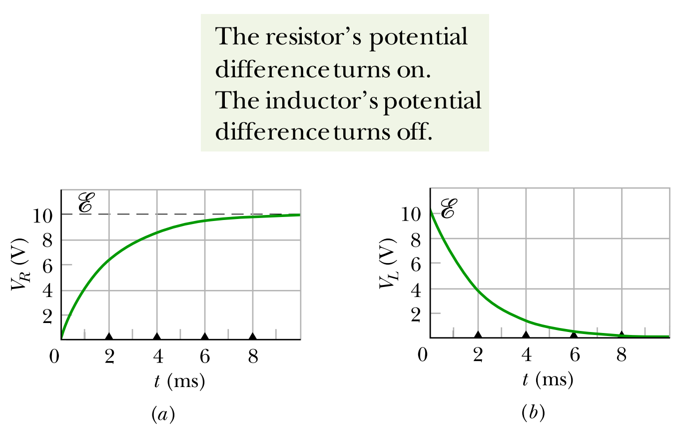


Figure 30.6.3 The variation with time of (a) V_R, the potential difference across the resistor in the circuit of Fig. 30.6.2, and (b) V_L, the potential difference across the inductor in that circuit. The small triangles represent successive intervals of one inductive time constant $\tau_L = L/R$. The figure is plotted for $R = 2000\ \Omega$, $L = 4.0$ H, and $\mathscr{E} = 10$ V.

To show that the quantity τ_L ($= L/R$) has the dimension of time (as it must, because the argument of the exponential function in Eq. 30.6.6 must be dimensionless), we convert from henries per ohm as follows:

$$1\frac{\mathrm{H}}{\Omega} = 1\frac{\mathrm{H}}{\Omega}\left(\frac{1\ \mathrm{V\cdot s}}{1\ \mathrm{H\cdot A}}\right)\left(\frac{1\Omega\cdot \mathrm{A}}{1\ \mathrm{V}}\right) = 1\ \mathrm{s}.$$

The first quantity in parentheses is a conversion factor based on Eq. 30.5.3, and the second one is a conversion factor based on the relation $V = iR$.

Time Constant. The physical significance of the time constant follows from Eq. 30.6.6. If we put $t = \tau_L = L/R$ in this equation, it reduces to

$$i = \frac{\mathscr{E}}{R}(1 - e^{-1}) = 0.63\frac{\mathscr{E}}{R}. \tag{30.6.8}$$

Thus, the time constant τ_L is the time it takes the current in the circuit to reach about 63% of its final equilibrium value $\mathscr{E}/R$. Since the potential difference V_R across the resistor is proportional to the current i, a graph of the increasing current versus time has the same shape as that of V_R in Fig. 30.6.3*a*.

Current Decay. If the switch S in Fig. 30.6.1 is closed on a long enough for the equilibrium current $\mathscr{E}/R$ to be established and then is thrown to b, the effect will be to remove the battery from the circuit. (The connection to b must actually be made an instant before the connection to a is broken. A switch that does this is called a *make-before-break* switch.) With the battery gone, the current through the resistor will decrease. However, it cannot drop immediately to zero but must decay to zero over time. The differential equation that governs the decay can be found by putting $\mathscr{E} = 0$ in Eq. 30.6.4:

$$L\frac{di}{dt} + iR = 0. \tag{30.6.9}$$

By analogy with Eqs. 27.4.9 and 27.4.10, the solution of this differential equation that satisfies the initial condition $i(0) = i_0 = \mathscr{E}/R$ is

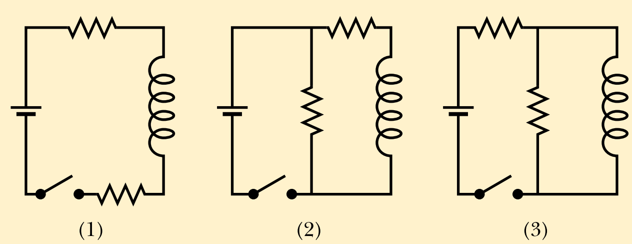
$$i = \frac{\mathscr{E}}{R}e^{-t/\tau_L} = i_0 e^{-t/\tau_L} \quad \text{(decay of current).} \tag{30.6.10}$$

We see that both current rise (Eq. 30.6.6) and current decay (Eq. 30.6.10) in an RL circuit are governed by the same inductive time constant, τ_L.

We have used i_0 in Eq. 30.6.10 to represent the current at time $t = 0$. In our case that happened to be $\mathscr{E}/R$, but it could be any other initial value.

Checkpoint 30.6.1

The figure shows three circuits with identical batteries, inductors, and resistors. Rank the circuits according to the current through the battery (a) just after the switch is closed and (b) a long time later, greatest first. (If you have trouble here, work through the next sample problem and then try again.)



Sample Problem 30.6.1 *RL* circuit, immediately after switching and after a long time

Figure 30.6.4*a* shows a circuit that contains three identical resistors with resistance $R = 9.0\ \Omega$, two identical inductors with inductance $L = 2.0$ mH, and an ideal battery with emf $\mathscr{E} = 18$ V.

(a) What is the current i through the battery just after the switch is closed?

KEY IDEA

Just after the switch is closed, the inductor acts to oppose a change in the current through it.

Calculations: Because the current through each inductor is zero before the switch is closed, it will also be zero just afterward. Thus, immediately after the switch is closed, the inductors act as broken wires, as indicated in Fig. 30.6.4*b*. We then have a single-loop circuit for which the loop rule gives us

$$\mathscr{E} - iR = 0.$$

Substituting given data, we find that

$$i = \frac{\mathscr{E}}{R} = \frac{18\text{ V}}{9.0\ \Omega} = 2.0\text{ A}. \qquad \text{(Answer)}$$

(b) What is the current i through the battery long after the switch has been closed?

KEY IDEA

Long after the switch has been closed, the currents in the circuit have reached their equilibrium values, and the inductors act as simple connecting wires, as indicated in Fig. 30.6.4*c*.

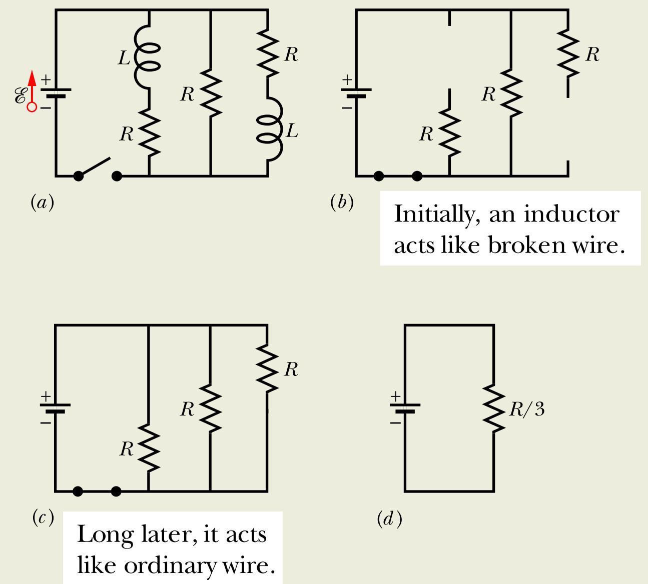


Figure 30.6.4 (*a*) A multiloop *RL* circuit with an open switch. (*b*) The equivalent circuit just after the switch has been closed. (*c*) The equivalent circuit a long time later. (*d*) The single-loop circuit that is equivalent to circuit (*c*).

Calculation: We now have a circuit with three identical resistors in parallel; from Eq. 27.2.6, their equivalent resistance is $R_{eq} = R/3 = (9.0\ \Omega)/3 = 3.0\ \Omega$. The equivalent circuit shown in Fig. 30.6.4*d* then yields the loop equation $\mathscr{E} - iR_{eq} = 0$, or

$$i = \frac{\mathscr{E}}{R_{eq}} = \frac{18\text{ V}}{3.0\ \Omega} = 6.0\text{ A}. \qquad \text{(Answer)}$$

Sample Problem 30.6.2 *RL* circuit, current during the transition

A solenoid has an inductance of 53 mH and a resistance of 0.37 Ω. If the solenoid is connected to a battery, how long will the current take to reach half its final equilibrium value? (This is a *real solenoid* because we are considering its small, but nonzero, internal resistance.)

KEY IDEA

We can mentally separate the solenoid into a resistance and an inductance that are wired in series with a battery, as in Fig. 30.6.2. Then application of the loop rule leads to Eq. 30.6.4, which has the solution of Eq. 30.6.6 for the current i in the circuit.

Calculations: According to that solution, current i increases exponentially from zero to its final equilibrium value of $\mathscr{E}/R$. Let t_0 be the time that current i takes to reach half its equilibrium value. Then Eq. 30.6.6 gives us

$$\frac{1}{2}\frac{\mathscr{E}}{R} = \frac{\mathscr{E}}{R}(1 - e^{-t_0/\tau_L}).$$

We solve for t_0 by canceling $\mathscr{E}/R$, isolating the exponential, and taking the natural logarithm of each side. We find

$$t_0 = \tau_L \ln 2 = \frac{L}{R}\ln 2 = \frac{53 \times 10^{-3}\text{ H}}{0.37\ \Omega}\ln 2$$

$$= 0.10\text{ s}. \qquad \text{(Answer)}$$

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30.7 ENERGY STORED IN A MAGNETIC FIELD

Learning Objectives

After reading this module, you should be able to . . .

30.7.1 Describe the derivation of the equation for the magnetic field energy of an inductor in an RL circuit with a constant emf source.

30.7.2 For an inductor in an RL circuit, apply the relationship between the magnetic field energy U, the inductance L, and the current i.

Key Idea

- If an inductor L carries a current i, the inductor's magnetic field stores an energy given by

$$U_B = \frac{1}{2}Li^2 \quad \text{(magnetic energy)}.$$

Energy Stored in a Magnetic Field

When we pull two charged particles of opposite signs away from each other, we say that the resulting electric potential energy is stored in the electric field of the particles. We get it back from the field by letting the particles move closer together again. In the same way we say energy is stored in a magnetic field, but now we deal with current instead of electric charges.

To derive a quantitative expression for that stored energy, consider again Fig. 30.6.2, which shows a source of emf $\mathscr{E}$ connected to a resistor R and an inductor L. Equation 30.6.4, restated here for convenience,

$$\mathscr{E} = L\frac{di}{dt} + iR, \tag{30.7.1}$$

is the differential equation that describes the growth of current in this circuit. Recall that this equation follows immediately from the loop rule and that the loop rule in turn is an expression of the principle of conservation of energy for single-loop circuits. If we multiply each side of Eq. 30.7.1 by i, we obtain

$$\mathscr{E}i = Li\frac{di}{dt} + i^2R, \tag{30.7.2}$$

which has the following physical interpretation in terms of the work done by the battery and the resulting energy transfers:

1. If a differential amount of charge dq passes through the battery of emf $\mathscr{E}$ in Fig. 30.6.2 in time dt, the battery does work on it in the amount $\mathscr{E}\,dq$. The rate at which the battery does work is $(\mathscr{E}\,dq)/dt$, or $\mathscr{E}i$. Thus, the left side of Eq. 30.7.2 represents the rate at which the emf device delivers energy to the rest of the circuit.
2. The rightmost term in Eq. 30.7.2 represents the rate at which energy appears as thermal energy in the resistor.
3. Energy that is delivered to the circuit but does not appear as thermal energy must, by the conservation-of-energy hypothesis, be stored in the magnetic field of the inductor. Because Eq. 30.7.2 represents the principle of conservation of energy for RL circuits, the middle term must represent the rate dU_B/dt at which magnetic potential energy U_B is stored in the magnetic field.

Thus

$$\frac{dU_B}{dt} = Li\frac{di}{dt}. \tag{30.7.3}$$

We can write this as

$$dU_B = Li\,di.$$

Integrating yields

$$\int_0^{U_B} dU_B = \int_0^i Li\,di$$

or

$$U_B = \tfrac{1}{2}Li^2 \quad \text{(magnetic energy)}, \tag{30.7.4}$$

which represents the total energy stored by an inductor L carrying a current i. Note the similarity in form between this expression for the energy stored in a magnetic field and the expression for the energy stored in an electric field by a capacitor with capacitance C and charge q; namely,

$$U_E = \frac{q^2}{2C}. \tag{30.7.5}$$

(The variable i^2 corresponds to q^2, and the constant L corresponds to $1/C$.)

Checkpoint 30.7.1

When we close a switch on an RL circuit, how does the magnetic field energy U_B depend on time:

(a) e^{-t/τ_L}, (b) $1 - e^{-t/\tau_L}$, (c) $(1 - e^{-t/\tau_L})^2$, (d) $(1 - e^{-t/\tau_L})e^{-t/\tau_L}$?

Sample Problem 30.7.1 Energy stored in a magnetic field

A coil has an inductance of 53 mH and a resistance of 0.35 Ω.

(a) If a 12 V emf is applied across the coil, how much energy is stored in the magnetic field after the current has built up to its equilibrium value?

KEY IDEA

The energy stored in the magnetic field of a coil at any time depends on the current through the coil at that time, according to Eq. 30.7.4 ($U_B = \frac{1}{2}Li^2$).

Calculations: Thus, to find the energy $U_{B\infty}$ stored at equilibrium, we must first find the equilibrium current. From Eq. 30.6.6, the equilibrium current is

$$i_\infty = \frac{\mathscr{E}}{R} = \frac{12\text{ V}}{0.35\,\Omega} = 34.3\text{ A}. \tag{30.7.6}$$

Then substitution yields

$$U_{B\infty} = \tfrac{1}{2}Li_\infty^2 = (\tfrac{1}{2})(53\times10^{-3}\text{ H})(34.3\text{ A})^2$$
$$= 31\text{ J}. \qquad \text{(Answer)}$$

(b) After how many time constants will half this equilibrium energy be stored in the magnetic field?

Calculations: Now we are being asked: At what time t will the relation

$$U_B = \tfrac{1}{2}U_{B\infty}$$

be satisfied? Using Eq. 30.7.4 twice allows us to rewrite this energy condition as

$$\tfrac{1}{2}Li^2 = (\tfrac{1}{2})\tfrac{1}{2}Li_\infty^2$$

or

$$i = \left(\frac{1}{\sqrt{2}}\right)i_\infty. \tag{30.7.7}$$

This equation tells us that, as the current increases from its initial value of 0 to its final value of i_∞, the magnetic field will have half its final stored energy when the current has increased to this value. In general, we know that i is given by Eq. 30.6.6, and here i_∞ (see Eq. 30.7.6) is $\mathscr{E}/R$; so Eq. 30.7.7 becomes

$$\frac{\mathscr{E}}{R}(1 - e^{-t/\tau_L}) = \frac{\mathscr{E}}{\sqrt{2}R}.$$

By canceling $\mathscr{E}/R$ and rearranging, we can write this as

$$e^{-t/\tau_L} = 1 - \frac{1}{\sqrt{2}} = 0.293,$$

which yields

$$\frac{t}{\tau_L} = -\ln 0.293 = 1.23$$

or

$$t \approx 1.2\tau_L. \qquad \text{(Answer)}$$

Thus, the energy stored in the magnetic field of the coil by the current will reach half its equilibrium value 1.2 time constants after the emf is applied.

30.8 ENERGY DENSITY OF A MAGNETIC FIELD

Learning Objectives

After reading this module, you should be able to . . .

30.8.1 Identify that energy is associated with any magnetic field.

30.8.2 Apply the relationship between energy density u_B of a magnetic field and the magnetic field magnitude B.

Key Idea

- If B is the magnitude of a magnetic field at any point (in an inductor or anywhere else), the density of stored magnetic energy at that point is

$$u_B = \frac{B^2}{2\mu_0} \quad \text{(magnetic energy density).}$$

Energy Density of a Magnetic Field

Consider a length l near the middle of a long solenoid of cross-sectional area A carrying current i; the volume associated with this length is Al. The energy U_B stored by the length l of the solenoid must lie entirely within this volume because the magnetic field outside such a solenoid is approximately zero. Moreover, the stored energy must be uniformly distributed within the solenoid because the magnetic field is (approximately) uniform everywhere inside.

Thus, the energy stored per unit volume of the field is

$$u_B = \frac{U_B}{Al}$$

or, since

$$U_B = \tfrac{1}{2}Li^2,$$

we have

$$u_B = \frac{Li^2}{2Al} = \frac{L}{l}\frac{i^2}{2A}. \tag{30.8.1}$$

Here L is the inductance of length l of the solenoid.

Substituting for L/l from Eq. 30.4.4, we find

$$u_B = \tfrac{1}{2}\mu_0 n^2 i^2, \tag{30.8.2}$$

where n is the number of turns per unit length. From Eq. 29.4.3 ($B = \mu_0 in$) we can write this *energy density* as

$$u_B = \frac{B^2}{2\mu_0} \quad \text{(magnetic energy density).} \tag{30.8.3}$$

This equation gives the density of stored energy at any point where the magnitude of the magnetic field is B. Even though we derived it by considering the special case of a solenoid, Eq. 30.8.3 holds for all magnetic fields, no matter how they are generated. The equation is comparable to Eq. 25.4.5,

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2, \tag{30.8.4}$$

which gives the energy density (in a vacuum) at any point in an electric field. Note that both u_B and u_E are proportional to the square of the appropriate field magnitude, B or E.

Checkpoint 30.8.1

The table lists the number of turns per unit length, current, and cross-sectional area for three solenoids. Rank the solenoids according to the magnetic energy density within them, greatest first.

Solenoid	Turns per Unit Length	Current	Area
a	$2n_1$	i_1	$2A_1$
b	n_1	$2i_1$	A_1
c	n_1	i_1	$6A_1$

30.9 MUTUAL INDUCTION

Learning Objectives

After reading this module, you should be able to . . .

30.9.1 Describe the mutual induction of two coils and sketch the arrangement.

30.9.2 Calculate the mutual inductance of one coil with respect to a second coil (or some second current that is changing).

30.9.3 Calculate the emf induced in one coil by a second coil in terms of the mutual inductance and the rate of change of the current in the second coil.

Key Idea

- If coils 1 and 2 are near each other, a changing current in either coil can induce an emf in the other. This mutual induction is described by

$$\mathscr{E}_2 = -M\frac{di_1}{dt}$$

and

$$\mathscr{E}_1 = -M\frac{di_2}{dt},$$

where M (measured in henries) is the mutual inductance.

Mutual Induction

In this section we return to the case of two interacting coils, which we first discussed in Module 30.1, and we treat it in a somewhat more formal manner. We saw earlier that if two coils are close together as in Fig. 30.1.2, a steady current i in one coil will set up a magnetic flux Φ through the other coil (*linking* the other coil). If we change i with time, an emf $\mathscr{E}$ given by Faraday's law appears in the second coil; we called this process *induction.* We could better have called it **mutual induction,** to suggest the mutual interaction of the two coils and to distinguish it from *self-induction*, in which only one coil is involved.

Let us look a little more quantitatively at mutual induction. Figure 30.9.1*a* shows two circular close-packed coils near each other and sharing a common central axis. With the variable resistor set at a particular resistance R, the battery produces a steady current i_1 in coil 1. This current creates a magnetic field represented by the lines of $\vec{B}_1$ in the figure. Coil 2 is connected to a sensitive meter but contains no battery; a magnetic flux Φ_{21} (the flux through coil 2 associated with the current in coil 1) links the N_2 turns of coil 2.

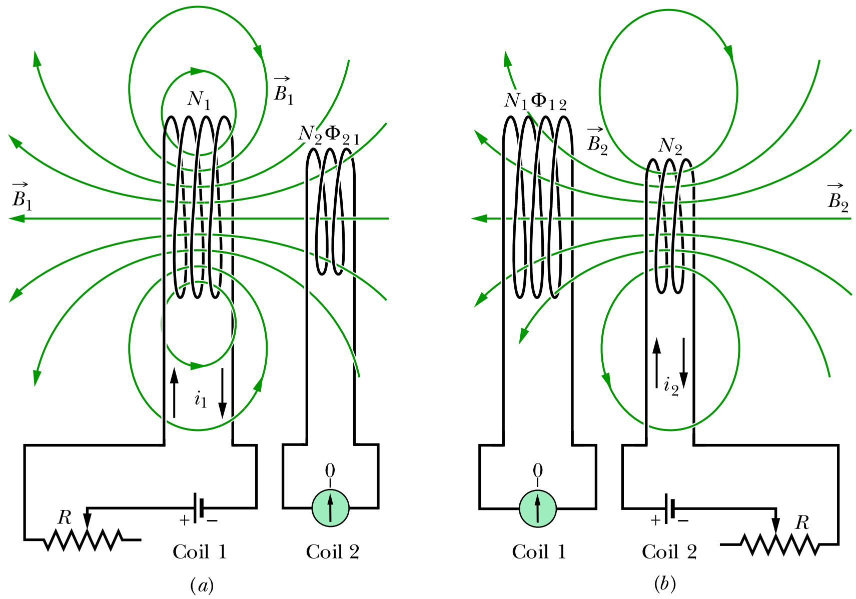

Figure 30.9.1 Mutual induction. (*a*) The magnetic field $\vec{B}_1$ produced by current i_1 in coil 1 extends through coil 2. If i_1 is varied (by varying resistance R), an emf is induced in coil 2 and current registers on the meter connected to coil 2. (*b*) The roles of the coils interchanged.

We define the mutual inductance M_{21} of coil 2 with respect to coil 1 as

$$M_{21} = \frac{N_2\Phi_{21}}{i_1}, \tag{30.9.1}$$

which has the same form as Eq. 30.4.1,

$$L = N\Phi/i, \tag{30.9.2}$$

the definition of inductance. We can recast Eq. 30.9.1 as

$$M_{21}i_1 = N_2\Phi_{21}. \tag{30.9.3}$$

If we cause i_1 to vary with time by varying R, we have

$$M_{21}\frac{di_1}{dt} = N_2\frac{d\Phi_{21}}{dt}. \tag{30.9.4}$$

The right side of this equation is, according to Faraday's law, just the magnitude of the emf $\mathscr{E}_2$ appearing in coil 2 due to the changing current in coil 1. Thus, with a minus sign to indicate direction,

$$\mathscr{E}_2 = -M_{21}\frac{di_1}{dt}, \tag{30.9.5}$$

which you should compare with Eq. 30.5.3 for self-induction ($\mathscr{E} = -L\,di/dt$).

Interchange. Let us now interchange the roles of coils 1 and 2, as in Fig. 30.9.1*b*; that is, we set up a current i_2 in coil 2 by means of a battery, and this produces a magnetic flux Φ_{12} that links coil 1. If we change i_2 with time by varying R, we then have, by the argument given above,

$$\mathscr{E}_1 = -M_{12}\frac{di_2}{dt}. \tag{30.9.6}$$

Thus, we see that the emf induced in either coil is proportional to the rate of change of current in the other coil. The proportionality constants M_{21} and M_{12} seem to be different. However, they turn out to be the same, although we cannot prove that fact here. Thus, we have

$$M_{21} = M_{12} = M, \tag{30.9.7}$$

and we can rewrite Eqs. 30.9.5 and 30.9.6 as

$$\mathscr{E}_2 = -M\frac{di_1}{dt} \tag{30.9.8}$$

and

$$\mathscr{E}_1 = -M\frac{di_2}{dt}. \tag{30.9.9}$$

Checkpoint 30.9.1

In Fig. 30.9.1*a*, consider the following three currents (in amperes and seconds) that we can set up in coil 1: (a) $i_a = 20.0$; (b) $i_b = 20t$; (c) $i_c = 10t$. Rank the currents according to the magnitude of the induced emf in coil 2, greatest first.

Review & Summary

Magnetic Flux The *magnetic flux* Φ_B through an area A in a magnetic field $\vec{B}$ is defined as

$$\Phi_B = \int \vec{B} \cdot d\vec{A}, \qquad (30.1.1)$$

where the integral is taken over the area. The SI unit of magnetic flux is the weber, where 1 Wb = 1 T·m^2. If $\vec{B}$ is perpendicular to the area and uniform over it, Eq. 30.1.1 becomes

$$\Phi_B = BA \quad (\vec{B} \perp A,\ \vec{B} \text{ uniform}). \qquad (30.1.2)$$

Faraday's Law of Induction If the magnetic flux Φ_B through an area bounded by a closed conducting loop changes with time, a current and an emf are produced in the loop; this process is called *induction*. The induced emf is

$$\mathscr{E} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's law)}. \qquad (30.1.4)$$

If the loop is replaced by a closely packed coil of N turns, the induced emf is

$$\mathscr{E} = -N\frac{d\Phi_B}{dt}. \qquad (30.1.5)$$

Lenz's Law An induced current has a direction such that the magnetic field *due to the current* opposes the change in the magnetic flux that induces the current. The induced emf has the same direction as the induced current.

Emf and the Induced Electric Field An emf is induced by a changing magnetic flux even if the loop through which the flux is changing is not a physical conductor but an imaginary line. The changing magnetic field induces an electric field $\vec{E}$ at every point of such a loop; the induced emf is related to $\vec{E}$ by

$$\mathscr{E} = \oint \vec{E} \cdot d\vec{s}, \qquad (30.3.4)$$

where the integration is taken around the loop. From Eq. 30.3.4 we can write Faraday's law in its most general form,

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's law)}. \qquad (30.3.5)$$

A changing magnetic field induces an electric field $\vec{E}$.

Inductors An **inductor** is a device that can be used to produce a known magnetic field in a specified region. If a current i is established through each of the N windings of an inductor, a magnetic flux Φ_B links those windings. The **inductance** L of the inductor is

$$L = \frac{N\Phi_B}{i} \quad \text{(inductance defined)}. \qquad (30.4.1)$$

The SI unit of inductance is the **henry** (H), where 1 henry = 1 H = 1 T·m^2/A. The inductance per unit length near the middle of a long solenoid of cross-sectional area A and n turns per unit length is

$$\frac{L}{l} = \mu_0 n^2 A \quad \text{(solenoid)}. \qquad (30.4.4)$$

Self-Induction If a current i in a coil changes with time, an emf is induced in the coil. This self-induced emf is

$$\mathscr{E}_L = -L\frac{di}{dt}. \qquad (30.5.3)$$

The direction of $\mathscr{E}_L$ is found from Lenz's law: The self-induced emf acts to oppose the change that produces it.

Series *RL* Circuits If a constant emf $\mathscr{E}$ is introduced into a single-loop circuit containing a resistance R and an inductance L, the current rises to an equilibrium value of $\mathscr{E}/R$:

$$i = \frac{\mathscr{E}}{R}(1 - e^{-t/\tau_L}) \quad \text{(rise of current)}. \qquad (30.6.6)$$

Here τ_L $(= L/R)$ is the **inductive time constant**. When the source of constant emf is removed, the current decays from a value i_0 according to

$$i = i_0 e^{-t/\tau_L} \quad \text{(decay of current)}. \qquad (30.6.10)$$

Magnetic Energy If an inductor L carries a current i, the inductor's magnetic field stores an energy given by

$$U_B = \tfrac{1}{2}Li^2 \quad \text{(magnetic energy)}. \qquad (30.7.4)$$

If B is the magnitude of a magnetic field at any point (in an inductor or anywhere else), the density of stored magnetic energy at that point is

$$u_B = \frac{B^2}{2\mu_0} \quad \text{(magnetic energy density)}. \qquad (30.8.3)$$

Mutual Induction If coils 1 and 2 are near each other, a changing current in either coil can induce an emf in the other. This mutual induction is described by

$$\mathscr{E}_2 = -M\frac{di_1}{dt} \qquad (30.9.8)$$

and

$$\mathscr{E}_1 = -M\frac{di_2}{dt}, \qquad (30.9.9)$$

where M (measured in henries) is the mutual inductance.

Questions

1 If the circular conductor in Fig. 30.1 undergoes thermal expansion while it is in a uniform magnetic field, a current is induced clockwise around it. Is the magnetic field directed into or out of the page?

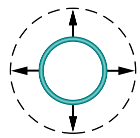

Figure 30.1 Question 1.

2 The wire loop in Fig. 30.2*a* is subjected, in turn, to six uniform magnetic fields, each directed parallel to the z axis, which is directed out of the plane of the figure. Figure 30.2*b* gives the z components B_z of the fields versus time t. (Plots 1 and 3 are parallel; so are plots 4 and 6. Plots 2 and 5 are parallel to the time axis.) Rank the six plots according to the emf induced in the loop, greatest clockwise emf first, greatest counterclockwise emf last.

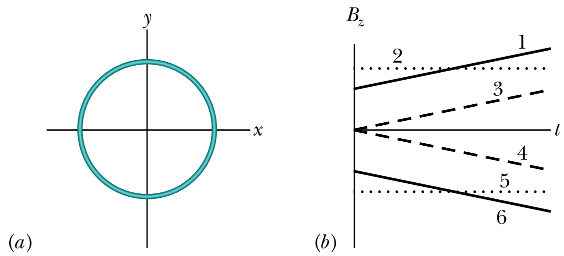

Figure 30.2 Question 2.

3 In Fig. 30.3, a long straight wire with current i passes (without touching) three rectangular wire loops with edge lengths L, $1.5L$, and $2L$. The loops are widely spaced (so as not to affect one another). Loops 1 and 3 are symmetric about the long wire. Rank the loops according to the size of the current induced in them if current i is (a) constant and (b) increasing, greatest first.

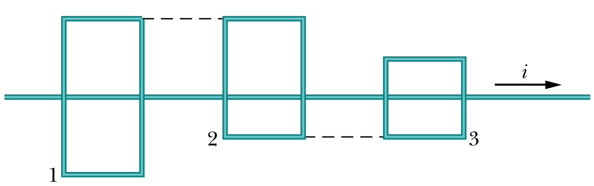

Figure 30.3 Question 3.

4 Figure 30.4 shows two circuits in which a conducting bar is slid at the same speed v through the same uniform magnetic field and along a **U**-shaped wire. The parallel lengths of the wire are separated by $2L$ in circuit 1 and by L in circuit 2. The current induced in circuit 1 is counterclockwise. (a) Is the magnetic field into or out of the page? (b) Is the current induced in circuit 2 clockwise or counterclockwise? (c) Is the emf induced in circuit 1 larger than, smaller than, or the same as that in circuit 2?

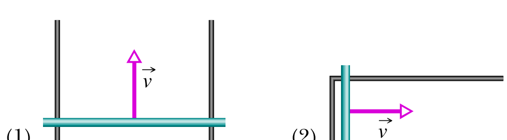

Figure 30.4 Question 4.

5 Figure 30.5 shows a circular region in which a decreasing uniform magnetic field is directed out of the page, as well as four concentric circular paths. Rank the paths according to the magnitude of $\oint \vec E \cdot d\vec s$ evaluated along them, greatest first.

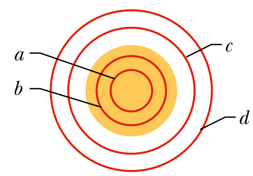

Figure 30.5 Question 5.

6 In Fig. 30.6, a wire loop has been bent so that it has three segments: segment *bc* (a quarter-circle), *ac* (a square corner), and *ab* (straight). Here are three choices for a magnetic field through the loop:

(1) $\vec B_1 = 3\hat i + 7\hat j - 5t\hat k$,

(2) $\vec B_2 = 5t\hat i - 4\hat j - 15\hat k$,

(3) $\vec B_3 = 2\hat i - 5t\hat j - 12\hat k$,

where $\vec B$ is in milliteslas and t is in seconds. Without written calculation, rank the choices according to (a) the work done per unit charge in setting up the induced current and (b) that induced current, greatest first. (c) For each choice, what is the direction of the induced current in the figure?

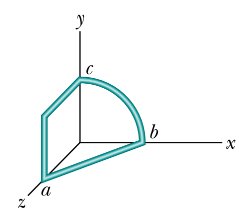

Figure 30.6 Question 6.

7 Figure 30.7 shows a circuit with two identical resistors and an ideal inductor. Is the current through the central resistor more than, less than, or the same as that through the other resistor (a) just after the closing of switch S, (b) a long time after that, (c) just after S is reopened a long time later, and (d) a long time after that?

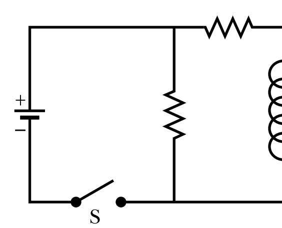

Figure 30.7 Question 7.

8 The switch in the circuit of Fig. 30.6.1 has been closed on a for a very long time when it is then thrown to b. The resulting current through the inductor is indicated in Fig. 30.8 for four sets of values for the resistance R and inductance L: (1) R_0 and L_0, (2) $2R_0$ and L_0, (3) R_0 and $2L_0$, (4) $2R_0$ and $2L_0$. Which set goes with which curve?

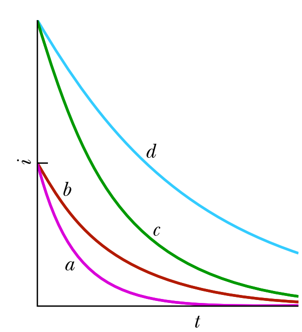

Figure 30.8 Question 8.

9 Figure 30.9 shows three circuits with identical batteries, inductors, and resistors. Rank the circuits, greatest first, according to the current through the resistor labeled R (a) long after the switch is closed, (b) just after the switch is reopened a long time later, and (c) long after it is reopened.

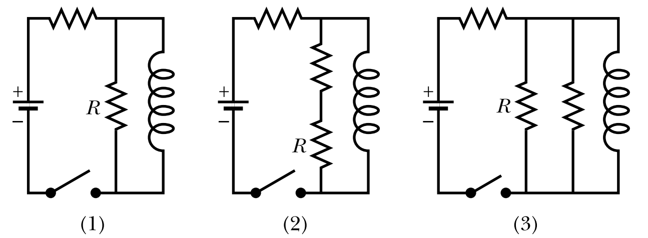

Figure 30.9 Question 9.

10 Figure 30.10 gives the variation with time of the potential difference V_R across a resistor in three circuits wired as shown in Fig. 30.6.2. The circuits contain the same resistance R and emf $\mathscr{E}$ but differ in the inductance L. Rank the circuits according to the value of L, greatest first.

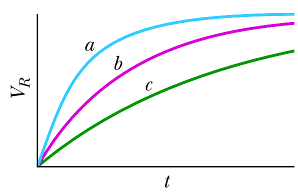

Figure 30.10 Question 10.

11 Figure 30.11 shows three situations in which a wire loop lies partially in a magnetic field. The magnitude of the field is either increasing or decreasing, as indicated. In each situation, a battery is part of the loop. In which situations are the induced emf and the battery emf in the same direction along the loop?

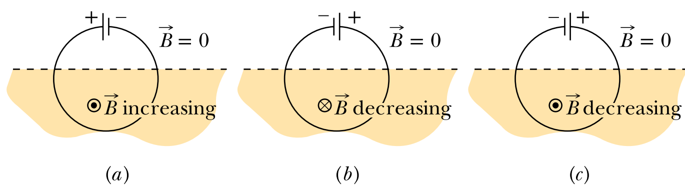

Figure 30.11 Question 11.

12 Figure 30.12 gives four situations in which we pull rectangular wire loops out of identical magnetic fields (directed into the page) at the same constant speed. The loops have edge lengths of either L or $2L$, as drawn. Rank the situations according to (a) the magnitude of the force required of us and (b) the rate at which energy is transferred from us to thermal energy of the loop, greatest first.

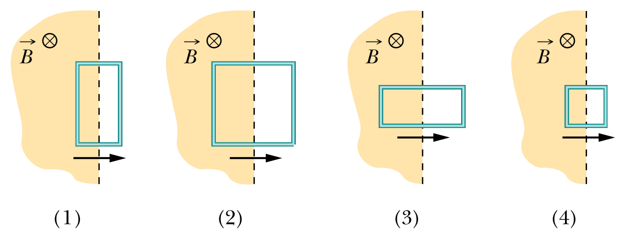

Figure 30.12 Question 12.

Problems

GO Tutoring problem available (at instructor's discretion) in *WileyPLUS*
SSM Worked-out solution available in Student Solutions Manual
E Easy M Medium H Hard
FCP Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com
CALC Requires calculus
BIO Biomedical application

Module 30.1 Faraday's Law and Lenz's Law

1 E In Fig. 30.13, a circular loop of wire 10 cm in diameter (seen edge-on) is placed with its normal $\vec{N}$ at an angle $\theta = 30°$ with the direction of a uniform magnetic field $\vec{B}$ of magnitude 0.50 T. The loop is then rotated such that $\vec{N}$ rotates in a cone about the field direction at the rate 100 rev/min; angle θ remains unchanged during the process. What is the emf induced in the loop?

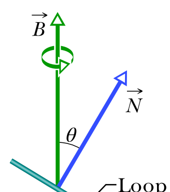

Figure 30.13 Problem 1.

2 E A certain elastic conducting material is stretched into a circular loop of 12.0 cm radius. It is placed with its plane perpendicular to a uniform 0.800 T magnetic field. When released, the radius of the loop starts to shrink at an instantaneous rate of 75.0 cm/s. What emf is induced in the loop at that instant?

3 E CALC SSM In Fig. 30.14, a 120-turn coil of radius 1.8 cm and resistance 5.3 Ω is coaxial with a solenoid of 220 turns/cm and diameter 3.2 cm. The solenoid current drops from 1.5 A to zero in time interval Δt = 25 ms. What current is induced in the coil during Δt?

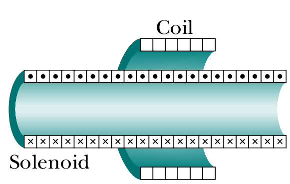

Figure 30.14 Problem 3.

4 E CALC A wire loop of radius 12 cm and resistance 8.5 Ω is located in a uniform magnetic field $\vec{B}$ that changes in magnitude as given in Fig. 30.15. The vertical axis scale is set by B_s = 0.50 T, and the horizontal axis scale is set by t_s = 6.00 s. The loop's plane is perpendicular to $\vec{B}$. What emf is induced in the loop during time intervals (a) 0 to 2.0 s, (b) 2.0 s to 4.0 s, and (c) 4.0 s to 6.0 s?

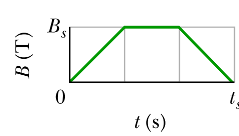

Figure 30.15 Problem 4.

5 E In Fig. 30.16, a wire forms a closed circular loop, of radius R = 2.0 m and resistance 4.0 Ω. The circle is centered on a long straight wire; at time t = 0, the current in the long straight wire is 5.0 A rightward. Thereafter, the current changes according to $i = 5.0\text{ A} - (2.0\text{ A/s}^2)t^2$. (The straight wire is insulated; so there is no electrical contact between it and the wire of the loop.) What is the magnitude of the current induced in the loop at times $t > 0$?

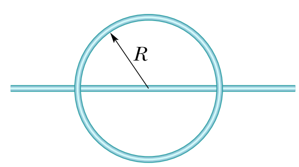

Figure 30.16 Problem 5.

6 E CALC Figure 30.17*a* shows a circuit consisting of an ideal battery with emf $\mathscr{E} = 6.00\,\mu$V, a resistance R, and a small wire loop of area 5.0 cm². For the time interval t = 10 s to t = 20 s, an external magnetic field is set up throughout the loop. The field is uniform, its direction is into the page in Fig. 30.17*a*, and the field magnitude is given by $B = at$, where B is in teslas, a is a constant, and t is in seconds. Figure 30.17b gives the current i in the circuit before, during, and after the external field is set up. The vertical axis scale is set by $i_s = 2.0$ mA. Find the constant a in the equation for the field magnitude.

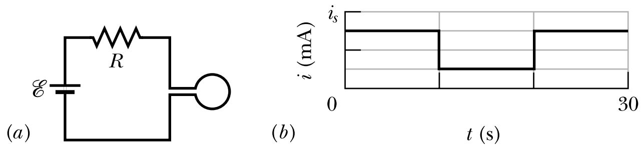

Figure 30.17 Problem 6.

7 E CALC In Fig. 30.18, the magnetic flux through the loop increases according to the relation $\Phi_B = 6.0t^2 + 7.0t$, where Φ_B is in milliwebers and t is in seconds. (a) What is the magnitude of the emf induced in the loop when $t = 2.0$ s? (b) Is the direction of the current through R to the right or left?

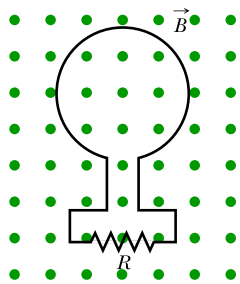

Figure 30.18 Problem 7.

8 E A uniform magnetic field $\vec{B}$ is perpendicular to the plane of a circular loop of diameter 10 cm formed from wire of diameter 2.5 mm and resistivity $1.69 \times 10^{-8}\ \Omega \cdot \text{m}$. At what rate must the magnitude of $\vec{B}$ change to induce a 10 A current in the loop?

9 E A small loop of area 6.8 mm^2 is placed inside a long solenoid that has 854 turns/cm and carries a sinusoidally varying current i of amplitude 1.28 A and angular frequency 212 rad/s. The central axes of the loop and solenoid coincide. What is the amplitude of the emf induced in the loop?

10 M CALC Figure 30.19 shows a closed loop of wire that consists of a pair of equal semicircles, of radius 3.7 cm, lying in mutually perpendicular planes. The loop was formed by folding a flat circular loop along a diameter until the two halves became perpendicular to each other. A uniform magnetic field $\vec{B}$ of magnitude 76 mT is directed perpendicular to the fold diameter and makes equal angles (of 45°) with the planes of the semicircles. The magnetic field is reduced to zero at a uniform rate during a time interval of 4.5 ms. During this interval, what are the (a) magnitude and (b) direction (clockwise or counterclockwise when viewed along the direction of $\vec{B}$) of the emf induced in the loop?

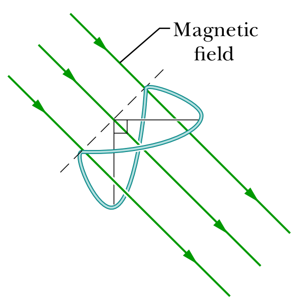

Figure 30.19 Problem 10.

11 M CALC A rectangular coil of N turns and of length a and width b is rotated at frequency f in a uniform magnetic field $\vec{B}$, as indicated in Fig. 30.20. The coil is connected to co-rotating cylinders, against which metal brushes slide to make contact. (a) Show that the emf induced in the coil is given (as a function of time t) by

$$\mathscr{E} = 2\pi f NabB \sin(2\pi ft) = \mathscr{E}_0 \sin(2\pi ft).$$

This is the principle of the commercial alternating-current generator. (b) What value of Nab gives an emf with $\mathscr{E}_0 = 150$ V when the loop is rotated at 60.0 rev/s in a uniform magnetic field of 0.500 T?

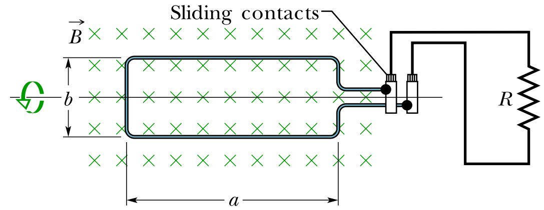

Figure 30.20 Problem 11.

12 M CALC In Fig. 30.21, a wire loop of lengths $L = 40.0$ cm and $W = 25.0$ cm lies in a magnetic field $\vec{B}$. What are the (a) magnitude $\mathscr{E}$ and (b) direction (clockwise or counterclockwise—or "none" if $\mathscr{E} = 0$) of the emf induced in the loop if $\vec{B} = (4.00 \times 10^{-2}\ \text{T/m})y\hat{k}$? What are (c) $\mathscr{E}$ and (d) direction if $\vec{B} = (6.00 \times 10^{-2}\ \text{T/s})t\hat{k}$? What are (e) $\mathscr{E}$ and (f) direction if $\vec{B} = (8.00 \times 10^{-2}\ \text{T/m} \cdot \text{s})yt\hat{k}$? What are (g) $\mathscr{E}$ and (h) direction if $\vec{B} = (3.00 \times 10^{-2}\ \text{T/m} \cdot \text{s})xt\hat{j}$? What are (i) $\mathscr{E}$ and (j) direction if $\vec{B} = (5.00 \times 10^{-2}\ \text{T/m} \cdot \text{s})yt\hat{i}$?

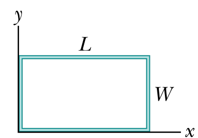

Figure 30.21 Problem 12.

13 M One hundred turns of (insulated) copper wire are wrapped around a wooden cylindrical core of cross-sectional area 1.20×10^{-3} m^2. The two ends of the wire are connected to a resistor. The total resistance in the circuit is 13.0 Ω. If an externally applied uniform longitudinal magnetic field in the core changes from 1.60 T in one direction to 1.60 T in the opposite direction, how much charge flows through a point in the circuit during the change?

14 M CALC GO In Fig. 30.22a, a uniform magnetic field $\vec{B}$ increases in magnitude with time t as given by Fig. 30.22b, where the vertical axis scale is set by $B_s = 9.0$ mT and the horizontal scale is set by $t_s = 3.0$ s. A circular conducting loop of area 8.0×10^{-4} m^2 lies in the field, in the plane of the page. The amount of charge q passing point A on the loop is given in Fig. 30.22c as a function of t, with the vertical axis scale set by $q_s = 6.0$ mC and the horizontal axis scale again set by $t_s = 3.0$ s. What is the loop's resistance?

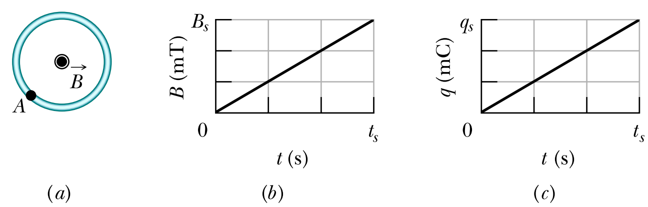

Figure 30.22 Problem 14.

15 M CALC GO A square wire loop with 2.00 m sides is perpendicular to a uniform magnetic field, with half the area of the loop in the field as shown in Fig. 30.23. The loop contains an ideal battery with emf $\mathscr{E} = 20.0$ V. If the magnitude of the field varies with time according to $B = 0.0420 - 0.870t$, with B in teslas and t in seconds, what are (a)

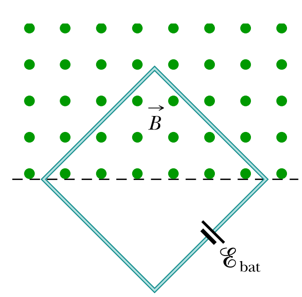

Figure 30.23 Problem 15.

the net emf in the circuit and (b) the direction of the (net) current around the loop?

16 M CALC GO Figure 30.24*a* shows a wire that forms a rectangle ($W = 20$ cm, $H = 30$ cm) and has a resistance of 5.0 mΩ. Its interior is split into three equal areas, with magnetic fields $\vec{B}_1$, $\vec{B}_2$, and $\vec{B}_3$. The fields are uniform within each region and directly out of or into the page as indicated. Figure 30.24*b* gives the change in the z components B_z of the three fields with time t; the vertical axis scale is set by $B_s = 4.0\ \mu$T and $B_b = -2.5B_s$, and the horizontal axis scale is set by $t_s = 2.0$ s. What are the (a) magnitude and (b) direction of the current induced in the wire?

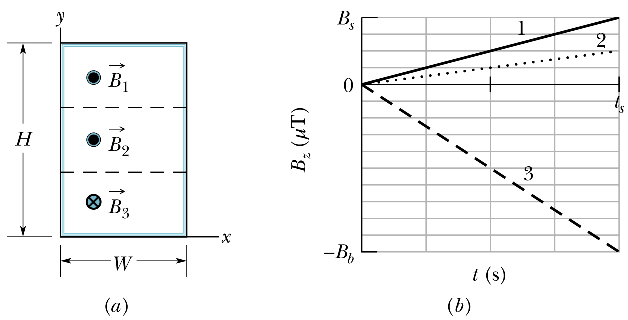

Figure 30.24 Problem 16.

17 M CALC A small circular loop of area 2.00 cm^2 is placed in the plane of, and concentric with, a large circular loop of radius 1.00 m. The current in the large loop is changed at a constant rate from 200 A to –200 A (a change in direction) in a time of 1.00 s, starting at $t = 0$. What is the magnitude of the magnetic field $\vec{B}$ at the center of the small loop due to the current in the large loop at (a) $t = 0$, (b) $t = 0.500$ s, and (c) $t = 1.00$ s? (d) From $t = 0$ to $t = 1.00$ s, is $\vec{B}$ reversed? Because the inner loop is small, assume $\vec{B}$ is uniform over its area. (e) What emf is induced in the small loop at $t = 0.500$ s?

18 M CALC In Fig. 30.25, two straight conducting rails form a right angle. A conducting bar in contact with the rails starts at the vertex at time $t = 0$ and moves with a constant velocity of 5.20 m/s along them. A magnetic field with $B = 0.350$ T is directed out of the page. Calculate (a) the flux through the triangle formed by the rails and bar at $t = 3.00$ s and (b) the emf around the triangle at that time. (c) If the emf is $\mathscr{E} = at^n$, where a and n are constants, what is the value of n?

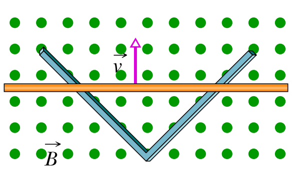

Figure 30.25 Problem 18.

19 M CALC An electric generator contains a coil of 100 turns of wire, each forming a rectangular loop 50.0 cm by 30.0 cm. The coil is placed entirely in a uniform magnetic field with magnitude $B = 3.50$ T and with $\vec{B}$ initially perpendicular to the coil's plane. What is the maximum value of the emf produced when the coil is spun at 1000 rev/min about an axis perpendicular to $\vec{B}$?

20 M At a certain place, Earth's magnetic field has magnitude $B = 0.590$ gauss and is inclined downward at an angle of 70.0° to the horizontal. A flat horizontal circular coil of wire with a radius of 10.0 cm has 1000 turns and a total resistance of 85.0 Ω. It is connected in series to a meter with 140 Ω resistance. The coil is flipped through a half-revolution about a diameter, so that it is again horizontal. How much charge flows through the meter during the flip?

21 M CALC In Fig. 30.26, a stiff wire bent into a semicircle of radius $a = 2.0$ cm is rotated at constant angular speed 40 rev/s in a uniform 20 mT magnetic field. What are the (a) frequency and (b) amplitude of the emf induced in the loop?

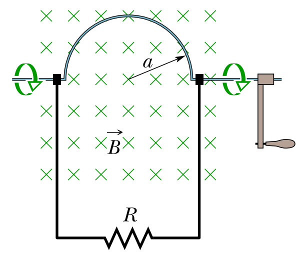

Figure 30.26 Problem 21.

22 M A rectangular loop (area = 0.15 m^2) turns in a uniform magnetic field, $B = 0.20$ T. When the angle between the field and the normal to the plane of the loop is $\pi/2$ rad and increasing at 0.60 rad/s, what emf is induced in the loop?

23 M CALC SSM Figure 30.27 shows two parallel loops of wire having a common axis. The smaller loop (radius r) is above the larger loop (radius R) by a distance $x \gg R$. Consequently, the magnetic field due to the counterclockwise current i in the larger loop is nearly uniform throughout the smaller loop. Suppose that x is increasing at the constant rate $dx/dt = v$. (a) Find an expression for the magnetic flux through the area of the smaller loop as a function of x. (*Hint:* See Eq. 29.5.3.) In the smaller loop, find (b) an expression for the induced emf and (c) the direction of the induced current.

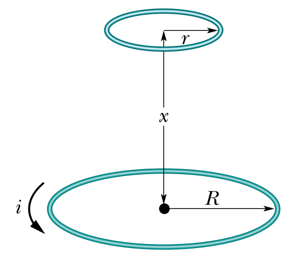

Figure 30.27 Problem 23.

24 M CALC A wire is bent into three circular segments, each of radius $r = 10$ cm, as shown in Fig. 30.28. Each segment is a quadrant of a circle, ab lying in the xy plane, bc lying in the yz plane, and ca lying in the zx plane. (a) If a uniform magnetic field $\vec{B}$ points in the positive x direction, what is the magnitude of the emf developed in the wire when B increases at the rate of 3.0 mT/s? (b) What is the direction of the current in segment bc?

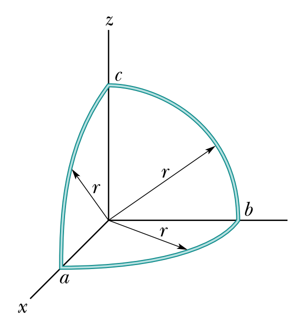

Figure 30.28 Problem 24.

25 H GO Two long, parallel copper wires of diameter 2.5 mm carry currents of 10 A in opposite directions. (a) Assuming that their central axes are 20 mm apart, calculate the magnetic flux per meter of wire that exists in the space between those axes. (b) What percentage of this flux lies inside the wires? (c) Repeat part (a) for parallel currents.

26 H CALC GO For the wire arrangement in Fig. 30.29, $a = 12.0$ cm and $b = 16.0$ cm. The current in the long straight wire is $i = 4.50t^2 - 10.0t$, where i is in amperes and t is in seconds. (a) Find the emf in the square loop at $t = 3.00$ s. (b) What is the direction of the induced current in the loop?

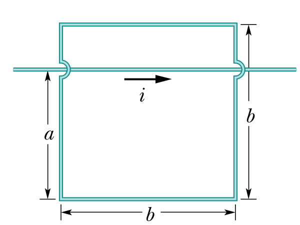

Figure 30.29 Problem 26.

27 H CALC As seen in Fig. 30.30, a square loop of wire has sides of length 2.0 cm. A magnetic field is directed out of the page; its magnitude is given by $B = 4.0t^2y$, where B is in teslas, t is in seconds, and y is in meters. At $t = 2.5$ s, what are the (a) magnitude and (b) direction of the emf induced in the loop?

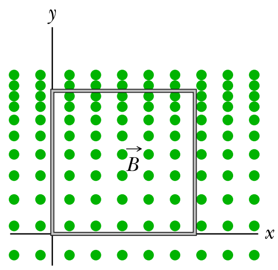

Figure 30.30 Problem 27.

28 H CALC GO In Fig. 30.31, a rectangular loop of wire with length $a = 2.2$ cm, width $b = 0.80$ cm, and resistance $R = 0.40$ mΩ is placed near an infinitely long wire carrying current $i = 4.7$ A. The loop is then moved away from the wire at constant speed $v = 3.2$ mm/s. When the center of the loop is at distance $r = 1.5b$, what are (a) the magnitude of the magnetic flux through the loop and (b) the current induced in the loop?

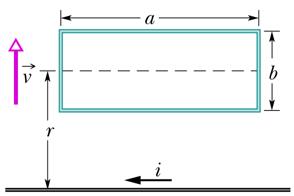

Figure 30.31 Problem 28.

Module 30.2 Induction and Energy Transfers

29 E In Fig. 30.32, a metal rod is forced to move with constant velocity $\vec{v}$ along two parallel metal rails, connected with a strip of metal at one end. A magnetic field of magnitude $B = 0.350$ T points out of the page. (a) If the rails are separated by $L = 25.0$ cm and the speed of the rod is 55.0 cm/s, what emf is generated? (b) If the rod has a resistance of 18.0 Ω and the rails and connector have negligible resistance, what is the current in the rod? (c) At what rate is energy being transferred to thermal energy?

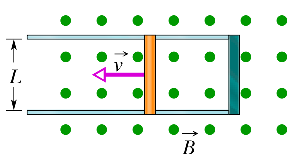

Figure 30.32 Problems 29 and 35.

30 E CALC In Fig. 30.33*a*, a circular loop of wire is concentric with a solenoid and lies in a plane perpendicular to the solenoid's central axis. The loop has radius 6.00 cm. The solenoid has radius 2.00 cm, consists of 8000 turns/m, and has a current i_{sol} varying with time t as given in Fig. 30.33*b*, where the vertical axis scale is set by $i_s = 1.00$ A and the horizontal axis scale is set by $t_s = 2.0$ s. Figure 30.33*c* shows, as a function of time, the energy E_{th} that is transferred to thermal energy of the loop; the vertical axis scale is set by $E_s = 100.0$ nJ. What is the loop's resistance?

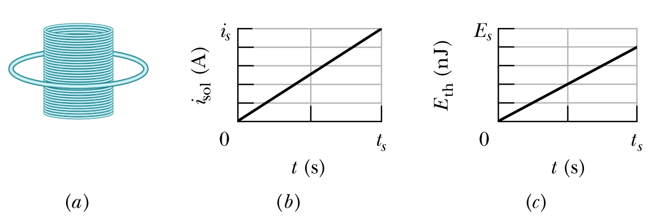

Figure 30.33 Problem 30.

31 E CALC SSM If 50.0 cm of copper wire (diameter = 1.00 mm) is formed into a circular loop and placed perpendicular to a uniform magnetic field that is increasing at the constant rate of 10.0 mT/s, at what rate is thermal energy generated in the loop?

32 E CALC A loop antenna of area 2.00 cm^2 and resistance 5.21 $\mu\Omega$ is perpendicular to a uniform magnetic field of magnitude 17.0 μT. The field magnitude drops to zero in 2.96 ms. How much thermal energy is produced in the loop by the change in field?

33 M CALC GO Figure 30.34 shows a rod of length $L = 10.0$ cm that is forced to move at constant speed $v = 5.00$ m/s along horizontal rails. The rod, rails, and connecting strip at the right form a conducting loop. The rod has resistance 0.400 Ω; the rest of the loop has negligible resistance. A current $i = 100$ A through the long straight wire at distance $a = 10.0$ mm from the loop sets up a (nonuniform) magnetic field through the loop. Find the (a) emf and (b) current induced in the loop. (c) At what rate is thermal energy generated in the rod? (d) What is the magnitude of the force that must be applied to the rod to make it move at constant speed? (e) At what rate does this force do work on the rod?

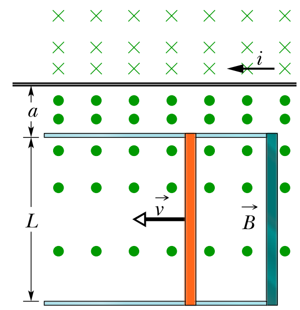

Figure 30.34 Problem 33.

34 M CALC In Fig. 30.35, a long rectangular conducting loop, of width L, resistance R, and mass m, is hung in a horizontal, uniform magnetic field $\vec{B}$ that is directed into the page and that exists only above line *aa*. The loop is then dropped; during its fall, it accelerates until it reaches a certain terminal speed v_t. Ignoring air drag, find an expression for v_t.

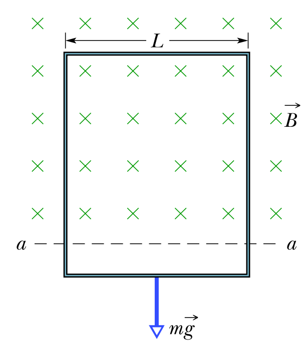

Figure 30.35 Problem 34.

35 M The conducting rod shown in Fig. 30.32 has length L and is being pulled along horizontal, frictionless conducting rails at a constant velocity $\vec{v}$. The rails are connected at one end with a metal strip. A uniform magnetic field $\vec{B}$, directed out of the page, fills the region in which the rod moves. Assume that $L = 10$ cm, $v = 5.0$ m/s, and $B = 1.2$ T. What are the (a) magnitude and (b) direction (up or down the page) of the emf induced in the rod? What are the (c) size and (d) direction of the current in the conducting loop? Assume that the resistance of the rod is 0.40 Ω and that the resistance of the rails and metal strip is negligibly small. (e) At what rate is thermal energy being generated in the rod? (f) What external force on the rod is needed to maintain $\vec{v}$? (g) At what rate does this force do work on the rod?

Module 30.3 Induced Electric Fields

36 E CALC Figure 30.36 shows two circular regions R_1 and R_2 with radii $r_1 = 20.0$ cm and $r_2 = 30.0$ cm. In R_1 there is a uniform magnetic field of magnitude $B_1 = 50.0$ mT directed into the page, and in R_2 there is a uniform magnetic field of magnitude $B_2 = 75.0$ mT directed out of the page (ignore

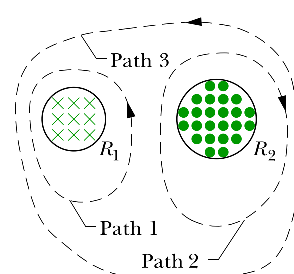

Figure 30.36 Problem 36.

fringing). Both fields are decreasing at the rate of 8.50 mT/s. Calculate $\oint \vec{E} \cdot d\vec{s}$ for (a) path 1, (b) path 2, and (c) path 3.

37 E CALC SSM A long solenoid has a diameter of 12.0 cm. When a current i exists in its windings, a uniform magnetic field of magnitude $B = 30.0$ mT is produced in its interior. By decreasing i, the field is caused to decrease at the rate of 6.50 mT/s. Calculate the magnitude of the induced electric field (a) 2.20 cm and (b) 8.20 cm from the axis of the solenoid.

38 M CALC GO A circular region in an xy plane is penetrated by a uniform magnetic field in the positive direction of the z axis. The field's magnitude B (in teslas) increases with time t (in seconds) according to $B = at$, where a is a constant. The magnitude E of the electric field set up by that increase in the magnetic field is given by Fig. 30.37 versus radial distance r; the vertical axis scale is set by $E_s = 300\ \mu$N/C, and the horizontal axis scale is set by $r_s = 4.00$ cm. Find a.

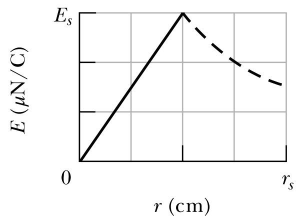

Figure 30.37 Problem 38.

39 M CALC The magnetic field of a cylindrical magnet that has a pole-face diameter of 3.3 cm can be varied sinusoidally between 29.6 T and 30.0 T at a frequency of 15 Hz. (The current in a wire wrapped around a permanent magnet is varied to give this variation in the net field.) At a radial distance of 1.6 cm, what is the amplitude of the electric field induced by the variation?

Module 30.4 Inductors and Inductance

40 E The inductance of a closely packed coil of 400 turns is 8.0 mH. Calculate the magnetic flux through the coil when the current is 5.0 mA.

41 E A circular coil has a 10.0 cm radius and consists of 30.0 closely wound turns of wire. An externally produced magnetic field of magnitude 2.60 mT is perpendicular to the coil. (a) If no current is in the coil, what magnetic flux links its turns? (b) When the current in the coil is 3.80 A in a certain direction, the net flux through the coil is found to vanish. What is the inductance of the coil?

42 M Figure 30.38 shows a copper strip of width $W = 16.0$ cm that has been bent to form a shape that consists of a tube of radius $R = 1.8$ cm plus two parallel flat extensions. Current $i = 35$ mA is distributed uniformly across the width so that the tube is effectively a one-turn solenoid. Assume that the magnetic field outside the tube is negligible and the field inside the tube is uniform. What are (a) the magnetic field magnitude inside the tube and (b) the inductance of the tube (excluding the flat extensions)?

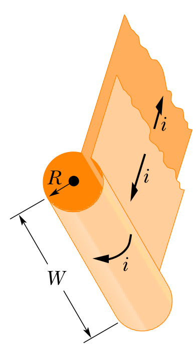

Figure 30.38 Problem 42.

43 M CALC GO Two identical long wires of radius $a = 1.53$ mm are parallel and carry identical currents in opposite directions. Their center-to-center separation is $d = 14.2$ cm. Neglect the flux within the wires but consider the flux in the region between the wires. What is the inductance per unit length of the wires?

Module 30.5 Self-Induction

44 E A 12 H inductor carries a current of 2.0 A. At what rate must the current be changed to produce a 60 V emf in the inductor?

45 E CALC At a given instant the current and self-induced emf in an inductor are directed as indicated in Fig. 30.39. (a) Is the current increasing or decreasing? (b) The induced emf is 17 V, and the rate of change of the current is 25 kA/s; find the inductance.

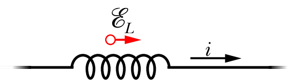

Figure 30.39 Problem 45.

46 M CALC The current i through a 4.6 H inductor varies with time t as shown by the graph of Fig. 30.40, where the vertical axis scale is set by $i_s = 8.0$ A and the horizontal axis scale is set by $t_s = 6.0$ ms. The inductor has a resistance of 12 Ω. Find the magnitude of the induced emf $\mathscr{E}$ during time intervals (a) 0 to 2 ms, (b) 2 ms to 5 ms, and (c) 5 ms to 6 ms. (Ignore the behavior at the ends of the intervals.)

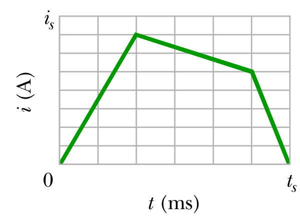

Figure 30.40 Problem 46.

47 M *Inductors in series.* Two inductors L_1 and L_2 are connected in series and are separated by a large distance so that the magnetic field of one cannot affect the other. (a) Show that the equivalent inductance is given by

$$L_{eq} = L_1 + L_2.$$

(*Hint:* Review the derivations for resistors in series and capacitors in series. Which is similar here?) (b) What is the generalization of (a) for N inductors in series?

48 M CALC *Inductors in parallel.* Two inductors L_1 and L_2 are connected in parallel and separated by a large distance so that the magnetic field of one cannot affect the other. (a) Show that the equivalent inductance is given by

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}.$$

(*Hint:* Review the derivations for resistors in parallel and capacitors in parallel. Which is similar here?) (b) What is the generalization of (a) for N inductors in parallel?

49 M The inductor arrangement of Fig. 30.41, with $L_1 = 30.0$ mH, $L_2 = 50.0$ mH, $L_3 = 20.0$ mH, and $L_4 = 15.0$ mH, is to be connected to a varying current source. What is the equivalent inductance of the arrangement? (First see Problems 47 and 48.)

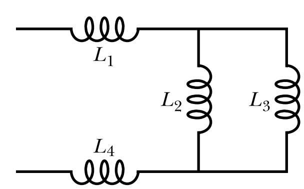

Figure 30.41 Problem 49.

Module 30.6 *RL* Circuits

50 E The current in an RL circuit builds up to one-third of its steady-state value in 5.00 s. Find the inductive time constant.

51 E The current in an RL circuit drops from 1.0 A to 10 mA in the first second following removal of the battery from the circuit. If L is 10 H, find the resistance R in the circuit.

52 E The switch in Fig. 30.6.1 is closed on a at time $t = 0$. What is the ratio $\mathscr{E}_L/\mathscr{E}$ of the inductor's self-induced emf to the battery's emf (a) just after $t = 0$ and (b) at $t = 2.00\tau_L$? (c) At what multiple of τ_L will $\mathscr{E}_L/\mathscr{E} = 0.500$?

53 E SSM A solenoid having an inductance of 6.30 μH is connected in series with a 1.20 kΩ resistor. (a) If a 14.0 V battery is connected across the pair, how long will it take for the current through the resistor to reach 80.0% of its final value? (b) What is the current through the resistor at time $t = 1.0\tau_L$?

54 E In Fig. 30.42, $\mathscr{E} = 100$ V, $R_1 = 10.0\ \Omega$, $R_2 = 20.0\ \Omega$, $R_3 = 30.0\ \Omega$, and $L = 2.00$ H. Immediately after switch S is closed, what are (a) i_1 and (b) i_2? (Let currents in the indicated directions have positive values and currents in the opposite directions have negative values.) A long time later, what are (c) i_1 and (d) i_2? The switch is then reopened. Just then, what are (e) i_1 and (f) i_2? A long time later, what are (g) i_1 and (h) i_2?

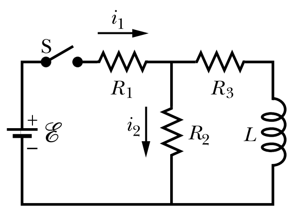

Figure 30.42 Problem 54.

55 E SSM A battery is connected to a series RL circuit at time $t = 0$. At what multiple of τ_L will the current be 0.100% less than its equilibrium value?

56 E CALC In Fig. 30.43, the inductor has 25 turns and the ideal battery has an emf of 16 V. Figure 30.44 gives the magnetic flux Φ through each turn versus the current i through the inductor. The vertical axis scale is set by $\Phi_s = 4.0 \times 10^{-4}$ T·m^2, and the horizontal axis scale is set by $i_s = 2.00$ A. If switch S is closed at time $t = 0$, at what rate di/dt will the current be changing at $t = 1.5\tau_L$?

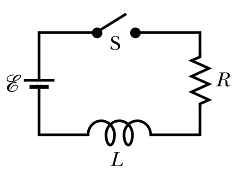

Figure 30.43 Problems 56, 80, and 83.

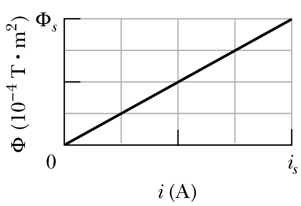

Figure 30.44 Problem 56.

57 M CALC GO In Fig. 30.45, $R = 15\ \Omega$, $L = 5.0$ H, the ideal battery has $\mathscr{E} = 10$ V, and the fuse in the upper branch is an ideal 3.0 A fuse. It has zero resistance as long as the current through it remains less than 3.0 A. If the current reaches 3.0 A, the fuse "blows" and thereafter has infinite resistance. Switch S is closed at time $t = 0$. (a) When does the fuse blow? (*Hint:* Equation 30.6.6 does not apply. Rethink Eq. 30.6.4.)(b) Sketch a graph of the current i through the inductor as a function of time. Mark the time at which the fuse blows.

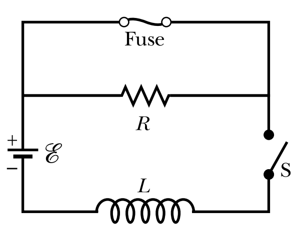

Figure 30.45 Problem 57.

58 M GO Suppose the emf of the battery in the circuit shown in Fig. 30.6.2 varies with time t so that the current is given by $i(t) = 3.0 + 5.0t$, where i is in amperes and t is in seconds. Take $R = 4.0\ \Omega$ and $L = 6.0$ H, and find an expression for the battery emf as a function of t. (*Hint:* Apply the loop rule.)

59 H SSM In Fig. 30.46, after switch S is closed at time $t = 0$, the emf of the source is automatically adjusted to maintain a constant current i through S. (a) Find the current through the inductor as a function of time. (b) At what time is the current through the resistor equal to the current through the inductor?

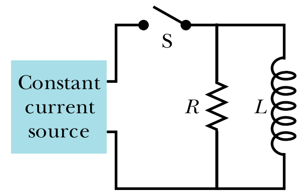

Figure 30.46 Problem 59.

60 H CALC A wooden toroidal core with a square cross section has an inner radius of 10 cm and an outer radius of 12 cm. It is wound with one layer of wire (of diameter 1.0 mm and resistance per meter 0.020 Ω/m). What are (a) the inductance and (b) the inductive time constant of the resulting toroid? Ignore the thickness of the insulation on the wire.

Module 30.7 Energy Stored in a Magnetic Field

61 E SSM A coil is connected in series with a 10.0 kΩ resistor. An ideal 50.0 V battery is applied across the two devices, and the current reaches a value of 2.00 mA after 5.00 ms. (a) Find the inductance of the coil. (b) How much energy is stored in the coil at this same moment?

62 E CALC A coil with an inductance of 2.0 H and a resistance of 10 Ω is suddenly connected to an ideal battery with $\mathscr{E} = 100$ V. At 0.10 s after the connection is made, what is the rate at which (a) energy is being stored in the magnetic field, (b) thermal energy is appearing in the resistance, and (c) energy is being delivered by the battery?

63 E CALC At $t = 0$, a battery is connected to a series arrangement of a resistor and an inductor. If the inductive time constant is 37.0 ms, at what time is the rate at which energy is dissipated in the resistor equal to the rate at which energy is stored in the inductor's magnetic field?

64 E At $t = 0$, a battery is connected to a series arrangement of a resistor and an inductor. At what multiple of the inductive time constant will the energy stored in the inductor's magnetic field be 0.500 its steady-state value?

65 M CALC GO For the circuit of Fig. 30.6.2, assume that $\mathscr{E} = 10.0$ V, $R = 6.70\ \Omega$, and $L = 5.50$ H. The ideal battery is connected at time $t = 0$. (a) How much energy is delivered by the battery during the first 2.00 s? (b) How much of this energy is stored in the magnetic field of the inductor? (c) How much of this energy is dissipated in the resistor?

Module 30.8 Energy Density of a Magnetic Field

66 E A circular loop of wire 50 mm in radius carries a current of 100 A. Find the (a) magnetic field strength and (b) energy density at the center of the loop.

67 E SSM A solenoid that is 85.0 cm long has a cross-sectional area of 17.0 cm^2. There are 950 turns of wire carrying a current of 6.60 A. (a) Calculate the energy density of the magnetic field inside the solenoid. (b) Find the total energy stored in the magnetic field there (neglect end effects).

68 E A toroidal inductor with an inductance of 90.0 mH encloses a volume of 0.0200 m^3. If the average energy density in the toroid is 70.0 J/m^3, what is the current through the inductor?

69 E What must be the magnitude of a uniform electric field if it is to have the same energy density as that possessed by a 0.50 T magnetic field?

70 M GO Figure 30.47*a* shows, in cross section, two wires that are straight, parallel, and very long. The ratio i_1/i_2 of the current carried by wire 1 to that carried by wire 2 is 1/3. Wire 1 is fixed in place. Wire 2 can be moved along the positive side of the x axis so as to change the magnetic energy density u_B set up by the two currents at the origin. Figure 30.47*b* gives u_B as a function of the position x of wire 2. The curve has an asymptote of $u_B = 1.96\ \text{nJ/m}^3$ as $x \to \infty$, and the horizontal axis scale is set by $x_s = 60.0$ cm. What is the value of (a) i_1 and (b) i_2?

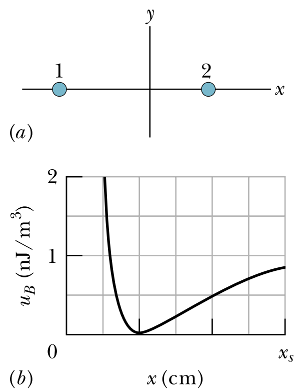

Figure 30.47 Problem 70.

71 M A length of copper wire carries a current of 10 A uniformly distributed through its cross section. Calculate the energy density of (a) the magnetic field and (b) the electric field at the surface of the wire. The wire diameter is 2.5 mm, and its resistance per unit length is 3.3 Ω/km.

Module 30.9 Mutual Induction

72 E Coil 1 has $L_1 = 25$ mH and $N_1 = 100$ turns. Coil 2 has $L_2 = 40$ mH and $N_2 = 200$ turns. The coils are fixed in place; their mutual inductance M is 3.0 mH. A 6.0 mA current in coil 1 is changing at the rate of 4.0 A/s. (a) What magnetic flux Φ_{12} links coil 1, and (b) what self-induced emf appears in that coil? (c) What magnetic flux Φ_{21} links coil 2, and (d) what mutually induced emf appears in that coil?

73 E SSM Two coils are at fixed locations. When coil 1 has no current and the current in coil 2 increases at the rate 15.0 A/s, the emf in coil 1 is 25.0 mV. (a) What is their mutual inductance? (b) When coil 2 has no current and coil 1 has a current of 3.60 A, what is the flux linkage in coil 2?

74 E Two solenoids are part of the spark coil of an automobile. When the current in one solenoid falls from 6.0 A to zero in 2.5 ms, an emf of 30 kV is induced in the other solenoid. What is the mutual inductance M of the solenoids?

75 M CALC A rectangular loop of N closely packed turns is positioned near a long straight wire as shown in Fig. 30.48. What is the mutual inductance M for the loop–wire combination if $N = 100$, $a = 1.0$ cm, $b = 8.0$ cm, and $l = 30$ cm?

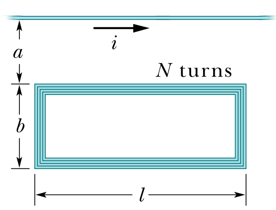

Figure 30.48 Problem 75.

76 M A coil C of N turns is placed around a long solenoid S of radius R and n turns per unit length, as in Fig. 30.49. (a) Show that the mutual inductance for the coil–solenoid combination is given by $M = \mu_0 \pi R^2 nN$. (b) Explain why M does not depend on the shape, size, or possible lack of close packing of the coil.

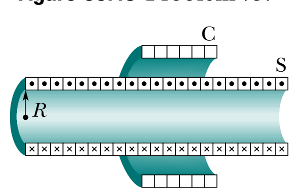

Figure 30.49 Problem 76.

77 M SSM Two coils connected as shown in Fig. 30.50 separately have inductances L_1 and L_2. Their mutual inductance is M. (a) Show that this combination can be replaced by a single coil of equivalent inductance given by

$$L_{eq} = L_1 + L_2 + 2M.$$

(b) How could the coils in Fig. 30.50 be reconnected to yield an equivalent inductance of

$$L_{eq} = L_1 + L_2 - 2M?$$

(This problem is an extension of Problem 47, but the requirement that the coils be far apart has been removed.)

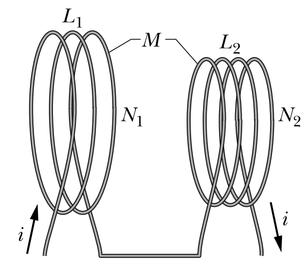

Figure 30.50 Problem 77.

Additional Problems

78 CALC At time $t = 0$, a 12.0 V potential difference is suddenly applied to the leads of a coil of inductance 23.0 mH and a certain resistance R. At time $t = 0.150$ ms, the current through the inductor is changing at the rate of 280 A/s. Evaluate R.

79 SSM In Fig. 30.51, the battery is ideal and $\mathscr{E} = 10$ V, $R_1 = 5.0\ \Omega$, $R_2 = 10\ \Omega$, and $L = 5.0$ H. Switch S is closed at time $t = 0$. Just afterwards, what are (a) i_1, (b) i_2, (c) the current i_S through the switch, (d) the potential difference V_2 across resistor 2, (e) the potential difference V_L across the inductor, and (f) the rate of change di_2/dt? A long time later, what are (g) i_1, (h) i_2, (i) i_S, (j) V_2, (k) V_L, and (l) di_2/dt?

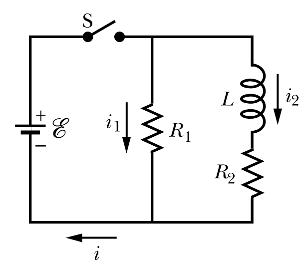

Figure 30.51 Problem 79.

80 In Fig. 30.43, $R = 4.0\ \text{k}\Omega$, $L = 8.0\ \mu\text{H}$, and the ideal battery has $\mathscr{E} = 20$ V. How long after switch S is closed is the current 2.0 mA?

81 CALC SSM Figure 30.52*a* shows a rectangular conducting loop of resistance $R = 0.020\ \Omega$, height $H = 1.5$ cm, and length $D = 2.5$ cm being pulled at constant speed $v = 40$ cm/s through two regions of uniform magnetic field. Figure 30.52*b* gives the current i induced in the loop as a function of the position x of the right side of the loop. The vertical axis scale is set by $i_s = 3.0\ \mu\text{A}$. For example, a current equal to i_s is induced clockwise as the loop enters region 1. What are the (a) magnitude and (b) direction (into or out of the page) of the magnetic field in region 1? What are the (c) magnitude and (d) direction of the magnetic field in region 2?

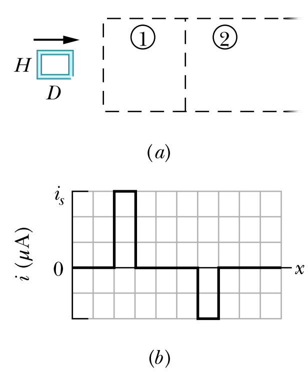

Figure 30.52 Problem 81.

82 A uniform magnetic field $\vec{B}$ is perpendicular to the plane of a circular wire loop of radius r. The magnitude of the field varies with time according to $B = B_0 e^{-t/\tau}$, where B_0 and τ are constants. Find an expression for the emf in the loop as a function of time.

83 Switch S in Fig. 30.43 is closed at time $t = 0$, initiating the buildup of current in the 15.0 mH inductor and the 20.0 Ω resistor. At what time is the emf across the inductor equal to the potential difference across the resistor?

84 CALC GO Figure 30.53*a* shows two concentric circular regions in which uniform magnetic fields can change. Region 1, with radius $r_1 = 1.0$ cm, has an outward magnetic field $\vec{B}_1$ that is increasing in magnitude. Region 2, with radius $r_2 = 2.0$ cm, has an outward magnetic field $\vec{B}_2$ that may also be changing. Imagine that a conducting ring of radius R is centered on the two regions and then the emf $\mathscr{E}$ around the ring is determined. Figure 30.53*b* gives emf $\mathscr{E}$ as a function of the square R^2 of the ring's radius, to the outer edge of region 2. The vertical axis scale is set by $\mathscr{E}_s = 20.0$ nV. What are the rates (a) dB_1/dt and (b) dB_2/dt? (c) Is the magnitude of $\vec{B}_2$ increasing, decreasing, or remaining constant?

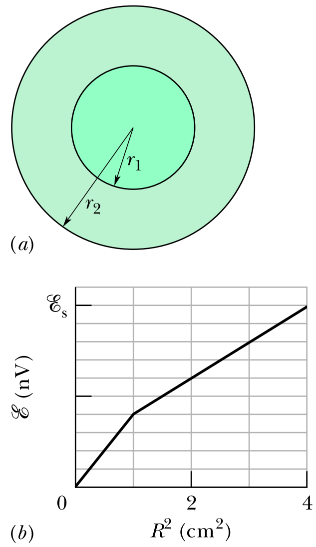

Figure 30.53 Problem 84.

85 CALC SSM Figure 30.54 shows a uniform magnetic field $\vec{B}$ confined to a cylindrical volume of radius R. The magnitude of $\vec{B}$ is decreasing at a constant rate of 10 mT/s. In unit-vector notation, what is the initial acceleration of an electron released at (a) point a (radial distance $r = 5.0$ cm), (b) point b ($r = 0$), and (c) point c ($r = 5.0$ cm)?

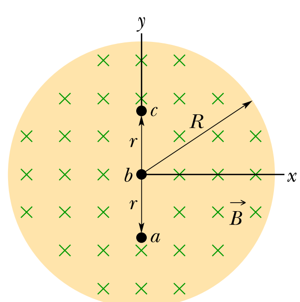

Figure 30.54 Problem 85.

86 GO In Fig. 30.55*a*, switch S has been closed on A long enough to establish a steady current in the inductor of inductance $L_1 = 5.00$ mH and the resistor of resistance $R_1 = 25.0\,\Omega$. Similarly, in Fig. 30.55*b*, switch S has been closed on A long enough to establish a steady current in the inductor of inductance $L_2 = 3.00$ mH and the resistor of resistance $R_2 = 30.0\ \Omega$. The ratio Φ_{02}/Φ_{01} of the magnetic flux through a turn in inductor 2 to that in inductor 1 is 1.50. At time $t = 0$, the two switches are closed on B. At what time t is the flux through a turn in the two inductors equal?

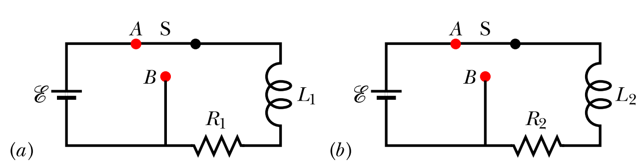

Figure 30.55 Problem 86.

87 SSM A square wire loop 20 cm on a side, with resistance 20 mΩ, has its plane normal to a uniform magnetic field of magnitude $B = 2.0$ T. If you pull two opposite sides of the loop away from each other, the other two sides automatically draw toward each other, reducing the area enclosed by the loop. If the area is reduced to zero in time $\Delta t = 0.20$ s, what are (a) the average emf and (b) the average current induced in the loop during Δt?

88 A coil with 150 turns has a magnetic flux of 50.0 nT $\cdot$ m^2 through each turn when the current is 2.00 mA. (a) What is the inductance of the coil? What are the (b) inductance and (c) flux through each turn when the current is increased to 4.00 mA? (d) What is the maximum emf $\mathscr{E}$ across the coil when the current through it is given by $i = (3.00\text{ mA})\cos(377t)$, with t in seconds?

89 A coil with an inductance of 2.0 H and a resistance of 10 Ω is suddenly connected to an ideal battery with $\mathscr{E} = 100$ V. (a) What is the equilibrium current? (b) How much energy is stored in the magnetic field when this current exists in the coil?

90 How long would it take, following the removal of the battery, for the potential difference across the resistor in an RL circuit (with $L = 2.00$ H, $R = 3.00\ \Omega$) to decay to 10.0% of its initial value?

91 SSM In the circuit of Fig. 30.56, $R_1 = 20$ kΩ, $R_2 = 20\ \Omega$, $L = 50$ mH, and the ideal battery has $\mathscr{E} = 40$ V. Switch S has been open for a long time when it is closed at time $t = 0$. Just after the switch is closed, what are (a) the current i_{bat} through the battery and (b) the rate di_{bat}/dt? At $t = 3.0\ \mu$s, what are (c) i_{bat} and (d) di_{bat}/dt? A long time later, what are (e) i_{bat} and (f) di_{bat}/dt?

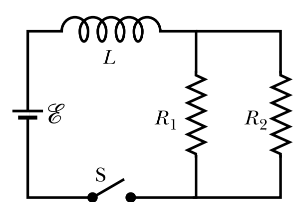

Figure 30.56 Problem 91.

92 The flux linkage through a certain coil of 0.75 Ω resistance would be 26 mWb if there were a current of 5.5 A in it. (a) Calculate the inductance of the coil. (b) If a 6.0 V ideal battery were suddenly connected across the coil, how long would it take for the current to rise from 0 to 2.5 A?

93 CALC *Fringing in a capacitor.* Prove that the electric field $\vec{E}$ in a charged parallel capacitor cannot drop abruptly to zero as is suggested at point a in Fig. 30.57 as we move perpendicular to the field along the horizontal arrow in the figure. To do this, apply Faraday's law to the rectangular path shown by the dashed lines. In actual capacitors fringing of the field lines always occurs, which means that the field approaches zero in a continuous and gradual way.

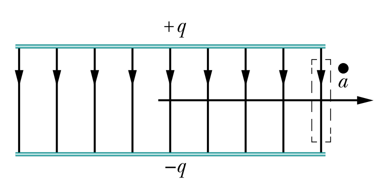

Figure 30.57 Problem 93.

94 CALC *Coaxial cable.* A long coaxial cable consists of two thin-walled concentric conducting cylinders with radii a and b. The inner cylinder A carries a steady current i, the outer cylinder B providing the return path. (a) Calculate the energy stored in the magnetic field between the cylinders for length l of the cable. (b) What is the stored energy per unit length of the cable if $a = 1.2$ mm, $b = 3.5$ mm, and $i = 2.7$ A?

95 CALC *Ballistic galvanometer.* Circuit 1 in Fig. 30.58 consists of an ammeter in series with a battery and coil 1. Circuit 2 consists of coil 2 and a ballistic galvanometer of resistance R; the galvanometer can measure the charge that moves through itself. When switch S is closed, the equilibrium current reading on the ammeter is i_f. The total charge sent through the galvanometer while the current circuit 2 reaches equilibrium is Q. Find the mutual inductance M between coils 1 and 2.

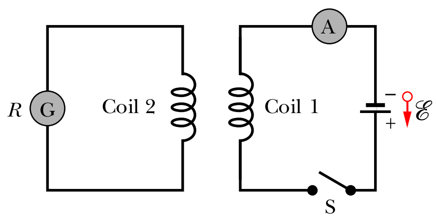

Figure 30.58 Problem 95.

96 CALC *Straight and triangle wires.* In Fig. 30.59, a long straight wire lies in the same plane as an equilateral triangle formed from a wire of length $3S$. The long wire is parallel to one side of the triangle and at distance d from the nearest vertex. What is the mutual inductance M of the wire and triangle?

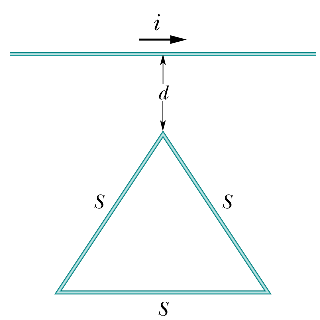

Figure 30.59 Problem 96.

97 CALC *Induction, large loop, small loop.* A small circular loop of area 2.00 cm^2 is placed in the plane of, and concentric with, a large circular loop of radius 1.00 m. The current in the large loop is changed uniformly from 200 A to −200 A (a change in direction) in a time of 1.00 s, beginning at $t = 0$. (a) What is the magnetic field at the center of the small circular loop due to the current in the large loop at $t = 0$, $t = 0.500$ s, and $t = 1.00$ s? (b) What is the magnitude of the emf induced in the small loop at $t = 0.500$ s? Because the inner loop is small, assume the magnetic field due to the outer loop is uniform over the area of the smaller loop.

98 CALC *Currents first equal.* Switch S in Fig. 30.60 is closed for time $t < 0$ and is opened at $t = 0$. When current i_1 through L_1 and R_1 and current i_2 through L_2 and R_2 are first equal to each other, what is their common value?

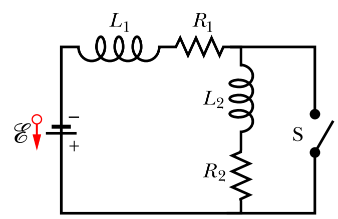

Figure 30.60 Problem 98.