CHAPTER 7

Kinetic Energy and Work

7.1 KINETIC ENERGY

Learning Objectives

After reading this module, you should be able to . . .

7.1.1 Apply the relationship between a particle's kinetic energy, mass, and speed.

7.1.2 Identify that kinetic energy is a scalar quantity.

Key Idea

- The kinetic energy K associated with the motion of a particle of mass m and speed v, where v is well below the speed of light, is

$$K = \tfrac{1}{2}mv^2 \quad \text{(kinetic energy).}$$

What Is Physics?

One of the fundamental goals of physics is to investigate something that everyone talks about: energy. The topic is obviously important. Indeed, our civilization is based on acquiring and effectively using energy.

For example, everyone knows that any type of motion requires energy: Flying across the Pacific Ocean requires it. Lifting material to the top floor of an office building or to an orbiting space station requires it. Throwing a fastball requires it. We spend a tremendous amount of money to acquire and use energy. Wars have been started because of energy resources. Wars have been ended because of a sudden, overpowering use of energy by one side. Everyone knows many examples of energy and its use, but what does the term *energy* really mean?

What Is Energy?

The term *energy* is so broad that a clear definition is difficult to write. Technically, energy is a scalar quantity associated with the state (or condition) of one or more objects. However, this definition is too vague to be of help to us now.

A looser definition might at least get us started. Energy is a number that we associate with a system of one or more objects. If a force changes one of the objects by, say, making it move, then the energy number changes. After countless experiments, scientists and engineers realized that if the scheme by which we assign energy numbers is planned carefully, the numbers can be used to predict the outcomes of experiments and, even more important, to build machines, such as flying machines. This success is based on a wonderful property of our universe: Energy can be transformed from one type to another and transferred from one object to another, but the total amount is always the same (energy is *conserved*). No exception to this *principle of energy conservation* has ever been found.

Money. Think of the many types of energy as being numbers representing money in many types of bank accounts. Rules have been made about what such money numbers mean and how they can be changed. You can transfer money numbers from one account to another or from one system to another, perhaps electronically with nothing material actually moving. However, the total amount (the total of all the money numbers) can always be accounted for: It is always conserved. In this chapter we focus on only one type of energy (*kinetic energy*) and on only one way in which energy can be transferred (*work*).

Kinetic Energy

Kinetic energy K is energy associated with the *state of motion* of an object. The faster the object moves, the greater is its kinetic energy. When the object is stationary, its kinetic energy is zero.

For an object of mass m whose speed v is well below the speed of light,

$$K = \tfrac{1}{2}mv^2 \quad \text{(kinetic energy).} \tag{7.1.1}$$

For example, a 3.0 kg duck flying past us at 2.0 m/s has a kinetic energy of 6.0 $\text{kg} \cdot \text{m}^2/\text{s}^2$; that is, we associate that number with the duck's motion.

The SI unit of kinetic energy (and all types of energy) is the **joule** (J), named for James Prescott Joule, an English scientist of the 1800s, and defined as

$$1 \text{ joule} = 1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2. \tag{7.1.2}$$

Thus, the flying duck has a kinetic energy of 6.0 J.

Checkpoint 7.1.1

The speed of a car (treat it as being a particle) increases from 5.0 m/s to 15.0 m/s. What is the ratio of the final kinetic energy K_f to the initial kinetic energy K_i?

Sample Problem 7.1.1 Kinetic energy, train crash

In 1896 in Waco, Texas, William Crush parked two locomotives at opposite ends of a 6.4-km-long track, fired them up, tied their throttles open, and then allowed them to crash head-on at full speed (Fig. 7.1.1) in front of 30,000 spectators. Hundreds of people were hurt by flying debris; several were killed. Assuming each locomotive weighed 1.2×10^6 N and its acceleration was a constant 0.26 m/s^2, what was the total kinetic energy of the two locomotives just before the collision? FCP

Courtesy of Library of Congress

Figure 7.1.1 The aftermath of an 1896 crash of two locomotives.

KEY IDEAS

(1) We need to find the kinetic energy of each locomotive with Eq. 7.1.1, but that means we need each locomotive's speed just before the collision and its mass. (2) Because we can assume each locomotive had constant acceleration, we can use the equations in Table 2.1.1 to find its speed v just before the collision.

Calculations: We choose Eq. 2.4.6 because we know values for all the variables except v:

$$v^2 = v_0^2 + 2a(x - x_0).$$

With $v_0 = 0$ and $x - x_0 = 3.2 \times 10^3$ m (half the initial separation), this yields

$$v^2 = 0 + 2(0.26\,\text{m/s}^2)(3.2 \times 10^3\,\text{m}),$$

or

$$v = 40.8\,\text{m/s} = 147\,\text{km/h}.$$

We can find the mass of each locomotive by dividing its given weight by g:

$$m = \frac{1.2 \times 10^6\,\text{N}}{9.8\,\text{m/s}^2} = 1.22 \times 10^5\,\text{kg}.$$

Now, using Eq. 7.1.1, we find the total kinetic energy of the two locomotives just before the collision as

$$K = 2(\tfrac{1}{2}mv^2) = (1.22 \times 10^5\,\text{kg})(40.8\,\text{m/s})^2$$
$$= 2.0 \times 10^8\,\text{J}. \qquad \text{(Answer)}$$

This collision was like an exploding bomb.

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7.2 WORK AND KINETIC ENERGY

Learning Objectives

After reading this module, you should be able to . . .

7.2.1 Apply the relationship between a force (magnitude and direction) and the work done on a particle by the force when the particle undergoes a displacement.

7.2.2 Calculate work by taking a dot product of the force vector and the displacement vector, in either magnitude-angle or unit-vector notation.

7.2.3 If multiple forces act on a particle, calculate the net work done by them.

7.2.4 Apply the work–kinetic energy theorem to relate the work done by a force (or the net work done by multiple forces) and the resulting change in kinetic energy.

Key Ideas

- Work W is energy transferred to or from an object via a force acting on the object. Energy transferred to the object is positive work, and from the object, negative work.
- The work done on a particle by a constant force $\vec{F}$ during displacement $\vec{d}$ is

$$W = Fd\cos\phi = \vec{F} \cdot \vec{d} \qquad \text{(work, constant force)},$$

in which ϕ is the constant angle between the directions of $\vec{F}$ and $\vec{d}$.

- Only the component of $\vec{F}$ that is along the displacement $\vec{d}$ can do work on the object.
- When two or more forces act on an object, their net work is the sum of the individual works done by the forces, which is also equal to the work that would be done on the object by the net force $\vec{F}_{\text{net}}$ of those forces.
- For a particle, a change ΔK in the kinetic energy equals the net work W done on the particle:

$$\Delta K = K_f - K_i = W \qquad \text{(work–kinetic energy theorem)},$$

in which K_i is the initial kinetic energy of the particle and K_f is the kinetic energy after the work is done. The equation rearranged gives us

$$K_f = K_i + W.$$

Work

If you accelerate an object to a greater speed by applying a force to the object, you increase the kinetic energy $K\ (= \frac{1}{2}mv^2)$ of the object. Similarly, if you decelerate the object to a lesser speed by applying a force, you decrease the kinetic energy of the object. We account for these changes in kinetic energy by saying that your force has transferred energy *to* the object from yourself or *from* the object to yourself. In such a transfer of energy via a force, **work** W is said to be *done on the object by the force.* More formally, we define work as follows:

> Work W is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.

"Work," then, is transferred energy; "doing work" is the act of transferring the energy. Work has the same units as energy and is a scalar quantity.

The term *transfer* can be misleading. It does not mean that anything material flows into or out of the object; that is, the transfer is not like a flow of water. Rather, it is like the electronic transfer of money between two bank accounts: The number in one account goes up while the number in the other account goes down, with nothing material passing between the two accounts.

Note that we are not concerned here with the common meaning of the word "work," which implies that *any* physical or mental labor is work. For example, if you push hard against a wall, you tire because of the continuously repeated muscle contractions that are required, and you are, in the common sense, working. However, such effort does not cause an energy transfer to or from the wall and thus is not work done on the wall as defined here.

To avoid confusion in this chapter, we shall use the symbol W only for work and shall represent a weight with its equivalent mg.

Work and Kinetic Energy

Finding an Expression for Work

Let us find an expression for work by considering a bead that can slide along a frictionless wire that is stretched along a horizontal x axis (Fig. 7.2.1). A constant force $\vec{F}$, directed at an angle ϕ to the wire, accelerates the bead along the wire. We can relate the force and the acceleration with Newton's second law, written for components along the x axis:

$$F_x = ma_x, \tag{7.2.1}$$

where m is the bead's mass. As the bead moves through a displacement $\vec{d}$, the force changes the bead's velocity from an initial value $\vec{v}_0$ to some other value $\vec{v}$. Because the force is constant, we know that the acceleration is also constant. Thus, we can use Eq. 2.4.6 to write, for components along the x axis,

$$v^2 = v_0^2 + 2a_x d. \tag{7.2.2}$$

Solving this equation for a_x, substituting into Eq. 7.2.1, and rearranging then give us

$$\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 = F_x d. \tag{7.2.3}$$

The first term is the kinetic energy K_f of the bead at the end of the displacement d, and the second term is the kinetic energy K_i of the bead at the start. Thus, the left side of Eq. 7.2.3 tells us the kinetic energy has been changed by the force, and the right side tells us the change is equal to $F_x d$. Therefore, the work W done on the bead by the force (the energy transfer due to the force) is

$$W = F_x d. \tag{7.2.4}$$

If we know values for F_x and d, we can use this equation to calculate the work W.

> To calculate the work a force does on an object as the object moves through some displacement, we use only the force component along the object's displacement. The force component perpendicular to the displacement does zero work.

From Fig. 7.2.1, we see that we can write F_x as $F\cos\phi$, where ϕ is the angle between the directions of the displacement $\vec{d}$ and the force $\vec{F}$. Thus,

$$W = Fd\cos\phi \quad \text{(work done by a constant force).} \tag{7.2.5}$$

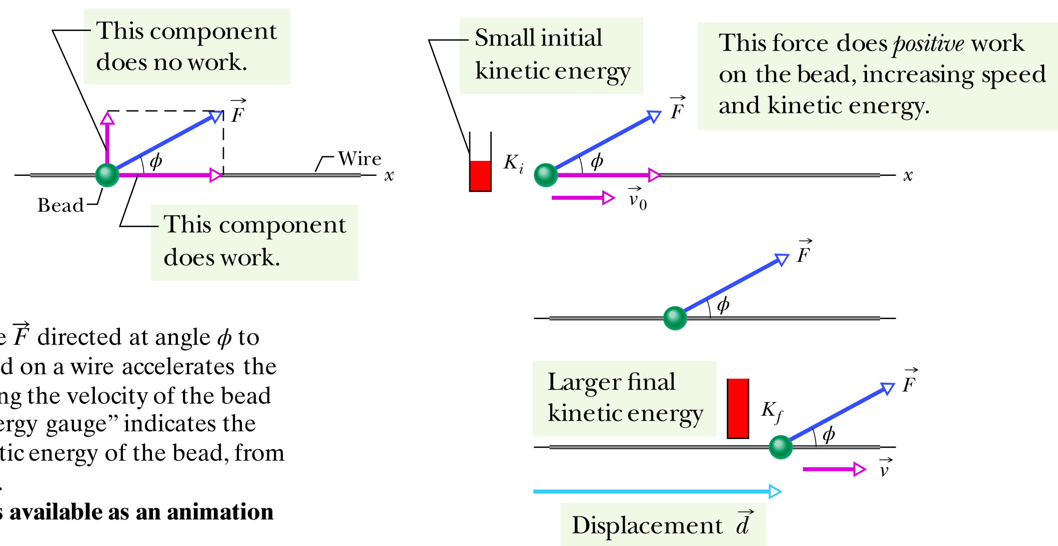

Figure 7.2.1 A constant force $\vec{F}$ directed at angle ϕ to the displacement $\vec{d}$ of a bead on a wire accelerates the bead along the wire, changing the velocity of the bead from $\vec{v}_0$ to $\vec{v}$. A "kinetic energy gauge" indicates the resulting change in the kinetic energy of the bead, from the value K_i to the value K_f.
In *WileyPLUS*, this figure is available as an animation with voiceover.

We can use the definition of the scalar (dot) product (Eq. 3.3.1) to write

$$W = \vec{F}\cdot\vec{d} \quad \text{(work done by a constant force),} \tag{7.2.6}$$

where F is the magnitude of $\vec{F}$. (You may wish to review the discussion of scalar products in Module 3.3.) Equation 7.2.6 is especially useful for calculating the work when $\vec{F}$ and $\vec{d}$ are given in unit-vector notation.

Cautions. There are two restrictions to using Eqs. 7.2.4 through 7.2.6 to calculate work done on an object by a force. First, the force must be a *constant force*; that is, it must not change in magnitude or direction as the object moves. (Later, we shall discuss what to do with a *variable force* that changes in magnitude.) Second, the object must be *particle-like*. This means that the object must be *rigid*; all parts of it must move together, in the same direction. In this chapter we consider only particle-like objects, such as the bed and its occupant being pushed in Fig. 7.2.2.

Signs for Work. The work done on an object by a force can be either positive work or negative work. For example, if angle ϕ in Eq. 7.2.5 is less than 90°, then $\cos\phi$ is positive and thus so is the work. However, if ϕ is greater than 90° (up to 180°), then $\cos\phi$ is negative and thus so is the work. (Can you see that the work is zero when $\phi = 90°$?) These results lead to a simple rule. To find the sign of the work done by a force, consider the force vector component that is parallel to the displacement:

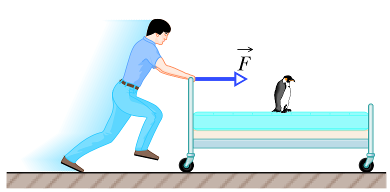

Figure 7.2.2 A contestant in a bed race. We can approximate the bed and its occupant as being a particle for the purpose of calculating the work done on them by the force applied by the contestant.

> A force does positive work when it has a vector component in the same direction as the displacement, and it does negative work when it has a vector component in the opposite direction. It does zero work when it has no such vector component.

Units for Work. Work has the SI unit of the joule, the same as kinetic energy. However, from Eqs. 7.2.4 and 7.2.5 we can see that an equivalent unit is the newton-meter ($N \cdot m$). The corresponding unit in the British system is the foot-pound ($ft \cdot lb$). Extending Eq. 7.1.2, we have

$$1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 1 \text{ N} \cdot \text{m} = 0.738 \text{ ft} \cdot \text{lb}. \tag{7.2.7}$$

Net Work. When two or more forces act on an object, the **net work** done on the object is the sum of the works done by the individual forces. We can calculate the net work in two ways. (1) We can find the work done by each force and then sum those works. (2) Alternatively, we can first find the net force $\vec{F}_{net}$ of those forces. Then we can use Eq. 7.2.5, substituting the magnitude F_{net} for F and also the angle between the directions of $\vec{F}_{net}$ and $\vec{d}$ for ϕ. Similarly, we can use Eq. 7.2.6 with $\vec{F}_{net}$ substituted for $\vec{F}$.

Work–Kinetic Energy Theorem

Equation 7.2.3 relates the change in kinetic energy of the bead (from an initial $K_i = \frac{1}{2}mv_0^2$ to a later $K_f = \frac{1}{2}mv^2$) to the work W ($= F_x d$) done on the bead. For such particle-like objects, we can generalize that equation. Let ΔK be the change in the kinetic energy of the object, and let W be the net work done on it. Then

$$\Delta K = K_f - K_i = W, \tag{7.2.8}$$

which says that

$$\begin{pmatrix}\text{change in the kinetic}\\ \text{energy of a particle}\end{pmatrix} = \begin{pmatrix}\text{net work done on}\\ \text{the particle}\end{pmatrix}.$$

We can also write

$$K_f = K_i + W, \tag{7.2.9}$$

which says that

$$\begin{pmatrix}\text{kinetic energy after}\\ \text{the net work is done}\end{pmatrix} = \begin{pmatrix}\text{kinetic energy}\\ \text{before the net work}\end{pmatrix} + \begin{pmatrix}\text{the net}\\ \text{work done}\end{pmatrix}.$$

These statements are known traditionally as the **work–kinetic energy theorem** for particles. They hold for both positive and negative work: If the net work done on a particle is positive, then the particle's kinetic energy increases by the amount of the work. If the net work done is negative, then the particle's kinetic energy decreases by the amount of the work.

For example, if the kinetic energy of a particle is initially 5 J and there is a net transfer of 2 J to the particle (positive net work), the final kinetic energy is 7 J. If, instead, there is a net transfer of 2 J from the particle (negative net work), the final kinetic energy is 3 J.

Checkpoint 7.2.1

A particle moves along an x axis. Does the kinetic energy of the particle increase, decrease, or remain the same if the particle's velocity changes (a) from -3 m/s to -2 m/s and (b) from -2 m/s to 2 m/s? (c) In each situation, is the work done on the particle positive, negative, or zero?

Sample Problem 7.2.1 Work done by two constant forces, industrial spies

Figure 7.2.3*a* shows two industrial spies sliding an initially stationary 225 kg floor safe a displacement $\vec{d}$ of magnitude 8.50 m. The push $\vec{F}_1$ of spy 001 is 12.0 N at an angle of 30.0° downward from the horizontal; the pull $\vec{F}_2$ of spy 002 is 10.0 N at 40.0° above the horizontal. The magnitudes and directions of these forces do not change as the safe moves, and the floor and safe make frictionless contact.

(a) What is the net work done on the safe by forces $\vec{F}_1$ and $\vec{F}_2$ during the displacement $\vec{d}$?

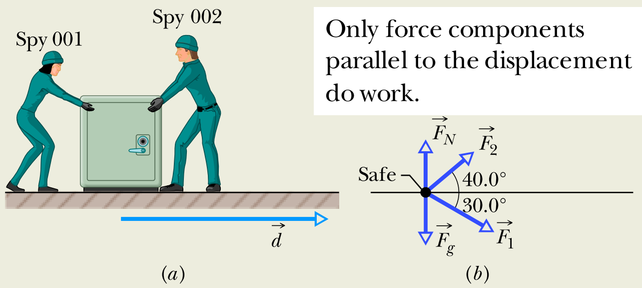

Figure 7.2.3 (*a*) Two spies move a floor safe through a displacement $\vec{d}$. (*b*) A free-body diagram for the safe.

KEY IDEAS

(1) The net work W done on the safe by the two forces is the sum of the works they do individually. (2) Because we can treat the safe as a particle and the forces are constant in both magnitude and direction, we can use either Eq. 7.2.5 ($W = Fd\cos\phi$) or Eq. 7.2.6 ($W = \vec{F}\cdot\vec{d}$) to calculate those works. Let's choose Eq. 7.2.5.

Calculations: From Eq. 7.2.5 and the free-body diagram for the safe in Fig. 7.2.3*b*, the work done by $\vec{F}_1$ is

$$W_1 = F_1 d\cos\phi_1 = (12.0\,\text{N})(8.50\,\text{m})(\cos 30.0°)$$
$$= 88.33\,\text{J},$$

and the work done by $\vec{F}_2$ is

$$W_2 = F_2 d\cos\phi_2 = (10.0\,\text{N})(8.50\,\text{m})(\cos 40.0°)$$
$$= 65.11\,\text{J}.$$

Thus, the net work W is

$$W = W_1 + W_2 = 88.33\,\text{J} + 65.11\,\text{J}$$
$$= 153.4\,\text{J} \approx 153\,\text{J}. \qquad \text{(Answer)}$$

During the 8.50 m displacement, therefore, the spies transfer 153 J of energy to the kinetic energy of the safe.

(b) During the displacement, what is the work W_g done on the safe by the gravitational force $\vec{F}_g$ and what is the work W_N done on the safe by the normal force $\vec{F}_N$ from the floor?

KEY IDEA

Because these forces are constant in both magnitude and direction, we can find the work they do with Eq. 7.2.5.

Calculations: Thus, with mg as the magnitude of the gravitational force, we write

$$W_g = mgd\cos 90° = mgd(0) = 0 \qquad \text{(Answer)}$$

and

$$W_N = F_N d\cos 90° = F_N d(0) = 0. \qquad \text{(Answer)}$$

We should have known this result. Because these forces are perpendicular to the displacement of the safe, they do zero work on the safe and do not transfer any energy to or from it.

(c) The safe is initially stationary. What is its speed v_f at the end of the 8.50 m displacement?

KEY IDEA

The speed of the safe changes because its kinetic energy is changed when energy is transferred to it by $\vec{F}_1$ and $\vec{F}_2$.

Calculations: We relate the speed to the work done by combining Eqs. 7.2.8 (the work–kinetic energy theorem) and 7.1.1 (the definition of kinetic energy):

$$W = K_f - K_i = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2.$$

The initial speed v_i is zero, and we now know that the work done is 153.4 J. Solving for v_f and then substituting known data, we find that

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(153.4\,\text{J})}{225\,\text{kg}}}$$
$$= 1.17\,\text{m/s}. \qquad \text{(Answer)}$$

Sample Problem 7.2.2 Work done by a constant force in unit-vector notation

During a storm, a crate of crepe is sliding across a slick, oily parking lot through a displacement $\vec{d} = (-3.0\,\text{m})\hat{\text{i}}$ while a steady wind pushes against the crate with a force $\vec{F} = (2.0\,\text{N})\hat{\text{i}} + (-6.0\,\text{N})\hat{\text{j}}$. The situation and coordinate axes are shown in Fig. 7.2.4.

(a) How much work does this force do on the crate during the displacement?

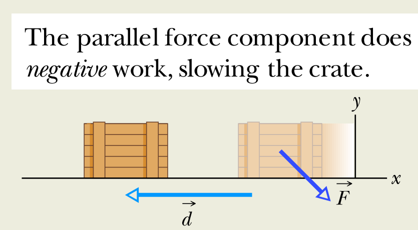

Figure 7.2.4 Force $\vec{F}$ slows a crate during displacement $\vec{d}$.

KEY IDEA

Because we can treat the crate as a particle and because the wind force is constant ("steady") in both magnitude and direction during the displacement, we can use either Eq. 7.2.5 ($W = Fd\cos\phi$) or Eq. 7.2.6 ($W = \vec{F}\cdot\vec{d}$) to calculate the work. Since we know $\vec{F}$ and $\vec{d}$ in unit-vector notation, we choose Eq. 7.2.6.

Calculations: We write

$$W = \vec{F} \cdot \vec{d} = [(2.0\text{ N})\hat{i} + (-6.0\text{ N})\hat{j}] \cdot [(-3.0\text{ m})\hat{i}].$$

Of the possible unit-vector dot products, only $\hat{i} \cdot \hat{i}$, $\hat{j} \cdot \hat{j}$, and $\hat{k} \cdot \hat{k}$ are nonzero (see Appendix E). Here we obtain

$$W = (2.0\text{ N})(-3.0\text{ m})\hat{i} \cdot \hat{i} + (-6.0\text{ N})(-3.0\text{ m})\hat{j} \cdot \hat{i}$$
$$= (-6.0\text{ J})(1) + 0 = -6.0\text{ J}. \qquad \text{(Answer)}$$

Thus, the force does a negative 6.0 J of work on the crate, transferring 6.0 J of energy from the kinetic energy of the crate.

(b) If the crate has a kinetic energy of 10 J at the beginning of displacement $\vec{d}$, what is its kinetic energy at the end of $\vec{d}$?

KEY IDEA

Because the force does negative work on the crate, it reduces the crate's kinetic energy.

Calculation: Using the work–kinetic energy theorem in the form of Eq. 7.2.9, we have

$$K_f = K_i + W = 10\text{ J} + (-6.0\text{ J}) = 4.0\text{ J}. \qquad \text{(Answer)}$$

Less kinetic energy means that the crate has been slowed.

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7.3 WORK DONE BY THE GRAVITATIONAL FORCE

Learning Objectives

After reading this module, you should be able to . . .

7.3.1 Calculate the work done by the gravitational force when an object is lifted or lowered.

7.3.2 Apply the work–kinetic energy theorem to situations where an object is lifted or lowered.

Key Ideas

- The work W_g done by the gravitational force $\vec{F}_g$ on a particle-like object of mass m as the object moves through a displacement $\vec{d}$ is given by

$$W_g = mgd\cos\phi,$$

in which ϕ is the angle between $\vec{F}_g$ and $\vec{d}$.

- The work W_a done by an applied force as a particle-like object is either lifted or lowered is related to the work W_g done by the gravitational force and the change ΔK in the object's kinetic energy by

$$\Delta K = K_f - K_i = W_a + W_g.$$

If $K_f = K_i$, then the equation reduces to

$$W_a = -W_g,$$

which tells us that the applied force transfers as much energy to the object as the gravitational force transfers from it.

Work Done by the Gravitational Force

We next examine the work done on an object by the gravitational force acting on it. Figure 7.3.1 shows a particle-like tomato of mass m that is thrown upward with initial speed v_0 and thus with initial kinetic energy $K_i = \frac{1}{2}mv_0^2$. As the tomato rises, it is slowed by a gravitational force $\vec{F}_g$; that is, the tomato's kinetic energy decreases because $\vec{F}_g$ does work on the tomato as it rises. Because we can treat the tomato as a particle, we can use Eq. 7.2.5 ($W = Fd\cos\phi$) to express the work done during a displacement $\vec{d}$. For the force magnitude F, we use mg as the magnitude of $\vec{F}_g$. Thus, the work W_g done by the gravitational force $\vec{F}_g$ is

$$W_g = mgd\cos\phi \qquad \text{(work done by gravitational force).} \qquad (7.3.1)$$

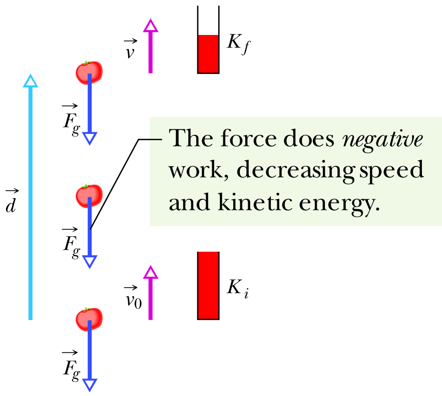

Figure 7.3.1 Because the gravitational force $\vec{F}_g$ acts on it, a particle-like tomato of mass m thrown upward slows from velocity $\vec{v}_0$ to velocity $\vec{v}$ during displacement $\vec{d}$. A kinetic energy gauge indicates the resulting change in the kinetic energy of the tomato, from $K_i = \frac{1}{2}mv_0^2$ to $K_f = \frac{1}{2}mv^2$.

For a rising object, force $\vec{F}_g$ is directed opposite the displacement $\vec{d}$, as indicated in Fig. 7.3.1. Thus, $\phi = 180°$ and

$$W_g = mgd\cos 180° = mgd(-1) = -mgd. \qquad (7.3.2)$$

The minus sign tells us that during the object's rise, the gravitational force acting on the object transfers energy in the amount mgd from the kinetic energy of the object. This is consistent with the slowing of the object as it rises.

After the object has reached its maximum height and is falling back down, the angle ϕ between force $\vec{F}_g$ and displacement $\vec{d}$ is zero. Thus,

$$W_g = mgd\cos 0° = mgd(+1) = +mgd. \qquad (7.3.3)$$

The plus sign tells us that the gravitational force now transfers energy in the amount mgd to the kinetic energy of the falling object (it speeds up, of course).

Work Done in Lifting and Lowering an Object

Now suppose we lift a particle-like object by applying a vertical force $\vec{F}$ to it. During the upward displacement, our applied force does positive work W_a on the object while the gravitational force does negative work W_g on it. Our applied force tends to transfer energy to the object while the gravitational force tends to transfer energy from it. By Eq. 7.2.8, the change ΔK in the kinetic energy of the object due to these two energy transfers is

$$\Delta K = K_f - K_i = W_a + W_g, \qquad (7.3.4)$$

in which K_f is the kinetic energy at the end of the displacement and K_i is that at the start of the displacement. This equation also applies if we lower the object, but then the gravitational force tends to transfer energy *to* the object while our force tends to transfer energy *from* it.

If an object is stationary before and after a lift (as when you lift a book from the floor to a shelf), then K_f and K_i are both zero, and Eq. 7.3.4 reduces to

$$W_a + W_g = 0$$

or

$$W_a = -W_g. \qquad (7.3.5)$$

Note that we get the same result if K_f and K_i are not zero but are still equal. Either way, the result means that the work done by the applied force is the negative of the work done by the gravitational force; that is, the applied force transfers the same amount of energy to the object as the gravitational force transfers from the object. Using Eq. 7.3.1, we can rewrite Eq. 7.3.5 as

$$W_a = -mgd\cos\phi \quad \text{(work done in lifting and lowering; } K_f = K_i\text{)}, \qquad (7.3.6)$$

with ϕ being the angle between $\vec{F}_g$ and $\vec{d}$. If the displacement is vertically upward (Fig. 7.3.2*a*), then $\phi = 180°$ and the work done by the applied force equals mgd. If the displacement is vertically downward (Fig. 7.3.2*b*), then $\phi = 0°$ and the work done by the applied force equals $-mgd$.

Equations 7.3.5 and 7.3.6 apply to any situation in which an object is lifted or lowered, with the object stationary before and after the lift. They are independent of the magnitude of the force used. For example, if you lift a mug from the floor to over your head, your force on the mug varies considerably during the lift. Still, because the mug is stationary before and after the lift, the work your force does on the mug is given by Eqs. 7.3.5 and 7.3.6, where, in Eq. 7.3.6, mg is the weight of the mug and d is the distance you lift it.

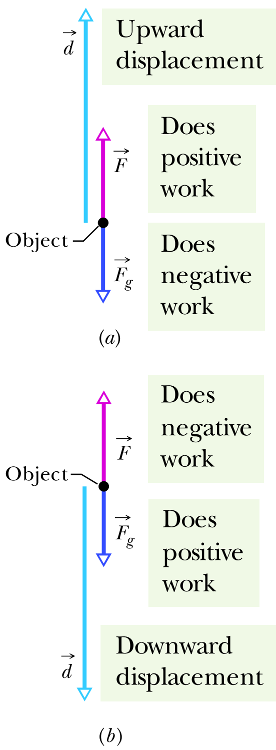

Figure 7.3.2 (*a*) An applied force $\vec{F}$ lifts an object. The object's displacement $\vec{d}$ makes an angle $\phi = 180°$ with the gravitational force $\vec{F}_g$ on the object. The applied force does positive work on the object. (*b*) An applied force $\vec{F}$ lowers an object. The displacement $\vec{d}$ of the object makes an angle $\phi = 0°$ with the gravitational force $\vec{F}_g$. The applied force does negative work on the object.

Sample Problem 7.3.1 Work in pulling a sleigh up a snowy slope

In this problem an object is pulled along a ramp but the object starts and ends at rest and thus has no overall change in its kinetic energy (that is important). Figure 7.3.3*a* shows the situation. A rope pulls a 200 kg sleigh (which you may know) up a slope at incline angle $\theta = 30°$, through distance $d = 20$ m. The sleigh and its contents have a total mass of 200 kg. The snowy slope is so slippery that we take it to be frictionless. How much work is done by each force acting on the sleigh?

KEY IDEAS

(1) During the motion, the forces are constant in magnitude and direction and thus we can calculate the work done by each with Eq. 7.2.5 ($W = Fd \cos \phi$) in which ϕ is the angle between the force and the displacement. We reach the same result with Eq. 7.2.6 ($W = \vec{F} \cdot \vec{d}$) in which we take a dot product of the force vector and displacement vector. (2) We can relate the net work done by the forces to the change in kinetic energy (or lack of a change, as here) with the work–kinetic energy theorem of Eq. 7.2.8 ($\Delta K = W$).

Calculations: The first thing to do with most physics problems involving forces is to draw a free-body diagram to organize our thoughts. For the sleigh, Fig. 7.3.3*b* is our free-body diagram, showing the gravitational force $\vec{F}_g$, the force $\vec{T}$ from the rope, and the normal force $\vec{F}_N$ from the slope.

Work W_N by the normal force. Let's start with this easy calculation. The normal force is perpendicular to the slope and thus also to the sleigh's displacement. Thus the normal force does not affect the sleigh's motion and does zero work. To be more formal, we can apply Eq. 7.2.5 to write

$$W_N = F_N d \cos 90° = 0. \qquad \text{(Answer)}$$

Work W_g by the gravitational force. We can find the work done by the gravitational force in either of two ways (you pick the more appealing way). From an earlier discussion about ramps (Sample Problem 5.3.2 and Fig. 5.3.6), we know that the component of the gravitational force along the slope has magnitude $mg \sin \theta$ and is directed down the slope. Thus the magnitude is

$$\begin{aligned} F_{gx} &= mg \sin \theta = (200\,\text{kg})(9.8\,\text{m/s}^2) \sin 30° \\ &= 980\,\text{N}. \end{aligned}$$

The angle ϕ between the displacement and this force component is 180°. So we can apply Eq. 7.2.5 to write

$$\begin{aligned} W_g &= F_{gx} d \cos 180° = (980\,\text{N})(20\,\text{m})(-1) \\ &= -1.96 \times 10^4\,\text{J}. \qquad \text{(Answer)} \end{aligned}$$

The negative result means that the gravitational force removes energy from the sleigh.

The second (equivalent) way to get this result is to use the full gravitational force $\vec{F}_g$ instead of a component. The angle between $\vec{F}_g$ and $\vec{d}$ is 120° (add the incline angle 30° to 90°). So, Eq. 7.2.5 gives us

$$\begin{aligned} W_g &= F_g d \cos 120° = mgd \cos 120° \\ &= (200\,\text{kg})(9.8\,\text{m/s}^2)(20\,\text{m}) \cos 120° \\ &= -1.96 \times 10^4\,\text{J}. \qquad \text{(Answer)} \end{aligned}$$

Work W_T by the rope's force. We have two ways of calculating this work. The quicker way is to use the work–kinetic energy theorem of Eq. 7.2.8 ($\Delta K = W$), where the net work W done by the forces is $W_N + W_g + W_T$ and the change ΔK in the kinetic energy is just zero (because the initial and final kinetic energies are the same—namely, zero). So, Eq. 7.2.8 gives us

$$0 = W_N + W_g + W_T = 0 - 1.96 \times 10^4\,\text{J} + W_T$$

and

$$W_T = 1.96 \times 10^4\,\text{J}. \qquad \text{(Answer)}$$

Instead of doing this, we can apply Newton's second law for motion along the x axis to find the magnitude F_T of the rope's force. Assuming that the acceleration along the slope is zero (except for the brief starting and stopping), we can write

$$F_{\text{net},x} = ma_x,$$

$$F_T - mg \sin 30° = m(0),$$

to find

$$F_T = mg \sin 30°.$$

This is the magnitude. Because the force and the displacement are both up the slope, the angle between those two vectors is zero. So, we can now write Eq. 7.2.5 to find the work done by the rope's force:

$$\begin{aligned} W_T &= F_T d \cos 0° = (mg \sin 30°) d \cos 0° \\ &= (200\,\text{kg})(9.8\,\text{m/s}^2)(\sin 30°)(20\,\text{m}) \cos 0° \\ &= 1.96 \times 10^4\,\text{J}. \qquad \text{(Answer)} \end{aligned}$$

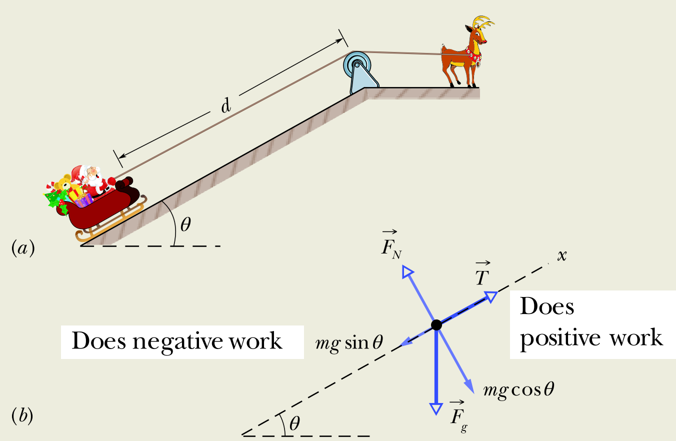

Figure 7.3.3 (*a*) A sleigh is pulled up a snowy slope. (*b*) The free-body diagram for the sleigh.

Sample Problem 7.3.2 Work done on an accelerating elevator cab

An elevator cab of mass $m = 500$ kg is descending with speed $v_i = 4.0$ m/s when its supporting cable begins to slip, allowing it to fall with constant acceleration $\vec{a} = \vec{g}/5$ (Fig. 7.3.4*a*).

(a) During the fall through a distance $d = 12$ m, what is the work W_g done on the cab by the gravitational force $\vec{F}_g$?

KEY IDEA

We can treat the cab as a particle and thus use Eq. 7.3.1 ($W_g = mgd\cos\phi$) to find the work W_g.

Calculation: From Fig. 7.3.4*b*, we see that the angle between the directions of $\vec{F}_g$ and the cab's displacement $\vec{d}$ is 0°. So,

$$W_g = mgd\cos 0° = (500\,\text{kg})(9.8\,\text{m/s}^2)(12\,\text{m})(1)$$
$$= 5.88 \times 10^4\,\text{J} \approx 59\,\text{kJ}. \qquad \text{(Answer)}$$

(b) During the 12 m fall, what is the work W_T done on the cab by the upward pull $\vec{T}$ of the elevator cable?

KEY IDEA

We can calculate work W_T with Eq. 7.2.5 ($W = Fd\cos\phi$) by first writing $F_{\text{net},y} = ma_y$ for the components in Fig. 7.3.4*b*.

Calculations: We get

$$T - F_g = ma. \qquad (7.3.7)$$

Solving for T, substituting mg for F_g, and then substituting the result in Eq. 7.2.5, we obtain

$$W_T = Td\cos\phi = m(a + g)d\cos\phi. \qquad (7.3.8)$$

Next, substituting $-g/5$ for the (downward) acceleration a and then 180° for the angle ϕ between the directions of forces $\vec{T}$ and $m\vec{g}$, we find

$$W_T = m\left(-\frac{g}{5} + g\right)d\cos\phi = \frac{4}{5}mgd\cos\phi$$
$$= \frac{4}{5}(500\,\text{kg})(9.8\,\text{m/s}^2)(12\,\text{m})\cos 180°$$
$$= -4.70 \times 10^4\,\text{J} \approx -47\,\text{kJ}. \qquad \text{(Answer)}$$

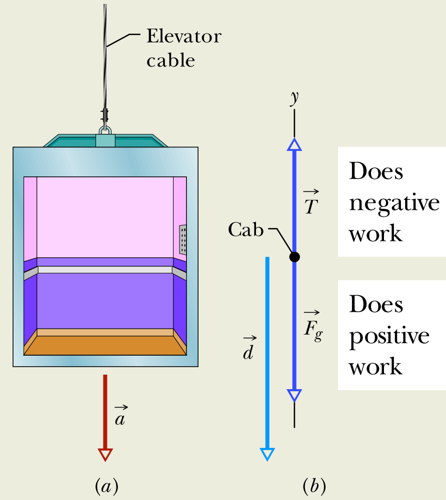

Figure 7.3.4 An elevator cab, descending with speed v_i, suddenly begins to accelerate downward. (*a*) It moves through a displacement $\vec{d}$ with constant acceleration $\vec{a} = \vec{g}/5$. (*b*) A free-body diagram for the cab, displacement included.

Caution: Note that W_T is not simply the negative of W_g because the cab accelerates during the fall. Thus, Eq. 7.3.5 (which assumes that the initial and final kinetic energies are equal) does not apply here.

(c) What is the net work W done on the cab during the fall?

Calculation: The net work is the sum of the works done by the forces acting on the cab:

$$W = W_g + W_T = 5.88 \times 10^4\,\text{J} - 4.70 \times 10^4\,\text{J}$$
$$= 1.18 \times 10^4\,\text{J} \approx 12\,\text{kJ}. \qquad \text{(Answer)}$$

(d) What is the cab's kinetic energy at the end of the 12 m fall?

KEY IDEA

The kinetic energy changes *because* of the net work done on the cab, according to Eq. 7.2.9 ($K_f = K_i + W$).

Calculation: From Eq. 7.1.1, we write the initial kinetic energy as $K_i = \frac{1}{2}mv_i^2$. We then write Eq. 7.2.9 as

$$K_f = K_i + W = \tfrac{1}{2}mv_i^2 + W$$
$$= \tfrac{1}{2}(500\,\text{kg})(4.0\,\text{m/s})^2 + 1.18 \times 10^4\,\text{J}$$
$$= 1.58 \times 10^4\,\text{J} \approx 16\,\text{kJ}. \qquad \text{(Answer)}$$

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Checkpoint 7.3.1

We do work W_1 in pulling some boxy fruit up along a frictionless ramp by a rope through a distance d. We then increase the angle of the ramp and again pull the boxy fruit up the ramp through the same distance d. Is our work greater than, less than, or the same as W_1?

7.4 WORK DONE BY A SPRING FORCE

Learning Objectives

After reading this module, you should be able to . . .

7.4.1 Apply the relationship (Hooke's law) between the force on an object due to a spring, the stretch or compression of the spring, and the spring constant of the spring.

7.4.2 Identify that a spring force is a variable force.

7.4.3 Calculate the work done on an object by a spring force by integrating the force from the initial position to the final position of the object or by using the known generic result of that integration.

7.4.4 Calculate work by graphically integrating on a graph of force versus position of the object.

7.4.5 Apply the work–kinetic energy theorem to situations in which an object is moved by a spring force.

Key Ideas

- The force $\vec{F}_s$ from a spring is

$$\vec{F}_s = -k\vec{d} \quad \text{(Hooke's law)},$$

where $\vec{d}$ is the displacement of the spring's free end from its position when the spring is in its relaxed state (neither compressed nor extended), and k is the spring constant (a measure of the spring's stiffness). If an x axis lies along the spring, with the origin at the location of the spring's free end when the spring is in its relaxed state, we can write

$$F_x = -kx \quad \text{(Hooke's law)}.$$

- A spring force is thus a variable force: It varies with the displacement of the spring's free end.

- If an object is attached to the spring's free end, the work W_s done on the object by the spring force when the object is moved from an initial position x_i to a final position x_f is

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2.$$

If $x_i = 0$ and $x_f = x$, then the equation becomes

$$W_s = -\tfrac{1}{2}kx^2.$$

Work Done by a Spring Force

We next want to examine the work done on a particle-like object by a particular type of *variable force*—namely, a **spring force**, the force from a spring. Many forces in nature have the same mathematical form as the spring force. Thus, by examining this one force, you can gain an understanding of many others.

The Spring Force

Figure 7.4.1*a* shows a spring in its **relaxed state**—that is, neither compressed nor extended. One end is fixed, and a particle-like object—a block, say—is attached to the other, free end. If we stretch the spring by pulling the block to the right as in Fig. 7.4.1*b*, the spring pulls on the block toward the left. (Because a spring force acts to restore the relaxed state, it is sometimes said to be a *restoring force*.) If we compress the spring by pushing the block to the left as in Fig. 7.4.1*c*, the spring now pushes on the block toward the right.

To a good approximation for many springs, the force $\vec{F}_s$ from a spring is proportional to the displacement $\vec{d}$ of the free end from its position when the spring is in the relaxed state. The *spring force* is given by

$$\vec{F}_s = -k\vec{d} \quad \text{(Hooke's law)}, \tag{7.4.1}$$

which is known as **Hooke's law** after Robert Hooke, an English scientist of the late 1600s. The minus sign in Eq. 7.4.1 indicates that the direction of the spring

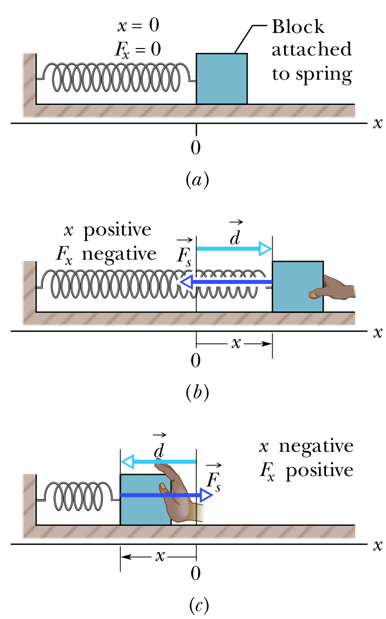

Figure 7.4.1 (*a*) A spring in its relaxed state. The origin of an x axis has been placed at the end of the spring that is attached to a block. (*b*) The block is displaced by $\vec{d}$, and the spring is stretched by a positive amount x. Note the restoring force $\vec{F}_s$ exerted by the spring. (*c*) The spring is compressed by a negative amount x. Again, note the restoring force.

force is always opposite the direction of the displacement of the spring's free end. The constant k is called the **spring constant** (or **force constant**) and is a measure of the stiffness of the spring. The larger k is, the stiffer the spring; that is, the larger k is, the stronger the spring's pull or push for a given displacement. The SI unit for k is the newton per meter.

In Fig. 7.4.1 an x axis has been placed parallel to the length of the spring, with the origin ($x = 0$) at the position of the free end when the spring is in its relaxed state. For this common arrangement, we can write Eq. 7.4.1 as

$$F_x = -kx \qquad \text{(Hooke's law)}, \tag{7.4.2}$$

where we have changed the subscript. If x is positive (the spring is stretched toward the right on the x axis), then F_x is negative (it is a pull toward the left). If x is negative (the spring is compressed toward the left), then F_x is positive (it is a push toward the right). Note that a spring force is a *variable force* because it is a function of x, the position of the free end. Thus F_x can be symbolized as $F(x)$. Also note that Hooke's law is a *linear* relationship between F_x and x.

The Work Done by a Spring Force

To find the work done by the spring force as the block in Fig. 7.4.1*a* moves, let us make two simplifying assumptions about the spring. (1) It is *massless;* that is, its mass is negligible relative to the block's mass. (2) It is an *ideal spring;* that is, it obeys Hooke's law exactly. Let us also assume that the contact between the block and the floor is frictionless and that the block is particle-like.

We give the block a rightward jerk to get it moving and then leave it alone. As the block moves rightward, the spring force F_x does work on the block, decreasing the kinetic energy and slowing the block. However, we *cannot* find this work by using Eq. 7.2.5 ($W = Fd\cos\phi$) because there is no one value of F to plug into that equation—the value of F increases as the block stretches the spring.

There is a neat way around this problem. (1) We break up the block's displacement into tiny segments that are so small that we can neglect the variation in F in each segment. (2) Then in each segment, the force has (approximately) a single value and thus we *can* use Eq. 7.2.5 to find the work in that segment. (3) Then we add up the work results for all the segments to get the total work. Well, that is our intent, but we don't really want to spend the next several days adding up a great many results and, besides, they would be only approximations. Instead, let's make the segments *infinitesimal* so that the error in each work result goes to zero. And then let's add up all the results by integration instead of by hand. Through the ease of calculus, we can do all this in minutes instead of days.

Let the block's initial position be x_i and its later position be x_f. Then divide the distance between those two positions into many segments, each of tiny length Δx. Label these segments, starting from x_i, as segments 1, 2, and so on. As the block moves through a segment, the spring force hardly varies because the segment is so short that x hardly varies. Thus, we can approximate the force magnitude as being constant within the segment. Label these magnitudes as F_{x1} in segment 1, F_{x2} in segment 2, and so on.

With the force now constant in each segment, we *can* find the work done within each segment by using Eq. 7.2.5. Here $\phi = 180°$, and so $\cos\phi = -1$. Then the work done is $-F_{x1}\,\Delta x$ in segment 1, $-F_{x2}\,\Delta x$ in segment 2, and so on. The net work W_s done by the spring, from x_i to x_f, is the sum of all these works:

$$W_s = \sum -F_{xj}\,\Delta x, \tag{7.4.3}$$

where j labels the segments. In the limit as Δx goes to zero, Eq. 7.4.3 becomes

$$W_s = \int_{x_i}^{x_f} -F_x\,dx. \qquad (7.4.4)$$

From Eq. 7.4.2, the force magnitude F_x is kx. Thus, substitution leads to

$$W_s = \int_{x_i}^{x_f} -kx\,dx = -k\int_{x_i}^{x_f} x\,dx$$
$$= (-\tfrac{1}{2}k)[x^2]_{x_i}^{x_f} = (-\tfrac{1}{2}k)(x_f^2 - x_i^2). \qquad (7.4.5)$$

Multiplied out, this yields

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \quad \text{(work by a spring force).} \qquad (7.4.6)$$

This work W_s done by the spring force can have a positive or negative value, depending on whether the *net* transfer of energy is to or from the block as the block moves from x_i to x_f. *Caution:* The final position x_f appears in the *second* term on the right side of Eq. 7.4.6. Therefore, Eq. 7.4.6 tells us:

Work W_s is positive if the block ends up closer to the relaxed position ($x = 0$) than it was initially. It is negative if the block ends up farther away from $x = 0$. It is zero if the block ends up at the same distance from $x = 0$.

If $x_i = 0$ and if we call the final position x, then Eq. 7.4.6 becomes

$$W_s = -\tfrac{1}{2}kx^2 \quad \text{(work by a spring force).} \qquad (7.4.7)$$

The Work Done by an Applied Force

Now suppose that we displace the block along the x axis while continuing to apply a force $\vec{F}_a$ to it. During the displacement, our applied force does work W_a on the block while the spring force does work W_s. By Eq. 7.2.8, the change ΔK in the kinetic energy of the block due to these two energy transfers is

$$\Delta K = K_f - K_i = W_a + W_s, \qquad (7.4.8)$$

in which K_f is the kinetic energy at the end of the displacement and K_i is that at the start of the displacement. If the block is stationary before and after the displacement, then K_f and K_i are both zero and Eq. 7.4.8 reduces to

$$W_a = -W_s. \qquad (7.4.9)$$

If a block that is attached to a spring is stationary before and after a displacement, then the work done on it by the applied force displacing it is the negative of the work done on it by the spring force.

Caution: If the block is not stationary before and after the displacement, then this statement is *not* true.

Checkpoint 7.4.1

For three situations, the initial and final positions, respectively, along the x axis for the block in Fig. 7.4.1 are (a) -3 cm, 2 cm; (b) 2 cm, 3 cm; and (c) -2 cm, 2 cm. In each situation, is the work done by the spring force on the block positive, negative, or zero?

Sample Problem 7.4.1 Work done by a spring to change kinetic energy

When a spring does work on an object, we *cannot* find the work by simply multiplying the spring force by the object's displacement. The reason is that there is no one value for the force—it changes. However, we can split the displacement up into an infinite number of tiny parts and then approximate the force in each as being constant. Integration sums the work done in all those parts. Here we use the generic result of the integration.

In Fig. 7.4.2, a cumin canister of mass $m = 0.40$ kg slides across a horizontal frictionless counter with speed $v = 0.50$ m/s. It then runs into and compresses a spring of spring constant $k = 750$ N/m. When the canister is momentarily stopped by the spring, by what distance d is the spring compressed?

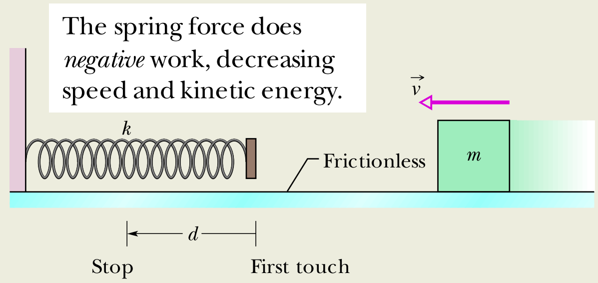

Figure 7.4.2 A canister moves toward a spring.

KEY IDEAS

1. The work W_s done on the canister by the spring force is related to the requested distance d by Eq. 7.4.7 ($W_s = -\frac{1}{2}kx^2$), with d replacing x.
2. The work W_s is also related to the kinetic energy of the canister by Eq. 7.2.8 ($K_f - K_i = W$).
3. The canister's kinetic energy has an initial value of $K = \frac{1}{2}mv^2$ and a value of zero when the canister is momentarily at rest.

Calculations: Putting the first two of these ideas together, we write the work–kinetic energy theorem for the canister as

$$K_f - K_i = -\tfrac{1}{2}kd^2.$$

Substituting according to the third key idea gives us this expression:

$$0 - \tfrac{1}{2}mv^2 = -\tfrac{1}{2}kd^2.$$

Simplifying, solving for d, and substituting known data then give us

$$d = v\sqrt{\frac{m}{k}} = (0.50\,\text{m/s})\sqrt{\frac{0.40\text{ kg}}{750\text{ N/m}}}$$

$$= 1.2 \times 10^{-2}\text{ m} = 1.2\text{ cm.} \qquad \text{(Answer)}$$

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7.5 WORK DONE BY A GENERAL VARIABLE FORCE

Learning Objectives

After reading this module, you should be able to . . .

7.5.1 Given a variable force as a function of position, calculate the work done by it on an object by integrating the function from the initial to the final position of the object, in one or more dimensions.

7.5.2 Given a graph of force versus position, calculate the work done by graphically integrating from the initial position to the final position of the object.

7.5.3 Convert a graph of acceleration versus position to a graph of force versus position.

7.5.4 Apply the work–kinetic energy theorem to situations where an object is moved by a variable force.

Key Ideas

- When the force $\vec{F}$ on a particle-like object depends on the position of the object, the work done by $\vec{F}$ on the object while the object moves from an initial position r_i with coordinates (x_i, y_i, z_i) to a final position r_f with coordinates (x_f, y_f, z_f) must be found by integrating the force. If we assume that component F_x may depend on x but not on y or z, component F_y may depend on y but not on x or z, and component F_z may depend on z but not on x or y, then the work is

$$W = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz.$$

- If $\vec{F}$ has only an x component, then this reduces to

$$W = \int_{x_i}^{x_f} F(x)\,dx.$$

Work Done by a General Variable Force

One-Dimensional Analysis

Let us return to the situation of Fig. 7.2.1 but now consider the force to be in the positive direction of the x axis and the force magnitude to vary with position x. Thus, as the bead (particle) moves, the magnitude $F(x)$ of the force doing work on it changes. Only the magnitude of this variable force changes, not its direction, and the magnitude at any position does not change with time.

Figure 7.5.1*a* shows a plot of such a *one-dimensional variable force*. We want an expression for the work done on the particle by this force as the particle moves from an initial point x_i to a final point x_f. However, we *cannot* use Eq. 7.2.5 ($W = Fd\cos\phi$) because it applies only for a constant force $\vec{F}$. Here, again, we shall use calculus. We divide the area under the curve of Fig. 7.5.1*a* into a number of narrow strips of width Δx (Fig. 7.5.1*b*). We choose Δx small enough to permit us to take the force $F(x)$ as being reasonably constant over that interval. We let $F_{j,\text{avg}}$ be the average value of $F(x)$ within the jth interval. Then in Fig. 7.5.1*b*, $F_{j,\text{avg}}$ is the height of the jth strip.

With $F_{j,\text{avg}}$ considered constant, the increment (small amount) of work ΔW_j done by the force in the jth interval is now approximately given by Eq. 7.2.5 and is

$$\Delta W_j = F_{j,\text{avg}}\,\Delta x. \tag{7.5.1}$$

In Fig. 7.5.1*b*, ΔW_j is then equal to the area of the jth rectangular, shaded strip.

To approximate the total work W done by the force as the particle moves from x_i to x_f, we add the areas of all the strips between x_i and x_f in Fig. 7.5.1*b*:

$$W = \sum \Delta W_j = \sum F_{j,\text{avg}}\Delta x. \tag{7.5.2}$$

Equation 7.5.2 is an approximation because the broken "skyline" formed by the tops of the rectangular strips in Fig. 7.5.1*b* only approximates the actual curve of $F(x)$.

We can make the approximation better by reducing the strip width Δx and using more strips (Fig. 7.5.1*c*). In the limit, we let the strip width approach zero; the number of strips then becomes infinitely large and we have, as an exact result,

$$W = \lim_{\Delta x \to 0} \sum F_{j,\text{avg}}\,\Delta x. \tag{7.5.3}$$

This limit is exactly what we mean by the integral of the function $F(x)$ between the limits x_i and x_f. Thus, Eq. 7.5.3 becomes

$$W = \int_{x_i}^{x_f} F(x)\,dx \quad \text{(work: variable force).} \tag{7.5.4}$$

If we know the function $F(x)$, we can substitute it into Eq. 7.5.4, introduce the proper limits of integration, carry out the integration, and thus find the work. (Appendix E contains a list of common integrals.) Geometrically, the work is equal to the area between the $F(x)$ curve and the x axis, between the limits x_i and x_f (shaded in Fig. 7.5.1*d*).

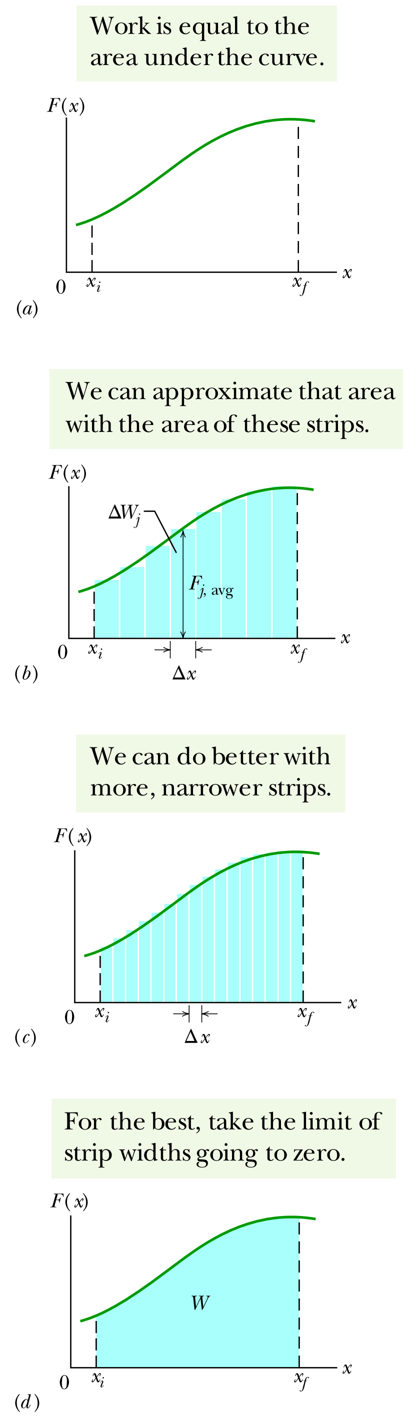

Figure 7.5.1 (*a*) A one-dimensional force $\vec{F}(x)$ plotted against the displacement x of a particle on which it acts. The particle moves from x_i to x_f. (*b*) Same as (*a*) but with the area under the curve divided into narrow strips. (*c*) Same as (*b*) but with the area divided into narrower strips. (*d*) The limiting case. The work done by the force is given by Eq. 7.5.4 and is represented by the shaded area between the curve and the x axis and between x_i and x_f.

Three-Dimensional Analysis

Consider now a particle that is acted on by a three-dimensional force

$$\vec{F} = F_x\hat{\mathrm{i}} + F_y\hat{\mathrm{j}} + F_z\hat{\mathrm{k}}, \tag{7.5.5}$$

in which the components F_x, F_y, and F_z can depend on the position of the particle; that is, they can be functions of that position. However, we make three simplifications: F_x may depend on x but not on y or z, F_y may depend on y but not on x or z, and F_z may depend on z but not on x or y. Now let the particle move through an incremental displacement

$$d\vec{r} = dx\hat{\mathrm{i}} + dy\hat{\mathrm{j}} + dz\hat{\mathrm{k}}. \tag{7.5.6}$$

The increment of work dW done on the particle by $\vec{F}$ during the displacement $d\vec{r}$ is, by Eq. 7.2.6,

$$dW = \vec{F} \cdot d\vec{r} = F_x\,dx + F_y\,dy + F_z\,dz. \tag{7.5.7}$$

The work W done by $\vec{F}$ while the particle moves from an initial position r_i having coordinates (x_i, y_i, z_i) to a final position r_f having coordinates (x_f, y_f, z_f) is then

$$W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz. \tag{7.5.8}$$

If $\vec{F}$ has only an x component, then the y and z terms in Eq. 7.5.8 are zero and the equation reduces to Eq. 7.5.4.

Work–Kinetic Energy Theorem with a Variable Force

Equation 7.5.4 gives the work done by a variable force on a particle in a one-dimensional situation. Let us now make certain that the work is equal to the change in kinetic energy, as the work–kinetic energy theorem states.

Consider a particle of mass m, moving along an x axis and acted on by a net force $F(x)$ that is directed along that axis. The work done on the particle by this force as the particle moves from position x_i to position x_f is given by Eq. 7.5.4 as

$$W = \int_{x_i}^{x_f} F(x)\,dx = \int_{x_i}^{x_f} ma\,dx, \tag{7.5.9}$$

in which we use Newton's second law to replace $F(x)$ with ma. We can write the quantity $ma\,dx$ in Eq. 7.5.9 as

$$ma\,dx = m\frac{dv}{dt}dx. \tag{7.5.10}$$

From the chain rule of calculus, we have

$$\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}v, \tag{7.5.11}$$

and Eq. 7.5.10 becomes

$$ma\,dx = m\frac{dv}{dx}v\,dx = mv\,dv. \tag{7.5.12}$$

Substituting Eq. 7.5.12 into Eq. 7.5.9 yields

$$\begin{aligned} W &= \int_{v_i}^{v_f} mv\,dv = m\int_{v_i}^{v_f} v\,dv \\ &= \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2. \end{aligned} \tag{7.5.13}$$

Note that when we change the variable from x to v we are required to express the limits on the integral in terms of the new variable. Note also that because the mass m is a constant, we are able to move it outside the integral.

Recognizing the terms on the right side of Eq. 7.5.13 as kinetic energies allows us to write this equation as

$$W = K_f - K_i = \Delta K,$$

which is the work–kinetic energy theorem.

Checkpoint 7.5.1

A particle moves along an x axis from $x = 0$ to $x = 2.0$ m as a force $\vec{F} = (3x^2\ \text{N})\hat{\text{i}}$ acts on it. How much work does the force do on the particle in that displacement?

Sample Problem 7.5.1 Epidural

In a procedure commonly used in childbirth, a surgeon or an anesthetist must run a needle through the skin on the patient's back (Fig. 7.5.2a), then through various tissue layers and into a narrow region called the epidural space that lies within the spinal canal surrounding the spinal cord. The needle is intended to deliver an anesthetic fluid. This tricky procedure requires much practice so that the doctor knows when the needle has reached the epidural space and not overshot it, a mistake that could result in serious complications. In the past, that practice has been done with actual patients. Now, however, new doctors can practice on virtual-reality simulations before injecting their first patient, allowing a doctor to learn how the force varies with a needle's penetration.

Figure 7.5.2b is a graph of the force magnitude F versus displacement x of the needle tip in a typical epidural procedure. (The line segments have been straightened somewhat from the original data.) As x increases from 0, the skin resists the needle, but at x = 8.0 mm the force is finally great enough to pierce the skin, and then the required force decreases. Similarly, the needle finally pierces the interspinous ligament at x = 18 mm and the relatively tough ligamentum flavum at x = 30 mm. The needle then enters the epidural space (where it is to deliver the anesthetic fluid), and the force drops sharply. A new doctor must learn this pattern of force versus displacement to recognize when to stop pushing on the needle. Thus, this is the pattern to be programmed into a virtual-reality simulation of epidural procedure. How much work W is done by the force exerted on the needle to get the needle to the epidural space at x = 30 mm?

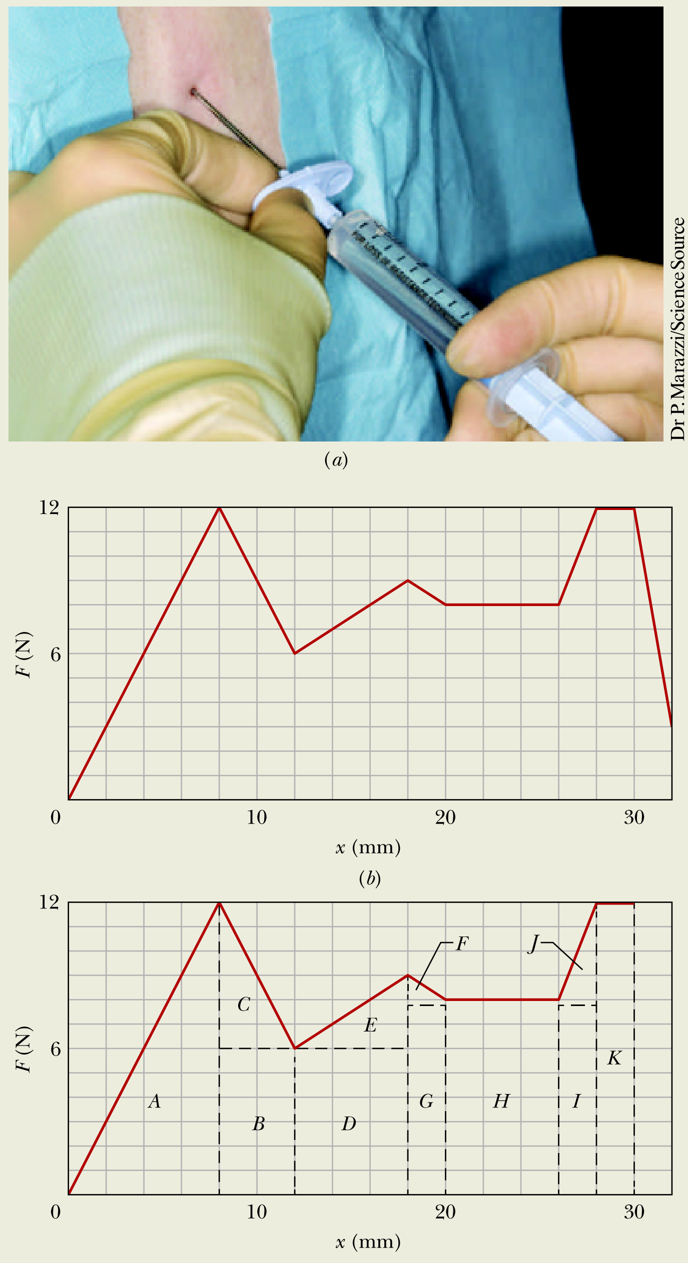

Figure 7.5.2 (a) Epidural injection. (b) The force magnitude F versus displacement x of the needle. (c) Splitting up the graph to find the area under the curve.

KEY IDEAS

(1) We can calculate the work W done by a variable force $F(x)$ by integrating the force versus position x. Equation 7.5.4 tells us that

$$W = \int_{x_i}^{x_f} F(x)\ dx.$$

We want the work done by the force during the displacement from $x_i = 0$ to $x_f = 0.030$ m.

(2) We can evaluate the integral by finding the area under the curve on the graph of Fig 7.5.2b.

Calculations: Because our graph consists of straight-line segments, we can find the area by splitting the region below the curve into rectangular and triangular regions, as shown in Fig. 7.5.2*c*. For example, the area in triangular region A is

$$\text{area}_A = \tfrac{1}{2}(0.0080\,\text{m})(12\,\text{N}) = 0.048\,\text{N}\cdot\text{m} = 0.048\,\text{J}.$$

Once we've calculated the areas for all the labeled regions in the figure, we find that the total work is

$$\begin{aligned} W &= (\text{sum of the areas of regions } A \text{ through } K) \\ &= 0.048 + 0.024 + 0.012 + 0.036 + 0.009 + 0.001 + 0.016 \\ &\quad + 0.048 + 0.016 + 0.004 + 0.024 \\ &= 0.238\,\text{J}. \end{aligned}$$

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7.6 POWER

Learning Objectives

After reading this module, you should be able to . . .

7.6.1 Apply the relationship between average power, the work done by a force, and the time interval in which that work is done.

7.6.2 Given the work as a function of time, find the instantaneous power.

7.6.3 Determine the instantaneous power by taking a dot product of the force vector and an object's velocity vector, in magnitude-angle and unit-vector notations.

Key Ideas

- The power due to a force is the *rate* at which that force does work on an object.
- If the force does work W during a time interval Δt, the average power due to the force over that time interval is

$$P_{\text{avg}} = \frac{W}{\Delta t}.$$

- Instantaneous power is the instantaneous rate of doing work:

$$P = \frac{dW}{dt}.$$

- For a force $\vec{F}$ at an angle ϕ to the direction of travel of the instantaneous velocity $\vec{v}$, the instantaneous power is

$$P = Fv\cos\phi = \vec{F}\cdot\vec{v}.$$

Power

The time rate at which work is done by a force is said to be the **power** due to the force. If a force does an amount of work W in an amount of time Δt, the **average power** due to the force during that time interval is

$$P_{\text{avg}} = \frac{W}{\Delta t} \quad \text{(average power)}. \tag{7.6.1}$$

The **instantaneous power** P is the instantaneous time rate of doing work, which we can write as

$$P = \frac{dW}{dt} \quad \text{(instantaneous power)}. \tag{7.6.2}$$

Suppose we know the work $W(t)$ done by a force as a function of time. Then to get the instantaneous power P at, say, time $t = 3.0$ s during the work, we would first take the time derivative of $W(t)$ and then evaluate the result for $t = 3.0$ s.

The SI unit of power is the joule per second. This unit is used so often that it has a special name, the **watt** (W), after James Watt, who greatly improved the rate at which steam engines could do work. In the British system, the unit of power is the foot-pound per second. Often the horsepower is used. These are related by

$$1 \text{ watt} = 1 \text{ W} = 1 \text{ J/s} = 0.738 \text{ ft} \cdot \text{lb/s} \tag{7.6.3}$$

and

$$1 \text{ horsepower} = 1 \text{ hp} = 550 \text{ ft} \cdot \text{lb/s} = 746 \text{ W}. \tag{7.6.4}$$

Inspection of Eq. 7.6.1 shows that work can be expressed as power multiplied by time, as in the common unit kilowatt-hour. Thus,

$$\begin{aligned} 1 \text{ kilowatt-hour} &= 1 \text{ kW} \cdot \text{h} = (10^3 \text{ W})(3600 \text{ s}) \\ &= 3.60 \times 10^6 \text{ J} = 3.60 \text{ MJ}. \end{aligned} \tag{7.6.5}$$

Figure 7.6.1 The power due to the truck's applied force on the trailing load is the rate at which that force does work on the load.

Perhaps because they appear on our utility bills, the watt and the kilowatt-hour have become identified as electrical units. They can be used equally well as units for other examples of power and energy. Thus, if you pick up a book from the floor and put it on a tabletop, you are free to report the work that you have done as, say, 4×10^{-6} kW · h (or more conveniently as 4 mW · h).

We can also express the rate at which a force does work on a particle (or particle-like object) in terms of that force and the particle's velocity. For a particle that is moving along a straight line (say, an x axis) and is acted on by a constant force $\vec{F}$ directed at some angle ϕ to that line, Eq. 7.6.2 becomes

$$P = \frac{dW}{dt} = \frac{F \cos \phi \, dx}{dt} = F \cos \phi \left(\frac{dx}{dt}\right),$$

or

$$P = Fv \cos \phi. \tag{7.6.6}$$

Reorganizing the right side of Eq. 7.6.6 as the dot product $\vec{F} \cdot \vec{v}$, we may also write the equation as

$$P = \vec{F} \cdot \vec{v} \quad \text{(instantaneous power)}. \tag{7.6.7}$$

For example, the truck in Fig. 7.6.1 exerts a force $\vec{F}$ on the trailing load, which has velocity $\vec{v}$ at some instant. The instantaneous power due to $\vec{F}$ is the rate at which $\vec{F}$ does work on the load at that instant and is given by Eqs. 7.6.6 and 7.6.7. Saying that this power is "the power of the truck" is often acceptable, but keep in mind what is meant: Power is the rate at which the applied *force* does work.

Checkpoint 7.6.1

A block moves with uniform circular motion because a cord tied to the block is anchored at the center of a circle. Is the power due to the force on the block from the cord positive, negative, or zero?

Sample Problem 7.6.1 Power, force, and velocity

Here we calculate an instantaneous work—that is, the rate at which work is being done at any given instant rather than averaged over a time interval. Figure 7.6.2 shows constant forces $\vec{F}_1$ and $\vec{F}_2$ acting on a box as the box slides rightward across a frictionless floor. Force $\vec{F}_1$ is horizontal, with magnitude 2.0 N; force $\vec{F}_2$ is angled upward by 60° to the floor and has magnitude 4.0 N. The speed v of the box at a certain instant is 3.0 m/s. What is the power due to each force acting on the box at that instant, and what is the net power? Is the net power changing at that instant?

KEY IDEA

We want an instantaneous power, not an average power over a time period. Also, we know the box's velocity (rather than the work done on it).

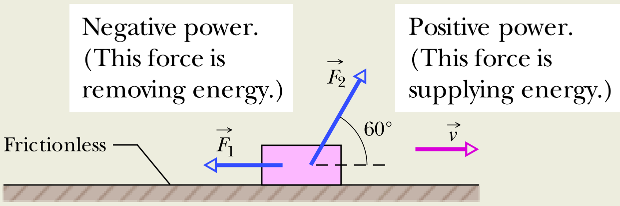

Figure 7.6.2 Two forces $\vec{F}_1$ and $\vec{F}_2$ act on a box that slides rightward across a frictionless floor. The velocity of the box is $\vec{v}$.

Calculation: We use Eq. 7.6.6 for each force. For force $\vec{F}_1$, at angle $\phi_1 = 180°$ to velocity $\vec{v}$, we have

$$P_1 = F_1 v \cos \phi_1 = (2.0\ \text{N})(3.0\ \text{m/s}) \cos 180° = -6.0\ \text{W}. \qquad \text{(Answer)}$$

This negative result tells us that force $\vec{F}_1$ is transferring energy *from* the box at the rate of 6.0 J/s.

For force $\vec{F}_2$, at angle $\phi_2 = 60°$ to velocity $\vec{v}$, we have

$$P_2 = F_2 v \cos \phi_2 = (4.0\ \text{N})(3.0\ \text{m/s}) \cos 60° = 6.0\ \text{W}. \qquad \text{(Answer)}$$

This positive result tells us that force $\vec{F}_2$ is transferring energy *to* the box at the rate of 6.0 J/s.

The net power is the sum of the individual powers (complete with their algebraic signs):

$$P_{net} = P_1 + P_2 = -6.0\ \text{W} + 6.0\ \text{W} = 0, \qquad \text{(Answer)}$$

which tells us that the net rate of transfer of energy to or from the box is zero. Thus, the kinetic energy ($K = \frac{1}{2}mv^2$) of the box is not changing, and so the speed of the box will remain at 3.0 m/s. With neither the forces $\vec{F}_1$ and $\vec{F}_2$ nor the velocity $\vec{v}$ changing, we see from Eq. 7.6.7 that P_1 and P_2 are constant and thus so is P_{net}.

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Review & Summary

Kinetic Energy The **kinetic energy** K associated with the motion of a particle of mass m and speed v, where v is well below the speed of light, is

$$K = \tfrac{1}{2}mv^2 \quad \text{(kinetic energy)}. \tag{7.1.1}$$

Work **Work** W is energy transferred to or from an object via a force acting on the object. Energy transferred to the object is positive work, and from the object, negative work.

Work Done by a Constant Force The work done on a particle by a constant force $\vec{F}$ during displacement $\vec{d}$ is

$$W = Fd \cos \phi = \vec{F} \cdot \vec{d} \quad \text{(work, constant force)}, \qquad (7.2.5, 7.2.6)$$

in which ϕ is the constant angle between the directions of $\vec{F}$ and $\vec{d}$. Only the component of $\vec{F}$ that is along the displacement $\vec{d}$ can do work on the object. When two or more forces act on an object, their **net work** is the sum of the individual works done by the forces, which is also equal to the work that would be done on the object by the net force $\vec{F}_{net}$ of those forces.

Work and Kinetic Energy For a particle, a change ΔK in the kinetic energy equals the net work W done on the particle:

$$\Delta K = K_f - K_i = W \quad \text{(work–kinetic energy theorem)}, \tag{7.2.8}$$

in which K_i is the initial kinetic energy of the particle and K_f is the kinetic energy after the work is done. Equation 7.2.8 rearranged gives us

$$K_f = K_i + W. \tag{7.2.9}$$

Work Done by the Gravitational Force The work W_g done by the gravitational force $\vec{F}_g$ on a particle-like object of mass m as the object moves through a displacement $\vec{d}$ is given by

$$W_g = mgd \cos \phi, \tag{7.3.1}$$

in which ϕ is the angle between $\vec{F}_g$ and $\vec{d}$.

Work Done in Lifting and Lowering an Object The work W_a done by an applied force as a particle-like object is either lifted or lowered is related to the work W_g done by the gravitational force and the change ΔK in the object's kinetic energy by

$$\Delta K = K_f - K_i = W_a + W_g. \tag{7.3.4}$$

If $K_f = K_i$, then Eq. 7.3.4 reduces to

$$W_a = -W_g, \tag{7.3.5}$$

which tells us that the applied force transfers as much energy to the object as the gravitational force transfers from it.

Spring Force The force $\vec{F}_s$ from a spring is

$$\vec{F}_s = -k\vec{d} \quad \text{(Hooke's law)}, \tag{7.4.1}$$

where $\vec{d}$ is the displacement of the spring's free end from its position when the spring is in its **relaxed state** (neither compressed nor extended), and k is the **spring constant** (a measure of the spring's stiffness). If an x axis lies along the spring, with the origin at the location of the spring's free end when the spring is in its relaxed state, Eq. 7.4.1 can be written as

$$F_x = -kx \quad \text{(Hooke's law)}. \tag{7.4.2}$$

A spring force is thus a variable force: It varies with the displacement of the spring's free end.

Work Done by a Spring Force If an object is attached to the spring's free end, the work W_s done on the object by the spring force when the object is moved from an initial position x_i to a final position x_f is

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2. \tag{7.4.6}$$

If $x_i = 0$ and $x_f = x$, then Eq. 7.4.6 becomes

$$W_s = -\tfrac{1}{2}kx^2. \tag{7.4.7}$$

Work Done by a Variable Force When the force $\vec{F}$ on a particle-like object depends on the position of the object, the work done by $\vec{F}$ on the object while the object moves from an initial position r_i with coordinates (x_i, y_i, z_i) to a final position r_f with coordinates (x_f, y_f, z_f) must be found by integrating the force. If we assume that component F_x may depend on x but not on y or z, component F_y may depend on y but not on x or z, and component F_z may depend on z but not on x or y, then the work is

$$W = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz. \tag{7.5.8}$$

If $\vec{F}$ has only an x component, then Eq. 7.5.8 reduces to

$$W = \int_{x_i}^{x_f} F(x)\,dx. \tag{7.5.4}$$

Power The **power** due to a force is the *rate* at which that force does work on an object. If the force does work W during a time interval Δt, the *average power* due to the force over that time interval is

$$P_{\text{avg}} = \frac{W}{\Delta t}. \tag{7.6.1}$$

Instantaneous power is the instantaneous rate of doing work:

$$P = \frac{dW}{dt}. \tag{7.6.2}$$

For a force $\vec{F}$ at an angle ϕ to the direction of travel of the instantaneous velocity $\vec{v}$, the instantaneous power is

$$P = Fv\cos\phi = \vec{F}\cdot\vec{v}. \tag{7.6.6, 7.6.7}$$

Questions

1 Rank the following velocities according to the kinetic energy a particle will have with each velocity, greatest first: (a) $\vec{v} = 4\hat{i} + 3\hat{j}$, (b) $\vec{v} = -4\hat{i} + 3\hat{j}$, (c) $\vec{v} = -3\hat{i} + 4\hat{j}$, (d) $\vec{v} = 3\hat{i} - 4\hat{j}$, (e) $\vec{v} = 5\hat{i}$, and (f) $v = 5$ m/s at 30° to the horizontal.

2 Figure 7.1*a* shows two horizontal forces that act on a block that is sliding to the right across a frictionless floor. Figure 7.1*b* shows three plots of the block's kinetic energy K versus time t. Which of the plots best corresponds to the following three situations: (a) $F_1 = F_2$, (b) $F_1 > F_2$, (c) $F_1 < F_2$?

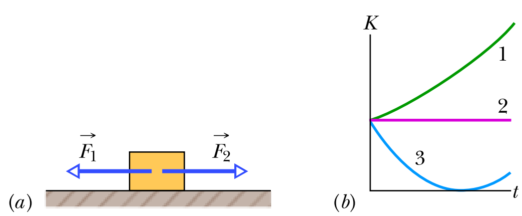

Figure 7.1 Question 2.

3 Is positive or negative work done by a constant force $\vec{F}$ on a particle during a straight-line displacement $\vec{d}$ if (a) the angle between $\vec{F}$ and $\vec{d}$ is 30°; (b) the angle is 100°; (c) $\vec{F} = 2\hat{i} - 3\hat{j}$ and $\vec{d} = -4\hat{i}$?

4 In three situations, a briefly applied horizontal force changes the velocity of a hockey puck that slides over frictionless ice. The overhead views of Fig. 7.2 indicate, for each situation, the puck's initial speed v_i, its final speed v_f, and the directions of the corresponding velocity vectors. Rank the situations according to the work done on the puck by the applied force, most positive first and most negative last.

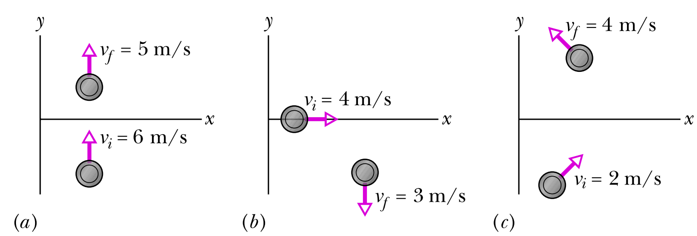

Figure 7.2 Question 4.

5 The graphs in Fig. 7.3 give the x component F_x of a force acting on a particle moving along an x axis. Rank them according to the work done by the force on the particle from $x = 0$ to $x = x_1$, from most positive work first to most negative work last.

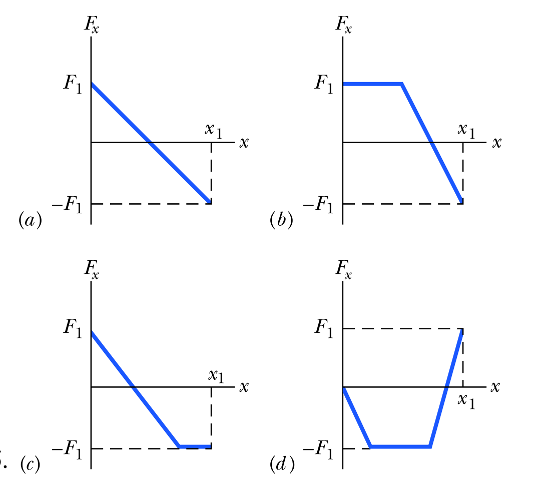

Figure 7.3 Question 5.

6 Figure 7.4 gives the x component F_x of a force that can act on a particle. If the particle begins at rest at $x = 0$, what is its coordinate when it has (a) its greatest kinetic energy, (b) its greatest speed, and (c) zero speed? (d) What is the particle's direction of travel after it reaches $x = 6$ m?

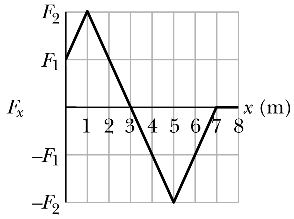

Figure 7.4 Question 6.

7 In Fig. 7.5, a greased pig has a choice of three frictionless slides along which to slide to the ground. Rank the slides according to how much work the gravitational force does on the pig during the descent, greatest first.

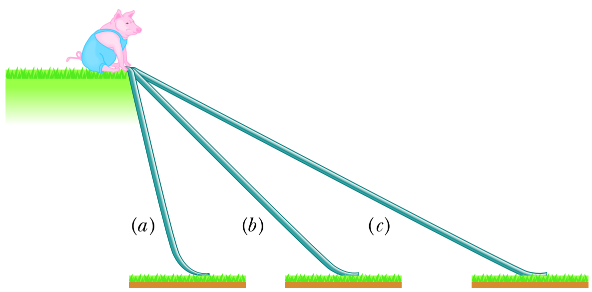

Figure 7.5 Question 7.

8 Figure 7.6*a* shows four situations in which a horizontal force acts on the same block, which is initially at rest. The force magnitudes are $F_2 = F_4 = 2F_1 = 2F_3$. The horizontal component v_x of the block's velocity is shown in Fig. 7.6*b* for the four situations. (a) Which plot in Fig. 7.6*b* best corresponds to which force in Fig. 7.6*a*? (b) Which plot in Fig. 7.6*c* (for kinetic energy K versus time t) best corresponds to which plot in Fig. 7.6*b*?

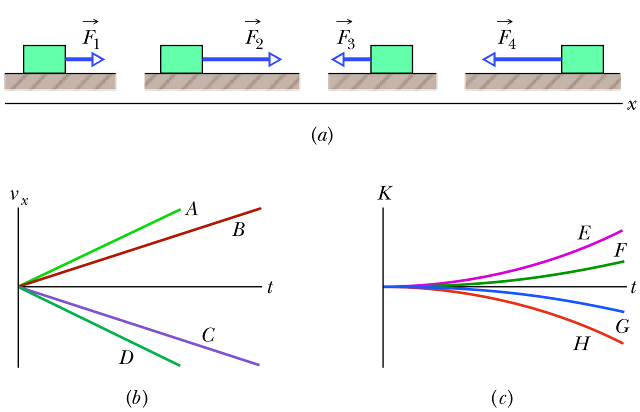

Figure 7.6 Question 8.

9 Spring A is stiffer than spring B ($k_A > k_B$). The spring force of which spring does more work if the springs are compressed (a) the same distance and (b) by the same applied force?

10 A glob of slime is launched or dropped from the edge of a cliff. Which of the graphs in Fig. 7.7 could possibly show how the kinetic energy of the glob changes during its flight?

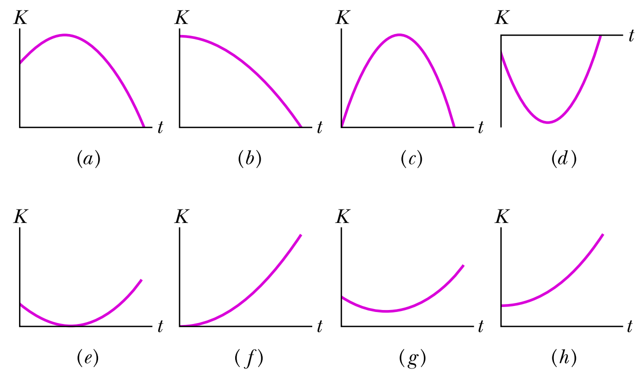

Figure 7.7 Question 10.

11 In three situations, a single force acts on a moving particle. Here are the velocities (at that instant) and the forces: (1) $\vec{v} = (-4\hat{i})$ m/s, $\vec{F} = (6\hat{i} - 20\hat{j})$ N; (2) $\vec{v} = (2\hat{i} - 3\hat{j})$ m/s, $\vec{F} = (-2\hat{j} + 7\hat{k})$ N; (3) $\vec{v} = (-3\hat{i} + \hat{j})$ m/s, $\vec{F} = (2\hat{i} + 6\hat{j})$ N. Rank the situations according to the rate at which energy is being transferred, greatest transfer to the particle ranked first, greatest transfer from the particle ranked last.

12 Figure 7.8 shows three arrangements of a block attached to identical springs that are in their relaxed state when the block is centered as shown. Rank the arrangements according to the magnitude of the net force on the block, largest first, when the block is displaced by distance d (a) to the right and (b) to the left. Rank the arrangements according to the work done on the block by the spring forces, greatest first, when the block is displaced by d (c) to the right and (d) to the left.

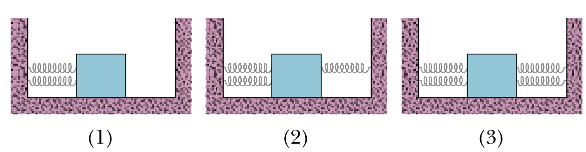

Figure 7.8 Question 12.

Problems

GO Tutoring problem available (at instructor's discretion) in *WileyPLUS*
SSM Worked-out solution available in Student Solutions Manual
E Easy M Medium H Hard
FCP Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com
CALC Requires calculus
BIO Biomedical application

Module 7.1 Kinetic Energy

1 E SSM A proton (mass $m = 1.67 \times 10^{-27}$ kg) is being accelerated along a straight line at 3.6×10^{15} m/s^2 in a machine. If the proton has an initial speed of 2.4×10^7 m/s and travels 3.5 cm, what then is (a) its speed and (b) the increase in its kinetic energy?

2 E If a Saturn V rocket with an Apollo spacecraft attached had a combined mass of 2.9×10^5 kg and reached a speed of 11.2 km/s, how much kinetic energy would it then have?

3 E FCP On August 10, 1972, a large meteorite skipped across the atmosphere above the western United States and western Canada, much like a stone skipping across water. The accompanying fireball was so bright that it could be seen in the daytime sky and was brighter than the usual meteorite trail. The meteorite's mass was about 4×10^6 kg; its speed was about 15 km/s. Had it entered the atmosphere vertically, it would have hit Earth's surface with about the same speed. (a) Calculate the meteorite's loss of kinetic energy (in joules) that would have been associated with the vertical impact. (b) Express the energy as a multiple of the explosive energy of 1 megaton of TNT, which is 4.2×10^{15} J. (c) The energy associated with the atomic bomb explosion over Hiroshima was equivalent to 13 kilotons of TNT. To how many Hiroshima bombs would the meteorite impact have been equivalent?

4 E FCP An explosion at ground level leaves a crater with a diameter that is proportional to the energy of the explosion raised to the $\frac{1}{3}$ power; an explosion of 1 megaton of TNT leaves a crater with a 1 km diameter. Below Lake Huron in Michigan there appears to be an ancient impact crater with a 50 km diameter. What was the kinetic energy associated with that impact, in terms of (a) megatons of TNT (1 megaton yields 4.2×10^{15} J) and (b) Hiroshima bomb equivalents (13 kilotons of TNT each)? (Ancient meteorite or comet impacts may have significantly altered the climate, killing off the dinosaurs and other life-forms.)

5 M A father racing his son has half the kinetic energy of the son, who has half the mass of the father. The father speeds up by 1.0 m/s and then has the same kinetic energy as the son. What are the original speeds of (a) the father and (b) the son?

6 M A bead with mass 1.8×10^{-2} kg is moving along a wire in the positive direction of an x axis. Beginning at time $t = 0$, when the bead passes through $x = 0$ with speed 12 m/s, a constant force acts on the bead. Figure 7.9 indicates the bead's position at these four times: $t_0 = 0$, $t_1 = 1.0$ s, $t_2 = 2.0$ s, and $t_3 = 3.0$ s. The bead momentarily stops at $t = 3.0$ s. What is the kinetic energy of the bead at $t = 10$ s?

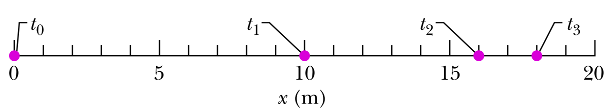

Figure 7.9 Problem 6.

Module 7.2 Work and Kinetic Energy

7 E A 3.0 kg body is at rest on a frictionless horizontal air track when a constant horizontal force $\vec{F}$ acting in the positive direction of an x axis along the track is applied to the body. A stroboscopic graph of the position of the body as it slides to the right is shown in Fig. 7.10. The force $\vec{F}$ is applied to the body at $t = 0$, and the graph records the position of the body at 0.50 s intervals. How much work is done on the body by the applied force $\vec{F}$ between $t = 0$ and $t = 2.0$ s?

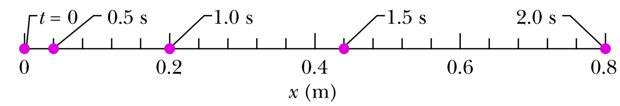

Figure 7.10 Problem 7.

8 E A ice block floating in a river is pushed through a displacement $\vec{d} = (15\text{ m})\hat{i} - (12\text{ m})\hat{j}$ along a straight embankment by rushing water, which exerts a force $\vec{F} = (210\text{ N})\hat{i} - (150\text{ N})\hat{j}$ on the block. How much work does the force do on the block during the displacement?

9 E The only force acting on a 2.0 kg canister that is moving in an xy plane has a magnitude of 5.0 N. The canister initially has a velocity of 4.0 m/s in the positive x direction and some time later has a velocity of 6.0 m/s in the positive y direction. How much work is done on the canister by the 5.0 N force during this time?

10 E A coin slides over a frictionless plane and across an xy coordinate system from the origin to a point with xy coordinates (3.0 m, 4.0 m) while a constant force acts on it. The force has magnitude 2.0 N and is directed at a counterclockwise angle of 100° from the positive direction of the x axis. How much work is done by the force on the coin during the displacement?

11 M A 12.0 N force with a fixed orientation does work on a particle as the particle moves through the three-dimensional displacement $\vec{d} = (2.00\hat{i} - 4.00\hat{j} + 3.00\hat{k})$ m. What is the angle between the force and the displacement if the change in the particle's kinetic energy is (a) +30.0 J and (b) −30.0 J?

12 M A can of bolts and nuts is pushed 2.00 m along an x axis by a broom along the greasy (frictionless) floor of a car repair shop in a version of shuffleboard. Figure 7.11 gives the work W done on the can by the constant horizontal force from the broom, versus the can's position x. The scale of the figure's vertical axis is set by $W_s = 6.0$ J. (a) What is the magnitude of that force? (b) If the can had an initial kinetic energy of 3.00 J, moving in the positive direction of the x axis, what is its kinetic energy at the end of the 2.00 m?

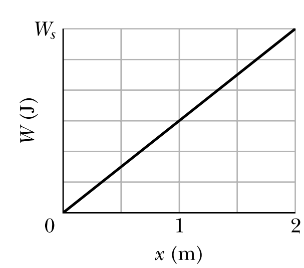

Figure 7.11 Problem 12.

13 M A luge and its rider, with a total mass of 85 kg, emerge from a downhill track onto a horizontal straight track with an initial speed of 37 m/s. If a force slows them to a stop at a constant rate of 2.0 m/s^2, (a) what magnitude F is required for the force, (b) what distance d do they travel while slowing, and (c) what work W is done on them by the force? What are (d) F, (e) d, and (f) W if they, instead, slow at 4.0 m/s^2?

14 M GO Figure 7.12 shows an overhead view of three horizontal forces acting on a cargo canister that was initially stationary but now moves across a frictionless floor. The force magnitudes are $F_1 = 3.00$ N, $F_2 = 4.00$ N, and $F_3 = 10.0$ N, and the indicated angles are $\theta_2 = 50.0°$ and $\theta_3 = 35.0°$. What is the net work done on the canister by the three forces during the first 4.00 m of displacement?

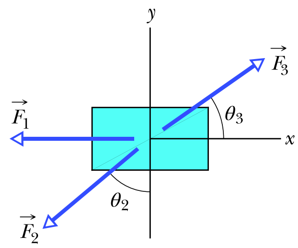

Figure 7.12 Problem 14.

15 M GO Figure 7.13 shows three forces applied to a trunk that moves leftward by 3.00 m over a frictionless floor. The force magnitudes are $F_1 = 5.00$ N, $F_2 = 9.00$ N, and $F_3 = 3.00$ N, and the indicated angle is $\theta = 60.0°$. During the displacement, (a) what is the net work done on the trunk by the three forces and (b) does the kinetic energy of the trunk increase or decrease?

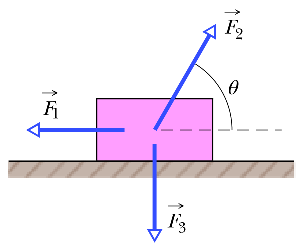

Figure 7.13 Problem 15.

16 M GO An 8.0 kg object is moving in the positive direction of an x axis. When it passes through $x = 0$, a constant force directed along the axis begins to act on it. Figure 7.14 gives its kinetic energy K versus position x as it moves from $x = 0$ to $x = 5.0$ m; $K_0 = 30.0$ J. The force continues to act. What is v when the object moves back through $x = -3.0$ m?

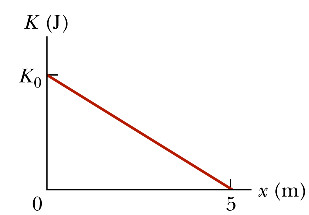

Figure 7.14 Problem 16.

Module 7.3 Work Done by the Gravitational Force

17 E SSM A helicopter lifts a 72 kg astronaut 15 m vertically from the ocean by means of a cable. The acceleration of the astronaut is $g/10$. How much work is done on the astronaut by (a) the force from the helicopter and (b) the gravitational force on her? Just before she reaches the helicopter, what are her (c) kinetic energy and (d) speed?

18 E BIO FCP (a) In 1975 the roof of Montreal's Velodrome, with a weight of 360 kN, was lifted by 10 cm so that it could be centered. How much work was done on the roof by the forces making the lift? (b) In 1960 a Tampa, Florida, mother reportedly raised one end of a car that had fallen onto her son when a jack failed. If her panic lift effectively raised 4000 N (about $\frac{1}{4}$ of the car's weight) by 5.0 cm, how much work did her force do on the car?

19 M GO In Fig. 7.15, a block of ice slides down a frictionless ramp at angle $\theta = 50°$ while an ice worker pulls on the block (via a rope) with a force $\vec{F}_r$ that has a magnitude of 50 N and is directed up the ramp. As the block slides through distance $d = 0.50$ m along the ramp, its kinetic energy increases by 80 J. How much greater would its kinetic energy have been if the rope had not been attached to the block?

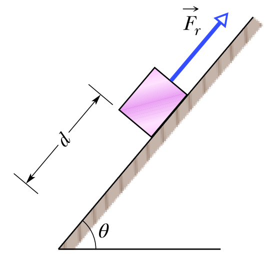

Figure 7.15 Problem 19.

20 M A block is sent up a frictionless ramp along which an x axis extends upward. Figure 7.16 gives the kinetic energy of the block as a function of position x; the scale of the figure's vertical axis is set by $K_s = 40.0$ J. If the block's initial speed is 4.00 m/s, what is the normal force on the block?

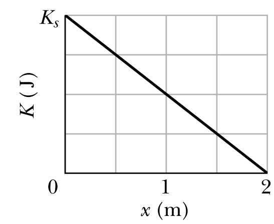

Figure 7.16 Problem 20.

21 M SSM A cord is used to vertically lower an initially stationary block of mass M at a constant downward acceleration of $g/4$. When the block has fallen a distance d, find (a) the work done by the cord's force on the block, (b) the work done by the gravitational force on the block, (c) the kinetic energy of the block, and (d) the speed of the block.

22 M A cave rescue team lifts an injured spelunker directly upward and out of a sinkhole by means of a motor-driven cable. The lift is performed in three stages, each requiring a vertical distance of 10.0 m: (a) the initially stationary spelunker is accelerated to a speed of 5.00 m/s; (b) he is then lifted at the constant speed of 5.00 m/s; (c) finally he is decelerated to zero speed. How much work is done on the 80.0 kg rescuee by the force lifting him during each stage?

23 M In Fig. 7.17, a constant force $\vec{F}_a$ of magnitude 82.0 N is applied to a 3.00 kg shoe box at angle $\phi = 53.0°$, causing the box to move up a frictionless ramp at constant speed. How much work is done on the box by $\vec{F}_a$ when the box has moved through vertical distance $h = 0.150$ m?

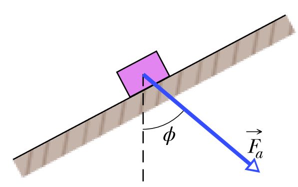

Figure 7.17 Problem 23.

24 M GO In Fig. 7.18, a horizontal force $\vec{F}_a$ of magnitude 20.0 N is applied to a 3.00 kg psychology book as the book slides a distance $d = 0.500$ m up a frictionless ramp at angle $\theta = 30.0°$. (a) During the displacement, what is the net work done on the book by $\vec{F}_a$, the gravitational force on the book, and the normal force on the book? (b) If the book has zero kinetic

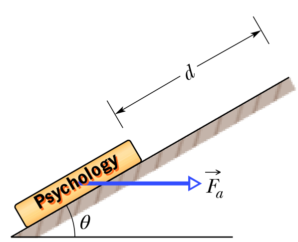

Figure 7.18 Problem 24.

energy at the start of the displacement, what is its speed at the end of the displacement?

25 H GO In Fig. 7.19, a 0.250 kg block of cheese lies on the floor of a 900 kg elevator cab that is being pulled upward by a cable through distance $d_1 = 2.40$ m and then through distance $d_2 = 10.5$ m. (a) Through d_1, if the normal force on the block from the floor has constant magnitude $F_N = 3.00$ N, how much work is done on the cab by the force from the cable? (b) Through d_2, if the work done on the cab by the (constant) force from the cable is 92.61 kJ, what is the magnitude of F_N?

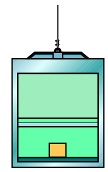

Figure 7.19 Problem 25.

Module 7.4 Work Done by a Spring Force

26 E In Fig. 7.4.1, we must apply a force of magnitude 80 N to hold the block stationary at $x = -2.0$ cm. From that position, we then slowly move the block so that our force does +4.0 J of work on the spring–block system; the block is then again stationary. What is the block's position? (*Hint:* There are two answers.)

27 E A spring and block are in the arrangement of Fig. 7.4.1. When the block is pulled out to $x = +4.0$ cm, we must apply a force of magnitude 360 N to hold it there. We pull the block to $x = 11$ cm and then release it. How much work does the spring do on the block as the block moves from $x_i = +5.0$ cm to (a) $x = +3.0$ cm, (b) $x = -3.0$ cm, (c) $x = -5.0$ cm, and (d) $x = -9.0$ cm?

28 E During spring semester at MIT, residents of the parallel buildings of the East Campus dorms battle one another with large catapults that are made with surgical hose mounted on a window frame. A balloon filled with dyed water is placed in a pouch attached to the hose, which is then stretched through the width of the room. Assume that the stretching of the hose obeys Hooke's law with a spring constant of 100 N/m. If the hose is stretched by 5.00 m and then released, how much work does the force from the hose do on the balloon in the pouch by the time the hose reaches its relaxed length?

29 M In the arrangement of Fig. 7.4.1, we gradually pull the block from $x = 0$ to $x = +3.0$ cm, where it is stationary. Figure 7.20 gives the work that our force does on the block. The scale of the figure's vertical axis is set by $W_s = 1.0$ J. We then pull the block out to $x = +5.0$ cm and release it from rest. How much work does the spring do on the block when the block moves from $x_i = +5.0$ cm to (a) $x = +4.0$ cm, (b) $x = -2.0$ cm, and (c) $x = -5.0$ cm?

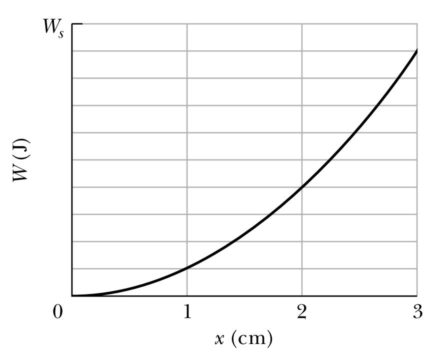

Figure 7.20 Problem 29.

30 M In Fig. 7.4.1*a*, a block of mass m lies on a horizontal frictionless surface and is attached to one end of a horizontal spring (spring constant k) whose other end is fixed. The block is initially at rest at the position where the spring is unstretched ($x = 0$) when a constant horizontal force $\vec{F}$ in the positive direction of the x axis is applied to it. A plot of the resulting kinetic energy of the block versus its position x is shown in Fig. 7.21. The scale of the figure's vertical axis is set by $K_s = 4.0$ J. (a) What is the magnitude of $\vec{F}$? (b) What is the value of k?

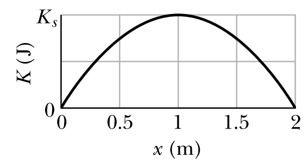

Figure 7.21 Problem 30.

31 M CALC SSM The only force acting on a 2.0 kg body as it moves along a positive x axis has an x component $F_x = -6x$ N, with x in meters. The velocity at $x = 3.0$ m is 8.0 m/s. (a) What is the velocity of the body at $x = 4.0$ m? (b) At what positive value of x will the body have a velocity of 5.0 m/s?

32 M Figure 7.22 gives spring force F_x versus position x for the spring–block arrangement of Fig. 7.4.1. The scale is set by $F_s = 160.0$ N. We release the block at $x = 12$ cm. How much work does the spring do on the block when the block moves from $x_i = +8.0$ cm to (a) $x = +5.0$ cm, (b) $x = -5.0$ cm, (c) $x = -8.0$ cm, and (d) $x = -10.0$ cm?

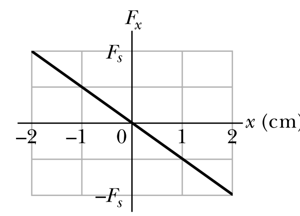

Figure 7.22 Problem 32.

33 H GO The block in Fig. 7.4.1*a* lies on a horizontal frictionless surface, and the spring constant is 50 N/m. Initially, the spring is at its relaxed length and the block is stationary at position $x = 0$. Then an applied force with a constant magnitude of 3.0 N pulls the block in the positive direction of the x axis, stretching the spring until the block stops. When that stopping point is reached, what are (a) the position of the block, (b) the work that has been done on the block by the applied force, and (c) the work that has been done on the block by the spring force? During the block's displacement, what are (d) the block's position when its kinetic energy is maximum and (e) the value of that maximum kinetic energy?

Module 7.5 Work Done by a General Variable Force

34 E CALC A 10 kg brick moves along an x axis. Its acceleration as a function of its position is shown in Fig. 7.23. The scale of the figure's vertical axis is set by $a_s = 20.0$ m/s^2. What is the net work performed on the brick by the force causing the acceleration as the brick moves from $x = 0$ to $x = 8.0$ m?

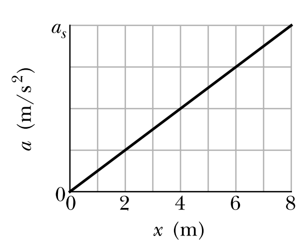

Figure 7.23 Problem 34.

35 E CALC SSM The force on a particle is directed along an x axis and given by $F = F_0(x/x_0 - 1)$. Find the work done by the force in moving the particle from $x = 0$ to $x = 2x_0$ by (a) plotting $F(x)$ and measuring the work from the graph and (b) integrating $F(x)$.

36 E CALC GO A 5.0 kg block moves in a straight line on a horizontal frictionless surface under the influence of a force that varies with position as shown in Fig. 7.24. The scale of the figure's vertical axis is set by $F_s = 10.0$ N. How much work is done by the force as the block moves from the origin to $x = 8.0$ m?

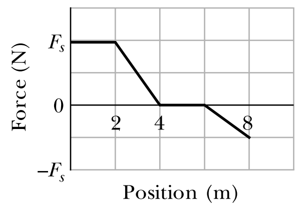

Figure 7.24 Problem 36.

37 M CALC GO Figure 7.25 gives the acceleration of a 2.00 kg particle as an applied force $\vec{F}_a$ moves it from rest along an x axis from $x = 0$ to $x = 9.0$ m. The scale of the figure's vertical axis is set by $a_s = 6.0\ \text{m/s}^2$. How much work has the force done on the particle when the particle reaches (a) $x = 4.0$ m, (b) $x = 7.0$ m, and (c) $x = 9.0$ m? What is the particle's speed and direction of travel when it reaches (d) $x = 4.0$ m, (e) $x = 7.0$ m, and (f) $x = 9.0$ m?

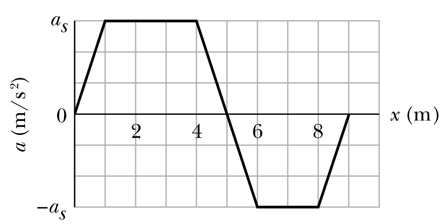

Figure 7.25 Problem 37.

38 M CALC A 1.5 kg block is initially at rest on a horizontal frictionless surface when a horizontal force along an x axis is applied to the block. The force is given by $\vec{F}(x) = (2.5 - x^2)\hat{i}$ N, where x is in meters and the initial position of the block is $x = 0$. (a) What is the kinetic energy of the block as it passes through $x = 2.0$ m? (b) What is the maximum kinetic energy of the block between $x = 0$ and $x = 2.0$ m?

39 M CALC GO A force $\vec{F} = (cx - 3.00x^2)\hat{i}$ acts on a particle as the particle moves along an x axis, with $\vec{F}$ in newtons, x in meters, and c a constant. At $x = 0$, the particle's kinetic energy is 20.0 J; at $x = 3.00$ m, it is 11.0 J. Find c.

40 M CALC A can of sardines is made to move along an x axis from $x = 0.25$ m to $x = 1.25$ m by a force with a magnitude given by $F = \exp(-4x^2)$, with x in meters and F in newtons. (Here exp is the exponential function.) How much work is done on the can by the force?

41 M CALC A single force acts on a 3.0 kg particle-like object whose position is given by $x = 3.0t - 4.0t^2 + 1.0t^3$, with x in meters and t in seconds. Find the work done by the force from $t = 0$ to $t = 4.0$ s.

42 H GO Figure 7.26 shows a cord attached to a cart that can slide along a frictionless horizontal rail aligned along an x axis. The left end of the cord is pulled over a pulley, of negligible mass and friction and at cord height $h = 1.20$ m, so the cart slides from $x_1 = 3.00$ m to $x_2 = 1.00$ m. During the move, the tension in the cord is a constant 25.0 N. What is the change in the kinetic energy of the cart during the move?

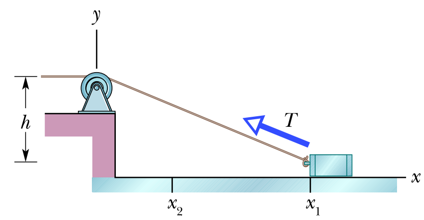

Figure 7.26 Problem 42.

Module 7.6 Power

43 E CALC SSM A force of 5.0 N acts on a 15 kg body initially at rest. Compute the work done by the force in (a) the first, (b) the second, and (c) the third seconds and (d) the instantaneous power due to the force at the end of the third second.

44 E A skier is pulled by a towrope up a frictionless ski slope that makes an angle of 12° with the horizontal. The rope moves parallel to the slope with a constant speed of 1.0 m/s. The force of the rope does 900 J of work on the skier as the skier moves a distance of 8.0 m up the incline. (a) If the rope moved with a constant speed of 2.0 m/s, how much work would the force of the rope do on the skier as the skier moved a distance of 8.0 m up the incline? At what rate is the force of the rope doing work on the skier when the rope moves with a speed of (b) 1.0 m/s and (c) 2.0 m/s?

45 E SSM A 100 kg block is pulled at a constant speed of 5.0 m/s across a horizontal floor by an applied force of 122 N directed 37° above the horizontal. What is the rate at which the force does work on the block?

46 E The loaded cab of an elevator has a mass of 3.0×10^3 kg and moves 210 m up the shaft in 23 s at constant speed. At what average rate does the force from the cable do work on the cab?

47 M A machine carries a 4.0 kg package from an initial position of $\vec{d}_i = (0.50\ \text{m})\hat{i} + (0.75\ \text{m})\hat{j} + (0.20\ \text{m})\hat{k}$ at $t = 0$ to a final position of $\vec{d}_f = (7.50\ \text{m})\hat{i} + (12.0\ \text{m})\hat{j} + (7.20\ \text{m})\hat{k}$ at $t = 12$ s. The constant force applied by the machine on the package is $\vec{F} = (2.00\ \text{N})\hat{i} + (4.00\ \text{N})\hat{j} + (6.00\ \text{N})\hat{k}$. For that displacement, find (a) the work done on the package by the machine's force and (b) the average power of the machine's force on the package.

48 M A 0.30 kg ladle sliding on a horizontal frictionless surface is attached to one end of a horizontal spring ($k = 500$ N/m) whose other end is fixed. The ladle has a kinetic energy of 10 J as it passes through its equilibrium position (the point at which the spring force is zero). (a) At what rate is the spring doing work on the ladle as the ladle passes through its equilibrium position? (b) At what rate is the spring doing work on the ladle when the spring is compressed 0.10 m and the ladle is moving away from the equilibrium position?

49 M SSM A fully loaded, slow-moving freight elevator has a cab with a total mass of 1200 kg, which is required to travel upward 54 m in 3.0 min, starting and ending at rest. The elevator's counterweight has a mass of only 950 kg, and so the elevator motor must help. What average power is required of the force the motor exerts on the cab via the cable?

50 M (a) At a certain instant, a particle-like object is acted on by a force $\vec{F} = (4.0\ \text{N})\hat{i} - (2.0\ \text{N})\hat{j} + (9.0\ \text{N})\hat{k}$ while the object's velocity is $\vec{v} = -(2.0 \text{ m/s})\hat{i} + (4.0 \text{ m/s})\hat{k}$. What is the instantaneous rate at which the force does work on the object? (b) At some other time, the velocity consists of only a y component. If the force is unchanged and the instantaneous power is -12 W, what is the velocity of the object?

51 **M** A force $\vec{F} = (3.00 \text{ N})\hat{i} + (7.00 \text{ N})\hat{j} + (7.00 \text{ N})\hat{k}$ acts on a 2.00 kg mobile object that moves from an initial position of $\vec{d}_i = (3.00 \text{ m})\hat{i} - (2.00 \text{ m})\hat{j} + (5.00 \text{ m})\hat{k}$ to a final position of $\vec{d}_f = -(5.00 \text{ m})\hat{i} + (4.00 \text{ m})\hat{j} + (7.00 \text{ m})\hat{k}$ in 4.00 s. Find (a) the work done on the object by the force in the 4.00 s interval, (b) the average power due to the force during that interval, and (c) the angle between vectors $\vec{d}_i$ and $\vec{d}_f$.

52 **H** **CALC** A funny car accelerates from rest through a measured track distance in time T with the engine operating at a constant power P. If the track crew can increase the engine power by a differential amount dP, what is the change in the time required for the run?

Additional Problems

53 Figure 7.27 shows a cold package of hot dogs sliding rightward across a frictionless floor through a distance $d = 20.0$ cm while three forces act on the package. Two of them are horizontal and have the magnitudes $F_1 = 5.00$ N and $F_2 = 1.00$ N; the third is angled down by $\theta = 60.0°$ and has the magnitude $F_3 = 4.00$ N. (a) For the 20.0 cm displacement, what is the *net* work done on the package by the three applied forces, the gravitational force on the package, and the normal force on the package? (b) If the package has a mass of 2.0 kg and an initial kinetic energy of 0, what is its speed at the end of the displacement?

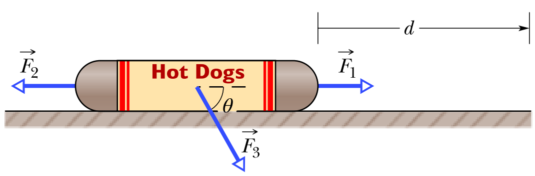

Figure 7.27 Problem 53.

54 **GO** The only force acting on a 2.0 kg body as the body moves along an x axis varies as shown in Fig. 7.28. The scale of the figure's vertical axis is set by $F_s = 4.0$ N. The velocity of the body at $x = 0$ is 4.0 m/s. (a) What is the kinetic energy of the body at $x = 3.0$ m? (b) At what value of x will the body have a kinetic energy of 8.0 J? (c) What is the maximum kinetic energy of the body between $x = 0$ and $x = 5.0$ m?

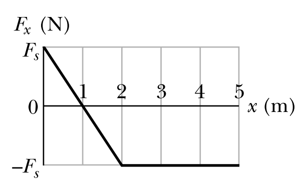

Figure 7.28 Problem 54.

55 **SSM** **BIO** A horse pulls a cart with a force of 40 lb at an angle of 30° above the horizontal and moves along at a speed of 6.0 mi/h. (a) How much work does the force do in 10 min? (b) What is the average power (in horsepower) of the force?

56 An initially stationary 2.0 kg object accelerates horizontally and uniformly to a speed of 10 m/s in 3.0 s. (a) In that 3.0 s interval, how much work is done on the object by the force accelerating it? What is the instantaneous power due to that force (b) at the end of the interval and (c) at the end of the first half of the interval?

57 A 230 kg crate hangs from the end of a rope of length $L = 12.0$ m. You push horizontally on the crate with a varying force $\vec{F}$ to move it distance $d = 4.00$ m to the side (Fig. 7.29). (a) What is the magnitude of $\vec{F}$ when the crate is in this final position? During the crate's displacement, what are (b) the total work done on it, (c) the work done by the gravitational force on the crate, and (d) the work done by the pull on the crate from the rope? (e) Knowing that the crate is motionless before and after its displacement, use the answers to (b), (c), and (d) to find the work your force $\vec{F}$ does on the crate. (f) Why is the work of your force not equal to the product of the horizontal displacement and the answer to (a)?

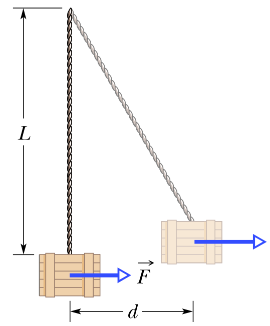

Figure 7.29 Problem 57.

58 To pull a 50 kg crate across a horizontal frictionless floor, a worker applies a force of 210 N, directed 20° above the horizontal. As the crate moves 3.0 m, what work is done on the crate by (a) the worker's force, (b) the gravitational force, and (c) the normal force? (d) What is the total work?

59 A force $\vec{F}_a$ is applied to a bead as the bead is moved along a straight wire through displacement +5.0 cm. The magnitude of $\vec{F}_a$ is set at a certain value, but the angle ϕ between $\vec{F}_a$ and the bead's displacement can be chosen. Figure 7.30 gives the work W done by $\vec{F}_a$ on the bead for a range of ϕ values; $W_0 = 25$ J. How much work is done by $\vec{F}_a$ if ϕ is (a) 64° and (b) 147°?

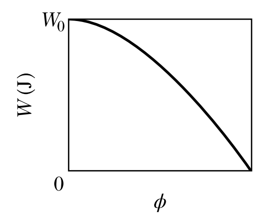

Figure 7.30 Problem 59.

60 A frightened child is restrained by her mother as the child slides down a frictionless playground slide. If the force on the child from the mother is 100 N up the slide, the child's kinetic energy increases by 30 J as she moves down the slide a distance of 1.8 m. (a) How much work is done on the child by the gravitational force during the 1.8 m descent? (b) If the child is not restrained by her mother, how much will the child's kinetic energy increase as she comes down the slide that same distance of 1.8 m?

61 **CALC** How much work is done by a force $\vec{F} = (2x \text{ N})\hat{i} + (3 \text{ N})\hat{j}$, with x in meters, that moves a particle from a position $\vec{r}_i = (2 \text{ m})\hat{i} + (3 \text{ m})\hat{j}$ to a position $\vec{r}_f = -(4 \text{ m})\hat{i} - (3 \text{ m})\hat{j}$?

62 A 250 g block is dropped onto a relaxed vertical spring that has a spring constant of $k = 2.5$ N/cm (Fig. 7.31). The block becomes attached to the spring and compresses the spring 12 cm before momentarily stopping. While the spring is being compressed, what work is done on the block by (a) the gravitational force on it and (b) the spring force? (c) What is the speed of the block just before it hits the spring? (Assume that friction is negligible.) (d) If the speed at impact is doubled, what is the maximum compression of the spring?

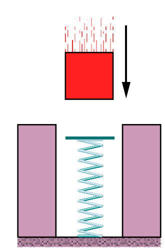

Figure 7.31 Problem 62.

63 **SSM** To push a 25.0 kg crate up a frictionless incline, angled at 25.0° to the horizontal, a worker exerts a force of 209 N parallel to the incline. As the crate slides 1.50 m, how much work is done on the crate by (a) the worker's applied force, (b) the gravitational force on the crate, and (c) the normal force exerted by the incline on the crate? (d) What is the total work done on the crate?

64 Boxes are transported from one location to another in a warehouse by means of a conveyor belt that moves with a constant speed of 0.50 m/s. At a certain location the conveyor belt moves for 2.0 m up an incline that makes an angle of 10° with the horizontal, then for 2.0 m horizontally, and finally for 2.0 m down an incline that makes an angle of 10° with the horizontal. Assume that a 2.0 kg box rides on the belt without slipping. At what rate is the force of the conveyor belt doing work on the box as the box moves (a) up the 10° incline, (b) horizontally, and (c) down the 10° incline?

65 In Fig. 7.32, a cord runs around two massless, frictionless pulleys. A canister with mass $m = 20$ kg hangs from one pulley, and you exert a force $\vec{F}$ on the free end of the cord. (a) What must be the magnitude of $\vec{F}$ if you are to lift the canister at a constant speed? (b) To lift the canister by 2.0 cm, how far must you pull the free end of the cord? During that lift, what is the work done on the canister by (c) your force (via the cord) and (d) the gravitational force? (*Hint:* When a cord loops around a pulley as shown, it pulls on the pulley with a net force that is twice the tension in the cord.)

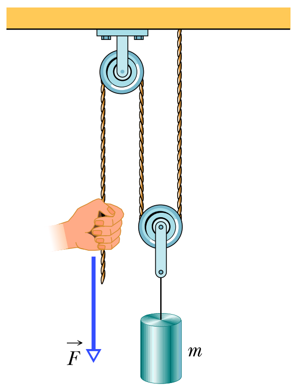

Figure 7.32 Problem 65.

66 If a car of mass 1200 kg is moving along a highway at 120 km/h, what is the car's kinetic energy as determined by someone standing alongside the highway?

67 SSM A spring with a pointer attached is hanging next to a scale marked in millimeters. Three different packages are hung from the spring, in turn, as shown in Fig. 7.33. (a) Which mark on the scale will the pointer indicate when no package is hung from the spring? (b) What is the weight W of the third package?

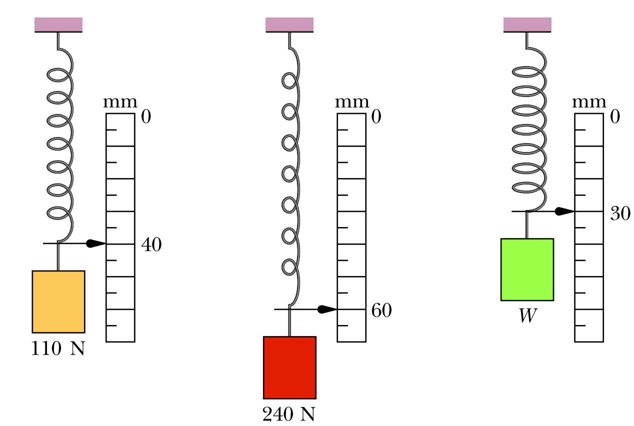

Figure 7.33 Problem 67.

68 An iceboat is at rest on a frictionless frozen lake when a sudden wind exerts a constant force of 200 N, toward the east, on the boat. Due to the angle of the sail, the wind causes the boat to slide in a straight line for a distance of 8.0 m in a direction 20° north of east. What is the kinetic energy of the iceboat at the end of that 8.0 m?

69 If a ski lift raises 100 passengers averaging 660 N in weight to a height of 150 m in 60.0 s, at constant speed, what average power is required of the force making the lift?

70 A force $\vec{F} = (4.0\text{ N})\hat{\text{i}} + c\hat{\text{j}}$ acts on a particle as the particle travels through displacement $\vec{d} = (3.0\text{ m})\hat{\text{i}} - (2.0\text{ m})\hat{\text{j}}$. (Other forces also act on the particle.) What is c if the work done on the particle by force $\vec{F}$ is (a) 0, (b) 17 J, and (c) −18 J?

71 *Kinetic energy.* If a vehicle with a mass of 1500 kg has a speed of 120 km/h, what is the vehicle's kinetic energy as determined by someone passing the vehicle at 140 km/h?

72 CALC *Work calculated by graphical integration.* In Fig. 7.34*b*, an 8.0 kg block slides along a frictionless floor as a force acts on it, starting at $x_1 = 0$ and ending at $x_3 = 6.5$ m. As the block moves, the magnitude and direction of the force vary according to the graph shown in Fig. 7.34*a*. For example, from $x = 0$ to $x = 1$ m, the force is positive (in the positive direction of the x axis) and increases in magnitude from 0 to 40 N. And from $x = 4$ m to $x = 5$ m, the force is negative and increases in magnitude from 0 to 20 N. The block's kinetic energy at x_1 is $K_1 = 280$ J. What is the block's speed at (a) $x_1 = 0$, (b) $x_2 = 4.0$ m, and (c) $x_3 = 6.5$ m?

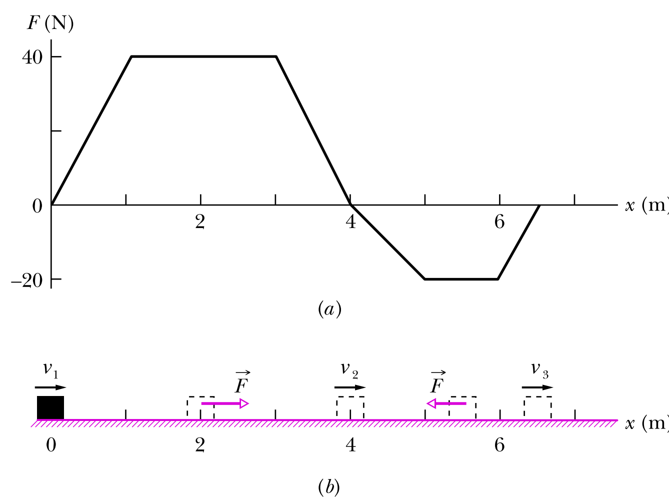

Figure 7.34 Problem 72.

73 *Brick load.* A load of bricks with mass $m = 420$ kg is to be lifted by a winch to a stationary position at height $h = 120$ m in 5.00 min. What must be the average power of the winch motion in kilowatts and horsepower?

74 BIO CALC *Hip fracture and body mass index.* Hip fracture due to a fall is a chronic problem, especially with older people and people subject to seizures. One research focus is on the correlation between fracture risk and weight, specifically, the body mass index (BMI). That index is defined as m/h^2, where m is the mass (in kilograms) and h is the height (in meters) of a person. Is a person with a higher BMI more or less likely to fracture a hip in a fall on a floor?

One way to measure the fracture risk is to measure the amount of energy absorbed as the hip impacts the floor and any covering in a sideways fall. During the impact and compression of the floor and covering, the force from the hip does work on the floor and covering. A larger amount of work implies a smaller amount of energy left to fracture the hip. In an experiment, a participant is held horizontally in a sling with the left hip 5.0 cm above a force plate with a floor covering. When the participant is dropped, measurements are made of the force magnitude F on the plate during impact and the plate's deflection d. Figure 7.35 gives idealized plots for two participants. For A: $m = 55.0$ kg, $h = 1.70$ m, peak force $F_A = 1400$ N, and maximum plate deflection $d_A = 2.00$ cm. For B: $m = 110$ kg, $h = 1.70$ m, $F_{B2} = 1600$ N, $d_{B2} = 6.00$ cm, and intermediate force $F_{B1} = 500$ N at deflection $d_{B1} = 4.00$ cm.

What is the BMI for (a) (lighter) participant A and (b) (heavier) participant B? (c) Which participant experiences the greater peak force from the plate, the lighter one or the heavier one? How much energy is absorbed by the plate and covering (how much work is done on the plate) for (d) participant A and (e) participant B? What is the absorbed energy per unit mass of (f) participant A and (g) participant B? (h) Do the results indicate higher plate absorption (and thus lower fracture risk for the participant) for the higher BMI or lower BMI? (i) Does this correlate with the peak force results?

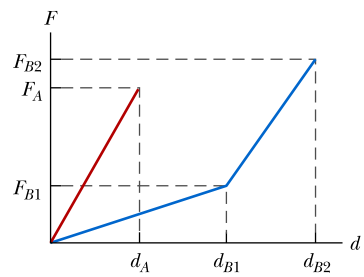

Figure 7.35 Problem 74.

75 *Car crash force from seat belt.* A car crashes head on into a wall and stops, with the front collapsing by 0.500 m. The 70 kg driver is firmly held to the seat by a seat belt and thus moves forward by 0.500 m during the crash. Assume that the force on the driver from the seat belt is constant during the crash. Use the work–kinetic energy theorem to find the magnitude of that force during the crash if the initial speed of the car is (a) 35 mi/h and (b) 70 mi/h? (c) If the initial speed is multiplied by 2, as here, by what multiplying factor is the force increased?

76 **CALC** *Work and power as functions of time.* A body of mass m accelerates uniformly from rest to speed v_f in time t_f. In terms of these symbols, at time t, what is (a) the work done on the body and (b) the power delivered to the body?

77 *Work, train observer, ground observer.* An object with mass m is initially stationary inside a train that moves at constant speed u along an x axis. A constant force then gives the object an acceleration a in the forward direction for time t. In terms of these given symbols, how much work is done by the force as measured by (a) an observer stationary inside the train and (b) an observer stationary alongside the track?

78 **CALC** *Work and power, graphical integration.* A single force acts on a 3.0 kg body that moves along an x axis. Figure 7.36 gives the velocity v versus time t due to the motion. What is the work done on the body (sign included) for the time intervals (a) 0 to 2.0 ms, (b) 2.0 to 5.0 ms, (c) 5.0 to 8.0 ms, and (d) 8.0 to 11 ms? What is the average power supplied to the body (sign included) for the time intervals (e) 0 to 2.0 ms, (f) 2.0 to 5.0 ms, (g) 5.0 to 8.0 ms, and (h) 8.0 to 11 ms?

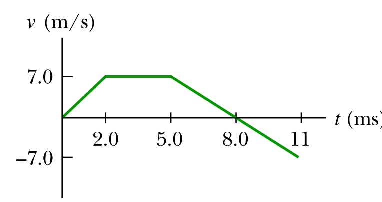

Figure 7.36 Problem 78.