CHAPTER 2

# Motion Along a Straight Line

## 2.1 POSITION, DISPLACEMENT, AND AVERAGE VELOCITY

### Learning Objectives

*After reading this module, you should be able to . . .*

2.1.1 Identify that if all parts of an object move in the same direction and at the same rate, we can treat the object as if it were a (point-like) particle. (This chapter is about the motion of such objects.)

2.1.2 Identify that the position of a particle is its location as read on a scaled axis, such as an $x$ axis.

2.1.3 Apply the relationship between a particle's displacement and its initial and final positions.

2.1.4 Apply the relationship between a particle's average velocity, its displacement, and the time interval for that displacement.

2.1.5 Apply the relationship between a particle's average speed, the total distance it moves, and the time interval for the motion.

2.1.6 Given a graph of a particle's position versus time, determine the average velocity between any two particular times.

### Key Ideas

- The position $x$ of a particle on an $x$ axis locates the particle with respect to the origin, or zero point, of the axis.
- The position is either positive or negative, according to which side of the origin the particle is on, or zero if the particle is at the origin. The positive direction on an axis is the direction of increasing positive numbers; the opposite direction is the negative direction on the axis.
- The displacement $\Delta x$ of a particle is the change in its position:

$$\Delta x = x_2 - x_1.$$

- Displacement is a vector quantity. It is positive if the particle has moved in the positive direction of the $x$ axis and negative if the particle has moved in the negative direction.
- When a particle has moved from position $x_1$ to position $x_2$ during a time interval $\Delta t = t_2 - t_1$, its average velocity during that interval is

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}.$$

- The algebraic sign of $v_{avg}$ indicates the direction of motion ($v_{avg}$ is a vector quantity). Average velocity does not depend on the actual distance a particle moves, but instead depends on its original and final positions.
- On a graph of $x$ versus $t$, the average velocity for a time interval $\Delta t$ is the slope of the straight line connecting the points on the curve that represent the two ends of the interval.
- The average speed $s_{avg}$ of a particle during a time interval $\Delta t$ depends on the total distance the particle moves in that time interval:

$$s_{avg} = \frac{\text{total distance}}{\Delta t}.$$

## What Is Physics?

One purpose of physics is to study the motion of objects—how fast they move, for example, and how far they move in a given amount of time. NASCAR engineers are fanatical about this aspect of physics as they determine the performance of their cars before and during a race. Geologists use this physics to measure tectonic-plate motion as they attempt to predict earthquakes. Medical researchers need this physics to map the blood flow through a patient when diagnosing a partially closed artery, and motorists use it to determine how they might slow sufficiently when their radar detector sounds a warning. There are countless other examples. In this chapter, we study the basic physics of motion where the object (race car, tectonic plate, blood cell, or any other object) moves along a single axis. Such motion is called *one-dimensional motion.*

## Motion

The world, and everything in it, moves. Even seemingly stationary things, such as a roadway, move with Earth's rotation, Earth's orbit around the Sun, the Sun's orbit around the center of the Milky Way galaxy, and that galaxy's migration relative to other galaxies. The classification and comparison of motions (called **kinematics**) is often challenging. What exactly do you measure, and how do you compare?

Before we attempt an answer, we shall examine some general properties of motion that is restricted in three ways.

1. The motion is along a straight line only. The line may be vertical, horizontal, or slanted, but it must be straight.
2. Forces (pushes and pulls) cause motion but will not be discussed until Chapter 5. In this chapter we discuss only the motion itself and changes in the motion. Does the moving object speed up, slow down, stop, or reverse direction? If the motion does change, how is time involved in the change?
3. The moving object is either a **particle** (by which we mean a point-like object such as an electron) or an object that moves like a particle (such that every portion moves in the same direction and at the same rate). A stiff pig slipping down a straight playground slide might be considered to be moving like a particle; however, a tumbling tumbleweed would not.

## Position and Displacement

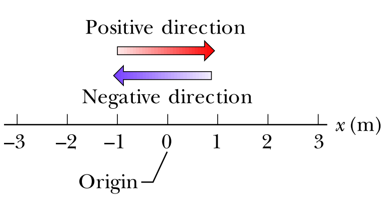



**Figure 2.1.1** Position is determined on an axis that is marked in units of length (here meters) and that extends indefinitely in opposite directions. The axis name, here $x$, is always on the positive side of the origin.

To locate an object means to find its position relative to some reference point, often the **origin** (or zero point) of an axis such as the $x$ axis in Fig. 2.1.1. The **positive direction** of the axis is in the direction of increasing numbers (coordinates), which is to the right in Fig. 2.1.1. The opposite is the **negative direction.**

For example, a particle might be located at $x = 5$ m, which means it is 5 m in the positive direction from the origin. If it were at $x = -5$ m, it would be just as far from the origin but in the opposite direction. On the axis, a coordinate of $-5$ m is less than a coordinate of $-1$ m, and both coordinates are less than a coordinate of $+5$ m. A plus sign for a coordinate need not be shown, but a minus sign must always be shown.

A change from position $x_1$ to position $x_2$ is called a **displacement** $\Delta x$, where

$$\Delta x = x_2 - x_1. \qquad (2.1.1)$$

(The symbol $\Delta$, the Greek uppercase delta, represents a change in a quantity, and it means the final value of that quantity minus the initial value.) When numbers are inserted for the position values $x_1$ and $x_2$ in Eq. 2.1.1, a displacement in the positive direction (to the right in Fig. 2.1.1) always comes out positive, and a displacement in the opposite direction (left in the figure) always comes out negative. For example, if the particle moves from $x_1 = 5$ m to $x_2 = 12$ m, then the displacement is $\Delta x = (12 \text{ m}) - (5 \text{ m}) = +7$ m. The positive result indicates that the motion is in the positive direction. If, instead, the particle moves from $x_1 = 5$ m to $x_2 = 1$ m, then $\Delta x = (1 \text{ m}) - (5 \text{ m}) = -4$ m. The negative result indicates that the motion is in the negative direction.

The actual number of meters covered for a trip is irrelevant; displacement involves only the original and final positions. For example, if the particle moves

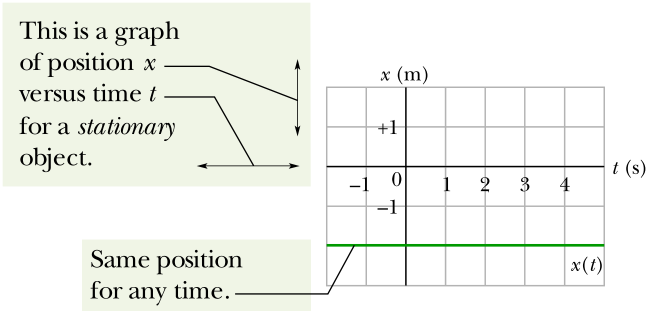



**Figure 2.1.2** The graph of $x(t)$ for an armadillo that is stationary at $x = -2$ m. The value of $x$ is $-2$ m for all times $t$.

from $x = 5$ m out to $x = 200$ m and then back to $x = 5$ m, the displacement from start to finish is $\Delta x = (5\text{ m}) - (5\text{ m}) = 0$.

***Signs.*** A plus sign for a displacement need not be shown, but a minus sign must always be shown. If we ignore the sign (and thus the direction) of a displacement, we are left with the **magnitude** (or absolute value) of the displacement. For example, a displacement of $\Delta x = -4$ m has a magnitude of 4 m.

Displacement is an example of a **vector quantity**, which is a quantity that has both a direction and a magnitude. We explore vectors more fully in Chapter 3, but here all we need is the idea that displacement has two features: (1) Its *magnitude* is the distance (such as the number of meters) between the original and final positions. (2) Its *direction*, from an original position to a final position, can be represented by a plus sign or a minus sign if the motion is along a single axis.

*Here is the first of many checkpoints where you can check your understanding with a bit of reasoning. The answers are in the back of the book.*

### Checkpoint 2.1.1

Here are three pairs of initial and final positions, respectively, along an $x$ axis. Which pairs give a negative displacement: (a) $-3$ m, $+5$ m; (b) $-3$ m, $-7$ m; (c) 7 m, $-3$ m?

## Average Velocity and Average Speed

A compact way to describe position is with a graph of position $x$ plotted as a function of time $t$—a graph of $x(t)$. (The notation $x(t)$ represents a function $x$ of $t$, not the product $x$ times $t$.) As a simple example, Fig. 2.1.2 shows the position function $x(t)$ for a stationary armadillo (which we treat as a particle) over a 7 s time interval. The animal's position stays at $x = -2$ m.

Figure 2.1.3 is more interesting, because it involves motion. The armadillo is apparently first noticed at $t = 0$ when it is at the position $x = -5$ m. It moves toward $x = 0$, passes through that point at $t = 3$ s, and then moves on to increasingly larger positive values of $x$. Figure 2.1.3 also depicts the straight-line motion of the armadillo (at three times) and is something like what you would see. The graph in Fig. 2.1.3 is more abstract, but it reveals how fast the armadillo moves.

Actually, several quantities are associated with the phrase "how fast." One of them is the **average velocity** $v_{avg}$, which is the ratio of the displacement $\Delta x$ that occurs during a particular time interval $\Delta t$ to that interval:

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}. \quad (2.1.2)$$

The notation means that the position is $x_1$ at time $t_1$ and then $x_2$ at time $t_2$. A common unit for $v_{avg}$ is the meter per second (m/s). You may see other units in the problems, but they are always in the form of length/time.

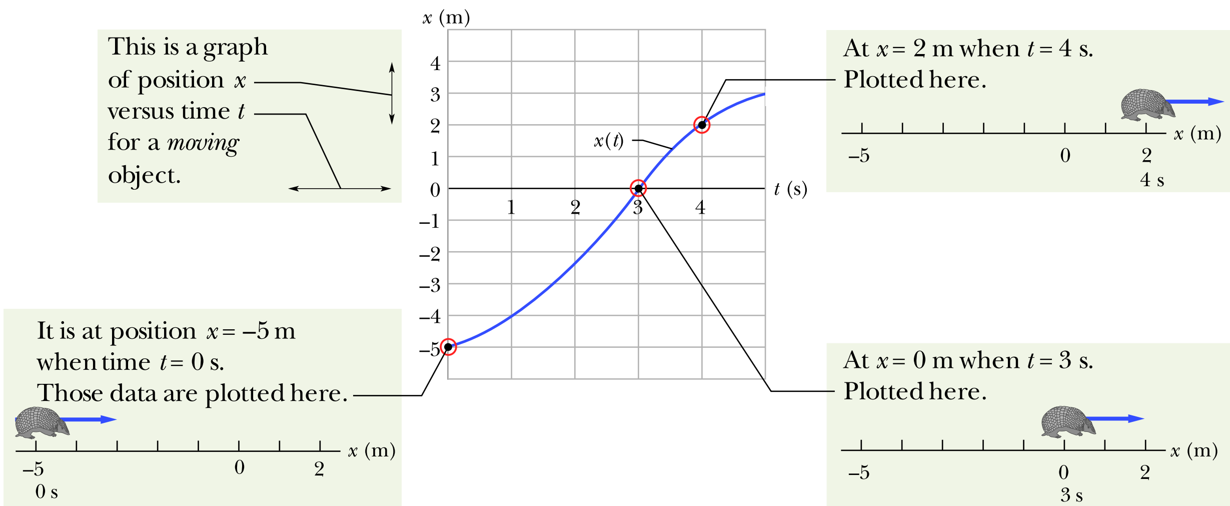


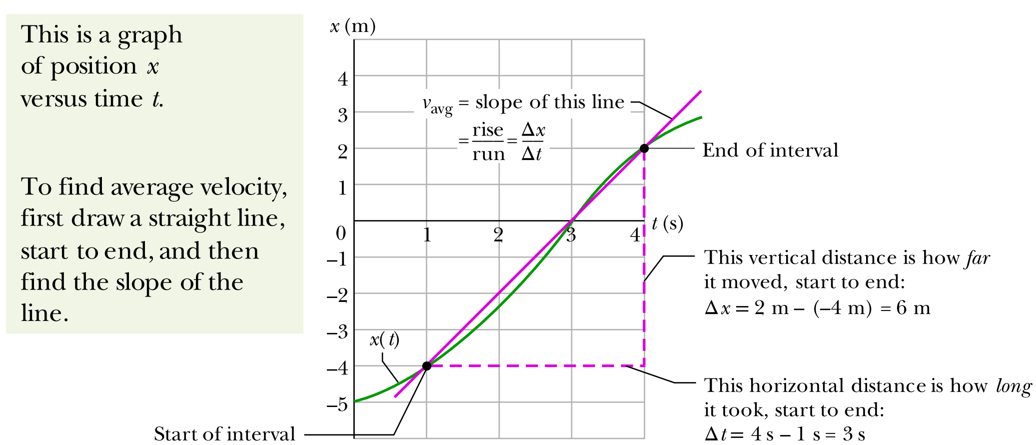

**Figure 2.1.3** The graph of $x(t)$ for a moving armadillo. The path associated with the graph is also shown, at three times.

***Graphs.*** On a graph of $x$ versus $t$, $v_{avg}$ is the **slope** of the straight line that connects two particular points on the $x(t)$ curve: one is the point that corresponds to $x_2$ and $t_2$, and the other is the point that corresponds to $x_1$ and $t_1$. Like displacement, $v_{avg}$ has both magnitude and direction (it is another vector quantity). Its magnitude is the magnitude of the line's slope. A positive $v_{avg}$ (and slope) tells us that the line slants upward to the right; a negative $v_{avg}$ (and slope) tells us that the line slants downward to the right. The average velocity $v_{avg}$ always has the same sign as the displacement $\Delta x$ because $\Delta t$ in Eq. 2.1.2 is always positive.

Figure 2.1.4 shows how to find $v_{avg}$ in Fig. 2.1.3 for the time interval $t = 1$ s to $t = 4$ s. We draw the straight line that connects the point on the position curve at the beginning of the interval and the point on the curve at the end of the interval. Then we find the slope $\Delta x/\Delta t$ of the straight line. For the given time interval, the average velocity is

$$v_{avg} = \frac{6\text{ m}}{3\text{ s}} = 2\text{ m/s}.$$

**Figure 2.1.4** Calculation of the average velocity between $t = 1$ s and $t = 4$ s as the slope of the line that connects the points on the $x(t)$ curve representing those times. The swirling icon indicates that a figure is available in *WileyPLUS* as an animation with voiceover.



**Average speed** $s_{avg}$ is a different way of describing "how fast" a particle moves. Whereas the average velocity involves the particle's displacement $\Delta x$, the average speed involves the total distance covered (for example, the number of meters moved), independent of direction; that is,

$$s_{avg} = \frac{\text{total distance}}{\Delta t}. \tag{2.1.3}$$

Because average speed does *not* include direction, it lacks any algebraic sign. Sometimes $s_{avg}$ is the same (except for the absence of a sign) as $v_{avg}$. However, the two can be quite different.

## Sample Problem 2.1.1 Average velocity

You get a lift from a car service to take you to a state park along a straight road due east (directly toward the east) for 10.0 km at an average velocity of 40.0 km/h. From the drop-off point, you jog along a straight path due east for 3.00 km, which takes 0.500 h.

(a) What is your overall displacement from your starting point to the point where your jog ends?

### KEY IDEA

For convenience, assume that you move in the positive direction of an $x$ axis, from a first position of $x_1 = 0$ to a second position of $x_2$ at the end of the jog. That second position must be at $x_2 = 10.0\text{ km} + 3.00\text{ km} = 13.0\text{ km}$. Then your displacement $\Delta x$ along the $x$ axis is the second position minus the first position.

***Calculation:*** From Eq. 2.1.1, we have

$$\Delta x = x_2 - x_1 = 13.0 - 0 = 13.0\text{ km}. \qquad \text{(Answer)}$$

Thus, your overall displacement is 13.0 km in the positive direction of the $x$ axis.

(b) What is the time interval $\Delta t$ from the beginning of your movement to the end of the jog?

### KEY IDEA

We already know the jogging time interval $\Delta t_{jog}$ (= 0.500 h), but we lack the time interval $\Delta t_{car}$ for the ride. However, we know that the displacement $\Delta x_{car}$ is 10.0 km and the average velocity $v_{avg,car}$ is 40.0 km/h. That average velocity is the ratio of that displacement to the time interval for the ride, so we can find that time interval.

***Calculations:*** We first write

$$v_{avg,car} = \frac{\Delta x_{car}}{\Delta t_{car}}.$$

Rearranging and substituting data then give us

$$\Delta t_{car} = \frac{\Delta x_{car}}{v_{avg,car}} = \frac{10.0\text{ km}}{40.0\text{ km/h}} = 0.250\text{ h}.$$

So,

$$\Delta t = \Delta t_{car} + \Delta t_{jog}$$
$$= 0.250\text{ h} + 0.500\text{ h} = 0.750\text{ h}. \qquad \text{(Answer)}$$

(c) What is your average velocity $v_{avg}$ from the starting point to the end of the jog? Find it both numerically and graphically.

### KEY IDEA

From Eq. 2.1.2 we know that $v_{avg}$ *for the entire trip* is the ratio of the displacement of 13.0 km *for the entire trip* to the time interval of 0.750 h *for the entire trip*.

***Calculation:*** Here we find

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{13.0\text{ km}}{0.750\text{ h}} = 17.3\text{ km/h}. \qquad \text{(Answer)}$$

To find $v_{avg}$ graphically, first we graph the function $x(t)$ as shown in Fig. 2.1.5, where the beginning and final points on the graph are the origin and the point labeled "Stop."

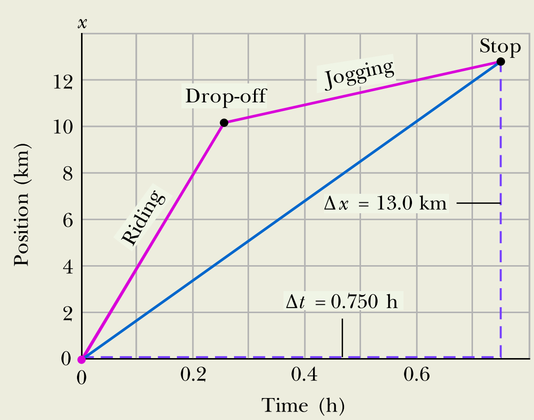



**Figure 2.1.5** The lines marked "Riding" and "Jogging" are the position–time plots for the riding and jogging stages. The slope of the straight line joining the origin and the point labeled "Stop" is the average velocity for the motion from start to stop.

Your average velocity is the slope of the straight line connecting those points; that is, $v_{avg}$ is the ratio of the *rise* ($\Delta x = 13.0$ km) to the *run* ($\Delta t = 0.750$ h), which gives us $v_{avg} = 17.3$ km/h.

(d) Suppose you then jog back to the drop-off point for another 0.500 h. What is your average *speed* from the beginning of your trip to that return?

**KEY IDEA**

Your average speed is the ratio of the total distance you covered to the total time interval you took.

***Calculation:*** The total distance is 10.0 km + 3.00 km + 3.00 km = 16.0 km. The total time interval is 0.250 h + 0.500 h + 0.500 h = 1.25 h. Thus, Eq. 2.1.3 gives us

$$s_{avg} = \frac{16.0 \text{ km}}{1.25 \text{ h}} = 12.8 \text{ km/h}. \qquad \text{(Answer)}$$

**WileyPLUS** Additional examples, video, and practice available at *WileyPLUS*

# 2.2 INSTANTANEOUS VELOCITY AND SPEED

## Learning Objectives

*After reading this module, you should be able to . . .*

**2.2.1** Given a particle's position as a function of time, calculate the instantaneous velocity for any particular time.

**2.2.2** Given a graph of a particle's position versus time, determine the instantaneous velocity for any particular time.

**2.2.3** Identify speed as the magnitude of the instantaneous velocity.

## Key Ideas

- The instantaneous velocity (or simply velocity) $v$ of a moving particle is

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt},$$

where $\Delta x = x_2 - x_1$ and $\Delta t = t_2 - t_1$.

- The instantaneous velocity (at a particular time) may be found as the slope (at that particular time) of the graph of $x$ versus $t$.
- Speed is the magnitude of instantaneous velocity.

## Instantaneous Velocity and Speed

You have now seen two ways to describe how fast something moves: average velocity and average speed, both of which are measured over a time interval $\Delta t$. However, the phrase "how fast" more commonly refers to how fast a particle is moving at a given instant—its **instantaneous velocity** (or simply **velocity**) $v$.

The velocity at any instant is obtained from the average velocity by shrinking the time interval $\Delta t$ closer and closer to 0. As $\Delta t$ dwindles, the average velocity approaches a limiting value, which is the velocity at that instant:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}. \qquad (2.2.1)$$

Note that $v$ is the rate at which position $x$ is changing with time at a given instant; that is, $v$ is the derivative of $x$ with respect to $t$. Also note that $v$ at any instant is the slope of the position–time curve at the point representing that instant. Velocity is another vector quantity and thus has an associated direction.

**Speed** is the magnitude of velocity; that is, speed is velocity that has been stripped of any indication of direction, either in words or via an algebraic sign. (*Caution:* Speed and average speed can be quite different.) A velocity of +5 m/s and one of −5 m/s both have an associated speed of 5 m/s. The speedometer in a car measures speed, not velocity (it cannot determine the direction).

### Checkpoint 2.2.1

The following equations give the position $x(t)$ of a particle in four situations (in each equation, $x$ is in meters, $t$ is in seconds, and $t > 0$): (1) $x = 3t - 2$; (2) $x = -4t^2 - 2$; (3) $x = 2/t^2$; and (4) $x = -2$. (a) In which situation is the velocity $v$ of the particle constant? (b) In which is $v$ in the negative $x$ direction?

## Sample Problem 2.2.1 Velocity and slope of $x$ versus $t$, elevator cab

Figure 2.2.1*a* is an $x(t)$ plot for an elevator cab that is initially stationary, then moves upward (which we take to be the positive direction of $x$), and then stops. Plot $v(t)$.

### KEY IDEA

We can find the velocity at any time from the slope of the $x(t)$ curve at that time.

***Calculations:*** The slope of $x(t)$, and so also the velocity, is zero in the intervals from 0 to 1 s and from 9 s on, so then the cab is stationary. During the interval $bc$, the slope is constant and nonzero, so then the cab moves with constant velocity. We calculate the slope of $x(t)$ then as

$$\frac{\Delta x}{\Delta t} = v = \frac{24\text{ m} - 4.0\text{ m}}{8.0\text{ s} - 3.0\text{ s}} = +4.0\text{ m/s}. \qquad (2.2.2)$$

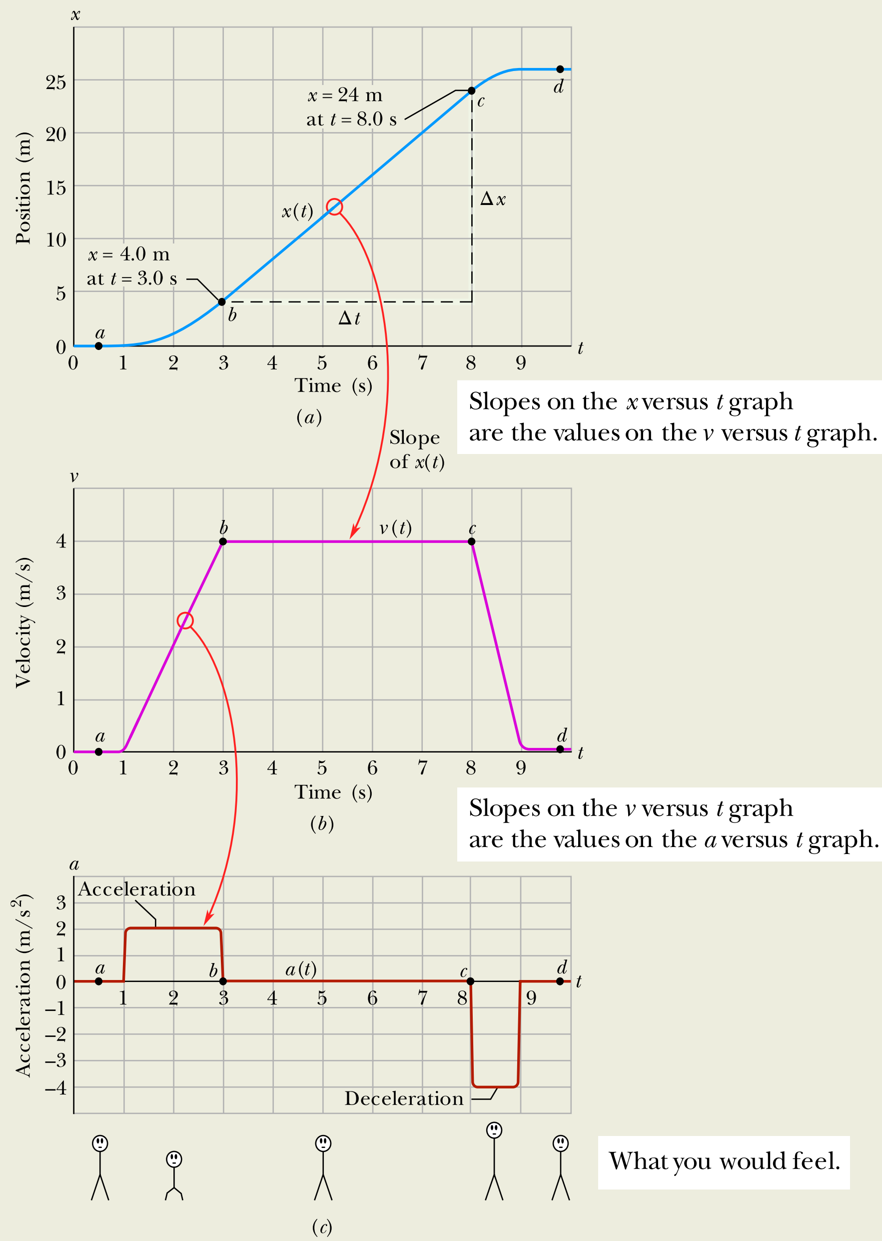



**Figure 2.2.1** (*a*) The $x(t)$ curve for an elevator cab that moves upward along an $x$ axis. (*b*) The $v(t)$ curve for the cab. Note that it is the derivative of the $x(t)$ curve ($v = dx/dt$). (*c*) The $a(t)$ curve for the cab. It is the derivative of the $v(t)$ curve ($a = dv/dt$). The stick figures along the bottom suggest how a passenger's body might feel during the accelerations.

The plus sign indicates that the cab is moving in the positive $x$ direction. These intervals (where $v = 0$ and $v = 4$ m/s) are plotted in Fig. 2.2.1*b*. In addition, as the cab initially begins to move and then later slows to a stop, $v$ varies as indicated in the intervals 1 s to 3 s and 8 s to 9 s. Thus, Fig. 2.2.1*b* is the required plot. (Figure 2.2.1*c* is considered in Module 2.3.)

Given a $v(t)$ graph such as Fig. 2.2.1*b*, we could "work backward" to produce the shape of the associated $x(t)$ graph (Fig. 2.2.1*a*). However, we would not know the actual values for $x$ at various times, because the $v(t)$ graph indicates only *changes* in $x$. To find such a change in $x$ during any interval, we must, in the language of calculus, calculate the area "under the curve" on the $v(t)$ graph for that interval. For example, during the interval 3 s to 8 s in which the cab has a velocity of 4.0 m/s, the change in $x$ is

$$\Delta x = (4.0\,\text{m/s})(8.0\,\text{s} - 3.0\,\text{s}) = +20\,\text{m}. \qquad (2.2.3)$$

(This area is positive because the $v(t)$ curve is above the $t$ axis.) Figure 2.2.1*a* shows that $x$ does indeed increase by 20 m in that interval. However, Fig. 2.2.1*b* does not tell us the *values* of $x$ at the beginning and end of the interval. For that, we need additional information, such as the value of $x$ at some instant.

**WileyPLUS** Additional examples, video, and practice available at *WileyPLUS*

# 2.3 ACCELERATION

## Learning Objectives

*After reading this module, you should be able to . . .*

**2.3.1** Apply the relationship between a particle's average acceleration, its change in velocity, and the time interval for that change.

**2.3.2** Given a particle's velocity as a function of time, calculate the instantaneous acceleration for any particular time.

**2.3.3** Given a graph of a particle's velocity versus time, determine the instantaneous acceleration for any particular time and the average acceleration between any two particular times.

## Key Ideas

- Average acceleration is the ratio of a change in velocity $\Delta v$ to the time interval $\Delta t$ in which the change occurs:

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t}.$$

The algebraic sign indicates the direction of $a_{\text{avg}}$.

- Instantaneous acceleration (or simply acceleration) $a$ is the first time derivative of velocity $v(t)$ and the second time derivative of position $x(t)$:

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}.$$

- On a graph of $v$ versus $t$, the acceleration $a$ at any time $t$ is the slope of the curve at the point that represents $t$.

## Acceleration

When a particle's velocity changes, the particle is said to undergo **acceleration** (or to accelerate). For motion along an axis, the **average acceleration** $a_{\text{avg}}$ over a time interval $\Delta t$ is

$$a_{\text{avg}} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}, \qquad (2.3.1)$$

where the particle has velocity $v_1$ at time $t_1$ and then velocity $v_2$ at time $t_2$. The **instantaneous acceleration** (or simply **acceleration**) is

$$a = \frac{dv}{dt}. \qquad (2.3.2)$$

In words, the acceleration of a particle at any instant is the rate at which its velocity is changing at that instant. Graphically, the acceleration at any point is the slope of the curve of $v(t)$ at that point. We can combine Eq. 2.3.2 with Eq. 2.2.1 to write

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}. \tag{2.3.3}$$

In words, the acceleration of a particle at any instant is the second derivative of its position $x(t)$ with respect to time.

A common unit of acceleration is the meter per second per second: $m/(s \cdot s)$ or $m/s^2$. Other units are in the form of length/(time · time) or length/time$^2$. Acceleration has both magnitude and direction (it is yet another vector quantity). Its algebraic sign represents its direction on an axis just as for displacement and velocity; that is, acceleration with a positive value is in the positive direction of an axis, and acceleration with a negative value is in the negative direction.

Figure 2.2.1 gives plots of the position, velocity, and acceleration of an elevator moving up a shaft. Compare the $a(t)$ curve with the $v(t)$ curve—each point on the $a(t)$ curve shows the derivative (slope) of the $v(t)$ curve at the corresponding time. When $v$ is constant (at either 0 or 4 m/s), the derivative is zero and so also is the acceleration. When the cab first begins to move, the $v(t)$ curve has a positive derivative (the slope is positive), which means that $a(t)$ is positive. When the cab slows to a stop, the derivative and slope of the $v(t)$ curve are negative; that is, $a(t)$ is negative.

Next compare the slopes of the $v(t)$ curve during the two acceleration periods. The slope associated with the cab's slowing down (commonly called "deceleration") is steeper because the cab stops in half the time it took to get up to speed. The steeper slope means that the magnitude of the deceleration is larger than that of the acceleration, as indicated in Fig. 2.2.1*c*.

***Sensations.*** The sensations you would feel while riding in the cab of Fig. 2.2.1 are indicated by the sketched figures at the bottom. When the cab first accelerates, you feel as though you are pressed downward; when later the cab is braked to a stop, you seem to be stretched upward. In between, you feel nothing special. In other words, your body reacts to accelerations (it is an accelerometer) but not to velocities (it is not a speedometer). When you are in a car traveling at 90 km/h or an airplane traveling at 900 km/h, you have no bodily awareness of the motion. However, if the car or plane quickly changes velocity, you may become keenly aware of the change, perhaps even frightened by it. Part of the thrill of an amusement park ride is due to the quick changes of velocity that you undergo (you pay for the accelerations, not for the speed). A more extreme example is shown in the photographs of Fig. 2.3.1, which were taken while a rocket sled was rapidly accelerated along a track and then rapidly braked to a stop. FCP

***g Units.*** Large accelerations are sometimes expressed in terms of $g$ units, with

$$1g = 9.8 \text{ m/s}^2 \quad (g \text{ unit}). \tag{2.3.4}$$

(As we shall discuss in Module 2.5, $g$ is the magnitude of the acceleration of a falling object near Earth's surface.) On a roller coaster, you may experience brief accelerations up to $3g$, which is $(3)(9.8 \text{ m/s}^2)$, or about $29 \text{ m/s}^2$, more than enough to justify the cost of the ride.

***Signs.*** In common language, the sign of an acceleration has a nonscientific meaning: Positive acceleration means that the speed of an object is increasing, and negative acceleration means that the speed is decreasing (the object is decelerating). In this book, however, the sign of an acceleration indicates a direction, not whether an object's speed is increasing or decreasing. For example, if a car with an initial velocity $v = -25$ m/s is braked to a stop in 5.0 s, then $a_{avg} = +5.0 \text{ m/s}^2$. The acceleration is *positive*, but the car's speed has decreased. The reason is the difference in signs: The direction of the acceleration is opposite that of the velocity.

Courtesy U.S. Air Force

**Figure 2.3.1** Colonel J. P. Stapp in a rocket sled as it is brought up to high speed (acceleration out of the page) and then very rapidly braked (acceleration into the page).

Here then is the proper way to interpret the signs:

If the signs of the velocity and acceleration of a particle are the same, the speed of the particle increases. If the signs are opposite, the speed decreases.

### Checkpoint 2.3.1

A wombat moves along an $x$ axis. What is the sign of its acceleration if it is moving (a) in the positive direction with increasing speed, (b) in the positive direction with decreasing speed, (c) in the negative direction with increasing speed, and (d) in the negative direction with decreasing speed?

## Sample Problem 2.3.1 Acceleration and *dv/dt*

A particle's position on the $x$ axis of Fig. 2.1.1 is given by

$$x = 4 - 27t + t^3,$$

with $x$ in meters and $t$ in seconds.

(a) Because position $x$ depends on time $t$, the particle must be moving. Find the particle's velocity function $v(t)$ and acceleration function $a(t)$.

### KEY IDEAS

(1) To get the velocity function $v(t)$, we differentiate the position function $x(t)$ with respect to time. (2) To get the acceleration function $a(t)$, we differentiate the velocity function $v(t)$ with respect to time.

***Calculations:*** Differentiating the position function, we find

$$v = -27 + 3t^2, \qquad \text{(Answer)}$$

with $v$ in meters per second. Differentiating the velocity function then gives us

$$a = +6t, \qquad \text{(Answer)}$$

with $a$ in meters per second squared.

(b) Is there ever a time when $v = 0$?

***Calculation:*** Setting $v(t) = 0$ yields

$$0 = -27 + 3t^2,$$

which has the solution

$$t = \pm 3 \text{ s.} \qquad \text{(Answer)}$$

Thus, the velocity is zero both 3 s before and 3 s after the clock reads 0.

(c) Describe the particle's motion for $t \geq 0$.

***Reasoning:*** We need to examine the expressions for $x(t)$, $v(t)$, and $a(t)$.

At $t = 0$, the particle is at $x(0) = +4$ m and is moving with a velocity of $v(0) = -27$ m/s—that is, in the negative direction of the $x$ axis. Its acceleration is $a(0) = 0$ because just then the particle's velocity is not changing (Fig. 2.3.2*a*).

For $0 < t < 3$ s, the particle still has a negative velocity, so it continues to move in the negative direction. However, its acceleration is no longer 0 but is increasing and positive. Because the signs of the velocity and the acceleration are opposite, the particle must be slowing (Fig. 2.3.2*b*).

Indeed, we already know that it stops momentarily at $t = 3$ s. Just then the particle is as far to the left of the origin in Fig. 2.1.1 as it will ever get. Substituting $t = 3$ s into the expression for $x(t)$, we find that the particle's position just then is $x = -50$ m (Fig. 2.3.2*c*). Its acceleration is still positive.

For $t > 3$ s, the particle moves to the right on the axis. Its acceleration remains positive and grows progressively larger in magnitude. The velocity is now positive, and it too grows progressively larger in magnitude (Fig. 2.3.2*d*).

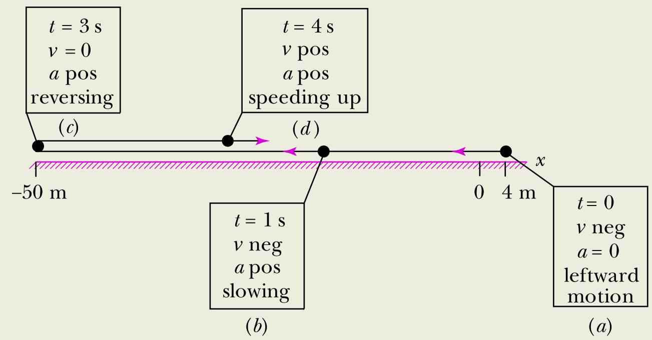



**Figure 2.3.2** Four stages of the particle's motion.

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# 2.4 CONSTANT ACCELERATION

## Learning Objectives

*After reading this module, you should be able to . . .*

**2.4.1** For constant acceleration, apply the relationships between position, displacement, velocity, acceleration, and elapsed time (Table 2.4.1).

**2.4.2** Calculate a particle's change in velocity by integrating its acceleration function with respect to time.

**2.4.3** Calculate a particle's change in position by integrating its velocity function with respect to time.

## Key Idea

- The following five equations describe the motion of a particle with constant acceleration:

$$v = v_0 + at, \qquad x - x_0 = v_0 t + \frac{1}{2}at^2,$$

$$v^2 = v_0^2 + 2a(x - x_0), \qquad x - x_0 = \frac{1}{2}(v_0 + v)t, \qquad x - x_0 = vt - \frac{1}{2}at^2.$$

These are *not* valid when the acceleration is not constant.

## Constant Acceleration: A Special Case

In many types of motion, the acceleration is either constant or approximately so. For example, you might accelerate a car at an approximately constant rate when a traffic light turns from red to green. Then graphs of your position, velocity,

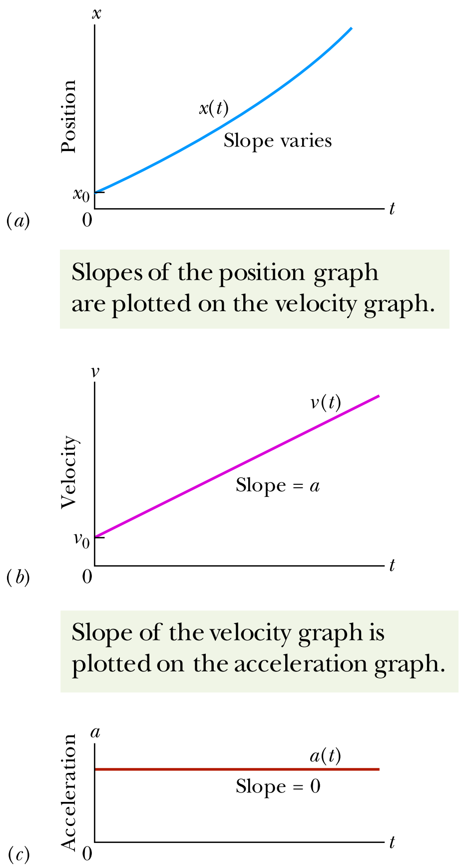


**Figure 2.4.1** (*a*) The position $x(t)$ of a particle moving with constant acceleration. (*b*) Its velocity $v(t)$, given at each point by the slope of the curve of $x(t)$. (*c*) Its (constant) acceleration, equal to the (constant) slope of the curve of $v(t)$.

and acceleration would resemble those in Fig. 2.4.1. (Note that $a(t)$ in Fig. 2.4.1*c* is constant, which requires that $v(t)$ in Fig. 2.4.1*b* have a constant slope.) Later when you brake the car to a stop, the acceleration (or deceleration in common language) might also be approximately constant.

Such cases are so common that a special set of equations has been derived for dealing with them. One approach to the derivation of these equations is given in this section. A second approach is given in the next section. Throughout both sections and later when you work on the homework problems, keep in mind that *these equations are valid only for constant acceleration (or situations in which you can approximate the acceleration as being constant)*.

***First Basic Equation.*** When the acceleration is constant, the average acceleration and instantaneous acceleration are equal and we can write Eq. 2.3.1, with some changes in notation, as

$$a = a_{avg} = \frac{v - v_0}{t - 0}.$$

Here $v_0$ is the velocity at time $t = 0$ and $v$ is the velocity at any later time $t$. We can recast this equation as

$$v = v_0 + at. \tag{2.4.1}$$

As a check, note that this equation reduces to $v = v_0$ for $t = 0$, as it must. As a further check, take the derivative of Eq. 2.4.1. Doing so yields $dv/dt = a$, which is the definition of $a$. Figure 2.4.1*b* shows a plot of Eq. 2.4.1, the $v(t)$ function; the function is linear and thus the plot is a straight line.

***Second Basic Equation.*** In a similar manner, we can rewrite Eq. 2.1.2 (with a few changes in notation) as

$$v_{avg} = \frac{x - x_0}{t - 0}$$

and then as

$$x = x_0 + v_{avg}t, \tag{2.4.2}$$

in which $x_0$ is the position of the particle at $t = 0$ and $v_{avg}$ is the average velocity between $t = 0$ and a later time $t$.

For the linear velocity function in Eq. 2.4.1, the *average* velocity over any time interval (say, from $t = 0$ to a later time $t$) is the average of the velocity at the beginning of the interval ($= v_0$) and the velocity at the end of the interval ($= v$). For the interval from $t = 0$ to the later time $t$ then, the average velocity is

$$v_{avg} = \tfrac{1}{2}(v_0 + v). \tag{2.4.3}$$

Substituting the right side of Eq. 2.4.1 for $v$ yields, after a little rearrangement,

$$v_{avg} = v_0 + \tfrac{1}{2}at. \tag{2.4.4}$$

Finally, substituting Eq. 2.4.4 into Eq. 2.4.2 yields

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2. \tag{2.4.5}$$

As a check, note that putting $t = 0$ yields $x = x_0$, as it must. As a further check, taking the derivative of Eq. 2.4.5 yields Eq. 2.4.1, again as it must. Figure 2.4.1*a* shows a plot of Eq. 2.4.5; the function is quadratic and thus the plot is curved.

***Three Other Equations.*** Equations 2.4.1 and 2.4.5 are the *basic equations for constant acceleration;* they can be used to solve any constant acceleration problem in this book. However, we can derive other equations that might prove useful in certain specific situations. First, note that as many as five quantities can possibly be involved in any problem about constant acceleration—namely, $x - x_0$, $v$, $t$, $a$, and $v_0$. Usually, one of these quantities is *not* involved in the problem, *either as a given or as an unknown.* We are then presented with three of the remaining quantities and asked to find the fourth.

Equations 2.4.1 and 2.4.5 each contain four of these quantities, but not the same four. In Eq. 2.4.1, the "missing ingredient" is the displacement $x - x_0$. In Eq. 2.4.5, it is the velocity $v$. These two equations can also be combined in three ways to yield three additional equations, each of which involves a different "missing variable." First, we can eliminate $t$ to obtain

$$v^2 = v_0^2 + 2a(x - x_0). \tag{2.4.6}$$

This equation is useful if we do not know $t$ and are not required to find it. Second, we can eliminate the acceleration $a$ between Eqs. 2.4.1 and 2.4.5 to produce an equation in which $a$ does not appear:

$$x - x_0 = \tfrac{1}{2}(v_0 + v)t. \tag{2.4.7}$$

Finally, we can eliminate $v_0$, obtaining

$$x - x_0 = vt - \tfrac{1}{2}at^2. \tag{2.4.8}$$

Note the subtle difference between this equation and Eq. 2.4.5. One involves the initial velocity $v_0$; the other involves the velocity $v$ at time $t$.

Table 2.4.1 lists the basic constant-acceleration equations (Eqs. 2.4.1 and 2.4.5) as well as the specialized equations that we have derived. To solve a simple constant-acceleration problem, you can usually use an equation from this list (*if* you have the list with you). Choose an equation for which the only unknown variable is the variable requested in the problem. A simpler plan is to remember only Eqs. 2.4.1 and 2.4.5, and then solve them as simultaneous equations whenever needed.

**Table 2.4.1 Equations for Motion with Constant Acceleration**[a]

| Equation Number | Equation | Missing Quantity |
|---|---|---|
| 2.4.1 | $v = v_0 + at$ | $x - x_0$ |
| 2.4.5 | $x - x_0 = v_0 t + \frac{1}{2}at^2$ | $v$ |
| 2.4.6 | $v^2 = v_0^2 + 2a(x - x_0)$ | $t$ |
| 2.4.7 | $x - x_0 = \frac{1}{2}(v_0 + v)t$ | $a$ |
| 2.4.8 | $x - x_0 = vt - \frac{1}{2}at^2$ | $v_0$ |

[a]Make sure that the acceleration is indeed constant before using the equations in this table.

### Checkpoint 2.4.1

The following equations give the position $x(t)$ of a particle in four situations: (1) $x = 3t - 4$; (2) $x = -5t^3 + 4t^2 + 6$; (3) $x = 2/t^2 - 4/t$; (4) $x = 5t^2 - 3$. To which of these situations do the equations of Table 2.4.1 apply?

## Sample Problem 2.4.1 Autonomous car passing slower car

In Fig. 2.4.2*a*, you are riding in a car controlled by an autonomous driving system and trail a slower car that you want to pass. Figure 2.4.2*b* shows the initial situation, with you in car $B$. Your system's radar detects the speed and location of slow car $A$. Both cars have length $L = 4.50$ m, speed $v_0 = 22.0$ m/s (49 mi/h, slower than the speed limit), and travel on a straight road with one lane in each direction. Your car initially trails $A$ by distance $3.00L$ when you ask it to pass the slow car. That would require you to move into the other lane where there can be an oncoming vehicle. Your system must determine the time required for passing $A$, to see if passing would be safe. So, the first step in the system's control is to calculate that passing time.

We want $B$ to pull into the other lane, accelerate at a constant $a = 3.50$ m/s$^2$ until it reaches a speed of $v = 27.0$ m/s (60 mi/h, the speed limit) and then, when it is at distance $3.00L$ ahead of $A$, pull back into the initial lane (it will then maintain 27.0 m/s). Assume that the lane changing takes negligible time. Figure 2.4.2*c* shows the situation at the onset of the acceleration, with the rear of car $B$ at $x_{B1} = 0$ and the rear of car $A$ at $x_{A1} = 4L$. Figure 2.4.2*d* shows the situation when car $B$ is about to pull back into the initial lane. Let $t_1$ and $d_1$ be the time required for the acceleration and the distance traveled during the acceleration. Let $t_2$ be the time from the end of the acceleration to when $B$ is ahead of $A$ by $3L$ and ready to pull back. We want the total time $t_{tot} = t_1 + t_2$. Here are the pieces in the calculation. What are the values of (a) $t_1$ and (b) $d_1$? (c) In terms of $L$, $v_0$, $t_1$, and $t_2$, what is the coordinate $x_{B2}$ of the rear of car $B$ when $B$ is ready to pull back? (d) In terms of $L$, $v_0$, $t_1$, and $t_2$, what is the coordinate $x_{A2}$ of the rear of car $A$ just then? (e) What is $x_{B2}$ in terms of $x_{A2}$ and $L$? Putting the pieces together, find the values of (f) $t_2$ and (g) $t_{tot}$.

## KEY IDEA

We can apply the equations of constant acceleration to both stages of passing: when car $B$ has acceleration $a = 3.50\,\text{m/s}^2$ and when it travels at constant speed (thus, with constant $a = 0$).

***Calculations:*** (a) In the passing lane, $B$ accelerates at the constant rate $a = 3.50\,\text{m/s}^2$ from initial speed $v_0 = 22.0\,\text{m/s}$ to final speed $v = 27.0$ m/s. From Eq. 2.4.1, we find the time $t_1$ required for the acceleration:

$$t_1 = \frac{v - v_0}{a} = \frac{(27.0\,\text{m/s}) - (22.0\,\text{m/s})}{3.50\,\text{m/s}^2}$$

$$= 1.4285\,\text{s} \approx 1.43\,\text{s}. \qquad \text{(Answer)}$$

(b) In Eq. 2.4.6, let $x - x_0$ be the distance $d_1$ traveled by $B$ during the acceleration. We can then write

$$v^2 = v_0^2 + 2ad_1$$

$$d_1 = \frac{v^2 - v_0^2}{2a} = \frac{(27.0\text{ m/s})^2 - (22.0\text{ m/s})^2}{2(3.50\text{ m/s}^2)}$$

$$= 35.0\,\text{m} \qquad \text{(Answer)}$$

(c) After the acceleration through displacement $d_1$ from its initial position of $x_{B1} = 0$, the rear of car $B$ moves at constant speed $v$ for the unknown time $t_2$. Its position is then

$$x_{B2} = d_1 + vt_2. \qquad \text{(Answer)}$$

(d) From its initial position of $x_{A1} = 4L$, the rear of car $A$ moves at constant speed $v_0$ for the total time $t_1 + t_2$. Thus, its position is then

$$x_{A2} = 4L + v_0(t_1 + t_2). \qquad \text{(Answer)}$$

(e) The rear of car $B$ is then $3L$ from the front of $A$ and thus $4L$ from the rear of $A$. So,

$$x_{B2} = x_{A2} + 4L. \qquad \text{(Answer)}$$

(f) Putting the pieces together, we find

$$x_{B2} = x_{A2} + 4L$$

$$d_1 + vt_2 = 4L + v_0(t_1 + t_2) + 4L$$

$$t_2(v - v_0) = 8L + v_0 t_1 - d_1$$

$$t_2 = \frac{8L + v_0 t_1 - d_1}{v - v_0}$$

$$= \frac{8(4.50) + (22.0\,\text{m/s})(1.4285\text{s}) - 35.0\,\text{m}}{(27.0\,\text{m/s}) - (22.0\,\text{m/s})}$$

$$= 6.4854\,\text{s} \approx 6.49\,\text{s}. \qquad \text{(Answer)}$$

(g) The total time is

$$t_{tot} = t_1 + t_2 = 1.4285\,\text{s} + 6.4854\,\text{s}$$

$$= 7.91\,\text{s}. \qquad \text{(Answer)}$$

As explored in one of the end-of-chapter problems, the next step for your car's control system is to detect the speed and distance of any oncoming car, to see if this much time is safe.

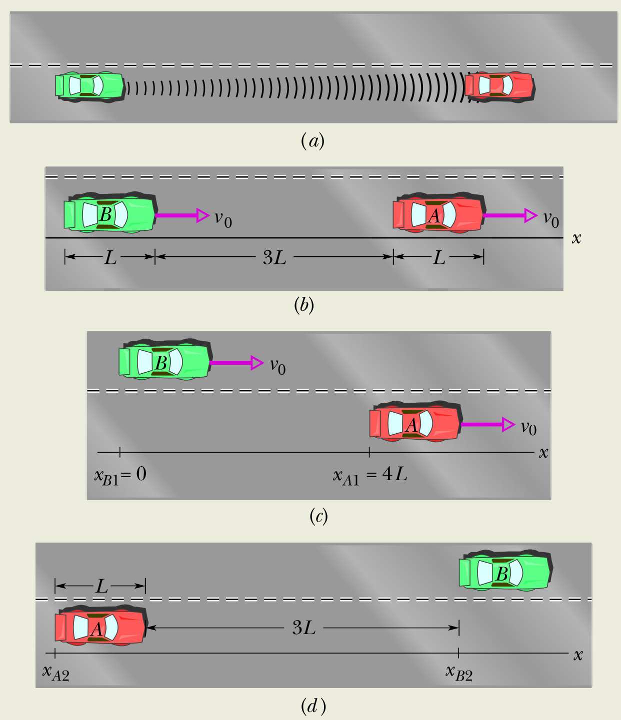


**Figure 2.4.2** (*a*) Trailing car's radar system detects distance and speed of lead car. (*b*) Initial situation. (*c*) Trailing car $B$ pulls into passing lane. (*d*) Car $B$ is about to pull back into initial lane.

## Another Look at Constant Acceleration*

The first two equations in Table 2.4.1 are the basic equations from which the others are derived. Those two can be obtained by integration of the acceleration with the condition that $a$ is constant. To find Eq. 2.4.1, we rewrite the definition of acceleration (Eq. 2.3.2) as

$$dv = a\ dt.$$

We next write the *indefinite integral* (or *antiderivative*) of both sides:

$$\int dv = \int a\ dt.$$

Since acceleration $a$ is a constant, it can be taken outside the integration. We obtain

$$\int dv = a \int dt$$

or

$$v = at + C. \tag{2.4.9}$$

To evaluate the constant of integration $C$, we let $t = 0$, at which time $v = v_0$. Substituting these values into Eq. 2.4.9 (which must hold for all values of $t$, including $t = 0$) yields

$$v_0 = (a)(0) + C = C.$$

Substituting this into Eq. 2.4.9 gives us Eq. 2.4.1.

To derive Eq. 2.4.5, we rewrite the definition of velocity (Eq. 2.2.1) as

$$dx = v\ dt$$

and then take the indefinite integral of both sides to obtain

$$\int dx = \int v\ dt.$$

Next, we substitute for $v$ with Eq. 2.4.1:

$$\int dx = \int (v_0 + at)\ dt.$$

Since $v_0$ is a constant, as is the acceleration $a$, this can be rewritten as

$$\int dx = v_0 \int dt + a \int t\ dt.$$

Integration now yields

$$x = v_0 t + \tfrac{1}{2}at^2 + C', \tag{2.4.10}$$

where $C'$ is another constant of integration. At time $t = 0$, we have $x = x_0$. Substituting these values in Eq. 2.4.10 yields $x_0 = C'$. Replacing $C'$ with $x_0$ in Eq. 2.4.10 gives us Eq. 2.4.5.

*This section is intended for students who have had integral calculus.

# 2.5 FREE-FALL ACCELERATION

## Learning Objectives

*After reading this module, you should be able to . . .*

**2.5.1** Identify that if a particle is in free flight (whether upward or downward) and if we can neglect the effects of air on its motion, the particle has a constant downward acceleration with a magnitude $g$ that we take to be 9.8 m/s$^2$.

**2.5.2** Apply the constant-acceleration equations (Table 2.4.1) to free-fall motion.

## Key Idea

- An important example of straight-line motion with constant acceleration is that of an object rising or falling freely near Earth's surface. The constant-acceleration equations describe this motion, but we make two changes in notation: (1) We refer the motion to the vertical $y$ axis with $+y$ vertically up; (2) we replace $a$ with $-g$, where $g$ is the magnitude of the free-fall acceleration. Near Earth's surface,

$$g = 9.8\ \text{m/s}^2 = 32\ \text{ft/s}^2.$$

## Free-Fall Acceleration

If you tossed an object either up or down and could somehow eliminate the effects of air on its flight, you would find that the object accelerates downward at a certain constant rate. That rate is called the **free-fall acceleration,** and its magnitude is represented by $g$. The acceleration is independent of the object's characteristics, such as mass, density, or shape; it is the same for all objects.

Two examples of free-fall acceleration are shown in Fig. 2.5.1, which is a series of stroboscopic photos of a feather and an apple. As these objects fall, they accelerate downward—both at the same rate $g$. Thus, their speeds increase at the same rate, and they fall together.

The value of $g$ varies slightly with latitude and with elevation. At sea level in Earth's midlatitudes the value is 9.8 m/s$^2$ (or 32 ft/s$^2$), which is what you should use as an exact number for the problems in this book unless otherwise noted.

The equations of motion in Table 2.4.1 for constant acceleration also apply to free fall near Earth's surface; that is, they apply to an object in vertical flight, either up or down, when the effects of the air can be neglected. However, note that for free fall: (1) The directions of motion are now along a vertical $y$ axis instead of the $x$ axis, with the positive direction of $y$ upward. (This is important for later chapters when combined horizontal and vertical motions are examined.) (2) The free-fall acceleration is negative—that is, downward on the $y$ axis, toward Earth's center—and so it has the value $-g$ in the equations.

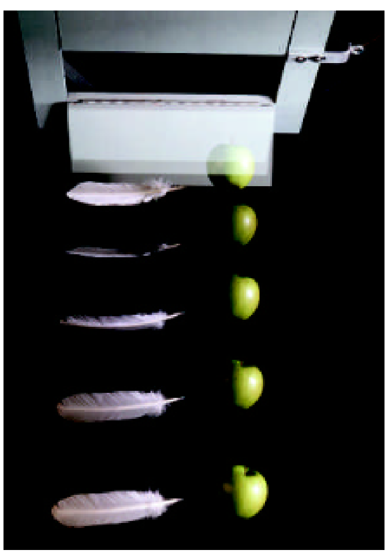



**Figure 2.5.1** A feather and an apple free fall in vacuum at the same magnitude of acceleration $g$. The acceleration increases the distance between successive images. In the absence of air, the feather and apple fall together.

The free-fall acceleration near Earth's surface is $a = -g = -9.8\ \text{m/s}^2$, and the *magnitude* of the acceleration is $g = 9.8\ \text{m/s}^2$. Do not substitute $-9.8\ \text{m/s}^2$ for $g$.

Suppose you toss a tomato directly upward with an initial (positive) velocity $v_0$ and then catch it when it returns to the release level. During its *free-fall flight* (from just after its release to just before it is caught), the equations of Table 2.4.1 apply to its motion. The acceleration is always $a = -g = -9.8\ \text{m/s}^2$, negative and thus downward. The velocity, however, changes, as indicated by Eqs. 2.4.1

and 2.4.6: During the ascent, the magnitude of the positive velocity decreases, until it momentarily becomes zero. Because the tomato has then stopped, it is at its maximum height. During the descent, the magnitude of the (now negative) velocity increases.

### Checkpoint 2.5.1

(a) If you toss a ball straight up, what is the sign of the ball's displacement for the ascent, from the release point to the highest point? (b) What is it for the descent, from the highest point back to the release point? (c) What is the ball's acceleration at its highest point?

## Sample Problem 2.5.1 Time for full up-down flight, baseball toss

In Fig. 2.5.2, a pitcher tosses a baseball up along a $y$ axis, with an initial speed of 12 m/s. FCP

(a) How long does the ball take to reach its maximum height?

### KEY IDEAS

(1) Once the ball leaves the pitcher and before it returns to his hand, its acceleration is the free-fall acceleration $a = -g$. Because this is constant, Table 2.4.1 applies to the motion. (2) The velocity $v$ at the maximum height must be 0.

***Calculation:*** Knowing $v$, $a$, and the initial velocity $v_0 =$ 12 m/s, and seeking $t$, we solve Eq. 2.4.1, which contains those four variables. This yields

$$t = \frac{v - v_0}{a} = \frac{0 - 12\text{ m/s}}{-9.8\text{ m/s}^2} = 1.2\text{ s.} \qquad \text{(Answer)}$$

(b) What is the ball's maximum height above its release point?

***Calculation:*** We can take the ball's release point to be $y_0 = 0$. We can then write Eq. 2.4.6 in $y$ notation, set $y - y_0 = y$ and $v = 0$ (at the maximum height), and solve for $y$. We get

$$y = \frac{v^2 - v_0^2}{2a} = \frac{0 - (12\text{ m/s})^2}{2(-9.8\text{ m/s}^2)} = 7.3\text{ m.} \qquad \text{(Answer)}$$

(c) How long does the ball take to reach a point 5.0 m above its release point?

***Calculations:*** We know $v_0$, $a = -g$, and displacement $y - y_0 = 5.0$ m, and we want $t$, so we choose Eq. 2.4.5. Rewriting it for $y$ and setting $y_0 = 0$ give us

$$y = v_0 t - \tfrac{1}{2}gt^2,$$

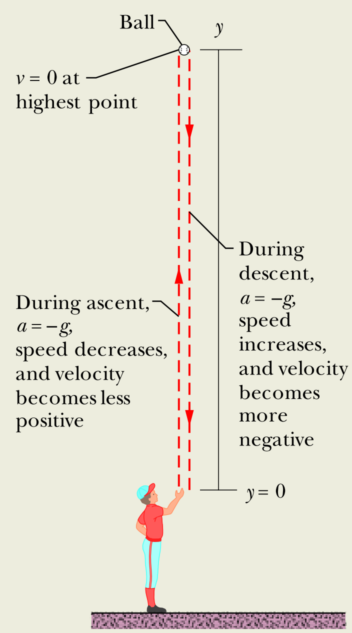


**Figure 2.5.2** A pitcher tosses a baseball straight up into the air. The equations of free fall apply for rising as well as for falling objects, provided any effects from the air can be neglected.

or

$$5.0\text{ m} = (12\text{ m/s})t - \left(\tfrac{1}{2}\right)(9.8\text{ m/s}^2)t^2.$$

If we temporarily omit the units (having noted that they are consistent), we can rewrite this as

$$4.9t^2 - 12t + 5.0 = 0.$$

Solving this quadratic equation for $t$ yields

$$t = 0.53\text{ s} \quad \text{and} \quad t = 1.9\text{ s.} \qquad \text{(Answer)}$$

There are two such times! This is not really surprising because the ball passes twice through $y = 5.0$ m, once on the way up and once on the way down.

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# 2.6 GRAPHICAL INTEGRATION IN MOTION ANALYSIS

## Learning Objectives

*After reading this module, you should be able to . . .*

**2.6.1** Determine a particle's change in velocity by graphical integration on a graph of acceleration versus time.

**2.6.2** Determine a particle's change in position by graphical integration on a graph of velocity versus time.

## Key Ideas

- On a graph of acceleration $a$ versus time $t$, the change in the velocity is given by

$$v_1 - v_0 = \int_{t_0}^{t_1} a\, dt.$$

The integral amounts to finding an area on the graph:

$$\int_{t_0}^{t_1} a\, dt = \begin{pmatrix}\text{area between acceleration curve}\\ \text{and time axis, from } t_0 \text{ to } t_1\end{pmatrix}.$$

- On a graph of velocity $v$ versus time $t$, the change in the position is given by

$$x_1 - x_0 = \int_{t_0}^{t_1} v\, dt,$$

where the integral can be taken from the graph as

$$\int_{t_0}^{t_1} v\, dt = \begin{pmatrix}\text{area between velocity curve}\\ \text{and time axis, from } t_0 \text{ to } t_1\end{pmatrix}.$$

## Graphical Integration in Motion Analysis

***Integrating Acceleration.*** When we have a graph of an object's acceleration $a$ versus time $t$, we can integrate on the graph to find the velocity at any given time. Because $a$ is defined as $a = dv/dt$, the Fundamental Theorem of Calculus tells us that

$$v_1 - v_0 = \int_{t_0}^{t_1} a\, dt. \tag{2.6.1}$$

The right side of the equation is a definite integral (it gives a numerical result rather than a function), $v_0$ is the velocity at time $t_0$, and $v_1$ is the velocity at later time $t_1$. The definite integral can be evaluated from an $a(t)$ graph, such as in Fig. 2.6.1$a$. In particular,

$$\int_{t_0}^{t_1} a\, dt = \begin{pmatrix}\text{area between acceleration curve}\\ \text{and time axis, from } t_0 \text{ to } t_1\end{pmatrix}. \tag{2.6.2}$$

If a unit of acceleration is 1 m/s$^2$ and a unit of time is 1 s, then the corresponding unit of area on the graph is

$$(1\ \text{m/s}^2)(1\ \text{s}) = 1\ \text{m/s},$$

which is (properly) a unit of velocity. When the acceleration curve is above the time axis, the area is positive; when the curve is below the time axis, the area is negative.

***Integrating Velocity.*** Similarly, because velocity $v$ is defined in terms of the position $x$ as $v = dx/dt$, then

$$x_1 - x_0 = \int_{t_0}^{t_1} v\, dt, \tag{2.6.3}$$

where $x_0$ is the position at time $t_0$ and $x_1$ is the position at time $t_1$. The definite integral on the right side of Eq. 2.6.3 can be evaluated from a $v(t)$ graph, like that shown in Fig. 2.6.1$b$. In particular,

$$\int_{t_0}^{t_1} v\, dt = \begin{pmatrix}\text{area between velocity curve}\\ \text{and time axis, from } t_0 \text{ to } t_1\end{pmatrix}. \tag{2.6.4}$$

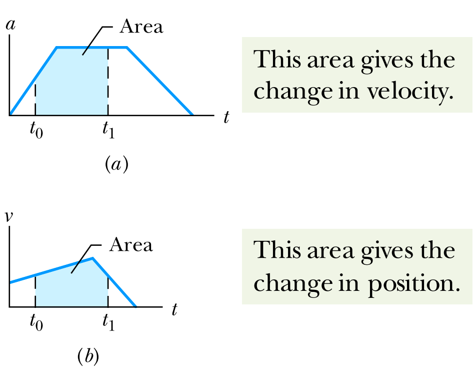


**Figure 2.6.1** The area between a plotted curve and the horizontal time axis, from time $t_0$ to time $t_1$, is indicated for ($a$) a graph of acceleration $a$ versus $t$ and ($b$) a graph of velocity $v$ versus $t$.

If the unit of velocity is 1 m/s and the unit of time is 1 s, then the corresponding unit of area on the graph is

$$(1\ \text{m/s})(1\ \text{s}) = 1\ \text{m},$$

which is (properly) a unit of position and displacement. Whether this area is positive or negative is determined as described for the $a(t)$ curve of Fig. 2.6.1*a*.

### Checkpoint 2.6.1

(a) To get the change in position function $\Delta x$ from a graph of velocity $v$ versus time $t$, do you graphically integrate the graph or find the slope of the graph? (b) Which do you do to get the acceleration?

## Sample Problem 2.6.1 Graphical integration *a* versus *t*, whiplash injury

"Whiplash injury" commonly occurs in a rear-end collision where a front car is hit from behind by a second car. In the 1970s, researchers concluded that the injury was due to the occupant's head being whipped back over the top of the seat as the car was slammed forward. As a result of this finding, head restraints were built into cars, yet neck injuries in rear-end collisions continued to occur.

In a recent test to study neck injury in rear-end collisions, a volunteer was strapped to a seat that was then moved abruptly to simulate a collision by a rear car moving at 10.5 km/h. Figure 2.6.2*a* gives the accelerations of the volunteer's torso and head during the collision, which began at time $t = 0$. The torso acceleration was delayed by 40 ms because during that time interval the seat back had to compress against the volunteer. The head acceleration was delayed by an additional 70 ms. What was the torso speed when the head began to accelerate? FCP

### KEY IDEA

We can calculate the torso speed at any time by finding an area on the torso $a(t)$ graph.

***Calculations:*** We know that the initial torso speed is $v_0 = 0$ at time $t_0 = 0$, at the start of the "collision." We want the torso speed $v_1$ at time $t_1 = 110$ ms, which is when the head begins to accelerate.

Combining Eqs. 2.6.1 and 2.6.2, we can write

$$v_1 - v_0 = \begin{pmatrix}\text{area between acceleration curve}\\ \text{and time axis, from } t_0 \text{ to } t_1\end{pmatrix}. \qquad (2.6.5)$$

For convenience, let us separate the area into three regions (Fig. 2.6.2*b*). From 0 to 40 ms, region $A$ has no area:

$$\text{area}_A = 0.$$

From 40 ms to 100 ms, region $B$ has the shape of a triangle, with area

$$\text{area}_B = \tfrac{1}{2}(0.060\ \text{s})(50\ \text{m/s}^2) = 1.5\ \text{m/s}.$$

From 100 ms to 110 ms, region $C$ has the shape of a rectangle, with area

$$\text{area}_C = (0.010\ \text{s})(50\ \text{m/s}^2) = 0.50\ \text{m/s}.$$

Substituting these values and $v_0 = 0$ into Eq. 2.6.5 gives us

$$v_1 - 0 = 0 + 1.5\ \text{m/s} + 0.50\ \text{m/s},$$

or

$$v_1 = 2.0\ \text{m/s} = 7.2\ \text{km/h}. \qquad \text{(Answer)}$$

***Comments:*** When the head is just starting to move forward, the torso already has a speed of 7.2 km/h. Researchers argue that it is this difference in speeds during the early stage of a rear-end collision that injures the neck. The backward whipping of the head happens later and could, especially if there is no head restraint, increase the injury.

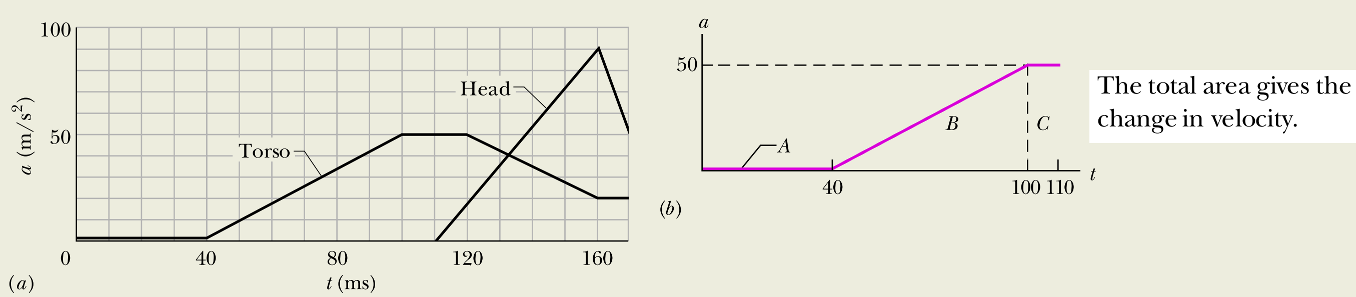


**Figure 2.6.2** (*a*) The $a(t)$ curve of the torso and head of a volunteer in a simulation of a rear-end collision. (*b*) Breaking up the region between the plotted curve and the time axis to calculate the area.

**WileyPLUS** Additional examples, video, and practice available at *WileyPLUS*

# Review & Summary

**Position** The *position* $x$ of a particle on an $x$ axis locates the particle with respect to the **origin,** or zero point, of the axis. The position is either positive or negative, according to which side of the origin the particle is on, or zero if the particle is at the origin. The **positive direction** on an axis is the direction of increasing positive numbers; the opposite direction is the **negative direction** on the axis.

**Displacement** The *displacement* $\Delta x$ of a particle is the change in its position:

$$\Delta x = x_2 - x_1. \quad (2.1.1)$$

Displacement is a vector quantity. It is positive if the particle has moved in the positive direction of the $x$ axis and negative if the particle has moved in the negative direction.

**Average Velocity** When a particle has moved from position $x_1$ to position $x_2$ during a time interval $\Delta t = t_2 - t_1$, its *average velocity* during that interval is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}. \quad (2.1.2)$$

The algebraic sign of $v_{\text{avg}}$ indicates the direction of motion ($v_{\text{avg}}$ is a vector quantity). Average velocity does not depend on the actual distance a particle moves, but instead depends on its original and final positions.

On a graph of $x$ versus $t$, the average velocity for a time interval $\Delta t$ is the slope of the straight line connecting the points on the curve that represent the two ends of the interval.

**Average Speed** The *average speed* $s_{\text{avg}}$ of a particle during a time interval $\Delta t$ depends on the total distance the particle moves in that time interval:

$$s_{\text{avg}} = \frac{\text{total distance}}{\Delta t}. \quad (2.1.3)$$

**Instantaneous Velocity** The *instantaneous velocity* (or simply **velocity**) $v$ of a moving particle is

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}, \quad (2.2.1)$$

where $\Delta x$ and $\Delta t$ are defined by Eq. 2.1.2. The instantaneous velocity (at a particular time) may be found as the slope (at that particular time) of the graph of $x$ versus $t$. **Speed** is the magnitude of instantaneous velocity.

**Average Acceleration** *Average acceleration* is the ratio of a change in velocity $\Delta v$ to the time interval $\Delta t$ in which the change occurs:

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t}. \quad (2.3.1)$$

The algebraic sign indicates the direction of $a_{\text{avg}}$.

**Instantaneous Acceleration** *Instantaneous acceleration* (or simply **acceleration**) $a$ is the first time derivative of velocity $v(t)$ and the second time derivative of position $x(t)$:

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}. \quad (2.3.2, 2.3.3)$$

On a graph of $v$ versus $t$, the acceleration $a$ at any time $t$ is the slope of the curve at the point that represents $t$.

**Constant Acceleration** The five equations in Table 2.4.1 describe the motion of a particle with constant acceleration:

$$v = v_0 + at, \quad (2.4.1)$$

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2, \quad (2.4.5)$$

$$v^2 = v_0^2 + 2a(x - x_0), \quad (2.4.6)$$

$$x - x_0 = \tfrac{1}{2}(v_0 + v)t, \quad (2.4.7)$$

$$x - x_0 = vt - \tfrac{1}{2}at^2. \quad (2.4.8)$$

These are *not* valid when the acceleration is not constant.

**Free-Fall Acceleration** An important example of straight-line motion with constant acceleration is that of an object rising or falling freely near Earth's surface. The constant-acceleration equations describe this motion, but we make two changes in notation: (1) We refer the motion to the vertical $y$ axis with $+y$ vertically *up*; (2) we replace $a$ with $-g$, where $g$ is the magnitude of the free-fall acceleration. Near Earth's surface, $g = 9.8\text{ m/s}^2$ ($= 32\text{ ft/s}^2$).

# Questions

**1** Figure 2.1 gives the velocity of a particle moving on an $x$ axis. What are (a) the initial and (b) the final directions of travel? (c) Does the particle stop momentarily? (d) Is the acceleration positive or negative? (e) Is it constant or varying?

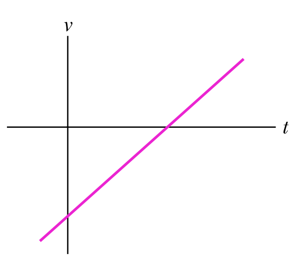


**Figure 2.1** Question 1.

**2** Figure 2.2 gives the acceleration $a(t)$ of a Chihuahua as it chases a German shepherd along an axis. In which of the time periods indicated does the Chihuahua move at constant speed?

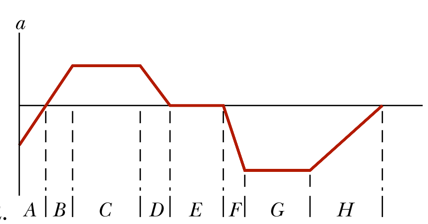


**Figure 2.2** Question 2.

**3** Figure 2.3 shows four paths along which objects move from a starting point to a final point, all in the same time interval. The paths pass over a grid of equally spaced straight lines. Rank the paths according to (a) the average velocity of the objects and (b) the average speed of the objects, greatest first.

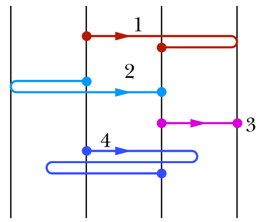


**Figure 2.3** Question 3.

**4** Figure 2.4 is a graph of a particle's position along an $x$ axis versus time. (a) At time $t = 0$, what is the sign of the particle's position? Is the particle's velocity positive, negative, or 0 at (b) $t = 1$ s, (c) $t = 2$ s, and (d) $t = 3$ s? (e) How many times does the particle go through the point $x = 0$?

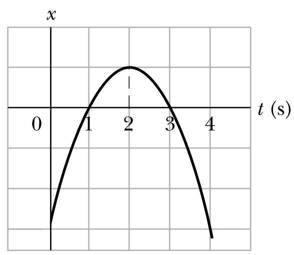


**Figure 2.4** Question 4.

**5** Figure 2.5 gives the velocity of a particle moving along an axis. Point 1 is at the highest point on the curve; point 4 is at the lowest point; and points 2 and 6 are at the same height. What is the direction of travel at (a) time $t = 0$ and (b) point 4? (c) At which of the six numbered points does the particle reverse its direction of travel? (d) Rank the six points according to the magnitude of the acceleration, greatest first.

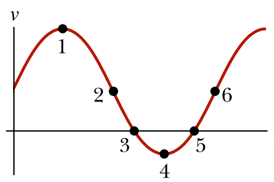


**Figure 2.5** Question 5.

**6** At $t = 0$, a particle moving along an $x$ axis is at position $x_0 = -20$ m. The signs of the particle's initial velocity $v_0$ (at time $t_0$) and constant acceleration $a$ are, respectively, for four situations: (1) +, +; (2) +, −; (3) −, +; (4) −, −. In which situations will the particle (a) stop momentarily, (b) pass through the origin, and (c) never pass through the origin?

**7** Hanging over the railing of a bridge, you drop an egg (no initial velocity) as you throw a second egg downward. Which curves in Fig. 2.6 give the velocity $v(t)$ for (a) the dropped egg and (b) the thrown egg? (Curves $A$ and $B$ are parallel; so are $C$, $D$, and $E$; so are $F$ and $G$.)

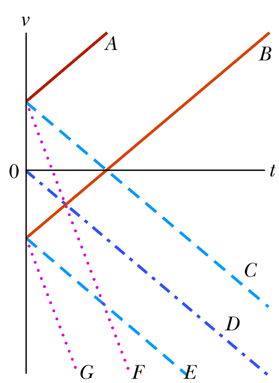


**Figure 2.6** Question 7.

**8** The following equations give the velocity $v(t)$ of a particle in four situations: (a) $v = 3$; (b) $v = 4t^2 + 2t - 6$; (c) $v = 3t - 4$; (d) $v = 5t^2 - 3$. To which of these situations do the equations of Table 2.4.1 apply?

**9** In Fig. 2.7, a cream tangerine is thrown directly upward past three evenly spaced windows of equal heights. Rank the windows according to (a) the average speed of the cream tangerine while passing them, (b) the time the cream tangerine takes to pass them, (c) the magnitude of the acceleration of the cream tangerine while passing them, and (d) the change $\Delta v$ in the speed of the cream tangerine during the passage, greatest first.

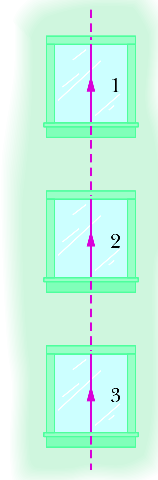


**Figure 2.7** Question 9.

**10** Suppose that a passenger intent on lunch during his first ride in a hot-air balloon accidently drops an apple over the side during the balloon's liftoff. At the moment of the apple's release, the balloon is accelerating upward with a magnitude of 4.0 m/s$^2$ and has an upward velocity of magnitude 2 m/s. What are the (a) magnitude and (b) direction of the acceleration of the apple just after it is released? (c) Just then, is the apple moving upward or downward, or is it stationary? (d) What is the magnitude of its velocity just then? (e) In the next few moments, does the speed of the apple increase, decrease, or remain constant?

**11** Figure 2.8 shows that a particle moving along an $x$ axis undergoes three periods of acceleration. Without written computation, rank the acceleration periods according to the increases they produce in the particle's velocity, greatest first.

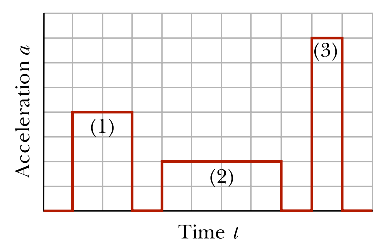


**Figure 2.8** Question 11.

# Problems

GO Tutoring problem available (at instructor's discretion) in *WileyPLUS*

SSM Worked-out solution available in Student Solutions Manual

E Easy M Medium H Hard

FCP Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

CALC Requires calculus

BIO Biomedical application

## Module 2.1 Position, Displacement, and Average Velocity

**1** E While driving a car at 90 km/h, how far do you move while your eyes shut for 0.50 s during a hard sneeze?

**2** E Compute your average velocity in the following two cases: (a) You walk 73.2 m at a speed of 1.22 m/s and then run 73.2 m at a speed of 3.05 m/s along a straight track. (b) You walk for 1.00 min at a speed of 1.22 m/s and then run for 1.00 min at 3.05 m/s along a straight track. (c) Graph $x$ versus $t$ for both cases and indicate how the average velocity is found on the graph.

**3** E SSM An automobile travels on a straight road for 40 km at 30 km/h. It then continues in the same direction for another 40 km at 60 km/h. (a) What is the average velocity of the car during the full 80 km trip? (Assume that it moves in the positive $x$ direction.) (b) What is the average speed? (c) Graph $x$ versus $t$ and indicate how the average velocity is found on the graph.

**4** E A car moves uphill at 40 km/h and then back downhill at 60 km/h. What is the average speed for the round trip?

**5** E CALC SSM The position of an object moving along an $x$ axis is given by $x = 3t - 4t^2 + t^3$, where $x$ is in meters and $t$ in seconds. Find the position of the object at the following values of $t$: (a) 1 s, (b) 2 s, (c) 3 s, and (d) 4 s. (e) What is the object's displacement between $t = 0$ and $t = 4$ s? (f) What is its average velocity for the time interval from $t = 2$ s to $t = 4$ s? (g) Graph $x$ versus $t$ for $0 \le t \le 4$ s and indicate how the answer for (f) can be found on the graph.

**6** E BIO The 1992 world speed record for a bicycle (human-powered vehicle) was set by Chris Huber. His time through the measured 200 m stretch was a sizzling 6.509 s, at which he commented, "Cogito ergo zoom!" (I think, therefore I go fast!). In 2001, Sam Whittingham beat Huber's record by 19.0 km/h. What was Whittingham's time through the 200 m?

**7** M Two trains, each having a speed of 30 km/h, are headed at each other on the same straight track. A bird that can fly 60 km/h flies off the front of one train when they are 60 km apart and heads directly for the other train. On reaching the other train, the (crazy) bird flies directly back to the first train, and so forth. What is the total distance the bird travels before the trains collide?

**8** M FCP GO *Panic escape*. Figure 2.9 shows a general situation in which a stream of people attempt to escape through an exit door that turns out to be locked. The people move toward the door at speed $v_s = 3.50$ m/s, are each $d = 0.25$ m in depth, and are separated by $L = 1.75$ m. The arrangement in Fig. 2.9 occurs at time $t = 0$. (a) At what average rate does the layer of people at the door increase? (b) At what time does the layer's depth reach 5.0 m? (The answers reveal how quickly such a situation becomes dangerous.)

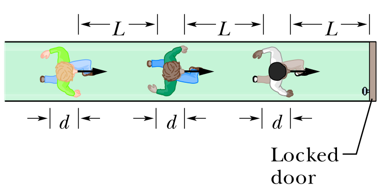


**Figure 2.9** Problem 8.

**9** M BIO In 1 km races, runner 1 on track 1 (with time 2 min, 27.95 s) appears to be faster than runner 2 on track 2 (2 min, 28.15 s). However, length $L_2$ of track 2 might be slightly greater than length $L_1$ of track 1. How large can $L_2 - L_1$ be for us still to conclude that runner 1 is faster?

**10** M FCP To set a speed record in a measured (straight-line) distance $d$, a race car must be driven first in one direction (in time $t_1$) and then in the opposite direction (in time $t_2$). (a) To eliminate the effects of the wind and obtain the car's speed $v_c$ in a windless situation, should we find the average of $d/t_1$ and $d/t_2$ (method 1) or should we divide $d$ by the average of $t_1$ and $t_2$? (b) What is the fractional difference in the two methods when a steady wind blows along the car's route and the ratio of the wind speed $v_w$ to the car's speed $v_c$ is 0.0240?

**11** M GO You are to drive 300 km to an interview. The interview is at 11:15 A.M. You plan to drive at 100 km/h, so you leave at 8:00 A.M. to allow some extra time. You drive at that speed for the first 100 km, but then construction work forces you to slow to 40 km/h for 40 km. What would be the least speed needed for the rest of the trip to arrive in time for the interview?

**12** H FCP *Traffic shock wave*. An abrupt slowdown in concentrated traffic can travel as a pulse, termed a *shock wave*, along the line of cars, either downstream (in the traffic direction) or upstream, or it can be stationary. Figure 2.10 shows a uniformly spaced line of cars moving at speed $v = 25.0$ m/s toward a uniformly spaced line of slow cars moving at speed $v_s = 5.00$ m/s. Assume that each faster car adds length $L = 12.0$ m (car length plus buffer zone) to the line of slow cars when it joins the line, and assume it slows abruptly at the last instant. (a) For what separation distance $d$ between the faster cars does the shock wave remain stationary? If the separation is twice that amount, what are the (b) speed and (c) direction (upstream or downstream) of the shock wave?

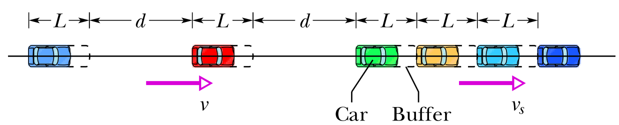


**Figure 2.10** Problem 12.

**13** H You drive on Interstate 10 from San Antonio to Houston, half the *time* at 55 km/h and the other half at 90 km/h. On the way back you travel half the *distance* at 55 km/h and the other half at 90 km/h. What is your average speed (a) from San Antonio to Houston, (b) from Houston back to San Antonio, and (c) for the entire trip? (d) What is your average velocity for the entire trip? (e) Sketch $x$ versus $t$ for (a), assuming the motion is all in the positive $x$ direction. Indicate how the average velocity can be found on the sketch.

## Module 2.2 Instantaneous Velocity and Speed

**14** E GO CALC An electron moving along the $x$ axis has a position given by $x = 16te^{-t}$ m, where $t$ is in seconds. How far is the electron from the origin when it momentarily stops?

**15** E GO CALC (a) If a particle's position is given by $x = 4 - 12t + 3t^2$ (where $t$ is in seconds and $x$ is in meters), what is its velocity at $t = 1$ s? (b) Is it moving in the positive or negative direction of $x$ just then? (c) What is its speed just then? (d) Is the speed increasing or decreasing just then? (Try answering the next two questions without further calculation.) (e) Is there ever an instant when the velocity is zero? If so, give the time $t$; if not, answer no. (f) Is there a time after $t = 3$ s when the particle is moving in the negative direction of $x$? If so, give the time $t$; if not, answer no.

**16** E CALC The position function $x(t)$ of a particle moving along an $x$ axis is $x = 4.0 - 6.0t^2$, with $x$ in meters and $t$ in seconds. (a) At what time and (b) where does the particle (momentarily) stop? At what (c) negative time and (d) positive time does the particle pass through the origin? (e) Graph $x$ versus $t$ for the range $-5$ s to $+5$ s. (f) To shift the curve rightward on the graph, should we include the term $+20t$ or the term $-20t$ in $x(t)$? (g) Does that inclusion increase or decrease the value of $x$ at which the particle momentarily stops?

**17** M CALC The position of a particle moving along the $x$ axis is given in centimeters by $x = 9.75 + 1.50t^3$, where $t$ is in seconds. Calculate (a) the average velocity during the time interval $t$ = 2.00 s to $t$ = 3.00 s; (b) the instantaneous velocity at $t$ = 2.00 s; (c) the instantaneous velocity at $t$ = 3.00 s; (d) the instantaneous velocity at $t$ = 2.50 s; and (e) the instantaneous velocity when the particle is midway between its positions at $t$ = 2.00 s and $t$ = 3.00 s. (f) Graph $x$ versus $t$ and indicate your answers graphically.

## Module 2.3 Acceleration

**18** E CALC The position of a particle moving along an $x$ axis is given by $x = 12t^2 - 2t^3$, where $x$ is in meters and $t$ is in seconds. Determine (a) the position, (b) the velocity, and (c) the acceleration of the particle at $t$ = 3.0 s. (d) What is the maximum positive coordinate reached by the particle and (e) at what time is it reached? (f) What is the maximum positive velocity reached by the particle and (g) at what time is it reached? (h) What is the acceleration of the particle at the instant the particle is not moving (other than at $t$ = 0)? (i) Determine the average velocity of the particle between $t$ = 0 and $t$ = 3 s.

**19** E SSM At a certain time a particle had a speed of 18 m/s in the positive $x$ direction, and 2.4 s later its speed was 30 m/s in the opposite direction. What is the average acceleration of the particle during this 2.4 s interval?

**20** E CALC (a) If the position of a particle is given by $x = 20t - 5t^3$, where $x$ is in meters and $t$ is in seconds, when, if ever, is the particle's velocity zero? (b) When is its acceleration $a$ zero? (c) For what time range (positive or negative) is $a$ negative? (d) Positive? (e) Graph $x(t)$, $v(t)$, and $a(t)$.

**21** M From $t$ = 0 to $t$ = 5.00 min, a man stands still, and from $t$ = 5.00 min to $t$ = 10.0 min, he walks briskly in a straight line at a constant speed of 2.20 m/s. What are (a) his average velocity $v_{avg}$ and (b) his average acceleration $a_{avg}$ in the time interval 2.00 min to 8.00 min? What are (c) $v_{avg}$ and (d) $a_{avg}$ in the time interval 3.00 min to 9.00 min? (e) Sketch $x$ versus $t$ and $v$ versus $t$, and indicate how the answers to (a) through (d) can be obtained from the graphs.

**22** M CALC The position of a particle moving along the $x$ axis depends on the time according to the equation $x = ct^2 - bt^3$, where $x$ is in meters and $t$ in seconds. What are the units of (a) constant $c$ and (b) constant $b$? Let their numerical values be 3.0 and 2.0, respectively. (c) At what time does the particle reach its maximum positive $x$ position? From $t$ = 0.0 s to $t$ = 4.0 s, (d) what distance does the particle move and (e) what is its displacement? Find its velocity at times (f) 1.0 s, (g) 2.0 s, (h) 3.0 s, and (i) 4.0 s. Find its acceleration at times (j) 1.0 s, (k) 2.0 s, (l) 3.0 s, and (m) 4.0 s.

## Module 2.4 Constant Acceleration

**23** E SSM An electron with an initial velocity $v_0 = 1.50 \times 10^5$ m/s enters a region of length $L$ = 1.00 cm where it is electrically accelerated (Fig. 2.11). It emerges with $v = 5.70 \times 10^6$ m/s. What is its acceleration, assumed constant?

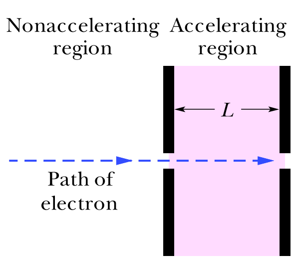


**Figure 2.11** Problem 23.

**24** E BIO FCP *Catapulting mushrooms.* Certain mushrooms launch their spores by a catapult mechanism. As water condenses from the air onto a spore that is attached to the mushroom, a drop grows on one side of the spore and a film grows on the other side. The spore is bent over by the drop's weight, but when the film reaches the drop, the drop's water suddenly spreads into the film and the spore springs upward so rapidly that it is slung off into the air. Typically, the spore reaches a speed of 1.6 m/s in a 5.0 $\mu$m launch; its speed is then reduced to zero in 1.0 mm by the air. Using those data and assuming constant accelerations, find the acceleration in terms of $g$ during (a) the launch and (b) the speed reduction.

**25** E An electric vehicle starts from rest and accelerates at a rate of 2.0 m/s$^2$ in a straight line until it reaches a speed of 20 m/s. The vehicle then slows at a constant rate of 1.0 m/s$^2$ until it stops. (a) How much time elapses from start to stop? (b) How far does the vehicle travel from start to stop?

**26** E A muon (an elementary particle) enters a region with a speed of $5.00 \times 10^6$ m/s and then is slowed at the rate of $1.25 \times 10^{14}$ m/s$^2$. (a) How far does the muon take to stop? (b) Graph $x$ versus $t$ and $v$ versus $t$ for the muon.

**27** E An electron has a constant acceleration of +3.2 m/s$^2$. At a certain instant its velocity is +9.6 m/s. What is its velocity (a) 2.5 s earlier and (b) 2.5 s later?

**28** E On a dry road, a car with good tires may be able to brake with a constant deceleration of 4.92 m/s$^2$. (a) How long does such a car, initially traveling at 24.6 m/s, take to stop? (b) How far does it travel in this time? (c) Graph $x$ versus $t$ and $v$ versus $t$ for the deceleration.

**29** E A certain elevator cab has a total run of 190 m and a maximum speed of 305 m/min, and it accelerates from rest and then back to rest at 1.22 m/s$^2$. (a) How far does the cab move while accelerating to full speed from rest? (b) How long does it take to make the nonstop 190 m run, starting and ending at rest?

**30** E The brakes on your car can slow you at a rate of 5.2 m/s$^2$. (a) If you are going 137 km/h and suddenly see a state trooper, what is the minimum time in which you can get your car under the 90 km/h speed limit? (The answer reveals the futility of braking to keep your high speed from being detected with a radar or laser gun.) (b) Graph $x$ versus $t$ and $v$ versus $t$ for such a slowing.

**31** E SSM Suppose a rocket ship in deep space moves with constant acceleration equal to 9.8 m/s$^2$, which gives the illusion of normal gravity during the flight. (a) If it starts from rest, how long will it take to acquire a speed one-tenth that of light, which travels at $3.0 \times 10^8$ m/s? (b) How far will it travel in so doing?

**32** E BIO FCP A world's land speed record was set by Colonel John P. Stapp when in March 1954 he rode a rocket-propelled sled that moved along a track at 1020 km/h. He and the sled were brought to a stop in 1.4 s. (See Fig. 2.3.1.) In terms of $g$, what acceleration did he experience while stopping?

**33** E SSM A car traveling 56.0 km/h is 24.0 m from a barrier when the driver slams on the brakes. The car hits the barrier 2.00 s later. (a) What is the magnitude of the car's constant acceleration before impact? (b) How fast is the car traveling at impact?

**34** M GO In Fig. 2.12, a red car and a green car, identical except for the color, move toward each other in adjacent lanes and parallel to an $x$ axis. At time $t$ = 0, the red car is at $x_r$ = 0 and the green car is at $x_g$ = 220 m. If the red car has a constant velocity of 20 km/h, the cars pass each other at $x = 44.5$ m, and if it has a constant velocity of 40 km/h, they pass each other at $x = 76.6$ m. What are (a) the initial velocity and (b) the constant acceleration of the green car?

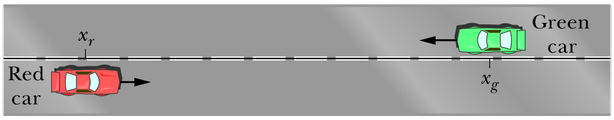


**Figure 2.12** Problems 34 and 35.

**35** M Figure 2.12 shows a red car and a green car that move toward each other. Figure 2.13 is a graph of their motion, showing the positions $x_{g0} = 270$ m and $x_{r0} = -35.0$ m at time $t = 0$. The green car has a constant speed of 20.0 m/s and the red car begins from rest. What is the acceleration magnitude of the red car?

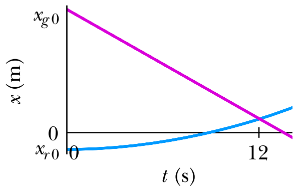


**Figure 2.13** Problem 35.

**36** M A car moves along an $x$ axis through a distance of 900 m, starting at rest (at $x = 0$) and ending at rest (at $x = 900$ m). Through the first $\frac{1}{4}$ of that distance, its acceleration is $+2.25\ \text{m/s}^2$. Through the rest of that distance, its acceleration is $-0.750\ \text{m/s}^2$. What are (a) its travel time through the 900 m and (b) its maximum speed? (c) Graph position $x$, velocity $v$, and acceleration $a$ versus time $t$ for the trip.

**37** M Figure 2.14 depicts the motion of a particle moving along an $x$ axis with a constant acceleration. The figure's vertical scaling is set by $x_s = 6.0$ m. What are the (a) magnitude and (b) direction of the particle's acceleration?

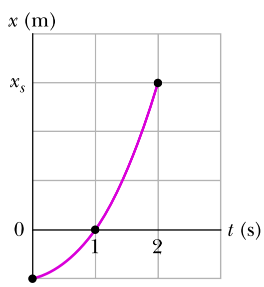


**Figure 2.14** Problem 37.

**38** M (a) If the maximum acceleration that is tolerable for passengers in a subway train is $1.34\ \text{m/s}^2$ and subway stations are located 806 m apart, what is the maximum speed a subway train can attain between stations? (b) What is the travel time between stations? (c) If a subway train stops for 20 s at each station, what is the maximum average speed of the train, from one start-up to the next? (d) Graph $x$, $v$, and $a$ versus $t$ for the interval from one start-up to the next.

**39** M Cars $A$ and $B$ move in the same direction in adjacent lanes. The position $x$ of car $A$ is given in Fig. 2.15, from time $t = 0$ to $t = 7.0$ s. The figure's vertical scaling is set by $x_s = 32.0$ m. At $t = 0$, car $B$ is at $x = 0$, with a velocity of 12 m/s and a negative constant acceleration $a_B$. (a) What must $a_B$ be such that the cars are (momentarily) side by side (momentarily at the same value of $x$) at $t = 4.0$ s? (b) For that value of $a_B$, how many times are the cars side by side? (c) Sketch the position $x$ of car $B$ versus time $t$ on Fig. 2.15.

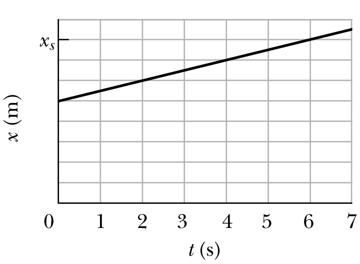


**Figure 2.15** Problem 39.

How many times will the cars be side by side if the magnitude of acceleration $a_B$ is (d) more than and (e) less than the answer to part (a)?

**40** M FCP You are driving toward a traffic signal when it turns yellow. Your speed is the legal speed limit of $v_0 = 55$ km/h; your best deceleration rate has the magnitude $a = 5.18\ \text{m/s}^2$. Your best reaction time to begin braking is $T = 0.75$ s. To avoid having the front of your car enter the intersection after the light turns red, should you brake to a stop or continue to move at 55 km/h if the distance to the intersection and the duration of the yellow light are (a) 40 m and 2.8 s, and (b) 32 m and 1.8 s? Give an answer of brake, continue, either (if either strategy works), or neither (if neither strategy works and the yellow duration is inappropriate).

**41** M GO As two trains move along a track, their conductors suddenly notice that they are headed toward each other. Figure 2.16 gives their velocities $v$ as functions of time $t$ as the conductors slow the trains. The figure's vertical scaling is set by $v_s = 40.0$ m/s. The slowing processes begin when the trains are 200 m apart. What is their separation when both trains have stopped?

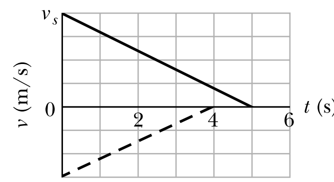


**Figure 2.16** Problem 41.

**42** H GO You are arguing over a cell phone while trailing an unmarked police car by 25 m; both your car and the police car are traveling at 110 km/h. Your argument diverts your attention from the police car for 2.0 s (long enough for you to look at the phone and yell, "I won't do that!"). At the beginning of that 2.0 s, the police officer begins braking suddenly at $5.0\ \text{m/s}^2$. (a) What is the separation between the two cars when your attention finally returns? Suppose that you take another 0.40 s to realize your danger and begin braking. (b) If you too brake at $5.0\ \text{m/s}^2$, what is your speed when you hit the police car?

**43** H GO When a high-speed passenger train traveling at 161 km/h rounds a bend, the engineer is shocked to see that a locomotive has improperly entered onto the track from a siding and is a distance $D = 676$ m ahead (Fig. 2.17). The locomotive is moving at 29.0 km/h. The engineer of the high-speed train immediately applies the brakes. (a) What must be the magnitude of the resulting constant deceleration if a collision is to be just avoided? (b) Assume that the engineer is at $x = 0$ when, at $t = 0$, he first spots the locomotive. Sketch $x(t)$ curves for the locomotive and high-speed train for the cases in which a collision is just avoided and is not quite avoided.

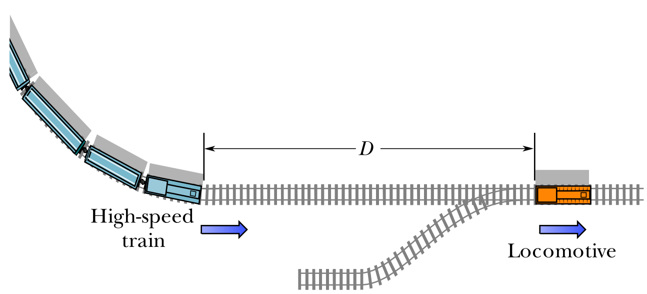


**Figure 2.17** Problem 43.

## Module 2.5 Free-Fall Acceleration

**44** E When startled, an armadillo will leap upward. Suppose it rises 0.544 m in the first 0.200 s. (a) What is its initial speed as it leaves the ground? (b) What is its speed at the height of 0.544 m? (c) How much higher does it go?

**45** E SSM (a) With what speed must a ball be thrown vertically from ground level to rise to a maximum height of 50 m? (b) How long will it be in the air? (c) Sketch graphs of $y$, $v$, and $a$ versus $t$ for the ball. On the first two graphs, indicate the time at which 50 m is reached.

**46** E Raindrops fall 1700 m from a cloud to the ground. (a) If they were not slowed by air resistance, how fast would the drops be moving when they struck the ground? (b) Would it be safe to walk outside during a rainstorm?

**47** E SSM At a construction site a pipe wrench struck the ground with a speed of 24 m/s. (a) From what height was it inadvertently dropped? (b) How long was it falling? (c) Sketch graphs of $y$, $v$, and $a$ versus $t$ for the wrench.

**48** E A hoodlum throws a stone vertically downward with an initial speed of 12.0 m/s from the roof of a building, 30.0 m above the ground. (a) How long does it take the stone to reach the ground? (b) What is the speed of the stone at impact?

**49** E SSM A hot-air balloon is ascending at the rate of 12 m/s and is 80 m above the ground when a package is dropped over the side. (a) How long does the package take to reach the ground? (b) With what speed does it hit the ground?

**50** M At time $t = 0$, apple 1 is dropped from a bridge onto a roadway beneath the bridge; somewhat later, apple 2 is thrown down from the same height. Figure 2.18 gives the vertical positions $y$ of the apples versus $t$ during the falling, until both apples have hit the roadway. The scaling is set by $t_s = 2.0$ s. With approximately what speed is apple 2 thrown down?

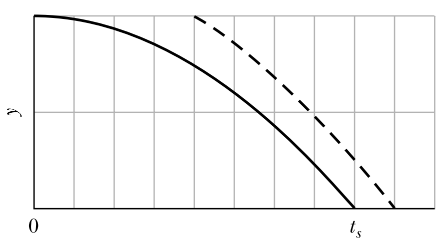


**Figure 2.18** Problem 50.

**51** M As a runaway scientific balloon ascends at 19.6 m/s, one of its instrument packages breaks free of a harness and free-falls. Figure 2.19 gives the vertical velocity of the package versus time, from before it breaks free to when it reaches the ground. (a) What maximum height above the break-free point does it rise? (b) How high is the break-free point above the ground?

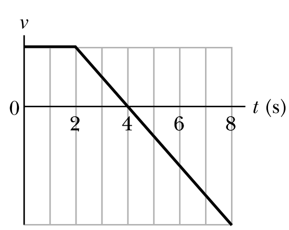


**Figure 2.19** Problem 51.

**52** M GO A bolt is dropped from a bridge under construction, falling 90 m to the valley below the bridge. (a) In how much time does it pass through the last 20% of its fall? What is its speed (b) when it begins that last 20% of its fall and (c) when it reaches the valley beneath the bridge?

**53** M SSM A key falls from a bridge that is 45 m above the water. It falls directly into a model boat, moving with constant velocity, that is 12 m from the point of impact when the key is released. What is the speed of the boat?

**54** M GO A stone is dropped into a river from a bridge 43.9 m above the water. Another stone is thrown vertically down 1.00 s after the first is dropped. The stones strike the water at the same time. (a) What is the initial speed of the second stone? (b) Plot velocity versus time on a graph for each stone, taking zero time as the instant the first stone is released.

**55** M SSM A ball of moist clay falls 15.0 m to the ground. It is in contact with the ground for 20.0 ms before stopping. (a) What is the magnitude of the average acceleration of the ball during the time it is in contact with the ground? (Treat the ball as a particle.) (b) Is the average acceleration up or down?

**56** M GO Figure 2.20 shows the speed $v$ versus height $y$ of a ball tossed directly upward, along a $y$ axis. Distance $d$ is 0.40 m. The speed at height $y_A$ is $v_A$. The speed at height $y_B$ is $\frac{1}{3}v_A$. What is speed $v_A$?

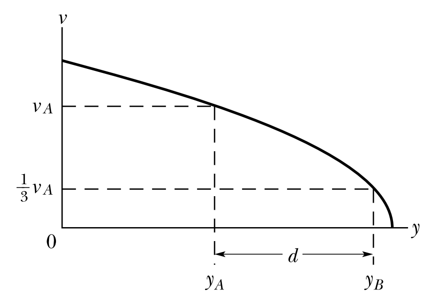


**Figure 2.20** Problem 56.

**57** M To test the quality of a tennis ball, you drop it onto the floor from a height of 4.00 m. It rebounds to a height of 2.00 m. If the ball is in contact with the floor for 12.0 ms, (a) what is the magnitude of its average acceleration during that contact and (b) is the average acceleration up or down?

**58** M An object falls a distance $h$ from rest. If it travels $0.50h$ in the last 1.00 s, find (a) the time and (b) the height of its fall. (c) Explain the physically unacceptable solution of the quadratic equation in $t$ that you obtain.

**59** M Water drips from the nozzle of a shower onto the floor 200 cm below. The drops fall at regular (equal) intervals of time, the first drop striking the floor at the instant the fourth drop begins to fall. When the first drop strikes the floor, how far below the nozzle are the (a) second and (b) third drops?

**60** M GO A rock is thrown vertically upward from ground level at time $t = 0$. At $t = 1.5$ s it passes the top of a tall tower, and 1.0 s later it reaches its maximum height. What is the height of the tower?

**61** H GO A steel ball is dropped from a building's roof and passes a window, taking 0.125 s to fall from the top to the bottom of the window, a distance of 1.20 m. It then falls to a sidewalk and bounces back past the window, moving from bottom to top in 0.125 s. Assume that the upward flight is an exact reverse of the fall. The time the ball spends below the bottom of the window is 2.00 s. How tall is the building?

**62** H BIO FCP A basketball player grabbing a rebound jumps 76.0 cm vertically. How much total time (ascent and descent) does the player spend (a) in the top 15.0 cm of this jump and (b) in the bottom 15.0 cm? (The player seems to hang in the air at the top.)

**63** H GO A drowsy cat spots a flowerpot that sails first up and then down past an open window. The pot is in view for a total of 0.50 s, and the top-to-bottom height of the window is 2.00 m. How high above the window top does the flowerpot go?

**64** H A ball is shot vertically upward from the surface of another planet. A plot of $y$ versus $t$ for the ball is shown in Fig. 2.21, where $y$ is the height of the ball above its starting point and $t = 0$ at the instant the ball is shot. The figure's vertical scaling is set by $y_s = 30.0$ m. What are the magnitudes of (a) the free-fall acceleration on the planet and (b) the initial velocity of the ball?

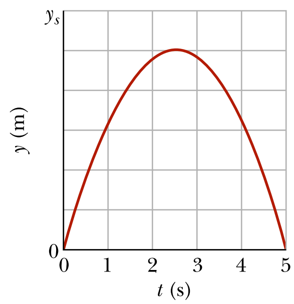

**Figure 2.21** Problem 64.

## Module 2.6 Graphical Integration in Motion Analysis

**65** E BIO CALC FCP Figure 2.6.2$a$ gives the acceleration of a volunteer's head and torso during a rear-end collision. At maximum head acceleration, what is the speed of (a) the head and (b) the torso?

**66** M BIO CALC FCP In a forward punch in karate, the fist begins at rest at the waist and is brought rapidly forward until the arm is fully extended. The speed $v(t)$ of the fist is given in Fig. 2.22 for someone skilled in karate. The vertical scaling is set by $v_s = 8.0$ m/s. How far has the fist moved at (a) time $t = 50$ ms and (b) when the speed of the fist is maximum?

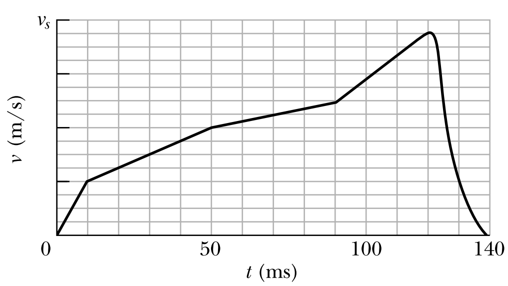

**Figure 2.22** Problem 66.

**67** M BIO CALC When a soccer ball is kicked toward a player and the player deflects the ball by "heading" it, the acceleration of the head during the collision can be significant. Figure 2.23 gives the measured acceleration $a(t)$ of a soccer player's head for a bare head and a helmeted head, starting from rest. The scaling on the vertical axis is set by $a_s = 200$ m/s$^2$. At time $t = 7.0$ ms, what is the difference in the speed acquired by the bare head and the speed acquired by the helmeted head?

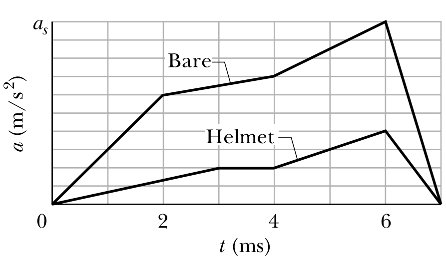

**Figure 2.23** Problem 67.

**68** M BIO CALC FCP A salamander of the genus *Hydromantes* captures prey by launching its tongue as a projectile: The skeletal part of the tongue is shot forward, unfolding the rest of the tongue, until the outer portion lands on the prey, sticking to it. Figure 2.24 shows the acceleration magnitude $a$ versus time $t$ for the acceleration phase of the launch in a typical situation. The indicated accelerations are $a_2 = 400$ m/s$^2$ and $a_1 = 100$ m/s$^2$. What is the outward speed of the tongue at the end of the acceleration phase?

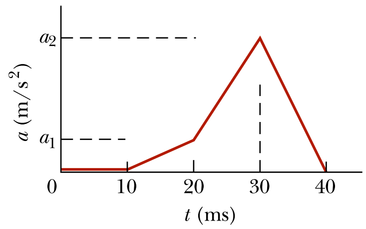

**Figure 2.24** Problem 68.

**69** M BIO CALC How far does the runner whose velocity–time graph is shown in Fig. 2.25 travel in 16 s? The figure's vertical scaling is set by $v_s = 8.0$ m/s.

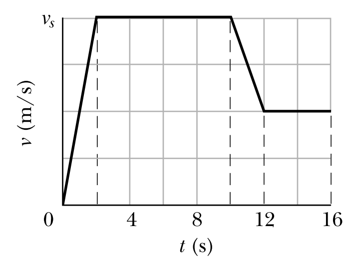

**Figure 2.25** Problem 69.

**70** H CALC Two particles move along an $x$ axis. The position of particle 1 is given by $x = 6.00t^2 + 3.00t + 2.00$ (in meters and seconds); the acceleration of particle 2 is given by $a = -8.00t$ (in meters per second squared and seconds) and, at $t = 0$, its velocity is 20 m/s. When the velocities of the particles match, what is their velocity?

## Additional Problems

**71** CALC In an arcade video game, a spot is programmed to move across the screen according to $x = 9.00t - 0.750t^3$, where $x$ is distance in centimeters measured from the left edge of the screen and $t$ is time in seconds. When the spot reaches a screen edge, at either $x = 0$ or $x = 15.0$ cm, $t$ is reset to 0 and the spot starts moving again according to $x(t)$. (a) At what time after starting is the spot instantaneously at rest? (b) At what value of $x$ does this occur? (c) What is the spot's acceleration (including sign) when this occurs? (d) Is it moving right or left just prior to coming to rest? (e) Just after? (f) At what time $t > 0$ does it first reach an edge of the screen?

**72** A rock is shot vertically upward from the edge of the top of a tall building. The rock reaches its maximum height above the top of the building 1.60 s after being shot. Then, after barely missing the edge of the building as it falls downward, the rock strikes the ground 6.00 s after it is launched. In SI units: (a) with what upward velocity is the rock shot, (b) what maximum height above the top of the building is reached by the rock, and (c) how tall is the building?

**73** GO At the instant the traffic light turns green, an automobile starts with a constant acceleration $a$ of 2.2 m/s$^2$. At the same instant a truck, traveling with a constant speed of 9.5 m/s, overtakes and passes the automobile. (a) How far beyond the traffic signal will the automobile overtake the truck? (b) How fast will the automobile be traveling at that instant?

**74** A pilot flies horizontally at 1300 km/h, at height $h = 35$ m above initially level ground. However, at time $t = 0$, the pilot begins to fly over ground sloping upward at angle $\theta = 4.3°$

(Fig. 2.26). If the pilot does not change the airplane's heading, at what time $t$ does the plane strike the ground?

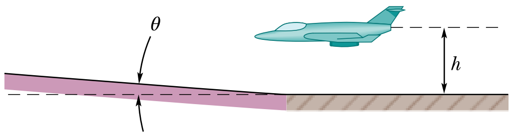


**Figure 2.26** Problem 74.

**75** GO To stop a car, first you require a certain reaction time to begin braking; then the car slows at a constant rate. Suppose that the total distance moved by your car during these two phases is 56.7 m when its initial speed is 80.5 km/h, and 24.4 m when its initial speed is 48.3 km/h. What are (a) your reaction time and (b) the magnitude of the acceleration?

**76** GO FCP Figure 2.27 shows part of a street where traffic flow is to be controlled to allow a *platoon* of cars to move smoothly along the street. Suppose that the platoon leaders have just reached intersection 2, where the green light appeared when they were distance $d$ from the intersection. They continue to travel at a certain speed $v_p$ (the speed limit) to reach intersection 3, where the green appears when they are distance $d$ from it. The intersections are separated by distances $D_{23}$ and $D_{12}$. (a) What should be the time delay of the onset of green at intersection 3 relative to that at intersection 2 to keep the platoon moving smoothly?

Suppose, instead, that the platoon had been stopped by a red light at intersection 1. When the green comes on there, the leaders require a certain time $t_r$ to respond to the change and an additional time to accelerate at some rate $a$ to the cruising speed $v_p$. (b) If the green at intersection 2 is to appear when the leaders are distance $d$ from that intersection, how long after the light at intersection 1 turns green should the light at intersection 2 turn green?

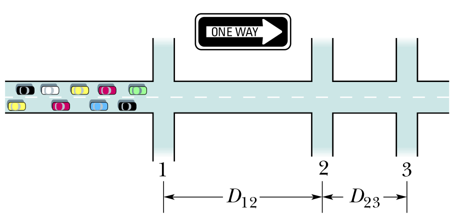


**Figure 2.27** Problem 76.

**77** SSM A hot rod can accelerate from 0 to 60 km/h in 5.4 s. (a) What is its average acceleration, in m/s$^2$, during this time? (b) How far will it travel during the 5.4 s, assuming its acceleration is constant? (c) From rest, how much time would it require to go a distance of 0.25 km if its acceleration could be maintained at the value in (a)?

**78** GO A red train traveling at 72 km/h and a green train traveling at 144 km/h are headed toward each other along a straight, level track. When they are 950 m apart, each engineer sees the other's train and applies the brakes. The brakes slow each train at the rate of 1.0 m/s$^2$. Is there a collision? If so, answer yes and give the speed of the red train and the speed of the green train at impact, respectively. If not, answer no and give the separation between the trains when they stop.

**79** GO At time $t = 0$, a rock climber accidentally allows a piton to fall freely from a high point on the rock wall to the valley below him. Then, after a short delay, his climbing partner, who is 10 m higher on the wall, throws a piton downward. The positions $y$ of the pitons versus $t$ during the falling are given in Fig. 2.28. With what speed is the second piton thrown?

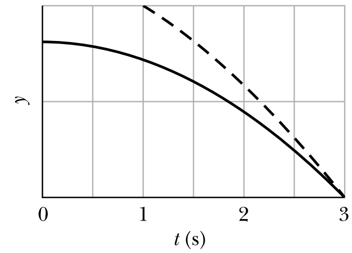


**Figure 2.28** Problem 79.

**80** A train started from rest and moved with constant acceleration. At one time it was traveling 30 m/s, and 160 m farther on it was traveling 50 m/s. Calculate (a) the acceleration, (b) the time required to travel the 160 m mentioned, (c) the time required to attain the speed of 30 m/s, and (d) the distance moved from rest to the time the train had a speed of 30 m/s. (e) Graph $x$ versus $t$ and $v$ versus $t$ for the train, from rest.

**81** CALC SSM A particle's acceleration along an $x$ axis is $a = 5.0t$, with $t$ in seconds and $a$ in meters per second squared. At $t = 2.0$ s, its velocity is +17 m/s. What is its velocity at $t = 4.0$ s?

**82** CALC Figure 2.29 gives the acceleration $a$ versus time $t$ for a particle moving along an $x$ axis. The $a$-axis scale is set by $a_s = 12.0$ m/s$^2$. At $t = -2.0$ s, the particle's velocity is 7.0 m/s. What is its velocity at $t = 6.0$ s?

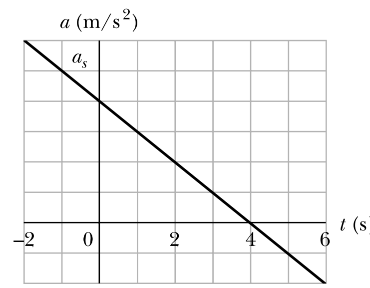


**Figure 2.29** Problem 82.

**83** BIO Figure 2.30 shows a simple device for measuring your reaction time. It consists of a cardboard strip marked with a scale and two large dots. A friend holds the strip *vertically,* with thumb and forefinger at the dot on the right in Fig. 2.30. You then position your thumb and forefinger at the other dot (on the left in Fig. 2.30), being careful not to touch the strip. Your friend releases the strip, and you try to pinch it as soon as possible after you see it begin to fall. The mark at the place where you pinch the strip gives your reaction time. (a) How far from the lower dot should you place the 50.0 ms mark? How much higher should you place the marks for (b) 100, (c) 150, (d) 200, and (e) 250 ms? (For example, should the 100 ms marker be 2 times as far from the dot as the 50 ms marker? If so, give an answer of 2 times. Can you find any pattern in the answers?)

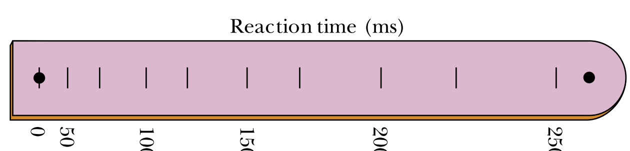


**Figure 2.30** Problem 83.

**84** BIO FCP A rocket-driven sled running on a straight, level track is used to investigate the effects of large accelerations on humans. One such sled can attain a speed of 1600 km/h in 1.8 s, starting from rest. Find (a) the acceleration (assumed constant) in terms of $g$ and (b) the distance traveled.

**85** *Fastball timing.* In professional baseball, the *pitching distance* of 60 feet 6 inches is the distance from the front of the pitcher's plate (or rubber) to the rear of the home plate. (a) Assuming that a 95 mi/h fastball travels that full distance horizontally, what is its flight time, which is the time a batter must judge if the ball is "hittable" and then swing the bat? (b) Research indicates that even an elite batter cannot track the ball for the full flight and yet many players have described seeing the ball–bat collision. One explanation is that the eyes track the ball in the early part of the flight and then undergo a *predictive saccade* in which they jump to an anticipated point later in the flight. A saccade suppresses vision for 20 ms. How far in feet does the fastball travel during that interval of no vision?

**86** *Measuring the free-fall acceleration.* At the National Physical Laboratory in England, a measurement of the free-fall acceleration $g$ was made by throwing a glass ball straight up in an evacuated tube and letting it return. Let $\Delta T_L$ in Fig. 2.31 be the time interval between the two passes of the ball across a certain lower level, $\Delta T_U$ the time interval between the two passages across an upper level, and $H$ the distance between the two levels. What is $g$ in terms of those quantities?

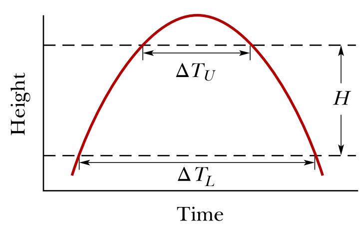


**Figure 2.31** Problem 86.

**87** CALC *Velocity versus time.* Figure 2.32 gives the velocity $v$ (m/s) versus time $t$ (s) for a particle moving along an $x$ axis. The area between the time axis and the plotted curve is given for the two portions of the graph. At $t = t_A$ (at one of the crossing points in the plotted figure), the particle's position is $x = 14$ m. What is its position at (a) $t = 0$ and (b) $t = t_B$?

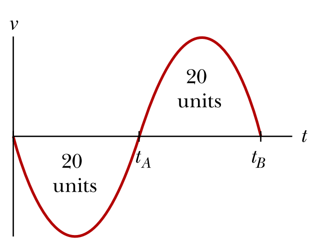


**Figure 2.32** Problem 87.

**88** CALC *Hockey puck on frozen lake.* At time $t = 0$, a hockey puck is sent sliding over a frozen lake, directly into a strong wind. Figure 2.33 gives the velocity $v$ of the puck versus time, as the puck moves along an $x$ axis, starting at $x_0 = 0$. At $t = 14$ s, what is its coordinate?

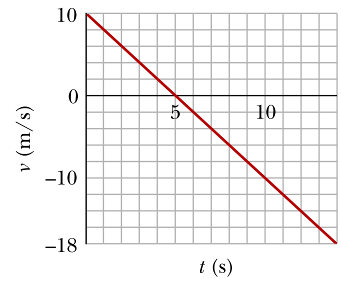


**Figure 2.33** Problem 88.

**89** *Seafloor spread.* Figure 2.34 is a plot of the age of ancient seafloor material, in millions of years, against the distance from a certain ocean ridge. Seafloor material extruded from that ridge moves away from it at approximately uniform speed. What is that speed in centimeters per year?

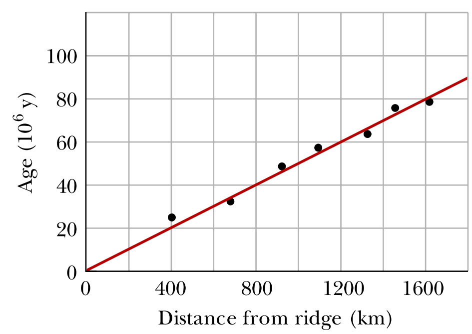


**Figure 2.34** Problem 89.

**90** *Braking, no reaction time.* Modern cars with a computer system using radar can eliminate the normal reaction time for a driver to recognize an upcoming danger and apply the brakes. For example, the system can detect the sudden stopping of a car in front of a driver by using radar signals that travel at the speed of light. Rapid processing then can almost immediately activate the braking. For a car traveling at $v = 31.3$ m/s (70 mi/h) and assuming a normal reaction time of 0.750 s, find the reduction in a car's stopping distance with such a computer system.

**91** *100 m dash.* The running event known as the 100 m dash consists of three stages. In the first, the runner accelerates to the maximum speed, which usually occurs at the 50 m to 70 m mark. That speed is then maintained until the last 10 m, when the runner slows. Consider three parts of the record-setting run by Usain Bolt in the 2008 Olympics: (a) from 10 m to 20 m, elapsed time of 1.02 s, (b) from 50 m to 60 m, elapsed time of 0.82 s, and (c) from 90 m to 100 m, elapsed time of 0.90 s. What was the average velocity for each part?

**92** *Drag race of car and motorcycle.* A popular web video shows a jet airplane, a car, and a motorcycle racing from rest along a runway (Fig. 2.35). Initially the motorcycle takes the lead, but then the jet takes the lead, and finally the car blows past the motorcycle. Consider the motorcycle–car race. The motorcycle's constant acceleration $a_m = 8.40\ \text{m/s}^2$ is greater than the car's constant acceleration $a_c = 5.60\ \text{m/s}^2$, but the motorcycle has an upper limit of $v_m = 58.8$ m/s to its speed while the car has an upper limit of $v_c = 106$ m/s. Let the car and motorcycle race in the positive direction of an $x$ axis, starting with their midpoints at $x = 0$ at $t = 0$. At what (a) time and (b) position are their midpoints again aligned?

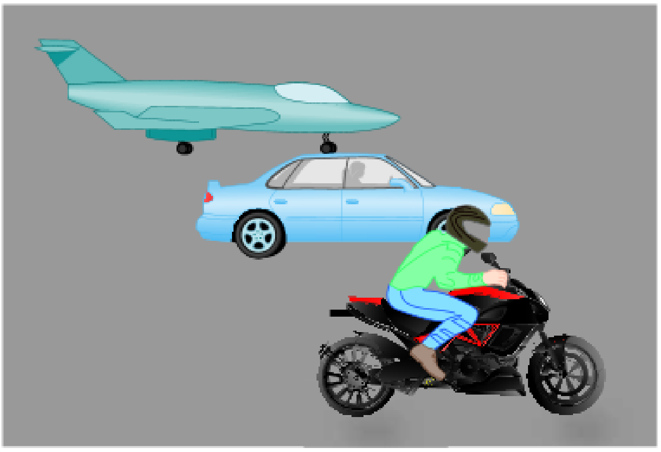

**Figure 2.35** Problem 92.

**93** *Speedy ants.* The silver ants of the Sahara Desert are the fastest ants in terms of their body length, which averages 7.92 mm. In the hottest part of the day, they run as fast as 0.855 m/s. In terms of body lengths per second, how fast do they run?

**94** *Car lengths in trailing.* When trailing a car on a highway, you are advised to maintain a trailing distance that is often quoted in terms of car lengths, such as in "stay back by 3 car lengths." Suppose the other car suddenly stops (it hits, say, a large stationary truck). Assume a car length $L$ is 4.50 m, your car speed $v_0$ is 31.3 m/s (70 mi/h), your trailing distance is $nL = 10.0L$, and the acceleration magnitude at which your car can brake is $8.50\ \text{m/s}^2$. What is your speed just before colliding with the other car if (a) your reaction time $t_r$ to start braking is 0.750 s and (b) automatic braking is immediately started by your car's radar system that continuously monitors the road? What is the minimum value for $n$ needed to avoid a collision with (c) the reaction time $t_r$ and (d) automatic braking?

**95** *Speed limits.* (a) The greatest speed limit in the United States is along the tolled section of Texas State Highway 130 where the limit is 85 mi/h. How much time would be saved in driving that 41 mi section at the speed limit instead of 60 mi/h? (b) The speed limit in residential areas is commonly 25 mi/h but some motorists drive at an average speed of 45 mi/h, perhaps by weaving through traffic and even driving through traffic lights that had just turned red. How much time would be saved in driving at that faster speed through 5.5 mi instead of the posted speed limit if the car does not stop at any intersections?

**96** *Autonomous car passing with following car.* Figure 2.36*a* gives an overhead view of three cars with the same length $L = 4.50$ m. Cars $A$ and $B$ are moving at $v_A = 22.0$ m/s (49 mi/h) along the right-hand lane of a long, straight road with two lanes in each direction, and car $C$ is moving along the passing lane at $v_C = 27.0$ m/s (60 mi/h) at initial distance $d$ behind $B$. Car $B$ is autonomous and is equipped with a computer control system using radar to detect the speeds and distances of the other two cars. At time $t = 0$, the front of car $B$ is $3.00L$ behind the rear of car $A$, which is at $x_{A1} = 0$ on the $x$ axis. We want $B$ to pull into the passing lane, speed up and pass $A$, and then pull back into the right-hand lane, $3.00L$ in front of $A$ and at the initial speed. The computer control system will allow 15.0 s for the maneuver and only if the front of $C$ will be no closer than $3.00L$ behind $A$ at the end of the maneuver, as in Fig. 2.36*b*. What is the least value of $d$ that the system will allow?

**97** *Freeway entrance ramps.* When freeways were first built in the United States in the 1950s, entrance ramps were often too short for an entering car to safely merge into existing traffic. Consider an aggressive car acceleration of $a = 4.0\ \text{m/s}^2$ and an initial car speed of $v_0 = 25$ mi/h as the car enters the entrance ramp of a freeway where other cars are moving at 55 mi/h. (a) If the ramp has a length of $d = 40$ yd, what is the car's speed $v$ in miles per hour as it attempts to merge? (b) What is the minimum length $d$ in yards needed for the car's speed to match the speed of the other cars?

**98** *Autonomous car passing with oncoming car.* Figure 2.37*a* gives an overhead view of three cars with the same length $L = 4.50$ m. Cars $A$ and $B$ are moving at $v_A = 22.0$ m/s (49 mi/h) along a long, straight road with one lane in each direction and car $C$ is oncoming at $v_C = 27.0$ m/s (60 mi/h) at initial distance $d$ in front of $B$. Car $B$ is autonomous and is equipped with a computer and radar control system to detect the speeds and distances of the other two cars. At time $t = 0$, the front of car $B$ is $3.00L$ behind the rear of car $A$, which is at $x_{A1} = 0$ on the $x$ axis. We want $B$ to pull into the other lane, speed up and pass $A$, and then pull back into the initial lane to be $3.00L$ in front of $A$ and at the initial speed (Fig. 2.37*b*). The computer control system will allow 15.0 s for the maneuver but only if the front of $C$ will be no closer than $10L$ in front of $B$ at the end of the maneuver, as shown in the figure. What is the least value of $d$ that the system will allow?

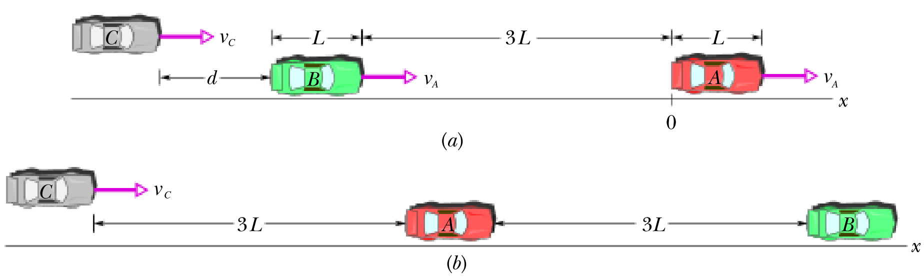


**Figure 2.36** Problem 96.

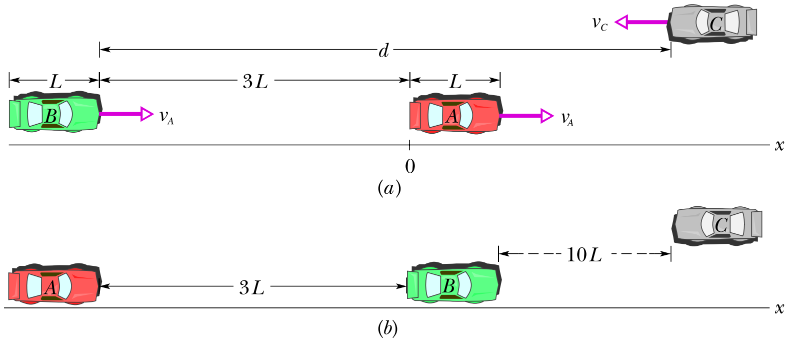


**Figure 2.37** Problem 98.

**99** *Record accelerations.* When Kitty O'Neil set the dragster records for the greatest speed and least elapsed time by reaching 392.54 mi/h in 3.72 s, what was her average acceleration in (a) meters per second squared and (b) $g$ units? When Eli Beeding, Jr., reached 72.5 mi/h in 0.0400 s on a rocket sled, what was his average acceleration in (c) meters per second squared and (d) $g$ units? For each person, assume the motion is in the positive direction of an $x$ axis.

**100** *Travel to a star.* How much time would be required for a starship to reach Proxima Centauri, the star closest to the Sun, at a distance of $L = 4.244$ light years (ly)? Assume that it starts from rest, maintains a comfortable acceleration magnitude of $1.000g$ for the first 0.0450 y and a deceleration (slowing) magnitude of $1.000g$ for the last 0.0450 y, and cruises at constant speed in between those periods.

**101** **CALC** *Bobsled acceleration.* In the start of a four-person bobsled race, two drivers (a pilot and a brakeman) are already on board while two pushers accelerate the sled along the ice by pushing against the ice with spiked shoes. After pushing for 50 m along a straight course, the pushers jump on board. The acceleration during the pushing largely determines the time to slide through the rest of the course and thus decides the winner with the least run time, which often depends on differences of 1.0 ms. Consider an $x$ axis along the 50 m, with the origin at the start position. If the position $x$ versus time $t$ in the pushing phase is given by $x = 0.3305t^2 + 4.2060t$ (in meters and seconds), then at the end of a 9.000 s push what are (a) the speed and (b) the acceleration?

**102** *Car-following stopping distance.* When you drive behind another car, what is the minimum distance you should keep between the cars to avoid a rear-end collision if the other car were to suddenly stop (it hits, say, a stationary truck)? Some drivers use a "2 second rule" while others use a "3 second rule." To apply such rules, pick out an object such as a tree alongside the road. When the front car passes it, begin to count off seconds. For the first rule, you want to pass that object at a count of 2 s, and for the second rule, 3 s. For the 2 s rule, what is the resulting car–car separation at a speed of (a) 15.6 m/s (35 mi/h, slow) and (b) 31.3 m/s (70 mi/h, fast)? For the 3 s rule, what is the car–car separation at a speed of (c) 15.6 m/s and (d) 31.3 m/s? To check if the results give safe trailing distances, find the stopping distance required of you at those initial speeds. Assume that your car's braking acceleration is $-8.50\ \text{m/s}^2$ and your reaction time to apply the brake upon seeing the danger is 0.750 s. What is your stopping distance at a speed of (e) 15.6 m/s and (f) 31.3 m/s? (g) For which is the 2 s rule adequate? (h) For which is the 3 s rule adequate?

**103** *Vehicle jerk indicating aggression.* One common form of aggressive driving is for a trailing driver to repeat a pattern of accelerating suddenly to come close to the car in front and then braking suddenly to avoid a collision. One way to monitor such behavior, either remotely or with an onboard computer system, is to measure *vehicle jerk*, where jerk is the physics term for the time rate of change of an object's acceleration along a straight path. Figure 2.38 is a graph of acceleration $a$ versus time $t$ in a typical situation for a car. Determine the jerk for each of the time periods: (a) gas pedal pushed down rapidly, 2.0 s interval, (b) gas pedal released, 1.5 s interval, (c) brake pedal pushed down rapidly, 1.5 s interval, (d) brake pedal released, 2.5 s interval.

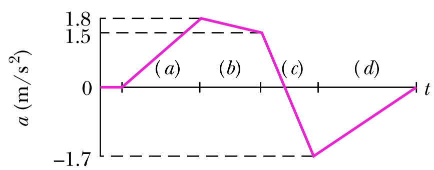


**Figure 2.38** Problem 103.

**104** *Metal baseball bat danger.* Wood bats are required in professional baseball but metal bats are sometimes allowed in youth and college baseball. One result is that the *exit speed* $v$ of the baseball off a metal bat can be greater. In one set of measurements under the same circumstances, $v = 50.98$ m/s off a wood bat and $v = 61.50$ m/s off a metal bat. Consider a ball hit directly toward the pitcher. The regulation distance between pitcher and batter is $\Delta x = 60$ ft 6 in. For those measured speeds, how much time $\Delta t$ does the ball take to reach the pitcher for (a) the wood bat and (b) the metal bat? (c) By what percentage would $\Delta t$ be reduced if professional baseball switched to metal bats? Because pitchers do not wear any protective equipment on face or body, the situation is already dangerous and the switch would add to that danger.

**105** *Falling wrench.* A worker drops a wrench down the elevator shaft of a tall building. (a) Where is the wrench 1.5 s later? (b) How fast is the wrench falling just then?

**106** *Crash acceleration.* A car crashes head on into a wall and stops, with the front collapsing by 0.500 m. The driver is firmly held to the seat by a seat belt and thus moves forward by 0.500 m during the crash. Assume that the acceleration is constant during the crash. What is the magnitude of the driver's acceleration in $g$ units if the initial speed of the car is (a) 35 mi/h and (b) 70 mi/h?

**107** *Billboard distraction.* Highway billboards have long been a possible source of driver distraction, especially the modern electronic billboards with moving parts or with flipping from one scene to another within a few seconds. If you are traveling at 31.3 m/s (70 mi/h), how far along the road do you move if you look at a colorful and animated billboard for (a) 0.20 s (a glancing look), (b) 0.80 s, and (c) 2.0 s? Answer in both meters and in yards (to give you a feel for how your travel would be along an American football field).

**108** BIO CALC *Remote fall detection.* Falling is a chronic danger to the elderly and people subject to seizure. Researchers search for ways to detect a fall remotely so that a caretaker can go to the victim quickly. One way is to use a computer system that analyzes the motions of someone on CCTV in real time. The system monitors the vertical velocity of someone and then calculates the vertical acceleration when that velocity changes. If the system detects a large negative (downward) acceleration followed by a briefer positive acceleration and accompanied by a sound burst for the onset of the positive acceleration, a signal is sent to a caretaker. Figure 2.39 gives an idealized graph of vertical velocity $v$ versus time $t$ as determined by the system: $t_1 = 1.0$ s, $t_2 = 2.5$ s, $t_3 = 3.0$ s, $t_4 = 4.0$ s, $v_1 = -7.0$ m/s. (The plot on a more realistic graph would be curved.) What are (a) the acceleration during the descent and (b) the upward acceleration during the impact with the floor?

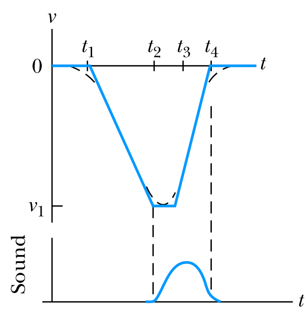


**Figure 2.39** Problem 108.

**109** *Ship speed in knots.* Before modern instrumentation, a ship's speed was measured with a line that had small knots tied along its length, separated by 47 feet 3 inches. The line was attached by three cords to a wood plate (a *clip log*) in the shape of a pie slice as shown in Fig. 2.40. One sailor threw the plate overboard and then allowed the force of the water against the plate to pull the line off a reel and through his hand so that he could detect the periodic passage of knots. Another sailor inverted a sandglass so that sand flowed from its upper chamber into the lower chamber in 28 s. During that interval the first sailor counted the number of knots passing through his hand. The result was the ship's speed in knots (abbreviated as kn). If 17 knots passed, what was the ship's speed in (a) knots, (b) miles per hour, and (c) kilometers per hour?

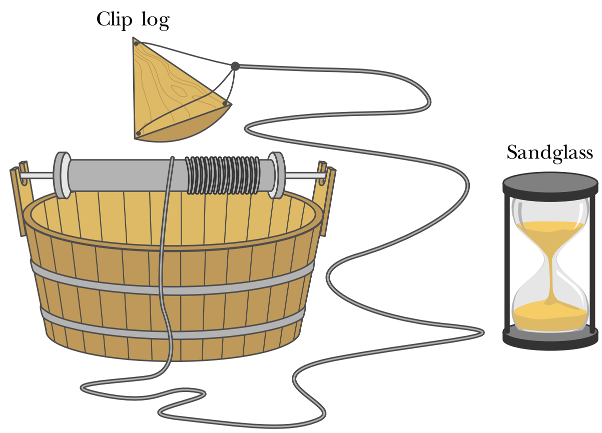


**Figure 2.40** Problem 109.