CHAPTER 25

Capacitance

25.1 CAPACITANCE

Learning Objectives

After reading this module, you should be able to . . .

25.1.1 Sketch a schematic diagram of a circuit with a parallel-plate capacitor, a battery, and an open or closed switch.

25.1.2 In a circuit with a battery, an open switch, and an uncharged capacitor, explain what happens to the conduction electrons when the switch is closed.

25.1.3 For a capacitor, apply the relationship between the magnitude of charge q on either plate ("the charge on the capacitor"), the potential difference V between the plates ("the potential across the capacitor"), and the capacitance C of the capacitor.

Key Ideas

- A capacitor consists of two isolated conductors (the plates) with charges $+q$ and $-q$. Its capacitance C is defined from

$$q = CV,$$

where V is the potential difference between the plates.

- When a circuit with a battery, an open switch, and an uncharged capacitor is completed by closing the switch, conduction electrons shift, leaving the capacitor plates with opposite charges.

What Is Physics?

One goal of physics is to provide the basic science for practical devices designed by engineers. The focus of this chapter is on one extremely common example—the capacitor, a device in which electrical energy can be stored. For example, the batteries in a camera store energy in the photoflash unit by charging a capacitor. The batteries can supply energy at only a modest rate, too slowly for the photoflash unit to emit a flash of light. However, once the capacitor is charged, it can supply energy at a much greater rate when the photoflash unit is triggered—enough energy to allow the unit to emit a burst of bright light.

The physics of capacitors can be generalized to other devices and to any situation involving electric fields. For example, Earth's atmospheric electric field is modeled by meteorologists as being produced by a huge spherical capacitor that partially discharges via lightning. The charge that skis collect as they slide along snow can be modeled as being stored in a capacitor that frequently discharges as sparks (which can be seen by nighttime skiers on dry snow).

Paul Silvermann/Fundamental Photographs

Figure 25.1.1 An assortment of capacitors.

The first step in our discussion of capacitors is to determine how much charge can be stored. This "how much" is called capacitance.

Capacitance

Figure 25.1.1 shows some of the many sizes and shapes of capacitors. Figure 25.1.2 shows the basic elements of *any* capacitor—two isolated conductors of any shape. No matter what their geometry, flat or not, we call these conductors *plates*.

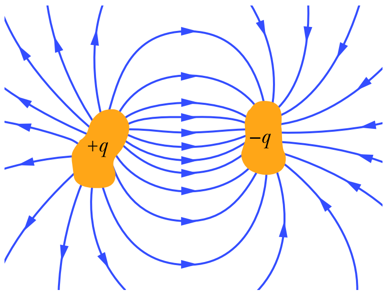


Figure 25.1.2 Two conductors, isolated electrically from each other and from their surroundings, form a *capacitor.* When the capacitor is charged, the charges on the conductors, or *plates* as they are called, have the same magnitude q but opposite signs.

Figure 25.1.3*a* shows a less general but more conventional arrangement, called a *parallel-plate capacitor,* consisting of two parallel conducting plates of area A separated by a distance d. The symbol we use to represent a capacitor (⊣⊢) is based on the structure of a parallel-plate capacitor but is used for capacitors of all geometries. We assume for the time being that no material medium (such as glass or plastic) is present in the region between the plates. In Module 25.5, we shall remove this restriction.

When a capacitor is *charged,* its plates have charges of equal magnitudes but opposite signs: $+q$ and $-q$. However, we refer to the *charge of a capacitor* as being q, the absolute value of these charges on the plates. (Note that q is not the net charge on the capacitor, which is zero.)

Because the plates are conductors, they are equipotential surfaces; all points on a plate are at the same electric potential. Moreover, there is a potential difference between the two plates. For historical reasons, we represent the absolute value of this potential difference with V rather than with the ΔV we used in previous notation.

The charge q and the potential difference V for a capacitor are proportional to each other; that is,

$$q = CV. \tag{25.1.1}$$

The proportionality constant C is called the **capacitance** of the capacitor. Its value depends only on the geometry of the plates and *not* on their charge or potential difference. The capacitance is a measure of how much charge must be put on the plates to produce a certain potential difference between them: The *greater the capacitance, the more charge is required.*

The SI unit of capacitance that follows from Eq. 25.1.1 is the coulomb per volt. This unit occurs so often that it is given a special name, the *farad* (F):

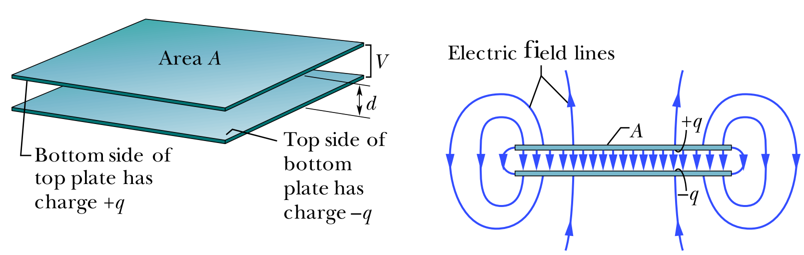
$$1 \text{ farad} = 1 \text{ F} = 1 \text{ coulomb per volt} = 1 \text{ C/V}. \tag{25.1.2}$$

As you will see, the farad is a very large unit. Submultiples of the farad, such as the microfarad ($1\ \mu\text{F} = 10^{-6}$ F) and the picofarad ($1 \text{ pF} = 10^{-12}$ F), are more convenient units in practice.

Charging a Capacitor

One way to charge a capacitor is to place it in an electric circuit with a battery. An *electric circuit* is a path through which charge can flow. A *battery* is a device that maintains a certain potential difference between its *terminals* (points at which charge can enter or leave the battery) by means of internal electrochemical reactions in which electric forces can move internal charge.

In Fig. 25.1.4*a*, a battery B, a switch S, an uncharged capacitor C, and interconnecting wires form a circuit. The same circuit is shown in the *schematic diagram* of Fig. 25.1.4*b*, in which the symbols for a battery, a switch, and a capacitor represent those devices. The battery maintains potential difference V between its terminals. The terminal of higher potential is labeled + and is often called the



(*a*) (*b*)

Figure 25.1.3 (*a*) A parallel-plate capacitor, made up of two plates of area A separated by a distance d. The charges on the facing plate surfaces have the same magnitude q but opposite signs. (*b*) As the field lines show, the electric field due to the charged plates is uniform in the central region between the plates. The field is not uniform at the edges of the plates, as indicated by the "fringing" of the field lines there.

positive terminal; the terminal of lower potential is labeled – and is often called the *negative* terminal.

The circuit shown in Figs. 25.1.4*a* and *b* is said to be *incomplete* because switch S is *open;* that is, the switch does not electrically connect the wires attached to it. When the switch is *closed,* electrically connecting those wires, the circuit is complete and charge can then flow through the switch and the wires. As we discussed in Chapter 21, the charge that can flow through a conductor, such as a wire, is that of electrons. When the circuit of Fig. 25.1.4 is completed, electrons are driven through the wires by an electric field that the battery sets up in the wires. The field drives electrons from capacitor plate h to the positive terminal of the battery; thus, plate h, losing electrons, becomes positively charged. The field drives just as many electrons from the negative terminal of the battery to capacitor plate l; thus, plate l, gaining electrons, becomes negatively charged *just as much* as plate h, losing electrons, becomes positively charged.

Initially, when the plates are uncharged, the potential difference between them is zero. As the plates become oppositely charged, that potential difference increases until it equals the potential difference V between the terminals of the battery. Then plate h and the positive terminal of the battery are at the same potential, and there is no longer an electric field in the wire between them. Similarly, plate l and the negative terminal reach the same potential, and there is then no electric field in the wire between them. Thus, with the field zero, there is no further drive of electrons. The capacitor is then said to be *fully charged,* with a potential difference V and charge q that are related by Eq. 25.1.1.

In this book we assume that during the charging of a capacitor and afterward, charge cannot pass from one plate to the other across the gap separating them. Also, we assume that a capacitor can retain (or *store*) charge indefinitely, until it is put into a circuit where it can be *discharged.*

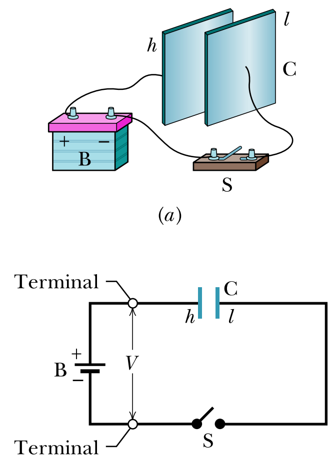


Figure 25.1.4 (*a*) Battery B, switch S, and plates h and l of capacitor C, connected in a circuit. (*b*) A schematic diagram with the *circuit elements* represented by their symbols.

Checkpoint 25.1.1

Does the capacitance C of a capacitor increase, decrease, or remain the same (a) when the charge q on it is doubled and (b) when the potential difference V across it is tripled?

25.2 CALCULATING THE CAPACITANCE

Learning Objectives

After reading this module, you should be able to . . .

25.2.1 Explain how Gauss' law is used to find the capacitance of a parallel-plate capacitor.

25.2.2 For a parallel-plate capacitor, a cylindrical capacitor, a spherical capacitor, and an isolated sphere, calculate the capacitance.

Key Ideas

- We generally determine the capacitance of a particular capacitor configuration by (1) assuming a charge q to have been placed on the plates, (2) finding the electric field $\vec{E}$ due to this charge, (3) evaluating the potential difference V between the plates, and (4) calculating C from $q = CV$. Some results are the following:
- A parallel-plate capacitor with flat parallel plates of area A and spacing d has capacitance

$$C = \frac{\varepsilon_0 A}{d}.$$

- A cylindrical capacitor (two long coaxial cylinders) of length L and radii a and b has capacitance

$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}.$$

- A spherical capacitor with concentric spherical plates of radii a and b has capacitance

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}.$$

- An isolated sphere of radius R has capacitance

$$C = 4\pi\varepsilon_0 R.$$

Calculating the Capacitance

Our goal here is to calculate the capacitance of a capacitor once we know its geometry. Because we shall consider a number of different geometries, it seems wise to develop a general plan to simplify the work. In brief our plan is as follows: (1) Assume a charge q on the plates; (2) calculate the electric field $\vec{E}$ between the plates in terms of this charge, using Gauss' law; (3) knowing $\vec{E}$, calculate the potential difference V between the plates from Eq. 24.2.4; (4) calculate C from Eq. 25.1.1.

Before we start, we can simplify the calculation of both the electric field and the potential difference by making certain assumptions. We discuss each in turn.

Calculating the Electric Field

To relate the electric field $\vec{E}$ between the plates of a capacitor to the charge q on either plate, we shall use Gauss' law:

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q. \tag{25.2.1}$$

Here q is the charge enclosed by a Gaussian surface and $\oint \vec{E} \cdot d\vec{A}$ is the net electric flux through that surface. In all cases that we shall consider, the Gaussian surface will be such that whenever there is an electric flux through it, $\vec{E}$ will have a uniform magnitude E and the vectors $\vec{E}$ and $d\vec{A}$ will be parallel. Equation 25.2.1 then reduces to

$$q = \varepsilon_0 EA \quad \text{(special case of Eq. 25.2.1)}, \tag{25.2.2}$$

in which A is the area of that part of the Gaussian surface through which there is a flux. For convenience, we shall always draw the Gaussian surface in such a way that it completely encloses the charge on the positive plate; see Fig. 25.2.1 for an example.

Calculating the Potential Difference

In the notation of Chapter 24 (Eq. 24.2.4), the potential difference between the plates of a capacitor is related to the field $\vec{E}$ by

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}, \tag{25.2.3}$$

in which the integral is to be evaluated along any path that starts on one plate and ends on the other. We shall always choose a path that follows an electric field line, from the negative plate to the positive plate. For this path, the vectors $\vec{E}$ and $d\vec{s}$ will have opposite directions; so the dot product $\vec{E} \cdot d\vec{s}$ will be equal to $-E\,ds$. Thus, the right side of Eq. 25.2.3 will then be positive. Letting V represent the difference $V_f - V_i$, we can then recast Eq. 25.2.3 as

$$V = \int_-^+ E\,ds \quad \text{(special case of Eq. 25.2.3)}, \tag{25.2.4}$$

in which the − and + remind us that our path of integration starts on the negative plate and ends on the positive plate.

We are now ready to apply Eqs. 25.2.2 and 25.2.4 to some particular cases.

We use Gauss' law to relate q and E. Then we integrate the E to get the potential difference.

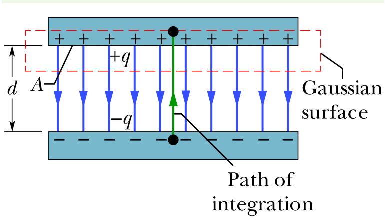


Figure 25.2.1 A charged parallel-plate capacitor. A Gaussian surface encloses the charge on the positive plate. The integration of Eq. 25.2.4 is taken along a path extending directly from the negative plate to the positive plate.

A Parallel-Plate Capacitor

We assume, as Fig. 25.2.1 suggests, that the plates of our parallel-plate capacitor are so large and so close together that we can neglect the fringing of the electric field at the edges of the plates, taking $\vec{E}$ to be constant throughout the region between the plates.

We draw a Gaussian surface that encloses just the charge q on the positive plate, as in Fig. 25.2.1. From Eq. 25.2.2 we can then write

$$q = \varepsilon_0 EA, \tag{25.2.5}$$

where A is the area of the plate.

Equation 25.2.4 yields

$$V = \int_{-}^{+} E\,ds = E\int_0^d ds = Ed. \tag{25.2.6}$$

In Eq. 25.2.6, E can be placed outside the integral because it is a constant; the second integral then is simply the plate separation d.

If we now substitute q from Eq. 25.2.5 and V from Eq. 25.2.6 into the relation $q = CV$ (Eq. 25.1.1), we find

$$C = \frac{\varepsilon_0 A}{d} \quad \text{(parallel-plate capacitor)}. \tag{25.2.7}$$

Thus, the capacitance does indeed depend only on geometrical factors—namely, the plate area A and the plate separation d. Note that C increases as we increase area A or decrease separation d.

As an aside, we point out that Eq. 25.2.7 suggests one of our reasons for writing the electrostatic constant in Coulomb's law in the form $1/4\pi\varepsilon_0$. If we had not done so, Eq. 25.2.7—which is used more often in engineering practice than Coulomb's law—would have been less simple in form. We note further that Eq. 25.2.7 permits us to express the permittivity constant ε_0 in a unit more appropriate for use in problems involving capacitors; namely,

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{F/m} = 8.85\ \text{pF/m}. \tag{25.2.8}$$

We have previously expressed this constant as

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2. \tag{25.2.9}$$

A Cylindrical Capacitor

Figure 25.2.2 shows, in cross section, a cylindrical capacitor of length L formed by two coaxial cylinders of radii a and b. We assume that $L \gg b$ so that we can neglect the fringing of the electric field that occurs at the ends of the cylinders. Each plate contains a charge of magnitude q.

As a Gaussian surface, we choose a cylinder of length L and radius r, closed by end caps and placed as is shown in Fig. 25.2.2. It is coaxial with the cylinders and encloses the central cylinder and thus also the charge q on that cylinder. Equation 25.2.2 then relates that charge and the field magnitude E as

$$q = \varepsilon_0 EA = \varepsilon_0 E(2\pi rL),$$

in which $2\pi rL$ is the area of the curved part of the Gaussian surface. There is no flux through the end caps. Solving for E yields

$$E = \frac{q}{2\pi\varepsilon_0 Lr}. \tag{25.2.10}$$

Substitution of this result into Eq. 25.2.4 yields

$$V = \int_{-}^{+} E\,ds = -\frac{q}{2\pi\varepsilon_0 L}\int_b^a \frac{dr}{r} = \frac{q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right), \tag{25.2.11}$$

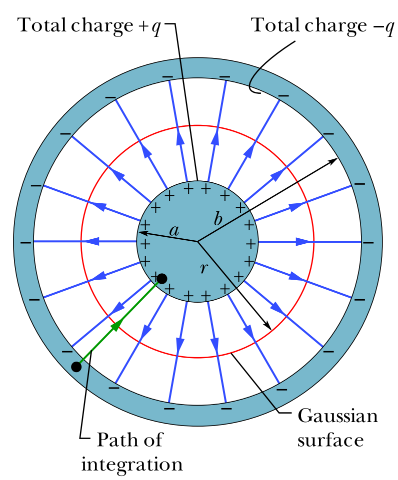


Figure 25.2.2 A cross section of a long cylindrical capacitor, showing a cylindrical Gaussian surface of radius r (that encloses the positive plate) and the radial path of integration along which Eq. 25.2.4 is to be applied. This figure also serves to illustrate a spherical capacitor in a cross section through its center.

where we have used the fact that here $ds = -dr$ (we integrated radially inward). From the relation $C = q/V$, we then have

$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)} \quad \text{(cylindrical capacitor).} \tag{25.2.12}$$

We see that the capacitance of a cylindrical capacitor, like that of a parallel-plate capacitor, depends only on geometrical factors, in this case the length L and the two radii b and a.

A Spherical Capacitor

Figure 25.2.2 can also serve as a central cross section of a capacitor that consists of two concentric spherical shells, of radii a and b. As a Gaussian surface we draw a sphere of radius r concentric with the two shells; then Eq. 25.2.2 yields

$$q = \varepsilon_0 EA = \varepsilon_0 E(4\pi r^2),$$

in which $4\pi r^2$ is the area of the spherical Gaussian surface. We solve this equation for E, obtaining

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}, \tag{25.2.13}$$

which we recognize as the expression for the electric field due to a uniform spherical charge distribution (Eq. 23.6.2).

If we substitute this expression into Eq. 25.2.4, we find

$$V = \int_-^+ E\,ds = -\frac{q}{4\pi\varepsilon_0}\int_b^a \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{q}{4\pi\varepsilon_0}\frac{b-a}{ab}, \tag{25.2.14}$$

where again we have substituted $-dr$ for ds. If we now substitute Eq. 25.2.14 into Eq. 25.1.1 and solve for C, we find

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} \quad \text{(spherical capacitor).} \tag{25.2.15}$$

An Isolated Sphere

We can assign a capacitance to a *single* isolated spherical conductor of radius R by assuming that the "missing plate" is a conducting sphere of infinite radius. After all, the field lines that leave the surface of a positively charged isolated conductor must end somewhere; the walls of the room in which the conductor is housed can serve effectively as our sphere of infinite radius.

To find the capacitance of the conductor, we first rewrite Eq. 25.2.15 as

$$C = 4\pi\varepsilon_0 \frac{a}{1 - a/b}.$$

If we then let $b \to \infty$ and substitute R for a, we find

$$C = 4\pi\varepsilon_0 R \quad \text{(isolated sphere).} \tag{25.2.16}$$

Note that this formula and the others we have derived for capacitance (Eqs. 25.2.7, 25.2.12, and 25.2.15) involve the constant ε_0 multiplied by a quantity that has the dimensions of a length.

Checkpoint 25.2.1

For capacitors charged by the same battery, does the charge stored by the capacitor increase, decrease, or remain the same in each of the following situations? (a) The plate separation of a parallel-plate capacitor is increased. (b) The radius of the inner cylinder of a cylindrical capacitor is increased. (c) The radius of the outer spherical shell of a spherical capacitor is increased.

Sample Problem 25.2.1 Charging the plates in a parallel-plate capacitor

In Fig. 25.2.3*a*, switch S is closed to connect the uncharged capacitor of capacitance $C = 0.25\ \mu F$ to the battery of potential difference $V = 12$ V. The lower capacitor plate has thickness $L = 0.50$ cm and face area $A = 2.0 \times 10^{-4}\ m^2$, and it consists of copper, in which the density of conduction electrons is $n = 8.49 \times 10^{28}$ electrons/m^3. From what depth d within the plate (Fig. 25.2.3*b*) must electrons move to the plate face as the capacitor becomes charged?

KEY IDEA

The charge collected on the plate is related to the capacitance and the potential difference across the capacitor by Eq. 25.1.1 ($q = CV$).

Calculations: Because the lower plate is connected to the negative terminal of the battery, conduction electrons move up to the face of the plate. From Eq. 25.1.1, the total charge magnitude that collects there is

$$q = CV = (0.25 \times 10^{-6}\ \text{F})(12\ \text{V}) = 3.0 \times 10^{-6}\ \text{C}.$$

Dividing this result by e gives us the number N of conduction electrons that come up to the face:

$$N = \frac{q}{e} = \frac{3.0 \times 10^{-6}\ \text{C}}{1.602 \times 10^{-19}\ \text{C}} = 1.873 \times 10^{13}\ \text{electrons.}$$

These electrons come from a volume that is the product of the face area A and the depth d we seek. Thus, from the density of conduction electrons (number per volume), we can write

$$n = \frac{N}{Ad},$$

or

$$d = \frac{N}{An} = \frac{1.873 \times 10^{13}\ \text{electrons}}{(2.0 \times 10^{-4}\ \text{m}^2)(8.49 \times 10^{28}\ \text{electrons/m}^3)}$$
$$= 1.1 \times 10^{-12}\ \text{m} = 1.1\ \text{pm}. \qquad \text{(Answer)}$$

We commonly say that electrons move from the battery to the negative face but, actually, the battery sets up an electric field in the wires and plate such that electrons very close to the plate face move up to the negative face.

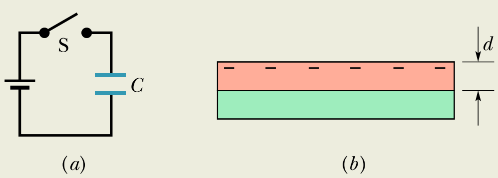


Figure 25.2.3 (*a*) A battery and capacitor circuit. (*b*) The lower capacitor plate.

25.3 CAPACITORS IN PARALLEL AND IN SERIES

Learning Objectives

After reading this module, you should be able to . . .

25.3.1 Sketch schematic diagrams for a battery and (a) three capacitors in parallel and (b) three capacitors in series.

25.3.2 Identify that capacitors in parallel have the same potential difference, which is the same value that their equivalent capacitor has.

25.3.3 Calculate the equivalent of parallel capacitors.

25.3.4 Identify that the total charge stored on parallel capacitors is the sum of the charges stored on the individual capacitors.

25.3.5 Identify that capacitors in series have the same charge, which is the same value that their equivalent capacitor has.

25.3.6 Calculate the equivalent of series capacitors.

25.3.7 Identify that the potential applied to capacitors in series is equal to the sum of the potentials across the individual capacitors.

25.3.8 For a circuit with a battery and some capacitors in parallel and some in series, simplify the circuit in steps by finding equivalent capacitors, until the charge and potential on the final equivalent capacitor can be determined, and then reverse the steps to find the charge and potential on the individual capacitors.

25.3.9 For a circuit with a battery, an open switch, and one or more uncharged capacitors, determine the amount of charge that moves through a point in the circuit when the switch is closed.

25.3.10 When a charged capacitor is connected in parallel to one or more uncharged capacitors, determine the charge and potential difference on each capacitor when equilibrium is reached.

Key Idea

- The equivalent capacitances C_{eq} of combinations of individual capacitors connected in parallel and in series can be found from

$$C_{eq} = \sum_{j=1}^{n} C_j \quad (n \text{ capacitors in parallel})$$

and

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j} \quad (n \text{ capacitors in series}).$$

Equivalent capacitances can be used to calculate the capacitances of more complicated series–parallel combinations.

Capacitors in Parallel and in Series

When there is a combination of capacitors in a circuit, we can sometimes replace that combination with an **equivalent capacitor**—that is, a single capacitor that has the same capacitance as the actual combination of capacitors. With such a replacement, we can simplify the circuit, affording easier solutions for unknown quantities of the circuit. Here we discuss two basic combinations of capacitors that allow such a replacement.

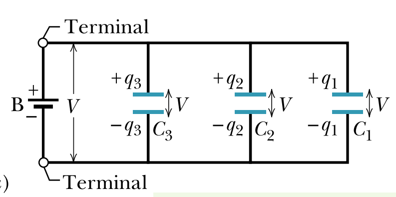
Capacitors in Parallel

Figure 25.3.1*a* shows an electric circuit in which three capacitors are connected *in parallel* to battery B. This description has little to do with how the capacitor plates are drawn. Rather, "in parallel" means that the capacitors are directly wired together at one plate and directly wired together at the other plate, and that the same potential difference V is applied across the two groups of wired-together plates. Thus, each capacitor has the same potential difference V, which produces charge on the capacitor. (In Fig. 25.3.1*a*, the applied potential V is maintained by the battery.) In general:

> When a potential difference V is applied across several capacitors connected in parallel, that potential difference V is applied across each capacitor. The total charge q stored on the capacitors is the sum of the charges stored on all the capacitors.

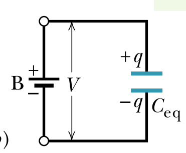
When we analyze a circuit of capacitors in parallel, we can simplify it with this mental replacement:

> Capacitors connected in parallel can be replaced with an equivalent capacitor that has the same *total* charge q and the same potential difference V as the actual capacitors.



(*a*)

Parallel capacitors and their equivalent have the same V ("par-V").



(*b*)

Figure 25.3.1 (*a*) Three capacitors connected in parallel to battery B. The battery maintains potential difference V across its terminals and thus across *each* capacitor. (*b*) The equivalent capacitor, with capacitance C_{eq}, replaces the parallel combination.

(You might remember this result with the nonsense word "par-V," which is close to "party," to mean "capacitors in parallel have the same V.") Figure 25.3.1*b* shows the equivalent capacitor (with equivalent capacitance C_{eq}) that has replaced the three capacitors (with actual capacitances C_1, C_2, and C_3) of Fig. 25.3.1*a*.

To derive an expression for C_{eq} in Fig. 25.3.1*b*, we first use Eq. 25.1.1 to find the charge on each actual capacitor:

$$q_1 = C_1V, \quad q_2 = C_2V, \quad \text{and} \quad q_3 = C_3V.$$

The total charge on the parallel combination of Fig. 25.3.1*a* is then

$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V.$$

The equivalent capacitance, with the same total charge q and applied potential difference V as the combination, is then

$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3,$$

a result that we can easily extend to any number n of capacitors, as

$$C_{eq} = \sum_{j=1}^{n} C_j \quad (n \text{ capacitors in parallel}). \tag{25.3.1}$$

Thus, to find the equivalent capacitance of a parallel combination, we simply add the individual capacitances.

Capacitors in Series

Figure 25.3.2*a* shows three capacitors connected *in series* to battery B. This description has little to do with how the capacitors are drawn. Rather, "in series" means that the capacitors are wired serially, one after the other, and that a potential difference V is applied across the two ends of the series. (In Fig. 25.3.2*a*, this potential difference V is maintained by battery B.) The potential differences that then exist across the capacitors in the series produce identical charges q on them.

> When a potential difference V is applied across several capacitors connected in series, the capacitors have identical charge q. The sum of the potential differences across all the capacitors is equal to the applied potential difference V.

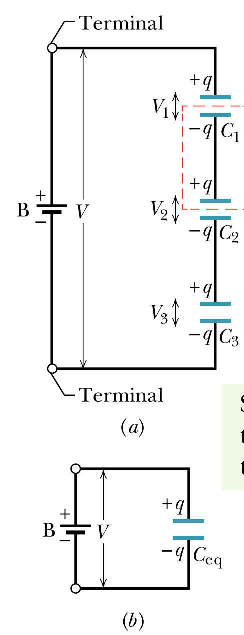
We can explain how the capacitors end up with identical charge by following a *chain reaction* of events, in which the charging of each capacitor causes the charging of the next capacitor. We start with capacitor 3 and work upward to capacitor 1. When the battery is first connected to the series of capacitors, it produces charge $-q$ on the bottom plate of capacitor 3. That charge then repels negative charge from the top plate of capacitor 3 (leaving it with charge $+q$). The repelled negative charge moves to the bottom plate of capacitor 2 (giving it charge $-q$). That charge on the bottom plate of capacitor 2 then repels negative charge from the top plate of capacitor 2 (leaving it with charge $+q$) to the bottom plate of capacitor 1 (giving it charge $-q$). Finally, the charge on the bottom plate of capacitor 1 helps move negative charge from the top plate of capacitor 1 to the battery, leaving that top plate with charge $+q$.

Here are two important points about capacitors in series:

1. When charge shifts from one capacitor to another in a capacitor series, it can move along only one route, such as from capacitor 3 to capacitor 2 in Fig. 25.3.2*a*. If there are additional routes, the capacitors are not in series.
2. The battery directly produces charges on only the two plates to which it is connected (the bottom plate of capacitor 3 and the top plate of capacitor 1 in Fig. 25.3.2*a*). Charges that are produced on the other plates are due merely to the shifting of charge already there. For example, in Fig. 25.3.2*a*, the part of the circuit enclosed by dashed lines is electrically isolated from the rest of the circuit. Thus, its charge can only be redistributed.

When we analyze a circuit of capacitors in series, we can simplify it with this mental replacement:

> Capacitors that are connected in series can be replaced with an equivalent capacitor that has the same charge q and the same *total* potential difference V as the actual series capacitors.



Series capacitors and their equivalent have the same q ("seri-q").

Figure 25.3.2 (*a*) Three capacitors connected in series to battery B. The battery maintains potential difference V between the top and bottom plates of the series combination. (*b*) The equivalent capacitor, with capacitance C_{eq}, replaces the series combination.

(You might remember this with the nonsense word "seri-q" to mean "capacitors in series have the same q.") Figure 25.3.2*b* shows the equivalent capacitor (with equivalent capacitance C_{eq}) that has replaced the three actual capacitors (with actual capacitances C_1, C_2, and C_3) of Fig. 25.3.2*a*.

To derive an expression for C_{eq} in Fig. 25.3.2*b*, we first use Eq. 25.1.1 to find the potential difference of each actual capacitor:

$$V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad \text{and} \quad V_3 = \frac{q}{C_3}.$$

The total potential difference V due to the battery is the sum

$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right).$$

The equivalent capacitance is then

$$C_{eq} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3},$$

or

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

We can easily extend this to any number n of capacitors as

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j} \quad (n \text{ capacitors in series}). \tag{25.3.2}$$

Using Eq. 25.3.2 you can show that the equivalent capacitance of a series of capacitances is always *less* than the least capacitance in the series.

Checkpoint 25.3.1

A battery of potential V stores charge q on a combination of two identical capacitors. What are the potential difference across and the charge on either capacitor if the capacitors are (a) in parallel and (b) in series?

Sample Problem 25.3.1 Capacitors in parallel and in series

(a) Find the equivalent capacitance for the combination of capacitances shown in Fig. 25.3.3*a*, across which potential difference V is applied. Assume

$$C_1 = 12.0\,\mu\text{F}, \quad C_2 = 5.30\,\mu\text{F}, \quad \text{and} \quad C_3 = 4.50\,\mu\text{F}.$$

KEY IDEA

Any capacitors connected in series can be replaced with their equivalent capacitor, and any capacitors connected in parallel can be replaced with their equivalent capacitor. Therefore, we should first check whether any of the capacitors in Fig. 25.3.3*a* are in parallel or series.

Finding equivalent capacitance: Capacitors 1 and 3 are connected one after the other, but are they in series? No. The potential V that is applied to the capacitors produces charge on the bottom plate of capacitor 3. That charge causes charge to shift from the top plate of capacitor 3. However, note that the shifting charge can move to the bottom plates of both capacitor 1 and capacitor 2. Because there is more than one route for the shifting charge, capacitor 3 is not in series with capacitor 1 (or capacitor 2). Any time you think you might have two capacitors in series, apply this check about the shifting charge.

Are capacitor 1 and capacitor 2 in parallel? Yes. Their top plates are directly wired together and their bottom plates are directly wired together, and electric potential is applied between the top-plate pair and the bottom-plate pair. Thus, capacitor 1 and capacitor 2 are in parallel, and Eq. 25.3.1 tells us that their equivalent capacitance C_{12} is

$$C_{12} = C_1 + C_2 = 12.0\,\mu\text{F} + 5.30\,\mu\text{F} = 17.3\,\mu\text{F}.$$

In Fig. 25.3.3*b*, we have replaced capacitors 1 and 2 with their equivalent capacitor, called capacitor 12 (say "one two" and not "twelve"). (The connections at points A and B are exactly the same in Figs. 25.3.3*a* and *b*.)

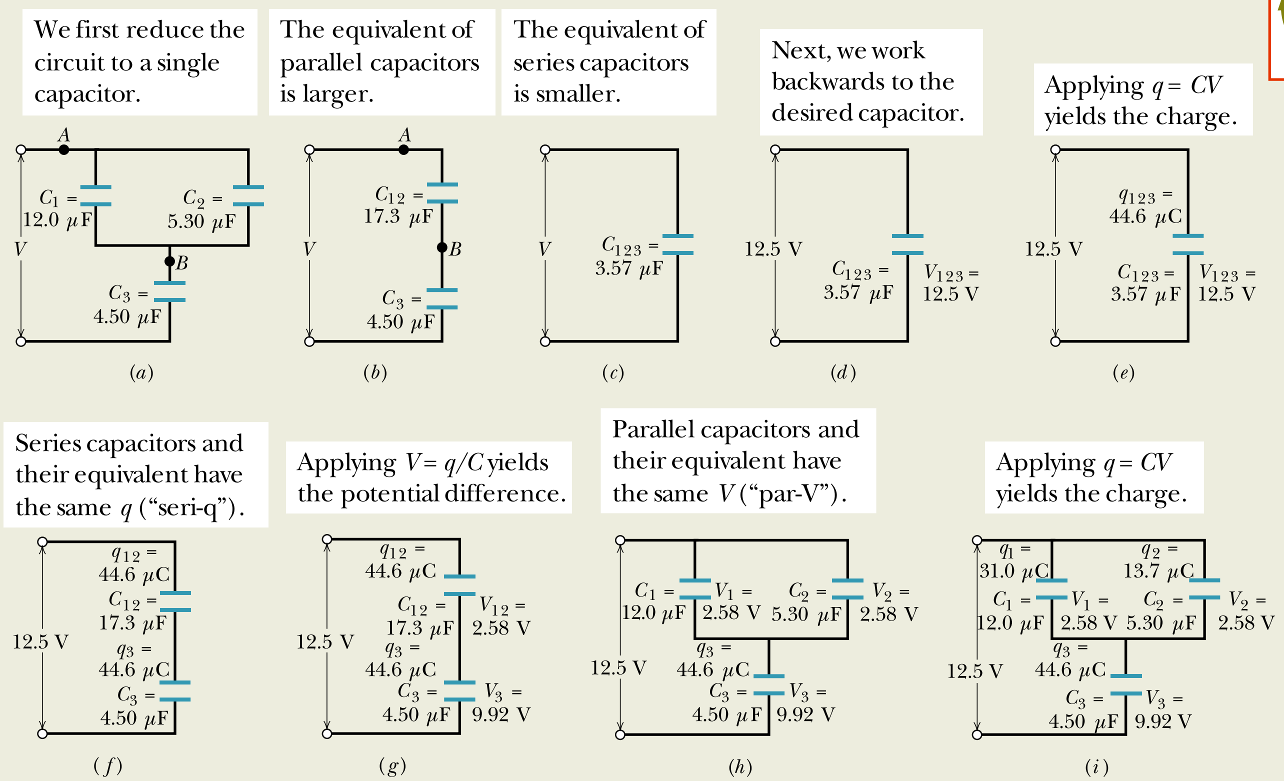


Figure 25.3.3 (*a*)–(*d*) Three capacitors are reduced to one equivalent capacitor. (*e*)–(*i*) Working backwards to get the charges.

Is capacitor 12 in series with capacitor 3? Again applying the test for series capacitances, we see that the charge that shifts from the top plate of capacitor 3 must entirely go to the bottom plate of capacitor 12. Thus, capacitor 12 and capacitor 3 are in series, and we can replace them with their equivalent C_{123} ("one two three"), as shown in Fig. 25.3.3*c*. From Eq. 25.3.2, we have

$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{1}{17.3\,\mu\text{F}} + \frac{1}{4.50\,\mu\text{F}} = 0.280\,\mu\text{F}^{-1},$$

from which

$$C_{123} = \frac{1}{0.280\,\mu\text{F}^{-1}} = 3.57\,\mu\text{F}. \qquad \text{(Answer)}$$

(b) The potential difference applied to the input terminals in Fig. 25.3.3*a* is $V = 12.5$ V. What is the charge on C_1?

KEY IDEAS

We now need to work backwards from the equivalent capacitance to get the charge on a particular capacitor. We have two techniques for such "backwards work": (1) Seri-q: Series capacitors have the same charge as their equivalent capacitor. (2) Par-V: Parallel capacitors have the same potential difference as their equivalent capacitor.

Working backwards: To get the charge q_1 on capacitor 1, we work backwards to that capacitor, starting with the equivalent capacitor 123. Because the given potential difference V (= 12.5 V) is applied across the actual combination of three capacitors in Fig. 25.3.3*a*, it is also applied across C_{123} in Figs. 25.3.3*d* and *e*. Thus, Eq. 25.1.1 ($q = CV$) gives us

$$q_{123} = C_{123}V = (3.57\,\mu\text{F})(12.5\text{ V}) = 44.6\,\mu\text{C}.$$

The series capacitors 12 and 3 in Fig. 25.3.3*b* each have the same charge as their equivalent capacitor 123 (Fig. 25.3.3*f*). Thus, capacitor 12 has charge $q_{12} = q_{123} = 44.6\,\mu$C. From Eq. 25.1.1 and Fig. 25.3.3*g*, the potential difference across capacitor 12 must be

$$V_{12} = \frac{q_{12}}{C_{12}} = \frac{44.6\,\mu\text{C}}{17.3\,\mu\text{F}} = 2.58\text{ V}.$$

The parallel capacitors 1 and 2 each have the same potential difference as their equivalent capacitor 12 (Fig. 25.3.3*h*). Thus, capacitor 1 has potential difference $V_1 = V_{12} = 2.58$ V, and, from Eq. 25.1.1 and Fig. 25.3.3*i*, the charge on capacitor 1 must be

$$q_1 = C_1V_1 = (12.0\,\mu\text{F})(2.58\text{ V}) = 31.0\,\mu\text{C}. \qquad \text{(Answer)}$$

Sample Problem 25.3.2 One capacitor charging up another capacitor

Capacitor 1, with $C_1 = 3.55\,\mu$F, is charged to a potential difference $V_0 = 6.30$ V, using a 6.30 V battery. The battery is then removed, and the capacitor is connected as in Fig. 25.3.4to an uncharged capacitor 2, with $C_2 = 8.95\,\mu$F. When switch S is closed, charge flows between the capacitors. Find the charge on each capacitor when equilibrium is reached.

KEY IDEAS

The situation here differs from the previous example because here an applied electric potential is *not* maintained across a combination of capacitors by a battery or some other source. Here, just after switch S is closed, the only applied electric potential is that of capacitor 1 on capacitor 2, and that potential is decreasing. Thus, the capacitors in Fig. 25.3.4 are not connected *in series;* and although they are drawn parallel, in this situation they are not *in parallel.*

As the electric potential across capacitor 1 decreases, that across capacitor 2 increases. Equilibrium is reached when the two potentials are equal because, with no potential difference between connected plates of the capacitors, there is no electric field within the connecting wires to move conduction electrons. The initial charge on capacitor 1 is then shared between the two capacitors.

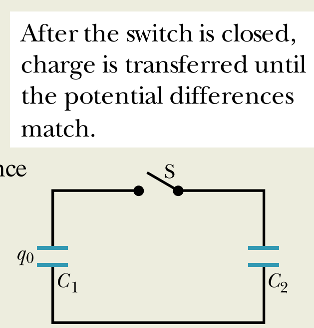


Figure 25.3.4 A potential difference V_0 is applied to capacitor 1 and the charging battery is removed. Switch S is then closed so that the charge on capacitor 1 is shared with capacitor 2.

Calculations: Initially, when capacitor 1 is connected to the battery, the charge it acquires is, from Eq. 25.1.1,

$$q_0 = C_1V_0 = (3.55 \times 10^{-6}\text{ F})(6.30\text{ V})$$
$$= 22.365 \times 10^{-6}\text{ C}.$$

When switch S in Fig. 25.3.4 is closed and capacitor 1 begins to charge capacitor 2, the electric potential and charge on capacitor 1 decrease and those on capacitor 2 increase until

$$V_1 = V_2 \quad \text{(equilibrium)}.$$

From Eq. 25.1.1, we can rewrite this as

$$\frac{q_1}{C_1} = \frac{q_2}{C_2} \quad \text{(equilibrium)}.$$

Because the total charge cannot magically change, the total after the transfer must be

$$q_1 + q_2 = q_0 \quad \text{(charge conservation)};$$

thus

$$q_2 = q_0 - q_1.$$

We can now rewrite the second equilibrium equation as

$$\frac{q_1}{C_1} = \frac{q_0 - q_1}{C_2}.$$

Solving this for q_1 and substituting given data, we find

$$q_1 = 6.35\,\mu\text{C}. \qquad \text{(Answer)}$$

The rest of the initial charge ($q_0 = 22.365\,\mu$C) must be on capacitor 2:

$$q_2 = 16.0\,\mu\text{C}. \qquad \text{(Answer)}$$

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25.4 ENERGY STORED IN AN ELECTRIC FIELD

Learning Objectives

After reading this module, you should be able to . . .

25.4.1 Explain how the work required to charge a capacitor results in the potential energy of the capacitor.

25.4.2 For a capacitor, apply the relationship between the potential energy U, the capacitance C, and the potential difference V.

25.4.3 For a capacitor, apply the relationship between the potential energy, the internal volume, and the internal energy density.

25.4.4 For any electric field, apply the relationship between the potential energy density u in the field and the field's magnitude E.

25.4.5 Explain the danger of sparks in airborne dust.

Key Ideas

- The electric potential energy U of a charged capacitor,

$$U = \frac{q^2}{2c} = \tfrac{1}{2}CV^2,$$

is equal to the work required to charge the capacitor. This energy can be associated with the capacitor's electric field $\vec{E}$.

- Every electric field, in a capacitor or from any other source, has an associated stored energy. In vacuum, the energy density u (potential energy per unit volume) in a field of magnitude E is

$$u = \tfrac{1}{2}\varepsilon_0 E^2.$$

Energy Stored in an Electric Field

Work must be done by an external agent to charge a capacitor. We can imagine doing the work ourselves by transferring electrons from one plate to the other, one by one. As the charges build, so does the electric field between the plates, which opposes the continued transfer. So, greater amounts of work are required. Actually, a battery does all this for us, at the expense of its stored chemical energy. We visualize the work as being stored as electric potential energy in the electric field between the plates.

Suppose that, at a given instant, a charge q' has been transferred from one plate of a capacitor to the other. The potential difference V' between the plates at that instant will be q'/C. If an extra increment of charge dq' is then transferred, the increment of work required will be, from Eq. 24.1.6,

$$dW = V'\,dq' = \frac{q'}{C}\,dq'.$$

The work required to bring the total capacitor charge up to a final value q is

$$W = \int dW = \frac{1}{C}\int_0^q q'\,dq' = \frac{q^2}{2C}.$$

This work is stored as potential energy U in the capacitor, so that

$$U = \frac{q^2}{2C} \quad \text{(potential energy).} \tag{25.4.1}$$

From Eq. 25.1.1, we can also write this as

$$U = \tfrac{1}{2}CV^2 \quad \text{(potential energy).} \tag{25.4.2}$$

Equations 25.4.1 and 25.4.2 hold no matter what the geometry of the capacitor is.

To gain some physical insight into energy storage, consider two parallel-plate capacitors that are identical except that capacitor 1 has twice the plate separation of capacitor 2. Then capacitor 1 has twice the volume between its plates and also, from Eq. 25.2.7, half the capacitance of capacitor 2. Equation 25.2.2 tells us that if both capacitors have the same charge q, the electric fields between their plates are identical. And Eq. 25.4.1 tells us that capacitor 1 has twice the stored potential energy of capacitor 2. Thus, of two otherwise identical capacitors with the same charge and same electric field, the one with twice the volume between its plates has twice the stored potential energy. Arguments like this tend to verify our earlier assumption:

> The potential energy of a charged capacitor may be viewed as being stored in the electric field between its plates.

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Figure 25.4.1 The result of a grain explosion.

Explosions in Airborne Dust

As we discussed in Module 24.8, making contact with certain materials, such as clothing, carpets, and even playground slides, can leave you with a significant electrical potential. You might become painfully aware of that potential if a spark leaps between you and a grounded object, such as a faucet. In many industries involving the production and transport of powder, such as in the cosmetic and food industries, such a spark can be disastrous. Although the powder in bulk may not burn at all, when individual powder grains are airborne and thus surrounded by oxygen, they can burn so fiercely that a cloud of the grains burns as an explosion. Figure 25.4.1 shows the result of such a grain explosion. Safety engineers cannot eliminate all possible sources of sparks in the powder industries. Instead, they attempt to keep the amount of energy available in the sparks below the threshold value U_t ($\approx$ 150 mJ) typically required to ignite airborne grains.

Energy Density

In a parallel-plate capacitor, neglecting fringing, the electric field has the same value at all points between the plates. Thus, the **energy density** u—that is, the potential energy per unit volume between the plates—should also be uniform. We can find u by dividing the total potential energy by the volume Ad of the space between the plates. Using Eq. 25.4.2, we obtain

$$u = \frac{U}{Ad} = \frac{CV^2}{2Ad}. \tag{25.4.3}$$

With Eq. 25.2.7 ($C = \varepsilon_0 A/d$), this result becomes

$$u = \tfrac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2. \tag{25.4.4}$$

However, from Eq. 24.6.5 ($E = -\Delta V/\Delta s$), V/d equals the electric field magnitude E; so

$$u = \tfrac{1}{2}\varepsilon_0 E^2 \qquad \text{(energy density)}. \tag{25.4.5}$$

Although we derived this result for the special case of an electric field of a parallel-plate capacitor, it holds for any electric field. If an electric field $\vec{E}$ exists at any point in space, that site has an electric potential energy with a density (amount per unit volume) given by Eq. 25.4.5.

Checkpoint 25.4.1

Capacitors 1 and 2 are air-filled and identical except that capacitor 1 has twice the plate separation as capacitor 2: $d_1 = 2d_2$. They have equal charges. (a) How do the magnitudes of the electric fields between the plates compare: $E_1 > E_2$, $E_1 < E_2$, or $E_1 = E_2$? (b) How does the volume Vol_1 of capacitor 1 (the volume between the plates) compare with the volume Vol_2 of capacitor 2: $\text{Vol}_1 = \text{Vol}_2$, $\text{Vol}_1 = 2(\text{Vol}_2)$, or $\text{Vol}_1 = (\text{Vol}_2)/2$? (c) How does the potential energy U_1 of capacitor 1 compare with the potential energy U_2 of capacitor 2: $U_1 = U_2$, $U_1 = 2U_2$, or $U_1 = U_2/2$?

Sample Problem 25.4.1 Fire with hospital gurney

Often, a burn victim is treated while lying on a gurney in an enclosed chamber filled with oxygen-enriched air, called a hyperbaric chamber (Fig. 25.4.2*a*). Once a treatment session is over, a hospital worker pulls the gurney and patient from the chamber onto a trolley, to be rolled away. On at least two occasions, the gurney caught fire at the end that was last to leave the chamber. Obviously, a burning gurney holding a patient already suffering from

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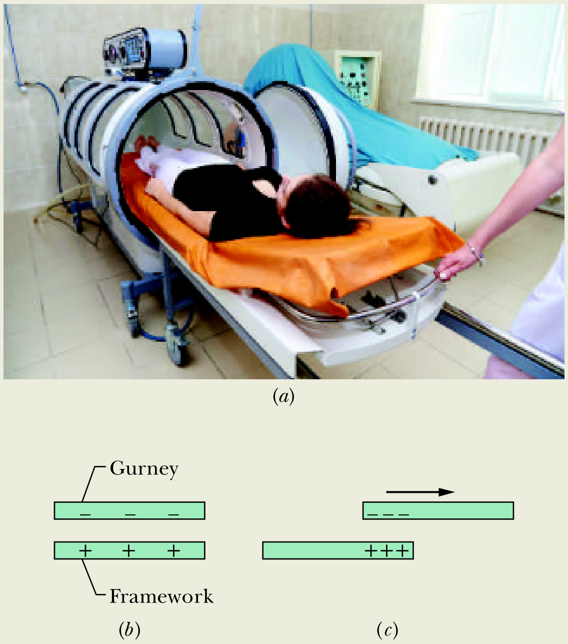

Figure 25.4.2 (*a*) A hyperbaric chamber. (*b*) A gurney and the chamber's metal framework form a capacitor that is charged by stray electrostatic charge. (*c*) As the gurney is pulled from the chamber, the charge crowds onto a smaller area.

burns is a dangerous situation, and obviously fires burn easily in air rich in oxygen, but the question remains: What caused the gurneys to catch fire?

Investigators realized that charge separation occurred between the patient's skin, the hospital gown on the patient, and the sheet on the gurney. They also found that the gurney and the part of the chamber's metal framework below the gurney formed a parallel-plate capacitor (Fig. 25.4.2*b*) of capacitance $C_i = 250$ pF. If the gurney discharged its excess charge and the associated energy by sparking, could the spark ignite the gurney? Measurements revealed that a spark could occur only if the potential difference V on the gurney–framework capacitor exceeded 2000 V and that a fire could start only if the capacitor's potential energy U exceeded 0.20 mJ. However, the potential difference on the gurney–framework capacitor was only $V_i = 600$ V, not enough to produce a spark.

(a) As the gurney was withdrawn from the chamber, the area of the gurney–framework overlap decreased (Fig. 25.4.2*c*). Thus, the plate area of the capacitor decreased from its initial value A_i. What was the potential difference V_f when the overlap plate area was $A_f = 0.10A_i$?

KEY IDEAS

(1) The potential difference V across a capacitor is related to the charge q and capacitance C according to $q = CV$. (2) As the gurney was withdrawn from the chamber, the charge q did not change. (3) The capacitance of a parallel-plate capacitor is related to the plate area: $C = \varepsilon_0 A/d$.

Calculations: From Eq. 25.1.1, the charge q was

$$q = C_i V_i = C_f V_f,$$

and so

$$V_f = \frac{C_i}{C_f} V_i.$$

We can now write

$$C_f = \frac{\varepsilon_0 A_f}{d} = \frac{\varepsilon_0 (0.10 A_i)}{d}$$
$$= 0.10 \frac{\varepsilon_0 A_i}{d} = 0.10 C_i.$$

Substituting this into our expressions for the potential differences gives us

$$V_f = \frac{C_i}{0.10 C_i} V_i = 10 V_i = (10)(600\text{ V})$$
$$= 6000\text{ V}.$$

As the gurney was withdrawn, the potential difference increased because the charge on the capacitor was crowded into a smaller plate area. The potential difference $V_f = 6000$ V was more than enough to produce a spark.

(b) What was the energy U_f of the gurney–framework capacitor when the plate area was $0.10A_i$?

KEY IDEA

The potential energy U stored in a capacitor is related to the capacitance C and potential difference V according to $U = \frac{1}{2}CV^2$.

Calculation: Using our result of $C_f = 0.10C_i$, we write

$$U_f = \tfrac{1}{2} C_f V_f^2 = \tfrac{1}{2}(0.10 C_i) V_f^2$$
$$= \tfrac{1}{2}(0.10)(250 \times 10^{-12}\text{ F})(6000\text{ V})^2$$
$$= 4.5 \times 10^{-4}\text{ J} = 0.45\text{ mJ}.$$

This was more than enough energy to ignite the gurney. The investigators concluded that the gurney fire was due to a spark produced by the gurney–framework capacitor as the charge became crowded into a smaller area while the gurney was being withdrawn from the chamber.

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25.5 CAPACITOR WITH A DIELECTRIC

Learning Objectives

After reading this module, you should be able to . . .

25.5.1 Identify that capacitance is increased if the space between the plates is filled with a dielectric material.

25.5.2 For a capacitor, calculate the capacitance with and without a dielectric.

25.5.3 For a region filled with a dielectric material with a given dielectric constant κ, identify that all electrostatic equations containing the permittivity constant ε_0 are modified by multiplying that constant by the dielectric constant to get $\kappa\varepsilon_0$.

25.5.4 Name some of the common dielectrics.

25.5.5 In adding a dielectric to a charged capacitor, distinguish the results for a capacitor (a) connected to a battery and (b) not connected to a battery.

25.5.6 Distinguish polar dielectrics from nonpolar dielectrics.

25.5.7 In adding a dielectric to a charged capacitor, explain what happens to the electric field between the plates in terms of what happens to the atoms in the dielectric.

Key Ideas

- If the space between the plates of a capacitor is completely filled with a dielectric material, the capacitance C in vacuum (or, effectively, in air) is multiplied by the material's dielectric constant κ, which is a number greater than 1.
- In a region that is completely filled by a dielectric, all electrostatic equations containing the permittivity constant ε_0 must be modified by replacing ε_0 with $\kappa\varepsilon_0$.
- When a dielectric material is placed in an external electric field, it develops an internal electric field that is oriented opposite the external field, thus reducing the magnitude of the electric field inside the material.
- When a dielectric material is placed in a capacitor with a fixed amount of charge on the surface, the net electric field between the plates is decreased.

Capacitor with a Dielectric

If you fill the space between the plates of a capacitor with a *dielectric,* which is an insulating material such as mineral oil or plastic, what happens to the capacitance? Michael Faraday—to whom the whole concept of capacitance is largely due and for whom the SI unit of capacitance is named—first looked into this matter in 1837. Using simple equipment much like that shown in Fig. 25.5.1, he found that the capacitance *increased* by a numerical factor κ, which he called the

Figure 25.5.1 The simple electrostatic apparatus used by Faraday. An assembled apparatus (second from left) forms a spherical capacitor consisting of a central brass ball and a concentric brass shell. Faraday placed dielectric materials in the space between the ball and the shell.

dielectric constant of the insulating material. Table 25.5.1 shows some dielectric materials and their dielectric constants. The dielectric constant of a vacuum is unity by definition. Because air is mostly empty space, its measured dielectric constant is only slightly greater than unity. Even common paper can significantly increase the capacitance of a capacitor, and some materials, such as strontium titanate, can increase the capacitance by more than two orders of magnitude.

Another effect of the introduction of a dielectric is to limit the potential difference that can be applied between the plates to a certain value V_{max}, called the *breakdown potential.* If this value is substantially exceeded, the dielectric material will break down and form a conducting path between the plates. Every dielectric material has a characteristic *dielectric strength,* which is the maximum value of the electric field that it can tolerate without breakdown. A few such values are listed in Table 25.5.1.

As we discussed just after Eq. 25.2.16, the capacitance of any capacitor can be written in the form

$$C = \varepsilon_0 \mathscr{L}, \tag{25.5.1}$$

in which $\mathscr{L}$ has the dimension of length. For example, $\mathscr{L} = A/d$ for a parallel-plate capacitor. Faraday's discovery was that, with a dielectric *completely* filling the space between the plates, Eq. 25.5.1 becomes

$$C = \kappa\varepsilon_0 \mathscr{L} = \kappa C_{air}, \tag{25.5.2}$$

where C_{air} is the value of the capacitance with only air between the plates. For example, if we fill a capacitor with strontium titanate, with a dielectric constant of 310, we multiply the capacitance by 310.

Figure 25.5.2 provides some insight into Faraday's experiments. In Fig. 25.5.2*a* the battery ensures that the potential difference V between the plates will remain constant. When a dielectric slab is inserted between the plates, the charge q on the plates increases by a factor of κ; the additional charge is delivered to the capacitor plates by the battery. In Fig. 25.5.2*b* there is no battery, and therefore the charge q must remain constant when the dielectric slab is inserted; then the potential difference V between the plates decreases by a factor of κ. Both these observations are consistent (through the relation $q = CV$) with the increase in capacitance caused by the dielectric.

Comparison of Eqs. 25.5.1 and 25.5.2 suggests that the effect of a dielectric can be summed up in more general terms:

In a region completely filled by a dielectric material of dielectric constant κ, all electrostatic equations containing the permittivity constant ε_0 are to be modified by replacing ε_0 with $\kappa\varepsilon_0$.

Table 25.5.1 Some Properties of Dielectrics[a]

Material	Dielectric Constant κ	Dielectric Strength (kV/mm)
Air (1 atm)	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer oil	4.5	
Pyrex	4.7	14
Ruby mica	5.4	
Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20°C)	80.4	
Water (25°C)	78.5	
Titania ceramic	130	
Strontium titanate	310	8

For a vacuum, κ = unity.

[a]Measured at room temperature, except for the water.

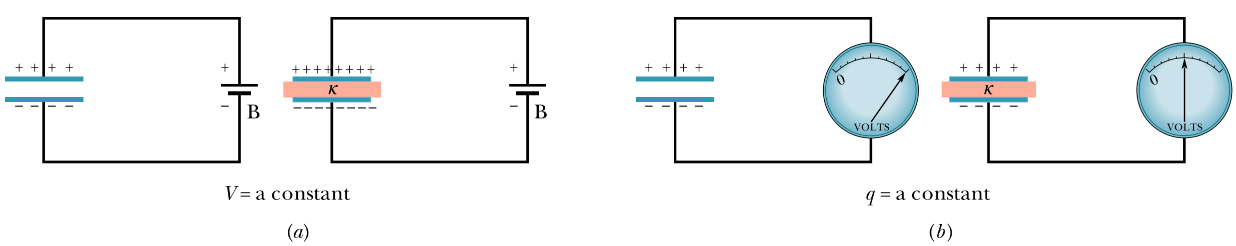

Figure 25.5.2 (*a*) If the potential difference between the plates of a capacitor is maintained, as by battery B, the effect of a dielectric is to increase the charge on the plates. (*b*) If the charge on the capacitor plates is maintained, as in this case, the effect of a dielectric is to reduce the potential difference between the plates. The scale shown is that of a *potentiometer,* a device used to measure potential difference (here, between the plates). A capacitor cannot discharge through a potentiometer.

Thus, the magnitude of the electric field produced by a point charge inside a dielectric is given by this modified form of Eq. 23.6.1:

$$E = \frac{1}{4\pi\kappa\varepsilon_0}\frac{q}{r^2}. \tag{25.5.3}$$

Also, the expression for the electric field just outside an isolated conductor immersed in a dielectric (see Eq. 23.3.1) becomes

$$E = \frac{\sigma}{\kappa\varepsilon_0}. \tag{25.5.4}$$

Because κ is always greater than unity, both these equations show that *for a fixed distribution of charges, the effect of a dielectric is to weaken the electric field* that would otherwise be present.

Sample Problem 25.5.1 Work and energy when a dielectric is inserted into a capacitor

A parallel-plate capacitor whose capacitance C is 13.5 pF is charged by a battery to a potential difference $V = 12.5$ V between its plates. The charging battery is now disconnected, and a porcelain slab ($\kappa = 6.50$) is slipped between the plates.

(a) What is the potential energy of the capacitor before the slab is inserted?

KEY IDEA

We can relate the potential energy U_i of the capacitor to the capacitance C and either the potential V (with Eq. 25.4.2) or the charge q (with Eq. 25.4.1):

$$U_i = \tfrac{1}{2}CV^2 = \frac{q^2}{2C}.$$

Calculation: Because we are given the initial potential V (= 12.5 V), we use Eq. 25.4.2 to find the initial stored energy:

$$U_i = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(13.5\times 10^{-12}\,\text{F})(12.5\,\text{V})^2$$
$$= 1.055\times 10^{-9}\,\text{J} = 1055\,\text{pJ} \approx 1100\,\text{pJ}. \quad \text{(Answer)}$$

(b) What is the potential energy of the capacitor–slab device after the slab is inserted?

KEY IDEA

Because the battery has been disconnected, the charge on the capacitor cannot change when the dielectric is inserted. However, the potential *does* change.

Calculations: Thus, we must now use Eq. 25.4.1 to write the final potential energy U_f, but now that the slab is within the capacitor, the capacitance is κC. We then have

$$U_f = \frac{q^2}{2\kappa C} = \frac{U_i}{\kappa} = \frac{1055\,\text{pJ}}{6.50}$$
$$= 162\,\text{pJ} \approx 160\,\text{pJ}. \quad \text{(Answer)}$$

When the slab is introduced, the potential energy decreases by a factor of κ.

The "missing" energy, in principle, would be apparent to the person who introduced the slab. The capacitor would exert a tiny tug on the slab and would do work on it, in amount

$$W = U_i - U_f = (1055 - 162)\,\text{pJ} = 893\,\text{pJ}.$$

If the slab were allowed to slide between the plates with no restraint and if there were no friction, the slab would oscillate back and forth between the plates with a (constant) mechanical energy of 893 pJ, and this system energy would transfer back and forth between kinetic energy of the moving slab and potential energy stored in the electric field.

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Dielectrics: An Atomic View

What happens, in atomic and molecular terms, when we put a dielectric in an electric field? There are two possibilities, depending on the type of molecule:

1. *Polar dielectrics.* The molecules of some dielectrics, like water, have permanent electric dipole moments. In such materials (called *polar dielectrics*), the electric dipoles tend to line up with an external electric field as in Fig. 25.5.3.

Because the molecules are continuously jostling each other as a result of their random thermal motion, this alignment is not complete, but it becomes more complete as the magnitude of the applied field is increased (or as the temperature, and thus the jostling, are decreased). The alignment of the electric dipoles produces an electric field that is directed opposite the applied field and is smaller in magnitude.

2. *Nonpolar dielectrics.* Regardless of whether they have permanent electric dipole moments, molecules acquire dipole moments by induction when placed in an external electric field. In Module 24.4 (see Fig. 24.4.2), we saw that this occurs because the external field tends to "stretch" the molecules, slightly separating the centers of negative and positive charge.

Figure 25.5.4*a* shows a nonpolar dielectric slab with no external electric field applied. In Fig. 25.5.4*b*, an electric field $\vec{E}_0$ is applied via a capacitor, whose plates are charged as shown. The result is a slight separation of the centers of the positive and negative charge distributions within the slab, producing positive charge on one face of the slab (due to the positive ends of dipoles there) and negative charge on the opposite face (due to the negative ends of dipoles there). The slab as a whole remains electrically neutral and—within the slab—there is no excess charge in any volume element.

Figure 25.5.4*c* shows that the induced surface charges on the faces produce an electric field $\vec{E}'$ in the direction opposite that of the applied electric field $\vec{E}_0$. The resultant field $\vec{E}$ inside the dielectric (the vector sum of fields $\vec{E}_0$ and $\vec{E}'$) has the direction of $\vec{E}_0$ but is smaller in magnitude.

Both the field $\vec{E}'$ produced by the surface charges in Fig. 25.5.4*c* and the electric field produced by the permanent electric dipoles in Fig. 25.5.3 act in the same way—they oppose the applied field $\vec{E}$. Thus, the effect of both polar and nonpolar dielectrics is to weaken any applied field within them, as between the plates of a capacitor.

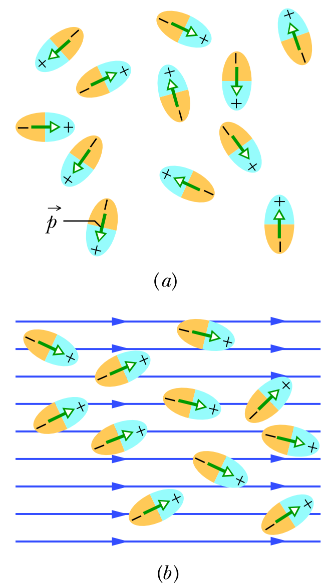
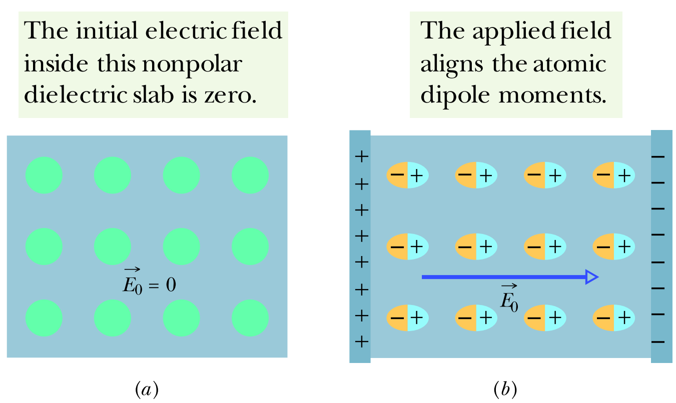


Figure 25.5.3 (*a*) Molecules with a permanent electric dipole moment, showing their random orientation in the absence of an external electric field. (*b*) An electric field is applied, producing partial alignment of the dipoles. Thermal agitation prevents complete alignment.



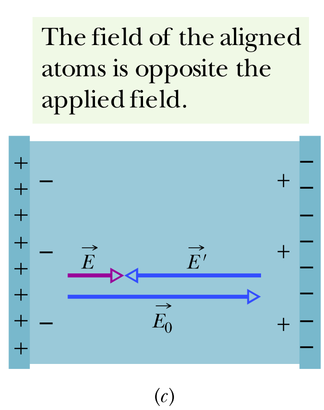


Figure 25.5.4 (*a*) A nonpolar dielectric slab. The circles represent the electrically neutral atoms within the slab. (*b*) An electric field is applied via charged capacitor plates; the field slightly stretches the atoms, separating the centers of positive and negative charge. (*c*) The separation produces surface charges on the slab faces. These charges set up a field $\vec{E}'$, which opposes the applied field $\vec{E}_0$. The resultant field $\vec{E}$ inside the dielectric (the vector sum of $\vec{E}_0$ and $\vec{E}'$) has the same direction as $\vec{E}_0$ but a smaller magnitude.

25.6 DIELECTRICS AND GAUSS' LAW

Learning Objectives

After reading this module, you should be able to . . .

25.6.1 In a capacitor with a dielectric, distinguish free charge from induced charge.

25.6.2 When a dielectric partially or fully fills the space in a capacitor, find the free charge, the induced charge, the electric field between the plates (if there is a gap, there is more than one field value), and the potential between the plates.

Key Ideas

- Inserting a dielectric into a capacitor causes induced charge to appear on the faces of the dielectric and weakens the electric field between the plates.
- The induced charge is less than the free charge on the plates.
- When a dielectric is present, Gauss' law may be generalized to

$$\varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q,$$

where q is the free charge. Any induced surface charge is accounted for by including the dielectric constant κ inside the integral.

Dielectrics and Gauss' Law

In our discussion of Gauss' law in Chapter 23, we assumed that the charges existed in a vacuum. Here we shall see how to modify and generalize that law if dielectric materials, such as those listed in Table 25.5.1, are present. Figure 25.6.1 shows a parallel-plate capacitor of plate area A, both with and without a dielectric. We assume that the charge q on the plates is the same in both situations. Note that the field between the plates induces charges on the faces of the dielectric by one of the methods described in Module 25.5.

For the situation of Fig. 25.6.1*a*, without a dielectric, we can find the electric field $\vec{E}_0$ between the plates as we did in Fig. 25.2.1: We enclose the charge $+q$ on the top plate with a Gaussian surface and then apply Gauss' law. Letting E_0 represent the magnitude of the field, we find

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 E_0 A = q, \tag{25.6.1}$$

or

$$E_0 = \frac{q}{\varepsilon_0 A}. \tag{25.6.2}$$

In Fig. 25.6.1*b*, with the dielectric in place, we can find the electric field between the plates (and within the dielectric) by using the same Gaussian surface. However, now the surface encloses two types of charge: It still encloses charge $+q$ on the top plate, but it now also encloses the induced charge $-q'$ on the top face of the dielectric. The charge on the conducting plate is said to be *free charge* because it can move if we change the electric potential of the plate; the induced

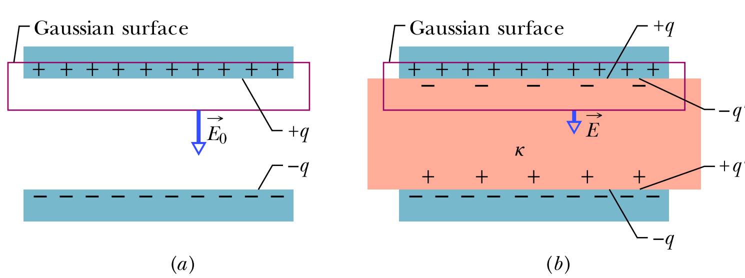


Figure 25.6.1 A parallel-plate capacitor (*a*) without and (*b*) with a dielectric slab inserted. The charge q on the plates is assumed to be the same in both cases.

charge on the surface of the dielectric is not free charge because it cannot move from that surface.

The net charge enclosed by the Gaussian surface in Fig. 25.6.1*b* is $q - q'$, so Gauss' law now gives

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 EA = q - q', \tag{25.6.3}$$

or

$$E = \frac{q - q'}{\varepsilon_0 A}. \tag{25.6.4}$$

The effect of the dielectric is to weaken the original field E_0 by a factor of κ; so we may write

$$E = \frac{E_0}{\kappa} = \frac{q}{\kappa \varepsilon_0 A}. \tag{25.6.5}$$

Comparison of Eqs. 25.6.4 and 25.6.5 shows that

$$q - q' = \frac{q}{\kappa}. \tag{25.6.6}$$

Equation 25.6.6 shows correctly that the magnitude q' of the induced surface charge is less than that of the free charge q and is zero if no dielectric is present (because then $\kappa = 1$ in Eq. 25.6.6).

By substituting for $q - q'$ from Eq. 25.6.6 in Eq. 25.6.3, we can write Gauss' law in the form

$$\varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q \quad \text{(Gauss' law with dielectric).} \tag{25.6.7}$$

This equation, although derived for a parallel-plate capacitor, is true generally and is the most general form in which Gauss' law can be written. Note:

1. The flux integral now involves $\kappa\vec{E}$, not just $\vec{E}$. (The vector $\varepsilon_0\kappa\vec{E}$ is sometimes called the *electric displacement* $\vec{D}$, so that Eq. 25.6.7 can be written in the form $\oint \vec{D} \cdot d\vec{A} = q$.)
2. The charge q enclosed by the Gaussian surface is now taken to be the *free charge only*. The induced surface charge is deliberately ignored on the right side of Eq. 25.6.7, having been taken fully into account by introducing the dielectric constant κ on the left side.
3. Equation 25.6.7 differs from Eq. 23.2.2, our original statement of Gauss' law, only in that ε_0 in the latter equation has been replaced by $\kappa\varepsilon_0$. We keep κ inside the integral of Eq. 25.6.7 to allow for cases in which κ is not constant over the entire Gaussian surface.

Sample Problem 25.6.1 Dielectric partially filling the gap in a capacitor

Figure 25.6.2 shows a parallel-plate capacitor of plate area A and plate separation d. A potential difference V_0 is applied between the plates by connecting a battery between them. The battery is then disconnected, and a dielectric slab of thickness b and dielectric constant κ is placed between the plates as shown. Assume $A = 115\ \text{cm}^2$, $d = 1.24$ cm, $V_0 = 85.5$ V, $b = 0.780$ cm, and $\kappa = 2.61$.

(a) What is the capacitance C_0 before the dielectric slab is inserted?

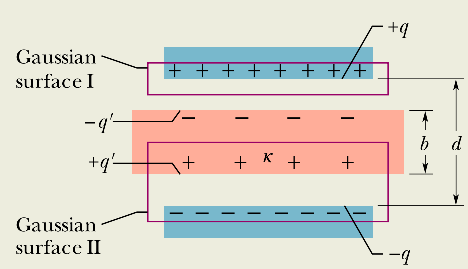


Figure 25.6.2 A parallel-plate capacitor containing a dielectric slab that only partially fills the space between the plates.

Calculation: From Eq. 25.2.7 we have

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{(8.85\times10^{-12}\ \text{F/m})(115\times10^{-4}\ \text{m}^2)}{1.24\times10^{-2}\ \text{m}}$$
$$= 8.21\times10^{-12}\ \text{F} = 8.21\ \text{pF}. \quad \text{(Answer)}$$

(b) What free charge appears on the plates?

Calculation: From Eq. 25.1.1,

$$q = C_0V_0 = (8.21\times10^{-12}\ \text{F})(85.5\ \text{V})$$
$$= 7.02\times10^{-10}\ \text{C} = 702\ \text{pC}. \quad \text{(Answer)}$$

Because the battery was disconnected before the slab was inserted, the free charge is unchanged.

(c) What is the electric field E_0 in the gaps between the plates and the dielectric slab?

KEY IDEA

We need to apply Gauss' law, in the form of Eq. 25.6.7, to Gaussian surface I in Fig. 25.6.2.

Calculations: That surface passes through the gap, and so it encloses *only* the free charge on the upper capacitor plate. Electric field pierces only the bottom of the Gaussian surface. Because there the area vector $d\vec{A}$ and the field vector $\vec{E}_0$ are both directed downward, the dot product in Eq. 25.6.7 becomes

$$\vec{E}_0\cdot d\vec{A} = E_0\,dA\cos 0° = E_0\,dA.$$

Equation 25.6.7 then becomes

$$\varepsilon_0\kappa E_0\oint dA = q.$$

The integration now simply gives the surface area A of the plate. Thus, we obtain

$$\varepsilon_0\kappa E_0A = q,$$

or

$$E_0 = \frac{q}{\varepsilon_0\kappa A}.$$

We must put $\kappa = 1$ here because Gaussian surface I does not pass through the dielectric. Thus, we have

$$E_0 = \frac{q}{\varepsilon_0\kappa A} = \frac{7.02\times10^{-10}\ \text{C}}{(8.85\times10^{-12}\ \text{F/m})(1)(115\times10^{-4}\ \text{m}^2)}$$
$$= 6900\ \text{V/m} = 6.90\ \text{kV/m}. \quad \text{(Answer)}$$

Note that the value of E_0 does not change when the slab is introduced because the amount of charge enclosed by Gaussian surface I in Fig. 25.6.2 does not change.

(d) What is the electric field E_1 in the dielectric slab?

KEY IDEA

Now we apply Gauss' law in the form of Eq. 25.6.7 to Gaussian surface II in Fig. 25.6.2.

Calculations: Only the free charge $-q$ is in Eq. 25.6.7, so

$$\varepsilon_0\oint\kappa\vec{E}_1\cdot d\vec{A} = -\varepsilon_0\kappa E_1A = -q. \qquad (25.6.8)$$

The first minus sign in this equation comes from the dot product $\vec{E}_1\cdot d\vec{A}$ along the top of the Gaussian surface because now the field vector $\vec{E}_1$ is directed downward and the area vector $d\vec{A}$ (which, as always, points outward from the interior of a closed Gaussian surface) is directed upward. With 180° between the vectors, the dot product is negative. Now $\kappa = 2.61$. Thus, Eq. 25.6.8 gives us

$$E_1 = \frac{q}{\varepsilon_0\kappa A} = \frac{E_0}{\kappa} = \frac{6.90\ \text{kV/m}}{2.61}$$
$$= 2.64\ \text{kV/m}. \quad \text{(Answer)}$$

(e) What is the potential difference V between the plates after the slab has been introduced?

KEY IDEA

We find V by integrating along a straight line directly from the bottom plate to the top plate.

Calculation: Within the dielectric, the path length is b and the electric field is E_1. Within the two gaps above and below the dielectric, the total path length is $d - b$ and the electric field is E_0. Equation 25.2.4 then yields

$$V = \int_-^+ E\,ds = E_0(d-b) + E_1b$$
$$= (6900\ \text{V/m})(0.0124\ \text{m} - 0.00780\ \text{m})$$
$$+ (2640\ \text{V/m})(0.00780\ \text{m})$$
$$= 52.3\ \text{V}. \quad \text{(Answer)}$$

This is less than the original potential difference of 85.5 V.

(f) What is the capacitance with the slab in place?

KEY IDEA

The capacitance C is related to q and V via Eq. 25.1.1.

Calculation: Taking q from (b) and V from (e), we have

$$C = \frac{q}{V} = \frac{7.02\times10^{-10}\ \text{C}}{52.3\ \text{V}}$$
$$= 1.34\times10^{-11}\ \text{F} = 13.4\ \text{pF}. \quad \text{(Answer)}$$

This is greater than the original capacitance of 8.21 pF.

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Checkpoint 25.6.1

We have two dielectric materials that will completely fill the gap between the plates of a charged, isolated capacitor. Dielectric 1 has a small dielectric constant; dielectric 2 has a larger dielectric constant. We insert dielectric 1 and then remove it. Then we insert dielectric 2. (a) How do the free charges compare in the two situations: $q_1 = q_2$, $q_1 > q_2$, or $q_1 < q_2$? (b) How do the induced charges compare: $q_1' = q_2'$, $q_1' > q_2'$, or $q_1' < q_2'$? (c) How do the potential differences between the plates compare: $V_1 = V_2$, $V_1 > V_2$, or $V_1 < V_2$?

Review & Summary

Capacitor; Capacitance A **capacitor** consists of two isolated conductors (the *plates*) with charges $+q$ and $-q$. Its **capacitance** C is defined from

$$q = CV, \tag{25.1.1}$$

where V is the potential difference between the plates.

Determining Capacitance We generally determine the capacitance of a particular capacitor configuration by (1) assuming a charge q to have been placed on the plates, (2) finding the electric field $\vec{E}$ due to this charge, (3) evaluating the potential difference V, and (4) calculating C from Eq. 25.1.1. Some specific results are the following:

A *parallel-plate capacitor* with flat parallel plates of area A and spacing d has capacitance

$$C = \frac{\varepsilon_0 A}{d}. \tag{25.2.7}$$

A *cylindrical capacitor* (two long coaxial cylinders) of length L and radii a and b has capacitance

$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}. \tag{25.2.12}$$

A *spherical capacitor* with concentric spherical plates of radii a and b has capacitance

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}. \tag{25.2.15}$$

An *isolated sphere* of radius R has capacitance

$$C = 4\pi\varepsilon_0 R. \tag{25.2.16}$$

Capacitors in Parallel and in Series The **equivalent capacitances** C_{eq} of combinations of individual capacitors connected in **parallel** and in **series** can be found from

$$C_{eq} = \sum_{j=1}^{n} C_j \quad (n \text{ capacitors in parallel}) \tag{25.3.1}$$

and

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j} \quad (n \text{ capacitors in series}). \tag{25.3.2}$$

Equivalent capacitances can be used to calculate the capacitances of more complicated series–parallel combinations.

Potential Energy and Energy Density The **electric potential energy** U of a charged capacitor,

$$U = \frac{q^2}{2C} = \tfrac{1}{2}CV^2, \tag{25.4.1, 25.4.2}$$

is equal to the work required to charge the capacitor. This energy can be associated with the capacitor's electric field $\vec{E}$. By extension we can associate stored energy with any electric field. In vacuum, the **energy density** u, or potential energy per unit volume, within an electric field of magnitude E is given by

$$u = \tfrac{1}{2}\varepsilon_0 E^2. \tag{25.4.5}$$

Capacitance with a Dielectric If the space between the plates of a capacitor is completely filled with a dielectric material, the capacitance C is increased by a factor κ, called the **dielectric constant,** which is characteristic of the material. In a region that is completely filled by a dielectric, all electrostatic equations containing ε_0 must be modified by replacing ε_0 with $\kappa\varepsilon_0$.

The effects of adding a dielectric can be understood physically in terms of the action of an electric field on the permanent or induced electric dipoles in the dielectric slab. The result is the formation of induced charges on the surfaces of the dielectric, which results in a weakening of the field within the dielectric for a given amount of free charge on the plates.

Gauss' Law with a Dielectric When a dielectric is present, Gauss' law may be generalized to

$$\varepsilon_0 \oint \kappa\vec{E}\cdot d\vec{A} = q. \tag{25.6.7}$$

Here q is the free charge; any induced surface charge is accounted for by including the dielectric constant κ inside the integral.

Questions

1 Figure 25.1 shows plots of charge versus potential difference for three parallel-plate capacitors that have the plate areas and separations given in the table. Which plot goes with which capacitor?

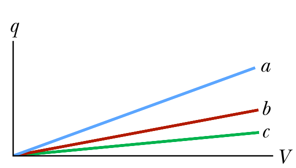

Figure 25.1 Question 1.

Capacitor	Area	Separation
1	A	d
2	$2A$	d
3	A	$2d$

2 What is C_{eq} of three capacitors, each of capacitance C, if they are connected to a battery (a) in series with one another and (b) in parallel? (c) In which arrangement is there more charge on the equivalent capacitance?

3 (a) In Fig. 25.2*a*, are capacitors 1 and 3 in series? (b) In the same figure, are capacitors 1 and 2 in parallel? (c) Rank the equivalent capacitances of the four circuits shown in Fig. 25.2, greatest first.

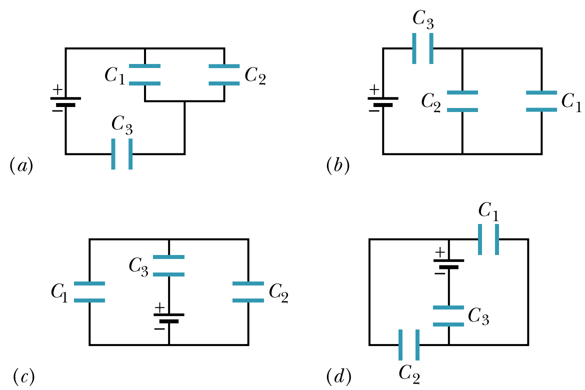

Figure 25.2 Question 3.

4 Figure 25.3 shows three circuits, each consisting of a switch and two capacitors, initially charged as indicated (top plate positive). After the switches have been closed, in which circuit (if any) will the charge on the left-hand capacitor (a) increase, (b) decrease, and (c) remain the same?

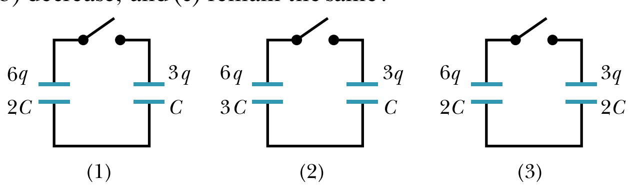

Figure 25.3 Question 4.

5 Initially, a single capacitance C_1 is wired to a battery. Then capacitance C_2 is added in parallel. Are (a) the potential difference across C_1 and (b) the charge q_1 on C_1 now more than, less than, or the same as previously? (c) Is the equivalent capacitance C_{12} of C_1 and C_2 more than, less than, or equal to C_1? (d) Is the charge stored on C_1 and C_2 together more than, less than, or equal to the charge stored previously on C_1?

6 Repeat Question 5 for C_2 added in series rather than in parallel.

7 For each circuit in Fig. 25.4, are the capacitors connected in series, in parallel, or in neither mode?

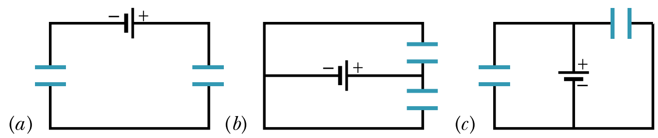

Figure 25.4 Question 7.

8 Figure 25.5 shows an open switch, a battery of potential difference V, a current-measuring meter A, and three identical uncharged capacitors of capacitance C. When the switch is closed and the circuit reaches equilibrium, what are (a) the potential difference across each capacitor and (b) the charge on the left plate of each capacitor? (c) During charging, what net charge passes through the meter?

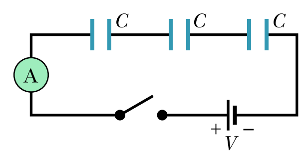

Figure 25.5 Question 8.

9 A parallel-plate capacitor is connected to a battery of electric potential difference V. If the plate separation is decreased, do the following quantities increase, decrease, or remain the same: (a) the capacitor's capacitance, (b) the potential difference across the capacitor, (c) the charge on the capacitor, (d) the energy stored by the capacitor, (e) the magnitude of the electric field between the plates, and (f) the energy density of that electric field?

10 When a dielectric slab is inserted between the plates of one of the two identical capacitors in Fig. 25.6, do the following properties of that capacitor increase, decrease, or remain the same: (a) capacitance, (b) charge, (c) potential difference, and (d) potential energy? (e) How about the same properties of the other capacitor?

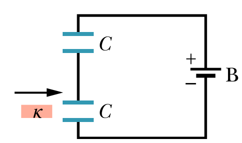

Figure 25.6 Question 10.

11 You are to connect capacitances C_1 and C_2, with $C_1 > C_2$, to a battery, first individually, then in series, and then in parallel. Rank those arrangements according to the amount of charge stored, greatest first.

Problems

GO Tutoring problem available (at instructor's discretion) in *WileyPLUS*
SSM Worked-out solution available in Student Solutions Manual
E Easy M Medium H Hard
FCP Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com
CALC Requires calculus
BIO Biomedical application

Module 25.1 Capacitance

1 E The two metal objects in Fig. 25.7 have net charges of +70 pC and –70 pC, which result in a 20 V potential difference between them. (a) What is the capacitance of the system? (b) If the charges are changed to +200 pC and –200 pC, what does the capacitance become? (c) What does the potential difference become?

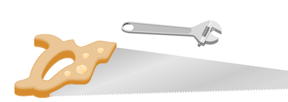

Figure 25.7 Problem 1.

2 E The capacitor in Fig. 25.8 has a capacitance of 25 μF and is initially uncharged. The battery provides a potential difference of 120 V. After switch S is closed, how much charge will pass through it?

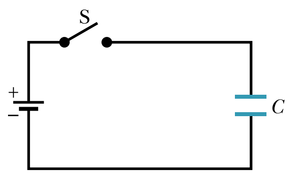

Figure 25.8 Problem 2.

Module 25.2 Calculating the Capacitance

3 E SSM A parallel-plate capacitor has circular plates of 8.20 cm radius and 1.30 mm separation. (a) Calculate the capacitance. (b) Find the charge for a potential difference of 120 V.

4 E The plates of a spherical capacitor have radii 38.0 mm and 40.0 mm. (a) Calculate the capacitance. (b) What must be the plate area of a parallel-plate capacitor with the same plate separation and capacitance?

5 E What is the capacitance of a drop that results when two mercury spheres, each of radius $R = 2.00$ mm, merge?

6 E You have two flat metal plates, each of area 1.00 m^2, with which to construct a parallel-plate capacitor. (a) If the capacitance of the device is to be 1.00 F, what must be the separation between the plates? (b) Could this capacitor actually be constructed?

7 E If an uncharged parallel-plate capacitor (capacitance C) is connected to a battery, one plate becomes negatively charged as electrons move to the plate face (area A). In Fig. 25.9, the depth d from which the electrons come in the plate in a particular capacitor is plotted against a range of values for the potential difference V of the battery. The density of conduction electrons in the copper plates is 8.49×10^{28} electrons/m^3. The vertical scale is set by $d_s = 1.00$ pm, and the horizontal scale is set by $V_s = 20.0$ V. What is the ratio C/A?

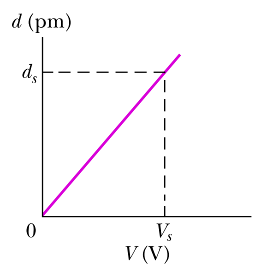

Figure 25.9 Problem 7.

Module 25.3 Capacitors in Parallel and in Series

8 E How many 1.00 μF capacitors must be connected in parallel to store a charge of 1.00 C with a potential of 110 V across the capacitors?

9 E Each of the uncharged capacitors in Fig. 25.10 has a capacitance of 25.0 μF. A potential difference of $V = 4200$ V is established when the switch is closed. How many coulombs of charge then pass through meter A?

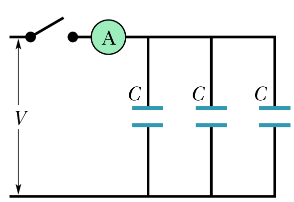

Figure 25.10 Problem 9.

10 E In Fig. 25.11, find the equivalent capacitance of the combination. Assume that C_1 is 10.0 μF, C_2 is 5.00 μF, and C_3 is 4.00 μF.

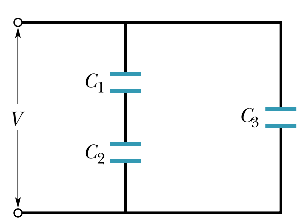

Figure 25.11 Problems 10 and 34.

11 E In Fig. 25.12, find the equivalent capacitance of the combination. Assume that $C_1 = 10.0\,\mu$F, $C_2 = 5.00\,\mu$F, and $C_3 = 4.00\,\mu$F.

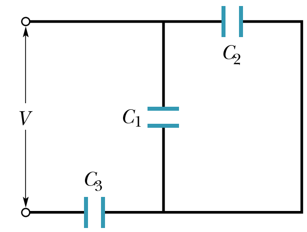

Figure 25.12 Problems 11, 17, and 38.

12 M Two parallel-plate capacitors, 6.0 μF each, are connected in parallel to a 10 V battery. One of the capacitors is then squeezed so that its plate separation is 50.0% of its initial value. Because of the squeezing, (a) how much additional charge is transferred to the capacitors by the battery and (b) what is the increase in the total charge stored on the capacitors?

13 M SSM A 100 pF capacitor is charged to a potential difference of 50 V, and the charging battery is disconnected. The capacitor is then connected in parallel with a second (initially uncharged) capacitor. If the potential difference across the first capacitor drops to 35 V, what is the capacitance of this second capacitor?

14 M GO In Fig. 25.13, the battery has a potential difference of $V = 10.0$ V and the five capacitors each have a capacitance of 10.0 μF. What is the charge on (a) capacitor 1 and (b) capacitor 2?

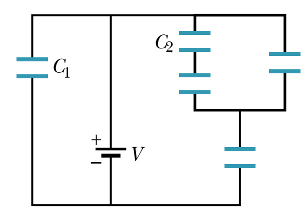

Figure 25.13 Problem 14.

15 M GO In Fig. 25.14, a 20.0 V battery is connected across capacitors of capacitances $C_1 = C_6 = 3.00\,\mu$F and $C_3 = C_5 = 2.00C_2 = 2.00C_4 = 4.00\,\mu$F. What are (a) the equivalent capacitance C_{eq} of the capacitors and (b) the charge stored by C_{eq}? What are (c) V_1 and (d) q_1 of capacitor 1, (e) V_2 and (f) q_2 of capacitor 2, and (g) V_3 and (h) q_3 of capacitor 3?

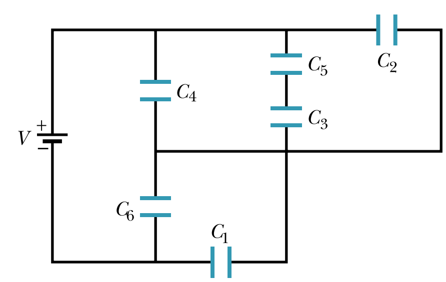

Figure 25.14 Problem 15.

16 M Plot 1 in Fig. 25.15*a* gives the charge q that can be stored on capacitor 1 versus the electric potential V set up across it. The vertical scale is set by $q_s = 16.0\,\mu$C, and the horizontal scale is set by $V_s = 2.0$ V. Plots 2 and 3 are similar plots for capacitors 2 and 3, respectively. Figure 25.15*b* shows a circuit with those three capacitors and a 6.0 V battery. What is the charge stored on capacitor 2 in that circuit?

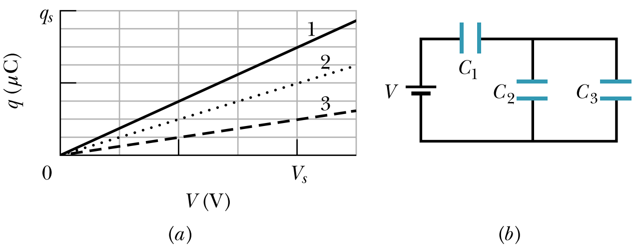

Figure 25.15 Problem 16.

17 M GO In Fig. 25.12, a potential difference of $V = 100.0$ V is applied across a capacitor arrangement with capacitances $C_1 = 10.0\,\mu$F, $C_2 = 5.00\,\mu$F, and $C_3 = 4.00\,\mu$F. If capacitor 3 undergoes electrical breakdown so that it becomes equivalent to conducting wire, what is the increase in (a) the charge on capacitor 1 and (b) the potential difference across capacitor 1?

18 M Figure 25.16 shows a circuit section of four air-filled capacitors that is connected to a larger circuit. The graph below the section shows the electric potential $V(x)$ as a function of position x along the lower part of the section, through capacitor 4. Similarly, the graph above the section shows the electric potential $V(x)$ as a function of position x along the upper part of the section, through capacitors 1, 2, and 3. Capacitor 3 has a capacitance of $0.80\,\mu$F. What are the capacitances of (a) capacitor 1 and (b) capacitor 2?

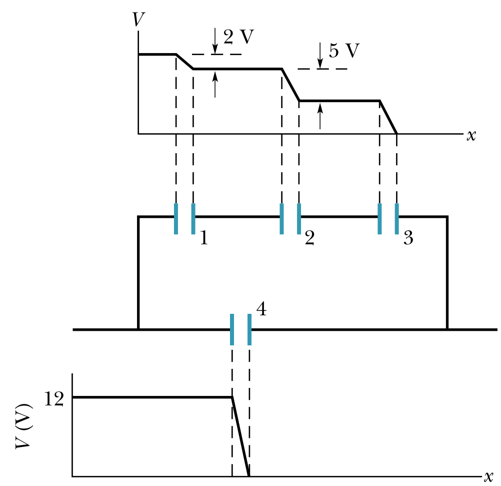

Figure 25.16 Problem 18.

19 M GO In Fig. 25.17, the battery has potential difference $V = 9.0$ V, $C_2 = 3.0\,\mu$F, $C_4 = 4.0\,\mu$F, and all the capacitors are initially uncharged. When switch S is closed, a total charge of $12\,\mu$C passes through point a and a total charge of $8.0\,\mu$C passes through point b. What are (a) C_1 and (b) C_3?

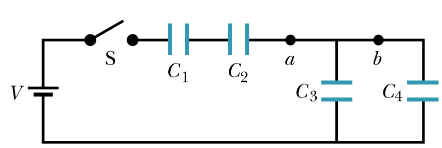

Figure 25.17 Problem 19.

20 M Figure 25.18 shows a variable "air gap" capacitor for manual tuning. Alternate plates are connected together; one group of plates is fixed in position, and the other group is capable of rotation. Consider a capacitor of $n = 8$ plates of alternating polarity, each plate having area $A = 1.25\ \text{cm}^2$ and separated from adjacent plates by distance $d = 3.40$ mm. What is the maximum capacitance of the device?

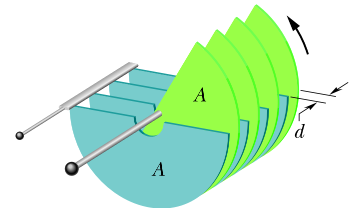

Figure 25.18 Problem 20.

21 M SSM In Fig. 25.19, the capacitances are $C_1 = 1.0\,\mu$F and $C_2 = 3.0\,\mu$F; both capacitors are charged to a potential difference of $V = 100$ V but with opposite polarity as shown. Switches S_1 and S_2 are now closed. (a) What is now the potential difference between points a and b? What now is the charge on capacitor (b) 1 and (c) 2?

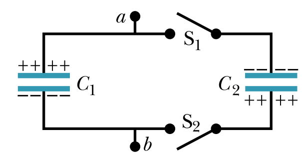

Figure 25.19 Problem 21.

22 M In Fig. 25.20, $V = 10$ V, $C_1 = 10\,\mu$F, and $C_2 = C_3 = 20\,\mu$F. Switch S is first thrown to the left side until capacitor 1 reaches equilibrium. Then the switch is thrown to the right. When equilibrium is again reached, how much charge is on capacitor 1?

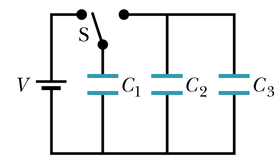

Figure 25.20 Problem 22.

23 M The capacitors in Fig. 25.21 are initially uncharged. The capacitances are $C_1 = 4.0\,\mu$F, $C_2 = 8.0\,\mu$F, and $C_3 = 12\,\mu$F, and the battery's potential difference is $V = 12$ V. When switch S is closed, how many electrons travel through (a) point a, (b) point b, (c) point c, and (d) point d? In the figure, do the electrons travel up or down through (e) point b and (f) point c?

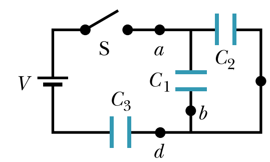

Figure 25.21 Problem 23.

24 M GO Figure 25.22 represents two air-filled cylindrical capacitors connected in series across a battery with potential $V = 10$ V. Capacitor 1 has an inner plate radius of 5.0 mm, an outer plate radius of 1.5 cm, and a length of 5.0 cm. Capacitor 2 has an inner plate radius of 2.5 mm, an outer plate radius of 1.0 cm, and a length of 9.0 cm. The outer plate of capacitor 2 is a conducting organic membrane that can be stretched, and the capacitor can be inflated to increase the plate separation. If the outer plate radius is increased to 2.5 cm by inflation, (a) how many electrons move through point P and (b) do they move toward or away from the battery?

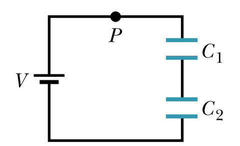

Figure 25.22 Problem 24.

25 M GO In Fig. 25.23, two parallel-plate capacitors (with air between the plates) are connected to a battery. Capacitor 1 has a plate area of $1.5\ \text{cm}^2$ and an electric field (between its plates) of magnitude 2000 V/m. Capacitor 2 has a plate area of $0.70\ \text{cm}^2$ and an electric field of magnitude 1500 V/m. What is the total charge on the two capacitors?

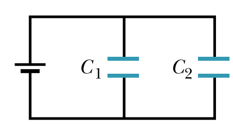

Figure 25.23 Problem 25.

26 H GO Capacitor 3 in Fig. 25.24a is a *variable capacitor* (its capacitance C_3 can be varied). Figure 25.24b gives the electric potential V_1 across capacitor 1 versus C_3. The horizontal scale is set by $C_{3s} = 12.0\ \mu$F. Electric potential V_1 approaches an asymptote of 10 V as $C_3 \to \infty$. What are (a) the electric potential V across the battery, (b) C_1, and (c) C_2?

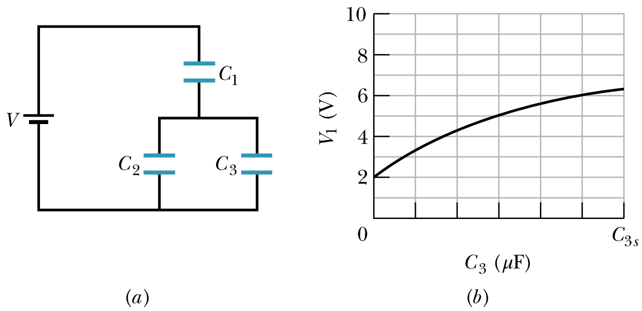

Figure 25.24 Problem 26.

27 H GO Figure 25.25 shows a 12.0 V battery and four uncharged capacitors of capacitances $C_1 = 1.00\ \mu$F, $C_2 = 2.00\ \mu$F, $C_3 = 3.00\ \mu$F, and $C_4 = 4.00\ \mu$F. If only switch S_1 is closed, what is the charge on (a) capacitor 1, (b) capacitor 2, (c) capacitor 3, and (d) capacitor 4? If both switches are closed, what is the charge on (e) capacitor 1, (f) capacitor 2, (g) capacitor 3, and (h) capacitor 4?

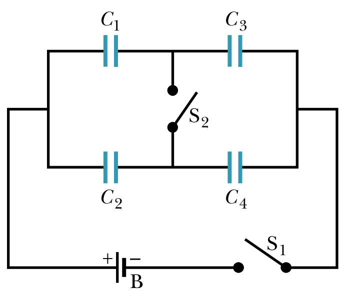

Figure 25.25 Problem 27.

28 H GO Figure 25.26 displays a 12.0 V battery and 3 uncharged capacitors of capacitances $C_1 = 4.00\ \mu$F, $C_2 = 6.00\ \mu$F, and $C_3 = 3.00\ \mu$F. The switch is thrown to the left side until capacitor 1 is fully charged. Then the switch is thrown to the right. What is the final charge on (a) capacitor 1, (b) capacitor 2, and (c) capacitor 3?

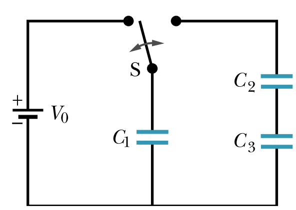

Figure 25.26 Problem 28.

Module 25.4 Energy Stored in an Electric Field

29 E What capacitance is required to store an energy of 10 kW · h at a potential difference of 1000 V?

30 E How much energy is stored in 1.00 m^3 of air due to the "fair weather" electric field of magnitude 150 V/m?

31 E SSM A 2.0 μF capacitor and a 4.0 μF capacitor are connected in parallel across a 300 V potential difference. Calculate the total energy stored in the capacitors.

32 E A parallel-plate air-filled capacitor having area 40 cm^2 and plate spacing 1.0 mm is charged to a potential difference of 600 V. Find (a) the capacitance, (b) the magnitude of the charge on each plate, (c) the stored energy, (d) the electric field between the plates, and (e) the energy density between the plates.

33 M A charged isolated metal sphere of diameter 10 cm has a potential of 8000 V relative to $V = 0$ at infinity. Calculate the energy density in the electric field near the surface of the sphere.

34 M In Fig. 25.11, a potential difference $V = 100$ V is applied across a capacitor arrangement with capacitances $C_1 = 10.0\ \mu$F, $C_2 = 5.00\ \mu$F, and $C_3 = 4.00\ \mu$F. What are (a) charge q_3, (b) potential difference V_3, and (c) stored energy U_3 for capacitor 3, (d) q_1, (e) V_1, and (f) U_1 for capacitor 1, and (g) q_2, (h) V_2, and (i) U_2 for capacitor 2?

35 M Assume that a stationary electron is a point of charge. What is the energy density u of its electric field at radial distances (a) $r = 1.00$ mm, (b) $r = 1.00\ \mu$m, (c) $r = 1.00$ nm, and (d) $r = 1.00$ pm? (e) What is u in the limit as $r \to 0$?

36 M FCP As a safety engineer, you must evaluate the practice of storing flammable conducting liquids in nonconducting containers. The company supplying a certain liquid has been using a squat, cylindrical plastic container of radius $r = 0.20$ m and filling it to height $h = 10$ cm, which is not the container's full interior height (Fig. 25.27). Your investigation reveals that during handling at the company, the exterior surface of the container commonly acquires a negative charge density of magnitude 2.0 μC/m^2 (approximately uniform). Because the liquid is a conducting material, the charge on the container induces charge separation within the liquid. (a) How much negative charge is induced in the center of the liquid's bulk? (b) Assume the capacitance of the central portion of the liquid relative to ground is 35 pF. What is the potential energy associated with the negative charge in that effective capacitor? (c) If a spark occurs between the ground and the central portion of the liquid (through the venting port), the potential energy can be fed into the spark. The minimum spark energy needed to ignite the liquid is 10 mJ. In this situation, can a spark ignite the liquid?

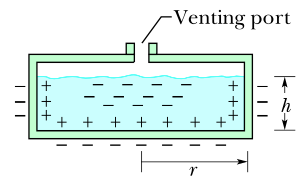

Figure 25.27 Problem 36.

37 M SSM The parallel plates in a capacitor, with a plate area of 8.50 cm^2 and an air-filled separation of 3.00 mm, are charged by a 6.00 V battery. They are then disconnected from the battery and pulled apart (without discharge) to a separation of 8.00 mm. Neglecting fringing, find (a) the potential difference between the plates, (b) the initial stored energy, (c) the final stored energy, and (d) the work required to separate the plates.

38 M In Fig. 25.12, a potential difference $V = 100$ V is applied across a capacitor arrangement with capacitances $C_1 = 10.0\ \mu$F, $C_2 = 5.00\ \mu$F, and $C_3 = 15.0\ \mu$F. What are (a) charge q_3, (b) potential difference V_3, and (c) stored energy U_3 for capacitor 3, (d) q_1, (e) V_1, and (f) U_1 for capacitor 1, and (g) q_2, (h) V_2, and (i) U_2 for capacitor 2?

39 M GO In Fig. 25.28, $C_1 = 10.0\ \mu$F, $C_2 = 20.0\ \mu$F, and $C_3 = 25.0\ \mu$F. If no capacitor can withstand a potential difference of more than 100 V without failure, what are (a) the magnitude of the maximum potential difference that can exist between points A and B and (b) the maximum energy that can be stored in the three-capacitor arrangement?

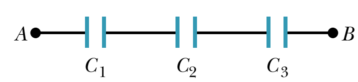

Figure 25.28 Problem 39.

Module 25.5 Capacitor with a Dielectric

40 E An air-filled parallel-plate capacitor has a capacitance of 1.3 pF. The separation of the plates is doubled, and wax is inserted between them. The new capacitance is 2.6 pF. Find the dielectric constant of the wax.

41 E SSM A coaxial cable used in a transmission line has an inner radius of 0.10 mm and an outer radius of 0.60 mm. Calculate the capacitance per meter for the cable. Assume that the space between the conductors is filled with polystyrene.

42 E A parallel-plate air-filled capacitor has a capacitance of 50 pF. (a) If each of its plates has an area of $0.35\ \text{m}^2$, what is the separation? (b) If the region between the plates is now filled with material having $\kappa = 5.6$, what is the capacitance?

43 E Given a 7.4 pF air-filled capacitor, you are asked to convert it to a capacitor that can store up to 7.4 μJ with a maximum potential difference of 652 V. Which dielectric in Table 25.5.1 should you use to fill the gap in the capacitor if you do not allow for a margin of error?

44 M You are asked to construct a capacitor having a capacitance near 1 nF and a breakdown potential in excess of 10 000 V. You think of using the sides of a tall Pyrex drinking glass as a dielectric, lining the inside and outside curved surfaces with aluminum foil to act as the plates. The glass is 15 cm tall with an inner radius of 3.6 cm and an outer radius of 3.8 cm. What are the (a) capacitance and (b) breakdown potential of this capacitor?

45 M A certain parallel-plate capacitor is filled with a dielectric for which $\kappa = 5.5$. The area of each plate is $0.034\ \text{m}^2$, and the plates are separated by 2.0 mm. The capacitor will fail (short out and burn up) if the electric field between the plates exceeds 200 kN/C. What is the maximum energy that can be stored in the capacitor?

46 M In Fig. 25.29, how much charge is stored on the parallel-plate capacitors by the 12.0 V battery? One is filled with air, and the other is filled with a dielectric for which $\kappa = 3.00$; both capacitors have a plate area of $5.00 \times 10^{-3}\ \text{m}^2$ and a plate separation of 2.00 mm.

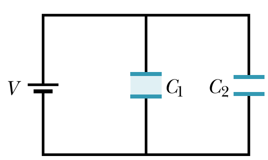

Figure 25.29 Problem 46.

47 M SSM A certain substance has a dielectric constant of 2.8 and a dielectric strength of 18 MV/m. If it is used as the dielectric material in a parallel-plate capacitor, what minimum area should the plates of the capacitor have to obtain a capacitance of $7.0 \times 10^{-2}\ \mu$F and to ensure that the capacitor will be able to withstand a potential difference of 4.0 kV?

48 M Figure 25.30 shows a parallel-plate capacitor with a plate area $A = 5.56\ \text{cm}^2$ and separation $d = 5.56$ mm. The left half of the gap is filled with material of dielectric constant $\kappa_1 = 7.00$; the right half is filled with material of dielectric constant $\kappa_2 = 12.0$. What is the capacitance?

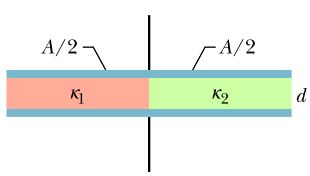

Figure 25.30 Problem 48.

49 M Figure 25.31 shows a parallel-plate capacitor with a plate area $A = 7.89\ \text{cm}^2$ and plate separation $d = 4.62$ mm. The top half of the gap is filled with material of dielectric constant $\kappa_1 = 11.0$; the bottom half is filled with material of dielectric constant $\kappa_2 = 12.0$. What is the capacitance?

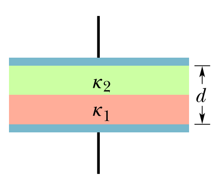

Figure 25.31 Problem 49.

50 M GO Figure 25.32 shows a parallel-plate capacitor of plate area $A = 10.5\ \text{cm}^2$ and plate separation $2d = 7.12$ mm. The left half of the gap is filled with material of dielectric constant $\kappa_1 = 21.0$; the top of the right half is filled with material of dielectric constant $\kappa_2 = 42.0$; the bottom of the right half is filled with material of dielectric constant $\kappa_3 = 58.0$. What is the capacitance?

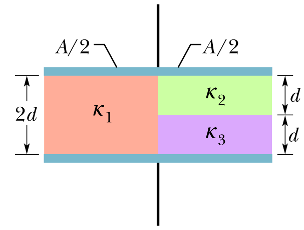

Figure 25.32 Problem 50.

Module 25.6 Dielectrics and Gauss' Law

51 E SSM A parallel-plate capacitor has a capacitance of 100 pF, a plate area of $100\ \text{cm}^2$, and a mica dielectric ($\kappa = 5.4$) completely filling the space between the plates. At 50 V potential difference, calculate (a) the electric field magnitude E in the mica, (b) the magnitude of the free charge on the plates, and (c) the magnitude of the induced surface charge on the mica.

52 E For the arrangement of Fig. 25.6.2, suppose that the battery remains connected while the dielectric slab is being introduced. Calculate (a) the capacitance, (b) the charge on the capacitor plates, (c) the electric field in the gap, and (d) the electric field in the slab, after the slab is in place.

53 M A parallel-plate capacitor has plates of area $0.12\ \text{m}^2$ and a separation of 1.2 cm. A battery charges the plates to a potential difference of 120 V and is then disconnected. A dielectric slab of thickness 4.0 mm and dielectric constant 4.8 is then placed symmetrically between the plates. (a) What is the capacitance before the slab is inserted? (b) What is the capacitance with the slab in place? What is the free charge q (c) before and (d) after the slab is inserted? What is the magnitude of the electric field (e) in the space between the plates and dielectric and (f) in the dielectric itself? (g) With the slab in place, what is the potential difference across the plates? (h) How much external work is involved in inserting the slab?

54 M Two parallel plates of area $100\ \text{cm}^2$ are given charges of equal magnitudes 8.9×10^{-7} C but opposite signs. The electric field within the dielectric material filling the space between the plates is 1.4×10^6 V/m. (a) Calculate the dielectric constant of the material. (b) Determine the magnitude of the charge induced on each dielectric surface.

55 M The space between two concentric conducting spherical shells of radii $b = 1.70$ cm and $a = 1.20$ cm is filled with a substance of dielectric constant $\kappa = 23.5$. A potential difference $V = 73.0$ V is applied across the inner and outer shells. Determine (a) the capacitance of the device, (b) the free charge q on the inner shell, and (c) the charge q' induced along the surface of the inner shell.

Additional Problems

56 In Fig. 25.33, the battery potential difference V is 10.0 V and each of the seven capacitors has capacitance 10.0 μF. What is the charge on (a) capacitor 1 and (b) capacitor 2?

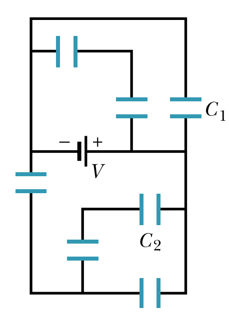

Figure 25.33 Problem 56.

57 SSM In Fig. 25.34, $V = 9.0$ V, $C_1 = C_2 = 30\ \mu$F, and $C_3 = C_4 = 15\ \mu$F. What is the charge on capacitor 4?

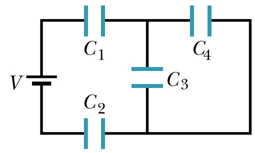

Figure 25.34 Problem 57.

58 (a) If $C = 50\ \mu$F in Fig. 25.35, what is the equivalent capacitance between points A and B? (*Hint:* First imagine that a battery is connected between those two points.) (b) Repeat for points A and D.

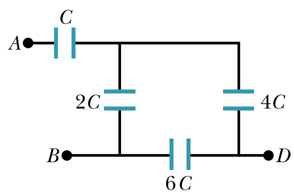

Figure 25.35 Problem 58.

59 In Fig. 25.36, $V = 12$ V, $C_1 = C_4 = 2.0\ \mu$F, $C_2 = 4.0\ \mu$F, and $C_3 = 1.0\ \mu$F. What is the charge on capacitor 4?

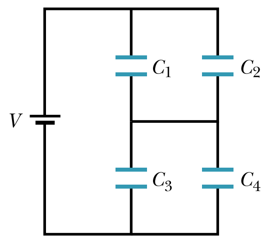

Figure 25.36 Problem 59.

60 **FCP** *The chocolate crumb mystery.* This story begins with Problem 60 in Chapter 23. As part of the investigation of the biscuit factory explosion, the electric potentials of the workers were measured as they emptied sacks of chocolate crumb powder into the loading bin, stirring up a cloud of the powder around themselves. Each worker had an electric potential of about 7.0 kV relative to the ground, which was taken as zero potential. (a) Assuming that each worker was effectively a capacitor with a typical capacitance of 200 pF, find the energy stored in that effective capacitor. If a single spark between the worker and any conducting object connected to the ground neutralized the worker, that energy would be transferred to the spark. According to measurements, a spark that could ignite a cloud of chocolate crumb powder, and thus set off an explosion, had to have an energy of at least 150 mJ. (b) Could a spark from a worker have set off an explosion in the cloud of powder in the loading bin? (The story continues with Problem 60 in Chapter 26.)

61 Figure 25.37 shows capacitor 1 ($C_1 = 8.00\ \mu$F), capacitor 2 ($C_2 = 6.00\ \mu$F), and capacitor 3 ($C_3 = 8.00\ \mu$F) connected to a 12.0 V battery. When switch S is closed so as to connect uncharged capacitor 4 ($C_4 = 6.00\ \mu$F), (a) how much charge passes through point P from the battery and (b) how much charge shows up on capacitor 4? (c) Explain the discrepancy in those two results.

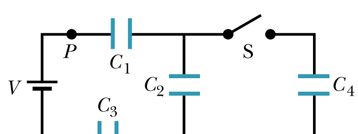

Figure 25.37 Problem 61.

62 Two air-filled, parallel-plate capacitors are to be connected to a 10 V battery, first individually, then in series, and then in parallel. In those arrangements, the energy stored in the capacitors turns out to be, listed least to greatest: 75 μJ, 100 μJ, 300 μJ, and 400 μJ. Of the two capacitors, what is the (a) smaller and (b) greater capacitance?

63 *Coaxial cable.* The inner and outer cylindrical conductors of a long coaxial cable have diameters $a = 0.15$ mm and $b = 2.1$ mm. What is the capacitance per unit length?

64 *Earth's capacitance.* What is the capacitance of Earth, viewed as an isolated conducting sphere of radius 6370 km?

65 *Energy outside conducting sphere.* An isolated conducting sphere has radius $R = 6.85$ cm and charge $q = 1.25$ nC. (a) How much potential energy is stored in the electric field? (b) What is the energy density at the surface of the sphere? (c) What is the radius R_0 of an imaginary spherical surface such that one-half of the stored potential energy lies within it?

66 *Shocking walk across a carpet*. On a day with low humidity, you can become charged by walking over certain carpets (there is charge transfer between the carpet and your shoes). If a spark jumps between your hand and a metal doorknob when the separation is about 5.0 mm, you were probably at a potential of 15 kV relative to the doorknob. To determine your accumulated charge q, make the rough approximation your body can be represented by a uniformly charged conducting sphere with radius $R = 25$ cm in radius and isolated from its surroundings. What is q?

67 *Force and electrostatic stress.* Module 8.3 relates force to potential energy: $|F| = dU/dx$. (a) Apply that relationship to a parallel-plate capacitor with charge q, plate area A, and plate separation x to find an expression for the magnitude of the force between the plates. (b) Evaluate the magnitude of that force for $q = 6.00\ \mu$C and $A = 2.50$ cm^2. (c) *Electrostatic stress* is the force per unit area $|F/A|$ on either plate. Find an expression for the stress in terms of ε_0 and the magnitude E of the electric field between the plates. (d) Evaluate the stress for a potential difference of 110 V and a plate separation of $x = 2.00$ mm.

68 **CALC** *Thermal expansion of a capacitor.* A capacitor is to be designed to operate, with constant capacitance, in an environment of fluctuating temperature. As shown in Fig. 25.38, the capacitor is a parallel-plate type with thin plastic "spacers" to keep the plates aligned. (a) Show that the rate of change of capacitance C with temperature T is given by

$$\frac{dC}{dT} = C\left(\frac{1}{A}\frac{dA}{dT} - \frac{1}{x}\frac{dx}{dT}\right),$$

where A is the plate area and x the plate separation, both of which vary with a temperature change. (b) If the plates are aluminum, what should be the coefficient of thermal expansion of the spacers in order that the capacitance not vary with temperature? (Neglect the effect of the spacers on the capacitance.)

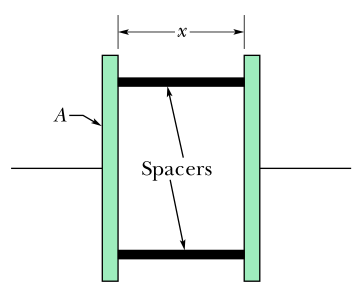

Figure 25.38 Problem 68.

69 *Capacitors and diodes.* An ideal diode allows negative charge (electrons) to move through it only in the direction opposite the schematic arrow in a circuit diagram. Figure 25.39 shows a circuit with two such ideal diodes and two identical capacitors C. A 100 V battery is connected across the input terminals a and b (the potential difference between them is 100 V). What is the output voltage V_{out} if the battery's positive terminal is connected to (a) a and then (b) b?

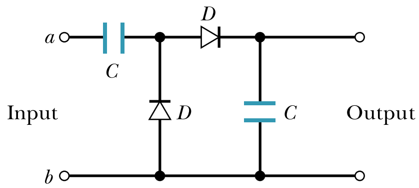

Figure 25.39 Problem 69.

70 BIO The ability of a capacitor to store potential energy is the basis of *defibrillator* devices, which are used by emergency teams to stop the fibrillation of heart attack victims (Fig. 25.40). In the portable version, a battery charges a capacitor to a high potential difference, storing a large amount of energy in less than a minute. The battery maintains only a modest potential difference; an electronic circuit repeatedly uses that potential difference to greatly increase the potential difference of the capacitor. The power, or rate of energy transfer, during this process is also modest. Conducting leads ("paddles") are placed on the victim's chest. When a control switch is closed, the capacitor sends a portion of its stored energy from paddle to paddle through the victim. (a) If a 70 μF capacitor in a defibrillator is charged to 5.0 kV, what is the stored potential energy? (b) If 23% of that energy is sent through the chest in 2.0 ms, what is the power of the pulse?

Iakov Filimonov/123 RF

Figure 25.40 Problem 70.

71 *Movable center section*. Figure 25.41 shows two capacitors in series, with a rigid center section that can be moved vertically, either upward or downward. (a) The plate area A is the same for the capacitors. In terms of A, a, b, and ε_0, what is the equivalent capacitance C? (b) Evaluate C for A = 2.0 cm^2, a = 7.0 mm, and b = 4.0 mm. (c) If the center section is moved downward (without touching the bottom plate), does C increase, decrease, or stay the same?

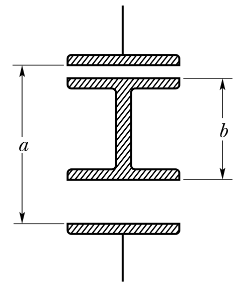

Figure 25.41 Problem 71.

72 *Spark danger in airborne dust.* A person walking through airborne dust in, say, a cosmetic plant can possibly become dangerously charged by contact with the floor and various objects that are touched. Safety engineers often calculate the danger threshold for the electric potential on a person by modeling the person as a spherical capacitor of radius R = 1.8 m. What electric potential corresponds to the threshold value U_t = 150 mJ of stored energy at which a spark could ignite the dust and set off an explosion?