

# Magnetic Fields Due to Currents

## 29.1 MAGNETIC FIELD DUE TO A CURRENT

### Learning Objectives

After reading this module, you should be able to . . .

- 29.1.1** Sketch a current-length element in a wire and indicate the direction of the magnetic field that it sets up at a given point near the wire.
- 29.1.2** For a given point near a wire and a given current-length element in the wire, determine the magnitude and direction of the magnetic field due to that element.
- 29.1.3** Identify the magnitude of the magnetic field set up by a current-length element at a point in line with the direction of that element.
- 29.1.4** For a point to one side of a long straight wire carrying current, apply the relationship between the magnetic field magnitude, the current, and the distance to the point.
- 29.1.5** For a point to one side of a long straight wire carrying current, use a right-hand rule to determine the direction of the field vector.

- 29.1.6** Identify that around a long straight wire carrying current, the magnetic field lines form circles.
- 29.1.7** For a point to one side of the end of a semi-infinite wire carrying current, apply the relationship between the magnetic field magnitude, the current, and the distance to the point.
- 29.1.8** For the center of curvature of a circular arc of wire carrying current, apply the relationship between the magnetic field magnitude, the current, the radius of curvature, and the angle subtended by the arc (in radians).
- 29.1.9** For a point to one side of a short straight wire carrying current, integrate the Biot–Savart law to find the magnetic field set up at the point by the current.

### Key Ideas

The magnetic field set up by a current-carrying conductor can be found from the Biot–Savart law. This law asserts that the contribution  $d\vec{B}$  to the field produced by a current-length element  $i d\vec{s}$  at a point  $P$  located a distance  $r$  from the current element is

$$d\vec{B} = \frac{\mu_0 i d\vec{s} \times \hat{r}}{4\pi r^2}, \quad (\text{Biot–Savart law}).$$

Here  $\hat{r}$  is a unit vector that points from the element toward  $P$ . The quantity  $\mu_0$ , called the permeability constant, has the value

$$4\pi \times 10^{-7} \text{ T}\cdot\text{m/A} \approx 1.26 \times 10^{-6} \text{ T}\cdot\text{m/A}.$$

For a long straight wire carrying a current  $i$ , the Biot–Savart law gives, for the magnitude of the magnetic field at a perpendicular distance  $R$  from the wire,

$$B = \frac{\mu_0 i}{2R} \quad (\text{long straight wire}).$$

The magnitude of the magnetic field at the center of a circular arc, of radius  $R$  and central angle  $\theta$  (in radians), carrying current  $i$ , is

$$B = \frac{\mu_0 i \theta}{4R} \quad (\text{at center of circular arc}).$$

## What Is Physics?

One basic observation of physics is that a moving charged particle produces a magnetic field around itself. Thus a current of moving charged particles produces a magnetic field around the current. This feature of *electromagnetism*, which is the combined study of electric and magnetic effects, came as a surprise to the people who discovered it. Surprise or not, this feature has become enormously

important in everyday life because it is the basis of countless electromagnetic devices. For example, a magnetic field is produced in maglev trains and other devices used to lift heavy loads.

Our first step in this chapter is to find the magnetic field due to the current in a very small section of current-carrying wire. Then we shall find the magnetic field due to the entire wire for several different arrangements of the wire.

## Calculating the Magnetic Field Due to a Current

Figure 29.1.1 shows a wire of arbitrary shape carrying a current  $i$ . We want to find the magnetic field  $\vec{B}$  at a nearby point  $P$ . We first mentally divide the wire into differential elements  $ds$  and then define for each element a length vector  $d\vec{s}$  that has length  $ds$  and whose direction is the direction of the current in  $ds$ . We can then define a differential *current-length element* to be  $i d\vec{s}$ ; we wish to calculate the field  $d\vec{B}$  produced at  $P$  by a typical current-length element. From experiment we find that magnetic fields, like electric fields, can be superimposed to find a net field. Thus, we can calculate the net field  $\vec{B}$  at  $P$  by summing, via integration, the contributions  $d\vec{B}$  from all the current-length elements. However, this summation is more challenging than the process associated with electric fields because of a complexity; whereas a charge element  $dq$  producing an electric field is a scalar, a current-length element  $i d\vec{s}$  producing a magnetic field is a vector, being the product of a scalar and a vector.

**Magnitude.** The magnitude of the field  $d\vec{B}$  produced at point  $P$  at distance  $r$  by a current-length element  $i d\vec{s}$  turns out to be

$$dB = \frac{\mu_0}{4\pi\epsilon_0} \frac{i ds \sin \hat{\imath}}{r^2}, \quad (29.1.1)$$

where  $\hat{\imath}$  is the angle between the directions of  $d\vec{s}$  and  $\vec{r}$ ; a unit vector that points from  $ds$  toward  $P$ . Symbol  $\mu_0$  is a constant, called the *permeability constant*, whose value is defined to be exactly

$$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \approx 1.26 \times 10^{-6} \text{ T} \cdot \text{m/A}. \quad (29.1.2)$$

**Direction.** The direction of  $d\vec{B}$ , shown as being into the page in Fig. 29.1.1, is that of the cross product  $d\vec{s} \times \vec{r}$ . We can therefore write Eq. 29.1.1 in vector form as

$$d\vec{B} = \frac{\mu_0}{4\pi\epsilon_0} \frac{i d\vec{s} \times \vec{r}}{r^2}, \quad (\text{Biot-Savart law}). \quad (29.1.3)$$

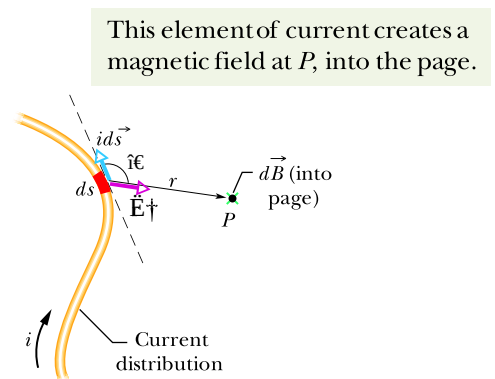
This vector equation and its scalar form, Eq. 29.1.1, are known as the **law of Biot and Savart** (rhymes with *le bon bazaar*). The law, which is experimentally deduced, is an inverse-square law. We shall use this law to calculate the net magnetic field  $\vec{B}$  produced at a point by various distributions of current.

Here is one easy distribution: If current in a wire is either directly toward or directly away from a point  $P$  of measurement, can you see from Eq. 29.1.1 that the magnetic field at  $P$  from the current is simply zero (the angle  $\hat{\imath}$  is either  $0^\circ$  for toward or  $180^\circ$  for away, and both result in  $\sin \hat{\imath} = 0$ )?

### Magnetic Field Due to a Current in a Long Straight Wire

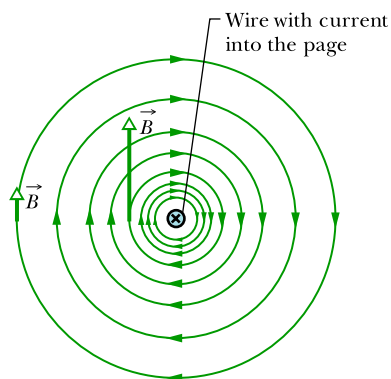
Shortly we shall use the law of Biot and Savart to prove that the magnitude of the magnetic field at a perpendicular distance  $R$  from a long (infinite) straight wire carrying a current  $i$  is given by

$$B = \frac{\mu_0}{2\pi R} i \quad (\text{long straight wire}). \quad (29.1.4)$$

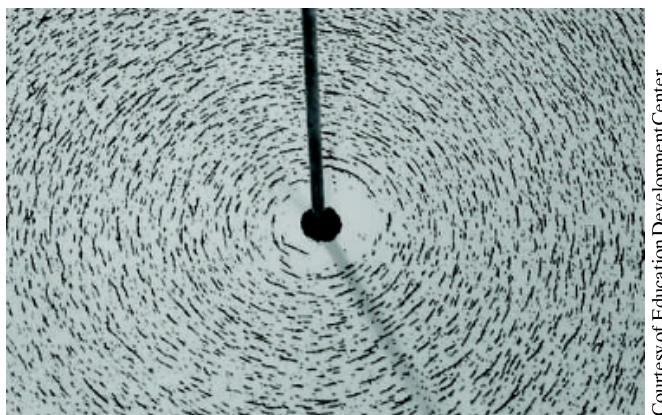


**Figure 29.1.1** A current-length element  $i d\vec{s}$  produces a differential magnetic field  $d\vec{B}$  at point  $P$ . The green  $\vec{A}$  (the tail of an arrow) at the dot for point  $P$  indicates that  $d\vec{B}$  is directed *into* the page there.

The magnetic field vector at any point is tangent to a circle.



**Figure 29.1.2** The magnetic field lines produced by a current in a long straight wire form concentric circles around the wire. Here the current is into the page, as indicated by the  $\odot$ .



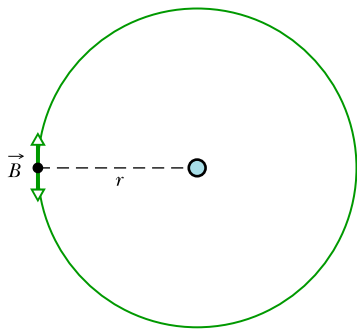
**Figure 29.1.3** Iron filings that have been sprinkled onto cardboard collect in concentric circles when current is sent through the central wire. The alignment, which is along magnetic field lines, is caused by the magnetic field produced by the current.

The field magnitude  $B$  in Eq. 29.1.4 depends only on the current and the perpendicular distance  $R$  of the point from the wire. We shall show in our derivation that the field lines of  $\vec{B}$  form concentric circles around the wire, as Fig. 29.1.2 shows and as the iron filings in Fig. 29.1.3 suggest. The increase in the spacing of the lines in Fig. 29.1.2 with increasing distance from the wire represents the  $1/R$  decrease in the magnitude of  $\vec{B}$  predicted by Eq. 29.1.4. The lengths of the two vectors  $\vec{B}$  in the figure also show the  $1/R$  decrease.

**Directions.** Plugging values into Eq. 29.1.4 to find the field magnitude  $B$  at a given radius is easy. What is difficult for many students is finding the direction of a field vector  $\vec{B}$  at a given point. The field lines form circles around a long straight wire, and the field vector at any point on a circle must be tangent to the circle. That means it must be perpendicular to a radial line extending to the point from the wire. But there are two possible directions for that perpendicular vector, as shown in Fig. 29.1.4. One is correct for current into the figure, and the other is correct for current out of the figure. How can you tell which is which? Here is a simple right-hand rule for telling which vector is correct:



**Curled “straight-hand rule:** Grasp the element in your right hand with your extended thumb pointing in the direction of the current. Your fingers will then naturally curl around in the direction of the magnetic field lines due to that element.

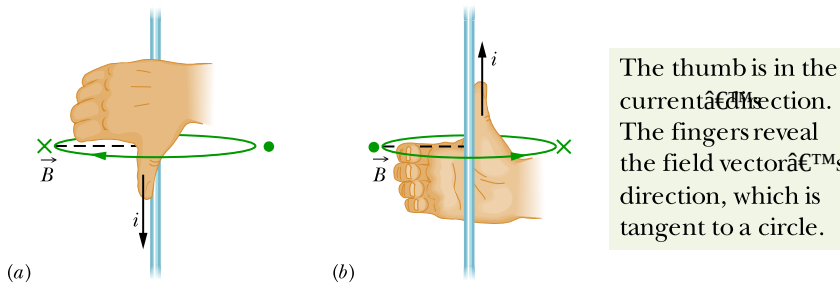


**Figure 29.1.4** The magnetic field vector  $\vec{B}$  is perpendicular to the radial line extending from a long straight wire with current, but which of the two perpendicular vectors is it?

The result of applying this right-hand rule to the current in the straight wire of Fig. 29.1.2 is shown in a side view in Fig. 29.1.5a. To determine the direction of the magnetic field  $\vec{B}$  set up at any particular point by this current, mentally wrap your right hand around the wire with your thumb in the direction of the current. Let your fingertips pass through the point; their direction is then the direction of the magnetic field at that point. In the view of Fig. 29.1.2,  $\vec{B}$  at any point is *tangent to a magnetic field line*; in the view of Fig. 29.1.5, it is *perpendicular to a dashed radial line connecting the point and the current*.

### Proof of Equation 29.1.4

Figure 29.1.6, which is just like Fig. 29.1.1 except that now the wire is straight and of infinite length, illustrates the task at hand. We seek the field  $\vec{B}$  at point  $P$ , a perpendicular distance  $R$  from the wire. The magnitude of the differential



The thumb is in the current's direction. The fingers reveal the field vector's direction, which is tangent to a circle.

**Figure 29.1.5** A right-hand rule gives the direction of the magnetic field due to a current in a wire. (a) The situation of Fig. 29.1.2, seen from the side. The magnetic field  $\vec{B}$  at any point to the left of the wire is perpendicular to the dashed radial line and directed into the page, in the direction of the fingertips, as indicated by the  $\times$ . (b) If the current is reversed,  $\vec{B}$  at any point to the left is still perpendicular to the dashed radial line but now is directed out of the page, as indicated by the dot.

magnetic field produced at  $P$  by the current-length element  $i d\vec{s}$  located a distance  $r$  from  $P$  is given by Eq. 29.1.1:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \sin \hat{\theta}}{r^2}.$$

The direction of  $d\vec{B}$  in Fig. 29.1.6 is that of the vector  $d\vec{s} \times \hat{r}$ , namely directly into the page.

Note that  $d\vec{B}$  at point  $P$  has this same direction for all the current-length elements into which the wire can be divided. Thus, we can find the magnitude of the magnetic field produced at  $P$  by the current-length elements in the upper half of the infinitely long wire by integrating  $dB$  in Eq. 29.1.1 from 0 to  $\infty$ .

Now consider a current-length element in the lower half of the wire, one that is as far below  $P$  as  $d\vec{s}$  is above  $P$ . By Eq. 29.1.3, the magnetic field produced at  $P$  by this current-length element has the same magnitude and direction as that from element  $i d\vec{s}$  in Fig. 29.1.6. Further, the magnetic field produced by the lower half of the wire is exactly the same as that produced by the upper half. To find the magnitude of the *total* magnetic field  $\vec{B}$  at  $P$ , we need only multiply the result of our integration by 2. We get

$$B = 2 \int_0^{\infty} dB = \frac{\mu_0}{2\pi} \int_0^{\infty} \frac{\sin \hat{\theta}}{r^2} ds. \quad (29.1.5)$$

The variables  $\hat{\theta}$ ,  $s$ , and  $r$  in this equation are not independent; Fig. 29.1.6 shows that they are related by

$$r = \sqrt{s^2 + R^2}$$

and

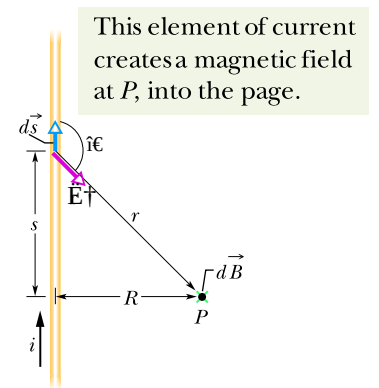
$$\sin \hat{\theta} = \sin(\hat{r} \cdot \hat{s}) = \frac{R}{\sqrt{s^2 + R^2}}.$$

With these substitutions and integral 19 in Appendix E, Eq. 29.1.5 becomes

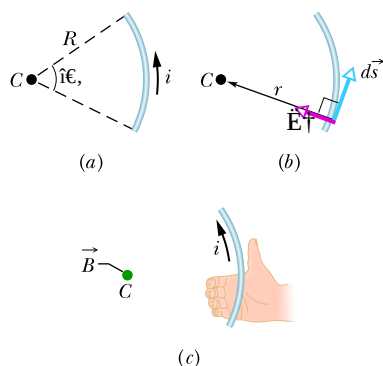
$$\begin{aligned} B &= \frac{\mu_0}{2\pi} \int_0^{\infty} \frac{R ds}{(s^2 + R^2)^{3/2}} \\ &= \frac{\mu_0}{2\pi R} \left[ \frac{s}{(s^2 + R^2)^{1/2}} \right]_0^{\infty} = \frac{\mu_0}{2\pi R}, \end{aligned} \quad (29.1.6)$$

as we wanted. Note that the magnetic field at  $P$  due to either the lower half or the upper half of the infinite wire in Fig. 29.1.6 is half this value; that is,

$$B = \frac{\mu_0}{4\pi R} \quad (\text{semi-infinite straight wire}). \quad (29.1.7)$$



**Figure 29.1.6** Calculating the magnetic field produced by a current  $i$  in a long straight wire. The field  $d\vec{B}$  at  $P$  associated with the current-length element  $i d\vec{s}$  is directed into the page, as shown.



The right-hand rule reveals the field  $\hat{\mathbf{B}}$ 's direction at the center.

**Figure 29.1.7** (a) A wire in the shape of a circular arc with center  $C$  carries current  $i$ . (b) For any element of wire along the arc, the angle between the directions of  $d\hat{\mathbf{s}}$  and  $r$  is  $90^\circ$ . (c) Determining the direction of the magnetic field at the center  $C$  due to the current in the wire; the field is out of the page, in the direction of the fingertips, as indicated by the colored dot at  $C$ .

### Magnetic Field Due to a Current in a Circular Arc of Wire

To find the magnetic field produced at a point by a current in a curved wire, we would again use Eq. 29.1.1 to write the magnitude of the field produced by a single current-length element, and we would again integrate to find the net field produced by all the current-length elements. That integration can be difficult, depending on the shape of the wire; it is fairly straightforward, however, when the wire is a circular arc and the point is the center of curvature.

Figure 29.1.7a shows such an arc-shaped wire with central angle  $\hat{\mathbf{I}}\bullet$ , radius  $R$ , and center  $C$ , carrying current  $i$ . At  $C$ , each current-length element  $i d\hat{\mathbf{s}}$  of the wire produces a magnetic field of magnitude  $dB$  given by Eq. 29.1.1. Moreover, as Fig. 29.1.7b shows, no matter where the element is located on the wire, the angle  $\hat{\mathbf{I}}\alpha$  between the vectors  $d\hat{\mathbf{s}}$  and  $r$  is  $90^\circ$  also,  $r = R$ . Thus, by substituting  $R$  for  $r$  and  $90^\circ$  for  $\hat{\mathbf{I}}\alpha$  in Eq. 29.1.1, we obtain

$$dB = \frac{\hat{\mathbf{I}}\mu_0 i ds \sin 90^\circ}{4\hat{\mathbf{I}}\pi R^2} = \frac{\hat{\mathbf{I}}\mu_0 i ds}{4\hat{\mathbf{I}}\pi R^2}. \quad (29.1.8)$$

The field at  $C$  due to each current-length element in the arc has this magnitude.

**Directions.** How about the direction of the differential field  $d\hat{\mathbf{B}}$  set up by an element? From above we know that the vector must be perpendicular to a radial line extending through point  $C$  from the element, either into the plane of Fig. 29.1.7a or out of it. To tell which direction is correct, we use the right-hand rule for any of the elements, as shown in Fig. 29.1.7c. Grasping the wire with the thumb in the direction of the current and bringing the fingers into the region near  $C$ , we see that the vector  $d\hat{\mathbf{B}}$  due to any of the differential elements is out of the plane of the figure, not into it.

**Total Field.** To find the total field at  $C$  due to all the elements on the arc, we need to add all the differential field vectors  $d\hat{\mathbf{B}}$ . However, because the vectors are all in the same direction, we do not need to find components. We just sum the magnitudes  $dB$  as given by Eq. 29.1.8. Since we have a vast number of those magnitudes, we sum via integration. We want the result to indicate how the total field depends on the angle  $\hat{\mathbf{I}}\bullet$  of the arc (rather than the arc length). So, in Eq. 29.1.8 we switch from  $ds$  to  $d\hat{\mathbf{I}}\bullet$  by using the identity  $ds = R d\hat{\mathbf{I}}\bullet$ . The summation by integration then becomes

$$B = \hat{\mathbf{I}}B = \hat{\mathbf{I}}\int_0^{\hat{\mathbf{I}}\bullet} \frac{\hat{\mathbf{I}}\mu_0 i R d\hat{\mathbf{I}}\bullet}{4\hat{\mathbf{I}}\pi R^2} = \frac{\hat{\mathbf{I}}\mu_0 i}{4\hat{\mathbf{I}}\pi R} \hat{\mathbf{I}}\bullet.$$

Integrating, we find that

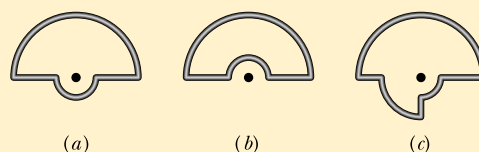
$$B = \frac{\hat{\mathbf{I}}\mu_0 i \hat{\mathbf{I}}\bullet}{4\hat{\mathbf{I}}\pi R} \quad (\text{at center of circular arc}). \quad (29.1.9)$$

**Heads Up.** Note that this equation gives us the magnetic field *only* at the center of curvature of a circular arc of current. When you insert data into the equation, you must be careful to express  $\hat{\mathbf{I}}\bullet$  in radians rather than degrees. For example, to find the magnitude of the magnetic field at the center of a full circle of current, you would substitute  $2\hat{\mathbf{I}}\pi$  rad for  $\hat{\mathbf{I}}\bullet$  in Eq. 29.1.9, finding

$$B = \frac{\hat{\mathbf{I}}\mu_0 i (2\hat{\mathbf{I}}\pi)}{4\hat{\mathbf{I}}\pi R} = \frac{\hat{\mathbf{I}}\mu_0 i}{2R} \quad (\text{at center of full circle}). \quad (29.1.10)$$

### Checkpoint 29.1.1

The figure shows three circuits consisting of concentric circular arcs (either half- or quarter-circles of radii  $r$ ,  $2r$ , and  $3r$ ). The circuits carry the same current. Rank them according to the magnitude of the magnetic field produced at the center of curvature (the dot), greatest first.





### Sample Problem 29.1.1 Magnetic field due to brain activity

Scientists, medical researchers, and physiologists would like to understand how the human brain works. For example, when you read this sentence, what part of your brain is activated? One of the recent research tools is *magnetoencephalography* (MEG), a procedure in which magnetic fields of a person's brain are monitored as the person performs a task such as reading (Fig. 29.1.8). The task activates part of the brain that processes reading, causing weak electrical pulses to be sent along conducting paths between brain cells. As with any other current, each pulse produces a magnetic field. The magnetic fields detected by MEG are probably produced by pulses along the walls of the fissures (crevices) on the brain surface. Detecting the fields requires extremely sensitive instruments called SQUIDS (superconducting quantum interference devices). In Fig. 29.1.9, what is the magnetic field at point  $P$  located a distance  $r = 2$  cm from a pulse on a fissure wall, with the current  $i = 10$  A along a conducting path of length  $ds = 1$  mm that is perpendicular to  $r$ ?



Figure 29.1.8 MEG apparatus.

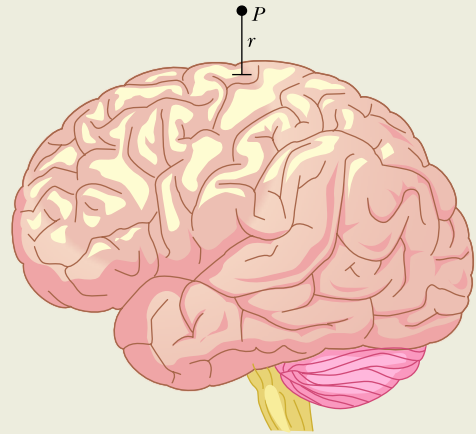


Figure 29.1.9 A pulse along a fissure wall on the brain surface produces a magnetic field at point  $P$  at distance  $r$ .

#### KEY IDEA

The path length is short enough that we can apply the Biot-Savart law directly to find the magnetic field.

**Calculation:** We write the Biot-Savart law (Eq. 29.1.1) as

$$\begin{aligned} B &= \frac{\mu_0}{4\pi} \frac{i \, ds \sin \theta}{r^2} \\ &= \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}) (10 \text{ A}) (1 \times 10^{-3} \text{ m}) \sin 90^\circ}{(2 \times 10^{-2} \text{ m})^2} \\ &= 2.5 \times 10^{-12} \text{ T} = 2.5 \text{ pT.} \end{aligned} \quad (\text{Answer})$$

SQUIDS can measure fields of magnitude less than 1 pT, but care must be taken to eliminate other sources of varying magnetic fields, such as an elevator in the building or even a truck in a nearby street.

### Sample Problem 29.1.2 Magnetic field off to the side of two long straight currents

Figure 29.1.10a shows two long parallel wires carrying currents  $i_1$  and  $i_2$  in opposite directions. What are the magnitude and direction of the net magnetic field at point  $P$ ? Assume the following values:  $i_1 = 15$  A,  $i_2 = 32$  A, and  $d = 5.3$  cm.

#### KEY IDEAS

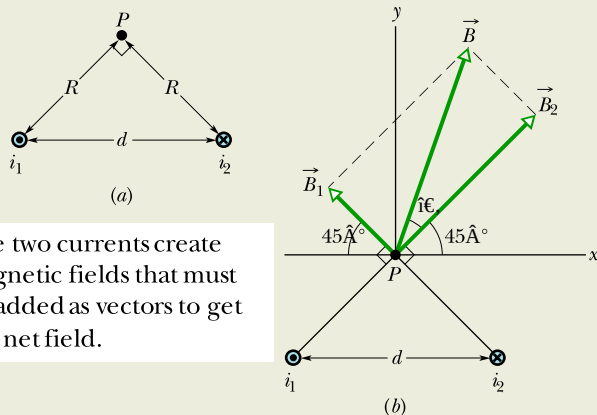
- (1) The net magnetic field  $\vec{B}$  at point  $P$  is the vector sum of the magnetic fields due to the currents in the two wires.
- (2) We can find the magnetic field due to any current by applying the Biot-Savart law to the current. For points near the current in a long straight wire, that law leads to Eq. 29.1.4.

**Finding the vectors:** In Fig. 29.1.10a, point  $P$  is distance  $R$  from both currents  $i_1$  and  $i_2$ . Thus, Eq. 29.1.4 tells us that at point  $P$  those currents produce magnetic fields  $\vec{B}_1$  and  $\vec{B}_2$  with magnitudes

$$B_1 = \frac{\mu_0 i_1}{2\pi R} \quad \text{and} \quad B_2 = \frac{\mu_0 i_2}{2\pi R}.$$

In the right triangle of Fig. 29.1.10a, note that the base angles (between sides  $R$  and  $d$ ) are both  $45^\circ$ . This allows us to write  $\cos 45^\circ = R/d$  and replace  $R$  with  $d \cos 45^\circ$ . Then the field magnitudes  $B_1$  and  $B_2$  become

$$B_1 = \frac{\mu_0 i_1}{2\pi d \cos 45^\circ} \quad \text{and} \quad B_2 = \frac{\mu_0 i_2}{2\pi d \cos 45^\circ}.$$



The two currents create magnetic fields that must be added as vectors to get the net field.

**Figure 29.1.10** (a) Two wires carry currents  $i_1$  and  $i_2$  in opposite directions (out of and into the page). Note the right angle at  $P$ . (b) The separate fields  $\vec{B}_1$  and  $\vec{B}_2$  are combined vectorially to yield the net field  $\vec{B}$ .

We want to combine  $\vec{B}_1$  and  $\vec{B}_2$  to find their vector sum, which is the net field  $\vec{B}$  at  $P$ . To find the directions of  $\vec{B}_1$  and  $\vec{B}_2$ , we apply the right-hand rule of Fig. 29.1.5 to each current in Fig. 29.1.10a. For wire 1, with current out of the page, we mentally grasp the wire with the right hand, with the thumb pointing out of the page. Then the curled fingers indicate that the field lines run counterclockwise. In particular, in the region of point  $P$ , they are directed upward to the left. Recall that the magnetic field at a point near a long, straight current-carrying wire must be directed perpendicular to a radial line between the point and the current. Thus,  $\vec{B}_1$  must be directed upward to the left as drawn in Fig. 29.1.10b.

(Note carefully the perpendicular symbol between vector  $\vec{B}_1$  and the line connecting point  $P$  and wire 1.)

Repeating this analysis for the current in wire 2, we find that  $\vec{B}_2$  is directed upward to the right as drawn in Fig. 29.1.10b.

**Adding the vectors:** We can now vectorially add  $\vec{B}_1$  and  $\vec{B}_2$  to find the net magnetic field  $\vec{B}$  at point  $P$ , either by using a vector-capable calculator or by resolving the vectors into components and then combining the components of  $\vec{B}$ . However, in Fig. 29.1.10b, there is a third method: Because  $\vec{B}_1$  and  $\vec{B}_2$  are perpendicular to each other, they form the legs of a right triangle, with  $\vec{B}$  as the hypotenuse. So,

$$\begin{aligned}
 B &= \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2\pi d} \sqrt{i_1^2 + i_2^2} \\
 &= \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(15 \text{ A})^2 + (32 \text{ A})^2}{(2\pi)(5.3 \times 10^{-2} \text{ m})(\cos 45^\circ)} \\
 &= 1.89 \times 10^{-4} \text{ T} \approx 190 \mu\text{T}. \quad (\text{Answer})
 \end{aligned}$$

The angle  $\theta$  between the directions of  $\vec{B}$  and  $\vec{B}_2$  in Fig. 29.1.10b follows from

$$\theta = \tan^{-1} \frac{B_1}{B_2},$$

which, with  $B_1$  and  $B_2$  as given above, yields

$$\theta = \tan^{-1} \frac{i_1}{i_2} = \tan^{-1} \frac{15 \text{ A}}{32 \text{ A}} = 25^\circ.$$

The angle between  $\vec{B}$  and the  $x$  axis shown in Fig. 29.1.10b is then

$$\theta + 45^\circ = 25^\circ + 45^\circ = 70^\circ. \quad (\text{Answer})$$

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## 29.2 FORCE BETWEEN TWO PARALLEL CURRENTS

### Learning Objectives

After reading this module, you should be able to . . .

**29.2.1** Given two parallel or antiparallel currents, find the magnetic field of the first current at the location of the second current and then find the force acting on that second current.

**29.2.2** Identify that parallel currents attract each other, and antiparallel currents repel each other.

**29.2.3** Describe how a rail gun works.

### Key Idea

Parallel wires carrying currents in the same direction attract each other, whereas parallel wires carrying currents in opposite directions repel each other. The magnitude of the force on a length  $L$  of either wire is

$$F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d},$$

where  $d$  is the wire separation, and  $i_a$  and  $i_b$  are the currents in the wires.

## Force Between Two Parallel Currents

Two long parallel wires carrying currents exert forces on each other. Figure 29.2.1 shows two such wires, separated by a distance  $d$  and carrying currents  $i_a$  and  $i_b$ . Let us analyze the forces on these wires due to each other.

We seek first the force on wire  $b$  in Fig. 29.2.1 due to the current in wire  $a$ . That current produces a magnetic field  $\vec{B}_a$ , and it is this magnetic field that actually causes the force we seek. To find the force, then, we need the magnitude and direction of the field  $\vec{B}_a$  at the site of wire  $b$ . The magnitude of  $\vec{B}_a$  at every point of wire  $b$  is, from Eq. 29.1.4,

$$B_a = \frac{\mu_0 i_a}{2\pi d} \quad (29.2.1)$$

The curled-right-hand rule tells us that the direction of  $\vec{B}_a$  at wire  $b$  is down, as Fig. 29.2.1 shows. Now that we have the field, we can find the force it produces on wire  $b$ . Equation 28.6.2 tells us that the force  $\vec{F}_{ba}$  on a length  $L$  of wire  $b$  due to the external magnetic field  $\vec{B}_a$  is

$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a \quad (29.2.2)$$

where  $\vec{L}$  is the length vector of the wire. In Fig. 29.2.1, vectors  $\vec{L}$  and  $\vec{B}_a$  are perpendicular to each other, and so with Eq. 29.2.1, we can write

$$F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d} \quad (29.2.3)$$

The direction of  $\vec{F}_{ba}$  is the direction of the cross product  $\vec{L} \times \vec{B}_a$ . Applying the right-hand rule for cross products to  $\vec{L}$  and  $\vec{B}_a$  in Fig. 29.2.1, we see that  $\vec{F}_{ba}$  is directly toward wire  $a$ , as shown.

The general procedure for finding the force on a current-carrying wire is this:



To find the force on a current-carrying wire due to a second current-carrying wire, first find the field due to the second wire at the site of the first wire. Then find the force on the first wire due to that field.

We could now use this procedure to compute the force on wire  $a$  due to the current in wire  $b$ . We would find that the force is directly toward wire  $b$ ; hence, the two wires with parallel currents attract each other. Similarly, if the two currents were antiparallel, we could show that the two wires repel each other. Thus,



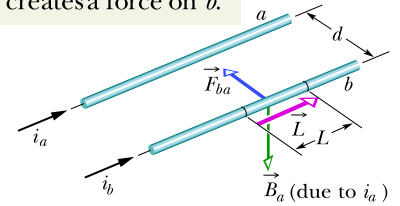
Parallel currents attract each other, and antiparallel currents repel each other.

The force acting between currents in parallel wires is the basis for the definition of the ampere, which is one of the seven SI base units. The definition, adopted in 1946, is this: The ampere is that constant current which, if maintained in two straight, parallel conductors of infinite length, of negligible circular cross section, and placed 1 m apart in vacuum, would produce on each of these conductors a force of magnitude  $2 \times 10^{-7}$  newton per meter of wire length.

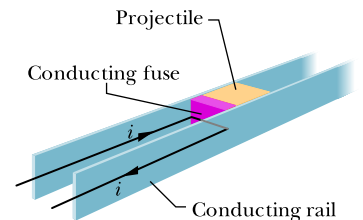
### Rail Gun

The basics of a rail gun are shown in Fig. 29.2.2a. A large current is sent out along one of two parallel conducting rails, across a conducting fuse (such as a narrow piece of copper) between the rails, and then back to the current source along the second rail. The projectile to be fired lies on the far side of the fuse and fits loosely between the rails. Immediately after the current begins, the fuse element melts and vaporizes, creating a conducting gas between the rails where the fuse had been.

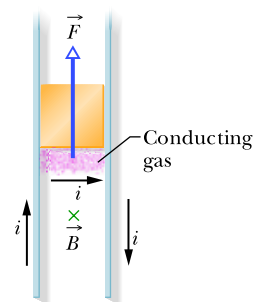
The field due to  $a$  at the position of  $b$  creates a force on  $b$ .



**Figure 29.2.1** Two parallel wires carrying currents in the same direction attract each other.  $\vec{B}_a$  is the magnetic field at wire  $b$  produced by the current in wire  $a$ .  $\vec{F}_{ba}$  is the resulting force acting on wire  $b$  because it carries current in  $\vec{B}_a$ .



(a)



(b)

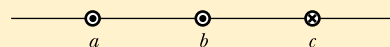
**Figure 29.2.2** (a) A rail gun, as a current  $i$  is set up in it. The current rapidly causes the conducting fuse to vaporize. (b) The current produces a magnetic field  $\vec{B}$  between the rails, and the field causes a force  $\vec{F}$  to act on the conducting gas, which is part of the current path. The gas propels the projectile along the rails, launching it.



The curled “straight-hand rule” of Fig. 29.1.5 reveals that the currents in the rails of Fig. 29.2*a* produce magnetic fields that are directed downward between the rails. The net magnetic field  $\vec{B}$  exerts a force  $\vec{F}$  on the gas due to the current  $i$  through the gas (Fig. 29.2*b*). With Eq. 29.2.2 and the right-hand rule for cross products, we find that  $\vec{F}$  points outward along the rails. As the gas is forced outward along the rails, it pushes the projectile, accelerating it by as much as  $5 \times 10^6 g$ , and then launches it with a speed of 10 km/s, all within 1 ms. Someday rail guns may be used to launch materials into space from mining operations on the Moon or an asteroid.

### Checkpoint 29.2.1

The figure here shows three long, straight, parallel, equally spaced wires with identical currents either into or out of the page. Rank the wires according to the magnitude of the force on each due to the currents in the other two wires, greatest first.



## 29.3 AMPERE’S LAW

### Learning Objectives

After reading this module, you should be able to . . .

**29.3.1** Apply Ampere’s Law to a loop that encircles current.

**29.3.2** With Ampere’s Law, use a right-hand rule for determining the algebraic sign of an encircled current.

**29.3.3** For more than one current within an Amperian loop, determine the net current to be used in Ampere’s Law.

**29.3.4** Apply Ampere’s Law to a long straight wire with current, to find the magnetic field magnitude inside and outside the wire, identifying that only the current encircled by the Amperian loop matters.

### Key Idea

Ampere’s Law states that

$$\oint_C \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \quad (\text{Ampere’s Law}).$$

The line integral in this equation is evaluated around a closed loop called an Amperian loop. The current  $i$  on the right side is the *net* current encircled by the loop.

### Ampere’s Law

We can find the net electric field due to *any* distribution of charges by first writing the differential electric field  $d\vec{E}$  due to a charge element and then summing the contributions of  $d\vec{E}$  from all the elements. However, if the distribution is complicated, we may have to use a computer. Recall, however, that if the distribution has planar, cylindrical, or spherical symmetry, we can apply Gauss’s Law to find the net electric field with considerably less effort.

Similarly, we can find the net magnetic field due to *any* distribution of currents by first writing the differential magnetic field  $d\vec{B}$  (Eq. 29.1.3) due to a current-length element and then summing the contributions of  $d\vec{B}$  from all the elements. Again we may have to use a computer for a complicated distribution. However, if the distribution has some symmetry, we may be able to apply **Ampere’s Law** to find the magnetic field with considerably less effort. This law, which can be derived from the Biot–Savart law, has traditionally been credited to André-Marie Ampère.

(1775–1836) for whom the SI unit of current is named. However, the law actually was advanced by English physicist James Clerk Maxwell. Ampere's law is

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \quad (\text{Ampere's law}). \quad (29.3.1)$$

The loop on the integral sign means that the scalar (dot) product  $\vec{B} \cdot d\vec{s}$  is to be integrated around a *closed* loop, called an *Amperian loop*. The current  $i_{\text{enc}}$  is the *net* current encircled by that closed loop.

To see the meaning of the scalar product  $\vec{B} \cdot d\vec{s}$  and its integral, let us first apply Ampere's law to the general situation of Fig. 29.3.1. The figure shows cross sections of three long straight wires that carry currents  $i_1$ ,  $i_2$ , and  $i_3$  either directly into or directly out of the page. An arbitrary Amperian loop lying in the plane of the page encircles two of the currents but not the third. The counterclockwise direction marked on the loop indicates the arbitrarily chosen direction of integration for Eq. 29.3.1.

To apply Ampere's law, we mentally divide the loop into differential vector elements  $d\vec{s}$  that are everywhere directed along the tangent to the loop in the direction of integration. Assume that at the location of the element  $d\vec{s}$  shown in Fig. 29.3.1, the net magnetic field due to the three currents is  $\vec{B}$ . Because the wires are perpendicular to the page, we know that the magnetic field at  $d\vec{s}$  due to each current is in the plane of Fig. 29.3.1; thus, their net magnetic field  $\vec{B}$  at  $d\vec{s}$  must also be in that plane. However, we do not know the orientation of  $\vec{B}$  within the plane. In Fig. 29.3.1,  $\vec{B}$  is arbitrarily drawn at an angle  $\hat{\imath}$  to the direction of  $d\vec{s}$ . The scalar product  $\vec{B} \cdot d\vec{s}$  on the left side of Eq. 29.3.1 is equal to  $B \cos \hat{\imath} ds$ . Thus, Ampere's law can be written as

$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos \hat{\imath} ds = \mu_0 i_{\text{enc}}. \quad (29.3.2)$$

We can now interpret the scalar product  $\vec{B} \cdot d\vec{s}$  as being the product of a length  $ds$  of the Amperian loop and the field component  $B \cos \hat{\imath}$  tangent to the loop. Then we can interpret the integration as being the summation of all such products around the entire loop.

**Signs.** When we can actually perform this integration, we do not need to know the direction of  $\vec{B}$  before integrating. Instead, we arbitrarily assume  $\vec{B}$  to be generally in the direction of integration (as in Fig. 29.3.1). Then we use the following curled-finger-right-hand rule to assign a plus sign or a minus sign to each of the currents that make up the net encircled current  $i_{\text{enc}}$ :



Curl your right hand around the Amperian loop, with the fingers pointing in the direction of integration. A current through the loop in the general direction of your outstretched thumb is assigned a plus sign, and a current generally in the opposite direction is assigned a minus sign.

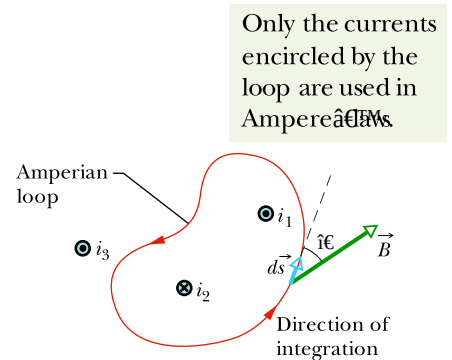
Finally, we solve Eq. 29.3.2 for the magnitude of  $\vec{B}$ . If  $B$  turns out positive, then the direction we assumed for  $\vec{B}$  is correct. If it turns out negative, we neglect the minus sign and redraw  $\vec{B}$  in the opposite direction.

**Net Current.** In Fig. 29.3.2 we apply the curled-finger-right-hand rule for Ampere's law to the situation of Fig. 29.3.1. With the indicated counterclockwise direction of integration, the net current encircled by the loop is

$$i_{\text{enc}} = i_1 - i_2.$$

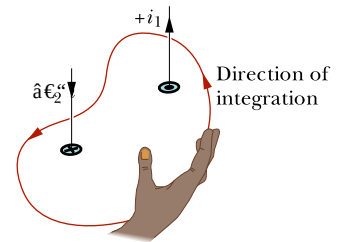
(Current  $i_3$  is not encircled by the loop.) We can then rewrite Eq. 29.3.2 as

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 (i_1 - i_2). \quad (29.3.3)$$



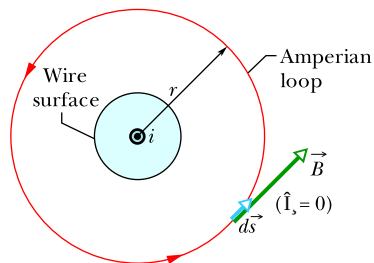
**Figure 29.3.1** Ampere's law applied to an arbitrary Amperian loop that encircles two long straight wires but excludes a third wire. Note the directions of the currents.

This is how to assign a sign to a current used in Ampere's law.



**Figure 29.3.2** A right-hand rule for Ampere's law, to determine the signs for currents encircled by an Amperian loop. The situation is that of Fig. 29.3.1.

All of the current is encircled and thus all is used in Ampere's law.



**Figure 29.3.3** Using Ampere's law to find the magnetic field that a current  $i$  produces outside a long straight wire of circular cross section. The Amperian loop is a concentric circle that lies outside the wire.

You might wonder why, since current  $i_3$  contributes to the magnetic-field magnitude  $B$  on the left side of Eq. 29.3.3, it is not needed on the right side. The answer is that the contributions of current  $i_3$  to the magnetic field cancel out because the integration in Eq. 29.3.3 is made around the full loop. In contrast, the contributions of an encircled current to the magnetic field do not cancel out.

We cannot solve Eq. 29.3.3 for the magnitude  $B$  of the magnetic field because for the situation of Fig. 29.3.1 we do not have enough information to simplify and solve the integral. However, we do know the outcome of the integration; it must be equal to  $\hat{\mathbf{i}}_1 \times \hat{\mathbf{i}}_2$ , the value of which is set by the net current passing through the loop.

We shall now apply Ampere's law to two situations in which symmetry does allow us to simplify and solve the integral, hence to find the magnetic field.

### Magnetic Field Outside a Long Straight Wire with Current

Figure 29.3.3 shows a long straight wire that carries current  $i$  directly out of the page. Equation 29.1.4 tells us that the magnetic field  $\vec{B}$  produced by the current has the same magnitude at all points that are the same distance  $r$  from the wire; that is, the field  $\vec{B}$  has cylindrical symmetry about the wire. We can take advantage of that symmetry to simplify the integral in Ampere's law (Eqs. 29.3.1 and 29.3.2) if we encircle the wire with a concentric circular Amperian loop of radius  $r$ , as in Fig. 29.3.3. The magnetic field then has the same magnitude  $B$  at every point on the loop. We shall integrate counterclockwise, so that  $d\vec{s}$  has the direction shown in Fig. 29.3.3.

We can further simplify the quantity  $B \cos \hat{\mathbf{i}}$  in Eq. 29.3.2 by noting that  $\vec{B}$  is tangent to the loop at every point along the loop, as is  $d\vec{s}$ . Thus,  $\vec{B}$  and  $d\vec{s}$  are either parallel or antiparallel at each point of the loop, and we shall arbitrarily assume the former. Then at every point the angle  $\hat{\mathbf{i}}$  between  $d\vec{s}$  and  $\vec{B}$  is  $0^\circ$ , so  $\cos \hat{\mathbf{i}} = \cos 0^\circ = 1$ . The integral in Eq. 29.3.2 then becomes

$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos \hat{\mathbf{i}} ds = B \oint ds = B(2\pi r).$$

Note that  $\oint ds$  is the summation of all the line segment lengths  $ds$  around the circular loop; that is, it simply gives the circumference  $2\pi r$  of the loop.

Our right-hand rule gives us a plus sign for the current of Fig. 29.3.3. The right side of Ampere's law becomes  $+\mu_0 i$ , and we then have

$$B(2\pi r) = \mu_0 i,$$

or

$$B = \frac{\mu_0 i}{2\pi r} \quad (\text{outside straight wire}). \quad (29.3.4)$$

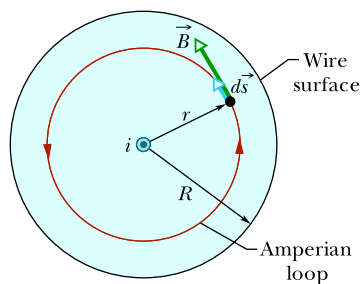
With a slight change in notation, this is Eq. 29.1.4, which we derived earlier using the law of Biot and Savart. In addition, because the magnitude  $B$  turned out positive, we know that the correct direction of  $\vec{B}$  must be the one shown in Fig. 29.3.3.

### Magnetic Field Inside a Long Straight Wire with Current

Figure 29.3.4 shows the cross section of a long straight wire of radius  $R$  that carries a uniformly distributed current  $i$  directly out of the page. Because the current is uniformly distributed over a cross section of the wire, the magnetic field  $\vec{B}$  produced by the current must be cylindrically symmetrical. Thus, to find the magnetic field at points inside the wire, we can again use an Amperian loop of radius  $r$ , as shown in Fig. 29.3.4, where now  $r < R$ . Symmetry again suggests that  $\vec{B}$  is tangent to the loop, as shown; so the left side of Ampere's law again yields

$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = B(2\pi r). \quad (29.3.5)$$

Only the current encircled by the loop is used in Ampere's law.



**Figure 29.3.4** Using Ampere's law to find the magnetic field that a current  $i$  produces inside a long straight wire of circular cross section. The current is uniformly distributed over the cross section of the wire and emerges from the page. An Amperian loop is drawn inside the wire.

Because the current is uniformly distributed, the current  $i_{\text{enc}}$  encircled by the loop is proportional to the area encircled by the loop; that is,

$$i_{\text{enc}} = i \frac{\pi r^2}{\pi R^2}. \quad (29.3.6)$$

Our right-hand rule tells us that  $i_{\text{enc}}$  gets a plus sign. Then Ampere's Law gives us

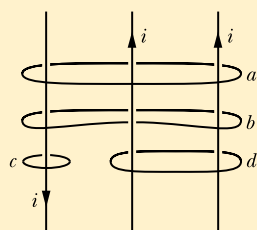
$$B(2\pi r) = \mu_0 i \frac{\pi r^2}{\pi R^2},$$

or 
$$B = \left( \frac{\mu_0 i}{2\pi R^2} \right) r \quad (\text{inside straight wire}). \quad (29.3.7)$$

Thus, inside the wire, the magnitude  $B$  of the magnetic field is proportional to  $r$ , is zero at the center, and is maximum at  $r = R$  (the surface). Note that Eqs. 29.3.4 and 29.3.7 give the same value for  $B$  at the surface.

### Checkpoint 29.3.1

The figure here shows three equal currents  $i$  (two parallel and one antiparallel) and four Amperian loops. Rank the loops according to the magnitude of  $\oint \vec{B} \cdot d\vec{s}$ , along each, greatest first.



### Sample Problem 29.3.1 Ampere's Law to find the field inside a long cylinder of current

Figure 29.3.5a shows the cross section of a long conducting cylinder with inner radius  $a = 2.0$  cm and outer radius  $b = 4.0$  cm. The cylinder carries a current out of the page, and the magnitude of the current density in the cross section is given by  $J = cr^2$ , with  $c = 3.0 \times 10^6$  A/m<sup>4</sup> and  $r$  in meters. What is the magnetic field  $\vec{B}$  at the dot in Fig. 29.3.5a, which is at radius  $r = 3.0$  cm from the central axis of the cylinder?

#### KEY IDEAS

The point at which we want to evaluate  $\vec{B}$  is inside the material of the conducting cylinder, between its inner and outer radii. We note that the current distribution has cylindrical symmetry (it is the same all around the cross section for any given radius). Thus, the symmetry allows us to use Ampere's Law to find  $\vec{B}$  at the point. We first draw the Amperian loop shown in Fig. 29.3.5b. The loop is concentric with the cylinder and has radius  $r = 3.0$  cm because we want to evaluate  $\vec{B}$  at that distance from the cylinder's central axis.

Next, we must compute the current  $i_{\text{enc}}$  that is encircled by the Amperian loop. However, we *cannot* set up a proportionality as in Eq. 29.3.6, because here the current is not uniformly distributed. Instead, we must integrate the current density magnitude from the cylinder's inner

radius  $a$  to the loop radius  $r$ , using the steps shown in Figs. 29.3.5c through h.

**Calculations:** We write the integral as

$$\begin{aligned} i_{\text{enc}} &= \int_a^r J dA = \int_a^r cr^2 (2\pi r dr) \\ &= 2\pi c \int_a^r r^3 dr = 2\pi c \left[ \frac{r^4}{4} \right]_a^r \\ &= \frac{\pi c (r^4 - a^4)}{2}. \end{aligned}$$

Note that in these steps we took the differential area  $dA$  to be the area of the thin ring in Figs. 29.3.5d and then replaced it with its equivalent, the product of the ring's circumference  $2\pi r$  and its thickness  $dr$ .

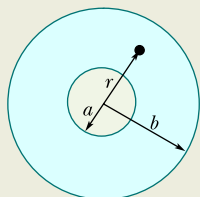
For the Amperian loop, the direction of integration indicated in Fig. 29.3.5b is (arbitrarily) clockwise. Applying the right-hand rule for Ampere's Law to that loop, we find that we should take  $i_{\text{enc}}$  as negative because the current is directed out of the page but our thumb is directed into the page.

We next evaluate the left side of Ampere's Law as we did in Fig. 29.3.4, and we again obtain Eq. 29.3.5. Then Ampere's Law,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}},$$

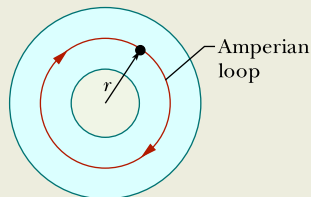


We want the magnetic field at the dot at radius  $r$ .



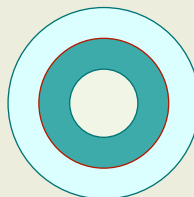
(a)

So, we put a concentric Amperian loop through the dot.



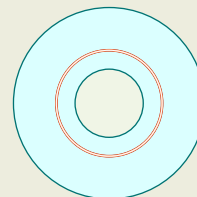
(b)

We need to find the current in the area encircled by the loop.



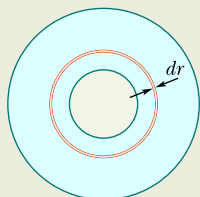
(c)

We start with a ring that is so thin that we can approximate the current density as being uniform within it.



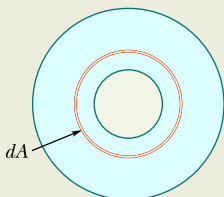
(d)

Its area  $dA$  is the product of the ring's circumference and the width  $dr$ .



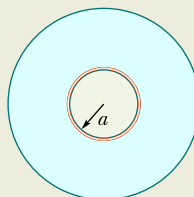
(e)

The current within the ring is the product of the current density  $J$  and the ring's area  $dA$ .



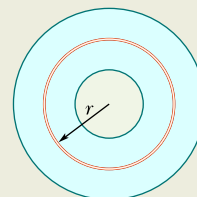
(f)

Our job is to sum the currents in all rings from this smallest one ...



(g)

... to this largest one, which has the same radius as the Amperian loop.



(h)

**Figure 29.3.5** (a)–(h) To find the magnetic field at a point within this conducting cylinder, we use a concentric Amperian loop through the point. We then need the current encircled by the loop. (c)–(h) Because the current density is nonuniform, we start with a thin ring and then sum (via integration) the currents in all such rings in the encircled area.

gives us

$$B(2\pi r) = \frac{\mu_0}{2} (r^4 - a^4).$$

Solving for  $B$  and substituting known data yield

$$\begin{aligned} B &= \frac{\mu_0}{4\pi} (r^4 - a^4) \\ &= \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(3.0 \times 10^{-6} \text{ A/m}^4)}{4(0.030 \text{ m})} \\ &\quad - \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(0.020 \text{ m})^4}{4(0.030 \text{ m})} \\ &= 2.0 \times 10^{-5} \text{ T}. \end{aligned}$$

Thus, the magnetic field  $\vec{B}$  at a point 3.0 cm from the central axis has magnitude

$$B = 2.0 \times 10^{-5} \text{ T} \quad (\text{Answer})$$

and forms magnetic field lines that are directed opposite our direction of integration, hence counterclockwise in Fig. 29.3.5.



## 29.4 SOLENOIDS AND TOROIDS

### Learning Objectives

After reading this module, you should be able to . . .

**29.4.1** Describe a solenoid and a toroid and sketch their magnetic field lines.

**29.4.2** Explain how Ampere's law is used to find the magnetic field inside a solenoid.

**29.4.3** Apply the relationship between a solenoid's internal magnetic field  $B$ , the current  $i$ , and the number of turns per unit length  $n$  of the solenoid.

**29.4.4** Explain how Ampere's law is used to find the magnetic field inside a toroid.

**29.4.5** Apply the relationship between a toroid's internal magnetic field  $B$ , the current  $i$ , the radius  $r$ , and the total number of turns  $N$ .

### Key Ideas

Inside a long solenoid carrying current  $i$ , at points not near its ends, the magnitude  $B$  of the magnetic field is

$$B = \mu_0 i n \quad (\text{ideal solenoid}),$$

where  $n$  is the number of turns per unit length.

At a point inside a toroid, the magnitude  $B$  of the magnetic field is

$$B = \frac{\mu_0 i N}{2\pi r} \quad (\text{toroid}),$$

where  $r$  is the distance from the center of the toroid to the point.

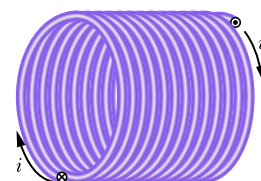
## Solenoids and Toroids

### Magnetic Field of a Solenoid

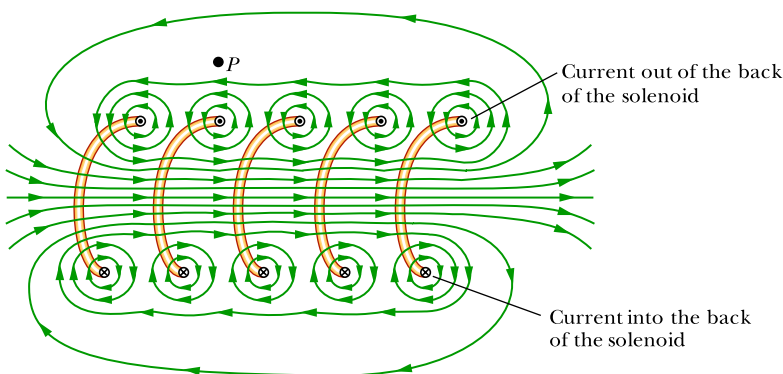
We now turn our attention to another situation in which Ampere's law proves useful. It concerns the magnetic field produced by the current in a long, tightly wound helical coil of wire. Such a coil is called a **solenoid** (Fig. 29.4.1). We assume that the length of the solenoid is much greater than the diameter.

Figure 29.4.2 shows a section through a portion of a stretched-out solenoid. The solenoid's magnetic field is the vector sum of the fields produced by the individual turns (*windings*) that make up the solenoid. For points very close to a turn, the wire behaves magnetically almost like a long straight wire, and the lines of  $B$  there are almost concentric circles. Figure 29.4.2 suggests that the field tends to cancel between adjacent turns. It also suggests that, at points inside the solenoid and reasonably far from the wire,  $B$  is approximately parallel to the (central) solenoid axis. In the limiting case of an *ideal solenoid*, which is infinitely long and consists of tightly packed (*close-packed*) turns of square wire, the field inside the coil is uniform and parallel to the solenoid axis.

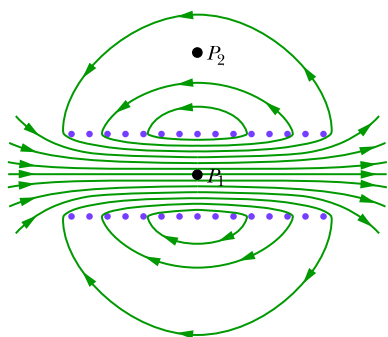
At points above the solenoid, such as  $P$  in Fig. 29.4.2, the magnetic field set up by the upper parts of the solenoid turns (these upper turns are marked  $\otimes$ ) is directed to the left (as drawn near  $P$ ) and tends to cancel the field set up at  $P$  by the lower parts of the turns (these lower turns are marked  $\odot$ ), which is directed to the right (not drawn). In the limiting case of an ideal solenoid, the magnetic field outside the solenoid is zero. Taking the external field to be zero is an excellent assumption for a real solenoid if its length is much greater than its diameter and if we consider external points such as point  $P$  that are not at either end of the solenoid. The direction of the magnetic field along the solenoid axis is given by a curled "straight-hand" rule: Grasp the solenoid with your right hand so that your fingers follow the direction of the current in the windings; your extended right thumb then points in the direction of the axial magnetic field.



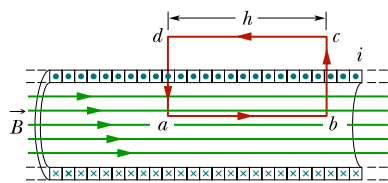
**Figure 29.4.1** A solenoid carrying current  $i$ .



**Figure 29.4.2** A vertical cross section through the central axis of a “stretched-out” solenoid. The back portions of five turns are shown, as are the magnetic field lines due to a current through the solenoid. Each turn produces circular magnetic field lines near itself. Near the solenoid’s axis, the field lines combine into a net magnetic field that is directed along the axis. The closely spaced field lines there indicate a strong magnetic field. Outside the solenoid the field lines are widely spaced; the field there is very weak.



**Figure 29.4.3** Magnetic field lines for a real solenoid of finite length. The field is strong and uniform at interior points such as  $P_1$  but relatively weak at external points such as  $P_2$ .



**Figure 29.4.4** Application of Ampere’s Law to a section of a long ideal solenoid carrying a current  $i$ . The Amperian loop is the rectangle  $abcd$ .

Figure 29.4.3 shows the lines of  $\vec{B}$  for a real solenoid. The spacing of these lines in the central region shows that the field inside the coil is fairly strong and uniform over the cross section of the coil. The external field, however, is relatively weak.

**Ampere’s Law.** Let us now apply Ampere’s Law,

$$\oint_C \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}}, \quad (29.4.1)$$

to the ideal solenoid of Fig. 29.4.4, where  $\vec{B}$  is uniform within the solenoid and zero outside it, using the rectangular Amperian loop  $abcd$ . We write  $\oint_C \vec{B} \cdot d\vec{s}$  as the sum of four integrals, one for each loop segment:

$$\oint_C \vec{B} \cdot d\vec{s} = \int_a^b \vec{B} \cdot d\vec{s} + \int_b^c \vec{B} \cdot d\vec{s} + \int_c^d \vec{B} \cdot d\vec{s} + \int_d^a \vec{B} \cdot d\vec{s}. \quad (29.4.2)$$

The first integral on the right of Eq. 29.4.2 is  $Bh$ , where  $B$  is the magnitude of the uniform field  $\vec{B}$  inside the solenoid and  $h$  is the (arbitrary) length of the segment from  $a$  to  $b$ . The second and fourth integrals are zero because for every element  $ds$  of these segments,  $\vec{B}$  either is perpendicular to  $ds$  or is zero, and thus the product  $\vec{B} \cdot d\vec{s}$  is zero. The third integral, which is taken along a segment that lies outside the solenoid, is zero because  $B = 0$  at all external points. Thus,  $\oint_C \vec{B} \cdot d\vec{s}$  for the entire rectangular loop has the value  $Bh$ .

**Net Current.** The net current  $i_{\text{enc}}$  encircled by the rectangular Amperian loop in Fig. 29.4.4 is not the same as the current  $i$  in the solenoid windings because the windings pass more than once through this loop. Let  $n$  be the number of turns per unit length of the solenoid; then the loop encloses  $nh$  turns and

$$i_{\text{enc}} = i(nh).$$

Ampere’s Law then gives us

$$Bh = \mu_0 i(nh),$$

$$\text{or} \quad B = \mu_0 i n \quad (\text{ideal solenoid}). \quad (29.4.3)$$

Although we derived Eq. 29.4.3 for an infinitely long ideal solenoid, it holds quite well for actual solenoids if we apply it only at interior points and well away from the solenoid ends. Equation 29.4.3 is consistent with the experimental fact that the magnetic field magnitude  $B$  within a solenoid does not depend on the diameter or the length of the solenoid and that  $B$  is uniform over the solenoidal cross section. A solenoid thus provides a practical way to set up a known uniform magnetic field for experimentation, just as a parallel-plate capacitor provides a practical way to set up a known uniform electric field.

### Magnetic Field of a Toroid

Figure 29.4.5a shows a **toroid**, which we may describe as a (hollow) solenoid that has been curved until its two ends meet, forming a sort of hollow bracelet. What magnetic field  $\vec{B}$  is set up inside the toroid (inside the hollow of the bracelet)? We can find out from Ampere's law and the symmetry of the bracelet.

From the symmetry, we see that the lines of  $\vec{B}$  form concentric circles inside the toroid, directed as shown in Fig. 29.4.5b. Let us choose a concentric circle of radius  $r$  as an Amperian loop and traverse it in the clockwise direction. Ampere's law (Eq. 29.3.1) yields

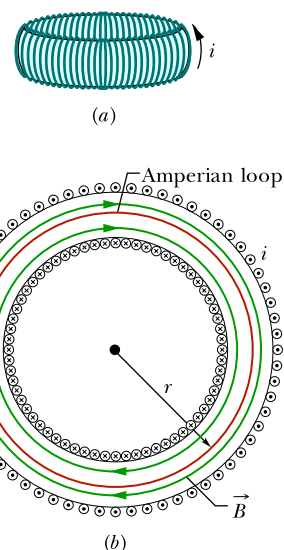
$$(B)(2\pi r) = \mu_0 iN,$$

where  $i$  is the current in the toroid windings (and is positive for those windings enclosed by the Amperian loop) and  $N$  is the total number of turns. This gives

$$B = \frac{\mu_0 i N}{2\pi r} \quad (\text{toroid}). \quad (29.4.4)$$

In contrast to the situation for a solenoid,  $B$  is not constant over the cross section of a toroid.

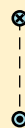
It is easy to show, with Ampere's law, that  $B = 0$  for points outside an ideal toroid (as if the toroid were made from an ideal solenoid). The direction of the magnetic field within a toroid follows from our curled-fingers-right-hand rule: Grasp the toroid with the fingers of your right hand curled in the direction of the current in the windings; your extended right thumb points in the direction of the magnetic field.



**Figure 29.4.5** (a) A toroid carrying a current  $i$ . (b) A horizontal cross section of the toroid. The interior magnetic field (inside the bracelet-shaped tube) can be found by applying Ampere's law with the Amperian loop shown.

#### Checkpoint 29.4.1

This figure indicates the directions of the current in one turn of a long, tightly packed solenoid that is horizontal. Is the direction of the magnetic field inside the solenoid leftward or rightward?



## 29.5 A CURRENT-CARRYING COIL AS A MAGNETIC DIPOLE

### Learning Objectives

After reading this module, you should be able to . . .

**29.5.1** Sketch the magnetic field lines of a flat coil that is carrying current.

**29.5.2** For a current-carrying coil, apply the relationship between the dipole moment magnitude  $\vec{\mu}$  and the coil's current  $i$ , number of turns  $N$ , and area per turn  $A$ .

**29.5.3** For a point along the central axis, apply the relationship between the magnetic field magnitude  $B$ , the magnetic moment  $\vec{\mu}$  and the distance  $z$  from the center of the coil.

## Key Idea

The magnetic field produced by a current-carrying coil, which is a magnetic dipole, at a point  $P$  located a distance  $z$  along the coil's perpendicular central axis is parallel to the axis and is given by

$$\vec{B}(z) = \frac{\mu_0 \vec{\mu}}{2\pi\epsilon z^3},$$

where  $\vec{\mu}$  is the dipole moment of the coil. This equation applies only when  $z$  is much greater than the dimensions of the coil.

## A Current-Carrying Coil as a Magnetic Dipole

So far we have examined the magnetic fields produced by current in a long straight wire, a solenoid, and a toroid. We turn our attention here to the field produced by a coil carrying a current. You saw in Module 28.8 that such a coil behaves as a magnetic dipole in that, if we place it in an external magnetic field  $\vec{B}$ , a torque  $\vec{\tau}$  given by

$$\vec{\tau} = \vec{\mu} \times \vec{B} \quad (29.5.1)$$

acts on it. Here  $\vec{\mu}$  is the magnetic dipole moment of the coil and has the magnitude  $NiA$ , where  $N$  is the number of turns,  $i$  is the current in each turn, and  $A$  is the area enclosed by each turn. (*Caution:* Don't confuse the magnetic dipole moment  $\vec{\mu}$  with the permeability constant  $\mu_0$ .)

Recall that the direction of  $\vec{\mu}$  is given by a curled “right-hand rule: Grasp the coil so that the fingers of your right hand curl around it in the direction of the current; your extended thumb then points in the direction of the dipole moment  $\vec{\mu}$ .

### Magnetic Field of a Coil

We turn now to the other aspect of a current-carrying coil as a magnetic dipole. What magnetic field does it produce at a point in the surrounding space? The problem does not have enough symmetry to make Ampere's law useful; so we must turn to the law of Biot and Savart. For simplicity, we first consider only a coil with a single circular loop and only points on its perpendicular central axis, which we take to be a  $z$  axis. We shall show that the magnitude of the magnetic field at such points is

$$B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}}, \quad (29.5.2)$$

in which  $R$  is the radius of the circular loop and  $z$  is the distance of the point in question from the center of the loop. Furthermore, the direction of the magnetic field  $\vec{B}$  is the same as the direction of the magnetic dipole moment  $\vec{\mu}$  of the loop.

**Large  $z$ .** For axial points far from the loop, we have  $z \gg R$  in Eq. 29.5.2. With that approximation, the equation reduces to

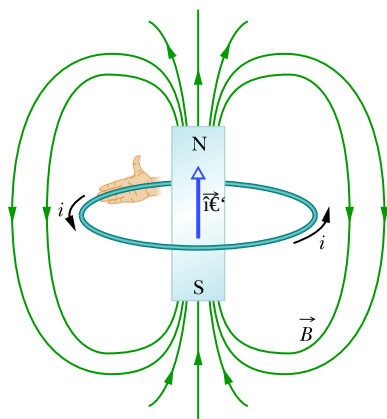
$$B(z) \approx \frac{\mu_0 R^2}{2z^3}.$$

Recalling that  $\pi R^2$  is the area  $A$  of the loop and extending our result to include a coil of  $N$  turns, we can write this equation as

$$B(z) = \frac{\mu_0 NiA}{2\pi\epsilon z^3}.$$

Further, because  $\vec{B}$  and  $\vec{\mu}$  have the same direction, we can write the equation in vector form, substituting from the identity  $\mu = NiA$ :

$$\vec{B}(z) = \frac{\mu_0 \vec{\mu}}{2\pi\epsilon z^3} \quad (\text{current-carrying coil}). \quad (29.5.3)$$

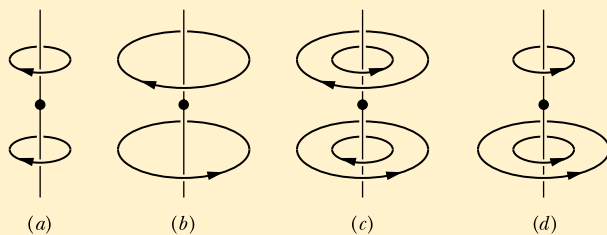


**Figure 29.5.1** A current loop produces a magnetic field like that of a bar magnet and thus has associated north and south poles. The magnetic dipole moment  $\vec{\mu}$  of the loop, its direction given by a curled “straight-hand rule,” points from the south pole to the north pole, in the direction of the field  $\vec{B}$  within the loop.

Thus, we have two ways in which we can regard a current-carrying coil as a magnetic dipole: (1) It experiences a torque when we place it in an external magnetic field; (2) it generates its own intrinsic magnetic field, given, for distant points along its axis, by Eq. 29.5.3. Figure 29.5.1 shows the magnetic field of a current loop; one side of the loop acts as a north pole (in the direction of  $\vec{\mu}$ ) and the other side as a south pole, as suggested by the lightly drawn magnet in the figure. If we were to place a current-carrying coil in an external magnetic field, it would tend to rotate just like a bar magnet would.

### Checkpoint 29.5.1

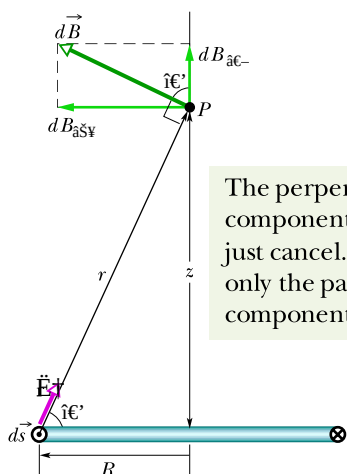
The figure here shows four arrangements of circular loops of radius  $r$  or  $2r$ , centered on vertical axes (perpendicular to the loops) and carrying identical currents in the directions indicated. Rank the arrangements according to the magnitude of the net magnetic field at the dot, midway between the loops on the central axis, greatest first.



### Proof of Equation 29.5.2

Figure 29.5.2 shows the back half of a circular loop of radius  $R$  carrying a current  $i$ . Consider a point  $P$  on the central axis of the loop, a distance  $z$  from its plane. Let us apply the law of Biot and Savart to a differential element  $ds$  of the loop, located at the left side of the loop. The length vector  $d\vec{s}$  for this element points perpendicularly out of the page. The angle  $\hat{\theta}$  between  $d\vec{s}$  and  $\vec{r}$  in Fig. 29.5.2 is  $90^\circ$ . The plane formed by these two vectors is perpendicular to the plane of the page and contains both  $\vec{r}$  and  $d\vec{s}$ . From the law of Biot and Savart (and the right-hand rule), the differential field  $d\vec{B}$  produced at point  $P$  by the current in this element is perpendicular to this plane and thus is directed in the plane of the figure, perpendicular to  $\vec{r}$  as indicated in Fig. 29.5.2.





**Figure 29.5.2** Cross section through a current loop of radius  $R$ . The plane of the loop is perpendicular to the page, and only the back half of the loop is shown. We use the law of Biot and Savart to find the magnetic field at point  $P$  on the central perpendicular axis of the loop.

The perpendicular components just cancel. We add only the parallel components.

Let us resolve  $d\vec{B}$  into two components:  $dB_{\parallel}$  along the axis of the loop and  $dB_{\perp}$  perpendicular to this axis. From the symmetry, the vector sum of all the perpendicular components  $dB_{\perp}$  due to all the loop elements  $ds$  is zero. This leaves only the axial (parallel) components  $dB_{\parallel}$  and we have

$$B = \oint dB_{\parallel}.$$

For the element  $ds$  in Fig. 29.5.2, the law of Biot and Savart (Eq. 29.1.1) tells us that the magnetic field at distance  $r$  is

$$dB = \frac{\mu_0 i ds \sin 90^\circ}{4\pi r^2}.$$

We also have

$$dB_{\parallel} = dB \cos \theta$$

Combining these two relations, we obtain

$$dB_{\parallel} = \frac{\mu_0 i \cos \theta ds}{4\pi r^2}. \quad (29.5.4)$$

Figure 29.5.2 shows that  $r$  and  $\theta$  are related to each other. Let us express each in terms of the variable  $z$ , the distance between point  $P$  and the center of the loop. The relations are

$$r = \sqrt{R^2 + z^2} \quad (29.5.5)$$

and

$$\cos \theta = \frac{R}{r} = \frac{R}{\sqrt{R^2 + z^2}}. \quad (29.5.6)$$

Substituting Eqs. 29.5.5 and 29.5.6 into Eq. 29.5.4, we find

$$dB_{\parallel} = \frac{\mu_0 i R}{4\pi (R^2 + z^2)^{3/2}} ds.$$

Note that  $i$ ,  $R$ , and  $z$  have the same values for all elements  $ds$  around the loop; so when we integrate this equation, we find that

$$\begin{aligned} B &= \oint dB_{\parallel} \\ &= \frac{\mu_0 i R}{4\pi (R^2 + z^2)^{3/2}} \oint ds, \end{aligned}$$

or, because  $\oint ds$  is simply the circumference  $2\pi R$  of the loop,

$$B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}}.$$

This is Eq. 29.5.2, the relation we sought to prove.

## Review & Summary

**The Biot-Savart Law** The magnetic field set up by a current-carrying conductor can be found from the *Biot-Savart law*. This law asserts that the contribution  $d\vec{B}$  to the field produced by a current-length element  $i d\vec{s}$  at a point  $P$  located a distance  $r$  from the current element is

$$d\vec{B} = \frac{\mu_0 i d\vec{s} \times \hat{r}}{4\pi r^2}, \quad (\text{Biot-Savart law}). \quad (29.1.3)$$

Here  $\hat{r}$  is a unit vector that points from the element toward  $P$ . The quantity  $\mu_0$ , called the permeability constant, has the value

$$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \approx 1.26 \times 10^{-6} \text{ T} \cdot \text{m/A}.$$

**Magnetic Field of a Long Straight Wire** For a long straight wire carrying a current  $i$ , the Biot-Savart law gives, for

the magnitude of the magnetic field at a perpendicular distance  $R$  from the wire,

$$B = \frac{\mu_0 i}{2\pi R} \quad (\text{long straight wire}). \quad (29.1.4)$$

**Magnetic Field of a Circular Arc** The magnitude of the magnetic field at the center of a circular arc, of radius  $R$  and central angle  $\hat{\Gamma}$  (in radians), carrying current  $i$ , is

$$B = \frac{\mu_0 i \hat{\Gamma}}{4\pi R} \quad (\text{at center of circular arc}). \quad (29.1.9)$$

**Force Between Parallel Currents** Parallel wires carrying currents in the same direction attract each other, whereas parallel wires carrying currents in opposite directions repel each other. The magnitude of the force on a length  $L$  of either wire is

$$F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d}, \quad (29.2.3)$$

where  $d$  is the wire separation, and  $i_a$  and  $i_b$  are the currents in the wires.

**Ampere's Law** Ampere's Law states that

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \quad (\text{Ampere's Law}). \quad (29.3.1)$$

The line integral in this equation is evaluated around a closed loop called an *Amperian loop*. The current  $i$  on the right side is

the *net* current encircled by the loop. For some current distributions, Eq. 29.3.1 is easier to use than Eq. 29.1.3 to calculate the magnetic field due to the currents.

**Fields of a Solenoid and a Toroid** Inside a *long solenoid* carrying current  $i$ , at points not near its ends, the magnitude  $B$  of the magnetic field is

$$B = \mu_0 n i \quad (\text{ideal solenoid}), \quad (29.4.3)$$

where  $n$  is the number of turns per unit length. Thus the internal magnetic field is uniform. Outside the solenoid, the magnetic field is approximately zero.

At a point inside a *toroid*, the magnitude  $B$  of the magnetic field is

$$B = \frac{\mu_0 N i}{2\pi r} \quad (\text{toroid}), \quad (29.4.4)$$

where  $r$  is the distance from the center of the toroid to the point.

**Field of a Magnetic Dipole** The magnetic field produced by a current-carrying coil, which is a *magnetic dipole*, at a point  $P$  located a distance  $z$  along the coil's perpendicular central axis is parallel to the axis and is given by

$$B(z) = \frac{\mu_0 \hat{\Gamma}}{2\pi z^3}, \quad (29.5.3)$$

where  $\hat{\Gamma}$  is the dipole moment of the coil. This equation applies only when  $z$  is much greater than the dimensions of the coil.

## Questions

**1** Figure 29.1 shows three circuits, each consisting of two radial lengths and two concentric circular arcs, one of radius  $r$  and the other of radius  $R > r$ . The circuits have the same current through them and the same angle between the two radial lengths. Rank the circuits according to the magnitude of the net magnetic field at the center, greatest first.

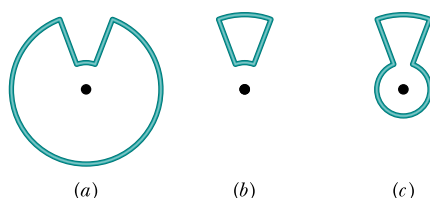


Figure 29.1 Question 1.

**2** Figure 29.2 represents a snapshot of the velocity vectors of four electrons near a wire carrying current  $i$ . The four velocities have the same magnitude; velocity  $\vec{v}_2$  is directed into the page. Electrons 1 and 2 are at the same distance from the wire, as are electrons 3 and 4. Rank the electrons according to the magnitudes of the magnetic forces on them due to current  $i$ , greatest first.

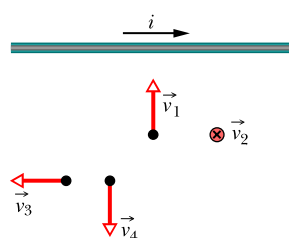


Figure 29.2 Question 2.

**3** Figure 29.3 shows four arrangements in which long parallel wires carry equal currents directly into or out of the page at the corners of identical squares. Rank the arrangements according to the magnitude of the net magnetic field at the center of the square, greatest first.

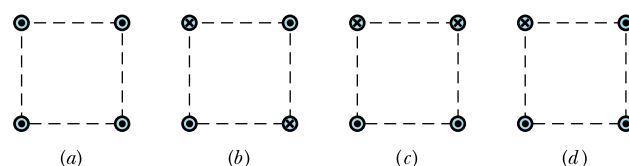


Figure 29.3 Question 3.

**4** Figure 29.4 shows cross sections of two long straight wires; the left-hand wire carries current  $i_1$  directly out of the page. If the net magnetic field due to the two currents is to be zero at point  $P$ , (a) should the direction of current  $i_2$  in the right-hand wire be directly into or out of the page, and (b) should  $i_2$  be greater than, less than, or equal to  $i_1$ ?



Figure 29.4 Question 4.

**5** Figure 29.5 shows three circuits consisting of straight radial lengths and concentric circular arcs (either half- or quarter-circles of radii  $r$ ,  $2r$ , and  $3r$ ). The circuits carry the same current. Rank them according to the magnitude of the magnetic field produced at the center of curvature (the dot), greatest first.

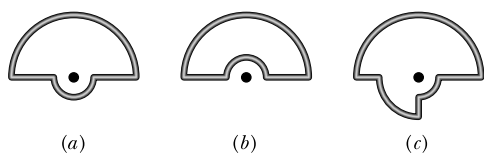


Figure 29.5 Question 5.

**6** Figure 29.6 gives, as a function of radial distance  $r$ , the magnitude  $B$  of the magnetic field inside and outside four wires ( $a$ ,  $b$ ,  $c$ , and  $d$ ), each of which carries a current that is uniformly distributed across the wire's cross section. Overlapping portions of the plots (drawn slightly separated) are indicated by double labels. Rank the wires according to (a) radius, (b) the magnitude of the magnetic field on the surface, and (c) the value of the current, greatest first. (d) Is the magnitude of the current density in wire  $a$  greater than, less than, or equal to that in wire  $c$ ?

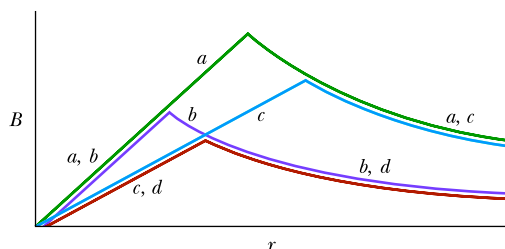


Figure 29.6 Question 6.

**7** Figure 29.7 shows four circular Amperian loops ( $a$ ,  $b$ ,  $c$ ,  $d$ ) concentric with a wire whose current is directed out of the page. The current is uniform across the wire's circular cross section (the shaded region). Rank the loops according to the magnitude of  $\oint \vec{B} \cdot d\vec{s}$  around each, greatest first.

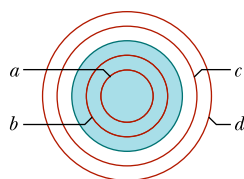


Figure 29.7 Question 7.

**8** Figure 29.8 shows four arrangements in which long, parallel, equally spaced wires carry equal currents directly into or out of the page. Rank the arrangements according to the magnitude of the net force on the central wire due to the currents in the other wires, greatest first.

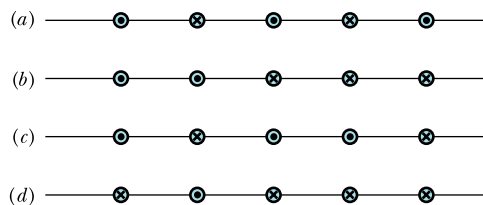


Figure 29.8 Question 8.

**9** Figure 29.9 shows four circular Amperian loops ( $a$ ,  $b$ ,  $c$ ,  $d$ ) and, in cross section, four long circular conductors (the shaded regions), all of which are concentric. Three of the conductors are hollow cylinders; the central conductor is a solid cylinder. The currents in the conductors are, from smallest radius to largest radius, 4 A out of the page, 9 A into the page, 5 A out of the page, and 3 A into the page. Rank the Amperian loops according to the magnitude of  $\oint \vec{B} \cdot d\vec{s}$  around each, greatest first.

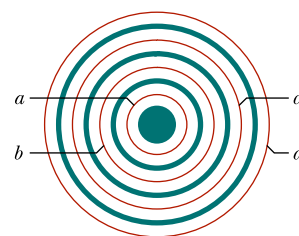


Figure 29.9 Question 9.

**10** Figure 29.10 shows four identical currents  $i$  and five Amperian paths ( $a$  through  $e$ ) encircling them. Rank the paths according to the value of  $\oint \vec{B} \cdot d\vec{s}$ , taken in the directions shown, most positive first.

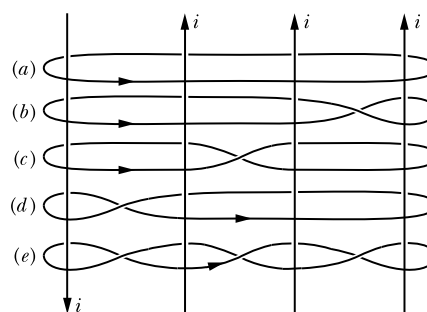


Figure 29.10 Question 10.

**11** Figure 29.11 shows three arrangements of three long straight wires carrying equal currents directly into or out of the page. (a) Rank the arrangements according to the magnitude of the net force on wire  $A$  due to the currents in the other wires, greatest first. (b) In arrangement 3, is the angle between the net force on wire  $A$  and the dashed line equal to, less than, or more than  $45^\circ$ ?

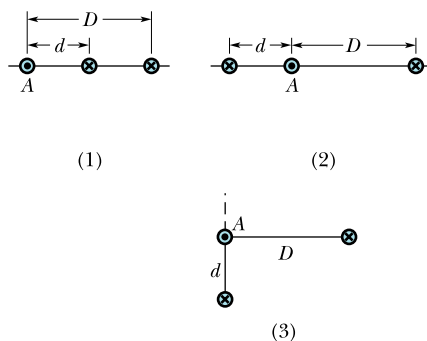


Figure 29.11 Question 11.

## Problems



Tutoring problem available (at instructor's discretion) in WileyPLUS



Worked-out solution available in Student Solutions Manual



Easy



Medium



Hard



Additional information available in *The Flying Circus of Physics* and at [flyingcircusofphysics.com](http://flyingcircusofphysics.com)



Requires calculus



Biomedical application

### Module 29.1 Magnetic Field Due to a Current

**1** **E** A surveyor is using a magnetic compass 6.1 m below a power line in which there is a steady current of 100 A. (a) What is the

magnetic field at the site of the compass due to the power line? (b) Will this field interfere seriously with the compass reading? The horizontal component of Earth's magnetic field at the site is  $20 \mu\text{T}$ .

**2 E** Figure 29.12a shows an element of length  $ds = 1.00 \hat{\text{m}}$  in a very long straight wire carrying current. The current in that element sets up a differential magnetic field  $d\vec{B}$  at points in the surrounding space. Figure 29.12b gives the magnitude  $dB$  of the field for points 2.5 cm from the element, as a function of angle  $\hat{\epsilon}$ , between the wire and a straight line to the point. The vertical scale is set by  $dB_s = 60.0 \text{ pT}$ . What is the magnitude of the magnetic field set up by the entire wire at perpendicular distance 2.5 cm from the wire?

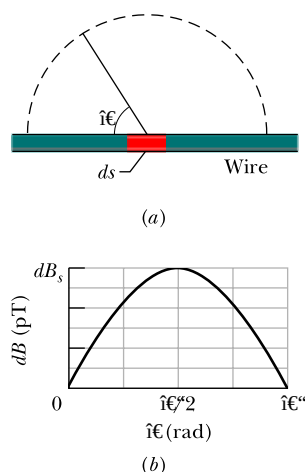


Figure 29.12 Problem 2.

**3 E SSM** At a certain location in the Philippines, Earth's magnetic field of  $39 \hat{\text{T}}$  is horizontal and directed due north. Suppose the net field is zero exactly 8.0 cm above a long, straight, horizontal wire that carries a constant current. What are the (a) magnitude and (b) direction of the current?

**4 E** A straight conductor carrying current  $i = 5.0 \text{ A}$  splits into identical semicircular arcs as shown in Fig. 29.13. What is the magnetic field at the center  $C$  of the resulting circular loop?

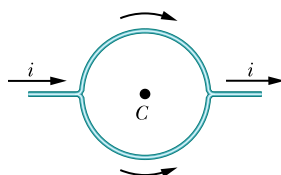


Figure 29.13 Problem 4.

**5 E** In Fig. 29.14, a current  $i = 10 \text{ A}$  is set up in a long hairpin conductor formed by bending a wire into a semicircle of radius  $R = 5.0 \text{ mm}$ . Point  $b$  is midway between the straight sections and so distant from the semicircle that each straight section can be approximated as being an infinite wire. What are the (a) magnitude and (b) direction (into or out of the page) of  $\vec{B}$  at  $a$  and the (c) magnitude and (d) direction of  $\vec{B}$  at  $b$ ?

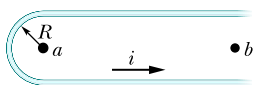


Figure 29.14 Problem 5.

**6 E** In Fig. 29.15, point  $P$  is at perpendicular distance  $R = 2.00 \text{ cm}$  from a very long straight wire carrying a current. The magnetic field  $\vec{B}$  set up at point  $P$  is due to contributions from all the identical current-length elements  $i ds$  along the wire. What is the distance  $s$  to the element making (a) the greatest contribution to field  $\vec{B}$  and (b) 10.0% of the greatest contribution?

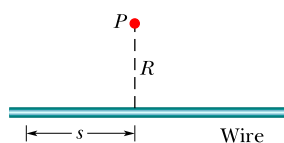


Figure 29.15 Problem 6.

**7 E GO** In Fig. 29.16, two circular arcs have radii  $a = 13.5 \text{ cm}$  and  $b = 10.7 \text{ cm}$ , subtend angle  $\hat{\epsilon} = 74.0^\circ$ , carry current  $i = 0.411 \text{ A}$ , and share the same center of curvature  $P$ . What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at  $P$ ?

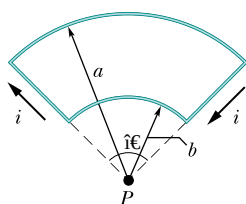


Figure 29.16 Problem 7.

**8 E** In Fig. 29.17, two semicircular arcs have radii  $R_2 = 7.80 \text{ cm}$  and  $R_1 = 3.15 \text{ cm}$ , carry current  $i = 0.281 \text{ A}$ , and have the same center of curvature  $C$ . What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at  $C$ ?

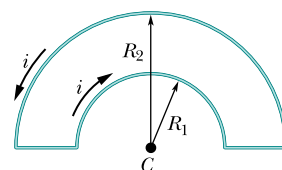


Figure 29.17 Problem 8.

**9 E SSM** Two long straight wires are parallel and 8.0 cm apart. They are to carry equal currents such that the magnetic field at a point halfway between them has magnitude  $300 \hat{\text{T}}$ . (a) Should the currents be in the same or opposite directions? (b) How much current is needed?

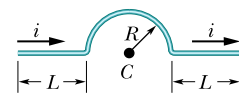


Figure 29.18 Problem 10.

**10 E** In Fig. 29.18, a wire forms a semicircle of radius  $R = 9.26 \text{ cm}$  and two (radial) straight segments each of length  $L = 13.1 \text{ cm}$ . The wire carries current  $i = 34.8 \text{ mA}$ . What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at the semicircle's center of curvature  $C$ ?

**11 E** In Fig. 29.19, two long straight wires are perpendicular to the page and separated by distance  $d_1 = 0.75 \text{ cm}$ . Wire 1 carries 6.5 A into the page. What are the (a) magnitude and (b) direction (into or out of the page) of the current in wire 2 if the net magnetic field due to the two currents is zero at point  $P$  located at distance  $d_2 = 1.50 \text{ cm}$  from wire 2?

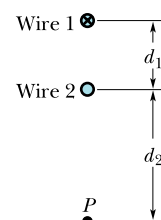


Figure 29.19 Problem 11.

**12 E** In Fig. 29.20, two long straight wires at separation  $d = 16.0 \text{ cm}$  carry currents  $i_1 = 3.61 \text{ mA}$  and  $i_2 = 3.00i_1$  out of the page. (a) Where on the  $x$  axis is the net magnetic field equal to zero? (b) If the two currents are doubled, is the zero-field point shifted toward wire 1, shifted toward wire 2, or unchanged?

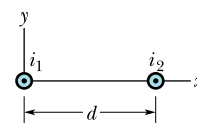


Figure 29.20 Problem 12.

**13 M CALC** In Fig. 29.21, point  $P_1$  is at distance  $R = 13.1 \text{ cm}$  on the perpendicular bisector of a straight wire of length  $L = 18.0 \text{ cm}$  carrying current  $i = 58.2 \text{ mA}$ . (Note that the wire is *not* long.) What is the magnitude of the magnetic field at  $P_1$  due to  $i$ ?

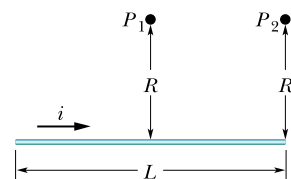


Figure 29.21 Problems 13 and 17.

**14 M** Equation 29.1.4 gives the magnitude  $B$  of the magnetic field set up by a current in an *infinitely long* straight wire, at a point  $P$  at perpendicular distance  $R$  from the wire. Suppose that point  $P$  is actually at perpendicular distance  $R$  from the midpoint of a wire with a *finite* length  $L$ . Using Eq. 29.1.4 to calculate  $B$  then results in a certain percentage error. What value must the ratio  $L/R$  exceed if the percentage error is to be less than 1.00%? That is, what  $L/R$  gives

$$\frac{(B \text{ from Eq. 29.1.4}) - (B \text{ actual})}{(B \text{ actual})} (100\%) = 1.00\%$$

**15 M** Figure 29.22 shows two current segments. The lower segment carries a current of  $i_1 = 0.40$  A and includes a semicircular arc with radius  $5.0$  cm, angle  $180^\circ$  and center point  $P$ . The upper segment carries current  $i_2 = 2i_1$  and includes a circular arc with radius  $4.0$  cm, angle  $120^\circ$  and the same center point  $P$ . What are the (a) magnitude and (b) direction of the net magnetic field  $\vec{B}$  at  $P$  for the indicated current directions? What are the (c) magnitude and (d) direction of  $\vec{B}$  if  $i_1$  is reversed?

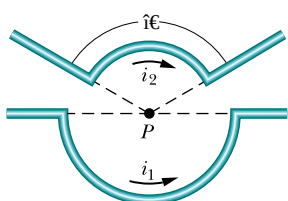


Figure 29.22 Problem 15.

**16 M GO** In Fig. 29.23, two concentric circular loops of wire carrying current in the same direction lie in the same plane. Loop 1 has radius  $1.50$  cm and carries  $4.00$  mA. Loop 2 has radius  $2.50$  cm and carries  $6.00$  mA. Loop 2 is to be rotated about a diameter while the net magnetic field  $\vec{B}$  set up by the two loops at their common center is measured. Through what angle must loop 2 be rotated so that the magnitude of that net field is  $100$  nT?



Figure 29.23 Problem 16.

**17 M CALC SSM** In Fig. 29.21, point  $P_2$  is at perpendicular distance  $R = 25.1$  cm from one end of a straight wire of length  $L = 13.6$  cm carrying current  $i = 0.693$  A. (Note that the wire is not long.) What is the magnitude of the magnetic field at  $P_2$ ?

**18 M** A current is set up in a wire loop consisting of a semicircle of radius  $4.00$  cm, a smaller concentric semicircle, and two radial straight lengths, all in the same plane. Figure 29.24a shows the arrangement but is not drawn to scale. The magnitude of the magnetic field produced at the center of curvature is  $47.25$   $\mu\text{T}$ . The smaller semicircle is then flipped over (rotated) until the loop is again entirely in the same plane (Fig. 29.24b). The magnetic field produced at the (same) center of curvature now has magnitude  $15.75$   $\mu\text{T}$ , and its direction is reversed from the initial magnetic field. What is the radius of the smaller semicircle?

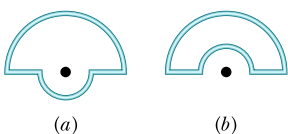


Figure 29.24 Problem 18.

**19 M** One long wire lies along an  $x$  axis and carries a current of  $30$  A in the positive  $x$  direction. A second long wire is perpendicular to the  $xy$  plane, passes through the point  $(0, 4.0$  m,  $0)$ , and carries a current of  $40$  A in the positive  $z$  direction. What is the magnitude of the resulting magnetic field at the point  $(0, 2.0$  m,  $0)$ ?

**20 M** In Fig. 29.25, part of a long insulated wire carrying current  $i = 5.78$  mA is bent into a circular section of radius  $R = 1.89$  cm. In unit-vector notation, what is the magnetic field at the center of curvature  $C$  if the circular section (a) lies in the plane of the page as shown and (b) is perpendicular to the plane of the page after being rotated  $90^\circ$  counterclockwise as indicated?

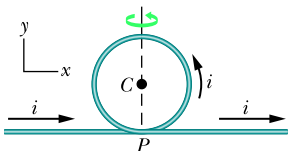


Figure 29.25 Problem 20.

**21 M GO** Figure 29.26 shows two very long straight wires (in cross section) that each carry a current of  $4.00$  A directly out of the page. Distance  $d_1 = 6.00$  m and distance  $d_2 = 4.00$  m. What is the magnitude of the net magnetic field at point  $P$ , which lies on a perpendicular bisector to the wires?

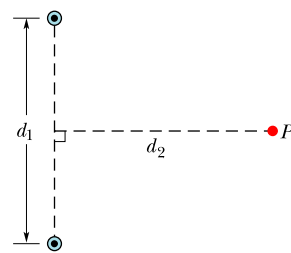


Figure 29.26 Problem 21.

**22 M GO** Figure 29.27a shows, in cross section, two long parallel wires carrying current and separated by distance  $L$ . The ratio  $i_1/i_2$  of their currents is  $4.00$ ; the directions of the currents are not indicated. Figure 29.27b shows the  $y$  component  $B_y$  of their net magnetic field along the  $x$  axis to the right of wire 2. The vertical scale is set by  $B_{ys} = 4.0$  nT, and the horizontal scale is set by  $x_s = 20.0$  cm. (a) At what value of  $x > 0$  is  $B_y$  maximum? (b) If  $i_2 = 3$  mA, what is the value of that maximum? What is the direction (into or out of the page) of (c)  $i_1$  and (d)  $i_2$ ?

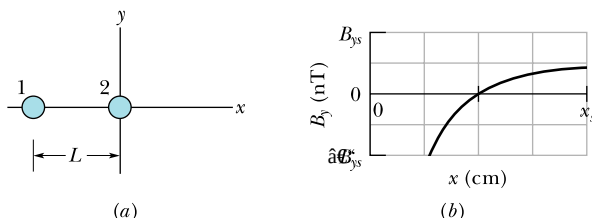


Figure 29.27 Problem 22.

**23 M** Figure 29.28 shows a snapshot of a proton moving at velocity  $\vec{v} = (200 \text{ m/s})\hat{j}$  toward a long straight wire with current  $i = 350$  mA. At the instant shown, the proton's distance from the wire is  $d = 2.89$  cm. In unit-vector notation, what is the magnetic force on the proton due to the current?

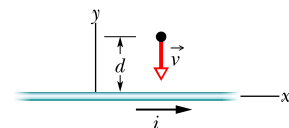


Figure 29.28 Problem 23.

**24 M GO** Figure 29.29 shows, in cross section, four thin wires that are parallel, straight, and very long. They carry identical currents in the directions indicated. Initially all four wires are at distance  $d = 15.0$  cm from the origin of the coordinate system, where they create a net magnetic field  $\vec{B}$ . (a) To what value of  $x$  must you move wire 1 along the  $x$  axis in order to rotate  $\vec{B}$  counterclockwise by  $30^\circ$ ? (b) With wire 1 in that new position, to what value of  $x$  must you move wire 3 along the  $x$  axis to rotate  $\vec{B}$  by  $30^\circ$  back to its initial orientation?

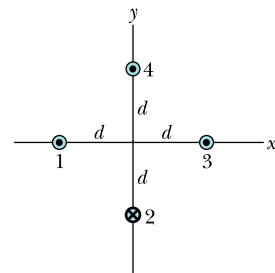


Figure 29.29 Problem 24.

**25 M SSM** A wire with current  $i = 3.00$  A is shown in Fig. 29.30. Two semi-infinite straight sections, both tangent to the same circle, are connected by a circular arc that has a central angle  $\hat{1}$ , and runs along the circumference of the circle. The arc

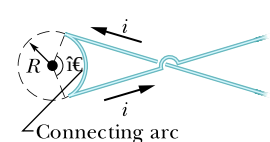


Figure 29.30 Problem 25.



and the two straight sections all lie in the same plane. If  $B = 0$  at the circle's center, what is  $\hat{\theta}$ ?

**26 M GO** In Fig. 29.31a, wire 1 consists of a circular arc and two radial lengths; it carries current  $i_1 = 0.50$  A in the direction indicated. Wire 2, shown in cross section, is long, straight, and perpendicular to the plane of the figure. Its distance from the center of the arc is equal to the radius  $R$  of the arc, and it carries a current  $i_2$  that can be varied. The two currents set up a net magnetic field  $\vec{B}$  at the center of the arc. Figure 29.31b gives the square of the field's magnitude  $B^2$  plotted versus the square of the current  $i_2^2$ . The vertical scale is set by  $B_s^2 = 10.0 \text{ A}^2 \cdot \text{T}^2$ . What angle is subtended by the arc?

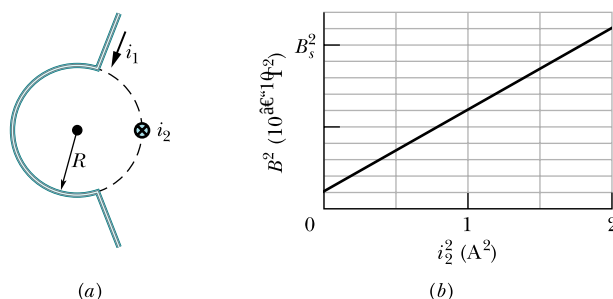


Figure 29.31 Problem 26.

**27 M** In Fig. 29.32, two long straight wires (shown in cross section) carry the currents  $i_1 = 30.0$  mA and  $i_2 = 40.0$  mA directly out of the page. They are equal distances from the origin, where they set up a magnetic field  $\vec{B}$ . To what value must current  $i_1$  be changed in order to rotate  $\vec{B}$   $20.0^\circ$  clockwise?

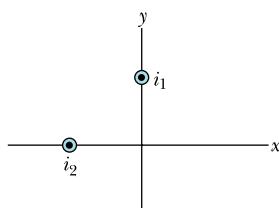


Figure 29.32 Problem 27.

**28 M GO** Figure 29.33a shows two wires, each carrying a current. Wire 1 consists of a circular arc of radius  $R$  and two radial lengths; it carries current  $i_1 = 2.0$  A in the direction indicated. Wire 2 is long and straight; it carries a current  $i_2$  that can be varied; and it is at distance  $R/2$  from the center of the arc. The net magnetic field  $\vec{B}$  due to the two currents is measured at the center of curvature of the arc. Figure 29.33b is a plot of the component of  $\vec{B}$  in the direction perpendicular to the figure as a function of current  $i_2$ . The horizontal scale is set by  $i_{2s} = 1.00$  A. What is the angle subtended by the arc?

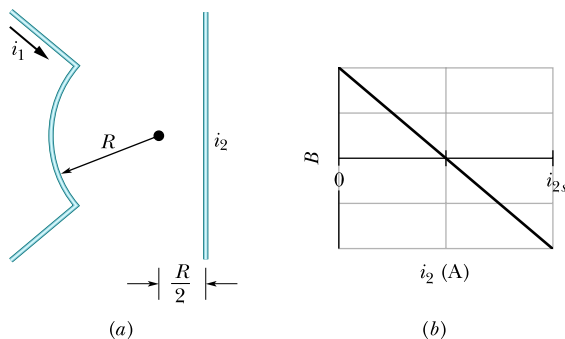


Figure 29.33 Problem 28.

**29 M SSM** In Fig. 29.34, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length  $a = 20$  cm. The currents are out of the page in wires

1 and 4 and into the page in wires 2 and 3, and each wire carries 20 A. In unit-vector notation, what is the net magnetic field at the square's center?

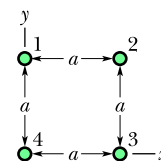


Figure 29.34 Problems 29, 37, and 40.

**30 H GO** Two long straight thin wires with current lie against an equally long plastic cylinder, at radius  $R = 20.0$  cm from the cylinder's central axis. Figure 29.35a shows, in cross section, the cylinder and wire 1 but not wire 2. With wire 2 fixed in place, wire 1 is moved around the cylinder, from angle  $\hat{\theta}_1 = 0^\circ$  to angle  $\hat{\theta}_1 = 180^\circ$  through the first and second quadrants of the  $xy$  coordinate system. The net magnetic field  $\vec{B}$  at the center of the cylinder is measured as a function of  $\hat{\theta}_1$ . Figure 29.35b gives the  $x$  component  $B_x$  of that field as a function of  $\hat{\theta}_1$  (the vertical scale is set by  $B_{xs} = 6.0 \text{ mT}$ ), and Fig. 29.35c gives the  $y$  component  $B_y$  (the vertical scale is set by  $B_{ys} = 4.0 \text{ mT}$ ). (a) At what angle  $\hat{\theta}_2$  is wire 2 located? What are the (b) size and (c) direction (into or out of the page) of the current in wire 1 and the (d) size and (e) direction of the current in wire 2?

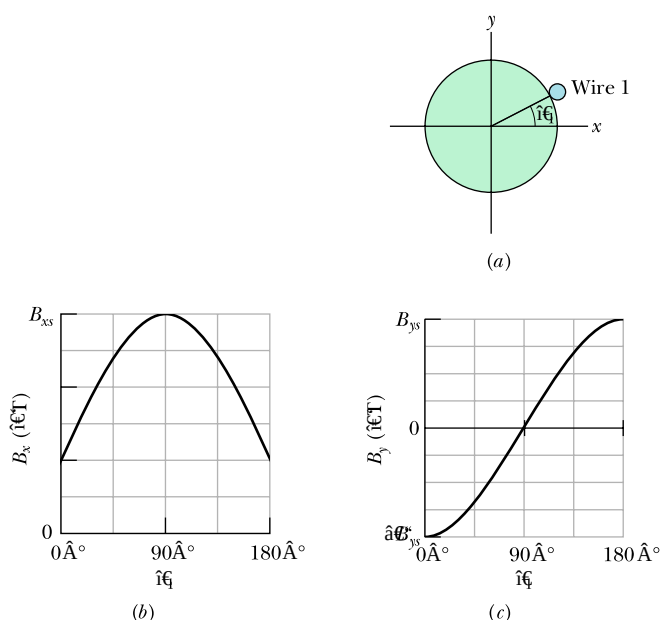


Figure 29.35 Problem 30.

**31 H** In Fig. 29.36, length  $a$  is 4.7 cm (short) and current  $i$  is 13 A. What are the (a) magnitude and (b) direction (into or out of the page) of the magnetic field at point  $P$ ?

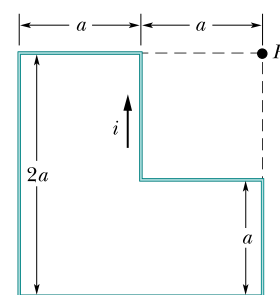


Figure 29.36 Problem 31.

**32 H GO** The current-carrying wire loop in Fig. 29.37a lies all in one plane and consists of a semicircle of radius 10.0 cm, a smaller semicircle with the same center, and two radial lengths. The smaller

semicircle is rotated out of that plane by angle  $\hat{i}$ , until it is perpendicular to the plane (Fig. 29.37b). Figure 29.37c gives the magnitude of the net magnetic field at the center of curvature versus angle  $\hat{i}$ . The vertical scale is set by  $B_a = 10.0 \hat{i}\text{T}$  and  $B_b = 12.0 \hat{i}\text{T}$ . What is the radius of the smaller semicircle?

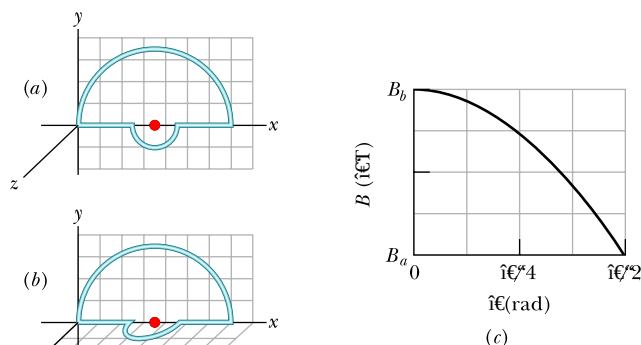


Figure 29.37 Problem 32.

**33 H CALC SSM** Figure 29.38 shows a cross section of a long thin ribbon of width  $w = 4.91$  cm that is carrying a uniformly distributed total current  $i = 4.61$  A into the page. In unit-vector notation, what is the magnetic field  $\vec{B}$  at a point  $P$  in the plane of the ribbon at a distance  $d = 2.16$  cm from its edge? (Hint: Imagine the ribbon as being constructed from many long, thin, parallel wires.)

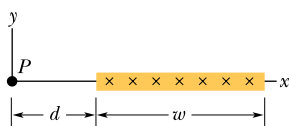


Figure 29.38 Problem 33.

**34 H GO** Figure 29.39 shows, in cross section, two long straight wires held against a plastic cylinder of radius 20.0 cm. Wire 1 carries current  $i_1 = 60.0$  mA out of the page and is fixed in place at the left side of the cylinder. Wire 2 carries current  $i_2 = 40.0$  mA out of the page and can be moved around the cylinder. At what (positive) angle  $\hat{i}_2$  should wire 2 be positioned such that, at the origin, the net magnetic field due to the two currents has magnitude  $80.0$  nT?

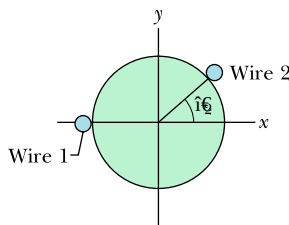


Figure 29.39 Problem 34.

### Module 29.2 Force Between Two Parallel Currents

**35 E SSM** Figure 29.40 shows wire 1 in cross section; the wire is long and straight, carries a current of  $4.00$  mA out of the page, and is at distance  $d_1 = 2.40$  cm from a surface. Wire 2, which is parallel to wire 1 and also long, is at horizontal distance  $d_2 = 5.00$  cm from wire 1 and carries a current of  $6.80$  mA into the page. What is the  $x$  component of the magnetic force per unit length on wire 2 due to wire 1?

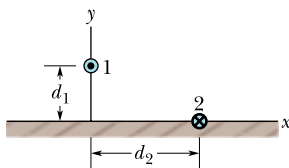


Figure 29.40 Problem 35.

**36 M** In Fig. 29.41, five long parallel wires in an  $xy$  plane are separated by distance  $d = 8.00$  cm, have lengths of  $10.0$  m, and carry identical currents of  $3.00$  A out of the page. Each wire experiences a magnetic force due to the currents in the other

wires. In unit-vector notation, what is the net magnetic force on (a) wire 1, (b) wire 2, (c) wire 3, (d) wire 4, and (e) wire 5?

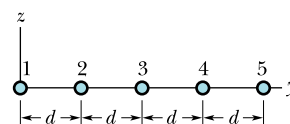


Figure 29.41 Problems 36 and 39.

**37 M GO** In Fig. 29.34, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length  $a = 13.5$  cm. Each wire carries  $7.50$  A, and the currents are out of the page in wires 1 and 4 and into the page in wires 2 and 3. In unit-vector notation, what is the net magnetic force per meter of wire length on wire 4?

**38 M GO** Figure 29.42a shows, in cross section, three current-carrying wires that are long, straight, and parallel to one another. Wires 1 and 2 are fixed in place on an  $x$  axis, with separation  $d$ . Wire 1 has a current of  $0.750$  A, but the direction of the current is not given. Wire 3, with a current of  $0.250$  A out of the page, can be moved along the  $x$  axis to the right of wire 2. As wire 3 is moved, the magnitude of the net magnetic force  $\vec{F}_2$  on wire 2 due to the currents in wires 1 and 3 changes. The  $x$  component of that force is  $F_{2x}$  and the value per unit length of wire 2 is  $F_{2x}/L_2$ . Figure 29.42b gives  $F_{2x}/L_2$  versus the position  $x$  of wire 3. The plot has an asymptote  $F_{2x}/L_2 = 0.627$  N/m as  $x \rightarrow \infty$ . The horizontal scale is set by  $x_s = 12.0$  cm. What are the (a) size and (b) direction (into or out of the page) of the current in wire 2?

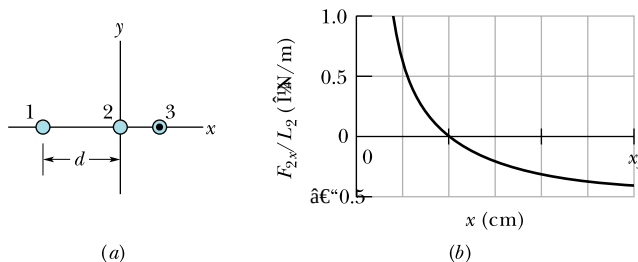


Figure 29.42 Problem 38.

**39 M GO** In Fig. 29.41, five long parallel wires in an  $xy$  plane are separated by distance  $d = 50.0$  cm. The currents into the page are  $i_1 = 2.00$  A,  $i_3 = 0.250$  A,  $i_4 = 4.00$  A, and  $i_5 = 2.00$  A; the current out of the page is  $i_2 = 4.00$  A. What is the magnitude of the net force per unit length acting on wire 3 due to the currents in the other wires?

**40 M** In Fig. 29.34, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length  $a = 8.50$  cm. Each wire carries  $15.0$  A, and all the currents are out of the page. In unit-vector notation, what is the net magnetic force per meter of wire length on wire 1?

**41 H** In Fig. 29.43, a long straight wire carries a current  $i_1 = 30.0$  A and a rectangular loop carries current  $i_2 = 20.0$  A. Take the dimensions to be  $a = 1.00$  cm,  $b = 8.00$  cm, and  $L = 30.0$  cm. In unit-vector notation, what is the net force on the loop due to  $i_1$ ?

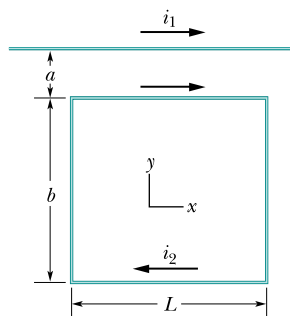


Figure 29.43 Problem 41.

### Module 29.3 Ampere's Law

**42 E** In a particular region there is a uniform current density of  $15$  A/m<sup>2</sup> in the positive  $z$  direction.

What is the value of  $\oint \vec{B} \cdot d\vec{s}$ , when that line integral is calculated along a closed path consisting of the three straight-line segments from  $(x, y, z)$  coordinates  $(4d, 0, 0)$  to  $(4d, 3d, 0)$  to  $(0, 0, 0)$  to  $(4d, 0, 0)$ , where  $d = 20$  cm?

- 43 E** Figure 29.44 shows a cross section across a diameter of a long cylindrical conductor of radius  $a = 2.00$  cm carrying uniform current  $170$  A. What is the magnitude of the current's magnetic field at radial distance (a)  $0$ , (b)  $1.00$  cm, (c)  $2.00$  cm (wire's surface), and (d)  $4.00$  cm?

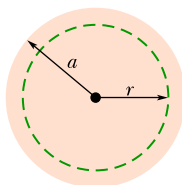


Figure 29.44  
Problem 43.

- 44 E** Figure 29.45 shows two closed paths wrapped around two conducting loops carrying currents  $i_1 = 5.0$  A and  $i_2 = 3.0$  A. What is the value of the integral  $\oint \vec{B} \cdot d\vec{s}$  for (a) path 1 and (b) path 2?

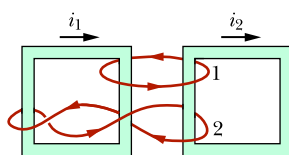


Figure 29.45 Problem 44.

- 45 E SSM** Each of the eight conductors in Fig. 29.46 carries  $2.0$  A of current into or out of the page. Two paths are indicated for the line integral  $\oint \vec{B} \cdot d\vec{s}$ . What is the value of the integral for (a) path 1 and (b) path 2?

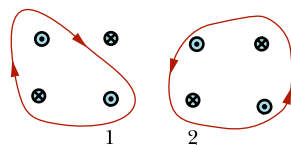


Figure 29.46 Problem 45.

- 46 E** Eight wires cut the page perpendicularly at the points shown in Fig. 29.47. A wire labeled with the integer  $k$  ( $k = 1, 2, \dots, 8$ ) carries the current  $ki$ , where  $i = 4.50$  mA. For those wires with odd  $k$ , the current is out of the page; for those with even  $k$ , it is into the page. Evaluate  $\oint \vec{B} \cdot d\vec{s}$  along the closed path indicated and in the direction shown.

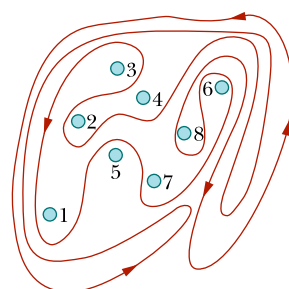


Figure 29.47 Problem 46.

- 47 M CALC** The current density  $\vec{J}$  inside a long, solid, cylindrical wire of radius  $a = 3.1$  mm is in the direction of the central axis, and its magnitude varies linearly with radial distance  $r$  from the axis according to  $J = J_0 r/a$ , where  $J_0 = 310$  A/m<sup>2</sup>. Find the magnitude of the magnetic field at (a)  $r = 0$ , (b)  $r = a/2$ , and (c)  $r = a$ .

- 48 M** In Fig. 29.48, a long circular pipe with outside radius  $R = 2.6$  cm carries a (uniformly distributed) current  $i = 8.00$  mA into the page. A wire runs parallel to the pipe at a distance of  $3.00R$  from center to center. Find the (a) magnitude and (b) direction (into or out of the page) of the current in the wire such that the net magnetic field at point  $P$  has the same magnitude as the net magnetic field at the center of the pipe but is in the opposite direction.

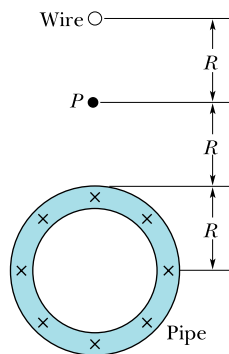


Figure 29.48  
Problem 48.

## Module 29.4 Solenoids and Toroids

- 49 E** A toroid having a square cross section,  $5.00$  cm on a side, and an inner radius of  $15.0$  cm has  $500$  turns and carries a current of  $0.800$  A. (It is made up of a square solenoid instead of a round one as in Fig. 29.4.1 bent into a doughnut shape.) What is the magnetic field inside the toroid at (a) the inner radius and (b) the outer radius?

- 50 E** A solenoid that is  $95.0$  cm long has a radius of  $2.00$  cm and a winding of  $1200$  turns; it carries a current of  $3.60$  A. Calculate the magnitude of the magnetic field inside the solenoid.

- 51 E** A  $200$ -turn solenoid having a length of  $25$  cm and a diameter of  $10$  cm carries a current of  $0.29$  A. Calculate the magnitude of the magnetic field  $\vec{B}$  inside the solenoid.

- 52 E** A solenoid  $1.30$  m long and  $2.60$  cm in diameter carries a current of  $18.0$  A. The magnetic field inside the solenoid is  $23.0$  mT. Find the length of the wire forming the solenoid.

- 53 M** A long solenoid has  $100$  turns/cm and carries current  $i$ . An electron moves within the solenoid in a circle of radius  $2.30$  cm perpendicular to the solenoid axis. The speed of the electron is  $0.0460c$  ( $c =$  speed of light). Find the current  $i$  in the solenoid.

- 54 M** An electron is shot into one end of a solenoid. As it enters the uniform magnetic field within the solenoid, its speed is  $800$  m/s and its velocity vector makes an angle of  $30^\circ$  with the central axis of the solenoid. The solenoid carries  $4.0$  A and has  $8000$  turns along its length. How many revolutions does the electron make along its helical path within the solenoid by the time it emerges from the solenoid at the opposite end? (In a real solenoid, where the field is not uniform at the two ends, the number of revolutions would be slightly less than the answer here.)

- 55 M SSM** A long solenoid with  $10.0$  turns/cm and a radius of  $7.00$  cm carries a current of  $20.0$  mA. A current of  $6.00$  A exists in a straight conductor located along the central axis of the solenoid. (a) At what radial distance from the axis will the direction of the resulting magnetic field be at  $45.0^\circ$  to the axial direction? (b) What is the magnitude of the magnetic field there?

## Module 29.5 A Current-Carrying Coil as a Magnetic Dipole

- 56 E** Figure 29.49 shows an arrangement known as a Helmholtz coil. It consists of two circular coaxial coils, each of  $200$  turns and radius  $R = 25.0$  cm, separated by a distance  $s = R$ . The two coils carry equal currents  $i = 12.2$  mA in the same direction. Find the magnitude of the net magnetic field at  $P$ , midway between the coils.

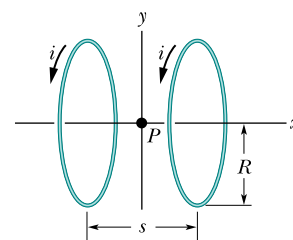


Figure 29.49  
Problems 56 and 87.

- 57 E SSM** A student makes a short electromagnet by winding  $300$  turns of wire around a wooden cylinder of diameter  $d = 5.0$  cm. The coil is connected to a battery producing a current of  $4.0$  A in the wire. (a) What is the magnitude of the magnetic dipole moment of this device? (b) At what axial distance  $z$  will the magnetic field have the magnitude  $5.0 \times 10^{-4}$  T (approximately one-tenth that of Earth's magnetic field)?

- 58 E** Figure 29.50a shows a length of wire carrying a current  $i$  and bent into a circular coil of one turn. In Fig. 29.50b the same

length of wire has been bent to give a coil of two turns, each of half the original radius. (a) If  $B_a$  and  $B_b$  are the magnitudes of the magnetic fields at the centers of the two coils, what is the ratio  $B_b/B_a$ ? (b) What is the ratio  $\hat{I}_b/\hat{I}_a$  of the dipole moment magnitudes of the coils?

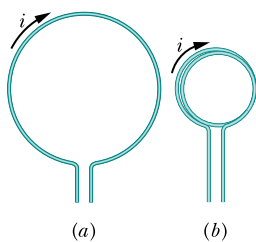


Figure 29.50 Problem 58.

**59 E SSM** What is the magnitude of the magnetic dipole moment  $\hat{I}$  of the solenoid described in Problem 51?

**60 M GO** In Fig. 29.51a, two circular loops, with different currents but the same radius of 4.0 cm, are centered on a y axis. They are initially separated by distance  $L = 3.0$  cm, with loop 2 positioned at the origin of the axis. The currents in the two loops produce a net magnetic field at the origin, with y component  $B_y$ . That component is to be measured as loop 2 is gradually moved in the positive direction of the y axis. Figure 29.51b gives  $B_y$  as a function of the position y of loop 2. The curve approaches an asymptote of  $B_y = 7.20 \hat{I}$  as  $y \rightarrow \infty$ . The horizontal scale is set by  $y_s = 10.0$  cm. What are (a) current  $i_1$  in loop 1 and (b) current  $i_2$  in loop 2?

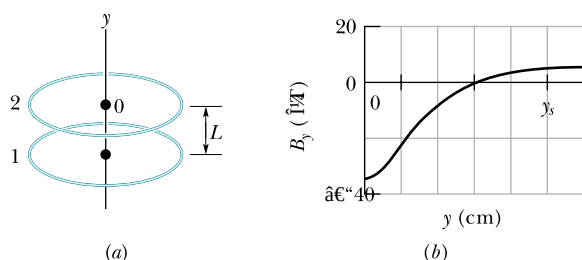


Figure 29.51 Problem 60.

**61 M** A circular loop of radius 12 cm carries a current of 15 A. A flat coil of radius 0.82 cm, having 50 turns and a current of 1.3 A, is concentric with the loop. The plane of the loop is perpendicular to the plane of the coil. Assume the loop's magnetic field is uniform across the coil. What is the magnitude of (a) the magnetic field produced by the loop at its center and (b) the torque on the coil due to the loop?

**62 M** In Fig. 29.52, current  $i = 56.2$  mA is set up in a loop having two radial lengths and two semicircles of radii  $a = 5.72$  cm and  $b = 9.36$  cm with a common center  $P$ . What are the (a) magnitude and (b) direction (into or out of the page) of the magnetic field at  $P$  and the (c) magnitude and (d) direction of the loop's magnetic dipole moment?

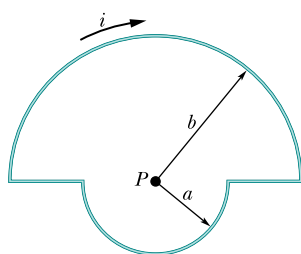


Figure 29.52 Problem 62.

**63 M** In Fig. 29.53, a conductor carries 6.0 A along the closed path  $abcdefgha$  running along 8 of the 12 edges of a cube of edge

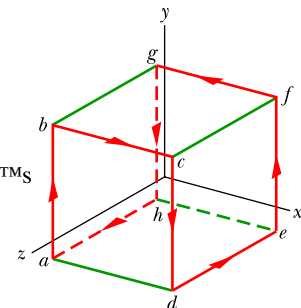


Figure 29.53 Problem 63.

length 10 cm. (a) Taking the path to be a combination of three square current loops ( $bcfgb$ ,  $abgha$ , and  $cdefc$ ), find the net magnetic moment of the path in unit-vector notation. (b) What is the magnitude of the net magnetic field at the  $xyz$  coordinates of  $(0, 5.0 \text{ m}, 0)$ ?

### Additional Problems

**64** In Fig. 29.54, a closed loop carries current  $i = 200$  mA. The loop consists of two radial straight wires and two concentric circular arcs of radii 2.00 m and 4.00 m. The angle  $\hat{I}$  is  $\pi/4$  rad. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at the center of curvature  $P$ ?

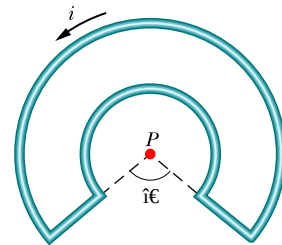


Figure 29.54 Problem 64.

**65** A cylindrical cable of radius 8.00 mm carries a current of 25.0 A, uniformly spread over its cross-sectional area. At what distance from the center of the wire is there a point within the wire where the magnetic field magnitude is 0.100 mT?

**66** Two long wires lie in an  $xy$  plane, and each carries a current in the positive direction of the  $x$  axis. Wire 1 is at  $y = 10.0$  cm and carries 6.00 A; wire 2 is at  $y = 5.00$  cm and carries 10.0 A. (a) In unit-vector notation, what is the net magnetic field  $\vec{B}$  at the origin? (b) At what value of  $y$  does  $\vec{B} = 0$ ? (c) If the current in wire 1 is reversed, at what value of  $y$  does  $\vec{B} = 0$ ?

**67** Two wires, both of length  $L$ , are formed into a circle and a square, and each carries current  $i$ . Show that the square produces a greater magnetic field at its center than the circle produces at its center.

**68** A long straight wire carries a current of 50 A. An electron, traveling at  $1.0 \times 10^7$  m/s, is 5.0 cm from the wire. What is the magnitude of the magnetic force on the electron if the electron velocity is directed (a) toward the wire, (b) parallel to the wire in the direction of the current, and (c) perpendicular to the two directions defined by (a) and (b)?

**69** Three long wires are parallel to a  $z$  axis, and each carries a current of 10 A in the positive  $z$  direction. Their points of intersection with the  $xy$  plane form an equilateral triangle with sides of 50 cm, as shown in Fig. 29.55. A fourth wire (wire  $b$ ) passes through the midpoint of the base of the triangle and is parallel to the other three wires. If the net magnetic force on wire  $a$  is zero, what are the (a) size and (b) direction ( $+z$  or  $-z$ ) of the current in wire  $b$ ?

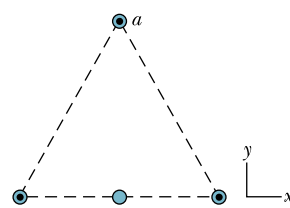


Figure 29.55 Problem 69.

**70** Figure 29.56 shows a closed loop with current  $i = 2.00$  A. The loop consists of a half-circle of radius 4.00 m, two quarter-circles each of radius 2.00 m, and three radial straight wires. What is the magnitude of the net magnetic field at the common center of the circular sections?

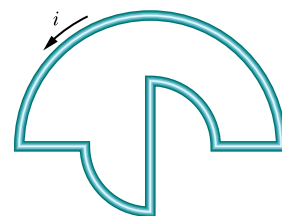


Figure 29.56 Problem 70.



**71** A 10-gauge bare copper wire (2.6 mm in diameter) can carry a current of 50 A without overheating. For this current, what is the magnitude of the magnetic field at the surface of the wire?

**72** A long vertical wire carries an unknown current. Coaxial with the wire is a long, thin, cylindrical conducting surface that carries a current of 30 mA upward. The cylindrical surface has a radius of 3.0 mm. If the magnitude of the magnetic field at a point 5.0 mm from the wire is  $1.0 \hat{\phi}$  T, what are the (a) size and (b) direction of the current in the wire?

**73** Figure 29.57 shows a cross section of a long cylindrical conductor of radius  $a = 4.00$  cm containing a long cylindrical hole of radius  $b = 1.50$  cm. The central axes of the cylinder and hole are parallel and are distance  $d = 2.00$  cm apart; current  $i = 5.25$  A is uniformly distributed over the tinted area. (a) What is the magnitude of the magnetic field at the center of the hole? (b) Discuss the two special cases  $b = 0$  and  $d = 0$ .

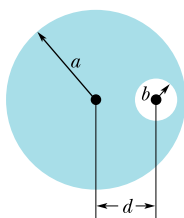


Figure 29.57  
Problem 73.

**74** The magnitude of the magnetic field at a point 88.0 cm from the central axis of a long straight wire is  $7.30 \hat{\phi}$  T. What is the current in the wire?

**75 SSM** Figure 29.58 shows a wire segment of length  $\ell = 3.0$  cm, centered at the origin, carrying current  $i = 2.0$  A in the positive  $y$  direction (as part of some complete circuit). To calculate the magnitude of the magnetic field  $\vec{B}$  produced by the segment at a point several meters from the origin, we can use  $B = \frac{\mu_0 i}{4\pi r^2} \sin \theta$  as the Biot-Savart law. This is because  $r$  and  $\theta$  are essentially constant over the segment. Calculate  $\vec{B}$  (in unit-vector notation) at the  $(x, y, z)$  coordinates (a)  $(0, 0, 5.0 \text{ m})$ , (b)  $(0, 6.0 \text{ m}, 0)$ , (c)  $(7.0 \text{ m}, 7.0 \text{ m}, 0)$ , and (d)  $(3.0 \text{ m}, 4.0 \text{ m}, 0)$ .

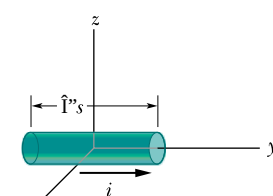


Figure 29.58 Problem 75.

**76 GO** Figure 29.59 shows, in cross section, two long parallel wires spaced by distance  $d = 10.0$  cm; each carries 100 A, out of the page in wire 1. Point  $P$  is on a perpendicular bisector of the line connecting the wires. In unit-vector notation, what is the net magnetic field at  $P$  if the current in wire 2 is (a) out of the page and (b) into the page?

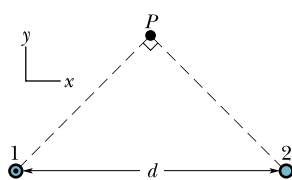


Figure 29.59 Problem 76.

**77** In Fig. 29.60, two infinitely long wires carry equal currents  $i$ . Each follows a  $90^\circ$  arc on the circumference of the same circle of radius  $R$ . Show that the magnetic field  $\vec{B}$  at the center of the circle is the same as the field  $\vec{B}$  a distance  $R$  below an infinite straight wire carrying a current  $i$  to the left.

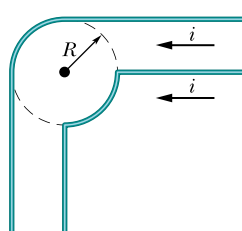


Figure 29.60  
Problem 77.

**78** A long wire carrying 100 A is perpendicular to the magnetic field lines of a uniform magnetic field of

magnitude 5.0 mT. At what distance from the wire is the net magnetic field equal to zero?

**79** A long, hollow, cylindrical conductor (with inner radius 2.0 mm and outer radius 4.0 mm) carries a current of 24 A distributed uniformly across its cross section. A long thin wire that is coaxial with the cylinder carries a current of 24 A in the opposite direction. What is the magnitude of the magnetic field (a) 1.0 mm, (b) 3.0 mm, and (c) 5.0 mm from the central axis of the wire and cylinder?

**80** A long wire is known to have a radius greater than 4.0 mm and to carry a current that is uniformly distributed over its cross section. The magnitude of the magnetic field due to that current is 0.28 mT at a point 4.0 mm from the axis of the wire, and 0.20 mT at a point 10 mm from the axis of the wire. What is the radius of the wire?

**81 CALC SSM** Figure 29.61 shows a cross section of an infinite conducting sheet carrying a current per unit  $x$ -length of  $i\hat{x}$ ; the current emerges perpendicularly out of the page. (a) Use the Biot-Savart law and symmetry to show that for all points  $P$  above the sheet and all points  $P'$  below it, the magnetic field  $\vec{B}$  is parallel to the sheet and directed as shown. (b) Use Ampère's law to prove that  $B = \frac{\mu_0 i}{2}$  at all points  $P$  and  $P'$ .

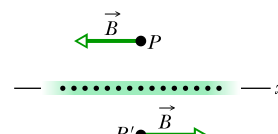


Figure 29.61 Problems 81 and 85.

**82** Figure 29.62 shows, in cross section, two long parallel wires that are separated by distance  $d = 18.6$  cm. Each carries 4.23 A, out of the page in wire 1 and into the page in wire 2. In unit-vector notation, what is the net magnetic field at point  $P$  at distance  $R = 34.2$  cm, due to the two currents?

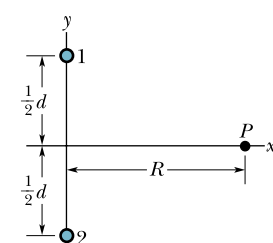


Figure 29.62 Problem 82.

**83 Transcranial magnetic stimulation.** Since 1985, research has been conducted on treating chronic depression, Parkinson's disease, and other brain malfunctions by applying pulsed magnetic fields from coils near the scalp to force neurons several centimeters deep to discharge (Fig. 29.63). Consider the simple situation of a flat coil with radius  $r = 5.0$  cm and number of turns  $N = 14$ . What is the peak magnetic field magnitude at the target distance  $z = 4.0$  cm along the central axis when the peak current is  $i = 4000$  A?



Figure 29.63 Problem 83.



**84 CALC** *Spinning charged disk.* A thin plastic disk of radius  $R$  has a charge  $Q$  uniformly spread over its surface. It is spinning with angular speed  $\omega$  about its central axis. (a) In terms of  $\omega$ ,  $Q$ , and the given symbols, what is the magnitude of the magnetic field at the center of the disk? (*Hint:* The rotating disk is equivalent to an array of current loops around the center.) (b) What is the magnetic dipole moment of the disk?

**85** *Solenoid as a cylinder.* Treat an ideal solenoid as a thin cylindrical conductor, whose current per unit length, measured parallel to the cylinder axis, is  $\lambda$ . By doing so, show that the magnitude of the magnetic field inside an ideal solenoid can be written as  $B = \mu_0 \lambda$ . This is the value of the *change* in  $B$  that you encounter as you move from inside the solenoid to outside, through the solenoid wall. Show that this same change occurs as you move through an infinite plane current sheet such as that of Fig. 29.61. Is this equality surprising?

**86** *Collecting charged particles in a toroid.* An interesting (and frustrating) effect occurs when one attempts to confine a collection of electrons and positive ions (a plasma) in the magnetic field of a toroid. Particles whose motion is perpendicular to the magnetic field will not execute circular paths because the field strength varies with radial distance from the axis of the toroid. This effect, which is shown (exaggerated) in Fig. 29.64, causes particles of opposite sign to drift in opposite directions parallel to the axis of the toroid. (a) What is the sign of the charge on the particle whose path is sketched in the figure? (b) If the particle

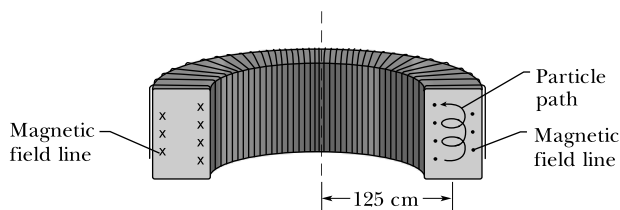


Figure 29.64 Problem 86.

path has a radius of curvature of 11.0 cm when its average radial distance from the axis of the toroid is 125 cm, what will be the radius of curvature when the particle is an average radial distance of 110 cm from the axis?

**87 CALC** *Helmholtz coils.* Figure 29.49 shows two circular coaxial coils of  $N$  turns and radius  $R$ . They have equal currents  $i$  and separation  $s$ . (a) Show that the first derivative of the magnitude of the net magnetic field of the coils ( $dB/dx$ ) vanishes at the midpoint  $P$  regardless of the value of  $s$ . Why would you expect this result from symmetry? (b) Show that the second derivative ( $d^2B/dx^2$ ) also vanishes at  $P$  if  $s = R$ . This accounts for the uniformity of  $B$  near  $P$  in that special case.

**88 CALC** Figure 29.65 is an idealized schematic drawing of a rail gun. Projectile  $P$  sits between two wide rails of circular cross section; a source of current sends current through the rails and through the (conducting) projectile (a fuse is not used). (a) Let  $w$  be the distance between the rails,  $R$  the radius of each rail, and  $i$  the current. Show that the force on the projectile is directed to the right along the rails and is given approximately by

$$F = \frac{i^2 \mu_0}{2\pi\epsilon} \ln \frac{w + R}{R}.$$

(b) If the projectile starts from the left end of the rails at rest, find the speed  $v$  at which it is expelled at the right. Assume that  $i = 450$  kA,  $w = 12$  mm,  $R = 6.7$  cm,  $L = 4.0$  m, and the projectile mass is 10 g.

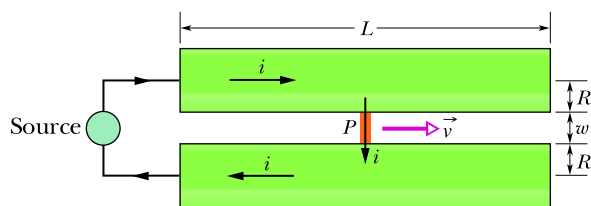


Figure 29.65 Problem 88.