To Do:

* try with larger dataset
* input budget for portfolio, return quantity (vs. percent) of each stock to purchase
* try with higher risk/higher return stocks

Intro #1: In this article I will summarize the process of optimizing an investment portfolio using the Markowitz approach. This is a common problem in finance: given a set of potential stocks to invest in, which combination of stocks and what quantity of each is best? In this approach, we define best to mean the portfolio that will maximize returns while minimizing risk. Of course we want to maximize our returns, but not necessarily by picking the stocks with the highest returns, as these may require taking greater risks with our investment than we are comfortable with. As always with these types of investments, returns are not guaranteed. Thus we focus on minimizing risk, while constraining the returns.

Intro #2: In this article I will define and solve a simple stock portfolio optimization problem using the Markowitz approach in Excel. This is a common problem in finance: given a set of potential stocks to invest in, which combination of stocks and what quantity of each is best? In this approach, we define best to mean the portfolio that will maximize returns while minimizing risk. Of course we want to maximize our returns, but not necessarily by picking the stocks with the highest returns, as these may require taking greater risks with our investment than we are comfortable with. As always with these types of investments, returns are not guaranteed. Thus we focus on minimizing risk, while constraining the returns.

The data can be easily downloaded from [Yahoo! Finance](https://finance.yahoo.com/), or feel free to download stock data for a company of your choice and substitute it in your portfolio. Once you have data on stock prices over a given number of periods, you need to calculate the returns for each period, as these values will be used to calculate the risk involved in various options for portfolio composition. The return is defined as:

rt = (pt / pt-1) - 1

I have used a small sample of stock data from Microsoft, Verison, and Walmart, but this method is easy enough to apply to a larger sample and/or to a larger set of stocks.

There are three main components to any optimization problem:

1. Objective function: a value we are trying to maximize or minimize
2. Variables: the components of our objective function that can vary
3. Constraints on the values those variables can take: for example, we may not want our variables to take on negative values, or if one variable represents a percent its value should not exceed 1.

There are two possible objective functions for a stock portfolio optimization problem: maximize profit and minimize risk. We could include both of them, but mult-objective function optimization problems are extremely complicated so we will pick one for the sake of solvability. In this case, we will minimize risk. We could also choose to set a minimum value on profit, so that we are guaranteed to earn a certain return on our investment. But that step comes when we set constraints on our variables, so we’ll put it off for now.

So we’ve decided that our objective function is to minimize risk. But how is risk quantified here? Risk is represented by the variance of the portfolio. The variance is calculated using the standard deviation of each stock in the portfolio as well as the covariance between the stocks. To calculate the portfolio’s variance we first form a covariance matrix for the stocks. This step is key to the Markowitz approach. Rather than considering the risk and expected returns for each individual stock, we consider their combined risk and expected return.

The variables in this problem are the quantities of all the stocks we are considering, as a percent of the total portfolio. We allow these percentages to vary to determine the value for each that minimizes the overal risk of the portfolio.

How will we constrain these variables? First, the percents must be non-negative and their sum must equal 1. If we want to make sure we invest at least some money in each stock, we can set the minimum percent for each to a small number such as 0.01. An additional constraint that I mentioned earlier is a minimum value for that the total return on the portfolio. The total return is calculated as the sum of the mean returns for each stock, weighted by the percent of each stock included in our final portfolio. Given that these stocks do not have very high mean returns (the highest, Microsoft, is only 2.46%), a reasonable expectation for minimum return may be just 1%.

How to quantify risk for a stock portfolio? We use the variance of the portfolio as a proxy for risk.

We begin with an Excel file that contains the stock prices of three companies - Microsoft, Walmart, and Verizon, over ten time periods.

From here we calculate returns for each fund for each period.

The next step is to calculate a covariance matrix for the returns of all three stocks. This matrix represents the risk of the portfolio.

Now we set up a table where our variables will be stored. The solver will change these values to find the optimal portfolio. In this problem the variables are the quantity of each stock to purchase, as a percent of the total portfolio.

What constraints do we need on our variables? Each percent must be non-negative, since we can’t have a negative percent. Also, the sum of the percents must equal 1, since these are the only stocks we’re including in our portfolio, so the total must be 100%.

We put this objective function, variables, and constraints into the solver and it finds the optimal portfolio. The final returns are quite low, but considering the mean returns for each individual stock, it’s not surprising. If we chose a group of stocks to choose from that had higher mean returns the overall return of our portfolio would be higher.

Assumptions: stocks are normally distributed

References

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