



PARALLELIZATION TECHNIQUES IN CONCURRENT DATA STRUCTURES AND ALGORITHMS

Diploma Thesis

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INTRODUCTION

Moore's law

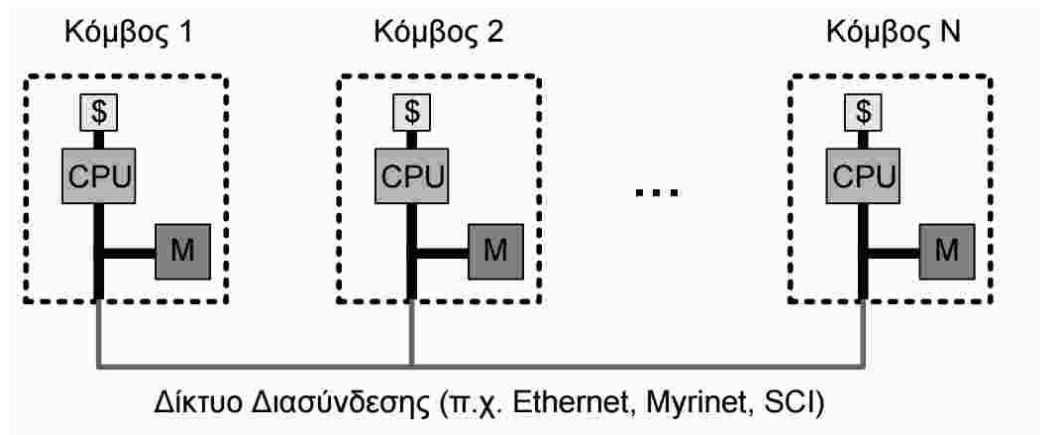
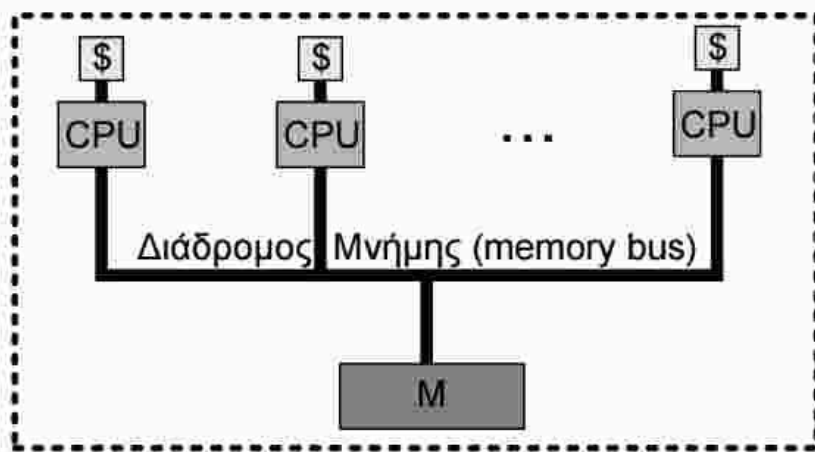
- the number of transistors in an integrated circuit will double approximately every two years

Parallel Architectures

- Shared memory architecture
- Distributed memory architecture
- Hybrid memory architecture

Shared memory architecture

- Uniform Memory Access (UMA)
- Non-Uniform Memory Access (NUMA)



Moore's law

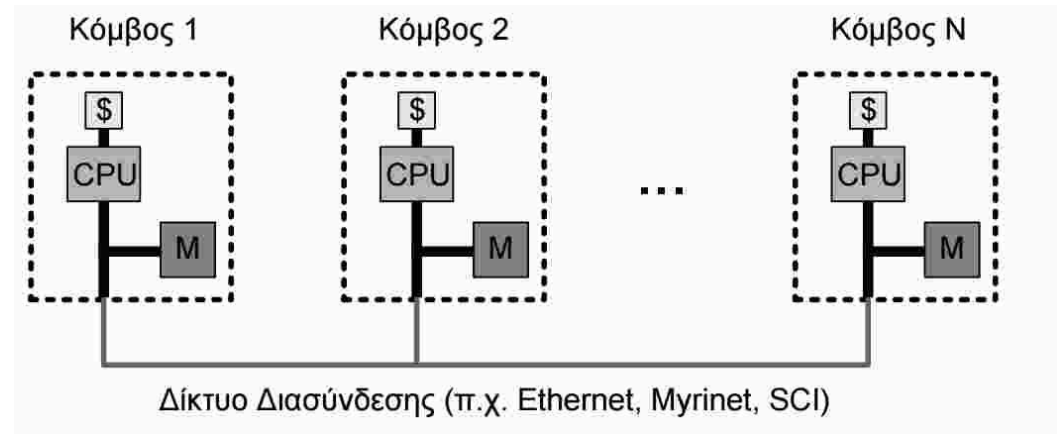
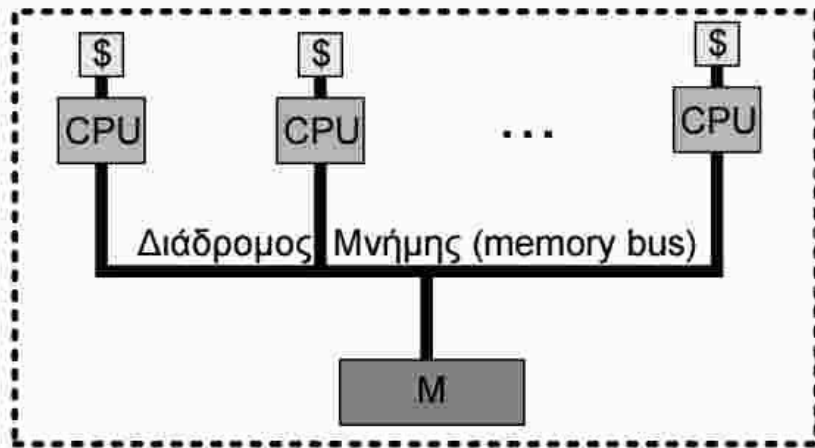
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Parallel Architectures

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Shared memory architecture

- Uniform Memory Access (UMA)
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Redesign:

- Data structures
- Algorithms

Synchronization

- Threads **communicate** with each other
- Threads execute operations in the **same shared** data
- Be executed as if running in isolation
- Particular segment of program: critical section

Synchronization techniques

- Mutual Exclusion (blocking scheme)
- Atomic operations (non-blocking scheme)
- **Transactional memory** (extracting instruction groups to atomic transactions)

Listing 1.1: Inconsistencies due to lack of synchronization

```
1 Thread 1  
2 var1 = counter;  
3 var1 += 1;  
4 counter = var1;
```

```
Thread 2  
var2 = counter;  
var2 -= 1;  
counter = var2;
```



CONCURRENT BINARY SEARCH TREES

Concurrent Data Structures

- Multiple threads access data simultaneously

Concurrent Search trees

- Locate specific values from within a set (key-value pairs)
- **BST**: no balance
- **AVL, Red-Black trees**: height balanced (rotations and recolorings for rebalance)

Techniques for constructing concurrent data structures

- Coarse-grained locking (mutual exclusion)
 - ✓ Lock granularity: the entire shared data are locked via a global lock
 - ✓ **Easy** to program
- Fine-grained locking
 - ✓ Multiple locks, small granularity are used to protect the smallest possible part of the data structure
 - ✓ **Hard** to program
 - ✓ Avoid deadlock: acquire locks in the same direction (global order)
- Lock-free programming (non-blocking scheme)
 - ✓ Programming without locks, **using atomic operations**: CAS, TAS TTAS

Internal trees

- Internal nodes (nodes with two children): store key-value pairs

External trees

- Internal nodes: used only for **routing** purposes
- **Key-value pairs** are stored only in the **leaves**

Bottom-up trees

- An update operation has two phases
- 1st: a traversal with a **top-down manner** until the desired node is reached
- 2nd: rebalancing the tree with a **bottom up manner**

Top-down trees

- There is only a **top down** traversal
- Performing **rebalances in advance** (generally more tree modifications are performed)
- Created to simplify fine-grained locking approaches

System configuration

Huawei platform (60-core, NUMA architecture)

- 4 sockets (Intel(R) Xeon(R) CPU E7-4880 v2 @ 2.50GHz)
- 15 cores per socket (30 threads with hyperthreading)
- 32KB L1 data cache per core
- 32KB L1 instruction cache per core
- 256KB L2 cache per core
- 38MB L3 cache per socket
- 1TB RAM

A naive approach

Coarse-grained locking

- AVL external bottom-up tree
- RBT external bottom-up tree
- RBT internal bottom-up tree
- RBT external top-down tree
- RBT internal top-down tree

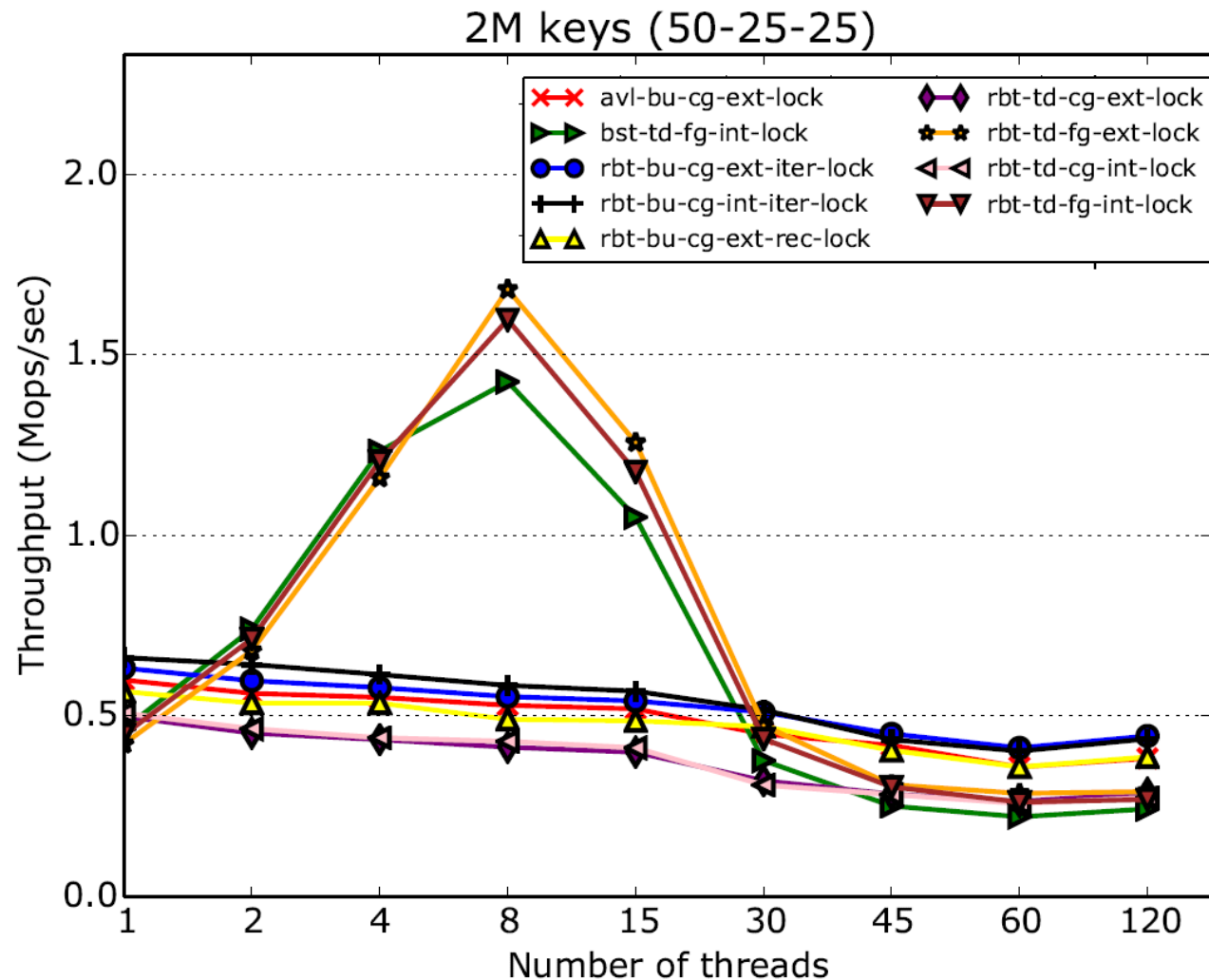
Run configurations

- Key ranges: 2K, 32K, 2000000 keys
- Different workloads ([lookup-insertion-deletion](#)) : 80-10-10, 50-25-25, 20-40-40

Fine-grained locking

- BST internal top-down tree
- RBT external top-down tree
- RBT internal top-down tree

Performance results



- Fine-grained locking scale up to 8 threads (**poorly**)
- 30 threads: NUMA effect **more than coarse-grained implementations**
- Coarse-grained locking provides no parallelism
- RBT external fine-grained: best throughput, since internal approach has to lock the whole subtree in deletion

A sophisticated approach

Bronson

- Lock-based
- Relaxed balanced AVL tree
 - ✓ Rebalances may be delayed
- Partial external tree
 - ✓ Leaving a deleted node when it has 2 children
- Version numbers to indicate if a write is in progress

Aravind

- Lock-free BST tree
- Atomic operations: CAS, bit-test-and-set (BST)
- Marking deleting edges instead of nodes
- Multiple keys can be removed in a single step

Draschler

- Lock-based
- BST internal tree
- Logical ordering
- Find successor in $O(1)$ (succ pointer)

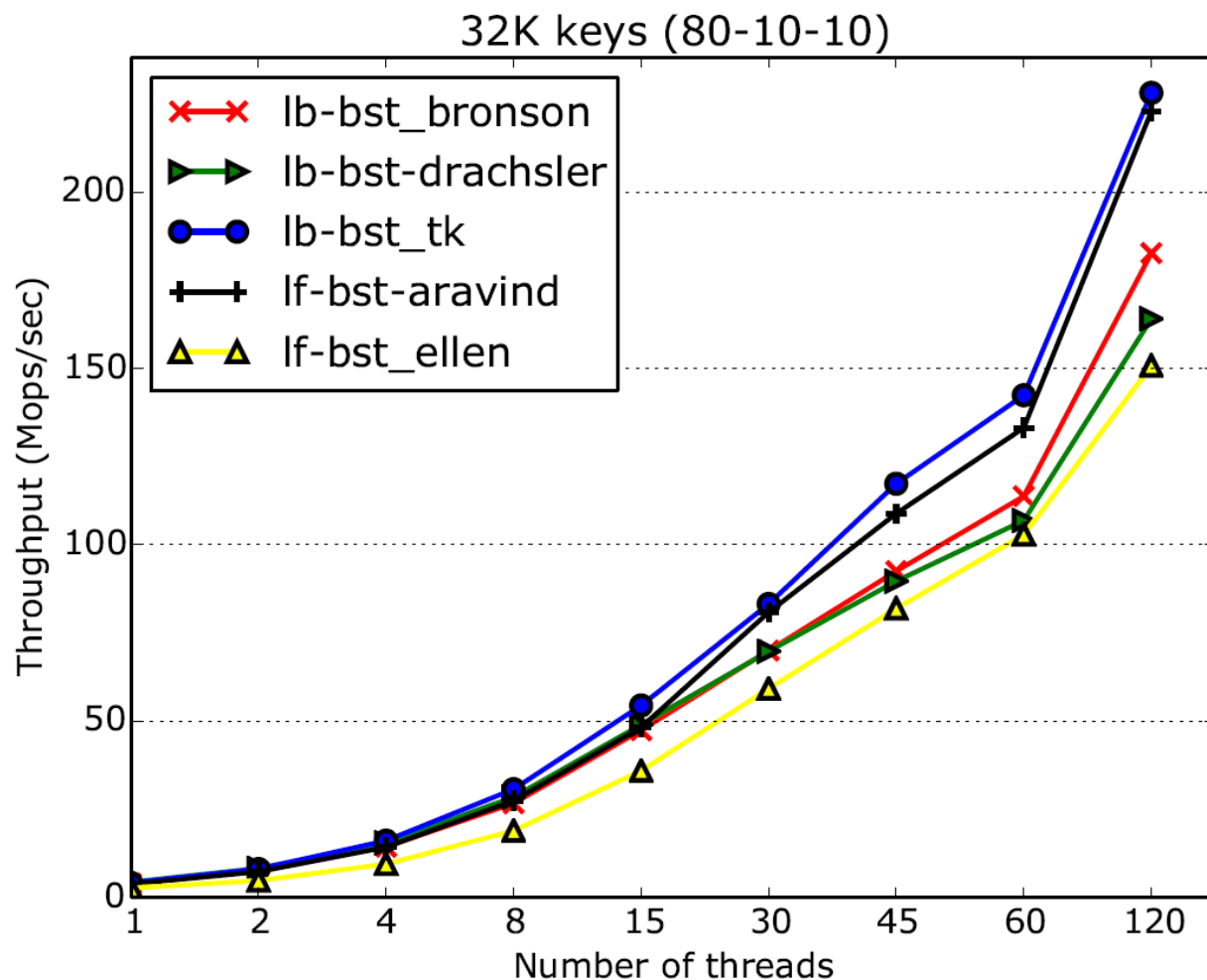
BST Ticket

- Lock-based external BST
- Version numbers to detect concurrent updates

Ellen

- Lock-free external BST tree
- Atomic operation: CAS
- Use explicit objects

Performance results



- BST Ticket, Aravind :better, Ellen, Draschler, Bronson: **more complex** synchronization
- Cache coherence: **limiting factor**, attempt to **minimize** the amount of **cache traffic**
- **Lock-based** and **lock-free** trees are close in terms of performance
- Lock-free: **robustness**, better applicable to oversubscriptions configurations
- Number of stores in a successful update should be **close to** an asynchronous, **sequential** algorithm

Conclusions

Naive lock-based implementations

- Very **low throughput**
- **Easy** to implement

More sophisticated implementations

- Really **hard** to implement
- Lock-based:
 - ✓ High performance
 - ✓ **No** robustness
- Lock-free:
 - ✓ High performance
 - ✓ Robustness

TRANSACTIONAL MEMORY

Transactional Memory

- a **group of instructions** is executed as an atomic transaction
- **easy** programming
- the underlying TM system executes the transaction in an **atomic way**
- the programmer **defines** the block of code to be executed as **transaction**
- If no conflicts are detected, the TM system persists the results (**transactional commit**)
- If conflicts are detected, the TM system rolls back the transaction. Modifications are discarded (**transactional abort**)

Three categories

- Software Transactional Memory (STM)
- Hardware Transactional Memory (HTM)
- Hybrid Transactional Memory (Hybrid TM)

Basic TM characteristics

- **Data versioning**
 - ✓ Eager versioning (undo-log to roll back the transaction)
 - ✓ Lazy versioning (write-buffer)
- **Conflict detection**
 - ✓ Pessimistic detection (detect conflicts early)
 - ✓ Optimistic detection (detect conflicts at commit time)
- **Conflict resolution** (resolution policy e.g. stall or abort the transaction)
- **Isolation**
 - ✓ Strong isolation (conflicts detected even if in a non-transactional code)
 - ✓ Weak isolation (transaction isolated only from other transactions)
- **Granularity** (unit of storage over which TM system detects conflicts)
 - ✓ Object granularity
 - ✓ Word granularity
 - ✓ Cache line granularity
- **Conflicts**
 - ✓ Data conflicts
 - ✓ Capacity aborts
 - ✓ Explicit abort
 - ✓ Other (e.g system calls)
- **Best effort**
 - ✓ No forward progress is guaranteed
 - ✓ A non-transactional fallback path is necessary

Intel's Haswell HTM

- HTM implementation (2013)

TSX Intel Interfaces

- Hardware Lock Elision (HLE)
 - ✓ two new instruction prefixes, used to denote the bounds of the critical section
 - ✓ run to hardware without TSX
- **Restricted Transactional Memory (RTM)**
 - ✓ **flexibility** to specify a fallback code path
 - ✓ does not run to hardware without TSX
 - ✓ 4 new instructions
 - ❑ XBEGIN
 - ❑ XEND
 - ❑ XTEST
 - ❑ XABORT

PARALLELIZING DIJKSTRA'S ALGORITHM

Dijkstra's algorithm

SSSP Problem

- Directed graph $G=(V,E)$
- **Non-negative** edges
- Source vertex s
- Observation: any subpath of any shortest path is itself a shortest path

Shortest path estimate $d(v)$

- Gradually converges to $\delta(v)$ through relaxations
- Relax(v,w): $d(w) = \min \{d(w), d(v) + w(v,w)\}$
can we find a better path ?

Three partitions of vertices

- Settled: $d(v) = \delta(v)$
- Queued: $d(v) > \delta(v)$ and $d(v) \neq \infty$
- Unreached: $d(v) = \infty$

Dijkstra's algorithm

Serial algorithm

```
Input      :  $G = (V, E)$ ,  $w : E \rightarrow \mathbb{R}^+$ ,  
              source vertex  $s$ , min  $Q$   
Output    : shortest distance array  $d$ ,  
              predecessor array  $\pi$   
foreach  $v \in V$  do  
     $d[v] \leftarrow \text{INF}$ ;  
     $\pi[v] \leftarrow \text{NIL}$ ;  
    Insert( $Q, v$ );  
end  
 $d[s] \leftarrow 0$ ;  
while  $Q \neq \emptyset$  do  
     $u \leftarrow \text{ExtractMin}(Q)$ ;  
    foreach  $v$  adjacent to  $u$  do  
         $sum \leftarrow d[u] + w(u, v)$ ;  
        if  $d[v] > sum$  then  
            DecreaseKey( $Q, v, sum$ );  
             $d[v] \leftarrow sum$ ;  
             $\pi[v] \leftarrow u$ ;  
        end  
    end  
end
```

- Min $d(v)$: min-priority queue
- Binary heap: $\Theta(m * \log n)$
- Fibonacci heap: $\Theta(m + n * \log n)$

Using binary heap

- Array implementation
- Maintain all but the settled vertices
- Min-heap property:
 $\forall i : d(\text{parent}(i)) \leq d(i)$

The four phases of the algorithm

Main thread

```
while  $Q \neq \emptyset$  do
   $u \leftarrow \text{ExtractMin}(Q);$  timer 1
   $done \leftarrow 0;$ 
  foreach  $v$  adjacent to  $u$  do
     $sum \leftarrow d[u] + w(u, v);$  timer 2
    Begin-Xact
    if  $d[v] > sum$  then timer 3
       $\text{DecreaseKey}(Q, v, sum);$ 
       $d[v] \leftarrow sum;$ 
       $\pi[v] \leftarrow u;$  timer 4
    End-Xact
  end
  Begin-Xact
   $done \leftarrow 1;$ 
  End-Xact
end
```

Timers:

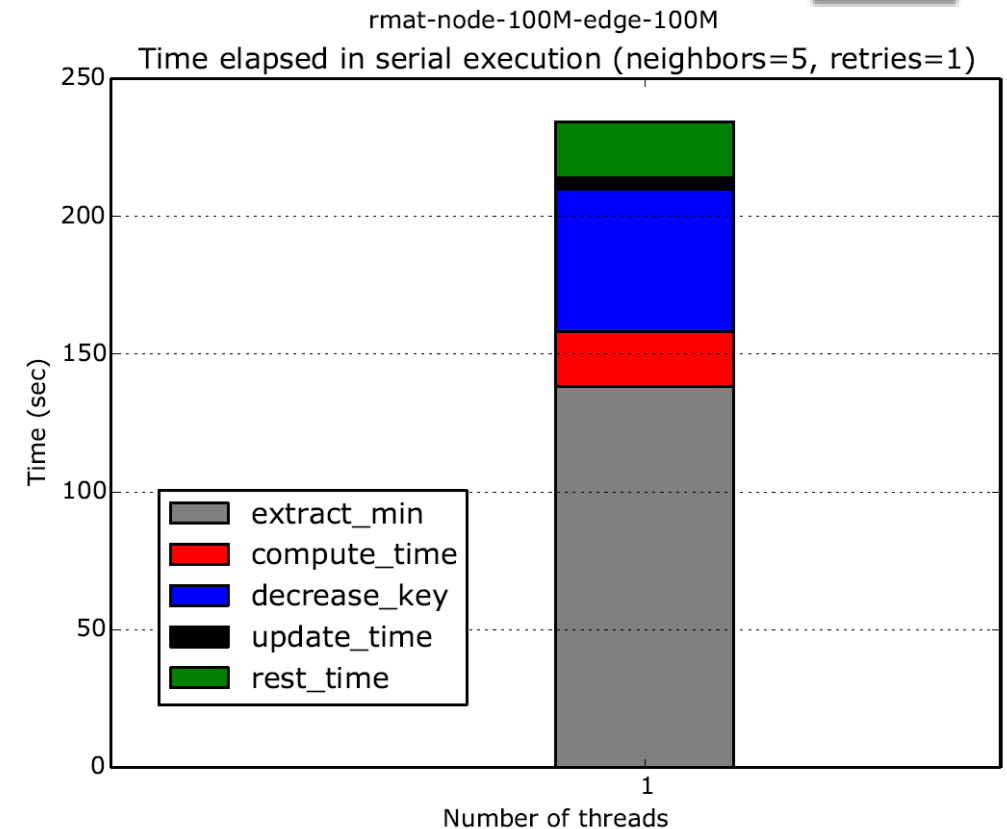
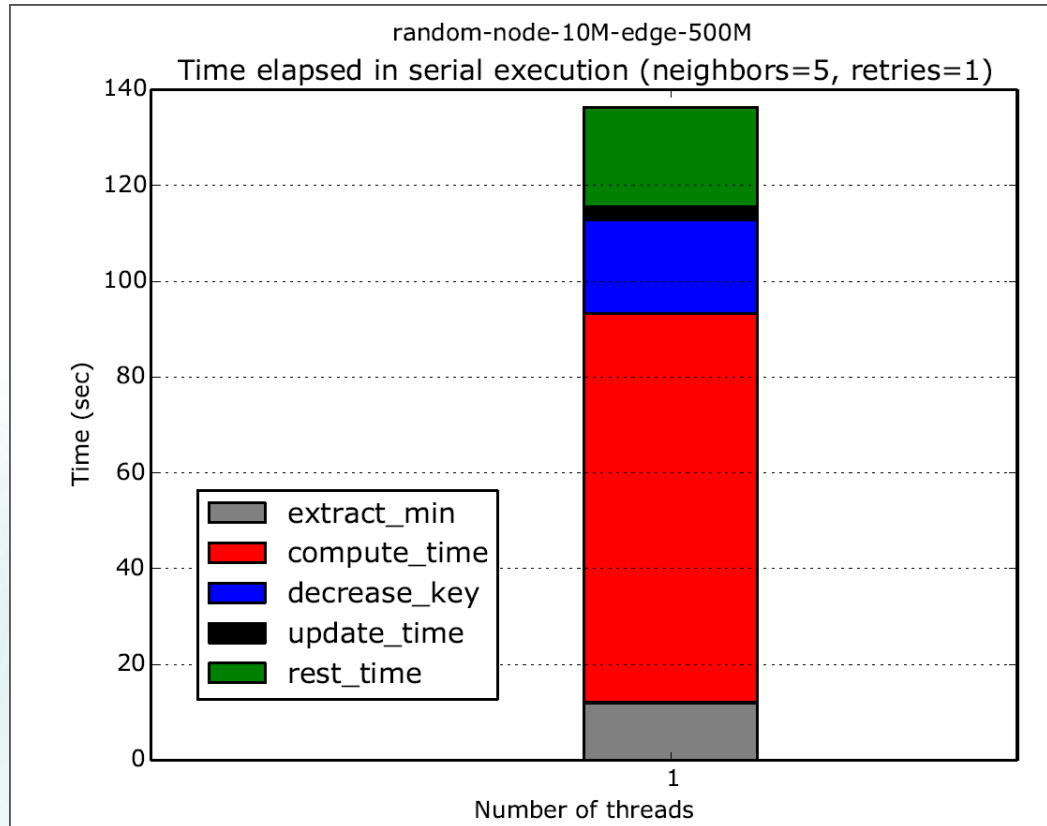
- Timer 1: [ExtractMin\(\)](#)
- Timer 2: [Compute](#) an estimate for distance
- Timer 3: [DecreaseKey\(\)](#)
- Timer 4: [Update](#) new distance (relax)

System configuration

Haci3 platform (28-core, NUMA architecture)

- 2 sockets (Intel(R) Xeon(R) CPU E5-2697 v3 @ 2.60GHz)
- 14 cores per socket (28 threads with hyperthreading)
- 32KB L1 data cache per core
- 32KB L1 instruction cache per core
- 256KB L2 cache per core
- 35MB L3 cache per socket
- 128GB RAM
- Hardware Transactional Memory:
 - ✓ lazy data versioning
 - ✓ eager conflict resolution
 - ✓ best effort HTM
 - ✓ strong isolation
 - ✓ cache line granularity
 - ✓ 4MB read set
 - ✓ 22KB write set

Experimentation in the serial algorithm

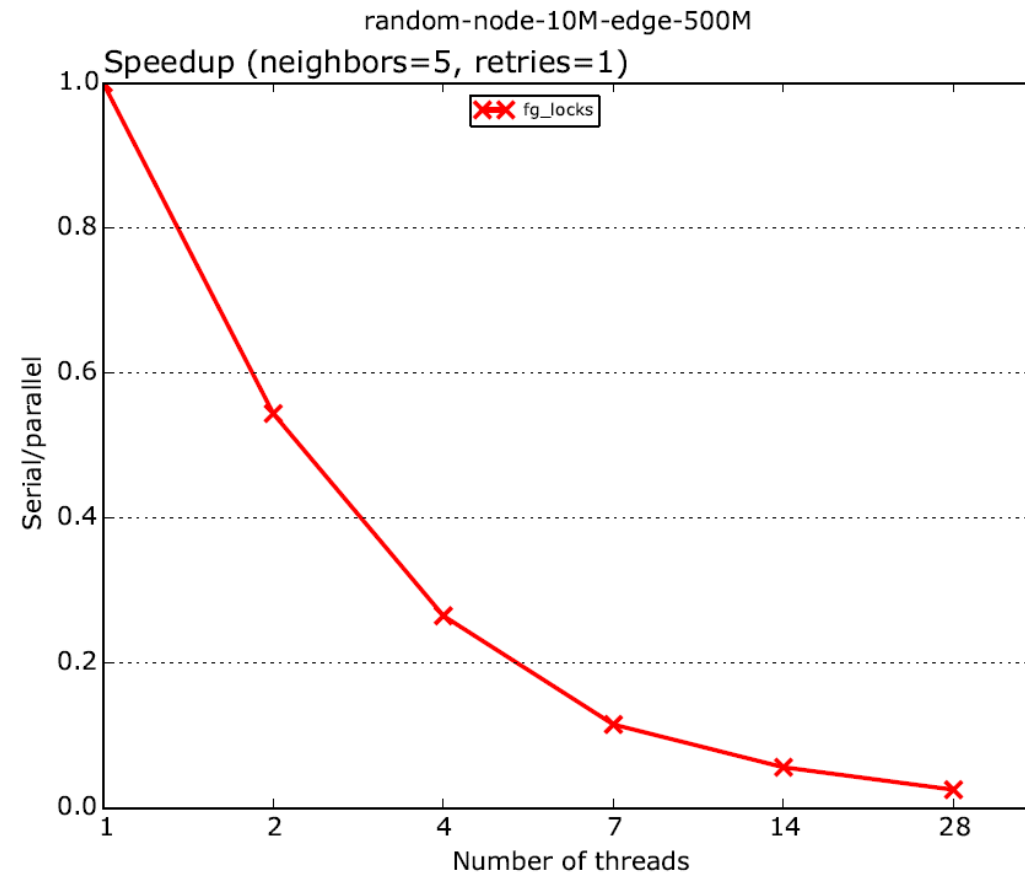


- Four phases: ExtractMin(), compute distance, DecreaseKey(), update distance
- Parallel part: very small percentage of the total runtime
- Can **not extract much parallelism**
- Dijkstra's algorithm: **inherently serial** algorithm

A naïve parallelization

algorithm

```
Input      :  $G = (V, E)$ ,  $w : E \rightarrow \mathbf{R}^+$ ,  
            source vertex  $s$ , min  $Q$   
Output     : shortest distance array  $d$ ,  
            predecessor array  $\pi$   
foreach  $v \in V$  do  
     $d[v] \leftarrow \text{INF}$ ;  
     $\pi[v] \leftarrow \text{NIL}$ ;  
    Insert( $Q, v$ );  
end  
 $d[s] \leftarrow 0$ ;  
while  $Q \neq \emptyset$  do  
    if (id == 0)  
         $u \leftarrow \text{ExtractMin}(Q)$ ;  
    BARRIER  
    Parallel For  
    foreach  $v$  adjacent to  $u$  do  
         $sum \leftarrow d[u] + w(u, v)$ ;  
        if  $d[v] > sum$  then  
            /* Fine-grained */ DecreaseKey( $Q, v, sum$ );  
             $d[v] \leftarrow sum$ ;  
             $\pi[v] \leftarrow u$ ;  
        end  
    end  
end
```

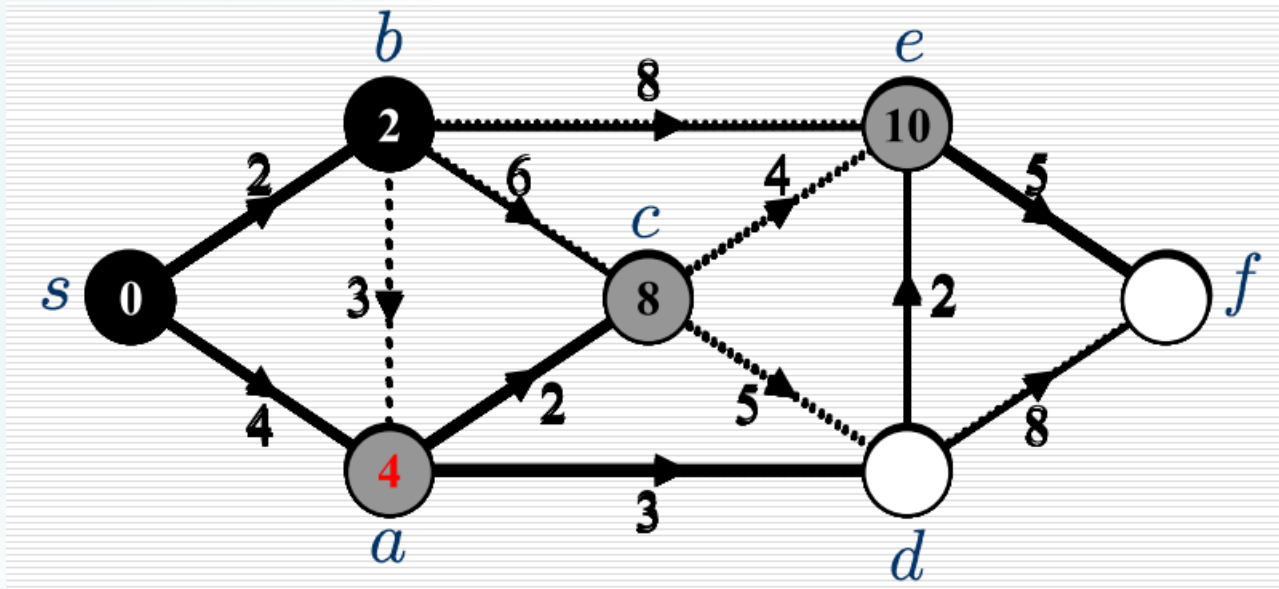


Thread	Time (sec)
1	78
2	145
4	298
7	687
14	1422
28	3184

Parallelizing Dijkstra's algorithm

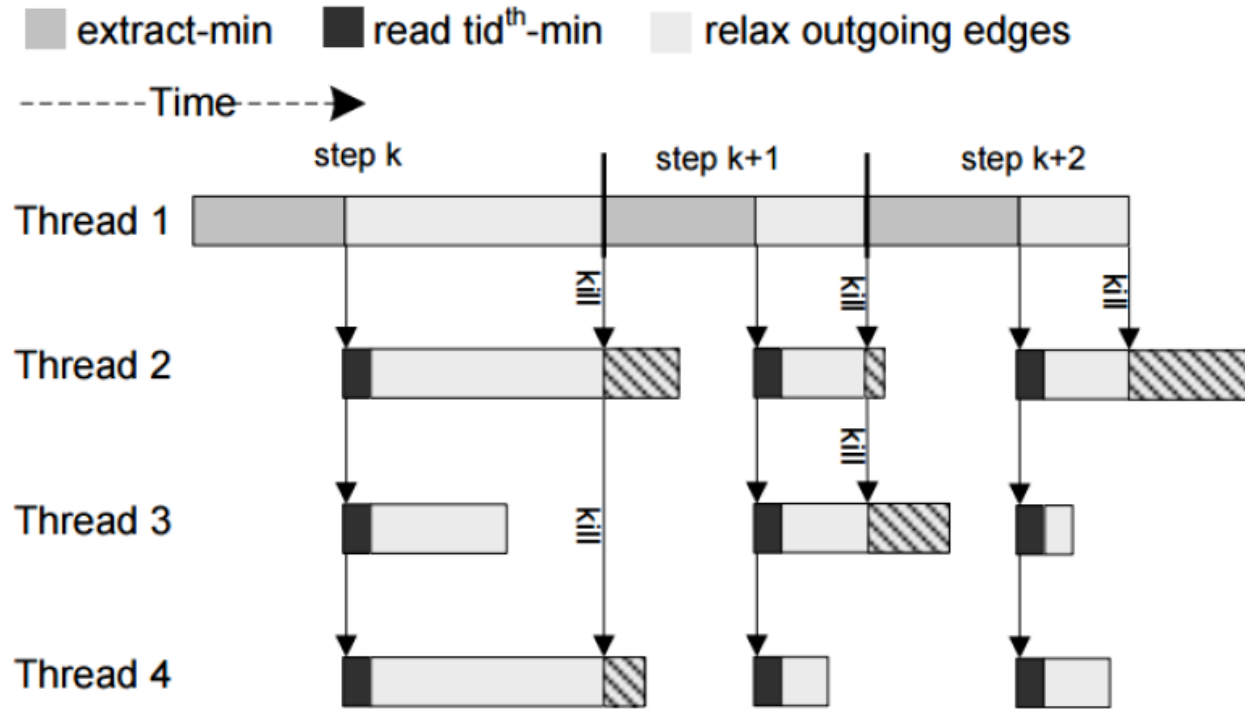
The idea

- In serial: relaxations are performed only from the extracted (settled) vertex
- Allow relaxations for the queued vertices (some of them might have obtained the optimal value)
 - ✓ If the vertex is **settled**, helper thread **will offload work** from the main thread
 - ✓ If the vertex has **not** obtained its **optimal** value, the updated (suboptimal) distances **will be corrected** by the main thread



2	5	16	21	33
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Execution pattern



- main thread extracts the min vertex and signals helper threads
- n helper threads read the next n vertices in the queue
- main thread stops all helper threads at the end of each iteration
- unfinished will be corrected in next steps

Parallelizing Dijkstra's algorithm

Main thread

```
while  $Q \neq \emptyset$  do
   $u \leftarrow \text{ExtractMin}(Q)$ ;
   $done \leftarrow 0$ ;
  foreach  $v$  adjacent to  $u$  do
     $sum \leftarrow d[u] + w(u, v)$ ;
    Begin-Xact
    if  $d[v] > sum$  then
      DecreaseKey( $Q, v, sum$ );
       $d[v] \leftarrow sum$ ;
       $\pi[v] \leftarrow u$ ;
    End-Xact
  end
  Begin-Xact
   $done \leftarrow 1$ ;
  End-Xact
end
```

Helper thread

```
while  $Q \neq \emptyset$  do
  while  $done = 1$  do ;
   $x \leftarrow \text{ReadMin}(Q, tid)$ 
   $stop \leftarrow 0$ 
  foreach  $y$  adjacent to  $x$  and while  $stop = 0$  do
    Begin-Xact
    if  $done = 0$  then
       $sum \leftarrow d[x] + w(x, y)$ 
      if  $d[y] > sum$  then
        DecreaseKey( $Q, y, sum$ )
         $d[y] \leftarrow sum$ 
         $\pi[y] \leftarrow x$ 
      else
         $stop \leftarrow 1$ 
    End-Xact
  end
end
```

- Enclose relaxation within a transaction

Parallelizing Dijkstra's algorithm

Main thread

```
while  $Q \neq \emptyset$  do
   $u \leftarrow \text{ExtractMin}(Q)$ ;
   $done \leftarrow 0$ ;
  foreach  $v$  adjacent to  $u$  do
     $sum \leftarrow d[u] + w(u, v)$ ;
    Begin-Xact
    if  $d[v] > sum$  then
      DecreaseKey( $Q, v, sum$ );
       $d[v] \leftarrow sum$ ;
       $\pi[v] \leftarrow u$ ;
    End-Xact
  end
  Begin-Xact
   $done \leftarrow 1$ ;
  End-Xact
end
```

Optimizations :

- Strong isolation HTM: “done” variable not in transactional mode
- Cache line granularity in HTM: **structure padding** in binary heap's vertices
- **Coarse-grained transactions**: more relaxations within a single transaction (less overhead)
- Best effort HTM: in fall back path helper threads can **never obtain the global lock** such that no to interfere in main thread's progress.

Experimentation in the parallel algorithm

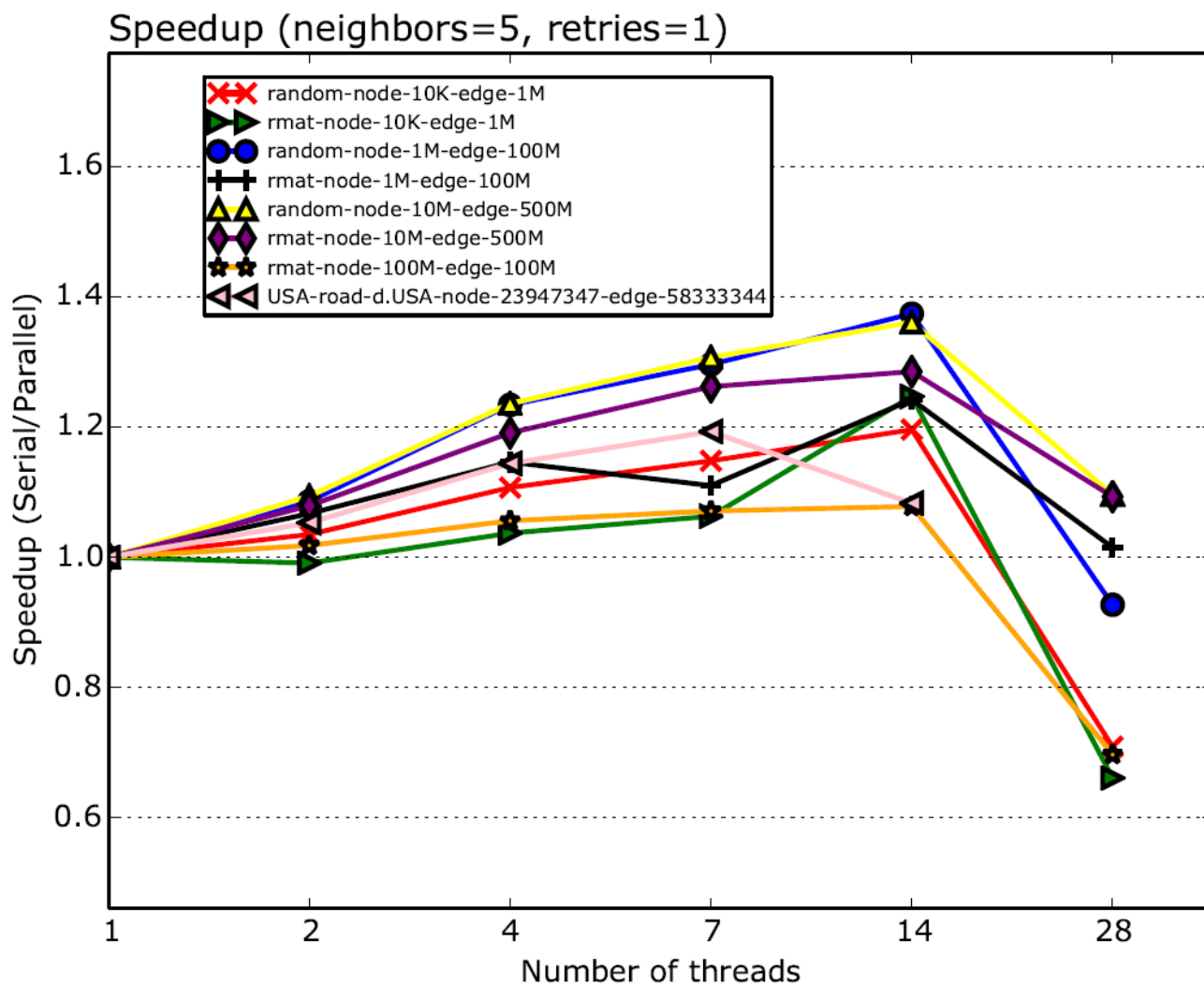
Number of retries in transactional mode before acquiring the global lock

- Main thread: 1 retry (goal: run as in the serial execution)
- Helper threads: can never acquire the global lock (always executing in transactional mode)

Number of edges examined within a single transaction (more coarse-grained)

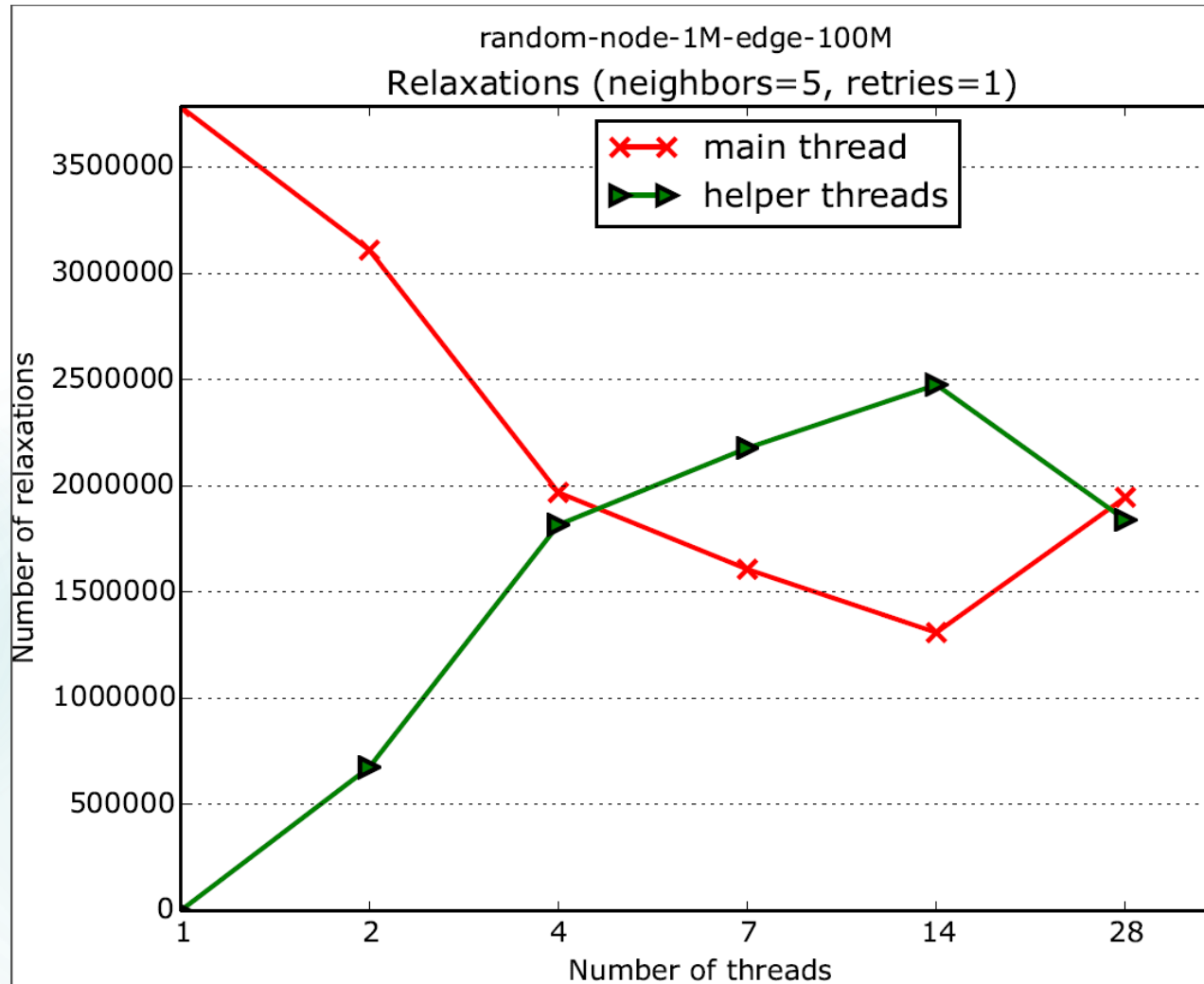
- Main thread: 5, 10 edges examined within a single transaction. A very coarse-grained transaction leads to capacity aborts, especially in large graphs.
- Helper threads: 1 edge per transaction (Helper threads have to perform many, small relaxations before they are stopped by the main thread).

Performance results



- Performance is strongly related to the density of the graph
- ✓ in dense graphs => more parallelism can be exposed
- ✓ sparse graphs => limited parallelism
- 28 threads: NUMA effect
- ✓ expensive cache line transfers from one socket to another => negatively influence scalability
- ✓ Future Work: redesign algorithm such that to scale even in the presence of non-uniformity.

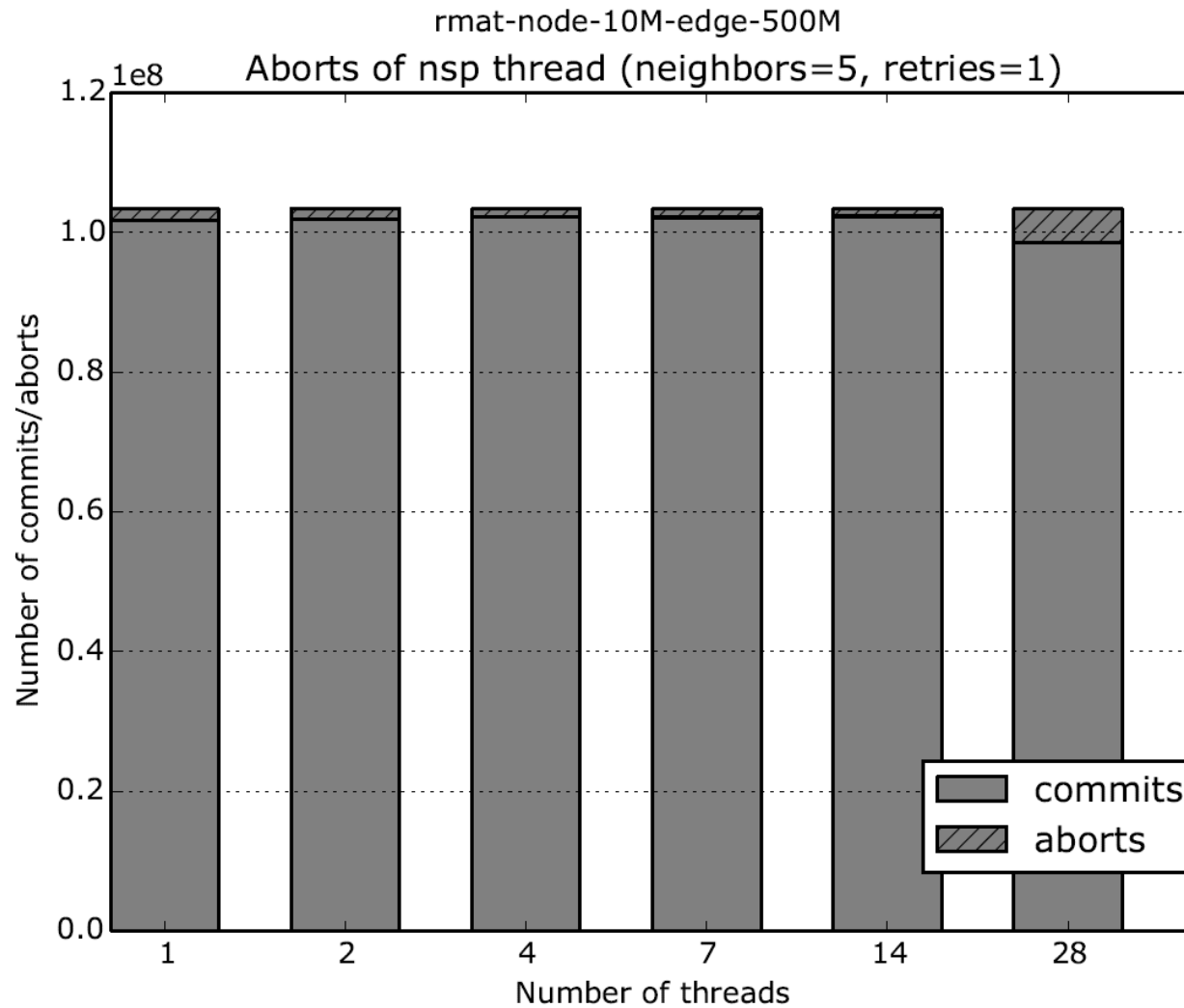
A closer look at the results



Relaxations

- Number of **threads increases** => **fewer relaxations** (main thread)
=> scalability
- **Gain in relaxations** => we believe that the algorithm can **give much better** performance
- ✓ time spent in **thread synchronization**
- ✓ delays due to **conflicting transactions**
- 28 threads: **NUMA effect**

A closer look at the results



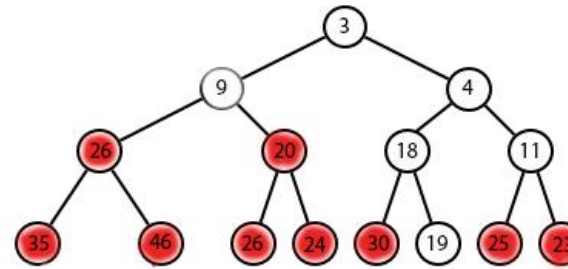
Aborts

- **Main** thread: suffers a really **low** number of **aborts**
- ✓ **Helper** threads do **not obstruct its progress** even not contributing to useful work
- Addition of **more threads** => do **not increase** the number of **aborts**
- ✓ If NUMA effect did not exist => **better scalability** for more helper threads

Employing skip list

Characteristics

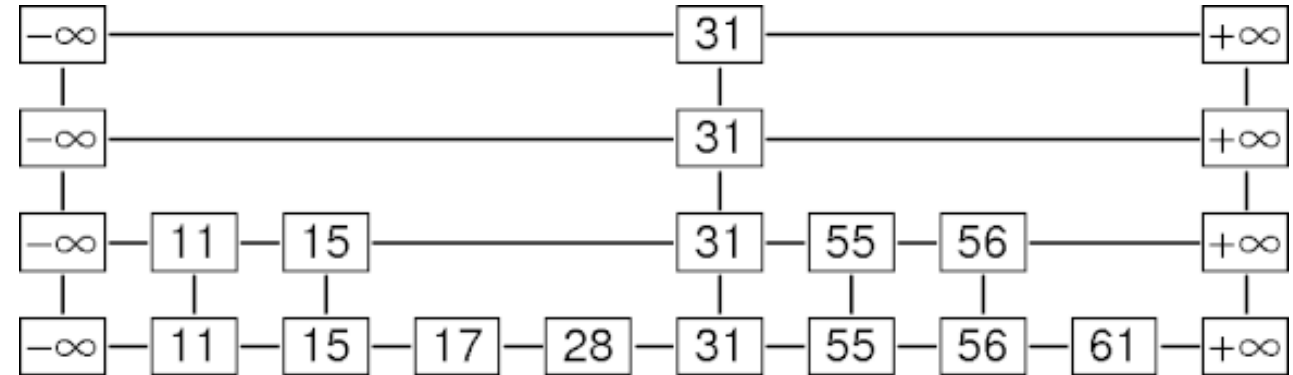
- Keys in **sorted** order
- It is build in $O(\log n)$ layers
- Sentinel elements (head, tail): in all layers
- Internal elements: **random height** between 1 and levelmax



3	4	9	11	18	19	20	23	24	25	26	26	30	35	46
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Skip list in Dijkstra's algorithm

- Serial: ExtractMin() in $O(1)$
- DecreaseKey() in $O(\log n)$
- Parallel: Threads **act** more **locally**, write only the previous and the next element, while **using binary heap** has **bigger contention window**, especially when proceeding higher in the heap.
- A more **concurrently friendly** structure



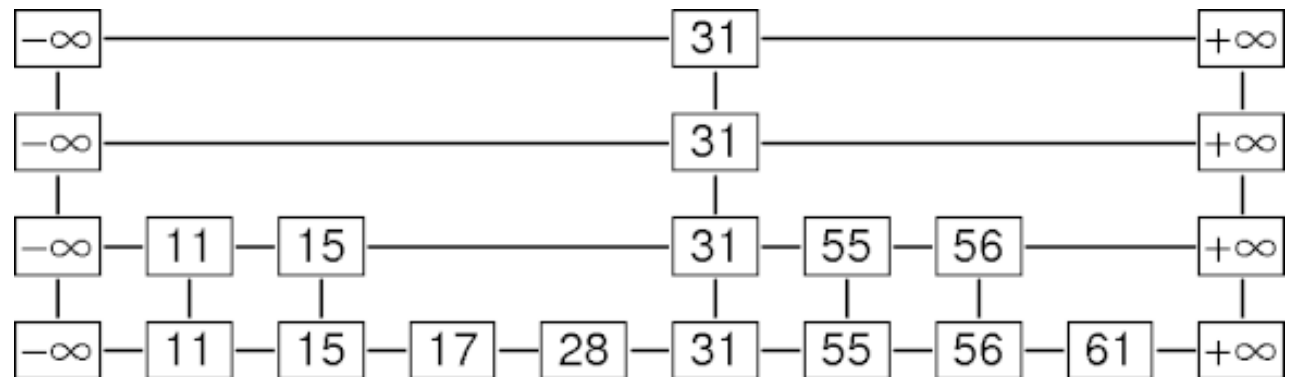
Employing skip list

During execution

- Simple skip list: an element for each vertex of the graph (**much memory**)
- Time consuming traversal (DecreaseKey()) => **too many steps**
- Too **many elements with the same key** (same distance from source)
- Execution time: extremely **large**
- **Low performance**

Solution => an **optimized** skip list

- Contains only elements with **discrete** keys
- **Internal nested list**: stores the ids of the vertices that have the same distance (key) from the source
- **Less memory**
- **Faster traversal**



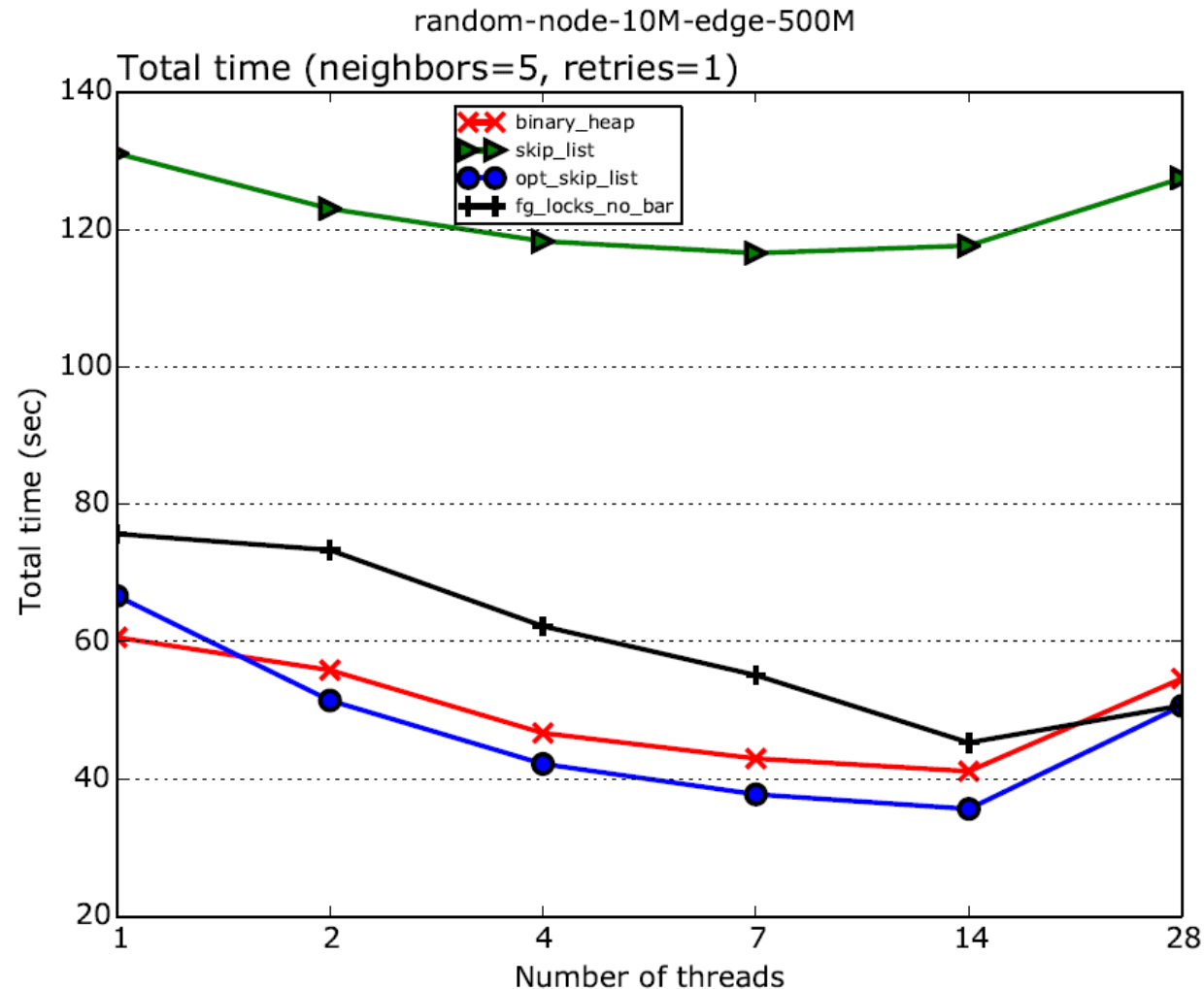
Comparing the serial execution



Total time

- Skip list: **less time consuming** ExtractMin() ($O(1)$)
- DecreaseKey(): too **many steps in the simple skip list**
- ✓ **Many elements with the same key** to be overtaken during traversal

Comparing the parallel execution

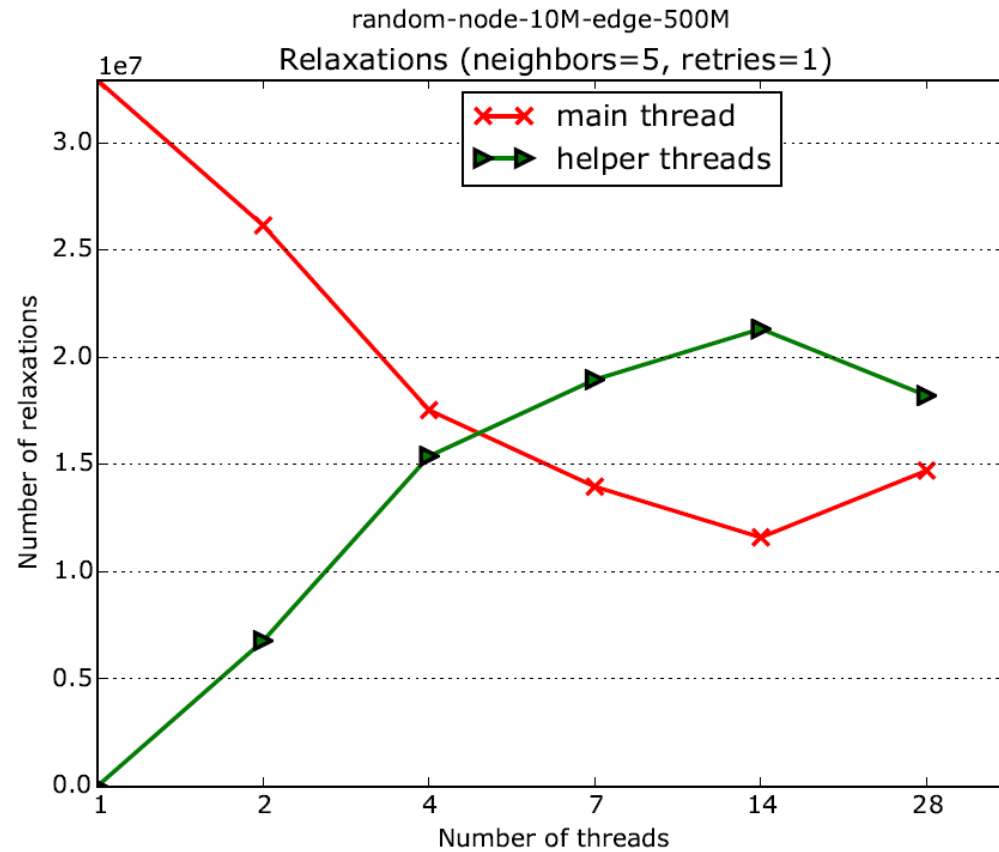


Total time

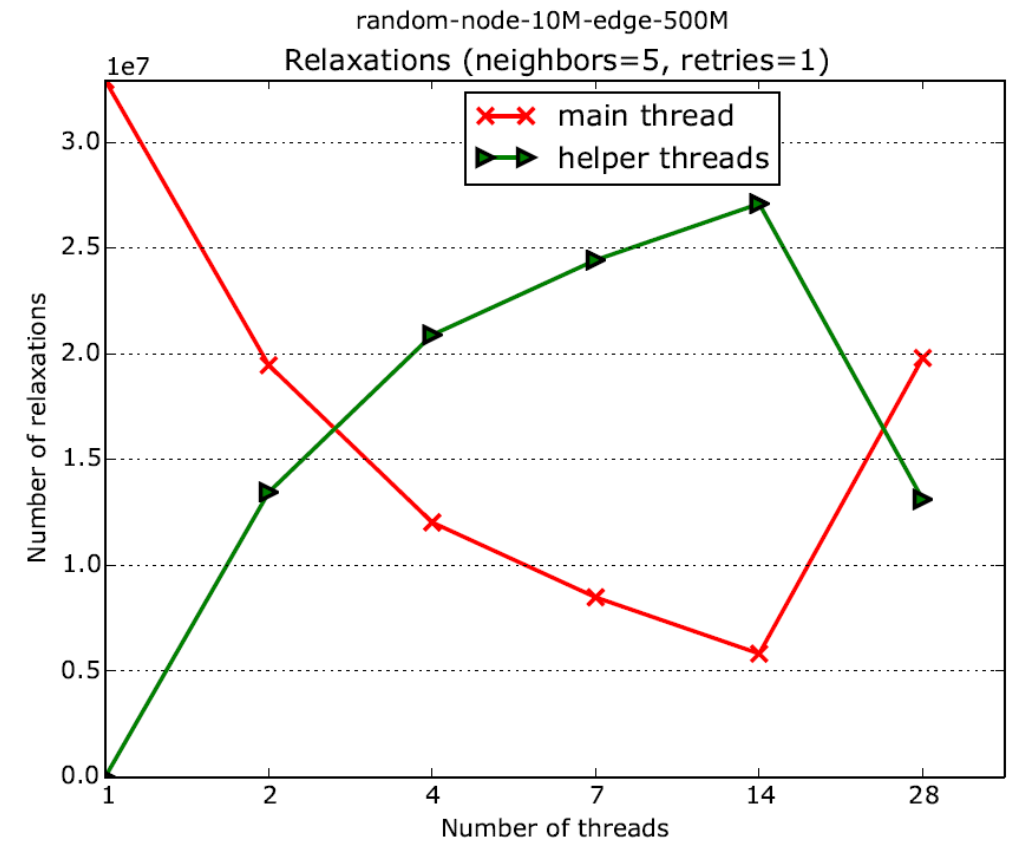
- Simple skip list: DecreaseKey() time consuming
- Optimized skip list: better scalability
- ✓ Helper threads act more locally and perform more relaxations
- ✓ Binary heap and opt skip list execution have comparable transactional aborts
- 28 threads: NUMA effect

* **Fine-grained_no_bar**: Fine-grained locking in Decreasekey(), that uses barriers for synchronization. We have removed the time spent in barriers to use it as a baseline.

Relaxations performed Binary heap

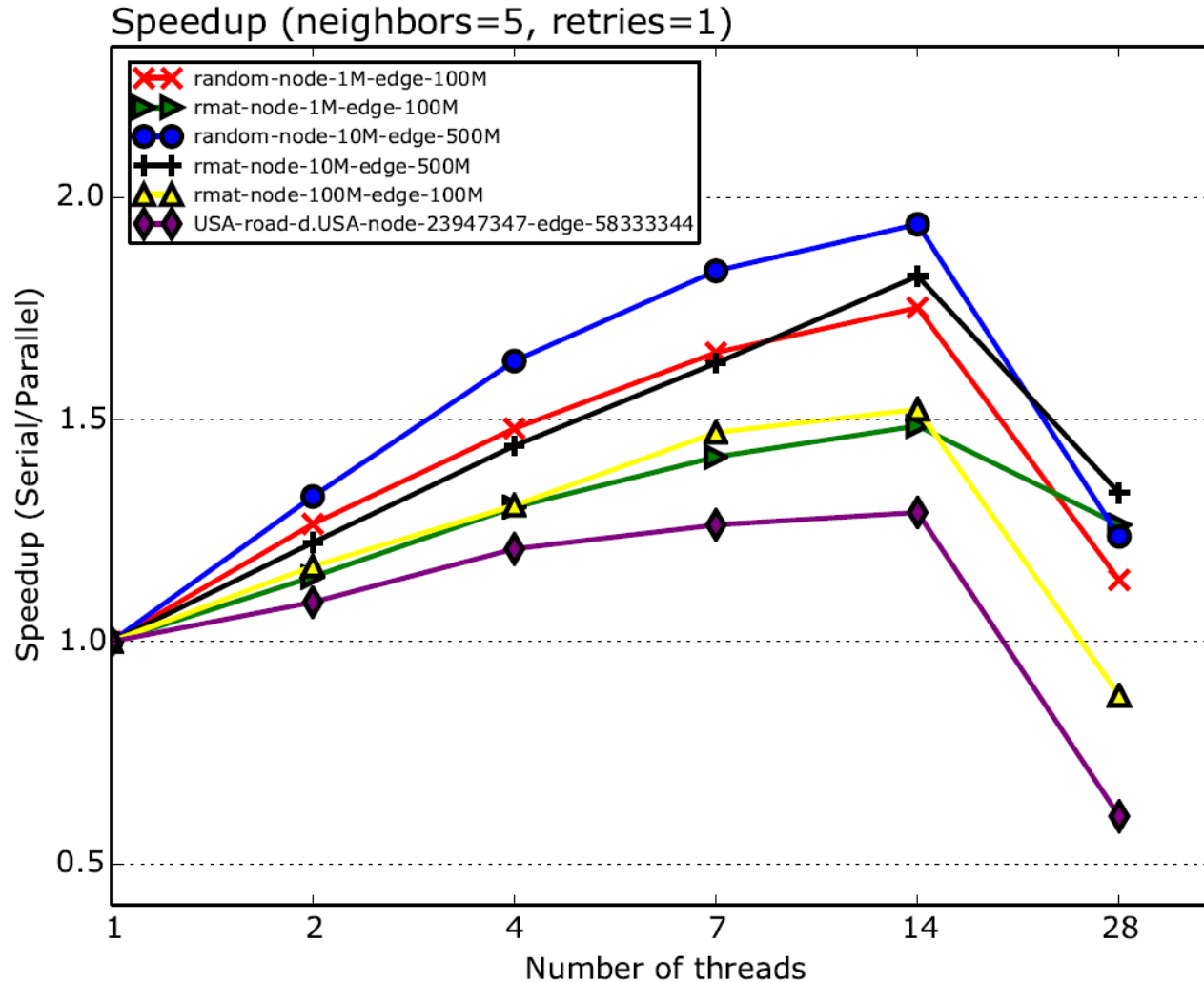


Skip list



✓ Binary heap and opt skip list execution have comparable transactional aborts

Performance results



Speedup

- Speedup related to the **density** of the graph
- **Better scalability**
- ✓ Helper threads offload **more work** from the main thread
- **Real USA network** => there is also a very **good performance** (estimates for **distances have the same values**, during the execution of the algorithm, with some strong probability **in real networks**, too).

- Bin_heap vs Skip_list: **comparable transactional aborts**, but **in the skip list** execution, helper threads perform **more relaxations** (offload more work).

A speedup approximation (14 threads)

Binary heap

Graph	Ideal Speedup	Speedup achieved
random-node-1M-edge-100M	4	1.45
rmat-node-1M-edge-100M	3.58	1.27
random-node-10M-edge-500M	3.74	1.46
rmat-node-10M-edge-500M	3.09	1.35
rmat-node-100M-edge-100M	1.22	1.1
USA-road-node-23M-edge-58M	1.91	1.08

Skip list

Graph	Ideal Speedup	Speedup achieved
random-node-1M-edge-100M	4.57	1.75
rmat-node-1M-edge-100M	5.11	1.49
random-node-10M-edge-500M	5.14	1.94
rmat-node-10M-edge-500M	4.74	1.82
rmat-node-100M-edge-100M	1.79	1.52
USA-road-node-23M-edge-58M	1.48	1.29

$$s = \frac{1 + d}{1 + a * d}$$

- a: is the ratio of the main thread's DecreaseKey() operations to those executed in the serial case
- d: the average out-degree of the vertices.

FUTURE WORK

Future Work

Concurrent Binary Search Trees

- Evaluate other lock-based and lock free implementations
- Use of [transactional memory](#) as synchronization mechanism in BSTs
- Evaluate other structures like [FIFO queues](#), [Hash tables](#), [priority queues](#)

Dijkstra's algorithm

- Examine [levelmax](#) of our [optimized](#) skip list
- Evaluate the algorithm to another HTM implementation with [different resolution policy](#)
- Explore adaptive schemes the number of helper threads will be [dynamically adjusted](#)
- Opt_skip_list: [extract multiple vertices](#) with the same (min) distance in a single step

QUESTIONS ???