(1a) Prove that the $(d-1)$-dimensional simplex $\triangle^{d-1}$ has $\binom{d}{k}$ faces of dimension $(k-1)$, for $0 \leq k \leq d$.

(1b) Using the simple form $n! \approx (\frac{n}{e})^n$ of Stirling's formula, show that $\varphi_d(x) := \log \binom{d}{xd}$ is asymptotically proportional to $-x \log x - (1-x) \log(1-x)$, for $x \in (0,1)$ and $d \to \infty$. Discuss the real function φ_d on $[0,1]$.

(2a) Prove that the d-dimensional cube $\square^d$ has $2^{d-k}\binom{d}{k}$ faces of dimension k, for $0 \leq k \leq d$.

(2b) Using the simple form $n! \approx (\frac{n}{e})^n$ of Stirling's formula, show that $\psi_d(x) := d(1-x) + \log \binom{d}{xd}$ is asymptotically proportional to $1 - x - x \log x - (1-x) \log(1-x)$, where $\log = \log_2$ denotes the binary logarithm. Find an approximation to the maximum of this function on $(0,1)$.