Discrete and Algorithmic Geometry

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Sheet 3

due on Monday, December 2, 2013

Writing

(1) Compute the polytopality range of the complete bipartite graph $K_{m,n}$ for $m, n \geq 3$.

(2) Prove or disprove: Any graph on n vertices (connected or not) arises as a subgraph of the 1-skeleton of a 4-dimensional polytope.

Software

(1) Inside your local copy of the `github` repository of this class, create a `polymake` extension named `dag2013-your-name` and copy the original `polymake` client
`apps/polytope/src/join_polytopes.cc`
to the repository:
`2013/coding/dag2013-your-name/apps/polytope/src/direct_sum.cc` .
Commit only this source file to the repository, and push it to `github`.

(2) Adapt this client to produce the direct sum of two polytopes:

$$P \oplus Q \;=\; \operatorname{conv}\big(P \times \{0\} \cup \{0\} \times Q\big),$$

where $0 \in P$, $0 \in Q$. Make sure your code is more legible and better formatted than the original, and test your client before pushing the final version!

(3) Use your new client to calculate the f-vector of $Q := \Pi_3 \oplus \Pi_3$, where Π_3 is the 3-dimensional permutahedron (with 24 vertices). Write down a single line of `polymake` code that achieves this calculation. How many facets does Q have? Does your answer agree with the formula for the number of facets we discussed in class?

Turning in your work

Put your answers into a .pdf file. To turn it in, use `gpg` and the public key `julian.gpg.pub` in the `github` repository to create an encrypted copy that is only readable by me. Then commit and push this encrypted file to the repository.