**Table 13.1 $z$-Transforms**

| $x(t)$ | $X(s)$ | $X(z)$ |
|---|---|---|
| $\delta(t) = \begin{cases} \frac{1}{\epsilon}, & t < \epsilon, \epsilon \to 0 \\ 0 & \text{otherwise} \end{cases}$ | $1$ | — |
| $\delta(t-a) = \begin{cases} \frac{1}{\epsilon}, & a < t < a + \epsilon, \epsilon \to 0 \\ 0 & \text{otherwise} \end{cases}$ | $e^{-as}$ | — |
| $\delta_o(t) = \begin{cases} 1 & t = 0, \\ 0 & t = kT, k \neq 0 \end{cases}$ | — | $1$ |
| $\delta_o(t - kT) = \begin{cases} 1 & t = kT, \\ 0 & t \neq kT \end{cases}$ | — | $z^{-k}$ |
| $u(t)$, unit step | $1/s$ | $\dfrac{z}{z-1}$ |
| $t$ | $1/s^2$ | $\dfrac{Tz}{(z-1)^2}$ |
| $e^{-at}$ | $\dfrac{1}{s+a}$ | $\dfrac{z}{z - e^{-aT}}$ |
| $1 - e^{-at}$ | $\dfrac{1}{s(s+a)}$ | $\dfrac{(1 - e^{-aT})z}{(z-1)(z - e^{-aT})}$ |
| $\sin(\omega t)$ | $\dfrac{\omega}{s^2 + \omega^2}$ | $\dfrac{z \sin(\omega T)}{z^2 - 2z\cos(\omega T) + 1}$ |
| $\cos(\omega t)$ | $\dfrac{s}{s^2 + \omega^2}$ | $\dfrac{z(z - \cos(\omega T))}{z^2 - 2z\cos(\omega T) + 1}$ |
| $e^{-at}\sin(\omega t)$ | $\dfrac{\omega}{(s+a)^2 + \omega^2}$ | $\dfrac{(ze^{-aT}\sin(\omega T))}{z^2 - 2ze^{-aT}\cos(\omega T) + e^{-2aT}}$ |
| $e^{-at}\cos(\omega t)$ | $\dfrac{s+a}{(s+a)^2 + \omega^2}$ | $\dfrac{z^2 - ze^{-aT}\cos(\omega T)}{z^2 - 2ze^{-aT}\cos(\omega T) + e^{-2aT}}$ |

Expanding into partial fractions, we have

$$G(s) = (1 - e^{-sT})\left(\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1}\right), \tag{13.14}$$

and the $z$-transform is

$$G(z) = Z\{G(s)\} = (1 - z^{-1})Z\left(\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1}\right). \tag{13.15}$$