Homework 5

FINA 6281-G

Due on April 19, 2019 at 1:30pm

You may work in groups of 3-4 and submit a single copy. Email solutions to bwhitcher@gwu.edu.

Problem 1. Read the magazine article from *The New Yorker* entitled "Blowing Up: How Nassim Taleb Turned the Inevitability of Disaster into an Investment Strategy," available on Blackboard. In a brief paragraph discuss one or more of the following topics:

- Do you personally identify more with Taleb or Niederhoffer? Why?
- How does this article relate to our discussion in class regarding risk vs. uncertainty?
- Which topics from behavioral finance are relevant to this story?
- What did you learn and/or what surprised you?

Problem 2. Answer the following questions and include an Excel or R file.

(a) The discrete-time version of the geometric Brownian motion model from Black-Scholes posits that stocks evolve according to the real-world dynamics

$$\begin{aligned} \Delta S_t^{(1)} &= S_t^{(1)}\mu_1 \ \Delta t + S_t^{(1)}\sigma_1\sqrt{\Delta t}Z_t^{(1)} \\ \Delta S_t^{(2)} &= S_t^{(2)}\mu_2 \ \Delta t + S_t^{(2)}\sigma_2\left(\rho\sqrt{\Delta t}Z_t^{(1)} + \sqrt{1-\rho^2}\sqrt{\Delta t}Z_t^{(2)}\right) \end{aligned}$$

The dataset in `stock_prices.csv` contains two years of historical end-of-trading-day prices ($\Delta t = 1/252$) for two non-dividend-paying stocks in chronological order from $t = t_0$ to $t = t_N$. Compute log-returns and then use the data to estimate $\hat{\sigma}_1$, $\hat{\sigma}_2$, and $\hat{\rho}$. (Drifts are notoriously difficult to estimate.)

(b) Use the fitted parameters and $\mu_1 = \mu_2 = 0$ to simulate at least 1,000 joint realizations of $\left(S_{t_N+\Delta t}^{(1)}, S_{t_N+\Delta t}^{(2)}\right)$ where t_N is the date of the most recent observation in the dataset and $\Delta t = 1/12$ (1 month). Use your simulations to answer the following questions:

(i) What is the probability that both stocks fall by at least 5%?

(ii) What is the 1-month 95% Value at Risk of a portfolio containing 1 share of $S^{(1)}$ and 10 shares of $S^{(2)}$?

(c) Is is possible to answer part (b) analytically (without simulation)? If so, how?

Problem 3.

- One can model default risk for asset i by comparing realizations of standard normal random variables $Z^{(i)}$ to a quantile $\Phi^{-1}(PD^{(i)})$ corresponding to some default probability $PD^{(i)}$.
- The $Z^{(i)}$ are standardized indications of credit quality: when $Z^{(i)}$ is low, default is likely; when $Z^{(i)}$ is high, default is unlikely.
- In the event of default, a loan will typically produce some recovery $R^{(i)} = 1 - LGD^{(i)}$ expressed as the percentage of face value recovered by the investor.
- If the default of one loan is statistically more or less likely conditional on the default of another loan, default events are not independent. In this case, we can model multivariate default risk with correlated variables $Z^{(i)}, Z^{(j)}$.

(a) Using the notation given above, write an expression for the value of a loan on its scheduled repayment date, with an amount due of \$100.

(b) Simulate the value of a portfolio containing two \$100 loans to different borrowers with default probabilities $PD^{(1)} = PD^{(2)} = 5\%$, zero recovery in case of default ($R^{(1)} = R^{(2)} = 0\%$), and credit quality variables having a correlation of 0.2.

(i) What is the probability of no default?

(ii) What is the probability of one default?

(iii) What is the probability of two defaults?

(iv) What is the conditional Value-at-Risk (CVaR) of this portfolio at the 90% confidence level (the average loss over the worst 10% of cases)?

(c) Repeat part (b) with $\rho = 0.8$. Explain the reason for the difference in results (i)-(iv).