FIT3139: Final project

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SPECIFICATION TABLE:

Base model	Euclidian TSP
Extension assumptions	Euclidian TSP with introduced human error.
Techniques showcased	Montecarlo simulation: there is a random chance the salesman doesn't choose the most optimal route. Markov chains: Implement weather states that affect cost on certain cities.
Modelling question 1	Does the implementation of human error prevent the model from ever reaching the optimal answer?
Modelling question 2	Does the random chance of a city costing more make the model more realistic? And if so, to what extent?

INTRODUCTION

The Travelling Salesman Problem (TSP) defines a situation where a 'salesman' needs to visit n number of cities via the shortest route possible, starting and ending at the same city. This is normally found using heuristic algorithms, in this case the simulated annealing (SA) algorithm will be used to find the most optimal tour.

However, the travelling salesman problem seems to occur in a perfect world that does not account for any problems or external factors that may occur, thus, this report will be exploring how adding said factors can affect how the most optimal tour is found, if it ever is. This will be achieved via the introduction of stochasticity in the form of weather states and human error. While the simulated annealing algorithm is already an example of the use of Monte Carlo, the addition of further randomness will allow us to analyse how a problem like this set in the real world could turn out. Furthermore, adding an unpredictable factor such as weather that affects road length into the mix will extend the model in a more realistic way and allow us to determine whether heuristic algorithms such as simulated annealing are able to still reach the most optimal tour.

MODEL DESCRIPTION

The travelling salesman problem can be of any size but for most cases in this report, unless specified otherwise, we will be using 4 different cities numbered 0, 1, 2 and 3. With cost from once city to the other as follows:

	0	**1**	**2**	**3**
0	0	10	15	20
1	10	0	35	25
2	15	35	0	30
3	20	25	30	0

The ideal tour is then found to be [0, 1, 3,2] (with cost of 10 + 25 + 30 + 15 = 80), which can also be written in any other way in which the loop remains in the same order.

While not many there are some assumptions:

- Each set of the n number of cities is called a tour.
- While tours are written as [0, 1, 2, 3] it is assumed that the salesman starts and ends at the first city (so four distances are added up).
- The salesman does not repeat cities in a singular tour.

As discussed previously this is the 'simple' version of the travelling salesman problem, which accounts for a static environment that will always remain the same (road costs will always remain the same) and does not consider human error. Our extended model, however, does take into account all these factors. So, there are a few more assumptions:

- The weather has three different states it can exists as:
    - F – Fine with probability $P(F)$. In which road cost stays the same.
    - R – Rainy with probability $P(R)$. In which certain roads (from any city to 1 and from any city to 2) not prepared for the rain get their cost multiplied by 1.5. Making the ideal tour in a rainy environment be [1, 3, 2, 0] -> 100.
    - S – Snowy with probability $P(S)$. In which certain roads (from any city to 0 and from any city to 3) not prepared for the snow get their cost multiplied by 2. Making the ideal tour in a snowy environment be [3, 2, 0, 1] -> 120.
- Given the simulated annealing algorithm, whenever a new solution is processed there is a 50% chance of the solution being rejected due to human error (this will be applied with and without the weather states).

In order to simulate the weather states, we'll be using Markov Chains, and in turn transition matrices, which there are a few of. First, we'll start with a world in which once it's a Fine day the state will not change (not very realistic). Rain and Snow both have the same chance of staying the same than to transition into a different state. If transitioning, they have a lower chance of transitioning into a Fine day.

So, the transition matrix is as follows:

$$\begin{array}{c} \\ F \\ R \\ S \end{array}\begin{array}{c} \begin{array}{ccc} F & R & S \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0.15 & 0.5 & 0.35 \\ 0.15 & 0.35 & 0.5 \end{bmatrix} \end{array}$$

And in canonical form (Fine is an absorption state and the rest are transient.):

$$\begin{array}{c} \\ S \\ R \\ F \end{array}\begin{array}{c} \begin{array}{ccc} S & R & F \end{array} \\ \begin{bmatrix} 0.5 & 0.35 & 0.15 \\ 0.35 & 0.5 & 0.15 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

In the above matrix while there is a low chance of ending on a Fine day, once it is the model essentially becomes the same as the base model. So, in order to further test our extension, we will also make use of an ergodic transition matrix for a more realistic model, such as:

$$\begin{array}{c} \\ F \\ R \\ S \end{array}\begin{array}{c} \begin{array}{ccc} F & R & S \end{array} \\ \begin{bmatrix} 0.90 & 0.05 & 0.05 \\ 0.15 & 0.5 & 0.35 \\ 0.15 & 0.35 & 0.5 \end{bmatrix} \end{array}$$

Where the chance of it being a fine day is still high, but now it can transition from Fine to any day, thus making it more real world like.

As for the Monte Carlo implementation simulated annealing already has one, in which you start with a starting tour, perturb this solution, and if this new solution's cost is less than the previous accept otherwise you accept if the exponent of the cost of the new solution minus the cost of the previous one over the current temperature is less than some random number. So:

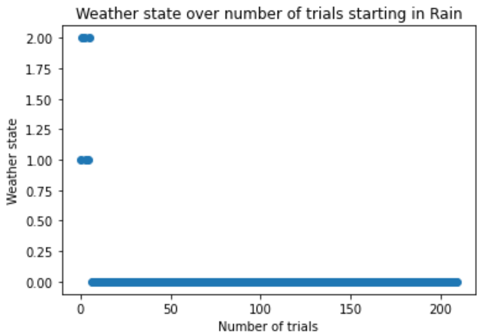
$$Accept\ with\ probability \begin{cases} 1\ if\ f(\tilde{x}) > f(x_i) \\ \exp\left(\dfrac{f(\tilde{x}) - f(x_i)}{T}\right) otherwise \end{cases}$$

However since we are adding a 50% chance of human error we will be first adding this twice (as humans can sometimes make a lot of mistakes) first if the cost of the new solution is better than the previous one, it also has to pass that 50-50 test, and if its not instead of the second equation a different random number is drawn and if its less than 0.5 it will be accepted. In this case, we will be using the following cities to simulate a larger model:

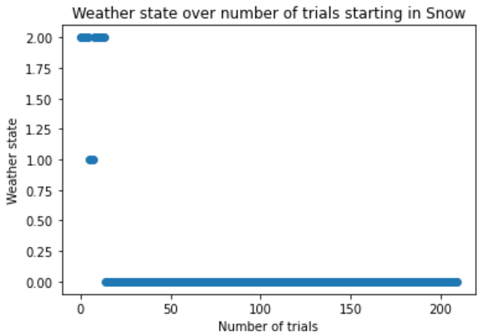
	0	**1**	**2**	**3**	**4**	**5**
0	0	2053	1155	3017	1385	1456
1	2053	0	1080	3415	939	2876
2	1155	1080	0	3940	285	856
3	3017	3415	3940	0	3975	1984
4	1385	939	285	3975	0	3278
5	1456	2876	856	1984	3278	0

RESULTS

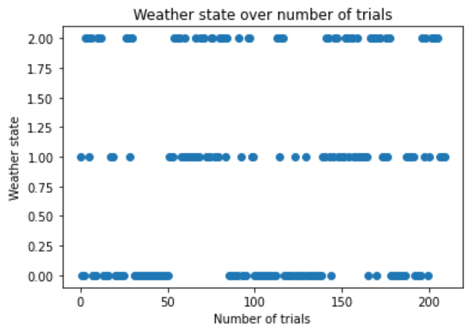
Before getting into the true analysing of results one must first understand why the absorption matrix is not as effective as the ergodic one in simulating a real-world model. As we can see from the 3 graphs below (the first 2 being the absorption matrix and the third being ergodic), it's clear that due to the 6.66666667 absorption time of both Rain and Snow and probability of absorption of 1 the model will always end up back in the Fine weather state, which is essentially just the base model. This is shown as in any trial ran with the absorption matrix the simulated annealing algorithm will always find the most optimal tour with 80 cost. Thus, this report will be focusing more on the use of the ergodic transition matrix, as shown in the scatter plot it shows what a more realistic 200 days would look like, with a lot of weather fluctuation.



Absorption matrix (starting Rainy)

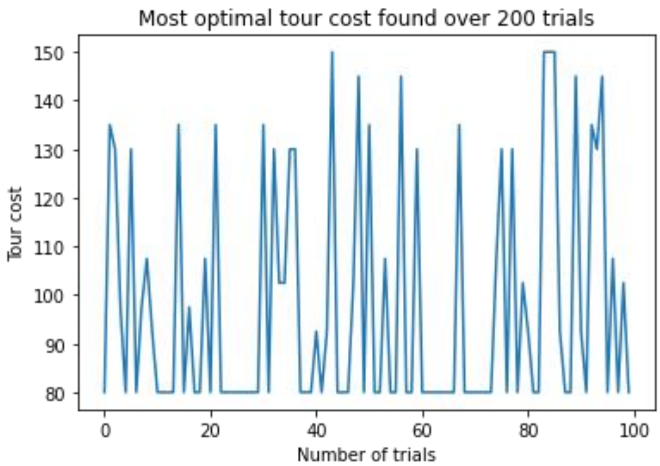


Absorption matrix (starting Snowy)



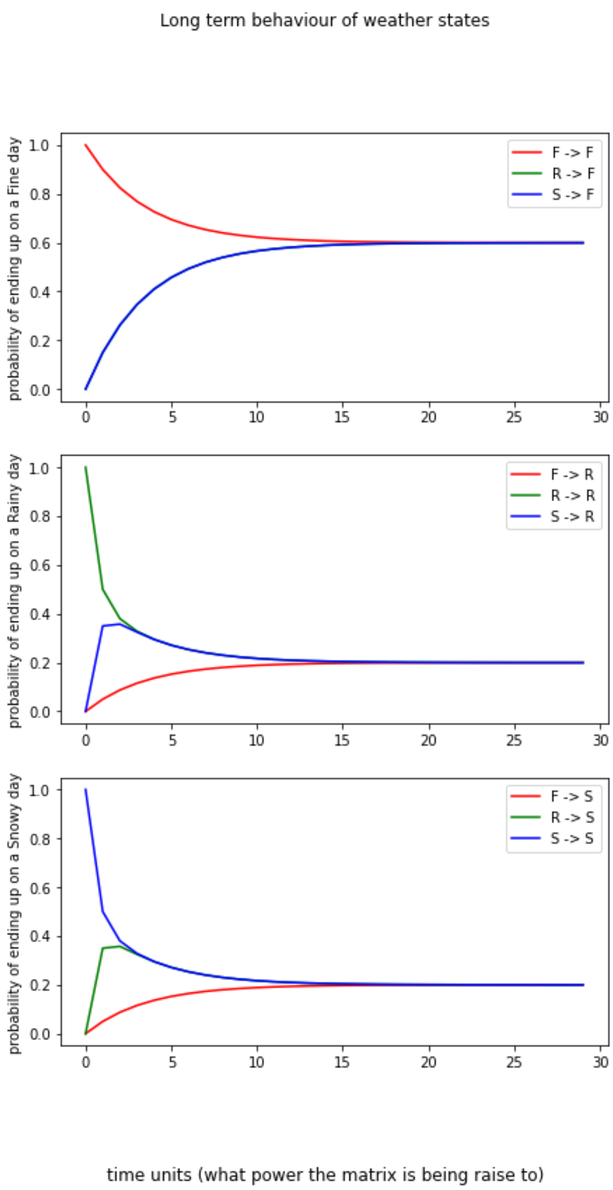
Ergodic Matrix (starting Rainy)

Because the ergodic chain does not have an absorption state it will always fluctuate between weather states, while still most often visiting the Fine state due to its high probability, most of the times it will transition out of this. Because of this, when running the simulated annealing algorithm, the weather is not always able to remain in the fine state when reaching the end of the algorithm, and since both Rain and Snow affect the road costs, this leads to the optimal tour sometimes not being found.



As seen in the graph above, when running a certain amount of trials, there is still clearly a majority of optimal tours that fall on the 80 cost (the quote-on-quote correct answer), with around 60% of tours falling on this line. This is most likely due to the high probability that Fine has to stay in that same state, so while it has a low probability of actually transitioning to fine once it does it will stay there for a while, as shown on the graph with long stretches when it hits 80, while the rest are mostly peaks. On the other hand, because Rain and Snow both have a 50% chance of transitioning to another state, even over 100 trials there are no tours found that are at the most optimal for Rain and Snow (100 and 120 respectively).

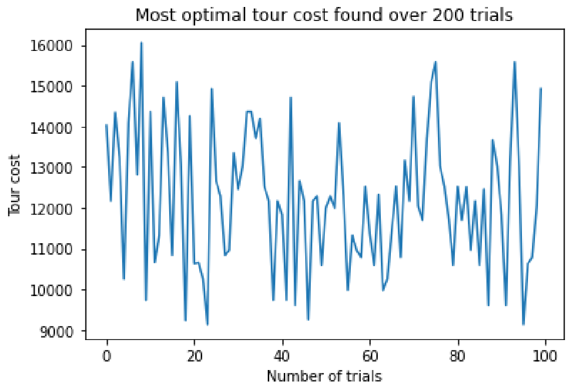
However, because we only did 100 trials we can never be sure that the data shown above is what will always happen over the long term. So, if we want to know what happens to the weather at any given time, we can look at the stationary distribution. We can do this in two different ways, by looking at the long-term behaviour of the transition matrix or by finding the eigenvector for eigenvalue 1 of the transition matrix.



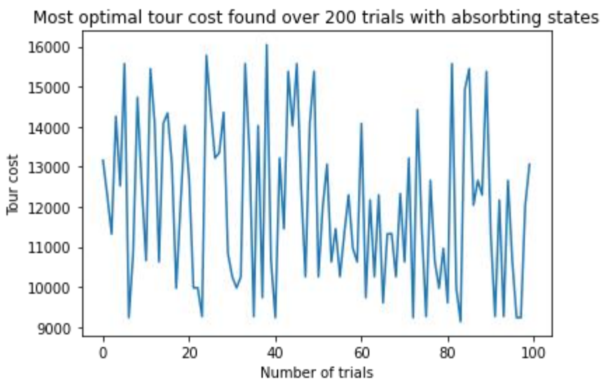
By raising the ergodic matrix to a very high power we are able to see how it will behave in the long term. As seen in the three graphs to the side, there is around a 60% chance of having a Fine day and a 20% chance each for a Rainy or Snowy day. This is reinforced by the eigenvector [0.6, 0.2, 0.2] that tells us the same.

If we look at the weather in any real city this might seem much more like what we would see than with the absorption matrix. So going back to one the main questions of the report, with the use of Markov Chains we can model a much more realistic travelling salesman problem, that can be applied to any similar real-world situations with more realistic results. However, in real life the weather is much more erratic and diverse, so a larger Markov chain implementation would be needed for more realism.

And, while the model does yield more realistic result, it is sometimes unable to return the most optimal tour, which at the end of the day is the objective of the model.

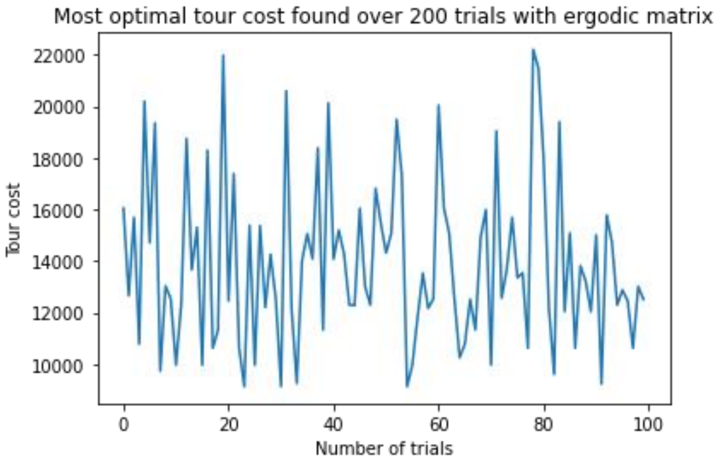


Base model



With absorbing transition matrix

After running the simulated annealing code for 100 trials, we can see that adding human error into the algorithm affects the model in a major way. While from time to time we can obtain the most optimal tour, as seen in both graphs, this is not very often. We can also see that in this case, the absorbing transition matrix does little to no effect on how the model behaves with this randomness included, as both graphs seem to behave very similarly.



With ergodic transition matrix

On the other hand, if we look at the model with the ergodic transition matrix, we can see that is much more erratic in which there are a lot more changes in tour cost than in the previous two graphs. This is due to the higher variety in weather states, as the multiplier will allow for a more diverse set of outputs than with the base model and the absorbing matrix model where the maximum cost will be capped at much less (16000 compared to 22000). This is becuase the base will always have this cap, and the absorbing model will be unable to stay too long in the Rainy and Snowy states to raise this maximum any further.

Overall we can conclude that weather states in the form of a Markov chain do have an effect on what is considered the most optimal tour, however with an absorbing transition matrix, most of the time the model is able to reach the true optimal tour, but when an ergodic matrix is used we are only able to do this around 60% of the time due to too not enough stability in the weather. This reinforces what previously thought that this model now reflects a more realistic set of events. We can also come to realise that human error, as hypothesised, will prevent the model from reaching the true optimal tour, with and without weather states. Due to the unpredictable nature of humans alone or combined with the weather the model is unable to reach the optimal route most of the time.

LIST OF ALGORITHM AND CONCEPTS

1. Euclidian Travelling Salesman Problem: The base model chosen.
2. Monte Carlo: To introduced human error (and in the SA algorithm)
3. Simulated Annealing (Heuristics): To solve the travelling salesman problem.
4. Markov Chains: To introduce weather into the base model via transition matrices (both absorbing and ergodic).