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Math 522

HW 4

3.25. Let $\mathscr{K}$ be the graph returned by the PDAG algorithm. We work by way of contradiction. Assume there exists a minimal loop $X_1 - X_2 - \cdots - X_k - X_1$ of length greater than or equal to 4. Recall that the graph $\mathscr{K}$ is acyclic. Thus, there must exist at least one directed edge in the loop (or else it would be a cycle). We then work by cases:

1. Assume there is an immorality in the path. Thus there exists $X_1 - \cdots - X_{i-1} \to X_i \leftarrow X_{i+1} - \cdots - X_k - X_1$.