

HW 10
2.6.2,3,14

2.6.2. For all the operations, the range and domain are both over the reals. It suffices to show that the second part of the definition is true for all operators.

1. Consider the operator $T(x_1, x_2) = (x_1)$. Then for some $\alpha, \beta \in \mathbb{R}$,

$$T(\alpha(x_1, x_2) + \beta(y_1, y_2)) = T((\alpha x_1, \alpha x_2) + (\beta y_1, \beta y_2)) \quad (1)$$

$$= T(\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2) \quad (2)$$

$$= (\alpha x_1 + \beta y_1, 0) \quad (3)$$

$$= (\alpha x_1, 0) + (\beta y_1, 0) \quad (4)$$

$$= \alpha(x_1, 0) + \beta(y_1, 0) \quad (5)$$

$$= \alpha T(x_1, x_2) + \beta T(y_1, y_2). \quad (6)$$