Cory Glover
2/4/20
Math 525

HW 11

7.15. Recall that $\sigma = (\mathbf{D} - \alpha\mathbf{A})^{-1}$. This means that

$$\sigma = (\mathbf{D} - \alpha\mathbf{A})^{-1} \tag{1}$$
$$\sigma^{-1} = \mathbf{D} - \alpha\mathbf{A} \tag{2}$$
$$\sigma^{-1}\mathbf{D}^{-1} = \mathbf{I} - \alpha\mathbf{A}\mathbf{D}^{-1} \tag{3}$$
$$(\mathbf{D}\sigma)^{-1} = \mathbf{I} - \alpha\mathbf{A}\mathbf{D}^{-1}. \tag{4}$$

Recall that the pagerank centrality is given by $\mathbf{x} = (\mathbf{I} - \alpha\mathbf{A}\mathbf{D}^{-1})^{-1}\mathbf{1}$. This means that

$$\mathbf{x} = (\mathbf{I} - \alpha\mathbf{A}\mathbf{D}^{-1})^{-1}\mathbf{1} \tag{5}$$
$$= \mathbf{D}\sigma\mathbf{1} \tag{6}$$
$$\mathbf{D}^{-1}\mathbf{x} = \sigma\mathbf{1}. \tag{7}$$

Then we see that $\sigma_i = \sum_j \sigma_{ij}$ is just the i^{th} entry of $\sigma\mathbf{1}$ since we are summing over the columns. Note that since D is diagonal, the D^{-1} is diagonal with diagonal entries $\frac{1}{k_i}$. So the i^{th} entry of $\mathbf{D}^{-1}\mathbf{x}$ is $\frac{1}{k_i}x_i$. So $\sigma_i = \frac{1}{k_i}x_i$. Hence σ_i is just the pagerank centrality of i divided by the degree of node i.

7.16.

1.

Group	e_r	a_r
Black	.129	.177
Hispanic	.0785	.147
White	.153	.247
Other	.008	.0605

So $Q = .251$.

2.

Group	e_r	a_r
Democrat	.125	.22
Independent	.03	.175
Republican	.03	.245

So $Q = .046$.

3. Based on the modularity values, it is clear that people associate themselves with people similar to themselves. This trend is strong in marriages than politics.