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Math 525
1/23/19

HW 6

1. A

2.

1. Recall from previous homework that $A\mathbf{1} = \mathbf{k}$ where $\mathbf{k}$ is the degree vector. Assume that every node has degree k. Then $\mathbf{k} = k\mathbf{1}$. So $A\mathbf{1} = k\mathbf{1}$. So $\mathbf{1}$ is an eigenvector of A.

2. We want to find $\mathbf{x}$ such that $\mathbf{x} = \alpha A\mathbf{x} + \beta\mathbf{1}$. Let $\mathbf{x} = \frac{\beta}{1-\alpha k}\mathbf{1}$. Then

$$\begin{aligned}
\alpha A(\frac{\beta}{1-\alpha k})\mathbf{1} + \beta\mathbf{1} &= \frac{\alpha\beta}{1-\alpha k}A\mathbf{1} + \beta\mathbf{1} && (1)\\
&= \frac{k\alpha\beta}{1-\alpha k} + \beta\mathbf{1} && (2)\\
&= \frac{k\alpha\beta + \beta - \beta\alpha k}{1-\alpha k}\mathbf{1} && (3)\\
&= \frac{\beta}{1-\alpha k}\mathbf{1} && (4)\\
&= \mathbf{x}. && (5)
\end{aligned}$$

   So the Katz centrality is $\mathbf{x} = \frac{\beta}{1-\alpha k}\mathbf{1}$.

3. Betweenness centrality

3. A network of basketball teams indicating wins as in arrows and loses as out arrows. Importance would be beating teams that are very good. For example, if you are a small school that wins a lot, but loses everytime to a very good school that plays more competitively, you are not as important. So eigenvector centrality is a better measure.

4.

1. $c(1) = 1/13, c(2) = 3/13, c(3) = 3/13, c(4) = 2/13, c(5) = 2/13, c(6) = 2/13$

2. $c(1) = .455, c(2) = .233, c(3) = .119, c(4) = .537, c(5) = .4343, c(6) = .4971$

3. $\alpha = .1 : c(1) = .149, c(2) = .181, c(3) = .179, c(4) = .163, c(5) = .164, c(6) = .164.$

   $\alpha = .3 : c(1) = .128, c(2) = .210, c(3) = .196, c(4) = .151, c(5) = .158, c(6) = .158.$

   $\alpha = .5 : c(1) = .124, c(2) = .241, c(3) = .207, c(4) = .140, c(5) = .149, c(6) = .149.$

   As α increases, the importance of the most connected nodes increases more and the less connected nodes goes down more.