Cory Glover
1/25/20
Math 525

1. C
2. B
3.

1. 7.3

(a) Consider Katz centrality $\mathbf{x} = \alpha A\mathbf{x} + \mathbf{1}$. Solving we get $(I - \alpha A)\mathbf{x} = \mathbf{1}$. If we can take an inverse (which we can with probability 1), we get that $\mathbf{x} = (I - \alpha A)^{-1}\mathbf{1}$. We then see we have a geometric series. So

$$\mathbf{x} = \Big(\sum_{i=0}^{\infty} (\alpha A)^r\Big)\mathbf{1} \tag{1}$$

$$= \mathbf{1} + \alpha A\mathbf{1} + \alpha^2 A^2 \mathbf{1} + \cdots . \tag{2}$$

(b) Recall that $A\mathbf{1} = \mathbf{k}$. So we have that $\mathbf{x} = \mathbf{1} + \alpha\mathbf{k} + \alpha^2 A\mathbf{k} + \cdots$. Thus, we get that $\mathbf{x} = \mathbf{1} + (\sum_{i=0}^{\infty} \alpha^{i+1} A^i)\mathbf{k} = \mathbf{1} + \alpha(\sum_{i=0}^{\infty} \alpha^i A^i)\mathbf{k} = \mathbf{1} + \frac{\alpha}{I - \alpha A}\mathbf{k}$. Solving for $\mathbf{k}$ we get

$$\frac{I - \alpha A}{\alpha}(\mathbf{x} - \mathbf{1}) = \mathbf{k} \tag{3}$$

2. 7.5: