Cory Glover
Math 540
1/9/20

HW 1.1
1.2.3,4,5,8

1.2.3. Let $x = (\xi_i) \in l^2$ and $y = (\eta_i) \in l^2$. Then the Cauchy-Schwarz inequality yields

$$\sum_{j=1}^{\infty} |\xi_j \eta_j| \leq \sqrt{\sum_{k=1}^{\infty} |\xi_k|^2} \sqrt{\sum_{m=1}^{\infty} |\eta_m|^2}.$$

Let $x = (\xi_1, ..., \xi_n)$ and $y = (1, 1, ..., 1)$ where y has length n. Then the Cauchy Schwarz inequality gives

$$\sum_{i=1}^{n} |\xi_i(1)| \leq \sqrt{\sum_{k=1}^{n} |\xi_k|^2} \sqrt{\sum_{k=1}^{n} 1^2} \tag{1}$$

$$\left(\sum_{i=1}^{n} |\xi_i|\right)^2 \leq n \sum_{k=1}^{n} |\xi_k|^2, \tag{2}$$

by squaring both sides and note that $\sum_{k=1}^{n} 1^2 = n$.

1.2.4. Consider the sequence $S_n = (\frac{1}{\log(n)})$. Since $\log(n)$ is increasing as $n \to \infty$, then S_n converges to 0. However, note that $\sum_{n=1}^{\infty} \frac{1}{n} < \sum_{n=1}^{\infty} \frac{1}{\log(n)^p}$ for all $p > 0$. Since $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, so does S_n for all $p \geq 1$. So $S_n \notin l^p$ for all $p \geq 1$.

1.2.5. Consider the sequence $S_n = (\frac{1}{n})$. It is commonly know that for $p = 1$, then $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (i.e. the harmonic series). However we see that the series $\sum_{n=1}^{\infty} 2^n (\frac{1}{2^n})^p = \sum_{n=1}^{\infty} 2^n (1-p)$. This series converges if and only if $p > 1$. If this series converges, then by the Cauchy test, $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges. Thus for all $p > 1$, $S_n \in l^p$. (This was found on accident on wikipedia while looking for convergence tests to check results. All work was done without referring to the result though.)

1.2.8. Let $A = \{0, 1\}$ and $B = \{-1, 0\}$. Then $D(A, B) = 0$ but $A \neq B$ since $-1 \notin A$ and $-1 \in B$.