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Math 522
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**2.9.** Let $G$ be an undirected graph. In this case, begin the breadth first search and view all the neighbors of a given node. We call this node $g^0$ and the set of its neighbors $g_n^0$. Then we check the each of the neighbors. As we travel to them, we label each one $g^1$ and the set of its neighbors $g_n^1$. If $g_n^0 \cap g_n^1 \neq \emptyset$, then there is a cycle. Once we have passed through all neighbors, we check the neighbors of the first node in $g_n^0$ and label this node $g_n^0$ and repeat the process. Further, if a node in is the neighborhood of the current node $g^i$ and is not the previous node $g^{i-1}$ (or $g^{i+1}$), then there is a cycle.

**3.1.** Let $X_1, X_2, X_3$ be random variables. We define the following properties:

$$P(X_1 = x_0) = .5 \tag{1}$$
$$P(X_1 = x_1) = .5 \tag{2}$$
$$P(X_2 = x_0) = .7 \tag{3}$$
$$P(X_2 = x_1) = .3 \tag{4}$$
$$P(X_3 = x_0) = .2 \tag{5}$$
$$P(X_3 = x_1) = .8. \tag{6}$$

As we can see, all the variables are independent. Then we define the probabilities

$$P(X_1 = x_0, X_2 = x_0 \mid X_3 = x_0) = .13 \tag{7}$$
$$P(X_1 = x_0, X_2 = x_0 \mid X_3 = x_1) = .02 \tag{8}$$
$$P(X_1 = x_1, X_2 = x_1 \mid X_3 = x_0) = .3 \tag{9}$$
$$P(X_1 = x_1, X_2 = x_1 \mid X_3 = x_1) = .05. \tag{10}$$

Thus we see that $X_1, X_2$ is dependent on $X_3$.

**3.2.**

1. Let $X_1, ..., X_n$ be random variables and $C$ be a class variable. Assume that $(X_i \perp \mathbf{X}_{-i} \mid C)$ for all $i$ where $\mathbf{X}_{-i} = \{X_1, ..., X_n\} - \{X_i\}$. Then

$$P(C, X_1, ..., X_n) = P(C)P(X_1 \mid C)P(X_2, ..., X_n \mid X_1, C) \tag{11}$$
$$= P(C)P(X_1 \mid C)P(X_2 \mid X_1, C)P(X_3, ..., X_n \mid X_1, X_2, C) \tag{12}$$
$$= P(C)\prod_{i=1}^{n} P(X_i \mid C, X_{i-1}, ..., X_1) \tag{13}$$
$$= P(C)\prod_{i=1}^{n} P(X_i \mid C), \tag{14}$$

where the last step used the assumption of independence.

2. Using the property proven in part (i), we see that

$$\frac{P(C = c_1 \mid X_1, ..., X_n)}{P(C = c_2 \mid X_1, ..., X_n)} = \frac{\frac{P(X_1, ..., X_n \mid C = c_1)P(C = c_1)}{P(X_1, ..., X_n)}}{\frac{P(X_1, ..., X_n \mid C = c_2)P(C = c_2)}{P(X_1, ..., X_n)}} \tag{15}$$
$$= \frac{P(X_1, ..., X_n \mid C = c_1)P(C = c_1)}{P(X_1, ..., X_n \mid C = c_2)P(C = c_2)} \tag{16}$$
$$= \frac{P(C = c_1)}{P(C = c_2)}\prod_{i=1}^{n} \frac{P(X_i \mid C = c_1)}{P(X_i \mid C = c_2)}. \tag{17}$$

1

**3.3.**

1. While having an earthquake does not eliminate the possibility of a burglary, having an earthquake causes the alarm to go off. So if an alarm goes off and there is an earthquake, it decreases the probability that a burglary set off the alarm because the earthquake did happen and was able to set the alarm off.

2. Assume that $P(a^1 \mid b^1, e^1) = P(a^1 \mid b^0, e^1) = 1$. Then

$$P(b^1 \mid a^1, e^1) = \frac{P(b^1)P(a^1, e^1 \mid b^1)}{P(b^0)P(a^1, e^1 \mid b^0) + P(b^1)P(a^1, e^1 \mid b^1)} \tag{18}$$

$$= \frac{P(b^1)P(a^1 \mid e^1, b^1)}{P(b^0)P(a^1 \mid e^1, b^0) + P(b^1)P(a^1 \mid e^1, b^1)} \tag{19}$$

$$= \frac{P(b^1)}{P(b^0) + P(b^1)} = P(b^1). \tag{20}$$

**3..**
**3.6.**

1. $P(t^1 \mid d^1) = P(t^1)$ since $D$ is not a parent of $T$. Relevant trails are $D->C->T$ where $D$ has negative influence on $C$ and $C$ has positive influence on $T$.

2. $P(d^1 \mid t^0) = P(d^1)$ since $T$ is not a parent of $D$. Relevant trails are $D->C->T$ where $D$ has negative influence on $C$ and $C$ has positive influence on $T$.

3. $P(h^1 \mid e^1, f^1) = P(h^1 \mid e^1)$ since $H$ is a parent of $E$ and not related to $F$. Relevant trails are $H-E-F$ where $H$ has a positive influence on $H$ and $F$ has a negative influence on $E$.

4. $P(c^1 \mid f^0) = P(c^1)$ since $F$ is not a parent of $C$. Relevant trails are $F-E-W-D-C$ and $F-E-H-D-C$ where $D$ has a negative influence on $C$, $D$ has a positive influence on $W$, $H$ has a positive influence on $D$, $H$ has a positive influence on $E$, $E$ has a positive influence on $W$, and $F$ has a negative influence on $E$.

5. $P(c^1 \mid h^0) = P(c^1)$ since $H$ is not a parent of $C$. Relevant trails are $H-D-C$ where $H$ has a positive influence on $D$ and $D$ has a negative influence on $C$.

6. $P(c^1 \mid h^0, f^0) = P(c^1 \mid h^0)$ since $H$ and $F$ are not parents of $C$. Relevant trails are $F-E-W-D-C$ and $F-E-H-D-C$ and $H-D-C$ where $D$ has a negative influence on $C$, $D$ has a positive influence on $W$, $H$ has a positive influence on $D$, $H$ has a positive influence on $E$, $E$ has a positive influence on $W$, and $F$ has a negative influence on $E$.

7. $P(d^1 \mid h^1, e^0) = P(d^1 \mid h^1)$ since $E$ is not a parent of $D$. Relevant trails are $H-D$ and $E-W-D$ and $E-H-D$ where $H$ has a positive influence on $D$, $E$ has a positive influence on $W$, $D$ has a positive influence on $W$, and $H$ has a positive influence on $E$.

8. $P(d^1 \mid e^1, f^0, w^1) = P(d^1 \mid e^1, f^0)$ since $E, F$ and $W$ are not parents of $D$. Relevant trails are $F-E-W-D$ and $F-E-H-D$ where $F$ has a negative influence on $E$, $E$ has a positive influence on $D$, $D$ has a positive influence on $W$, $H$ has a positive influence on $E$ and $H$ has a positive influence on $D$.

9. $P(t^1 \mid w^1, f^0) = P(t^1 \mid w^1)$ since $F$ is not a parent of $W$. Relevant trails are $F-E-W-D-C-T$ and $F-E-H-D-C-T$ where $F$ has a positive influence on $E$, $E$ has a positive influence on $W$, $D$ has a positive influence on $W$, $D$ has a negative influence on $C$, $C$ has a positive influence on $T$, $H$ has a positive influence on $D$ and $H$ has a positive influence on $E$.