Cory Glover
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Math 525

HW 12

8.1.

1. $O(n^2)$
2. $O(n)$

8.3.

1. $O(n^2)$
2. $O(n)$ since there are m multiplications and m additions and $m \propto n$.
3. Note that $Bv = Av - \frac{1}{2m}Kv$ where $K_{ij} = k_i k_j$. We also note that $K_i v = k_i k_1 v_1 + k_i k_2 v_2 + \cdots + k_i k_n v_n = k_i(\sum_{j=1}^n k_j v_j)$ where K_i is the i^{th} row of K. We then devise the following algorithm:
   (a) Compute Av.
   (b) Calculate each k_i.
   (c) Calculate $\sum_{i=1}^n k_i v_i$.
   (d) Calculate each $k_i(\sum_{j=1}^n k_j v_j)$.
   (e) Add $(Av)_i$ to $k_i(\sum_{j=1}^n k_j v_j)$ for each i.

   We analyze each step. From part (2) we know that step one has complexity $O(n)$. The second step has complexity $O(m/n)$ for each node, so in total has complexity $O(n)$. The third step has n multiplications and n additions and is thus $O(n)$. The fourth step is n multiplications so it has $O(n)$. The last step has n additions so it is $O(n)$. So overall, the algorithm has complexity $O(n)$.