

HW 11

7.15. Recall that $\sigma = (\mathbf{D} - \alpha\mathbf{A})^{-1}$. This means that

$$\sigma = (\mathbf{D} - \alpha\mathbf{A})^{-1} \quad (1)$$

$$\sigma^{-1} = \mathbf{D} - \alpha\mathbf{A} \quad (2)$$

$$\sigma^{-1}\mathbf{D}^{-1} = \mathbf{I} - \alpha\mathbf{A}\mathbf{D}^{-1} \quad (3)$$

$$(\mathbf{D}\sigma)^{-1} = \mathbf{I} - \alpha\mathbf{A}\mathbf{D}^{-1}. \quad (4)$$

Recall that the pagerank centrality is given by $\mathbf{x} = (\mathbf{I} - \alpha\mathbf{A}\mathbf{D}^{-1})^{-1}\mathbf{1}$. This means that

$$\mathbf{x} = (\mathbf{I} - \alpha\mathbf{A}\mathbf{D}^{-1})^{-1}\mathbf{1} \quad (5)$$

$$= \mathbf{D}\sigma\mathbf{1} \quad (6)$$

$$\mathbf{D}^{-1}\mathbf{x} = \sigma\mathbf{1}. \quad (7)$$

Then we see that $\sigma_i = \sum_j \sigma_{ij}$ is just the i^{th} entry of $\sigma\mathbf{1}$ since we are summing over the columns. Note that since D is diagonal, the D^{-1} is diagonal with diagonal entries $\frac{1}{k_i}$. So the i^{th} entry of $\mathbf{D}^{-1}\mathbf{x}$ is $\frac{1}{k_i}x_i$. So $\sigma_i = \frac{1}{k_i}x_i$. Hence σ_i is just the pagerank centrality of i divided by the degree of node i .

7.16.

	Group	e_r	a_r	
	Black	.129	.177	
1.	Hispanic	.0785	.147	So $Q = .251$.
	White	.153	.247	
	Other	.008	.0605	

	Group	e_r	a_r	
	Democrat	.125	.22	
2.	Independent	.03	.175	So $Q = .046$.
	Republican	.03	.245	

3. Based on the modularity values, it is clear that people associate themselves with people similar to themselves. This trend is strong in marriages than politics.