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Math 540

HW 9// 2.5.2,9.10

2.5.2. Let X be a discrete metric space with infinitely many points. Consider the sequence $(x_1, x_2, x_3, ...) \in X$. Note that for any x_i and x_{i+1}, then $d(x_i, x_{i+1}) = 1$. Thus, we choose an arbitrary subsequence $(x_i, x_{i+1}, x_{i+2}, ...)$ which may or may not be infinite. Then for $\epsilon = \frac{1}{2}$, then for every k with x_k an element of the subsequence, $d(x_k, x_{k+1}) = 1 > \epsilon$. So the subsequence does not converge. Since the subsequence was arbitrary, X is not compact.

2.5.9. Let X be a compact metric space and that $M \subset X$ is closed. Let $(x_n) \in M$. Since X is compact, we know that (x_n) has a subsequence (x_{n_k}) which converges to some x. Since (x_{n_k}) is a subsequence of (x_n), each element of (x_{n_k}) is in M. Further, since M is closed, $x \in M$. Thus, since (x_n) was arbitrary, M must be compact.

2.5.10. Let X and Y be metric spaces. Let X be compact and let $T \colon X \to Y$ be bijective and continous. We want to show that T is a homeomorphism (that is T has a continuous inverse). Note that since X is compact, $T(X)$ is compact and therefore Y is compact since T is bijective. Consider the function T^{-1}. Let (y_n) be a sequence in Y. Since Y is compact, there exists a convergent subsequence (y_{n_k}) which converges to some y. Then for $\epsilon > 0$, there exists $k > N$ such that $\|y_{n_k} - y\| < \epsilon$. Since T is bijective, we know then there exists a convergent subsequence (x_{n_k}) such that $(y_{n_k}) = T((x_{n_k}))$ and $y = T(x)$. So $\|T((x_{n_k})) - T(x)\| < \epsilon$.