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Math 522
1/16/20

2.9. Let G be an undirected graph. In this case, begin the breadth first search and view all the neighbors of a given node. We call this node g^0 and the set of its neighbors g_n^0. Then we check the each of the neighbors. As we travel to them, we label each one g^1 and the set of its neighbors g_n^1. If $g_n^0 \cap g_n^1 \neq \emptyset$, then there is a cycle. Once we have passed through all neighbors, we check the neighbors of the first node in g_n^0 and label this node g_n^0 and repeat the process. Further, if a node in is the neighborhood of the current node g^i and is not the previous node g^{i-1} (or g^{i+1}), then there is a cycle.

3.1. Let X_1, X_2, X_3 be random variables. We define the following properties:

$$P(X_1 = x_0) = .5 \tag{1}$$
$$P(X_1 = x_1) = .5 \tag{2}$$
$$P(X_2 = x_0) = .7 \tag{3}$$
$$P(X_2 = x_1) = .3 \tag{4}$$
$$P(X_3 = x_0) = .2 \tag{5}$$
$$P(X_3 = x_1) = .8. \tag{6}$$

As we can see, all the variables are independent. Then we define the probabilities

$$P(X_1 = x_0, X_2 = x_0 \mid X_3 = x_0) = .13 \tag{7}$$
$$P(X_1 = x_0, X_2 = x_0 \mid X_3 = x_1) = .02 \tag{8}$$
$$P(X_1 = x_1, X_2 = x_1 \mid X_3 = x_0) = .3 \tag{9}$$
$$P(X_1 = x_1, X_2 = x_1 \mid X_3 = x_1) = .05. \tag{10}$$

Thus we see that X_1, X_2 is dependent on X_3.

3.2.

1. Let $X_1, ..., X_n$ be random variables and C be a class variable. Assume that $(X_i \perp \mathbf{X}_{-i} \mid C)$ for all i where $\mathbf{X}_{-i} = \{X_1, ..., X_n\} - \{X_i\}$. Then

$$\begin{aligned} P(C, X_1, ..., X_n) &= P(C)P(X_1 \mid C)P(X_2, ..., X_n \mid X_1, C) && (11)\\ &= P(C)P(X_1 \mid C)P(X_2 \mid X_1, C)P(X_3, ..., X_n \mid X_1, X_2, C) && (12)\\ &= P(C)\prod_{i=1}^{n} P(X_i \mid C, X_{i-1}, ..., X_1) && (13)\\ &= P(C)\prod_{i=1}^{n} P(X_i \mid C), && (14)\end{aligned}$$

where the last step used the assumption of independence.

2. Using the property proven in part (i), we see that

$$\begin{aligned} \frac{P(C = c_1 \mid X_1, ..., X_n)}{P(C = c_2 \mid X_1, ..., X_n)} &= \frac{\frac{P(X_1, ..., X_n \mid C = c_1)P(C = c_1)}{P(X_1, ..., X_n)}}{\frac{P(X_1, ..., X_n \mid C = c_2)P(C = c_2)}{P(X_1, ..., X_n)}} && (15)\\ &= \frac{P(X_1, ..., X_n \mid C = c_1)P(C = c_1)}{P(X_1, ..., X_n \mid C = c_2)P(C = c_2)} && (16)\\ &= \frac{P(C = c_1)}{P(C = c_2)}\prod_{i=1}^{n}\frac{P(X_i \mid C = c_1)}{P(X_i \mid C = c_2)}. && (17)\end{aligned}$$

3.3.

1. While having an earthquake does not eliminate the possibility of a burglary, having an earthquake causes the alarm to go off. So if an alarm goes off and there is an earthquake, it decreases the probability that a burglary set off the alarm because the earthquake did happen and was able to set the alarm off.

2. Assume that $P(a^1 \mid b^1, e^1) = P(a^1 \mid b^0, e^1) = 1$. Then

$$P(b^1 \mid a^1, e^1) = \frac{P(b^1)P(a^1, e^1 \mid b^1)}{P(b^0)P(a^1, e^1 \mid b^0) + P(b^1)P(a^1, e^1 \mid b^1)} \tag{18}$$

$$= \frac{P(b^1)P(a^1 \mid e^1, b^1)}{P(b^0)P(a^1 \mid e^1, b^0) + P(b^1)P(a^1 \mid e^1, b^1)} \tag{19}$$

$$= \frac{P(b^1)}{P(b^0) + P(b^1)} = P(b^1). \tag{20}$$

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