Cory Glover
1/16/20
Math 540

HW 8
2.3.3,8,10
2.4.2,4,6

2.3.3. Let X be l^∞ and let Y be the subset of all sequences with only finitely many nonzero terms. We first show that Y is a subspace of l^∞. We first note that the zero sequence has no nonzero terms, and thus is in Y. So Y is nonempty. Let $x = (x_i)$ and $y = (y_i)$ be elements of Y. Let x_n be the last nonzero term of x and let y_m be the last nonzero term of y. Then $\alpha x + \beta y$ has a last nonzero term at either n or m (αx_n or βy_m respectively). Thus, $x + y \in Y$. So Y is a subspace.

However consider the sequence $x_n = (1, 1, 0, ...)$ where the first n entries of each tuple is 1 and the rest are zero. Then for $\epsilon > 0$, there exists an N such that when $n > N$, $\|x_n - (1, 1, 1, ...)\| \leq \epsilon$. Since $(1, 1, 1, ...) \notin Y$, we see that Y is not closed, and thus is not a closed subspace.

2.3.8. Let X be a normed space where absolute convergence implies convergence. Let $(x_n) \in X$ be a Cauchy sequence. Then for $\epsilon > 0$, there exists an N such that when $m, n > N$, then $\|x_n - x_m\| < \epsilon$. So for some $x \in X$ and $\epsilon' > 0$, then there exists an N such that when $n > N$,

$$\begin{aligned} \|x_n - x\| &= \|x_n - x_m + x_m - x\| && (1) \\ &\leq \|x_n - x_m\| + \|x_m - x\| && (2) \end{aligned}$$