DEFINITION	
	Metric
	Math 540: Linear Analysis (Midterm 1)
Definition	
	Metric Space
	Math 540: Linear Analysis (Midterm 1)
	WATH 540. DINEAR ANALISIS (MIDTERM 1)
DEFINITION	
	Supspace (of a metric space)
	/
	Math 540: Linear Analysis (Midterm 1)

The function d is a metric on X, defined on $X\times X$ such that for all $x,y,z\in X$ we have:

- $1.\ d$ is finite, real-valued, and non-negative,
- 2. d(x,y) = 0 if and only if x = y,
- 3. $d(x,z) \le d(x,y) + d(y,z)$,
- 4. d(x, y) = d(y, x).

A metric space is a set X with an associated metric d. Denoted (X, d).

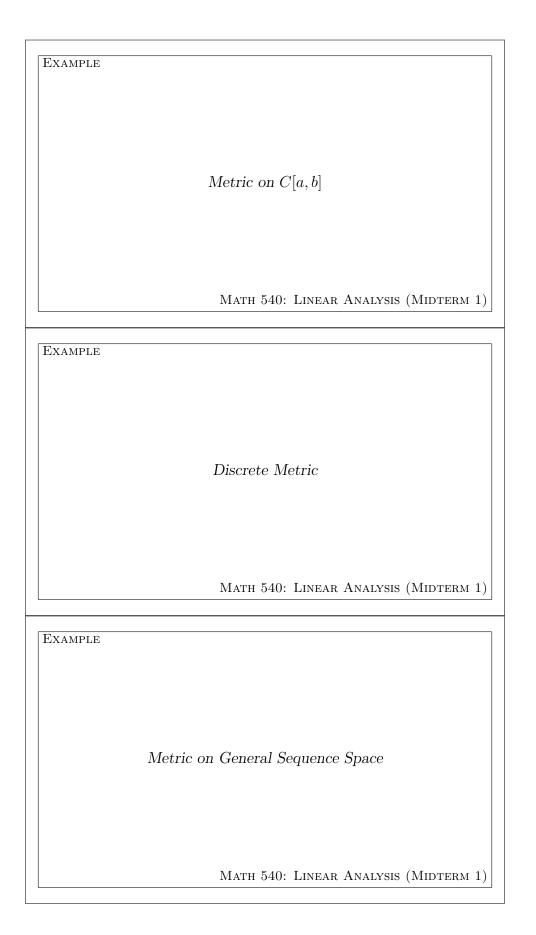
A subset $Y \subset X$ of a metric space (X,d) where the associated metric is $d|_{Y\times Y}$, the metric on X restricted to $Y\times Y$. This metric is said to be induced.

Example	
	Metric on \mathbb{R}^n (\mathbb{C}^n)
	Math 540: Linear Analysis (Midterm 1)
EXAMPLE	
	Metric on l^{∞}
	Math 540: Linear Analysis (Midterm 1)
DEFINITION	
DEFINITION	
	100
	l^{∞}
	Math 540: Linear Analysis (Midterm 1)

$$d(\land, \land) = \sqrt{|x_1 - y_1|^2 + \dots + |x_n - y_n|^2}$$

Let $x = (x_i)$ and $y = (y_i)$. Then $d(x, y) = \sup_i |x_i - y_i|$.

The set of all bounded sequences of complex numbers.



Let $x(t),y(t)\in C[a,b].$ Then $d(x,y)=\max_{t\in [a,b]}|x(t)-y(t)|.$

Let
$$x, y \in X$$
. Then $d(x, y) = \begin{cases} 1 & x = y \\ 0 & x \neq y \end{cases}$.

Let X be a set of sequences (not necessarily bounded). Then on metric is

$$d(x,y) = \sum_{j=1}^{\infty} \frac{1}{2^j} \frac{|x_j - y_j|}{1 + |x_j - y_j|}.$$

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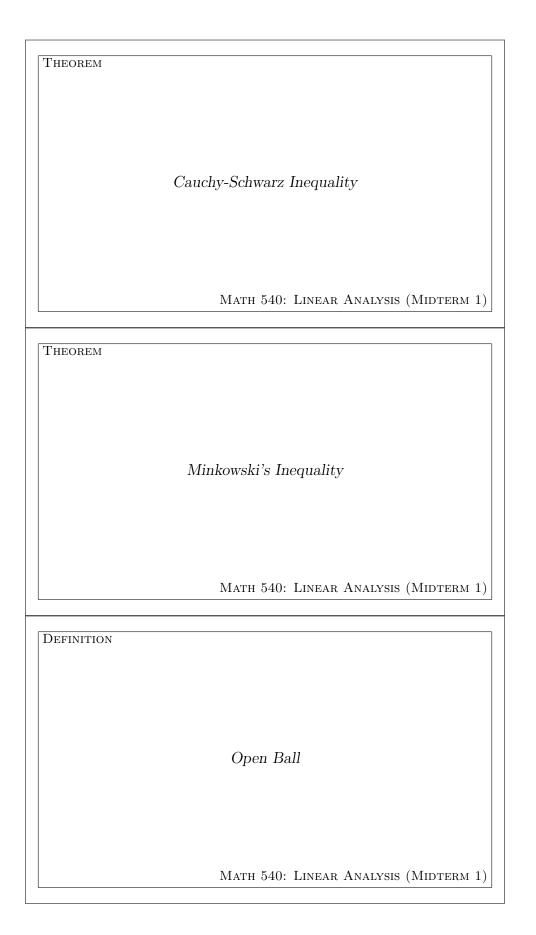
Let $x(t), y(t) \in B(A)$. Then $d(x,y) = \sup_{t \in A} |x(t) - y(t)|.$

Let $(x), (y) \in l^p$. Then

$$d(x,y) = \left(\sum_{j=1}^{\infty} |x_j - y_j|^p\right)^{1/p}.$$

Let p > 1 and define q by $\frac{1}{p} + \frac{1}{q} = 1$. Then

$$\sum_{j=1}^{\infty} |x_j y_j| \le \left(\sum_{j=1}^{\infty} |x_i|^p\right)^{1/p} \left(\sum_{j=1}^{\infty} |y_j|^q\right)^{1/q}.$$



Hölder Inequality for p=2:

$$\sum_{j=1}^{\infty}|x_jy_j|\leq\sqrt{\sum_{j=1}^{\infty}|x_i|^2}\sqrt{\sum_{j=1}^{\infty}|y_j|^2}.$$

Let $p \geq 1$. Then

$$\left(\sum_{j=1}^{\infty} |x_i + y_i|^p\right)^{1/p} = \left(\sum_{j=1}^{\infty} |x_i|^p\right)^{1/p} + \left(\sum_{j=1}^{\infty} |y_i|^p\right)^{1/p}.$$

Given a point $x_0 \in X$ and a real number r > 0, then a ball $B(x_0; r) = \{x \in X \colon d(x, x_0) < r\}.$

D	
DEFINITION	
	Sphere
	Math 540: Linear Analysis (Midterm 1)
Definition	
	Closed Ball
	Closed Dan
	Math 540: Linear Analysis (Midterm 1)
	MAIH 540. LINEAR ANALYSIS (MIDTERM 1)
Definition	
DEFINITION	
	Open Set
	Math 540: Linear Analysis (Midterm 1)

Given a point $x_0 \in X$ and a real number r > 0, then a sphere $S(x_0; r) = \{x \in X : d(x, x_0) = r\}.$

Given a point $x_0 \in X$ and a real number r > 0, then a closed ball is $\widetilde{B}(x_0;r) = \{x \in X \colon d(x,x_0) \leq r\}.$

Let $M \subseteq X$ be a subset of X. Then M is an open set if for every $x \in M$, there exists an open ball $B(x;r) \in M$ for some r > 0. (If $r = \epsilon$, then this is an ϵ -neighborhood).

DEFINITION	
	Closed Set
	Math 540: Linear Analysis (Midterm 1)
	` /
DEFINITION	
	Interior of a set M
	MARIA 540. LINDAD ANALYZIZ (MIDRIDIN 1)
	Math 540: Linear Analysis (Midterm 1)
DEFINITION	
	Topology
	101000)
	Math 540: Linear Analysis (Midterm 1)



An interior point x of M is a point x where M is a neighborhood containing x. The interior of M, M^0 , is the set of all interior points of M. This is the largest open set contained in M.

A collection $\mathcal T$ of open subsets of X. It satisfies the following properties:

- 1. $\emptyset \in \mathscr{T}, X \in \mathscr{T}$
- 2. The union of any members of ${\mathscr T}$ is a member of ${\mathscr T}$
- 3. The intersection of finitely many members of \mathscr{T} is a member of \mathscr{T} .

DEFINITION	
DEFINITION	
	Continuous Mapping
	Math 540: Linear Analysis (Midterm 1)
Theorem	
THEOREM	
	Continuous Mapping Theorem
	Math 540: Linear Analysis (Midterm 1)
DEFINITION	
	Accumulation Points and Closure

Let X=(X,d) and $Y=(Y,\widetilde{d})$ be metrics spaces. A mapping $T\colon X\to Y$ is continuous at x_0 if for every $\epsilon>0$, there exists a $\delta>0$ such that $\widetilde{T}(Tx,Tx_0)<\epsilon$ for all $x\in X$ satisfying $d(x,x_0)<\delta$. T is continuous if it is continuous for all $x\in X$.

Let X=(X,d) and $Y=(Y,\widetilde{d})$ be a metric spaces and let $T\colon X\to Y$. Then T is continuous if and only if the inverse image of any open set in Y is an open set in X.

A point $x_0 \in X$ is an accumulation point of M is for every neighborhood of x_0 , there is at least one point $y \in M$ distinct from x_0 . The set consisting of all the points in M and the accumulation points of M is the closure of M, denoted \overline{M} .

DEFINITION	
	Dense Set and Separable Space
	Dense Set and Separable Space
	Math 540: Linear Analysis (Midterm 1)
Example	
	Examples of Separable Sets
	Math 540: Linear Analysis (Midterm 1)
Example	
	Unexample of separable sets
	Onexample of separable sets

A subset of M of a metric space is dense in X if $\overline{M} = X$. X is said to be separable if it has a countable subset which is dense in X.

- 1. \mathbb{R}^n
- 2. \mathbb{C}^n
- 3. A discrete metric space X is separable if and only if X is countable.
- 4. The space l^p for $1 \le p < \infty$ is separable.

DEFINITION	
	Convergence of a sequence, limit
	Math 540: Linear Analysis (Midterm 1)
L	THIN OLD BINDING (MIDIBINE 1)
DEFINITION	
	Bounded Set (in a metric space)
	Dounded Set (in a metric space)
	Marry 540. Lynnan Avaryorg (Minmony 1)
	Math 540: Linear Analysis (Midterm 1)
THEOREM	
	Boundedness, limit on metric spaces
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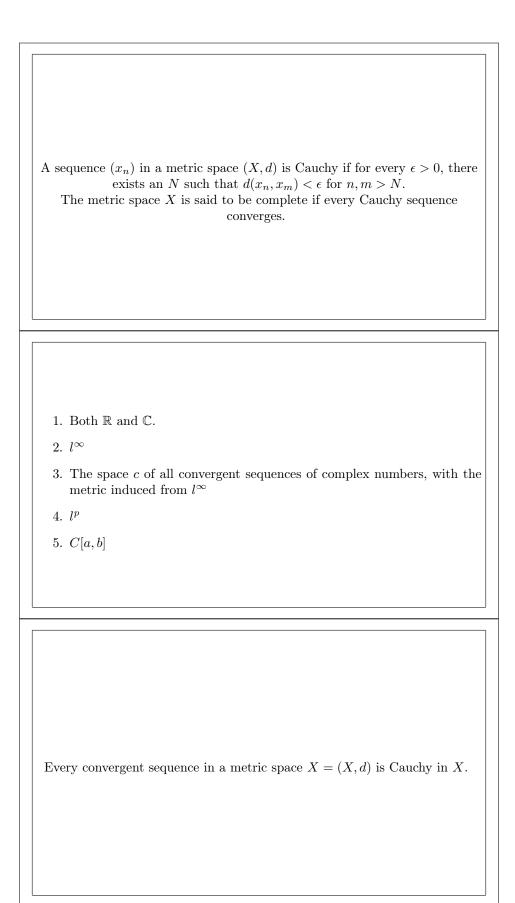
A sequence $(x_n) \in X$ converges if there exists some $x \in X$ such that for $\epsilon > 0$, there exists N > 0 such that when n > N, then $d(x_n, x) < \epsilon$. The limit of (x_n) is said to be x.

A set M where $\sup_{x,y\in M} d(x,y) < \infty$.

Let X = (X, d) be a metric space. Then

- 1. A convergent sequence in X is bounded and its limit is unique.
- 2. If $x_n \to x$ and $y_n \to y$, then $d(x_n, y_n) \to d(x, y)$.

DEFINITION
Cauchy Sequences and Completeness
coulding sequences and completeness
Math 540: Linear Analysis (Midterm 1)
MINITI 610. BINEMI TIMBISIS (MIBIBINI 1)
Example
Examples of Complete Metric Space
Examples of Complete Metric Space
MARIL 540. LINEAR ANALYSIS (MIRRERN 1)
Math 540: Linear Analysis (Midterm 1)
Theorem
THEOREM
Convergent Sequences and Cauchy Sequences
Convergent bequences and Cauchy bequences
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Math 540: Linear Analysis (Midterm 1)



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THEOREM	
	Closure and Closed Sets
	Closure and Closed Sens
	Math 540: Linear Analysis (Midterm 1)
Theorem	
	Complete Subspace
	Math 540: Linear Analysis (Midterm 1)
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Theorem	
	Continuous mapping (convergence)
	Math 540: Linear Analysis (Midterm 1)

Let M be a nonempty subset of a metric space (X,d) and \overline{M} its closure. Then

- 1. $x \in \overline{M}$ if and only if there is a sequence (x_n) in M such that $x_n \to x$,
- 2. M is closed if and only if having $x_n \in M$, then $x_n \to x$ implies that $x \in M$.

A subspace M of a complete metric space X is complete if and only if the set M is closed in X.

A mapping $T\colon X\to Y$ of a metric space X=(X,d) into a metric space $Y=(Y,\widetilde{d})$ is continuous at a point $x_0\in X$ if and only if $x_n\to x_0$ implies $Tx_n\to Tx_0$.

Тирове	
THEOREM	
	Uniform Convergence
	Math 540: Linear Analysis (Midterm 1)
D	
Example	
	Unexamples of Complete Metric Spaces
	Chexamples of Complete Metric Spaces
	Math 540: Linear Analysis (Midterm 1)
D	
DEFINTION	
	Isometric Mapping, Isometric Spaces
	nomenie mapping, nomenie opaces

Convergence $x_m \to x$ in the space C[a,b] is uniform convergence.

- 1. Q
- 2. Polynomials
- 3. Continuous functions

Let
$$X=(X,d)$$
 and $\widetilde{X}=(\widetilde{X},\widetilde{d})$ be metric spaces. Then

- 1. A mapping T of X into \widetilde{X} is said to be isometric or an isometry if T preserves distances, that is, if for all $x, y \in X$, $\widetilde{d}(Tx, Ty) = d(x, y)$,
- 2. The space X is said to be isometric with the space \widetilde{X} if there exists a bijective isometry of X onto \widetilde{X} .

Тнеокем	
	Metric Space Completion Theorem
	Math 540: Linear Analysis (Midterm 1)
DEFINITION	
	Vector Space
	Math 540: Linear Analysis (Midterm 1)
EXAMPLE	
	Examples of Vector Spaces
	Math 540: Linear Analysis (Midterm 1)

For a metric space X=(X,d) there exists a complete metric space $\hat{X} = (\hat{X}, \hat{d})$ which has a subspace W that is isometric with X and is dense in \hat{X} . This space \hat{X} is unique up to isometries. A nonempty set X over a field K with addition and multiplication. 1. \mathbb{R}^n 2. \mathbb{C}^n 3. C[a,b]4. l^2

DEFINITION	
	Hamel Basis
	Math 540: Linear Analysis (Midterm 1)
DEFINITION	
	Normed space, Banach Space
	Math 540: Linear Analysis (Midterm 1)
DEFINITION	
	Norm
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	Math 540: Linear Analysis (Midterm 1)

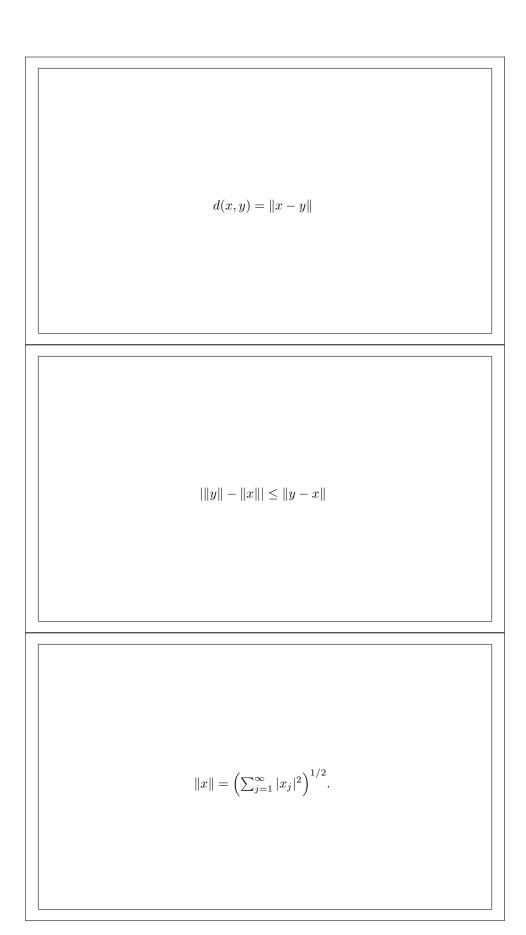
Let X be a vector space (not necessarily finite dimensional). Then a linearly independent subset of X which spans X is called a Hamel basis.

A normed space X is a vector space with a norm. A Banach space is a complete normed space.

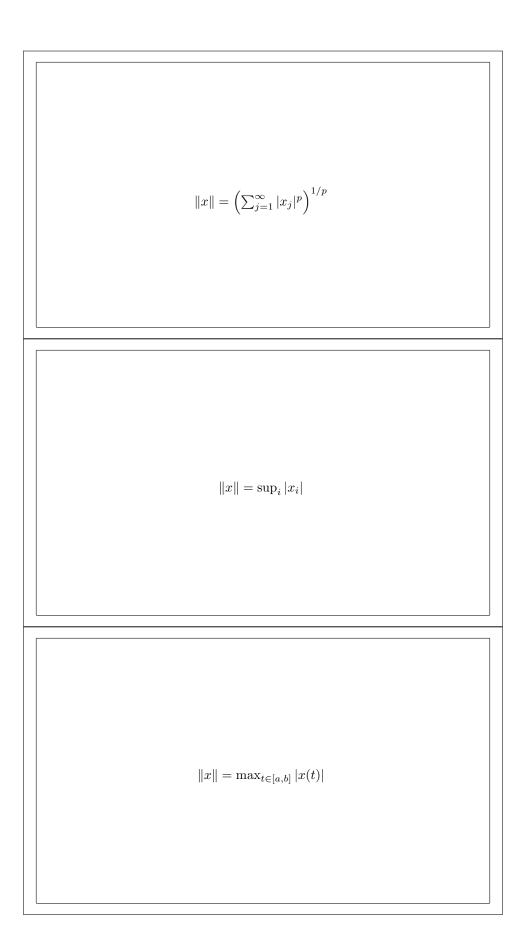
Let X be a vector space. Then $\|\cdot\|$ is a norm on X if it is a real-valued function such that

- 1. $||x|| \ge 0$
- $2. ||x+y|| \le ||x|| + ||y||$
- 3. ||x|| = 0 if and only if x = 0
- 4. $\|\alpha x\| = |\alpha| \|x\|$.

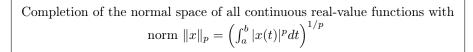
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DEFINITION	
	Metric Induced by a norm
	Medic induced by a norm
	Math 540: Linear Analysis (Midterm 1)
DEFINITION	
	Reverse triangle inequality
	recverse mangle inequality
	Math 540: Linear Analysis (Midterm 1)
EVANDLE	
Example	
	Norm on \mathbb{R}^n and \mathbb{C}^n
	Math 540: Linear Analysis (Midterm 1)



EXAMPLE	
	NT. In
	Norm on l^p
	Math 540: Linear Analysis (Midterm 1)
Example	
EXAMPLE	
	Norm on l^{∞}
	Math 540: Linear Analysis (Midterm 1)
EXAMPLE	
	Name on Cl - 1]
	Norm on $C[a,b]$
	Math 540: Linear Analysis (Midterm 1)



EXAMPLE	
	L^p
	Math 540: Linear Analysis (Midterm 1)
THEOREM	
	Translation-Invariance
	Translation invariance
	Math 540: Linear Analysis (Midterm 1)
Тнеогем	
	Subspace of a Banach Space
	Math 540: Linear Analysis (Midterm 1)



A metric d induced by a norm on a normed space X satisfies:

$$d(x+a, y+a) = d(x, y)$$

$$d(\alpha x, \alpha y) = |\alpha| d(x, y)$$

for all $x, y, a \in X$ and every scalar α .

A subspace Y of a Banach space X is complete if and only if the set Y is closed in X.

DEFINITION	
	Schauder Basis
	Math 540: Linear Analysis (Midterm 1)
THEOREM	
	Completion of Normed Spaces
	Math 540: Linear Analysis (Midterm 1)
Тнеопем	
	Linear combinations
	Math 540: Linear Analysis (Midterm 1)

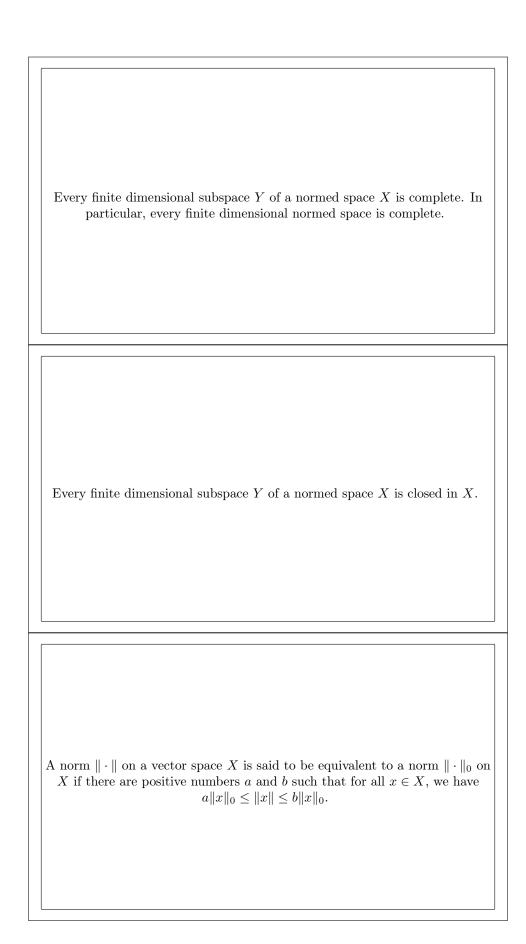
If a normed space X contains a sequence (e-n) with the property that for every $x \in X$ there is a unique sequence of scalars (α_n) such that $||x - (\alpha_1 e_1 + \dots + \alpha_n e_n)|| \to 0$ as $n \to \infty$, then (e_n) is a Schauder basis for X.

Let $X = (X, \|\cdot\|)$ be a normed space. Then there is a Banach space \hat{X} and an isometry A from X onto a subspace W of \hat{X} which is dense in \hat{X} . The space \hat{X} is unique, except for isometries.

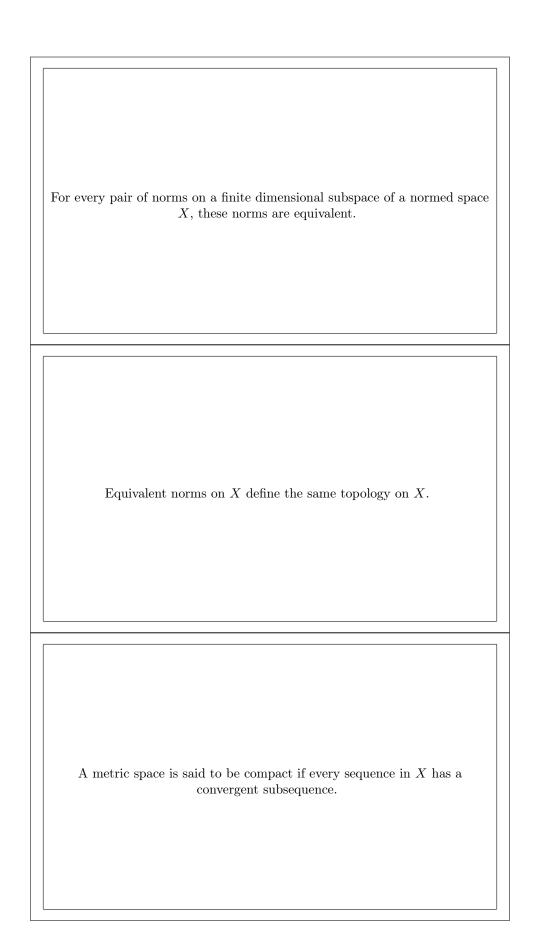
Let $\{x_1, ..., x_n\}$ be a linearly independent set of vectors in a normed space X (of any dimension). Then there is a number c > 0 such that for every choice of scalars $\alpha_1, ..., \alpha_n$ we have

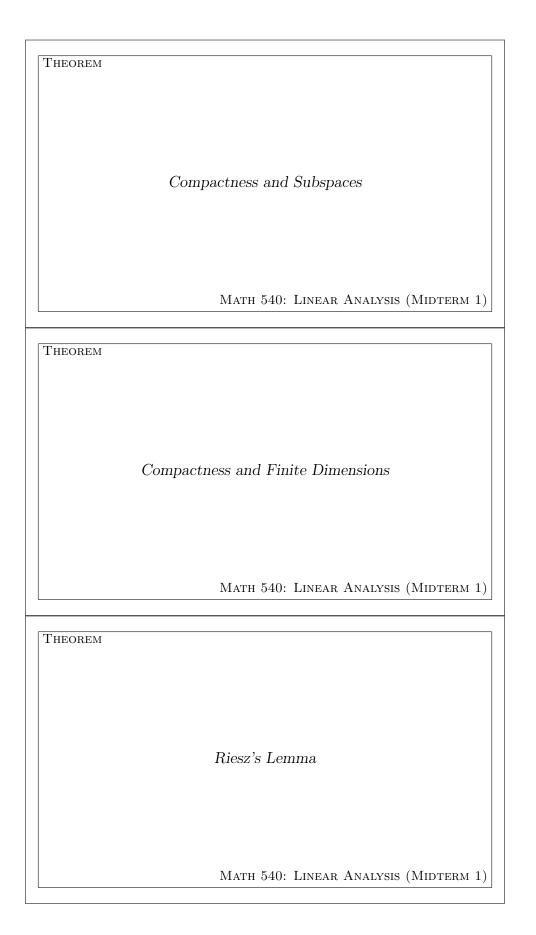
$$\|\alpha_1 x_1 + \dots + \alpha_n x_n\| \ge c(|\alpha_1| + \dots + |\alpha_n|).$$

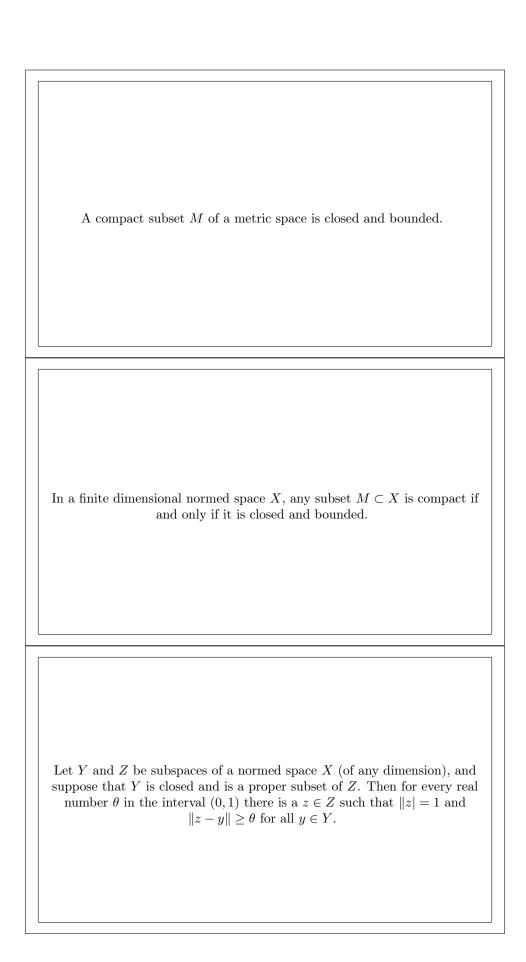
Theorem	
Finite Dimensional subspaces and completeness	
Math 540: Linear Analysis (Midti	ERM 1)
Theorem	
Finite Dimensional and Closed	
Math 540: Linear Analysis (Midti	ERM 1)
DEFINITION	
DEFINITION	
Equivalent Norms	
Math 540: Linear Analysis (Midti	ERM 1)

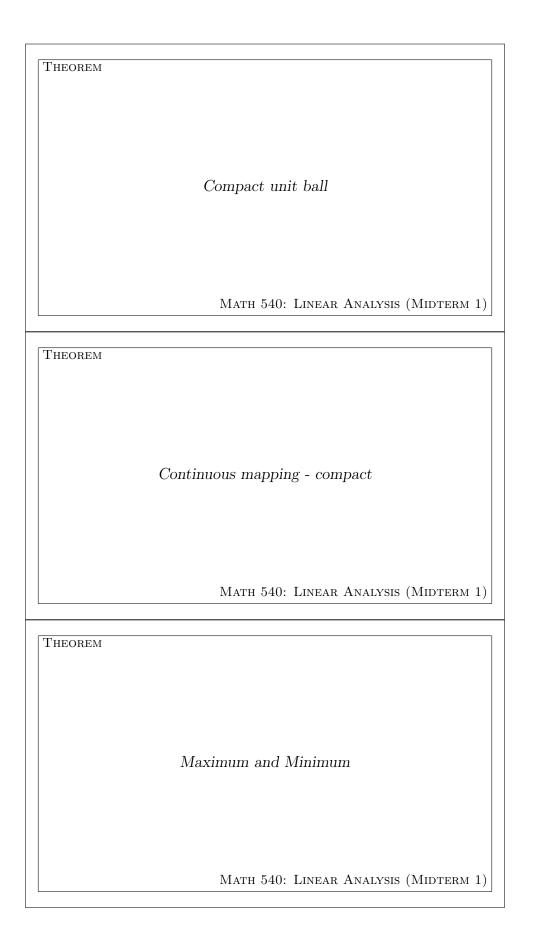


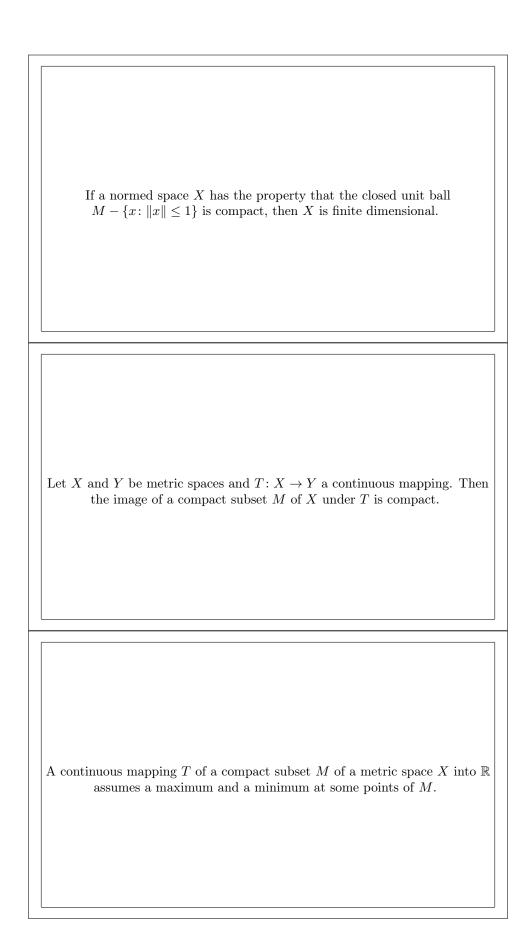
THEOREM	
	Finite Dimensional: Equivalent Norms
	Math 540: Linear Analysis (Midterm 1)
Тнеокем	
	Topology: Equivalent norms
	Math 540: Linear Analysis (Midterm 1)
DEFINITION	
	Compactness
	Math 540: Linear Analysis (Midterm 1)











EXAMPLE	
	Examples of Linear Operators
	Math 540: Linear Analysis (Midterm 1)
THEOREM	
	Range and Null Space
	Math 540: Linear Analysis (Midterm 1)
THEOREM	
THEOREM	
	Inverse Operator
	Math 540: Linear Analysis (Midterm 1)

- 1. Identity operator
- 2. Zero operator
- 3. Differentiation
- 4. Integration
- 5. Multiplication by t
- 6. Elementary vector algebra (cross and dot product)
- 7. Matrices

Let T be a linear operator. Then

- 1. $\mathcal{R}(T)$ is a vector space
- 2. $\mathcal{N}(t)$ is a vector space
- 3. If $dim(\mathcal{D}(T)) = n < \infty$, then $\mathcal{R}(T) \le n$.

Let T be a linear operator. Then

- 1. T^{-1} exists if and only if Tx = 0 implies that x = 0.
- 2. If T^{-1} exists, it is a linear operator.
- 3. If $\dim\mathscr{D}(T)=n<\infty$ and T^{-1} exists, then $\dim(\mathscr{R}(T))=\dim(\mathscr{D}(T)).$

THEOREM	
	Inverse of product
	Math 540: Linear Analysis (Midterm 1)
DEFINITION	
	Bounded Linear Operator
	Math 540: Linear Analysis (Midterm 1)
DEFINITION	
	Norm of an operator
	Math 540: Linear Analysis (Midterm 1)

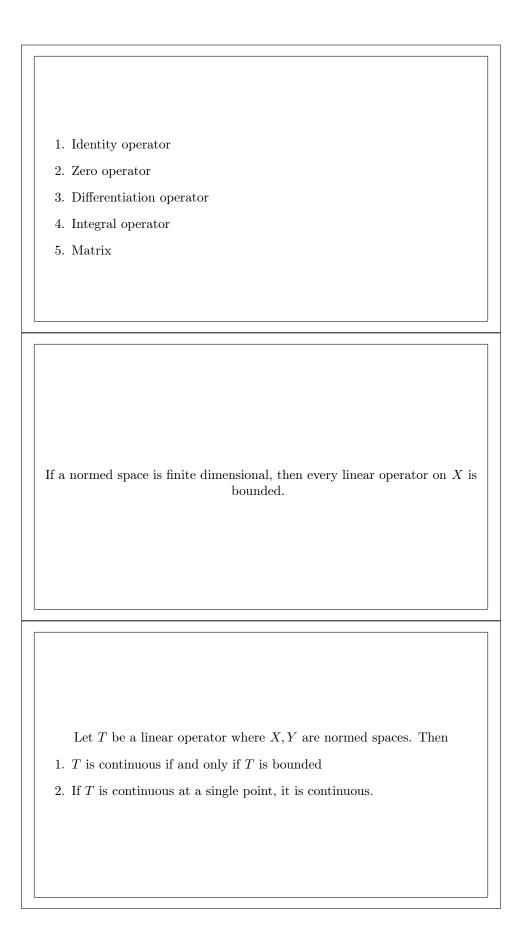
Let $T\colon X\to Y$ and $S\colon Y\to Z$ be bijective linera operators, where X,Y and Z are vector spaces. Then the inverse $(ST)^{-1}\colon Z\to X$ of the product ST exists and is $(ST)^{-1}=T^{-1}S^{-1}$.

Let X and Y be normed spaces and $T\colon \mathscr{D}(T)\to Y$ a linear operator, where $\mathscr{D}(T)\subset X$. The operator T is said to be bounded if there is a real number c such that

$$||Tx|| \le c||x||.$$

$$||T|| = \sup_{x \in \mathscr{D}(T), x \neq 0} \frac{||Tx||}{||x||} = \sup_{x \in \mathscr{D}(T), ||x|| = 1} ||Tx||.$$

EXAMPLE	
	Examples of Bounded Linear Operators
	MATH 540: LINEAR ANALYSIS (MIDTERM 1)
	,
THEOREM	
	Finite dimensional bounded oeprators
	Timile difficusional bounded ocpiators
	Math 540: Linear Analysis (Midterm 1)
Тнеокем	
	Continuity and boundedness of operators
	continuity and boundedness of operations
	Marry 540. Lavren Averyga (Marrows 1)
	Math 540: Linear Analysis (Midterm 1)



THEOREM	
	Continuity, null space
	Math 540: Linear Analysis (Midterm 1)
THEOREM	
	Bounded Linear Extension
	Math 540: Linear Analysis (Midterm 1)
DEFINITION	
	Linear Functional
	Math 540: Linear Analysis (Midterm 1)

Let T be a bounded linear operator. Then,

- 1. $x_n \to x$ implies that $Tx_n \to Tx$.
- 2. The null space $\mathcal{N}(T)$ is closed.

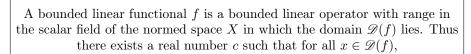
Let $T\colon \mathscr{D}(T)\to T$ be a bounded linear operator, where $\mathscr{D}(T)$ lies in a normed space X and Y is a Banach space. Then T has an extension

$$\widetilde{T} \colon \overline{\mathscr{D}(T)} \to Y$$

where \widetilde{T} is a bounded linear operator of norm $\|\widetilde{T}\| = \|T\|.$

A linear functional f is a linear operator with domain in a vector space X and range in the scalar field K of X.

DEFINITION	
	Bounded Linear Functional
	MATH 540: LINEAR ANALYSIS (MIDTERM 1)
DEFINITION	
	Norm of a Linear Functional
	Math 540: Linear Analysis (Midterm 1)
Theorem	
THEOREM	
~	
Cont	inuity and boundedness - Linear Functional
İ	
	Math 540: Linear Analysis (Midterm 1)



$$|f(x)| \le c||x||.$$

$$||f|| = \sup_{x \in \mathscr{D}(f), ||x|| = 1} |f(x)|.$$

A linear functional f with domain $\mathscr{D}(f)$ in a normed space is continuous if and only if f is bounded.

EXAMPLE	
	Linear Functional Examples
	Math 540: Linear Analysis (Midterm 1)
DEFINITION	
	Algebraic Dual Space
	Math 540: Linear Analysis (Midterm 1)
DEFINITION	
	Second Algebraic Dual Space
	Math 540: Linear Analysis (Midterm 1)

 Norm Dot product Definite integral 	
The set of all linear functionals defined on a vector space X . Denoted X^{i}	**.
The set of linear functionals on X^* .	

Definition
Canonical Mapping
Math 540: Linear Analysis (Midterm 1)
Definition
BELIATION
Algebraically Reflexive
Math 540: Linear Analysis (Midterm 1)

