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Math 522
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2.9. Let G be an undirected graph. In this case, begin the breadth first search and view all the neighbors of a given node. We call this node g^0 and the set of its neighbors g_n^0. Then we check the each of the neighbors. As we travel to them, we label each one g^1 and the set of its neighbors g_n^1. If $g_n^0 \cap g_n^1 \neq \emptyset$, then there is a cycle. Once we have passed through all neighbors, we check the neighbors of the first node in g_n^0 and label this node g_n^0 and repeat the process. Further, if a node in is the neighborhood of the current node g^i and is not the previous node g^{i-1} (or g^{i+1}), then there is a cycle.

3.1. Let X_1, X_2, X_3 be random variables. We define the following properties:

$$P(X_1 = x_0) = .5 \tag{1}$$
$$P(X_1 = x_1) = .5 \tag{2}$$
$$P(X_2 = x_0) = .7 \tag{3}$$
$$P(X_2 = x_1) = .3 \tag{4}$$
$$P(X_3 = x_0) = .2 \tag{5}$$
$$P(X_3 = x_1) = .8. \tag{6}$$

As we can see, all the variables are independent. Then we define the probabilities

$$P(X_1 = x_0, X_2 = x_0 \mid X_3 = x_0) = .13 \tag{7}$$
$$P(X_1 = x_0, X_2 = x_0 \mid X_3 = x_1) = .02 \tag{8}$$
$$P(X_1 = x_1, X_2 = x_1 \mid X_3 = x_0) = .3 \tag{9}$$
$$P(X_1 = x_1, X_2 = x_1 \mid X_3 = x_1) = .05. \tag{10}$$

Thus we see that X_1, X_2 is dependent on X_3.

3.2.

1. Let $X_1, ..., X_n$ be random variables and C be a class variable. Assume that $(X_i \perp \mathbf{X}_{-i} \mid C)$ for all i where $\mathbf{X}_{-i} = \{X_1, ..., X_n\} - \{X_i\}$. Then

$$\begin{aligned} P(C, X_1, ..., X_n) &= P(C)P(X_1 \mid C)P(X_2, ..., X_n \mid X_1, C) && (11)\\ &= P(C)P(X_1 \mid C)P(X_2 \mid X_1, C)P(X_3, ..., X_n \mid X_1, X_2, C) && (12)\\ &= P(C)\prod_{i=1}^{n} P(X_i \mid C, X_{i-1}, ..., X_1) && (13)\\ &= P(C)\prod_{i=1}^{n} P(X_i \mid C), && (14)\end{aligned}$$

   where the last step used the assumption of independence.

2. Using the property proven in part (i), we see that

$$\begin{aligned} \frac{P(C = c_1 \mid X_1, ..., X_n)}{P(C = c_2 \mid X_1, ..., X_n)} &= \frac{\frac{P(X_1,...,X_n \mid C=c_1)P(C=c_1)}{P(X_1,...,X_n)}}{\frac{P(X_1,...,X_n \mid C=c_2)P(C=c_2)}{P(X_1,...,X_n)}} && (15)\\ &= \frac{P(X_1, ..., X_n \mid C = c_1)P(C = c_1)}{P(X_1, ..., X_n \mid C = c_2)P(C = c_2)} && (16)\\ &= \frac{P(C = c_1)}{P(C = c_2)}\prod_{i=1}^{n}\frac{P(X_i \mid C = c_1)}{P(X_i \mid C = c_2)}. && (17)\end{aligned}$$

3.3.

1. While having an earthquake does not eliminate the possibility of a burglary, having an earthquake causes the alarm to go off. So if an alarm goes off and there is an earthquake, it decreases the probability that a burglary set off the alarm because the earthquake did happen and was able to set the alarm off.

2. Assume that $P(a^1 \mid b^1, e^1) = P(a^1 \mid b^0, e^1) = 1$. Then

$$P(b^1 \mid a^1, e^1) = \frac{P(b^1)P(a^1, e^1 \mid b^1)}{P(b^0)P(a^1, e^1 \mid b^0) + P(b^1)P(a^1, e^1 \mid b^1)} \tag{18}$$

$$= \frac{P(b^1)P(a^1 \mid e^1, b^1)}{P(b^0)P(a^1 \mid e^1, b^0) + P(b^1)P(a^1 \mid e^1, b^1)} \tag{19}$$

$$= \frac{P(b^1)}{P(b^0) + P(b^1)} = P(b^1). \tag{20}$$

3.5. Consider the case where Z is the probability that a basketball team wins a game, X is the probability all the teams players are in good health, and Y is the probability the team has a lot of money. We see that $P(z_1) < P(z_1 \mid x_1)$ since if the players are in good health, they will likely play better. Similarly, $P(z_1) < P(z_1 \mid y_1)$ since if a team has money, it has more resources to buy better players and train them better. Further we see that $P(x_1 \mid z_1) < P(x_1 \mid z_1, y_1)$ since if the team has more money, they can spend more money on keeping the players in good health. Lastly, $P(y_1 \mid z_1) < P(y_1 \mid x_1, z_1)$ since if a team is in better health, they are spending less money on medical treatment and thus have more money.

3.6.

1. $P(t^1 \mid d^1) = P(t^1)$ since D is not a parent of T. Relevant trails are $D- > C- > T$ where D has negative influence on C and C has positive influence on T.

2. $P(d^1 \mid t^0) = P(d^1)$ since T is not a parent of D. Relevant trails are $D- > C- > T$ where D has negative influence on C and C has positive influence on T.

3. $P(h^1 \mid e^1, f^1) = P(h^1 \mid e^1)$ since H is a parent of E and not related to F. Relevant trails are $H - E - F$ where H has a positive influence on H and F has a negative influence on E.

4. $P(c^1 \mid f^0) = P(c^1)$ since F is not a parent of C. Relevant trails are $F - E - W - D - C$ and $F - E - H - D - C$ where D has a negative influence on C, D has a positive influence on W, H has a positive influence on D, H has a positive influence on E, E has a positive influence on W, and F has a negative influence on E.

5. $P(c^1 \mid h^0) = P(c^1)$ since H is not a parent of C. Relevant trails are $H - D - C$ where H has a positive influence on D and D has a negative influence on C.

6. $P(c^1 \mid h^0, f^0) = P(c^1 \mid h^0)$ since H and F are not parents of C. Relevant trails are $F - E - W - D - C$ and $F - E - H - D - C$ and $H - D - C$ where D has a negative influence on C, D has a positive influence on W, H has a positive influence on D, H has a positive influence on E, E has a positive influence on W, and F has a negative influence on E.

7. $P(d^1 \mid h^1, e^0) = P(d^1 \mid h^1)$ since E is not a parent of D. Relevant trails are $H - D$ and $E - W - D$ and $E - H - D$ where H has a positive influence on D, E has a positive influence on W, D has a positive influence on W, and H has a positive influence on E.

8. $P(d^1 \mid e^1, f^0, w^1) = P(d^1 \mid e^1, f^0)$ since E, F and W are not parents of D. Relevant trails are $F - E - W - D$ and $F - E - H - D$ where F has a negative influence on E, E has a positive influence on D, D has a positive influence on W, H has a positive influence on E and H has a positive influence on D.

9. $P(t^1 \mid w^1, f^0) = P(t^1 \mid w^1)$ since F is not a parent of W. Relevant trails are $F - E - W - D - C - T$ and $F - E - H - D - C - T$ where F has a positive influence on E, E has a positive influence on W, D has a positive influence on W, D has a negative influence on C, C has a positive influence on T, H has a positive influence on D and H has a positive influence on E.