Problem 1.17 Name the zero vector for each of these vector spaces.

(a) The space of three polynomials under the natural operations (i.e. $a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3$) has zero vector $0x^0 + 0x^1 + 0x^2 + 0x^3$.

(b) The space of 2×3 matrices $\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \end{pmatrix}$ has the zero vector $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

(c) The space of $\{f : [0..1] \to \mathbb{R} \mid f \text{ is continuous}\}$ has the constant function $f(x) = 0$ as its zero vector.

(d) The space of functions $f : \mathbb{N} \to \mathbb{R}$ has the constant function $f(n) = 0$ as its zero vector.

Problem 1.18 Find the additive inverse, in the vector space, of the vector.

(a) In $\mathcal{P}_3$, the vector $\mathbf{v} = -3 - 2x + x^2$.

$$\begin{aligned} \mathbf{v} + \mathbf{v^{-1}} &= \mathbf{0} \\ \begin{pmatrix} -3 \\ -2 \\ 1 \end{pmatrix} + \mathbf{v^{-1}} &= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \\ \mathbf{v^{-1}} &= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} -3 \\ -2 \\ 1 \end{pmatrix} \\ \mathbf{v^{-1}} &= \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} \end{aligned}$$

(b) In the 2×2 space, vector $\mathbf{v} = \begin{pmatrix} 1 & -1 \\ 0 & 3 \end{pmatrix}$.

$$\begin{aligned} \mathbf{v} + \mathbf{v^{-1}} &= \mathbf{0} \\ \begin{pmatrix} 1 & -1 \\ 0 & 3 \end{pmatrix} + \mathbf{v^{-1}} &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \mathbf{v^{-1}} &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 1 & -1 \\ 0 & 3 \end{pmatrix} \\ \mathbf{v^{-1}} &= \begin{pmatrix} -1 & 1 \\ 0 & -3 \end{pmatrix} \end{aligned}$$

(c) In the $\{ae^x + be^{-x} \mid a, b \in \mathbb{R}\}$, the space of real variable x under the natrual operations, the vector $\mathbf{v} = 3e^x - 2e^{-x}$

Here the zero vector $\mathbf{0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

So:

$$\begin{aligned} \mathbf{v} + \mathbf{v^{-1}} &= \mathbf{0} \\ \begin{pmatrix} 3 \\ -2 \end{pmatrix} + \mathbf{v^{-1}} &= \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ \mathbf{v^{-1}} &= \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix} \\ \mathbf{v^{-1}} &= \begin{pmatrix} -3 \\ 2 \end{pmatrix} \end{aligned}$$