



Southwest Center  
for Arithmetic Geometry

## ARIZONA WINTER SCHOOL 2018

Department of Mathematics  
The University of Arizona®

Deadline to apply for funding:  
November 10, 2017

<http://swc.math.arizona.edu>

## IWASAWA THEORY

**John Coates**

*Classical algebraic Iwasawa theory*

**David Loeffler and Sarah Zerbes**

*Euler systems*

**Romyar Sharifi**

*Modular curves and cyclotomic fields*

**Christopher Skinner**

*Iwasawa theory, modular forms,  
and elliptic curves*

TUCSON, MARCH 3-7, 2018

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University of Arizona

# Arizona Winter School 2018

## Iwasawa Theory

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Notes By: Caleb McWhorter

March 2018

## Contents

**Part I**

## **Talk Notes**

# 1 Name: Lecture Title

## 1.1 Lecture 1

### 1.1.1 Lecture Name

$$[F : \mathbb{Q}] < \infty, F_\infty/F, \Gamma = (F_\infty/F)$$

$$\Gamma \xrightarrow{\sim} \mathbb{Z}_p, \Gamma_n \xrightarrow{\sim} p^n \mathbb{Z}_p,$$

$$F_\infty^{\Gamma_n} = F_n, (F_n/F) = \mathbb{Z}/p^n \mathbb{Z}$$

$$F = F_0 \subset F_1 \subset \cdots \subset F_n \subset \cdots F_\infty = \cup F_n$$

Facet 1.  $F_\infty/F$

Classical Iwasawa Theory:  $p$ -adic behavior of ideal class groups and units in  $F_\infty/F$ , and interpretation via global class field theory.

$$\zeta(s) = \prod_p (1 - p^{-s})^{-1}$$

(a) Complex zeros of  $\zeta(s) \leftrightarrow$  distribution of prime numbers.

(b)  $\zeta(1-n) \in \mathbb{Q} (n = 2, 4, 6, \dots)$

Kummer  $\leftrightarrow$  class number of  $\mathbb{Q}(\mu_p)^+$

Leopoldt-Kubota:  $p$ -adic analogue of  $\zeta(s)$ .

Iwasawa: zeroes of  $p$ -adic analogue of  $\zeta(s) \leftrightarrow$  classical Iwasawa Theory of  $\mathbb{Q}(\mu_p^\infty)^+/\mathbb{Q}(\mu_p)^+$ .

## **2   Name: Lecture Title**

### **2.1   Lecture 1**

#### **2.1.1   Lecture Name**

### **3   Name: Lecture Title**

#### **3.1   Lecture 1**

##### **3.1.1   Lecture Name**

## **4   Name: Lecture Title**

### **4.1   Lecture 1**

#### **4.1.1   Lecture Name**



## **5   Name: Lecture Title**

### **5.1   Lecture 1**

#### **5.1.1   Lecture Name**

## **6   Name: Lecture Title**

### **6.1   Lecture 1**

#### **6.1.1   Lecture Name**

## **7   Name: Lecture Title**

### **7.1   Lecture 1**

#### **7.1.1   Lecture Name**

## **8   Name: Lecture Title**

### **8.1   Lecture 1**

#### **8.1.1   Lecture Name**

## **9   Name: Lecture Title**

### **9.1   Lecture 1**

#### **9.1.1   Lecture Name**

## **10   Name: Lecture Title**

### **10.1   Lecture 1**

#### **10.1.1   Lecture Name**

## **11   Name: Lecture Title**

### **11.1   Lecture 1**

#### **11.1.1   Lecture Name**

## **12   Name: Lecture Title**

### **12.1   Lecture 1**

#### **12.1.1   Lecture Name**



## **13   Name: Lecture Title**

### **13.1   Lecture 1**

#### **13.1.1   Lecture Name**

## **14   Name: Lecture Title**

### **14.1   Lecture 1**

#### **14.1.1   Lecture Name**

## **15   Name: Lecture Title**

### **15.1   Lecture 1**

#### **15.1.1   Lecture Name**

## **16   Name: Lecture Title**

### **16.1   Lecture 1**

#### **16.1.1   Lecture Name**

## **17   Name: Lecture Title**

### **17.1   Lecture 1**

#### **17.1.1   Lecture Name**

## **18   Name: Lecture Title**

### **18.1   Lecture 1**

#### **18.1.1   Lecture Name**

**Part II**

## **Course/Project Outlines & Lecture Notes**