



Syracuse University

# MATH 830: Reflection Groups and Coxeter Groups

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Fall 2017

Last Updated: October 6, 2018

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## 0 Introduction

**MAT 830:** Basic facts about Coxeter groups, including the classification of finite Coxeter groups, root systems, exchange conditions, and Bruhat ordering.

**Textbook:** *Reflection Groups and Coxeter Groups*, James E. Humphreys

## 1 Reflections and Rotations

### 1.1 Reflections and Rotations in Euclidean Space

Let  $\mathbb{R}^n$  denote Euclidean space and  $\langle \cdot, \cdot \rangle$  denote the standard inner product. Let  $0 \neq \alpha \in \mathbb{R}^n$  and let  $\mathcal{H}_\alpha$  denote the hyperplane of all vectors orthogonal to  $\alpha$ . We create a function  $s_\alpha : \mathbb{R}^n \rightarrow \mathbb{R}^n$  on  $\mathbb{R}^n$  as follows:  $s_\alpha(\lambda) := \lambda - 2 \frac{\langle \lambda, \alpha \rangle}{\langle \alpha, \alpha \rangle} \alpha$ . The function  $s_\alpha$  has the following properties:

- (i) If  $\lambda \in \mathcal{H}_\alpha$ , then  $s_\alpha(\lambda) = \lambda$ .
- (ii)  $s_\alpha(\alpha) = -\alpha$ . That is,  $s_\alpha$  is the reflection about  $\mathcal{H}_\alpha$ .
- (iii)