Syracuse University

MATH 830:
Reflection Groups and Coxeter Groups

Lecturer: Dr. Mark Kleiner
Notes By: Caleb McWhorter

Fall 2017

Contents

0 Introduction

MAT 830: Basic facts about Coxeter groups, including the classification of finite Coxeter groups, root systems, exchange conditions, and Bruhat ordering.

Textbook: *Reflection Groups and Coxeter Groups*, James E. Humphreys

1 Reflections and Rotations

1.1 Reflections and Rotations in Euclidean Space

Let $\mathbb{R}^n$ denote Euclidean space and $\langle\ ,\ \rangle$ denote the standard inner product. Let $0 \neq \alpha \in \mathbb{R}^n$ and let $\mathscr{H}_\alpha$ denote the hyperplane of all vectors orthogonal to α. We create a function $s_\alpha : \mathbb{R}^n \to \mathbb{R}^n$ on $\mathbb{R}^n$ as follows: $s_\alpha(\lambda) := \lambda - 2\dfrac{\langle \lambda\ ,\ \alpha\rangle}{\langle \alpha\ ,\ \alpha\rangle}\,\alpha$. The function s_α has the following properties:

(i) If $\lambda \in \mathscr{H}_\alpha$, then $s_\alpha(\lambda) = \lambda$.

(ii) $s_\alpha(\alpha) = -\alpha$. That is, s_α is the reflection about $\mathscr{H}_\alpha$.

(iii)