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MATH 108
Fall 2022
HW 15: Due 11/22

"There is hardly any theory which is more elementary [than linear algebra], in spite of the fact that generations of professors and textbook writers have obscured its simplicity by preposterous calculations with matrices."

–Jean Dieudonné

Problem 1. (10pt) Let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^4$ be the following vectors: $\mathbf{u} = \begin{pmatrix} 1 \\ -2 \\ 3 \\ 4 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 10 \\ 8 \\ -2 \\ 3 \end{pmatrix}$.

Compute the following:

(a) $-2\mathbf{u}$

(b) $\mathbf{v} - \mathbf{u}$

(c) $\mathbf{u} \cdot \mathbf{v}$

(d) $\mathbf{v}^T$

Solution.

(a)

$$-2\mathbf{u} = -2 \begin{pmatrix} 1 \\ -2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ -6 \\ -8 \end{pmatrix}$$

(b)

$$\mathbf{v} - \mathbf{u} = \begin{pmatrix} 10 \\ 8 \\ -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 10-1 \\ 8-(-2) \\ -2-3 \\ 3-4 \end{pmatrix} = \begin{pmatrix} 9 \\ 10 \\ -5 \\ -1 \end{pmatrix}$$

(c)

$$\begin{pmatrix} 1 \\ -2 \\ 3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 10 \\ 8 \\ -2 \\ 3 \end{pmatrix} = 1(10) + (-2)8 + 3(-2) + 4(3) = 10 - 16 - 6 + 12 = 0$$

Note: The fac that $\mathbf{u} \cdot \mathbf{v} = 0$ shows that $\mathbf{u}$ and $\mathbf{v}$ are perpendicular.

(d)

$$\begin{pmatrix} 10 \\ 8 \\ -2 \\ 3 \end{pmatrix}^T = \begin{pmatrix} 10 & 8 & -2 & 3 \end{pmatrix}$$

Problem 2. (10pt) Define the following matrices:

$$A = \begin{pmatrix} 3 & 6 & 2 \\ -9 & 3 & -8 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & -4 & 2 \\ 1 & -8 & -3 \end{pmatrix}$$

Compute the following:

(a) $3B$

(b) $A + B$

(c) A^T

(d) $A^T B$

Solution.

(a)

$$3\begin{pmatrix} 1 & -4 & 2 \\ 1 & -8 & -3 \end{pmatrix} = \begin{pmatrix} 3 & -12 & 6 \\ 3 & -24 & -9 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 3 & 6 & 2 \\ -9 & 3 & -8 \end{pmatrix} + \begin{pmatrix} 1 & -4 & 2 \\ 1 & -8 & -3 \end{pmatrix} = \begin{pmatrix} 3+1 & 6-4 & 2+2 \\ -9+1 & 3-8 & -8+(-3) \end{pmatrix} = \begin{pmatrix} 4 & 2 & 4 \\ -8 & -5 & -11 \end{pmatrix}$$

(c)

$$\begin{pmatrix} 3 & 6 & 2 \\ -9 & 3 & -8 \end{pmatrix}^T = \begin{pmatrix} 3 & -9 \\ 6 & 3 \\ 2 & -8 \end{pmatrix}$$

(d)

$$\begin{aligned}
\begin{pmatrix} 3 & 6 & 2 \\ -9 & 3 & -8 \end{pmatrix}^T \begin{pmatrix} 1 & -4 & 2 \\ 1 & -8 & -3 \end{pmatrix} &= \begin{pmatrix} 3 & -9 \\ 6 & 3 \\ 2 & -8 \end{pmatrix} \begin{pmatrix} 1 & -4 & 2 \\ 1 & -8 & -3 \end{pmatrix} \\
&= \begin{pmatrix} 3(1)+(-9)1 & 3(-4)-9(-8) & 3(2)-9(-3) \\ 6(1)+3(1) & 6(-4)+3(-8) & 6(2)+3(-3) \\ 2(1)-8(1) & 2(-4)-8(-8) & 2(2)-8(-3) \end{pmatrix} \\
&= \begin{pmatrix} 3-9 & -12+72 & 6+27 \\ 6+3 & -24-24 & 12-9 \\ 2-8 & -8+64 & 4+24 \end{pmatrix} \\
&= \begin{pmatrix} -6 & 60 & 33 \\ 9 & -48 & 3 \\ -6 & 56 & 28 \end{pmatrix}
\end{aligned}$$

Problem 3. (10pt) Define the following matrices:

$$A = \begin{pmatrix} 1 & 0 \\ -2 & 3 \\ 0 & -1 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 0 & -1 & 5 \end{pmatrix}, \qquad C = \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix}$$

Only one of the following is computable: CA, AB, or AC. For those products that cannot be computed, explain why. For the product that can be computed, compute the product.

Solution. The matrix A has dimension 3×2. The matrix B has dimension 1×4. The matrix C has dimension 2×2. If P is a $m \times n$ matrix and Q is a $a \times b$ matrix, then PQ can be multiplied only if $n = a$. If so, then the resulting matrix, PQ, has dimension $m \times b$. But then neither AB nor BA can be formed. Similarly, neither BC nor CB can be formed. While CA cannot be formed, we can form the product AC. Then we have...

$$\begin{aligned} AC &= \begin{pmatrix} 1 & 0 \\ -2 & 3 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 1(2) + 0(-1) & 1(0) + 0(3) \\ -2(2) + 3(-1) & -2(0) + 3(3) \\ 0(2) + (-1)(-1) & 0(0) + (-1)3 \end{pmatrix} \\ &= \begin{pmatrix} 2+0 & 0+0 \\ -4-3 & 0+9 \\ 0+1 & 0-3 \end{pmatrix} \\ &= \begin{pmatrix} 2 & 0 \\ -7 & 9 \\ 1 & -3 \end{pmatrix} \end{aligned}$$