Name: ____________________________________
MATH 308
Fall 2021
HW 12: Due 11/12

"People think that computer science is the art of geniuses but the actual reality is the opposite, just many people doing things that build on each other, like a wall of mini stones."

–Donald Knuth

Problem 1. (10pt) Define a relation $\sim$ on $\mathbb{N} \times \mathbb{N}$ via $(x, y) \sim (a, b)$ if and only if $x - y = a - b$.

(a) Is $(3, 1) \sim (2, 5)$? Explain.

(b) Is $(7, 3) \sim (5, 1)$? Explain.

(c) Show that $\sim$ is an equivalence relation on X.

(d) Find at least 3 elements in each of the equivalence classes $[(1, 1)]$ and $[(3, 5)]$.

Problem 2. (10pt) Define a relation on $\mathbb{R}$ via $x \sim y$ if and only if $x \leq y$. Prove or disprove whether $\sim$ is an equivalence relation on $\mathbb{R}$.

Problem 3. (10pt) Define a relation on $\mathbb{R}^2$ via $(x, y) \sim (a, b)$ if and only if (x, y) and (a, b) are the same distance from the origin.

(a) Prove that $\sim$ is an equivalence relation.

(b) Describe the equivalence classes graphically.

(c) Describe graphically how the equivalence classes partition $\mathbb{R}^2$.

Problem 4. (10pt) Define a relation on $\mathbb{Z}$ via $a \sim b$ if and only if a and b have the same parity, i.e. a and b are either both even or they are both odd.

(a) Show that $\sim$ is an equivalence relation.

(b) Describe all the equivalence classes, i.e. determine the set $\mathbb{Z}/\sim$.

Problem 5. (10pt) Prove that if X is a set and S is a nonempty subset of X, then $\{S, X \setminus S\}$ is a partition of X.

Problem 6. (10pt) Let X be a nonempty set. Every equivalence relation $\sim$ on X gives rise to a partition on X. Moreover, every partition on X gives rise to an equivalence relation $\sim$ on X. We proved the first statement in class. Suppose that $\{X_i\}_{i\in\mathcal{I}}$ is a partition of X. Show that this partition induces an equivalence relation $X/\sim$ given by $a \sim b$ if and only if $a, b \in X_i$ for some $i \in \mathcal{I}$.