

Name: _____

MATH 308

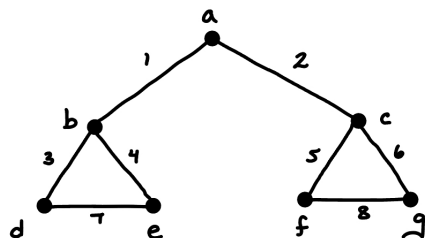
Fall 2023

HW 18: Due 12/12

"It has been said that geometry is the art of applying good reasoning to bad diagrams."

–Richard J. Trudeau

Problem 1. (10pt) Consider the graph G shown below.

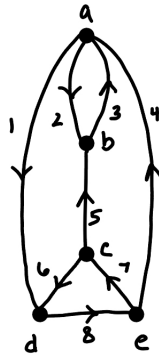


- (a) Is the graph G connected? Explain.
- (b) Is $d7e4b1a2c5f8g$ a trail? Explain. Is it a path? Explain.
- (c) Is $c5f8g6c2a$ a path? Explain. Is this walk closed? Explain.
- (d) Does this graph have a circuit? Explain.
- (e) Let A_G denote the adjacency matrix of G . Given the following:

$$A_G^{10} = \begin{pmatrix} 860 & 746 & 746 & 681 & 681 & 681 & 681 \\ 746 & 1282 & 940 & 884 & 884 & 543 & 543 \\ 746 & 940 & 1282 & 543 & 543 & 884 & 884 \\ 681 & 884 & 543 & 743 & 742 & 401 & 401 \\ 681 & 884 & 543 & 742 & 743 & 401 & 401 \\ 681 & 543 & 884 & 401 & 401 & 743 & 742 \\ 681 & 543 & 884 & 401 & 401 & 742 & 743 \end{pmatrix}$$

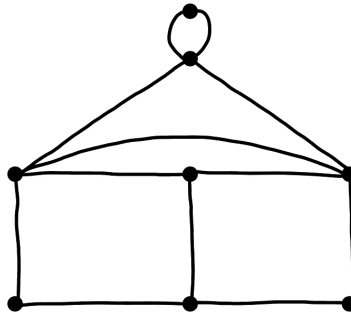
How many closed walks are there starting at a ? Explain.

Problem 2. (10pt) Consider the graph G shown below.



- (a) Does there exist a Hamiltonian circuit for this graph? Explain.
- (b) Does there exist an Euler circuit for this graph? Explain.
- (c) Find the adjacency matrix for this graph.
- (d) Find the number of walks from a to b of length 4. Be sure to justify your answer.

Problem 3. (10pt) Consider the graph G shown below.



- (a) Does there exist an Euler trail for this graph? If so, find one. If not, explain why.
- (b) Does there exist an Euler circuit for this graph? If so, find one. If not, explain why.
- (c) Does there exist a Hamiltonian circuit for this graph? If so, find one. If not, explain why.

Problem 4. (10pt) Showing all your work and fully justifying your reasoning, respond to the following:

- (a) Does there exist a tree with 2023 vertices and 2024 edges? Explain.
- (b) Does a graph with five vertices and four edges have to be a tree? Explain.
- (c) Find two non-isomorphic trees with five vertices. Be sure to explain why they cannot be isomorphic.