Name: *Caleb McWhorter — Solutions*
MATH 108
Spring 2022
Written HW 11: Due 05/02

"The laws of probability, so true in general, so fallacious in particular."
–Edward Gibbon

Problem 1. (10pt) The probabilities of several events in a finite probability space are given below:

$$\begin{aligned} P(A) &= 0.10 \quad & P(A \text{ and } B) &= 0.06 \\ P(B) &= 0.34 \quad & P(A \text{ and } C) &= 0.77 \\ P(C) &= 0.81 \quad & P(B \text{ or } C) &= 0.25 \\ P(D) &= 0.50 \quad & P(B \text{ and } D) &= 0.00 \end{aligned}$$

(a) Find $P(A \text{ or } B)$.

(b) Assuming A and D are independent events, find $P(A \text{ and } D)$.

(c) Find $P(A \mid B)$.

(d) Find $P(B \text{ and } C)$.

(e) Are C and D disjoint? Explain.

(f) Are A and C independent? Explain.

(g) Are B and D independent? Explain.

Solution.

(a)
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 0.10 + 0.34 - 0.06 = 0.38$$

(b)
$$P(A \text{ and } D) = P(A) \cdot P(D) = 0.10 \cdot 0.50 = 0.05$$

(c)
$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{0.06}{0.34} = 0.1765$$

(d)
$$\begin{aligned} P(B \text{ or } C) &= P(B) + P(C) - P(B \text{ and } C) \\ 0.25 &= 0.34 + 0.81 - P(B \text{ and } C) \\ P(B \text{ and } C) &= 0.90 \end{aligned}$$

(e) If C and D were disjoint, then $P(C \text{ or } D) = P(C) + P(D) = 0.81 + 0.50 = 1.31 > 1$, which is impossible. Therefore, C and D cannot be disjoint.

(f) If A and C were independent, then $P(A \text{ and } C) = P(A) \cdot P(C)$. But $P(A \text{ and } C) = 0.77$ while $P(A) \cdot P(C) = 0.10 \cdot 0.81 = 0.081$. Therefore, A and C are not independent.

(g) We see that $P(B \text{ and } D) = 0.00$. Therefore, B and D are disjoint. But then B and D cannot be independent because disjoint events are never independent.

Problem 2. (10pt) Of the most recent hit 'scamster' shows, people were surveyed about whether they had watched *The Dropout* or *Inventing Anna*. Fifty people were surveyed with twenty-one of them saying that they had seen *The Dropout*, thirty-seven of them saying that they had seen *Inventing Anna*, and twelve of them saying that they had watched both. Suppose you select one of these fifty people at random.

(a) Find the probability that they had seen *The Dropout*.

(b) Find the probability that they had only seen *Inventing Anna*.

(c) Find the probability that they had seen neither show.

(d) Find the probability that they had seen *The Dropout* given that they had seen *Inventing Anna*.

Solution.

(a)

$$P(\text{Dropout}) = \frac{9+12}{50} = \frac{21}{50} \approx 0.42$$

(b)

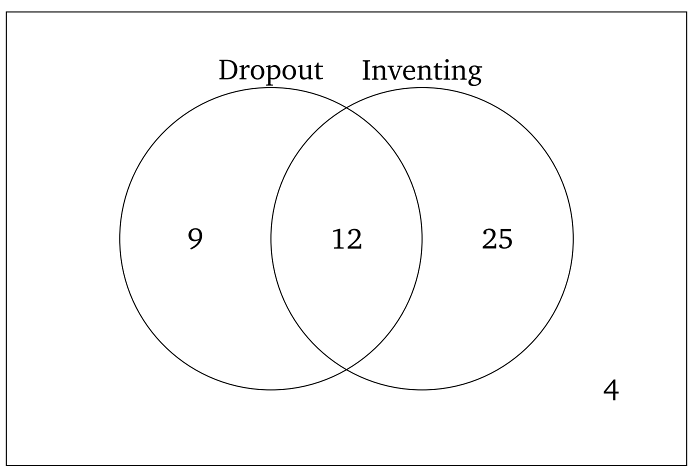
$$P(\text{Only Inventing}) = \frac{25}{50} \approx 0.50$$

(c)

$$P(\text{Neither}) = \frac{4}{50} \approx 0.08$$

(d)

$$P(\text{Dropout} \mid \text{Inventing}) = \frac{12}{12+25} = \frac{12}{37} \approx 0.3243$$



Problem 3. (10pt) Suppose a blood test for a common genetic marker correctly identifies when a person has the marker 97.4% of the time. The test incorrectly indicates that a person has the marker when they do not 6.5% of the time. It is estimated that 78% of the population possesses the genetic marker.

(a) What is the probability that a person that tests positive for the marker has the genetic marker?

(b) What is the probability that a person being tested for the marker will test positive or have the marker?

(c) What is the probability that a person that does not possess the marker will test negative for the marker?

(d) Given that a person tests for the marker, what is the probability that they actually have the marker?

Solution.

(a)

$$P(H \mid +) = \frac{0.75972}{0.75972 + 0.0143} = \frac{0.75972}{0.77402} = 0.981525$$

(b)

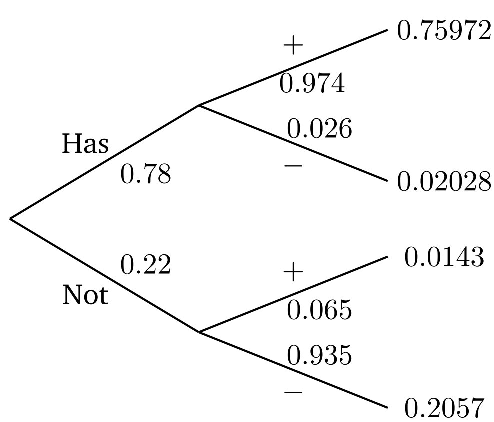
$$P(+ \text{ or } H) = 0.75972 + 0.02028 + 0.0143 = 0.7943$$

(c)

$$P(- \mid N) = \frac{0.2057}{0.0143 + 0.2057} = \frac{0.2057}{0.22} = 0.935$$

(d)

$$P(H \mid +) = \frac{0.75972}{0.75972 + 0.0143} = \frac{0.75972}{0.77402} = 0.981525$$



Problem 4. (10pt) You are playing a game where you roll a die. If you roll a number three or less, you win nothing. If you roll a four or a five, you win \$1. But if you roll a six, you win \$3.

(a) Find the probability that if you roll the die twice that you win nothing.

(b) Find the average amount you expect to win per game.

(c) If you had to pay \$2 to play this game, should you play this game? Explain.

Solution.

(a) To win nothing, you must roll a one, two, or three. The probability of this is $P(X = 1, 2, 3) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$. Because the dice rolls are independent, we have

$$P(\text{nothing twice}) = P(\text{nothing}) \cdot P(\text{nothing}) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

(b) The amount you expect to win/lose on average is the expected value. This is...

$$\mu = \sum_i x_i p_i = 0 \cdot \frac{1}{6} + 0 \cdot \frac{1}{6} + 0 \cdot \frac{1}{6} + 1 \cdot \frac{1}{6} + 1 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} = 0 + 0 + 0 + \frac{1}{6} + \frac{1}{6} + \frac{3}{6} = \frac{5}{6} \approx 0.83$$

(c) By (b), the amount you expect to win on average is \$0.83. However, you must pay \$2 each time to play the game. Then, on average, you expect to win $\$0.83 - \$2 = -\$1.17$ per game. But then you are losing money, on average, each game. Therefore, you should not play this game.