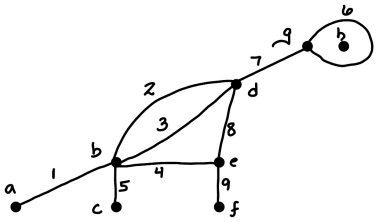
Name: _____

MATH 308
Fall 2023
HW 17: Due 12/12

"Geometric diagrams are to geometers what board and pieces are to chess masters: visual aids, helpful but not indispensable."
–Richard J. Trudeau

Problem 1. (10pt) Consider the graph G given below.



(a) Is b adjacent to f? Explain.

(b) Are 4 and 8 parallel? Explain.

(c) Are there isolated vertices? Explain.

(d) Is the graph simple? Explain.

(e) Is the graph connected? Explain.

(f) What are the endpoints of 5?

(g) is g incident to 3? Explain.

(h) What is $\deg(g)$?

(i) What is $\deg(b)$?

(j) What is the degree of G?

Problem 2. (10pt) Being sure to show all your work and fully justify your answers, complete the following:

(a) Draw a simple graph with three vertices that has a unique isolated vertex.

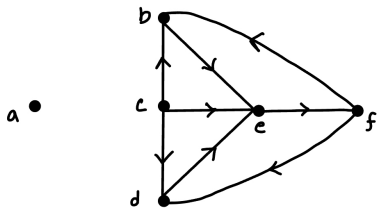
(b) Draw the graph K_6.

(c) Draw the graph $K_{5,2}$.

(d) Draw the undirected graph given by the following adjacency matrix:

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 2 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Problem 3. (10pt) Consider the graph G below.



(a) Is e adjacent to c? Explain. Is c adjacent to e? Explain.

(b) Find the adjacency matrix of the graph.

(c) Find the in and out degree of each vertex.

(d) Does G have any sources or sinks? Explain.

Problem 4. (10pt) The adjacency matrix for an undirected graph G is given below.

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

(a) Find $|V(G)|$ and $|E(G)|$.

(b) Are there any loops in G? Explain.

(c) Does G have parallel edges? Explain.

(d) Find the degree of G.

(e) How many connected components does G have? Explain.