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MATH 308

Fall 2023

HW 5: Due 10/05

*“No one shall expel us from the paradise
which Cantor has created for us.”*

–David Hilbert

Problem 1. (10pt) Define the following sets:

- (a) The set of positive odd numbers less than 50 that are not prime.
- (b) The set of real-valued solutions to $\frac{x^2-4}{x+2} = 0$.
- (c) The set of integer solutions to $\sqrt{2x-1} + 4 = 12$.
- (d) The set of English sentences containing a homonym.
- (e) The set of linear functions with positive y -intercept.

For each of the sets described above, do the following:

- (i) Determine if the set is empty or nonempty. If the set is nonempty, give an element and non-element of the set.
- (ii) Determine whether the set is finite or infinite. If it is finite, state its cardinality.
- (iii) If the set is finite, enumerate all its elements. If the set is infinite, give the set using set-builder notation.

Solution.

(a)

- (i)
- (ii)
- (iii)

(b)

- (i)
- (ii)
- (iii)

(c)

- (i)
- (ii)
- (iii)

(d)

- (i)

(ii)

(iii)

(e)

(i)

(ii)

(iii)

— *Continued Space for Problem 1* —

Problem 2. (10pt) For each of the sets given below, describe the sets in words. Also for each set, give an example of an element and non-element of the set.

(a) $\{3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, \dots\}$

(b) $\{0, 1, 4, 9, 16, 25, 36, 49, 81, 100, \dots\}$

(c) $\{f(x) : f(x) \text{ function}, (\exists x_0 \in \mathbb{R})[f(x_0) = 0]\}$

(d) $\{f(x) : f(x) \text{ function}, f(x) > 0 \text{ for all } x \in \mathbb{R}\}$

(e) $\{f(x, y) : f(2, 3) = 0\}$

Solution.

(a)

(b)

(c)

(d)

(e)

Problem 3. (10pt) Define the following sets:

$$A = \{-10, -9, -8, \dots, 8, 9, 10\}$$

$$B = \{-10, -8, -6, \dots, 6, 8, 10\}$$

$$C = \{-9, -7, -5, \dots, 5, 7, 9\}$$

$$D = \{-10, -5, 0, 5, 10\}$$

$$E = \{-4, -1, 1, 2, 3, 5, 7\}$$

$$F = \{-10, -9, -2, -1, 1, 5, 6, 9\}$$

Consider each of the sets above as coming from the universal set $\mathcal{U} := A$. Compute the following:

(a) $B \cup C$

(d) $F \Delta C$

(b) $B \cap C$

(e) E^c

(c) $E \setminus D$

(f) $(C \cup E) - B$

Solution.

(a)

(b)

(c)

(d)

(e)

(f)

Problem 4. (10pt) Define the following sets:

$$A := (-10, 10)$$

$$B := [0, 3]$$

$$C := (-1, 15]$$

$$D := (-20, -3] \cup [4, 12)$$

Consider each of the sets above as coming from the universal set $\mathbb{R} = (-\infty, \infty)$. Compute the following:

(a) D^c

(b) $B \cap C$

(c) $A \Delta C$

(d) $C - B$

(e) $(A \cap D) \cup C$

Solution.

(a) $D^c =$

(b) $B \cap C =$

(c) $A \Delta C =$

(d) $C - B =$

(e) $(A \cap D) \cup C =$