

Name: \_\_\_\_\_

MATH 308

Fall 2021

HW 2: Due 09/24

*“Some people are immune to good advice.”*

*– Saul Goodman, Breaking Bad*

**Problem 1.** (10pt) Determine if the following sentences are predicates. If the sentence is a predicate, mark it ‘T’; otherwise, mark the sentence ‘F.’ [Let the universal set be  $\mathbb{R}$ .]

(a) \_\_\_\_\_:  $x$  is odd.

(b) \_\_\_\_\_:  $x^2 + x + 1$

(c) \_\_\_\_\_:  $P(x): x^2 + 1 < 0$

(d) \_\_\_\_\_:  $Q(x): x$  is an integer

(e) \_\_\_\_\_:  $R(x, y): x^2 < y^3$

**Problem 2.** (10pt) Give an original example of a predicate having more than one variable.

**Problem 3.** (10pt) Let  $P(x)$  be the predicate  $P(x): 1 \leq 2^x \leq 100$ . Suppose that the domain is the nonnegative integers. What is the truth set for  $P(x)$ ? What is the truth set if the domain were instead the set of real numbers?

**Problem 4.** (10pt) Defining appropriate propositional functions and variables, write the following English sentences using logical symbols and the functions/variables that you defined.

- (a) Every cloud has a silver lining.
- (b) All that glitters is not gold.
- (c) Every human is guilty of all the good that they did not do.
- (d) None but the brave deserve the fair.

**Problem 5.** (10pt) Define the following predicates:

- (i)  $P(x): x > 0$
- (ii)  $Q(x): x$  is even
- (iii)  $R(x): x$  is a perfect square
- (iv)  $S(x): x$  is divisible by 4
- (v)  $T(x): x$  is divisible by 5

Write the following in symbolic form:

- (a) Any perfect square is positive.
- (b) If an integer is divisible by 4, then the integer is even.
- (c) No even integer is divisible by 5.

Write the following in the form of an English sentence:

- (d)  $\forall x (S(x) \rightarrow Q(x))$
- (e)  $\exists x (S(x) \wedge \neg R(x))$

**Problem 6.** (10pt) Let  $P(x)$  be the predicate  $x^2 = x$ . Determine if the following statements are true or false:

(a) \_\_\_\_\_:  $P(0)$

(b) \_\_\_\_\_:  $P(-1)$

(c) \_\_\_\_\_:  $\forall x, P(x)$

(d) \_\_\_\_\_:  $\exists x, P(x)$

(e) \_\_\_\_\_:  $\exists!x, P(x)$

**Problem 7.** (12pt) Let  $P(x)$ ,  $Q(x)$ ,  $R(x)$  denote the predicates  $1 - 2x = 7$ ,  $x^2 = 9$ , and  $x^2 > 9$ , respectively. Determine whether the following propositions are true or false. If the statement is true, explain why. If the statement is false, give a counterexample.

- (a)  $(\forall x)(P(x) \wedge Q(x))$
- (b)  $(\exists x)(P(x) \wedge Q(x))$
- (c)  $(\forall x)(P(x) \rightarrow Q(x))$
- (d)  $(\forall x)(P(x) \rightarrow R(x))$
- (e)  $(\exists x)(P(x) \vee R(x))$
- (f)  $(\exists!x)(P(x) \wedge Q(x))$

**Problem 8.** (10pt) What well-known property does the following proposition represent:  $\forall x \forall y \forall z [x + (y + z) = (x + y) + z]$ .



**Problem 9.** (10pt) Determine if the following statements are true or false:

(a) \_\_\_\_\_:  $\exists x \exists y (xy = 1)$

(b) \_\_\_\_\_:  $\exists x \forall y (xy = 1)$

(c) \_\_\_\_\_:  $\forall x \exists y (xy = 1)$

(d) \_\_\_\_\_:  $\forall x \forall y (x^2 + y = 1)$

(e) \_\_\_\_\_:  $\forall x \exists y (x^2 + y = 1)$

**Problem 10.** (10pt) Negate the following proposition:

$$\forall x \exists y (P(x, y) \wedge Q(x, y) \rightarrow R(x, y))$$

**Problem 11.** (10pt) One way of stating the definition for a function  $f(x)$  to have limit  $L$  at  $x$ , i.e.  $\lim_{x \rightarrow a} f(x) = L$ , is as follows:

$$(\forall \epsilon > 0)(\exists \delta > 0)(\forall x \in \mathbb{R})[0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon]$$

Give a definition for a function  $f(x)$  to not have a limit at  $x = a$  by negating the statement above. Your answer should not contain any negations.

**Problem 12.** (10pt) The universal and existential quantifier do not necessarily ‘distribute’ over  $\wedge$  and  $\vee$ . One of the following ‘equivalences’ is not correct:

$$\forall x (P(x) \wedge Q(x)) \iff \forall x P(x) \wedge \forall x Q(x)$$

$$\forall x (P(x) \vee Q(x)) \iff \forall x P(x) \vee \forall x Q(x)$$

Determine which one is always true and state it. For the one that is false, give an example to show that it is false.