Name: *Caleb McWhorter — Solutions*
MATH 101
Spring 2024
HW 3: Due 01/31

"And I knew exactly what to do. . . but in a much more real sense, I had no idea what to do."
— Michael Scott, The Office

Problem 1. (10pts) Showing all your work, simplify the following as much as possible (express any denominators using negative powers):

(a) $\dfrac{x^5y^3}{x^3y^9}$

(b) $\dfrac{(x^2y^{-3})^4}{x^0y^2}$

(c) $\dfrac{(x^8y^3)^0xy^7}{(x^2)^3y}$

Solution.

(a)

$$\frac{x^5y^3}{x^3y^9} = \frac{\cancel{x^2}\,\cancel{y^3}}{\cancel{x^3}\,\cancel{y^6}} = \frac{x^2}{y^6} = x^2y^{-6}$$

(b)

$$\frac{(x^2y^{-3})^4}{x^0y^2} = \frac{x^8y^{-12}}{1\cdot y^2} = \frac{x^8}{y^{12}y^2} = \frac{x^8}{y^{14}} = x^8y^{-14}$$

(c)

$$\frac{(x^8y^3)^0xy^7}{(x^2)^3y} = \frac{1\cdot xy^7}{x^6y} = \frac{\cancel{xy}^{\,1}y^6}{\cancel{x}^{\,5}\cancel{y}} = \frac{y^6}{x^5} = y^6x^{-5}$$

Problem 2. (10pts) Showing all your work, simplify the following as much as possible (do not express your answer using any negative powers):

(a) $\dfrac{x^{-2}yz^6}{xy^{-6}z^5}$

(b) $\dfrac{(xy^{-2})^{-1}}{x^3y^{-7}}$

(c) $\left(\dfrac{x^5y^{-4}}{(x^{-4}y^3)^{-8}}\right)^0$

Solution.

(a)

$$\frac{x^{-2}yz^6}{xy^{-6}z^5} = \frac{y^6yz^6}{x^2xz^5} = \frac{y^7z^6}{x^3z^5} = \frac{y^7z^{\cancel{6}^{1}}}{x^3\cancel{z^5}} = \frac{y^7z}{x^3}$$

(b)

$$\frac{(xy^{-2})^{-1}}{x^3y^{-7}} = \frac{x^{-1}y^2}{x^3y^{-7}} = \frac{y^7y^2}{x^1x^3} = \frac{y^9}{x^4}$$

(c)

$$\left(\frac{x^5y^{-4}}{(x^{-4}y^3)^{-8}}\right)^0 = 1$$

Problem 3. (10pts) Showing all your work, simplify the following as much as possible (do not express your answer using any negative powers):

$$\frac{\left((xyz)^5 xz^{-4}\right)^2 x^{-5}}{xy^{-3}z^{-2}}$$

Solution.

$$\frac{\left((xyz)^5 xz^{-4}\right)^2 x^{-5}}{xy^{-3}z^{-2}}$$

$$\frac{\left(x^5y^5z^5 xz^{-4}\right)^2 x^{-5}}{xy^{-3}z^{-2}}$$

$$\frac{\left(x^6y^5z^1\right)^2 x^{-5}}{xy^{-3}z^{-2}}$$

$$\frac{x^{12}y^{10}z^2x^{-5}}{xy^{-3}z^{-2}}$$

$$\frac{x^7y^{10}z^2y^3z^2}{x}$$

$$\frac{x^7y^{13}z^4}{x}$$

$$\frac{\cancel{x^7}^{6}y^{13}z^4}{\cancel{x}}$$

$$x^6y^{13}z^4$$