Name: _____

MATH 308
Fall 2022
HW 2: Due 09/13

"Science is simply common sense at its best; that is, rigidly accurate in observation, and merciless to fallacy in logic."

– Thomas Huxley

Problem 1. (10pt) Show that $P \wedge (\neg Q \vee R) \to (P \wedge R) \vee \neg Q$ is a tautology by constructing its truth table. Using words, explain why this expression is a tautology without making reference to its truth table.

Problem 2. (10pt) Let n be a fixed natural number. Consider the statement, "If n is even, then n is not prime or n is 2."

(a) By defining appropriate propositions, write the given statement as a logical expression.

(b) Write the converse of your answer in (a). Then write this logical expression in words. Is this statement always true? Explain.

(c) Write the contrapositive of your answer in (a). Then write this logical expression in words. Is this statement always true? Explain.

Problem 3. (10pt) Suppose the statement, "If this metal is glowing red, then its temperature must be at least 460°C," is true. Indicate whether the following statements are true or false. No justification is necessary.

(a) If the temperate of the metal is at least 460°C, then it is glowing red.

(b) If the temperature of the metal is less than 460°C, then it is not glowing red.

(c) The metal will only glow red if its temperature is at least 460°C.

(d) If the metal is now glowing red, then its temperature is less than 460°C.

(e) A necessary condition for the metal to glow red is that its temperature be at least 460°C.

(f) A sufficient condition for the metal to glow red is that its temperature is at least 460°C.

Problem 4. (10pt) Suppose one defines a logical connective '$\boxplus$' by saying that $P \boxplus Q$ is true if and only if *both* P and Q are true.

(a) Construct the truth table for $P \boxplus Q$.

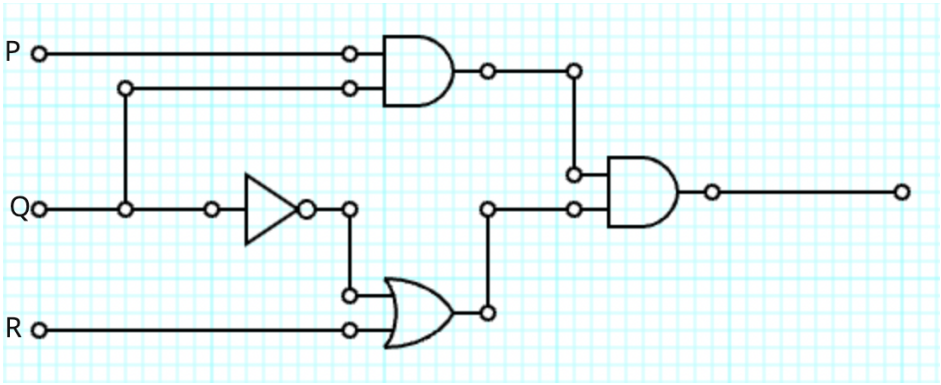
(b) Using a truth table, show that $(P \boxplus Q) \equiv (P \leftrightarrow Q) \wedge P$ and $(P \boxplus Q) \equiv (P \leftrightarrow Q) \wedge Q$.

(c) Without referencing a truth table, explain the logical equivalences in (b).

(d) Using the result from (b)—but without referencing a truth table—explain why $\big((P \boxplus Q)\big) \leftrightarrow \big((P \leftrightarrow Q) \wedge P\big)$ and $\big((P \boxplus Q)\big) \leftrightarrow \big((P \leftrightarrow Q) \wedge P\big)$ are tautologies.

(e) Without using a truth table, explain why $P \boxplus Q \equiv Q \boxplus P$.

Problem 5. (10pt) Find a Boolean expression that represents the following circuit and construct its input/output table.



Problem 6. (10pt) Draw a circuit diagram corresponding to the Boolean expression $\neg(P \vee Q) \wedge Q$.

Problem 7. (10pt) Suppose a certain tax exemption applies to individuals that make less than than $55,000 a year, or have three dependents and have at least $6,000 in charitable donations. Define appropriate propositions and write the condition for this tax exemption as a Boolean expression. Then construct a circuit diagram corresponding to this Boolean expression.

Problem 8. (10pt) Watch at least one of the following videos:

- Exploring How Computers Work
- How Do Computers Remember?

Then as thoroughly as possible, comment on what you observed and learned from the video. Be sure to remark as much as possible on how these videos connect to the course content.