

Name: \_\_\_\_\_

**MATH 308**

**Fall 2021**

**HW 6: Due 10/08**

*“So I was not born with a whole lot of natural talent. But I work hard  
and I never give up. That is my gift. That is my ninja way!”*

*–Rock Lee, Naturo*

**Problem 1.** (20pt) Describe all sets (if any) with. . .

- (a) no proper subsets.
- (b) one proper subset.
- (c) two proper subsets.

**Problem 2.** (20pt) The symmetric difference of two sets  $A$  and  $B$ , denoted  $A\Delta B$ , is defined by  $A\Delta B := (A \setminus B) \cup (B \setminus A)$ .

- (a) Describe  $A\Delta B$  in words.
- (b) Show that  $A\Delta B = (A \cup B) - (A \cap B)$ .
- (c) Prove that the symmetric difference is commutative.
- (d) Prove that if  $A\Delta B = \emptyset$ , then  $A = B$ . Is the converse true?

**Problem 3.** (20pt) Let  $A, B$  be sets with a common universal set  $\mathcal{U}$ . Prove the following:

(a)  $A - (A - B) = A \cap B$

(b)  $A \subseteq B$  if and only if  $B^c \supseteq A^c$

**Problem 4.** (10pt) If  $A \subseteq U$  and  $B \subseteq V$ , is  $A \times B \subseteq U \times V$ ? Justify your answer.

**Problem 5.** (10pt) Suppose that  $X$  and  $Y$  are sets with a common universal set  $\mathcal{U}$ . Show that  $X = Y$  if and only if  $(X \cap Y^c) \cup (X^c \cap Y) = \emptyset$ .

**Problem 6.** (20pt) Prove or disprove:

(a)  $(A \cup B) \setminus B = A$

(b)  $A \cap (B \setminus C) = (A \cap B) - (A \cap C)$

(c)  $A \cap (B \setminus C) = (A \cap B) \setminus C$

(d)  $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

**Problem 7.** (20pt) Express the following sets as an interval, collection of intervals, or well known set (prove your answer):

(a)  $\bigcap_{n \geq 1} \left[0, 1 + \frac{1}{n}\right)$

(b)  $\bigcup_{n \geq 1} \left[0, 1 + \frac{1}{n}\right)$

(c)  $\bigcup_{n \in \mathbb{Z}} \bigcap_{m \geq 1} \left(n - \frac{1}{m}, n + \frac{1}{m}\right)$