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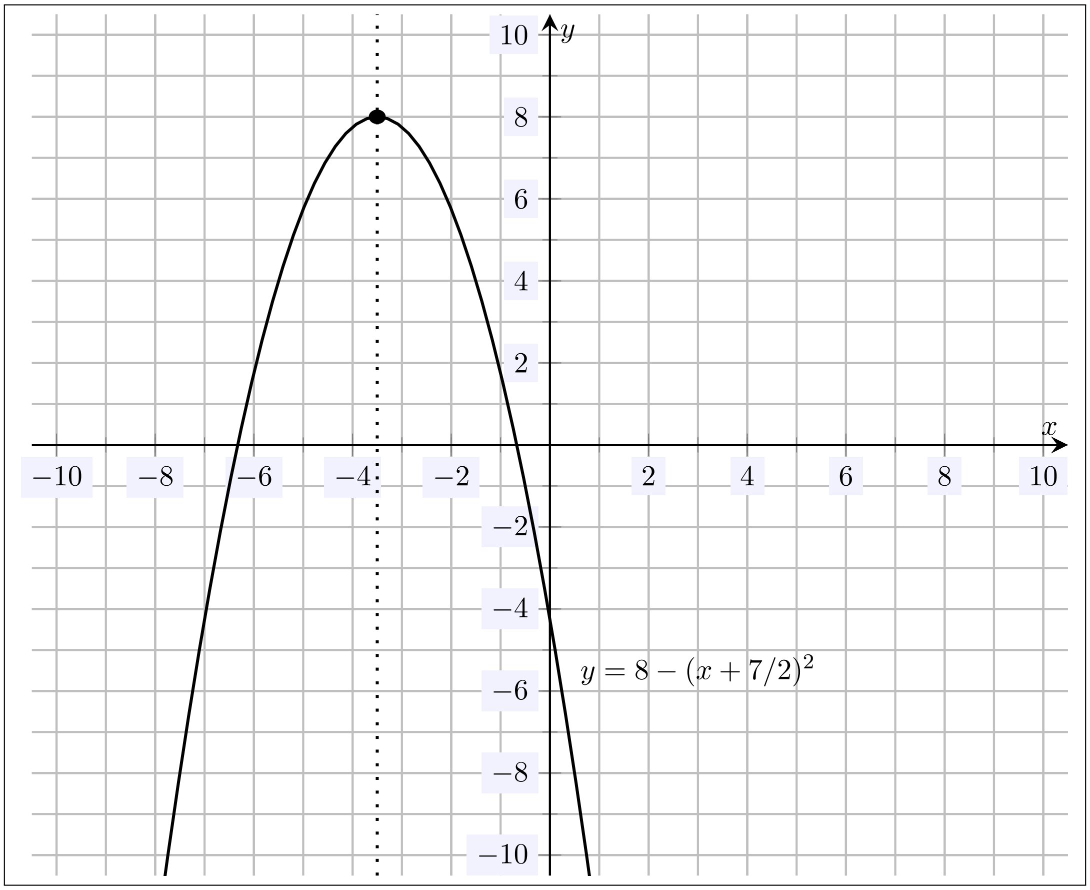
MATH 101
Spring 2022
HW 8: Due 03/29

"You only live once, but if you do it right, once is enough."
–Mae West

Problem 1. (10pt) Give a rough sketch of the quadratic function $y = 8 - (x + 7/2)^2$. Your sketch should include the vertex and axis of symmetry.



Solution. We place the quadratic function in vertex form, i.e. $y = (x - p)^2 + q$. We have...

$$y = 8 - (x + 7/2)^2 = -(x + 7/2)^2 + 8 = -\left(x - \frac{-7}{2}\right)^2 + 8$$

Therefore, we have vertex $(-7/2, 8)$. The axis of symmetry is then $x = -\frac{7}{2}$. Because $a = -1 < 0$, the parabola opens downwards. This gives us the sketch above.

Problem 2. (10pt) Find the vertex form of the function $f(x) = 8x^2 + 24x + 13$ both by completing the square and using the 'evaluation-method.'

Solution. Completing the square, we have...

$$\begin{aligned} 8x^2 + 24x + 13 &= 8\left(x^2 + 3x + \frac{13}{8}\right) \\ &= 8\left(x^2 + 3x + \frac{9}{4} - \frac{9}{4} + \frac{13}{8}\right) \\ &= 8\left(\left(x^2 + 3x + \frac{9}{4}\right) - \frac{9}{4} + \frac{13}{8}\right) \\ &= 8\left(\left(x + \frac{3}{2}\right)^2 - \frac{18}{8} + \frac{13}{8}\right) \\ &= 8\left(\left(x + \frac{3}{2}\right)^2 - \frac{5}{8}\right) \\ &= 8\left(x + \frac{3}{2}\right)^2 - 5 \end{aligned}$$

Using the 'evaluation-method', we know the vertex occurs at $x = -\frac{b}{2a} = -\frac{24}{2(8)} = -\frac{24}{16} = -\frac{3}{2}$. We then have...

$$f(-3/2) = 8\left(-\frac{3}{2}\right)^2 + 24\left(-\frac{3}{2}\right) + 13 = 18 - 36 + 13 = -5$$

Therefore, the vertex is $(-3/2, -5)$. Because we have $a = 8$, this gives...

$$f(x) = a(x-p)^2 + q = 8\left(x - \frac{-3}{2}\right)^2 + (-5) = 8\left(x + \frac{3}{2}\right)^2 - 5$$

Problem 3. (10pt) Consider the function $f(x) = (x-8)^2 - 27$.

(a) Determine if the given parabola opens upwards or downwards.

(b) Is the parabola convex or concave?

(c) Does the function $f(x)$ have a maximum or a minimum?

(d) Find the vertex and axis of symmetry.

(e) Find the maximum/minimum value of $f(x)$.

Solution.

(a) Because $a = 1 > 0$, the parabola opens upwards.

(b) Because $a = 1 > 0$, the parabola opens upwards so that it is convex.

(c) Because the parabola opens upwards, we know that the quadratic function has a minimum.

(d) Because $f(x) = (x-8)^2 - 27$ is in vertex form, we know that the vertex is $(8, -27)$. Therefore, the axis of symmetry is $x = 8$.

(e) Because $f(x) = (x-8)^2 - 27$ is in vertex form, we know that the minimum value is the y-coordinate of the vertex, which is -27.

Problem 4. (10pt) Consider the function $f(x) = x^2 + 6x + 3$.

(a) Find the vertex form of $f(x)$.

(b) Determine if the given parabola opens upwards or downwards.

(c) Is the parabola convex or concave?

(d) Does the function $f(x)$ have a maximum or a minimum? Find this value.

(e) Find the vertex and axis of symmetry.

Solution.

(a) Completing the square, we have...

$$x^2 + 6x + 3 = x^2 + 6x + 9 - 9 + 3 = (x^2 + 6x + 9) + (-9 + 3) = (x + 3)^2 - 6$$

Alternatively, using the 'evaluation-method', we know the vertex occurs when $x = -\frac{b}{2a} = -\frac{6}{2(1)} = -3$. We have $f(-3) = (-3)^2 + 6(-3) + 3 = 9 - 18 + 3 = -6$. Because $a = 1$, we know that $f(x) = a(x - p)^2 + q = 1(x - (-3))^2 + (-6) = (x + 3)^2 - 6$.

(b) Because $a = 1 > 0$, the parabola opens upwards, i.e. it is convex.

(c) Because $a = 1 > 0$, the parabola opens upwards and is therefore convex.

(d) Because the parabola opens upwards, the quadratic function has a minimum. We know the vertex form of the parabola, $f(x) = (x + 3)^2 - 6$. Therefore, the vertex is $(-3, -6)$. This implies that the minimum value is -6—the y-coordinate of the vertex.

(e) We know the vertex form of the parabola, $f(x) = (x + 3)^2 - 6$. Therefore, the vertex is $(-3, -6)$. This implies that the axis of symmetry is $x = -3$.