

Name: _____

MATH 308

Fall 2021

HW 5: Due 10/08

“Penny, while I subscribe to the many worlds theory which posits the existence of an infinite number of Sheldons in an infinite number of universes—I assure you that in none of them am I dancing.”

– Sheldon Cooper, Big Bang Theory

Problem 1. (10pt) List at least 3 elements from each of the following sets:

(a) $\{n \in \mathbb{N} : \exists k \in \mathbb{N}, n = 6k\}$

(b) $\{x \in \mathbb{R} : \exists y \in \mathbb{R}, x = y^2\}$

(c) $\{m \in \mathbb{N} : \sqrt[3]{m} \in \mathbb{N}\}$

(d) $\{q \in \mathbb{Q} : 4q + 1 \in \mathbb{N}\}$

(e) $\{a \in \mathbb{N} : \exists b \exists c, b, c \in \mathbb{N}, a^2 + b^2 = c^2\}$

Problem 2. (10pt) Use the set-builder notation to give a set equal to each of the following sets:

- (a) $\{1, 4, 9, 16, 25, 36, 49, 64, \dots\}$
- (b) $\{0, \pm 3, \pm 6, \pm 9, \pm 12, \pm 15, \dots\}$
- (c) The set of rational numbers between 0 and 1.
- (d) The set of functions passing through the point $(6, 5)$.
- (e) The set of differentiable functions with a horizontal tangent line at $x = 1$.

Problem 3. (10pt) Let $\mathcal{U} = \{1, 2, 3, \{1\}, \{2\}, \{1, 2\}\}$. Let $A = \{2, 1, 2\}$ and $B = \{1\}$.

(a) Is $A \in \mathcal{U}$? Explain.

(b) Is $A \subseteq \mathcal{U}$? Explain.

(c) Is $B \in \mathcal{U}$? Explain.

(d) Is $B \subseteq \mathcal{U}$? Explain.

Problem 4. (20pt) Define the following sets:

$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$B = \{1, 3, 5, 7, 9\}$$

$$C = \{2, 4, 6, 8, 10\}$$

$$D = \{2, 3, 5, 7\}$$

$$E = \{4, 8, 9\}$$

$$F = \{1, 2, \{3\}\}$$

Compute the following sets:

(a) $A \cap B$

(b) $C \cup D$

(c) $D \cap E$

(d) $D \setminus B$

(e) $B \setminus A$

(f) $B \times C$

(g) $(D \cap F) \cup (B \cap E)$

In addition, answer the following:

(h) Is $F \subseteq A$? Explain.

(i) Is $B \cap F = \{1, 3\}$? Explain.

(j) Is A a universal set for B, C, D, E, F ? If it is, compute D^c . If not, explain why.

Problem 5. (10pt) Compute each of the following sets:

(a) $\mathcal{P}(\emptyset)$

(b) $\mathcal{P}(\{1, \{1\}\})$

(c) $\mathcal{P}(\{1, e, \pi\})$

(d) $\mathcal{P}(\{1\} \times \{a, b\})$

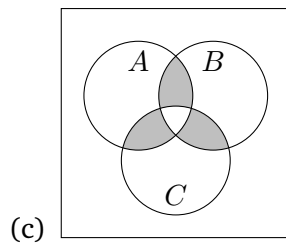
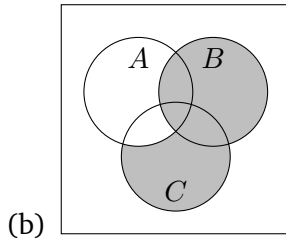
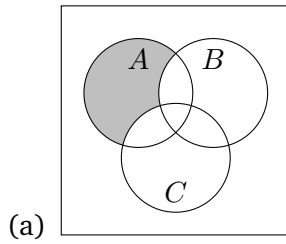
Problem 6. (10pt) Suppose A, B are sets with a common universal set \mathcal{U} . Denote each of the following sets with a Venn diagram:

(a) $A \cap B^c$

(b) $(A \cup B)^c$

(c) $(A \cup B) \setminus (A \cap B)$

Problem 7. (10pt) Suppose A, B, C are sets with a common universal set \mathcal{U} . For each of the Venn diagrams, write down the shaded sets.



Problem 8. (10pt) Let $A = \{b, c\}$. Suppose that $A \cup B = \{a, b, c, e\}$ and $B \cup C = \{a, c, d, e, f\}$. From this information can we determine the sets A, B, C ? Explain. If not, what is the minimal additional information (in terms of unions and intersections of the sets alone) would uniquely determine the three sets?