

MAT 308: Exam 3
Fall – 2023
12/14/2023
'∞' Minutes

Name: _____

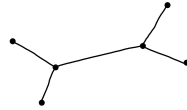
Write your name on the appropriate line on the exam cover sheet. This exam contains 11 pages (including this cover page) and 10 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total:	100	

1. (10 points) There have been numerous studies showing that physicians, medical staff, and patients have trouble understanding and interpreting conditional probabilities. One study showed that physicians had serious misunderstandings about the following hypothetical statistics problem: the probability of breast cancer is 2% for women of a particular age group participating in a routine screening. The medical procedure used to identify this breast cancer is 97% accurate at identifying cancer when it is present and misidentifies a woman as having breast cancer when do not 2% of the time.
- (a) Find the probability that a woman in this age group will test positive for breast cancer.
 - (b) Find the probability that a woman in this age group will test positive for breast cancer or have breast cancer.
 - (c) Find the probability that a woman in this age group will have breast cancer and test positive.
 - (d) If a woman in this age group tests positive for breast cancer, what is the probability that she actually has cancer?

2. (10 points) Showing all your work and fully justifying your reasoning, respond to the following:

(a) Is the graph shown below a tree? Explain.



- (b) What is the degree of a tree with n vertices?
- (c) Are all trees with the same number of vertices isomorphic? If so, explain why. If not, give a counterexample.
- (d) Are there always unique paths between distinct vertices in a tree with $n \geq 1$ vertices? If so, explain why. If not, give a counterexample.

3. (10 points) Define a *pseudo tripartite* graph on (m, n, p) vertices, denoted $K_{m,n,p}$, as follows:

- (i) $K_{m,n,p}$ has vertices v_1, v_2, \dots, v_m , w_1, w_2, \dots, w_n , and x_1, x_2, \dots, x_p .
- (ii) There exists a unique edge connecting v_i and w_j for all $i = 1, \dots, m$ and $j = 1, \dots, n$ and there exists a unique edge connecting w_r and x_s for all $r = 1, \dots, n$ and $s = 1, \dots, p$.
- (iii) There exists no edge between v_i and v_j for all i, j . There exists no edge between w_i and w_j for all i, j . There exists no edge between x_i and x_j for all i, j .

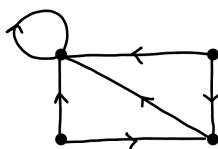
Given the definition above, complete the following:

- (a) Sketch $K_{3,2,4}$.
- (b) What is $|V(K_{m,n,p})|$? Explain.
- (c) What is $|E(K_{m,n,p})|$? Explain.
- (d) Is $K_{m,n,p}$ connected? Explain.
- (e) Can $K_{m,n,p}$ be isomorphic to $K_{r,s}$ for any $m, n, p, r, s \geq 1$? Explain.

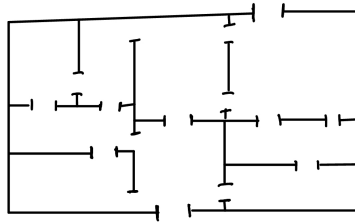
4. (10 points) Let G be the graph whose adjacency matrix, A_G , is given below.

$$\begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

- (a) Sketch G .
- (b) If G' has adjacency matrix $A_{G'}$ and $A_G = A_{G'}$, must it be that G and G' are isomorphic?
- (c) Can G be isomorphic to the graph below? Explain.

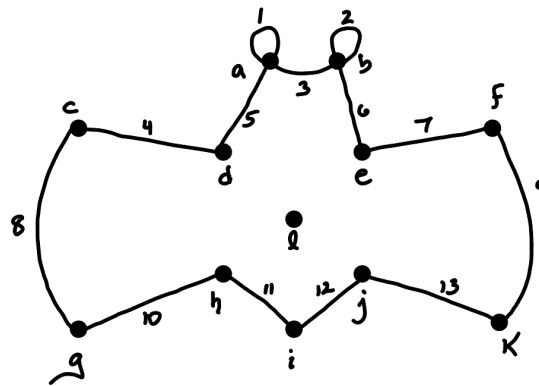


5. (10 points) Consider the floor plan shown below.



- (a) Is it possible to start at the front door and walk through each door of the house once and end at the back door? Explain.
- (b) Is it possible to find a 'path' that starts and ends in the same room such that you walk through each room in the house once? Explain.
- (c) Is it possible to find a 'path' that starts and ends in the same room such that you walk through each door in the house once? Explain.

6. (10 points) Consider the graph G show below.



- | | |
|--|---|
| (a) What is $ V(G) $? | (g) Is the graph connected? Explain. |
| (b) What is $ E(G) $? | (h) Are there isolated vertices? Explain. |
| (c) Are there loops? Explain. | (i) Is c incident to 4? Explain. |
| (d) Are there parallel edges? Explain. | (j) What is $\deg(b)$? |
| (e) Are 7 and 9 adjacent? Explain. | (k) What is the degree of the graph? |
| (f) Are h and j adjacent? Explain. | |

7. (10 points) Showing all your work and fully justifying your reasoning, complete the following:
- (a) Sketch the bipartite graph $K_{6,2}$.
 - (b) Is a complete graph with n vertices, K_n , connected for all n ? Explain.
 - (c) Is every bipartite graph simple? Explain.
 - (d) For $n > 2$, can K_n be isomorphic to $K_{r,s}$ for some r, s ? Explain.

8. (10 points) A gambler is playing a dice game using a specially made six-sided die. This die has atypical probabilities. The probabilities for this die are partially given below.

n	1	2	3	4	5	6
$P(n)$	$\frac{3}{15}$		$\frac{4}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

If the player rolls a 1 or 2, they must pay \$3 or \$2, respectively. If the player rolls a 3, they win/lose nothing. If the player rolls a 4, 5, or 6, they win \$1, \$2, or \$80, respectively.

- (a) Find $P(2)$.
- (b) Find the amount that one wins 'on average' playing this game, i.e. the expected value.
- (c) If one must pay \$20 upfront and then \$2 each time they wish to roll the die, should one play this game? Explain.

9. (10 points) Let G be a graph with the adjacency matrix, A_G , given below.

$$\begin{pmatrix} 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

- (a) Is G directed or undirected? Explain.
- (b) How many vertices and edges does G have? Explain.
- (c) Does G have isolated vertices? Explain.
- (d) How many connected components does G have? Explain.
- (e) Does G have any loops? If so, how many. If not, explain why.
- (f) Find $\deg^+ v_4$ and $\deg^- v_4$.

10. (10 points) The RMS Titanic was a British passenger ship that sank on its maiden voyage across the Atlantic Ocean after striking an iceberg—killing the majority of its 2,224 passengers. A summary of the mortality statistics for the Titanic are given below.

	Men	Women	Boys	Girls	Total
Survived	338	316	29	27	710
Died	1352	109	35	18	1514
Total	1690	425	64	45	2224

	Survived	Died	Total
First Class Men	57	118	175
First Class Women	140	4	144
First Class Children	5	1	6
Second Class Men	14	154	168
Second Class Women	80	13	93
Second Class Children	24	0	24
Third Class Men	75	387	462
Third Class Women	76	89	165
Third Class Children	27	52	79
Crew (Men)	192	693	885
Crew (Women)	20	3	23
Total	710	1514	2224

- Find the probability that a randomly selected person aboard the Titanic survived or was a child.
- Find the probability that a randomly selected person aboard the Titanic was a female that survived the sinking.
- Find the probability that a randomly selected person aboard the Titanic was a crew member.
- Find the probability that a randomly selected person aboard the Titanic was a woman, if they survived the sinking.
- Based on the data given, determine whether class was tied to, i.e. dependent on, survival expectancy. Explain how you came to your conclusion based on the data using probability.