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MATH 101

Fall 2022

HW 12: Due 11/07

Problem 1. (10pt) Consider the line $2x - 5y = 4$.

   (a) Is $(-2, 0)$ on the line? Explain.

   (b) Is $(-3, -2)$ on the line? Explain.

   (c) Showing all your work, find two points, distinct from $(-2, 0)$ and $(-3, -2)$, on the given line.

Solution.

   (a) We have. . .

$$2x - 5y = 4$$

$$2(-2) - 5(0) \overset{?}{=} 4$$

$$-4 - 0 \overset{?}{=} 4$$

$$-4 \neq 4$$

$$\text{✗}$$

Therefore, $(-2, 0)$ is not on the line $2x - 5y = 4$.

   (b) We have. . .

$$2x - 5y = 4$$

$$2(-3) - 5(-2) \overset{?}{=} 4$$

$$-6 + 10 \overset{?}{=} 4$$

$$4 = 4$$

$$\text{✓}$$

Therefore, $(-3, -2)$ is on the line $2x - 5y = 4$.

   (c) Any (x, y) which satisfy the equation $2x - 5y = 4$ are on the line. If $x = 0$, we have $-5y = 4$ so that $y = -\frac{4}{5}$. Therefore, $(0, -\frac{4}{5})$ is a point on the line. Similarly, if $y = 0$, we have $2x = 4$ so that $x = \frac{4}{2} = 2$. Therefore, $(2, 0)$ is on the line.

Problem 2. (10pt) Consider the line $-3x - 5y = 10$.

    (a) Find the slope of the line.

    (b) Find the y-intercept of the line.

    (c) Find this line as a linear function, $f(x)$.

    (d) Using your $f(x)$ from (c), find a point on the line distinct from the y-intercept.

Solution. First, observe that...

$$-3x - 5y = 10$$

$$-5y = 3x + 10$$

$$y = -\frac{3}{5}x - 2$$

    (a) Because $y = -\frac{3}{5}x - 2$ is of the form $f(x) = mx + b$ with $m = -\frac{3}{5}$ and $b = -2$, the slope of the line is $-\frac{3}{5}$.

    (b) Because $y = -\frac{3}{5}x - 2$ is of the form $f(x) = mx + b$ with $m = -\frac{3}{5}$ and $b = -2$, the y-intercept of the line is -2, i.e. $(0, -2)$.

    (c) From the work above, we have $f(x) = -\frac{3}{5}x - 2$.

    (d) If $x = x_0$, then $(x_0, f(x_0))$ is a point on the line. For instance, suppose that $x = 5$, then we have...

$$f(5) = -\frac{3}{5} \cdot 5 - 2 = -3 - 2 = -5$$

Therefore, $(5, -5)$ is a point on the line.