

Name: _____

MATH 308

Fall 2022

HW 1: Due 09/08

*"I mean not homework. It's not work if
you love it."*

–Alex Dunphy, Modern Family

Problem 1. (10pt) Determine if each of the following are propositions. If the example is a proposition, state its truth value with a brief justification. If the example is *not* a proposition, briefly explain why:

(a) Have you been watching "The Rings of Power"?

(b) $|9 - 17| > 10$

(c) $x^2 + x - 2 = 0$

(d) The novel *Ulysses* was written by James Joyce.

(e) The sixth digit of e is 1.

Solution.

(a)

(b)

(c)

(d)

(e)

Problem 2. (10pt) For each of the following, either define appropriate primitive propositions (using P , Q , R , etc.) and write the ‘statement’ using logical connectives, or give an English sentence for the given primitives and ‘translate’ the logical ‘sentence’ into an English sentence:

- (a) Either he is lying and isn’t coming, or we are at the wrong place.
- (b) $(P \wedge \neg Q) \rightarrow R$
- (c) If you exercise and eat healthy, then you will live a long life.
- (d) $P \vee (\neg P \wedge Q)$

Solution.

- (a)
- (b)
- (c)
- (d)

Problem 3. (10pt) Consider the following compound statement: $(P \vee \neg Q) \rightarrow (\neg P \wedge Q) \vee \neg Q$

- (a) Determine whether the given compound statement is a tautology. Be sure to justify your response.
- (b) Using a truth table, show that the *negation* of the given compound statement is logically equivalent to $P \wedge Q$.
- (c) Show that the *negation* of the given compound statement is logically equivalent to $P \wedge Q$ by simplifying the given compound statement.

Solution.

- (a)
- (b)
- (c)

Problem 4. (10pt) Consider the statement, “if $x = 3$, then $x^2 = 9$.”

- (a) Determine the truth value of this statement with an explanation.
- (b) Rewrite the given statement by defining appropriate primitive propositions and logical connectives.
- (c) Find the negation, converse, and contrapositive of your result from (b).
- (d) Rewrite your answers from (c) as English sentences. Then determine the truth value, with explanation, of each of the statements.

Solution.

- (a)
- (b)
- (c)
- (d)