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MATH 108
Fall 2022
HW 11: Due 11/01

"There are three types of lies: lies, damn lies, and statistics."
–Benjamin Disraeli

Problem 1. (10pt) Suppose Kevin and Cristine work at a company where the salaries are normally distributed. Kevin's salary corresponds to a z-score of $z_K = -0.47$ and Cristine's salary corresponds to a z-score of $z_C = 1.62$.

(a) Who makes less money? Explain.

(b) What percent of workers at the company many less than Cristine? Explain.

(c) What percent of workers at the company make more than Kevin? Explain.

(d) If the salary have distribution $N(\$68327, \$7419)$, find the amount Kevin and Cristine make.

Solution.

(a) Because Kevin's z-score is negative, he makes less than the average salary at the company. Because Cristine's z-score is positive, she makes more than the average salary at the company. Therefore, Cristine makes more money.

(b) We have...

$$z_C = 1.62 \rightsquigarrow 0.9474$$

Therefore, 94.74% of workers at the company makes less than Cristine.

(c) We have...

$$z_K = -0.47 \rightsquigarrow 0.3192$$

But then we have $1 - 0.3192 = 0.6808$. Therefore, 68.08% of workers at the company make more than Kevin.

(d) We have...

$$\begin{aligned} z_K &= -0.47 & \qquad z_C &= 1.62 \\ \frac{K - 68327}{7419} &= -0.47 & \qquad \frac{C - 68327}{7419} &= 1.62 \\ K - 68327 &= -3486.93 & \qquad C - 68327 &= 12018.78 \\ K &= 64840.07 & \qquad C &= 80345.78 \end{aligned}$$

Therefore, Kevin makes \$64,840.07 and Cristine makes \$80,345.78.

Problem 2. (10pt) Suppose Bill wants to purchase a house. Based on his salary, the most expensive house he can afford is \$380,000. Looking up housing data for his county on a government website, he finds that the local housing prices indicate that the housing costs are approximately normally distributed with the average house cost approximately \$450,000 and standard deviation \$52,000. What percent of houses in his area can Bill afford?

Solution. We have...

$$z_{\text{Bill}} = \frac{380000 - 450000}{52000} = \frac{-70000}{52000} \approx -1.35 \rightsquigarrow 0.0885$$

Therefore, Bill can only afford approximately 8.9% of the houses in his county.

Problem 3. (10pt) A highly desired university receives more applicants than they could ever hope to accept. In fact, they receive so many applications, it is overly burdensome to thoroughly examine each application. They are only able to screen approximately 45% of the applications that they receive. To reduce the number of applications, they will not screen applicants that receive below a certain score on an entrance exam. The entrance exam is known to have scores that are normally distributed with mean 340 and standard deviation 32. What is the minimum score you can receive to have this school actually screen your application?

Solution. The university will only look at the applicants whose exam scores are in the top 45% of applicants, i.e. those students who scored better than at least 55% of applicants. But then the worst possible exam score, say S, will have a z-score corresponding to '0.55 area to its left.' But then we have $z_S \approx 0.125$. But then we have...

$$\begin{aligned} z_S &= 0.125 \\ \frac{S - 340}{32} &= 0.125 \\ S - 340 &= 4 \\ S &= 344 \end{aligned}$$

Therefore, one must score at least 344 on the entrance exam to have their application considered.