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MATH 101
Fall 2022
HW 18: Due 11/14

"Poetry is a precise a thing as geometry."
–Gustave Flaubert

Problem 1. (10pt) Solve the following equation and verify your solution:

$$4(x-2) = 5 - 9x$$

Solution. We have...

$$4(x-2) = 5 - 9x$$

$$4x - 8 = 5 - 9x$$

$$13x - 8 = 5$$

$$13x = 13$$

$$x = 1$$

We can check/verify the solution:

$$4(x-2) = 5 - 9x$$

$$4(1-2) \stackrel{?}{=} 5 - 9(1)$$

$$4(-1) \stackrel{?}{=} 5 - 9$$

$$-4 = -4$$

✓

Problem 2. (10pt) Solve the following equation and check your solution:

$$\frac{x-3}{1-x} = 5$$

Solution. We have...

$$\frac{x-3}{1-x} = 5$$

$$x - 3 = 5(1-x)$$

$$x - 3 = 5 - 5x$$

$$6x - 3 = 5$$

$$6x = 8$$

$$x = \frac{8}{6}$$

$$x = \frac{4}{3}$$

We can check/verify this solution:

$$\frac{x-3}{1-x} = 5$$

$$\frac{\frac{4}{3}-3}{1-\frac{4}{3}} \stackrel{?}{=} 5$$

$$\frac{\frac{4}{3}-\frac{9}{3}}{\frac{3}{3}-\frac{4}{3}} \stackrel{?}{=} 5$$

$$\frac{-\frac{5}{3}}{-\frac{1}{3}} \stackrel{?}{=} 5$$

$$-\frac{5}{3} \cdot -\frac{3}{1} \stackrel{?}{=} 5$$

$$5 = 5$$

✓

Problem 3. (10pt) If $f(x) = 5 - 3x$ and $g(x) = -3(x + 8)$, will there be a solution to $f(x) = g(x)$? Explain.

Solution. There is a solution of an equation $f(x) = g(x)$ if and only if the graphs of $f(x)$ and $g(x)$ intersect at the point $\big(x, f(x)\big) = \big(x, g(x)\big)$. Both $f(x)$ and $g(x)$ are linear functions. Observe that $f(x) = 5 - 3x$ is a linear function with slope $m = -3$. The function $g(x) = -3(x + 8) = -3x - 24$ is a linear function with slope $m = -3$. But then the linear functions $f(x)$ and $g(x)$ have the same slope so that they are parallel. But then the lines do not intersect so that the equation $f(x) = g(x)$ cannot have a solution.