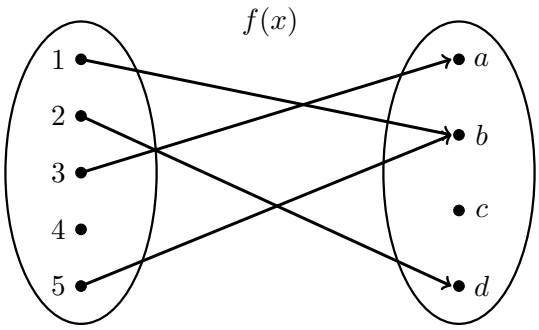
Name: _____

MATH 308
Fall 2022
HW 8: Due 10/13

"The difference between mathematicians and physicists is that after physicists prove a big result they think it is fantastic but after mathematicians prove a big result they think it is trivial."

–Lucien Szpiro

Problem 1. (10pt) Consider the relation $f(x)$ given below.



(a) Explain why $f(x)$ is not a function.

(b) Add an arrow to the diagram so that $f(x)$ is a surjective function.

(c) Identify the domain, codomain, and range for $f(x)$.

(d) Is $f(x)$ an injective function? Explain why or why not.

Problem 2. (10pt) Complete the proof of the proposition stated below by filling in the blanks.

Proposition. Let $f : X \to Y$ be a function and $B \subseteq Y$. Then $X \setminus f^{-1}(B) \subseteq f^{-1}(Y \setminus B)$.

Proof. We know that if $X \setminus f^{-1}(B) = \varnothing$, then $X \setminus f^{-1}(B) \subseteq f^{-1}(Y \setminus B)$. Assume that $X \setminus f^{-1}(B) \neq \varnothing$. To show that $X \setminus f^{-1}(B) \subseteq f^{-1}(Y \setminus B)$, we need to show that if _____, then _____.

Let $x \in X \setminus f^{-1}(B)$. But then we know that _____ and $x \notin$ _____. Because $x \notin$ _____, we know that $f(x) \notin$ _____. It is clear that $f(x) \in Y$. But then _____ and $f(x) \notin$ _____. This shows that $f(x) \in$ _____.

This shows that $f(x)$ is in the preimage of $Y \setminus B$. But then we know that $x \in$ _____.

But then if $x \in$ _____, then $x \in$ _____. Therefore, $X \setminus f^{-1}(B) \subseteq f^{-1}(Y \setminus B)$.

Problem 3. (10pt) Let $f : \mathbb{R} \to \mathbb{R}$ be the function given by $x \mapsto x^2 + 3x - 7$.

(a) Without referencing the graph of f, use the definition of decreasing to show that $f(x)$ is not a decreasing function on $\mathbb{R}$ by giving a counterexample.

(b) Determine whether or not $3 \in$ im f. If $3 \in$ im f, find an element in the preimage of 3. If $3 \notin$ im f, explain why.

(c) Is $f^{-1}(x)$ a function? Explain why or why not by referencing the graph of $f(x)$. Give an additional explanation of why or why not using your response in (b).