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MATH 101
Fall 2021
HW 19: Due 12/10

"It's about time somebody stood up to Auntie Eleanor. But you, not me. Oh, god! She can't ever know I was here."
– Oliver T'sien, Crazy Rich Asians

Problem 1. (10pt) Suppose you invest \$1500 in an account which gains 6% annual interest compounded monthly.

(a) Determine the amount of money in the account after 8 years.

(b) How long until the account has \$4000?

(c) How much should you place in the account if you want to have \$2000 saved after 3 years?

Solution.

(a)

$$\begin{aligned} F &= P\left(1+\frac{r}{k}\right)^{kt} \\ F &= 1500\left(1+\frac{0.06}{12}\right)^{12\cdot 8} \\ F &= 1500(1.005)^{96} \\ F &= \$2,421.21 \end{aligned}$$

(b)

$$\begin{aligned} F &= P\left(1+\frac{r}{k}\right)^{kt} \\ 4000 &= 1500\left(1+\frac{0.06}{12}\right)^{12\cdot t} \\ 4000 &= 1500(1.005)^{12t} \\ 2.6667 &= (1.005)^{12t} \\ \ln(2.6667) &= \ln(1.005)^{12t} \\ 12t\ln(1.005) &= \ln(2.6667) \\ t &= \frac{\ln(2.6667)}{12\ln(1.005)} \approx 16.3882 \text{ years} \end{aligned}$$

(c)

$$\begin{aligned} F &= P\left(1+\frac{r}{k}\right)^{kt} \\ 2000 &= P\left(1+\frac{0.06}{12}\right)^{12\cdot 3} \\ 4000 &= P(1.005)^{36} \\ 1.19668P &= 4000 \\ P &= \$3,342.58 \end{aligned}$$

Problem 2. (10pt) Suppose you invest \$430 in an account which gains 4% annual interest compounded semiannually.

(a) Determine the amount of money in the account after 2 years.

(b) How long until the account has \$1000?

(c) How much should you place in the account if you want to have \$600 saved after 5 years?

Solution.

(a)

$$\begin{aligned} F &= P\left(1+\frac{r}{k}\right)^{kt} \\ F &= 430\left(1+\frac{0.04}{2}\right)^{2\cdot 2} \\ F &= 430(1.02)^4 \\ F &= \$465.45 \end{aligned}$$

(b)

$$\begin{aligned} F &= P\left(1+\frac{r}{k}\right)^{kt} \\ 1000 &= 450\left(1+\frac{0.04}{2}\right)^{2\cdot t} \\ 1000 &= 450(1.02)^{2t} \\ 2.2222 &= (1.02)^{2t} \\ \ln(2.2222) &= \ln(1.02)^{2t} \\ 2t\ln(1.02) &= \ln(2.2222) \\ t &= \frac{\ln(2.2222)}{2\ln(1.02)} \approx 20.1614 \text{ years} \end{aligned}$$

(c)

$$\begin{aligned} F &= P\left(1+\frac{r}{k}\right)^{kt} \\ 600 &= P\left(1+\frac{0.04}{2}\right)^{2\cdot 5} \\ 600 &= P(1.02)^{10} \\ 1.21899P &= 600 \\ P &= \$492.21 \end{aligned}$$

Problem 3. (10pt) Suppose you invest \$600 in an account which gains 5% annual interest compounded continuously.

(a) Determine the amount of money in the account after 3 years.

(b) How long until the account has \$800?

(c) How much should you place in the account if you want to have \$900 saved after 10 years?

Solution.

(a)

$$\begin{aligned} F &= Pe^{rt} \\ F &= 600e^{0.05\cdot 3} \\ F &= 600e^{0.15} \\ F &= 600(1.16183) \\ F &= \$697.10 \end{aligned}$$

(b)

$$\begin{aligned} F &= Pe^{rt} \\ 800 &= 600e^{0.05t} \\ 1.3333 &= e^{0.05t} \\ \ln(1.3333) &= \ln e^{0.05t} \\ 0.05t &= \ln(1.3333) \\ t &= \frac{\ln(1.3333)}{0.05} \approx 5.753 \text{ years} \end{aligned}$$

(c)

$$\begin{aligned} F &= Pe^{rt} \\ 900 &= Pe^{0.05\cdot 10} \\ 900 &= Pe^{0.5} \\ 1.64872P &= 900 \\ P &= \$545.88 \end{aligned}$$

Problem 4. (10pt) Suppose you invest $3000 in an account which gains 2% annual interest compounded continuously.

(a) Determine the amount of money in the account after 7 years.

(b) How long until the account has $3500?

(c) How much should you place in the account if you want to have $3700 saved after 4 years?

Solution.

(a)

$$\begin{aligned} F &= Pe^{rt} \\ F &= 3000e^{0.02\cdot 7} \\ F &= 3000e^{0.14} \\ F &= 3000(1.15027) \\ F &= \$3,450.82 \end{aligned}$$

(b)

$$\begin{aligned} F &= Pe^{rt} \\ 3500 &= 3000e^{0.02t} \\ 1.1667 &= e^{0.02t} \\ \ln(1.1667) &= \ln e^{0.02t} \\ 0.02t &= \ln(1.1667) \\ t &= \frac{\ln(1.1667)}{0.02} \approx 7.709 \text{ years} \end{aligned}$$

(c)

$$\begin{aligned} F &= Pe^{rt} \\ 3700 &= Pe^{0.02\cdot 4} \\ 3700 &= Pe^{0.08} \\ 1.08329P &= 900 \\ P &= \$830.81 \end{aligned}$$