Quiz 1. *True/False*: $9/3 + 2(3^2 + 10) - 8 + 4 \cdot 3 = 45$

Solution. The statement is *false*. To see this, we can simply follow the order of operations—using PEMDAS as a guide:

$$\begin{aligned}
9/3 + 2(3^2 + 10) - 8 + 4 \cdot 3 &\stackrel{?}{=} 55 \\
9/3 + 2(9 + 10) - 8 + 4 \cdot 3 &\stackrel{?}{=} 55 \\
9/3 + 2(19) - 8 + 4 \cdot 3 &\stackrel{?}{=} 55 \\
3 + 2(19) - 8 + 4 \cdot 3 &\stackrel{?}{=} 55 \\
3 + 38 - 8 + 4 \cdot 3 &\stackrel{?}{=} 55 \\
3 + 38 - 8 + 12 &\stackrel{?}{=} 55 \\
41 - 8 + 12 &\stackrel{?}{=} 55 \\
33 + 12 &\stackrel{?}{=} 55 \\
45 &\neq 55
\end{aligned}$$

Quiz 2. *True/False*: $\gcd(2^3 \cdot 3 \cdot 5, 2 \cdot 3^2 \cdot 7) = 2^3 \cdot 3^2 \cdot 5 \cdot 7$.

Solution. The statement is *false*. Remember given a prime factorization of the numbers, we find the gcd by choosing the *smallest* powers of each prime that appears in the factorizations. So we should have $\gcd(2^3 \cdot 3 \cdot 5, 2 \cdot 3^2 \cdot 7) = 2 \cdot 3$. Instead, the largest power of each prime that appears in the factorizations was chosen which is how we compute the lcm. Therefore, we have $\operatorname{lcm}(2^3 \cdot 3 \cdot 5, 2 \cdot 3^2 \cdot 7) = 2^3 \cdot 3^2 \cdot 5 \cdot 7$.

Quiz 3. *True/False*: $\dfrac{\frac{3}{10}}{\frac{12}{5}} = \dfrac{1}{8}$

Solution. The statement is *true*. Note that division by a nonzero number is the same as multiplying by its reciprocal. So we have

$$\frac{\frac{3}{10}}{\frac{12}{5}} = \frac{3}{10} \cdot \frac{5}{12} = \frac{\cancel{3}^1}{\cancel{10}^2} \cdot \frac{\cancel{5}^1}{\cancel{12}^4} = \frac{1}{8}$$

One can also rewrite the problem as. . .

$$\frac{\frac{3}{10}}{\frac{12}{5}} = \frac{3}{10} \div \frac{12}{5}$$

But then to divide, we multiply by the reciprocal and proceed as in the solution above.

Quiz 4. *True/False*: The number $0.\overline{19}$ is rational.

Solution. The statement is *true*. Any real number with a decimal expansion that either terminates or repeats is a rational and hence can be expressed as a/b, where a and b are integers and $b \neq 0$. Moreover, every rational number, i.e. the a/b's, have a decimal expansion that either terminates or repeats. We can even find a rational expression for $0.\overline{19}$:

$$\begin{array}{rrcl} & 100r & = & 19.191919191919\ldots \\ - & r & = & 0.191919191919\ldots \\ \hline & 99r & = & 19 \end{array}$$

But then $r = 0.\overline{19} = \dfrac{19}{99}$.

Quiz 5. *True/False*: $\sqrt[3]{2^8 \cdot 3^3 \cdot 5^1 \cdot 7^5} = 2^2 \cdot 3^1 \cdot 7\sqrt[3]{2^2 \cdot 5^1 \cdot 7^2}$

Solution. The statement is *true*. There are two ways to think about this. First, we should write out the numbers and group them into threes and pull out/leave the terms appropriately:

$$\sqrt[3]{2^8 \cdot 3^3 \cdot 5^1 \cdot 7^5} = \sqrt[3]{\underline{2 \cdot 2 \cdot 2} \cdot \underline{2 \cdot 2 \cdot 2} \cdot 2 \cdot 2 \cdot \underline{3 \cdot 3 \cdot 3} \cdot 5 \cdot \underline{7 \cdot 7 \cdot 7} \cdot 7 \cdot 7} = 2^2 \cdot 3^1 \cdot 7\sqrt[3]{2^2 \cdot 5 \cdot 7^2}$$

Alternatively, we can use division. We know that $8/3$ is 2 with remainder 2, $3/3$ is 1 with remainder 0, $1/3$ is 0 with remainder 1, and $5/3$ is 1 with remainder 2. So we can pull out two 3's with 2 remaining, one 3 with 0 remaining, no 5's with 1 remaining, and two 7's with 2 remaining, which gives:

$$\sqrt[3]{2^8 \cdot 3^3 \cdot 5^1 \cdot 7^5} = 2^2 \cdot 3^1 \cdot 7\sqrt[3]{2^2 \cdot 5^1 \cdot 7^2}$$

Quiz 6. *True/False*: 57 increased by 127% is $57(1.27)$.

Solution. The statement is *false*. To find 127% of 57, we would multiply 57 by the percent written as a decimal. This would be $57(1.27)$. However, to increase or decrease a number by a percentage, we compute the number $\#(1 \pm \%)$, where we add if we are increasing, subtract if we are decreasing, $\#$ is the number, and % is the percentage written as a decimal. So to increase 57 by 127%, we need to compute $57(1 + 1.27) = 57(2.27)$.

Quiz 7. *True/False*: If $f(x) = x + 1$ and $g(x) = x^2$, then $(f \circ g)(2) = 9$.

Solution. The statement is *false*. Recall that $(f \circ g)(2) = f(g(2))$. First, we compute $g(2)$. We have $g(2) = 2^2 = 4$. Then we have $f(g(2)) = f(4)$ and we compute $f(4)$: $f(4) = 4 + 1 = 5$.

Quiz 8. *True/False*: The point $(1, 3)$ is on the graph of $f(x) = 2x - 5$.

Solution. The statement is *false*. We have the point $(x, y) = (1, 3)$. If this point is on the graph of $f(x)$, then these x and y satisfy the equation for $f(x)$. We can check this:

$$\begin{aligned} f(x) &= 2x - 5 \\ 3 &= 2(2) - 5 \\ 3 &= 4 - 5 \\ 3 &\neq -1 \end{aligned}$$

Therefore, the point $(1, 3)$ is not on the graph of $f(x)$. Alternatively, if $x = 1$, then the corresponding point on the graph of $f(x)$ would have y-value $f(1) = 2(1) - 5 = 2 - 5 = -3$. Then the point $(1, -3)$ is on the graph of $f(x)$. But then $(1, 3)$ is not on the graph of $f(x)$.

Quiz 9. *True/False*: If $f^{-1}(3) = 9$, then $f(3) = 9$.

Solution. The statement is *false*. Recall that $f^{-1}(y) = x$ if and only if $f(x) = y$; that is, f^{-1} asks the question, 'what do I plug into f to get this number.' So if $f^{-1}(3) = 9$, this means we should be able to plug in 9 into $f(x)$ and obtain 3, i.e. $f(9) = 3$. But then $f(3) = 9$ is not necessarily true.

Quiz 10. *True/False*: To find the x-intercept, you find $f(0)$.

Solution. The statement is *false*. Recall that an x-intercept is where a function intersects the x-axis. But then the y-value must be zero. But then because $y = f(x)$, we have $f(x) = 0$. Whereas if we wanted to find a y-intercept, we would recall that along the x-axis, $x = 0$ so that we would need to find $f(0)$. So finding x-intercepts involves solving $f(x) = 0$, whereas finding y-intercepts involves evaluating $f(0)$.

Quiz 11. *True/False*: The lines $y = \frac{2}{3}x + 5$ and $3x + 2y = -6$ are perpendicular.

Solution. The statement is *true*. The line $y = \frac{2}{3}x + 5$ has slope $m = \frac{2}{3}$. Solving for y in the second line, we have $y = -3 - \frac{3}{2}x$. This line has slope $m = -\frac{3}{2}$. The negative reciprocal of $\frac{2}{3}$ is $-\frac{3}{2}$. Therefore, the lines are perpendicular.

Quiz 12. All lines perpendicular to $y = 4$ are of the form $x = \#$.

Solution. The statement is *true*. The line $y = 4$ is horizontal. For a line to be perpendicular to a horizontal line, the line must be vertical. But all vertical lines are of the form $x = \#$.

Quiz 13. *True/False*: Any line with slope 0 must be of the form $y = \#$.

Solution. The statement is *true*. All vertical lines 'look like' $y = mx + b$ for some m, b. If the slope is 0, then $m = 0$. But then $y = \#$.

Quiz 14. *True/False*: All functions have inverses.

Solution. The statement is *false*. All constant functions, i.e. $f(x) = \#$, do not have inverses. Constant functions are functions—every input has exactly one output (even if they all happen to be the same). However, you cannot 'tell' what x gave you $\#$. Alternatively, $f(x) = \#$ fails the horizontal line test. [Recall that a function has an inverse if and only if it passes the horizontal line test.]

Quiz 15. *True/False*: The quadratic function $y = 5x + 3 - x^2$ opens downwards, is concave, and has a maximum.

Solution. The statement is *true*. Writing the quadratic function in standard form, i.e. $y = ax^2 + bx + c$, we have $y = -x^2 + 5x + 3$. Therefore, for this quadratic function, $a = -1$, $b = 5$, and $c = 3$. Because $a = -1 < 0$, the quadratic function opens downwards, i.e. is concave (down), and has a maximum.

Quiz 16. *True/False*: The quadratic function $f(x) = 2(x + 2)^2 + 4$ has vertex $(2, 4)$.

Solution. The statement is *false*. The x-coordinate of the vertex is the x-value that makes the square term zero. In this case, $x = -2$ would make $2(x + 2)^2$ zero. Then we would be left with $y = 4$, which is the y-coordinate of the vertex. Therefore, the vertex is $(-2, 4)$. Alternatively, the 'proper' vertex form of a quadratic function is $y = A(x - B) + C$. The vertex is (B, C). Writing the 'proper' vertex form of the quadratic function $y = 2(x+2)^2+4$, we have $y = 2(x- \ -2)^2+4$. Therefore, the vertex form is $(-2, 4)$. Finally, one could expand this out: $y = 2(x + 2)^2 + 4 = 2(x^2 + 4x + 4) + 4 = 2x^2 + 8x + 8 + 4 = 2x^2 + 8x + 12$. The x-coordinate of the vertex is $x = \frac{-b}{2a} = \frac{-8}{2(2)} = -2$. Then the y-coordinate of the vertex is $y(-2) = 2(-2)^2 + 8(-2) + 12 = 8 - 16 + 12 = 4$. Therefore, the vertex is $(-2, 4)$.

Quiz 17. *True/False*: The quadratic function $y = x^2 - 4x - 12$ factors as $(x - 6)(x + 2)$.

Solution. Solution. The statement is *true*. One way of seeing this would be to expand $(x-6)(x+2)$,

$$(x - 6)(x + 2) = x^2 + 2x - 6x - 12 = x^2 - 4x - 12.$$

Alternatively, we can factor the polynomial $x^2 - 4x - 12$. First, we find the factors of 12, which are only $1, 12$, and $2, 6$, and $3, 4$. Because the 12 is negative, the factors must have opposite signs.

$$\begin{aligned}
1, -12 &: \; -11 \\
-1, 12 &: \quad 11 \\
2, -6 &: \; -4 \\
-2, 6 &: \quad 4 \\
3, -4 &: \; -1 \\
-3, 4 &: \quad 1
\end{aligned}$$

We want these signed factors to add to -4. Therefore, we want 'factors' $2, -6$. Therefore,

$$x^2 - 4x - 12 = (x + 2)(x - 6)$$

Quiz 18. *True/False*: Let D be the discriminant of a quadratic polynomial. If $D = -4$, then the polynomial factors 'nicely.'

Solution. The statement is *false*. Recall that a quadratic polynomial factors 'nicely' if and only if its discriminant is a perfect square. However, $D = -4$ is *not* a perfect square. The number 4 is a perfect square because $2^2 = 4$. But there is no real number whose square is -4. Therefore, the quadratic polynomial must not factor 'nicely.' Note that if $D < 0$, then the quadratic polynomial factors over $\mathbb{C}$.

Quiz 19. *True/False*: $x^2 - 2x - 1 = \big(x - (1 + \sqrt{2})\big)\big(x - (1 - \sqrt{2})\big)$

Solution. The statement is *true*. For the quadratic function $x^2 - 2x - 1$, we have $a = 1$, $b = -2$, and $c = -1$. We can compute the discriminant to find $D = b^2 - 4ac = (-2)^2 - 4(1)(-1) = 4 + 4 = 8$. Because $D = 8$ is not a perfect square, the quadratic polynomial $x^2 - 2x - 1$ does not factor 'nicely.' However, all quadratic functions are factorable. To find the factorization, we find the roots of $x^2 - 2x - 1$, i.e. the solutions to $x^2 - 2x - 1 = 0$, using the quadratic formula. We have...

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} \\
&= \frac{2 \pm \sqrt{4 + 4}}{2} \\
&= \frac{2 \pm \sqrt{8}}{2} \\
&= \frac{2 \pm \sqrt{4 \cdot 2}}{2} \\
&= \frac{2 \pm 2\sqrt{2}}{2} \\
&= 1 \pm \sqrt{2}
\end{aligned}$$

Then we have roots $r_1 = 1 + \sqrt{2}$ and $r_2 = 1 - \sqrt{2}$. Therefore, the factorization is

$$x^2 - 2x - 1 = a(x - r_1)(x - r_2) = \big(x - (1 + \sqrt{2})\big)\big(x - (1 - \sqrt{2})\big)$$

Quiz 20. *True/False*: The quadratic formula can be used to solve $4x - 5x + 1 = 0$.

Solution. The statement is *false*. The quadratic formula can be used to solve *quadratic* equations. The equation $4x - 5x + 1 = 0$ is linear. However, if the equation were $4x^2 - 5x + 1 = 0$, then this quadratic formula could be used to solve this equation. We would have $a = 4, b = -5, c = 1$. Then…

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)1}}{2(4)} \\ &= \frac{5 \pm \sqrt{25 - 16}}{8} \\ &= \frac{5 \pm \sqrt{9}}{8} \\ &= \frac{5 \pm 3}{8} \end{aligned}$$

But then either $x = (5 + 3)/8 = 8/8 = 1$ or $x = (5 - 3)/8 = 2/8 = 1/4$.

Quiz 21. *True/False*: The vertical asymptotes of $f(x) = \dfrac{(x-3)(x+2)}{(x+1)(x-3)}$ are $x = -1$ and $x = 3$.

Solution. The statement is *false*. We first find the domain. The domain of a rational function is where the denominator is not 0. The denominator is 0 if $(x + 1)(x - 3) = 0$. But then $x + 1 = 0$, i.e. $x = -1$, or $x - 3 = 0$, i.e. $x = 3$. Therefore, the domain is all real numbers such that $x \neq -1, 3$. We can then cancel any common factors in the numerator and denominator. Then

$$\frac{(x-3)(x+2)}{(x+1)(x-3)} = \frac{\cancel{(x-3)}(x+2)}{(x+1)\cancel{(x-3)}} = \frac{x+2}{x+1}$$

The vertical asymptotes are then the values where the denominator vanishes in this reduced function. But then $x + 1 = 0$, i.e. $x = -1$. Therefore, the only vertical asymptote is $x = -1$.

Quiz 22. *True/False*: The function $y = -5(1/3)^{-x}$ is increasing.
Solution. The statement is *false*. We write the function in the form $y = Ab^x$. We have…

$$y = -5\left(\frac{1}{3}\right)^{-x} = -5\left(\left(\frac{1}{3}\right)^{-1}\right)^{x} = -5(3^x)$$

But then because we have $b = 3 > 1$ and $A = -5 < 0$, the function is decreasing.

Quiz 23. *True/False*: If $4^{-x} = 16$, then $x = 2$.

Solution. The statement is *false*. We have...

$$\begin{aligned} 4^{-x} &= 16 \\ 4^{-x} &= 4^2 \end{aligned}$$

Because the bases are the same on both sides, we must have $-x = 2$, i.e. $x = -2$.

Quiz 24. *True/False*: If $2^{x^2} = 16$, then $x = \pm 2$.

Solution. The statement is *true*. We have...

$$\begin{aligned} 2^{x^2} &= 16 \\ 2^{x^2} &= 2^4 \end{aligned}$$

Because the bases are the same on both sides, we must have $x^2 = 4$. But then $x = \pm\sqrt{4} = \pm 2$.

Quiz 25. *True/False*: If you invest \$500 in an account receiving 5% annual interest compounded quarterly, then the amount in the account after 3 years is $500(1 + 0.05)^3$.

Solution. The statement is *false*. Recall that the formula for the future value of an investment in a principal that is accruing interest that is being compounded discretely is...

$$F = P\left(1 + \frac{r}{k}\right)^{kt}$$

where F is the future value, P is the principal, r is the annual interest rate (as a decimal), k is the rate of compounding, and t is the time (in years). Here, we have $P = 500$, $r = 0.05$, $k = 4$, and $t = 3$. But then we have

$$F = 500\left(1 + \frac{0.05}{4}\right)^{4 \cdot 3}$$

Quiz 26. *True/False*: The greater the rate of compounding, the greater the amount of interest earned.

Solution. The statement is *true*. The intuitive reason is that the greater the rate of compounding, the more frequently interest is being added to the account. But then at the next compounding, the interest is being applied to a large total of money, thus accruing more interest.

To prove this rigorously is more difficult: suppose that you have compounding rates K, k with $K > k$, fixed interest rate r, fixed principal P, and that $t > 0$. Because $K > k$, we know that $K/k > 1$. By the Bernoulli Inequality, we know that $(1 + x)^y \geq 1 + yx$ for $y \geq 1$ and $x \geq -1$. [Note that this inequality is strict if $x \neq 0$ and $y \neq 0, 1$.] But then we have...

$$\left(1 + \frac{r}{K}\right)^{K/k} > 1 + \frac{K}{k} \cdot \frac{r}{K} = 1 + \frac{r}{k}$$

But using the fact that $1 + r/K > 1$ and $K/k > 1$, we have...

$$\begin{aligned}
\left(1 + \frac{r}{K}\right)^{K/k} &> 1 + \frac{r}{k} \\
\left(1 + \frac{r}{K}\right)^{K} &> \left(1 + \frac{r}{k}\right)^{k} \\
\left(1 + \frac{r}{K}\right)^{Kt} &> \left(1 + \frac{r}{k}\right)^{kt} \\
P\left(1 + \frac{r}{K}\right)^{Kt} &> P\left(1 + \frac{r}{k}\right)^{kt} \\
F_K &> F_k
\end{aligned}$$

But then the amount of money after time t compounding at rate K, F_K, is greater than the amount of money after time t compounding at rate k, F_k.

Quiz 27. *True/False*: The point $(-2, 1)$ is the intersection point of...

$$\begin{aligned}
3x - y &= -7 \\
x + 2y &= 4
\end{aligned}$$

Solution. The statement is *false*. For a point to be an intersection point for a system of equations, it must satisfy all of the equations. We check the first equation:

$$\begin{aligned}
3x - y &\stackrel{?}{=} -7 \\
3(-2) - 1 &\stackrel{?}{=} -7 \\
-6 - 1 &\stackrel{?}{=} -7 \\
-7 &= -7
\end{aligned}$$

Therefore, $(-2, 1)$ satisfies the first equation. However, $(-2, 1)$ need also satisfy the second equation also.

$$\begin{aligned}
x + 2y &\stackrel{?}{=} 4 \\
-2 - 2(1) &\stackrel{?}{=} 4 \\
-2 - 2 &\stackrel{?}{=} 4 \\
-4 &\neq 4
\end{aligned}$$

Therefore, $(-2, 1)$ is not an intersection point for the system of equations.