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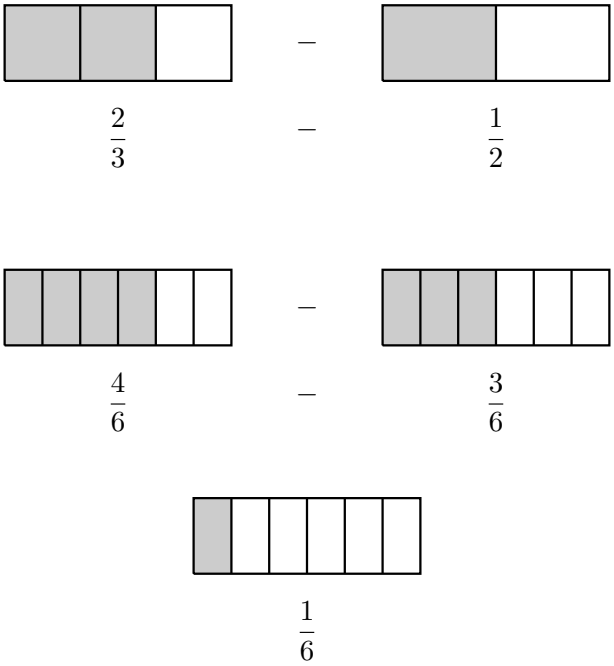
MATH 101
Spring 2022
HW 2: Due 02/10

"A book hasn't caused me this much trouble since Where's Waldo went to that barber pole factory."
–Tracy Jordan, 30 Rock

Problem 1. (10pt) Represent each step in the following computation using fractions of a rectangle:

$$\frac{2}{3} - \frac{1}{2}$$

Solution.



Problem 2. (10pt) Showing all your work, reduce the following rational numbers:

(a) $\frac{10}{14}$

(b) $\frac{18}{30}$

(c) $\frac{11}{17}$

(d) $\frac{36}{15}$

Solution.

(a)

$$\frac{10}{14} = \frac{2 \cdot 5}{2 \cdot 7} = \frac{\cancel{2} \cdot 5}{\cancel{2} \cdot 7} = \frac{5}{7}$$

(b)

$$\frac{18}{30} = \frac{2 \cdot 3^2}{2 \cdot 3 \cdot 5} = \frac{\cancel{2} \cdot 3^{\cancel{2}^{1}}}{\cancel{2} \cdot \cancel{3} \cdot 5} = \frac{3}{5}$$

(c)

$$\frac{11}{17} = \frac{11}{17}$$

(d)

$$\frac{36}{15} = \frac{2^2 \cdot 3^2}{3 \cdot 5} = \frac{2^2 \cdot 3^{\cancel{2}^{1}}}{\cancel{3} \cdot 5} = \frac{12}{5}$$

Problem 3. (10pt) Showing all your work and reducing as much as possible, perform the following computations:

(a) $\frac{2}{3} - \frac{5}{7}$

(b) $\frac{5}{6} + \frac{7}{2}$

(c) $\frac{3}{4} - \frac{11}{6}$

(d) $\frac{1}{2} - \frac{1}{3} + \frac{3}{4}$

Solution.

(a)
$$\frac{2}{3} - \frac{5}{7} = \frac{14}{21} - \frac{15}{21} = -\frac{1}{21}$$

(b)
$$\frac{5}{6} + \frac{7}{2} = \frac{5}{6} + \frac{21}{6} = \frac{26}{6} = \frac{13}{3}$$

(c)
$$\frac{3}{4} - \frac{11}{6} = \frac{9}{12} - \frac{22}{12} = -\frac{13}{12}$$

(d)
$$\frac{1}{2} - \frac{1}{3} + \frac{3}{4} = \frac{6}{12} - \frac{4}{12} + \frac{9}{12} = \frac{11}{12}$$

Problem 4. (10pt) Showing all your work and reducing as much as possible, perform the following computations:

(a) $\frac{7}{6} \cdot \frac{12}{3}$

(b) $-\frac{3}{4} \cdot \frac{12}{27}$

(c) $\frac{6}{35} \cdot \frac{14}{15}$

(d) $-\frac{10}{7} \cdot -\frac{5}{3}$

Solution.

(a)

$$\frac{7}{6} \cdot \frac{12}{3} = \frac{7}{\cancel{6}} \cdot \frac{\cancel{12}^{2}}{3} = \frac{14}{3}$$

(b)

$$-\frac{3}{4} \cdot \frac{12}{27} = -\frac{\cancel{3}}{\cancel{4}} \cdot \frac{\cancel{12}^{\cancel{3}^{1}}}{\cancel{27}^{\cancel{9}^{1}}} = -\frac{1}{3}$$

(c)

$$\frac{6}{35} \cdot \frac{14}{15} = \frac{\cancel{6}^{2}}{\cancel{35}^{5}} \cdot \frac{\cancel{14}^{2}}{\cancel{15}^{5}} = \frac{4}{25}$$

(d)

$$-\frac{10}{7} \cdot -\frac{5}{3} = \frac{50}{21}$$

Problem 5. (10pt) Showing all your work and reducing as much as possible, perform the following computations:

(a) $\dfrac{\frac{4}{5}}{\frac{2}{15}}$

(b) $\dfrac{\frac{3}{4}}{\frac{5}{2}}$

(c) $\dfrac{\frac{3}{7}}{\frac{5}{6}}$

(d) $\dfrac{\frac{14}{33}}{\frac{10}{21}}$

Solution.

(a)
$$\frac{\frac{4}{5}}{\frac{2}{15}} = \frac{4}{5} \cdot \frac{15}{2} = \frac{\cancel{4}^{2}}{\cancel{5}} \cdot \frac{\cancel{15}^{3}}{\cancel{2}} = 6$$

(b)
$$\frac{\frac{3}{4}}{\frac{5}{2}} = \frac{3}{4} \cdot \frac{2}{5} = \frac{3}{\cancel{4}^{2}} \cdot \frac{\cancel{2}}{5} = \frac{3}{10}$$

(c)
$$\frac{\frac{3}{7}}{\frac{5}{6}} = \frac{3}{7} \cdot \frac{6}{5} = \frac{18}{35}$$

(d)
$$\frac{\frac{14}{33}}{\frac{10}{21}} = \frac{14}{33} \cdot \frac{21}{10} = \frac{\cancel{14}^{7}}{\cancel{33}^{11}} \cdot \frac{\cancel{21}^{7}}{\cancel{10}^{5}} = \frac{49}{55}$$

Problem 6. (10pt) Convert the following mixed fraction to an improper fraction:

$$3\,\frac{4}{7}$$

Solution.

$$3\,\frac{4}{7} = \frac{3(7)+4}{7} = \frac{21+4}{7} = \frac{25}{7}$$

Problem 7. (10pt) Convert the following improper fraction to a mixed number:

$$\frac{27}{5}$$

Solution.

$$\frac{27}{5} = \frac{25+2}{5} = \frac{25}{5} + \frac{2}{5} = 5 + \frac{2}{5} = 5\,\frac{2}{5}$$

Problem 8. (10pt) Showing all your work, represent the following fractions as a decimal:

(a) $\frac{5}{8}$

(b) $\frac{4}{11}$

Solution.

(a)

$$\begin{array}{r} 0.625 \\ 8\overline{)\,5.000} \\ \underline{4.8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

$$\frac{5}{8} = 0.625$$

(b)

$$\begin{array}{r} 0.\overline{36} \\ 11\overline{)\,4.00} \\ \underline{3.3} \\ 70 \\ \underline{66} \\ 4 \end{array}$$

$$\frac{4}{11} = 0.\overline{36}$$

Problem 9. (10pt) Convert the following decimal to a fraction—reducing as much as possible:

$$0.45$$

Solution.

$$0.45 = \frac{45}{100} = \frac{3^2 \cdot \cancel{5}}{2^2 \cdot \cancel{5}^{\cancel{2}^1}} = \frac{9}{20}$$

Problem 10. (10pt) Convert the following decimal to a fraction—reducing as much as possible:

$$0.6363636\overline{63}$$

Solution.

$$\begin{array}{rrcc} & 100N & = & 63.6363636\overline{63} \\ - & N & = & 0.636363636\overline{63} \\ \hline & 99N & = & 63 \\ & N & = & \frac{63}{99} \\ & N & = & \frac{7}{11} \end{array}$$

$$0.\overline{63} = \frac{7}{11}$$