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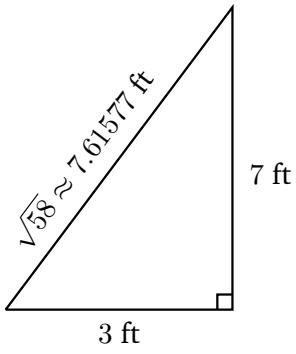
MATH 100
Fall 2023
HW 3: Due 09/18

"Fire can't go through doors, stupid. It's not a ghost."
–Ben Chang, Community

Problem 1. (10pt) Consider the triangle given below:



(a) Find the perimeter of the triangle.

(b) Find the area of the triangle.

(c) If the lengths of the legs in the triangle were mislabeled as being in feet when they should have been in meters, convert your answer in (b) to square meters.

Solution.

(a) To find the perimeter of the triangle, we first need to find the length of the hypotenuse. Using the Pythagorean Theorem, $a^2 + b^2 = c^2$, we have...

$$c^2 = a^2 + b^2$$
$$c^2 = (3 \text{ ft})^2 + (7 \text{ ft})^2$$
$$c^2 = 9 \text{ ft}^2 + 49 \text{ ft}^2$$
$$c^2 = 58 \text{ ft}^2$$
$$c = \sqrt{58} \text{ ft} \approx 7.61577 \text{ ft}$$

We know that the perimeter of the triangle is the sum of the lengths of its sides, i.e. $P = a + b + c$. But then...

$$P = a + b + c = 3 \text{ ft} + 7 \text{ ft} + 7.61577 \text{ ft} = 17.61577 \text{ ft}$$

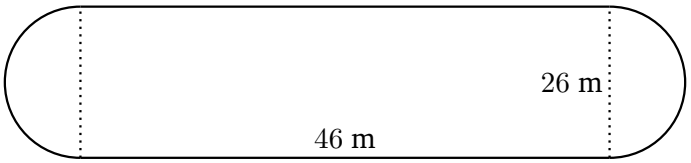
(b) We know that the area of a triangle is $A = \frac{1}{2} \cdot \text{base} \cdot \text{height}$. But then we have $A = \frac{1}{2} \cdot 3 \text{ ft} \cdot 7 \text{ ft} = \frac{21}{2} \text{ ft}^2 = 10.5 \text{ ft}^2$.

(c) We have...

$$\frac{10.5 \text{ ft}^2 \,\|\, 1 \text{ m} \mid 1 \text{ m}}{\| \, 3.28084 \text{ ft} \mid 3.28084 \text{ ft}} = 0.975482 \text{ m}^2$$

Alternatively, we can convert each of the lengths of legs of the triangle to meters. The legs with length 3 ft and 7 ft in meters are 0.9143 m and 2.1336 m, respectively. But then the area of the triangle is $A = \frac{1}{2} \cdot 0.9143 \text{ m} \cdot 2.1336 \text{ m} = 0.975375 \text{ m}^2$, as obtained above.

Problem 2. (10pt) Consider the 'track' shown below:



(a) Find the perimeter of the track.

(b) Find the area of the track.

(c) If you scale the track's size by a factor of two, what is the new perimeter and area?

(d) Suppose you were going to tile the interior rectangular portion of the track with special 2 m $\times$ 2 m tiles. How many would you need?

Solution.

(a)

(b)

(c)

(d)

Problem 3. (10pt) A whiskey barrel is approximately cylindrical in shape. Suppose that an American Oak whiskey barrel is 18 in across and 30 in tall.

(a) Estimate the volume of the barrel.

(b) If one cubic inch is 16.3871 ml, find the volume of the barrel in milliliters.

(c) You know expensive whiskeys can fetch \$450 per bottle, i.e. 750 ml. Use this to estimate the value of such a barrel filled with expensive whiskey if the barrel itself also has a value of \$250.

Solution.

(a) Because the barrel is approximately a cylinder, the volume of the barrel should be approximately the volume of the corresponding cylinder. We know that the volume of a cylinder is $V = \pi r^2 h$, where r is the radius of the cylinder and h is the height of the cylinder. Because the diameter of the barrel is 18 in, the radius of the barrel is $\frac{18 \text{ in}}{2} = 9$ in. But then we have...

$$V_{\text{barrel}} \approx \pi r^2 h = \pi \cdot (9 \text{ in})^2 \cdot 30 \text{ in} = \pi \cdot 81 \text{ in}^2 \cdot 30 \text{ in} \approx 7634.07 \text{ in}^3$$

(b) Converting our answer from (a), we have...

$$\frac{7634.07 \text{ in}^3 \quad \| \quad 16.3871 \text{ ml}}{\| \quad 1 \text{ in}^3} = 125{,}100.27 \text{ ml}$$

(c) We know that the value of the whiskey in the barrel is the number of bottles the barrel contains times the value of a bottle of whiskey. The number of bottles the barrel contains is...

$$\text{Number Bottles} = \frac{\text{Volume Barrel}}{\text{Bottle Volume}} = \frac{125100.27 \text{ ml}}{750 \text{ ml/bottle}} = 166.80036 \text{ bottles} \approx 166.8 \text{ bottles}$$

But then the value of the whiskey in the barrel is...

$$\text{Whiskey Value} = \text{Cost per Bottle} \cdot \text{Number Bottles} = \$450/\text{bottle} \cdot 166.8 \text{ bottles} = \$75,060$$

But then the total value of the barrel of whiskey is...

$$\text{Total Value} = \text{Whiskey Value} + \text{Barrel Value} = \$75,060 + \$250 = \$75,310$$