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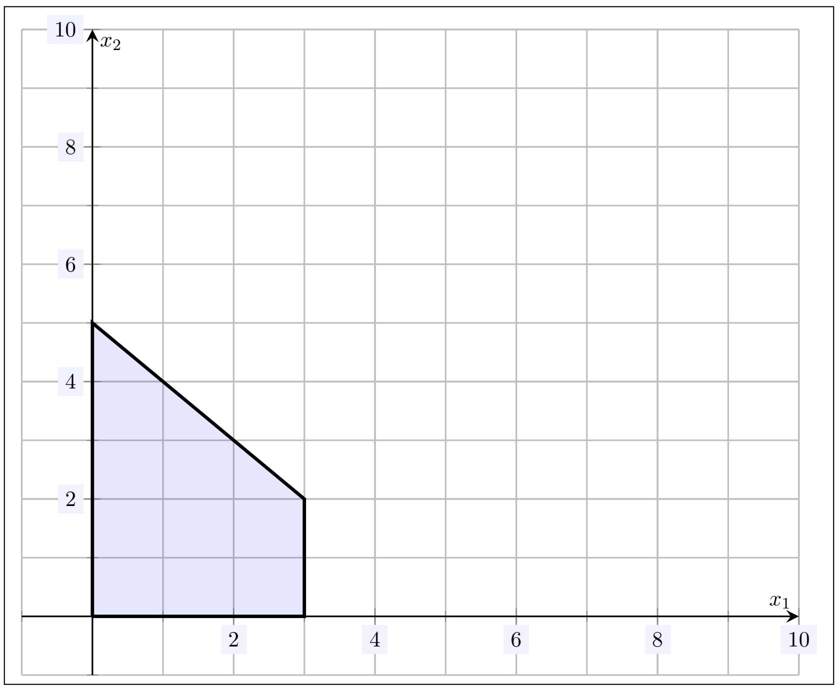
MATH 108
Fall 2022
HW 17: Due 12/01

"There is no branch of mathematics, however abstract, which may not some day be applied to phenomena of the real world."
–Nikolai Lobachevsky

Problem 1. (10pt) Showing all your work and as accurately as possible, plot the region given by the inequalities below:

$$\begin{aligned} x_1 + x_2 &\leq 5 \\ x_1 &\leq 3 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Is the region bounded or unbounded?



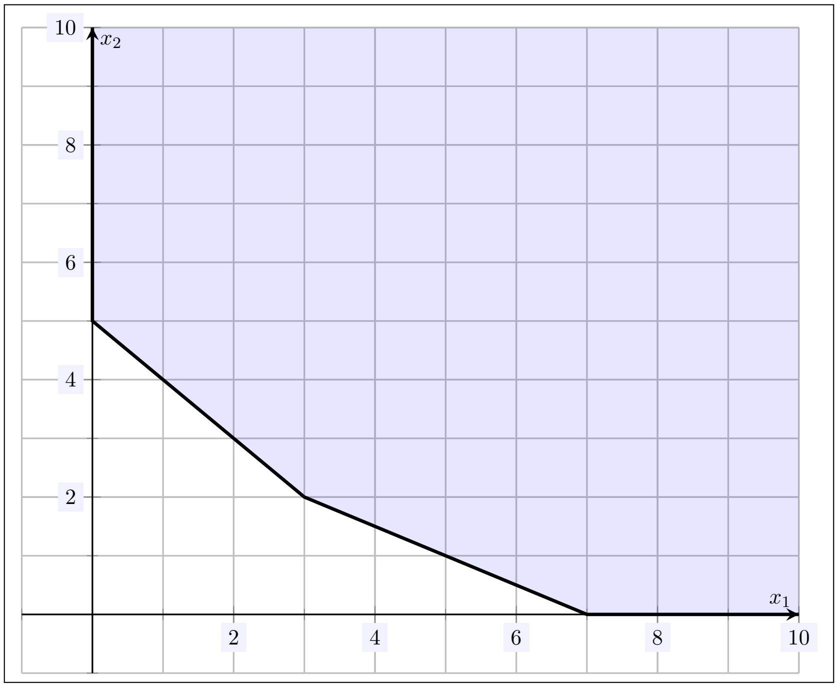
Solution. First, observe that if we 'solve' $x_1 + x_2 \leq 5$ for x_2, we obtain $x_2 \leq 5 - x_1$. The line $x_2 = 5 - x_1$ is a line with y-intercept $(0, 5)$ and slope -1. Because $x_2 \leq 5 - x_1$, we need shade below this line. The line $x_1 = 3$ is a vertical line with $x_1 = 3$ for all points on the line. Because $x_1 \leq 3$, we need shade to the left of this line. The line $x_1 = 0$ is the y-axis. Because $x_1 \geq 0$, we need shade to the right of the y-axis. The line $x_2 = 0$ is the x-axis. Because $x_2 \geq 0$, we need shade above the x-axis. [Note: Together, the inequalities $x_1, x_2 \geq 0$ simply state that the region must be in Quadrant I.] Clearly, we need find the intersection of the lines $x_2 = 5 - x_1$ and $x_1 = 3$. But if $x_1 = 3$, then $x_2 = 5 - x_1 = 5 - 3 = 2$. Therefore, these intersect at the point $(x_1, x_2) = (3, 2)$. Combining all this data gives the region shaded above.

Because we can clearly draw a 'ball' around this region, e.g. the circle at the origin with radius 10, the region is bounded.

Problem 2. (10pt) Showing all your work and as accurately as possible, plot the region given by the inequalities below:

$$x_1 + 2x_2 \geq 7$$
$$x_1 + x_2 \geq 5$$
$$x_1, x_2 \geq 0$$

Is the region bounded or unbounded?



Solution. First, observe that if we 'solve' $x_1 + 2x_2 \geq 7$ for x_2, we obtain $x_2 \geq \frac{7}{2} - \frac{1}{2}\,x_1$. The line $x_2 = \frac{7}{2} - \frac{1}{2}\,x_1$ has y-intercept $\frac{7}{2}$ and slope $-\frac{1}{2}$. Because $x_2 \geq \frac{7}{2} - \frac{1}{2}\,x_1$, we need shade above this line. 'Solving' $x_1 + x_2 \geq 5$ for x_2, we obtain $x_2 \geq 5 - x_1$. The line $x_2 = 5 - x_1$ has y-intercept 5 and slope -1. Because $x_2 \geq 5 - x_1$, we need shade above this line. The line $x_1 = 0$ is the y-axis. Because $x_1 \geq 0$, we need shade to the right of the y-axis. The line $x_2 = 0$ is the x-axis. Because $x_2 \geq 0$, we need shade above the x-axis. [Note: Together, the inequalities $x_1, x_2 \geq 0$ simply state that the region must be in Quadrant I.] Sketching these lines, we clearly need find the intersection of $x_2 = \frac{7}{2} - \frac{1}{2}\,x_1$ and $x_2 = 5 - x_1$. But then we have $\frac{7}{2} - \frac{1}{2}\,x_1 = 5 - x_1$. Multiplying both sides by 2, we obtain $7 - x_1 = 10 - 2x_1$. But then we have $7 + x_1 = 10$ so that $x_1 = 3$. Using this in the line $x_2 = 5 - x_1$, we have $x_2 = 5 - 3 = 2$. Therefore, the intersection of $x_2 = \frac{7}{2} - \frac{1}{2}\,x_1$ and $x_2 = 5 - x_1$ is $(3, 2)$. Clearly, we also need the x-intercept of $x_2 = \frac{7}{2} - \frac{1}{2}\,x_1$. At the x-intercept, $x_2 = 0$. But then $0 = \frac{7}{2} - \frac{1}{2}\,x_1$ so that $\frac{1}{2}\,x_1 = \frac{7}{2}$. This implies that $x_1 = 7$. Therefore, the line $x_2 = \frac{7}{2} - \frac{1}{2}\,x_1$ has x-intercept $(7, 0)$. Combining all this information gives the region shaded above.

Because there are points, (x, y) with arbitrarily large coordinates, the region is unbounded. That is, there is no 'ball' of fixed size which can enclose the region. Therefore, the region is unbounded.