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MATH 101

Spring 2022

HW 1: Due 02/08

“Windows are the eyes to the house.”

–Andy Dwyer, Parks & Recreation

Problem 1. (10pt) Give the definition of a real number. Also, give at least five original examples of a real number.

A real number is ‘any’ number which is expressible as a decimal, e.g.

$$0 = 0.0$$

$$1 = 1.0$$

$$-5 = -5.0$$

$$\frac{1}{2} = 0.5$$

$$-\frac{1}{10} = -0.1$$

$$0.11958904771$$

$$\sqrt{2} = 1.414213562373095 \dots$$

$$\pi = 3.141592653589793 \dots$$

$$e = 2.718281828495045 \dots$$

$$\gamma = 0.577215664901532 \dots$$

Problem 2. (10pt) Give the definition of a rational number. Also, give at least five original examples of a rational number.

A rational number is a real number of the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$, e.g.

$$0 = \frac{0}{1}$$

$$2 = \frac{2}{1}$$

$$-5 = -\frac{5}{1}$$

$$\frac{1}{2}$$

$$-\frac{5}{7}$$

$$\frac{20}{100}$$

Problem 3. (10pt) Find the prime factorizations of the following integers:

(a) 54

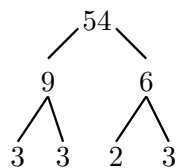
(b) 97

(c) 168

(d) 184

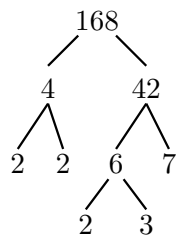
Solution.

(a) $54 = 2 \cdot 3^3$

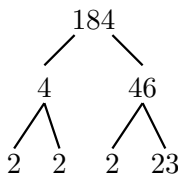


(b) $97 = 97$ (Already prime)

(c) $168 = 2^3 \cdot 3 \cdot 7$



(d) $184 = 2^3 \cdot 23$



Problem 4. (10pt) Without using a calculator, answer the following:

- (a) Does 2 divide 2346? Explain.
- (b) Does 3 divide 596012? Explain.
- (c) Does 4 divide 990140? Explain.
- (d) Does 5 divide 1431? Explain.
- (e) Does 9 divide 70155? Explain.

Problem 5. (10pt) Using the ‘square root method,’ show that 157 is prime.

Problem 6. (10pt) By listing out all the divisors of the given numbers, compute the following:

(a) $\gcd(12, 15)$

(b) $\gcd(20, 22)$

(c) $\gcd(36, 60)$

(d) $\gcd(20, 100)$

Problem 7. (10pt) By listing out sufficiently many multiples of the given integers, compute the following:

(a) $\text{lcm}(24, 36)$

(b) $\text{lcm}(12, 15)$

(c) $\text{lcm}(12, 18)$

(d) $\text{lcm}(36, 48)$

Problem 8. (10pt) By finding prime factorizations, compute the following:

(a) $\gcd(12, 15)$

(b) $\gcd(20, 22)$

(c) $\gcd(36, 60)$

(d) $\gcd(20, 100)$

Problem 9. (10pt) By finding prime factorizations, compute the following:

(a) $\text{lcm}(24, 36)$

(b) $\text{lcm}(12, 15)$

(c) $\text{lcm}(12, 18)$

(d) $\text{lcm}(36, 48)$

Problem 10. (10pt) Compute the following:

(a) $\gcd(2^3 \cdot 3^1 \cdot 5^3 \cdot 11^5, 2^2 \cdot 3^3 \cdot 5 \cdot 7)$

(b) $\text{lcm}(2^3 \cdot 3^1 \cdot 5^3 \cdot 11^5, 2^2 \cdot 3^3 \cdot 5 \cdot 7)$

(c) $\gcd(2^{10} \cdot 5^5 \cdot 13, 3^5 \cdot 5^1 \cdot 11^2)$

(d) $\text{lcm}(2^{10} \cdot 5^5 \cdot 13, 3^5 \cdot 5^1 \cdot 11^2)$