Name: ______________________________

MATH 308

Fall 2021

HW 5: Due 10/08

"Penny, while I subscribe to the many worlds theory which posits the existence of an infinite number of Sheldons in an infinite number of universes—I assure you that in none of them am I dancing."

–Sheldon Cooper, Big Bang Theory

Problem 1. (10pt) List at least 3 elements from each of the following sets:

(a) $\{n \in \mathbb{N} \colon \exists k \in \mathbb{N}, n = 6k\}$

(b) $\{x \in \mathbb{R} \colon \exists y \in \mathbb{R}, x = y^2\}$

(c) $\{m \in \mathbb{N} \colon \sqrt[3]{m} \in \mathbb{N}\}$

(d) $\{q \in \mathbb{Q} \colon 4q + 1 \in \mathbb{N}\}$

(e) $\{a \in \mathbb{N} \colon \exists b\, \exists c,\ b, c \in \mathbb{N}, a^2 + b^2 = c^2\}$

Problem 2. (10pt) Use the set-builder notation to give a set equal to each of the following sets:

(a) $\{1, 4, 9, 16, 25, 36, 49, 64, \ldots\}$

(b) $\{0, \pm 3, \pm 6, \pm 9, \pm 12, \pm 15, \ldots\}$

(c) The set of rational numbers between 0 and 1.

(d) The set of functions passing through the point $(6, 5)$.

(e) The set of differentiable functions with a horizontal tangent line at $x = 1$.

Problem 3. (10pt) Let $\mathscr{U} = \{1, 2, 3, \{1\}, \{2\}, \{1, 2\}\}$. Let $A = \{2, 1, 2\}$ and $B = \{1\}$.

(a) Is $A \in \mathscr{U}$? Explain.

(b) Is $A \subseteq \mathscr{U}$? Explain.

(c) Is $B \in \mathscr{U}$? Explain.

(d) Is $B \subseteq \mathscr{U}$? Explain.

Problem 4. (20pt) Define the following sets:

$$\begin{aligned} A &= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \\ B &= \{1, 3, 5, 7, 9\} \\ C &= \{2, 4, 6, 8, 10\} \\ D &= \{2, 3, 5, 7\} \\ E &= \{4, 8, 9\} \\ F &= \{1, 2, \{3\}\} \end{aligned}$$

Compute the following sets:

(a) $A \cap B$

(b) $C \cup D$

(c) $D \cap E$

(d) $D \setminus B$

(e) $B \setminus A$

(f) $B \times C$

(g) $(D \cap F) \cup (B \cap E)$

In addition, answer the following:

(h) Is $F \subseteq A$? Explain.

(i) Is $B \cap F = \{1, 3\}$? Explain.

(j) Is A a universal set for B, C, D, E, F? If it is, compute D^c. If not, explain why.

Problem 5. (10pt) Compute each of the following sets:

(a) $\mathscr{P}(\varnothing)$

(b) $\mathscr{P}(\{1, \{1\}\})$

(c) $\mathscr{P}(\{1, e, \pi\})$

(d) $\mathscr{P}(\{1\} \times \{a, b\})$

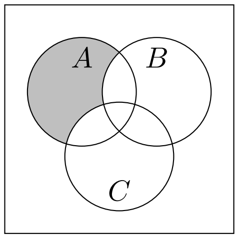
Problem 6. (10pt) Suppose A, B are sets with a common universal set $\mathscr{U}$. Denote each of the following sets with a Venn diagram:

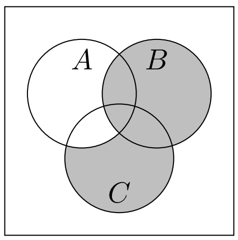
(a) $A \cap B^c$

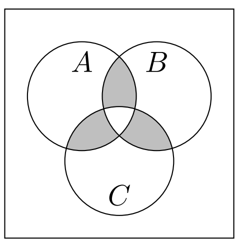
(b) $(A \cup B)^c$

(c) $(A \cup B) \setminus (A \cap B)$

Problem 7. (10pt) Suppose A, B, C are sets with a common universal set $\mathscr{U}$. For each of the Venn diagrams, write down the shaded sets.

(a) 

(b) 

(c) 

Problem 8. (10pt) Let $A = \{b, c\}$. Suppose that $A \cup B = \{a, b, c, e\}$ and $B \cup C = \{a, c, d, e, f\}$. From this information can we determine the sets A, B, C? Explain. If not, what is the minimal additional information (in terms of unions and intersections of the sets alone) would uniquely determine the three sets?