Quiz 1. *True/False*: If you had a bill of \$25.77 and were going to pay a tip of 20%, the total amount you would pay could be computed by finding $25.77(1.20)$.

Solution. The statement is *true*. Recall to calculate a percentage of a number N, we compute $N \cdot \%$, where N is the number and $\%$ is the percentage (written as a decimal). For instance, to compute 57% of 23, we compute $23(0.57) = 13.11$. To compute 172% of 150, we compute $150(1.72) = 258$. However, to compute a $\%$ percent increase or decrease of a number N, we compute $N(1 \pm \%)$, where N is the number, $\%$ is the percentage as a decimal, and we choose plus for increase and negative for decrease. For instance, to compute a 75% decrease of 13, we compute $13(1 - 0.75) = 13(0.25) = 3.25$. To compute a 115% increase of 120, we compute $120(1+1.15) = 120(2.15) = 258$. Here, we are increasing 25.77 by 20%, so we compute $25.77(1 + 0.20) = 25.77(1.20)$.

Quiz 2. *True/False*: The amount of concrete in tons, C, used to repair r roads remaining in a storage facility is given by $C(r) = 450.7 - 16.3r$. Because this function is linear, we can interpret the slope of $C(r)$ as saying that each road uses approximately 16.3 tons of concrete to repair.

Solution. The statement is *true*. The slope of the linear function $C(r) = 450.7 - 16.3r$ is...

$$m = -16.3 = -\frac{16.3}{1} = \frac{-16.3}{1}$$

Thinking of this slope as $\frac{\Delta\text{output}}{\Delta\text{input}}$, we can see that for each one increase in r, i.e. one additional road, there is a decrease by 16.3 tons in the amount of concrete remaining. Therefore, we can summarize this as that each road requires approximately 16.3 tons of concrete to repair.

Quiz 3. *True/False*: A company sells a product for $\$5.75$ per item. Each item costs approximately $\$1.37$ to manufacture and is produced in a machine that costs $\$87.50$ to operate. Given this data, we have $R(x) = 5.75$ and $C(x) = (1.37 + 87.50)x = 88.88x$.

Solution. The statement is *false*. If one sells x items, the revenue is $R(x) = 5.75 \cdot 7 = 5.75x$. Therefore, $R(x)$ is correct. However, we know that $C(x) = \text{VC} + \text{FC}$. The fixed costs are the machine operation costs, i.e. $\text{FC} = \$87.50$. The variable costs are the $\$1.37$ cost per item. If x items are produced, then the manufacture costs are $\text{VC} = 1.37 \cdot x = 1.37x$. Therefore, $C(x) = \text{VC} + \text{FC} = 1.37x + 87.50$.

Quiz 4. *True/False*: If the following matrix represents an augmented matrix in RREF, then the corresponding system has solution $x_1 = 5$, $x_2 = -3$, and $x_3 = 7$.

$$\begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Solution. The statement is *false*. Examining the equation corresponding to the last row, we see that $0 = 1$, which is impossible. Therefore, the original system of equations was inconsistent. But then the original system of equations has no solution.

Quiz 5. *True/False*: You can perform the following multiplication:

$$\begin{pmatrix} 1 & -1 & 0 & 5 & 3 \\ 0 & 4 & -2 & 6 & 1 \end{pmatrix} \begin{pmatrix} 3 & -2 \\ 3 & 8 \\ 4 & 0 \\ 2 & -1 \\ 0 & 5 \end{pmatrix}$$

Solution. The statement is *true*. Recall that you can multiply a $m \times n$ matrix with a $p \times q$ matrix if $n = p$. If so, you obtain a $m \times q$ matrix. The first matrix is 2×5 while the second matrix is 5×2. But because $5 = 5$, we can multiply these matrix to obtain a 2×2 matrix. One can check that the product is...

$$\begin{pmatrix} 10 & 0 \\ 16 & 31 \end{pmatrix}$$

Quiz 6. *True/False*: The matrix $\begin{pmatrix} -2 & 8 \\ -2 & 6 \end{pmatrix}$ has an inverse.

Solution. The statement is *true*. Recall that a matrix has an inverse if and only if the determinant of the matrix is *not* zero. We have...

$$\begin{pmatrix} -2 & 8 \\ -2 & 6 \end{pmatrix} = -2(6) - 8(-2) = -12 + 16 = 4 \neq 0$$

Therefore, the matrix is invertible. Recalling that if A is a 2×2 matrix (given below) that is invertible, we have...

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$A^{-1} = \frac{1}{\det A} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Therefore,

$$\begin{pmatrix} -2 & 8 \\ -2 & 6 \end{pmatrix}^{-1} = \frac{1}{4} \begin{pmatrix} 6 & -8 \\ 2 & -2 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} & -2 \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

Quiz 7. *True/False*: The point $(1, -3)$ satisfies the following system of inequalities:

$$\begin{aligned} x + y &\leq 0 \\ x - 2y &\leq 5 \end{aligned}$$

Solution. The statement is *false*. If a point satisfies a system of inequalities, it satisfies each of the inequalities individually—which we can check:

$$\begin{aligned} x + y &\leq 0 & \qquad x - 2y &\overset{?}{\leq} 5 \\ 1 + (-3) &\overset{?}{\leq} 0 & 1 - 2(-3) &\overset{?}{\leq} 5 \\ -2 &\leq 0 \checkmark & 1 + 6 &\overset{?}{\leq} 5 \\ & & 7 &\not\leq 5 \times \end{aligned}$$

Because $(1, -3)$ does not satisfy all the inequalities, $(1, -3)$ does not satisfy the system of inequalities.

Quiz 8. *True/False*: To maximize $z = 5x + 6y$ subject to $2x + 3y \leq 6$, $-6x + y \leq 20$, and $x, y \geq 0$, the initial simplex tableau is...

$$\begin{array}{cccc|c} 2 & 3 & 1 & 0 & 6 \\ -6 & 1 & 0 & 1 & 20 \\ \hline -5 & -6 & 0 & 0 & 0 \end{array}$$

Solution. The statement is *true*. First, note that the problem is in standard form. For each inequality, we introduce a slack variable so that we have...

$$\begin{aligned} 2x + 3y + s_1 \qquad &= 6 \\ -6x + y \qquad + s_2 &= 20 \end{aligned}$$

Moving all the variables to the left side in $z = 5x + 6y$, we have $z - 5x - 6y = 0$. Aligning the equations, we have...

$$\begin{aligned} 2x + 3y + s_1 \qquad &= 6 \\ -6x + y \qquad + s_2 &= 20 \\ z - 5x - 6y \qquad\qquad &= 0 \end{aligned}$$

This gives us the initial simplex tableau...

$$\begin{array}{cccc|c} 2 & 3 & 1 & 0 & 6 \\ -6 & 1 & 0 & 1 & 20 \\ \hline -5 & -6 & 0 & 0 & 0 \end{array}$$

Quiz 9. *True/False*: Given the following minimization problem:

$$\begin{aligned} \min w = x_1 + 2x_2 + 3x_3 \\ x_1 + x_2 + x_3 \geq 4 \\ x_1 - x_2 + x_3 \geq 6 \\ -x_1 + x_2 - x_3 \geq 8 \end{aligned}$$

the dual problem is...

$$\begin{pmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 2 \\ 1 & 1 & -1 & 3 \\ 4 & 6 & 8 & 0 \end{pmatrix}$$

Solution. The statement is *false*. First, observe that the problem is in standard form. Given a minimization problem in standard form, we first align all the inequalities with the function as the bottom equation:

$$\begin{aligned} x_1 + x_2 + x_3 \geq 4 \\ x_1 - x_2 + x_3 \geq 6 \\ -x_1 + x_2 - x_3 \geq 8 \\ x_1 + 2x_3 + 3x_3 = 0 \end{aligned}$$

From this, we form the matrix...

$$\begin{pmatrix} 1 & 1 & 1 & 4 \\ 1 & -1 & 1 & 6 \\ -1 & 1 & -1 & 8 \\ 1 & 2 & 3 & 0 \end{pmatrix}$$

We then take the transpose of this matrix...

$$\begin{pmatrix} 1 & 1 & 1 & 4 \\ 1 & -1 & 1 & 6 \\ -1 & 1 & -1 & 8 \\ 1 & 2 & 3 & 0 \end{pmatrix}^T = \begin{pmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 2 \\ 1 & 1 & -1 & 3 \\ 4 & 6 & 8 & 0 \end{pmatrix}$$

But then the corresponding maximization problem (the dual problem) is...

$$\begin{aligned} \max z = 4y_1 + 6y_2 + 8y_3 \\ y_1 + y_2 - y_3 \leq 1 \\ y_1 - y_2 + y_3 \leq 2 \\ y_1 + y_2 - y_3 \leq 3 \end{aligned}$$

Quiz 10. *True/False*: If \$4,000 is placed into an account that earns 5% interest, compounded quarterly, then the amount of money in the account after 8 years is...

$$4000\left(1+\frac{0.05}{4}\right)^{32}$$

Solution. The statement is *true*. We know that if P dollars is placed into an account earning an annual interest rate r, compounded k times per year, then the amount of money in the account after t years, F, is given by...

$$F = P\left(1+\frac{r}{k}\right)^{kt}$$

We have $P = 4000$, $r = 0.05$, $k = 4$, and $t = 8$ so that we have...

$$F = 4000\left(1+\frac{0.05}{4}\right)^{4\cdot 8} = 4000\left(1+\frac{0.05}{4}\right)^{32}$$

Quiz 11. *True/False*: An ordinary annuity is a series of equal payments, paid at equal intervals of time with payments occurring at the start of the payment period.

Solution. The statement is *false*. In an ordinary (simple) annuity, payments are made at the *end* of each payment period and the payment periods are the same as the interest periods (otherwise, it is a general ordinary annuity). If the payments are made at the *start* of each payment period and the payment periods are the same as the interest periods, then it is a (simple) annuity due (otherwise, it is a general annuity due). We can summarize this in the following chart:

	Payment at Start of Period	Payment at End of Period
Payment Period = Compounding Period	Ordinary Annuity Due	Ordinary (Simple) Annuity
Payment Period ≠ Compounding Period	General Annuity Due	General Annuity

Quiz 12. *True/False*: If you have a \$5,000 loan at 5.4% annual interest, compounded monthly, that you pay over 5 years using a series of monthly payments of \$95.28, then the amount you still owe on the loan after 2 years is...

$$P = 5000\, a_{\overline{36}|0.0045} = 3160.11$$

Solution. The statement is *false*. We know that the amount due on an (ordinary) amortized loan is given by $P = R\, a_{\overline{n-m}|i}$. We have $R = 95.28$, $i_p = r/k = 0.054/12 = 0.0045$, $n = kt = 12 \cdot 5 = 60$, and $m = kt_0 = 12 \cdot 2 = 24$. Then we have...

$$P = 95.28\, a_{\overline{36}|0.0045} = 3160.11$$

Quiz 13. *True/False*: In an amortized loan, a portion of each payment goes towards paying off the principal, with a portion of the payment going towards the interest. Over time, the portion of the payment going towards the interest decreases.

Solution. The statement is *true*. Because the amount owed on the loan is at first 'large', the amount of the interest is 'large' at first. Therefore, more of each payment is going towards the interest than later in the loan when the interest is less because the interest is being applied to a smaller amount (because portions of the loan have been paid off).