

Name: _____

MATH 308

Fall 2022

HW 12: Due 11/04

*“Algebra is the intellectual instrument which has been created for
rendering clear the quantitative aspects of the world.”*

–Alfred North Whitehead

Problem 1. (10pt) Showing all your work, complete the following:

- (a) Find the last digit of 3^{300} .
- (b) Find the last two digits of 13^{100} .
- (c) Fermat’s Little Theorem states that if p is prime, then $a^p \equiv a \pmod{p}$. Verify this claim when $p = 5$ and $a = 3$.
- (d) A generalization of Fermat’s Little Theorem states that $a^{\varphi(n)} \equiv 1 \pmod{n}$ if a is coprime to n , where $\varphi(n)$ is the Euler Phi function. Verify this claim when $n = 3$ and $a = 8$.

Problem 2. (10pt) Showing all your work, compute the following:

- (a) Compute 147 modulo 3.
- (b) Compute 147 modulo 3 by writing $147 = 1 \cdot 100 + 4 \cdot 10 + 7 \cdot 1$.
- (c) Compute $a_2a_1a_0$ modulo 3 by writing $a_2a_1a_0 = a_2 \cdot 100 + a_1 \cdot 10 + a_0 \cdot 1$. When is $a_2a_1a_0$ divisible by 3? Explain.
- (d) Using the previous parts, give a necessary and sufficient condition for an integer to be divisible by 3.

Problem 3. (10pt) Use the Chinese Remainder Theorem to solve the following system of linear congruences

$$2x \equiv 1 \pmod{3}$$

$$x - 3 \equiv 0 \pmod{4}$$

$$3x + 2 \equiv 4 \pmod{5}$$

Problem 4. (10pt) Show that there are no integer solutions to $x^3 + 7y^2 = 5$.