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MATH 101

Summer 2022

HW 1: Due 05/24

*"It is easy to forget now, how effervescent and free we all felt that summer."*

*—Anna Godbersen*

**Problem 1.** (10pt) Showing all your work and simplifying as much as possible, compute each of the following:

(a)  $15/5(1 + 2)$

(b)  $20/(4(2 + 3))$

(c)  $\frac{4^2/2 - 8 + 3 \cdot -2}{(12 - 4)(4 - 5)}$

(d)  $\frac{8 - 12}{-2^2} + 7 \cdot 12/2$

(e)  $4(-1)^3 - 2(-1)^3 + 6 \cdot 5/4$

**Solution.**

(a)

$$15/5(1 + 2) = 15/5(3) = 3(3) = 9$$

(b)

$$20/(4(2 + 3)) = 20/(4(5)) = 20/20 = 1$$

(c)

$$\frac{4^2/2 - 8 + 3 \cdot -2}{(12 - 4)(4 - 5)} = \frac{4^2/2 - 8 + 3 \cdot -2}{8 \cdot -1} = \frac{16/2 - 8 - 6}{-8} = \frac{8 - 8 - 6}{-8} = \frac{-6}{-8} = \frac{6}{8} = \frac{3}{4}$$

(d)

$$\frac{8 - 12}{-2^2} + 7 \cdot 12/2 = \frac{-4}{-4} + 7 \cdot 6 = 1 + 42 = 43$$

(e)

$$4(-1)^3 - 2(-1)^3 + 6 \cdot 5/4 = 4(-1) - 2(-1) + 30/4 = -4 + 2 + \frac{15}{2} = -2 + \frac{15}{2} = \frac{-4}{2} + \frac{15}{2} = \frac{11}{2}$$

**Problem 2.** (10pt) Showing all your work, find the prime factorizations of the following integers:

(a) 90

(b) 141

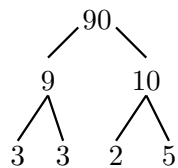
(c) 149

(d) 27

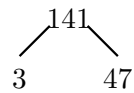
(e) 185

**Solution.**

(a)  $90 = 2 \cdot 3^2 \cdot 5$

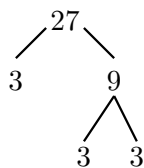


(b)  $141 = 3 \cdot 47$

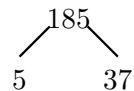


(c)  $149 = 149^1$

(d)  $27 = 3^3$



(e)  $185 = 5 \cdot 37$



**Problem 3.** (10pt) Compute each of the following by finding the divisors/multiples of the given integers:

(a)  $\gcd(18, 24)$

(b)  $\gcd(60, 125)$

(c)  $\text{lcm}(14, 20)$

(d)  $\text{lcm}(10, 21)$

**Solution.**

(a)

18: 1, 2, 3, 6, 9, 18

24: 1, 2, 3, 4, 6, 8, 12, 24

Therefore,  $\gcd(18, 24) = 6$ .

(b)

60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

125: 1, 5, 25, 125

Therefore,  $\gcd(60, 125) = 5$ .

(c)

14: 14, 28, 42, 56, 70, 84, 98, 112, 126, **140**

20: 20, 40, 60, 80, 100, 120, **140**, 160, 180, 200

Therefore,  $\text{lcm}(14, 20) = 140$ .

(d)

10: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140, 150, 160, 170, 180, 190, 200, **210**

21: 21, 42, 63, 84, 105, 126, 147, 168, 189, **210**

Therefore,  $\text{lcm}(10, 21) = 210$ .

**Problem 4.** (10pt) Use the prime factorizations of the given integers to compute each of the following:

(a)  $\gcd(142, 200)$

(b)  $\text{lcm}(72, 204)$

(c)  $\gcd(2^{11} \cdot 3^8 \cdot 7^2 \cdot 17^4, 2^5 \cdot 3^2 \cdot 5^6 \cdot 11^{30})$

(d)  $\text{lcm}(2^{11} \cdot 3^8 \cdot 7^2 \cdot 17^4, 2^5 \cdot 3^2 \cdot 5^6 \cdot 11^{30})$

**Solution.**

(a)

$$\gcd(142, 200) = \gcd(2 \cdot 71, 2^3 \cdot 5^2) = 2$$

(b)

$$\text{lcm}(72, 204) = \text{lcm}(2^3 \cdot 3^2, 2^2 \cdot 3 \cdot 17) = 2^3 \cdot 3^2 \cdot 17 = 1224$$

(c)

$$\gcd(2^{11} \cdot 3^8 \cdot 7^2 \cdot 17^4, 2^5 \cdot 3^2 \cdot 5^6 \cdot 11^{30}) = 2^5 \cdot 3^2 = 288$$

(d)

$$\begin{aligned} \text{lcm}(2^{11} \cdot 3^8 \cdot 7^2 \cdot 17^4, 2^5 \cdot 3^2 \cdot 5^6 \cdot 11^{30}) &= 2^{11} \cdot 3^8 \cdot 5^6 \cdot 7^2 \cdot 11^{30} \cdot 17^4 \\ &= 14993131026926250123829433260382033579359008000000 \end{aligned}$$

**Problem 5.** (10pt) For each of the following, either convert the rational number from an improper fraction to a proper fraction or vice versa:

(a)  $5\frac{6}{7}$

(b)  $\frac{35}{3}$

(c)  $-9\frac{3}{4}$

(d)  $-\frac{26}{5}$

**Solution.**

(a)

$$5\frac{6}{7} = 5 + \frac{6}{7} = \frac{35}{7} + \frac{6}{7} = \frac{41}{7}$$

(b)

$$\begin{array}{r} 11.\overline{6} \\ 3 \overline{) 35.0} \\ \underline{3} \phantom{0} \\ 05 \phantom{0} \\ \underline{3} \phantom{0} \\ 2.0 \phantom{0} \\ \underline{1.8} \phantom{0} \\ 2 \phantom{0} \end{array}$$

Then we have  $35 - 3(11) = 35 - 33 = 2$  so that  $\frac{35}{3} = 3\frac{2}{3}$ .

(c)

$$-9\frac{3}{4} = -\left(9 + \frac{3}{4}\right) = -\left(\frac{36}{4} + \frac{3}{4}\right) = -\frac{39}{4}$$

(d)

$$\begin{array}{r} 5.2 \\ 5 \overline{) 26.0} \\ \underline{25} \phantom{0} \\ 1.0 \phantom{0} \\ \underline{1.0} \phantom{0} \\ 0 \phantom{0} \end{array}$$

Then we have  $26 - 5(5) = 26 - 25 = 1$  so that  $-\frac{26}{5} = -5\frac{1}{5}$ .

**Problem 6.** (10pt) Completely reduce the following rational numbers, showing all your work:

(a)  $\frac{15}{33}$

(b)  $-\frac{140}{90}$

(c)  $\frac{210}{308}$

(d)  $\frac{10}{21}$

**Solution.**

(a)

$$\frac{15}{33} = \frac{\cancel{3} \cdot 5}{\cancel{3} \cdot 11} = \frac{5}{11}$$

(b)

$$-\frac{140}{90} = -\frac{14 \cdot \cancel{10}}{9 \cdot \cancel{10}} = -\frac{14}{9}$$

(c)

$$\frac{210}{308} = \frac{21 \cdot 10}{4 \cdot 77} = \frac{3 \cdot \cancel{7} \cdot 10}{4 \cdot \cancel{7} \cdot 11} = \frac{3 \cdot \cancel{10^5}}{\cancel{4}^2 \cdot 11} = \frac{3 \cdot 5}{2 \cdot 11} = \frac{15}{22}$$

(d)

$$\frac{10}{21} = \frac{2 \cdot 5}{3 \cdot 7} = \frac{10}{21}$$

**Problem 7.** (10pt) Simplifying as much as possible and showing all your work, compute the following:

(a)  $\frac{12}{15} - \frac{5}{9}$

(b)  $\frac{1}{6} + \frac{7}{12}$

(c)  $-\frac{5}{12} + \frac{7}{18}$

(d)  $2 + \frac{1}{3} - \frac{5}{2}$

**Solution.**

(a)

$$\frac{12}{15} - \frac{5}{9} = \frac{36}{45} - \frac{25}{45} = \frac{36 - 25}{45} = \frac{11}{45}$$

(b)

$$\frac{1}{6} + \frac{7}{12} = \frac{2}{12} + \frac{7}{12} = \frac{2 + 7}{12} = \frac{9}{12} = \frac{3}{4}$$

(c)

$$-\frac{5}{12} + \frac{7}{18} = -\frac{15}{36} + \frac{14}{36} = \frac{-15 + 14}{36} = -\frac{1}{36}$$

(d)

$$2 + \frac{1}{3} - \frac{5}{2} = \frac{12}{6} + \frac{2}{6} - \frac{15}{6} = \frac{12 + 2 - 15}{6} = -\frac{1}{6}$$

**Problem 8.** (10pt) Simplifying as much as possible and showing all your work, compute the following:

(a)  $\frac{15}{14} \cdot \frac{7}{33}$

(b)  $\frac{\frac{5}{6}}{\frac{7}{15}}$

(c)  $\frac{19}{4} \cdot -\frac{10}{9}$

(d)  $\frac{\frac{2}{45}}{\frac{20}{21}}$

**Solution.**

(a)

$$\frac{15}{14} \cdot \frac{7}{33} = \frac{\cancel{3} \cdot 5}{2 \cdot \cancel{7}} \cdot \frac{\cancel{7}}{\cancel{3} \cdot 11} = \frac{5}{22}$$

(b)

$$\frac{\frac{5}{6}}{\frac{7}{15}} = \frac{5}{6} \cdot \frac{15}{7} = \frac{5}{\cancel{6}^2} \cdot \frac{\cancel{15}^5}{7} = \frac{25}{14}$$

(c)

$$\frac{19}{4} \cdot -\frac{10}{9} = \frac{19}{\cancel{4}^2} \cdot -\frac{\cancel{10}^5}{9} = -\frac{95}{18}$$

(d)

$$\frac{\frac{2}{45}}{\frac{20}{21}} = \frac{2}{45} \cdot \frac{21}{20} = \frac{\cancel{2}}{\cancel{3} \cdot 3 \cdot 5} \cdot \frac{\cancel{3} \cdot 7}{\cancel{2} \cdot 2 \cdot 5} = \frac{7}{150}$$



**Problem 9.** (10pt) Showing all your work, convert the following rational numbers to decimals:

(a)  $\frac{4}{9}$

(b)  $\frac{7}{20}$

(c)  $\frac{2}{11}$

**Solution.**

(a)

$$\begin{array}{r} 0.\overline{4} \\ 9 \overline{) 4.0} \\ \underline{3.6} \\ 4 \end{array}$$

(b)

$$\begin{array}{r} 0.35 \\ 20 \overline{) 7.00} \\ \underline{6.0} \\ 1.00 \\ \underline{1.00} \\ 0 \end{array}$$

(c)

$$\begin{array}{r} 0.\overline{18} \\ 11 \overline{) 2.00} \\ \underline{1.1} \\ 90 \\ \underline{88} \\ 2 \end{array}$$

**Problem 10.** (10pt) Showing all your work, convert the following decimals to rational numbers:

(a) 0.7

(b) 0.125

(c)  $0.121212\overline{12}$

**Solution.**

(a)

$$0.7 = \frac{7}{10}$$

(b)

$$0.125 = \frac{125}{1000} = \frac{3}{40}$$

(c)

$$\begin{array}{rcl} 100N & = & 12.12121212\overline{12} \\ - N & = & 0.12121212\overline{12} \\ \hline 99N & = & 12 \\ N & = & \frac{12}{99} \\ N & = & \frac{4}{33} \end{array}$$

$$0.\overline{12} = \frac{4}{33}$$