Name: ______________________________

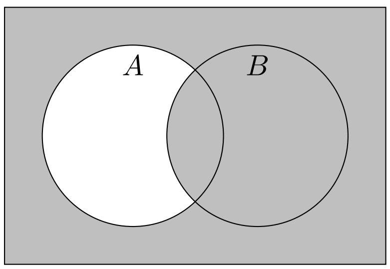
MATH 308
Fall 2023
HW 7: Due 10/05

"Since, as is well known, God helps those who help themselves, presumably the Devil helps all those, and only those, who don't help themselves. Does the Devil help himself?"

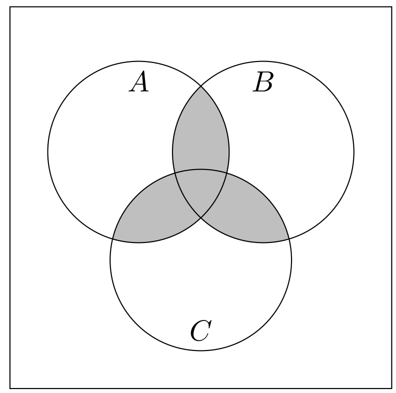
–Douglas Hofstadter

Problem 1. (10pt) For each of the following, if a Venn diagram is given, then express the shaded region as a set or set operation, and if a set operation is given, express the given set with a Venn diagram:

(a)



(c)



(b) $(A \cap B) \cup (A \cup B)^c$

(d) $(A \setminus C) \cap B$

Problem 2. (10pt) We shall create a new mathematical term: let A, B be sets. We say A is a *pseudo-subset* of B, written $A \sqsubset B$, if there is an element of A that is also an element of B and also an element of A that is not an element of B.

(a) We know if S is a set, then $\varnothing \subseteq S$. Is the same true for *pseudo-subsets*? That is, do we have $\varnothing \sqsubset S$ for all sets S? Explain.

(b) If A is a *pseudo-subset* of B, are A and B disjoint? Explain.

(c) Express the definition of being a *pseudo-subset* as a quantified logical statement.

(d) If what it means for $A \not\sqsubset B$ by negating your expression in (c). Write this quantified statement as a complete English sentence.

Problem 3. (10pt) Below is a partial proof of the fact that if A, B, C are sets, then $A \cap (B - C) = (A \cap B) - (A \cap C)$. By filling in the missing portions, complete the partial proof below so that it is a correct, logically sound proof with 'no gaps.'

Proposition. If A, B, C are sets, then $A \cap (B - C) = (A \cap B) - (A \cap C)$.

Proof. To prove that $A \cap (B - C) = (A \cap B) - (A \cap C)$, we need to show ________________ and

________________.

If $A \cap (B - C) = \varnothing$ or $(A \cap B) - (A \cap C) = \varnothing$, then $\varnothing = A \cap (B - C) \subseteq (A \cap B) - (A \cap C)$ and $\varnothing = (A \cap B) - (A \cap C) \subseteq A \cap (B - C)$, respectively. Assume neither $A \cap (B - C)$ nor $(A \cap B) - (A \cap C)$ are empty.

$A \cap (B - C) \subseteq (A \cap B) - (A \cap C)$: Let $x \in$ ________________. Then ________________ and

________________. Because $x \in B - C$, we know that ________________ and ________________.

But then $x \in A$ and $x \in B$ so that ________________. Now $x \in A$ but $x \notin$ ________________ so that

$x \notin$ ________________. This shows that $x \in (A \cap B) - (A \cap C)$. Therefore, ________________.

$(A \cap B) - (A \cap C) \subseteq A \cap (B - C)$: Let $x \in (A \cap B) - (A \cap C)$. Then $x \in$ ________________

and $x \notin A \cap C$. Because $x \in A \cap B$, we know that $x \in$ ________________ and $x \in$ ________________.

Because $x \notin A \cap C$, we know that $x \notin$ ________________ or $x \notin$ ________________. But because

$x \in A$, it must be that $x \notin$ ________________. Because $x \in B$ and $x \notin C$, we know that

$x \in$ ________________. But then $x \in$ ________________ and $x \in$ ________________, so that

$x \in A \cap (B - C)$. Therefore, ________________.

Because ________________ and ________________, we know that $A \cap (B - C) = (A \cap B) - (A \cap C)$. $\square$

Problem 4. (10pt) Let A, B be sets, not necessarily nonempty. Complete the following parts:

(a) Is possible for $A - B = B - A$? Explain.

(b) If $A \nsubseteq B$, does this imply that A is a proper subset of B? Explain.

(c) If A, B are not disjoint, does this imply there is an element $x \in A$ and $x \in B$? Explain.

(d) Is it possible for $A \subseteq A^c$? Explain.