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MATH 101
Spring 2024
HW 5: Due 02/07

"I don't wanna have to bring this up... But it's my turn to take a selfish."
— David Rose, Schitt's Creek

Problem 1. (10pts) Express each of the following decimal numbers as a rational number in simplest form and express each of the rational numbers as a decimal number:

(a) $\frac{1}{11}$

(b) 1.12

(c) $\frac{71}{5}$

Solution.

(a)

$$\begin{array}{r} 0.\overline{09} \\ 11\,\big)\overline{\,1.00} \\ \underline{99} \\ 1 \end{array}$$

(b)

$$1.12 = \frac{112}{100} = \frac{4 \cdot 28}{4 \cdot 25} = \frac{\cancel{4} \cdot 28}{\cancel{4} \cdot 25} = \frac{28}{25}$$

(c)

$$\begin{array}{r} 14.2 \\ 5\,\big)\overline{\,71.0} \\ \underline{5} \\ 21 \\ \underline{20} \\ 1.0 \\ \underline{1.0} \\ 0 \end{array}$$

Problem 2. (10pts) Showing all your work, express the number $0.\overline{123}$ as a rational number.

Solution. Suppose that $N = 0.\overline{123} = 0.123123123123\overline{123}$. We have...

$$\begin{array}{rcc} & 1000N & = & 123.123123123123\overline{123} \\ - & N & = & 0.123123123123\overline{123} \\ \hline & 999N & = & 123 \\ & N & = & \frac{123}{999} \\ & N & = & \frac{\cancel{3}\cdot 41}{\cancel{3}\cdot 333} \\ & N & = & \frac{41}{333} \end{array}$$

$$0.\overline{123} = \frac{41}{333}$$

Problem 3. (10pts) Perform the following operations in $\mathbb{C}$:

(a) $(6-8i)+(4+2i)$

(b) $(13-i)-(15-8i)$

(c) $(5+i)(6-2i)$

(d) $\dfrac{1+2i}{3+i}$

Solution.

(a)

$$(6-8i)+(4+2i)=(6+4)+(-8i+2i)=10-6i$$

(b)

$$(13-i)-(15-8i)=(13-15)+\big(-i-(-8i)\big)=-2+7i$$

(c)

$$(5+i)(6-2i)=5\cdot 6+5\cdot -2i+i\cdot 6+i\cdot -2i=30-10i+6i-2i^2=30-4i-2(-1)=32-4i$$

(d)

$$\frac{1+2i}{3+i}=\frac{1+2i}{3+i}\cdot\frac{3-i}{3-i}=\frac{(1+2i)(3-i)}{3^2+1^2}=\frac{3-i+6i-2i^2}{9+1}=\frac{3+5i-2(-1)}{10}=\frac{5+5i}{10}=\frac{1}{2}+\frac{1}{2}\,i$$

Problem 4. (10pts) Every quadratic equation $ax^2 + bx + c = 0$ has exactly two (not necessarily distinct) solutions when the solutions are allowed to be complex numbers. For instance, the equation $2x^2 - 20x + 68 = 0$ has as its solutions $5 \pm 3i$. Verify that $5 - 3i$ is a solution to this equation.

Solution. We have. . .

$$2x^2 - 20x + 68\Big|_{x=5-3i}$$

$$2(5-3i)^2 - 20(5-3i) + 68$$

$$2(5-3i)(5-3i) + (-100+60i) + 68$$

$$2(25 - 15i - 15i + 9i^2) + (-100+60i) + 68$$

$$2\big(25 - 30i + 9(-1)\big) + (-100+60i) + 68$$

$$2(16-30i) + (-100+60i) + 68$$

$$(32-60i) + (-100+60i) + 68$$

$$(32 - 100 + 68) + (-60i + 60i)$$

$$0 + 0i$$

$$0$$