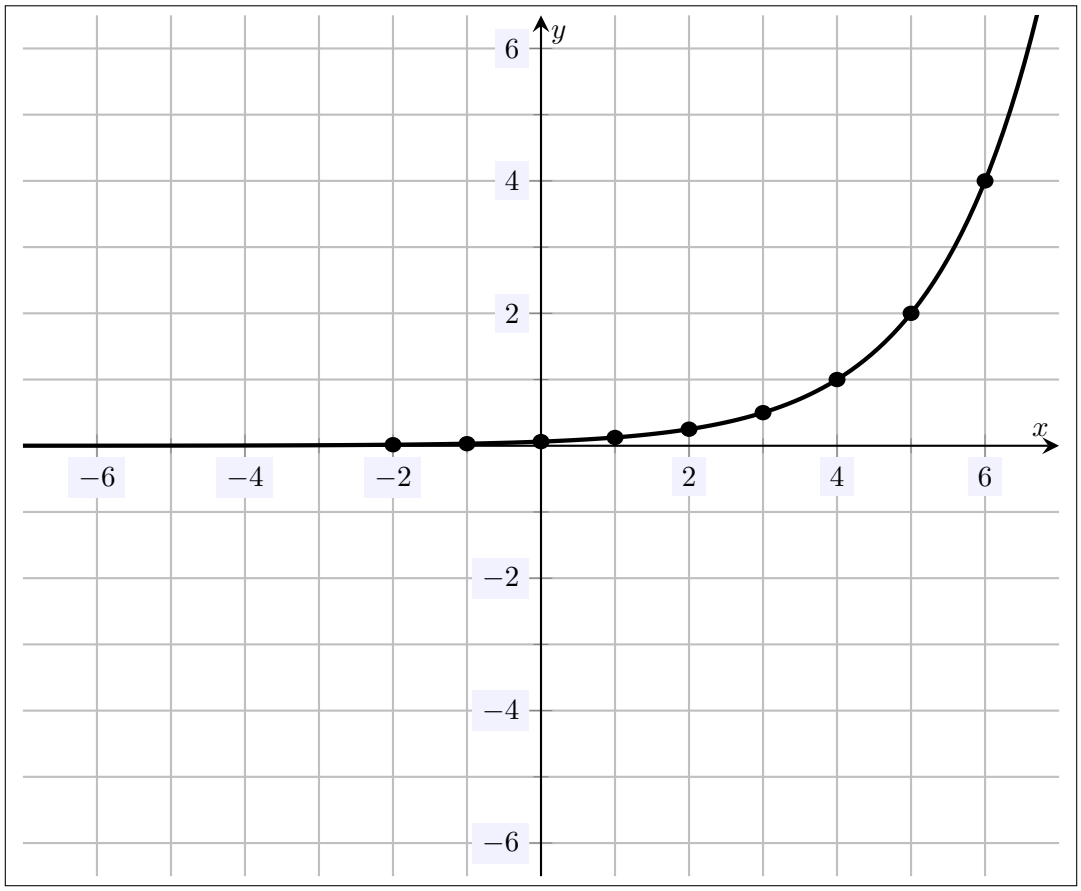
Name: *Caleb McWhorter — Solutions*
MATH 101
Fall 2021
HW 15: Due 11/16

"He is a self-made man and worships his creator."

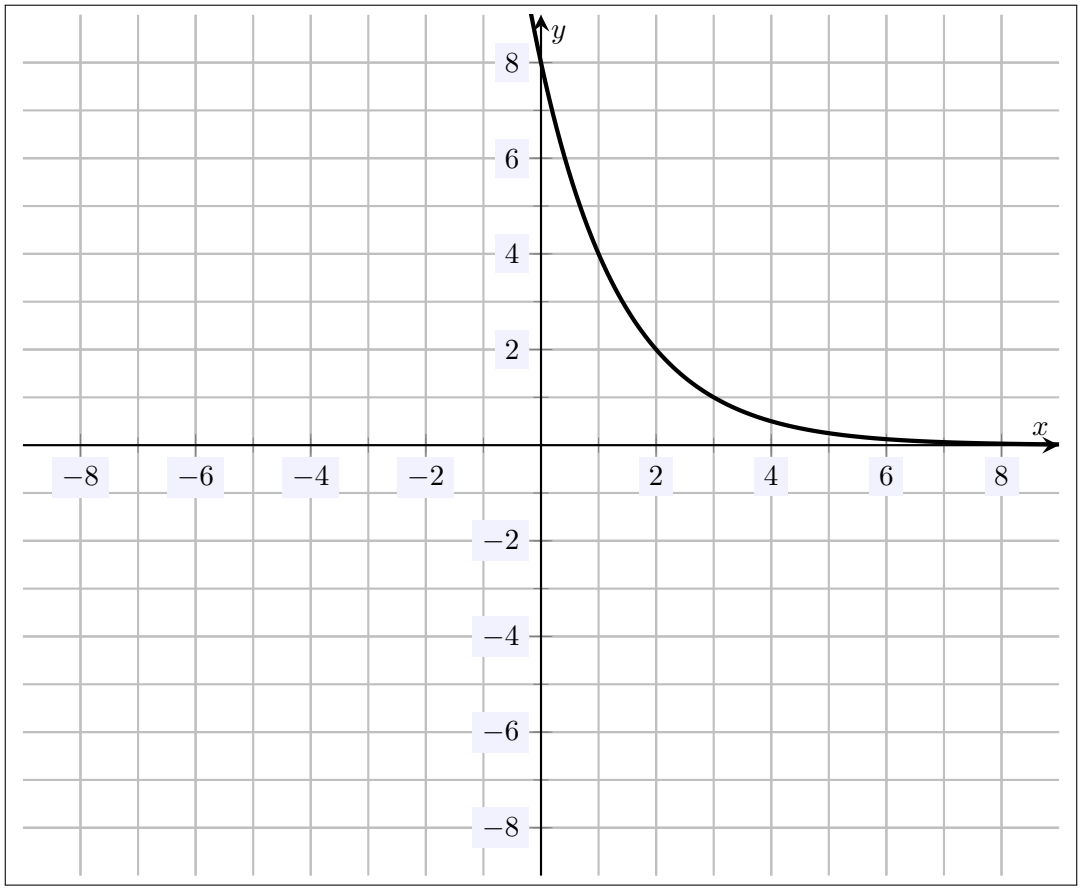
–Henry Clapp

Problem 1. (10pt) As accurately as possible, plot the function $f(x) = \frac{1}{4}\left(2^{x-2}\right)$.



x	-2	-1	0	1	2	3	4	5	6
$f(x)$	$\frac{1}{64}$	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4

Problem 2. (10pt) Sketch a graph of the function $y = 4(2^{1-x})$.



Writing y in the form Ab^x, we have...

$$y = 4(2^{1-x}) = 4(2 \cdot 2^{-x}) = 8(2^{-x}) = 8(2^{-1})^x = 8\left(\frac{1}{2}\right)^x$$

Because $b = \frac{1}{2}$, where $0 < b < 1$, and $A = 8 > 0$, the function is decreasing. We have y-intercept with y-coordinate...

$$y(0) = 4(2^{1-0}) = 4(2) = 8,$$

i.e. the y-intercept is $(0, 8)$.

Problem 3. (10pt) Consider the function $y = -9(2^{-2x})$.

(a) Is the function increasing or decreasing? Explain.

(b) Find the y-intercept of this function.

(c) What are the x-intercepts and zeros for this function?

(d) Find $y(-1)$.

Solution.

(a) We write y in the form Ab^x:

$$y = -9(2^{-2x}) = -9(2^{-2})^x = -9\left(\frac{1}{2^2}\right)^x = -9\left(\frac{1}{4}\right)^x$$

Because $b = \frac{1}{4}$, where $0 < b < 1$, and $A = -9 < 0$, the function is increasing.

(b) The y-intercept occurs when $x = 0$, where then y is...

$$y = -9(2^{-2(0)}) = -9(2^0) = -9 \cdot 1 = -9$$

Therefore, the y-intercept is $(0, -9)$.

(c) The function $y = -9(2^{-2x})$ is always negative because $A = -9 < 0$. Therefore, there are no x-intercepts (and hence zeros) for the function $y = -9(2^{-2x})$.

(d)

$$y(-1) = -9(2^{-2(-1)}) = -9(2^2) = -9(4) = -36$$

Problem 4. (10pt) Consider the function $y = 3^{1-x} - 9$.

(a) Is the function increasing or decreasing? Explain.

(b) Find the y-intercept of this function.

(c) What are the x-intercepts and zeros for this function?

(d) Find $y(2)$.

Solution.

(a) We write y in the form $Ab^x + C$:

$$y = 3^{1-x} - 9 = 3 \cdot 3^{-x} - 9 = 3(3^{-1})^x - 9 = 3\left(\frac{1}{3}\right)^x - 9$$

Because $b = \frac{1}{3}$, where $0 < b < 1$, and $A = 3 > 0$, the function is decreasing.

(b) The y-intercept occurs when $x = 0$, where then y is...

$$y(0) = 3^{1-0} - 9 = 3^1 - 9 = 3 - 9 = -6$$

Therefore, the y-intercept is $(0, -6)$.

(c)

$$3^{1-x} - 9 = 0$$

$$3^{1-x} = 9$$

$$3^{1-x} = 3^2$$

Because the bases on each side are the same, we must have $1 - x = 2$. But then $x = -1$. Therefore, the only zero is $x = -1$. This corresponds to an x-intercept of $(-1, 0)$.

(d)

$$y(2) = 3^{1-2} - 9 = 3^{-1} - 9 = \frac{1}{3} - 9 = \frac{1}{3} - \frac{27}{3} = -\frac{26}{3}$$