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MATH 108
Spring 2023
HW 12: Due 03/31

"Taking on a challenge is a lot like riding a horse, isn't it? If you're comfortable while you're doing it, you're probably doing it wrong."
–Ted Lasso, Ted Lasso

Problem 1. (10pt) Determine whether $(x_1, x_2) = (-1, 2)$ is a solution to the following system of equations:

$$4x_1 - 5x_2 = -14$$

$$3x_1 + 7x_2 = 17$$

Solution. We test whether $(x_1, x_2) = (-1, 2)$ satisfies *both* equations, i.e. whether $x_1 = -1$ and $x_2 = 2$ satisfy both equations.

$$\begin{array}{rcr} 4x_1 - 5x_2 = -14 & \qquad & 3x_1 + 7x_2 = 17 \\ 4(-1) - 5(2) \stackrel{?}{=} -14 & & 3(-1) + 7(2) \stackrel{?}{=} 17 \\ -4 - 10 \stackrel{?}{=} -14 & & -3 + 14 \stackrel{?}{=} 17 \\ -14 = -14 & & 11 \neq 17 \\ \checkmark & & ✗ \end{array}$$

Because $(x_1, x_2) = (-1, 2)$ does not satisfy *both* equations, it is not a solution to the system of equations. Note that if one directly solves for x_1, x_2, we find that the solution is $(x_1, x_2) = \left(\frac{17}{10}, \frac{104}{25}\right) \approx (1.7, 4.16)$.

Problem 2. (10pt) Show that $(x_1, x_2) = (3, 1)$ is a solution to the following system of equations:

$$x_1 + 6x_2 = 9$$

$$-5x_1 + 4x_2 = -11$$

Also, writing this system of equations as $A\mathbf{x} = \mathbf{b}$, determine A, $\mathbf{b}$, and the solution vector to this system.

Solution. To show that $(x_1, x_2) = (3, 1)$ is a solution to the system of equations, we show that $(x_1, x_2) = (3, 1)$ satisfy the equations, i.e. that $x_1 = 3$ and $x_2 = 1$ satisfy the system of equations.

$$\begin{array}{rcr} x_1 + 6x_2 = 9 & \qquad & -5x_1 + 4x_2 = -11 \\ 3 + 6(1) \stackrel{?}{=} 9 & & -5(3) + 4(1) \stackrel{?}{=} -11 \\ 3 + 6 \stackrel{?}{=} 9 & & -15 + 4 \stackrel{?}{=} -11 \\ 9 = 9 & & -11 = -11 \\ \checkmark & & \checkmark \end{array}$$

Because $(x_1, x_2) = (3, 1)$ satisfies *both* equations, it is a solution to the system of equations.

Recall that that when written in vector form, i.e. $A\mathbf{x} = \mathbf{b}$, the matrix A is the coefficient matrix (written column-by-column in the same order as the variable vector), $\mathbf{x}$ is the variable vector, and $\mathbf{b}$ is the constant vector. Because the variables in the equations are already properly aligned, we have...

$$A = \begin{pmatrix} 1 & 6 \\ -5 & 4 \end{pmatrix} \qquad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} 9 \\ -11 \end{pmatrix}$$

Therefore, we can write the system of equations as...

$$\begin{pmatrix} 1 & 6 \\ -5 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 9 \\ -11 \end{pmatrix}$$