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MATH 100
Fall 2022
HW 15: Due 11/07

"Life is a school of probability."
– Walter Bagehot

Problem 1. (10pt) Researchers surveyed 320 people on which superpower from comics they found most interesting/best. A table of their findings are found below.

	Child	Adult	Total
Flight	30	36	66
Mind Reading	15	60	75
Super Strength	40	14	54
Total	85	110	195

(a) Find the probability that a person chose flight as the best superpower.

(b) Find the probability that a person surveyed was a child.

(c) Find the probability that a person was a child and chose super strength as the most interesting super power.

(d) Find the probability that a person chose mind reading or was an adult.

(e) Find the probability that a person that chose mind reading was an adult.

Solution.

(a)
$$P(\text{flight}) = \frac{66}{195} \approx 0.3385$$

(b)
$$P(\text{child}) = \frac{85}{195} \approx 0.4359$$

(c)
$$P(\text{child and super strength}) = \frac{40}{195} \approx 0.2051$$

(d)
$$P(\text{mind reading or adult}) = \frac{75 + 110 - 60}{195} = \frac{125}{195} \approx 0.6410$$

(e)
$$P(\text{adult} \mid \text{mind reading}) = \frac{60}{75} \approx 0.80$$

Problem 2. (10pt) Consider the breakdown of students majors at a college by gender.

	School of Business	School of Education	Arts & Social Sciences	STEM
Male	80	85	75	92
Female	75	80	90	94
Unspecified	8	15	22	1

(a) Find the probability that a randomly selected person from this school was in education.

(b) Find the probability that a randomly selected person from this school was male.

(c) Find the probability that a randomly selected person from this school was a female and in the School of Business.

(d) Find the probability that a randomly selected person from this school was in STEM or had unspecified gender.

(e) Find the probability that a person was in the school of Arts & Social Sciences given that their gender was unspecified.

Solution. First, we should find the totals in each row and column:

	School of Business	School of Education	Arts & Social Sciences	STEM	Total
Male	80	85	75	92	332
Female	75	80	90	94	339
Unspecified	8	15	22	1	46
Total	163	180	187	187	717

(a)
$$P(\text{education}) = \frac{180}{717} \approx 0.2510$$

(b)
$$P(\text{male}) = \frac{332}{717} \approx 0.4630$$

(c)
$$P(\text{female and business}) = \frac{75}{717} \approx 0.1046$$

(d)
$$P(\text{STEM or unspecified}) = \frac{187 + 46 - 1}{717} = \frac{232}{717} \approx 0.3236$$

(e)
$$P(\text{Arts \& Social} \mid \text{unspecified}) = \frac{22}{46} \approx 0.4783$$

Problem 3. (10pt) Is it always true that for events A and B, $P(A \text{ and } B) = P(A) \cdot P(B)$? Explain using an original example.

Solution. No. It is true that if A and B are independent events that $P(A \text{ and } B) = P(A) \cdot P(B)$. However, this is not generally true for arbitrary events A and B. Generally, we have $P(A \text{ and } B) = P(A)P(B \mid A)$ or $P(A \text{ and } B) = P(B)P(A \mid B)$. To see why $P(A \text{ and } B)$ may be different than $P(A) \cdot P(B)$, consider the events A and B, where A is passing a course and B is failing a course. Clearly, you cannot pass and fail a course at the same time. So $P(A \text{ and } B) = 0$. However, unless $P(A) = 0$ or $P(B) = 0$, then $P(A) \cdot P(B) \neq 0$. Because A and B are disjoint events (they cannot happen at the same time), they cannot be independent. So we should not have expected that $P(A \text{ and } B) = P(A) \cdot P(B)$.