Quiz 1. *True/False*: The following is a truth table for $P \to Q$:

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	F
F	F	F

Solution. The statement is *false*. The correct truth table should be...

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

One way to think about this is as follows: imagine P is a guarantee. Namely, we promise that if P happens, Q must happen. For instance, P could represent the statement, "You do not tamper with your hardware," and Q could be the statement, "I will replace your broken computer." So $P \to Q$ is then the statement, "If you do not tamper with your hardware, then I will replace your broken computer." If both P and Q are true, then this should be true—because I promised to replace the computer if you left it alone. If P is true and Q is false, then the statement should be false because I broke my promise. However, my promise holds true whenever P is false. Why? Because you broke our agreement by tampering with the hardware. So while I may or may not replace the computer, my promise has not been broken in either case, i.e. it remains true. In an implication $P \to Q$, if P is false, then the statement $P \to Q$ is *always* true.

Quiz 2. *True/False*: $\forall x, \exists y,\ x^2 + y = 4$

Solution. The statement is *true*. The statement says that for all x there is a y such that $x^2 + y = 4$. If this is true (which it is), we need to prove it. Fix an x, say x_0. We need to find a y such that $x_0^2 + y = 4$. Define $y_0 := 4 - x_0^2$. But then we have

$$x_0^2 + y_0 = x_0^2 + (4 - x_0^2) = 4,$$

as desired.

Quiz 3. *True/False*: $\neg\left(\forall x, \exists y, P(x,y) \vee \neg Q(x,y)\right) = \exists x, \forall y, \neg P(x,y) \wedge Q(x,y)$

Solution. The statement is *true*. We can simply compute the negation step-by-step:

$$\begin{aligned}
\neg\left(\forall x, \exists y, P(x,y) \vee \neg Q(x,y)\right) &\equiv \exists x, \neg(\exists y, P(x,y) \vee \neg Q(x,y)) \\
&\equiv \exists x, \forall y, \neg(P(x,y) \vee \neg Q(x,y)) \\
&\equiv \exists x, \forall y, \neg P(x,y) \wedge \neg(\neg Q(x,y)) \\
&\equiv \exists x, \forall y, \neg P(x,y) \wedge Q(x,y)
\end{aligned}$$

Quiz 4. *True/False*: To prove $P \Rightarrow Q$, you can prove $Q \Rightarrow P$.

Solution. The statement is *false*. The converse of $P \Rightarrow Q$ is $Q \Rightarrow P$. The converse of a logical statement is not necessarily logically equivalent to the original statement. So proving the converse does not necessarily prove the original statement. However, the contrapositive of $P \Rightarrow Q$, which is $\neg Q \Rightarrow \neg P$, is logically equivalent to $P \Rightarrow Q$. Therefore, to prove $P \Rightarrow Q$, one only need prove $\neg Q \Rightarrow \neg P$. This is called proof by contrapositive.