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MATH 101
Spring 2024
HW 17: Due 04/15

"I can talk to animals. Well, not talk to them; I can take commands from them."
— Kenneth Parcell, 30 Rock

Problem 1. (10pts) Complete the square in $x^2 + 8x + 6$ to find the vertex form of this quadratic function.

Solution. We have...

$$x^2 + 8x + 6$$

$$x^2 + 8x + \left(\frac{8}{2}\right)^2 - \left(\frac{8}{2}\right)^2 + 6$$

$$x^2 + 8x + 4^2 - 4^2 + 6$$

$$(x^2 + 8x + 16) - 16 + 6$$

$$(x + 4)^2 - 10$$

Therefore, the vertex form is $(x + 4)^2 - 10$, which implies that the vertex is $(-4, -10)$. Because $a = 1 > 0$, this quadratic function opens upwards.

Problem 2. (10pts) Use the 'evaluation method' to find the vertex form of the quadratic function $2x^2 + 12x - 14$.

Solution. A quadratic function $f(x) = ax^2 + bx + c$ has vertex located at $x_0 = -\frac{b}{2a}$. The y-coordinate of the vertex is then $y_0 = f(x_0)$. For the quadratic function $2x^2 + 12x - 14$, we have $a = 2$, $b = 12$, and $c = -14$. Now $x_0 = -\frac{b}{2a} = -\frac{12}{2(2)} = -\frac{12}{4} = -3$. The y-coordinate of the vertex is then $f(-3) = 2(-3)^2 + 12(-3) - 14 = 2(9) + 12(-3) - 14 = 18 - 36 - 14 = -32$. Therefore, the vertex is $(-3, -32)$. We know that for this quadratic function, $a = 2$. The vertex form of a quadratic function is $a(x - P)^2 + Q$, where (P, Q) is the vertex. Therefore, the vertex form of this quadratic function is...

$$2x^2 + 12x - 14 = 2\big(x - (-3)\big)^2 + (-32) = 2(x + 3)^2 - 32$$

Problem 3. (10pts) Use completing the square to solve the following quadratic equation:

$$x(x-6)=7$$

Solution. We have...

$$x(x-6)=7$$

$$x^2-6x=7$$

$$x^2-6x+\left(\frac{-6}{2}\right)^2=7+\left(\frac{-6}{2}\right)^2$$

$$x^2-6x+(-3)^2=7+(-3)^2$$

$$x^2-6x+9=7+9$$

$$(x-3)^2=16$$

$$\sqrt{(x-3)^2}=\sqrt{16}$$

$$x-3=\pm 4$$

$$x=3\pm 4$$

Therefore, the solutions are $x=3-4=-1$ and $x=3+4=7$.

Problem 4. (10pts) Use the quadratic formula to solve the following quadratic equation:

$$x(5-x)=3$$

Solution. First, observe that this equation is equivalent to...

$$x(5-x)=3$$

$$5x-x^2=3$$

$$0=x^2-5x+3$$

To find the solutions for this quadratic equation (and hence the original), we need to find the zeros of x^2-5x+3. This is a quadratic function with $a=1$, $b=-5$, and $c=3$. But then we have...

$$\begin{aligned} x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a} \\ &= \frac{-(-5)\pm\sqrt{(-5)^2-4(1)3}}{2(1)} \\ &= \frac{5\pm\sqrt{25-12}}{2} \\ &= \frac{5\pm\sqrt{13}}{2} \end{aligned}$$

Therefore, the solutions to the original equation are $x=\frac{5-\sqrt{13}}{2}\approx 0.697224$ and $x=\frac{5+\sqrt{13}}{2}\approx 4.30278$.