Name: _____

MATH 308
Fall 2023
HW 10: Due 11/10

"Matrices act. They don't just sit there."
–Gilbert Strang

Problem 1. (10pt) Showing all your work, compute the following:

(a) $\sum_{k=0}^{5}(5k-3)$

(b) $\sum_{\substack{k=-2 \\ k\neq 0}}^{3} \frac{k+1}{k}$

(c) $\prod_{j=1}^{4} 2j$

(d) $\prod_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right)$ [Hint: Combine terms, then split the product.]

(e) $\sum_{k=1}^{\infty}\frac{1}{k^2+3k}$ [Hint: Use partial fractions, then write out some terms.]

— Continued Space for Problem 1 —

Problem 2. (10pt) Define $\mathbf{u} = \langle 2, 0, -1, 3 \rangle$ and $\mathbf{v} = \langle 1, -1, 5, 6 \rangle$. Showing all your work, complete the following:

(a) $\mathbf{u} - 2\mathbf{v}$

(b) $\|\mathbf{u} - 2\mathbf{v}\|$

(c) $\mathbf{u} \cdot \mathbf{v}$

(d) If $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, then $\mathbf{x} \cdot \mathbf{y} = \|\mathbf{x}\| \, \|\mathbf{y}\| \cos\theta$, where θ is the angle between $\mathbf{x}$ and $\mathbf{y}$. Using this fact, compute the angle between $\mathbf{u}$ and $\mathbf{v}$.

Problem 3. (10pt) Define the following:

$$A = \begin{pmatrix} 0 & -2 \\ 6 & 5 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 0 & -1 & 3 \\ 5 & 1 & 0 & 4 \end{pmatrix}, \qquad C = \begin{pmatrix} 2 & 1 \\ -1 & 0 \\ 4 & 1 \\ -1 & 1 \end{pmatrix}, \qquad \mathbf{u} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Showing all your work, compute the following:

(a) $BC - 2A$

(b) CB

(c) $B^T\mathbf{u}$

Problem 4. (10pt) A *neural network* is a computational model resembling how the human brain works and they are used to create predictive models in data science. There are many types of neural networks: feed-forward neural networks, recurrent neural networks, convolutional neural networks, etc.

(a) Watch 3Blue1Brown's "But what is a neural network?" and then comment about what you learned and how it relates to the course material.

(b) Using sigmoid function $\sigma(x) = \frac{1}{1+e^{-x}}$, bias vectors $\mathbf{b}_1 = \begin{pmatrix} 1.5 \\ -0.4 \end{pmatrix}$ and $\mathbf{b}_2 = \begin{pmatrix} 0.3 \\ 2.0 \end{pmatrix}$, and initial input $\mathbf{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$, compute the output of the single hidden layer neural network given below.