Name: ______________________________________

MATH 308

Fall 2021

HW 19: Due 12/15

"Controlling complexity is the essence of computer programming."

–Brian Kernighan

Problem 1. (10pt) Prove that if $g(x)$ is $O(f(x))$, then $f(x)$ is $\Omega(g(x))$.

Problem 2. (10pt) Prove that if $f(x)$ is $O(g(x))$ and $c \in \mathbb{R} \setminus \{0\}$, then $cf(x)$ is $O(g(x))$.

Problem 3. (10pt) Finding appropriate constants, show that $f(x) = 3x^4 + x^3 - 2x^2 + 6$ is $O(x^4)$.

Problem 4. (10pt) Let $n \in \mathbb{Z}_{\geq 0}$. Find the number of operations (additions, subtractions, multiplications, and divisions) the following algorithm requires. What is the time complexity of the algorithm?

```
for i = 1 to n
  for j = 1 to i
  print(2n - i^2 j);
```

Problem 5. (10pt) Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, where $a_i \in \mathbb{R}$, be a polynomial.

(a) Compute the number of operations (additions, subtractions, multiplications, and divisions) required to compute $f(x_0)$ for some $x_0 \in \mathbb{R}$ the 'traditional way.'

(b) Horner's Method says to write $f(x)$ as...

$$a_0 + x\Big(a_1 + x\big(a_2 + x\big(a_3 + \cdots + x(a_{n-1} + a_n x)\big)\big)\Big)$$

Writing $f(x)$ as above, compute the number of operations required to compute $f(x_0)$.