MAT 101: Exam 3 **Name:** _____
Fall – 2023
12/13/2023
85 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 11 pages (including this cover page) and 10 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total:	100	

1. (10 points) Without using a calculator and showing all your work, solve the following system of equations:

$$\begin{cases} 4x + 2y = -4 \\ 6x - 5y = 18 \end{cases}$$

2. (10 points) Consider the quadratic function $f(x) = 9 - 2(x+5)^2$.

   (a) Find the vertex of $f(x)$ and axis of symmetry.

   (b) Does $f(x)$ open upwards or downwards?

   (c) Is $f(x)$ convex or concave?

   (d) Does $f(x)$ have a maximum or a minimum? Find whichever value exists.

   (e) Find a, b, c for the standard form for $f(x)$.

3. (10 points) Without using a calculator and showing all your work, find the vertex form for $2x^2 + 4x + 1$.

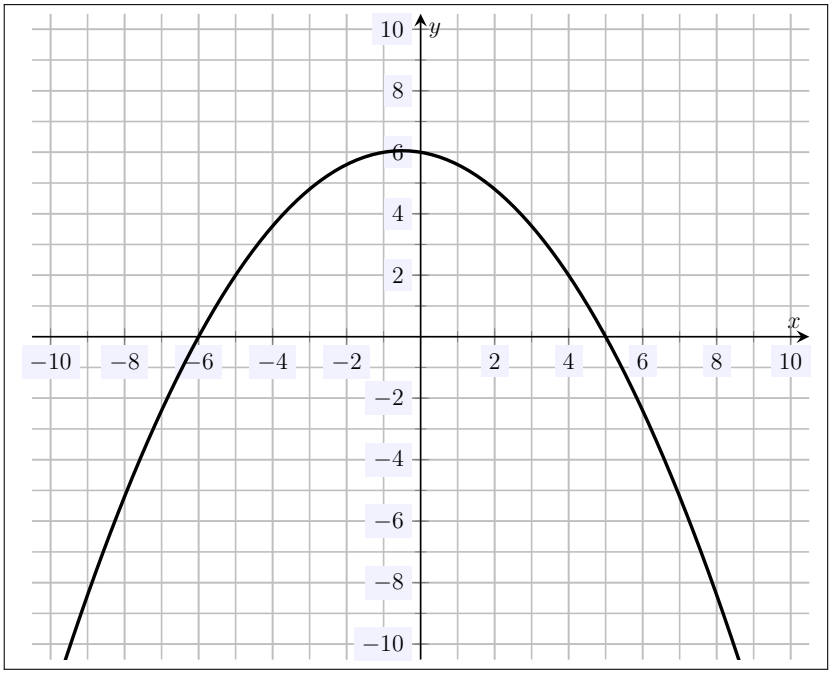
4. (10 points) Without using a calculator and showing all your work, factor the following polynomials as much as possible (if they cannot be factored, state so):

   (a) $3x^2 + 36x - 84$

   (b) $16x^4 - 1$

5. (10 points) Without using a calculator and showing all your work, factor the polynomial $6x^2 - 11x - 7$ as much as possible. If it cannot be factored, state so and explain why.

6. (10 points) Find the equation of the quadratic function plotted below.



7. (10 points) Without using a calculator and showing all your work, find the exact solution(s) to the following equation:

$$x(x+3) = 18$$

8. (10 points) Without using a calculator and showing all your work, find the exact solution(s) to the following equation by using the quadratic formula:

$$\frac{5x-1}{x} = \frac{6x}{x-1}$$

9. (10 points) Consider the polynomial $p(x) = 9x^3(x+1)(x-3)^2(x+6)^4(x-7)^9$.

   (a) Find the degree of $p(x)$.

   (b) How many *distinct* roots does $p(x)$ have?

   (c) Find the roots of $p(x)$ along with their multiplicities.

   (d) Do the 'ends' of $p(x)$ 'point' in the same direction or opposite? Explain.

   (e) Does $p(x)$ have a maximum, minimum, both, or neither?

10. (10 points) Find the equation of the polynomial $p(x)$ plotted below, if $\deg p(x) = 6$.