Name: _____

MATH 308
Fall 2023
HW 13: Due 11/10

"The difference between mathematicians and physicists is that after physicists prove a big result they think it is fantastic but after mathematicians prove a big result they think it is trivial."

–Lucien Szpiro

Problem 1. (10pt) Showing all your work, compute the following:

(a) $45 - 69 \mod 27$

(b) $115 + 82 \mod 6$

(c) $11 \cdot 17 \mod 3$

(d) $2^{100} \mod 5$

(e) $-17 \cdot 14 \mod 8$

Problem 2. (10pt) Showing all your work, compute the following:

(a) $\phi(143)$

(b) $\phi(64)$

(c) $\phi(660)$

Problem 3. (10pt) Showing all your work, complete the following:

(a) The number of digits in 96758^{2023}.

(b) What is the remainder when 19^{115} is divided by 5.

(c) What are the last two digits of 178^{996}?

Problem 4. (10pt) Consider the congruence $18x + 27 \equiv 5 \mod 31$.

(a) Explain why the given congruence has a solution.

(b) Explain why 18^{-1} exists mod 31.

(c) Solve the congruence and give at least three explicit solutions.

(d) Verify that one of your solutions in (c) is correct.