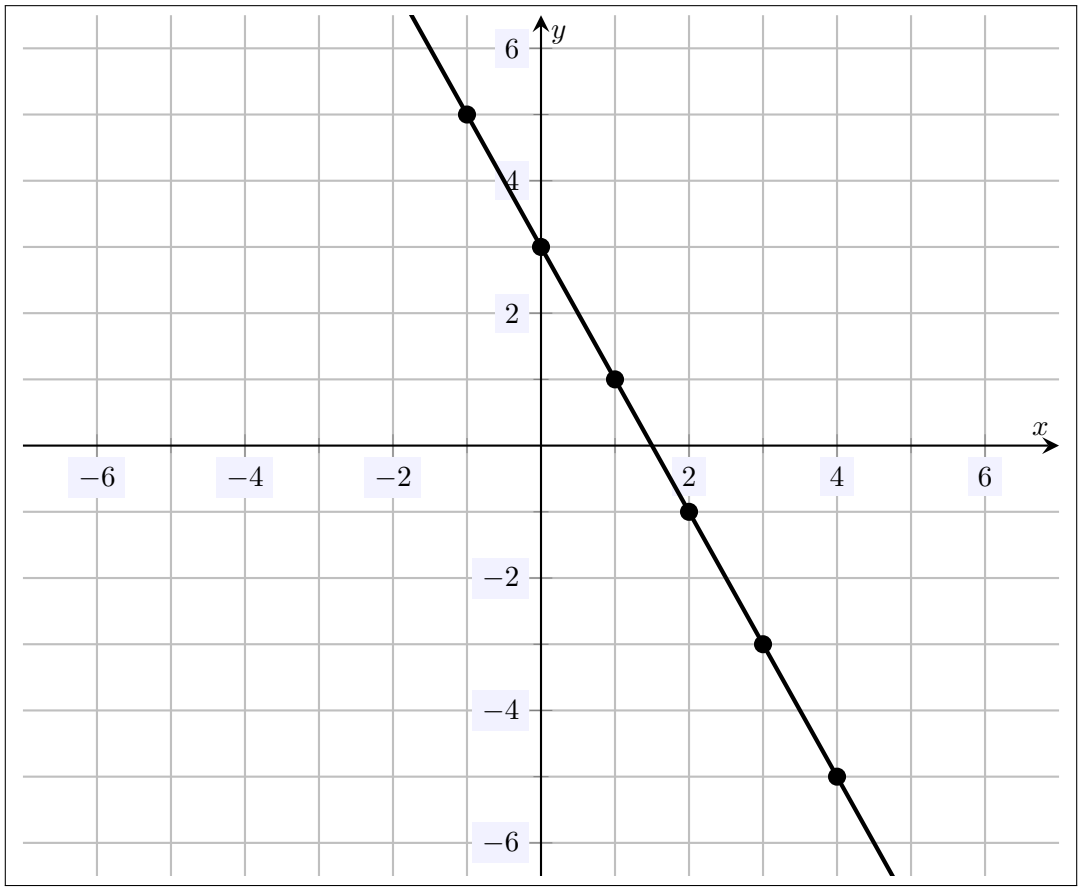
Name: *Caleb McWhorter — Solutions*
MATH 100
Fall 2021
HW 5: Due 10/06

"Martini. Gin, not vodka. Obviously. Stirred for 10 seconds while glancing at an unopened bottle of vermouth."
– Gary 'Eggsy' Unwin, The Kingsman

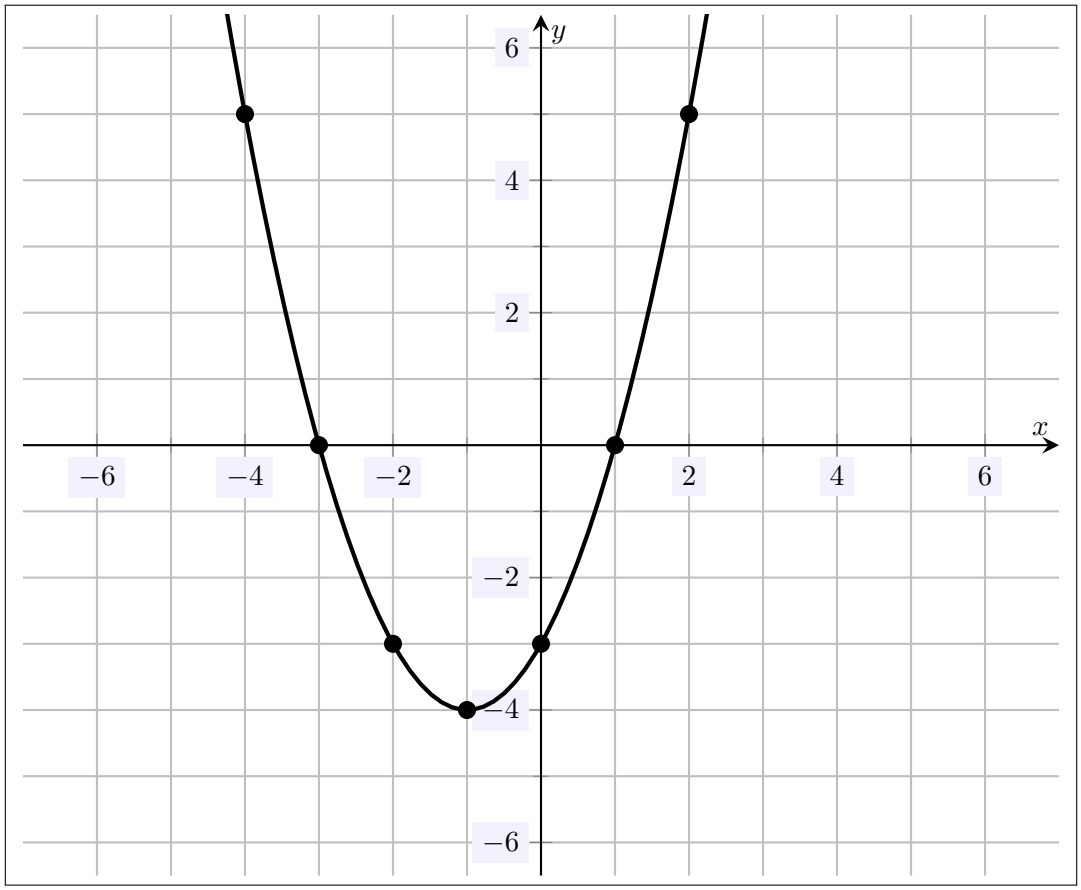
Problem 1. (10pt) Plot the function $f(x) := 3 - 2x$, being as accurate as possible.



Solution. To sketch the line, we should find around 8–12 equally spaces points on the curve and connect them "smoothly."

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	11	9	7	5	3	1	-1	-3	-5

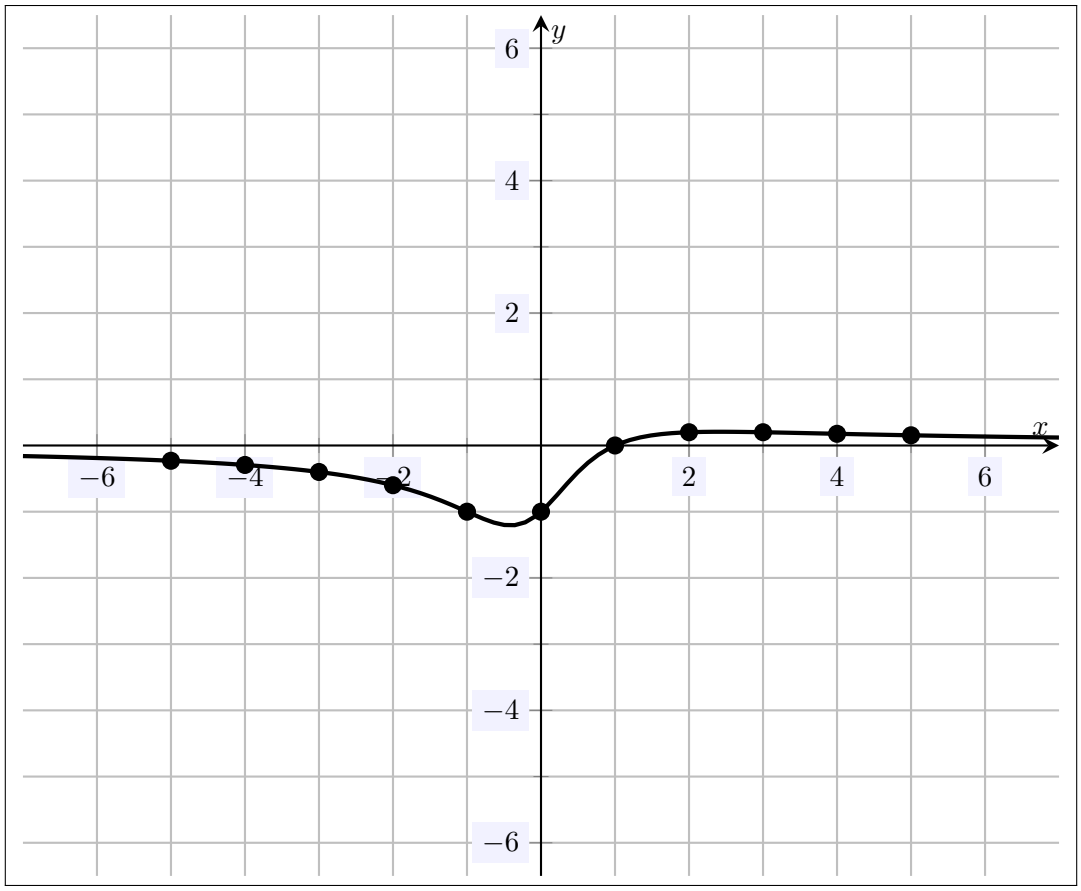
Problem 2. (10pt) Plot the function $f(x) := x^2 + 2x - 3$, being as accurate as possible.



Solution. To sketch the line, we should find around 8–12 equally spaces points on the curve and connect them "smoothly."

x	-5	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	12	5	0	-3	-4	-3	0	5	12	21

Problem 3. (10pt) Plot the function $f(x) := \dfrac{x-1}{x^2+1}$, being as accurate as possible.



Solution. To sketch the line, we should find around 8–12 equally spaces points on the curve and connect them "smoothly."

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f(x)$	$-\frac{3}{13}$	$-\frac{5}{17}$	$-\frac{2}{5}$	$-\frac{3}{5}$	-1	-1	0	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{3}{17}$	$\frac{2}{13}$

Problem 4. (10pt) Let $f(x) := 4x - 7$.

(a) Find $f(1)$.

$$f(1) = 4(1) - 7 = 4 - 7 = -3$$

(b) What value(s) for x make the output of $f(x)$ twice the output from (a)?

The output from (a) was -3. So twice this would be -6. We want x so that $f(x) = -6$. But then

$$\begin{aligned} 4x - 7 &= -6 \\ 4x &= 1 \\ x &= \frac{1}{4} \end{aligned}$$

(c) Is $(1, 1)$ on the graph of $f(x)$? Explain.

We have $f(1) = 4(1) - 7 = 4 - 7 = -3$ so that $(1, -3)$ is a point on the graph. But this is not the point $(1, 1)$. Therefore, $(1, 1)$ is not a point on the graph of $f(x)$. Alternatively, because $y = 4x - 7$, we check $1 \stackrel{?}{=} 4(1) - 7 = -3$, which is false so that $(1, 1)$ is not on the graph of $f(x)$.

(d) Is $(3, 5)$ on the graph of $f(x)$? Explain.

We have $f(3) = 4(3) - 7 = 12 - 7 = 5$ so that $(3, 5)$ is a point on the graph. But this is exactly the given point. Alternatively, because $y = 4x - 7$, we check $5 \stackrel{?}{=} 4(3) - 7 = 5$ so that $(3, 5)$ is on the graph of $f(x)$.

Problem 5. (10pt) Define the following functions:

$$f(x) := x - x^3$$
$$g(x) := x^2 - 3x + 1$$
$$h(x) := x^4 + 1$$

Determine if the functions $f(x)$, $g(x)$ and $h(x)$ are even functions, odd functions, or neither. Be sure to justify your answer.

Solution. *First, observe that...*

$$f(-x) = -x - (-x)^3 = -x - (-x^3) = -x + x^3 = -(x - x^3) = -f(x)$$

Therefore, $f(x)$ is odd because $f(-x) = -f(x)$. [Hence, $f(x)$ will be symmetric about the origin.] Next, observe that...

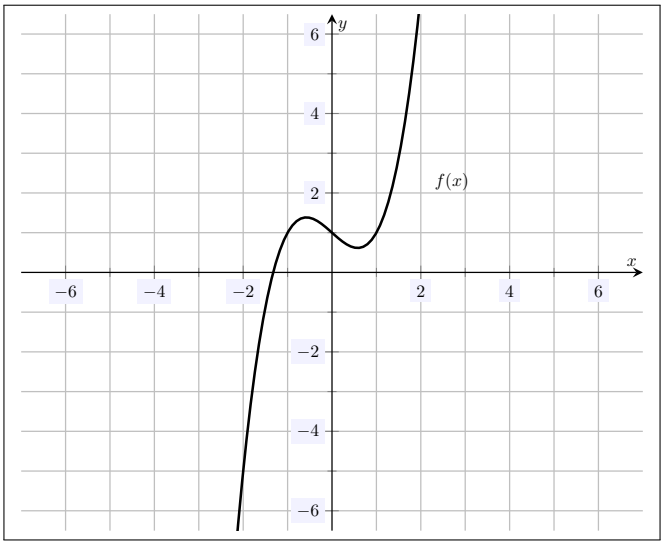
$$g(-x) = (-x)^2 - 3(-x) + 1 = x^2 + 3x + 1$$

But then $g(-x) \neq -g(x)$ and $g(-x) \neq g(x)$; therefore, $g(x)$ is neither odd nor even. Finally, observe that...

$$h(-x) = (-x)^4 + 1 = x^4 + 1 = h(x)$$

Therefore, $h(x)$ is even because $h(-x) = h(x)$. [Hence, $h(x)$ will be symmetric about the y-axis.]

Problem 6. (10pt) Consider the function $f(x)$ plotted below.



(a) What is $f(1-)$?

From the graph, we see that $f(-1) = 1$.

(b) Is the point $(1, 1)$ on the graph of $f(x)$? Explain.

Because $f(x)$ *passes through the point* $(1, 1)$*, the point* $(1, 1)$ *is on the graph of* $f(x)$.

(c) Is the point $(-2, 3)$ on the graph of $f(x)$? Explain.

We can see that $f(x)$ *does not pass through the point* $(-2, 3)$*. Therefore,* $(-2, 3)$ *is not on the graph of* $f(x)$.

(d) Is the function $f(x)$ even, odd, or neither. Explain.

Because $f(x)$ *is not symmetric across the* y*-axis and also not symmetric through the origin,* $f(x)$ *can be neither even nor odd.*