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MATH 101
Spring 2024
HW 14: Due 03/27

"You think you're pretty clever, don't you? I happen to know that every word in your book was published years ago. Perhaps you've read the dictionary!"
— Dick Solomon, Third Rock from the Sun

Problem 1. (10pts) Find the inverse of the linear function $\ell(x) = 6x - 1$. Use this inverse function to solve the equation $\ell(x) = 10$.

Solution. Writing $\ell(x) = 6x - 1$ using y, we have $y = 6x - 1$. To find the inverse of this linear function, we reverse the roles of x and y and solve for y. But then we have...

$$x = 6y - 1$$
$$x + 1 = 6y$$
$$y = \frac{x+1}{6}$$

Therefore, $\ell^{-1}(x) = \dfrac{x+1}{6}$. We can use this to solve the equation $\ell(x) = 10$ by using the fact that $(\ell^{-1} \circ \ell)(x) = x$:

$$\ell(x) = 10$$
$$\ell^{-1}\big(\ell(x)\big) = \ell^{-1}(10)$$
$$x = \ell^{-1}(10)$$
$$x = \frac{10+1}{6}$$
$$x = \frac{11}{6}$$

We can validate this solution:

$$\ell\left(\frac{11}{6}\right) = 6 \cdot \frac{11}{6} - 1 = 11 - 1 = 10$$

Problem 2. (10pts) Explain why the lines $\ell_1(x) = 5x - 1$ and $\ell_2(x) = 2 - 3x$ intersect. Find their point of intersection.

Solution. The slope of $\ell_1(x)$ is $m_1 = 5$ and the slope of ℓ_2 is $m_2 = -3$. Because $m_1 = 5 \neq -3 = m_2$, we know that the lines are not parallel. Therefore, the lines must intersect. If x_0 is the x-coordinate of their intersection, we know that $\ell_1(x_0) = \ell_2(x_0)$ (because their y-coordinate must be the same). But then...

$$\ell_1(x_0) = \ell_2(x_0)$$

$$5x_0 - 1 = 2 - 3x_0$$

$$8x_0 = 3$$

$$x_0 = \frac{3}{8}$$

We then have...

$$\ell_1\left(\frac{3}{8}\right) = 5 \cdot \frac{3}{8} - 1 = \frac{15}{8} - 1 = \frac{7}{8}$$

Therefore, the lines intersect at the point $\left(\frac{3}{8}, \frac{7}{8}\right)$.

Problem 3. (10pts) Find the x and y-intercept for the line $y = \dfrac{6x-11}{3}$.

Solution. The y-intercept of a curve is the point where the curve intersects the y-axis. The y-axis is the line where $x = 0$. But then...

$$y = \frac{6(0)-11}{3} = \frac{0-11}{3} = -\frac{11}{3}$$

Therefore, the y-intercept is $-\frac{11}{3}$, i.e. the point $\left(0, -\frac{11}{3}\right)$.

The x-intercept of a curve is the point where the curve intersects the x-axis. The x-axis is the line where $y = 0$. But then...

$$0 = \frac{6x-11}{3}$$

$$0 = 6x - 11$$

$$6x = 11$$

$$x = \frac{11}{6}$$

Therefore, the x-intercept is $\frac{11}{6}$, i.e. the point $\left(\frac{11}{6}, 0\right)$.

Problem 4. (10pts) Let $\ell(x)$ be the linear function given by $\ell(x) = 5x + c$, where c is some constant. Find the value of c such that $\ell(x)$ contains the point $(5, -4)$. What is the x-intercept of this line?

Solution. If $\ell(x)$ contains the point $(5, -4)$, when $x = 5$ then $y = -4$. But then...

$$\ell(x) = 5x + c$$

$$\ell(5) = 5(5) + c$$

$$-4 = 25 + c$$

$$c = -29$$

Therefore, $\ell(x) = 5x - 29$.

The x-intercept of a curve is the point where the curve intersects the x-axis. The x-axis is the line where $y = 0$. But then if x_0 is a y-intercept of $\ell(x)$, we have...

$$\ell(x_0) = 5x_0 - 29$$

$$0 = 5x_0 - 29$$

$$5x_0 = 29$$

$$x_0 = \frac{29}{5}$$

Therefore, the x-intercept of $\ell(x)$ is $x_0 = \frac{29}{5}$, i.e. the point $\left(\frac{29}{5}, 0\right)$.