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MATH 108
Spring 2022
Written HW 7: Due 03/28

"There's no such thing as a free lunch."
–Milton Friedman

Problem 1. (10pt) Ms. Currant invests a principal of \$3,500 in an account with 8.5% per year simple interest.

(a) How much interest has accumulated after 20 months?

(b) How much is the investment worth after 20 months?

(c) How long until the investment is worth \$4,200?

(d) How long until the investment has doubled in value?

Solution.

(a) We know that $I = Prt$, where $P = 3500$ is the principal, $r = 0.085$ is the annual interest rate, and $t = 20/12$ is the time. Then we have...

$$I = Prt = 3500 \cdot 0.085 \cdot \frac{20}{12} = \$495.8333 \approx \$495.83$$

(b) This is the future value. We know that $F = P(1 + rt)$, where $P = 3500$ is the principal, $r = 0.085$ is the annual interest rate, and $t = 20/12$ is the time. Then we have...

$$F = P(1 + rt) = 3500 \left(1 + 0.085\frac{20}{12}\right) = \$3995.8333 \approx \$3,995.83$$

Alternatively, the total value of the investment will be its initial value (the principal) plus the interest accumulated in the 20 months. We know the interest accumulated from (a). Therefore, $F = 3500 + 495.833 = 3995.833 \approx \$3,995.83$.

(c) We know that $F = P(1 + rt)$. We want to know the time t such that $F = \$4200$. But then we have...

$$\begin{aligned} F &= P(1 + rt) \\ 4200 &= 3500(1 + 0.085t) \\ 1.20 &= 1 + 0.085t \\ 0.20 &= 00.085t \\ t &= 2.35294 \text{ years} \end{aligned}$$

(d) This is the doubling time. We know that $t_D = \dfrac{1}{r}$. But then we have...

$$t_D = \frac{1}{r} = \frac{1}{0.085} = 11.7647$$

Problem 2. (10pt) Colonel Tumeric takes out a short-term loan of \$680. The bank issues a 9% discount loan for 90 days.

(a) What is the maturity?

(b) What are the discount and proceeds?

(c) How much is owed after 90 days and how much is paid in total?

(d) What are the nominal and effective interest rates?

Solution.

(a) The maturity would be the original loan amount, which is \$680.

(b) We know that $D = Mrt$, where $M = 680$ is the maturity, $r = 0.09$ is the annual interest rate, and $t = \frac{90}{365}$ is the time. But then...

$$D = Mrt = 680 \cdot 0.09 \cdot \frac{90}{365} = 15.0904 \approx \$15.09$$

The proceeds are then loan amount, i.e. the maturity, lessened by the discount:

$$P = M - D = 680 - 15.09 = \$664.91$$

(c) After 90 days, the full amount of the loan—the \$680—is due. One has already paid the interest up-front. In total, one has paid the loan amount, which is \$680, and the interest, which is \$15.09, for a total of $\$680 + \$15.09 = \$695.09$. Alternatively, the total amount paid on the loan is...

$$F = P(1 + rt) = 680 \left(1 + 0.09 \cdot \frac{90}{365}\right) = 695.0904 \approx \$695.09$$

(d) The nominal interest rate is the advertised 9%. The effective interest rate is...

$$r_{\text{eff}} = \frac{r}{1 - rt} = \frac{0.09}{1 - 0.09 \cdot \frac{90}{365}} = 0.09204 \approx 9.2\%$$

Problem 3. (10pt) Professor Mauve invests her money with an investment startup that promises interest returns of 3.5% per year, compounded semiannually. Suppose she initially invests \$8,000.

(a) How much is the investment worth in 3 years?

(b) How long until the investment is worth \$10,000?

(c) How much should she have invested to have \$10,000 after 3 years?

(d) Find the effective interest rate.

Solution.

(a) This is the future value. We know that $F = P\left(1+\frac{r}{k}\right)^{kt}$, where $P = 8000$ is the principal, $r = 0.035$ is the annual interest rate, $k = 2$ is the number of compounds per year, and $t = 3$ is the number of years. Then we have...

$$F = P\left(1+\frac{r}{k}\right)^{kt} = 8000\left(1+\frac{0.035}{2}\right)^{2\cdot 3} = 8000(1.109702) = 8877.6188 \approx \$8,877.62$$

(b) We know that...

$$t = \frac{\ln(F/P)}{k\ln\left(1+\frac{r}{k}\right)} = \frac{\ln(10000/8000)}{2\ln\left(1+\frac{0.035}{2}\right)} = \frac{0.223144}{0.0346973} = 6.43115 \approx 6.43 \text{ years}$$

(c) We know that...

$$P = \frac{F}{\left(1+\frac{r}{k}\right)^{kt}} = \frac{10000}{\left(1+\frac{0.035}{2}\right)^{2\cdot 3}} = \frac{10000}{1.109702} = 9011.4254 \approx \$9,011.43$$

(d) We know that...

$$r_{\text{eff}} = \left(1+\frac{r}{k}\right)^2 - 1 = \left(1+\frac{0.032}{2}\right)^2 - 1 = 0.03531 \approx 3.53\%$$

Problem 4. (10pt) Mrs. Cobalt takes out a loan of \$400,000 at a yearly interest rate of 1.5%, compounded continuously.

(a) How much is owed on the loan after 3 years?

(b) How long until \$500,000 is owed on the loan?

(c) How much should the loan have been for if she planned on paying \$600,000 after 5 years?

(d) Find the effective interest rate.

Solution.

(a) We know that...

$$F = Pe^{rt} = 400000e^{0.015\cdot 3} = 400000(1.046028) = \$418,411.14$$

(b) We know that...

$$t = \frac{\ln(F/P)}{r} = \frac{\ln(500000/400000)}{0.015} = \frac{0.223144}{0.015} = 14.8762 \approx 14.88 \text{ years}$$

(c) We know that...

$$P = \frac{F}{e^{rt}} = \frac{600000}{e^{0.015\cdot 5}} = \frac{600000}{1.07788} = 556646.0918 \approx \$556,646.09$$

(d) We know that...

$$r_{\text{eff}} = e^r - 1 = e^{0.015} - 1 = 0.0151131 \approx 1.51\%$$