Name: ______MATH 308

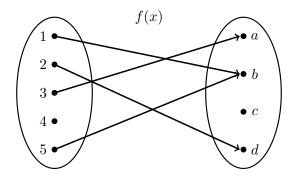
Fall 2022

HW 8: Due 10/13

"The difference between mathematicians and physicists is that after physicists prove a big result they think it is fantastic but after mathematicians prove a big result they think it is trivial."

-Lucien Szpiro

Problem 1. (10pt) Consider the relation f(x) given below.



- (a) Explain why f(x) is not a function.
- (b) Add an arrow to the diagram so that f(x) is a surjective function.
- (c) Identify the domain, codomain, and range for f(x).
- (d) Is f(x) an injective function? Explain why or why not.

Problem 2. (10pt) Complete the proof of the proposition stated below by filling in the blanks. **Proposition.** Let $f: X \to Y$ be a function and $B \subseteq Y$. Then $X \setminus f^{-1}(B) \subseteq f^{-1}(Y \setminus B)$. *Proof.* We know that if $X \setminus f^{-1}(B) = \emptyset$, then $X \setminus f^{-1}(B) \subseteq f^{-1}(Y \setminus B)$. Assume that $X \setminus f^{-1}(B) \neq \emptyset$. To show that $X \setminus f^{-1}(B) \subseteq f^{-1}(Y \setminus B)$, we need to show that if ________, then ______. Because $x \notin$ ________, we know that $f(x) \notin$ _______. It is clear that $f(x) \in Y$. But then _______. This shows that $f(x) \in$ _______. But then if $x \in$ _______, then $x \in$ _______. Therefore, $X \setminus f^{-1}(B) \subseteq f^{-1}(Y \setminus B)$.

Problem 3. (10pt) Let $f: \mathbb{R} \to \mathbb{R}$ be the function given by $x \mapsto x^2 + 3x - 7$.

- (a) Without referencing the graph of f, use the definition of decreasing to show that f(x) is not a decreasing function on \mathbb{R} by giving a counterexample.
- (b) Determine whether or not $3 \in \text{im } f$. If $3 \in \text{im } f$, find an element in the preimage of 3. If $3 \notin \text{im } f$, explain why.
- (c) Is $f^{-1}(x)$ a function? Explain why or why not by referencing the graph of f(x). Give an additional explanation of why or why not using your response in (b).