Name: ________________________________

MATH 308

Fall 2023

HW 3: Due 09/21

"Insanity is often the logic of an accurate mind overtasked."
–Oliver Wendell Holmes, Sr.

Problem 1. (10pt) Let the universe for x be the set of integers. Let $P(x)$ be the predicate $P(x)\colon 10-x<5$ and $Q(x)$ be the predicate $Q(x)\colon$ x is a positive even integer less than 10.

(a) Find at least two values for which $P(x)$ is true and two values for which $P(x)$ is false. Do the same for $Q(x)$.

(b) Find the truth set for $P(x)$ and find the truth set for $Q(x)$.

(c) Is it true that there is a unique x in the domain such that $P(x) \wedge Q(x)$ is true? Explain.

(d) Would your answer in (c) change if the universe were instead the set of integers greater than 6? Explain.

Problem 2. (10pt) Let the universe, $\mathcal{U}$, for m, n, j, k be the set of integers. Define the following predicates:

$$\begin{aligned} P(m)&\colon m \text{ is even} \\ Q(n)&\colon n \text{ is odd} \\ R(j)&\colon j \text{ is a perfect square} \\ S(k)&\colon k \text{ prime} \\ W(\ell)&\colon 1 \leq \ell \leq 10 \end{aligned}$$

Write the open sentences below as complete English sentences as 'simply' as possible and then determine whether the statement is true or false. If the statement is true, explain why. If not, give a counterexample.

(a) $(\exists x)\big(Q(x) \wedge R(x)\big)$

(b) $(\forall x)\big(P(x) \vee Q(x)\big)$

(c) $(\exists! x)\big(P(x) \wedge S(x)\big)$

(d) $(\forall x)\big(P(x) \to S(x)\big)$

(e) $(\exists x)\big(R(x) \wedge W(x)\big)$

Problem 3. (10pt) By defining appropriate universes and predicates, quantify the open sentences below. Indicate whether the resulting statement is true or false. No justification is necessary.

(a) For exists an integer n such that $5 - 6n = 10$.

(b) For all real numbers y, there exists x such that $2x + 3y = 4$.

(c) There exists x such that for any integer y, $2x + 3y = 4$.

(d) For all real numbers, if x is nonnegative, then x has a well defined square root.

(e) Multiplication of real numbers is commutative.

Problem 4. (10pt) Let $P(x)$ be the predicate $R(x)$: $x^2 + x < 6$ and let $S(x)$ be the predicate $S(x)$: $-3 < x < 2$.

(a) Write $\forall x\big(R(x) \to S(x)\big)$ as a complete English sentence.

(b) Write the contrapositive, converse, and negation of the open sentence in (a) as complete English sentences.