MAT 108: Exam 3 **Name:** _____
Spring – 2023
05/03/2023
85 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 13 pages (including this cover page) and 12 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
Total:	120	

1. (10 points) Define the following vectors:

$$\mathbf{u} = \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} \qquad \mathbf{v} = \begin{pmatrix} -4 \\ 1 \\ 5 \end{pmatrix}$$

Showing all your work, compute the following:

(a) $-2\mathbf{v}$

(b) $\mathbf{v} - 3\mathbf{u}$

(c) $\mathbf{u} \cdot \mathbf{v}$

2. (10 points) Define the following matrices:

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ 0 & 6 & -2 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 8 \\ -2 & 0 \\ 0 & 3 \\ 9 & -6 \end{pmatrix} \qquad C = \begin{pmatrix} 2 & -7 & -9 \\ 3 & 6 & 5 \\ 8 & -2 & -3 \end{pmatrix}$$

(a) Compute B^T.

(b) Showing all your work, compute $-2C$.

(c) Showing all your work, compute $C - A$.

(d) Explain why one cannot form the product AB.

3. (10 points) Consider the system of linear equations shown below:

$$2x + 3y = 1$$
$$4x + 5y = 5$$

Let A be the coefficient matrix and $\mathbf{b}$ be the constant vector associated to the system of equations above.

(a) Write the system of equations above in the form $A\mathbf{x} = \mathbf{b}$.

(b) Explain why A^{-1} exists and find A^{-1}.

(c) Use A^{-1} to find the solution to the system of equations above.

4. (10 points) The matrix below is the initial augmented matrix coming from a system of linear equations. Find the original system of equations.

$$\begin{pmatrix} 3 & -1 & 5 & 2 & 8 \\ 7 & 0 & 2 & 8 & -11 \\ 4 & 1 & -1 & 6 & 15 \end{pmatrix}$$

5. (10 points) The matrix shown below is the RREF of an augmented matrix coming from a system of linear equations. Does the corresponding system of equations have a solution? If so, find the solution(s). If not, explain why.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 1 & 3 \end{pmatrix}$$

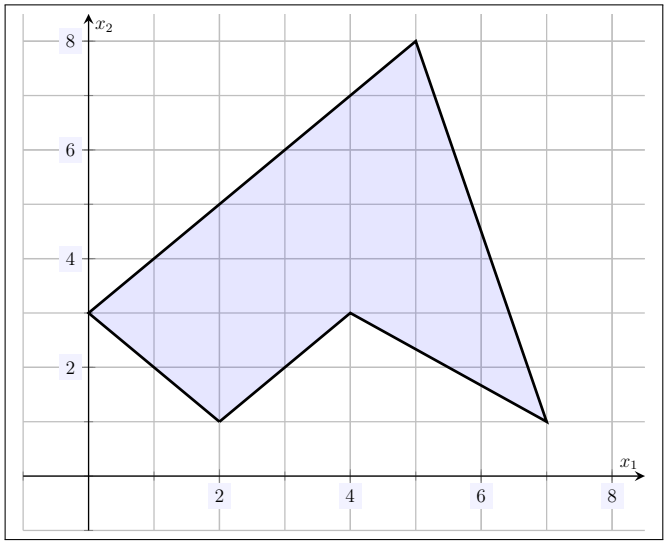
6. (10 points) The matrix shown below is the REF of an augmented matrix coming from a system of linear equations. Does the corresponding system of equations have a solution? If so, find the solution(s). If not, explain why.

$$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 9 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

7. (10 points) The matrix shown below is the RREF of an augmented matrix coming from a system of linear equations. Does the corresponding system of equations have a solution? If so, find the solution(s). If not, explain why.

$$\begin{pmatrix} 1 & 2 & 0 & 5 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

8. (10 points) Consider the function $f(x_1, x_2) = 3x_1 - x_2$ on the region shown below.



Find the maximum and minimum values of $f(x_1, x_2)$ on the region above—if they exist. Be sure to fully justify your answer.

9. (10 points) Find the initial simplex tableau corresponding to the maximization problem shown below.

$$\max z = 2.1x_1 + 6.8x_2 - 4.9x_3$$
$$1.1x_1 + 4.8x_2 - 9.0x_3 \leq 17.5$$
$$2.8x_1 + 15.8x_3 \leq 19.4$$
$$7.4x_1 - 5.6x_2 + 6.8x_3 \geq -18.7$$
$$3.1x_1 + 8.8x_2 - 3.1x_3 \geq 14.9$$
$$x_1, x_2, x_3 \geq 0$$

10. (10 points) Below is the initial simplex tableau corresponding to some maximization problem. Find the corresponding maximization problem.

$$\begin{array}{rrrrrrrrr}
3 & -1 & 2 & -1 & 1 & 0 & 0 & 0 & 22 \\
1 & 0 & 6 & 9 & 0 & 1 & 0 & 0 & 15 \\
-2 & 1 & 1 & -1 & 0 & 0 & -1 & 0 & 4 \\
1 & 1 & 1 & 3 & 0 & 0 & 0 & 1 & 16 \\
-4 & 3 & -5 & 1 & 0 & 0 & 0 & 0 & 0
\end{array}$$

11. (10 points) Below is the final simplex tableau corresponding to some maximization problem. Find the solution to the original optimization problem.

$$\begin{array}{cccrrrrrr}
0 & 0 & 1 & 0.77 & 0.13 & -0.04 & 0.04 & 0 & 16.21 \\
0 & 1 & 0 & 1.34 & 0.14 & 0.05 & -0.01 & 0 & 22.78 \\
1 & 0 & 0 & 1.78 & 0.06 & 0.07 & 0.08 & 0 & 45.55 \\
0 & 0 & 0 & -7.69 & -0.2 & -0.79 & -0.66 & 1 & 24.34 \\
0 & 0 & 0 & 12.58 & 0.49 & 0.04 & 0.55 & 0 & 220.34
\end{array}$$

12. (10 points) Find the dual problem to the minimization problem shown below.

$$\begin{gathered}\min z = 3x_1 + x_2 + 5x_3 \\ x_1 + x_2 - x_3 \geq 9 \\ 2x_1 - 5x_2 + x_3 \geq 11 \\ x_1 - x_3 \geq 6 \\ x_2 + x_3 \geq 3 \\ x_1, x_2, x_3 \geq 0\end{gathered}$$