Name: _____

MATH 308
Fall 2021
HW 6: Due 10/08

"So I was not born with a whole lot of natural talent. But I work hard and I never give up. That is my gift. That is my ninja way!"
–Rock Lee, Naturo

Problem 1. (20pt) Describe all sets (if any) with...

(a) no proper subsets.

(b) one proper subset.

(c) two proper subsets.

Problem 2. (20pt) The symmetric difference of two sets A and B, denoted $A\Delta B$, is defined by $A\Delta B := (A \setminus B) \cup (B \setminus A)$.

(a) Describe $A\Delta B$ in words.

(b) Show that $A\Delta B = (A \cup B) - (A \cap B)$.

(c) Prove that the symmetric difference is commutative.

(d) Prove that if $A\Delta B = \varnothing$, then $A = B$. Is the converse true?

Problem 3. (20pt) Let A, B be sets with a common universal set $\mathscr{U}$. Prove the following:

(a) $A - (A - B) = A \cap B$

(b) $A \subseteq B$ if and only if $B^c \supseteq A^c$

Problem 4. (10pt) If $A \subseteq U$ and $B \subseteq V$, is $A \times B \subseteq U \times V$? Justify your answer.

Problem 5. (10pt) Suppose that X and Y are sets with a common universal set $\mathscr{U}$. Show that $X = Y$ if and only if $(X \cap Y^c) \cup (X^c \cap Y) = \varnothing$.

Problem 6. (20pt) Prove or disprove:

(a) $(A \cup B) \setminus B = A$

(b) $A \cap (B \setminus C) = (A \cap B) - (A \cap C)$

(c) $A \cap (B \setminus C) = (A \cap B) \setminus C$

(d) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

Problem 7. (20pt) Express the following sets as an interval, collection of intervals, or well known set (prove your answer):

(a) $\bigcap_{n\geq 1}\left[0, 1+\frac{1}{n}\right)$

(b) $\bigcup_{n\geq 1}\left[0, 1+\frac{1}{n}\right)$

(c) $\bigcup_{n\in\mathbb{Z}}\bigcap_{m\geq 1}\left(n-\frac{1}{m}, n+\frac{1}{m}\right)$