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MATH 107
Winter 2022
HW 1: Due 01/04

"I know three things will never be believed—the true, the probable, and the logical."
–John Steinbeck

Problem 1. (10pt) Define the following propositions:

P: Bill took a Math course.
Q: Susan is not a Biology major.
R: Bill is a senior.
S: Susan is a Sophomore.

Write the following logical propositions as a complete English sentence:

(a) $P \wedge Q$

(b) $P \vee R$

(c) $S \wedge \neg Q$

(d) $R \to P$

Solution.

(a) The proposition $P \wedge Q$ is the statement, "Bill took a Math course and Susan is not a Biology major."

(b) The proposition $P \vee R$ is the statement, "Bill took a Math course or Bill is a senior."

(c) We know that $\neg Q$ is the statement, "Susan is a Biology major." Therefore, the proposition $S \wedge \neg Q$ is the statement, "Susan is a Sophomore and Susan is a Biology major."

(d) We know that $R \to P$ is the statement, "If Bill is a Senior, then Bill took a Math course."

Problem 2. (10pt) Define the following logical statements:

P: The plant receives sunlight.
Q: The plant lives.

Write the following as complete English sentences:

(a) $P \to Q$

(b) The inverse of $P \to Q$

(c) The converse of $P \to Q$

(d) The contrapositive of $P \to Q$

Solution.

(a) The proposition $P \to Q$ is the statement, "If the plant receives sunlight, then the plant lives."

(b) Given an implication $A \to B$, the inverse of this implication is $\neg A \to \neg B$. Then the inverse of $P \to Q$ is $\neg P \to \neg Q$. We know that $\neg P$ is the statement, "The plant does not receive sunlight," and $\neg Q$ is the statement, "The plant does not live," i.e. the statement, "The plant dies." Therefore, the inverse of $P \to Q$ is the statement, "If the plant does not receive sunlight, then the plant dies."

(c) Given an implication $A \to B$, the converse of this implication is $B \to A$. Then the converse of $P \to Q$ is $Q \to P$. But then the converse of $P \to Q$ is the statement, "If the plant lives, then the plant receives sunlight."

(d) Given an implication $A \to B$, the contrapositive of this implication is $\neg B \to \neg A$. Then the contrapositive of $P \to Q$ is $\neg Q \to \neg P$. We know that $\neg Q$ is the statement, "The plant does not live," i.e. the statement, "The plant dies," and $\neg P$ is the statement, "The plant does not receive sunlight." Therefore, the contrapositive of $P \to Q$ is the statement, "If the plant dies, then the plant is not receiving sunlight."

Problem 3. (10pt) Construct the truth table for the following:

(a) $\neg(P \wedge Q) \to P$

(b) $(P \vee \neg R) \wedge (Q \vee P)$

Solution.

(a)

P	Q	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg(P \wedge Q) \to P$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	F
F	F	F	T	F

(b)

P	Q	R	$\neg R$	$P \vee \neg R$	$Q \vee P$	$(P \vee \neg R) \wedge (Q \vee P)$
T	T	T	F	T	T	T
T	T	F	T	T	T	T
T	F	T	F	T	T	T
T	F	F	T	T	T	T
F	T	T	F	F	T	F
F	T	F	T	T	T	T
F	F	T	F	F	T	F
F	F	F	T	T	F	F

Problem 4. (10pt) Show $\neg(P \vee \neg Q)$ is logically equivalent to $Q \wedge \neg P$.

Solution. To show two logical expressions are logically equivalent, we show that they have the same logical outputs for the same inputs, i.e. equivalent columns. We have...

P	Q	$\neg Q$	$P \vee \neg Q$	$\neg(P \vee \neg Q)$	$\neg P$	$Q \wedge \neg P$
T	T	F	T	F	F	F
T	F	T	T	F	F	F
F	T	F	F	T	T	T
F	F	T	T	F	T	F

Because the fifth and seventh columns corresponding to $\neg(P \vee \neg Q)$ and $Q \wedge \neg P$, respectively, we know that $\neg(P \vee \neg Q)$ is logically equivalent to $Q \wedge \neg P$, i.e. $\neg(P \vee \neg Q) \equiv Q \wedge \neg P$.

Alternatively, we can use the properties of logic to show that $\neg(P \vee \neg Q)$ is logically equivalent to $Q \wedge \neg P$:

$$\neg(P \vee \neg Q) \equiv \neg P \wedge \neg(\neg Q) \equiv \neg P \wedge Q \equiv Q \wedge \neg P$$

Problem 5. (10pt) Defining appropriate propositions, write the following using the defined propositions and logical connectives: *"Jennifer has her license or if she does not have her license, then she is under 18."*

Solution. Define P to be the statement, "Jennifer has her license." Define Q to be the statement, "Jennifer is under 18." Then the statement, "Jennifer has her license or if she does not have her license, then she is under 18," can be represented by $P \vee (\neg P \to Q)$.