MAT 100: Exam 1
Fall – 2022
10/07/2022
85 Minutes

Name: *Caleb M^{c}Whorter — Solutions*

Write your name on the appropriate line on the exam cover sheet. This exam contains 16 pages (including this cover page) and 15 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
14	10	
15	10	
Total:	150	

1. (10 points) Showing all your work and finding an exact answer, compute the following:

   (a) $3(2^4) - 12$

   (b) $\dfrac{(-2)^3 - 5 + 3 \cdot 6}{-5}$

   (c) $\dfrac{5 \cdot 4 - 4 \cdot 3}{5 - 1}$

Solution.

(a)

$$3(2^4) - 12 = 3(16) - 12 = 48 - 12 = 36$$

(b)

$$\frac{(-2)^3 - 5 + 3 \cdot 6}{-5} = \frac{-8 - 5 + 3\dot{6}}{-5} = \frac{-8 - 5 + 18}{-5} = \frac{-13 + 18}{-5} = \frac{5}{-5} = -1$$

(c)

$$\frac{5 \cdot 4 - 4 \cdot 3}{5 - 1} = \frac{20 - 12}{5 - 1} = \frac{8}{4} = 2$$

2. (10 points) Define the following sets:

$$A = \{-3,\ 1,\ 2,\ 3,\ 5,\ 10,\ 20,\ 30,\ 40,\ 50\}$$
$$B = \{-3,\ 3,\ 10,\ 50\}$$
$$C = \{2,\ 5,\ 20,\ 40,\ 50\}$$
$$D = \{-3,\ 1,\ 5\}$$
$$E = \{1,\ 2,\ 3,\ 5\}$$

Consider all these sets as subsets of A. Compute the following:

(a) D^c

(b) $C \cup D$

(c) $B \setminus C$

(d) $D \cap E$

(e) $|E|$

Solution.

(a)
$$D^c = \{2,\ 3,\ 10,\ 20,\ 30,\ 40,\ 50\}$$

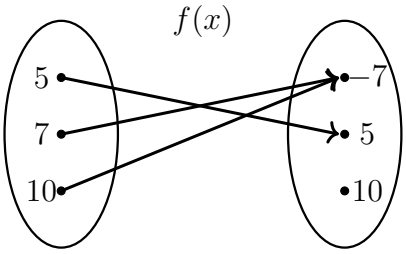
(b)
$$C \cup D = \{-3,\ 1,\ 2,\ 5,\ 20,\ 40,\ 50\}$$

(c)
$$B \setminus C = \{-3,\ 3,\ 10\}$$

(d)
$$D \cap E = \{1,\ 5\}$$

(e)
$$|E| = 4$$

3. (10 points) Determine whether the relation below is a function of not. Be sure to fully justify your response. If the relation is a function, find its domain, codomain, and range.



Solution. *Because each input has a unique output, $f(x)$ is a function; that is, given any of the possible inputs—namely 5, 7, 10—we know the output—namely $f(5) = 5$, $f(7) = -7$, and $f(10) = -7$. The fact that $f(7) = -7$ and $f(10) = -7$ does not matter because the outputs need not be different from each other, only that the given the input, there is a unique output.*

The domain of $f(x)$ is the set $\{5, 7, 10\}$, the codomain of $f(x)$ is $\{-7, 5, 10\}$, and the range of $f(x)$ is $\{-7, 5\}$.

4. (10 points) Explain why $f(x,y) = x^2 - y + 1$ is a function. Then showing all your work, find $f(0,0)$, $f(0,-4)$, $f(3,6)$, and $f(-2,10)$.

Solution. *For each of the inputs, (x,y), there is a unique output for $f(x,y)$, namely the one obtained by evaluating f at (x,y) and following order of operations. We have...*

$$f(0,0) = 0^2 - 0 + 1 = 0 - 0 + 1 = 0 + 1 = 1$$

$$f(0,-4) = 0^2 - (-4) + 1 = 0 - (-4) + 1 = 4 + 1 = 5$$

$$f(3,6) = 3^2 - 6 + 1 = 9 - 6 + 1 = 3 + 1 = 4$$

$$f(-2,10) = (-2)^2 - 10 + 1 = 4 - 10 + 1 = -6 + 1 = -5$$

5. (10 points) Showing all your work, find the following:

   (a) 56% of 920

   (b) 150% of 60

   (c) 1% of 840

Solution.

(a)

$$920(0.56) = 515.2$$

(b)

$$60(1.50) = 90$$

(c)

$$840(0.01) = 8.40$$

6. (10 points) Showing all your work, compute the following:

   (a) 150 increased by 42%

   (b) 245 decreased by 20%

   (c) 660 increased by 125%

Solution.

(a)

$$150(1 + 0.42) = 150(1.42) = 213$$

(b)

$$245(1 - 0.20) = 245(0.80) = 196$$

(c)

$$660(1 + 1.25) = 660(2.25) = 1485$$

7. (10 points) Find the average value of the following numbers: $-7, -4.2, -1, 0, 2, 6, 10, 12$.

Solution. *We have...*

$$\begin{aligned}
\text{Avg}(\{-7, -4.2, -1, 0, 2, 6, 10, 12\}) &= \frac{\sum \text{ values}}{\#\text{ numbers}} \\
&= \frac{-7 + (-4.2) + (-1) + 0 + 2 + 6 + 10 + 12}{8} \\
&= \frac{17.8}{8} \\
&= 2.225
\end{aligned}$$

8. (10 points) Suppose a student's course grade consists of the following weights:

Homework	40%	Exam 2	15%
Quizzes	5%	Final Exam	17%
Exam 1	8%	Project	15%

Suppose also that a student had a 85.6% homework average, 80% quiz average, 92% on exam 1, 87% on exam 2, 79% on the final, and 95% on the project. Compute the student's course average to the nearest tenth of a percent.

Solution. *We have...*

$$\begin{aligned}\text{Course Average} &= \sum \text{value} \cdot \text{weight} \\ &= 40\%(0.856) + 5\%(0.80) + 8\%(0.92) + 15\%(0.87) + 17\%(0.79) + 15\%(0.95) \\ &= 34.24 + 4.0 + 7.36 + 13.05 + 13.43 + 14.25 \\ &\approx 86.3\%\end{aligned}$$

9. (10 points) Suppose you take the courses shown below and receive the given letter grades. Showing all your work, compute your GPA to the nearest thousandth. The university's letter grade system is shown on the right below.

Course	Credits	Letter Grade
Eastern Europe	3	B−
Modern American Literature	3	C+
Calculus II	4	A
Chemistry I	4	A−
Introduction to College Life	1	A
Freshman Seminar	3	B

A	4.0	C+	2.3
A–	3.7	C	2.0
B+	3.3	C–	1.7
B	3.0	D	1.0
B–	2.7	F	0.0

Solution. *We have...*

$$\begin{aligned}
\text{GPA} &= \frac{\sum \text{value} \cdot \text{credits}}{\text{total credits}} \\
&= \frac{2.7(3) + 2.3(3) + 4.0(4) + 3.7(4) + 4.0(1) + 3.0(3)}{3+3+4+4+1+3} \\
&= \frac{8.1 + 6.9 + 16 + 14.8 + 4 + 9}{18} \\
&= \frac{58.8}{18} \\
&\approx 3.267
\end{aligned}$$

10. (10 points) Showing all your work, convert 648 in^2 to square feet. Recall that there are twelve inches in every foot.

Solution. *We have...*

$$\frac{648\ \text{in}^2 \left| \begin{array}{r} 1\ \text{ft} \\ \hline 12\ \text{in} \end{array} \right| \begin{array}{r} 1\ \text{ft} \\ \hline 12\ \text{in} \end{array}}{} = 4.5\ \text{ft}^2$$

11. (10 points) Showing all your work, convert 9.5 km/hr to feet per minute. Note that 1 m is 3.28084 ft.

Solution. *We have...*

$$\frac{9.5 \text{ km}}{1 \text{ hr}} \left| \frac{1000 \text{ m}}{1 \text{ km}} \right| \frac{3.28084 \text{ ft}}{1 \text{ m}} \left| \frac{1 \text{ hr}}{60 \text{ min}} \right. = 519.466 \text{ ft/min}$$

12. (10 points) Suppose you are covering a wall with black and white checker tiles. The wall measures 40 ft across and is 9 ft tall. If each tile is 6 in by 6 in, how many tiles will you need to cover the wall? Suppose also that you can put tiles on the wall at a rate of 3 tiles per minute. How long will it take you to tile the wall? For both questions, be sure to show all your work and fully justify your answer.

Solution. *Note that it takes 2 tiles to cover a 1 ft wide area. Then it will require* $2 \cdot 40 = 80$ *tiles to cover the width of the wall. However, this line of tiles only covers up to a height of 6 in. Another layer, requiring an additional 80 tiles for a total of* $80 + 80 = 160$ *tiles, will cover the width up to 1 ft high. Another 8 ft, i.e. 8 layers, will then cover the entire wall. This will then use an additional* $8 \cdot 160 = 1280$ *tiles for a total of* $160 + 1280 = 1440$ *tiles, i.e. 720 black tiles and 720 white tiles. Alternatively, the area of the wall is* $A = lw = 40\,ft \cdot 9\,ft = 360\,ft^2$*. Each tile is 6 in by 6 in, i.e. 0.5 ft by 0.5 ft. The area of each tile is then* $(0.5\,ft)(0.5\,ft) = 0.25\,ft^2$*. Therefore, it will take* $360\,ft^2/0.25\,ft^2 = 1440$ *tiles, i.e. 720 black tiles and 720 white tiles.*

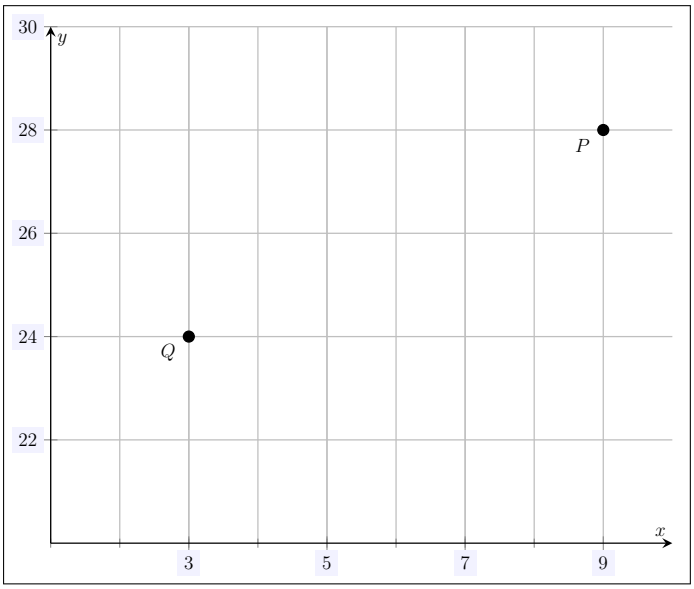
To find the total amount of time required to tile the wall, we have that we tile at a rate of 3 tiles per minute with a total of 1,440 tiles to place. This will require $1440\,tiles/(3\,tiles/min) = 480$ *min, i.e. 8 hours.*

13. (10 points) Showing all your work, find the volume of a cylinder that is measures 4 in across the bottom and is 11 in tall.

Solution. *Note that because the cylinder measures 4 in across the entire bottom, its diameter is 4 in. But then the radius of the cylinder is 2 in. The height is 11 in. But then we have...*

$$V = \pi r^2 h = \pi(2\ in)^2 \cdot 11\ in = pi \cdot 4\ in^2 \cdot 11\ in = 44\pi\ in^3 \approx 138.23\ in^3$$

14. A city is laid out in a grid structure with street corners meeting at right angles, as shown below with the street labeled. Suppose you are standing at point P and your friend is standing at point Q.



(a) (4 points) How many blocks are you from your friend 'as the crow flies'?

(b) (4 points) How many blocks are you from your friend using the taxicab metric?

(c) (2 points) Supposing you can walk a block in 4 min and given your answer in (b), how long do you estimate that it will take you to walk to your friend?

Solution.

(a) This is the Euclidean distance from P to Q:

$$\sqrt{(3-9)^2+(24-28)^2} = \sqrt{(-6)^2+(-4)^2} = \sqrt{36+16} = \sqrt{52} \approx 7.21 \text{ blocks}$$

(b) Using the taxicab/Manhattan metric, we have...

$$|3-9| + |24-28| = |-6| + |-4| = 6+4 = 10$$

(c) Given that we are in a city, the taxicab/Manhattan metric seems to be the more useful metric of distance. By (b), we are a total 10 blocks from our friend by (b). We walk a block in 4 min, i.e. we walk at a pace of 4 min/block. But then, it should take approximately 10 *blocks* $\cdot\, 4$ *min/block* $= 40$ *minutes to walk to our friend.*

15. (10 points) Showing all your work and fully explaining your approximation, estimate how many pet dogs there are in the United States.

Solution. *Answers will vary, but here is a possible approach:*

I approximate from my own experience that about 1 in every 10 people that I know own a dog. They typically own one dog but occasionally they will own two or three. I then estimate that of the 1 in 10 people that own a dog, they own 1.5 dogs on average. The population of the United States is approximately 330 million. Therefore, the estimated amount of pet dogs in the United States is then...

$$\frac{1}{10} \cdot 330000000 \cdot 1.5 = 49,500,000$$