**Quiz 1.** *True/False*: Prunella is trying to find the cardinality of the set $\{1, 2, 1, 4, 1, 8\}$. She counts how many numbers are in the set and finds that there are six numbers. Therefore, the cardinality of the set is 6.

**Solution.** The statement is *false*. The cardinality (or 'size') of a set is the number of elements in a set. However, the order of elements of a set does not matter nor do repeats within a set. Therefore, the set $\{1, 2, 1, 4, 1, 8\}$ is the same as the set $\{1, 2, 4, 8\}$. The cardinality of this set is clearly 4. We may then more clearly define the cardinality of a set to be the number of *distinct* elements of a set. Prunella is mistaken believing that the repeated 1s count towards the cardinality.

**Quiz 2.** *True/False*: If $A$ is a set with $5$ elements and $B$ is a set with $3$ elements, then $A - B$ is a set with 2 elements.

**Solution.** The statement is *false*. It may be possible for some sets. For instance, if $A = \{a, b, c, d, e\}$ and $B = \{a, c, e\}$, then $A - B = \{b, d\}$. Then $|A| = 5$, $|B| = 3$, and $|A - B| = 2$. However, this is not true for *all* sets. For instance, if $A = \{a, b, c, d, e\}$ and $B = \{-5, 6, \text{'nice'}\}$, then $A - B = \{a, b, c, d, e\}$. But then in this case, $|A| = 5$, $|B| = 3$, and $|A - B| = 5$. The cardinality of $A - B$ depends on how many elements of $A$ have been 'removed' because they were elements of $B$. This could be 1, 2, or 3 elements depending on the cardinality of $A \cap B$.

**Quiz 3.** *True/False*: The number of ways of choosing three distinct candle sticks from a collection of five to arrange on a mantle is ${}_5P_3 = 5 \cdot 4 \cdot 3 = 60$ possible choices of arrangements.

**Solution.** The statement is *true*. We can count this directly. There are 5 possible candlesticks to choose for the far left position. This leaves 4 possible choices for the rightmost candlestick and then finally 3 possible choices for the middle candlestick. But then in total there are $5 \cdot 4 \cdot 3 = 60$ total possible arrangements. Alternatively, we know the number of ways of arranging $k$ objects from a collection of $n$ distinct objects, with repetition not allowed, where the order of the arrangement matters, is given by ${}_nP_k$. Here, we have $n = 5$ and $k = 3$. But then we know the number of possible arrangements is ${}_5P_3 = \frac{5!}{(5-3)!} = \dfrac{5!}{2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 5 \cdot 4 \cdot 3 = 60$.

**Quiz 4.** *True/False*: The number of possible ways of guessing the correct answers to a 10 question True/False exam is ${}_{10}C_{10}$.

**Solution.** The statement is *false*. We know that the order of the answer choices matters. We know also that ${}_{10}C_{10}$ represents a combination. For combinations, order is unimportant. Therefore, it is highly unlikely (but not strictly speaking impossible) that ${}_{10}C_{10}$ gives the correct count. We know each question has 2 possible answers. One must answer the first question, and the second, and the third, etc. There are 10 questions. Therefore, there are $2 \cdot 2 \cdot \cdots \cdot 2 = 2^{10} = 1024$ total possible number of ways of answering (guessing) the answers for this collection of 10 true/false questions.

**Quiz 5.** *True/False*: Harriet lives In Alkonost, AZ. There it is sunny 90% of the time. Harriet is planning her weekend. She can expect that there is a $0.90 \cdot 0.90 = 0.81$ probability, i.e. $81\%$ chance, that it is sunny both days.

**Solution.** The statement is *false*. If $A$ and $B$ are events, then we know that $P(A \text{ and } B) = P(A) \cdot P(B)$, if $A$ and $B$ are independent. Recall two events are independent if and only if the occurrence or non-occurrence of an event changes the probability that the other event occurs/does not occur. If $A$ and $B$ are not independent, it may not be true that $P(A \text{ and } B) = P(A) \cdot P(B)$. Let $A$ be the event that it is sunny on Saturday and $B$ be the event that it is sunny on Sunday. Clearly, $A$ and $B$ are not independent events. For instance, if it is sunny/rainy one day, it is more/less likely to be sunny/rainy the next. Therefore, it may not be the case that there is a $0.90 \cdot 0.90 = 0.81$ probability, i.e. $81\%$ chance, that it is sunny both days. Generally, $P(A \text{ and } B) = P(A)P(B \mid A) = P(B)P(A \mid B)$, whether or not $A$ and $B$ are independent.

**Quiz 6.** *True/False*: At a community college, 45% of students have some experience with Excel, 55% of students have some experience with Word, and 70% of students have experience with at least one of them. Therefore, 15% of students have experience only with Excel.

**Solution.** The statement is *true*. If $A$ and $B$ are events, we know that $P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$. But then we know that $P(\text{Excel or Word}) = P(\text{Excel}) + P(\text{Word}) - P(\text{Excel \& Word})$. But then we have $0.70 = 0.45 + 0.55 - P(\text{Excel \& Word})$. Then $0.70 = 1.00 - P(\text{Excel \& Word})$ so that $P(\text{Excel \& Word}) = 0.30$. Finally, because every person that knows Excel either knows word or does not (and these are mutually exclusive), we know that $0.45 = P(\text{Excel}) = P(\text{Only Excel}) + P(\text{Excel and Word}) = P(\text{Only Excel}) + 0.30$. But this shows that $P(\text{Only Excel}) = 0.15$. Therefore, 15% of students have experience only with Excel.