Name: _____
MATH 108
Spring 2022
Written HW 3: Due 02/16

"This is not a dictatorship. This is America. Give me liberty, or give me meth."

–Frank Gallagher, Shameless

Problem 1. (10pt) Determine if the point $(x_1, x_2) = (-1, 3)$ is a solution to the system of equations below. Be sure to fully justify your answer.

$$\begin{aligned} x^2 + xy + y &= 1 \\ x^3 - y^3 &= -26 \end{aligned}$$

Problem 2. (10pt) Determine if the linear system of equations below has none, one, or infinitely many solutions. Be sure to fully justify your answer.

$$2x - y = -2$$
$$3x + 5y = 10$$

Problem 3. (10pt) Find the coefficient matrix, solution vector, and augmented matrix associated with the system of equations below.

$$\begin{aligned} 5x_1 + x_2 - 6x_3 &= 19 \\ 3x_2 - 2x_3 &= -6 \\ 9x_1 + 8x_3 &= 5 \end{aligned}$$

Problem 4. (10pt) Write the system of equations associated to the augmented matrix below.

$$\begin{pmatrix} 6 & 1 & -5 & -7 \\ 4 & 0 & -1 & 9 \\ 1 & 1 & 1 & 4 \end{pmatrix}$$

Problem 5. (10pt) Find all the pivot positions in the augmented matrix below. Also, determine if the system of equations is consistent or not.

$$\begin{pmatrix} 1 & 4 & 6 & -2 & 5 \\ 0 & 0 & -1 & 7 & 12 \\ 0 & 0 & 0 & -9 & 5 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Problem 6. (10pt) The matrix below represents a reduced-row echelon form of augmented matrix for a system of equations. Determine the solutions to this original system of equations.

$$\begin{pmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{pmatrix}$$

Problem 7. (10pt) Solve the following system of equations using elimination. Then solve the system of equations again by creating an augmented matrix and find its reduced-row echelon form.

$$\begin{aligned} x - 3y &= -9 \\ -2x + y &= 8 \end{aligned}$$

Problem 8. (10pt) Use WolframAlpha's `RowReduce` to find the solution to the following system of equations:

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 1 \\ x_1 - 2x_2 + 3x_3 - 4x_4 &= 2 \\ 10x_1 + 3x_2 - 5x_3 - 2x_4 &= 3 \\ -2x_1 - 4x_2 + 6x_3 + 8x_4 &= 4 \end{aligned}$$