Quiz 1. *True/False*: If $x \in A$ but $x \notin B$, then $x \notin A \cup B$.

Solution. The statement is *false*. Recall that if S is a set, then $x \in S$ means that x is an element of S. For example, suppose $S = \{1, 2, 3, 4, 5\}$. If $x = 1$, then $x \in S$. However, if $x = 9$ then x is not in S, i.e. $x \notin S$. Recall also that $A \cup B$ is the set of elements that are either in A or in B—including the possibility that it might be in both! Because $x \in A$, even though $x \notin B$, we know that $x \in A \cup B$ because x is in A or B—after all, it's in A! As a concrete example, take $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$. Then we know that $A \cup B = \{1, 2, 3, 4\}$. Now if $x = 1$, then $x \in A$ and $x \notin B$. But we can see that $x \in A \cup B$.

Quiz 2. *True/False*: A person's salary at their first job is a function of their number of years of schooling.

Solution. The statement is *false*. Recall that a relation is a function if for each input, there is only one possible output, i.e. $f(x)$ is a function if given each x, there is one and only one possible output $f(x)$. In this example, we are wondering whether a person's income, I, is a function of their number of years of school, n. So is $I(n)$ a function? Then there would be one and only one possible output, i.e. salary, given some number of years of schooling. For instance, if $n = 4$ years (of high school, college, etc.), then the person's salary, $I(4)$, would have only one possible value. But we know that there are many people with the same number of years of schooling that have the same salary! For instance, there will be many people that graduate together (most having the same number of years of schooling) and will have widely varying salaries. Therefore, $I(n)$ cannot be a function, i.e. we cannot exactly predict someone's income from their number of years of school. However, one could try to perform statistical analysis on this problem, e.g. what does the *average* person make if they have n years of schooling.

Quiz 3. *True/False*: A company is bulk ordering parts for their production line. The order is for $\$256,478.33$ and the processing company charges a 1% surcharge on an order. Therefore, the total they will be charged (before tax) for the goods is $\$256478.33(1.10) \approx \$282126.16$.

Solution. The statement is *false*. Because the company is charging a surcharge, the price is going up. We know the final bill will then be 1%, i.e. we need to compute $\$256,478.33$ increased by 1%. To compute a percentage increase/decrease of a number N by P%, we compute $N(1 \pm P_d)$, where N is the number, we choose '$+$' if we are computing an increase and '$-$' if we are computing a decrease, and P_d is the percentage written as a decimal. In our case, writing 1% as a decimal, we have 0.01. But then we have total $\$256\,478.33(1 + 0.01) = 256\,478.33(1.01) \approx \$259,043.11$.

Quiz 4. *True/False*: If there are three exams in a class. You received an 85% on the first, 91% on the second, and 78% on the last. The exams are weighted such that the last is worth three times as much as the other two. Then your exam average is given by $85\left(\frac{1}{5}\right) + 91\left(\frac{1}{5}\right) + 78\left(\frac{3}{5}\right) = 82\%$.

Solution. The statement is *true*. If this were an ordinary average, we could simply add up the grades and divide by the number of grades: $\frac{85\%+91\%+78\%}{3} = \frac{254\%}{3} \approx 84.7\%$. We can view this as a weighted average by algebraic manipulation and see that the weight is then one over the number of grades: $\frac{85\%+91\%+78\%}{3} = 85\%(\frac{1}{3}) + 91\%(\frac{1}{3}) + 78\%(\frac{1}{3}) \approx 28.33\% + 30.33\% + 26.0\% \approx 84.7\%$. In a weighted average, we add up each of the grades times their weight. For an ordinary average, this weight is simply $\frac{1}{n}$, where n is the number of objects. Here, Exam 3 is worth three times as much as the other two. If we split the grade into five parts, the weights of Exam 1, Exam 2, and Exam 3 are $1/5$, $1/5$, and $3/5$, respectively. Then the exam average is $85\%(\frac{1}{5}) + 91\%(\frac{1}{5}) + 78\%(\frac{3}{5}) = 17.0\% + 18.2\%. + 46.8\% = 82\%$.

Quiz 5. *True/False*: If 1 meter is 39.3701 inches, then to convert 5 m^2 to in^2, one computes $5 \cdot 39.3701 \approx 196.85$ in^2.

Solution. The statement is *false*. To convert this, we have...

$$\begin{array}{c|c|c} 5 \text{ m}^2 & 39.3701 \text{ in} & 39.3701 \text{ in} \\ \hline & 1 \text{ m} & 1 \text{ m} \end{array} = 5 \cdot 39.3701 \text{ in} \cdot 39.3701 \text{ in} = (5 \cdot 39.3701^2) \text{ in}^2 \approx 7750.02 \text{ in}^2$$

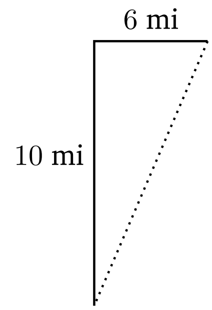
As given, the individual only converted one of the units of meters to inches—neglecting to convert the other unit of meters to inches. So the given computation, $5 \cdot 39.3701 \approx 196.85$ in^2, was actually $5 \text{ m} \cdot 39.3701 \text{ in} \approx 196.85$ m in, which does not have the correct units.

Quiz 6. *True/False*: Suppose you have an hourly wage job. Then if you work twice as much for twice as much pay, you will make twice as much money.

Solution. The statement is *false*. Suppose that you made \$10/hr and work 8 hours a week. Then you make \$10/hr $\cdot$ 8 hours $=$ \$80. Now if you double each of those, you make \$20/hr and work 16 hours per week. But then you make \$20/hr $\cdot$ 16 hours $=$ \$320, which is four times the amount—not double the amount. Generally, suppose you make r dollars an hour and work h hours a week. Then you make $P = r \cdot h$ dollars a week. Now suppose you double your pay and hours a week. Then you make $(2r)(2h) = (2 \cdot 2)(rh) = 4P$ dollars a week, which is four times the amount—not double.

Quiz 7. *True/False*: If you travel 6 mi west and 10 mi south, then you have traveled approximately 11.7 mi from your original location—as the crow flies.

Solution. The statement is *true*. The distance 'as the crow flies', is the straight line distance from one point to another. Drawing this out, we have. . .



The distance we want is the dotted line—the hypotenuse of the triangle sketched above. But by the Pythagorean Theorem, we have. . .

$$d = \sqrt{a^2 + b^2} = \sqrt{(6 \text{ mi})^2 + (10 \text{ mi})^2} = \sqrt{36 \text{ mi}^2 + 100 \text{ mi}^2} = \sqrt{136 \text{ mi}^2} \approx 11.66 \text{ mi}^2.$$

Quiz 8. *True/False*: 'Any' function which has a constant rate of change can be represented by a linear function.

Solution. The statement is *true*. Suppose the rate of change were 5 and the current value is 2. After one step in time, the value is $2 + 1(5) = 2 + 5 = 7$. After another step in time, the value is $7+5 = 12$, or $2+2(5) = 2+10 = 12$. Generally, after n steps, the value is $2+n \cdot 5 = 5n+2$, which is a linear function. Generally, if we start with initial value y_0 and have a constant rate of change m, after x steps, we have $y = y_0 + x \cdot m = mx + y_0$. This is a linear function with $y = y$, $x = x$, $m = m$, and $b = y_0$. But then we see that 'any' function which changes at a constant rate is a linear function. We know that a linear function $y = mx + b$ has a constant rate of change—the slope m. Therefore, a function is linear if and only if it has a constant rate of change.

Quiz 9. *True/False*: The y-intercept of a 'real-world' function always has an interpretation in the context of the problem.

Solution. The statement is *false*. We can always compute a y-intercept—it occurs when the function input is 0. Therefore, the y-intercept of $f(x)$ is $f(0)$ (or $\big(0, f(0)\big)$).However, this does not mean the y-intercept has an interpretation in the context of the problem. Suppose $M(t)$ represents the amount of money in your checking account in t days. The y-intercept is $M(0) = \$8271$, which means you have \$8,271 in your checking account right now—which makes sense. Now suppose $W(t)$ represents your weight at time t. The y-intercept is $W(0) = -15$, so at the 'start' time, you have weight -15, which is nonsense. [Note that a negative value does not mean it does not have an interpretation in the problem context. For instance, return to the exam where $M(t)$ represents the amount of money in your checking account in t days. If $M(0) = -540$, one could interpret this as meaning you have $-\$540$ in your checking account right now, i.e. right now your checking account is $540 overdrawn.] Whether or not a 'real-world' function has an interpretable y-intercept depends on the function, the intercept, and how one is interpreting the values.

Quiz 10. *True/False*: Janet is waiting in line for the copier. She notices that the faculty member ahead of her already has 10 copies printed and that in the last 2 minutes, 6 copies have been made. Then the amount of copies made can be modeled by $C(t) = 3t + 10$.

Solution. The statement is *true*. We shall assume that the copier is making copies at a constant rate. Then we know that $C(t) = mt + b$ for some m, b. We know that the copier makes 6 copies in 2 minutes; therefore, the copier makes $6/2 = 3$ copies per minute. We know then that $m = 3$. So we have $C(t) = 3t + b$. We know after 2 minutes, a total of $10 + 6 = 16$ copies have been made. But then we have...

$$\begin{aligned} C(t) &= 3t + b \\ C(2) &= 3(2) + b \\ 16 &= 6 + b \\ b &= 10 \end{aligned}$$

Therefore, we know that the number of copies made after t minutes is given by $C(t) = 3t + 10$. [Equivalently, one could have said at 0 minutes that only 10 copies have been made. But then we know that $10 = C(0) = 3(0) + b = b$, arriving again at $C(t) = 3t + 10$.]

Quiz 11. *True/False*: If the CPI last year was $255.43 and the CPI this year was $271.11, then the inflation rate was approximately 61%.

Solution. The statement is *false*. We know that the ratio of this year's CPI to last year's CPI is...

$$\frac{\text{Current CPI}}{\text{Previous CPI}} = \frac{\$271.11}{\$255.43} = 1.06139 = 1 + 0.06139$$

Interpreting this as a percentage increase (because the ratio is greater than 1), we can see that the CPI from last year increased by 6.139% to this year's CPI. Therefore, the inflation rate was 6.139%.

Quiz 12. *True/False*: If $5000 accumulates interest at a 4.2% annual interest rate, compounded monthly, then the amount after 6 years is...

$$\$5000 \left(1 + \frac{0.042}{12}\right)^{12 \cdot 6} \approx \$6430.15$$

Solution. The statement is *true*. We know that P dollars gaining interest at an annual interest rate r each year, compounded k times per year for t years is given by $P\left(1+\frac{r}{k}\right)^{kt}$. Here we begin with an initial value of $P = \$5000$. The annual interest rate is 4.2%, i.e. $r = 0.042$. The interest is compounded monthly, i.e. 12 times per year. Then we know that $k = 12$. The money is earning interest for 6 years, so that $t = 6$. Therefore, the amount of money after this time is...

$$\$5000\left(1+\frac{0.042}{12}\right)^{12\cdot 6} \approx \$5000(1.0035)^{72} \approx \$5000(1.28603) \approx \$6430.15$$

Quiz 13. *True/False*: If you take out a loan for $2,300 at 5.6% annual interest, compounded quarterly, for a period of 3 years, then the amount you pay in interest is $417.59.

Solution. The statement is *true*. We know that P dollars gaining interest at an annual interest rate r each year, compounded k times per year for t years is given by $P\left(1+\frac{r}{k}\right)^{kt}$. Here we begin with an initial value of $P = \$2300$. The annual interest rate is 5.6%, i.e. $r = 0.056$. The interest is compounded quarterly, i.e. 4 times per year. Then we know that $k = 4$. The money is earning interest for 3 years, so that $t = 3$. Therefore, the amount of money after this time is...

$$\$2300\left(1+\frac{0.056}{4}\right)^{4\cdot 3} = \$2300\,(1.014)^{12} = \$2300(1.1815591) \approx \$2717.59$$

Because the loan was only for $2,300, the interest on the loan is $\$2717.59 - \$2300 = \$417.59$.

Quiz 14. *True/False*: If A and B are events, then $P(A \text{ and } B) = P(A) \cdot P(B)$.

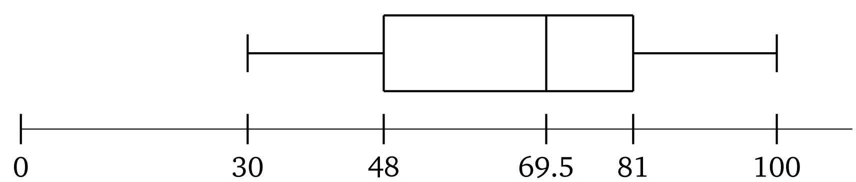
Solution. The statement is *false*. It is true that if A and B are independent events that $P(A \text{ and } B) = P(A) \cdot P(B)$. However, this is not generally true for arbitrary events A and B. Generally, we have $P(A \text{ and } B) = P(A)P(B \mid A)$ or $P(A \text{ and } B) = P(B)P(A \mid B)$. To see why $P(A \text{ and } B)$ may be different than $P(A) \cdot P(B)$, consider the events A and B, where A is passing a course and B is failing a course. Clearly, you cannot pass and fail a course at the same time. So $P(A \text{ and } B) = 0$. However, unless $P(A) = 0$ or $P(B) = 0$, then $P(A) \cdot P(B) \neq 0$. Because A and B are disjoint events (they cannot happen at the same time), they cannot be independent. So we should not have expected that $P(A \text{ and } B) = P(A) \cdot P(B)$.

Quiz 15. *True/False*: If A and B are independent events with $P(A) = 0.40$, $P(B) = 0.20$, then $P(A \text{ or } B) = P(A) + P(B) = 0.40 + 0.20 = 0.60$.

Solution. The statement is *false*. We know that A and B are disjoint events, then $P(A \text{ or } B) = P(A) + P(B)$. However, if A and B are not disjoint, this may not be true. Generally, if A and B are events, then $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. If A and B are disjoint, then $P(A \text{ and } B) = 0$ and the formula reduces to the disjoint case. Now because A and B are independent, we know $P(A \text{ and } B) = P(A) \cdot P(B) = 0.40 \cdot 0.20 = 0.08$. But then we have $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 0.40 + 0.20 - 0.08 = 0.52$.

Quiz 16. *True/False*: The 5-number summary is the minimum, Q1, mean, Q3, and maximum.

Solution. The statement is *false*. The five numbers included in the 5-number summary are: minimum, Q1, the *median*, Q3, and the maximum. The purpose of the 5-number summary is to try to 'break-up' and show the 'spread' of the data about the median. The 5-number summary then breaks up the data into four 'chunks', each containing 25% of the data values—though each quarter of the data need not be 'spread' out equally. We can then visualize this spread with a box plot, e.g. the one below



Quiz 17. *True/False*: Suppose x comes from a normal distribution, $N(\mu, \sigma)$. Then the larger the value of $|z_x|$, the more 'unusual' the value.

Solution. The statement is *true*. Recall that the z-score for x is given by $z_x = \frac{x-\mu}{\sigma}$. But then because $\sigma \geq 0$, $|z_x| = \left|\frac{x-\mu}{\sigma}\right| = \frac{|x-\mu|}{|\sigma|} = \frac{|x-\mu|}{\sigma}$. The value of $|x - \mu|$ is how far x is from the mean, μ. But then $|z_x| = \frac{|x-\mu|}{\sigma}$ measures the number of standard deviations x is from the mean. The sign of z_x indicates the direction, i.e. $z_x < 0$ implies x is less than the mean, $z_x = 0$ implies x is equal to the mean, and $z_x > 0$ implies that x is greater than the mean. Then the greater $|z_x|$ is, the farther x is above/below the mean, where values 'occur less frequently.' Hence, the larger the value of $|z_x|$, the more 'unusual' the value.

Quiz 18. *True/False*: If you take simple random samples of size 15 from a distribution with mean 130 and standard deviation 25, the resulting distribution of sample means is $N(130, 25/\sqrt{15}) \approx N(130, 6.45497)$.

Solution. The statement is *false*. If a random variable X has a distribution with mean μ and (finite) standard deviation σ, the Central Limit Theorem (CLT) says that the distribution of sample means, $\overline{X}$, of size n has distribution $N(\mu, \sigma/\sqrt{n})$ so long as at least one of the following criterion are met: the underlying distribution is normal, i.e. you are sampling from $N(\mu, \sigma)$, or the sample size is 'sufficiently large' (e.g., $n \geq 30$). We were not told that the underlying distribution was normal. Because the sample size is $15 < 30$, the sample size is not sufficiently large enough to meet the criterion of the Central Limit Theorem. Therefore, it is not necessarily the case that the distribution of $\overline{X}$ is $N(130, 25/\sqrt{15}) \approx N(130, 6.45497)$.