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MATH 101
Fall 2023
HW 12: Due 11/06

"If you think education is expensive, try ignorance."
–Jeff Rich

Problem 1. (10pt) Consider the linear function $\ell(x) = \frac{13-11x}{5}$.

(a) Find the slope of this function.

(b) Find the y-intercept of this function.

(c) Find the x-intercept of this function.

(d) Does the graph of this function contain the point $(6, -8)$? Explain.

Solution.

(a) Given a linear function $y = mx + b$, we know that m is the slope and b is the y-intercept. We have $\ell(x) = \frac{13-11x}{5} = \frac{13}{5} - \frac{11}{5}\,x$. Therefore, the slope is $m = -\frac{11}{5}$.

(b) From (a), we know that $\ell(x) = \frac{13-11x}{5} = \frac{13}{5} - \frac{11}{5}\,x$. Therefore, the y-intercept is $\frac{13}{5}$, i.e. the point $(0, \frac{13}{5})$. Equivalently, we know the y-intercept is the value of the function when $x = 0$. But then we have $\ell(0) = \frac{13-11(0)}{5} = \frac{13-0}{5} = \frac{13}{5}$, i.e. the y-intercept is $(0, \frac{13}{5})$.

(c) The x-intercept(s) occurs at the x-value(s) when the output is 0. But then...

$$\ell(x) = 0\frac{13 - 11x}{5} = 013 - 11x = 013 = 11xx = \frac{13}{11} \approx 1.18182$$

That is, the x-intercept is the point $(\frac{13}{11}, 0)$.

(d) If the graph of ℓ contains the point $(6, -8)$, then $\ell(6) = -8$. But we have...

$$\ell(6) = \frac{13 - 11(6)}{5} = \frac{13 - 66}{5} = -\frac{53}{5} \approx -10.6 \neq -8$$

Therefore, the graph of ℓ does not contain the point $(6, -8)$. Alternatively, the graph of ℓ contains the point $(6, -8)$ if the point satisfies the equation of ℓ. But then...

$$\begin{aligned} \ell(x) &= \frac{13 - 11x}{5} \\ \ell(6) &\stackrel{?}{=} \frac{13 - 11(6)}{5} \\ -8 &\stackrel{?}{=} \frac{13 - 66}{5} \\ -8 &\neq -\frac{53}{5} \end{aligned}$$

Therefore, the graph of ℓ does not contain the point $(6, -8)$.

Problem 2. (10pt) Solve the following equation and verify that your solution is correct:

$$9 - 3(x + 1) = \frac{6 - x}{2}$$

Solution. We have...

$$9 - 3(x + 1) = \frac{6 - x}{2}$$

$$9 - 3x - 3 = \frac{6 - x}{2}$$

$$6 - 3x = \frac{6 - x}{2}$$

$$2(6 - 3x) = 2\left(\frac{6 - x}{2}\right)$$

$$12 - 6x = 6 - x$$

$$12 = 6 + 5x$$

$$5x = 6$$

$$x = \frac{6}{5}$$

We can now verify this solution:

$$9 - 3(x + 1) = \frac{6 - x}{2}$$

$$9 - 3\left(\frac{6}{5} + 1\right) \stackrel{?}{=} \frac{6 - \frac{6}{5}}{2}$$

$$9 - 3 \cdot \frac{11}{5} \stackrel{?}{=} \frac{\frac{24}{5}}{2}$$

$$9 - \frac{33}{5} \stackrel{?}{=} \frac{24}{5} \cdot \frac{1}{2}$$

$$\frac{12}{5} = \frac{12}{5}$$

Therefore, the solution $x = \frac{6}{5}$ is correct.

Problem 3. (10pt) Solve the following equation:

$$5\sqrt{2}\,x + 8 = -3(1 - 2x)$$

Solution. We have...

$$5\sqrt{2}\,x + 8 = -3(1 - 2x)$$

$$5\sqrt{2}\,x + 8 = -3 + 6x$$

$$5\sqrt{2}\,x - 6x + 8 = -3$$

$$5\sqrt{2}\,x - 6x = -11$$

$$(5\sqrt{2} - 6)x = -11$$

$$x = \frac{-11}{5\sqrt{2} - 6}$$

$$x = \frac{11}{6 - 5\sqrt{2}} \approx -10.2701$$

We can also numerically verify this solution:

$$5\sqrt{2}\,x + 8 = -3(1 - 2x)$$

$$5\sqrt{2}(-10.2701) + 8 \stackrel{?}{=} -3(1 - 2 \cdot -10.2701)$$

$$-72.6206 + 8 \stackrel{?}{=} -3(1 - (-20.5402))$$

$$-64.6206 \stackrel{?}{=} -3(21.5402)$$

$$-64.6206 = -64.6206$$

Problem 4. (10pt) Find the equation of the line perpendicular to the line $y = \pi$ with x-intercept $\sqrt{2}$.

Solution. The line $y = \pi$ is horizontal. Because the line in question is perpendicular to a horizontal line, the line must be vertical. Therefore, the line has the form $x = a$ for some number a. We know that the x-intercept of the line is $\sqrt{2}$, I.e. the point $(\sqrt{2}, 0)$. But then it must be that $x = \sqrt{2}$.