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MATH 108
Fall 2022
HW 10: Due 10/27

“Most people use statistics like a drunk man uses a lamppost—more for support than illumination.”

—Andrew Lang

Problem 1. (10pt) Suppose you have a random variable X that has distribution $N(567.10, 64.30)$. Find the following:

- (a) $P(X = 567.10)$
- (b) $P(X \leq 490)$
- (c) $P(X \geq 490)$
- (d) $P(490 \leq X \leq 715)$

Solution.

- (a) Because normal distributions are continuous and $P(X = x_0) = 0$. for continuous distributions, we have $P(X = 567.10) = 0$.

- (b) We have...

$$z_{490} = \frac{490 - 567.10}{64.30} = \frac{-77.10}{64.30} \approx -1.20 \rightsquigarrow 0.1151$$

Therefore, $P(X \leq 490) = 0.1151$.

- (c) We know that $P(X \leq 490) = 0.1151$. Therefore, $P(X \geq 490) = 1 - P(X \leq 490) = 1 - 0.1151 = 0.8849$.

- (d) We know that $P(490 \leq X \leq 715) = P(X \leq 715) - P(X \leq 490)$. We know that $P(X \leq 490) = 0.1151$. We only need find $P(X \leq 715)$. We have...

$$z_{715} = \frac{715 - 567.10}{64.30} = \frac{147.9}{64.30} \approx 2.30 \rightsquigarrow 0.9893$$

Therefore, we have...

$$P(490 \leq X \leq 715) = P(X \leq 715) - P(X \leq 490) = 0.9893 - 0.1151 = 0.8742$$

Problem 2. (10pt) Alice took the SAT and received a score of 1350. Bob took the ACT and received a score of 27. Suppose that the SAT was normally distributed with mean 1050 and standard deviation 180, while the ACT was normally distributed with mean 20.3 and standard deviation 5.9. Who did better on their exam? Explain.

Solution. We can find the z -score for Alice and Bob to create a ‘standardized’ way to compare their scores:

$$z_{\text{Alice}} = \frac{1350 - 1050}{180} = \frac{300}{180} \approx 1.67$$

$$z_{\text{Bob}} = \frac{27 - 20.3}{5.9} = \frac{6.7}{5.9} \approx 1.14$$

Because both z -scores are positive, we know that they each did better than the average score for each of their exams. However, because Alice’s z -score is larger than Bob’s z -score, her exam score was more standard deviations above the mean than Bob’s, i.e. her exam score was more ‘unusual.’ Therefore, Alice did better on her exam.