Name: ____
MATH 308
Fall 2021
HW 10: Due 11/05

"There is only one problem with common sense; it's not very common."
–Milt Bryce

Problem 1. (10pt) Determine if the following functions are injective, surjective, and/or bijective. Which of the functions have an inverse function? [No formal proofs required.]

(a) $f : \mathbb{R} \to [0, 1]$ defined by $f(x) = \sin^2 x$.

(b) $g : [0, \frac{\pi}{2}] \to [0, 1]$ defined by $g(x) = \cos x$.

(c) $h : \mathbb{N} \to \mathbb{Z}$ given by $h(n) = 3^n$.

(d) $j : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ given by $j(n, m) = (n - m + 3)^2$.

Problem 2. (10pt) Show that the function $f : \mathbb{R} \setminus \{-1\} \to \mathbb{R} \setminus \{3\}$ given by $f(x) = \dfrac{3x-5}{x+1}$ is a bijection. Explain why this implies f is invertible and then find the inverse for $f(x)$.

Problem 3. (10pt) Let $S \subseteq \mathbb{R}$ and $f, g : S \to \mathbb{R}$ be monotone increasing functions.

(a) Prove that $f + g$ is a monotone increasing function.

(b) If f and $f + g$ are increasing on S, then is g necessarily increasing on S? Prove this statement or give a counterexample.

Problem 4. (10pt) Let $f : X \to Y$ and let $A, B \in \mathcal{P}(X)$.

(a) Prove that $f(A \cup B) = f(A) \cup f(B)$.

(b) Is it true that $f(A \cap B) = f(A) \cap f(B)$? Prove or give a counterexample.

Problem 5. (10pt) Let $f : X \to Y$ and $A, B \subseteq Y$. Prove that $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$.

Problem 6. (10pt) For each of the following, find a function $f : \mathbb{N} \to \mathbb{Z}$ with the following properties:

(a) f is injective but not surjective

(b) f is surjective but not injective

(c) f is neither surjective nor injective

(d) f is a bijection