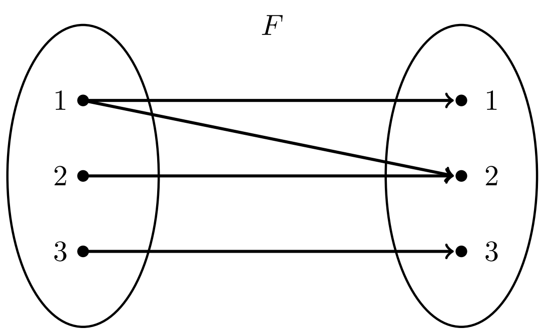
Name: *Caleb McWhorter — Solutions*
MATH 101
Spring 2024
HW 9: Due 02/28

"[Penny] God, you know, four years I lived with him. Four years—that's like as long as high school. [Sheldon] It took you four years to get through high school?"

— Penny & Sheldon Cooper, Big Bang Theory

Problem 1. (10pts) Determine whether the relations F and G shown below are functions. Be sure to fully justify your answer.



x	G
1	1
2	1
3	1
4	1
5	1

Solution. The relation F is *not* a function because $F(1)$ has more than one possible output, i.e. $F(1)$ is not well defined. However, the relation G is a function because each input has only one possible output—even though all the outputs for G are the same.

Problem 2. (10pts) Determine whether the relations $f(x) = 16 - x^2$ and $g(x,y) = \dfrac{x+y}{x-y}$ are functions. Be sure to fully justify your answer. Also, find $f(5)$ and $g(4,5)$.

Solution. The relation $f(x)$ is a function: for each input x, there is only one possible output—namely, the one found by evaluating f at x. The relation $g(x,y)$ is a function: for each input (x,y), there is only one possible output—namely, the one found by evaluating g at the given x, y. We have...

$$f(5) = 16 - 5^2 = 16 - 25 = -9$$

$$g(4,5) = \frac{4+5}{4-5} = \frac{9}{-1} = -9$$

Problem 3. (10pts) Suppose $f(x)$ and $g(x)$ are the functions given below.

$$f(x) = 1 - 4x$$

$$g(x) = x^2 + 1$$

Compute the following:

(a) $6f(1) - g(2) = 6\big(1 - 4(1)\big) - (2^2 + 1) = 6(-3) - (4 + 1) = -18 - 5 = -23$

(b) $(f + g)(1) = f(1) + g(1) = \big(1 - 4(1)\big) + (1^2 + 1) = -3 + 2 = -1$

(c) $(f - g)(0) = f(0) - g(0) = \big(1 - 4(0)\big) - (0^2 + 1) = 1 - 1 = 0$

(d) $(fg)(2) = f(2) \cdot g(2) = \big(1 - 4(2)\big) \cdot (2^2 + 1) = -7 \cdot 5 = -35$

(e) $(f \circ g)(-1) = f\big(g(-1)\big) = f\big((-1)^2 + 1\big) = f(1 + 1) = f(2) = 1 - 4(2) = 1 - 8 = -7$

(f) $(g \circ f)(-1) = g\big(f(-1)\big) = g\big(1 - 4(-1)\big) = g(1 + 4) = g(5) = 5^2 + 1 = 25 + 1 = 26$