Name: _____

MATH 308
Fall 2022
HW 16: Due 12/06

"Algebra is the metaphysics of arithmetic."

–John Ray

Problem 1. (10pt) Let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^4$ be defined by $\mathbf{u} = \begin{pmatrix} 1 \\ 0 \\ -3 \\ 2 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 4 \\ -1 \\ 6 \\ -5 \end{pmatrix}$. Showing all your work, compute the following:

(a) $-6\mathbf{u}$

(b) $\mathbf{v} - \mathbf{u}$

(c) $\mathbf{u} + 2\mathbf{v}$

(d) $\mathbf{u} \cdot \mathbf{v}$

Problem 2. (10pt) Define matrices A, B, C as follows:

$$A = \begin{pmatrix} 1 & 0 & -4 \\ -2 & 3 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 2 & -2 \\ 5 & 1 & 4 \end{pmatrix}, \qquad C = \begin{pmatrix} 2 & 0 \\ -1 & 6 \\ 5 & 3 \end{pmatrix}$$

Showing all your work, compute the following:

(a) $4A$

(b) $A - B$

(c) $3A + B$

(d) AC

(e) B^T

Problem 3. (10pt) Define matrices A, B, C as follows:

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 3 \end{pmatrix}, \qquad B = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}, \qquad C = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Showing all your work and explaining your reasoning, answer the following:

(a) What is B^2?

(b) If CA is defined, compute it. If not, explain why.

(c) What is a_{23}? What is b_{21}?

(d) If $M = AC$, without explicitly computing AC, what is m_{23}?

Problem 4. (10pt) If A, B are matrices, is it true $(A+B)^2 = A^2 + 2AB + B^2$? If so, explain why. If not, explain why not.