Name: *Caleb McWhorter — Solutions*
MATH 100
Fall 2021
HW 6: Due 10/13

"Good rain knows the best time to fall."
–VIP, Squid Game

Problem 1. (10pt) Consider the function $\ell(x) = 5x + 2$.

(a) What is the graph of the function $\ell(x)$.

(b) Find two points on the graph of $\ell(x)$.

(c) Is the point $(2, 5)$ on the graph of $\ell(x)$? Explain.

(d) Is the point $(-1, -3)$ on the graph of $\ell(x)$? Explain.

Solution.

(a) Because the function $\ell(x)$ is of the form $mx + b$, $\ell(x)$ is a linear function. But then the graph of $\ell(x)$ is a line.

(b) To find points on $\ell(x)$, we choose x-values and find the corresponding y-values. If $x = 0$, then $\ell(0) = 5(0) + 2 = 0 + 2 = 2$. Therefore, $(0, 2)$ is a point on the graph of $\ell(x)$. If $x = 1$, then $\ell(1) = 5(1) + 2 = 5 + 2 = 7$. Therefore, $(1, 7)$ is also a point on the graph of $\ell(x)$.

(c) The point $(2, 5)$ has x-value 2. Then if $(2, 5)$ is on the graph of $\ell(x)$, it must be that $\ell(2) = 5$. We check this. Observe $\ell(2) = 5(2) + 2 = 10 + 2 = 12 \neq 5$. Therefore, $(2, 5)$ is not on the graph of $\ell(x)$.

(d) The point $(-1, -3)$ has x-value -1. Then if $(-1, -3)$ is on the graph of $\ell(x)$, it must be that $\ell(-1) = -3$. We check this. Observe $\ell(-1) = 5(-1) + 2 = -5 + 2 = -3$. Therefore, $(-1, -3)$ is on the graph of $\ell(x)$.

Problem 2. (10pt) Do the points $(5,2)$, $(1,-1)$, and $(-3,4)$ lie along a line? Explain.

Solution. The slope of a line is constant. Therefore, if all the points lie along a line, the slope calculated through each of these points must be the same. Observe...

$$m_1 = \frac{2-(-1)}{5-1} = \frac{2+1}{5-1} = \frac{3}{4}$$

$$m_2 = \frac{-1-4}{1-(-3)} = \frac{-1-4}{1+3} = \frac{-5}{4}$$

Because the slopes are not the same, these points do not all lie along a line.

Problem 3. (10pt) Consider the line given by the function $\ell(x) = 1 - 3x$.

(a) Write this line in the form $Ax + By = C$ for some A, B, C.

(b) What is the slope of this line?

(c) Find the y-intercept of the line.

(d) Find the x-intercept of the line.

Solution.

(a) We replace $\ell(x)$ with y. Then we have $y = 1 - 3x$. Then we have $3x + y = 1$. But then this is a line of the form $Ax + By = C$, where $A = 3$, $B = 1$, and $C = 1$.

(b) Examining $\ell(x) = 1 - 3x$, we see that the slope of this line is -3.

(c) The y-intercept is where the curve passes through the y-axis. This is where $x = 0$. But we have $\ell(0) = 1 - 3(0) = 1 - 0 = 1$. Therefore, the y-intercept is the point $(0, 1)$.

(d) The x-intercept is where the curve passes through the x-axis. This is where $y = 0$, i.e. $\ell(x) = 0$. But then

$$\begin{aligned} \ell(x) &= 0 \\ 1 - 3x &= 0 \\ 3x &= 1 \\ x &= \frac{1}{3} \end{aligned}$$

Therefore, the x-intercept is the point $(\frac{1}{3}, 0)$.

Problem 4. (10pt) Are the following lines parallel, perpendicular, or neither? Explain.

$$y = 2x - 5$$
$$2y = 4x - 8$$

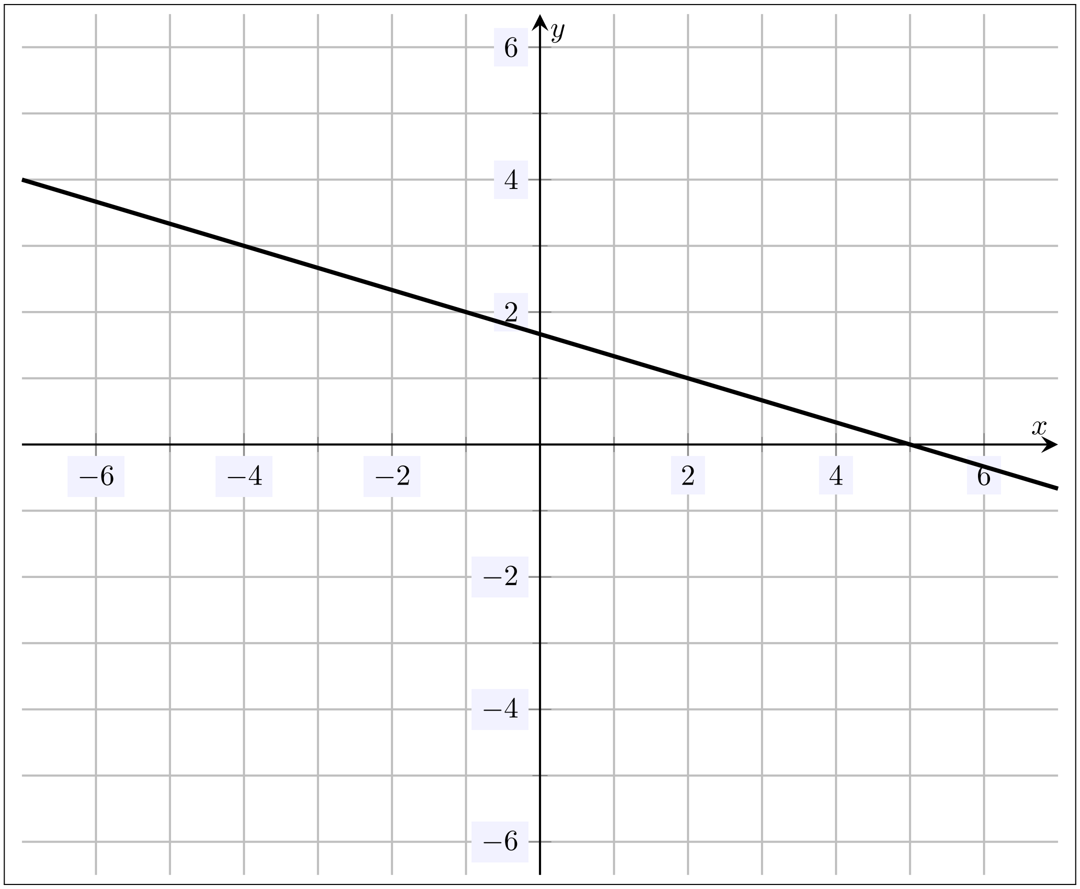
Solution. We want each line in the form $y = mx + b$. The first line is already in this form. For the second line, we divide both sides by 2 to obtain $y = 2x - 4$. The slope of the first line is $m = 2$. The slope of the second line is $m = 2$. Because the lines have equal slope, the lines are parallel.

Problem 5. (10pt) Are the following lines parallel, perpendicular, or neither? Explain.

$$y = \frac{2}{3}x + 7$$
$$2x + 3y = 5$$

Solution. We want each line in the form $y = mx + b$. The first line is already in this form. For the second line, we solve for y. We have $y = \frac{5}{3} - \frac{2}{3}x$. The slope of the first line is $m = \frac{2}{3}$. The slope of the second line is $m = -\frac{2}{3}$. Perpendicular lines have negative reciprocal slopes. The negative reciprocal of $\frac{2}{3}$ is $-\frac{3}{2}$. Because the slopes are not negative reciprocals, the lines are not perpendicular.

Problem 6. (10pt) Find the equation of the line plotted below.



Solution. We see that the line passes through the points $(-4,3),(2,1),(5,0)$. We compute the slope of the line using two of the points:

$$m = \frac{0-1}{5-2} = \frac{-1}{3} = -\frac{1}{3}$$

Now we use the fact that the line must be of the form $y = mx + b$ (because the line is not vertical). Using the point $(5,0)$, i.e. $x = 5$ and $y = 0$, we find

$$\begin{aligned} y &= mx + b \\ y &= -\frac{1}{3}x + b \\ 0 &= -\frac{1}{3} \cdot 5 + b \\ 0 &= -\frac{5}{3} + b \\ b &= -\frac{5}{3} \end{aligned}$$

Therefore, the equation of the line is $y = -\frac{1}{3}x - \frac{5}{3} = \frac{-x-5}{3} = -\frac{x+5}{3}$.

Problem 7. (10pt) Solve the following equation for y:

$$5x - 3y = 12$$

Solution. We have...

$$\begin{aligned} 5x - 3y &= 12 \\ -3y &= 12 - 5x \\ y &= \frac{12}{-3} - \frac{5}{-3}\,x \\ y &= -4 + \frac{5}{3}\,x \\ y &= \frac{5}{3}\,x - 4 \end{aligned}$$