Name: <u>Caleb McWhorter — Solutions</u>

MATH 101 Summer 2022 HW 9: Due 06/09 "Science is organized knowledge. Wisdom is organized life."

–Immanuel Kant

Problem 1. (10pt) Find the domain of the rational function below. What are the vertical asymptotes of the given rational function? Also, simplify the rational function.

$$\frac{x^2 - 36}{x^2 - 2x - 24}$$

Solution. The domain of a rational function $\frac{p(x)}{q(x)}$ are the values for which $q(x) \neq 0$. We have...

$$q(x) = 0$$

 $x^2 - 2x - 24 = 0$
 $(x+4)(x-6) = 0$
 $x+4 = 0 \text{ or } x-6 = 0$
 $x = -4 \text{ or } x = 6$

Therefore, the domain of the function is $x \neq -4$, 6, i.e. all real numbers except for x = -4 and x = 6. For $x \neq -4$, 6, we have...

$$\frac{x^2 - 36}{x^2 - 2x - 24} = \frac{(x - 6)(x + 6)}{(x + 4)(x - 6)} = \frac{\cancel{(x - 6)}(x + 6)}{\cancel{(x + 4)}\cancel{(x - 6)}} = \frac{x + 6}{x + 4}$$

Therefore, the vertical asymptotes of $\frac{x^2-36}{x^2-2x-24}$ are x=-4. Because $\frac{x+6}{x+4}$ evaluated at x=6 is $\frac{6+6}{6+4}=\frac{12}{10}=\frac{6}{5}$, the value x=6 corresponds to the hole $(6,\frac{6}{5})$ for $\frac{x^2-36}{x^2-2x-24}$.

 $\label{eq:def:Domain: } \begin{subarray}{ll} \beg$

Problem 2. (10pt) Simplifying as much as possible, compute the following:

$$\frac{3x+1}{x^2-1} - \frac{x+5}{x+1}$$

$$\frac{3x+1}{x^2-1} - \frac{x+5}{x+1} = \frac{3x+1}{(x-1)(x+1)} - \frac{x+5}{x+1}$$

$$= \frac{3x+1}{(x-1)(x+1)} - \frac{(x-1)(x+5)}{(x-1)(x+1)}$$

$$= \frac{3x+1}{(x-1)(x+1)} - \frac{x^2+5x-x-5}{(x-1)(x+1)}$$

$$= \frac{3x+1-(x^2+4x-5)}{(x-1)(x+1)}$$

$$= \frac{3x+1-x^2-4x+5}{(x-1)(x+1)}$$

$$= \frac{-x^2-x+6}{(x-1)(x+1)}$$

$$= \frac{-(x^2+x-6)}{(x-1)(x+1)}$$

$$= \frac{-(x+3)(x-2)}{(x-1)(x+1)}$$

Problem 3. (10pt) Simplifying as much as possible, compute the following:

$$\frac{7-x}{x^2+8x+12} + \frac{x}{x^2+x-30}$$

$$\frac{7-x}{x^2+8x+12} + \frac{x}{x^2+x-30} = \frac{7-x}{(x+2)(x+6)} + \frac{x}{(x+6)(x-5)}$$

$$= \frac{(7-x)(x-5)}{(x+2)(x+6)(x-5)} + \frac{x(x+2)}{(x+6)(x-5)(x+2)}$$

$$= \frac{7x-35-x^2+5x}{(x+2)(x+6)(x-5)} + \frac{x^2+2x}{(x+6)(x-5)(x+2)}$$

$$= \frac{-x^2+12x-35}{(x+2)(x+6)(x-5)} + \frac{x^2+2x}{(x+6)(x-5)(x+2)}$$

$$= \frac{-x^2+12x-35+x^2+2x}{(x+2)(x+6)(x-5)}$$

$$= \frac{14x-35}{(x+2)(x+6)(x-5)}$$

$$= \frac{7(2x-5)}{(x+2)(x+6)(x-5)}$$

Problem 4. (10pt) Simplifying as much as possible, compute the following:

$$\frac{x^2-4}{x^3-9x} \cdot \frac{x^2-2x-3}{x^2-3x-10}$$

$$\frac{x^2 - 4}{x^3 - 9x} \cdot \frac{x^2 - 2x - 3}{x^2 - 3x - 10} = \frac{(x - 2)(x + 2)}{x(x^2 - 9)} \cdot \frac{(x - 3)(x + 1)}{(x - 5)(x + 2)}$$

$$= \frac{(x - 2)(x + 2)}{x(x - 3)(x + 3)} \cdot \frac{(x - 3)(x + 1)}{(x - 5)(x + 2)}$$

$$= \frac{(x - 2)\cancel{(x + 2)}}{\cancel{x}\cancel{(x - 3)}\cancel{(x + 3)}} \cdot \frac{\cancel{(x - 3)}\cancel{(x + 1)}}{\cancel{(x - 5)}\cancel{(x + 2)}}$$

$$= \frac{(x - 2)\cancel{(x + 1)}}{x(x + 3)(x - 5)}$$

Problem 5. (10pt) Simplifying as much as possible, compute the following:

$$\frac{2x^2 + 8x}{x^2 - 6x - 7}$$

$$\frac{x^2 + 9x + 20}{x^2 - 4x - 5} = \frac{2x^2 + 8x}{x^2 - 6x - 7} \cdot \frac{x^2 - 4x - 5}{x^2 + 9x + 20}$$

$$= \frac{2x(x+4)}{(x-7)(x+1)} \cdot \frac{(x-5)(x+1)}{(x+4)(x+5)}$$

$$= \frac{2x(x+4)}{(x-7)(x+1)} \cdot \frac{(x-5)(x+1)}{(x+4)(x+5)}$$

$$= \frac{2x(x-5)}{(x-7)(x+5)}$$

Problem 6. (10pt) Simplifying as much as possible, compute the following:

$$\frac{x^2 + 2x - 3}{x^2 + 11x + 10}$$

$$\frac{x^2 + 8x - 9}{x^2 + 8x - 20}$$

$$\frac{x^2 + 2x - 3}{x^2 + 11x + 10}$$

$$\frac{x^2 + 8x - 9}{x^2 + 8x - 20} = \frac{x^2 + 2x - 3}{x^2 + 11x + 10} \cdot \frac{x^2 + 8x - 20}{x^2 + 8x - 9}$$

$$= \frac{(x+3)(x-1)}{(x+1)(x+10)} \cdot \frac{(x-2)(x+10)}{(x+9)(x-1)}$$

$$= \frac{(x+3)(x-1)}{(x+1)(x+10)} \cdot \frac{(x-2)(x+10)}{(x+9)(x-1)}$$

$$= \frac{(x+3)(x-2)}{(x+1)(x+9)}$$

Problem 7. (10pt) Fully justifying your answer, determine if the point (-1,3) is a solution to the following system of equations:

$$\begin{cases} 4x - 7y = -8 \\ -3x + 5y = 5 \end{cases}$$

Solution. The point (-1,3) is a solution to the system of equations if and only if it satisfies both of the equations. We check this:

$$4x - 7y = -8$$

$$4(-1) - 7(3) \stackrel{?}{=} -8$$

$$-4 - 21 \stackrel{?}{=} -8$$

$$-25 \neq -8$$
X

and

$$-3x + 5y = 5$$
 $-3(-1) + 5(3) \stackrel{?}{=} 5$
 $3 + 15 \stackrel{?}{=} 5$
 $18 \neq 5$

Because (-1,3) satisfies neither equation, (-1,3) is not a solution to the system of equations.

Problem 8. (10pt) Show that the following system of equations has a solution:

$$6x - 3y = 11$$

$$2x + 5y = 12$$

Solution. This a system of linear equations. The system will have a solution if and only if the lines intersect. But this will only happen if they are not parallel. We find the slopes of each line:

$$6x - 3y = 11$$

$$-3y = -6x + 11$$

$$y = 2x - \frac{11}{3}$$

$$2x + 5y = 12$$

$$5y = -2x + 12$$

$$y = -\frac{2}{5}x + \frac{12}{5}$$

The slope of the first line is $m_1=2$ while the slope of the second line is $m_2=-\frac{2}{5}$. Because $m_1\neq m_2$, the lines are not parallel. But then the lines intersect so that there is a solution to the system of equations.

Problem 9. (10pt) Solve the following system of equations and verify that your solution is valid:

$$\begin{cases} 6x + 4y = 0\\ -12x + 6y = -7 \end{cases}$$

Solution. This is a system of linear equations. Assuming that there is a solution, it can be found with substitution or elimination. If we use substitution, we can solve for y in the first equation. This yields 4y = -6x so that $y = -\frac{3}{2}x$. Using this in the second equation, we have...

$$-12x + 6y = -7$$

$$-12x + 6 \cdot -\frac{3}{2}x = -7$$

$$-12x - 9x = -7$$

$$-21x = -7$$

$$x = \frac{1}{3}$$

But then we have $y=-\frac{3}{2}\,x=-\frac{3}{2}\cdot\frac{1}{3}=-\frac{1}{2}.$ Therefore, the solution is $(\frac{1}{3},-\frac{1}{2}).$

Using elimination, suppose we eliminate x. Multiplying the first equation by 2 and adding this to the second equation, we find

$$12x + 8y = 0$$
$$-12x + 6y = -7$$
$$14y = -7$$
$$y = -\frac{1}{2}$$

Using this in the first equation, we find

$$6x + 4y = 0$$

$$6x + 4 \cdot -\frac{1}{2} = 0$$

$$6x - 2 = 0$$

$$6x = 2$$

$$x = \frac{1}{3}$$

Therefore, the solution is $(\frac{1}{3}, -\frac{1}{2})$. We verify this in each of the two equations:

$$6x + 4y = 0 -12x + 6y = -7$$

$$6 \cdot \frac{1}{3} + 4 \cdot -\frac{1}{2} \stackrel{?}{=} 0 -12 \cdot \frac{1}{3} + 6 \cdot -\frac{1}{2} \stackrel{?}{=} -7$$

$$2 - 2 \stackrel{?}{=} 0 -4 - 3 \stackrel{?}{=} -7$$

$$0 = 0 -7 = -7$$

Therefore, $(\frac{1}{3}, -\frac{1}{2})$ is the solution to the given system of equations.

Problem 10. (10pt) Solve the following system of equations and explain whether your solution is the only one possible:

$$\frac{1}{2}x + 4y = -1$$

$$\frac{1}{3}x - 5y = 7$$

Solution. This is a system of linear equations. Assuming that there is a solution, it can be found with substitution or elimination. If we use substitution, we can solve for x in the first equation. This yields $\frac{1}{2}x = -4y - 1$ so that x = -8y - 2. Using this in the second equation, we have...

$$\frac{1}{3}x - 5y = 7$$

$$\frac{1}{3} \cdot (-8y - 2) - 5y = 7$$

$$-8y - 2 - 15y = 21$$

$$-23y = 23$$

$$y = -1$$

But then we have x = -8y - 2 = -8(-1) - 2 = 8 - 2 = 6. Therefore, the solution is (6, -1).

Using elimination, suppose we eliminate x. Multiplying the first equation by 2, the second equation by -3, and adding these equations, we find

$$x + 8y = -2$$
$$-x + 15y = -21$$
$$23y = -23$$
$$y = -1$$

Using this in the second equation, we find

$$\frac{1}{3}x - 5y = 7$$

$$x - 15y = 21$$

$$x - 15(-1) = 21$$

$$x + 15 = 21$$

$$x = 6$$

Therefore, the solution is (6, -1). We verify this in each of the two equations:

$$\frac{1}{2}x + 4y = -1
\frac{1}{3}x - 5y = 7$$

$$\frac{1}{2} \cdot 6 + 4 \cdot -1 \stackrel{?}{=} -1
3 - 4 \stackrel{?}{=} -1
-1 = -1$$

$$\frac{1}{3}x - 5y = 7$$

$$\frac{1}{3} \cdot 6 - 5 \cdot -1 \stackrel{?}{=} 7$$

$$2 + 5 \stackrel{?}{=} 7$$

$$7 = 7$$

Therefore, (6, -1) is the solution to the given system of equations.