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MATH 101

Fall 2021

HW 14: Due 11/09

“Science is a way of thinking much more than it is a body of knowledge.”

–Carl Sagan

Problem 1. (10pt) Compute the following, being sure to show all your work and simplifying as much as possible:

$$\frac{5}{x-1} - \frac{4}{x+3}$$

Solution.

$$\begin{aligned}\frac{5}{x-1} - \frac{4}{x+3} &= \frac{5(x+3)}{(x-1)(x+3)} - \frac{4(x-1)}{(x-1)(x+3)} \\ &= \frac{5(x+3) - 4(x-1)}{(x-1)(x+3)} \\ &= \frac{5x + 15 - 4x + 4}{(x-1)(x+3)} \\ &= \frac{x + 19}{(x-1)(x+3)}\end{aligned}$$

Problem 2. (10pt) Compute the following, being sure to show all your work and simplifying as much as possible:

$$\frac{2}{x+1} + \frac{1}{x^2 - x - 2}$$

Solution.

$$\begin{aligned}\frac{2}{x+1} + \frac{1}{x^2 - x - 2} &= \frac{2}{x+1} + \frac{1}{(x-2)(x+1)} \\&= \frac{2(x-2)}{(x+1)(x-2)} + \frac{1}{(x-2)(x+1)} \\&= \frac{2(x-2) + 1}{(x-2)(x+1)} \\&= \frac{2x - 4 + 1}{(x-2)(x+1)} \\&= \frac{2x - 3}{(x-2)(x+1)}\end{aligned}$$

Problem 3. (10pt) Compute the following, being sure to show all your work and simplifying as much as possible:

$$\frac{x}{x+6} + \frac{x-2}{x+3}$$

Solution.

$$\begin{aligned}\frac{x}{x+6} + \frac{x-2}{x+3} &= \frac{x(x+3)}{(x+6)(x+3)} + \frac{(x-2)(x+6)}{(x+3)(x+6)} \\ &= \frac{x(x+3) + (x-2)(x+6)}{(x+3)(x+6)} \\ &= \frac{(x^2 + 3x) + (x^2 + 6x - 2x - 12)}{(x+3)(x+6)} \\ &= \frac{x^2 + 3x + x^2 + 4x - 12}{(x+3)(x+6)} \\ &= \frac{2x^2 + 7x - 12}{(x+3)(x+6)}\end{aligned}$$

Problem 4. (10pt) Compute the following, being sure to show all your work and simplifying as much as possible:

$$\frac{x-2}{x^2+x-2} - \frac{x}{x^2-6x+5}$$

Solution.

$$\begin{aligned}\frac{x-2}{x^2+x-2} - \frac{x}{x^2+6x+5} &= \frac{x-2}{(x-1)(x+2)} + \frac{x}{(x-1)(x-5)} \\&= \frac{(x-2)(x-5)}{(x-1)(x+2)(x-5)} + \frac{x(x+2)}{(x-1)(x-5)(x+2)} \\&= \frac{(x-2)(x-5) + x(x+2)}{(x-1)(x+2)(x+5)} \\&= \frac{(x^2-5x-2x+10) + (x^2+2x)}{(x-1)(x+2)(x+5)} \\&= \frac{x^2-7x+10+x^2+2x}{(x-1)(x+2)(x+5)} \\&= \frac{2x^2-5x+10}{(x-1)(x+2)(x+5)}\end{aligned}$$

Problem 5. (10pt) Compute the following, being sure to show all your work and simplifying as much as possible:

$$\frac{2}{x+3} \cdot \frac{x^2 - 2x - 15}{x-1}$$

Solution.

$$\begin{aligned} \frac{2}{x+3} \cdot \frac{x^2 - 2x - 15}{x-1} &= \frac{2}{x+3} \cdot \frac{(x-5)(x+3)}{x-1} \\ &= \frac{2}{\cancel{x+3}} \cdot \frac{(x-5)\cancel{(x+3)}}{x-1} \\ &= \frac{2(x-5)}{x-1} \end{aligned}$$

Problem 6. (10pt) Compute the following, being sure to show all your work and simplifying as much as possible:

$$\frac{x+3}{x^2-4x-12} \cdot \frac{x^2-4}{2x^2+5x-3}$$

Solution.

$$\begin{aligned} \frac{x+3}{x^2-4x-12} \cdot \frac{x^2-4}{2x^2+5x-3} &= \frac{x+3}{(x-6)(x+2)} \cdot \frac{(x-2)(x+2)}{(2x-1)(x+3)} \\ &= \frac{\cancel{x+3}}{(x-6)\cancel{(x+2)}} \cdot \frac{(x-2)\cancel{(x+2)}}{(2x-1)\cancel{(x+3)}} \\ &= \frac{x-2}{(x-6)(2x-1)} \end{aligned}$$

Problem 7. (10pt) Compute the following, being sure to show all your work and simplifying as much as possible:

$$\frac{\frac{x}{x+1}}{\frac{x^2+4x}{x^2+3x+2}}$$

Solution.

$$\begin{aligned}\frac{\frac{x}{x+1}}{\frac{x^2+4x}{x^2+3x+2}} &= \frac{x}{x+1} \cdot \frac{x^2+3x+2}{x^2+4x} \\ &= \frac{x}{x+1} \cdot \frac{(x+1)(x+2)}{x(x+4)} \\ &= \frac{\cancel{x}}{\cancel{x+1}} \cdot \frac{(\cancel{x+1})(x+2)}{\cancel{x}(x+4)} \\ &= \frac{x+2}{x+4}\end{aligned}$$

Problem 8. (10pt) Compute the following, being sure to show all your work and simplifying as much as possible:

$$\frac{\frac{x^2 + x - 6}{x^2 + 2x - 24}}{\frac{2x^2 + 9x + 9}{x^2 + x - 30}}$$

Solution.

$$\begin{aligned} \frac{\frac{x^2 + x - 6}{x^2 + 2x - 24}}{\frac{2x^2 + 9x + 9}{x^2 + x - 30}} &= \frac{x^2 + x - 6}{x^2 + 2x - 24} \cdot \frac{x^2 + x - 30}{2x^2 + 9x + 9} \\ &= \frac{(x - 2)(x + 3)}{(x - 4)(x + 6)} \cdot \frac{(x - 5)(x + 6)}{(2x + 3)(x + 3)} \\ &= \frac{(x - 2)\cancel{(x + 3)}}{(x - 4)\cancel{(x + 6)}} \cdot \frac{(x - 5)\cancel{(x + 6)}}{(2x + 3)\cancel{(x + 3)}} \\ &= \frac{(x - 2)(x - 5)}{(x - 4)(2x + 3)} \end{aligned}$$