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MATH 101
Fall 2022
HW 8: Due 10/17

"I hate Algebra."

–John H. Conway

Problem 1. (10pt) Determine whether the point $(-6, -2)$ is on the graph of $f(x) = 8 - \frac{5}{3}x$. Determine also whether the point $(12, -12)$ is on the graph of $f(x)$. For each, explain why or why not.

Solution. If $(-6, -2)$ is a point on the graph of $f(x)$, then we know that $f(-6) = -2$. But we have...

$$f(-6) = 8 - \frac{5}{3} \cdot -6 = 8 - 5(-2) = 8 + 10 = 18$$

Because $18 \neq -2$, we know that $(-6, -2)$ is not a point on the graph of $f(x)$.

If the point $(12, -12)$ is on the graph of $f(x)$, then we know that $f(12) = -12$. But we have...

$$f(12) = 8 - \frac{5}{3} \cdot 12 = 8 - 5(4) = 8 - 20 = -12$$

Therefore, $(12, -12)$ is a point on the graph of $f(x)$.

Problem 2. (10pt) Suppose $f(x)$ and $g(x)$ are the functions given below.

x	-3	-2	-1	0	1	2	3
$f(x)$	4	2	0	-5	1	2	4
$g(x)$	2	1	-1	1	-2	3	-3
$h(x)$	-12	4	10	-2	4	-4	0

Compute the following:

(a) $(f+h)(-1) = f(-1) + h(-1) = 0 + 10 = 10$

(b) $(h-g)(2) = h(2) - g(2) = -4 - 3 = -7$

(c) $(5f)(2) = 5f(2) = 5(2) = 10$

(d) $\left(\frac{h}{g}\right)(-3) = \frac{h(-3)}{g(-3)} = \frac{-12}{2} = -6$

(e) $f(0)\,h(1) = -5 \cdot 4 = -20$

(f) $g\big(2 - h(1)\big) = g(2-4) = g(-2) = 1$

(g) $(f \circ g)(-3) = f\big(g(-3)\big) = f(2) = 2$

(h) $(g \circ h)(3) = g\big(h(3)\big) = g(0) = 1$

(i) $(h \circ g)(3) = h\big(g(3)\big) = h(-3) = -12$

(j) $(f \circ g \circ h)(0) = f\Big(g\big(h(0)\big)\Big) = f\big(g(-2)\big) = f(1) = 1$

Problem 3. (10pt) Suppose $f(x)$ and $g(x)$ are the functions given below.

$$f(x) = 2 - x$$

$$g(x) = x^2 - 3x + 2$$

Compute the following:

(a) $f(-4) = 2 - (-4) = 2 + 4 = 6$

(b) $g(2) = 2^2 - 3(2) + 2 = 4 - 6 + 2 = 0$

(c) $2f(1) - g(3) = 2(1) - 2 = 2 - 2 = 0$

(d) $f(x) - g(x) = (2 - x) - (x^2 - 3x + 2) = 2 - x - x^2 + 3x - 2 = -x^2 + 2x$

(e) $f(x)\,g(x) = (2 - x)(x^2 - 3x + 2) = 2x^2 - 6x + 4 - x^3 - 3x^2 - 2x = -x^3 + 5x^2 - 8x + 4$

(f) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{2 - x}{x^2 - 3x + 2}$

(g) $(f \circ g)(0) = f\big(g(0)\big) = f(2) = 0$

(h) $(g \circ f)(0) = g\big(f(0)\big) = g(2) = 0$

(i) $(f \circ g)(x) = f\big(g(x)\big) = f(x^2 - 3x + 2) = 2 - (x^2 - 3x + 2) = 2 - x^2 + 3x - 2 = -x^2 + 3x$

(j) $(g \circ f)(x) = g\big(f(x)\big) = g(2 - x) = (2 - x)^2 - 3(2 - x) + 2 = (4 - 4x + x^2) + (-6 + 3x) + 2 = x^2 - x$

Problem 4. (10pt) Suppose $f(x)$ and $g(x)$ are functions.

(a) Explain what it means for $f(2) = g(2)$ graphically.

(b) Explain what $f(x)$ and $g(x)$ intersecting at the point $(-1, 7)$ means algebraically.

Solution.

(a) We know that $\big(x, f(x)\big)$ and $\big(x, g(x)\big)$ are points on the graph of $f(x)$ and $g(x)$, respectively. But then $\big(2, f(2)\big)$ is a point on the graph of $f(x)$ and $\big(2, g(2)\big)$ is a point on the graph of $g(x)$. But because $f(2) = g(2)$, we know that $\big(2, f(2)\big) = \big(2, g(x)\big)$. Therefore, if $f(2) = g(2)$, the graphs of $f(x)$ and $g(x)$ intersect when $x = 2$.

(b) If $f(x)$ and $g(x)$ intersect at the point $(-1, 7)$, then we know that $(-1, 7)$ is a point on the graph of $f(x)$ and $g(x)$. But then when $x = -1$, we know that $y = 7$. Therefore, $f(-1) = 7$ and $g(-1) = 7$. But then we know that $f(-1) = g(-1)$.