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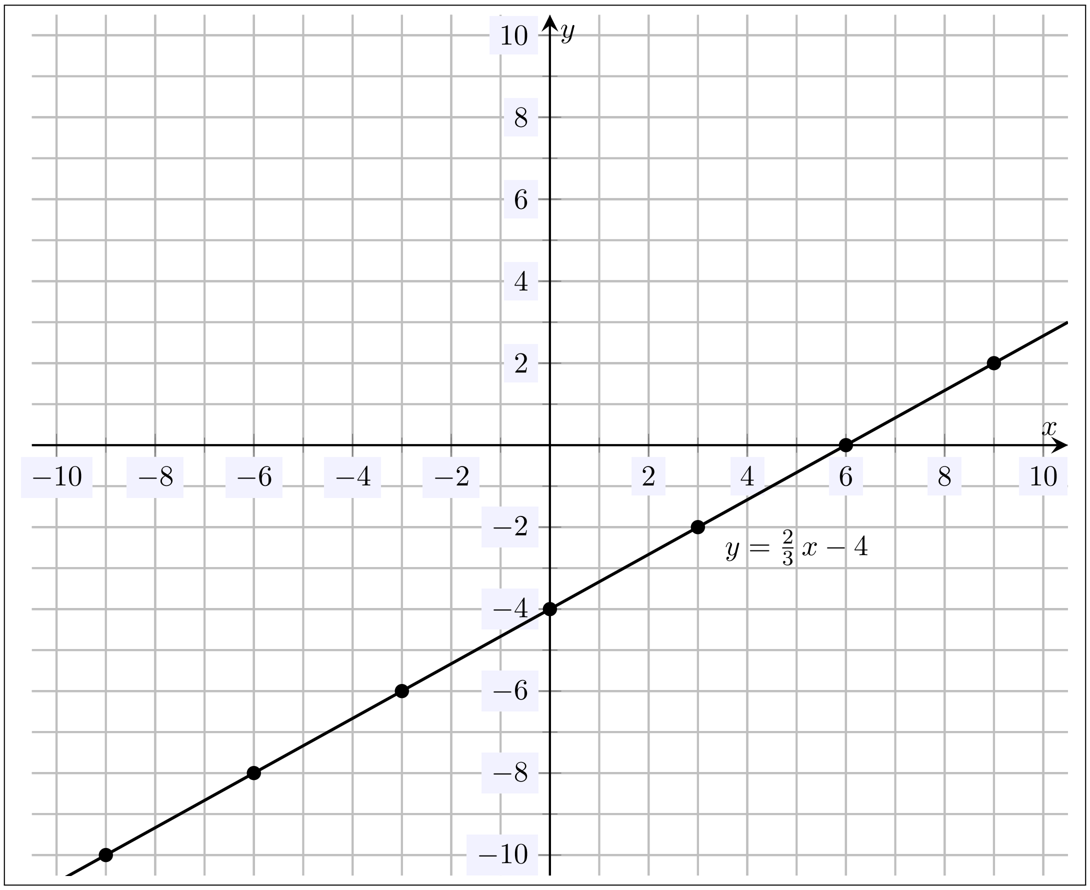
MATH 101
Spring 2022
HW 7: Due 03/03

"Today is a good day to try."
– Quasimodo, The Hunchback of Notre Dame

Problem 1. (10pt) Being as accurate as possible, sketch the graph of the line $2x - 3y = 12$.



We can solve for y in the equation $2x - 3y = 12$:

$$\begin{aligned} 2x - 3y &= 12 \\ -3y &= -2x + 12 \\ y &= \frac{2}{3}x - 4 \end{aligned}$$

This line has slope $m = \frac{2}{3}$ and y-intercept $b = -4$ (technically, $(0, -4)$). We can then use the 'slope' method of plotting: interpreting the slope $m = \frac{2}{3}$ as $\frac{\Delta y}{\Delta x}$, we see that for each increase of 3 in x results in an increase of 2 in y. Alternatively, writing $m = \frac{2}{3} = \frac{-2}{-3}$, each decrease of 3 in x results in a decrease of 2 in y. Using this, we can create a series of points to smoothly connect in the plot above.

Problem 2. (10pt) Consider the linear function $f(x) = 5 - \frac{3}{4}\,x$.

(a) Find the slope of this linear function.

(b) Interpret the slope two different ways.

(c) Is the linear function increasing, decreasing, or constant? Explain.

(d) Determine the y-intercept for $f(x)$.

(e) Determine the x-intercept for $f(x)$.

Solution.

(a) Because this is a linear function, it must be able to be put in the form $y = mx + b$, where m is the slope. Here, we have $y = f(x)$, $x = x$, $m = -\frac{3}{4}$, and $b = 5$. Therefore, the slope is $m = -\frac{3}{4}$.

(b) Interpreting the slope $m = -\frac{3}{4} = \frac{-3}{4}$ as $\frac{\Delta y}{\Delta x}$, we see that for every increase of 4 in x, there is a corresponding decrease of 3 in y. Alternatively, writing $m = -\frac{3}{4} = \frac{3}{-4}$, we see that for every decrease of 4 in x, we see a corresponding increase of 3 in y. Finally, another 'immediate' interpretation is given by writing $m = -\frac{3}{4} = -0.75 = \frac{-0.75}{1}$. Then for every increase of 1 in x, we see a corresponding decrease of 0.75 in y. Alternatively, writing $m = -\frac{3}{4} = -0.75 = \frac{0.75}{-1}$. Then for every decrease of 1 in x, we see a corresponding increase of 0.75 in y.

(c) Because $m = -\frac{3}{4} < 0$, we see that this linear function is decreasing in x.

(d) Because this is a linear function, it must be able to be put in the form $y = mx + b$, where b is the y-intercept (technically, $(0, b)$). Here, we have $y = f(x)$, $x = x$, $m = -\frac{3}{4}$, and $b = 5$. Therefore, the y-intercept is $b = 5$—technically, $(0, 5)$.

(e) The x-intercept occurs when $f(x)$ passes through the x-axis, i.e. when the output is 0. But then $f(x) = 0$. We can then solve for x:

$$\begin{aligned} f(x) &= 0 \\ 5 - \frac{3}{4}\,x &= 0 \\ \frac{3}{4}\,x &= 5 \\ \frac{4}{3} \cdot \frac{3}{4}\,x &= 5 \cdot \frac{4}{3} \\ x &= \frac{20}{3} \end{aligned}$$

Therefore, the x-intercept is $\frac{20}{3}$—technically, $(\frac{20}{3}, 0)$.

Problem 3. (10pt) Showing all your work, find the equation of the line perpendicular to $y = 5 - 3x$ that passes through the point $(1, -4)$.

Solution. Because the line is perpendicular to the line $y = 5 - 3x$—which is 'sloped', we know that the line is not vertical. Therefore, the line must have the form $y = mx + b$. Because the line is perpendicular to the line $y = 5 - 3x$, the slope of the line must be the negative reciprocal of the slope of the line $y = 5 - 3x$. The slope of the line $y = 5 - 3x$ is -3. The negative reciprocal of this is $-(\frac{1}{-3}) = \frac{1}{3}$. Therefore, we have $m = \frac{1}{3}$ so that $y = \frac{1}{3}x + b$. But we know that the line passes through the point $(1, -4)$, i.e. when $x = 1$, we know $y = -4$. But then...

$$\begin{aligned}
y &= \frac{1}{3}\,x + b \\
-4 &= \frac{1}{3} \cdot 1 + b \\
-4 &= \frac{1}{3} + b \\
b &= -4 - \frac{1}{3} \\
b &= -\frac{12}{3} - \frac{1}{3} \\
b &= -\frac{13}{3}
\end{aligned}$$

Therefore, the equation of the line is $y = \frac{1}{3}x - \frac{13}{3}$.

$$\boxed{y = \frac{1}{3}x - \frac{13}{3}}$$

Problem 4. (10pt) Showing all your work, solve the following linear equation, be sure to verify that your solution satisfies the equation:

$$5x - 6 = 1 - 7x$$

Solution.

$$\begin{aligned} 5x - 6 &= 1 - 7x \\ 12x - 6 &= 1 \\ 12x &= 7 \\ x &= \frac{7}{12} \end{aligned}$$

We can check this solution:

$$\begin{aligned} 5x - 6 &= 1 - 7x \\ 5 \cdot \frac{7}{12} - 6 &\stackrel{?}{=} 1 - 7\frac{7}{12} \\ \frac{35}{12} - 6 &\stackrel{?}{=} 1 - \frac{49}{12} \\ \frac{35}{12} - \frac{72}{12} &\stackrel{?}{=} \frac{12}{12} - \frac{49}{12} \\ -\frac{37}{12} &= -\frac{37}{12} \\ &\checkmark \end{aligned}$$

Problem 5. (10pt) Water is flowing into a 'rectangular' box with side lengths 2 ft, 4 ft, and 5 ft at a rate of 3.4 ft^3/min. Currently, the box contains 16 ft^3 of water. Let $W(t)$ denote the amount of water in the box t minutes from now.

(a) Explain why $W(t)$ is linear.

(b) Find $W(t)$.

(c) What do the slope and y-intercept of $W(t)$ represent in context?

(d) Determine when the box will begin to overflow.

Solution.

(a) The water is flowing into the container at a constant rate. Because the rate of change is constant, we know that $W(t)$ must be linear.

(b) Clearly, $W(t)$ is not a vertical line. Therefore, $W(t)$ must have the form $W(t) = mt + b$. We know at time $t = 0$ that the box contains 16 ft^3 of water. But then $16 = W(0) = m(0) + b = 0 + b = b$ so that $b = 0$. We know also that the water is flowing into the box at a rate of 3.4 ft^3 per minute. But then we know that $m = 3.4$ so that $W(t) = 3.4t + 16$.

(c) We have $W(t) = 3.4t + 16$. The slope is $m = 3.4$, which is the rate of flow of water into the box. The y-intercept is 16 (properly, $(0, 16)$), which corresponds to the fact that at the 'start' (the initial time) the box contains 16 ft^3 of water.

(d) The box has volume $V = \ell wh = 2 \cdot 4 \cdot 5 = 40$ ft^3. The box will overflow once the amount of water, $W(t)$, is 40 ft^3. But then we have...

$$\begin{aligned} W(t) &= 40 \\ 3.4t + 16 &= 40 \\ 3.4t &= 24 \\ t &= 7.059 \text{ min} \end{aligned}$$

Therefore, the box will begin to overflow after 7.059 minutes, i.e. 7 minutes and 3.54 seconds.