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MATH 108

Spring 2023

HW 8: Due 03/06

*“Laura, clear out the rest of my day! I have to push a boulder up a hill and then have it roll over me time and time again with no regard for my well-being.”*

*–Princess Carolyn, BoJack Horseman*

**Problem 1.** (10pt) Kelsey is gambling at a casino. She is playing a game where you roll two die. If you roll two 6's, you win \$100. If you the dice and the numbers on both die are four or greater (but not two 6's), you win \$10. If the numbers on both die are less than 3, you lose \$8. Otherwise, you win nothing. You must pay \$5 as a 'buy-in' each round to play. Find the amount that you win/lose 'on average.' Should one play this game?

**Solution.** Rolling two die, there are  $6 \cdot 6 = 36$  total outcomes for the values on the two die. There is only one way to roll two 6's—by rolling two 6's. If you want to roll a four or bigger on each die, there are 3 possibilities for each die (4, 5, or 6) so that there are  $3 \cdot 3 = 9$  such combinations. But this includes two 6's on each die. Therefore, there are  $9 - 1 = 8$  total ways to roll the dice and have a four or greater on each die. If a number on a die is less than 3, there are 2 possibilities (1 or 2). Therefore, there are  $2 \cdot 2 = 4$  total combinations where both numbers on the die are less than 3. But then we have...

$$P(\text{two 6's}) = \frac{1}{36}$$

$$P(\text{Four or Great (Not Boxcars)}) = \frac{8}{36} = \frac{2}{9}$$

$$P(\text{Less than 3}) = \frac{4}{36} = \frac{1}{9}$$

$$P(\text{Anything Else}) = 1 - \frac{1}{36} - \frac{8}{36} - \frac{4}{36} = \frac{23}{36}$$

We can create a table of the possible outcomes, their probability, and their net expected payout (that is, the payout minus the \$5 fee to play). We have...

Outcome	Two 6's	Four or Greater (Not Boxcars)	Less than 3	Anything Else
Probability	$\frac{1}{36}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{23}{36}$
Value	\$95	\$5	-\$13	-\$5

We can then compute the expected payout:

$$EX = \sum xP(x) = \$95 \cdot \frac{1}{36} + \$5 \cdot \frac{2}{9} - \$13 \cdot \frac{1}{9} - \$5 \cdot \frac{23}{36} \approx \$2.639 + \$1.111 - \$1.444 - \$3.194 = -\$0.888 \approx -\$0.89$$

Because the expected value is negative, on average, you lose \$0.89 each game by playing this game. Therefore, you should not play this game.

**Problem 2.** (10pt) Find the least square regression line for the points:  $(1, 3), (3, 5), (1, 2), (2, 2)$ . Show all your work.

**Solution.** First, observe that we have  $n = 4$  points. Examining the  $x$  and  $y$  values, we have...

$$\bar{x} = \frac{1 + 3 + 1 + 2}{4} = \frac{7}{4} \approx 1.75$$

$$\bar{y} = \frac{3 + 5 + 2 + 2}{4} = \frac{12}{4} = 3$$

But then we have...

$x_i$	$y_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$y_i - \bar{y}$	$(y_i - \bar{y})^2$	$(x_i - \bar{x})(y_i - \bar{y})$
1	3	-0.75	0.5625	0	0	0
3	5	1.25	1.5625	2	4	2.5
1	2	-0.75	0.5625	-1	1	0.75
2	2	0.25	0.0625	-1	1	-0.25
Total:		2.75	Total:	6	3	

But then we have...

$$s_x^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2 = \frac{1}{3} \cdot 2.75 = 0.916667 \implies s_x = \sqrt{0.916667} = 0.957427$$

$$s_y^2 = \frac{1}{n-1} \sum (y_i - \bar{y})^2 = \frac{1}{3} \cdot 6 = 2 \implies s_y = \sqrt{2} = 1.41421$$

$$r = \frac{1}{n-1} \sum \left( \frac{x_i - \bar{x}}{s_x} \right) \left( \frac{y_i - \bar{y}}{s_y} \right) = \frac{1}{n-1} \cdot \frac{1}{s_x} \cdot \frac{1}{s_y} \sum (x_i - \bar{x})(y_i - \bar{y}) = \frac{1}{3} \cdot \frac{1}{0.957427} \cdot \frac{1}{1.41421} \cdot 3 = 0.738551$$

This allows us to compute the coefficients for our model:

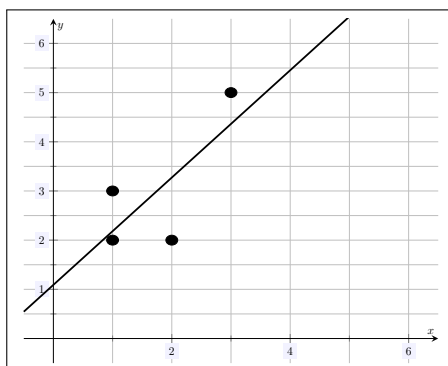
$$b_1 = r \frac{s_y}{s_x} = 0.738551 \cdot \frac{1.41421}{0.957427} = 1.09091$$

$$b_0 = \bar{y} - b_1 \bar{x} = 3 - 1.09091(1.75) = 1.09091$$

Therefore, the least square regression line is...

$$\hat{y} = b_1 x + b_0 = 1.09091x + 1.09091$$

This linear regression has  $r^2$  value 0.545458 and is shown with the data points below:



**Problem 3.** (10pt) Given the following information below, find the least square regression line. Show all your work.

$$n = 200$$

$$\bar{x} = 4.42726, \quad \sigma_x^2 = 10.6639$$

$$\bar{y} = 46.5248, \quad \sigma_y^2 = 1053.77$$

$$R = 0.962639$$

**Solution.** We have...

$$\sigma_x^2 = 10.6639 \implies \sigma_x = \sqrt{10.6639} = 3.26556$$

$$\sigma_y^2 = 1053.77 \implies \sigma_y = \sqrt{1053.77} = 32.4618$$

But then we can compute the model coefficients:

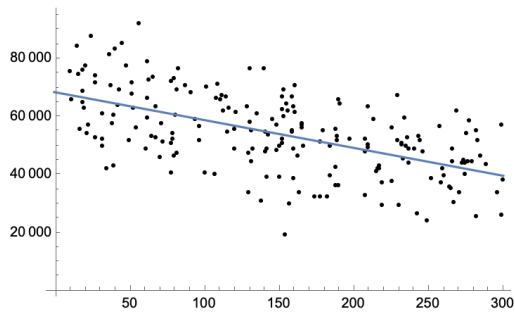
$$b_1 = r \frac{\sigma_y}{\sigma_x} = 0.962639 \cdot \frac{32.4618}{3.26556} = 9.56926$$

$$b_0 = \bar{y} - b_1 \bar{x} = 46.5248 - 9.56926(4.42726) = 4.1592$$

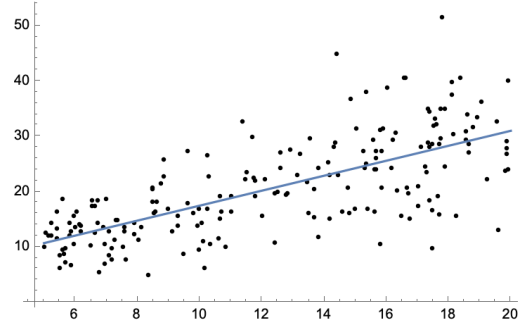
Therefore, the least square regression line is...

$$\hat{y} = b_1 x + b_0 = 9.56926x + 4.1592$$

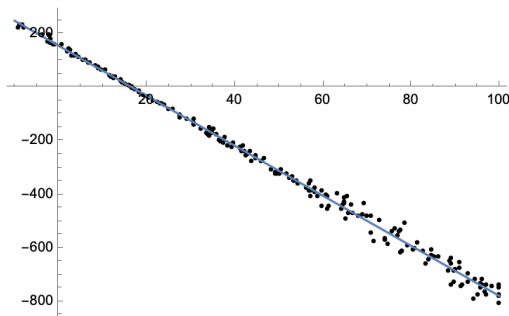
**Problem 4.** (10pt) Match each regression coefficient to its corresponding graph.



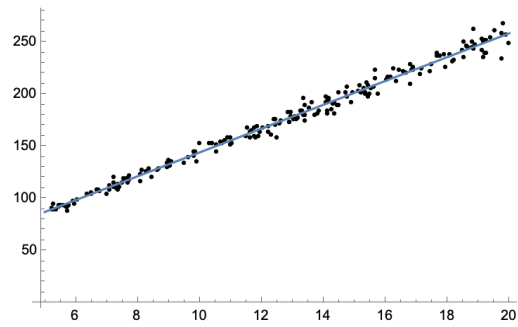
(a)



(b)



(c)



(d)

- (i) (b) :  $R = 0.836288$
- (ii) (c) :  $R = -0.998836$
- (iii) (d) :  $R = 0.997066$
- (iv) (a) :  $R = -0.759531$