Name: *Caleb McWhorter — Solutions*
MATH 100
Fall 2022
HW 6: Due 10/03

"If you fish can catch nothing, you have still caught a lesson."
–Matshona Dhliwayo

Problem 1. (10pt) You are standing at the corner of 6th and 40th while your friend is standing at the corner of 2nd and 59th.

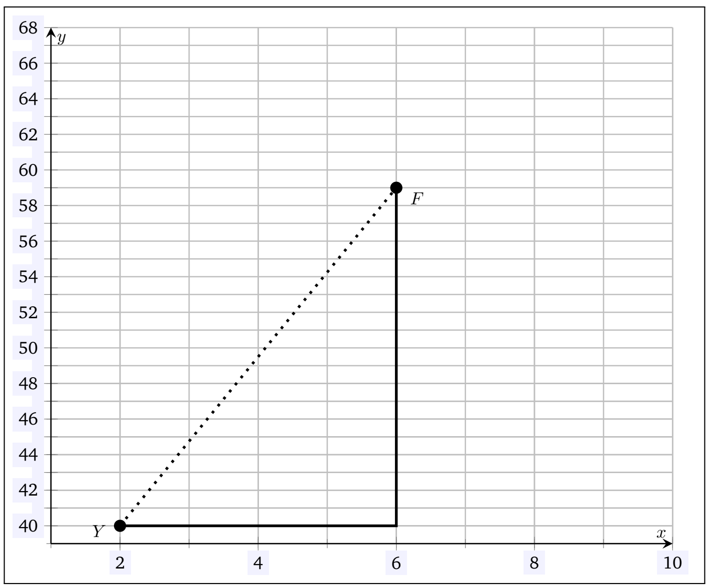
(a) How many blocks are you from your friend?

(b) How many blocks are you from your friend 'as the crow flies'?

(c) What is your Euclidean distance between you and your friend?

(d) What is your Manhattan distance between you and your friend?

Solution. We can draw a picture of this to aid in the solutions:



(a) Whether we go 6 blocks East and then 19 blocks North, 19 blocks North then 6 blocks East or any combination 'in-between', you must travel $4 + 19 = 23$ total blocks; that is, you must travel $(6 - 2) + (59 - 40) = 4 + 19 = 23$ blocks

(b) This is the 'straight line distance (the dotted line above). This is the length of the hypotenuse of a right angled triangle with legs 4 and 19. But then the distance is $\sqrt{4^2 + 19^2} = \sqrt{16 + 361} = \sqrt{377} \approx 19.4165$ blocks.

(c) This is the straight line distance that we computed in (b). Being more formal, we have...

$$d = \sqrt{(6-2)^2 + (59-40)^2} = \sqrt{4^2 + 19^2} = \sqrt{16 + 361} = \sqrt{377} \approx 19.4165 \text{ blocks}$$

(d) This is the 'block' distance we computed in (a). Being more formal, we have...

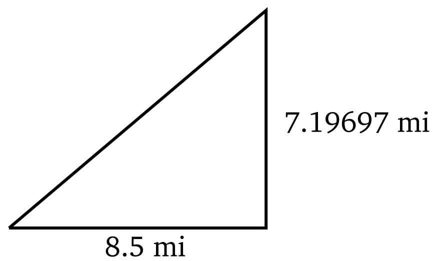
$$|6 - 2| + |59 - 40| = |4| + |19| = 4 + 19 = 23 \text{ blocks}$$

Problem 2. (10pt) You see a plane at cruising altitude (approximately 38,000 ft) flying over a nearby airport. You know that the airport is 8.5 mi away. How far is the plane from you at this moment? [5,280 ft = 1 mi]

Solution. First, we compute the cruising altitude to miles:

$$\begin{array}{r|c} 38000\text{ ft} & 1\text{ mi} \\ \hline & 5280\text{ ft} \end{array} = 7.19697\text{ mi}$$

Drawing a picture, we can see the distance is the hypotenuse of a right angled triangle. We can then use the Pythagorean Theorem.



$$\sqrt{(8.5\text{ mi})^2 + (7.19697\text{ mi})^2} = \sqrt{72.25\text{ mi}^2 + 51.7964\text{ mi}^2} = \sqrt{124.046\text{ mi}^2} \approx 11.1376\text{ mi}$$

Problem 3. (10pt) Suppose you are designing a custom fish tank. The fish tank will be a rectangular box will be 36 in x 18 in x 19 in. The top will be open so that the tank can be accessed.

(a) If you have to cover the exposed sides with a special plastic coating, what is the area of plastic coating that is needed per tank?

(b) How much water, in gallons, can the tank hold? [1 ft^3 = 7.48052 gallons]

(c) Suppose you are going to put a volcano in the tank, which will have the shape of a right circular cone. If you put the largest possible volcano in the tank, how much water will it then take to fill the tank?

Solution.

(a) This is a rectangular prism. However, because the top is open, it does not contribute to the area because there is no surface there. Moreover, because the bottom of the tank sits on a table, it also does not need to be covered. Therefore, the exposed area (surface area) is. . .

$$2lh + 2wh + 0lw = 2(36 \text{ in})(19 \text{ in}) + 2(18 \text{ in})(19 \text{ in}) = 1368 \text{ in}^2 + 684 \text{ in} = 2052 \text{ in}^2$$

(b) This is the volume of the tank. This is $V = lwh = (36 \text{ in})(18 \text{ in})(19 \text{ in}) = 12312 \text{ in}^3$. We now need to convert this to gallons:

$$\begin{array}{r|r|r|r|r} 12312 \text{ in}^3 & 1 \text{ ft} & 1 \text{ ft} & 1 \text{ ft} & 7.48052 \text{ gallons} \\ \hline & 12 \text{ in} & 12 \text{ in} & 12 \text{ in} & 1 \text{ ft}^3 \end{array} = 53.2987 \text{ gallons}$$

(c) Because the base of the volcano has to fit in the width or length of the tank, it can have a diameter of at most 18 in (the smaller of 36 in and 18 in). This is the diameter of the circular base. Therefore, it has a radius of $18/2 = 9$ in. The largest height of the tank is 19 in. Therefore, the volume of the largest possible volcano is $V = \frac{\pi}{3} r^2 h = \frac{\pi}{3} \cdot (9 \text{ in})^2 \cdot 19 \text{ in} \approx 1611.64 \text{ in}^3$. Because the volcano will take up this volume, the remaining volume of the tank can be filled with water. This volume is $12312 \text{ in}^3 - 1611.64 \text{ in}^3 = 10700.4 \text{ in}^3$. In gallons, this is. . .

$$\begin{array}{r|r|r|r|r} 10700.4 \text{ in}^3 & 1 \text{ ft} & 1 \text{ ft} & 1 \text{ ft} & 7.48052 \text{ gallons} \\ \hline & 12 \text{ in} & 12 \text{ in} & 12 \text{ in} & 1 \text{ ft}^3 \end{array} = 46.3221 \text{ gallons}$$

Problem 4. (10pt) Suppose a courtyard at some hedge fund is approximately elliptical in shape. It is approximately 50 ft 'the long way' and 35 ft 'the short way.'

(a) What is the area of the courtyard?

(b) If someone can push mow approximately 320 square foot of lawn per minute, how long does it take to mow the courtyard?

(c) If every 1,000 square foot has to be fertilized with 10 lbs of fertilizer and the fertilizer costs \$2.08 per pound, how much does it cost to fertilize the courtyard?

Solution.

(a) Because the area is elliptical, we know that $A = \pi ab$, where a, b are the length of the semimajor/semiminor axis, respectively. Because the courtyard is 50 ft 'the long way', the semimajor axis is $50\text{ ft}/2 = 25\text{ ft}$. Because the courtyard is 35 ft 'the short way', the semiminor axis is $35\text{ ft}/2 = 17.5\text{ ft}$. Therefore, the area of the courtyard is $A = \pi ab = \pi(25\text{ ft})(17.5\text{ ft}) = 437.5\pi\text{ ft}^2 \approx 1374.4468\text{ ft}^2$.

(b) We know net change, C, is related to a constant rate, r, and time, t, via $C = rt$. We know that the net change of mowed lawn we want is approximately 1374.4468 ft^2. The rate at which the lawn can be mowed is 320 ft^2/min. But then we have...

$$C = rt$$

$$1374.4468\text{ ft}^2 = (320\text{ ft}^2/\text{min})t$$

$$t \approx 4.295\text{ min}$$

(c) We know that the courtyard is approximately 1374.4468 ft^2. Each 1,000 square foot must contain 10 lbs of fertilizer. But then the amount of pounds of fertilizer required per square foot is $10\text{ lb}/1000\text{ ft}^2 = 0.01\text{ lb}/\text{ft}^2$. Using $C = rt$, we know that the courtyard requires approximately $1374.4468\text{ ft}^2 \cdot 0.01\text{ lb}/\text{ft}^2 = 13.744468$ lb of fertilizer. Each pound of fertilizer costs \$2.08. Therefore, again using $C = rt$, the total cost of 13.744468 lb of fertilizer is $13.744468\text{ lb} \cdot \$2.08/\text{lb} \approx \$28.59$.