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MATH 101
Spring 2022
HW 11: Due 04/12

"When you are dissatisfied and would like to go back to youth, think of Algebra"

— Will Rogers

Problem 1. (10pt) Determine whether $(x, y) = (5, -2)$ is a solution to the following system of equations:

$$2x + 6y = -2$$
$$3x - 2y = 11$$

Solution. The point $(5, -2)$ is a solution to the system of equations if and only if it satisfies both of the equations. We check this:

$$2x + 6y = -2$$
$$2(5) + 6(-2) \stackrel{?}{=} -2$$
$$10 - 12 \stackrel{?}{=} -2$$
$$-2 = -2$$
$$\checkmark$$

and

$$3x - 2y = 11$$
$$3(5) - 2(-2) \stackrel{?}{=} 11$$
$$15 + 4 \stackrel{?}{=} 11$$
$$19 = 11$$
$$\text{✗}$$

Therefore, $(5, -2)$ is *not* a solution to the system of equations.

Problem 2. (10pt) Use substitution to solve the following system of equations:

$$3x - 5y = -29$$
$$2x - y = -10$$

Solution. We solve for y in the second equation:

$$2x - y = -10$$

$$-y = -2x - 10$$

$$y = 2x + 10$$

Using this in the first equation, we have...

$$3x - 5y = -29$$

$$3x - 5(2x + 10) = -29$$

$$3x - 10x - 50 = -29$$

$$-7x = 21$$

$$x = -3$$

But then $y = 2x + 10 = 2(-3) + 10 = -6 + 10 = 4$. Therefore, the solution is $(x, y) = (-3, 4)$.

Problem 3. (10pt) Use elimination to solve the following system of equations:

$$5x + 6y = 4$$
$$4x - 3y = 11$$

Solution. Multiplying the second equation by 2, we have. . .

$$5x + 6y = 4$$
$$8x - 6y = 22$$

Adding these equations, we have. . .

$$13x = 26$$

$$x = 2$$

Using this in the first equation, we have. . .

$$5x + 6y = 4$$

$$5(2) + 6y = 4$$

$$10 + 6y = 4$$

$$6y = -6$$

$$y = -1$$

Therefore, we have solution $(x, y) = (2, -1)$.

Problem 4. (10pt) Determine whether the following system of equations has a solution:

$$-2x + 3y = 5$$
$$4x - 6y = -30$$

Solution. The system of linear equations has a solution if and only if the lines are not parallel. We solve for y in both equations to determine the slopes. We have

$$-2x + 3y = 5$$

$$3y = 2x + 5$$

$$y = \frac{2}{3}x + \frac{5}{3}$$

This line has slope $m_1 = \frac{2}{3}$. We also have...

$$4x - 6y = -30$$

$$-6y = -4x - 30$$

$$y = \frac{2}{3}x + 5$$

This lines has slope $m_2 = \frac{2}{3}$. Because $m_1 = m_2$, the lines are parallel. [Observe also that the lines are distinct.] Therefore, the system of equations has no solution.