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**MATH 308**
**Fall 2023**
**HW 1: Due 09/07**

*"And I knew exactly what to do... but in a much more real sense, I had no idea what to do."*
*–Michael Scott, The Office*

**Problem 1.** (10pt) Determine if each of the following are propositions. If the example is a proposition, state its truth value with a brief justification. If the example is *not* a proposition, briefly explain why:

(a) $3^2 - 15 = 6$

(b) The statement in (c) is false.

(c) George Orwell wrote *A Remembrance of Things Past*.

(d) There is intelligent life in the universe.

(e) $x - 3 \leq 10$

**Solution.**

(a) This is a proposition—either $3^2 - 15 = 6$ or $3^3 - 15 \neq 6$. In fact, because $3^2 - 15 = 9 - 15 = -6 \neq 6$, the proposition is false.

(b) This is a proposition. So long as (c) is a proposition, then the statement in (c) is either true or false. But this will mean the statement in (b) is either false or true, respectively. Because (c) is a proposition, we know that the statement in (b) is a proposition. Because (c) is false, the statement in (b) is true.

(c) This is a proposition—either George Orwell wrote *A Remembrance of Things Past* or he did not. In fact, *A Remembrance of Things Past* was written by Marcel Proust. Therefore, the proposition is false.

(d) This is a proposition—either there is 'intelligent' life in the universe or there is not. If we consider human life to be 'intelligent' life,[1] then the proposition is true.

(e) This is *not* a proposition. There is no definite truth value as the veracity of the inequality depends on the value of $x$. For instance, if $x = -6$, then $x - 3 = -6 - 3 = -9 \leq 10$, so that the expression is true. However, if $x = 20$, then $x - 3 = 20 - 3 = 17 \not\leq 10$, so that the expression is false.

[1] There are strong arguments to suggest otherwise...

**Problem 2.** (10pt) For each of the following, either define appropriate primitive propositions (using $P$, $Q$, $R$, etc.) and write the 'statement' using logical connectives, or give an English sentence for the given primitives and 'translate' the logical 'sentence' into an English sentence:

(a) $P \to (\neg Q \vee R)$

(b) You will succeed, if you believe and work hard.

(c) $Q \wedge (\neg P \vee Q)$

(d) I pay rent, or I lose my job and starve.

**Solution.**

(a) There are many possible solutions, depending on the choices for the propositions $P$, $Q$, and $R$. For instance,

(b)

(c)

(d)

**Problem 3.** (10pt) Consider the following compound statement: $\neg(P \to \neg Q) \wedge \neg Q$

(a) Determine whether the given compound statement is a tautology, contradiction, or neither. Be sure to justify your response.

(b) Using a truth table, show that the first part of the given compound statement, i.e. $\neg(P \to \neg Q)$, is logically equivalent to $P \wedge Q$.

(c) By 'simplifying' the expression $\neg\big(P \vee \neg(P \wedge Q)\big)$, show that this compound statement is logically equivalent to the compound statement given at the start of the problem.

**Solution.**

(a)

(b)

(c)

**Problem 4.** (10pt) Fix a real number $x$. Consider the statement, "if $x^2 > 4$, then $x > 2$'

(a) Determine the truth value of this statement with an explanation.

(b) Rewrite the given statement by defining appropriate primitive propositions and logical connectives.

(c) Find the negation, converse, and contrapositive of your result from (b).

(d) Rewrite your answers from (c) as English sentences. Then determine the truth value, with explanation, of each of the statements.

**Solution.**

(a)

(b)

(c)

(d)