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**MATH 100**
**Fall 2022**
**HW 8: Due 10/17**

*"Every single line means something."*
*–Jean-Michel Basquiat*

**Problem 1.** (10pt) Find the equation of the line with slope $-\frac{2}{3}$ that passes through the point $(-9, 10)$.

**Solution.** Because this is not a vertical line, we know that the line has the form $y = mx + b$ for some $m$ and $b$, where $m$ is the slope and $b$ is the $y$-intercept. We know that the slope, $m$, is $m = -\frac{2}{3}$. Therefore, we know that $y = -\frac{2}{3}\,x + b$. However, the line contains the point $(-9, 10)$, i.e. when $x = -9$, we know that $y = 10$. But then we have…

$$y = -\frac{2}{3}\,x + b$$

$$10 = -\frac{2}{3} \cdot -9 + b$$

$$10 = 6 + b$$

$$b = 4$$

Therefore, we know that…

$$y = -\frac{2}{3}\,x + 4$$

**Problem 2.** (10pt) Find the equation of the line passing through the points $(-5, 8)$ and $(7, 8)$.

**Solution.** Clearly, this is not a vertical line. Therefore, we know that the line has the form $y = mx + b$ for some $m$ and $b$, where $m$ is the slope and $b$ is the $y$-intercept. We know that...

$$m = \frac{\Delta y}{\Delta x} = \frac{8 - 8}{-5 - 7} = \frac{0}{-12} = 0$$

Then we know that $y = 0 \cdot x + b = b$. But because $(-5, 8)$ is on the line, we know that when $x = -5$, we have $y = 8$. Using this in $y = b$, we have $8 = b$. Therefore, we have...

$$y = 8$$

**Problem 3.** (10pt) Let $\ell(x) = 18.2 - 13.7x$. Find the slope and $y$-intercept of this function.

**Solution.** If $f(x)$ is a linear function, it has the form $f(x) = mx + b$, where $m$ is the slope and the $y$-intercept is $b$. Writing $\ell(x)$ in this form, we have $\ell(x) = -13.7x + 18.2$. Therefore, the slope is $m = -13.7$ and the $y$-intercept is $b = 18.2$ (or more generally, $(0, 18.2)$).

**Problem 4.** (10pt) Let $\ell(x) = 57.6x - 1654.8$. Explain why $\ell(x)$ is a linear function. Find the $y$-intercept and $x$-intercept of this function.

**Solution.** We know any function of the form $f(x) = mx + b$ is a linear function. Writing $\ell(x)$ in this form, we have $\ell(x) = 57.6x + (-1654.8)$. Therefore, $\ell(x)$ has the form $f(x) = mx + b$ with $m = 57.6$ and $b = -1654.8$.

We now find the $y$-intercept. We know the $y$-intercept occurs when the input, $x$, is 0. But we have...

$$\ell(0) = 57.6 \cdot 0 - 1654.8 = -1654.8$$

Therefore, the $y$-intercept is $-1654.8$ (or more generally, $(0, -1654.8)$).

We now find the $x$-intercept. We know the $x$-intercept occurs when the output, $\ell(x)$, is 0. But then we have...

$$\begin{aligned} \ell(x) &= 57.6x - 1654.8 \\ 0 &= 57.6x - 1654.8 \\ 57.6x &= 1654.8 \\ x &\approx 28.7292 \end{aligned}$$

Therefore, the $x$-intercept is $28.7292$ (or more generally, $(28.7292, 0)$).

**Problem 5.** (10pt) Suppose you work an hourly job where you are paid \$17.50 an hour. You have already made \$288.75 this week. Let $W$ represent the wages you have been paid by working an addition $h$ hours this week.

(a) Explain why $W$ is a linear function of $h$.

(b) Explain why $W(h) = 17.50h + 288.75$.

(c) What is the slope and what does it represent?

(d) What is the $y$-intercept and what does it represent?

**Solution.**

(a) The amount of money that you have only changes because you are working. Because you are paid a constant rate of \$17.50/hour, the rate at which your net money changes is constant. But a function with a constant rate of change is a linear function. Therefore, $W$ is a linear function of $h$.

(b) We know that the rate of change of $W$ is \$17.50/hour. So after working $h$ hours, you have added $\$17.50h$ to your account. Because you started with \$288.75, the total amount you have after working $h$ hours is then $\$17.50h + 288.75$. Therefore, $W(h) = 17.50h + 288.75$.

Alternatively, because $W(h)$ is linear, we know that $W(h) = mh + b$ for some $m, b$. We know that the rate of change of $W(h)$ is as a result of your hourly pay. Therefore, $m = 17.50$ so that $W(h) = 17.50h + b$. We know after working zero hours, you have \$288.75. But then we know that $288.75 = W(0) = 17.50(0) + b = b$. Therefore, $W(h) = 17.50h + 288.75$.

(c) The slope of a linear function is its rate of change. We know the rate of change of your money is a result of your hourly pay. Therefore, the slope represents your hourly pay.

(d) The $y$-intercept is $W(0) = 17.50(0) + 288.50 = 288.50$. This is the amount of money you initially have. Therefore, the $y$-intercept represents the initial \$288.75 you begin with.

**Problem 6.** (10pt) Let $M$ represent the total amount of money in your account $d$ days from now. Suppose that right now you have $15,000 in your account and that you spend $530 a day.

(a) Find $M(d)$.

(b) What are the slope and $y$-intercept of $M(d)$? What do they represent?

(c) Find the $x$-intercept of $M(d)$.

(d) Interpret your answer in (c).

**Solution.**

(a) Because your money only increases/decreases as a result of your spending and you are spending money at a constant rate, we know that $M(d)$ is a linear function. Therefore, we know that $M(d) = md + b$ for some $m, b$. Because you are spending $530 per day, we know that $m = -530$. Then we know that $M(d) = -530d + b$. We know you initially have $15,000. But then $15000 = M(0) = -530(0) + b = b$. Therefore, $M(d) = -530d + 15000$.

(b) The slope of $M(d) = -530d + 15000$ is $-530$. This is the rate of change of $M$. This represents the amount you spend per day. The $y$-intercept of $M(d)$ is $M(0) = -530(0) + 15000 = 15000$. This is the amount of money that you have on day zero, i.e. the amount of money you initially have.

(c) The $x$-intercept is the value(s) where the output is 0, i.e. the values of $d$ such that $M(d) = 0$. But then we have...

$$
\begin{aligned}
M(d) &= -530d + 15000 \\
0 &= -530d + 15000 \\
530d &= 15000 \\
d &= 28.3019
\end{aligned}
$$

Therefore, the $x$-intercept is 28.3019 days, i.e. $(0, 28.3019)$.

(d) The $x$-intercept is when the output is zero, i.e. $M(d) = 0$. But then the amount of money that you have is $0. Therefore, the $x$-intercept implies that you run out of money after 28.3 days.