Quiz 1. *True/False*: There must be a solution to the equation $x\left(e^{2x}-2\right)=15$.

Solution. The statement is *true*. The equation $x\left(e^{2x}-2\right)=15$ has a solution if and only if the equation $x\left(e^{2x}-2\right)-15=0$ has a solution. But the equation $x\left(e^{2x}-2\right)-15=0$ has a solution if and only if the function $f(x)=x\left(e^{2x}-2\right)-15$ has a root. The function $f(x)$ is a continuous function—being the composition, sum/difference, and product of continuous functions. We know that $f(0)=0(1-2)-15=-15<0$. Furthermore, there are clearly values for which $f(x)$ is positive because $\lim_{x\to\infty} f(x)=\infty$. But then by the Intermediate Value Theorem, there must be a value $x_0\in[-15,\infty)$ such that $f(x_0)=0$. This proves that $f(x)$ has a root; equivalently, that $x\left(e^{2x}-2\right)=15$ has a solution.