Name: _____

MATH 308
Fall 2021
HW 15: Due 11/22

"It is impossible to be a mathematician without being a poet in soul."
–Sofia Kovalevskaya

Problem 1. (10pt) Perform the following computations modulo 3:

(a) $1234 + 2345$

(b) $1784 \cdot 2021$

(c) 1996^{1997}

(d) 2^{2000}

Problem 2. (10pt) Prove that an integer N is divisible by 3 if and only if its the sum of its digits is divisible by 3.

Problem 3. (10pt) Prove that for all $n, m \in \mathbb{Z}_{\geq 0}$ that $101^n - 77^m$ is divisible by 4.

Problem 4. (10pt) Find the ones digit of 2^{98} and the tens digit of 7^{100}.

Problem 5. (10pt) For the following congruences, find a solution or explains why none exists.

(a) $2x \equiv 3 \mod 7$

(b) $6x \equiv 5 \mod 8$

(c) $4x \equiv 8 \mod 22$

Problem 6. (10pt) Use the Chinese Remainder Theorem to find the solutions modulo 60 to...

$$\begin{aligned} x &\equiv 3 \mod 4 \\ x &\equiv 2 \mod 3 \\ x &\equiv 4 \mod 5 \end{aligned}$$