Name: ______________________________

MATH 308
Fall 2022
HW 3: Due 09/15

"If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is."

–George Polya

Problem 1. (10pt) Let the universe, $\mathcal{U}$, for m, n, j, k be the set of integers. Define the following predicates:

$$\begin{aligned} P(m)&: m \text{ is even} \\ Q(n)&: n \text{ is a perfect square} \\ R(j)&: j \text{ is divisible by 3} \\ S(k)&: k \text{ is divisible by 6} \\ W(\ell)&: 1 < \ell \leq 8 \end{aligned}$$

Write the open sentences below as complete English sentences as 'simply' as possible and then determine whether the statement is true or false. If the statement is true, explain why. If not, give a counterexample.

(a) $(\exists! n)\big(Q(n) \wedge W(n)\big)$

(b) $(\forall m)\big(P(m) \vee R(m)\big)$

(c) $(\forall m)\big(\neg R(m) \rightarrow \neg S(m)\big)$

(d) $(\forall m)(\exists n)\big(S(m) \rightarrow [(m = 6n) \wedge P(n)]\big)$

Problem 2. (10pt) By defining appropriate universes and predicates, quantify the open sentences below. Indicate whether the resulting statement is true or false. No justification is necessary.

(a) For all integers m, there exists an integer n such that $m = n + 1$.

(b) For all integers n, if n is divisible by 5 then the 1's digit of n is either 0 or 5.

(c) Nonzero real numbers have a unique multiplicative inverse.

(d) Given any pair of distinct integers, there is another integer between them.

(e) Everybody has problems.

Problem 3. (10pt) Being as clear and detailed as possible, explain why $\exists x\, P(x) \wedge \exists x\, Q(x)$ does not imply $\exists x\, [P(x) \wedge Q(x)]$.

Problem 4. (10pt) Let $P(x)$ be the predicate $R(x)$: x is a rectangle and let $S(x)$ be the predicate $S(x)$: x is a square.

(a) Write $\forall x\big(R(x) \to S(x)\big)$ as a complete English sentence.

(b) Write the contrapositive, converse, and negation of the open sentence in (a) as complete English sentences.