MATH 308
Fall 2023
HW 4: Due 09/21

"Most of the mistakes in thinking are inadequacies of perception rather than mistakes of logic."

–Edward de Bono

Problem 1. (10pt) Suppose that $P(x)$ is a predicate with nonempty universe $\mathcal{U}$. Being sure to justify your answer, explain whether the following statements are true or false.

(a) There are values of x for which $\forall x, P(x)$ is true and values for which the statement is false.

(b) If $\forall x, P(x)$ is false, then the statement $\exists x, \neg P(x)$ must be true.

(c) If $\exists x\, P(x)$ is true, then so is $\forall x\, P(x)$.

Problem 2. (10pt) Showing all your work and simplifying your logical expression as much as possible, negate the following quantified open statements:

(a) $\forall x \, \exists y \, \forall z \, P(x, y, z)$

(b) $\exists x \big(\neg P(x) \wedge Q(x) \big)$

(c) $\forall x \big(\neg P(x) \rightarrow Q(x) \big)$

(d) $\exists y \, \forall x \big(P(x, y) \wedge \neg Q(x, y) \big)$

(e) $\exists x \big(P(x) \rightarrow (1 < x \vee x \geq 3) \big)$

Problem 3. (10pt) Being as clear and as detailed as possible, explain why $\forall x\, P(x) \wedge \forall x\, Q(x)$ does not imply $\forall x\, [P(x) \wedge Q(x)]$.

Problem 4. (10pt) A certain computer program has n as an integer variable. Suppose that A is an array of 10 integers values, i.e. A is a 'list' of the integer values $A[1], A[2], \ldots, A[10]$. Write the following as quantified open statements using $A[k]$:

(a) Every entry in the array is greater than 100.

(b) There is an entry in the array that is negative.

(c) The value $A[10]$ is the largest value in the array.

(d) There are two entries in the array that are the same.

Problem 5. (10pt) A function $f(x)$ is said to be *uniformly continuous* if for all $\epsilon > 0$, there exists $\delta > 0$ such that $|f(x) - f(y)| < \epsilon$ whenever $|x - y| < \delta$.

(a) Write the definition of uniform continuity as a quantified statement.

(b) Find a quantified statement for a function to be *not* uniformly continuous by negating your answer in (a).

(c) Write your answer in (b) as an English sentence.