Name: ______________________________
MATH 308
Fall 2023
HW 8: Due 10/12

"The study of Mathematics, like the Nile, begins in minuteness but ends in magnificence."

–Charles Caleb Colton

Problem 1. (10pt) Let $A = \{2, 6, 8, 10\}$, B be the set of nonnegative numbers less than 10, and C be the set of perfect squares less than 10. Define $f : A \to \mathbb{Z}$ and $g : B \setminus C$ via $x \to \frac{15(x+8)}{x}$ and $x \mapsto \frac{5(x^2-16x+88)}{4}$, respectfully. Fully justifying your answer, determine whether $f = g$.

Problem 2. (10pt) Define the following real-valued functions:

$$f(x) = 2x - 1 \qquad j(x) = \frac{x-1}{x+2}$$
$$g(x) = x^2 + x + 1 \qquad k(x) = \sin(\pi x)$$
$$h(x) = x2^x \qquad \ell(x) = 1 - x^2$$

Showing all your work, for each of the following, either compute the function or find a general rule for the given function operation:

(a) $(f + g)(0)$

(b) $(j - \ell)(2)$

(c) $(gk)(5)$

(d) $\left(\frac{f}{j}\right)(3)$

(e) $(h \circ k)(1)$

(f) $(2f + \ell)(x)$

(g) $(fg)(x)$

(h) $\left(\frac{h}{f}\right)(x)$

(i) $(k \circ \ell)(x)$

(j) $(\ell \circ g \circ f)(x)$

Problem 3. (10pt) Let $f : \mathbb{R} \to \mathbb{R}$ be given by $x \mapsto x^2 + 4x - 5$.

(a) Determine $f(-5)$.

(b) Compute $f([0, 1])$.

(c) Is $16 \in \operatorname{im} f$? Explain.

(d) Determine $f^{-1}(0)$.

(e) Find the domain, codomain, and range for $f(x)$.

Problem 4. (10pt) Being sure to justify your answer, complete the following:

(a) Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = 5 - x^2$. Is f an increasing function? Explain. Is f decreasing function? Explain.

(b) Let $g : \mathbb{R} \to \mathbb{R}$ be given by $g(x) = 5x - 8$. Is g a positive function? Explain. Is g a negative function? Explain.

(c) Let g be as in (b) and define $A = [2, \infty)$ and $B = (\infty, 0)$. Is $g\big|_A$ a positive function? Explain. Is $g\big|_B$ a negative function? Explain.

(d) Let $h : \mathbb{R} \to \mathbb{R}$ be given by...

$$h(x) = \begin{cases} 1 - x, & x < 2 \\ 3x + 5, & x \geq 2 \end{cases}$$

Find the largest possible interval $S \subseteq \mathbb{R}$ such that $h\big|_S$ is a nondecreasing function. Is h monotone on S? Is h strictly monotone on S?