

Name: _____

MATH 308

Fall 2021

HW 6: Due 10/08

*“So I was not born with a whole lot of natural talent. But I work hard
and I never give up. That is my gift. That is my ninja way!”*

–Rock Lee, Naturo

Problem 1. (20pt) Describe all sets (if any) with. . .

- (a) no proper subsets.
- (b) one proper subset.
- (c) two proper subsets.

Problem 2. (20pt) The symmetric difference of two sets A and B , denoted $A\Delta B$, is defined by $A\Delta B := (A \setminus B) \cup (B \setminus A)$.

- (a) Describe $A\Delta B$ in words.
- (b) Show that $A\Delta B = (A \cup B) - (A \cap B)$.
- (c) Prove that the symmetric difference is commutative.
- (d) Prove that if $A\Delta B = \emptyset$, then $A = B$. Is the converse true?

Problem 3. (20pt) Let A, B be sets with a common universal set \mathcal{U} . Prove the following:

(a) $A - (A - B) = A \cap B$

(b) $A \subseteq B$ if and only if $A^c \supseteq B^c$

Problem 4. (10pt) If $A \subseteq U$ and $B \subseteq V$, is $A \times B \subseteq U \times V$? Justify your answer.

Problem 5. (10pt) Suppose that X and Y are sets with a common universal set \mathcal{U} . Show that $X = Y$ if and only if $(X \cap Y^c) \cup (X^c \cap Y) = \emptyset$.

Problem 6. (20pt) Prove or disprove:

(a) $(A \cup B) \setminus B = A$

(b) $A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$

(c) $A \cap (B \setminus C) = (A \cap B) \setminus C$

(d) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

Problem 7. (20pt) Express the following sets as an interval, collection of intervals, or well known set (prove your answer):

(a) $\bigcap_{n \geq 1} \left[0, 1 + \frac{1}{n}\right)$

(b) $\bigcup_{n \geq 1} \left[0, 1 + \frac{1}{n}\right)$

(c) $\bigcup_{n \in \mathbb{Z}} \bigcap_{m \geq 1} \left(n - \frac{1}{m}, n + \frac{1}{m}\right)$