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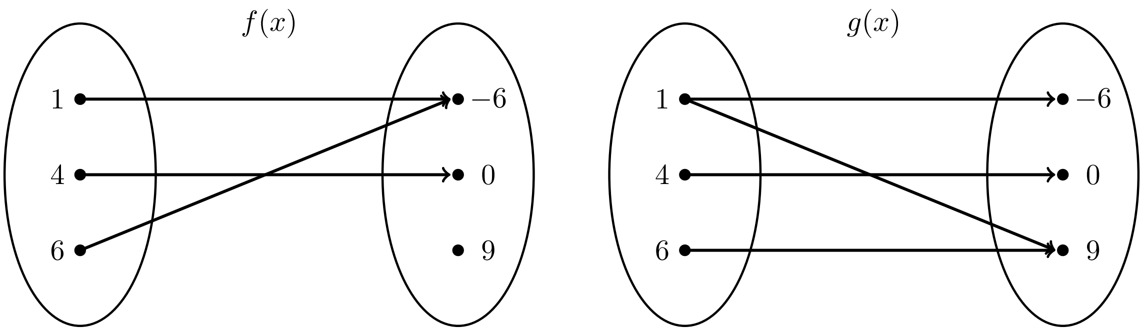
MATH 101
Fall 2021
HW 5: Due 10/08

"The economy stinks, bees are dying, and movies are pretty much all sequels now."

–Winston Saint-Marie Schmidt, New Girl

Problem 1. (10pt) Determine if the relations $f(x)$ and $g(x)$ shown below are functions. Explain why or why not.



The relation $f(x)$ is a function—for each input, there is precisely one output. It does not matter that two of the inputs (namely 1 and 6) both map to -6 under $f(x)$. The relation $g(x)$ is not a function—the input 1 maps to both -6 and 9, i.e. for this input, there is more than one output.

Problem 2. (10pt) Determine if the relations $f(x)$ and $g(x)$ shown below are functions. Explain why or why not.

x	$f(x)$
1	8
2	-7
3	8
4	8
5	10

x	$g(x)$
5	2
6	π
8	1.87
9	-9
5	3

The relation $f(x)$ is a function—for each input, there is precisely one output. It does not matter that three of the inputs (namely 1, 3, and 4) get mapped to 8 under $f(x)$. The relation $g(x)$ is not a function—for the input 5, there is more than one output.

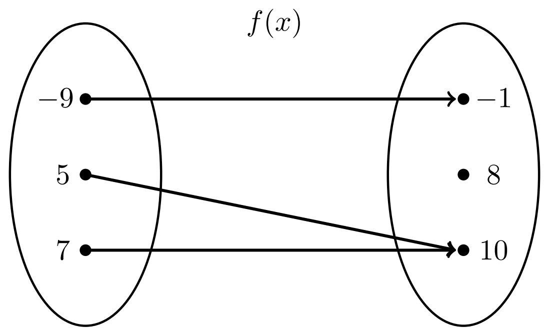
Problem 3. (10pt) Determine if the relations $f(x)$ and $g(x)$ shown below are functions. Explain why or why not.

$$f(x) = 2.54x + 91$$

$$g(x) = x^3 - x + 1$$

Both relations $f(x)$ and $g(x)$ are functions—for each input, there is one output. Namely, the output is the value obtained after plugging in the value for x and following order of operations.

Problem 4. (10pt) Suppose $f(x)$ is the function given below.



(a) What is the domain of $f(x)$?

(b) What is the codomain of $f(x)$?

(c) What is the range of $f(x)$?

Solution.

(a) The domain of $f(x)$ *is* $\{-9, 5, 7\}$.

(b) The codomain of $f(x)$ *is* $\{-1, 8, 10\}$.

(c) The range of $f(x)$ *is* $\{-1, 10\}$.

Problem 5. (10pt) Suppose $f(x)$ and $g(x)$ are the functions given below.

x	-2	0	1	3	4	5	10
$f(x)$	5	-3.1	π	5	$3/2$	14	0
$g(x)$	6	4	6.6	-15	4	9	2

Compute the following:

(a) $f(1) = \pi$

(b) $g(0) = 4$

(c) $(f+g)(5) = f(5) + g(5) = 14 + 9 = 23$

(d) $(f-g)(-2) = f(-2) - g(-2) = 5 - 6 = -1$

(e) $(6f)(1) = 6f(1) = 6(\pi) = 6\pi$

(f) $\left(\frac{f}{g}\right)(10) = \frac{f(10)}{g(10)} = \frac{0}{2} = 0$

(g) $f(4)\,g(5) = \frac{3}{2} \cdot 9 = \frac{27}{2}$

(h) $f(2 - g(0)) = f(2-4) = f(-2) = 5$

(i) $(f \circ g)(0) = f(g(0)) = f(4) = \frac{3}{2}$

(j) $(g \circ f)(3) = g(f(3)) = g(5) = 9$

Problem 6. (10pt) Suppose $f(x)$ and $g(x)$ are the functions given below.

$$f(x) = 3x - 1$$

$$g(x) = x^2 + x + 1$$

Compute the following:

(a) $f(1) = 3(1) - 1 = 3 - 1 = 2$

(b) $g(0) = 0^2 + 0 + 1 = 0 + 0 + 1 = 1$

(c) $f(1) - 2g(1) = \big(3(1) - 1\big) - 2\big(1^2 + 1 + 1\big) = 2 - 2(3) = 2 - 6 = -4$

(d) $f(x) - g(x) = (3x - 1) - (x^2 + x + 1) = 3x - 1 - x^2 - x - 1 = -x^2 + 2x - 2$

(e) $f(x)\,g(x) = (3x - 1)(x^2 + x + 1) = 3x^3 + 3x^2 + 3x - x^2 - x - 1 = 3x^3 + 2x^2 + 2x - 1$

(f) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{3x - 1}{x^2 + x + 1}$

(g) $(g \circ f)(1) = g(f(1)) = g(2) = 2^2 + 2 + 1 = 4 + 2 + 1 = 7$

(h) $f(g(0)) = f(1) = 2$

(i) $(f \circ g)(x) = f(g(x)) = f(x^2 + x + 1) = 3(x^2 + x + 1) - 1 = 3x^2 + 3x + 3 - 1 = 3x^2 + 3x + 2$

(j) $(g \circ f)(x) = g(f(x)) = g(3x - 1) = (3x - 1)^2 + (3x - 1) + 1 = (9x^2 - 6x + 1) + (3x - 1) + 1 = 9x^2 - 3x + 1$