Name: *Caleb McWhorter — Solutions*
MATH 100
Fall 2023
HW 10: Due 10/30

"The superior man understands what is right; the inferior man understands what will sell."
–Confucius

Problem 1. (10pt) Lee H. is taking out a loan to help expand his liquor store *Tequila Mockingbird*. Lee decides to borrow \$60,000 at 12.6% annual interest, compounded monthly. The loan will be repaid with equal end of the month payments over a period of 3 years.

(a) What will Lee's monthly payment be?

(b) How much does Lee pay in total for this loan?

(c) How much does Lee pay in interest for this loan?

Solution. Because Lee will make require monthly payments, this is an annuity.

(a) For a simple annuity immediate, we know that the monthly payments, R, are given by $R = \dfrac{P}{\dfrac{1-(1+i_p)^{-\text{PM}}}{i_p}}$, where i_p is the interest per period, P is the loan amount, and PM is the number of payments. We know that Lee will make monthly payments for 3 years—a total of $\text{PM} = 12 \cdot 3 = 36$ payments. We know that the interest per period is $i_p = \frac{r}{k}$, where r is the nominal interest rate and k is the number of interest compounds per year. But then $i_p = \frac{0.126}{12} = 0.0105$. We know the loan amount is $P = \$60{,}000$. Therefore, Lee's monthly payment is...

$$R = \frac{P}{\dfrac{1-(1+i_p)^{-\text{PM}}}{i_p}} = \frac{\$60{,}000}{\dfrac{1-(1+0.0105)^{-36}}{0.0105}} = \frac{\$60{,}000}{\dfrac{1-(1.0105)^{-36}}{0.0105}} = \frac{\$60{,}000}{\dfrac{1-0.68658223}{0.0105}} = \frac{\$60{,}000}{\dfrac{0.31341777}{0.0105}} = \frac{\$60{,}000}{29.849311} = \$2{,}010.10$$

(b) Lee makes 36 monthly payments of \$2,010.10. Therefore, Lee pays a total of $36 \cdot \$2{,}010.10 = \$72{,}363.60$ on this loan.

(c) Lee only pays back the original loan amount, \$60,000, and any interest on the loan. Because Lee pays a total of \$72,363.60 on this loan, he must pay $\$72{,}363.60 - \$60{,}000 = \$12{,}363.60$ in interest on this loan.

Problem 2. (10pt) A product has cost function $C(q) = 12.67q + 16200$ and revenue function $R(q) = 29.99q$.

(a) What are the fixed costs?

(b) How much does it cost to produce each product? How much does each product sell for?

(c) Find the break-even point.

(d) What is the minimum number of items that must be made/sold in order to make a profit?

Solution.

(a) We know that the fixed costs are the costs incurred in production regardless of the amount produced. But then the fixed costs are the costs even when nothing is produced. These costs are $C(0) = 12.67(0) + 16200 = 0 + 16200 = 16200$. Therefore, the fixed costs are \$16,200.

(b) Because the cost and revenue functions are linear functions, the cost to produce each product and the amount each product sells for is the rate of change of these functions, i.e. the slope. The slope of $C(q)$ is 12.67 and the slope of $R(q)$ is 29.99. Therefore, it costs \$12.67 to produce each product and each product sells for \$29.99.

(c) The break-even point is the production/sale level at which the total cost is the total revenue. But then...

$$\begin{aligned} C(q) &= R(q) \\ 12.67q + 16200 &= 29.99q \\ 16200 &= 17.32q \\ q &= \frac{16200}{17.32} \\ q &\approx 935.335 \end{aligned}$$

Therefore, the break-even point occurs at a production/sales level of 935.335.

(d) The minimum number of items that must be made/sold to make a profit is 936 items.

Problem 3. (10pt) You rent a small studio apartment in NYC for \$3,380 per month to produce social media content. Between hiring actors, purchasing props, travel costs, etc., it costs approximately \$510 to produce a video. However, each video typically makes \$870 in ad revenue and sponsorship deals. Let $C(v)$ and $R(v)$ denote the cost and revenue function to produce v videos.

(a) Explain why $C(v)$ and $R(v)$ are approximately linear.

(b) Find $C(v)$ and $R(v)$.

(c) What is the minimum number of videos you have to produce to make a profit each month?

Solution.

(a) The cost function, $C(v)$, is linear because it costs a constant amount of \$510 to produce each video. The revenue function, $R(v)$, is linear because you make a constant amount of \$870 per video.

(b) Because it costs \$510 to produce each video, the cost incurred from video-making is $510v$, where v is the number of videos. However, you must still rent the studio for \$3,380 per month. Therefore, the total costs to produce v videos each month is $C(v) = 510v + 3380$. Because you only make money from the videos and each video makes \$870, the revenue from making v videos is $R(v) = 870v$.

(c) To find the amount of videos we must make to turn a profit each month, we first find the break-even point, i.e. the number of videos such that $C(v) = R(v)$:

$$\begin{aligned} C(v) &= R(v) \\ 510v + 3380 &= 870v \\ 3380 &= 360v \\ v &= \frac{3380}{360} \\ v &\approx 9.38889 \end{aligned}$$

Therefore, you must make at least 10 videos each month to make a profit.