Name: *Caleb McWhorter — Solutions*
MATH 100
Fall 2023
HW 6: Due 10/02

"Pitter patter, let's get at'er."
–Wayne, Letterkenny

Problem 1. (10pt) For each of the following, describe whether the given dependent variable is a function of the independent variable:

(a) Independent: the number of days since you purchased your car.
Dependent: the milage for your car.

(b) Independent: the number of people in a specific room at noon.
Dependent: the day of the week.

(c) Independent: the day of the year.
Dependent: the sunrise time.

(d) Independent: your laptop battery percentage.
Dependent: the time remaining until your laptop runs out of power.

Solution.

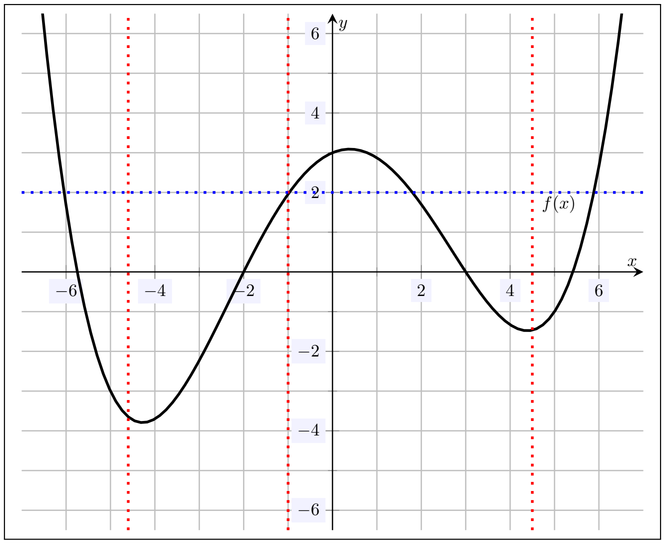
(a) One's car milage is a function of the number of days since they purchased the car. Given a number of days since you purchased it, there can only be one possible milage for the car.

(b) The day of the week is not a function of the number of people in a room at noon. There can be the same number of people in a room at noon on many different days. For example, if one works during the day, there is likely no one in any room of the house at noon from Monday to Friday. But then given that there are no people in some room at noon, one cannot determine if the day of the week is Monday, Tuesday, Wednesday, etc.

(c) The sunrise time is a function of the day of the year. Given any day of there year, there is only one possible sunrise time.

(d) The time remaining until your laptop runs out of power is not a function of your laptop battery percentage. For instance, each time you fully charge the computer, the battery percentage is 100%. However, each time you use the laptop starting a full charge, the battery may last different amount of time until running out of power based on usage, temperature, etc.

Problem 2. (10pt) Consider the relation plotted below:



(a) Is the relation, $f(x)$, plotted above a function? Explain.

(b) Find the y-intercept.

(c) Find the x-intercepts.

(d) Find the value of $f(6)$.

(e) Find any x-values for which $f(x) = 2$.

Solution.

(a) The relation, $f(x)$, plotted above passes the Vertical Line Test; that is, every vertical line intersects the relation, $f(x)$, at most once. [See some sample vertical lines in red on the graph above.] Therefore, $f(x)$ is a function.

(b) The y-intercept is the point (if any) where the curve intersects the y-axis. Examining the graph, we see that the y-intercept is $y = 3$, i.e. the point $(0, 3)$.

(c) The x-intercepts are the points (if any) where the curve intersects the x-axis. Examining the graph, we see that the x-intercepts are $x \approx -5.7469$, -2, 3, and 5.41357, i.e. the points $(-5.7469, 0)$, $(-2, 0)$, $(3, 0)$, and $(5.41357, 0)$.

(d) Examining the graph, we can see the point $(6, 2.65709)$ on the graph of $f(x)$. Therefore, $f(6) = 2.65709$.

(e) Solutions to the equation $f(x) = 2$ will correspond to points where the relation $f(x)$ intersects the line $y = 2$—sketched in blue above. We see the curve intersects the line at $y = 2$ at $(-6.04388, 2)$, $(-0.972562, 2)$, $(1.79901, 2)$, and $(5.88411, 2)$. Therefore, the x-values for which $f(x) = 2$ are $x = -6.04388$, -0.972562, 1.79901, and 5.88411.

Problem 3. (10pt) Define $f(x)$ to be the relation given by $f(x) := 2.7x + 14.9$.

(a) Is $f(x)$ a function? Explain.

(b) Find $f(9)$.

(c) Is there an x_0 so that $f(x_0) = 20$? If so, find it. If not, explain why.

(d) Find the y-intercept for $f(x)$.

(e) Find any x-intercepts for $f(x)$.

Solution.

(a) The relation $f(x)$ is a function. Given an input, x, there is only one possible output for $f(x)$—namely the one obtained by evaluating $f(x)$ at x.

(b) We have...

$$f(9) = 2.7(9) + 14.9 = 24.3 + 14.9 = 39.2$$

(c) If there were such an x_0, we would have $f(x_0) = 20$. But then...

$$\begin{aligned} f(x_0) &= 20 \\ 2.7x_0 + 14.9 &= 20 \\ 2.7x &= 5.1 \\ x &\approx 1.8889 \end{aligned}$$

As each step above is reversible, we know that $f(1.8889) \approx 20$.

(d) The y-intercept for $f(x)$ is the value at which the function intercepts the y-axis, i.e. its value at $x = 0$. But we have $f(0) = 2.7(0) + 14.9 = 0 + 14.9 = 14.9$. Therefore, the y-intercept is $y = 14.9$, i.e. the point $(0, 14.9)$.

(e) The x-intercepts for $f(x)$ are the value(s) (if any) where $f(x) = 0$. If x_0 is such a value, then we have...

$$\begin{aligned} f(x_0) &= 0 \\ 2.7x_0 + 14.9 &= 0 \\ 2.7x &= -14.9 \\ x &\approx -5.51852 \end{aligned}$$

As each step above is reversible, we know that $f(-5.51852) \approx 0$. Therefore, the only x-intercept for $f(x)$ is $x = -5.51852$, i.e. the point $(-5.51852, 0)$.

Problem 4. (10pt) Let $f(x)$ and $g(x)$ be the functions given by the values in the table below.

x	-2	-1	0	1	2
$f(x)$	4	5	-1	6	0
$g(x)$	3	-2	7	0	-1

Compute the following:

(a) $f(-2) - g(1)$

(b) $(f + g)(0)$

(c) $(fg)(-1)$

(d) $(f \circ g)(2)$

(e) $(g \circ f)(2)$

Solution.

(a)

$$f(-2) - g(1) = 4 - 0 = 4$$

(b)

$$(f + g)(0) = f(0) + g(0) = -1 + 7 = 6$$

(c)

$$(fg)(-1) = f(-1)g(-1) = 5 \cdot -2 = -10$$

(d)

$$(f \circ g)(2) = f\big(g(2)\big) = f(-1) = 5$$

(e)

$$(g \circ f)(2) = g\big(f(2)\big) = g(0) = -1$$