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MATH 101

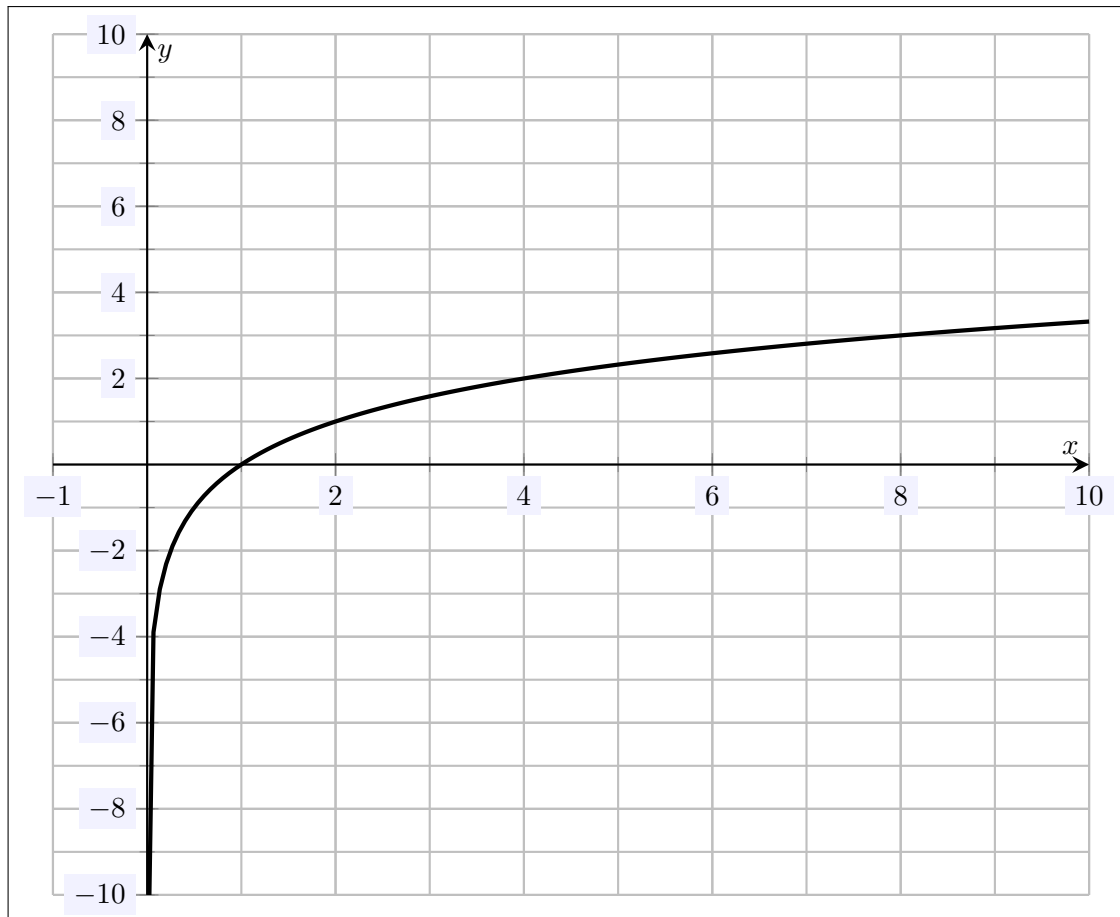
Fall 2021

HW 17: Due 11/23

"I was on the street. This guy waved to me, and he came up to me and said, 'I'm sorry. I thought you were someone else.' And I said, 'I am.'"

—Demetri Martin

Problem 1. (10pt) Sketch the function $y = \log_2 x$.



Problem 2. (10pt) Compute the following:

(a) $\log_4 4 - \log_6 1$

(b) $\log_5 25$

(c) $\log_3 \frac{1}{81}$

(d) $\log_9 \sqrt{3}$

(e) $\ln e^{2/3}$

Solution.

(a)

$$\log_4 4 - \log_6 1 = 1 - 0 = 1$$

(b)

$$\log_5 25 = \log_5 5^2 = 2$$

(c)

$$\log_3 \frac{1}{81} = \log_3 (81^{-1}) = \log_3 ((3^4)^{-1}) = \log_3 (3^{-4}) = -4$$

(d)

$$\log_9 \sqrt{3} = \log_9 (3^{1/2}) = \log_9 ((9^{1/2})^{1/2}) = \log_9 (9^{1/4}) = \frac{1}{4}$$

(e)

$$\ln e^{2/3} = \log_e e^{2/3} = \frac{2}{3}$$

Problem 3. (10pt) Expand the following logarithm completely by expressing it as a sum or difference of logs. Your answer should not include any exponents.

$$\log_3 \left(\frac{\sqrt[6]{x}}{3y^4} \right)$$

Solution.

$$\begin{aligned} \log_3 \left(\frac{\sqrt[6]{x}}{3y^4} \right) &= \log_3 \sqrt[6]{x} - \log_3(3y^4) \\ &= \log_3 x^{1/6} - \log_3 y^4 - \log_3 3 \\ &= \frac{1}{6} \log_3 x - 4 \log_3 y - 1 \end{aligned}$$

Problem 4. (10pt) Rewrite the expression below as a single logarithm.

$$\frac{1}{2} \ln x - \ln 1 + 3 \ln(x + 2) - \ln(1 - x)$$

Solution.

$$\frac{1}{2} \ln x - \ln 1 + 3 \ln(x + 2) - \ln(1 - x) = \ln x^{1/2} - 0 + \ln(x + 2)^3 - \ln(1 - x)$$

$$= \ln \sqrt{x} + \ln \left(\frac{(x + 2)^3}{1 - x} \right)$$

$$= \ln \left(\frac{\sqrt{x}(x + 2)^3}{1 - x} \right)$$