

Name: _____

MATH 308

Fall 2023

HW 8: Due 10/12

“The study of Mathematics, like the Nile, begins in minuteness but ends in magnificence.”

– Charles Caleb Colton

Problem 1. (10pt) Let $A = \{2, 6, 8, 10\}$, B be the set of nonnegative numbers less than 10, and C be the set of perfect squares less than 10. Define $f : A \rightarrow \mathbb{Z}$ and $g : B \setminus C$ via $x \mapsto \frac{15(x+8)}{x}$ and $x \mapsto \frac{5(x^2-16x+88)}{4}$, respectfully. Fully justifying your answer, determine whether $f \equiv g$.

Problem 2. (10pt) Define the following real-valued functions:

$$\begin{aligned} f(x) &= 2x - 1 & j(x) &= \frac{x - 1}{x + 2} \\ g(x) &= x^2 + x + 1 & k(x) &= \sin(\pi x) \\ h(x) &= x2^x & \ell(x) &= 1 - x^2 \end{aligned}$$

Showing all your work, for each of the following, either compute the function at the specified value or find a general rule for the given function operation:

- (a) $(f + g)(0)$
- (b) $(j - \ell)(2)$
- (c) $(gk)(5)$
- (d) $\left(\frac{f}{j}\right)(3)$
- (e) $(h \circ k)(1)$
- (f) $(2f + \ell)(x)$
- (g) $(fg)(x)$
- (h) $\left(\frac{h}{f}\right)(x)$
- (i) $(k \circ \ell)(x)$
- (j) $(\ell \circ g \circ f)(x)$

Problem 3. (10pt) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be given by $x \mapsto x^2 + 4x - 5$.

- (a) Determine $f(-5)$.
- (b) Compute $f([0, 1])$.
- (c) Is $16 \in \text{im } f$? Explain.
- (d) Determine $f^{-1}(0)$.
- (e) Find the domain, codomain, and range for $f(x)$.

Problem 4. (10pt) Being sure to justify your answer, complete the following:

- (a) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = 5 - x^2$. Is f an increasing function? Explain. Is f a decreasing function? Explain.
- (b) Let $g : \mathbb{R} \rightarrow \mathbb{R}$ be given by $g(x) = 5x - 8$. Is g a positive function? Explain. Is g a negative function? Explain.
- (c) Let g be as in (b) and define $A = [2, \infty)$ and $B = (-\infty, 0)$. Is $g|_A$ a positive function? Explain. Is $g|_B$ a negative function? Explain.
- (d) Let $h : \mathbb{R} \rightarrow \mathbb{R}$ be given by...

$$h(x) = \begin{cases} 1 - x, & x < 2 \\ 3x + 5, & x \geq 2 \end{cases}$$

Find the largest possible interval $S \subseteq \mathbb{R}$ such that $h|_S$ is a nondecreasing function. Is h monotone on S ? Is h strictly monotone on S ?