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MATH 101
Spring 2022
HW 1: Due 02/08

"Windows are the eyes to the house."
–Andy Dwyer, Parks & Recreation

Problem 1. (10pt) Give the definition of a real number. Also, give at least five original examples of a real number.

A real number is 'any' number which is expressible as a decimal, e.g.

$$0 = 0.0$$

$$1 = 1.0$$

$$-5 = -5.0$$

$$\frac{1}{2} = 0.5$$

$$-\frac{1}{10} = -0.1$$

$$0.11958904771$$

$$\sqrt{2} = 1.414213562373095\ldots$$

$$\pi = 3.141592653589793\ldots$$

$$e = 2.718281828495045\ldots$$

$$\gamma = 0.577215664901532\ldots$$

Problem 2. (10pt) Give the definition of a rational number. Also, give at least five original examples of a rational number.

A rational number is a real number of the form $\frac{a}{b}$*, where* a *and* b *are integers and* $b \neq 0$*, e.g.*

$$0 = \frac{0}{1}$$

$$2 = \frac{2}{1}$$

$$-5 = -\frac{5}{1}$$

$$\frac{1}{2}$$

$$-\frac{5}{7}$$

$$\frac{20}{100}$$

Problem 3. (10pt) Find the prime factorizations of the following integers:

(a) 54

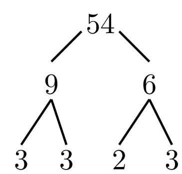
(b) 97

(c) 168

(d) 184

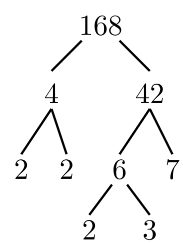
Solution.

(a) $54 = 2 \cdot 3^3$

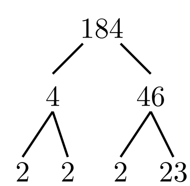


(b) $97 = 97$ (Already prime)

(c) $168 = 2^3 \cdot 3 \cdot 7$



(d) $184 = 2^3 \cdot 23$



Problem 4. (10pt) Without using a calculator, answer the following:

(a) Does 2 divide 2346? Explain.

(b) Does 3 divide 596012? Explain.

(c) Does 4 divide 990140? Explain.

(d) Does 5 divide 1431? Explain.

(e) Does 9 divide 70155? Explain.

Solution.

(a) We know that an integer is divisible by 2 if and only if the integer is even. Because 2346 is even, we know that it is divisible by 3. In fact, $2346 = 1173(2)$.

(b) We know that an integer is divisible by 3 if and only if the sum of the digits is divisible by 3. Because $5 + 9 + 6 + 0 + 1 + 2 = 23$, which is not divisible by 3, the integer 569012 is not divisible by 3. In fact, $596012/3 \approx 198670.667$.

(c) We know that an integer is divisible by 4 if and only if the last two digits of the integer is divisible by 4. Because 40 is divisibly by 4, we know that 990140 is divisible by 4. In fact, $990140 = 247535(4)$.

(d) We know that an integer is divisible by 5 if and only if the integer ends in a 0 or a 5. Because 1431 ends in a 1, we know that 1431 is not divisible by 5. In fact, $1431/5 = 286.2$.

(e) We know that an integer is divisible by 9 if and only if the sum of the digits is divisible by 9. Because $7 + 0 + 1 + 5 + 5 = 18$, which is divisible by 9, the integer 70155 is divisible by 9. In fact, $70155 = 7795(9)$.

Problem 5. (10pt) Using the 'square root method,' show that 157 is prime.

Solution. We know that if a positive integer N is composite that it has a factor at most $\sqrt{N}$. Observe that $\sqrt{157} \approx 12.53$. Therefore, if 157 is composite, i.e. not prime, then it must have a prime divisor between 2 and 12. We check to see if any of the prime numbers between 2 and 12, i.e. 2, 3, 5, 7, and 11, divide 157:

Prime	2	3	5	7	11
Divisible	✗	✗	✗	✗	✗

But then 157 does not have a prime divisor ≤ 12. Therefore, 157 cannot be composite, i.e. 157 is prime.

Problem 6. (10pt) By listing out all the divisors of the given numbers, compute the following:

(a) $\gcd(12, 15)$

(b) $\gcd(20, 22)$

(c) $\gcd(36, 60)$

(d) $\gcd(20, 100)$

Solution.

(a)

12: 1, 2, **3**, 4, 6, 12
15: 1, **3**, 5, 15

Therefore, $\gcd(12, 15) = 3$.

(b)

20: 1, **2**, 4, 5, 10, 20
22: 1, **2**, 11, 22

Therefore, $\gcd(20, 22) = 2$.

(c)

36: 1, 2, 3, 4, 6, 9, **12**, 18, 36
60: 1, 2, 3, 4, 5, 6, 10, **12**, 15, 20, 30, 60

Therefore, $\gcd(36, 60) = 12$.

(d)

20: 1, 2, 4, 5, 10, **20**
100: 1, 2, 4, 5, 10, **20**, 25, 50, 100

Therefore, $\gcd(20, 100) = 20$.

Problem 7. (10pt) By listing out sufficiently many multiples of the given integers, compute the following:

(a) $\operatorname{lcm}(24, 36)$

(b) $\operatorname{lcm}(12, 15)$

(c) $\operatorname{lcm}(12, 18)$

(d) $\operatorname{lcm}(36, 48)$

Solution.

(a)

24: 24, 48, **72**, 96, 120, 144, 168, 192 ...
36: 36, **72**, 108, 144, 180, 216, 252, 288, ...

Therefore, $\operatorname{lcm}(24, 36) = 72$.

(b)

12: 12, 24, 36, 48, **60**, 72, 84, 96, ...
15: 15, 30, 45, **60**, 75, 90, 105, 120, ...

Therefore, $\operatorname{lcm}(12, 15) = 60$.

(c)

12: 12, 24, **36**, 48, 60, 72, 84, 96, ...
18: 18, **36**, 54, 72, 90, 108, 126, 144, ...

Therefore, $\operatorname{lcm}(12, 18) = 36$.

(d)

36: 36, 72, 108, **144**, 180, 216, 252, 288, ...
48: 48, 96, **144**, 192, 240, 288, 336, 384, ...

Therefore, $\operatorname{lcm}(36, 48) = 144$.

Problem 8. (10pt) By finding prime factorizations, compute the following:

(a) $\gcd(12, 15)$

(b) $\gcd(20, 22)$

(c) $\gcd(36, 60)$

(d) $\gcd(20, 100)$

Solution.

(a)
$$\gcd(12, 15) = \gcd(2^2 \cdot 3, 3 \cdot 5) = 3^1 = 3$$

(b)
$$\gcd(20, 22) = \gcd(2^2 \cdot 5, 2 \cdot 11) = 2^1 = 2$$

(c)
$$\gcd(36, 60) = \gcd(2^2 \cdot 3^2, 2^2 \cdot 3 \cdot 5) = 2^2 \cdot 3 = 12$$

(d)
$$\gcd(20, 100) = \gcd(2^2 \cdot 5, 2^2 \cdot 5^2) = 2^2 \cdot 5 = 20$$

Problem 9. (10pt) By finding prime factorizations, compute the following:

(a) $\operatorname{lcm}(24, 36)$

(b) $\operatorname{lcm}(12, 15)$

(c) $\operatorname{lcm}(12, 18)$

(d) $\operatorname{lcm}(36, 48)$

Solution.

(a)
$$\operatorname{lcm}(24, 36) = \operatorname{lcm}(2^3 \cdot 3, 2^2 \cdot 3^2) = 2^3 \cdot 3^2 = 72$$

(b)
$$\operatorname{lcm}(12, 15) = \operatorname{lcm}(2^2 \cdot 3, 3 \cdot 5) = 2^2 \cdot 3 \cdot 5 = 60$$

(c)
$$\operatorname{lcm}(12, 18) = \operatorname{lcm}(2^2 \cdot 3, 2 \cdot 3^2) = 2^2 \cdot 3^2 = 36$$

(d)
$$\operatorname{lcm}(36, 48) = \operatorname{lcm}(2^2 \cdot 3^2, 2^4 \cdot 3^1) = 2^4 \cdot 3^2 = 144$$

Problem 10. (10pt) Compute the following:

(a) $\gcd(2^3 \cdot 3^1 \cdot 5^3 \cdot 11^5, 2^2 \cdot 3^3 \cdot 5 \cdot 7)$

(b) $\operatorname{lcm}(2^3 \cdot 3^1 \cdot 5^3 \cdot 11^5, 2^2 \cdot 3^3 \cdot 5 \cdot 7)$

(c) $\gcd(2^{10} \cdot 5^5 \cdot 13, 3^5 \cdot 5^1 \cdot 11^2)$

(d) $\operatorname{lcm}(2^{10} \cdot 5^5 \cdot 13, 3^5 \cdot 5^1 \cdot 11^2)$

Solution.

(a)

$$\gcd(2^3 \cdot 3^1 \cdot 5^3 \cdot 11^5, 2^2 \cdot 3^3 \cdot 5 \cdot 7) = 2^2 \cdot 3 \cdot 5 = 60$$

(b)

$$\operatorname{lcm}(2^3 \cdot 3^1 \cdot 5^3 \cdot 11^5, 2^2 \cdot 3^3 \cdot 5 \cdot 7) = 2^3 \cdot 3^3 \cdot 5^3 \cdot 7 \cdot 11^5 = 30\ 438\ 639\ 000$$

(c)

$$\gcd(2^{10} \cdot 5^5 \cdot 13, 3^5 \cdot 5^1 \cdot 11^2) = 5$$

(d)

$$\operatorname{lcm}(2^{10} \cdot 5^5 \cdot 13, 3^5 \cdot 5^1 \cdot 11^2) = 2^{10} \cdot 3^5 \cdot 5^5 \cdot 11^2 \cdot 13 = 1\ 223\ 164\ 800\ 000$$