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MATH 108
Spring 2024
HW 11: Due 03/06

"The universe is basically an animal. It grazes on the ordinary. It creates infinite idiots just to eat them."
— Rick Sanchez, Rick and Morty

Problem 1. (10pts) Suppose that the number of people served at the DMV is uniformly distributed over the 8 hours that they are open (9 am to 5 pm).

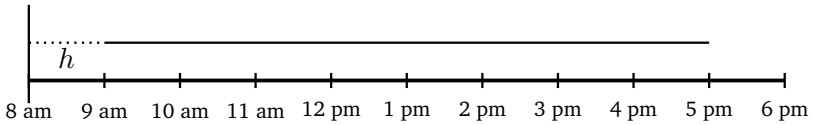
(a) What percentage of people are served before noon?

(b) What percentage of people are served after noon?

(c) What percentage of people are served between 11 am and 3 pm?

(d) At what time has the DMV served 90% of the people they will assist that day?

Solution. We can draw a sketch of this scenario:



We want to choose the height h so the area under the uniform distribution is 1. But this area is a rectangle. We know $A = \ell h$. So $1 = 8h$, which implies $h = \frac{1}{8}$.

(a) This is the area under the curve before noon:

$$P(t \leq 12) = 3 \cdot \frac{1}{8} = \frac{3}{8} \approx 0.375$$

(b)

$$P(t \geq 12) = 1 - P(\leq 12) = 1 - \frac{3}{8} = \frac{5}{8} \approx 0.625$$

(c)

$$P(11 \leq t \leq 3) = 4 \cdot \frac{1}{8} = \frac{4}{8} = \frac{1}{2} \approx 0.50$$

(d) We want a time, T, such that $P(t \leq T) = 0.90$. But then...

$$0.90 = P(t \leq T) = T \cdot \frac{1}{8} = \frac{T}{8}$$

This implies that $T = 7.2$, i.e. 7.2 hours after opening. We know 0.20 hours is $60 \cdot 0.20 = 12$ minutes. Seven hours after 8 am is 3 pm. But then the DMV has served 90% of people by 3:12 pm.

Problem 2. (10pts) Suppose you have a normal distribution with mean $\mu = 870$ and standard deviation $\sigma = 113$. Showing all your work, compute the following:

(a) $P(X = 900)$

(b) $P(X < 950)$

(c) $P(X > 950)$

(d) $P(800 < X < 950)$

Solution.

(a) In a continuous distribution, e.g. the normal distribution, $P(X = \#) = 0$. Therefore, we have...

$$P(X = 900) = 0$$

(b) We have...

$$z_{950} = \frac{950 - 870}{113} = \frac{80}{113} \approx 0.71 \rightsquigarrow 0.7611$$

Therefore, $P(X < 950) = 0.7611$.

(c) We have...

$$P(X > 950) = 1 - P(X < 950) = 1 - 0.7611 = 0.2389$$

(d) We have...

$$z_{800} = \frac{800 - 870}{113} = \frac{-70}{113} = -0.62 \rightsquigarrow 0.2676$$

But then...

$$P(800 < X < 950) = P(X < 950) - P(X < 800) = 0.7611 - 0.2676 = 0.4935$$