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MATH 100
Fall 2022
HW 7: Due 10/12

"He knows nothing, and he thinks he knows everything. That clearly points to a political career."
–George Bernard Shaw

Problem 1. (10pt) Define what it means to be a linear function. Then give an example of a linear function and evaluate it at some value.

Solution. A linear function is 'any' function which has a constant rate of change. Moreover, 'any' function which has a constant rate of change is a linear function. 'Any' function of one variable which has a constant can be written in the form $f(x) = mx + b$ for some real numbers m and b. For instance, choosing $m = -2$ and $b = 11$, we have...

$$f(x) = -2x + 11 = 11 - 2x$$

We can evaluate this function for any real number x. For instance, evaluating this at $x = 3$, we have...

$$f(3) = 11 - 2(3) = 11 - 6 = 5$$

Problem 2. (10pt) Consider the function $f(x) = 121.5 - 11.6x$.

(a) Explain why $f(x)$ is linear.

(b) Find the slope and y-intercept of $f(x)$.

(c) Find $f(x)$ when $x = 7.2$

(a) The given function $f(x) = 121.5 - 11.6x = -11.6x + 121.5$ can be written in the form $y = mx + b$, where $m = -11.6$ and $b = 121.5$. Therefore, $f(x)$ is linear.

(b) Because $f(x) = -11.6x + 121.5$ has the form $y = mx + b$ with $m = -11.6$ and $b = 121.5$, we know that the slope is $m = -11.6$ and that the y-intercept is $b = 121.5$.

(c) We have...

$$f(7.2) = -11.6(7.2) + 121.5 = -83.52 + 121.5 = 37.98$$

Problem 3. (10pt) Find the equation of the linear function which passes through the points $(-4, 10)$ and $(6, -8)$.

Solution. Because the function is linear, we know that it has the form $f(x) = mx + b$. To find the equation of the line, we need a point and a slope. We know that the linear function contains the point $(-4, 10)$. To find the slope, m, we use the fact that this is the ratio of change in y and x:

$$m = \frac{\Delta y}{\Delta x} = \frac{10 - (-8)}{-4 - 6} = \frac{10 + 8}{-10} = \frac{18}{-10} = -1.8$$

But then we know that $f(x) = mx + b = -1.8x + b$. Because the linear fucntion contains the point $(-4, 10)$, we know that when $x = -4$ that $f(x) = 10$. Therefore, we have...

$$f(x) = -1.8x + b$$

$$10 = -1.8(-4) + b$$

$$10 = 7.2 + b$$

$$b = 2.8$$

Therefore, we know that $f(x) = -1.8x + 2.8$.

Problem 4. (10pt) Find the equation of the linear function with slope -15 and y-intercept 19.

Solution. Because the function is linear, we know that it has the form $f(x) = mx + b$. To find the equation of the line, we need a point and a slope. Because the line has y-intercept is 19, we know that the line contains the point $(0, 19)$. Because the slope is -15, we know that $m = -15$. Therefore, we know that $f(x) = mx + b = -15x + b$. Because the linear function contains the point $(0, 19)$, we know when $x = 0$ that $f(x) = 19$. But then we have...

$$f(x) = -15x + b$$

$$19 = -15(0) + b$$

$$b = 19$$

Therefore, we know that $f(x) = -15x + 19$.