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MATH 101
Fall 2022
HW 21: Due 12/05

"Don't judge each day by the harvest you reap but by the seeds that you plant."
–Robert Louis Stevenson

Problem 1. (10pt) Solve the following quadratic equation:

$$x^2 - 18 = 0$$

Solution. We have...

$$\begin{aligned} x^2 - 18 &= 0 \\ x^2 &= 18 \\ \sqrt{x^2} &= \pm\sqrt{18} \\ x &= \pm\sqrt{9 \cdot 2} \\ x &= \pm 3\sqrt{2} \end{aligned}$$

OR

$$\begin{aligned} x^2 - 18 &= 0 \\ (x - \sqrt{18})(x + \sqrt{18}) &= 0 \\ (x - \sqrt{9 \cdot 2})(x + \sqrt{9 \cdot 2}) &= 0 \\ (x - 3\sqrt{2})(x + 3\sqrt{2}) &= 0 \end{aligned}$$

Therefore, either $x - 3\sqrt{2} = 0$, which implies $x = 3\sqrt{2}$, or $x + 3\sqrt{2} = 0$, which implies $x = -3\sqrt{2}$.

OR

Because $x^2 - 18 = x^2 + 0x - 18$, this is a quadratic function with $a = 1$, $b = 0$, and $c = -18$. Therefore, we have...

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-0 \pm \sqrt{0^2 - 4(1)(-18)}}{2(1)} \\ &= \frac{\pm\sqrt{4(18)}}{2} \\ &= \frac{\pm 2\sqrt{18}}{2} \\ &= \pm\sqrt{18} \\ &= \pm\sqrt{9 \cdot 2} \\ &= \pm 3\sqrt{2} \end{aligned}$$

Problem 2. (10pt) Use completing the square to solve the following quadratic equation:

$$2x^2 = 5x + 3$$

Solution. We have...

$$
\begin{aligned}
2x^2 - 5x - 3 &= 0 \\
2\left(x^2 - \frac{5}{2}x - \frac{3}{2}\right) &= 0 \\
2\left(x^2 - \frac{5}{2}x + \left(\frac{1}{2}\cdot\frac{5}{2}\right)^2 - \left(\frac{1}{2}\cdot\frac{5}{2}\right)^2 - \frac{3}{2}\right) &= 0 \\
2\left(x^2 - \frac{5}{2}x + \frac{25}{4} - \frac{25}{4} - \frac{3}{2}\right) &= 0 \\
2\left(\left(x^2 - \frac{5}{2}x + \frac{25}{4}\right) - \frac{25}{4} - \frac{3}{2}\right) &= 0 \\
2\left(\left(x - \frac{5}{2}\right)^2 - \frac{25}{4} - \frac{6}{4}\right) &= 0 \\
2\left(\left(x - \frac{5}{2}\right)^2 - \frac{31}{4}\right) &= 0 \\
2\left(x - \frac{5}{2}\right)^2 - \frac{31}{2} &= 0 \\
2\left(x - \frac{5}{2}\right)^2 &= \frac{31}{2} \\
\left(x - \frac{5}{2}\right)^2 &= \frac{31}{4} \\
\sqrt{\left(x - \frac{5}{2}\right)^2} &= \pm\sqrt{\frac{31}{4}} \\
x - \frac{5}{2} &= \pm\frac{\sqrt{31}}{2} \\
x &= \frac{5}{2} \pm \frac{\sqrt{31}}{2} \\
x &= \frac{5 \pm \sqrt{31}}{2}
\end{aligned}
$$

Problem 3. (10pt) Use the discriminant of $f(x) = x^2 - 10x + 19$ to explain why there are no 'nice' solutions to $f(x) = 0$. Then use the quadratic formula to find the solutions to $f(x) = 0$.

Solution. If $f(x)$ is a quadratic function, we know there are 'nice' solutions to $f(x) = 0$ if and only if the discriminant of $f(x)$ is a perfect square. If $f(x) = ax^2 + bx + c$, the discriminant of $f(x)$ is $D = b^2 - 4ac$. But because $f(x) = x^2 - 10x + 19$ has $a = 1$, $b = -10$, and $c = 19$, we have...

$$D = b^2 - 4ac = (-10)^2 - 4(1)19 = 100 - 76 = 24$$

Because $D = 24$ is not a perfect square (observe $4^2 = 16 < 24 < 25 = 5^2$), we know that there are no 'nice' solutions to $f(x) = 0$.

However, we can find the solutions to $f(x) = 0$ using the quadratic formula. Because $f(x) = x^2 - 10x + 19$ has $a = 1$, $b = -10$, and $c = 19$, we have...

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)19}}{2(1)} \\ &= \frac{10 \pm \sqrt{100 - 76}}{2} \\ &= \frac{10 \pm \sqrt{24}}{2} \\ &= \frac{10 \pm \sqrt{4 \cdot 6}}{2} \\ &= \frac{10 \pm 2\sqrt{6}}{2} \\ &= 5 \pm \sqrt{6} \end{aligned}$$