Name: *Caleb McWhorter — Solutions*
MATH 308
Fall 2021
HW 3: Due 09/24

"The problem with people who only want what they can't have is that once they have what they want, they don't want it anymore."
–John Michael Dorian (J.D.), Scrubs

Problem 1. (10pt) Watch the video New Math (Tom Lehrer). Who was Tom Lehrer? What other songs is he famous for? How does the video relate to the material?

Solution. Any 'solution' here is fine. Although, it would be worth noting that the video does contain the same method of arithmetic in other bases that we discussed in class.

Problem 2. (10pt) Convert the following integers to binary:

(a) 33

(b) 156

Solution. First, observe

$$\begin{aligned} 2^0 &= 1 & 2^4 &= 16 \\ 2^1 &= 2 & 2^5 &= 32 \\ 2^2 &= 4 & 2^6 &= 64 \\ 2^3 &= 8 & 2^7 &= 128 \end{aligned}$$

(a) We have...

$$33 = 32 + 1 = 1 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 100001_2$$

(b) We have...

$$156 = 128 + 16 + 8 + 4 = 1 \cdot 2^7 + 0 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 = 10011100_2$$

Problem 3. (10pt) Convert the following binary integers to base-10 integers:

(a) 101101_2

(b) 10110100_2

Solution. First, observe

$$\begin{aligned} 2^0 &= 1 & 2^4 &= 16 \\ 2^1 &= 2 & 2^5 &= 32 \\ 2^2 &= 4 & 2^6 &= 64 \\ 2^3 &= 8 & 2^7 &= 128 \end{aligned}$$

(a) We have...

$$101101_2 = 1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 32 + 8 + 4 + 1 = 45$$

(b) We have...

$$10110100_2 = 1 \cdot 2^7 + 0 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 = 128 + 32 + 16 + 4 = 180$$

Problem 4. (10pt) Convert the following integers to hexadecimal:

(a) 59

(b) 200

Solution. First, observe $16^0 = 1$, $16^1 = 16$, $16^2 = 256$ and

$$\begin{array}{cccc} 16 \cdot 0 = 0 & 16 \cdot 1 = 16 & 16 \cdot 2 = 32 & 16 \cdot 3 = 48 \\ 16 \cdot 4 = 64 & 16 \cdot 5 = 80 & 16 \cdot 6 = 96 & 16 \cdot 7 = 112 \\ 16 \cdot 8 = 128 & 16 \cdot 9 = 144 & 16 \cdot 10 = 160 & 16 \cdot 11 = 176 \\ 16 \cdot 12 = 192 & 16 \cdot 13 = 208 & 16 \cdot 14 = 224 & 16 \cdot 15 = 240 \end{array}$$

Also, recall

$$\begin{array}{cccc} 0 = 0 & 1 = 1 & 2 = 2 & 3 = 3 \\ 4 = 1 & 5 = 5 & 6 = 1 & 7 = 7 \\ 8 = 1 & 9 = 9 & 10 = A & 11 = B \\ 12 = C & 2 = D & 14 = E & 15 = F \end{array}$$

(a) We have...

$$59 = 48 + 11 = 3 \cdot 16^1 + 11 \cdot 16^0 = \text{3B}$$

(b) We have...

$$200 = 192 + 8 = 12 \cdot 16^1 + 8 \cdot 16^0 = \text{C8}$$

Problem 5. (10pt) Convert the following hexadecimal numbers to base-10:

(a) F1

(b) 5BF

Solution. First, observe $16^0 = 1$, $16^1 = 16$, $16^2 = 256$ and

$$\begin{array}{cccc} 16 \cdot 0 = 0 & 16 \cdot 1 = 16 & 16 \cdot 2 = 32 & 16 \cdot 3 = 48 \\ 16 \cdot 4 = 64 & 16 \cdot 5 = 80 & 16 \cdot 6 = 96 & 16 \cdot 7 = 112 \\ 16 \cdot 8 = 128 & 16 \cdot 9 = 144 & 16 \cdot 10 = 160 & 16 \cdot 11 = 176 \\ 16 \cdot 12 = 192 & 16 \cdot 13 = 208 & 16 \cdot 14 = 224 & 16 \cdot 15 = 240 \end{array}$$

Also, recall

$$\begin{array}{cccc} 0 = 0 & 1 = 1 & 2 = 2 & 3 = 3 \\ 4 = 1 & 5 = 5 & 6 = 1 & 7 = 7 \\ 8 = 1 & 9 = 9 & 10 = A & 11 = B \\ 12 = C & 2 = D & 14 = E & 15 = F \end{array}$$

(a) We have...

$$F1 = 15 \cdot 16^1 + 1 \cdot 16^0 = 240 + 1 = 241$$

(b) We have...

$$5BF = 5 \cdot 16^2 + 11 \cdot 16^1 + 15 \cdot 16^0 = 1280 + 176 + 15 = 1471$$

Problem 6. (10pt) Perform the following operations in binary:

(a) $101_2 + 11_2$

(b) $11011_2 + 11101_2$

Solution.

(a)

$$\begin{array}{rrrr} & {}^{1}1 & {}^{1}0 & 1 \\ & & 1 & 1 \\ \hline 1 & 0 & 0 & 0 \end{array}$$

(b)

$$\begin{array}{rrrrrr} & {}^{1}1 & {}^{1}1 & {}^{1}0 & {}^{1}1 & 1 \\ & 1 & 1 & 1 & 0 & 1 \\ \hline 1 & 1 & 1 & 0 & 0 & 0 \end{array}$$