Quiz 1. *True/False*: The integer 131313 is prime.

Solution. The statement is *false*. We know that an integer N is divisible by 3 if and only if the sum of its digits is divisible by 3. We know that $1+3+1+3+1+3 = 12$ is divisible by 3. Therefore, 131313 cannot be prime. In fact, $131313 = 3 \cdot 43771 = 3 \cdot 7 \cdot 13 \cdot 37$.

Quiz 2. *True/False*: Every rational number can be written as $\frac{a}{b}$, where $\gcd(a,b) = 1$.

Solution. The statement is *true*. By definition, a rational number r is a number of the form $\frac{a}{b}$, where a, b are integers and $b \neq 0$. Therefore, we can clearly write every rational number in the form $\frac{a}{b}$. Now can we impose the restriction that $\gcd(a,b) = 1$? Yes! By cancelling common factors from a, b, we can assure that the fraction is reduced, i.e. $\gcd(a,b) = 1$. In fact, we can always divide the numerator and denominator by $\gcd(a,b)$. After, we have $\gcd(a,b) = 1$. For instance, take $\frac{10}{15}$. We have $\gcd(10,15) = 5$. But then $\frac{10}{15} \cdot \frac{1/5}{1/5} = \frac{2}{3}$ is reduced.

Quiz 3. *True/False*: $\dfrac{(x^2)^3 x^5}{x^4} = x^6$

Solution. The statement is *false*. Recall that $x^a \cdot x^b = x^{a+b}$, $(x^a)^b = x^{ab}$, and $\frac{x^a}{x^b} = x^{a-b}$. We then have…

$$\frac{(x^2)^3 x^5}{x^4} = \frac{x^6 \cdot x^5}{x^4} = \frac{x^{11}}{x^4} = x^7$$

The mistake made was adding the powers in $(x^2)^3$ to obtain x^5 rather than multiplying the powers to obtain the correct x^6.

Quiz 4. *True/False*: $\sqrt{\sqrt[3]{x^2}} = x^{2/5}$

Solution. The statement is *false*. Recall that $\sqrt[n]{x^m} = x^{m/n}$ and $(x^a)^b = x^{ab}$. We then have…

$$\sqrt{\sqrt[3]{x^2}} = \sqrt{x^{2/3}} = (x^{2/3})^{1/2} = x^{\frac{2}{3} \cdot \frac{1}{2}} = x^{1/3} = \sqrt[3]{x}$$

The mistake made was adding the denominators rather than multiplying the powers correctly, i.e. $\sqrt{\sqrt[3]{x^2}} = \left((x^2)^{1/3}\right)^{1/2} = (x^2)^{1/5} = x^{2/5}$, which is incorrect.

Quiz 5. *True/False*: $(1-3i)(2+5i) = 17 - i$

Solution. The statement is *true*. Recall that $i^2 = -1$. Then we have…

$$(1-3i)(2+5i) = 1(2) + 1(5i) - 3i(2) - 3i(5i) = 2 + 5i - 6i - 15i^2 = 2 - i - 15(-1) = 2 - i + 15 = 17 - i$$

Quiz 6. *True/False*: If one increases 76 by 5% five times sequentially, the result is $76(1 + 0.25) = 76(1.25) = 95$.

Solution. The statement is *false*. If we want to compute N increased or decreased by a % a total of n times, we compute $N \cdot (1 \pm \%_d)^n$, where $\%_d$ is the percentage written as a decimal, n is the number of times we apply the percentage increase/decrease, and we choose '$+$' if it is a percentage increase and choose '$-$' if it is a percentage decrease. Then to compute 76 decreased by 5% consecutively five times, we need take $N = 76$, $\%_d = 0.05$, and choose '$+$'. Therefore, we have...

$$N \cdot (1 \pm \%_d)^n = 76(1 + 0.05)^5 = 76(1.05)^5 = 76(1.27628) = 96.9974$$

From the $76(1.27628)$ portion from the computation above, we can see that increasing a number by 5% consecutively five times actually results in a 27.628% increase in the original number's value because $1 + 0.27628 = 1.27628$. The mistake made in the quiz is thinking that repeated percentage increases or decreases are additive. An increase of 5% five times *does not* result in a $5 \cdot 5\% = 25\%$ increase, which was the percentage decrease computed in the quiz statement.

Quiz 7. *True/False*: The real number $0.3 \cdot 10^1$ is in scientific notation.

Solution. The statement is *false*. A real number is in scientific notation if it is expressed as $r \cdot 10^k$, where $1 \leq r < 10$ is a real number and k is an integer. In this case, we have $r = 0.3$ and $k = 1$. While $k = 1$ is an integer, $r \ngeq 1$. Properly writing this number in scientific notation, we have $3.0 \cdot 10^0$.

Quiz 8. *True/False*: Suppose one picks up one end of a slinky, leaving the other end flat on a table, and pulls the end into the air without stretching the slinky greatly or tilting the end (so that it remains parallel to the table). Knowing the diameter of the slinky and the height one pulled it into the air is sufficient to compute the volume contained within the slinky.

Solution. The statement is *true*. Recall that Cavalieri's Principle states that if two figures have the same height and cross sectional areas at every point along their height, they have the same volume. So long as we do not greatly stretch the slinky, once stretched, every cross section of the slinky will have the same area as cross section of the original slinky. Using Cavalieri's Principle, the stretched slinky will then have the same volume as a cylinder with the same diameter as the original slinky and as tall as the stretched slinky. If the diameter is d, then $r = \frac{d}{2}$. But then the volume is $V = A_{\text{base}}h = \pi r^2 h = \pi \left(\frac{d}{2}\right)^2 h = \frac{\pi d^2 h}{4}$.