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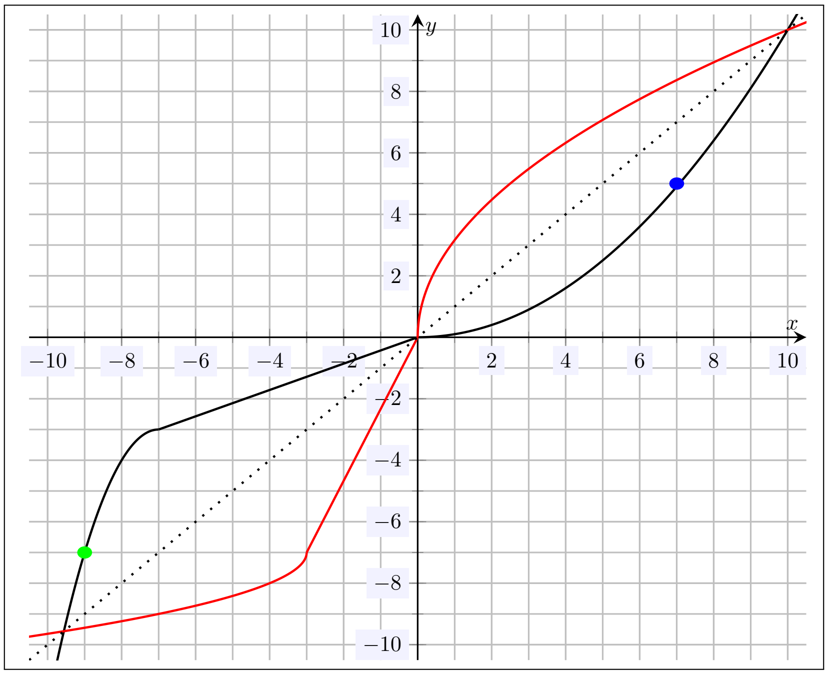
MATH 101
Fall 2023
HW 10: Due 10/30

"The study of mathematics, like the Nile, begins in minuteness but ends in magnificence."
–Charles Caleb Colton

Problem 1. (10pt) Consider the relation f plotted below.



(a) Compute $f(7)$ and $f(-9)$.

(b) Is $f(x)$ a function? Explain.

(c) Does $f(x)$ have an inverse? If so, sketch the inverse. If not, explain why.

Solution.

(a) Because the plot contains the points $(7, 5)$ and $(-9, -7)$, shown in plot above in blue and green, respectively, we can see that $f(7) = 5$ and $f(-9) = -7$.

(b) Yes, $f(x)$ is a function because it passes the vertical line test; that is, every vertical line intersects the relation at most once.

(c) Yes, the function $f(x)$ has an inverse because it passes the horizontal line test; that is, every horizontal line intersects the function at most once. We know that the graph of the inverse function for $f(x)$ is the reflection of $f(x)$ through the line $y = x$. We sketch this above in red. The line $y = x$ is sketched as a dotted black line.

Problem 2. (10pt) Showing all your work, verify that $g(x) = 4x + 9$ is the inverse function for $f(x) = \frac{x-9}{4}$. Also, compute $g(-2)$. What does the value of $g(-2)$ tell you about the function $f(x)$?

Solution. We know that $g = f^{-1}$ if $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$. We verify this:

$$\begin{aligned}(f \circ g)(x) &= f\big(g(x)\big) \\ &= f(4x+9) \\ &= \frac{(4x+9)-9}{4} \\ &= \frac{4x}{4} \\ &= x\end{aligned} \qquad \begin{aligned}(g \circ f)(x) &= g\big(f(x)\big) \\ &= g\left(\frac{x-9}{4}\right) \\ &= 4\left(\frac{x-9}{4}\right)+9 \\ &= (x-9)+9 \\ &= x\end{aligned}$$

Therefore, g is the inverse of f, i.e. $g = f^{-1}$.

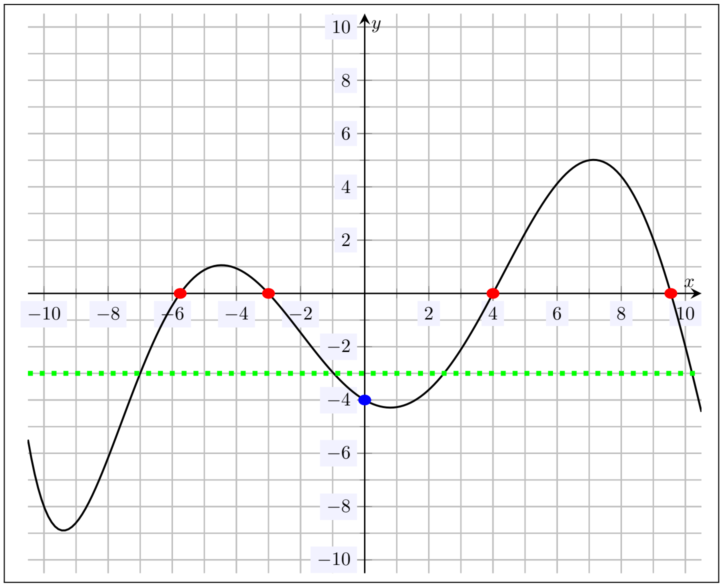
We have...

$$g(-2) = 4(-2) + 9 = -8 + 9 = 1$$

Of course, because $g = f^{-1}$, this tells us that $f^{-1}(-2) = 1$, i.e. $f(1) = -2$. We can verify this:

$$f(1) = \frac{1-9}{4} = \frac{-8}{4} = -2$$

Problem 3. (10pt) A relation ϕ is plotted below.



Using the plot above, answer the following:

(a) Compute $\phi(9)$.

(b) Find the y-intercept for $\phi(x)$.

(c) Find the x-intercepts for $\phi(x)$.

(d) As accurately as possible, compute the preimage of -3, i.e. $\phi^{-1}(-3)$.

(e) Explain why (d) implies that ϕ does not have an inverse function.

Solution.

(a) Because the plot contains the point $(9, 2)$, we can see that $\phi(9) = 2$.

(b) The y-intercept is where the relation intersects the y-axis. We can see from the plot, shown in blue, that the y-intercept is the point $(0, -4)$, i.e. the y-intercept is -4.

(c) The x-intercept(s) are the point(s) where the relation intersects the x-axis. We can see from the plot, shown in red, that the x-intercepts are $(-5.75251, 0)$, $(-3, 0)$, $(4, 0)$, and $(9.54717, 0)$, i.e. the x-intercepts are $-5.75251, -3, 4, 9.54717$.

(d) The value(s) of $\phi^{-1}(-3)$ are the x-values such that $\phi(x) = -3$. Graphically, these are x-values of the points on the curve that intersect the line $y = -3$, shown in green in the plot. Examining the plot, we can see that these are -7, -0.968256, 2.47042, and 10.2091, i.e. $\phi^{-1}(3) = \{-7, -0.968256, 2.47042, 10.2091\}$.

(e) We can see that the horizontal line at $y = -3$ intersects $\phi(x)$ more than once. Therefore, ϕ fails the horizontal line test. Therefore, ϕ does not have an inverse function. Alternatively, from (d), we see that $\phi^{-1}(-3)$ cannot be well-defined because there are 4 possible values for ϕ^{-1} as a function.