Quiz 1. *True/False*: The integer 45 has prime factorization $45 = 3 \cdot 15$, which shows that 3 and 15 are divisors of 45. Furthermore, we know that 1 is a multiple of 45.

Solution. The statement is *false*. While it is true that $45 = 3 \cdot 15$ is a *factorization* of 45, it is not a *prime factorization* of 45 because $15 = 3 \cdot 5$. The prime factorization of 45 is $45 = 3^2 \cdot 5$. It is true that if $45 = 3 \cdot 15$, then 3 and 15 are divisors of 45. Finally, while 1 is a divisor of 45 because $45 = 45 \cdot 1$, 1 is not a multiple of 45 because there is not an integer k such that $1 = 45k$.

Quiz 2. *True/False*: $\frac{\frac{2a}{b}}{\frac{4a}{bc}} = 8c$

Solution. The statement is *false*. We have…

$$\frac{\frac{2a}{b}}{\frac{4a}{bc}} = \frac{2a}{b} \cdot \frac{bc}{4a} = \frac{\cancel{2}\cancel{a}}{\cancel{b}} \cdot \frac{\cancel{b}c}{\cancel{4}2\cancel{a}} = \frac{c}{2}$$

Quiz 3. *True/False*: The expression $\dfrac{(xy^3)^{-2}}{(x^{-3}y^8)^2}$ when fully simplified is $\dfrac{x^4}{y^{22}}$.

Solution. The statement is *true*. We have…

$$\frac{(xy^3)^{-2}}{(x^{-3}y^8)^2} = \frac{x^{-2}y^{-6}}{x^{-6}y^{16}} = \frac{x^6}{x^2y^6y^{16}} = \frac{x^6}{x^2y^{22}} = \frac{x^4}{y^{22}}$$

Quiz 4. *True/False*: $\left(\dfrac{(x^2y^3)^4}{x^{-3}y^8}\right)^{-1/2} = \dfrac{1}{y^2\sqrt[11]{x^2}}$

Solution. The statement is *true*. We have…

$$\left(\frac{(x^2y^3)^4}{x^{-3}y^8}\right)^{-1/2} = \left(\frac{x^{-3}y^8}{(x^2y^3)^4}\right)^{1/2} = \left(\frac{x^{-3}y^8}{x^8y^{12}}\right)^{1/2} = \left(\frac{y^8}{x^3x^8y^{12}}\right)^{1/2} = \left(\frac{y^8}{x^{11}y^{12}}\right)^{1/2} = \left(\frac{1}{x^{11}y^4}\right)^{1/2} = \frac{1}{x^{11/2}y^{4/2}} = \frac{1}{y^2\sqrt{x^{11}}}$$

Therefore, the quiz statement is false. The quiz statement has $\sqrt[11]{x^2} = x^{2/11}$ instead of $\sqrt{x^{11}} = x^{11/2}$.

Quiz 5. *True/False*: The real number $0.123412341234\ldots$ is a rational number; therefore, one can find integers a, b such that $\frac{a}{b} = 0.123412341234\ldots$.

Solution. The statement is *true*. A rational number is a real number of the form $\frac{a}{b}$, where a, b are integers and $b \neq 0$. Equivalently, a rational number is a real number whose decimal expansion either terminates or repeats. Because the decimal expansion of $0.123412341234\ldots$ repeats, it must be that $0.123412341234\ldots$ is rational. Therefore, there must be integers a, b such that $\frac{a}{b} = 0.123412341234\ldots$. In fact, if $N = 0.123412341234\ldots$, we have...

$$\begin{array}{rrcc} & 10000N & = & 1234.12341234123412\overline{1234} \\ - & N & = & 0.12341234123412\overline{1234} \\ \hline & 9999N & = & 1234 \\ & N & = & \frac{1234}{9999} \end{array}$$

Quiz 6. *True/False*: Suppose a course has grade components of homework (50%), quizzes (10%), a midterm (20%), and a final (20%). If you had a 80% homework average, 75% quiz average, and received a 60% on the midterm, then your average is...

$$0.50(80\%) + 0.10(75\%) + 0.20(60\%) = 40\% + 7.5\% + 12\% = 59.5\%$$

Solution. The statement is *false*. One's course average is a weighted average where each percentage earned is weighted by the components worth. But then...

$$\text{Course Average} = \frac{\sum w_i x_i}{\sum w_i} = \frac{0.50 \cdot 0.80 + 0.10 \cdot 0.75 + 0.20 \cdot 0.60}{0.50 + 0.10 + 0.20} = \frac{0.40 + 0.075 + 0.12}{0.80} = \frac{0.595}{0.80} = 0.74375$$

Quiz 7. *True/False*: The real number $0.1 \cdot 10^3$ is in scientific notation.

Solution. The statement is *false*. A number in scientific notation is a real number in the form $R \cdot 10^n$, where $1 \leq |R| < 10$ and n is an integer. Observe that the given number is of the form $R \cdot 10^n$ with $R = 0.1$ and $n = 3$. But because $R = 0.1 < 1$, this number is not in scientific notation. Correctly written in scientific notation, the number $0.1 \cdot 10^3 = 0.1 \cdot 1000 = 100$ is $1 \cdot 10^2$.

Quiz 8. *True/False*: The surface area of a box that is open at the top with dimensions 1 ft $\times$ 8 in $\times$ 5 in is SA $= 12 \cdot 8 + 2(8 \cdot 5) + 2(12 \cdot 5) = 296$ in^2.

Solution. The statement is *true*. We know that the surface area of a 'box' is SA $= 2\ell w + 2\ell h + 2wh$. Because the box is open at the top, there is no surface area at the top of the box. The top of the box has surface area ℓw. But then the surface area of the described box is SA $= 2\ell w + 2\ell h + 2wh - \ell w = \ell w + 2\ell h + 2wh$. When one computes lengths, areas, volumes, etc., one need be sure that one is consistent with units. So we either have $\ell = 12$ in, $w = 8$ in, and $h = 5$ in or $\ell = 1$ ft, $w = \frac{8}{12}$ ft, and $h = \frac{5}{12}$ ft. In the former case, we have...

$$\text{SA} = \ell w + 2\ell h + 2wh = 12 \text{ in} \cdot 8 \text{ in} + 2(12 \text{ in})5 \text{ in} + 2(8 \text{ in})5 \text{ in} = 96 \text{ in}^2 + 120 \text{ in}^2 + 80 \text{ in}^2 = 296 \text{ in}^2$$

In the latter case, we have...

$$\text{SA} = \ell w + 2\ell h + 2wh = 1\text{ ft} \cdot \frac{8}{12}\text{ ft} + 2(1\text{ ft}) \cdot \frac{5}{12}\text{ ft} + 2\left(\frac{8}{12}\text{ ft}\right)\frac{5}{12}\text{ ft} = \frac{2}{3}\text{ ft}^2 + \frac{5}{6}\text{ ft}^2 + \frac{5}{9}\text{ ft}^2 = \frac{37}{18}\text{ ft}^2 \approx 2.05556\text{ ft}^2$$

We can then convert this to square inches: $\frac{37}{18}\text{ ft}^2 = \frac{37}{18}\text{ ft}^2 \cdot \frac{12\text{ in}}{1\text{ ft}} \cdot \frac{12\text{ in}}{1\text{ ft}} = 296\text{ in}^2$.

Quiz 9. *True/False*: The relation with domain $\mathbb{R}^3$ and codomain $\mathbb{R}$ given by $f(x, y, z) = x^2yz - yz^2 + 6$ is a function.

Solution. The statement is *true*. For each given input (x, y, z), there is only one possible output—namely the one obtained by 'plugging in' for x, y, z and following order of operations. For instance, $f(1, -1, 6) = 1^2(-1)6 - (-1)6^2 + 6 = -6 + 36 + 6 = 36$.

Quiz 10. *True/False*: If ψ is a function and $\psi(4) = 10 = \psi(-2)$, then ψ^{-1} exists and $\psi^{-1}(10) = 4$.

Solution. The statement is *false*. Recall that $\psi^{-1}(10)$ is the collection of values, x, such that $\psi(x) = 10$. Certainly, $x = 4$ is such a value because $\psi(4) = 10$. However, $x = -2$ is also a possible value because $\psi(-2) = 10$. But then we know that $\psi^{-1}(10)$ cannot be well-defined as a function because ψ does not have a single possible value for $\psi^{-1}(10)$.

Quiz 11. *True/False*: The point $(-\frac{1}{2}, 3)$ is on the graph of $f(x) = 4x + 5$.

Solution. The statement is *true*. If the point $(-\frac{1}{2}, 3)$ is on the graph of $f(-\frac{1}{2}) = 3$. We know that $f\left(-\frac{1}{2}\right) = 4 \cdot -\frac{1}{2} + 5 = -2 + 3 = 3$. Therefore, $(-\frac{1}{2}, 3)$ is on the graph of $f(x)$. Alternatively, if the point $(-\frac{1}{2}, 3)$ is on the graph of $f(x)$, then it satisfies the equation given by $f(x)$. But then...

$$\begin{aligned} f(x) &= 4x + 5 \\ f\left(-\frac{1}{2}\right) &\stackrel{?}{=} 4 \cdot -\frac{1}{2} + 5 \\ 3 &\stackrel{?}{=} -2 + 5 \\ 3 &= 3 \end{aligned}$$

Therefore, $(-\frac{1}{2}, 3)$ is on the graph of $f(x)$.

Quiz 12. *True/False*: There exists a function, f, with x-intercepts $-1, 0, 1$ such that f^{-1} exists.

Solution. The statement is *false*. If $f(x)$ is a function with x-intercepts $-1, 0, 1$, then $f(-1) = 0$, $f(0) = 0$, and $f(1) = 0$. Recall that $f^{-1}(y)$ is the set of x-values for which $f(x) = y$. Observe then that f^{-1} cannot be a function because $f^{-1}(0)$ is not well-defined because $f(-1) = f(0) = f(1) = 0$; that is, $f^{-1}(0)$ could be $-1, 0, 1$ so that $f^{-1}(0)$ is not well-defined.

Quiz 13. *True/False*: If you are driving down the highway at 65 mph from Albany to NYC, then your distance from NYC is given by a linear function.

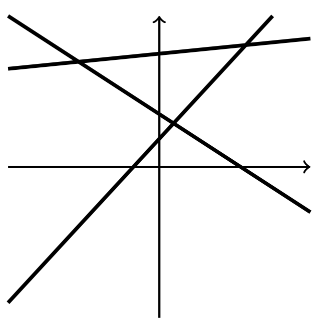
Solution. The statement is *true*. A linear function satisfies at least one of the following: (i) $f(x)$ has the form $y = mx + b$, (ii) a function with a constant rate of change, or (iii) a function whose graph is a line. Because you are driving at a constant rate of speed, your distance to NYC is decreasing at a constant rate. But then your distance to NYC must be a linear function. Alternatively, let $D(t)$ is your distance to NYC in t hours and let your initial distance to NYC be D_0 miles. Then $D(t) = D_0 - 65t$. But then $D(t)$ has the form $y = mx + b$ with $y = D(t)$, $x = t$, $m = -65$, and $b = D_0$. Therefore, $D(t)$ is a linear function.

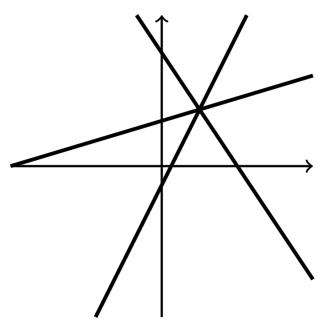
Quiz 14. *True/False*: There exists a horizontal line that is perpendicular to $y = 5x - 3$.

Solution. The statement is *false*. A line perpendicular to a horizontal line must be vertical. All vertical lines have the form $x = x_0$ for some number x_0. Clearly, $y = 5x - 3$ does not have the form $x = x_0$ so that it cannot be perpendicular to a horizontal line. Alternatively, perpendicular lines have negative reciprocal slopes. A line perpendicular to $y = 5x - 3$, which has slope 5, must have slope $-\frac{1}{5}$. But horizontal lines, i.e. lines of the form $y = y_0$ for some y_0 (which can be written $y = 0x + y_0$), have slope 0. As $0 \neq -\frac{1}{5}$, $y = 5x - 3$ cannot be perpendicular to a horizontal line.

Quiz 15. *True/False*: Three lines, none of which are parallel to the others, must intersect at a distinct point.

Solution. The statement is *false*. Certainly, because each line is not parallel to any of the other two lines, each pair of lines intersect. However, this only means that each pair of lines need intersect *not* that all the lines intersect at the *same* point. But then either of the two possibilities shown below are possible, so that it need not be the case that all the lines intersect at the same point.





Quiz 16. *True/False*: The quadratic function $f(x) = 6 - (x + 2)^2$ is convex and has vertex $(2, 6)$.

Solution. The statement is *false*. The vertex form of a quadratic function $f(x) = ax^2 + bx + c$ is $f(x)$ written in the form $f(x) = a_0(x - P)^2 + Q$, where $a_0 = a$ and (P, Q) is the vertex of $f(x)$. Observe that $f(x) = 6 - (x + 2)^2 = -(x + 2)^2 + 6 = -\big(x - (-2)\big)^2 + 6$. But then $(P, Q) = (-2, 6)$, so that the vertex is $(-2, 6)$, and $a = -1 < 0$. We know a quadratic function $f(x) = ax^2 + bx + c$ is convex if $a > 0$ and is concave if $a < 0$. Therefore, $f(x)$ is a quadratic function with vertex $(-2, 6)$ and is concave. The given statement incorrectly identifies the x-coordinate of the vertex as 2, rather than -2 (the x-value that makes the $(x + 2)^2$ term vanish) and also mistakes $a = 1$ rather than $a = -1$ so that the quadratic function is identified as being convex rather than concave.

Quiz 17. *True/False*: Let $ax^2 + bx + c$ be a quadratic function. If $x_0 = -\frac{b}{2a}$, then the vertex occurs at $\big(x_0, f(x_0)\big)$ and the minimum or maximum output of $f(x)$ is $f(x_0)$—depending on the value of a.

Solution. The statement is *true*. The location of the vertex is given by $x_0 = -\frac{b}{2a}$. The y-coordinate of the vertex must be given by the function value at x_0 which is $f(x_0)$. But then the vertex is $\big(x_0, f(x_0)\big)$. We know that the y-coordinate of the vertex is the either the minimum or maximum output of f depending on whether f opens upwards or downwards, respectively. If $a > 0$, then the parabola opens upwards and $f(x_0)$ is a minimum. If $a < 0$, then the parabola opens downwards and $f(x_0)$ is a maximum. We can see that the vertex is $\big(x_0, f(x_0)\big)$ via the following argument: note that

$$\begin{aligned}
f(x_0) &= f\left(-\frac{b}{2a}\right) \\
&= a\left(-\frac{b}{2a}\right)^2 + b\left(-\frac{b}{2a}\right) + c \\
&= a \cdot \frac{b^2}{4a^2} - \frac{b^2}{2a} + c \\
&= \frac{b^2}{4a} - \frac{b^2}{2a} + c \\
&= \frac{b^2}{4a} - \frac{2b^2}{4a} + \frac{4ac}{4a} \\
&= \frac{b^2 - 2b^2 + 4ac}{4a} \\
&= \frac{-b^2 + 4ac}{4a} \\
&= \frac{4ac - b^2}{4a}
\end{aligned}$$

But then we have. . .

$$\begin{aligned}
ax^2 + bx + c &= a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) \\
&= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{2^2a^2} - \frac{b^2}{2^2a^2} + \frac{c}{a}\right)
\end{aligned}$$

$$
\begin{aligned}
&= a\left(\left(x+\frac{b}{2a}\right)^2 - \frac{b^2}{2^2a^2} + \frac{c}{a}\right)\\
&= a\left(x+\frac{b}{2a}\right)^2 - \frac{b^2}{2^2a} + c\\
&= a\left(x-\frac{-b}{2a}\right)^2 - \frac{b^2}{4a} + c\\
&= a\left(x-\frac{-b}{2a}\right)^2 - \frac{b^2}{4a} + \frac{4ac}{4a}\\
&= a\left(x-\frac{-b}{2a}\right)^2 + \frac{4ac-b^2}{4a}\\
&= a\left(x-\frac{-b}{2a}\right)^2 + f(x_0)
\end{aligned}
$$

We see that this is the vertex form of the quadratic function $f(x)$ with vertex $(-\frac{b}{2a}, f(x_0)) = \big(x_0, f(x_0)\big)$.

Quiz 18. *True/False*: Let $f(x) = ax^2 + bx + c$ be a quadratic function. There will only be a distinct solution to the equation $f(x) = 0$ if the discriminant of $f(x)$ is zero.

Solution. The statement is *true*. Recall that the discriminant of a quadratic function $f(x) = ax^2 + bx + c$ is $D = b^2 - 4ac$. If $D < 0$, then $f(x)$ has two distinct, complex solutions. If $D > 0$, then $f(x)$ has two distinct real solutions. If $D = 0$, then D has one distinct, rational solution. The exact solution(s) to a quadratic function are given by the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-b \pm \sqrt{D}}{2a}$$

From the quadratic formula, we can also see that the nature of the roots depends only on D:

$$
\begin{aligned}
D < 0\colon x &= \frac{-b \pm \sqrt{D}}{2a} \implies x = \frac{-b \pm \sqrt{|D|}\, i}{2a} = \frac{-b}{2a} \pm \frac{\sqrt{|D|}}{2a}\, i\\
D > 0\colon x &= \frac{-b \pm \sqrt{D}}{2a} \implies x = \frac{-b}{2a} \pm \frac{1}{2a}\sqrt{D}\\
D = 0\colon x &= \frac{-b \pm \sqrt{D}}{2a} \implies x = \frac{-b}{2a}
\end{aligned}
$$

Quiz 19. *True/False*: For any polynomial $ax^2 + bx + c$, there exists a factorization $(x - r_1)(x - r_2)$, where r_1, r_2 are rational numbers.

Solution. The statement is *false*. Observe that there can never be an expression of the form $(x - r_1)(x - r_2)$ for the quadratic function $2x^2 + 3x + 1$ because $a = 2$ while $(x - r_1)(x - r_2) = x^2 - (r_1 + r_2)x + r_1r_2$ has $a = 1$. Furthermore, if every quadratic function could be expressed as

$(x - r_1)(x - r_2)$, every quadratic function would have rational roots r_1, r_2. However, we know that not all quadratic functions have rational roots—or even real roots. We can determine the nature of the roots from the discriminant of $ax^2 + bx + c$, which is $D = b^2 - 4ac$. If $D < 0$, then $f(x)$ has two distinct, complex solutions. If $D > 0$, then $f(x)$ has two distinct real solutions. If $D = 0$, then D has one distinct, rational solution. Therefore, the polynomial $ax^2 + bx + c$ will have an expression of the form $(x - r_1)(x - r_2)$ if and only if $D > 0$ and $a = 1$.