Name: *Caleb McWhorter — Solutions*
MATH 108
Fall 2023
HW 8: Due 10/17

"The consequences of an act affect the probability of its occurring again."
–B.F. Skinner

Problem 1. (10pt) The probabilities of several events in a finite probability space are given below:

$$\begin{aligned} P(A) &= 0.45 & P(D) &= 0.10 \\ P(B) &= 0.20 & P(A \text{ and } C) &= 0.01 \\ P(C) &= 0.85 & P(B \text{ and } C) &= 0.10 \end{aligned}$$

(a) Assuming that A and B are independent, find $P(A \text{ or } B)$.

(b) Assuming C and D are disjoint, find $P(C \text{ or } D)$.

(c) Are B and C disjoint? Explain.

(d) Are A and C independent? Explain.

(e) Find $P(B \mid C)$.

Solution.

(a) We know that $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. If A, B are independent, then $P(A \text{ and } B) = P(A)P(B)$. But then $P(A \text{ and } B) = P(A)P(B) = 0.45 \cdot 0.20 = 0.09$. But then...

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 0.45 + 0.20 - 0.09 = 0.56$$

(b) Because C and D are disjoint, $P(C \text{ and } D) = 0$. But then...

$$P(C \text{ or } D) = P(C) + P(D) - P(C \text{ and } D) = 0.85 + 0.10 - 0 = 0.95$$

Generally, if events E and F are disjoint, $P(E \text{ or } F) = P(E) + P(F)$.

(c) If B and C were disjoint, then $P(B \text{ or } C) = P(B)+P(C)$. But then $P(B \text{ or } C) = 0.20+0.85 = 1.05 > 1$, which is impossible. Therefore, B and C cannot be disjoint.

(d) If A and C were independent, then $P(A \text{ and } C) = P(A)P(C)$. But then $P(A \text{ and } C) = 0.01 \neq 0.3825 = 0.45 \cdot 0.85 = P(A)P(C)$. Therefore, A and C are not independent.

(e) We know that...

$$P(B \mid C) = \frac{P(B \text{ and } C)}{P(B)} = \frac{0.10}{0.20} = 0.50$$

Problem 2. (10pt) A statistician is examining tax rebates for small businesses in the area. She finds that of the 227 small businesses in the county, 109 qualified for a state tax rebate, 80 qualified for a federal tax rebate, and 38 qualified for both.

(a) Find the probability that a randomly selected local small business qualified for a state or federal tax rebate.

(b) Find the probability that a randomly selected local small business qualified for a state and federal tax rebate.

(c) Find the probability that a randomly selected local small business qualified for neither a state nor a federal tax rebate.

(d) Find the probability that a randomly selected local small business qualified for only a state tax rebate.

(e) Find the probability that a randomly selected local small business that qualified for a state tax rebate also qualified for a federal tax rebate.

Solution.

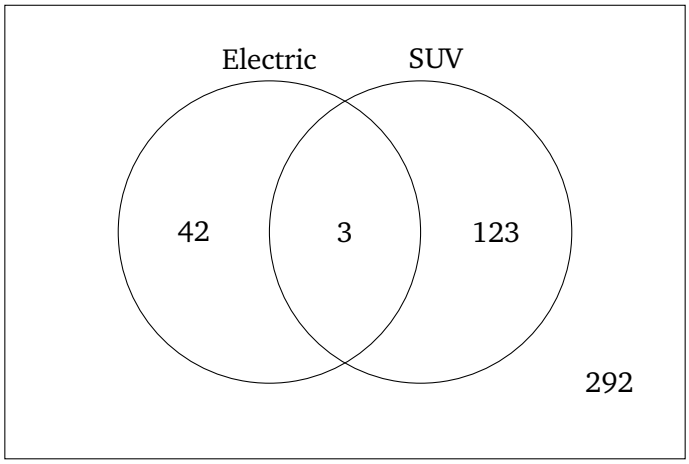
(a)

(b)

(c)

(d)

(e)



Problem 3. (10pt) A large accounting class has 156 students. A chart summarizing the pass/fail/withdraw results for students, broken down by class, is given below.

	Pass	Fail	Withdraw
Freshmen	41	14	6
Sophomore	56	11	3
Junior	18	3	1
Senior	3	0	0

Given the data above, answer the following:

(a) Find the probability that a randomly selected student failed the course.

(b) Find the probability that a randomly selected student was a sophomore or withdrew from the course.

(c) Find the probability that a randomly selected student was a junior and failed the course.

(d) Find the probability that a randomly selected freshman failed the course.

(e) Are freshmen status and failing the course independent events? Explain.

Solution. First, we should find the totals for each row/column:

	Pass	Fail	Withdraw	*Total*
Freshmen	41	14	6	61
Sophomore	56	11	3	70
Junior	18	3	1	22
Senior	3	0	0	3
Total	118	28	10	156

(a)
$$P(\text{failed}) = \frac{28}{156} = \frac{7}{39} \approx 0.1795$$

(b)
$$P(\text{soph or withd}) = \frac{70 + 10 - 3}{156} = \frac{77}{156} \approx 0.4936$$

(c)
$$P(\text{junior and fail}) = \frac{3}{156} = \frac{1}{52} \approx 0.0192$$

(d)
$$P(\text{fail} \mid \text{fresh}) = \frac{P(\text{fail and fresh})}{P(\text{fresh})} = \frac{14}{61} \approx 0.2295$$

(e) If they were independent, then $P(\text{fresh and fail}) = P(\text{fresh})P(\text{fail})$. But $P(\text{fresh and fail}) = \frac{14}{156} = 0.0897 \neq 0.0702 = \frac{61}{156} \cdot \frac{28}{156} = P(\text{fresh})P(\text{fail})$. Therefore, they are not independent. Alternatively, if they were independent, then $P(\text{fail} \mid \text{fresh}) = P(\text{fail})$. But by (a) and (d), $P(\text{fail} \mid \text{fresh}) = 0.2295 \neq 0.1795 = P(\text{fail})$.

Problem 4. (10pt) Administrators at a college are examining job placement for their graduates. Only 4% of their graduates are Computer Science majors. They find that 85% of their computer science majors obtain a job within 6 months of graduating. For all other majors at the college, 70% of their graduates find a job within 6 months of graduating.

(a) Find the percentage of graduates that received a job within 6 months of graduating.

(b) Find the percentage of graduates that were a computer science major and obtained a job within 6 months of graduating.

(c) Find the percentage of graduates that obtained a job within 6 months of graduating or were not a computer science major.

(d) Of the graduates that obtained a job within 6 months of graduating, what percentage were computer science majors?

Solution.

(a)

(b)

(c)

(d)