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MATH 108
Spring 2024
HW 13: Due 03/25

"That's the thing. I don't think I believe in deep down. I kinda think that all you are is just the things that you do."

— Diane Nguyen, BoJack Horseman

Problem 1. (10pts) Psychologists are examining how students cope with failure. They find estimates that 20% of individuals will fail at least one course during their lifetime. Suppose that you take a simple random sample of nine students.

(a) What is the probability that exactly 4 of them have failed a course?

(b) What is the probability that at least one of them has failed a course?

(c) What is the probability that less than 5 of them failed a course?

(d) What is the probability that exactly 8 of them failed a course?

Solution. Because this is a simple random sample, the samples are independent. There are a fixed number of students sampled. Each student either passed or failed a course with a fixed probability. Therefore, the number of persons in the sample that failed a course is given by a binomial distribution $B(n,p) = B(9, 0.20)$.

(a)

$$P(X = 4) = 0.0661$$

(b)

$$P(X \geq 1) = 1 - P(X = 0) = 1 - 0.1342 = 0.8658$$

(c)

$$\begin{aligned} P(X < 5) &= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) \\ &= 0.1342 + 0.3020 + 0.3020 + 0.1762 + 0.0661 \\ &= 0.9805 \end{aligned}$$

(d)

$$P(X = 8) \approx 0.$$

Problem 2. (10pts) A state senate race has grown increasing tense as scandal has struck the incumbents office, resulting a huge drop in their support. It is currently estimated that only 10% of voters currently support the incumbent. Suppose pollsters take a simple random sample of 15 potential voters.

(a) What is the probability that more than 8 of them support the incumbent?

(b) What is the probability that less than 4 of them support the incumbent?

(c) What is the probability that at least one of them support the incumbent?

Solution. Because the sample is a simple random sample, the samples are independent. There is a fixed number of people surveyed. Each person surveyed either supports the candidate or not with a fixed probability. Therefore, the number of people supporting the candidate is given by a binomial distribution $B(n, p) = B(15, 0.10)$.

(a)

$$P(X > 8) \approx 0.0$$

(b)

$$P(X < 4) = P(X = 0)+P(X = 1)+P(X = 2)+P(X = 3) = 0.2059+0.3432+0.2669+0.1285 = 0.9445$$

(c)

$$P(X \geq 1) = 1 - P(X = 0) = 1 - 0.2059 = 0.7941$$