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MATH 101
Fall 2021
HW 13: Due 11/09

"Science and technology revolutionize our lives, but memory, tradition, and myth frame our response."

–Arthur M. Schlesinger

Problem 1. (10pt) Showing all your work, find the domain, vertical asymptotes, and zeros of the following function:

$$f(x) = \frac{x-5}{x+3}$$

Solution. Observe that the numerator and denominator are already factored. The domain is the set of real numbers where the denominator is not zero. But if $x + 3 = 0$, then $x = -3$. Therefore, the domain is the set of real numbers such that $x \neq -3$. This also implies that the only vertical asymptote is the line $x = -3$. The zeros are the set of values such that the numerator is 0. But then $x - 5 = 0$. This implies that $x = 5$. Therefore, the only zero is $x = 5$.

$$\boxed{\begin{aligned} \text{Domain: } & x \in \mathbb{R}, x \neq -3 \\ \text{Vertical Asymptotes: } & x = -3 \\ \text{Zeros: } & x = 5 \end{aligned}}$$

Problem 2. (10pt) Showing all your work, find the domain, vertical asymptotes, and zeros of the following function:

$$g(x) = \frac{x^2 + 5x + 6}{x - 1}$$

Solution. First, we factor the numerator and the denominator:

$$g(x) = \frac{x^2 + 5x + 6}{x - 1} = \frac{(x+1)(x+5)}{x - 1}$$

The domain is the set of real numbers where the denominator is not zero. But if $x - 1 = 0$, then $x = 1$. Therefore, the domain is the set of real numbers such that $x \neq 1$. This also implies that the only vertical asymptote is the line $x = 1$. The zeros are the set of values such that the numerator is 0. But then $(x+1)(x+5) = 0$. This implies that either $x + 1 = 0$, i.e. $x = -1$, or $x + 5 = 0$, i.e. $x = -5$. Therefore, the zeros are $x = -5, -1$.

$$\boxed{\begin{array}{rl} \text{Domain:} & x \in \mathbb{R}, x \neq 1 \\ \text{Vertical Asymptotes:} & x = 1 \\ \text{Zeros:} & x = -5, -1 \end{array}}$$

Problem 3. (10pt) Showing all your work, find the domain, vertical asymptotes, and zeros of the following function:

$$h(x) = \frac{x + 10}{x^2 - 2x - 8}$$

Solution. First, we factor the numerator and the denominator:

$$h(x) = \frac{x + 10}{x^2 - 2x - 8} = \frac{x + 10}{(x - 4)(x + 2)}$$

The domain is the set of real numbers where the denominator is not zero. But if $(x-4)(x+2) = 0$, then either $x - 4 = 0$, i.e. $x = 4$, or $x + 2 = 0$, i.e. $x = -2$. Therefore, the domain is the set of real numbers such that $x \neq -2, 4$. This also implies that the only vertical asymptotes are the lines $x = -2$ and $x = 4$. The zeros are the set of values such that the numerator is 0. But then $x+10 = 0$, i.e. $x = -10$. Therefore, the only zero is $x = -10$.

$$\boxed{\begin{array}{rl} \text{Domain:} & x \in \mathbb{R}, x \neq -2, 4 \\ \text{Vertical Asymptotes:} & x = -2, x = 4 \\ \text{Zeros:} & x = -10 \end{array}}$$

Problem 4. (10pt) Showing all your work, find the domain, vertical asymptotes, and zeros of the following function:

$$j(x) = \frac{x^2 + 3x - 4}{x^2 - 4x + 3}$$

Solution. First, we factor the numerator and the denominator:

$$j(x) = \frac{x^2 + 3x - 4}{x^2 - 4x + 3} = \frac{(x+4)(x-1)}{(x-3)(x-1)}$$

The domain is the set of real numbers where the denominator is not zero. But if $(x-3)(x-1) = 0$, then either $x - 3 = 0$, i.e. $x = 3$, or $x - 1 = 0$, i.e. $x = 1$. Therefore, the domain is the set of real numbers such that $x \neq 1, 3$. Now that the domain has been found and there are terms to cancel, we simplify the expression for $j(x)$.

$$j(x) = \frac{(x+4)\cancel{(x-1)}}{(x-3)\cancel{(x-1)}} = \frac{x+4}{x-3}$$

The vertical asymptotes are where the denominator is 0. But then $x - 3 = 0$, i.e. $x = 3$. Therefore, the only vertical asymptote is $x = 3$. The zeros are the set of values such that the numerator is 0. But then $x + 4 = 0$, i.e. $x = -4$. Therefore, the only zero is $x = -4$.

$$\boxed{\begin{array}{r l} \text{Domain:} & x \in \mathbb{R}, x \neq 1, 3 \\ \text{Vertical Asymptotes:} & x = 3 \\ \text{Zeros:} & x = -4 \end{array}}$$