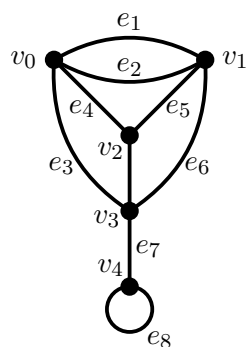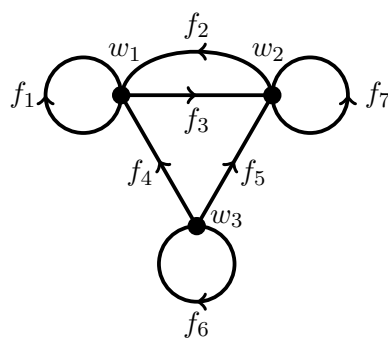*"The origins of graph theory are humble, even frivolous."*

$-Norman\ Biggs$

Problem 1. (10pt) Consider the graphs G and H given below.

G

H

(a) Find $\deg v_2$, $\deg v_4$, and $\deg G$.

(b) Is G connected? Explain. Is G simple? Explain.

(c) Find $\deg^+ w_1$ and $\deg^- w_1$ as well as $\deg^+ w_2$ and $\deg^- w_2$.

(d) Find a trail in H that is not a path.

(e) Does H have any sources or sinks? Explain.

Problem 2. (10pt) Suppose you have two graphs, G and H, where G is undirected and H may be undirected or directed. The adjacency matrices of G and H are given below.

$$\underbrace{\begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{pmatrix}}_{G} \qquad \underbrace{\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 & 0 \end{pmatrix}}_{H}$$

(a) Draw the graph of G.

(b) Does G have a loop? Justify your answer using *only* the adjacency matrix of G.

(c) Draw the graph of H.

(d) Is H undirected or directed? Justify your answer using *only* the adjacency matrix of H.

(e) Do G or H have multiple edges? Explain your answer using only the adjacency matrix of G and H, respectively.

Problem 3. (10pt) Showing all your work and fully justifying your responses, complete the following:

(a) Draw the graph of K_5. How many vertices and edges does K_5 have? For $n \geq 1$, what are $|V(K_n)|$ and $|E(K_n)|$?

(b) Draw the graph of $K_{3,5}$. How many vertices and edges does $K_{3,5}$ have? For $m, n \geq 1$, what are $|V(K_{m,n})|$ and $|E(K_{m,n})|$?

(c) If G is a simple graph, the complement of G, denoted $\widetilde{G}$, is a graph with $V(G) = V(\widetilde{G})$ and two vertices are adjacent in $\widetilde{G}$ if and only if they are not adjacent in G. Find the complement of the graph given below. Is G connected? Is $\widetilde{G}$ connected?
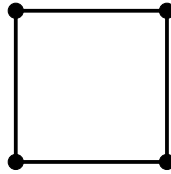
Problem 4. (10pt) Below are three graphs: G_1, G_2, and G_3. Determine which, if any, of the graphs are isomorphic. If two given graphs are isomorphic, show that they are isomorphic. If two graphs are not isomorphic, give at least two reasons why they are not isomorphic.

G_1

G_2

G_3