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MATH 100
Fall 2021
HW 12: Due 11/15

"Reality continues to ruin my life."
–Bill Watterson

Problem 1. (10pt) Showing all your work, find the domain, vertical asymptotes, and zeros of the following function:

$$f(x) = \frac{x+6}{x-1}$$

Solution. Observe that the numerator and denominator are already factored. The domain is the set of real numbers where the denominator is not zero. But if $x - 1 = 0$, then $x = 1$. Therefore, the domain is the set of real numbers such that $x \neq 1$. This also implies that the only vertical asymptote is the line $x = 1$. The zeros are the set of values such that the numerator is 0. But then $x + 6 = 0$. This implies that $x = -6$. Therefore, the only zero is $x = -6$.

$$\boxed{\begin{aligned} \text{Domain: } & x \in \mathbb{R}, x \neq 1 \\ \text{Vertical Asymptotes: } & x = 1 \\ \text{Zeros: } & x = -6 \end{aligned}}$$

Problem 2. (10pt) Showing all your work, find the domain, vertical asymptotes, and zeros of the following function:

$$g(x) = \frac{x^2 - 2x - 8}{x + 3}$$

Solution. First, we factor the numerator and the denominator:

$$g(x) = \frac{x^2 - 2x - 8}{x + 3} = \frac{(x-4)(x+2)}{x+3}$$

The domain is the set of real numbers where the denominator is not zero. But if $x + 3 = 0$, then $x = -3$. Therefore, the domain is the set of real numbers such that $x \neq -3$. This also implies that the only vertical asymptote is the line $x = -3$. The zeros are the set of values such that the numerator is 0. But then $(x-4)(x+2) = 0$. This implies that either $x - 4 = 0$, i.e. $x = 4$, or $x + 2 = 0$, i.e. $x = -2$. Therefore, the zeros are $x = -2, 4$.

Domain:	$x \in \mathbb{R}, x \neq -3$
Vertical Asymptotes:	$x = -3$
Zeros:	$x = -2, 4$

Problem 3. (10pt) Showing all your work, find the domain, vertical asymptotes, and zeros of the following function:

$$h(x) = \frac{x-5}{x^2+8x+12}$$

Solution. First, we factor the numerator and the denominator:

$$h(x) = \frac{x-5}{x^2+8x+12} = \frac{x-5}{(x+2)(x+6)}$$

The domain is the set of real numbers where the denominator is not zero. But if $(x+2)(x+6)=0$, then either $x+2=0$, i.e. $x=-2$, or $x+6=0$, i.e. $x=-6$. Therefore, the domain is the set of real numbers such that $x \neq -6, -2$. This also implies that the only vertical asymptotes are the lines $x=-6$ and $x=-2$. The zeros are the set of values such that the numerator is 0. But then $x-5=0$. Therefore, the only zero is $x=5$.

$$\boxed{\begin{array}{rl} \text{Domain: } & x \in \mathbb{R}, x \neq -6, -2 \\ \text{Vertical Asymptotes: } & x=-6, x=-2 \\ \text{Zeros: } & x=5 \end{array}}$$

Problem 4. (10pt) Showing all your work, find the domain, vertical asymptotes, and zeros of the following function:

$$j(x) = \frac{x^2 - x - 12}{x^2 - 3x - 18}$$

Solution. First, we factor the numerator and the denominator:

$$j(x) = \frac{x^2 - x - 12}{x^2 - 3x - 18} = \frac{(x-4)(x+3)}{(x-6)(x+3)}$$

The domain is the set of real numbers where the denominator is not zero. But if $(x-6)(x+3) = 0$, then either $x - 6 = 0$, i.e. $x = 6$, or $x + 3 = 0$, i.e. $x = -3$. Therefore, the domain is the set of real numbers such that $x \neq -3, 6$. Now that the domain has been found and there are terms to cancel, we simplify the expression for $j(x)$.

$$j(x) = \frac{(x-4)\cancel{(x+3)}}{(x-6)\cancel{(x+3)}} = \frac{x-4}{x-6}$$

The vertical asymptotes are where the denominator is 0. But then $x - 6 = 0$, i.e. $x = 6$. Therefore, the only vertical asymptote is $x = 6$. The zeros are the set of values such that the numerator is 0. But then $x - 4 = 0$, i.e. $x = 4$. Therefore, the only zero is $x = 4$.

$$\boxed{\begin{array}{rl} \text{Domain:} & x \in \mathbb{R}, x \neq -3, 6 \\ \text{Vertical Asymptotes:} & x = 6 \\ \text{Zeros:} & x = 4 \end{array}}$$