

MAT 104: Exam 1
Spring – 2023
03/03/2023
85 Minutes

Name: _____

Write your name on the appropriate line on the exam cover sheet. This exam contains 16 pages (including this cover page) and 15 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
14	10	
15	10	
Total:	150	

1. (10 points) Suppose that $f(x)$ is a function with domain $[-10, 10)$ and $g(x)$ is a function with domain $(-5, 15]$. Several outputs for the functions $f(x)$ and $g(x)$ are given below.

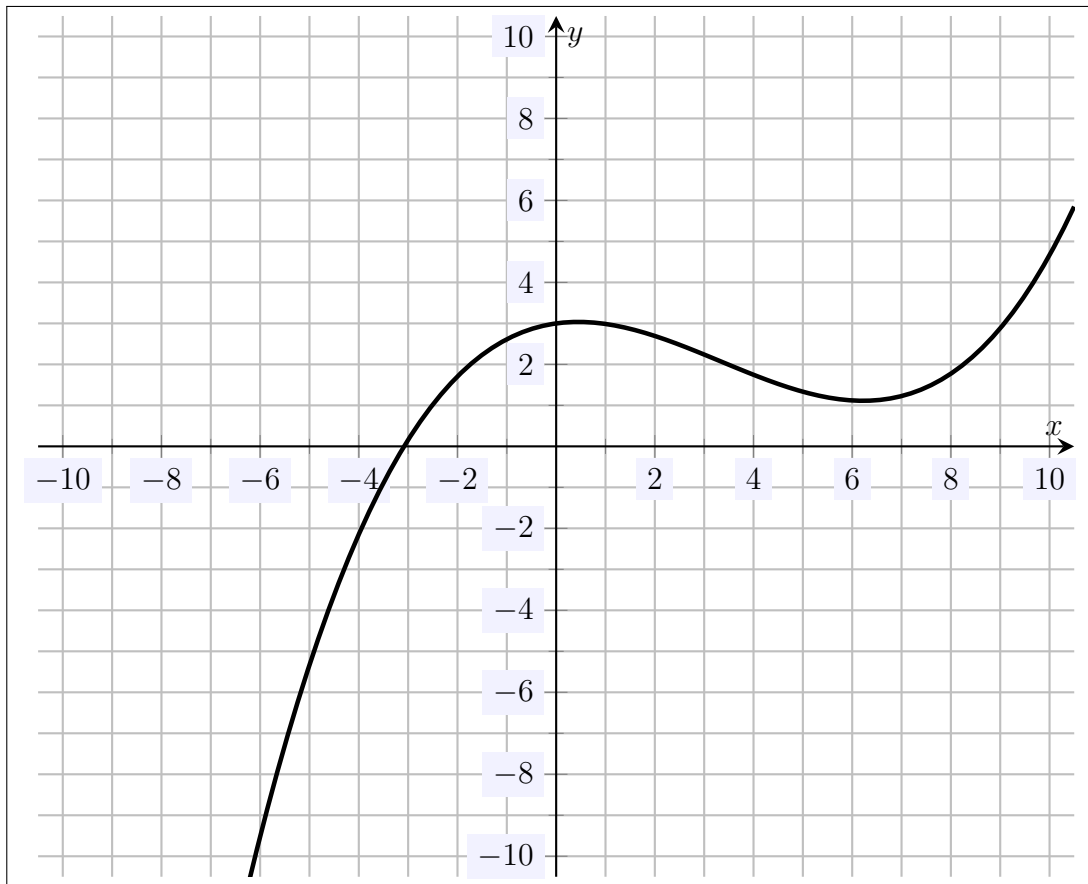
x	0	1	2	3	5
$f(x)$	3	0	2	4	4
$g(x)$	3	5	1	2	0

- (a) What is the domain of $f - g$?
- (b) Given the information above, what is the largest possible domain for $\frac{f}{g}$?
- (c) Find $(f + g)(2)$.
- (d) Find $(fg)(0)$.
- (e) Find $(g \circ f)(0)$.

2. (10 points) Let $f(x)$ be an exponential function with $f(-2) = 12$ and $f(3) = \frac{3}{8}$. Showing all your work, find $f(x)$.

3. (10 points) Showing all your work, find the inverse of the function $f(x) = \log_3(x^2) - 4$.

4. (10 points) A function $f(x)$ is plotted below. As accurately as possible, sketch the function $4 - f(x + 3)$ on the graph below.



5. (10 points) Let $f(x) = 3 - 2x$ and $g(x) = x^2 + x - 1$. Showing all your work and simplifying as much as possible, find the following:

- (a) $f(g(0))$
- (b) $(2f - g)(0)$
- (c) $(g - f)(x)$
- (d) $(f \circ g)(x)$
- (e) $(g \circ f)(x)$

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6. (10 points) Showing all your work, find the inverse of the function $f(x) = 3e^{1-2x}$.

7. (10 points) Let $f(x)$ be the function $f(x) = \frac{2^{3x+1}}{5^{1-x}}$.

- (a) Write $f(x)$ in the form ab^x for some a, b .
- (b) Is $f(x)$ increasing or decreasing?
- (c) Is $f(x)$ concave up or concave down?

8. (10 points) Showing all your work, find the exact solution to the following:

$$\log_2(50 - e^{x+1}) + 5 = 10$$

9. (10 points) Showing all your work, write the following as a sum of terms involving only $\log_2(x)$, $\log_2(y)$, and possibly a constant:

$$\log_2 \left(\frac{16\sqrt[3]{x}}{y^{-5}} \right)$$

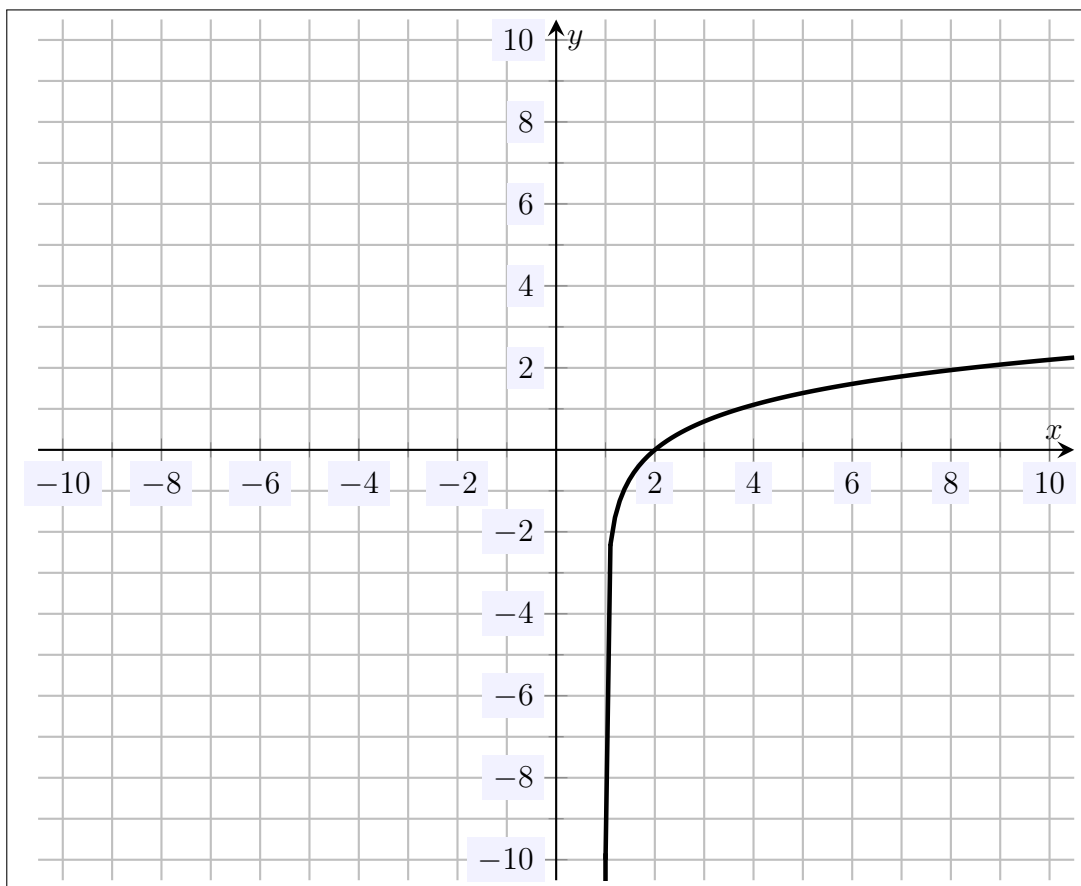
10. (10 points) Let $f(x) = 2 - e^x$ and $g(x) = 6 \log_5(2 - x)$.

- (a) What is the domain of $f(x)$?
- (b) What is the range of $f(x)$?
- (c) What is the domain of $g(x)$?
- (d) What is the range of $g(x)$?

11. (10 points) Showing all your work, find the exact solution to the following:

$$3^{2x} = 5 \cdot 2^x$$

12. (10 points) A relation $f(x)$ is plotted below.



- (a) Using the plot above, explain why $f(x)$ is a function.
- (b) Using the plot above, explain why $f^{-1}(x)$ exists.
- (c) Sketch the function $f^{-1}(x)$ on the plot above.

13. (10 points) Showing all your work, write each of the following as a single logarithm involving no negative powers:

(a) $\ln(x) + 3\ln(y)$

(b) $\log_5(x) - \log_5(y^{-2})$

(c) $4\log_3(x) - \frac{1}{2}\log_3(y) + 2$

14. (10 points) Showing all your work, compute the following “by hand”:

(a) $\ln(e^{3/2})$

(b) $\log_5(\sqrt{5})$

(c) $\log_4\left(\frac{1}{64}\right)$

(d) $\log_9(3)$

(e) $\log_8(128)$

15. (10 points) If one invests P dollars at an annual interest rate r (written as a decimal), compounded monthly, then the amount of money in the account after t years, $M(t)$, is given by...

$$M(t) = P \left(1 + \frac{r}{12} \right)^{12t}$$

Suppose that you invest \$8,000 at an annual interest rate of 6.2%, compounded monthly.

- (a) Find the value of the investment after 5 years.
- (b) Find how long until the investment is worth \$20,000.