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MATH 101
Fall 2021
HW 9: Due 10/29

"Laziness is nothing more than the habit of resting before you get tired."

–Jules Renard

Problem 1. (10pt) Find the vertex form of the quadratic function $y = x^2 + 6x + 4$.

Solution. The x-coefficient is 6. We have $(\frac{1}{2} \cdot 6)^2 = 3^2 = 9$. Then we have. . .

$$y = x^2 + 6x + 4$$

$$y = x^2 + 6x + (9 - 9) + 4$$

$$y = (x^2 + 6x + 9) - 9 + 4$$

$$y = (x + 3)^2 - 5$$

Problem 2. (10pt) Find the vertex form of the quadratic function $y = x^2 - 6x - 7$.

Solution. The x-coefficient is -6. We have $(\frac{1}{2} \cdot -6)^2 = (-3)^2 = 9$. Then we have...

$$y = x^2 - 6x - 7$$

$$y = x^2 - 6x + (9 - 9) - 7$$

$$y = (x^2 - 6x + 9) - 9 - 7$$

$$y = (x - 3)^2 - 16$$

Problem 3. (10pt) Find the vertex form of the quadratic function $y = 4x^2 - 4x + 7$.

Solution. We factor out the 4. This gives us $y = 4(x^2 - x + 7/4)$. The x-coefficient is -1. We have $(\frac{1}{2} \cdot -1)^2 = (-1/2)^2 = 1/4$. Then we have...

$$y = 4\left(x^2 - x + \frac{7}{4}\right)$$

$$y = 4\left(x^2 - x + \left(\frac{1}{4} - \frac{1}{4}\right) + \frac{7}{4}\right)$$

$$y = 4\left(\left(x^2 - x + \frac{1}{4}\right) - \frac{1}{4} + \frac{7}{4}\right)$$

$$y = 4\left(\left(x - \frac{1}{2}\right)^2 + \frac{6}{4}\right)$$

$$y = 4\left(x - \frac{1}{2}\right)^2 + 6$$

Problem 4. (10pt) Consider the quadratic function $f(x) = x^2 + 14x - 9$.

(a) Determine if the parabola opens upwards or downwards.

(b) Is the parabola convex or concave?

(c) Does the parabola have a maximum or minimum?

(d) Find the vertex and axis of symmetry.

(e) Find the maximum/minimum value of $f(x)$.

Solution.

(a) Because $a = 1 > 0$, the parabola opens upwards, i.e. the parabola is convex.

(b) Because the parabola opens upwards, it is convex.

(c) Because the parabola opens upwards, the vertex is a minimum.

(d) The vertex occurs when $x = -\frac{b}{2a} = -\frac{14}{2(1)} = -7$. But then the axis of symmetry is $x = -7$. We have

$$y(-7) = (-7)^2 + 14(-7) - 9 = 49 - 98 - 9 = -58$$

Therefore, the vertex is $(-7, -58)$. Alternatively, putting the parabola in vertex form:

$$y = x^2 + 14x - 9$$

$$y = x^2 + 14x + 49 - 49 - 9$$

$$y = (x - 7)^2 - 58$$

we can easily see that the vertex is $(-7, -58)$ and that the axis of symmetry is $x = -7$.

(e) Because the parabola opens upwards, the parabola has a minimum. The minimum occurs at the vertex. The vertex is $(-7, -58)$. Therefore, the maximum value is -58.

Problem 5. (10pt) Consider the quadratic function $f(x) = -2x^2 + 3x + 1$.

(a) Determine if the parabola opens upwards or downwards.

(b) Is the parabola convex or concave?

(c) Does the parabola have a maximum or minimum?

(d) Find the vertex and axis of symmetry.

(e) Find the maximum/minimum value of $f(x)$.

Solution.

(a) Because $a = -2 < 0$, the parabola opens downwards, i.e. the parabola is concave.

(b) Because the parabola opens downwards, it is concave.

(c) Because the parabola opens downwards, the vertex is a maximum.

(d) The vertex occurs when $x = -\frac{b}{2a} = -\frac{3}{2(-2)} = \frac{3}{4}$. But then the axis of symmetry is $x = \frac{3}{4}$. We have

$$y\left(\frac{3}{4}\right) = -2\left(\frac{3}{4}\right)^2 + 3\left(\frac{3}{4}\right) + 1 = -2 \cdot \frac{9}{16} + \frac{9}{4} + 1 = -\frac{18}{16} + \frac{36}{4} + \frac{16}{16} = \frac{-18 + 36 + 16}{16} = \frac{34}{16} = \frac{17}{8}$$

Therefore, the vertex is $(\frac{3}{4}, \frac{17}{8})$. Alternatively, putting the parabola in vertex form:

$$y = -2x^2 + 3x + 1$$

$$y = -2(x^2 - \frac{3}{2}x - \frac{1}{2}$$

$$y = -2\left(x^2 - \frac{3}{2}x + \frac{9}{16} - \frac{9}{16}\right)$$

$$y = -2\left(\left(x - \frac{3}{4}\right)^2 - \frac{17}{16}\right)$$

$$y = -2\left(x - \frac{3}{4}\right)^2 + \frac{17}{4}$$

we can easily see that the vertex is $(\frac{3}{4}, \frac{17}{8})$ and that the axis of symmetry is $x = \frac{3}{4}$.

(e) Because the parabola opens downwards, the parabola has a maximum. The maximum occurs at the vertex. The vertex is $(\frac{3}{4}, \frac{17}{8})$. Therefore, the maximum value is $\frac{17}{8}$.