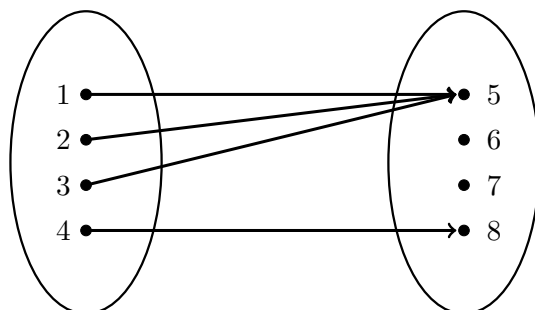


Quiz 1. *True/False:* The following relation represents a function:



Solution. The statement is *true*. A relation is a function if for every input, i.e. every value in the domain, there is one output. For each of the values 1, 2, 3, 4, we know the value of the output. It does not matter that several of the values get ‘sent’ to the same value—only that we know what value they go to.

Quiz 2. *True/False:* If $C(x)$ is a cost function, then the y -intercept of $C(x)$ is the fixed costs.

Solution. The statement is *true*. The y -intercept of a function is the value where $x = 0$ (which means the function is intersecting the y -axis). But then we are looking at the value of $C(0)$, which represents the costs of producing zero items. But any costs associated with producing zero items must be a ‘baseline’ cost for production, i.e. the fixed costs.

Quiz 3. *True/False:* The point $(1, 1)$ is the solution to the system of equations:

$$2x + y = 3$$

$$x - y = 1$$

Solution. The statement is *false*. The simplest way of seeing this is to check if $(x, y) = (1, 1)$ is a solution to the system, i.e. check if $x = 1$ and $y = 1$ satisfies both equations: $2(1) + 1 = 3$ but $1 - 1 = 0 \neq 1$. Therefore, $(1, 1)$ is not the solution to the system of equations. We can also determine if $(1, 1)$ is a solution to the system of equations by solving the system:

$$\begin{array}{rcl} 2x + y & = & 3 \\ x - y & = & 1 \\ \hline 3x & = & 4 \\ x & = & 4/3 \end{array}$$

Then certainly, we cannot have $x = 1$. We then can find that $y = x - 1 = 4/3 - 1 = 1/3$. Then the correct solution is $(x, y) = (4/3, 1/3)$.

Quiz 4. *True/False:* If $P(x)$ is a profit function, then $P(0)$ is a breakeven point.

Solution. The statement is *false*. Recall that a breakeven point is when revenue, $R(x)$, equals cost, $C(x)$. But this is $R(x) = C(x)$. This is the same as $R(x) - C(x) = 0$. But $P(x) = R(x) - C(x)$ so that $P(x) = 0$. Alternatively, a breakeven point can be defined as a x value when the profit is 0, i.e. $P(x) = 0$. However, this does not mean that $x = 0$, as in the problem statement.

Quiz 5. *True/False:* Consider the following system of equations:

$$\begin{aligned}2x + 3y &= -1 \\ x - 2y &= 5\end{aligned}$$

Then the associated augmented matrix is

$$\left(\begin{array}{cc|c}2 & 3 & -1 \\ 1 & -2 & 5\end{array}\right)$$

Solution. The statement is *true*. Remember, the augmented matrix is the matrix consisting of all the coefficients for the variables ('in order') in each equation for each of the equations ('in order'). Then the to right of the rightmost column, we add (augment) the matrix with the solution vector, i.e. the vector of the solutions for the equations ('in order'). Applying this to the given system of equations, we obtain exactly

$$\left(\begin{array}{cc|c}2 & 3 & -1 \\ 1 & -2 & 5\end{array}\right)$$

Quiz 6. *True/False:* A system of solutions to the system of equations given by the augmented matrix

$$\left(\begin{array}{ccccc}1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & -3 & -1 \\ 0 & 0 & 1 & 0 & 0\end{array}\right)$$

is $x_3 = 0$, $x_4 = \text{free}$, $x_1 = 2 - x_4$, and $x_2 = 3x_4 - 1$.

Solution. The statement is *true*. First, observe that the matrix is in reduced row echelon form so that we should be able to read off the solutions. Because the matrix is augmented, each column (except the last) of the matrix corresponds to one of the variables and the last column corresponds to the solution to each of the corresponding equations. Writing out the corresponding system of equations, we have

$$\begin{aligned}1x_1 + 0x_2 + 0x_3 + 1x_4 &= 2 \\ 0x_1 + 1x_2 + 0x_3 - 3x_4 &= -1 \\ 0x_1 + 0x_2 + 1x_3 + 0x_4 &= 0\end{aligned}$$

which of course is equivalent to...

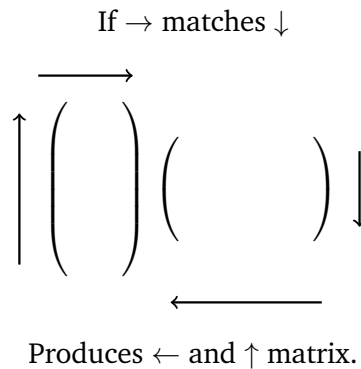
$$\begin{aligned}x_1 + x_4 &= 2 \\ x_2 - 3x_4 &= -1 \\ x_3 &= 0\end{aligned}$$

From this system, we see that $x_3 = 0$. Letting x_4 be free, we see that $x_1 = 2 - x_4$ and $x_2 = 3x_4 - 1$. But then a system of solutions to the equations is

$$\begin{aligned}x_1 &= 2 - x_4 \\x_2 &= 3x_4 - 1 \\x_3 &= 0 \\x_4 &: \text{free}\end{aligned}$$

Quiz 7. *True/False:* If A is a 5×3 matrix and B is a 3×6 matrix, you can compute AB , which will be a 3×6 matrix.

Solution. The statement is *false*. Recall that a $n \times m$ matrix A and a $p \times q$ matrix B can be multiplied if $m = p$. If so, they form a $n \times q$ matrix. So while we can multiply A and B , they form a 5×6 matrix.



Quiz 8. *True/False:* If $\det A = 0$, then A does not have an inverse.

Solution. The statement is *true*. If $\det A = 0$, then A does not have an inverse. Conversely, it is true that if A does not have an inverse that $\det A = 0$. Therefore, if A has an inverse, then $\det A \neq 0$ and that if $\det A \neq 0$, then A has an inverse.

Quiz 9. *True/False:* For any matrix A , if we have $Ax = b$, then $x = A^{-1}b$.

Solution. The statement is *false*. For A^{-1} to exist, the matrix A needs to be invertible. If A is invertible, then A^{-1} exists. But then we have

$$\begin{aligned}Ax &= b \\A^{-1}Ax &= A^{-1}b \\I_n x &= A^{-1}b \\x &= A^{-1}b\end{aligned}$$

Not all matrices are invertible.

Quiz 10. *True/False:* Any function on a nonempty region given by a collection of inequalities has a maximum and minimum value which occur at a corner point of the region.

Solution. The statement is *true*. This is the Fundamental Theorem of Linear Programming.

Quiz 11. *True/False:* The pivot position is given by the entry whose column contains the most negative entry in the bottom row and the smallest positive ratio in its row.

Solution. The statement is *true*. This is the procedure to choose the pivot position. For instance, in the tableau below, the most negative entry in the bottom row is in the third column. Using the third column and examining the ratios in each rows, we see that the second row has the smallest *positive* ration.

$$\begin{array}{ccccc|c}
 5 & -2 & 3 & 1 & -4 & 9 & 9/3 = 3 \\
 -1 & 0 & \textcircled{5} & 6 & 2 & 10 & 10/5 = 2 \\
 6 & 1 & 2 & 0 & -5 & -20 & -20/2 = -10 \\
 \hline
 8 & 0 & -4 & 1 & -3 & 0 &
 \end{array}$$

Therefore, the circled entry is the pivot position.

Quiz 12. *True/False:* Every linear programming minimization problem in standard form has a dual maximization problem.

Solution. The statement is *true*. This is sometimes known as the Fundamental Theorem of Duality.

Quiz 13. *True/False:* A least square regression line for a set of data $\{(x_i, y_i)\}_{i=1}^n$ is a line of the form $\hat{y} = b_1x + b_0$ which minimizes $\sum(y_i - \hat{y}_i)$.

Solution. The statement is *false*. The *least square* regression line minimizes the sum of the squares of the errors, i.e. the *least square* regression line minimizes the sum

$$\sum (y_i - \hat{y}_i)^2$$

Quiz 14. *True/False:* If a company invests \$8000 in an account that pays a 7.5% annual interest, then the value in the amount in the account after 4 months is $8000 \left(1 + 7.5 \cdot \frac{4}{12}\right)$.

Solution. The statement is *false*. The formula is $P(1 + rt)$, where P is the principal amount, t is the time (in units matching that of the rate), and r is the rate. The principal is \$8000, the time is $\frac{4}{12}$ (that is, 4/12 year, or 4 months of the 12 months that make up 1 year—the units of the rate), while the rate is 7.5%. However, percentages must always be expressed as decimals. So this should be

$$8000 \left(1 + 0.075 \cdot \frac{4}{12}\right)$$

Quiz 15. *True/False:* If you invest \$13,000 in an account which pays 5.3% interest, compounded semiannually, over a period of 6 years, then the amount of money in the account after this time is...

$$13000 \left(1 + \frac{0.053}{2} \right)^{2 \cdot 6}$$

Solution. The statement is *true*. The formula for the future value of an amount accruing interest compounded at discrete time intervals is $F = P(1 + r/n)^{nt}$, where P is the principal amount, r is the interest rate (as a decimal), n is the number of compounding per interest period, and t is the time. Here, $P = 13000$, $r = 0.053$, $n = 2$ (because it is semiannually), and $t = 6$, which gives us...

$$F = 13000 \left(1 + \frac{0.053}{2} \right)^{2 \cdot 6}$$

Quiz 16. *True/False:* Disjoint events can be independent.

Solution. The statement is *false*. If events are disjoint, then the events cannot happen at the same time. But then if one of the events occurs, the other cannot. If the other event is not impossible, then the occurrence or non-occurrence of one event affects the probability of the other occurring, which cannot for independent events.

Quiz 17. *True/False:* If $P(A) = 0.40$ and $P(B) = 0.30$, then $P(A \text{ and } B) = P(A) \cdot P(B) = 0.40 \cdot 0.30 = 0.12$.

Solution. The statement is *false*. The formula $P(A \text{ and } B) = P(A) \cdot P(B)$ is only valid if A and B are independent events, which we are not told here. Generally, we have $P(A \text{ and } B) = P(A)P(B | A) = P(B)P(A | B)$.

Quiz 18. *True/False:* From a normal distribution, the more ‘extreme’ the value, the greater the magnitude of the z -score.

Solution. The statement is *true*. Recall that

$$z = \frac{x - \mu}{\sigma}$$

But then the z -score measures the number of standard deviations above/below the mean a particular value is. But the greater this measure, the more extreme the value must be.

Quiz 19. *True/False:* Groups vary less than individuals.

Solution. The statement is *true*. The Central Theorem Limit states that if you randomly sample from a population with mean μ and finite standard deviation σ , then the group means are distributed as $N(\mu, \sigma/\sqrt{n})$ if the original distribution is normally distributed (which we do not know here) or if n is sufficiently large ($n \geq 30$ is a common standard). Notice that this distribution has smaller standard deviation than the original population, i.e. less variation. Therefore, on average, groups vary less than individuals.

Quiz 20. *True/False:* If you randomly sample from a population with mean μ and finite standard deviation σ using group size of $n = 17$, then the distribution of the group mean values is $N(\mu, \sigma/\sqrt{n})$.

Solution. The statement is *false*. This is close to the Central Limit Theorem. The Central Theorem Limit states that if you randomly sample from a population with mean μ and finite standard deviation σ , then the group means are distributed as $N(\mu, \sigma/\sqrt{n})$ if the original distribution is normally distributed (which we do not know here) or if n is sufficiently large ($n \geq 30$ is a common standard). However, here the group size is too small.