Name: *Caleb McWhorter — Solutions*
MATH 100
Fall 2022
HW 20: Due 12/05

"The only normal people are the ones you don't know very well."
–Alfred Adler

Problem 1. (10pt) Given the binomial distributions below, find the probability of the event(s) indicated.

(a) $B(0.40, 7)$, $P(X = 4)$

(b) $B(0.25, 12)$, $P(X < 3)$

(c) $B(0.50, 4)$, $P(X \geq 1)$

(d) $B(0.10, 15)$, $P(X > 5)$

(e) $B(0.15, 8)$, $P(2 < X \leq 6)$

Solution.

(a)

$$P(X = 4) = 0.0256$$

(b)

$$P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2) = 0.0317 + 0.1267 + 0.2323 = 0.3907$$

(c)

$$P(X \geq 1) = 1 - P(X = 0) = 1 - 0.0625 = 0.9375$$

(d)

$$\begin{aligned} P(X > 5) &= 1 - P(X \leq 4) \\ &= 1 - P(X = 0) - P(X = 1) - P(X = 2) - P(X = 3) - P(X = 4) \\ &= 1 - 0.2059 - 0.3432 - 0.2669 - 0.1285 \\ &= 0.0555 \end{aligned}$$

(e)

$$\begin{aligned} P(2 < X \leq 6) &= P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6) \\ &= 0.0839 + 0.0185 + 0.0026 + 0.0002 \\ &= 0.1052 \end{aligned}$$

Problem 2. (10pt) Suppose at a school that 15% of students do not receive any scholarships. If you take a simple random sample of 14 students, find the probability that...

(a) exactly four of the students did not receive any scholarships.

(b) five or more of the students did not receive any scholarships.

(c) at least one of the students did not receive any scholarships.

(d) less than 3 of the students did not receive any scholarships.

(e) none of the students received a scholarship.

Solution. Because each student either received a scholarship or not, the probability of not receiving a scholarship is fixed with $p = 0.15$, we take a fixed number of $n = 14$ students, and each sample was taken independently, this is a binomial distribution with $B(n, p) = B(14, 0.15)$.

(a) This is...

$$P(X = 4) = 0.0998$$

(b) This is...

$$\begin{aligned} P(X \geq 5) &= P(X = 5) + P(X = 6) + P(X = 7) + P(X = 8) + P(X = 9) + \cdots + P(X = 14) \\ &= 0.0352 + 0.0093 + 0.0019 + 0.0003 + 0.0000 + \cdots + 0.0000 \\ &= 0.0467 \end{aligned}$$

(c) This is...

$$P(X \geq 1) = 1 - P(X = 0) = 1 - 0.1028 = 0.8972$$

(d) This is ...

$$P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2) = 0.1028 + 0.2539 + 0.2912 = 0.6479$$

(e) If none of them received a scholarship, then all of them did not receive a scholarship. This is...

$$P(X = 14) = 0.000$$

Problem 3. (10pt) A city is considering whether or not to build a park in a newly acquired lot. It is estimated that 53% of the population is in favor of building the park. The city conducts a survey of its residents consisting of 300 people. What is the approximate probability that between 140 and 170 people surveyed are in favor of the park?

Solution. Because each person surveyed either is in favor of the park or not, the percentage chance of being in favor of the park is fixed at $p = 0.53$, there is a fixed survey of $n = 300$ people, and the survey respondents are assumed to be independent of each other, this is a binomial distribution with $B(n, p) = B(300, 0.53)$. However, because $np = 300(0.53) = 159 \geq 10$ and $n(1-p) = 300(1-0.53) = 300(0.47) = 141 \geq 10$, we can approximate the binomial distribution $B(n, p)$ with $N(np, \sqrt{np(1-p)})$. But then we have $B(300, 0.53) \approx N\big(300(0.53), \sqrt{300 \cdot 0.53 \cdot (1-0.53)}\big) = N(159, 8.6447)$. But then we have...

$$z_{140} = \frac{140 - 159}{8.6447} = \frac{-19}{8.6447} \approx -2.20 \rightsquigarrow 0.0139$$

$$z_{170} = \frac{170 - 159}{8.6447} = \frac{11}{8.6447} \approx 1.27 \rightsquigarrow 0.8980$$

But then we know that...

$$P(140 \leq X \leq 170) \approx P(X \leq 170) - P(140 \leq X) = 0.8980 - 0.0139 = 0.8841$$