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MATH 108
Fall 2021
HW 2: Due 09/28

"Listen, I'm not the nicest guy in the universe, because I'm the smartest, and being nice is something stupid people do to hedge their bets."
–Rick Sanchez, Rick & Morty

Problem 1. (10pt) Solve the following system of equations:

$$\begin{aligned} 2x - y &= 5 \\ x + y &= 1 \end{aligned}$$

Solution. *Adding the equations, we have...*

$$\begin{aligned} 3x &= 6 \\ x &= 2 \end{aligned}$$

But then using $x = 2$ in the second equation, we have

$$\begin{aligned} 2 + y &= 1 \\ y &= -1 \end{aligned}$$

Therefore, the solution is $(x, y) = (2, -1)$, i.e. $x = 2$ and $y = -1$.

Problem 2. (10pt) Solve the following system of equations:

$$\begin{aligned} 4x - 3y &= 10 \\ 6x + 5y &= -4 \end{aligned}$$

Solution. *First, we multiply the first equation by 3 and the second equation by* -2 *to obtain...*

$$\begin{aligned} 12x - 9y &= 30 \\ -12x - 10y &= 8 \end{aligned}$$

Adding these equations, we get

$$\begin{aligned} -19y &= 38 \\ y &= -2 \end{aligned}$$

Using $y = -2$ *in the first equation, we have...*

$$\begin{aligned} 4x - 3(-2) &= 10 \\ 4x + 6 &= 10 \\ 4x &= 4 \\ x &= 1 \end{aligned}$$

Therefore, the solution is $(x, y) = (1, -2)$*, i.e.* $x = 1$ *and* $y = -2$*.*

Problem 3. (10pt) Solve the following system of equations:

$$\begin{aligned} \frac{1}{2}x - \frac{1}{6}y &= -1 \\ x - \frac{1}{3}y &= -2 \end{aligned}$$

Solution. *To make the equations simpler to solve, we clear denominators by multiplying the first equation by 6 and the second by 3. Then we have*

$$\begin{aligned} 3x - y &= -6 \\ 3x - y &= -6 \end{aligned}$$

But then these are the same line! Therefore, any point satisfying one equation will satisfy the other. So the set of solutions will be the every point on the line $3x - y = -6$*, i.e* $y = 3x + 6$*. So the set of solutions is* $(x, y) = (x, 3x + 6)$ *for any* $x \in \mathbb{R}$*.*

Problem 4. (10pt) Solve the following system of equations:

$$\begin{aligned} 6x - 5y &= 1 \\ 3x + 10y &= 3 \end{aligned}$$

Solution. *We multiply the first equation by 2 to obtain...*

$$\begin{aligned} 12x - 10y &= 2 \\ 3x + 10y &= 3 \end{aligned}$$

Adding the equations, we have...

$$\begin{aligned} 15x &= 5 \\ x &= \frac{1}{3} \end{aligned}$$

Using $x = 1/3$ *in the first equation, we have*

$$\begin{aligned} 6\left(\frac{1}{3}\right) - 5y &= 1 \\ 2 - 5y &= 1 \\ 5y &= 1 \\ y &= \frac{1}{5} \end{aligned}$$

Therefore, the solution is $(x, y) = (\frac{1}{3}, \frac{1}{5})$*, i.e.* $x = \frac{1}{3}$ *and* $y = \frac{1}{5}$.

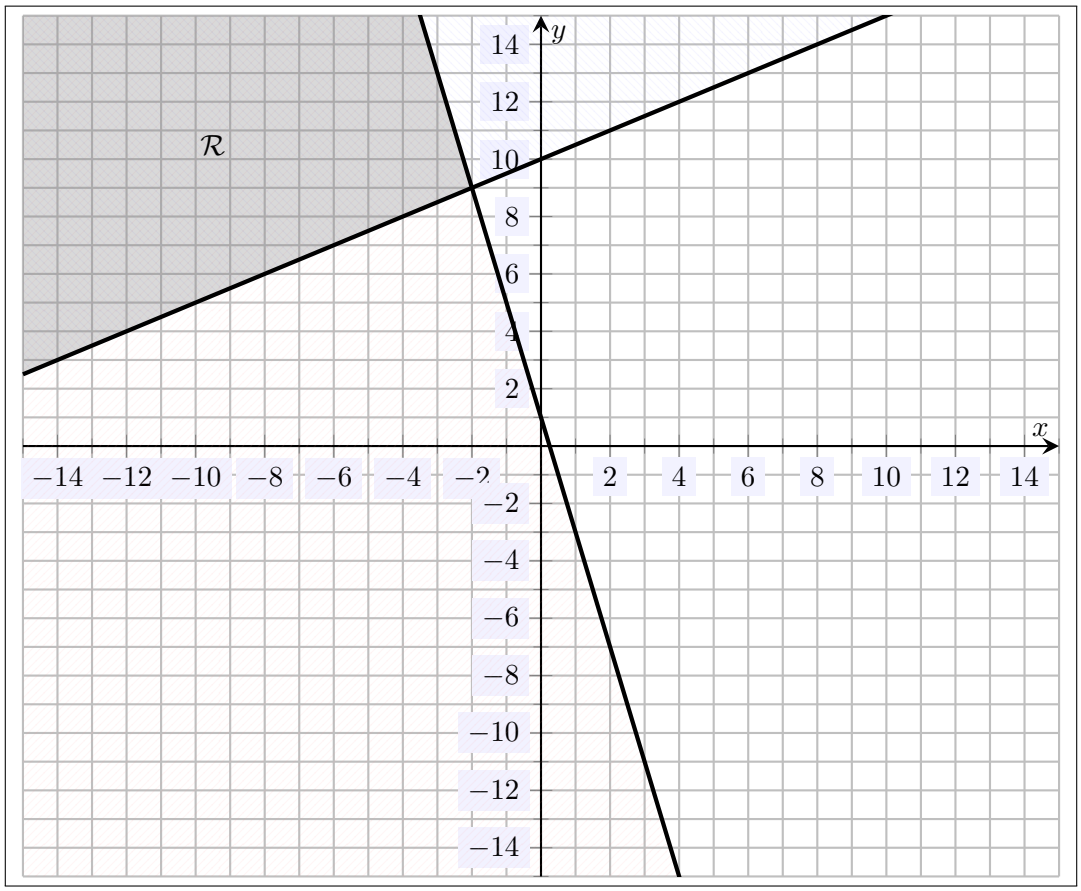
Problem 5. (10pt) Plot the solution set to the system of inequalities:

$$4x + y \leq 1$$
$$x - 2y \leq -20$$

Solution. Solving for y in each, we have

$$y \leq 1 - 4x$$
$$\frac{1}{2}x + 10 \leq y$$

Notice each line is solid. For the first line, we shade beneath the line. For the second line, we shade above the line.



Problem 6. (10pt) Plot the solution set to the system of inequalities:

$$x \geq 1$$
$$-3x + y \geq -4$$

Solution. Solving for y in the second, we have

$$x \geq 1$$
$$y \geq 3x - 4$$

Notice each line is solid. For the first line, we shade to the right of the line. For the second line, we shade above the line.