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MATH 108
Spring 2022
Written HW 12: Due 05/09

"Statistics are no substitute for judgement."

$-Henry\ Clay$

Problem 1. (10pt) An education researcher is examining the effect of the new state educational standards on student outcomes. Examining the SAT scores from students this past year, they find that the SAT scores were approximately normally distributed with mean 1070 and standard deviation 80.

  (a) Find the probability that a student scores below 1000 on the SAT.

  (b) Find the probability that a student scores greater than 1200 on the SAT.

  (c) Find the probability that a student scores between 1000 and 1200 on the SAT.

  (d) What is the probability that two randomly selected students both score between 1000 and 1200 on the SAT? What assumptions are required for this computation?

Solution.

  (a)
$$z_{1000} = \frac{1000 - 1070}{80} = \frac{-70}{80} = -0.875 \rightsquigarrow \frac{0.1922 + 0.1894}{2} = 0.1908$$
  But then $P(X \leq 1000) = 0.1908$.

  (b)
$$z_{1200} = \frac{1200 - 1070}{80} = \frac{130}{80} = 1.625 \rightsquigarrow \frac{0.9474 + 0.9484}{2} = 0.9479$$
  But then $P(X \geq 1200) = 1 - P(X \leq 1200) = 1 - 0.9479 = 0.0521$.

  (c)
$$P(1000 \leq X \leq 1200) = 0.9479 - 0.1908 = 0.7571$$

  (d) Because the underlying distribution is normal, the sampling distribution (using a sample size of two people) for the average score is $N(\mu, \sigma/\sqrt{2}) = N(1070, 80/\sqrt{2}) = N(1070, 56.5685)$ by the Central Limit Theorem. But then we have
$$z_{1200} = \frac{1200 - 1070}{56.5685} = \frac{130}{56.5685} = 2.30 \rightsquigarrow 0.9893$$
$$z_{1000} = \frac{1000 - 1070}{56.5685} = \frac{-70}{56.5685} = -1.24 \rightsquigarrow 0.1075$$

  Therefore, $P(1000 \leq \overline{X} \leq 1200) = 0.9893 - 0.1075 = 0.8818$.

Problem 2. (10pt) At Lower Creek High School, 15% of students are in the National Honor Society (NHS). Suppose you take twelve students at random from the high school.

(a) What is the probability that exactly four of them are in the NHS?

(b) What is the probability that between two and five of them are in the NHS?

(c) What is the probability that none of them are in the NHS?

(d) What is the probability that at least one of them are in the NHS?

Solution.

(a)
$$P(X = 4) = 0.0683$$

(b)
$$P(2 \leq X \leq 5) = 0.2924 + 0.1720 + 0.0683 + 0.0193 = 0.5520$$

(c)
$$P(X = 0) = 0.1422$$

(d)
$$P(X \geq 1) = 1 - P(X = 0) = 1 - 0.1422 = 0.8578$$