Name: _____

MATH 307
Spring 2023
HW 1: Due 02/13 (14)

"In learning you will teach, and in teaching you will learn."
–Phil Collins

Problem 1. (10pt) Let $\mathcal{U} = \{-10, -9, \ldots, 9, 10\}$. Define the following subsets of $\mathcal{U}$:

$$A = \{-2, 0, 5, 10\}$$
$$B = \text{even numbers in } \mathcal{U}$$
$$C = \{-9, -7, -5, -3, -1, 1, 3, 5, 7, 9\}$$
$$D = \text{positive prime numbers in } \mathcal{U}$$
$$E = \{-5, -4, \ldots, 4, 5\}$$

Using the sets defined above, answer the following:

(a) $A \cap B$

(b) $B \cup E$

(c) $E - A$

(d) B^c

(e) $|D|$

Problem 2. (10pt) Define the following sets:

$$A = \text{set of multiples of 3}$$
$$B = \text{set of divisors of 30}$$
$$C = \text{set of even numbers less than 10}$$

Using the sets defined above, answer the following:

(a) List the elements of B.

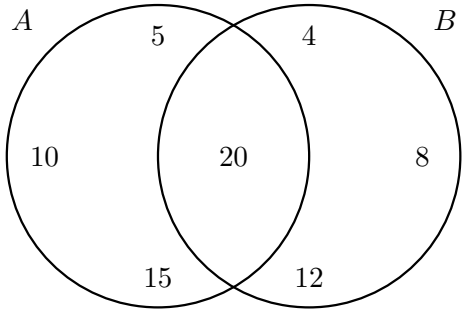
(b) Give the largest element of A less than 50 and the largest negative element of A.

(c) What are the elements of $B - A$?

(d) What are the elements of $A \cap B$?

(e) Are B and C disjoint? Explain.

Problem 3. (10pt) Look at the Venn diagram given below:



Use this diagram to answer the following:

(a) Assuming only a few of the elements of A and B are given in the diagram above, describe what the sets A and B likely represent.

(b) Place the numbers 25, 16, and 40 in appropriate places in the given Venn diagram.

(c) Using words, explain what numbers go in the same region of the Venn diagram in which 20 is found.

(d) Using words, explain what numbers go in the same region of the Venn diagram in which 4, 8, and 12 are found.

(e) What numbers would be placed outside of both the regions A and B? Give an example.

Problem 4. (10pt) You are working with a student named Lucy. You give her the following sets: $A = \{a, b, c, d, a\}$ and $B = \{c, d, e, f\}$.

(a) Lucy states that the cardinality of A is 5. Explain why Lucy is wrong. How might you correct her?

(b) You ask Lucy to find $A \cup B$ and she states that this is $\{c, d\}$. What has Lucy done wrong?

(c) Cameron overhears Lucy's answer in (b) and shouts that the answer is $\{a, b, e, f\}$. How has Cameron misunderstood the mathematical word *or* in this context?

(d) Both Lucy and Cameron state that you cannot find $A - B$ because they are filled with letters and you cannot subtract letters. Explain what they have misunderstood about sets.