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MATH 101
Fall 2022
HW 17: Due 11/14

"Geometry is not true, it is advantageous."

$-$ *Henri Poincaré*

Problem 1. (10pt) Explain why the lines $y = 2(2 - x)$ and $y = \frac{7}{3}x + 17$ intersect at the point $(-3, 10)$.

Solution. First, we check that each line contains the point $(-3, 10)$. If the point $(-3, 10)$ is on a line, then $x = -3$ and $y = 10$ satisfies the equations. But we have...

$$y = 2(2 - x) \qquad\qquad y = \frac{7}{3}x + 17$$

$$10 \stackrel{?}{=} 2\big(2 - (-3)\big) \qquad\qquad 10 \stackrel{?}{=} \frac{7}{3} \cdot -3 + 17$$

$$10 \stackrel{?}{=} 2(2 + 3) \qquad\qquad 10 \stackrel{?}{=} -7 + 17$$

$$10 \stackrel{?}{=} 2(5) \qquad\qquad 10 = 10$$

$$10 = 10 \qquad\qquad\qquad \checkmark$$

$$\checkmark$$

Therefore, $(-3, 10)$ is on each of the lines. But then $(-3, 10)$ is an intersection point for the lines $y = 2(2 - x)$ and $y = \frac{7}{3}x + 17$.

Problem 2. (10pt) Consider the following lines:

$$\ell_1 : y = \frac{2}{3}x + 3$$

$$\ell_2 : y = 14 - 3x$$

Explain why these lines intersect and find their point of intersection.

Solution. The slope of the first line is $m_1 = \frac{2}{3}$. The slope of the second line is $m_2 = -3$. Because $m_1 \neq m_2$, the lines are not parallel. But then the lines must intersect. We can find the x-coordinate of their intersection:

$$\frac{2}{3}x + 3 = 14 - 3x$$

$$\frac{2}{3}x + 3x + 3 = 14$$

$$\frac{2}{3}x + 3x = 11$$

$$\frac{2}{3}x + \frac{9}{3}x = 11$$

$$\frac{11}{3}x = 11$$

$$\frac{3}{11} \cdot \frac{11}{3}x = \frac{3}{11} \cdot 11$$

$$x = 3$$

Using either of the equations for the line, for instance the second one, we know that $y = 14 - 3x = 14 - 3(3) = 14 - 9 = 5$. Therefore, the point of intersection is $(3, 5)$.

Problem 3. (10pt) Find the point of intersection between the following lines:

$$\ell_1 : y = 4x - 13$$

$$\ell_2 : 5x + 4y = -10$$

Solution. First, we put each line into the form $y = mx + b$. Observe the first line already has this form: $y = 4x - 13$. For the second line, we have...

$$5x + 4y = -10$$

$$4y = -5x - 10$$

$$y = -\frac{5}{4}x - \frac{10}{4}$$

$$y = -\frac{5}{4}x - \frac{5}{2}$$

But then the x-coordinate of the intersection is...

$$4x - 13 = -\frac{5}{4}x - \frac{5}{2}$$

$$4(4x - 13) = 4 \cdot -\frac{5}{4}x - 4 \cdot \frac{5}{2}$$

$$16x - 52 = -5x - 10$$

$$21x - 52 = -10$$

$$21x = 42$$

$$x = 2$$

Then using either of the equations of the line, for instance the first one, we have $y = 4x - 13 = 4(2) - 13 = 8 - 13 = -5$. Therefore, the point of intersection is $(2, -5)$.