

Name: \_\_\_\_\_

MATH 361

Spring 2024

HW 1: Due 02/01

*“And I knew exactly what to do... but in a much more real sense, I had no idea what to do.”*

— Michael Scott, *The Office*

**Problem 1.** (10pts) Showing all your work and fully justifying your reasoning, compute the following:

(a)  $\lim_{x \rightarrow -6} \frac{x^2 + 3x - 18}{x^2 + 5x - 6}$

(c)  $\frac{d}{dx} \ln(x \cos x)$

(b)  $\lim_{n \rightarrow \infty} \frac{2n^2 - 35n + 17}{6n^2 + 19n - 49}$

(d)  $\int_0^1 \frac{x}{x+1} dx$

**Problem 2.** (10pts) Recall that a sequence  $\{a_n\}$  is increasing if  $a_{n+1} \geq a_n$  for all  $n$  and the sequence is decreasing if  $a_{n+1} \leq a_n$  for all  $n$ . A sequence  $\{a_n\}$  is called bounded if there exists  $M \in \mathbb{R}$  such that  $|a_n| \leq M$  for all  $n$ . The *Monotone Convergence Theorem* states the following: if  $\{a_n\}$  is a bounded sequence that is either increasing or decreasing, i.e. is ‘monotone’, then  $\{a_n\}$  converges. Now consider the sequence with  $a_0 = 2$  and given recursively via. . .

$$a_{n+1} = \frac{1}{2} \left( a_n + \frac{5}{a_n} \right)$$

- (a) Compute  $a_1, a_2, a_3$ .
- (b) Compare your values in (a) to  $\sqrt{5}$ . What might you conjecture?
- (c) Explain why the Monotone Convergence Theorem implies that  $\{a_n\}$  has a limit.
- (d) By (c), we know  $L := \lim_{n \rightarrow \infty} a_n$  exists. Taking the limit in both sides of the recursive definition for  $\{a_n\}$ , show that  $L = \sqrt{5}$ .

**Problem 3.** (10pts) The Intermediate Value Theorem states the following: if  $f(x)$  is continuous on  $[a, b]$  and  $f(a) < c < f(b)$ , then there exists an  $x_0 \in (a, b)$  such that  $f(x_0) = c$ . Consider the function  $f(x) = x^2 - 3x + 4$  on the interval  $[-1, 5]$ .

- (a) Give a sketch of  $f(x)$  on the interval  $[-1, 5]$ .
- (b) Explain why  $f(x)$  is continuous.
- (c) Explain why there is a  $x_0 \in [-1, 5]$  such that  $f(x_0) = 4$ .
- (d) Find the  $x_0 \in [-1, 5]$  such that  $f(x_0) = 4$ .

**Problem 4.** (10pts) The Mean Value Theorem states the following: if  $f(x)$  is continuous on  $[a, b]$  and differentiable on  $(a, b)$ , then there exists  $c \in (a, b)$  such that  $f(b) - f(a) = f'(c)(b - a)$ . Consider the function  $f(x) = x^3 + x^2 - 4x - 5$ . Find the values  $c \in [-1, 4]$  that satisfy the Mean Value Theorem.