Name: ____
MATH 308
Fall 2021
HW 8: Due 10/18

"I was never that great at math, but next to nothing is higher than nothing, right?"

–Dr. Gregory House, House

Problem 1. (20pt) Prove $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$.

Problem 2. (20pt) Let $\{a_n\}_{n\in\mathbb{N}}$ be the sequence with $a_1 = 1$, $a_2 = 8$, and $a_n = a_{n-1} + 2a_{n-2}$ for $n \geq 3$. Prove that $a_n = 3 \cdot 2^{n-1} + 2(-1)^n$ for all $n \in \mathbb{N}$.

Problem 3. (20pt) Prove that for $n \geq 4$, $n^3 < 3^n$.

Problem 4. (20pt) Recall that an integer m is divisible by 3 if $m = 3q$ for some $q \in \mathbb{Z}$. Prove that $7^n - 4^n$ is divisible by 3 for all $n \in \mathbb{Z}_{\geq 0}$.

Problem 5. (20pt) Prove that $\mathbb{Z} = \{3x + 2y \colon x, y \in \mathbb{Z}\}$.