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MATH 108
Spring 2022
Written HW 1: Due 02/07

"Leslie, I typed your symptoms into the thing up here and it says you could have network connectivity problems."
–Andy Dwyer, Parks and Recreation

Problem 1. (10pt) Compute the following:

(a) 36% of 657.30

(b) 97% of 450

(c) 154% of 78.56

(d) 220% of 11.2

Solution.

(a)
$$657.30(0.36) = 236.628$$

(b)
$$450(0.97) = 436.50$$

(c)
$$78.56(1.54) = 120.982$$

(d)
$$11.2(2.20) = 24.64$$

Problem 2. (10pt) Compute the following:

(a) 54 increased by 75%

(b) 1640 decreased by 22%

(c) 81 increased by 280%

(d) 771 decreased by 95%

Solution.

(a)
$$54(1 + 0.75) = 54(1.75) = 94.50$$

(b)
$$1640(1 - 0.22) = 1640(0.78) = 1279.20$$

(c)
$$81(1 + 2.80) = 81(3.80) = 307.80$$

(d)
$$771(1 - 0.95) = 771(0.05) = 38.55$$

Problem 3. (10pt) Convert the following:

(a) €120 to USD [$1 USD = €0.88]

(b) 50 km/h to miles per second [1 km = 0.621371 mi]

(c) €5/m^2 to USD/ft^2 [$1 USD = €0.88; 1 m = 3.28084 ft]

Solution.

(a)

$$\begin{array}{c|c} \text{€}120 & \$1 \\ \hline & \text{€}0.88 \end{array} = \$136.3636$$

(b)

$$\begin{array}{c|c|c|c} 50\text{ km} & 0.621371\text{ mi} & 1\text{ hr} & 1\text{ min} \\ \hline 1\text{ hr} & 1\text{ km} & 60\text{ min} & 60\text{ s} \end{array} = 0.008630\text{ mps}$$

(c)

$$\begin{array}{c|c|c|c} \text{€}5 & 1\text{ m} & 1\text{ m} & \$1 \\ \hline 1\text{ m}^2 & 3.28084\text{ ft} & 3.28084\text{ ft} & \text{€}0.88 \end{array} = \$0.5279/\text{ft}^2$$

Problem 4. (10pt) Given the following tables, do $f(x)$ and $g(x)$ represent functions? Explain.

x	$f(x)$
1	2
2	4
3	6
4	8
1	10

x	$g(x)$
3	3
4	0
6	4
7	5
8	6

Solution. The relation $f(x)$ does *not* represent a function. Observe that the input $x = 1$ has two possible outputs. Therefore, $f(x)$ cannot be a function because not all possible inputs have only one possible output. On the other hand, the relation $g(x)$ is a function—for each possible input, there is only one possible output.

$$\boxed{\begin{array}{r} f(x) \text{ is } \textit{not} \text{ a function.} \\ g(x) \text{ is a function.} \end{array}}$$

Problem 5. (10pt) Is the relation $f(x) = 576.10 - 14.39x$ a function of x? Explain.

Solution. Yes, the relation $f(x) = 576.10 - 14.39x$ is a function of x because for each possible input x, there is only one possible output—namely the one obtained by evaluating $f(x)$ at x and following order of operations.

Problem 6. (10pt) Is the relation $f(x, y, z) = 45.1x - 36.0y + 1.2z$ a function of x, y, z? Explain.

Solution. Yes, the relation $f(x, y, z) = 45.1x - 36.0y + 1.2z$ is a function of x, y, z because for each possible input x, y, z, there is only one possible output—namely the one obtained by evaluating $f(x)$ at x, y, z and following order of operations.

Problem 7. (10pt) For each of the following, indicate whether the function is linear (T), or not (F).

(a) ____*T*____: $y = 4.4x + 50.9$

(b) ____*F*____: $f(x) = x^2 - 2x + 1$

(c) ____*T*____: $w = \frac{5}{6}p + 14$

(d) ____*F*____: $g(t) = \dfrac{t}{t+1}$

(e) ____*T*____: $r = 16.8(b + 8.3)$

(f) ____*F*____: $h(x) = 6.8x(2.2x + 4.8)$

Remark. It should be routine to see that (a) and (c) are linear and that (b) and (d) are not linear. To see why (e) is linear, observe $r = 16.8(b+8.3) = 16.8b+139.44$, which is routinely verified to be linear. To see why (f) is not linear, observe that $h(x) = 6.8x(2.2x + 4.8) = 14.96x^2 + 32.64x$, which is routinely verified not to be linear.

Problem 8. (10pt) For each of the following, indicate whether the function is linear (L), affine linear (A), or neither (N).

(a) $\underline{\quad L \quad}$: $f(x, y, z) = 99.15x + 67.45y - 1.44z$

(b) $\underline{\quad N \quad}$: $g(x, y) = 45.34x^2 + 34.1y^2 + 16.1x - 96.0y$

(c) $\underline{\quad A \quad}$: $h(x_1, x_2, x_3) = 4.5x_1 + 6.1x_2 - 8.1x_3 + 8.9$

Problem 9. (10pt) Assume the numbers below represent the slope for some linear function. For each of the given slopes, indicate whether the function is increasing or decreasing and interpret the given slope in at least two different ways:

(a) $m = 5$

(b) $m = -3$

(c) $m = \dfrac{2}{3}$

(d) $m = -\dfrac{5}{6}$

(e) $m = 4.67$

Solution.

(a) Because $m = 5 > 0$, the function is increasing. We have $m = 5 = \frac{5}{1} = \frac{-5}{-1}$. We can interpret this as every increase of 1 in the input results in an increase of 5 in the output or that every decrease of 1 in the input results in a decrease of 5 in the output.

(b) Because $m = -3 < 0$, the function is decreasing. We have $m = -3 = -\frac{3}{1} = \frac{-3}{1} = \frac{3}{-1}$. We can interpret this as every increase of 1 in the input results in a decrease of 3 in the output or that every decrease of 1 in the input results in an increase of 3 in the output.

(c) Because $m = \frac{2}{3} > 0$, the function is increasing. We have $m = \frac{2}{3} = \frac{-2}{-3}$. We can interpret this as every increase of 3 in the input results in an increase of 2 in the output or that every decrease of 3 in the input results in a decrease of 2 in the output. Alternatively, using the fact that $m = \frac{2}{3} \approx 0.6667 = \frac{0.6667}{1} = \frac{-0.6667}{-1}$, we can interpret this as saying every increase of 1 in the input results in an increase of 0.6667 in the output or that every decrease of 1 in the input results in a decrease of 0.6667 in the output.

(d) Because $m = -\frac{5}{6} < 0$, the function is decrease. We have $m = -\frac{5}{6} = \frac{-5}{6} = \frac{5}{-6}$. We can interpret this as every increase of 6 in the input results in a decrease of 5 in the output or that every decrease of 6 in the input results in an increase of 5 in the output. Alternatively, using the fact that $m = -\frac{5}{6} \approx -0.8333 = \frac{-0.8333}{1} = \frac{0.8333}{-1}$, we can interpret this as saying every increase of 1 in the input results in a decrease of 0.8333 in the output or that every decrease of 1 in the input results in an increase of 0.8333 in the output.

(e) Because $m = 4.67 > 0$, the function is increasing. We have $m = 4.67 = \frac{4.67}{1} = \frac{-4.67}{-1}$. We can interpret this as every increase of 1 in the input results in an increase of 4.67 in the output or that every decrease of 1 in the input results in a decrease of 4.67 in the output.

Problem 10. (10pt) Jon is paid a base salary of $56,000 each year. However, he also earns a commission of 2% of the total amount of sales he makes each year.

(a) Explain why Jon's yearly income is a linear function of his sales.

(b) Find a function, $I(s)$, that gives Jon's yearly income, I, in terms of his total sales, s.

(c) What is the y-intercept for this function? What does it represent?

(d) What is the slope for this function? What does it represent?

Solution.

(a) Because Jon's salary has a constant rate of change, his yearly income is a linear function of his sales.

(b) We know Jon earns $56,000, regardless of his sales. For each s dollars in sales he makes, he receives 2% of this. But this amount is $0.02s$. Therefore, we have

$$I(s) = 0.02s + 56000$$

(c) We know the y-intercept occurs when the input(s) are zero. But then we have...

$$I(0) = 0.02(0) + 56000 = 56000$$

Therefore, the y-input is $(0, 56000)$ or 56000. This is the amount Jon earns when he makes $s = 0$ in sales. Therefore, the y-intercept is represents Jon's base yearly salary.

(d) We know that $I(s) = 0.02s + 56000$. This function is linear with slope $m = 0.02$. Interpreting this as $\frac{\Delta\text{Output}}{\Delta\text{Input}}$, we see that for every dollar in sales Jon makes, his yearly income increases by 0.02. Therefore, the slope represents Jon's commission rate.

Problem 11. (10pt) Aiyana is a statistician. She models that the number of traffic accidents at a particular city intersection can be modeled by $A(c) = 0.002c - 1.3$, where A is the number of accidents and c is the number of cars that pass through the intersection each month.

(a) Is the model $A(c)$ linear? Explain.

(b) Find the y-intercept for this function. If possible, interpret the intercept in context.

(c) Find the slope of $A(c)$. If possible, interpret this slope in context.

Solution.

(a) Yes, the function $A(c)$ is linear because $A(c)$ has the form $y = mx + b$ with $x = c$, $m = 0.002$, and $b = -1.3$, we know that $A(c)$ is a linear function.

(b) We know the y-intercept occurs when the input(s) are zero. But then we have...

$$A(0) = 0.002(0) - 1.3 = -1.3$$

Therefore, the y-intercept is $(0, -1.3)$ or -1.3. This is the number of accidents that occur when zero cars pass through the intersection that month. But because the number of accidents occurring at the intersection must be nonnegative and $-1.3 < 0$, this y-intercept does not have an (obvious) contextual interpretation.

(c) Because $A(c)$ has the form $y = mx + b$ with $m = 0.002$, we know that the slope is 0.002. We interpret this as $\frac{\Delta\text{Output}}{\Delta\text{Input}}$. Using the fact that $0.002 = \frac{0.002}{1}$ and scaling by 1000, i.e. $\frac{0.002}{1} \cdot \frac{1000}{1000} = \frac{2}{1000}$, we see that for every 1000 cars passing through the intersection, there are 2 accidents.

Problem 12. (10pt) Consider the linear function $\ell(x, y) = 56.4x - 5.6y$.

(a) Explain why this function is linear.

(b) Find $\ell(10.3, 7.1)$.

(c) What is the slope 'in the x-direction'? Interpret this slope and indicate whether ℓ is increasing or decreasing with respect to x.

(d) What is the slope 'in the y-direction'? Interpret this slope and indicate whether ℓ is increasing or decreasing with respect to x.

Solution.

(a) This function is linear because it has the form $f(x_1, x_2) = a_1 x_1 + a_2 x_2$ with $x = x_1$, $y = x_2$, $a_1 = 56.4$, and $a_2 = -5.6$, i.e. it is linear in each variable.

(b) We have...

$$\ell(10.3, 7.1) = 56.4(10.3) - 5.6(7.1) = 580.92 - 39.76 = 541.16$$

(c) The 'slope in the x-direction' is $a_1 = 56.4$. Because $a_1 > 0$, the function is increasing in x. Interpreting this as $56.4 = \frac{56.4}{1}$, we see that for every increase of 1 in x, $\ell(x, y)$ increases by 56.4.

(d) The 'slope in the y-direction' is $a_2 = -5.6$. Because $a_2 < 0$, the function is decreasing in y. Interpreting this as $-5.6 = \frac{-5.6}{1}$, we see that for every increase of 1 in x, $\ell(x, y)$ decreases by 5.6.