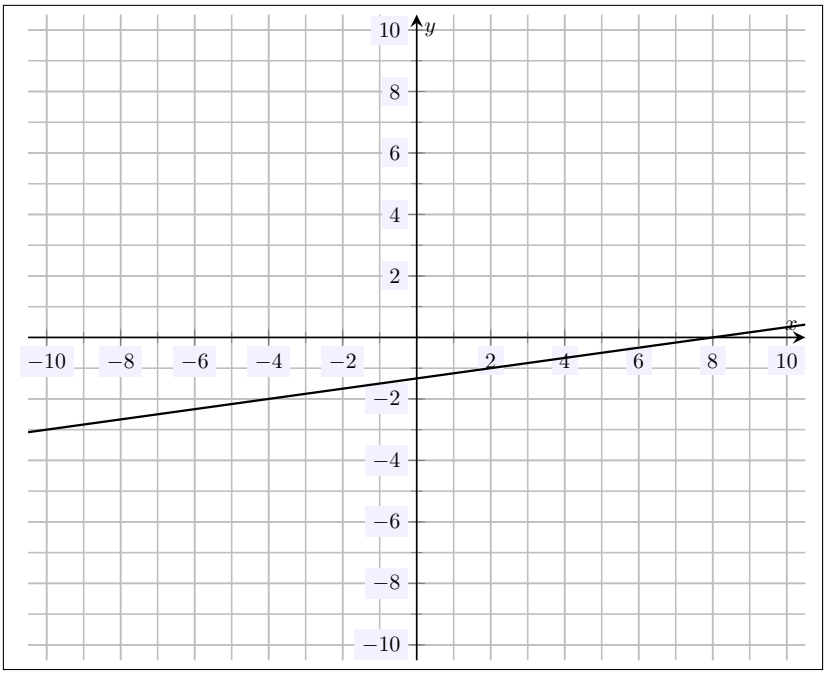
Name: *Caleb McWhorter — Solutions*
MATH 101
Spring 2024
HW 12: Due 03/20

"Mitchell's mother has a problem. . . with me. Last Christmas, for example. She gave me a piece of exercise equipment and a lettuce dryer. So, to recap, I gave her a gorgeous pair of diamond earrings and she gave me a hint."
— Cameron Tucker, Modern Family

Problem 1. (10pts) Find the equation of the line plotted below.



Solution. Because the line is not vertical, we know that it must have the form $y = mx + b$ for some m, b. We can see that the points $(-4, -2)$ and $(2, -1)$ lie exactly on the line. The slope of the line is then. . .

$$m = \frac{\Delta y}{\Delta x} = \frac{-2 - (-1)}{-4 - 2} = \frac{-2 + 1}{-4 + -2} = \frac{-1}{-6} = \frac{1}{6}$$

But because the line contains the point $(2, -1)$, when $x = 2$ we know that $y = -1$. Therefore, . . .

$$\begin{aligned} y &= mx + b \\ y &= \frac{1}{6}\,x + b \\ -1 &= \frac{1}{6} \cdot 2 + b \\ -1 &= \frac{1}{3} + b \\ b &= -1 - \frac{1}{3} \\ b &= -\frac{4}{3} \end{aligned}$$

Therefore, the equation of the line is $y = \frac{1}{6}\,x - \frac{4}{3} = \frac{x-8}{6}$.

Problem 2. (10pts) Find the equation of the following lines:

(a) The line through $(-1, 1)$ and $(6, -2)$.

(b) The line containing $(8, -1)$ with slope $\frac{4}{3}$.

(c) The line with y-intercept 5 and slope -6.

Solution.

(a) The slope of the line must be...

$$m = \frac{\Delta y}{\Delta x} = \frac{1-(-2)}{-1-6} = \frac{1+2}{-1+(-6)} = \frac{3}{-7} = -\frac{3}{7}$$

Then using the point-slope formula, $y = y_0 + m(x - x_0)$, the equation of the line is...

$$y = 1 - \frac{3}{7}\big(x-(-1)\big) = 1 - \frac{3}{7}(x+1) = 1 - \frac{3}{7}x - \frac{3}{7} = -\frac{3}{7}x + \frac{4}{7} = \frac{-3x+4}{7}$$

(b) Using the point-slope formula, $y = y_0 + m(x - x_0)$, the equation of the line is...

$$y = -1 + \frac{4}{3}(x-8) = -1 + \frac{4}{3}x - \frac{32}{3} = \frac{4}{3}x - \frac{35}{3} = \frac{4x-35}{3}$$

(c) Using the slope-intercept form, $y = mx + b$, we have...

$$y = -6x + 5$$

Problem 3. (10pts) Find the equation of the line with x-intercept -4 and y-intercept 6.

Solution. Because the line has x-intercept -4, it contains the point $(-4, 0)$. Because the line has y-intercept 6, it contains the point $(0, 6)$. But then the slope of the line is...

$$m = \frac{\Delta y}{\Delta x} = \frac{6-0}{0-(-4)} = \frac{6-0}{0+4} = \frac{6}{4} = \frac{3}{2}$$

Then using the slope-intercept form of the line, $y = mx + b$, the line must be...

$$y = \frac{3}{2}\,x + 6 = \frac{3x+12}{2}$$