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MATH 101
Spring 2024
HW 13: Due 03/25

"All right, let's not just dismiss that out of hand. What about a bomb? In my experience with the Air Force, that was very often the right answer. Very, very often."

— General Mark R. Naird, Space Force

Problem 1. (10pts) Consider the linear function $\ell(x) = \dfrac{5x-7}{-3}$.

(a) Find the slope of this function.

(b) Find the y-intercept of this function.

(c) Find the x-intercept of this function.

(d) Does the graph of this function contain the point $(2, 0)$? Explain.

Solution. We have $\ell(x) = \dfrac{5x-7}{-3} = -\dfrac{5}{3}\,x + \dfrac{7}{3}$.

(a) The slope of $\ell(x)$ is $m = -\frac{5}{3}$.

(b) The y-intercept of $\ell(x)$ is $b = \frac{7}{3}$.

(c) The x-intercept is the x-value where $\ell(x) = 0$. But then...

$$\ell(x) = 0$$

$$\frac{5x-7}{-3} = 0$$

$$5x - 7 = 0$$

$$5x = 7$$

$$x = \frac{7}{5}$$

(d) If the line $\ell(x)$ contains the point $(2, 0)$, then $\ell(2) = 0$. But we know that the x-intercept of the line $\ell(x)$ is $x = \frac{7}{5}$ from (c). Therefore, the line does not contain the point $(2, 0)$. Alternatively, we have $\ell(2) = \frac{5(2)-7}{-3} = \frac{10-7}{-3} = \frac{3}{-3} = -1 \neq 0$.

Problem 2. (10pts) Solve the following equation and verify that your solution is correct:

$$-2(5-x)+15=\frac{8x+1}{2}$$

Solution.

$$-2(5-x)+15=\frac{8x+1}{2}$$

$$2\cdot(-2(5-x)+15)=2\left(\frac{8x+1}{2}\right)$$

$$-4(5-x)+30=8x+1$$

$$-20+4x+30=8x+1$$

$$4x+10=8x+1$$

$$9=4x$$

$$x=\frac{9}{4}$$

Now we verify this solution:

$$-2(5-x)+15=\frac{8x+1}{2}$$

$$-2\left(5-\frac{9}{4}\right)+15\stackrel{?}{=}\frac{8\cdot\frac{9}{4}+1}{2}$$

$$-2\cdot\frac{11}{4}+15\stackrel{?}{=}\frac{18+1}{2}$$

$$-\frac{11}{2}+15\stackrel{?}{=}\frac{19}{2}$$

$$\frac{19}{2}=\frac{19}{2}$$

Problem 3. (10pts) Solve the following equation:

$$\sqrt{2}\,(x - \sqrt{8}) = \pi x + 5$$

Solution.

$$\sqrt{2}\,(x - \sqrt{8}) = \pi x + 5$$

$$\sqrt{2}\,x - \sqrt{16} = \pi x + 5$$

$$\sqrt{2}\,x - 4 = \pi x + 5$$

$$\sqrt{2}\,x - \pi x = 9$$

$$(\sqrt{2} - \pi)x = 9$$

$$x = \frac{9}{\sqrt{2} - \pi}$$

Problem 4. (10pts) Find the x-intercept of the line perpendicular to the line $y = \frac{2}{3}x + 5$ that contains the point $(-1, 6)$.

Solution. Because the line $y = \frac{2}{3}\,x + 5$ is not horizontal, the line perpendicular to it is not vertical. Perpendicular lines have negative reciprocal slopes. The slope of $y = \frac{2}{3}\,x + 5$ is $\frac{2}{3}$. Therefore, the slope of the line in question is $m = -\frac{3}{2}$. The line contains the point $(-1, 6)$. Using the point-slope form, we have...

$$y = y_0 + m(x - x_0)$$

$$y = 6 - \frac{3}{2}\big(x - (-1)\big)$$

$$y = 6 - \frac{3}{2}\,x - \frac{3}{2}$$

$$y = -\frac{3}{2}\,x + \frac{9}{2}$$

$$y = \frac{3x + 9}{2}$$