Name: _____

MATH 308
Fall 2022
HW 9: Due 10/13

"It's fine to work on any problem, so long as it generates interesting mathematics along the way—even if you don't solve it at the end of the day."

–Andrew Wiles

Problem 1. (10pt) Suppose that you have a function $f : \mathbb{R} \to \mathbb{R}$ that is strictly increasing.

(a) Explain why f must be an injective function.

(b) If f is merely increasing, does f have to be an injection? Explain why or give a counterexample.

(c) Does f have to be surjective? Explain why or give a counterexample.

Problem 2. (10pt) Consider the function $f : \mathbb{R}^{\geq 2} \to \mathbb{R}$ given by $f(x) = \sqrt{x-2}$.

(a) Solve the equation $\sqrt{x-2} = \sqrt{y-2}$ for y.

(b) Using your work in (a), explain why this shows that $f(x)$ is injective.

(c) Is $f(x)$ surjective? If $f(x)$ is surjective, explain why. If $f(x)$ is not surjective, find an element of the codomain not in the image of $f(x)$.

Problem 3. (10pt) Let A, B be nonempty sets. Find a bijective function from $A \times B$ to the set $B \times A$. Be sure to explain why your function is bijective. Does this mean that $A \times B$ and $B \times A$ are the same sets? Explain why or why not.