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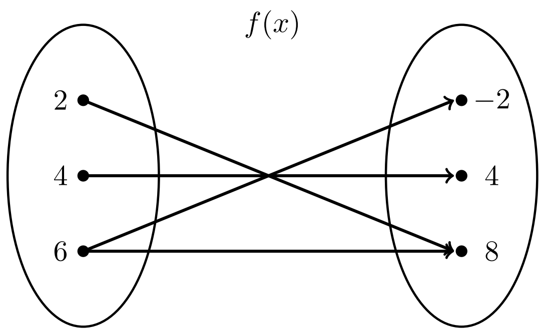
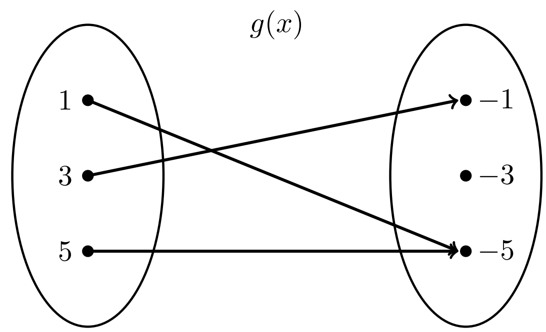
MATH 101
Fall 2022
HW 6: Due 10/12

"Good judgement comes from experience, and a lot of that comes from bad judgement."

–Will Rogers

Problem 1. (10pt) Determine if the relations $f(x)$ and $g(x)$ shown below are functions. Explain why or why not.

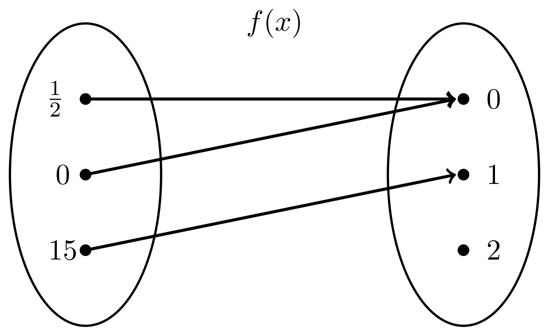




Solution. We know that the relation $f(x)$ is *not* a function. While we know that $f(2) = 8$ and $f(4) = 4$, $f(6)$ has two possible outputs; that is, $f(6)$ is not well defined because $f(6) \in \{-2, 8\}$.

On the other hand, $g(x)$ is a function because for each possible input, there is only one possible output. In fact, we know that $f(1) = -5$, $f(3) = -1$, and $f(5) = -5$.

Problem 2. (10pt) Suppose $f(x)$ is the function given below.



(a) Explain why $f(x)$ is a function.

(b) Find the value of $f(x)$ on each value in its domain.

(c) What is the domain of $f(x)$?

(d) What is the codomain of $f(x)$?

(e) What is the range of $f(x)$?

Solution.

(a) For each of the three possible outputs, namely $\frac{1}{2}$, 0, and 15, there is a single possible output. Therefore, $f(x)$ is a function. In fact, we know that $f(\frac{1}{2}) = 0$, $f(0) = 0$, and $f(15) = 1$.

(b) We know that...

$$f\left(\frac{1}{2}\right) = 0$$

$$f(0) = 0$$

$$f(15) = 1$$

(c) The domain of $f(x)$ is the set of possible inputs. Therefore, the domain is $\{\frac{1}{2}, 0, 15\}$.

(d) The codomain of $f(x)$ is the set of *possible* outputs. Therefore, the codomain is $\{0, 1, 2\}$.

(e) The range of $f(x)$ is the set of *actual* outputs. Therefore, the range is $\{0, 1\}$.

Problem 3. (10pt) Explain why $f(x,y) = 2x^2 - y^3 + 6$ is a function. Then find $f(0,0)$, $f(3,-1)$, $f(-3,2)$, and $f(1,1)$.

Solution. We know that $f(x,y)$ is a function because for each input, i.e. each (x,y), there is only one possible output—namely, the one obtained by 'plugging in' x and y into $f(x,y)$ and following order of operations. We know that...

$$f(0,0) = 2(0^2) - 0^3 + 6 = 2(0) - 0 + 6 = 0 - 0 + 6 = 6$$

$$f(3,-1) = 2(3^2) - (-1)^3 + 6 = 2(9) - (-1) + 6 = 18 + 1 + 6 = 25$$

$$f(-3,2) = 2(-3)^2 - 2^3 + 6 = 2(9) - 8 + 6 = 18 - 8 + 6 = 16$$

$$f(1,1) = 2(1^2) - 1^3 + 6 = 2(1) - 1 + 6 = 2 - 1 + 6 = 7$$

Problem 4. (10pt) Give an example of a 'real world' relationship between two variables which does represent a functional relationship. Also, give an example of a 'real world' relationship between two variables which *does not* represent a functional relationship. Be sure to fully explain your responses.

Answers will vary.