

Name: \_\_\_\_\_

MATH 308

Fall 2022

HW 11: Due 11/04

*“There are five elementary arithmetical operations: addition, subtraction, multiplication, division, and modular forms.”*

*–Martin Eichler*

**Problem 1.** (10pt) Showing all your steps, compute the following:

(a)  $(15 + 14) \bmod 6$

(f)  $14(5) \bmod 6$

(b)  $(8 - 17) \bmod 5$

(g)  $2(3) \bmod 7$

(c)  $-(1 + 8) \bmod 3$

(h)  $-7(4) \bmod 9$

(d)  $(20 - 11) \bmod 8$

(i)  $(-3)^3 \bmod 4$

(e)  $(9 + 7) \bmod 4$

(j)  $6^2 \bmod 5$

**Problem 2.** (10pt) Consider arithmetic modulo 4.

- (a) List two positive elements and two negative elements of  $[0]$  and  $[3]$ .
- (b) Choose elements  $x \in [1]$  and  $y \in [3]$  with  $x, y > 10$  and show that  $[x] + [y] = [0]$ ; that is, use the division algorithm to write  $x = 4m + r_x$  and  $y = 4n + r_y$  and show  $[x] + [y] = [r_x] + [r_y] = [0]$ .
- (c) Choose elements  $x, y \in [2]$  with  $x, y > 10$  and show that  $[x] \cdot [y] = [0]$ ; that is, use the division algorithm to write  $x = 4m + r_x$  and  $y = 4n + r_y$  and show  $[x] \cdot [y] = [r_x] \cdot [r_y] = [0]$ .

**Problem 3.** (10pt) Showing all your work, complete the following:

- (a) Compute  $\phi(7)$ ,  $\phi(11)$ , and  $\phi(131)$ .
- (b) Compute  $\phi(8)$ ,  $\phi(9)$ , and  $\phi(49)$ .
- (c) Compute  $\phi(360)$ .
- (d) How many integers  $0, 1, 2, \dots, 359$  are invertible modulo 360? Explain.

**Problem 4.** (10pt) Being sure to fully justify your responses, answer the following:

- (a) Is 7 invertible modulo 15? Explain.
- (b) Prove your claim in (a) by finding an inverse for 7 modulo 15 or showing that there is no inverse of 7 modulo 15.
- (c) Is 2 invertible modulo 6? Explain.
- (d) Prove your claim in (c) by finding an inverse for 2 modulo 6 or showing that there is no inverse of 2 modulo 6.