

Name: Caleb McWhorter — Solutions

MATH 101

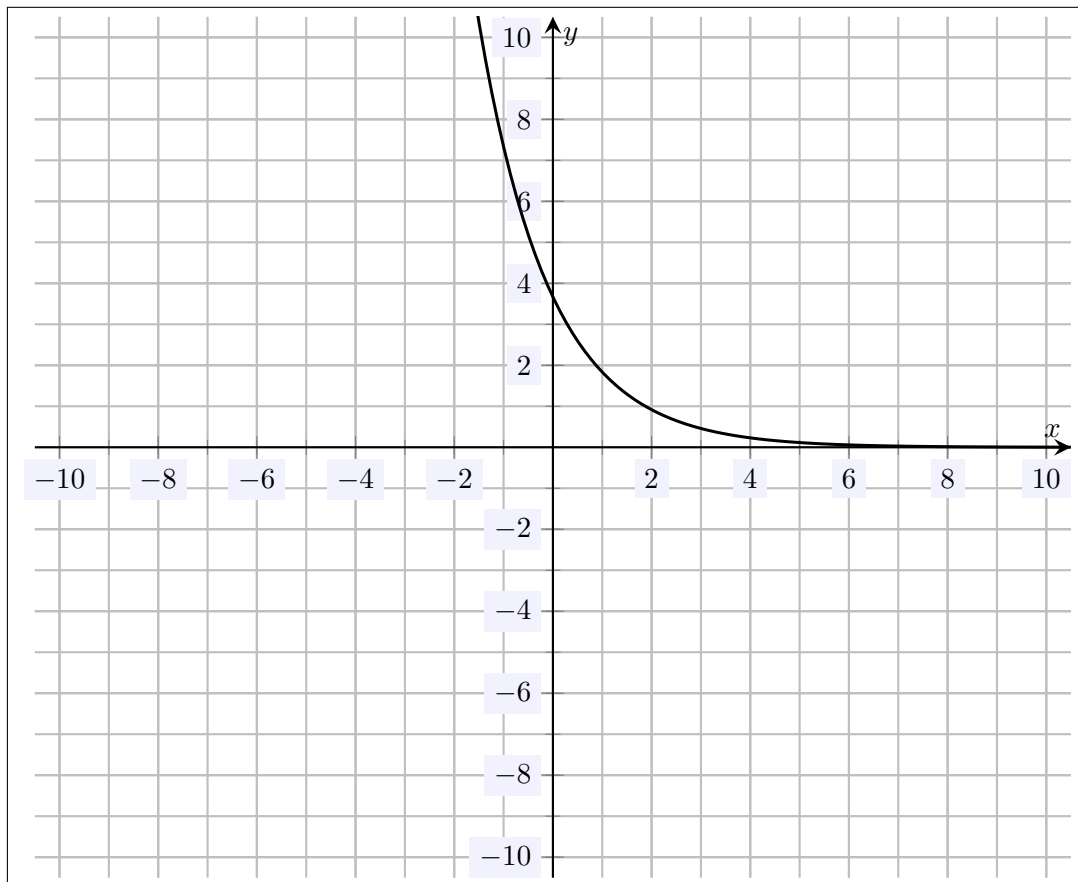
Summer 2022

HW 10: Due 06/14

*"Somewhere, something incredible is waiting to be known."*

*— Carl Sagan*

**Problem 1.** (10pt) Sketch the function  $f(x) = \frac{11}{3} \left(\frac{1}{2}\right)^x$  as accurately as possible on the graph below.



**Solution.** The function  $f(x) = \frac{11}{3} \left(\frac{1}{2}\right)^x$  is in the form  $Ab^{cx}$ . Because  $b = \frac{1}{2} < 1$ ,  $c = 1 > 0$ , and  $a = \frac{11}{3} > 0$ , we know that the function  $f(x)$  is decreasing. Because  $a > 0$ , we know that  $f(x)$  is always positive. We know also that the  $y$ -intercept is given by  $f(0) = \frac{11}{3} \left(\frac{1}{2}\right)^0 = \frac{11}{3} \cdot 1 = \frac{11}{3}$ , so that the  $y$ -intercept is  $(0, \frac{11}{3})$ . Putting this information gives the sketch above.

**Problem 2.** (10pt) Showing all your work, determine whether the following functions are increasing or decreasing:

(a)  $-5(2)^{-\frac{1}{5}x}$

(b)  $\frac{7}{8}\left(\frac{5}{6}\right)^{4x}$

(c)  $17\left(\frac{5}{4}\right)^{-x}$

(d)  $-10\left(\frac{1}{3}\right)^{-5x}$

**Solution.**

(a) The function  $-5(2)^{-\frac{1}{5}x}$  is in the form  $Ab^{cx}$ . Because  $b = 2 > 1$ ,  $c = -\frac{1}{5} < 0$ , and  $a = -5 < 0$ , we know that the function is increasing.

(b) The function  $\frac{7}{8}\left(\frac{5}{6}\right)^{4x}$  is in the form  $Ab^{cx}$ . Because  $b = \frac{5}{6} < 1$ ,  $c = 4 > 0$ , and  $a = \frac{7}{8} > 0$ , we know that the function is decreasing.

(c) The function  $17\left(\frac{5}{4}\right)^{-x}$  is in the form  $Ab^{cx}$ . Because  $b = \frac{5}{4} > 1$ ,  $c = -1 < 0$ , and  $a = 17 > 0$ , we know that the function is decreasing.

(d) The function  $-10\left(\frac{1}{3}\right)^{-5x}$  is in the form  $Ab^{cx}$ . Because  $b = \frac{1}{3} < 1$ ,  $c = -5 < 0$ , and  $a = -10 < 0$ , we know that the function is decreasing.

**Problem 3.** (10pt) Showing all your work, solve the following equation:

$$2^{3x} = 4$$

**Solution.**

$$2^{3x} = 4$$

$$2^{3x} = 2^2$$

$$3x = 2$$

$$x = \frac{2}{3}$$

**Problem 4.** (10pt) Showing all your work, solve the following equation:

$$7(4^{1-x}) = \frac{7}{16}$$

**Solution.**

$$7(4^{1-x}) = \frac{7}{16}$$

$$\frac{1}{7} \cdot 7(4^{1-x}) = \frac{7}{16} \cdot \frac{1}{7}$$

$$4^{1-x} = \frac{1}{16}$$

$$2^{2(1-x)} = 2^{-4}$$

$$2(1-x) = -4$$

$$1-x = -2$$

$$x = 3$$

**Problem 5.** (10pt) Showing all your work, solve the following equation:

$$\frac{1}{3^x} = 27^{\frac{4x+10}{3}}$$

**Solution.**

$$\frac{1}{3^x} = 27^{\frac{4x+10}{3}}$$

$$3^{-x} = 3^{3 \cdot \frac{4x+10}{3}}$$

$$-x = 4x + 10$$

$$-5x = 10$$

$$x = -2$$

**Problem 6.** (10pt) Showing all your work, solve the following equation:

$$5^{x-2} + 6 = 11$$

**Solution.**

$$5^{x-2} + 6 = 11$$

$$5^{x-2} = 5^1$$

$$x - 2 = 1$$

$$x = 3$$

**Problem 7.** (10pt) Showing all your work, solve the following equation:

$$\frac{1}{4^x} = 1024$$

**Solution.**

$$\frac{1}{4^x} = 1024$$

$$4^{-x} = 4^5$$

$$-x = 5$$

$$x = -5$$

**Problem 8.** (10pt) Showing all your work, solve the following equation:

$$\left(\frac{2}{3}\right)^{5x-7} = 1$$

**Solution.**

$$\left(\frac{2}{3}\right)^{5x-7} = 1$$

$$\left(\frac{2}{3}\right)^{5x-7} = \left(\frac{2}{3}\right)^0$$

$$5x - 7 = 0$$

$$5x = 7$$

$$x = \frac{7}{5}$$



**Problem 9.** (10pt) Suppose you invest \$5,000 in an account which earns 4.6% annual interest, compounded quarterly. How much will be in the account after 3 years?

**Solution.** If  $P$  dollars are placed in an account which earns yearly interest at a rate  $r$ , compounded  $k$  times per year, then the amount of money in the account after  $t$  years,  $F$ , is given by

$$F = P \left( 1 + \frac{r}{k} \right)^{kt}$$

Here, we have  $P = 5000$ ,  $r = 0.046$ ,  $k = 4$ , and  $t = 3$ . But then we have...

$$F = P \left( 1 + \frac{r}{k} \right)^{kt} = 5000 \left( 1 + \frac{0.046}{4} \right)^{4 \cdot 3} = 5000(1.0115)^{12} = 5000(1.14707191) = 5735.36$$

Therefore, the account will have \$5,735.36.

**Problem 10.** (10pt) If you take out a loan for \$1,200 at a 5.5% annual interest, compounded continuously, how much is owed after 2 years? How much of this amount is interest?

**Solution.** If  $P$  dollars are taken out on a loan with interest rate  $r$ , then the amount of money owed on the loan after  $t$  years,  $F$ , is given by

$$F = Pe^{rt}$$

Here, we have  $P = 1200$ ,  $r = 0.055$ , and  $t = 2$ . Then...

$$F = Pe^{rt} = 1200e^{0.055 \cdot 2} = 1200e^{0.11} = 1200(1.11627807) = 1339.53$$

Therefore, you will owe \$1,339.53 on the loan after 2 years.