Name: *Caleb McWhorter — Solutions*
MATH 108
Spring 2023
HW 7: Due 03/01

"Life is a school of probability."
– Walter Bagehot

Problem 1. (10pt) The probabilities of several events in a finite probability space are given below:

$$\begin{aligned} P(A) &= 0.30 \quad P(B \text{ and } C) = 0.15 \\ P(B) &= 0.55 \quad P(D \mid A) = 0.10 \\ P(C) &= 0.75 \quad P(B \text{ and } D) = 0.01 \\ P(D) &= 0.25 \quad P(C \text{ and } D) = 0.00 \end{aligned}$$

(a) Assuming that A and B are independent, find $P(A \text{ or } B)$.

(b) Find $P(C \mid B)$.

(c) Are A and C disjoint? Explain.

(d) Are B and D independent? Explain.

(e) Find $P(A \text{ and } D)$.

Solution.

(a) Because A and B are independent, we know that $P(A \text{ and } B) = P(A) \cdot P(B) = 0.30 \cdot 0.55 = 0.165$. But then we have...

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 0.30 + 0.55 - 0.165 = 0.685$$

(b) We have...

$$P(C \mid B) = \frac{P(C \text{ and } B)}{P(B)} = \frac{P(C \text{ and } B)}{P(B)} = \frac{0.15}{0.55} \approx 0.2727$$

(c) If A and C were disjoint, then $P(A \text{ or } C) = P(A) + P(C)$. But then $P(A \text{ or } C) = P(A) + P(C) = 0.30 + 0.75 = 1.05$, which is impossible. Therefore, it is not possible for A and C to be disjoint.

(d) If B and D were independent, then $P(B \text{ and } D) = P(B) \cdot P(D)$. We have $P(B \text{ and } D) = P(B) \cdot P(D) = 0.55 \cdot 0.25 = 0.1375$. However, we know that $P(B \text{ and } D) = 0.01$. Because $0.1375 \neq 0.01$, we know that B and D are not independent.

(e) We have...

$$P(A \text{ and } D) = P(A) \cdot P(D \mid A) = 0.30 \cdot 0.10 = 0.03$$

Problem 2. (10pt) At a small community college, the number of times students accessed tutoring resources available to them during a semester is summarized in the table below.

	Never	1 – 2	3 – 5	> 5	Total
Freshman	72	86	45	17	220
Sophomore	93	32	55	41	221
Junior	63	44	46	19	172
Senior	97	9	15	11	132
Total	325	171	161	88	745

Based on the table above and showing all your work, answer the following:

(a) Find the probability that a randomly selected student never went to tutoring.

(b) Find the probability that a randomly selected student is a Junior.

(c) Find the probability that a randomly selected student went to tutoring once to two times or was a Sophomore.

(d) Find the probability that a randomly selected Senior went to tutoring more than five times.

(e) If a student was a Freshman, find the probability that they went to tutoring at most five times.

Solution. We first need to find the totals for each row/column, which we include below on the table above.

(a) We have...

$$P(\text{no tutoring}) = \frac{325}{745} = \frac{65}{149} \approx 0.4362$$

(b) We have...

$$P(\text{Junior}) = \frac{172}{745} \approx 0.2309$$

(c) We have...

$$P(\text{Tutoring 1 – 2 or Sophomore}) = \frac{86+32+44+9+93+55+41}{745} = \frac{171+221-32}{745} = \frac{360}{745} = \frac{72}{149} \approx 0.4832$$

(d) We have...

$$P(\leq 5 \mid \text{Senior}) = \frac{P(\leq 5 \text{ and Senior})}{P(\text{Senior})} = \frac{97+9+15}{132} = \frac{121}{132} = \frac{11}{12} \approx 0.9167$$

(e) We have...

$$P(\leq 5 \mid \text{Freshman}) = \frac{P(\leq 5 \text{ and Freshman})}{P(\text{Freshman})} = \frac{72+86+45}{220} = \frac{203}{220} \approx 0.9227$$

Problem 3. (10pt) At a local car dealership, there are 460 cars in the lot. Of these cars, 45 of them are electric vehicles and 126 of them are SUVs. There are also 3 which are electric SUVs.

(a) Find the probability that a randomly selected car was an electric vehicle.

(b) Find the probability that a randomly selected car was an electric vehicle or an SUV.

(c) Find the probability that a randomly selected car was neither an electric vehicle nor an SUV.

(d) If a randomly selected car was an SUV, what is the probability that it was electric?

(e) Find the probability that a randomly selected car was a non-SUV, electric vehicle.

Solution.

(a) We have...

$$P(\text{electric}) = \frac{42+3}{460} = \frac{45}{460} = \frac{9}{92} \approx 0.0978$$

(b) We have...

$$P(\text{electric or SUV}) = \frac{42+3+123}{460} = \frac{45+126-3}{460} = \frac{168}{460} = \frac{42}{115} \approx 0.3652$$

(c) We have...

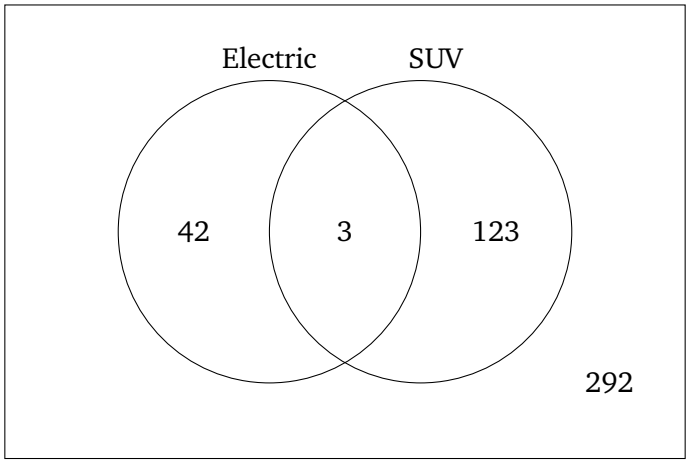
$$P(\text{not electric nor SUV}) = \frac{460-45-126+3}{460} = \frac{292}{460} = \frac{73}{115} \approx 0.6348$$

(d) We have...

$$P(\text{electric} \mid \text{SUV}) = \frac{P(\text{electric SUV})}{P(\text{SUV})} = \frac{3}{3+123} = \frac{3}{126} = \frac{1}{42} \approx 0.0238$$

(e) We have...

$$P(\text{non-SUV, electric}) = \frac{42}{460} = \frac{21}{230} \approx 0.0913$$



Problem 4. (10pt) You randomly assign delivery trucks routes to make their deliveries around the community. Of these routes, 40% of them are highways, while the rest of them are surface streets. If a delivery truck takes the highway, they make their deliveries on-time 90% of the time. If a delivery truck takes surface streets, they make their delivery on-time 85% of the time.

(a) What percent of deliveries are on-time?

(b) What percent of deliveries are on-time or use surface streets?

(c) What percent of deliveries use highways?

(d) What percent of deliveries that use the highway are on-time?

(e) What percent of deliveries that are on-time use the highway?

Solution.

(a) We have...

$$P(\text{on-time}) = 0.36 + 0.51 = 0.87$$

(b) We have...

$$P(\text{on-time or surface street}) = 0.36 + 0.51 + 0.09 = 0.96$$

(c) We have...

$$P(\text{highways}) = 0.36 + 0.04 = 0.40$$

(d) We have...

$$P(\text{on-time} \mid \text{highway}) = \frac{P(\text{on-time and highway})}{P(\text{highway})} = \frac{0.36}{0.36 + 0.04} = \frac{0.36}{0.40} \approx 0.90$$

(e) We have...

$$P(\text{highway} \mid \text{on-time}) = \frac{P(\text{highway and on-time})}{P(\text{on-time})} = \frac{0.36}{0.36 + 0.51} = \frac{0.36}{0.87} \approx 0.4138$$