

MAT 101: Exam 1
Spring – 2022
03/10/2022
85 Minutes

Name: _____

Write your name on the appropriate line on the exam cover sheet. This exam contains 20 pages (including this cover page) and 20 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work.

Question	Points	Score
1	10	
2	8	
3	8	
4	8	
5	8	
6	8	
7	6	
8	8	
9	4	
10	8	
11	8	
12	8	
13	8	
14	8	
15	6	
16	8	
17	6	
18	6	
19	8	
20	8	
Total:	150	

1. (10 points) Mark each of the following statements as True (T) or False (F).

(a) _____: $4^{100} + 4^{100} + 4^{100} + 4^{100} = 4^{101}$.

(b) _____: The number 1 is a multiple of 12.

(c) _____: The number 4 has three divisors.

(d) _____: The number 1 is prime.

(e) _____: Every integer greater than 1 is a product of prime numbers.

(f) _____: For all real numbers x , we know $x^0 = 1$.

(g) _____: There is no rational number equal to π .

(h) _____: The number $31.2 \cdot 10^5$ is in scientific notation.

(i) _____: Two lines with positive slopes can never be perpendicular.

(j) _____: The line $y = x + 6$ is parallel to the line $y = 7 - x$.

2. (8 points) Find the prime factorization for each of the following integers:

(a) 126

(b) 37

(c) 120

(d) 141

3. (8 points) Compute the following:

(a) $\gcd(28, 70)$

(b) $\text{lcm}(28, 70)$

(c) $\gcd(2^{500} \cdot 3^{98} \cdot 11^{82} \cdot 53^{17}, 2^{200} \cdot 3^{50} \cdot 7^{60} \cdot 13^{300})$

(d) $\text{lcm}(2^{500} \cdot 3^{98} \cdot 11^{82} \cdot 53^{17}, 2^{200} \cdot 3^{50} \cdot 7^{60} \cdot 13^{300})$

4. (8 points) Showing all your work and simplifying as much as possible, compute the following:

(a) $\frac{5}{12} - \frac{3}{4}$

(b) $\frac{11}{6} + \frac{4}{15}$

(c) $\frac{12}{55} \cdot \frac{5}{6}$

(d) $\frac{\frac{20}{21}}{\frac{8}{7}}$

5. (8 points) Showing all your work and being sure to use no negative powers, simplify the following as much as possible:

(a) $\left(\frac{x^3(x^2y^5)^0}{y^7} \right)^{-2}$

(b) $\frac{(x^2y^3)^3}{x^{10}y^{-5}}$

(c) $(\sqrt{x^5}y^{-3})^4$

(d) $\left(\frac{x^6}{y^5} \right)^{-1/3}$

6. (8 points) Simplify the following radical expressions:

(a) $\sqrt{36}$

(b) $\sqrt[3]{64}$

(c) $\sqrt{2^5 \cdot 3^2 \cdot 5}$

(d) $\sqrt[4]{2^3 \cdot 3^9 \cdot 5^4}$

7. (6 points) Showing all your work and simplifying as much as possible, rationalize the following:

(a) $\frac{4}{\sqrt{6}}$

(b) $\frac{1}{\sqrt[3]{7}}$

(c) $\frac{1}{3 - \sqrt{5}}$

8. (8 points) Showing all your work, convert 5 km/s^2 to miles per square minute. Note that $1 \text{ km} = 1000 \text{ m}$, $1 \text{ ft} = 0.3048 \text{ m}$, and $60 \text{ s} = 1 \text{ min}$.

9. (4 points) Complete the following:

(a) Convert the following number in scientific notation to an ordinary decimal number: $-4.73 \cdot 10^{-6}$

(b) Convert the following ordinary decimal number to scientific notation: 0.054

10. (8 points) Showing all your work, compute the following:

(a) 97 decreased by 60%

(b) 573 increased by 142%

(c) 71% of 140

11. (8 points) Compute the following, being sure to show all your work and to write your answer in the form $a + bi$:

(a) $(3i)^3$

(b) $(6 + 4i) - (4 - 4i)$

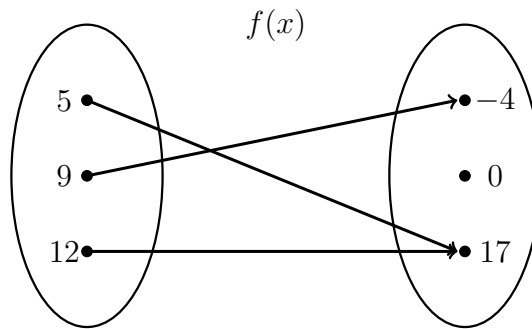
(c) $(1 + i)(3 - 2i)$

(d) $\frac{6 + i}{1 - i}$

12. (8 points) Suppose a student has a 90% participation average, 75% quiz average, 84% homework average, 86% on the midterm, and 66% on the final exam, given that their course grade is computed using the weights below, find their course average:

Participation:	5%
Quizzes:	15%
Homework:	45%
Midterm:	15%
Final:	20%

13. (8 points) Answer the following:

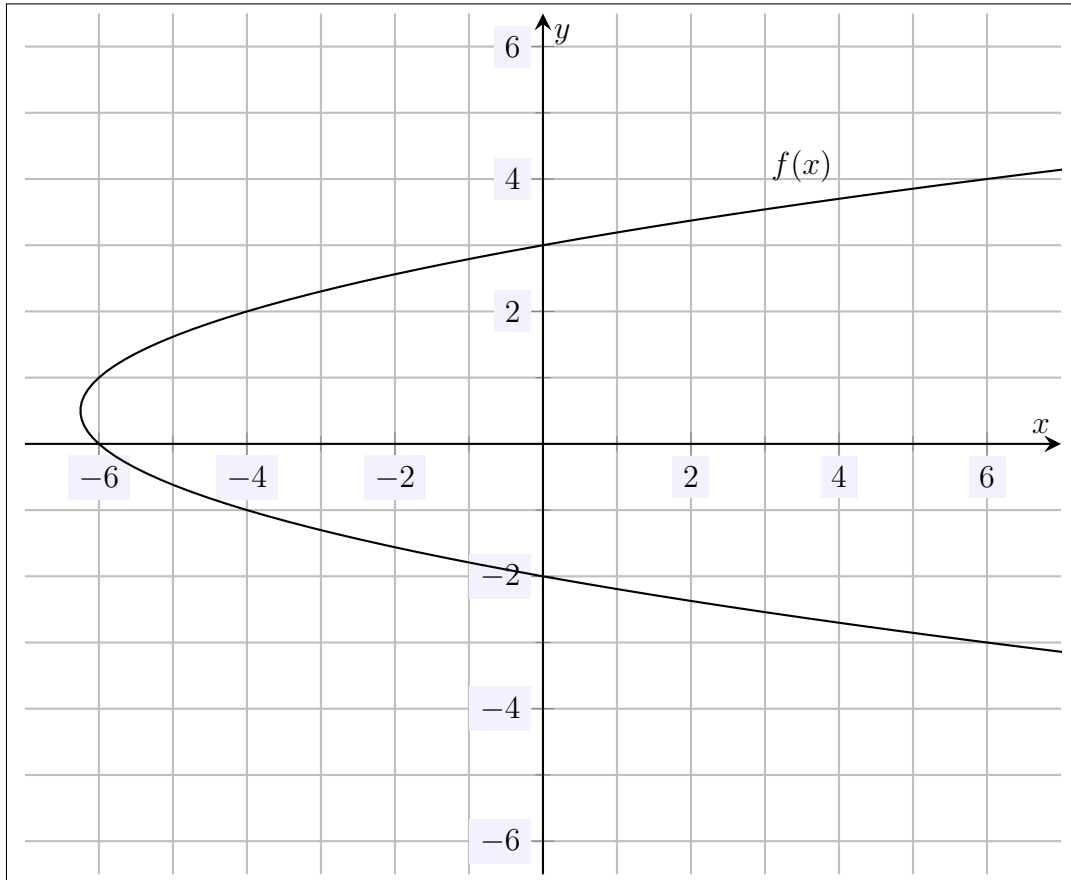


(a) Explain why the relation $f(x)$ above is a function.

(b) Find the domain, codomain, and range of the function $f(x)$.

(c) Is the relation $g(x) = 17x - x^3$ a function? Explain.

14. (8 points) Consider the relation $f(x)$ plotted below.



(a) Is the relation $f(x)$ plotted above a function? Explain.

(b) Does the relation above have an inverse function? Explain.

(c) Find the y -intercepts of the relation plotted above.

(d) Find the x -intercepts of the relation plotted above.

15. (6 points) Consider the functions given in the table below.

x	-2	-1	0	1	2
$f(x)$	5	-2	-5	-3	2
$g(x)$	6	2	10	7	-5

Compute the following:

(a) $g(2) =$

(b) $(f - g)(0) =$

(c) $(fg)(1) =$

(d) $\left(\frac{g}{f}\right)(0) =$

(e) $(f \circ g)(-1) =$

(f) $(g \circ f)(-1) =$

16. (8 points) Consider the function $\ell(x) = 6 - 2x$.

(a) Is $\ell(x)$ linear? Explain.

(b) Find the slope of $\ell(x)$.

(c) Find the y -intercept of $\ell(x)$.

(d) Is the point $(-1, 4)$ on the graph of $\ell(x)$? Explain.

17. (6 points) Find the equation of the line perpendicular to $y = 6 - 2x$ at its x -intercept.

18. (6 points) Solve the following equation and then verify your solution:

$$3x - 14 = 8 - \frac{2}{3}x$$

19. (8 points) You are driving home from university at 55 mph. Your home is 650 miles from your university. Assuming you left the university 2 hours ago and that you drive at a constant speed, find your distance from your home, $D(t)$, as function of time t , in hours.

20. (8 points) You rent an apartment in NYC, which you paid a \$50 application fee to apply for. The rent is \$2500/month. Therefore, the amount you have paid, $R(t)$, to rent the apartment t months after moving in is given by $R(t) = 2500t + 50$.

(a) Without knowing $R(t)$, how do you know that $R(t)$ is linear?

(b) What is the slope of $R(t)$ and what does it represent in the problem context?

(c) What is the y -intercept of $R(t)$ and what does it represent in the problem context?