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MATH 101

Spring 2022

HW 12: Due 04/28

"You must do the things you think you cannot do."

—Eleanor Roosevelt

Problem 1. (10pt) Consider the function $f(x) = -5 \left(\frac{4}{9}\right)^x$.

- (a) Is this function exponential? Explain. If it is exponential, find A , b , and c .
- (b) Find $f(-2)$.
- (c) Find the x and y -intercepts for $f(x)$. If there are none, state so.

Solution.

- (a) The function $f(x)$ has the form $y = Ab^{cx}$ with $A = -5$, $b = \frac{4}{9}$, and $c = 1$. Therefore, $f(x)$ is an exponential function.
- (b) We have...

$$f(-2) = -5 \left(\frac{4}{9}\right)^{-2} = -5 \left(\frac{9}{4}\right)^2 = -5 \left(\frac{81}{16}\right) = -\frac{405}{16}$$

- (c) The x -intercept(s) occurs when $f(x) = 0$. But then

$$\begin{aligned} -5 \left(\frac{4}{9}\right)^x &= 0 \\ \left(\frac{4}{9}\right)^x &= 0 \end{aligned}$$

But $\left(\frac{4}{9}\right)^x > 0$ for all x . Therefore, there are no x -intercepts. The y -intercept occurs when $x = 0$. But then we have...

$$f(0) = -5 \left(\frac{4}{9}\right)^0 = -5 \cdot 1 = -5$$

Therefore, the y -intercept is -5 , i.e. the point $(0, -5)$.

Problem 2. (10pt) Determine whether the following exponential functions are increasing or decreasing. Explain your answer for each.

(a) $y = 5(0.3)^x$

(b) $f(x) = -6(7^x)$

(c) $r = 9\left(\frac{3}{2}\right)^{-2t}$

(d) $g(x) = -7\left(\frac{12}{11}\right)^{x/2}$

Solution.

(a) We have $A = 5 > 0$, $b = 0.3 < 1$, and $c = 1 > 0$. Therefore, y is decreasing.

(b) We have $A = -6 < 0$, $b = 7 > 1$, and $c = 1 > 0$. Therefore, $f(x)$ is decreasing.

(c) We have $A = 9 > 0$, $b = \frac{3}{2} > 1$, and $c = -2 < 0$. Therefore, r is decreasing.

(d) We have $A = -7 < 0$, $b = \frac{12}{11} > 1$, and $c = \frac{1}{2} > 0$. Therefore, $g(x)$ is decreasing.

Problem 3. (10pt) Write the following functions in the form $y = Ab^x$:

(a) $y = 11(2^{3x})$

(b) $y = -8\left(\frac{7}{3}\right)^{-x}$

(c) $y = 6(7^{2x+1})$

Solution.

(a)

$$y = 11(2^{3x}) = 11((2^3)^x) = 11(8^x)$$

(b)

$$y = -8\left(\frac{7}{3}\right)^{-x} = y = -8\left(\left(\frac{7}{3}\right)^{-1}\right)^x = -8\left(\frac{3}{7}\right)^x$$

(c)

$$y = 6(7^{2x+1}) = 6(7^{2x} \cdot 7^1) = 42(7^{2x}) = 42(7^2)^x = 42(49^x)$$

Problem 4. (10pt) Solve the following exponential equations:

(a) $4^{3-x} = \frac{1}{64}$

(b) $5(3^x) + 7 = 52$

(c) $16^{2x} = 4^{8x-1}$

Solution.

(a)

$$4^{3-x} = \frac{1}{64}$$

$$4^{3-x} = \frac{1}{4^3}$$

$$4^{3-x} = 4^{-3}$$

$$3 - x = -3$$

$$x = 6$$

(b)

$$5(3^x) + 7 = 52$$

$$5(3^x) = 45$$

$$3^x = 9$$

$$3^x = 3^2$$

$$x = 2$$

(c)

$$16^{2x} = 4^{8x-1}$$

$$(4^2)^{2x} = 4^{8x-1}$$

$$4^{4x} = 4^{8x-1}$$

$$4x = 8x - 1$$

$$4x = 1$$

$$x = \frac{1}{4}$$