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MATH 101

Fall 2021

HW 3: Due 09/24

*“People make fun of the guy who stays home every night doing nothing. But the truth is that guy is a genius.”*

*– Ted Mosby, How I Met Your Mother*

**Problem 1.** (2pt) What are the approximate values of  $\pi$  and  $e$ ?

We have  $\pi \approx 3.141592654$  and  $e \approx 2.718281828$

**Problem 2.** (2pt) You may have recalled in school that  $\pi \approx \frac{22}{7}$ . Is it possible to find integers  $a$  and  $b$  such that  $\pi = \frac{a}{b}$ ? Explain.

No, it is not possible to find integers  $a, b$  so that  $\pi = \frac{a}{b}$ . The number  $\pi$  is irrational, which means it cannot be expressed as a ratio of integers.

**Problem 3.** (8pt) Express the following rational numbers as a decimal. Show all your work.

(a)  $\frac{5}{4} : 4 \overline{) 5.00}^{1.25}$ . Then  $\frac{5}{4} = 1.25$ .

$$\begin{array}{r} 4 \overline{) 5.00} \\ \underline{4} \phantom{00} \\ 1.0 \phantom{0} \\ \underline{8} \phantom{00} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

(c)  $\frac{170}{9} : 9 \overline{) 170.0}^{18.\bar{8}}$ . Then  $\frac{170}{9} = 18.\bar{8}$ .

$$\begin{array}{r} 9 \overline{) 170.0} \\ \underline{9} \phantom{00} \\ 80 \\ \underline{72} \\ 8.0 \\ \underline{7.2} \\ 8 \end{array}$$

(b)  $-\frac{1}{8} : 8 \overline{) 1.000}^{0.125}$ . Then  $-\frac{1}{8} = 0.125$ .

$$\begin{array}{r} 8 \overline{) 1.000} \\ \underline{8} \phantom{000} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

(d)  $\frac{13}{99} : 99 \overline{) 13.00}^{0.\bar{13}}$ . Then  $\frac{13}{99} = 0.\bar{13}$ .

$$\begin{array}{r} 99 \overline{) 13.00} \\ \underline{9.9} \phantom{00} \\ 3.10 \\ \underline{2.97} \\ 13 \end{array}$$

**Problem 4.** (8pt) Express the following decimals as rational numbers, reducing your rational expression as much as possible and showing all your work:

(a)  $3.0 = \frac{3}{1}$

(b)  $-1.5 = -1\frac{1}{2} = -\frac{3}{2}$

(c)  $1.25 = 1 + 0.25 = \frac{4}{4} + \frac{1}{4} = \frac{5}{4}$

(d)  $-0.94 = -\frac{94}{100} = -\frac{47}{50}$

**Problem 5.** (9pt) Express the following decimals as a rational number, reducing your rational expression as much as possible and showing all your work:

(a)  $0.7777\overline{7}$

$$\begin{array}{rcl} 10N & = & 7.7777\overline{7} \\ N & = & 0.7777\overline{7} \\ \hline 9N & = & 7 \\ N & = & \frac{7}{9} \end{array}$$

(b)  $0.212121\overline{21}$

$$\begin{array}{rcl} 100N & = & 21.212121\overline{21} \\ N & = & 0.212121\overline{21} \\ \hline 99N & = & 21 \\ N & = & \frac{21}{99} \\ N & = & \frac{7}{33} \end{array}$$

(c)  $0.25555\overline{5}$

Note that  $0.25555\overline{5} = 0.2 + 0.05555\overline{5} = \frac{2}{10} + 0.05555\overline{5} = \frac{1}{5} + 0.05555\overline{5}$ .

$$\begin{array}{rcl} 100N & = & 5.55555\overline{5} \\ N & = & 0.05555\overline{5} \\ \hline 99N & = & 5.5 \\ 99N & = & \frac{55}{10} \\ 99N & = & \frac{11}{2} \\ N & = & \frac{11}{99(2)} \\ N & = & \frac{1}{18} \end{array}$$

Then  $0.25555\overline{5} = \frac{1}{5} + \frac{1}{18} = \frac{18}{90} + \frac{5}{90} = \frac{23}{90}$ .

**Problem 6.** (3pt) Suppose two paints have to be mixed in a 5 : 6 ratio. If you want to use all of the second paint you have in the mix and you have 5.3 gallons of the paint left, how many gallons of the first paint should you add to the mix?

*Suppose  $x$  is the number of gallons needed to be used. Then we have*

$$\begin{aligned}\frac{5}{6} &= \frac{x}{5.3} \\ x &= \frac{5}{6} \cdot 5.3 \\ x &= 4.42\end{aligned}$$

*Therefore, you should use 4.42 gallons of the first paint.*

**Problem 7.** (8pt) Find the following:

(a)  $40\% \text{ of } 60 = 60(0.40) = 24$

(b)  $17\% \text{ of } 55 = 55(0.17) = 9.35$

(c)  $120 \text{ increased by } 12\% = 120(1 + 0.12) = 120(1.12) = 134.4$

(d)  $89 \text{ decreased by } 5\% = 89(1 - 0.05) = 89(0.95) = 84.55$

**Problem 8.** (6pt) Water is flowing into a tank at a rate of 12 gallons per minute. The tank can hold 1500 gallons and currently contains 86 gallons of water.

(a) How many minutes until the tank overflows?

*There are currently 86 gallons of water in the tank. Then there is  $1500 - 86 = 1414$  gallons left in the tank. But then it will take  $\frac{1414}{12} = 117.83$  minutes until the tank overflows, i.e. approximately 1 hour and 58 minutes.*

(b) If the time is currently 5:00 pm, at what time will the tank overflow?

*It will take 117.83 minutes for the tank to overflow. Then this is  $\frac{117.83}{60} = 1.96$  hours, i.e. 1 hour and another 0.96 hours. We know 0.96 hours is  $60(0.96) = 57.6 \approx 58$  minutes. Then it will take 1 hour and 58 minutes for the tank to overflow. But then the tank will overflow at 6:58 pm.*

**Problem 9.** (6pt) The travel distance between Sparkill, NY and Boston, MA is 203 miles. The speed limit on this highway is 65 mph.

- (a) How long will it take to travel from Sparkill, NY to Boston, MA using this highway?

*Let  $x$  be the time you spend driving at 65 mph. Then*

$$65x = 203$$

$$x = \frac{203}{65} \text{ hours}$$

$$x = 3.12 \text{ hours}$$

*Therefore, it will take 3.12 hours, i.e. 3 hours and 7 minutes to make the drive at this speed.*

- (b) Is it possible to make the drive in 3 hours or less without speeding? Explain.

*No. From (a), we know that driving at this speed it will take over 3 hours to make the drive. To make it in less time, one would have to go over 65 mph, i.e. one would have to speed.*

**Problem 10.** (8pt) Convert the following:

- (a) 165 lbs to kg [1 kg = 2.205 lb]

$$\frac{165 \text{ lb}}{2.205 \text{ lb}} = 74.83 \text{ kg}$$

- (b) 16 km to miles [1 mi = 1.609 km]

$$\frac{16 \text{ km}}{1.609 \text{ km}} = 9.94 \text{ mi}$$

- (c) 3.6 ft/s to mph [5280 ft = 1 mi]

$$\frac{3.6 \text{ ft}}{1 \text{ s}} \times \frac{1 \text{ mi}}{5280 \text{ ft}} \times \frac{60 \text{ s}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ hr}} = 2.45 \text{ mph}$$

- (d) 9.8 m/s<sup>2</sup> to ft/hr<sup>2</sup> [1 m = 3.2808 ft]

$$\frac{9.8 \text{ m}}{1 \text{ s}^2} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{3.2808 \text{ ft}}{1 \text{ m}} \times \frac{(60 \text{ s})^2}{(1 \text{ min})^2} \times \frac{(60 \text{ min})^2}{(1 \text{ hr})^2} = 416687846400 \text{ ft/hr}^2$$

**Problem 11.** (8pt) Write the following numbers as complex numbers:

(a)  $5 = 5 + 0i$

(b)  $\sqrt{-4} = \sqrt{4}i = 2i = 0 + 2i$

(c)  $\sqrt{16} = 4 = 4 + 0i$

(d)  $1 - \sqrt{-24} = 1 - \sqrt{24}i = 1 - \sqrt{4 \cdot 6}i = 1 - 2\sqrt{6}i$

**Problem 12.** (12pt) Compute the following:

(a)  $(1 + 3i) + (-6 + 7i) = (1 - 6) + (3 + 7)i = -5 + 10i$

(b)  $2i - (8 - 3i) = 2i - 8 + 3i = -8 + 5i$

(c)  $(1 - i)(6 + i) = 6 + i - 6i - i^2 = 6 - 5i - (-1) = 6 - 5i + 1 = 7 - 5i$

(d)  $(7 + 9i)^2 = (7 + 9i)(7 + 9i) = 49 + 63i + 63i + 81i^2 = 49 + 126i - 81 = -32 + 126i$

(e)  $\frac{10 + i}{1 - i} = \frac{10 + i}{1 - i} \cdot \frac{1 + i}{1 + i} = \frac{(10 + i)(1 + i)}{(1 - i)(1 + i)} = \frac{10 + 10i + i + i^2}{1 + i - i - i^2} = \frac{9 + 11i}{1 - (-1)} = \frac{9 + 11i}{2} = \frac{9}{2} + \frac{11}{2}i$

(f)  $\frac{16 - 12i}{(2i)^2} = \frac{16 - 12i}{4i^2} = \frac{16 - 12i}{4(-1)} = \frac{16 - 12i}{-4} = -4 + 3i$