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MATH 108
Spring 2024
HW 4: Due 02/07

"Oh my! It smells like Granda's house at Christmas. That's when we found her dead on the toilet."
— Kenneth Parcell, 30 Rock

Problem 1. (10pts) Eileen Bach sells propane and propane accessories. She wants to start a YouTube channel where she reviews grills. Of course, she will then have to regularly purchase grills. She wants to review a grill per week and post it to her channel. Eileen estimates that the average grill will cost her \$720. After she is done, she thinks that she will be able to re-sell the grill at a 40% discount. She plans on saving for 3 months worth of reviews by making a single deposit into an account that earns 1.13% annual interest, compounded every other month for a period of a year and a half.

(a) At the end of the month, how much should Eileen estimate that she has net spent on grills?

(b) How much should she deposit into the account?

Solution.

(a) She will need to purchase a grill per week. So Eileen will need to purchase 4 grills per month. This will cost an average of $\$720 \cdot 4 = \$2{,}880$. She then resells the grill for 40% off; that is, she recovers 60% of the value of the grills she purchased. But then she only actually loses, i.e. spends, 40% of the \$2,880. Therefore, on average, she spends a net of \$1,152 each month.

(b) Eileen wants to save for three months of reviews. From (a), each month should cost her a net of \$1,152, on average. Therefore, she wants to save $\$1{,}152 \cdot 3 = \$3{,}456$. So we want to know how much she should invest now, i.e. the principal P, so that after $t = 1.5$ years of earning interest at a nominal annual interest rate of $r = 0.013$, compounded $k = 6$ times per year, she will have a future amount of $F = \$3{,}456$. This is...

$$P = \frac{F}{\left(1 + \frac{r}{k}\right)^{kt}} = \frac{\$3{,}456}{\left(1 + \frac{0.0113}{6}\right)^{6 \cdot 1.5}} = \frac{\$3{,}456}{1.0018833333^9} = \frac{\$3{,}456}{1.0170782497} = \$3{,}397.97$$

Problem 2. (10pts) Susan Flaye has taken out a loan to afford the best possible broom she can to join her local adult Quidditch league. The loan was for \$870 at 9.55% annual interest, compounded quarterly. She has not made any payments on the loan for the past 2 years. Though Susan has performed fantastically on her team—leading them to over 13 victories—how much does Susan currently owe on her loan?

Solution. Because no payments are made on the loan and it is simply accruing discrete compounded interest, this is a discrete compounded interest problem. The interest has a nominal interest rate of $r = 0.0955$ and is compounded $k = 4$ times per year. We want to know the future value, F, of the principal $P = \$870$ after having earned interest for $t = 2$ years. This is...

$$F = P\left(1 + \frac{r}{k}\right)^{kt} = \$870\left(1 + \frac{0.0955}{4}\right)^{4\cdot 2} = \$870(1.023875)^{8} = \$870(1.2077457323) = \$1{,}050.74$$

Therefore, after 2 years, Susan owes \$1,050.74.

Problem 3. (10pts) Ty Coon is saving to build a roller coaster park. Though he has investors and can take out loans, he wants to have at least \$26 million saved to bring to the table on his own when the park opens. Ty will deposit money into an account that earns 2.9% annual interest, compounded continuously. The money will sit for 3 years while the park is being constructed. What is the minimum amount that Ty should deposit now to have at least \$26 million at the end of the three years?

Solution. This is a continuous compound interest problem. We know that the nominal annual interest rate is $r = 0.029$. We want to know the principal, P, that Ty should deposit now such that the future value of this deposit, F, after 3 years is \$26 million. This is...

$$P = \frac{F}{e^{rt}} = \frac{\$26{,}000{,}000}{e^{0.029\cdot 3}} = \frac{\$26{,}000{,}000}{e^{0.087}} = \frac{\$26{,}000{,}000}{1.09089667972} = \$23{,}833{,}604.49$$

Therefore, Ty should deposit \$23,833,604.49 right now.

Problem 4. (10pts) Justin Caese has invested in his future by purchasing the world's largest Pog collection. He currently estimates that the collection is worth \$5,600 and that the value increases each month by 1.17%.

(a) How much is the collection worth in 10 years?

(b) How long until the collection is worth \$100,000?

Solution. The collection's value is increasing in value every month but is not being 'artificially' altered. Therefore, this is a discrete compounded interest problem. We can view this as having annual interest which is compounded monthly with an interest rate per period of 1.17%. Then the nominal interest would be $12 \cdot 1.17\% = 14.04\%$.

Solution.

(a) We want to know the future value, F, of this collection after 10 years. This is a total of $12 \cdot 10 = 120$ monthly increases of 1.17%. Computing this as an iterative percentage increase, we have...

$$P(1+i_p)^n = \$5{,}600(1+0.0117)^{120} = \$5{,}600(1.0117)^{120} = \$5{,}600(4.03840619) = \$22{,}615.07$$

Alternatively, treating this as a discrete compounded interest problem with $r = 14.04\%$, $k = 12$, and $t = 10$, we have...

$$F = P\left(1+\frac{r}{k}\right)^{kt} = \$5{,}600\left(1+\frac{0.1404}{12}\right)^{12\cdot 10} = \$5,600(1.0117)^{120} = \$5{,}600(4.03840619) = \$22{,}615.07$$

Therefore, after 10 years, the collection will be worth \$22,615.07.

(b) We want to know the amount of time that it takes the initial value of $P = \$5{,}600$ to grow to a future value of $F = \$100{,}000$. Treating this as as a discrete compounded interest problem with $r = 14.04\%$, $k = 12$, and $t = 10$, we have...

$$t = \frac{\ln(F/P)}{k\ln\left(1+\frac{r}{k}\right)} = \frac{\ln(17.85714286)}{12\ln\left(1+\dfrac{0.1404}{12}\right)} = \frac{\ln(17.85714286)}{12\ln(1.0117)} = \frac{2.8824035884}{0.13958501} = 20.65 \text{ years}$$

Therefore, it will take 20.65 years to reach a value of \$100,000.