MAT 100: Exam 1
Fall – 2021
10/20/2021
85 Minutes

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Write your name on the appropriate line on the exam cover sheet. This exam contains 13 pages (including this cover page) and 15 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work.

Question	Points	Score
1	8	
2	8	
3	8	
4	6	
5	6	
6	4	
7	4	
8	6	
9	8	
10	6	
11	6	
12	4	
13	10	
14	8	
15	8	
Total:	100	

1. (8 points) Compute the following:

   (a) $4 + 4 - 5 \cdot 0 + 1 - 1 = 4 + 4 - 0 + 1 - 1 = 8$

   (b) $12/4 \cdot 3 - 1 = 3 \cdot 3 - 1 = 9 - 1 = 8$

   (c) $-5 - 6 + 2 \cdot 3^2 = -5 - 6 + 2 \cdot 9 = -5 - 6 + 18 = 7$

   (d) $10/5 - 6(3 - 4)^3 = 10/5 - 6(-1)^3 = 10/5 - 6(-1) = 2 + 6 = 8$

2. (8 points) Simplify the following, being sure to have no negative exponents in your expression.

(a) $\dfrac{x^{-6}}{x^{-3}} = \dfrac{x^3}{x^6} = \dfrac{1}{x^3}$

(b) $\dfrac{(xy^2)^2}{x^2y^3} = \dfrac{x^2y^4}{x^2y^3} = y$

(c) $\dfrac{x^6y^{-3}}{x^3y^2} = \dfrac{x^6}{x^3y^2y^3} = \dfrac{x^3}{y^5}$

(d) $\left(\dfrac{y^3}{x^2}\right)^{-2} = \left(\dfrac{x^2}{y^3}\right)^2 = \dfrac{x^4}{y^6}$

3. (8 points) Compute the following, being sure to simplify your answer completely:

(a) $4 + \frac{1}{5} = \frac{20}{5} + \frac{1}{5} = \frac{21}{5}$

(b) $\frac{3}{10} - \frac{5}{4} = \frac{6}{20} - \frac{25}{20} = \frac{6-25}{20} = -\frac{19}{20}$

(c) $\frac{6}{35} \cdot \frac{7}{15} = \frac{2}{5} \cdot \frac{1}{5} = \frac{2}{25}$

(d) $\frac{-\frac{7}{5}}{\frac{21}{10}} = -\frac{7}{5} \cdot \frac{10}{21} = -\frac{1}{1} \cdot \frac{2}{3} = -\frac{2}{3}$

4. (6 points) Write a mathematical expression that computes the following:

   (a) 33% of 48 $48(0.33)$

   (b) 92% of 172 $172(0.92)$

   (c) 121% of 16 $16(1.21)$

5. (6 points) Write a mathematical expression that computes the following:

   (a) 71 decreased by 90% $71(1 - 0.90) = 71(0.10)$

   (b) 88 increased by 30% $88(1 + 0.30) = 88(1.30)$

   (c) 55 increased by 190% $55(1 + 1.90) = 55(2.90)$

6. (4 points) Convert the following decimal numbers to scientific notation:

   (a) $7400 = 7.4 \cdot 10^3$

   (b) $0.002 = 2.0 \cdot 10^{-3}$

7. (4 points) Convert the following numbers in scientific notation to decimal notation:

   (a) $8.0 \cdot 10^{-6} = 0.000008$

   (b) $1.65 \cdot 10^3 = 1650$

8. (6 points) Find the prime factorizations of the following integers:

   (a) $210 = 2 \cdot 3 \cdot 5 \cdot 7$

   (b) $125 = 5^3$

   (c) $88 = 2^3 \cdot 11$

9. (8 points) Compute the following:

   (a) $\gcd(14, 21) = \gcd(2 \cdot 7, 3 \cdot 7) = 7$

   (b) $\text{lcm}(6, 20) = \text{lcm}(2 \cdot 3, 2^2 \cdot 5) = 2^2 \cdot 3 \cdot 5 = 60$

   (c) $\gcd(252, 9720) = \gcd(2^2 \cdot 3^2 \cdot 7, 2^3 \cdot 3^5 \cdot 5) = 2^2 \cdot 3^2$

   (d) $\text{lcm}(252, 9720) = \text{lcm}(2^2 \cdot 3^2 \cdot 7, 2^3 \cdot 3^5 \cdot 5) = 2^3 \cdot 3^5 \cdot 5 \cdot 7$

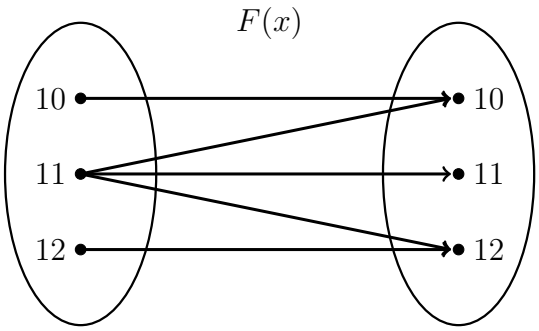
10. (6 points) Simplify the following as much as possible:

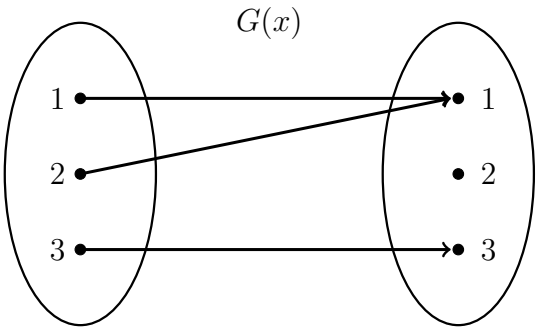
(a) $\sqrt{50} = \sqrt{2 \cdot 5^2} = 5\sqrt{2}$

(b) $\sqrt{80} = \sqrt{2^4 \cdot 5} = 2^2\sqrt{5} = 4\sqrt{5}$

(c) $\sqrt[3]{2^2 \cdot 3^6 \cdot 5} = 3^2\sqrt[3]{2^2 \cdot 5} = 9\sqrt[3]{4 \cdot 5} = 9\sqrt[3]{20}$

11. (6 points) Consider the following relations below:





x	$H(x)$
1	1
2	2
3	3
4	4
1	5

x	$J(x)$
5	-1
6	-2
7	-3
8	-4
9	-6

$$K(x) := 0.782x - 1283$$

$$L(x) := 2x(x^2+1)(x^4+1)$$

Determine if each of the relations given above is a function. If the relation is a function, write 'T' (True) and if the relation is *not* a function, write 'F' (False):

(a) ___F___: $F(x)$

(b) ___T___: $G(x)$

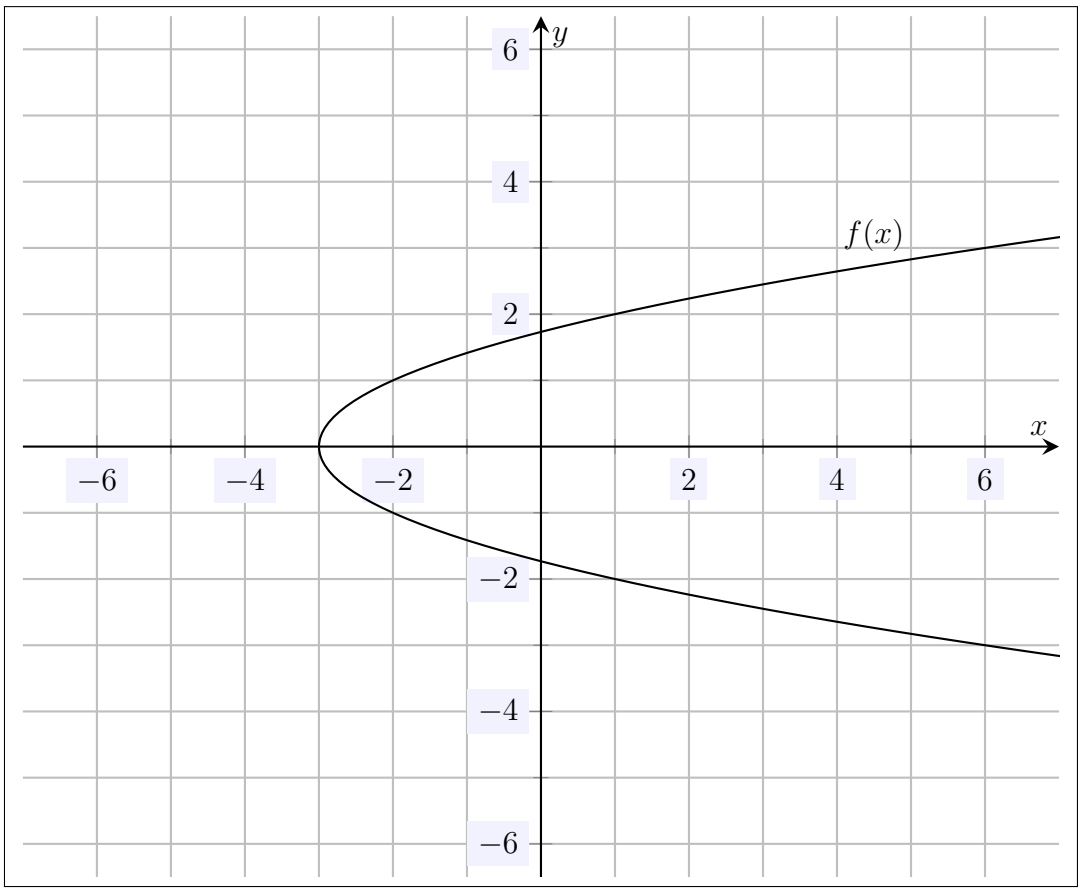
(c) ___F___: $H(x)$

(d) ___T___: $J(x)$

(e) ___T___: $K(x)$

(f) ___T___: $L(x)$

12. (4 points) Consider the relation plotted below.



(a) Is the relation plotted above a function? Explain.

The relation $f(x)$ above is not a function because it fails the vertical line test.

(b) Does the relation above have an inverse function? Explain.

The relation $f(x)$ above has an inverse function because it passes the horizontal line test.

13. (10 points) Consider the functions given in the table below.

x	0	1	2	3	4
$f(x)$	1	4	0	-2	6
$g(x)$	2	3	5	5	7

Compute the following:

(a) $g(3) = 5$

(b) $g(2) - f(3) = 5 - (-2) = 5 + 2 = 7$

(c) $f(0)\, g(1) = 1 \cdot 3 = 3$

(d) $(f + g)(4) = f(4) + g(4) = 6 + 7 = 13$

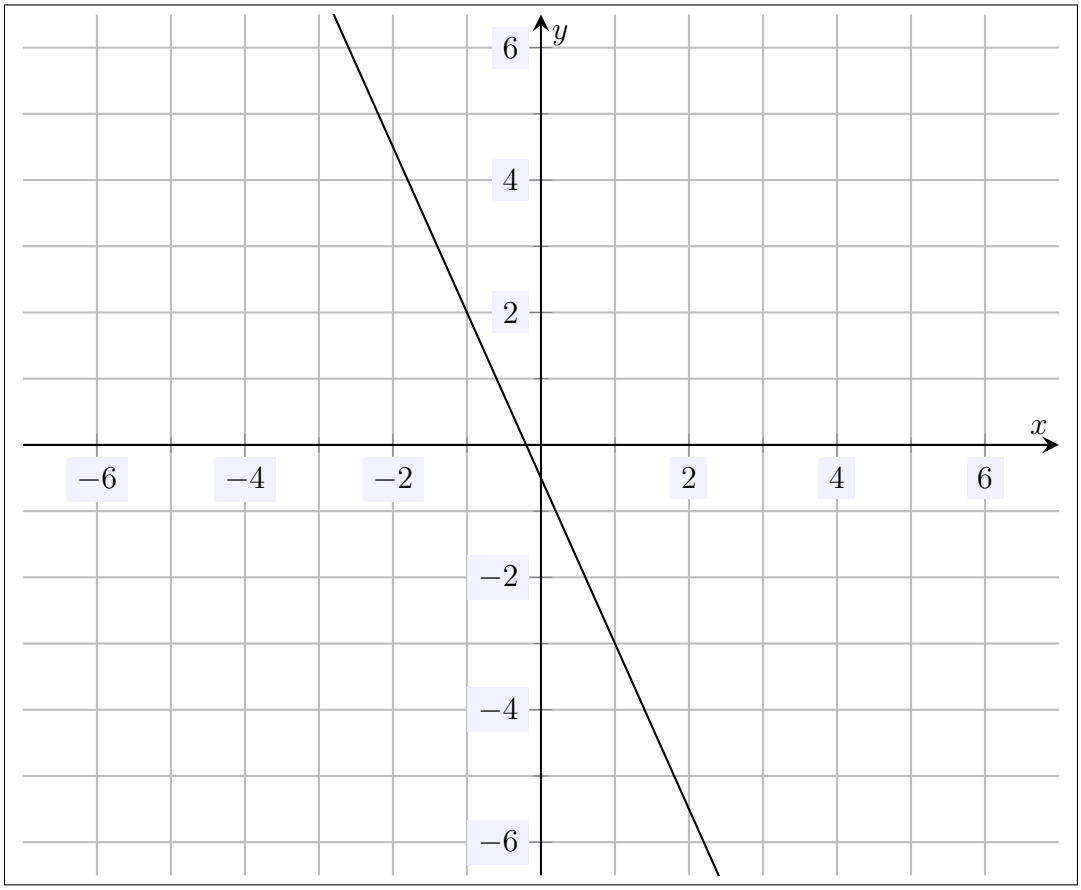
(e) $(f \circ g)(1) = f(g(1)) = f(3) = -2$

(f) $(g \circ f)(1) = g(f(1)) = g(4) = 7$

(g) x-intercept of $f(x)$: $(2, 0)$

(h) y-intercept of $g(x)$: $(0, 2)$

14. (8 points) Find the equation of the line shown below:



The line is not vertical so that it has the form $y = mx + b$*. The line passes through the point* $(-1, 2)$ *and* $(1, -3)$*. But then the slope is...*

$$m = \frac{2-(-3)}{-1-1} = \frac{2+3}{-1-1} = \frac{5}{-2} = -\frac{5}{2}$$

Then we know $y = -\frac{5}{2}x + b$*. But the line passes through the point* $(-1, 2)$*, i.e. contains the point where* $x = -1$ *and* $y = 2$*, so that...*

$$\begin{aligned} y &= -\frac{5}{2}x + b \\ 2 &= -\frac{5}{2} \cdot -1 + b \\ 2 &= \frac{5}{2} + b \\ b &= 2 - \frac{5}{2} \\ b &= \frac{4}{2} - \frac{5}{2} \\ b &= -\frac{1}{2} \end{aligned}$$

But then the equation of the line is $y = -\frac{5}{2}x - \frac{1}{2}$.

15. (8 points) Find the equation of the line that is perpendicular to the line $y = \frac{1}{2}x + 1$ that passes through the point $(2, 1)$.

The line $y = \frac{1}{2}x + 1$ is not horizontal so that a line perpendicular to it will not be vertical. Then our line must have the form $y = mx + b$. Because the line is perpendicular to the line $y = \frac{1}{2}x + 1$, the slope of our line is the negative reciprocal of the slope of the line $y = \frac{1}{2}x + 1$. The slope of the line $y = \frac{1}{2}x + 1$ is $\frac{1}{2}$, so the slope of our line is $-\frac{2}{1} = -2$, i.e. $m = -2$. Then we know $y = -2x + b$. But the line contains the point $(2, 1)$, i.e. the point where $x = 2$ and $y = 1$. But then

$$\begin{aligned} y &= -2x + b \\ 1 &= -2(2) + b \\ 1 &= -4 + b \\ b &= 5 \end{aligned}$$

Therefore, the equation of the line is $y = -2x + 5$.