Name: _____
MATH 308
Fall 2022
HW 7: Due 09/29

"'Obvious' is the most dangerous word in mathematics."

–E.T. Bell

Problem 1. (10pt) Determine whether each of the following relations is a function. If the relation is a function, determine its image.

(a) $\{(x, y) \colon x, y \in \mathbb{Z}, y = x^2 + 5\}$ as a relation from $\mathbb{Z}$ to $\mathbb{Z}$

(b) $\{(x, y) \colon x, y \in \mathbb{R}, y = x^2\}$ as a relation from $\mathbb{R}$ to $\mathbb{R}$

(c) $\{(x, y) \colon x, y \in \mathbb{Z}, y^2 = x\}$ as a relation from $\mathbb{R}$ to $\mathbb{R}$

(d) $\{(x, y) \colon x, y \in \mathbb{Z}, y = 2x + 3\}$ as a relation from $\mathbb{Z}$ to $\mathbb{Z}$

(e) $\{(x, y) \colon x, y \in \mathbb{Z}, x^2 + y^2 = 4\}$ as a relation from $\mathbb{R}$ to $\mathbb{R}$

Problem 2. (10pt) Define $A = \{3, 6, 9\}$ and $B = \{3x \colon x \in \mathbb{Z}\} - \{x \in \mathbb{Z} \colon x \leq 0, x > 10\}$. Let $f : A \to \mathbb{Z}$ be given by $f(x) = 2x + 1$ and $g : B \to \mathbb{Z}$ be defined by $g(x) = x^3 - 18x^2 + 101x - 161$. Show that $f = g$.

Problem 3. (10pt) Let $f : \mathbb{N} \to \mathbb{R}$ be given by $f(n) = 1 - n$ and $g : \mathbb{N} \to \mathbb{R}$ be given by $g(n) = \frac{n}{n+1}$. For each of the following, either find a rule for the given function or evaluate the given function:

(a) $(fg)(1)$

(b) $(f + g)(n)$

(c) $(g \circ f)(5)$

(d) $(6f)(-3)$

(e) $\left(\frac{f}{g}\right)(n)$

Problem 4. (10pt) Let $f : A \to \mathbb{R}$ be given by $f(x) = |x + 1|$, where $|\cdot|$ denotes the absolute value. For each of the following, find the image of A under f—no justification is necessary:

(a) $A = [1, 6]$

(b) $A = (-3, 4]$

(c) $A = \mathbb{N}$

(d) $A = \mathbb{Z}$

(e) $A = \mathbb{R}$