

Name: \_\_\_\_\_

MATH 308

Fall 2022

HW 7: Due 09/29

“‘Obvious’ is the most dangerous word  
in mathematics.”

—E.T. Bell

**Problem 1.** (10pt) Determine whether each of the following relations is a function. If the relation is a function, determine its image.

(a)  $\{(x, y) : x, y \in \mathbb{Z}, y = x^2 + 5\}$  as a relation from  $\mathbb{Z}$  to  $\mathbb{Z}$

(b)  $\{(x, y) : x, y \in \mathbb{R}, y = x^2\}$  as a relation from  $\mathbb{R}$  to  $\mathbb{R}$

(c)  $\{(x, y) : x, y \in \mathbb{Z}, y^2 = x\}$  as a relation from  $\mathbb{R}$  to  $\mathbb{R}$

(d)  $\{(x, y) : x, y \in \mathbb{Z}, y = 2x + 3\}$  as a relation from  $\mathbb{Z}$  to  $\mathbb{Z}$

(e)  $\{(x, y) : x, y \in \mathbb{Z}, x^2 + y^2 = 4\}$  as a relation from  $\mathbb{R}$  to  $\mathbb{R}$

**Problem 2.** (10pt) Define  $A = \{3, 6, 9\}$  and  $B = \{3x : x \in \mathbb{Z}\} - \{x \in \mathbb{Z} : x \leq 0, x > 10\}$ . Let  $f : A \rightarrow \mathbb{Z}$  be given by  $f(x) = 2x + 1$  and  $g : B \rightarrow \mathbb{Z}$  be defined by  $g(x) = x^3 - 18x^2 + 101x - 161$ . Show that  $f = g$ .

**Problem 3.** (10pt) Let  $f : \mathbb{N} \rightarrow \mathbb{R}$  be given by  $f(n) = 1 - n$  and  $g : \mathbb{N} \rightarrow \mathbb{R}$  be given by  $g(n) = \frac{n}{n+1}$ . For each of the following, either find a rule for the given function or evaluate the given function:

(a)  $(fg)(1)$

(b)  $(f + g)(n)$

(c)  $(g \circ f)(5)$

(d)  $(6f)(-3)$

(e)  $\left(\frac{f}{g}\right)(n)$

**Problem 4.** (10pt) Let  $f : A \rightarrow \mathbb{R}$  be given by  $f(x) = |x + 1|$ , where  $|\cdot|$  denotes the absolute value. For each of the following, find the image of  $A$  under  $f$ —no justification is necessary:

- (a)  $A = [1, 6]$
- (b)  $A = (-3, 4]$
- (c)  $A = \mathbb{N}$
- (d)  $A = \mathbb{Z}$
- (e)  $A = \mathbb{R}$