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MATH 108
Fall 2022
HW 8: Due 10/25

"I'll never let go, Jack. I'll never let go."
–Rose DeWitt Bukater, Titanic
(Shortly before letting go. . .)

Problem 1. (10pt) Below are two charts giving information on survival/death for the April 15, 1912 sinking of the Titanic.

	Men	Women	Boys	Girls	Total
Survived	338	316	29	27	710
Died	1352	109	35	18	1514
Total	1690	425	64	45	2224

	Survived	Died	Total
First Class Men	57	118	175
First Class Women	140	4	144
First Class Children	5	1	6
Second Class Men	14	154	168
Second Class Women	80	13	93
Second Class Children	24	0	24
Third Class Men	75	387	462
Third Class Women	76	89	165
Third Class Children	27	52	79
Crew (Men)	192	693	885
Crew (Women)	20	3	23
Total	710	1514	2224

(a) Find the probability that a randomly selected person died.

(b) Find the probability that a randomly selected person was a woman that died.

(c) Find the probability that a person that died was in first class.

(d) Find the probability that a person died or was in first class.

(e) Find the probability that a person was a crew member.

Solution.

(a)
$$P(\text{died}) = \frac{1514}{2224} \approx 0.680755$$

(b)
$$P(\text{woman and died}) = \frac{109}{2224} \approx 0.0490108$$

(c)
$$P(\text{first class} \mid \text{died}) = \frac{118+4+1}{1514} = \frac{123}{1514} \approx 0.0812417$$

(d)
$$P(\text{died or first class}) = \frac{(1514+175+144+6)-(118+4+1)}{2224} = \frac{1839-123}{2224} = \frac{1716}{2224} \approx 0.771583$$

(e)
$$P(\text{crew}) = \frac{885+23}{2224} = \frac{908}{2224} \approx 0.408273$$

Problem 2. (10pt) Some construction companies work exclusively with large businesses constructing office spaces, warehouses, etc. Others work in residential construction, and some work in both. Suppose that 47% of construction companies work in business construction, 35% of companies work in residential construction, and 12% work in either area.

(a) Find the percentage of companies that work in business or residential construction.

(b) Find the percentage of companies that work in neither business nor residential construction.

(c) Find the percentage of companies that work exclusively in business construction.

(d) Find the percentage of companies that work in residential construction that also do business construction.

(e) Find the percentage of companies that work exclusively in business or residential construction.

Solution.

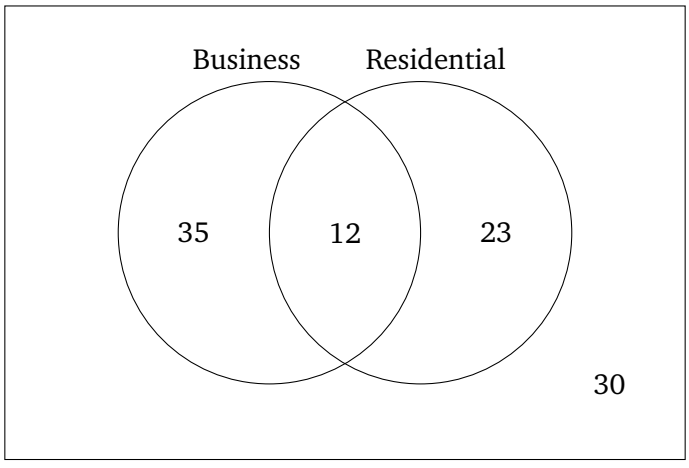
(a)
$$P(\text{business or residential}) = 35\% + 12\% + 23\% = 70\%$$

(b)
$$P(\text{neither business nor residential}) = 30\%$$

(c)
$$P(\text{only business}) = 35\%$$

(d)
$$P(\text{business} \mid \text{residential}) = \frac{12\%}{12\% + 23\%} = \frac{12\%}{35\%} \approx 0.342857 = 34.3\%$$

(e)
$$P(\text{only business or only residential}) = 35\% + 23\% = 58\%$$



Problem 3. (10pt) Suppose a disease only occurs in approximately 2% of the population. A test is developed that detects this disease in those that have it 97% of the time, and only incorrectly identifies those that do not have the disease as having it 4% of the time.

(a) Find the probability that a person tests positive for the disease.

(b) Assuming independence, what is the probability that you test positive twice if you do not have the disease?

(c) Find the probability that a person has the disease or tests positive.

(d) Find the probability that a person that tests positive for the disease actually has the disease.

(e) Does your response in (d) contradict the fact that the test is '97% accurate'? Explain.

Solution.

(a)
$$P(+) = 0.0194 + 0.0392 = 0.0586$$

(b)
$$P(+ \text{ twice} \mid \text{Not}) = P(+ \mid \text{Not}) \cdot P(+ \mid \text{Not}) = \frac{0.0392}{0.98} \cdot \frac{0.0392}{0.98} = 0.0016$$

(c)
$$P(\text{has or } +) = 0.0194 + 0.0006 + 0.0392 = 0.0592$$

(d)
$$P(\text{has} \mid +) = \frac{0.0194}{0.03292 + 0.0194} = \frac{0.0194}{0.05232} = 0.370795$$

(e) No. We know that the test correctly identifies those with the disease as having the disease 97% of the time. In that sense, the test is 97% accurate. Similarly, it correctly identifies a person without the disease as not having it 96% of the time. However accurate the test is, because the disease is 'rare', if a positive test occurs it is more likely that it is one of these 'rare' errors rather than actually being an indication of the disease. The test is '97% accurate' as advertised. It is simply that a positive test result is more likely indicative of a 'rare' error than a 'rare' disease.