Name: *Caleb McWhorter — Solutions*
MATH 108
Fall 2021
HW 3: Due 09/28

"This year I'm lovin' someone who deserves me. Me."
–Suzanne 'Crazy Eyes' Warren, Orange is the New Black

Problem 1. (10pt) Let $C(x)$ be the cost function given by $C(x) := 3.50x + 15$.

(a) Find the total cost in producing 100 items.

$$C(100) = 3.50(100) + 15 = 350 + 15 = 365$$

(b) If the company makes 100 items, what is the average cost of production per item?

$$\frac{C(100)}{100} = \frac{365}{100} = 3.65$$

(c) What is the production cost per item?

Because the function is linear, this is the slope of the cost function, which is 3.50.

(d) Find the y-intercept for $C(x)$.

$$C(0) = 3.50(0) + 15 = 0 + 15 = 15$$

Therefore, the y-intercept is $(0, 15)$.

(e) Interpret your answer from (d).

The y-intercept occurs when $x = 0$, i.e. when we produce 0 items. But then any cost associated with producing nothing must be the fixed costs.

Problem 2. (10pt) Let $R(x)$ be the revenue function given by $R(x) := 15.99x$.

(a) What is the price per item that the company sets?

Because the revenue function is a linear function, the price per item is the slope of the line, which is 15.99.

$$\$15.99$$

(b) How much revenue is gained by selling 150 items?

$$R(150) = 15.99(150) = \$2398.50$$

(c) How many items would the company need to sell to make at least $2,000 in revenue?

$$\begin{aligned} R(x) &= 2000 \\ 15.99x &= 2000 \\ x &= 125.078 \end{aligned}$$

The company cannot sell 125.078 items (presumably). Then the must sell either 125 or 126 items. Selling less would result in less revenue. Therefore, the company needs to sell at least 126 items.

Problem 3. (10pt) Let the revenue and cost functions for a company be given by $R(x) := 24.99x$ and $C(x) = 11.20x + 560$.

(a) Find the profit function, $P(x)$.

$$P(x) = R(x) - C(x) = 24.99x - (11.20x + 560) = 24.99x - 11.20x - 560 = 13.79x - 560$$

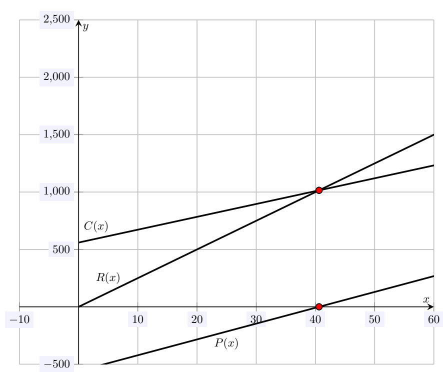
(b) Find the breakeven point.

$$\begin{aligned} P(x) &= 0 \\ 13.79x - 560 &= 0 \\ 13.79x &= 560 \\ x &= 40.609 \end{aligned}$$

(c) What does the breakeven point represent on the graph of $P(x)$?

The breakeven point is the point where profit is 0. But then $P(x) = 0$. *Then this must be a* x*-intercept for* $P(x)$.

(d) Sketch the functions $R(x), C(x)$ and $P(x)$ along with the breakeven point.



Problem 4. (10pt) A fine dining restaurant orders high-quality salmon for their menu. When bought in bulk, each salmon costs \$11.99 and there is a flat delivery fee of \$210. To turn profit on the fish orders, the restaurant marks the price up by 80%. What is the smallest number of salmon they have to order and sell to make a profit on their salmon sales?

Solution. *First, we find the revenue function. We know that the salmon costs \$11.99. The restaurant will mark this up by 80%. Therefore, the price of the salmon will be* $\$11.99(1.8) = \$21.582$*. Then the revenue function is* $R(x) = 21.582x$.

Now we need to find the cost function. We know they are charged a delivery of \$210—the fixed cost. Each salmon costs \$11.99, so the variable cost is $11.99x$*. Therefore, the (total) cost function is* $C(x) = 11.99x + 210$.

To find the point at which they start to turn a profit on salmon sales, we need to find the breakeven point. We can do this either by finding when $R(x) = C(x)$*, or we can find the profit function,* $P(x)$*, and then find when* $P(x) = 0$*. We choose the latter. We know* $P(x) = R(x) - C(x)$*. But then*

$$P(x) = R(x) - C(x) = 21.582x - (11.99x + 210) = 21.582x - 11.99x - 210 = 9.592x - 210$$

Now we find when $P(x) = 0$*:*

$$\begin{aligned} P(x) &= 0 \\ 9.592x - 210 &= 0 \\ 9.592x &= 210 \\ x &= 21.89 \end{aligned}$$

Because the restaurant cannot sell 21.89 *salmon, they must sell either 21 or 22 to break even. However, selling less fish clearly makes less money and* $x = 21.89$ *is the exact point where they break even, they must sell at least 22 salmon.*