

Name: \_\_\_\_\_

MATH 308

Fall 2023

HW 5: Due 10/05

*"No one shall expel us from the paradise  
which Cantor has created for us."*

*–David Hilbert*

**Problem 1.** (10pt) Define the following sets:

- (a) The set of positive odd numbers less than 50 that are not prime.
- (b) The set of real-valued solutions to  $\frac{x^2-4}{x+2} = 0$ .
- (c) The set of integer solutions to  $\sqrt{2x-1} + 4 = 12$ .
- (d) The set of English sentences containing a homonym.
- (e) The set of linear functions with positive  $y$ -intercept.

For each of the sets described above, do the following:

- (i) Determine if the set is empty or nonempty. If the set is nonempty, give an element and non-element of the set.
- (ii) Determine whether the set is finite or infinite. If it is finite, state its cardinality.
- (iii) If the set is finite, enumerate all its elements. If the set is infinite, give the set using set-builder notation.

— *Continued Space for Problem 1* —

**Problem 2.** (10pt) For each of the sets given below, describe the sets in words. Also for each set, give an example of an element and non-element of the set.

(a)  $\{3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, \dots\}$

(b)  $\{0, 1, 4, 9, 16, 25, 36, 49, 81, 100, \dots\}$

(c)  $\{f(x) : f(x) \text{ function}, (\exists x_0 \in \mathbb{R})[f(x_0) = 0]\}$

(d)  $\{f(x) : f(x) \text{ function}, f(x) > 0 \text{ for all } x \in \mathbb{R}\}$

(e)  $\{f(x, y) : f(2, 3) = 0\}$

**Problem 3.** (10pt) Define the following sets:

$$A = \{-10, -9, -8, \dots, 8, 9, 10\}$$

$$B = \{-10, -8, -6, \dots, 6, 8, 10\}$$

$$C = \{-9, -7, -5, \dots, 5, 7, 9\}$$

$$D = \{-10, -5, 0, 5, 10\}$$

$$E = \{-4, -1, 1, 2, 3, 5, 7\}$$

$$F = \{-10, -9, -2, -1, 1, 5, 6, 9\}$$

Consider each of the sets above as coming from the universal set  $\mathcal{U} := A$ . Compute the following:

(a)  $B \cup C$

(d)  $F \Delta C$

(b)  $B \cap C$

(e)  $E^c$

(c)  $E \setminus D$

(f)  $(C \cup E) - B$

**Problem 4.** (10pt) Define the following sets:

$$A := (-10, 10)$$

$$B := [0, 3]$$

$$C := (-1, 15]$$

$$D := (-20, -3] \cup [4, 12)$$

Consider each of the sets above as coming from the universal set  $\mathbb{R} = (-\infty, \infty)$ . Compute the following:

(a)  $D^c$

(b)  $B \cap C$

(c)  $A \Delta C$

(d)  $C - B$

(e)  $(A \cap D) \cup C$