Quiz 1. *True/False*: $9/3 + 2(3^2 + 10) - 8 + 4 \cdot 3 = 45$

Solution. The statement is *false*. To see this, we can simply follow the order of operations—using PEMDAS as a guide:

$$\begin{aligned}
9/3 + 2(3^2 + 10) - 8 + 4 \cdot 3 &\stackrel{?}{=} 55 \\
9/3 + 2(9 + 10) - 8 + 4 \cdot 3 &\stackrel{?}{=} 55 \\
9/3 + 2(19) - 8 + 4 \cdot 3 &\stackrel{?}{=} 55 \\
3 + 2(19) - 8 + 4 \cdot 3 &\stackrel{?}{=} 55 \\
3 + 38 - 8 + 4 \cdot 3 &\stackrel{?}{=} 55 \\
3 + 38 - 8 + 12 &\stackrel{?}{=} 55 \\
41 - 8 + 12 &\stackrel{?}{=} 55 \\
33 + 12 &\stackrel{?}{=} 55 \\
45 &\neq 55
\end{aligned}$$

Quiz 2. *True/False*: $\gcd(2^3 \cdot 3 \cdot 5, 2 \cdot 3^2 \cdot 7) = 2^3 \cdot 3^2 \cdot 5 \cdot 7$.

Solution. The statement is *false*. Remember given a prime factorization of the numbers, we find the gcd by choosing the *smallest* powers of each prime that appears in the factorizations. So we should have $\gcd(2^3 \cdot 3 \cdot 5, 2 \cdot 3^2 \cdot 7) = 2 \cdot 3$. Instead, the largest power of each prime that appears in the factorizations was chosen which is how we compute the lcm. Therefore, we have $\operatorname{lcm}(2^3 \cdot 3 \cdot 5, 2 \cdot 3^2 \cdot 7) = 2^3 \cdot 3^2 \cdot 5 \cdot 7$.

Quiz 3. *True/False*: $\dfrac{\frac{3}{10}}{\frac{12}{5}} = \dfrac{1}{8}$

Solution. The statement is *true*. Note that division by a nonzero number is the same as multiplying by its reciprocal. So we have

$$\frac{\frac{3}{10}}{\frac{12}{5}} = \frac{3}{10} \cdot \frac{5}{12} = \frac{\cancel{3}^1}{\cancel{10}^2} \cdot \frac{\cancel{5}^1}{\cancel{12}^4} = \frac{1}{8}$$

One can also rewrite the problem as...

$$\frac{\frac{3}{10}}{\frac{12}{5}} = \frac{3}{10} \div \frac{12}{5}$$

But then to divide, we multiply by the reciprocal and proceed as in the solution above.

Quiz 4. *True/False*: The number $0.\overline{19}$ is rational.

Solution. The statement is *true*. Any real number with a decimal expansion that either terminates or repeats is a rational and hence can be expressed as a/b, where a and b are integers and $b \neq 0$. Moreover, every rational number, i.e. the a/b's, have a decimal expansion that either terminates or repeats. We can even find a rational expression for $0.\overline{19}$:

$$\begin{array}{rrcr} & 100r & = & 19.191919191919\ldots \\ - & r & = & 0.191919191919\ldots \\ \hline & 99r & = & 19 \end{array}$$

But then $r = 0.\overline{19} = \dfrac{19}{99}$.

Quiz 5. *True/False*: $\sqrt[3]{2^8 \cdot 3^3 \cdot 5^1 \cdot 7^5} = 2^2 \cdot 3^1 \cdot 7\sqrt[3]{2^2 \cdot 5^1 \cdot 7^2}$

Solution. The statement is *true*. There are two ways to think about this. First, we should write out the numbers and group them into threes and pull out/leave the terms appropriately:

$$\sqrt[3]{2^8 \cdot 3^3 \cdot 5^1 \cdot 7^5} = \sqrt[3]{\underline{2 \cdot 2 \cdot 2} \cdot \underline{2 \cdot 2 \cdot 2} \cdot 2 \cdot 2 \cdot \underline{3 \cdot 3 \cdot 3} \cdot 5 \cdot \underline{7 \cdot 7 \cdot 7} \cdot 7 \cdot 7} = 2^2 \cdot 3^1 \cdot 7\sqrt[3]{2^2 \cdot 5 \cdot 7^2}$$

Alternatively, we can use division. We know that $8/3$ is 2 with remainder 2, $3/3$ is 1 with remainder 0, $1/3$ is 0 with remainder 1, and $5/3$ is 1 with remainder 2. So we can pull out two 3's with 2 remaining, one 3 with 0 remaining, no 5's with 1 remaining, and two 7's with 2 remaining, which gives:

$$\sqrt[3]{2^8 \cdot 3^3 \cdot 5^1 \cdot 7^5} = 2^2 \cdot 3^1 \cdot 7\sqrt[3]{2^2 \cdot 5^1 \cdot 7^2}$$

Quiz 6. *True/False*: 57 increased by 127% is $57(1.27)$.

Solution. The statement is *false*. To find 127% of 57, we would multiply 57 by the percent written as a decimal. This would be $57(1.27)$. However, to increase or decrease a number by a percentage, we compute the number $\#(1 \pm \%)$, where we add if we are increasing, subtract if we are decreasing, $\#$ is the number, and % is the percentage written as a decimal. So to increase 57 by 127%, we need to compute $57(1 + 1.27) = 57(2.27)$.

Quiz 7. *True/False*: If $f(x) = x + 1$ and $g(x) = x^2$, then $(f \circ g)(2) = 9$.

Solution. The statement is *false*. Recall that $(f \circ g)(2) = f(g(2))$. First, we compute $g(2)$. We have $g(2) = 2^2 = 4$. Then we have $f(g(2)) = f(4)$ and we compute $f(4)$: $f(4) = 4 + 1 = 5$.

Quiz 8. *True/False*: The point $(1, 3)$ is on the graph of $f(x) = 2x - 5$.

Solution. The statement is *false*. We have the point $(x, y) = (1, 3)$. If this point is on the graph of $f(x)$, then these x and y satisfy the equation for $f(x)$. We can check this:

$$\begin{aligned} f(x) &= 2x - 5 \\ 3 &= 2(2) - 5 \\ 3 &= 4 - 5 \\ 3 &\neq -1 \end{aligned}$$

Therefore, the point $(1, 3)$ is not on the graph of $f(x)$. Alternatively, if $x = 1$, then the corresponding point on the graph of $f(x)$ would have y-value $f(1) = 2(1) - 5 = 2 - 5 = -3$. Then the point $(1, -3)$ is on the graph of $f(x)$. But then $(1, 3)$ is not on the graph of $f(x)$.

Quiz 9. *True/False*: If $f^{-1}(3) = 9$, then $f(3) = 9$.

Solution. The statement is *false*. Recall that $f^{-1}(y) = x$ if and only if $f(x) = y$; that is, f^{-1} asks the question, 'what do I plug into f to get this number.' So if $f^{-1}(3) = 9$, this means we should be able to plug in 9 into $f(x)$ and obtain 3, i.e. $f(9) = 3$. But then $f(3) = 9$ is not necessarily true.

Quiz 10. *True/False*: To find the x-intercept, you find $f(0)$.

Solution. The statement is *false*. Recall that an x-intercept is where a function intersects the x-axis. But then the y-value must be zero. But then because $y = f(x)$, we have $f(x) = 0$. Whereas if we wanted to find a y-intercept, we would recall that along the x-axis, $x = 0$ so that we would need to find $f(0)$. So finding x-intercepts involves solving $f(x) = 0$, whereas finding y-intercepts involves evaluating $f(0)$.

Quiz 11. *True/False*: The lines $y = \frac{2}{3}x + 5$ and $3x + 2y = -6$ are perpendicular.

Solution. The statement is *true*. The line $y = \frac{2}{3}x + 5$ has slope $m = \frac{2}{3}$. Solving for y in the second line, we have $y = -3 - \frac{3}{2}x$. This line has slope $m = -\frac{3}{2}$. The negative reciprocal of $\frac{2}{3}$ is $-\frac{3}{2}$. Therefore, the lines are perpendicular.

Quiz 12. All lines perpendicular to $y = 4$ are of the form $x = \#$.

Solution. The statement is *true*. The line $y = 4$ is horizontal. For a line to be perpendicular to a horizontal line, the line must be vertical. But all vertical lines are of the form $x = \#$.

Quiz 13. *True/False*: Any line with slope 0 must be of the form $y = \#$.

Solution. The statement is *true*. All vertical lines 'look like' $y = mx + b$ for some m, b. If the slope is 0, then $m = 0$. But then $y = \#$.

Quiz 14. *True/False*: All functions have inverses.

Solution. The statement is *false*. All constant functions, i.e. $f(x) = \#$, do not have inverses. Constant functions are functions—every input has exactly one output (even if they all happen to be the same). However, you cannot 'tell' what x gave you $\#$. Alternatively, $f(x) = \#$ fails the horizontal line test. [Recall that a function has an inverse if and only if it passes the horizontal line test.]

Quiz 15. *True/False*: The quadratic function $y = 5x + 3 - x^2$ opens downwards, is concave, and has a maximum.

Solution. The statement is *true*. Writing the quadratic function in standard form, i.e. $y = ax^2 + bx + c$, we have $y = -x^2 + 5x + 3$. Therefore, for this quadratic function, $a = -1$, $b = 5$, and $c = 3$. Because $a = -1 < 0$, the quadratic function opens downwards, i.e. is concave (down), and has a maximum.

Quiz 16. *True/False*: The quadratic function $f(x) = 2(x + 2)^2 + 4$ has vertex $(2, 4)$.

Solution. The statement is *false*. The x-coordinate of the vertex is the x-value that makes the square term zero. In this case, $x = -2$ would make $2(x + 2)^2$ zero. Then we would be left with $y = 4$, which is the y-coordinate of the vertex. Therefore, the vertex is $(-2, 4)$. Alternatively, the 'proper' vertex form of a quadratic function is $y = A(x - B) + C$. The vertex is (B, C). Writing the 'proper' vertex form of the quadratic function $y = 2(x+2)^2+4$, we have $y = 2(x- \; -2)^2+4$. Therefore, the vertex form is $(-2, 4)$. Finally, one could expand this out: $y = 2(x + 2)^2 + 4 = 2(x^2 + 4x + 4) + 4 = 2x^2 + 8x + 8 + 4 = 2x^2 + 8x + 12$. The x-coordinate of the vertex is $x = \frac{-b}{2a} = \frac{-8}{2(2)} = -2$. Then the y-coordinate of the vertex is $y(-2) = 2(-2)^2 + 8(-2) + 12 = 8 - 16 + 12 = 4$. Therefore, the vertex is $(-2, 4)$.

Quiz 17. *True/False*: The quadratic function $y = x^2 - 4x - 12$ factors as $(x - 6)(x + 2)$.

Solution. Solution. The statement is *true*. One way of seeing this would be to expand $(x-6)(x+2)$,

$$(x - 6)(x + 2) = x^2 + 2x - 6x - 12 = x^2 - 4x - 12.$$

Alternatively, we can factor the polynomial $x^2 - 4x - 12$. First, we find the factors of 12, which are only $1, 12$, and $2, 6$, and $3, 4$. Because the 12 is negative, the factors must have opposite signs.

$$\begin{aligned}
1, -12&\colon \; -11 \\
-1, 12&\colon \quad 11 \\
2, -6&\colon \; -4 \\
-2, 6&\colon \quad 4 \\
3, -4&\colon \; -1 \\
-3, 4&\colon \quad 1
\end{aligned}$$

We want these signed factors to add to -4. Therefore, we want 'factors' $2, -6$. Therefore,

$$x^2 - 4x - 12 = (x + 2)(x - 6)$$

Quiz 18. *True/False*: Let D be the discriminant of a quadratic polynomial. If $D = -4$, then the polynomial factors 'nicely.'

Solution. The statement is *false*. Recall that a quadratic polynomial factors 'nicely' if and only if its discriminant is a perfect square. However, $D = -4$ is *not* a perfect square. The number 4 is a perfect square because $2^2 = 4$. But there is no real number whose square is -4. Therefore, the quadratic polynomial must not factor 'nicely.' Note that if $D < 0$, then the quadratic polynomial factors over $\mathbb{C}$.

Quiz 19. *True/False*: $x^2 - 2x - 1 = \big(x - (1 + \sqrt{2})\big)\big(x - (1 - \sqrt{2})\big)$

Solution. The statement is *true*. For the quadratic function $x^2 - 2x - 1$, we have $a = 1$, $b = -2$, and $c = -1$. We can compute the discriminant to find $D = b^2 - 4ac = (-2)^2 - 4(1)(-1) = 4 + 4 = 8$. Because $D = 8$ is not a perfect square, the quadratic polynomial $x^2 - 2x - 1$ does not factor 'nicely.' However, all quadratic functions are factorable. To find the factorization, we find the roots of $x^2 - 2x - 1$, i.e. the solutions to $x^2 - 2x - 1 = 0$, using the quadratic formula. We have…

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} \\
&= \frac{2 \pm \sqrt{4 + 4}}{2} \\
&= \frac{2 \pm \sqrt{8}}{2} \\
&= \frac{2 \pm \sqrt{4 \cdot 2}}{2} \\
&= \frac{2 \pm 2\sqrt{2}}{2} \\
&= 1 \pm \sqrt{2}
\end{aligned}$$

Then we have roots $r_1 = 1 + \sqrt{2}$ and $r_2 = 1 - \sqrt{2}$. Therefore, the factorization is

$$x^2 - 2x - 1 = a(x - r_1)(x - r_2) = \big(x - (1 + \sqrt{2})\big)\big(x - (1 - \sqrt{2})\big)$$