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MATH 101
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HW 6: Due 06/02

"The fact that we live at the bottom of a deep gravity well, on the surface of a gas covered planet going around a nuclear fireball 90 million miles away and think this to be normal is obviously some indication of how skewed our perspective tends to be."

–Douglas Adams

Problem 1. (10pt) Determine whether the following lines are parallel, perpendicular, or neither. Be sure to justify your answer.

$$\ell_1\colon y = \frac{2}{3}\,x + 5$$

$$\ell_2\colon 3x - 2y = 8$$

Solution. Putting each line in the form $y = mx + b$, we have...

$$y = \frac{2}{3}\,x + 5 \qquad\qquad \begin{aligned} 3x - 2y &= 8 \\ -2y &= -3x + 8 \\ y &= \frac{3}{2}\,x - 4 \end{aligned}$$

The slope of the first line is $m_1 = \frac{2}{3}$ and the slope of the second line is $m_2 = \frac{3}{2}$. Because $m_1 \neq m_2$, the lines are not parallel. Therefore, the lines intersect. Because the negative reciprocal of m_1 is $-\frac{3}{2} \neq m_2$, the lines are not perpendicular. Therefore, the lines are neither parallel nor perpendicular.

Problem 2. (10pt) Determine whether the following lines are parallel, perpendicular, or neither. Be sure to justify your answer.

$$\ell_1\colon\; -5x + 6y = 6$$

$$\ell_2\colon\; 5x + 6y = -12$$

Solution. Putting each line in the form $y = mx + b$, we have...

$$\begin{aligned} -5x + 6y &= 6 & \qquad 5x + 6y &= -12 \\ 6y &= 5x + 6 & 6y &= -5x - 12 \\ y &= \frac{5}{6}x + 1 & y &= -\frac{5}{6}x - 2 \end{aligned}$$

The slope of the first line is $m_1 = \frac{5}{6}$ and the slope of the second line is $m_2 = -\frac{5}{6}$. Because $m_1 \neq m_2$, the lines are not parallel. Therefore, the lines intersect. Because the negative reciprocal of m_1 is $-\frac{6}{5} \neq m_2$, the lines are not perpendicular. Therefore, the lines are neither parallel nor perpendicular.

Problem 3. (10pt) Find the equation of the line with x-intercept $(6, 0)$ and passing through the point $(-1, 10)$.

Solution. Because the desired line is not vertical, we know that it has the form $y = mx+b$. Because the line passes through the points $(6, 0)$ and $(-1, 10)$, we have. . .

$$m = \frac{0 - 10}{6 - (-1)} = \frac{-10}{7} = -\frac{10}{7}$$

We know that $y = -\frac{10}{7}\,x + b$. But the line contains the point $(6, 0)$ so that. . .

$$\begin{aligned} y &= -\frac{10}{7}\,x + b \\ 0 &= -\frac{10}{7} \cdot 6 + b \\ 0 &= -\frac{60}{7} + b \\ b &= \frac{60}{7} \end{aligned}$$

Therefore, the line is $y = -\frac{10}{7}\,x + \frac{60}{7} = \dfrac{60 - 10x}{7}$.

Problem 4. (10pt) Find the equation of the line perpendicular to the line $2x - 3y = 5$ that passes through the origin.

Solution. Because the desired line is not vertical, we know that it has the form $y = mx+b$. Because the line is perpendicular to the line $2x - 3y = 5$, its slope is the negative reciprocal of the slope of the line $2x - 3y = 5$. We know...

$$2x - 3y = 5$$

$$-3y = -2x + 5$$

$$y = \frac{2}{3}\,x - \frac{5}{3}$$

so that the line has slope $\frac{2}{3}$. Therefore, the slope of the desired line is $m = -\frac{3}{2}$. We then know that $y = -\frac{3}{2}x + b$. Because the line contains the origin, i.e. the point $(0, 0)$, we have...

$$y = -\frac{3}{2}\,x + b$$

$$0 = -\frac{3}{2} \cdot 0 + b$$

$$b = 0$$

Therefore, the line is $y = -\frac{3}{2}\,x$.

Problem 5. (10pt) Find the equation of the line that contains $(1, -1)$ and is parallel to the line $3x + y = 11$.

Solution. Because the desired line is not vertical, we know that the line has the form $y = mx + b$. The desired line is parallel to the line $3x + y = 11$, implying that they have the same slope. We know…

$$3x + y = 11$$

$$y = -3x + 11$$

so that the line has slope -3. Therefore, the slope of the desired line is $m = -3$. We then have $y = -3x + b$. The line contains the point $(1, -1)$ so that when $x = 1$, we know that $y = -1$. But then…

$$y = -3x + b$$

$$-1 = -3(1) + b$$

$$-1 = -3 + b$$

$$b = 2$$

Therefore, $y = -3x + 2$.

Problem 6. (10pt) Showing all your work, solve the following equation and verify that your solution is correct:

$$5x - 7 = 7 - 2x$$

Solution. We have...

$$\begin{aligned} 5x - 7 &= 7 - 2x \\ 7x - 7 &= 7 \\ 7x &= 14 \\ x &= 2 \end{aligned}$$

We verify the solution:

$$\begin{gathered} 5x - 7 = 7 - 2x \\ 5(2) - 7 \stackrel{?}{=} 7 - 2(2) \\ 10 - 7 \stackrel{?}{=} 7 - 4 \\ 3 = 3 \\ \checkmark \end{gathered}$$

Problem 7. (10pt) Showing all your work, solve the following equation and verify that your solution is correct:

$$2(1-x) = 6x + 11$$

Solution. We have...

$$\begin{aligned} 2(1-x) &= 6x + 11 \\ 2 - 2x &= 6x + 11 \\ 2 &= 8x + 11 \\ -9 &= 8x \\ x &= -\frac{9}{8} \end{aligned}$$

We verify the solution:

$$\begin{aligned} 2(1-x) &= 6x + 11 \\ 2\left(1 - \frac{-9}{8}\right) &\stackrel{?}{=} 6 \cdot -\frac{9}{8} + 11 \\ 2\left(\frac{8}{8} - \frac{-9}{8}\right) &\stackrel{?}{=} 3 \cdot -\frac{9}{4} + 11 \\ 2 \cdot \frac{17}{8} &\stackrel{?}{=} -\frac{27}{4} + 11 \\ \frac{17}{4} &\stackrel{?}{=} -\frac{27}{4} + \frac{44}{4} \\ \frac{17}{4} &= \frac{17}{4} \\ &\checkmark \end{aligned}$$

Problem 8. (10pt) Showing all your work, solve the following equation and verify that your solution is correct:

$$\frac{x-1}{x+3} = 5$$

Solution. We have...

$$\begin{aligned} \frac{x-1}{x+3} &= 5 \\ x-1 &= 5(x+3) \\ x-1 &= 5x+15 \\ -1 &= 4x+15 \\ -16 &= 4x \\ x &= -4 \end{aligned}$$

We verify the solution:

$$\begin{aligned} \frac{x-1}{x+3} &= 5 \\ \frac{-4-1}{-4+3} &\stackrel{?}{=} 5 \\ \frac{-5}{-1} &\stackrel{?}{=} 5 \\ 5 &= 5 \\ &\checkmark \end{aligned}$$

Problem 9. (10pt) Suppose you sell automobiles. You earn a weekly baseline salary of \$820 per week and make 3% commission on your sales. Let $I(s)$ denote your weekly income if you make s dollars in sales.

(a) Explain why $I(s)$ is linear.

(b) Find $I(s)$.

(c) Find an interpret the slope and y-intercept of $I(s)$ in context, if possible.

(d) How much in sales do you have to make in a given week to have made \$1,500?

Solution.

(a) The only money earned comes from the baseline salary and commission. Because you get paid a constant baseline salary and earn a constant commission rate, the total amount you make each week is constant. Therefore, the amount of money you earn each week after s dollars in sales, $I(s)$, is linear.

(b) Each week, you make \$820. If you sell s dollars, you earn 3% commission, i.e. 3% of the total sales value. This is $0.03 \cdot s = 0.03s$. Therefore, you make $0.03s + 820$ each week, i.e. $I(s) = 0.03s + 820$.

(c) Because $I(s) = 0.03s + 820$ is in the form $y = mx + b$, we have $m = 0.03$ and $b = 820$. The slope, $m = 0.03$, is the commission you make on s dollars in sales. The y-intercept, $b = 820$ or $(0, 820)$, represents the amount you are paid each week—regardless of the amount in sales you make.

(d) If you sell s dollars in automobiles, you make $I(s)$ total that week. But then we want $I(s) = 1500$. Then we have…

$$\begin{aligned} I(s) &= 1500 \\ 0.03s + 820 &= 1500 \\ 0.03s &= 680 \\ s &= 22666.67 \end{aligned}$$

Therefore, to make \$1,500 in a week, you have to sell \$22,666.67 in automobiles.

Problem 10. (10pt) The amount of people, on average, that have entered a store t hours after it has opened, $P(t)$, can be modeled by $P(t) = 30.5t - 4$.

(a) What does $P(t)$ being linear imply about the rate that people enter the store?

(b) Find an interpret the slope and y-intercept of $I(s)$ in context, if possible.

(c) Find $P(2)$ and interpret the value.

(d) How long after opening until 400 people have entered the store?

Solution.

(a) Because the amount of people having entered the store t hours after opening, $P(t)$, on average is linear, we know that the rate of people entering the store is constant, on average. Because $P(t) = 30.5t - 4$ has the form $y = mx + b$ with $m = 30.5$ and $b = -4$, we know that, on average, 30.5 people enter the store every hour.

(b) Because $P(t) = 30.5t - 4$ has the form $y = mx + b$ with $m = 30.5$ and $b = -4$, we know that the y-intercept is -4, i.e $(0, -4)$. This would imply that when $t = 0$, zero hours from when the store opens, i.e. at opening, -4 people will enter the store, on average. As it is impossible for there to be -4 people entering the store on average (unless one wants this to mean, on average, 4 people leave the store at opening), there is no in-context interpretation for the y-intercept.

(c) We have $P(2) = 30.5(2) - 4 = 61 - 4 = 57$; that is, two hours after the store opens, 57 people have entered the store, on average.

(d) If 400 people have entered the store, then $P(t) = 400$. But then we have...

$$\begin{aligned} P(t) &= 400 \\ 30.5t - 4 &= 400 \\ 30.5t &= 404 \\ t &= 13.25 \end{aligned}$$

Therefore, on average, after 13.25 hours after opening, i.e. 13 hours and 15 minutes after opening, 400 people will have entered the store.