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MATH 101
Fall 2021
HW 20: Due 12/10

"Give me some of your tots!"
–Napoleon Dynamite, Napoleon Dynamite

Problem 1. (10pt) Determine if the system of equations below has a solution. If it does, find it; if not, explain why.

$$\begin{aligned} x + y &= 5 \\ x - y &= 9 \end{aligned}$$

Solution. This a system of linear equations. The system will have a solution if and only if the lines intersect. But this will only happen if they are not parallel. So we find the slopes of each line.

$$\begin{aligned} x + y &= 5 \qquad & x - y &= 9 \\ y &= -x + 5 & y &= x - 9 \end{aligned}$$

The slope of the first line is $m_1 = -1$ while the slope of the second line is $m_2 = 1$. Because $m_1 \neq m_2$, the lines are not parallel. But then the lines intersect so that there is a solution to the system of equations. Now we find the solution by using both substitution and elimination.
If we use substitution, we can solve for y in the first equation. This yields $y = 5 - x$. Using this in the second equation, we have...

$$\begin{aligned} x - y &= 9 \\ x - (5 - x) &= 9 \\ x - 5 + x &= 9 \\ 2x - 5 &= 9 \\ 2x &= 14 \\ x &= 7 \end{aligned}$$

But then we have $y = 5 - 7 = -2$. Therefore, the solution is $(7, -2)$.

Using elimination, suppose we eliminate y. Adding the equations, we find...

$$\begin{array}{r} x + y = 5 \\ \underline{x - y = 9} \\ 2x = 14 \\ x = 7 \end{array}$$

Using this in the first equation, we find

$$\begin{aligned} x + y &= 5 \\ 7 + y &= 5 \\ y &= -2 \end{aligned}$$

Therefore, the solution is $(7, -2)$.

Problem 2. (10pt) Determine if the system of equations below has a solution. If it does, find it; if not, explain why.

$$\begin{aligned} 15x - 6y &= 10 \\ -5x + 2y &= -8 \end{aligned}$$

Solution. This a system of linear equations. The system will have a solution if and only if the lines intersect. But this will only happen if they are not parallel. So we find the slopes of each line.

$$\begin{aligned} 15x - 6y &= 10 & \qquad -5x + 2y &= -8 \\ -6y &= -15x + 10 & 2y &= 5x - 8 \\ y &= \frac{5}{2}x + \frac{5}{3} & y &= \frac{5}{2}x - 4 \end{aligned}$$

The slope of the first line is $m_1 = \frac{5}{2}$ while the slope of the second line is $m_2 = \frac{5}{2}$. Because $m_1 = m_2$, the lines are parallel. But then the lines do not intersect so that there is no solution to the system of equations.

Problem 3. (10pt) Determine if the system of equations below has a solution. If it does, find it; if not, explain why.

$$\begin{aligned} 5x + 3y &= 7 \\ 3x - 2y &= -11 \end{aligned}$$

Solution. This a system of linear equations. The system will have a solution if and only if the lines intersect. But this will only happen if they are not parallel. So we find the slopes of each line.

$$\begin{aligned} 5x + 3y &= 7 & \qquad 3x - 2y &= -11 \\ 3y &= -5x + 7 & -2y &= -3x - 11 \\ y &= -\frac{5}{3}\,x + \frac{7}{3} & y &= \frac{3}{2}\,x + \frac{11}{2} \end{aligned}$$

The slope of the first line is $m_1 = -\frac{5}{3}$ while the slope of the second line is $m_2 = \frac{3}{2}$. Because $m_1 \neq m_2$, the lines are not parallel. But then the lines intersect so that there is a solution to the system of equations. Now we find the solution by using both substitution and elimination.
If we use substitution, we can solve for y in the first equation. This yields $y = -\frac{5}{3}\,x + \frac{7}{3}$. Using this in the second equation, we have…

$$\begin{aligned} 3x - 2y &= -11 \\ 3x - 2\left(-\frac{5}{3}\,x + \frac{7}{3}\right) &= -11 \\ 3x + \frac{10}{3}\,x - \frac{14}{3} &= -11 \\ 3\left(3x + \frac{10}{3}\,x - \frac{14}{3}\right) &= -11 \cdot 3 \\ 9x + 10x - 14 &= -33 \\ 19x - 14 &= -33 \\ 19x &= -19 \\ x &= -1 \end{aligned}$$

But then we have $y = -\frac{5}{3} \cdot -1 + \frac{7}{3} = \frac{5}{3} + \frac{7}{3} = \frac{12}{3} = 4$. Therefore, the solution is $(-1, 4)$.

Using elimination, suppose we eliminate y. Multiplying the first equation by 2 and the second equation by 3 and adding, we find

$$\begin{array}{r} 10x + 6y = 14 \\ 9x - 6y = -33 \\ \hline 19x = -19 \\ x = -1 \end{array}$$

Using this in the first equation, we find

$$\begin{aligned} 5x + 3y &= 7 \\ 5(-1) + 3y &= 7 \\ 3y - 5 &= 7 \\ 3y &= 12 \\ y &= 4 \end{aligned}$$

Therefore, the solution is $(-1, 4)$.

Problem 4. (10pt) Determine if the system of equations below has a solution. If it does, find it; if not, explain why.

$$5x - 6y = 3$$
$$2x + 3y = 3$$

Solution. This a system of linear equations. The system will have a solution if and only if the lines intersect. But this will only happen if they are not parallel. So we find the slopes of each line.

$$\begin{aligned} 5x - 6y &= 3 \\ -6y &= -5x + 3 \\ y &= \frac{5}{6}\,x - \frac{1}{2} \end{aligned} \qquad\qquad \begin{aligned} 2x + 3y &= 3 \\ 3y &= -2x + 3 \\ y &= -\frac{2}{3}\,x + 1 \end{aligned}$$

The slope of the first line is $m_1 = \frac{5}{6}$ while the slope of the second line is $m_2 = \frac{2}{3}$. Because $m_1 \neq m_2$, the lines are not parallel. But then the lines intersect so that there is a solution to the system of equations. Now we find the solution by using both substitution and elimination.
If we use substitution, we can solve for y in the first equation. This yields $y = \frac{5}{6}\,x - \frac{1}{2}$. Using this in the second equation, we have…

$$\begin{aligned} 2x + 3y &= 3 \\ 2x + 3\left(\frac{5}{6}\,x - \frac{1}{2}\right) &= 3 \\ 2x + \frac{5}{2}\,x - \frac{3}{2} &= 3 \\ 2\left(2x + \frac{5}{2}\,x - \frac{3}{2}\right) &= 3 \cdot 2 \\ 4x + 5x - 3 &= 6 \\ 9x - 3 &= 6 \\ 9x &= 9 \\ x &= 1 \end{aligned}$$

But then we have $y = \frac{5}{6} \cdot 1 - \frac{1}{2} = \frac{5}{6} - \frac{1}{2} = \frac{5}{6} - \frac{3}{6} = \frac{2}{6} = \frac{1}{3}$. Therefore, the solution is $(1, \frac{1}{3})$.

Using elimination, suppose we eliminate y. Multiplying the second equation by 2 and adding, we find

$$\begin{aligned} 5x - 6y &= 3 \\ \underline{4x + 6y} &\underline{\,= 6} \\ 9x &= 9 \\ x &= 1 \end{aligned}$$

Using this in the first equation, we find

$$\begin{aligned} 5x - 6y &= 3 \\ 5(1) - 6y &= 3 \\ 5 - 6y &= 3 \\ -6y &= -2 \\ y &= \frac{1}{3} \end{aligned}$$

Therefore, the solution is $(1, \frac{1}{3})$.