Name: _____

MATH 308
Fall 2022
HW 11: Due 11/04

"There are five elementary arithmetical operations: addition, subtraction, multiplication, division, and modular forms."
–Martin Eichler

Problem 1. (10pt) Showing all your steps, compute the following:

(a) $(15 + 14) \mod 6$

(b) $(8 - 17) \mod 5$

(c) $-(1 + 8) \mod 3$

(d) $(20 - 11) \mod 8$

(e) $(9 + 7) \mod 4$

(f) $14(5) \mod 6$

(g) $2(3) \mod 7$

(h) $-7(4) \mod 9$

(i) $(-3)^3 \mod 4$

(j) $6^2 \mod 5$

Problem 2. (10pt) Consider arithmetic modulo 4.

(a) List two positive elements and two negative elements of $[0]$ and $[3]$.

(b) Choose elements $x \in [1]$ and $y \in [3]$ with $x, y > 10$ and show that $[x]+[y] = [0]$; that is, use the division algorithm to write $x = 4m + r_x$ and $y = 4n + r_y$ and show $[x] + [y] = [r_x] + [r_y] = [0]$.

(c) Choose elements $x, y \in [2]$ with $x, y > 10$ and show that $[x] \cdot [y] = [0]$; that is, use the division algorithm to write $x = 4m + r_x$ and $y = 4n + r_y$ and show $[x] \cdot [y] = [r_x] \cdot [r_y] = [0]$.

Problem 3. (10pt) Showing all your work, complete the following:

(a) Compute $\phi(7)$, $\phi(11)$, and $\phi(131)$.

(b) Compute $\phi(8)$, $\phi(9)$, and $\phi(49)$.

(c) Compute $\phi(360)$.

(d) How many integers $0, 1, 2, \ldots, 359$ are invertible modulo 360? Explain.

Problem 4. (10pt) Being sure to fully justify your responses, answer the following:

(a) Is 7 invertible modulo 15? Explain.

(b) Prove your claim in (a) by finding an inverse for 7 modulo 15 or showing that there is no inverse of 7 modulo 15.

(c) Is 2 invertible modulo 6? Explain.

(d) Prove your claim in (c) by finding an inverse for 2 modulo 6 or showing that there is no inverse of 2 modulo 6.