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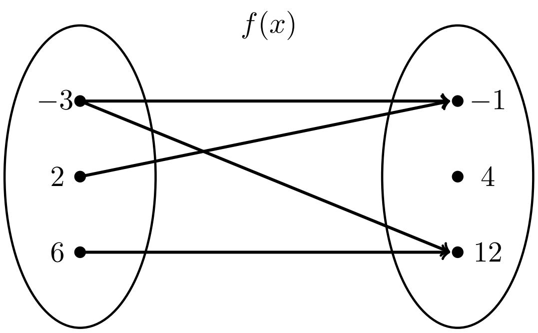
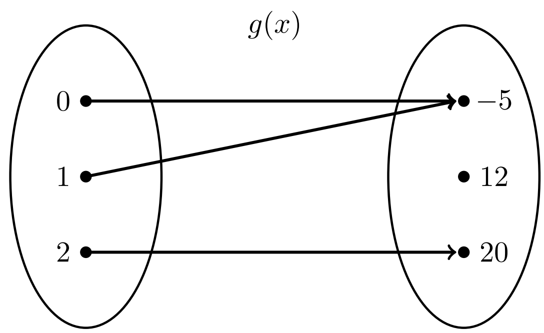
MATH 101
Spring 2022
HW 6: Due 02/24

"Honey, it's just the way your brain was hardwired. Plenty of great, intelligent, funny, interesting, and creative people have struggled with the same things you struggle with."

–Leslie Bennett, Euphoria

Problem 1. (10pt) Determine if the relations $f(x)$ and $g(x)$ shown below are functions. Explain why or why not. If the relation is a function, determine its domain, codomain, and range.





Solution. The relation $f(x)$ is *not* a function—$f(-3)$ is not well defined, i.e. the input -3 has two possible outputs. If it were a function, it would have domain $\{-3, 2, 6\}$, codomain $\{-1, 4, 12\}$, and range $\{-1, 12\}$.

The relation $g(x)$ is a function—each possible input has only one possible output. The domain of $g(x)$ is $\{0, 1, 2\}$, the codomain is $\{-5, 12, 20\}$, and the range is $\{-5, 20\}$.

Problem 2. (10pt) Determine if the relations $f(x)$ and $g(x)$ shown below are functions. Explain why or why not. If the relation is a function, compute the functions value at $x = 10$.

$$f(x) = 47.3 - 17.9x$$

$$g(x) = 2x^2 + 5x - 6$$

Solution. Both the relations $f(x)$ and $g(x)$ are functions—for each input, there is only one possible output. Namely, the output is obtained by evaluating at $x = x_0$ and following order of operations. Now we find $f(10)$ and $g(10)$:

$$f(10) = 47.3 - 17.9(10) = 47.3 - 1.79 = -131.7$$

$$g(10) = 2(10^2) + 5(10) - 6 = 2(100) + 5(10) - 6 = 200 + 50 - 6 = 244$$

Problem 3. (10pt) Suppose $f(x)$ and $g(x)$ are the functions given below.

x	-3	-2	-1	0	1	2	3
$f(x)$	5	2	0	-1	-2	-4	-5
$g(x)$	1	1	5	2	-3	-3	4
$h(x)$	-6	7	1	-2	0	1	-1

Compute the following:

(a) $(f+g)(3) = f(3) + g(3) = -5 + 4 = -1$

(b) $(f-g)(-1) = f(-1) - g(-1) = 0 - 5 = -5$

(c) $(5h)(1) = 5h(1) = 5(0) = 0$

(d) $\left(\frac{h}{g}\right)(-3) = \frac{h(-3)}{g(-3)} = \frac{-6}{1} = -6$

(e) $f(2)\,h(-2) = -4 \cdot 7 = -28$

(f) $h(-1 - f(0)) = h(-1 - (-1)) = h(0) = -2$

(g) $(g \circ f)(-2) = g(f(-2)) = g(2) = -3$

(h) $(h \circ g)(1) = h(g(1)) = h(-3) = -6$

(i) $(g \circ h)(1) = g(h(1)) = g(0) = 2$

(j) $(g \circ f \circ h)(-1) = g(f(h(-1))) = g(f(1)) = g(-2) = 1$

Problem 4. (10pt) Suppose $f(x)$ and $g(x)$ are the functions given below.

$$f(x) = 3x - 10$$

$$g(x) = 2x^2 - x + 5$$

Compute the following:

(a) $f(3) = 3(3) - 10 = 9 - 10 = -1$

(b) $g(-2) = 2(-2)^2 - (-2) + 5 = 2(4) + 2 + 5 = 8 + 2 + 5 = 15$

(c) $5f(6) - g(1) = 5(3 \cdot 6 - 10) - (2 \cdot 1^2 - 1 + 5) = 40 - 6 = 34$

(d) $f(x) - g(x) = (3x - 10) - (2x^2 - x + 5) = 3x - 10 - 2x^2 + x - 5 = -2x^2 + 4x - 15$

(e) $f(x)\,g(x) = (3x-10)(2x^2-x+5) = 6x^3 - 3x^2 + 15x - 20x^2 + 10x - 50 = 6x^3 - 23x^2 + 25x - 50$

(f) $\left(\dfrac{f}{g}\right)(x) = \dfrac{3x - 10}{2x^2 - x + 5}$

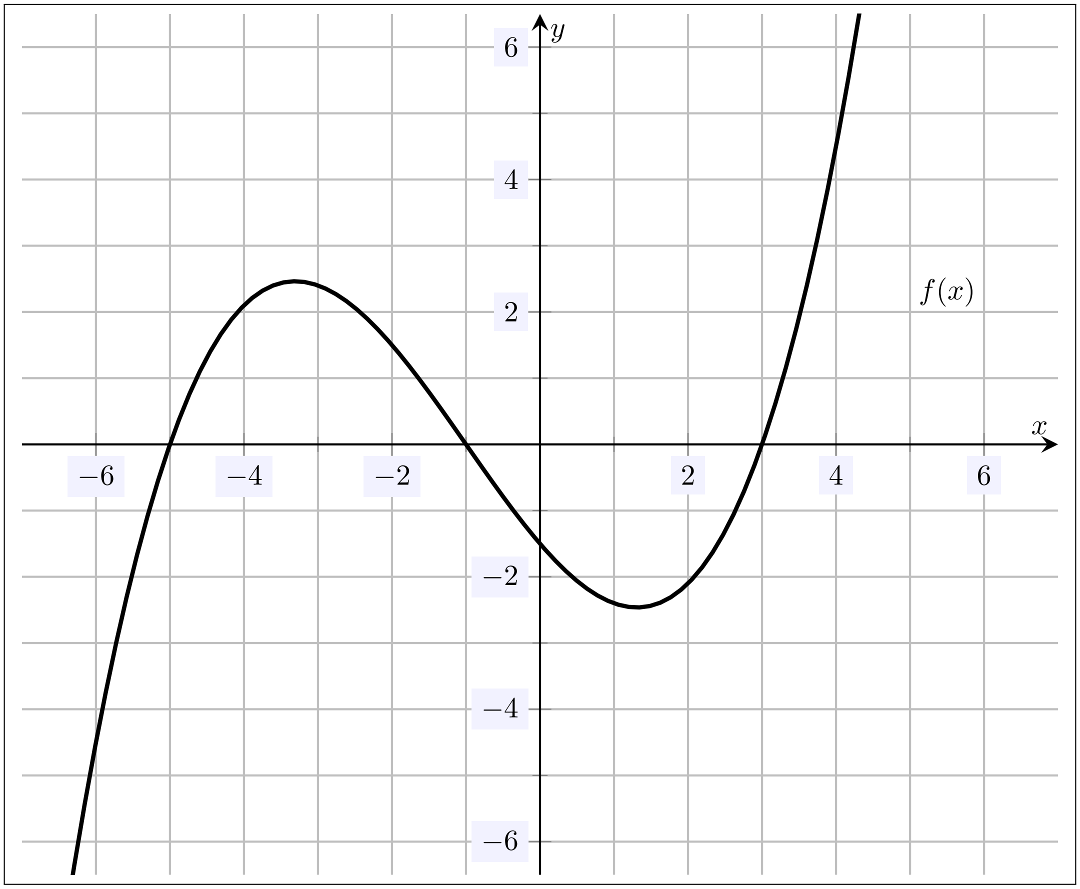
(g) $(f \circ g)(0) = f(g(0)) = f(2 \cdot 0^2 - 0 + 5) = f(5) = 3(5) - 10 = 5$

(h) $(g \circ f)(3) = g(f(3)) = g(3 \cdot 3 - 10) = g(-1) = 2(-1)^2 - (-1) + 5 = 2 + 1 + 5 = 8$

(i) $(f \circ g)(x) = f(g(x)) = f(2x^2 - x + 5) = 3(2x^2 - x + 5) - 10 = 6x^2 - 3x + 15 - 10 = 6x^2 - 3x + 5$

(j) $(g \circ f)(x) = g(f(x)) = g(3x - 10) = 2(3x - 10)^2 - (3x - 10) + 5 = 18x^2 - 123x + 215$

Problem 5. (10pt) Determine if the relation below is a function or not. If it is a function, explain why. If it is not a function, explain why. Determine also whether the relation has an inverse function. If it has an inverse function, explain why. If it does not have an inverse function, explain why not.



Solution. The relation *is* a function because it passes the vertical line test, i.e. every vertical line intersects the curve in at most one point. However, the relation *does not* have an inverse because it fails the horizontal line test, i.e. not every horizontal line intersects the curve in at most one point. For instance, the horizontal line $y = 0$ intersects the curve at the point $(-5, 0)$, $(-1, 0)$, $(3, 0)$. This implies that $f^{-1}(0)$ is not well defined so that $f^{-1}(x)$ cannot be a function.

Problem 6. (10pt) Determine whether the point $(2, -1)$ is on the graph of $f(x) = 2x^2 - 5x + 3$. Determine also whether the point $(1, 0)$ is on the graph of $f(x)$. For each, explain why or why not.

Solution. If $(x, y) = (2, -1)$ is on the curve $f(x) = 2x^2 - 5x + 3$, then when $x = 2$, $y = -1$, i.e. $f(2) = -1$, which we check:

$$f(2) = 2(2^2) - 5(2) + 3 = 2 \cdot 4 - 5(2) + 3 = 8 - 10 + 3 = 1$$

Because $f(2) = 1 \neq -1$, the point $(2, -1)$ is *not* on the graph of $f(x)$. We repeat the same process for $(x, y) = (1, 0)$:

$$f(1) = 2(1^2) - 5(1) + 3 = 2 \cdot 1 - 5(1) + 3 = 2 - 5 + 3 = 0$$

Therefore, $(1, 0)$ *is* on the graph of $f(x)$.