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MATH 308
Fall 2022
HW 5: Due 09/22

"To choose one sock from each of infinitely many pairs of socks requires the Axiom of Choice, but for shoes the Axiom is not needed."
–Bertrand Russell

Problem 1. (10pt) For each of the sets described below, either give the set by enumerating all its elements (if possible) or give the set using set-builder notation. Also for each set, give an element and non-element of the set.

(a) The set of integer multiples of 8.

(b) The set of negative solutions to $(x-4)(x+1)(x+6)=0$.

(c) The set of nonnegative rational numbers less than 1.

(d) The set of real numbers with a real-valued square root.

(e) The set of integer cubes with absolute value less than 100.

Solution.

(a)

(b)

(c)

(d)

(e)

Problem 2. (10pt) For each of the sets given below, describe the sets in words. Also for each set, give an example of an element and non-element of the set.

(a) $\{2, 3, 5, 7, 11, 13, \ldots\}$

(b) $\{\ldots, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, 16, \ldots\}$

(c) $\{n \in \mathbb{N} \colon n^2 = 30 - n\}$

(d) $\{k \in \mathbb{Z} \colon (3k + 1)/5 \in \mathbb{Z}\}$

(e) $\{n \in \mathbb{N} \colon (\exists k \in \mathbb{N})(n = 3k + 1)\}$

Solution.

(a)

(b)

(c)

(d)

(e)

Problem 3. (10pt) Define the following sets:

$$A = \{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10\}$$
$$B = \{2,\ 4,\ 6,\ 8,\ 10\}$$
$$C = \{1,\ 3,\ 5,\ 7,\ 9\}$$
$$D = \{2,\ 3,\ 5,\ 7\}$$
$$E = \{1,\ 2,\ 4,\ 8,\ 10\}$$
$$F = \{3,\ 5,\ 8,\ 9,\ 10\}$$

Consider each of the sets above as coming from the universal set $\mathcal{U} := A$. Compute the following:

(a) D^c

(b) $B \cup C$

(c) $C \cup (B \cap D)$

(d) $E \setminus F$

(e) $E \Delta F$

(f) $(B \cup C)^c$

Solution.

(a)
$$D^c = \{1,\ 4,\ 6,\ 8,\ 9,\ 10\}$$

(b)
$$B \cup C = \{1,\ 2,\ 3,\ 4,\ ,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10\} = A$$

(c)
$$C \cup (B \cap D) = \{1,\ 3,\ 5,\ 7,\ 9\} \cup \big(\{2,\ 4,\ 6,\ 8,\ 10\} \cap \{2,\ 3,\ 5,\ 7\}\big) = \{1,\ 3,\ 5,\ 7,\ 9\} \cup \{2\} = \{1,\ 2,\ 3,\ 5,\ 7,\ 9\}$$

(d)
$$E \setminus F = \{1,\ 2,\ 4\}$$

(e)
$$E \Delta F = \{1,\ 2,\ 3,\ 4,\ 5,\ 9\}$$

(f)
$$(B \cup C)^c = \big(\{2,\ 4,\ 6,\ 8,\ 10\} \cup \{1,\ 3,\ 5,\ 7,\ 9\}\big)^c = \{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10\}^c = \varnothing$$

Problem 4. (10pt) Let the universal set of discourse be the set of integers. Define the following sets:

$A =$ set of even integers
$B =$ set of odd integers
$C =$ set of prime integers
$D =$ set of square integers
$E =$ set of nonnegative integers
$F =$ set of positive integers
$G =$ set of integers strictly between 0 and 20
$H =$ set of integers that are a multiple of 5

Compute the sets below. When giving your solution, either enumerate all the elements of the resulting set (if possible), give the set using set-builder notation, or give the set using some 'standard' notation.

(a) B^c

(b) $A \cup B$

(c) $A \cap C$

(d) $B \cap C$

(e) $G - D$

(f) $E \Delta F$

(g) $C \cap H$

(h) $D \cap E^c$

(i) D^c

Solution.

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)