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MATH 101
Spring 2022
HW 10: Due 04/12

"Algebra is the metaphysics of arithmetic."
–John Ray

Problem 1. (10pt) Use completing the square to solve the following equation:

$$x(2x+7)=15$$

Solution.

$$\begin{aligned}
x(2x+7)&=15\\
2x^2+7x&=15\\
2x^2+7x-15&=0\\
2\left(x^2+\frac{7}{2}x-\frac{15}{2}\right)&=0\\
2\left(x^2+\frac{7}{2}x+\frac{49}{16}-\frac{49}{16}-\frac{15}{2}\right)&=0\\
2\left(\left(x+\frac{7}{4}\right)^2-\frac{49}{16}-\frac{120}{16}\right)&=0\\
2\left(\left(x+\frac{7}{4}\right)^2-\frac{169}{16}\right)&=0\\
2\left(x+\frac{7}{4}\right)^2-\frac{169}{8}&=0\\
2\left(x+\frac{7}{4}\right)^2&=\frac{169}{8}\\
\left(x+\frac{7}{4}\right)^2&=\frac{169}{16}\\
x+\frac{7}{4}&=\pm\sqrt{\frac{169}{16}}\\
x+\frac{7}{4}&=\pm\frac{13}{4}\\
x&=-\frac{7}{4}\pm\frac{13}{4}
\end{aligned}$$

Therefore, $x=-\frac{7}{4}-\frac{13}{4}=-\frac{20}{4}=-5$ or $x=-\frac{7}{4}+\frac{13}{4}=\frac{6}{4}=\frac{3}{3}$.

Problem 2. (10pt) Use factoring to solve the following equation:

$$2x = 80 - x^2$$

Solution.

$$2x = 80 - x^2$$

$$x^2 + 2x - 80 = 0$$

80	
$1 \cdot -80$	-79
$-1 \cdot 80$	79
$2 \cdot -40$	-38
$-2 \cdot 40$	38
$4 \cdot -20$	-16
$-4 \cdot 20$	16
$5 \cdot -16$	-9
$-5 \cdot 16$	9
$8 \cdot -10$	-2
$-8 \cdot 10$	2

$$2x = 80 - x^2$$

$$x^2 + 2x - 80 = 0$$

$$(x - 8)(x + 10) = 0$$

But then either $x - 8 = 0$, i.e. $x = 8$, or $x + 10 = 0$, i.e. $x = -10$.

Problem 3. (10pt) Use the quadratic equation to solve the following equation:

$$2x^2 - 1 = 2x$$

Solution.

$$2x^2 - 1 = 2x$$

$$2x^2 - 2x - 1 = 0$$

Then we have...

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{2 \pm \sqrt{4 + 8}}{4}$$

$$x = \frac{2 \pm \sqrt{12}}{4}$$

$$x = \frac{2 \pm \sqrt{4 \cdot 3}}{4}$$

$$x = \frac{2 \pm 2\sqrt{3}}{4}$$

$$x = \frac{1 \pm \sqrt{3}}{2}$$

Therefore, the solutions are $x = \frac{1 - \sqrt{3}}{2}$ and $x = \frac{1 + \sqrt{3}}{2}$.

Problem 4. (10pt) Use the quadratic formula to factor $280x^2 + 6x - 315$.

Solution. Solving the equation $280x^2 + 6x - 315 = 0$, we have...

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(280)(-315)}}{2(280)}$$

$$x = \frac{-6 \pm \sqrt{36 + 352800}}{560}$$

$$x = \frac{-6 \pm \sqrt{352836}}{560}$$

$$x = \frac{-6 \pm 594}{560}$$

Therefore, the solutions are $x = \frac{-6 - 594}{560} = \frac{-600}{560} = -\frac{15}{14}$ or $x = \frac{-6 + 594}{560} = \frac{588}{560} = \frac{21}{20}$. Therefore, we have...

$$280x^2 + 6x - 315 = a(x - r_1)(x - r_2) = 280\left(x - \frac{-15}{14}\right)\left(x - \frac{21}{20}\right) = 280\left(x + \frac{15}{14}\right)\left(x - \frac{21}{20}\right)$$