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MATH 101
Spring 2024
HW 4: Due 02/05

"Now we're going to do the most human thing of all: attempt something futile with a ton of unearned confidence and fail spectacularly!"
— Michael, The Good Place

Problem 1. (10pts) Showing all your work, compute the following "without a calculator":

(a) $\sqrt[3]{64}$

(b) $36^{-3/2}$

(c) $\left(\frac{1}{8}\right)^{-1/3}$

Solution.

(a)
$$\sqrt[4]{64} = \sqrt[4]{2^6} = 2^1\sqrt[4]{2^2} = 2\sqrt[4]{4} \quad \left(= 2\cdot 4^{1/4} = 2\cdot (2^2)^{1/4} = 2\cdot 2^{2/4} = 2\cdot 2^{1/2} = 2\sqrt{2}\right)$$

(b)
$$36^{-3/2} = \left(36^{1/2}\right)^{-3} = \left(\sqrt{36}\right)^{-3} = 6^{-3} = \frac{1}{6^3} = \frac{1}{216}$$

(c)
$$\left(\frac{1}{8}\right)^{-1/3} = \left(\frac{8}{1}\right)^{1/3} = 8^{1/3} = \sqrt[3]{8} = \sqrt[3]{2^3} = 2^1\sqrt[3]{2^0} = 2\cdot 1 = 2$$

Problem 2. (10pts) Simplify the following:

(a) $\sqrt{x^{10}y^4}$

(b) $\sqrt[3]{x^3y^5z^{12}}$

(c) $\sqrt{\sqrt[3]{y^6}}$

Solution.

(a)

$$\sqrt{x^{10}y^4} = x^5y^2\sqrt{x^0y^0} = x^5y^2$$

(b)

$$\sqrt[3]{x^3y^5z^{12}} = x^1y^1z^4\sqrt[3]{x^0y^2z^0} = xyz^4\sqrt[3]{y^2}$$

(c)

$$\sqrt{\sqrt[3]{y^6}} = \left(\left(y^6\right)^{1/3}\right)^{1/2} = \left(y^6\right)^{1/6} = \sqrt[6]{y^6} = y^1\sqrt[6]{y^0} = y$$

Problem 3. (10pts) Simplify the following:

(a) $x^2y\sqrt{x^8y^5}$

(b) $\dfrac{x^2y}{\sqrt[3]{xy^6}}$

(c) $\dfrac{\sqrt{x^2y}}{\sqrt[3]{x^3y^5}}$

Solution.

(a)

$$x^2y\sqrt{x^8y^5} = x^2y \cdot x^4y^2\sqrt{x^0y^1} = x^2y \cdot x^4y^2\sqrt{y} = x^6y^3\sqrt{y}$$

(b)

$$\frac{x^2y}{\sqrt[3]{xy^6}} = \frac{x^2y}{y^2\sqrt[3]{xy^0}} = \frac{x^2y}{y^2\sqrt[3]{x}} = \frac{x^2}{y^2\sqrt[3]{x}} = \frac{x^2}{y^2x^{1/3}} = \frac{x^{2-\frac{1}{3}}}{y^2} = \frac{x^{5/3}}{y^2} = \frac{\sqrt[3]{x^5}}{y^2}$$

(c)

$$\frac{\sqrt{x^2y}}{\sqrt[3]{x^3y^5}} = \frac{x^{2/2}y^{1/2}}{x^{3/3}y^{5/3}} = \frac{xy^{1/2}}{x^1y^{5/3}} = \frac{y^{1/2}}{y^{5/3}} = y^{\frac{1}{2}-\frac{5}{3}} = y^{-7/6} = \frac{1}{y^{7/6}} = \frac{1}{\sqrt[6]{y^7}}$$

Problem 4. (10pts) Simplify the following:

$$\left(\frac{\sqrt{x^8y^6}}{x^{-3}\sqrt{xy}}\right)^{-3/2}$$

Solution.

$$\left(\frac{\sqrt{x^8y^6}}{x^{-3}\sqrt{xy}}\right)^{-3/2}$$

$$\left(\frac{x^{8/2}y^{6/2}}{x^{-3}x^{1/2}y^{1/2}}\right)^{-3/2}$$

$$\left(\frac{x^4y^3}{x^{-3+\frac{1}{2}}y^{1/2}}\right)^{-3/2}$$

$$\left(\frac{x^4y^3}{x^{-5/2}y^{1/2}}\right)^{-3/2}$$

$$\left(\frac{x^4x^{5/2}y^3}{y^{1/2}}\right)^{-3/2}$$

$$\left(x^{4+\frac{5}{2}}y^{3-\frac{1}{2}}\right)^{-3/2}$$

$$\left(x^{13/2}y^{5/2}\right)^{-3/2}$$

$$x^{-39/4}y^{-15/4}$$

$$\frac{1}{x^{39/4}y^{15/4}}$$

$$\frac{1}{\sqrt[4]{x^{39}y^{15}}}$$