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MATH 108
Fall 2023
HW 2: Due 09/12

"You only have to do a few things right in your life so long as you don't do too many things wrong."
–Warren Buffett

Problem 1. (10pt) Suppose that the revenue and cost function for a certain item are given by $R(q) = 45.99q$ and $C(q) = 11.13q + 576000$, respectively.

(a) How much does the company sell each item for? How much does it cost to make each item?

(b) What are the fixed costs for the production of this good?

(c) What is the profit or loss if the company produces and sells ten-thousand of these items?

(d) What is the break-even point? At least many items does this company need to sell in order to make a profit on this item?

Solution.

(a) Observe that both $R(q)$ and $C(q)$ are linear. Because $R(q)$ is linear, the price of each item is the slope of the revenue function, $R(q)$. Therefore, each item sells for \$45.99. Because $C(q)$ is linear, the cost to make each item is the slope of the cost function, $C(q)$. Therefore, the cost to make each item is \$11.13.

(b) The fixed costs are the costs regardless of the level of production. But then we know that the fixed costs are given by $C(0)$. We have $C(0) = 11.13(0) + 576000 = 576000$. Therefore, the fixed costs are \$576,000.

(c) We know...

$$R(10000) = 45.99(10000) = 459900C(10000) \quad = 11.13(10000) + 576000 = 111300 + 576000 = 687300$$

Because the revenue, \$459,900, is less than the costs, \$687,300, the company is experiencing a loss. In fact, we know that $P(10000) = \$459900 - \$687300 = -\$227400$, i.e. the company is experiencing a \$227,400 loss.

(d) To find the break-even point, we can solve $R(q) = C(q)$. But then we have...

$$\begin{aligned} R(q) &= C(q) \\ 45.99q &= 11.13q + 576000 \\ 34.86q &= 5760000 \\ q &= 16523.2 \end{aligned}$$

But then in order to turn a profit, the company must produce/sell at least 16,524 items.

Problem 2. (10pt) Howard just started a small business cleaning service called *Grossbusters*. For now, he is renting a truck for \$1,550 per month. On average, he charges \$110 per cleaning and uses approximately \$4.86 in supplies per cleaning.

(a) What are the fixed and variable costs for Howard's cleaning service?

(b) Find the cost function for Howard's business.

(c) Find the revenue function for Howard's business.

(d) Find the break-even point for Howard's business. What is the minimal amount of cleanings Howard must book per month to make a profit?

(e) How many cleanings must Howard book each month to make a monthly profit of \$8,000 (translating to a yearly profit of \$96,000)? Does this seem feasible?

Solution.

(a) The fixed costs are the \$1,550 per month to rent the truck, while the variable costs are the \$4.86 used in supplies per cleaning, i.e. $4.86q$.

(b) We know $C(q) = \text{V.C.} + \text{F.C.}$. But then $C(q) = 4.86q + 1550$.

(c) We know that he charges \$110 per cleaning. Therefore, $R(q) = 110q$.

(d) To find the break-even point, one solves $R(q) = C(q)$ or computes $P(q)$ then solves $P(q) = 0$. In this instance, we use the latter method. First, we have...

$$P(q) = R(q) - C(q) = 110q - (4.86q + 1550) = 110q - 4.86q - 1550 = 105.14q - 1550$$

But then solving $P(q) = 0$, we have...

$$\begin{aligned} P(q) &= 0 \\ 105.14q - 1550 &= 0 \\ 105.14q &= 1550 \\ q &= 14.7422 \end{aligned}$$

Therefore, Howard must perform at least 15 cleanings per month to make a profit.

(e) We need find q such that $P(q) = 8000$. We found $P(q)$ in (d). But then, we have...

$$\begin{aligned} P(q) &= 8000 \\ 105.14q - 1550 &= 8000 \\ 105.14q &= 9550 \\ q &= 90.83 \end{aligned}$$

Therefore, to make a profit of at least \$8,000 per month, Howard must perform at least 91 cleanings per month. Working every day (assuming a month of 30 days), this yields 3.033 cleanings per day with an average time of approximately 2 hours 38 minutes per cleaning (assuming an 8 hour work day). If Howard only works 8 hours a day on weekdays, then this is 4.55 cleanings per day with an average time of 1 hour 45 minutes per cleaning. In either case, this seems feasible—so long as he can actually make the bookings.

Problem 3. (10pt) Suppose a company produces two items, q_1 and q_2, and has a cost function given by $C(q_1, q_2) = 56.20q_1 + 19.45q_2 + 7192$.

(a) What are the fixed costs for producing these two items?

(b) What is the total cost associated with producing 30 of the first item and 65 of the second item?

(c) How much does it cost to produce the first item? How much does it cost to produce the second item?

Solution.

(a) We know that the fixed costs are given by $C(0, 0) = 56.20(0)+19.45(0)+7192 = 0+0+7192 = 7192$. Therefore, the fixed costs are \$7,192.

(b) We have...

$$C(30, 65) = 56.20(30) + 19.45(65) + 7192 = 1686 + 1264.25 + 7192 = 10142.25$$

Therefore, the total cost to produce 30 of the first item and 65 of the second item is \$10,142.25. From the work above, we can also see that it costs \$1,686 to produce the first item and \$1264.25 to produce the second item—ignoring the fixed costs.

(c) Because $C(q_1, q_2)$ is (affine) linear, we know that the cost to produce each item is the 'slope' in the 'direction' of each variable. Therefore, it costs \$56.20 per item to produce the first item and \$19.45 per item to produce the second item.

Problem 4. (10pt) Suppose that you have a revenue function given by $R(q) = 120q$ and a cost function given by $C(q) = 70q + 1600$.

(a) What are the revenue and cost at a production/sale level of 80 units?

(b) Without finding the profit function, find the break-even point for the production/sale of this item.

(c) Find the profit function, $P(q)$.

(d) Compute $P(80)$. Explain how you could use (a) to find $P(80)$.

Solution.

(a) We have...

$$\begin{aligned} R(80) &= 120(80) = 9600 \\ C(80) &= 70(80) + 1600 = 7200 \end{aligned}$$

Therefore, at a production/sale level of 80 units, the revenue is \$9,600 and the costs are \$7,200.

(b) To find the break-even point, we solve $R(q) = C(q)$. But then, we have...

$$\begin{aligned} R(q) &= C(q) \\ 120q &= 70q + 1600 \\ 50q &= 1600 \\ q &= 32 \end{aligned}$$

Therefore, the break-even point is a production/sale level of 32 units.

(c) We have...

$$P(q) = R(q) - C(q) = 120q - (70q + 1600) = 120q - 70q - 1600 = 50q - 1600$$

(d) From (c), we know that $P(q) = 50q - 1600$. But then, we have...

$$P(80) = 50(80) - 1600 = 4000 - 1600 = 2400$$

Therefore, at a production/sale level of 80 units, the profits are \$2,400. Observe that we could have found this in (a). From (a), we know that at a production/sale level of 80 units, the revenue and cost are \$9,600 and \$7,200, respectively. But then the profit is $\$9600 - \$7200 = \$2400$.