MAT 101: Exam 1
**Fall – 2023**
10/11/2023
85 Minutes

Name: _____

Write your name on the appropriate line on the exam cover sheet. This exam contains 11 pages (including this cover page) and 10 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total:	100	

1. (10 points) Consider the relation f shown below.
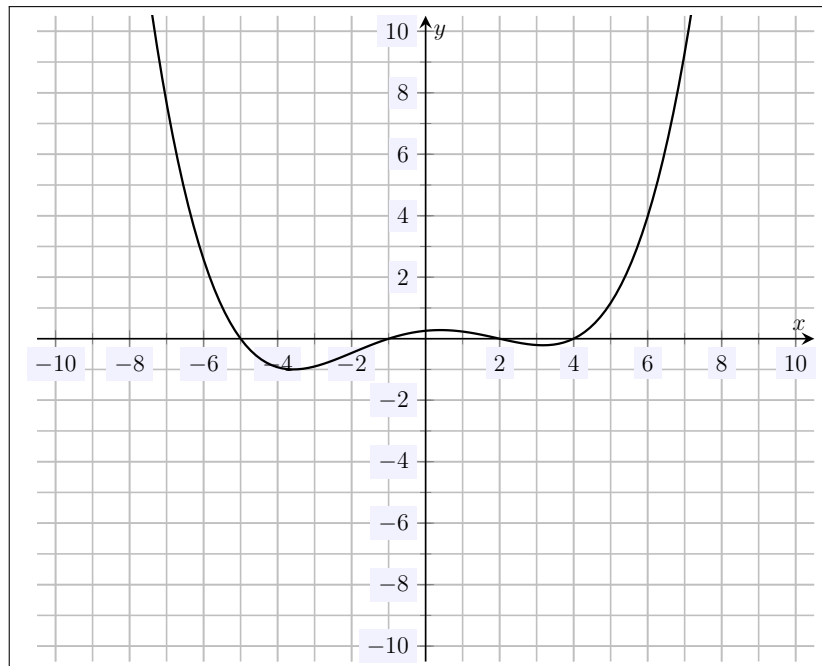
(a) Is the relation shown above a function of x? Explain.

(b) Assuming the relation is a function of x, does the relation above have an inverse that is a function of y? Explain.

(c) Find $f(6)$.

(d) Find the x-intercepts of $f(x)$.

(e) Is there an x such that $f(x) = 2$? Explain.

2. (10 points) Consider invertible functions f, g, whose values at several specified x-values are given below. Find the following:

x	-6	2	0	5	9
f	1	5	2	-6	3
g	0	2	7	1	6

(a) $(f + g)(9)$

(b) $(f \circ g)(0)$

(c) $\left(\frac{g}{f}\right)(2)$

(d) y-intercept of $f(x)$

(e) An x-intercept of $g(x)$

3. (10 points) Let $f(x) = x^2 + 2x - 1$, $g(x) = 3x + 8$, and c be a constant. Showing all your work and simplifying as much as possible, compute the following:

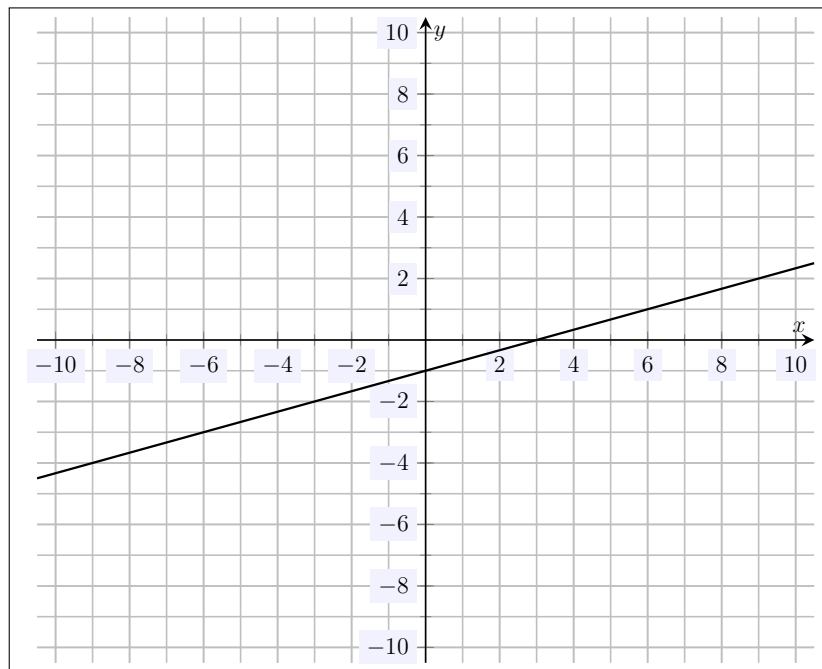   (a) $(fg)(4)$

   (b) $f(-2) - g(1)$

   (c) $(f - g)(2)$

   (d) $(f \circ g)(0)$

   (e) $(g \circ f)(c)$

4. (10 points) Consider the function $\ell(x) = 4x - 6$.

    (a) Is $\ell(x)$ linear? Explain.

    (b) Find the slope and y-intercept of $\ell(x)$.

    (c) Compute $\ell(\frac{17}{2})$.

    (d) Is there an x such that $\ell(x) = 10$? Explain.

    (e) Find the x-intercept of $\ell(x)$.

5. (10 points) Find the equation of the line that has y-intercept 5 that is parallel to the line shown below.

6. (10 points) Find the equation of the line with x-intercept -6 that passes through the point of intersection of $y = 5x - 1$ and $y = 6 - 2x$.

7. (10 points) Consider the lines $\ell_1(x) = 6x - 17$ and $\ell_2(x) = 8 - 11x$.

    (a) Determine whether the given lines are parallel, perpendicular, or neither. Justify your answer.

    (b) Do the lines intersect? If not, explain why. If so, find their point of intersection.

8. (10 points) Consider the function given by $f(x) = 11 - 9x$.

   (a) Explain why $f^{-1}(x)$ exists.

   (b) Find $f^{-1}(x)$.

   (c) Use f^{-1} to solve the equation $f(x) = \frac{17}{9}$.

9. (10 points) An *arithmetic sequence* is a list of numbers where the difference between one number and the next is always the same. For instance, 2, 6, 10, 14, 18, ... is an arithmetic sequence because the difference between sequential terms is always 4, while the sequence 1, 2, 3, 5, 7, 10, 13, ... is *not* an arithmetic sequence because the difference between sequential terms is not constant. Let S be the sequence 34, 57, 80, 103, 127, ...

   (a) Find a function $S(n)$ that gives the nth term of the sequence.

   (b) Find the 835th term of the sequence.

   (c) Is 3,500 a term of this sequence? Explain.

10. (10 points) A cleaning service does not have their prices listed on their website but the site does mention they charge a fixed amount per hour. You make some calls and have one friend that used their service and paid $212.50 for a 3 hour cleaning while another friend paid $400 for a 6 hour cleaning. Let $C(h)$ be the cost the service will charge for h hours of cleaning.

  (a) Explain why $C(h)$ is linear.

  (b) Find $C(h)$.

  (c) Interpret the slope and y-intercept for $C(h)$.

  (d) How many hours of cleaning can you get for $950?