Name: *Caleb McWhorter — Solutions*
MATH 101
Fall 2021
HW 11: Due 11/05

"Every brilliant experiment, like every great work of art, starts with an act of imagination."

–Jonah Lehrer

Problem 1. (10pt) Solve the equation $x^2 + 4x = 32$ by completing the square. Show all your work.

Solution. We have...

$$\begin{aligned}
x^2 + 4x &= 32 \\
x^2 + 4x - 32 &= 0 \\
x^2 + 4x + (4 - 4) - 32 &= 0 \\
(x^2 + 4x + 4) - 4 - 32 &= 0 \\
(x + 2)^2 - 36 &= 0 \\
(x + 2)^2 &= 36 \\
\sqrt{(x + 2)^2} &= \pm\sqrt{36} \\
x + 2 &= \pm 6 \\
x &= -2 \pm 6
\end{aligned}$$

Therefore, we have $x = -2 + 6 = 4$ or $x = -2 - 6 = -8$, i.e. $x = -8, 4$.

Problem 2. (10pt) Solve the equation $3 - 2x^2 = 5x$ by completing the square. Show all your work.

Solution. We have...

$$\begin{aligned}
3 - 2x^2 &= 5x \\
2x^2 + 5x - 3 &= 0 \\
2\left(x^2 + \frac{5}{2}x - \frac{3}{2}\right) &= 0 \\
2\left(x^2 + \frac{5}{2}x + \frac{25}{16} - \frac{25}{16} - \frac{3}{2}\right) &= 0 \\
2\left(\left(x + \frac{5}{4}\right)^2 - \frac{49}{16}\right) &= 0 \\
2\left(x + \frac{5}{2}\right)^2 - \frac{49}{8} &= 0 \\
2\left(x + \frac{5}{2}\right)^2 &= \frac{49}{8} \\
\left(x + \frac{5}{2}\right)^2 &= \frac{49}{16} \\
\sqrt{\left(x + \frac{5}{2}\right)^2} &= \pm\sqrt{\frac{49}{16}} \\
x + \frac{5}{2} &= \pm\frac{7}{4} \\
x &= -\frac{5}{2} \pm \frac{7}{4}
\end{aligned}$$

Therefore, $x = -\frac{5}{2} + \frac{7}{4} = \frac{2}{4} = \frac{1}{2}$ or $x = -\frac{5}{2} - \frac{7}{4} = \frac{-12}{4} = -3$, i.e. $x = -3, \frac{1}{2}$

Problem 3. (10pt) Solve the equation $x^2 + 4x = 5$ by factoring. Show all your work.

Solution. We have...

$$x^2 + 4x = 5$$

$$x^2 + 4x - 5 = 0$$

$$(x + 5)(x - 1) = 0$$

Then either $x + 5 = 0$, i.e. $x = -5$, or $x - 1 = 0$, i.e. $x = 1$. Therefore, $x = -5, 1$.

Problem 4. (10pt) Solve the equation $x^2 + 16 = 8x$ by factoring. Show all your work.

Solution. We have...

$$x^2 + 16 = 8x$$

$$x^2 - 8x + 16 = 0$$

$$(x - 2)(x - 8) = 0$$

Then either $x - 2 = 0$, i.e. $x = 2$, or $x - 8 = 0$, i.e. $x = 8$. Therefore, $x = 2, 8$.

Problem 5. (10pt) Solve the equation $x^2 = x + 72$ by using the quadratic formula. Show all your work.

Solution. First, we move everything to the left side: $x^2 - x - 72 = 0$. Then...

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-72)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{1 + 288}}{2}$$

$$x = \frac{1 \pm \sqrt{289}}{2}$$

$$x = \frac{1 \pm 17}{2}$$

Then either $x = \frac{1+17}{2} = \frac{18}{2} = 9$ or $x = \frac{1-17}{2} = \frac{-16}{2} = -8$. Therefore, $x = -8, 9$.

Problem 6. (10pt) Solve the equation $x^2 - 4x + 1 = 0$ by using the quadratic formula. Show all your work.

Solution. We have...

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 - 4)}}{2}$$

$$x = \frac{4 \pm \sqrt{12)}}{2}$$

$$x = \frac{4 \pm \sqrt{4 \cdot 3)}}{2}$$

$$x = \frac{4 \pm 2\sqrt{3}}{2}$$

$$x = 2 \pm \sqrt{3}$$

Then either $x = 2 + \sqrt{3}$ or $x = 2 - \sqrt{3}$. Therefore, $x = 2 - \sqrt{3}, 2 + \sqrt{3}$.