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**MATH 108**
**Spring 2023**
**HW 8: Due 03/06**

*"Laura, clear out the rest of my day! I have to push a boulder up a hill and then have it roll over me time and time again with no regard for my well-being."*

*–Princess Carolyn, BoJack Horseman*

**Problem 1.** (10pt) Kelsey is gambling at a casino. She is playing a game where you roll two die. If you roll two 6's, you win \$100. If you the dice and the numbers on both die are four or greater (but not two 6's), you win \$10. If the numbers on both die are less than 3, you lose \$8. Otherwise, you win nothing. You must pay \$5 as a 'buy-in' each round to play. Find the amount that you win/lose 'on average.' Should one play this game?

**Solution.** Rolling two die, there are $6 \cdot 6 = 36$ total outcomes for the values on the two die. There is only one way to roll two 6's—by rolling two 6's. If you want to roll a four or bigger on each die, there are 3 possibilities for each die (4, 5, or 6) so that there are $3 \cdot 3 = 9$ such combinations. But this includes two 6's on each die. Therefore, there are $9 - 1 = 8$ total ways to roll the dice and have a four or greater on each die. If a number on a die is less than 3, there are 2 possibilities (1 or 2). Therefore, there are $2 \cdot 2 = 4$ total combinations where both numbers on the die are less than 3. But then we have...

$$\begin{aligned} P(\text{two 6's}) &= \frac{1}{36} \\ P(\text{Four or Great (Not Boxcars)}) &= \frac{8}{36} = \frac{2}{9} \\ P(\text{Less than 3}) &= \frac{4}{36} = \frac{1}{9} \\ P(\text{Anything Else}) &= 1 - \frac{1}{36} - \frac{8}{36} - \frac{4}{36} = \frac{23}{36} \end{aligned}$$

We can create a table of the possible outcomes, their probability, and their net expected payout (that is, the payout minus the \$5 fee to play). We have...

| Outcome | Two 6's | Four or Greater (Not Boxcars) | Less than 3 | Anything Else |
|---|---|---|---|---|
| Probability | $\frac{1}{36}$ | $\frac{2}{9}$ | $\frac{1}{9}$ | $\frac{23}{36}$ |
| Value | \$95 | \$5 | $-$\$13 | $-$\$5 |

We can then compute the expected payout:

$$EX = \sum xP(x) = \$95 \cdot \frac{1}{36} + \$5 \cdot \frac{2}{9} - \$13 \cdot \frac{1}{9} - \$5 \cdot \frac{23}{36} \approx \$2.639 + \$1.111 - \$1.444 - \$3.194 = -\$0.888 \approx -\$0.89$$

Because the expected value is negative, on average, you lose \$0.89 each game by playing this game. Therefore, you should not play this game.

**Problem 2.** (10pt) Find the least square regression line for the points: $(1,3),(3,5),(1,2),(2,2)$. Show all your work.

**Solution.** First, observe that we have $n=4$ points. Examining the $x$ and $y$ values, we have...

$$\overline{x} = \frac{1+3+1+2}{4} = \frac{7}{4} \approx 1.75$$

$$\overline{y} = \frac{3+5+2+2}{4} = \frac{12}{4} = 3$$

But then we have...

| $x_i$ | $y_i$ | $x_i - \overline{x}$ | $(x_i - \overline{x})^2$ | $y_i - \overline{y}$ | $(y_i - \overline{y})^2$ | $(x_i - \overline{x})(y_i - \overline{y})$ |
|---|---|---|---|---|---|---|
| 1 | 3 | $-0.75$ | 0.5625 | 0 | 0 | 0 |
| 3 | 5 | 1.25 | 1.5625 | 2 | 4 | 2.5 |
| 1 | 2 | $-0.75$ | 0.5625 | $-1$ | 1 | 0.75 |
| 2 | 2 | 0.25 | 0.0625 | $-1$ | 1 | $-0.25$ |
| | | Total: | 2.75 | Total: | 6 | 3 |

But then we have...

$$s_x^2 = \frac{1}{n-1}\sum(x_i - \overline{x})^2 = \frac{1}{3}\cdot 2.75 = 0.916667 \implies s_x = \sqrt{0.916667} = 0.957427$$

$$s_y^2 = \frac{1}{n-1}\sum(y_i - \overline{y})^2 = \frac{1}{3}\cdot 6 = 2 \implies s_y = \sqrt{2} = 1.41421$$

$$r = \frac{1}{n-1}\sum\left(\frac{x_i-\overline{x}}{s_x}\right)\left(\frac{y_i-\overline{y}}{s_y}\right) = \frac{1}{n-1}\cdot\frac{1}{s_x}\cdot\frac{1}{s_y}\sum(x_i-\overline{x})(y_i-\overline{y}) = \frac{1}{3}\cdot\frac{1}{0.957427}\cdot\frac{1}{1.41421}\cdot 3 = 0.738551$$

This allows us to compute the coefficients for our model:

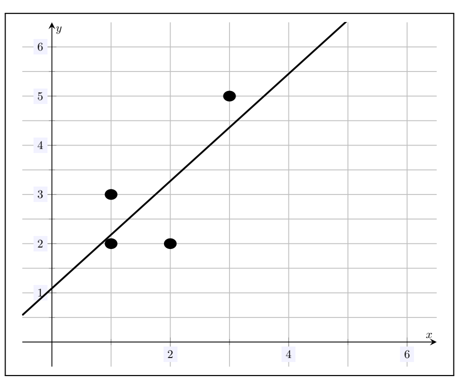
$$b_1 = r\,\frac{s_y}{s_x} = 0.738551\cdot\frac{1.41421}{0.957427} = 1.09091$$

$$b_0 = \overline{y} - b_1\overline{x} = 3 - 0.796018(1.75) = 1.09091$$

Therefore, the least square regression line is...

$$\widehat{y} = b_1 x + b_0 = 1.09091x + 1.09091$$

This linear regression has $r^2$ value $0.545458$ and is shown with the data points below:



**Problem 3.** (10pt) Given the following information below, find the least square regression line. Show all your work.

$$\begin{aligned} &n = 200 \\ &\overline{x} = 4.42726, \quad \sigma_x^2 &= 10.6639 \\ &\overline{y} = 46.5248, \quad \sigma_y^2 &= 1053.77 \\ &R = 0.962639 \end{aligned}$$

**Solution.** We have...

$$\sigma_x^2 = 10.6639 \Longrightarrow \sigma_x = \sqrt{10.6639} = 3.26556$$

$$\sigma_y^2 = 1053.77 \Longrightarrow \sigma_y = \sqrt{1053.77} = 32.4618$$

But then we can compute the model coefficients:

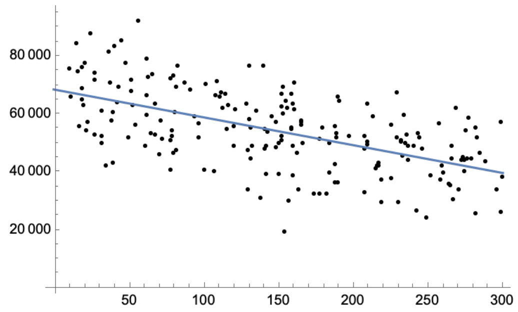
$$b_1 = r\,\frac{\sigma_y}{\sigma_x} = 0.962639 \cdot \frac{32.4618}{3.26556} = 9.56926$$

$$b_0 = \overline{y} - b_1\overline{x} = 46.5248 - 9.56926(10.6639) = -55.5208$$

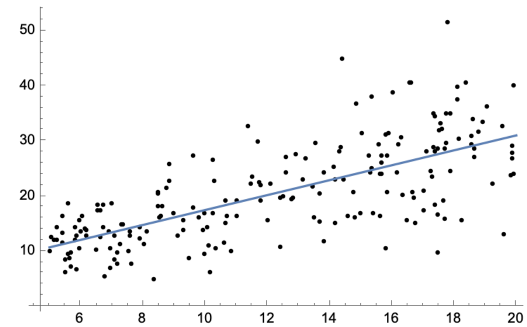
Therefore, the least square regression line is...

$$\widehat{y} = b_1 x + b_0 = 9.56926x - 55.5208$$

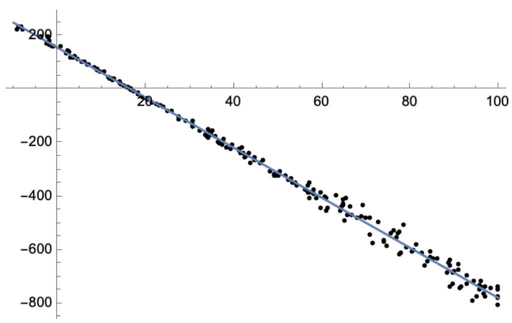
**Problem 4.** (10pt) Match each regression coefficient to its corresponding graph.



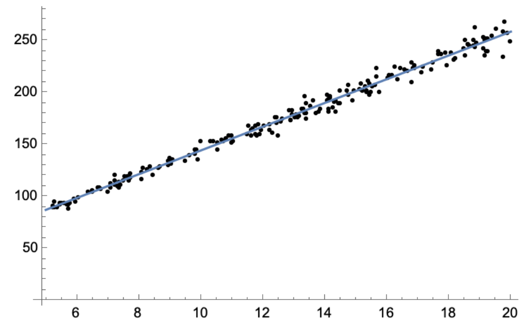
(a)



(b)



(c)



(d)

(i) __*(b)*__ : $R = 0.836288$

(ii) __*(c)*__ : $R = -0.998836$

(iii) __*(d)*__ : $R = 0.997066$

(iv) __*(a)*__ : $R = -0.759531$