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MATH 101
Spring 2022
HW 9: Due 04/12

"Algebra is the intellectual instrument which has been created for rendering clear the quantitative aspects of the world."
–Alfred Whitehead

Problem 1. (10pt) Showing all your work, factor the following completely:

(a) $x^2 - 14x + 48$

(b) $2x^2 + 14x - 120$

Solution.

(a)

$$\begin{array}{rr} \multicolumn{2}{c}{\underline{\mathbf{48}}} \\ 1 \cdot 48 & 49 \\ -1 \cdot -48 & -49 \\ 2 \cdot 24 & 26 \\ -2 \cdot -24 & -26 \\ 3 \cdot 16 & 19 \\ -3 \cdot -16 & -19 \\ 4 \cdot 12 & 16 \\ -4 \cdot -12 & -16 \\ 6 \cdot 8 & 14 \\ \hline \multicolumn{1}{|r}{-6 \cdot -8} & \multicolumn{1}{r|}{-14} \\ \hline \end{array}$$

Therefore,

$$x^2 - 14x + 48 = (x-6)(x-8)$$

(b) Note, we have $2x^2 + 14x - 120 = 2(x^2 + 7x - 60)$. Then we have. . .

$$\begin{array}{rrcrr} \multicolumn{5}{c}{\underline{\mathbf{60}}} \\ 1 \cdot -60 & -59 & & 4 \cdot -15 & -11 \\ -1 \cdot 60 & 59 & & -4 \cdot 15 & 11 \\ 2 \cdot -30 & -28 & & 5 \cdot -12 & -7 \\ \cline{4-5} -2 \cdot 30 & 28 & & \multicolumn{1}{|r}{-5 \cdot 12} & \multicolumn{1}{r|}{7} \\ \cline{4-5} 3 \cdot -20 & -17 & & 6 \cdot -10 & -4 \\ -3 \cdot 20 & 17 & & -6 \cdot 10 & 4 \end{array}$$

Therefore,

$$2x^2 + 14x - 120 = 2(x^2 + 7x - 60) = 2(x-5)(x+12)$$

Problem 2. (10pt) Showing all your work, factor the following completely:

(a) $x^2 - 19x$

(b) $25 - 9x^2$

Solution.

(a)

$$x^2 - 19x = x(x - 19)$$

(b) This is a difference of perfect squares:

$$25 - 9x^2 = (5 - 3x)(5 + 3x)$$

Problem 3. (10pt) Showing all your work, factor the following completely:

$$6x^2 - x - 12$$

Solution.

$$\begin{array}{c} \underline{\mathbf{12}} \\ 1 \cdot -12 \\ -1 \cdot 12 \\ 2 \cdot -6 \\ -2 \cdot 6 \\ 3 \cdot -4 \\ -3 \cdot 4 \end{array}$$

Then as $6 = 1 \cdot 6$ or $6 = 2 \cdot 3$, we have...

$$\begin{array}{ccc} 1,-72 & & 6,-12 \\ 1,6 & & 6,1 \\ & 1 \cdot -12 & \\ 2,3 & & 3,2 \\ 2,-36 & & 3,-24 \end{array} \qquad \begin{array}{ccc} -1,72 & & -6,12 \\ 1,6 & & 6,1 \\ & -1 \cdot 12 & \\ 2,3 & & 3,2 \\ -2,36 & & -3,24 \end{array}$$

$$\begin{array}{ccc} 2,-36 & & 12,-6 \\ 1,6 & & 6,1 \\ & 2 \cdot -6 & \\ 2,3 & & 3,2 \\ 4,-18 & & 6,-12 \end{array} \qquad \begin{array}{ccc} -2,36 & & -12,6 \\ 1,6 & & 6,1 \\ & -2 \cdot 6 & \\ 2,3 & & 3,2 \\ -4,18 & & -6,12 \end{array}$$

$$\begin{array}{ccc} 3,-24 & & 18,-4 \\ 1,6 & & 6,1 \\ & 3 \cdot -4 & \\ 2,3 & & 3,2 \\ 6,-12 & & 9,-8 \end{array} \qquad \begin{array}{ccc} -3,24 & & -18,4 \\ 1,6 & & 6,1 \\ & \boxed{-3 \cdot 4} & \\ 2,3 & & 3,2 \\ -6,12 & & \boxed{-9,8} \end{array}$$

Therefore,

$$6x^2 - x - 12 = (3x+4)(2x-3)$$

Problem 4. (10pt) Use the discriminant of $f(x) = x^2 - 2x + 5$ to determine whether the quadratic function factors over the integers, reals, or complex numbers.

Solution. We know that $D = b^2 - 4ac$. We have $a = 1$, $b = -2$, and $c = 5$. Therefore,

$$D = b^2 - 4ac = (-2)^2 - 4(1)5 = 4 - 20 = -16$$

Because $D < 0$, we know that $f(x)$ does not factor over the integers or the real numbers. However, $f(x)$ does factor over the complex numbers. In fact, we have...

$$f(x) = x^2 - 2x + 5 = \big(x - (1 + 2i)\big)\big(x - (1 - 2i)\big)$$