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MATH 108
Fall 2023
HW 10: Due 10/24

"Think of how stupid the average person is, and realize half of them are stupider than that."
–George Carlin

Problem 1. (10pt) Crybaby Cannon and Buzz Sawyer are amazing wrestlers in their leagues. They are arguing about who is the better wrestler. The win percentages for both players' league are normally distributed. The average win percentage for The Cannon's league is 72.4% with standard deviation 6.2%. The average win percentage for The Buzz's league is 81.5% with standard deviation 2.1%. Crybaby has won 88.5% of his matches, while Buzz has only won 87.6% of his matches.

(a) Give an argument why Crybaby Cannon could be considered the better wrestler.

(b) Give an argument why Buzz Sawyer could be considered the better wrestler.

Solution.

(a) One can argue that because the Crybaby Cannon has a win percentage of only 88.5%, while Buzz Sawyer has a win percentage of 87.6%, Crybaby Cannon should be considered the better wrestler because he has a higher win percentage.

(b) One can argue that the players should be judged relative to the other wrestlers in their league. For instance, a 100% win percentage against amateur wrestlers likely is 'worth less' than a win percentage of even 80% against professional wrestlers. To compare the 'unusualness' of values in a normal distribution, one can use the z-score. We have...

$$z_{\text{Crybaby}} = \frac{88.5\% - 72.4\%}{6.2\%} = \frac{16.1\%}{6.2\%} \approx 2.60 z_{\text{Buzz}} \quad = \frac{87.6\% - 81.5\%}{2.1\%} = \frac{6.1\%}{2.1\%} \approx 2.90$$

Because Buzz's z-score is larger, Buzz's win percentage is more 'unusual' (relative to his league). So he is much more above average (relative to his league) than the Crybaby Cannon is in his league. Buzz could be considered the better wrestler using this argument.

Problem 2. (10pt) Let D be a distribution such that $D \sim N(67.2, 4.7)$. Suppose that X is a random value sampled from the distribution D. Showing all your work, compute the following:

(a) $P(X < 73)$

(b) $P(X = 63)$

(c) $P(X > 61)$

(d) $P(61 < X < 73)$

(e) The probability that X is exactly the average value for the distribution D.

(f) The value X such that X is the smallest value to be in the largest 7% of values for this distribution.

Solution.

(a)

$$z_{73} = \frac{x-\mu}{\sigma} = \frac{73-67.2}{4.7} = \frac{5.8}{4.7} \approx 1.23 \rightsquigarrow 0.8907$$

(b) For any continuous distribution, $P(X = \#) = 0$. But then $P(X = 63) = 0$.

(c)

$$z_{61} = \frac{x-\mu}{\sigma} = \frac{61-67.2}{4.7} = \frac{-6.2}{4.7} \approx -1.32 \rightsquigarrow 0.0934$$

But then $P(X < 61) = 0.0934$, which implies $P(X > 61) = 1 - P(X < 61) = 1 - 0.0934 = 0.9066$.

(d) We have...

$$P(61 < X < 73) = P(X < 73) - P(X < 61) = 0.8907 - 0.0934 = 0.7973$$

(e) For any continuous distribution, $P(X = \#) = 0$. But then $P(X = 67.2) = 0$.

(f) If X is the smallest value that is in the largest 7% of values, then it is greater than 93% of the values in the distribution. But then $P(x < X) = 0.93$. This implies that $z_X \rightsquigarrow 0.93$. But then it must be that $z_X \approx 1.475$. But we know that...

$$\begin{aligned} z_X &= \frac{X-\mu}{\sigma} \\ 1.475 &= \frac{X-67.2}{4.7} \\ X - 67.2 &= 6.9325 \\ X &\approx 74.13 \end{aligned}$$