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MATH 100

Fall 2023

HW 11: Due 12/06

"The 50-50-90 rule: anytime you have a 50-50 chance of getting something right, there's a 90% probability you'll get it wrong."

—Andy Rooney

Problem 1. (10pt) A community college is trying to decide how to advertise to prospective students at a local high school. The college decides to investigate students' interest in various STEM disciplines. The chart below summarizes the number of students that indicated they were interest in a particular STEM discipline, broken down by HS year.

	Math	Physics	Chemistry	Biology	Exercise Science	Total
Junior	2	5	8	14	16	45
Senior	3	4	9	13	11	40
Total	5	9	17	27	27	85

- (a) Find the percentage of students that are interested in only Exercise Science.
- (b) Find the percentage of students that are Seniors interested in Physics.
- (c) Find the percentage of students interest in Math or that were Juniors.
- (d) Find the percentage of those interested in Exercise Science that were Seniors.
- (e) Are the 'events' of being a Senior and interested in Biology independent? Explain.

Solution.

(a)

$$P(\text{Only Exercise Science}) = \frac{27}{85} \approx 0.3176$$

(b)

$$P(\text{Senior and Physics}) = \frac{4}{85} \approx 0.0471$$

(c)

$$P(\text{Math or Juniors}) = P(\text{Math}) + P(\text{Juniors}) - P(\text{Math and Juniors}) = \frac{5}{85} + \frac{45}{85} - \frac{2}{85} = \frac{5 + 45 - 2}{85} = \frac{48}{85} \approx 0.5647$$

(d)

$$P(\text{Senior} \mid \text{Exercise Science}) = \frac{P(\text{Senior \& Exercise Science})}{P(\text{Exercise Science})} = \frac{11/85}{27/85} = \frac{11}{27} \approx 0.4074$$

(e) If they were independent, then $P(\text{Senior \& Biology}) = P(\text{Senior})P(\text{Biology})$. But we have...

$$P(\text{Senior \& Biology}) = \frac{13}{85} \approx 0.1529$$

$$P(\text{Senior})P(\text{Biology}) = \frac{40}{85} \cdot \frac{27}{85} = \frac{1080}{7225} \approx 0.1495$$

Therefore, the 'events' of being a senior and the 'event' of being interested in Biology are not independent.

Problem 2. (10pt) There are two main production lines at a manufacturing warehouse. The main line handles 60% of the production at the warehouse, while the secondary handles the rest. The first has a defect rate of 4% while the second has a defect rate of 2%.

- Find the probability that the warehouse produces a defective product.
- Find the probability that a product from the warehouse was made from the secondary line or defective.
- Find the probability that a product from the warehouse was made from the main line and was not defective.
- Find the probability that if a product was defective that it was produced from the main line.
- Are the events of a product being made on the secondary line and the event of a product being defective disjoint? Explain.

Solution.

(a)

$$P(\text{Defective}) = 0.0240 + 0.0080 = 0.0320$$

(b)

$$P(\text{Second or Defective}) = 0.0240 + 0.3920 + 0.0080 = 0.4240$$

(c)

$$P(\text{Main and Works}) = 0.5760$$

(d)

$$P(\text{Main} \mid \text{Defect}) = \frac{P(\text{Main and Defect})}{P(\text{Defect})} = \frac{0.0240}{0.0240 + 0.0080} = \frac{0.0240}{0.0320} = 0.75$$

- (e) No, disjoint events cannot occur at the same time. There are defective products made on the secondary line. In fact, $P(\text{Secondary} \& \text{Defect}) = 0.0080$.

