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MATH 101
Fall 2022
HW 22: Due 12/07

"Try to learn something about everything and everything about something."
–Thomas Huxley

Problem 1. (10pt) Factor each of the following quadratic functions:

(a) $x^2 - 64$

(b) $2x^2 - 4x - 6$

(c) $x^2 - 2x - 35$

(d) $x^2 + 8x + 16$

(e) $27 + 6x - x^2$

Solution.

(a)
$$x^2 - 64 = (x - 8)(x + 8)$$

(b)
$$2x^2 - 4x - 6 = 2(x^2 - 2x - 3) = 2(x - 3)(x + 1)$$

(c)
$$x^2 - 2x - 35 = (x - 7)(x + 5)$$

(d)
$$x^2 + 8x + 16 = (x + 4)(x + 4) = (x + 4)^2$$

(e)
$$27 + 6x - x^2 = -(x^2 - 6x - 27) = -(x - 9)(x + 3)$$

Problem 2. (10pt) Showing all your work, use the discriminant to show that $10x^2+33x-7$ factors 'nicely' then factor the polynomial.

Solution. The discriminant of a quadratic function $f(x)=ax^2+bx+c$ is $D=b^2-4ac$. The function $f(x)=10x^2+33x-7$ is quadratic with $a=10$, $b=33$, and $c=-7$. But then $D=b^2-4ac=33^2-4(10)(-7)=1089+280=1369=37^2$. A quadratic function factors 'nicely' if and only if the discriminant is a square. Because $D=1369=37^2$, the polynomial $10x^2+33x-7$ factors 'nicely.' To factor this polynomial, we find the roots of $10x^2+33x-7$ using the quadratic formula, i.e. the solutions to $10x^2+33x-7=0$:

$$\begin{aligned} x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a} \\ &= \frac{-33\pm\sqrt{33^2-4(10)(-7)}}{2(10)} \\ &= \frac{-33\pm\sqrt{1089+280}}{20} \\ &= \frac{-33\pm\sqrt{1369}}{20} \\ &= \frac{-33\pm 37}{20} \end{aligned}$$

Therefore, the roots are $x=\frac{-33-37}{20}=\frac{-70}{20}=-\frac{7}{2}$ and $x=\frac{-33+37}{20}=\frac{4}{20}=\frac{1}{5}$. Now recall given a quadratic function ax^2+bx+c with roots r_1 and r_2, the function factors as $a(x-r_1)(x-r_2)$. But then we have...

$$10x^2+33x-7=10\left(x-\frac{-7}{2}\right)\left(x-\frac{1}{5}\right)=10\left(x+\frac{7}{2}\right)\left(x-\frac{1}{5}\right)=2\left(x+\frac{7}{2}\right)\cdot 5\left(x-\frac{1}{5}\right)=(2x+7)(5x-1)$$

Problem 3. (10pt) Showing all your work, solve the following equation using factoring and then verify your solution:

$$x = 30 - x^2$$

Solution. We have...

$$x = 30 - x^2$$

$$x^2 + x - 30 = 0$$

$$(x + 6)(x - 5) = 0$$

But then either $x + 6 = 0$, which implies $x = -6$, or $x - 5 = 0$, which implies $x = 5$. Therefore, the solutions are $x = -6, 5$. We can verify the solution $x = -6$:

$$\begin{aligned} x &= 30 - x^2 \\ -6 &\stackrel{?}{=} 30 - (-6)^2 \\ -6 &\stackrel{?}{=} 30 - 36 \\ -6 &= -6 \\ &\checkmark \end{aligned}$$

and the solution $x = 5$:

$$\begin{aligned} x &= 30 - x^2 \\ 5 &\stackrel{?}{=} 30 - 5^2 \\ 5 &\stackrel{?}{=} 30 - 25 \\ 5 &= 5 \\ &\checkmark \end{aligned}$$