

Name: \_\_\_\_\_

MATH 308

Fall 2023

HW 1: Due 09/07

*“And I knew exactly what to do... but in a much more real sense, I had no idea what to do.”*

*–Michael Scott, The Office*

**Problem 1.** (10pt) Determine if each of the following are propositions. If the example is a proposition, state its truth value with a brief justification. If the example is *not* a proposition, briefly explain why:

(a)  $3^2 - 15 = 6$

(b) The statement in (c) is false.

(c) George Orwell wrote *A Remembrance of Things Past*.

(d) There is intelligent life in the universe.

(e)  $x - 3 \leq 10$

**Problem 2.** (10pt) For each of the following, either define appropriate primitive propositions (using  $P$ ,  $Q$ ,  $R$ , etc.) and write the 'statement' using logical connectives, or give an English sentence for the given primitives and 'translate' the logical 'sentence' into an English sentence:

(a)  $P \rightarrow (\neg Q \vee R)$

(b) You will succeed, if you believe and work hard.

(c)  $Q \wedge (\neg P \vee Q)$

(d) I pay rent, or I lose my job and starve.

**Problem 3.** (10pt) Consider the following compound statement:  $\neg(P \rightarrow \neg Q) \wedge \neg Q$

- (a) Determine whether the given compound statement is a tautology, contradiction, or neither. Be sure to justify your response.
- (b) Using a truth table, show that the first part of the given compound statement, i.e.  $\neg(P \rightarrow \neg Q)$ , is logically equivalent to  $P \wedge Q$ .
- (c) By ‘simplifying’ the expression  $\neg(P \vee \neg(P \wedge Q))$ , show that this compound statement is logically equivalent to the compound statement given at the start of the problem.

**Problem 4.** (10pt) Fix a real number  $x$ . Consider the statement, “if  $x^2 > 4$ , then  $x > 2$ ”

- (a) Determine the truth value of this statement with an explanation.
- (b) Rewrite the given statement by defining appropriate primitive propositions and logical connectives.
- (c) Find the negation, converse, and contrapositive of your result from (b).
- (d) Rewrite your answers from (c) as English sentences. Then determine the truth value, with explanation, of each of the statements.