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MATH 108
Fall 2021
HW 8: Due 11/09

"I did not attend his funeral, but I sent a nice letter saying I approved of it."
–Mark Twain

Problem 1. (10pt) Find the least square regression line for the points: $(1,1),(1,0),(2,3),(3,4)$. Show all your work.

Solution. We have 4 points so that $n = 4$.
First, we compute the x and y averages—$\overline{x}$ and $\overline{y}$, respectively.

$$\overline{x} = \frac{\sum x_i}{n} = \frac{1+1+2+3}{4} = \frac{7}{4} \approx 1.75$$
$$\overline{y} = \frac{\sum y_i}{n} = \frac{1+0+3+4}{4} = \frac{8}{4} \approx 2.00$$

Now we compute s_x, s_y, r: Then we have

x	y	$x_i - \overline{x}$	$(x_i - \overline{x})^2$	$y_i - \overline{y}$	$(y_i - \overline{y})^2$
1	1	-0.75	0.5625	-1	1
1	0	-0.75	0.5625	-2	4
2	3	0.25	0.0625	1	1
3	4	1.25	1.5625	2	4
	Total:		2.75		10

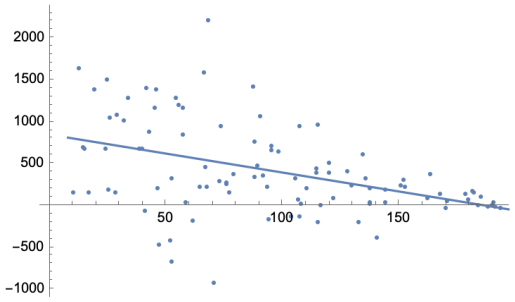
$$s_x^2 = \frac{1}{n-1}\sum (x_i - \overline{x})^2 = \frac{1}{4-1} \cdot 2.75 = 0.9167 s_y^2 \quad = \frac{1}{n-1}\sum (y_i - \overline{y})^2 = \frac{1}{4-1} \cdot 10 = 3.3333$$

Therefore, $s_x = \sqrt{0.9167} = 0.9574$ and $s_y = \sqrt{3.3333} = 1.8257$.

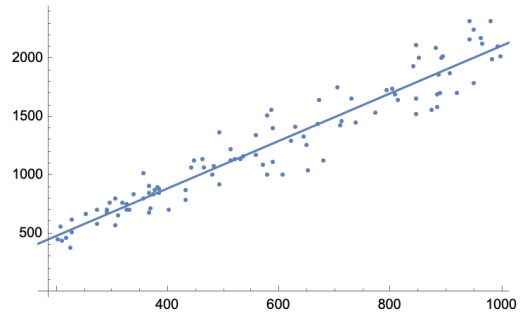
Problem 2. (10pt) Given the following information below, find the least square regression line. Show all your work.

$$
\begin{aligned}
n &= 11 \\
\overline{x} &= 3.45, \quad \sigma_x^2 = 7.073 \\
\overline{y} &= 6.81, \quad \sigma_y^2 = 5.371 \\
R &= 0.802
\end{aligned}
$$

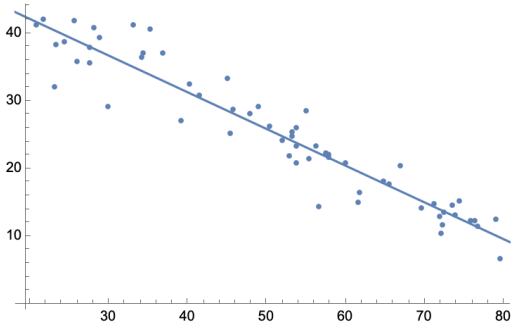
Problem 3. (10pt) Match each regression coefficient to its corresponding graph.



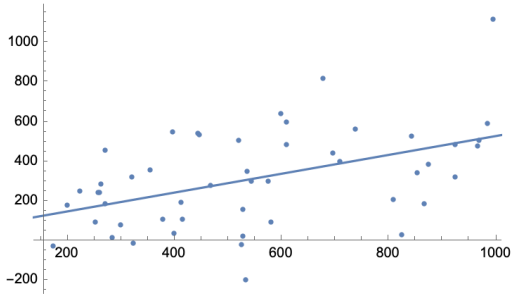
(a)



(b)



(c)



(d)

(i) _____: $R = -0.9529$

(ii) _____: $R = -0.4354$

(iii) _____: $R = 0.4759$

(iv) _____: $R = 0.9573$

Problem 4. (10pt) The lengths (in cm) of twenty snakes are taken 6 months after hatching and 2 years after hatching. The data is given below.

$$(41.2, 163.6), (18.1, 68.9), (42.3, 151.6), (13.2, 43.9), (45.8, 189.5),$$
$$(42.7, 180.5), (24.4, 92.8), (49.0, 166.), (24.6, 101.1), (18.9, 77.5),$$
$$(16.3, 63.6), (36.3, 142.2), (32.2, 124.3), (36.3, 121.), (24.7, 77.8),$$
$$(40.1, 139.7), (22.3, 72.8), (42.4, 182.2), (21.4, 73.), (12.3, 53.1)$$

A linear regression for this data was found to be $\widehat{y} = 3.9x - 3.1$ with $R = 0.9381$.

(a) Was the linear regression a good fit for the data? Explain.

(b) Find the residual for the data point $(41.2, 163.6)$. Was the model under or over prediction for the length of the snake? Explain.

(c) Given this data and model, predict the length of a snake after 2 years that measures 32.7 cm 6 months after hatching.

(d) Should this model be used to predict the length of a snake which is 65 cm six months after hatching? Explain.