MAT 101: Exam 1
Fall – 2023
10/11/2023
85 Minutes

Name: _Caleb McWhorter — Solutions_

Write your name on the appropriate line on the exam cover sheet. This exam contains 21 pages (including this cover page) and 20 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

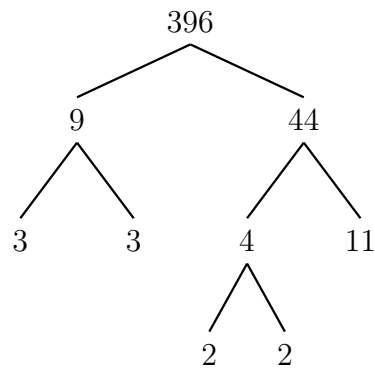| Question	Points	Score	Question	Points	Score
1	5		11	5	
2	5		12	5	
3	5		13	5	
4	5		14	5	
5	5		15	5	
6	5		16	5	
7	5		17	5	
8	5		18	5	
9	5		19	5	
10	5		20	5	
Points:			/100		

1. (5 points) Find the prime factorizations of the following:
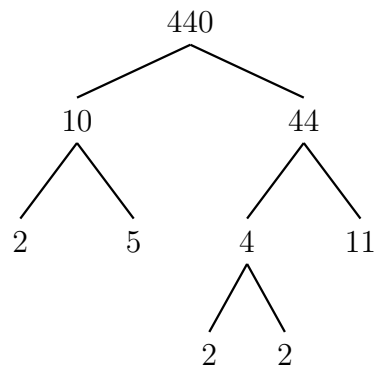
    (a) 396

    (b) 440

    Solution.

    (a) $396 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 11 = 2^2 \cdot 3^2 \cdot 11$

    (b) $440 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 11 = 2^3 \cdot 5 \cdot 11$

2. (5 points) Showing all your work, compute the following:

    (a) $\gcd(36, 54)$

    (b) $\text{lcm}(36, 54)$

Solution.

    (a) $\gcd(36, 54) = \gcd(2^2 \cdot 3^2, 2 \cdot 3^3) = 2^1 \cdot 3^2 = 18$

    (b) $\text{lcm}(36, 54) = \text{lcm}(2^2 \cdot 3^2, 2 \cdot 3^3) = 2^2 \cdot 3^3 = 108$

3. (5 points) Showing all your work, compute the following:

   (a) $\gcd\left(2^{40} \cdot 3^{90} \cdot 5^{30} \cdot 17^5, \ 2^{50} \cdot 3^{80} \cdot 11^{20} \cdot 17^5\right)$

   (b) $\operatorname{lcm}\left(2^{40} \cdot 3^{90} \cdot 5^{30} \cdot 17^5, \ 2^{50} \cdot 3^{80} \cdot 11^{20} \cdot 17^5\right)$

   Solution.

   (a) $\gcd\left(2^{40} \cdot 3^{90} \cdot 5^{30} \cdot 17^5, \ 2^{50} \cdot 3^{80} \cdot 11^{20} \cdot 17^5\right) = 2^{40} \cdot 3^{80} \cdot 5^0 \cdot 11^0 \cdot 17^5$

   $$2^{40} \cdot 3^{80} \cdot 5^0 \cdot 11^0 \cdot 17^5 = 230\,751\,647\,806\,918\,333\,113\,422\,816\,410\,774\,688\,737\,564\,386\,243\,989\,471\,232$$

   (b) $\operatorname{lcm}\left(2^{40} \cdot 3^{90} \cdot 5^{30} \cdot 17^5, \ 2^{50} \cdot 3^{80} \cdot 11^{20} \cdot 17^5\right) = 2^{50} \cdot 3^{90} \cdot 5^{30} \cdot 11^{20} \cdot 17^5$

$2^{50} \cdot 3^{90} \cdot 5^{30} \cdot 11^{20} \cdot 17^5 = 8\,742\,006\,963\,728\,917\,196\,307\,854\,228\,116\,466\,398\,107\,923\,957\,214\,655\,628\,007\,881\,328\,399\,108\,538\,368\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000$

4. (5 points) Showing all your work and simplifying as much as possible, compute the following:

(a) $\dfrac{99}{140} - \dfrac{29}{42}$

(b) $\dfrac{7}{15} - \dfrac{6}{10} + \dfrac{5}{6}$

Solution.

(a)

$$
\begin{aligned}
\frac{99}{140} - \frac{29}{42} &= \frac{99}{140} \cdot \frac{3}{3} - \frac{29}{42} \cdot \frac{10}{10} \\
&= \frac{297}{420} - \frac{290}{420} \\
&= \frac{297 - 290}{420} \\
&= \frac{7}{420} \\
&= \frac{7}{420} \cdot \frac{1/7}{1/7} \\
&= \frac{1}{60}
\end{aligned}
$$

(b)

$$
\begin{aligned}
\frac{7}{15} - \frac{6}{10} + \frac{5}{6} &= \frac{7}{15} \cdot \frac{2}{2} - \frac{6}{10} \cdot \frac{3}{3} + \frac{5}{6} \cdot \frac{5}{5} \\
&= \frac{14}{30} - \frac{18}{30} + \frac{25}{30} \\
&= \frac{14 - 18 + 25}{30} \\
&= \frac{21}{30} \\
&= \frac{21}{30} \cdot \frac{1/3}{1/3} \\
&= \frac{7}{10}
\end{aligned}
$$

5. (5 points) Showing all your work and simplifying as much as possible, compute the following:

   (a) $\dfrac{20}{66} \cdot \dfrac{117}{15}$

   (b) $\dfrac{\dfrac{33}{12}}{\dfrac{45}{26}}$

Solution.

(a)
$$\frac{20}{66} \cdot \frac{117}{15} = \frac{\cancel{20}^{\,4}}{\cancel{66}^{\,22}} \cdot \frac{\cancel{117}^{\,39}}{\cancel{15}^{\,3}} = \frac{\cancel{4}^{\,2}}{\cancel{22}^{\,11}} \cdot \frac{\cancel{39}^{\,13}}{\cancel{3}^{\,1}} = \frac{2}{11} \cdot \frac{13}{1} = \frac{26}{11}$$

(b)
$$\frac{\dfrac{33}{12}}{\dfrac{45}{26}} = \frac{33}{12} \cdot \frac{26}{45} = \frac{\cancel{33}^{\,11}}{\cancel{12}^{\,6}} \cdot \frac{\cancel{26}^{\,13}}{\cancel{45}^{\,15}} = \frac{11}{6} \cdot \frac{13}{15} = \frac{143}{90}$$

6. (5 points) Showing all your work and simplifying as much as possible, convert the given improper fractions to a proper fraction and the given proper fraction to an improper fraction:

   (a) $-\frac{129}{7}$

   (b) $-11\frac{7}{10}$

   Solution.

   (a) $-\frac{129}{7} = -18\frac{3}{7}$

$$
\begin{array}{r}
18 \\
7\overline{)129} \\
7 \\
\hline
59 \\
56 \\
\hline
3
\end{array}
$$

   (b) $-11\frac{7}{10} = -\frac{117}{10}$

$$
-11\frac{7}{10} = -\left(11\frac{7}{10}\right) = -\left(\frac{10 \cdot 11 + 7}{10}\right) = -\frac{110 + 7}{10} = -\frac{117}{10}
$$

7. (5 points) Showing all your work, simplify the following as much as possible:

$$\frac{x^{-5}y^6}{x^3y^{-2}(x^3y^5)^{-2}}\left(\frac{xy\big((x^3y^{-5})^2\big)^{-4}}{x^0y^8(x^{-5}y^6)^{-12}}\right)^0$$

Solution.

$$\frac{x^{-5}y^6}{x^3y^{-2}(x^3y^5)^{-2}}\left(\frac{xy\big((x^3y^{-5})^2\big)^{-4}}{x^0y^8(x^{-5}y^6)^{-12}}\right)^0$$

$$\frac{x^{-5}y^6}{x^3y^{-2}(x^3y^5)^{-2}}\cdot 1$$

$$\frac{x^{-5}y^6}{x^3y^{-2}(x^3y^5)^{-2}}$$

$$\frac{x^{-5}y^6}{x^3y^{-2}\cdot x^{-6}y^{-10}}$$

$$\frac{x^{-5}y^6}{x^{-3}y^{-12}}$$

$$\frac{x^3y^6y^{12}}{x^5}$$

$$\frac{x^3y^{18}}{x^5}$$

$$\frac{y^{18}}{x^2}$$

8. (5 points) Showing all your work, simplify the following as much as possible:

$$\sqrt{\frac{9(a^6b^5)^{1/3}}{a^{-2}b}}$$

Solution.

$$\sqrt{\frac{9(a^6b^5)^{1/3}}{a^{-2}b}}$$

$$\sqrt{\frac{9a^{6/3}b^{5/3}}{a^{-2}b}}$$

$$\sqrt{\frac{9a^2b^{5/3}}{a^{-2}b}}$$

$$\sqrt{\frac{9a^2a^2b^{5/3}}{b}}$$

$$\sqrt{\frac{9a^4b^{5/3}}{b}}$$

$$\sqrt{9a^4b^{5/3-1}}$$

$$\sqrt{9a^4b^{5/3-3/3}}$$

$$\sqrt{9a^4b^{2/3}}$$

$$\left(9a^4b^{2/3}\right)^{1/2}$$

$$9^{1/2}a^{4/2}b^{2/3 \cdot 1/2}$$

$$3a^2b^{1/3}$$

$$3a^2\sqrt[3]{b}$$

9. (5 points) Showing all your work, simplify the following as much as possible:

   (a) $\sqrt{24}$

   (b) $\sqrt[3]{24}$

Solution.

   (a) $\sqrt{24} = \sqrt{4 \cdot 6} = 2\sqrt{6}$ *or* $\sqrt{24} = \sqrt{2^3 \cdot 3} = 2\sqrt{2^1 \cdot 3} = 2\sqrt{6}$

   (b) $\sqrt[3]{8 \cdot 3} = 2\sqrt[3]{3}$ *or* $\sqrt[3]{24} = \sqrt[3]{2^3 \cdot 3} = 2^1 \sqrt[3]{3} = 2\sqrt[3]{3}$

10. (5 points) Showing all your work and simplifying as much as possible, convert the following decimal number $0.\overline{23}$ to a fraction.

   Solution. Suppose that $N = 0.\overline{23} = 0.232323232323232323\overline{23}$. We have...

$$
\begin{array}{rcl}
100N & = & 23.232323232323232323\overline{23} \\
- \quad N & = & 0.232323232323232323\overline{23} \\
\hline
99N & = & 23 \\
N & = & \frac{23}{99}
\end{array}
$$

$$0.\overline{23} = \frac{23}{99}$$

11. (5 points) Showing all your work and simplifying as much as possible, compute the following:

(a) $(8 - 6i) - (5 - 9i)$

(b) $\dfrac{1 - 4i}{-4 + 5i}$

Solution.

(a)

$$(8 - 6i) - (5 - 9i) = 8 - 6i - 5 + 9i = (8 - 5) + (-6i + 9i) = 3 + 3i$$

(b)

$$\frac{1 - 4i}{-4 + 5i} = \frac{1 - 4i}{-4 + 5i} \cdot \frac{-4 - 5i}{-4 - 5i}$$

$$\frac{(1 - 4i)(-4 - 5i)}{(-4 + 5i)(-4 - 5i)}$$

$$\frac{-4 - 5i + 16i + 20i^2}{16 + 20i - 20i - 25i^2}$$

$$\frac{-4 - 5i + 16i + 20(-1)}{16 + 20i - 20i - 25(-1)}$$

$$\frac{(-4 - 20) + (-5i + 16i)}{(16 + 25) + (20i - 20i)}$$

$$\frac{-24 + 11i}{41}$$

$$-\frac{24}{41} + \frac{11}{41}i$$

12. (5 points) Showing all your work, compute the following:

   (a) 67% of 7690

   (b) 0.1% of 4500

   Solution.

   (a)
   $$67\% \text{ of } 7690 = 7690(0.67) = 5152.3$$

   (b)
   $$0.1\% \text{ of } 4500 = 4500(0.001) = 4.5$$

13. (5 points) Showing all your work, compute the following:

    (a) 95 increased by 108%

    (b) 720 decreased by 35%

Solution.

    (a)
$$95 \text{ increased by } 108\% = 95(1 + 1.08) = 95(2.08) = 197.6$$

    (b)
$$720 \text{ decreased by } 35\% = 720(1 - 0.35) = 720(0.65) = 468$$

14. (5 points) Given the following course grade components, weights, and student scores, compute the student's course average.

Grade Component	Component Value	Student Grade
Participation	5%	85%
Homework	40%	81%
Project	15%	74%
Midterm	15%	92%
Final	25%	86%

Solution.

$$\text{Course Average} = \frac{\sum \text{weight} \cdot \text{value}}{\sum \text{weights}}$$

$$= \frac{0.05 \cdot 0.85 + 0.40 \cdot 0.81 + 0.15 \cdot 0.74 + 0.15 \cdot 0.92 + 0.25 \cdot 0.86}{0.05 + 0.40 + 0.15 + 0.15 + 0.25}$$

$$= \frac{0.0425 + 0.324 + 0.111 + 0.138 + 0.215}{1}$$

$$= \frac{0.8305}{1}$$

$$= 0.8305$$

Therefore, the student's course average is 83.05%.

15. (5 points) Suppose you received the following grades this semester:

Course	Credits	Grade
BIO 151: Essentials of Anatomy & Physiology	4	B–
KIN 202: Motor Development & Learning	3	B+
SPM 214: Sports Psychology	3	A
CA 219: Modern Movies (1950 – Present)	3	C
ENG 207: Writing about World Mythology	3	A–

Given the following grade values, compute your semester GPA.

Grade	Values	Grade	Values
A	4.0	C+	2.3
A–	3.7	C	2.0
B+	3.3	C–	1.7
B	3.0	D	1.0
B–	2.7	F	0

Solution.

$$
\begin{aligned}
\text{GPA} &= \frac{\sum \text{weight} \cdot \text{value}}{\sum \text{weights}} \\[2mm]
&= \frac{\sum \text{credit} \cdot \text{grade value}}{\sum \text{credits}} \\[2mm]
&= \frac{4 \cdot 2.7 + 3 \cdot 3.3 + 3 \cdot 4.0 + 3 \cdot 2.0 + 3 \cdot 3.7}{4 + 3 + 3 + 3 + 3} \\[2mm]
&= \frac{10.8 + 9.9 + 12.0 + 6.0 + 11.1}{4 + 3 + 3 + 3 + 3} \\[2mm]
&= \frac{49.8}{16} \\[2mm]
&\approx 3.113
\end{aligned}
$$

16. (5 points) Convert the given decimal number to scientific notation and the given number in scientific notation to a decimal number:

   (a) $1.4567 \cdot 10^2$

   (b) 0.0000065

   Solution.

   (a) $1.457 \cdot 10^2 = 145.7$

   (b) $0.0000065 = 6.5 \cdot 10^{-6}$

17. (5 points) Showing all your work, compute the following:

    (a) 15 quarts to liters [1 quart = 4 cups; 1 cup = 8 fl oz; 29.57 ml = 1 fl oz]

    (b) 9.8 m/s^2 to feet per square minute [1 m = 3.28084 ft]

Solution.

  (a)

$$\frac{15 \text{ quarts}}{} \left\Vert \frac{4 \text{ cups}}{1 \text{ quart}} \right| \frac{8 \text{ fl oz}}{1 \text{ cup}} \left| \frac{29.57 \text{ ml}}{1 \text{ fl oz}} \right| \frac{1 \text{ L}}{1000 \text{ ml}} = 14.1936 \text{ L}$$

  (b)

$$\frac{9.8 \text{ m}}{1 \text{ s}^2} \left\Vert \frac{3.28084 \text{ ft}}{1 \text{ m}} \right| \frac{60 \text{ s}}{1 \text{ min}} \left| \frac{60 \text{ s}}{1 \text{ min}} \right. = 115{,}748.0352 \text{ ft/min}^2$$

18. (5 points) A lighthouse is located 7 mi due West and 3 mi due South of you. You will walk a straight path to the lighthouse at a rate of 2.5 mph. How long will it take you to walk to the lighthouse?

Solution. First, we sketch a picture representing the situation:
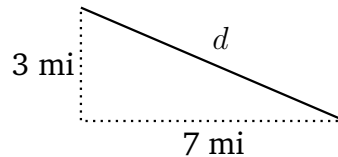
We travel the solid line. Using the Pythagorean Theorem, the distance we will travel, d, is. . .

$$d^2 = a^2 + b^2 = (3\text{ mi})^2 + (7\text{ mi})^2 = 9\text{ mi}^2 + 49\text{ mi}^2 = 58\text{ mi}^2 \implies d = \sqrt{58\text{ mi}^2} \approx 7.61577\text{ mi}$$

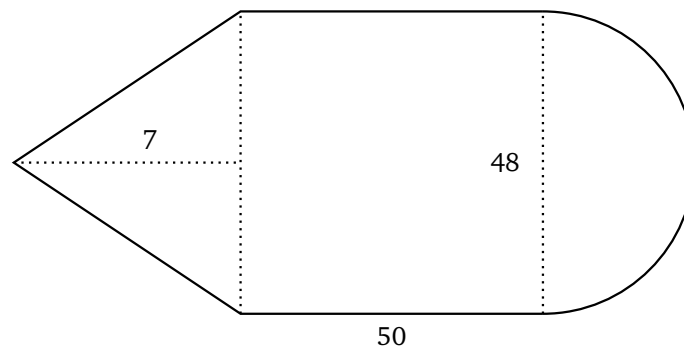But we know that $d = vt$. Therefore, we have. . .

$$d = vt$$

$$7.61577\text{ mi} = 2.5\text{ mph} \cdot t$$
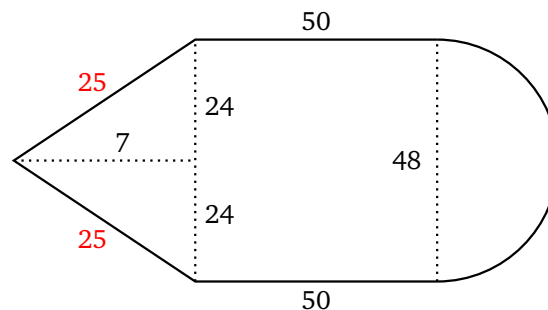
$$t = \frac{7.61577\text{ mi}}{2.5\text{ mph}}$$

$$t = 3.046308\text{ hrs}$$

Therefore, it will take 3.046308 hours to walk to the lighthouse, i.e. 3 hours, 2 minutes, and 46.7 seconds to walk to the lighthouse.

19. (5 points) Find the perimeter and area of the figure below.

Solution. We can use the symmetry of the diagram to fill it a bit more information:

We need to find the hypotenuse, c, of the triangle. We can use the Pythagorean Theorem:

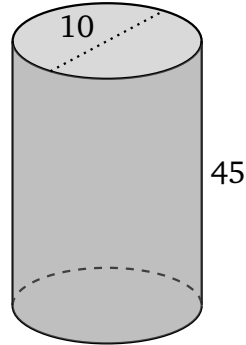$$c^2 = a^2 + b^2 = 7^2 + 24^2 = 49 + 576 = 625 \Longrightarrow c = \sqrt{625} = 25$$

We know that a perimeter of a circle is $C = \pi d$. The perimeter of the half circle is then $\frac{1}{2}C = \frac{1}{2}(\pi d) = \frac{d}{2}\pi = r\pi$. But then the perimeter is. . .

$$P = 50 + 25 + 25 + 50 + 24\pi = 150 + 24\pi \approx 225.398$$

The area of the region is. . .

$$A = A_{\triangle} + A_{\square} + \frac{1}{2}A_{\bigcirc}$$

$$= \frac{1}{2}bh + \ell w + \frac{1}{2} \cdot \pi r^2$$

$$= \frac{1}{2} \cdot 48 \cdot 7 + 50 \cdot 48 + \frac{1}{2}(24)^2\pi$$

$$= 168 + 2400 + 288\pi$$

$$= 2568 + 288\pi$$

$$\approx 3{,}472.78$$

20. (5 points) Compute the volume and surface area of the figure below.

Solution. Examining the cylinder above, we see that the cylinder has radius $\frac{10}{2} = 5$. We have...

$$V = \pi r^2 h$$

$$V = \pi (5^2) 45$$

$$V = \pi \cdot 25 \cdot 45$$

$$V = 1125\pi$$

$$V \approx 3534.29$$

The surface area of a cylinder is...

$$\text{S.A.} = 2\pi r^2 + 2\pi r h$$

$$= 2\pi (5^2) + 2\pi (5) 45$$

$$= 50\pi + 450\pi$$

$$= 500\pi$$

$$\approx 1{,}570.8$$