

Quiz 1. True/False: The integer 131313 is prime.

Solution. The statement is *false*. We know that an integer N is divisible by 3 if and only if the sum of its digits is divisible by 3. We know that $1 + 3 + 1 + 3 + 1 + 3 = 12$ is divisible by 3. Therefore, 131313 cannot be prime. In fact, $131313 = 3 \cdot 43771 = 3 \cdot 7 \cdot 13 \cdot 37$.

Quiz 2. True/False: Every rational number can be written as $\frac{a}{b}$, where $\gcd(a, b) = 1$.

Solution. The statement is *true*. By definition, a rational number r is a number of the form $\frac{a}{b}$, where a, b are integers and $b \neq 0$. Therefore, we can clearly write every rational number in the form $\frac{a}{b}$. Now can we impose the restriction that $\gcd(a, b) = 1$? Yes! By cancelling common factors from a, b , we can assure that the fraction is reduced, i.e. $\gcd(a, b) = 1$. In fact, we can always divide the numerator and denominator by $\gcd(a, b)$. After, we have $\gcd(a, b) = 1$. For instance, take $\frac{10}{15}$. We have $\gcd(10, 15) = 5$. But then $\frac{10}{15} \cdot \frac{1/5}{1/5} = \frac{2}{3}$ is reduced.

Quiz 3. True/False: $\frac{(x^2)^3 x^5}{x^4} = x^6$

Solution. The statement is *false*. Recall that $x^a \cdot x^b = x^{a+b}$, $(x^a)^b = x^{ab}$, and $\frac{x^a}{x^b} = x^{a-b}$. We then have...

$$\frac{(x^2)^3 x^5}{x^4} = \frac{x^6 \cdot x^5}{x^4} = \frac{x^{11}}{x^4} = x^7$$

The mistake made was adding the powers in $(x^2)^3$ to obtain x^5 rather than multiplying the powers to obtain the correct x^6 .

Quiz 4. True/False: $\sqrt{\sqrt[3]{x^2}} = x^{2/5}$

Solution. The statement is *false*. Recall that $\sqrt[n]{x^m} = x^{m/n}$ and $(x^a)^b = x^{ab}$. We then have...

$$\sqrt{\sqrt[3]{x^2}} = \sqrt{x^{2/3}} = (x^{2/3})^{1/2} = x^{\frac{2}{3} \cdot \frac{1}{2}} = x^{1/3} = \sqrt[3]{x}$$

The mistake made was adding the denominators rather than multiplying the powers correctly, i.e. $\sqrt{\sqrt[3]{x^2}} = ((x^2)^{1/3})^{1/2} = (x^2)^{1/5} = x^{2/5}$, which is incorrect.

Quiz 5. True/False: $(1 - 3i)(2 + 5i) = 17 - i$

Solution. The statement is *true*. Recall that $i^2 = -1$. Then we have...

$$(1 - 3i)(2 + 5i) = 1(2) + 1(5i) - 3i(2) - 3i(5i) = 2 + 5i - 6i - 15i^2 = 2 - i - 15(-1) = 2 - i + 15 = 17 - i$$

Quiz 6. True/False: If one increases 76 by 5% five times sequentially, the result is $76(1 + 0.25) = 76(1.25) = 95$.

Solution. The statement is *false*. If we want to compute N increased or decreased by a % a total of n times, we compute $N \cdot (1 \pm \%_d)^n$, where $\%_d$ is the percentage written as a decimal, n is the number of times we apply the percentage increase/decrease, and we choose '+' if it is a percentage increase and choose '-' if it is a percentage decrease. Then to compute 76 decreased by 5% consecutively five times, we need take $N = 76$, $\%_d = 0.05$, and choose '+'. Therefore, we have...

$$N \cdot (1 \pm \%_d)^n = 76(1 + 0.05)^5 = 76(1.05)^5 = 76(1.27628) = 96.9974$$

From the $76(1.27628)$ portion from the computation above, we can see that increasing a number by 5% consecutively five times actually results in a 27.628% increase in the original number's value because $1 + 0.27628 = 1.27628$. The mistake made in the quiz is thinking that repeated percentage increases or decreases are additive. An increase of 5% five times *does not* result in a $5 \cdot 5\% = 25\%$ increase, which was the percentage decrease computed in the quiz statement.