Quiz 1. *True/False*: The function $f(x) = 9 - 5x$ is a linear function with slope 5 and y-intercept 9.

Solution. The statement is *false*. We know a function of the form $f(x) = mx + b$ is a linear function with slope m and y-intercept b. Because we have $f(x) = 9 - 5x = -5x + 9$, we have $m = -5$, i.e. slope -5, and y-intercept 9, i.e. $(0, 9)$. But then the slope is -5, not the given value of 5.

Quiz 2. *True/False*: If $f(x) = 2x - 1$ and $g(x) = 3 - x$, then $(f \circ g)(0) = f(0)g(0) = -1 \cdot 3 = -3$.

Solution. The statement is *false*. First, note that $f(0) = 2(0) - 1 = -1$, $g(0) = 3 - 0 = 3$, and $f(3) = 2(3) - 1 = 6 - 1 = 5$. What was given was function multiplication, i.e. what was computed was $(fg)(0) = f(0)g(0) = -1 \cdot 3 = -3$. What was originally written was function composition. We have $(f \circ g)(0) = f\big(g(0)\big) = f(3) = 5$.

Quiz 3. *True/False*: Compared to the graph of $f(x)$, the graph of $5 - 3f(x + 2)$ is stretched by a factor of 3, then shifted to the right by 2 and up by 5.

Solution. The statement is *false*. We know that $f(x + 2)$ is the graph of $f(x)$ shifted 2 to the *left*. The graph of $-3f(x + 2)$ is then the graph of $f(x)$ shifted two to the left, stretched by a factor of 3, and reflected across the x-axis. Finally, the graph of $5 - 3f(x + 2)$ is the graph of $f(x)$ shifted two to the left, stretched by a factor of 3, reflected across the x-axis, then shifted upwards by 5.

Quiz 4. *True/False*: The function $f(x) = 4(5^{-x})$ is a concave up, decreasing, exponential function.

Solution. The statement is *true*. A function of the form $f(x) = Ab^x$ is an exponential function. We can summarize whether $f(x)$ is increasing or decreasing and concave up or down as follows: But

	$0 < b < 1$	$b > 1$
$A > 0$	Decreasing, Concave Up	Increasing, Concave Up
$A < 0$	Increasing, Concave Down	Decreasing, Concave Down

we have $f(x) = 4(5^{-x}) = 4(5^{-1})^x = 4\left(\dfrac{1}{5}\right)^x$. Therefore, $f(x)$ is exponential with $A = 4 > 0$ and $0 < b = \frac{1}{5} < 1$. Therefore, $f(x)$ is a decreasing, concave up, exponential function.

Quiz 5. *True/False*: The function $f(x) = 5(2^{1-2x})$ is equal to the function $g(x) = 10\left(\dfrac{1}{4}\right)^x$.

Solution. The statement is *true*. Observe that we have. . .

$$f(x) = 5(2^{1-2x}) = 5 \cdot 2^1 \cdot 2^{-2x} = 10 \cdot 2^{-2x} = 10(2^{-2})^x = 10\left(\frac{1}{2^2}\right)^x = 10\left(\frac{1}{4}\right)^x = g(x)$$

Quiz 6. *True/False*: $\log_5(4^{-3}) = -3$

Solution. The statement is *false*. Recall that $\log_b(y)$ represents the power of b that yields y; that is, $\log_b(y) = x$ if and only if $b^x = y$. Then clearly $\log_b(b^n) = n$ because $b^n = b^n$. Notice then that in the case of $\log_b(b^n)$, the logarithmic and exponential functions 'undo' each other. However, the base of the logarithm and the base of the exponential function need to match. In the case of $\log_5(4^{-3})$, $b = 5 \neq 4$ so that these do not 'undo' each other. In fact, we have $\log_5(4^{-3}) \approx -2.58406$ because $5^{-2.58406} \approx 4^{-3} = \frac{1}{64}$. One case use $\log_b(b^n) = n$ in the computation of $log_5(4^{-3}) = -3$ if one uses the change of base formula: $\log_b(y) = \frac{\log_a(y)}{\log_a(b)}$. In this case, we have...

$$\log_5(4^{-3}) = \frac{\log_4(4^{-3})}{\log_4(5)} = \frac{-3}{\log_4(5)} \approx \frac{-3}{1.160964} \approx -2.58406$$

Quiz 7. *True/False*: $\ln\left(\dfrac{x^5}{\sqrt[3]{y}}\right) = 5\ln(x) - \frac{1}{3}\ln(y)$

Solution. The statement is *true*. Recall that $\log_b(x^n) = n\log_b(x)$ and $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$; that is, for logarithms, you can turn powers into coefficients (and vice versa) and quotients into differences (and vice versa). But then we have...

$$\ln\left(\frac{x^5}{\sqrt[3]{y}}\right) = \ln\left(\frac{x^5}{y^{1/3}}\right) = \ln(x^5) - \ln(y^{1/3}) = 5\ln(x) - \frac{1}{3}\ln(y)$$

Quiz 8. *True/False*: If $2^{\sqrt{x}} - 5 = 3$, then $x = 9$.

Solution. The statement is *true*. One way of being somewhat convinced is to substitute $x = 9$:

$$\left(2^{\sqrt{x}} - 5\right)\Bigg|_{x=9} = 2^{\sqrt{9}} - 5 = 2^3 - 5 = 8 - 5 = 3$$

However, all this shows is that *if* $x = 9$, then $2^{\sqrt{x}} - 5 = 3$. We need to show that $2^{\sqrt{x}} - 5 = 3$, then it must be the case that $x = 9$; that is, we need to solve the equation $2^{\sqrt{x}} - 5 = 3$ for x. We have...

$$2^{\sqrt{x}} - 5 = 3$$

$$2^{\sqrt{x}} = 8$$

$$\log_2\left(2^{\sqrt{x}}\right) = \log_2(8)$$

$$\sqrt{x} = 3$$

$$(\sqrt{x})^2 = 3^2$$

$$x = 9$$

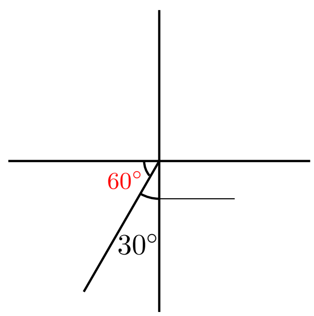
Quiz 9. *True/False*: $\tan(\theta)\cot(\theta) = 1$

Solution. The statement is *true*. Recall that $\cot(\theta) = \frac{1}{\tan\theta}$. But then we have...

$$\tan(\theta)\cot(\theta) = \tan(\theta) \cdot \frac{1}{\tan\theta} = 1$$

Quiz 10. *True/False*: The reference angle for the angle that is $30°$ clockwise from the negative y-axis is $240°$.

Solution. The statement is *false*. A reference angle is always an angle 'in' Quadrant I; that is, a reference angle θ is always such that $0 \leq \theta \leq \frac{\pi}{2}$, i.e. $0 \leq \theta \leq 90°$. Therefore, it is impossible to have a reference angle of $240°$. We can see in the diagram below that an angle that is $30°$ clockwise from the negative y-axis below.



This is indeed an angle of $240°$ with the positive x-axis (coming from $270° - 30° = 240°$). However, the smallest possible angle this ray makes with the x-axis is $60°$. Therefore, the reference angle is $60°$ (represented in red in the diagram above).

Quiz 11. *True/False*: Because we have $\tan(\theta + 2\pi) = \tan(\theta)$ for all $\theta \in \mathbb{R}$, the period of $\tan\theta$ is 2π.
Solution. The statement is *false*. The period of a function $f(x)$ (if it exists) is the *smallest* positive value P such that $f(x + P) = f(x)$ for all x. While it is true that $\tan(\theta + 2\pi) = \tan(\theta)$ for all $\theta \in \mathbb{R}$, this is not necessarily the *smallest* possible value such that this is true. Observe that...

$$\tan(\theta + \pi) = \frac{\sin(\theta + \pi)}{\cos(\theta + \pi)} = \frac{-\sin(\theta)}{-\cos(\theta)} = \frac{\sin(\theta)}{\cos(\theta)} = \tan(\theta)$$

But then the period is at most π. In fact, the period of tangent is π. Therefore, $\tan(\theta + \pi) = \tan(\theta)$ for all $\theta \in \mathbb{R}$.[1]

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[1]Note: We have only shown that the period is at most π. To show that the period is π, we need to show that there can be no smaller value, say P, such that $\tan(\theta + P) = \tan(\theta)$. Suppose that $\tan(\theta + P) = \tan(\theta)$. Then using the angle sum formula for tangent, we then have $\tan(\theta) = \tan(\theta + P) = \frac{\tan(\theta)+\tan(P)}{1-\tan(\theta)\tan(P)}$. But this gives $\tan(\theta) + \tan(P) = \tan(\theta) - \tan^2(\theta)\tan(P)$. But then we have $\tan(P)\left(\tan^2(\theta) + 1\right) = 0$. If $\tan^2(\theta) + 1 = 0$, then $(\tan(\theta))^2 = -1$, which is impossible. But then it must be $\tan(P) = 0$. This implies that $P = k\pi$ for some integer k. The smallest (positive) solution is clearly when $k = 1$, which gives $P = \pi$.

Quiz 12. *True/False*: $\cos^2(\theta) = \sin(\theta)\,(\csc(\theta) - \sin(\theta))$

Solution. The statement is *true*. Starting with the right hand side, we have...

$$\begin{aligned}\sin(\theta)\,(\csc(\theta) - \sin(\theta)) &= \sin(\theta)\left(\frac{1}{\sin(\theta)} - \sin(\theta)\right)\\ &= \frac{\sin(\theta)}{\sin(\theta)} - \sin^2(\theta)\\ &= 1 - \sin^2(\theta)\\ &= \cos^2(\theta)\end{aligned}$$

where for the last equality we have used the fact that $\sin^2(\theta) + \cos^2(\theta) = 1$, i.e. $\cos^2(\theta) = 1 - \sin^2(\theta)$.

Quiz 13. *True/False*: There are only two solutions to the equation $\tan\theta = \sqrt{3}$.

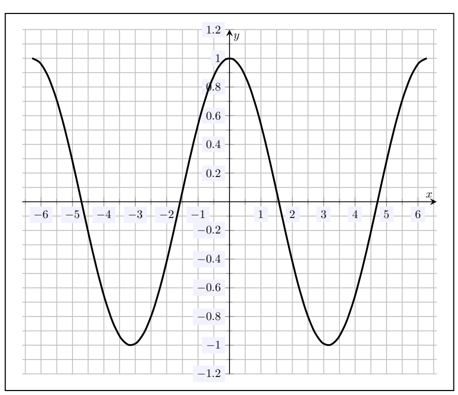
Solution. The statement is *false*. We know that $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$ and $\tan\left(\frac{4\pi}{3}\right) = \sqrt{3}$. There are then at least two solutions. However, the period of $\tan(\theta)$ is 2π. Then any rotation of $\frac{\pi}{3}$ by any multiple of π radians counterclockwise or clockwise will also be a solution of the equation. For instance, all of the following are solutions to the equation $\tan(\theta) = \sqrt{3}$.

$$\frac{\pi}{3} - 2\pi = -\frac{5\pi}{3} \qquad \frac{\pi}{3} + 2\pi = \frac{7\pi}{3}$$

$$\frac{\pi}{3} - \pi = -\frac{2\pi}{3} \qquad \frac{\pi}{3} + \pi = \frac{4\pi}{3}$$

Quiz 14. *True/False*: The function $f(x) = \cos x$ has a well-defined 'global' inverse.

Solution. The statement is *false*. This is immediately false because $\cos(0) = 1$ and $\cos(2\pi) = 1$ so that $\cos^{-1}(1)$ is not well defined. Alternatively, observe that the graph of $f(x) = \cos x$ fails the horizontal line test so that it cannot have a global inverse.



Therefore, $f(x) = \cos x$ can only have an inverse on a restricted domain. In fact, the entirety of the range of $f(x) = \cos x$ is seen on the interval $[0, \pi]$ and $f(x) = \cos x$ is one-to-one on this interval. Therefore, $\cos^{-1}(x)$ is well defined on this interval.

Quiz 15. *True/False*: To solve the equation $\sqrt{2}(\sqrt{2}\cos x - 1) = 0$, we divide by $\sqrt{2}$, add 1, then divide by $\sqrt{2}$ to obtain $\cos x = \frac{1}{\sqrt{2}}$. Because the period of $\cos x$ is 2π, we find the solutions in $[0, 2\pi]$, which are $x = \frac{\pi}{4}$ and $x = \frac{7\pi}{4}$. But then because the period of $\cos x$ is 2π, the solutions are $x = \frac{\pi}{4} \pm 2k\pi$ and $x = \frac{7\pi}{4} \pm 2k\pi$.

Solution. The statement is *true*. We can simply solve this equation:

$$\sqrt{2}(\sqrt{2}\cos x - 1) = 0$$

$$\sqrt{2}\cos x - 1 = 0$$

$$\sqrt{2}\cos x = 1$$

$$\cos x = \frac{1}{\sqrt{2}}$$

We know that $\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ and $\cos\left(\frac{7\pi}{4}\right) = \frac{1}{\sqrt{2}}$. But because the period of $\cos x$ is 2π, we know the solutions are $\frac{\pi}{4} \pm 2k\pi$ and $\frac{7\pi}{4} \pm 2k\pi$, where k is an integer.