Name: _____

MATH 100
Fall 2021
HW 2: Due 09/22

"...when I first read Aristotle, I thought it was pronounced like
'Chipotle' ...wait a minute, is it 'Chip-o-tottle'?"
—Eleanor Shellstrop, The Good Place

Problem 1. (5pt) Give the definition of a rational number.

Problem 2. (15pt) Find the prime factorization for each of the following integers:

(a) 105

(b) 137

(c) 137

(d) 525

(e) 1320

Problem 3. (10pt)

   (a) By listing out all of the divisors of 24 and 60, find $\gcd(24, 60)$.

   (b) By listing out at least the first 10 multiples of both 8 and 15, find $\operatorname{lcm}(8, 15)$.

Problem 4. (12pt) Use the 'prime factorization method' to find the following:

   (a) $\gcd(48, 80) =$

   (b) $\gcd(10050, 483000) = \gcd(2 \cdot 3 \cdot 5^2 \cdot 67, 2^3 \cdot 3 \cdot 5^3 \cdot 7 \cdot 23) =$

   (c) $\gcd(30, 132, 245) = \gcd(2 \cdot 3 \cdot 5, 2^2 \cdot 3 \cdot 11, 5 \cdot 7^2) =$

   (d) $\operatorname{lcm}(18, 30) =$

   (e) $\operatorname{lcm}(462, 4200) = \operatorname{lcm}(2 \cdot 3 \cdot 7 \cdot 11, 2^3 \cdot 3 \cdot 5^2 \cdot 7) =$

   (f) $\operatorname{lcm}(30, 105, 44) = \operatorname{lcm}(2 \cdot 3 \cdot 5, 3 \cdot 5 \cdot 7, 2^2 \cdot 11) =$

Problem 5. (4pt) Use WolframAlpha to find the following:

(a) $\gcd(1050, 29398) =$

(b) $\gcd(6\,300\,020, 9\,119\,019) =$

(c) $\text{lcm}(4918, 24287) =$

(d) $\text{lcm}(705\,299\,330, 342\,828\,558) =$

Problem 6. (8pt) For each of the following rational numbers, if it is a mixed number, write is as an improper fraction, and if the number is an improper fraction, write it as a mixed number.

(a) $1\frac{3}{5} =$

(b) $\frac{29}{4} =$

(c) $-2\frac{3}{7} =$

(d) $\frac{101}{7} =$

Problem 7. (5pt) Fully reduce the following rationals:

(a) $\dfrac{18}{24} =$

(b) $\dfrac{80}{72} =$

(c) $\dfrac{18}{9} =$

(d) $\dfrac{23}{52} =$

(e) $\dfrac{35}{120} =$

Problem 8. (10pt) Compute the following, being sure to simplify the answer completely:

(a) $\dfrac{3}{4} + \dfrac{2}{5} =$

(b) $\dfrac{5}{6} - \dfrac{7}{4} =$

(c) $\dfrac{1}{50} + \dfrac{201}{450} =$

(d) $-\dfrac{5}{6} + \dfrac{17}{9} =$

(e) $\dfrac{3}{4} - \dfrac{5}{2} + \dfrac{7}{12} =$

Problem 9. (10pt) Compute the following, being sure to simplify the answer completely:

(a) $\dfrac{2}{3} \cdot -\dfrac{7}{5} =$

(b) $\dfrac{5}{2} \cdot \dfrac{11}{20} =$

(c) $\dfrac{7}{10} \cdot \dfrac{2}{21} =$

(d) $\dfrac{\frac{2}{11}}{\frac{9}{10}} =$

(e) $\dfrac{4}{15} \div \dfrac{3}{5} =$

Problem 10. (10pt) Simplify the following expressions as much as possible:

(a) $\sqrt{90} =$

(b) $\sqrt{200} =$

(c) $\sqrt[3]{360} =$

(d) $\sqrt{\dfrac{45}{4}} =$

(e) $\sqrt[4]{2^8 \cdot 3^5 \cdot 5 \cdot 7} =$

Problem 11. (8pt) Rationalize the following:

(a) $-\dfrac{6}{\sqrt{3}} =$

(b) $\dfrac{1}{\sqrt{7}} =$

(c) $\sqrt{\dfrac{2}{3}} =$

(d) $\sqrt[3]{\dfrac{4}{8}} =$

Problem 12. (20pt) Determine if the following numbers are a natural number, integer, rational, irrational, or real. Place a check in the each spot that applies.

	Natural	Integer	Rational	Irrational	Real
6	_____	_____	_____	_____	_____
-5	_____	_____	_____	_____	_____
0	_____	_____	_____	_____	_____
$3/2$	_____	_____	_____	_____	_____
$24/8$	_____	_____	_____	_____	_____
$0.\overline{174}$	_____	_____	_____	_____	_____
$\sqrt{25}$	_____	_____	_____	_____	_____
1.65	_____	_____	_____	_____	_____
$\sqrt{2}$	_____	_____	_____	_____	_____
π	_____	_____	_____	_____	_____

Problem 13. (8pt) Use WolframAlpha to complete the following parts:

(a) Find the prime factorization of 853365401315989073434698.

(b) Determine whether the number 37124508045065437 is prime.

(c) Recall that if n is *not* a prime number, then it must have at least one prime factor p with $p \leq \sqrt{n}$. Therefore, if n is prime, you only need to check divisibility for numbers up to $\sqrt{n}$. If you have not found a factor by then, the integer n must be prime. For instance, if you are looking for prime factors of 137, you only need check the numbers up to $\sqrt{137} \approx 11.705$, i.e. up to 11. If you have not found a prime factor by then, the number 137 is prime. If you were checking whether the integer from (b) was prime by hand, up to what integer do you need to check?

(d) Suppose you could check whether one integer was a factor of another in one second, regardless of the sizes of the number. How long would it then take to show that the number in (b) was prime by hand? [Hint: You can enter 'XXXX seconds' in WolframAlpha and it will tell you how long this is in more reasonable units.]