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MATH 101
Spring 2024
HW 15: Due 04/08

"Sometimes I think I have felt everything I'm ever gonna feel. And from here on out, I'm not gonna feel anything new. Just lesser versions of what I've already felt."

— Theodore Twombly, Her

Problem 1. (10pts) Solve the following system of equations:

$$2x + y = 1$$
$$3x - y = -11$$

Solution. There are two elementary approaches:

Substitution. Solving for y in the first equation, we have $y = 1 - 2x$. Using this in the second equation, we have...

$$3x - y = -11$$

$$3x - (1 - 2x) = -11$$

$$3x - 1 + 2x = -11$$

$$5x = -10$$

$$x = -2$$

But then we have $y = 1 - 2x = 1 - 2(-2) = 1 + 4 = 5$. Therefore, the lines intersect at the point $(x, y) = (-2, 5)$.

Elimination. We add the two equations to eliminate y. This obtains $5x = -10$, which immediately implies $x = -2$. Using this in the first equation, we have...

$$2x + y = 1$$

$$2(-2) + y = 1$$

$$-4 + y = 1$$

$$y = 5$$

Therefore, the lines intersect at $(x, y) = (-2, 5)$.

Problem 2. (10pts) Solve the following system of equations:

$$\begin{cases} 4x + 6y = 6 \\ 5x + 2y = 13 \end{cases}$$

Solution. There are two elementary approaches:

Substitution. Solving for x in the first equation, we have...

$$\begin{aligned} 4x + 6y &= 6 \\ 4x &= 6 - 6y \\ x &= \frac{6 - 6y}{4} \\ x &= \frac{3 - 3y}{2} \end{aligned}$$

Using this in the second equation, we have...

$$\begin{aligned} 5x + 2y &= 13 \\ 5 \cdot \frac{3 - 3y}{2} + 2y &= 13 \\ \frac{15 - 15y}{2} + 2y &= 13 \\ 2\left(\frac{15 - 15y}{2} + 2y\right) &= 2 \cdot 13 \\ 15 - 15y + 4y &= 26 \\ 15 - 11y &= 26 \\ -11y &= 11 \\ y &= -1 \end{aligned}$$

But then we have $x = \frac{3-3(-1)}{2} = \frac{3+3}{2} = \frac{6}{2} = 3$. Therefore, the lines intersect at the point $(x, y) = (3, -1)$.

Elimination. We multiply the second equation by -3. This gives us the following system of equations:

$$\begin{cases} 4x + 6y = 6 \\ -15x - 6y = -39 \end{cases}$$

Adding these equations, we obtain $-11x = -33$. But then $x = \frac{-33}{-11} = 3$. Using this in the first equation, we have...

$$\begin{aligned} 4x + 6y &= 6 \\ 4(3) + 6y &= 6 \\ 12 + 6y &= 6 \\ 6y &= -6 \\ y &= -1 \end{aligned}$$

Therefore, the lines intersect at $(x, y) = (3, -1)$.