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MATH 108
Fall 2022
HW 13: Due 11/07

*“People believe the only alternative to
randomness is intelligence design.”
—Richard Dawkins*

Problem 1. (10pt) Suppose that at a small college there is a 20% chance that a student is a business major. You performing a survey of student satisfaction of the college’s new vision and you take a sample of 13 students.

- (a) What is the probability that exactly 4 students in the survey are business majors?
- (b) What is the probability that three or less of the students are business majors?
- (c) What is the probability that less than three of the students are business majors?
- (d) What is the probability that at least one of the students is a business major?

Solution. This is a binomial distribution with $n = 13$ and $p = 0.20$, i.e. $B(13, 0.20)$.

- (a) We have...

$$P(X = 4) = 0.1535$$

- (b) We have...

$$P(X \leq 3) = P(X = 3) + P(X = 2) + P(X = 1) + P(X = 0) = 0.2457 + 0.2680 + 0.1787 + 0.0550 = 0.7474$$

- (c) We have...

$$P(X < 3) = P(X = 2) + P(X = 1) + P(X = 0) = 0.2680 + 0.1787 + 0.0550 = 0.5017$$

- (d) We have...

$$P(X \geq 1) = 1 - P(X = 0) = 1 - 0.0550 = 0.9450$$

Problem 2. (10pt) You and your friends are all ‘serial late arrivals’, i.e. you always tend to be late for things. There is an 80% chance that you and your friends are late for events. Suppose you and 6 of your friends are invited to a party.

- (a) What is the probability that exactly four of you are late?
- (b) What is the probability that all of you are late?
- (c) What is the probability that more than three of you are on time?
- (d) What is the probability that none of you are late?

Solution. Because $p = 0.80$ is not ‘available’ on the binomial chart, we rephrase the questions in terms of the number of individuals on-time and its probability: $1 - 0.80 = 0.20$. Therefore, noting that you and your friends makes a total of 7 people, we have binomial distribution $N(7, 0.20)$.

- (a) If exactly four of you are late, then exactly 3 are on-time. Therefore, we have...

$$P(4 \text{ late}) = P(X = 2) = 0.1147$$

- (b) If everyone is late, then no one is on-time. Therefore, we have...

$$P(\text{all late}) = P(X = 0) = 0.2097$$

- (c) As this question is phrased in terms of being on-time, we have...

$$P(X > 3) = 0.0287 + 0.0043 + 0.0004 + 0.0000 = 0.0334$$

- (d) If none of you are late, then all of you are on-time. Therefore, we have...

$$P(X = 7) \approx 0.0000$$

[Note: The exact value is $P(X = 7) = (0.20)^7 = 0.0000128$.]