

Name: Caleb McWhorter — Solutions

MATH 101

Spring 2022

HW 10: Due 04/12

*“Algebra is the metaphysics of arithmetic.”*

*—John Ray*

**Problem 1.** (10pt) Use completing the square to solve the following equation:

$$x(2x + 7) = 15$$

**Solution.**

$$x(2x + 7) = 15$$

$$2x^2 + 7x = 15$$

$$2x^2 + 7x - 15 = 0$$

$$2\left(x^2 + \frac{7}{2}x - \frac{15}{2}\right) = 0$$

$$2\left(x^2 + \frac{7}{2}x + \frac{49}{16} - \frac{49}{16} - \frac{15}{2}\right) = 0$$

$$2\left(\left(x + \frac{7}{4}\right)^2 - \frac{49}{16} - \frac{120}{16}\right) = 0$$

$$2\left(\left(x + \frac{7}{4}\right)^2 - \frac{169}{16}\right) = 0$$

$$2\left(x + \frac{7}{4}\right)^2 - \frac{169}{8} = 0$$

$$2\left(x + \frac{7}{4}\right)^2 = \frac{169}{8}$$

$$\left(x + \frac{7}{4}\right)^2 = \frac{169}{16}$$

$$x + \frac{7}{4} = \pm\sqrt{\frac{169}{16}}$$

$$x + \frac{7}{4} = \pm\frac{13}{4}$$

$$x = -\frac{7}{4} \pm \frac{13}{4}$$

Therefore,  $x = -\frac{7}{4} - \frac{13}{4} = -\frac{20}{4} = -5$  or  $x = -\frac{7}{4} + \frac{13}{4} = \frac{6}{4} = \frac{3}{2}$ .

**Problem 2.** (10pt) Use factoring to solve the following equation:

$$2x = 80 - x^2$$

**Solution.**

$$2x = 80 - x^2$$

$$x^2 + 2x - 80 = 0$$

**80**

$$1 \cdot -80 \quad -79$$

$$-1 \cdot 80 \quad 79$$

$$2 \cdot -40 \quad -38$$

$$-2 \cdot 40 \quad 38$$

$$4 \cdot -20 \quad -16$$

$$-4 \cdot 20 \quad 16$$

$$5 \cdot -16 \quad -9$$

$$-5 \cdot 16 \quad 9$$

$$8 \cdot -10 \quad -2$$

$-8 \cdot 10$	$2$
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$$2x = 80 - x^2$$

$$x^2 + 2x - 80 = 0$$

$$(x - 8)(x + 10) = 0$$

But then either  $x - 8 = 0$ , i.e.  $x = 8$ , or  $x + 10 = 0$ , i.e.  $x = -10$ .

**Problem 3.** (10pt) Use the quadratic equation to solve the following equation:

$$2x^2 - 1 = 2x$$

**Solution.**

$$2x^2 - 1 = 2x$$

$$2x^2 - 2x - 1 = 0$$

Then we have...

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{2 \pm \sqrt{4 + 8}}{4}$$

$$x = \frac{2 \pm \sqrt{12}}{4}$$

$$x = \frac{2 \pm \sqrt{4 \cdot 3}}{4}$$

$$x = \frac{2 \pm 2\sqrt{3}}{4}$$

$$x = \frac{1 \pm \sqrt{3}}{2}$$

Therefore, the solutions are  $x = \frac{1 - \sqrt{3}}{2}$  and  $x = \frac{1 + \sqrt{3}}{2}$ .

**Problem 4.** (10pt) Use the quadratic formula to factor  $280x^2 + 6x - 315$ .

**Solution.** Solving the equation  $280x^2 + 6x - 315 = 0$ , we have...

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(280)(-315)}}{2(280)}$$

$$x = \frac{-6 \pm \sqrt{36 + 352800}}{560}$$

$$x = \frac{-6 \pm \sqrt{352836}}{560}$$

$$x = \frac{-6 \pm 594}{560}$$

Therefore, the solutions are  $x = \frac{-6 - 594}{560} = \frac{-600}{560} = -\frac{15}{14}$  or  $x = \frac{-6 + 594}{560} = \frac{588}{560} = \frac{21}{20}$ .  
Therefore, we have...

$$280x^2 + 6x - 315 = a(x - r_1)(x - r_2) = 280 \left( x - \frac{-15}{14} \right) \left( x - \frac{21}{20} \right) = 280 \left( x + \frac{15}{14} \right) \left( x - \frac{21}{20} \right)$$