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MATH 101
Summer 2022
HW 8: Due 06/08

"I do not know what I may appear to the world, but to myself I seem to have been only like a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."

–Issac Newton

Problem 1. (10pt) Use the discriminant to explain why the quadratic function $f(x) = x^2 - 4x + 13$ does not factor 'nicely.' Does the function factor 'nicely' over the complex numbers? Explain.

Solution. The discriminant of a polynomial $ax^2 + bx + c$ is $D = b^2 - 4ac$. The polynomial factors 'nicely' only if $|D|$ is a square, and factors over the real numbers if and only if $D \geq 0$. For $f(x) = x^2 - 4x + 13$, we have...

$$D = b^2 - 4ac = (-4)^2 - 4(1)(13) = 16 - 52 = -36$$

Because $|D| = 36 = 6^2$ is a square, $f(x)$ factors 'nicely.' However, because $D < 0$, we know that $f(x)$ factors 'nicely' only over the complex numbers. In fact, we have...

$$f(x) = x^2 - 4x + 13 = \big(x - (2 - 3i)\big)\big(x - (2 + 3i)\big)$$

Problem 2. (10pt) Find the factorization of $x^2+9x-36$ the 'traditional' way. Then use the quadratic formula to factor $x^2 + 9x - 36$. Confirm that your factorization is correct.

Solution.

36	
$1 \cdot -36$	-35
$-1 \cdot 36\ 35$	
$2 \cdot -18$	-16
$-2 \cdot 18$	16
$3 \cdot -12$	-9
$-3 \cdot 12$	9
$4 \cdot -9$	-5
$-4 \cdot 9$	5
$6 \cdot -6$	0

Therefore,

$$x^2 + 9x - 36 = (x-3)(x+12)$$

Alternatively, using the quadratic formula, we solve $x^2 + 9x - 36 = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-9 \pm \sqrt{9^2 - 4(1)(-36)}}{2(1)}$$

$$x = \frac{-9 \pm \sqrt{81 + 144}}{2}$$

$$x = \frac{-9 \pm \sqrt{225}}{2}$$

$$x = \frac{-9 \pm 15}{2}$$

Therefore, $x = \frac{-9+15}{2} = \frac{6}{2} = 3$ or $x = \frac{-9-15}{2} = \frac{-24}{2} = -12$. We then have...

$$x^2 + 9x - 36 = a(x - r_1)(x - r_2) = 1\big(x-3\big)\big(x - (-12)\big) = (x-3)(x+12)$$

We can check these factorizations by expanding the factorization:

$$(x-3)(x+12) = x^2 + 12x - 3x - 36 = x^2 + 9x - 36$$

Problem 3. (10pt) Use the quadratic formula to factor $2x^2 - 4x - 12$.

Solution. We know $2x^2 - 4x - 12$ is of the form $ax^2 + bx + c$ with $a = 2$, $b = -4$, and $c = -12$. Now we find the roots of $2x^2 - 4x - 12$, i.e. solve the equation $2x^2 - 4x - 12 = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-12)}}{2(2)}$$

$$x = \frac{4 \pm \sqrt{16 + 96}}{4}$$

$$x = \frac{4 \pm \sqrt{112}}{4}$$

$$x = \frac{4 \pm \sqrt{16 \cdot 7}}{4}$$

$$x = \frac{4 \pm 4\sqrt{7}}{4}$$

$$x = 1 \pm \sqrt{7}$$

Therefore, we have. . .

$$2x^2 - 4x - 12 = a(x - r_1)(x - r_2) = 2\big(x - (1 - \sqrt{7})\big)\big(x - (1 + \sqrt{7})\big)$$

Problem 4. (10pt) Use the quadratic formula to factor $x^2 - 10x + 34$.

Solution. We know $x^2 - 10x + 34$ is of the form $ax^2 + bx + c$ with $a = 1$, $b = -10$, and $c = 34$. Now we find the roots of $x^2 - 10x + 34$, i.e. solve the equation $x^2 - 10x + 34 = 0$:

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
x &= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(34)}}{2(1)} \\
x &= \frac{10 \pm \sqrt{100 - 136}}{2} \\
x &= \frac{10 \pm \sqrt{-36}}{2} \\
x &= \frac{10 \pm \sqrt{36}\, i}{2} \\
x &= \frac{10 \pm 6i}{2} \\
x &= 5 \pm 3i
\end{aligned}$$

Therefore, we have...

$$x^2 - 10x + 34 = a(x - r_1)(x - r_2) = \big(x - (5 - 3i)\big)\big(x - (5 + 3i)\big)$$

Problem 5. (10pt) Use the quadratic formula to factor $60x^2 - 2615x + 24200$.

Solution. We know $60x^2 - 2615x + 24200$ is of the form $ax^2 + bx + c$ with $a = 60$, $b = -2615$, and $c = 24200$. Now we find the roots of $60x^2 - 2615x + 24200$, i.e. solve the equation $60x^2 - 2615x + 24200 = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2615) \pm \sqrt{(-2615)^2 - 4(60)(24200)}}{2(60)}$$

$$x = \frac{2615 \pm \sqrt{6838225 - 5808000}}{120}$$

$$x = \frac{2615 \pm \sqrt{1030225}}{120}$$

$$x = \frac{2615 \pm 1015}{120}$$

$$x = \frac{523 \pm 203}{24}$$

Then $x = \dfrac{523 - 203}{24} = \dfrac{320}{24} = \dfrac{40}{3}$ or $x = \dfrac{523 + 203}{24} = \dfrac{726}{24} = \dfrac{121}{4}$. Therefore, we have...

$$\begin{aligned} 60x^2 - 2615x + 24200 &= a(x - r_1)(x - r_2) \\ &= 60\left(x - \frac{40}{3}\right)\left(x - \frac{121}{4}\right) \\ &= 5 \cdot 3\left(x - \frac{40}{3}\right) \cdot 4\left(x - \frac{121}{4}\right) \\ &= 5(3x - 40)(4x - 121) \end{aligned}$$

Problem 6. (10pt) Showing all your work, solve the following equation:

$$9x - x^2 = -10$$

Solution.

$$9x - x^2 = -10$$

$$x^2 - 9x - 10 = 0$$

$$(x+1)(x-10) = 0$$

$$x + 1 = 0 \text{ or } x - 10 = 0$$

$$x = -1 \text{ or } x = 10$$

Problem 7. (10pt) Showing all your work, solve the following equation:

$$2(x^2 - 3) = -11x$$

Solution.

$$2(x^2 - 3) = -11x$$

$$2x^2 - 6 = -11x$$

$$2x^2 + 11x - 6 = 0$$

$$(2x - 1)(x + 6) = 0$$

$$2x - 1 = 0 \text{ or } x + 6 = 0$$

$$x = \tfrac{1}{2} \text{ or } x = -6$$

Problem 8. (10pt) Showing all your work, solve the following equation:

$$x^2 = 6x - 7$$

Solution. We have...

$$\begin{aligned} x^2 &= 6x - 7 \\ x^2 - 6x + 7 &= 0 \end{aligned}$$

We know $x^2 - 6x + 7$ has the form $ax^2 + bx + c$ with $a = 1$, $b = -6$, and $c = 7$. Then using the quadratic formula, we then have...

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)} \\ x &= \frac{6 \pm \sqrt{36 - 28}}{2} \\ x &= \frac{6 \pm \sqrt{8}}{2} \\ x &= \frac{6 \pm \sqrt{4 \cdot 2}}{2} \\ x &= \frac{6 \pm 2\sqrt{2}}{2} \\ x &= 3 \pm \sqrt{2} \end{aligned}$$

Problem 9. (10pt) Showing all your work, solve the following equation:

$$x(2-x) = 2$$

Solution. We have...

$$\begin{aligned} x(2-x) &= 2 \\ 2x - x^2 &= 2 \\ x^2 - 2x + 2 &= 0 \end{aligned}$$

We know $x^2 - 2x + 2$ has the form $ax^2 + bx + c$ with $a = 1$, $b = -2$, and $c = 2$. Then using the quadratic formula, we then have...

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} \\ x &= \frac{2 \pm \sqrt{4-8}}{2} \\ x &= \frac{2 \pm \sqrt{-4}}{2} \\ x &= \frac{2 \pm \sqrt{4}\, i}{2} \\ x &= \frac{2 \pm 2i}{2} \\ x &= 1 \pm i \end{aligned}$$

Problem 10. (10pt) Showing all your work, solve the following equation:

$$\frac{x+1}{x-3} = x+1$$

Solution.

$$\frac{x+1}{x-3} = x+1$$

$$x+1 = (x-3)(x+1)$$

$$x+1 = x^2 + x - 3x - 3$$

$$x+1 = x^2 - 2x - 3$$

$$x^2 - 3x - 4 = 0$$

$$(x+1)(x-4) = 0$$

$$x+1 = 0 \text{ or } x-4 = 0$$

$$x = -1 \text{ or } x = 4$$