

Name: \_\_\_\_\_

MATH 308

Fall 2021

HW 1: Due 09/24

*"I'm not superstitious, but I am a little  
stitious."*

*–Michael Scott, The Office*

**Problem 1.** (10pt) Determine if the following sentences are propositions. If the sentence is a proposition, mark it 'T'; otherwise, mark the sentence 'F.'

(a) \_\_\_\_\_:  $|3 - 5| > 10$

(b) \_\_\_\_\_: I just started watching 'The Chair.'

(c) \_\_\_\_\_: The universe is infinite.

(d) \_\_\_\_\_:  $n + 1$  is odd.

(e) \_\_\_\_\_: Why are you doing this homework?

**Problem 2.** (10pt) Give an original example of a proposition.

**Problem 3.** (10pt) Give an original non-example of a proposition.

**Problem 4.** (5pt) Determine if the following propositions are true (T) or false (F).

- (a) \_\_\_\_\_: If  $n$  is an integer, then  $2n$  is even.
- (b) \_\_\_\_\_: Every prime number is odd.
- (c) \_\_\_\_\_:  $x^2 + 1 > 0$
- (d) \_\_\_\_\_: It will either rain tomorrow or not.
- (e) \_\_\_\_\_: If  $x^2 = 9$ , then  $x = 3$ ,

**Problem 5.** (10pt) Negate the following sentences:

- (a)  $2 \cdot 2 = 4$  or  $3 \cdot 3 = 6$
- (b) Everyone in the room has taken a Mathematic course.
- (c) She speaks German and English.
- (d)  $x > 1$  and  $x$  is an integer.
- (e) If you study for the exam, then you will pass.

**Problem 6.** (10pt) Negate each of the following propositional formulas  $P$  by finding a formula logically equivalent to  $P$  in which the negation applies only to individual atoms.

(a)  $P \vee (\neg Q)$

(b)  $\neg Q \rightarrow \neg P$

(c)  $(P \vee Q) \wedge (\neg P \vee \neg Q)$

(d)  $P \wedge Q \rightarrow P \vee Q$

(e)  $P \vee (Q \Leftrightarrow R)$

**Problem 7.** (10pt) Express the proposition “ $P$  unless  $Q$ ” in terms of the propositions  $P$  and  $Q$  and the logical symbols  $\neg, \wedge, \vee, \rightarrow$ . [Unless can mean many things, here it means “if not.”]

**Problem 8.** (10pt) Recall that the ‘exclusive or’, denoted  $\underline{\vee}$ , was defined by  $P \underline{\vee} Q \Leftrightarrow (P \vee Q) \wedge \neg(P \wedge Q)$ . Show that  $P \underline{\vee} Q$  is logically equivalent to  $P \leftrightarrow \neg Q$ .



**Problem 9.** (10pt) Compute the truth tables for the following compound propositions. In each case, indicate whether the propositional formula is a tautology, contradiction, or neither.

(a)  $(P \wedge Q) \wedge (R \wedge \neg Q)$

(b)  $(P \leftrightarrow Q) \leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$

(c)  $(P \rightarrow T_0) \wedge (F_0 \rightarrow Q)$

**Problem 10.** (10pt) Determine if the logical symbol  $\rightarrow$  is associative. Be sure to fully justify your answer.

**Problem 11.** (10pt) Give the converse/contrapositive of the following statements.

(a)  $P \rightarrow Q$

(b) If it is snowing outside, then it is cold.

**Problem 12.** (10pt) Determine if the following argument is logical. Explain.

$$\begin{array}{l} P \rightarrow R \\ \neg P \rightarrow Q \\ Q \rightarrow S \\ \hline \therefore \neg R \rightarrow S \end{array}$$