Name: _Caleb McWhorter — Solutions_
MATH 101
Fall 2021
HW 18: Due 11/23

Problem 1. (10pt) Solve the equation $2(4^{2x}) = 8$ using logarithms.

Solution.

$$2(4^{2x}) = 8$$

$$4^{2x} = 4$$

$$\log_4 4^{2x} = \log_4 4$$

$$2x = 1$$

$$x = \frac{1}{2}$$

Problem 2. (10pt) Solve the equation $6(9^{x-1}) = 5$ using logarithms.

Solution.

$$6(9^{x-1}) = 5$$

$$9^{x-1} = \frac{5}{6}$$

$$\log_9 9^{x-1} = \log_9\left(\frac{5}{6}\right)$$

$$x - 1 = \log_9\left(\frac{5}{6}\right)$$

$$x = \log_9\left(\frac{5}{6}\right) + 1 \approx 0.917022$$

Problem 3. (10pt) Solve the equation $\log_2(x+5) = 4$.

Solution.

$$\log_2(x+5) = 4$$

$$2^{\log_2(x+5)} = 2^4$$

$$x + 5 = 16$$

$$x = 11$$

Problem 4. (10pt) Solve the equation $\log_{10}(x+2)+\log_{10}(x+5) = 1$. Be sure to check that all your solutions are valid.

Solution.

$$\log_{10}(x + 2) + \log_{10}(x + 5) = 1$$

$$\log_{10}\big((x + 2)(x + 5)\big) = 1$$

$$10^{\log_{10}\big((x+2)(x+5)\big)} = 10^1$$

$$(x + 2)(x + 5) = 10$$

$$x^2 + 5x + 2x + 10 = 10$$

$$x^2 + 7x + 10 = 10$$

$$x^2 + 7x = 0$$

$$x(x + 7) = 0$$

But then either $x = 0$ or $x + 7 = 0$, i.e. $x = -7$. However, observe that. . .

$$\log_{10}(x + 2) + \log_{10}(x + 5)\Big|_{x=0} = \log_{10}(2) + \log_{10}(5) = \log_{10}(2 \cdot 5) = \log_{10}(10) = 1$$

$$\log_{10}(x + 2) + \log_{10}(x + 5)\Big|_{x=-7} = \log_{10}(-5) + \log_{10}(-2) \Longleftarrow \text{undefined!}$$

Therefore, the only solution is $x = 0$.