Name: <u>Caleb McWhorter — Solutions</u>

MATH 100

Fall 2021 "Laziness is nothing more than the habit of resting before you get

HW 9: Due 10/27

-Jules Renard

Problem 1. (10pt) Find the vertex form of the quadratic function $y = x^2 + 4x + 6$.

Solution. The x-coefficient is 4. We have $(\frac{1}{2} \cdot 4)^2 = 2^2 = 4$. Then we have...

$$y = x^2 + 4x + 6$$

$$y = x^2 + 4x + (4 - 4) + 6$$

$$y = (x^2 + 4x + 4) - 4 + 6$$

$$y = (x+2)^2 + 2$$

Problem 2. (10pt) Find the vertex form of the quadratic function $y = x^2 + 4x - 5$.

Solution. The x-coefficient is 4. We have $(\frac{1}{2} \cdot 4)^2 = 2^2 = 4$. Then we have...

$$y = x^2 + 4x - 5$$

$$y = x^2 + 4x + (4 - 4) - 5$$

$$y = (x^2 + 4x + 4) - 4 - 5$$

$$y = (x+2)^2 - 9$$

Problem 3. (10pt) Find the vertex form of the quadratic function $y = 2x^2 - 4x + 8$.

Solution. We factor out the 2. This gives us $y=2(x^2-2x+4)$. The x-coefficient is -2. We have $(\frac{1}{2}\cdot -2)^2=(-1)^2=1$. Then we have...

$$y = 2(x^{2} - 2x + 4)$$

$$y = 2(x^{2} - 2x + (1 - 1) + 4)$$

$$y = 2((x^{2} - 2x + 1) - 1 + 4)$$

$$y = 2((x - 1)^{2} + 3)$$

$$y = 2(x - 1)^{2} + 6$$

Problem 4. (10pt) Consider the quadratic function $f(x) = x^2 - 8x + 12$.

- (a) Determine if the parabola opens upwards or downwards.
- (b) Is the parabola convex or concave?
- (c) Does the parabola have a maximum or minimum?
- (d) Find the vertex and axis of symmetry.
- (e) Find the maximum/minimum value of f(x).

Solution.

- (a) Because a = 1 > 0, the parabola opens upwards, i.e. the parabola is convex.
- (b) Because the parabola opens upwards, it is convex.
- (c) Because the parabola opens upwards, the vertex is a minimum.
- (d) The vertex occurs when $x=-\frac{b}{2a}=-\frac{-8}{2(1)}=\frac{8}{2}=4$. But then the axis of symmetry is x=4. We have

$$y(4) = 4^2 - 8(4) + 12 = 16 - 32 + 12 = -4$$

Therefore, the vertex is (4, -4). Alternatively, putting the parabola in vertex form:

$$y = x^{2} - 8x + 12$$

$$y = x^{2} - 8x + 16 - 16 + 12$$

$$y = (x - 4)^{2} - 4$$

we can easily see that the vertex is (4, -4) and that the axis of symmetry is x = 4.

(e) Because the parabola opens upwards, the parabola has a minimum. The minimum occurs at the vertex. The vertex is (4, -4). Therefore, the maximum value is -4.

Problem 5. (10pt) Consider the quadratic function $f(x) = -2x^2 - 4x + 4$.

- (a) Determine if the parabola opens upwards or downwards.
- (b) Is the parabola convex or concave?
- (c) Does the parabola have a maximum or minimum?
- (d) Find the vertex and axis of symmetry.
- (e) Find the maximum/minimum value of f(x).

Solution.

- (a) Because a = -2 < 0, the parabola opens downwards, i.e. the parabola is concave.
- (b) Because the parabola opens downwards, it is concave.
- (c) Because the parabola opens downwards, the vertex is a maximum.
- (d) The vertex occurs when $x=-\frac{b}{2a}=-\frac{-4}{2(-2)}=-\frac{4}{4}=-1$. But then the axis of symmetry is x=-1. We have

$$y(-1) = -2(-1)^2 - 4(-1) + 4 = -2 + 4 + 4 = 6$$

Therefore, the vertex is (-1,6). Alternatively, putting the parabola in vertex form:

$$y = -2x^{2} - 4x + 4$$

$$y = -2(x^{2} + 2x - 2)$$

$$y = -2(x^{2} + 2x + 1 - 1 - 2)$$

$$y = -2((x+1)^{2} - 3)$$

$$y = -2(x+1)^{2} + 6$$

we can easily see that the vertex is (-1,6) and that the axis of symmetry is x=-1.

(e) Because the parabola opens downwards, the parabola has a maximum. The maximum occurs at the vertex. The vertex is (-1,6). Therefore, the maximum value is 6.