MAT 101: Exam 1 **Name:** _____
Fall – 2022
10/07/2022
85 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 21 pages (including this cover page) and 20 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
14	10	
15	10	
16	10	
17	10	
18	10	
19	10	
20	10	
Total:	200	

1. (10 points) Mark each of the following statements as True (T) or False (F).

   (a) _____: The number 1 is prime.

   (b) _____: Every rational number is a real number.

   (c) _____: The number $0.5 \cdot 10^{-3}$ is in scientific notation.

   (d) _____: Every real number is a rational number.

   (e) _____: The number 6 has four positive divisors.

   (f) _____: Every integer greater than 1 is either prime or can be written as a product of primes.

   (g) _____: $\left(\sqrt[20]{\pi^5}\right)^4 = \pi$

   (h) _____: A positive number decreased by 20% results in the same number as finding 80% of the number.

   (i) _____: All real numbers have a square root that is a real number.

   (j) _____: A real number is rational if and only if it has a decimal expansion that repeats.

2. (10 points) Showing all your work, find the prime factorizations of the following integers—if the integer is prime, simply state this:

   (a) 49

   (b) 54

   (c) 75

   (d) 131

   (e) 621

3. Without performing any explicit computation, e.g. saying $6/2 = 3$ is an integer or $7/2 = 3.5$ is not an integer, justify the following statements:
   (a) (3 points) The integer 5,549,787,074,121 is divisible by 3.
   (b) (3 points) The integer 13,513,205,490,217,689,368 is not divisible by 5.
   (c) (4 points) The integer 202,202,022 is divisible by 6.

4. Showing any necessary work, answer the following:
   (a) (3 points) Find at least three multiples of 12.
   (b) (3 points) Find at least two divisors of 140.
   (c) (4 points) Find the least common multiple of the smallest and largest prime factor of 30.

5. Showing any necessary work, find the following:
   (a) (2 points) $\mathrm{lcm}(20, 66)$
   (b) (2 points) $\gcd(20, 66)$
   (c) (3 points) $\gcd(2^{500} \cdot 3^{600} \cdot 7^{800} \cdot 11^{900},\ 2^{400} \cdot 3^{800} \cdot 5^{600} \cdot 13^{700})$
   (d) (3 points) $\mathrm{lcm}(2^{500} \cdot 3^{600} \cdot 7^{800} \cdot 11^{900},\ 2^{400} \cdot 3^{800} \cdot 5^{600} \cdot 13^{700})$

6. Showing all your work and reducing as much as possible, compute the following (be sure to give an exact value as your answer):

   (a) (3 points) $\frac{3}{10} - \frac{11}{15}$

   (b) (3 points) $\frac{6}{55} \cdot -\frac{22}{21}$

   (c) (4 points) $\dfrac{\frac{20}{21}}{\frac{10}{49}}$

7. Showing all your work, either convert the given rational number to a decimal value or the given decimal number to a rational number—in either case, simplify as much as possible:
   (a) (5 points) 0.18
   (b) (5 points) $\frac{19}{11}$

8. (10 points) Showing all your work, express the decimal number $0.1717171\overline{717}$ as a fully reduced rational number.

9. Showing all your work, simplify the expressions given below. Your answer should have each variable occurring at most once and should contain no negative powers.

   (a) (3 points) $\dfrac{(xy^5)^{-2}}{x^{-8}(xy)^6y^5}$

   (b) (3 points) $\left(\dfrac{x(x^{10}y^3)^0}{y^5}\right)^{-2}$

   (c) (4 points) $\dfrac{x\left(x^3(y^{-2}z)^{-2}\right)^{-1}}{y^{-5}z^2x^{-1}}$

10. Showing all your work, simplify the expressions given below, being sure to give an exact value:
    (a) (3 points) $\sqrt{24}$
    (b) (3 points) $\sqrt[3]{54}$
    (c) (4 points) $\sqrt[5]{2^3 \cdot 3^5 \cdot 5^{10} \cdot 7^6}$

11. Showing all your work, simplify the expressions given below. Your answer should have each variable occurring at most once, contain no negative powers, and contain no fractional powers:

    (a) (3 points) $\dfrac{\sqrt{x^4y^{11}}}{x\sqrt{y}}$

    (b) (3 points) $\left(\dfrac{x^5y^8}{x^{-2}y^5}\right)^{-1/3}$

    (c) (4 points) $\dfrac{\sqrt[5]{x^{20}y^6}}{x^{10}\sqrt{y^3}}$

12. Showing all your work and being sure to give an exact answer that is simplified as much as possible, rationalize the following:

    (a) (4 points) $\dfrac{4}{\sqrt{6}}$

    (b) (4 points) $\dfrac{6}{5+\sqrt{2}}$

    (c) (2 points) $\dfrac{-5}{\sqrt[3]{7}}$

13. Showing all your work, compute the following:
    (a) (3 points) 74% of 1450
    (b) (3 points) 180% of 22
    (c) (4 points) 1% of 60

14. Showing all your work, compute the following:
    (a) (3 points) 550 increased by 40%
    (b) (3 points) 1200 decreased by 70%
    (c) (4 points) 70 increased by 150%

15. (10 points) Suppose a student's course grade consists of the following weights:

Homework	50%	Exam 2	12%
Quizzes	5%	Final Exam	15%
Exam 1	8%	Project	10%

Suppose also that a student had a 80% homework average, 84.3% quiz average, 91% on exam 1, 95% on exam 2, 78% on the final, and 89% on the project. Compute the student's course average to the nearest tenth of a percent.

16. (10 points) A certain student has a current GPA of 3.351 based on 86 credits. This semester, they take a number of courses with the courses, credits, and their letter grade in the course shown below.

Course	Credits	Grade
Galois Representations	4	A
Advanced Music Theory	3	A−
Creative Writing	3	C
Business Ethics	3	B+
Common Law	3	B−

A	4.0	C+	2.3
A–	3.7	C	2.0
B+	3.3	C–	1.7
B	3.0	D	1.0
B–	2.7	F	0.0

Using the university's letter grade values show on the right above, what is the students current GPA to the nearest thousandth? [Be sure to show all your work.]

17. Showing all your work, convert the following numbers in scientific notation to decimal values:
    (a) (3 points) $1.7 \cdot 10^{-6}$
    (b) (3 points) $8.912 \cdot 10^{8}$
    (c) (4 points) $-4.3 \cdot 10^{0}$

18. Showing all your work, convert the following decimal numbers to scientific notation:
    (a) (3 points) 0.0000000044
    (b) (3 points) 15650000
    (c) (4 points) -7.8

19. (10 points) Showing all your work, convert 9.8 km/hr^2 to ft per second. Note that 1 m is 3.28084 ft.

20. (10 points) A rectangular wall is 80 ft long and 15 ft wide. A painter uses paint cans that contain enough paint to cover 84 ft^2 of a surface. The painter is also able to paint 230 ft^2 of a surface every hour. What is the minimum number of paint cans they need to bring to finish the job and how long will it take them to then paint the wall? Be sure to fully justify your answers.