Name: *Caleb McWhorter — Solutions*
MATH 101
Fall 2022
HW 2: Due 09/19

"I do not have time for this; I do not have time for you!"
–Anna Delvey, Inventing Anna

Problem 1. (10pt) Showing all your work, find the exact value of the following—being sure to simplify as much as possible:

(a) $(-2)^3 \cdot 3^{-2}$

(b) $\left(-\frac{9}{5}\right)^{-2}$

(c) $\frac{(2^3)^2 \cdot 5^0}{4^3 \cdot 3^{-1}}$

Solution.

(a)

$$(-2)^3 \cdot 3^{-2} = -8 \cdot \frac{1}{3^2} = -8 \cdot \frac{1}{9} = -\frac{8}{9}$$

(b)

$$\left(-\frac{9}{5}\right)^{-2} = \left(-\frac{5}{9}\right)^{2} = \frac{25}{81}$$

(c)

$$\frac{(2^3)^2 \cdot 5^0}{4^3 \cdot 3^{-1}} = \frac{8^2 \cdot 1}{64 \cdot 1/3} = \frac{8^2 \cdot 3}{8^2} = 3$$

Problem 2. (10pt) Showing all your work, find the exact value of the following—being sure to simplify as much as possible:

(a) $\sqrt{300}$

(b) $\sqrt{2^5 \cdot 3^2 \cdot 5^3}$

(c) $\sqrt[3]{2^4 \cdot 3^5 \cdot 5^6}$

Solution.

(a)

$$\sqrt{300} = \sqrt{2^2 \cdot 3^1 \cdot 5^2} = 2 \cdot 5\sqrt{3} = 10\sqrt{3}$$

(b)

$$\sqrt{2^5 \cdot 3^2 \cdot 5^3} = \sqrt{2^4 \cdot 2 \cdot 3^2 \cdot 5^2 \cdot 5} = 2^2 \cdot 3 \cdot 5\sqrt{2 \cdot 5} = 60\sqrt{10}$$

(c)

$$\sqrt[3]{2^4 \cdot 3^5 \cdot 5^6} = \sqrt[3]{2^3 \cdot 2^1 \cdot 3^3 \cdot 3^2 \cdot 5^6} = 2 \cdot 3 \cdot 5^2 \sqrt[3]{2 \cdot 3^2} = 150\sqrt[3]{18}$$

Problem 3. (10pt) Showing all your work, simplify the following as much as possible—being sure that your answer involves no negative powers and all variables appear only once:

(a) $(x^0y^3)^2 \cdot (x^4y^{-5})^{-2}$

(b) $\dfrac{x^4y^0z^6}{(xz^2)^{-3}}$

(c) $\left(\dfrac{x^6y^{-3}z}{(xz^2)^3y^{-5}}\right)^{-2}$

Solution.

(a)

$$(x^0y^3)^2 \cdot (x^4y^{-5})^{-2} = (x^0y^6) \cdot (x^{-8}y^{10}) = x^{-8}y^{16} = \frac{y^{16}}{x^8}$$

(b)

$$\frac{x^4y^0z^6}{(xz^2)^{-3}} = \frac{x^4 \cdot 1 \cdot z^6}{x^{-3} \cdot z^{-6}} = (x^4z^6) \cdot (x^3z^6) = x^7z^{12}$$

(c)

$$\left(\frac{x^6y^{-3}z}{(xz^2)^3y^{-5}}\right)^{-2} = \left(\frac{(xz^2)^3y^{-5}}{x^6y^{-3}z}\right)^{2} = \frac{(xz^2)^6y^{-10}}{x^{12}y^{-6}z^2} = \frac{x^6z^{12}y^{-10}}{x^{12}y^{-6}z^2} = \frac{x^6z^{12}y^6}{x^{12}y^{10}z^2} = \frac{z^{10}}{x^6y^4}$$

Problem 4. (10pt) Showing all your work, simplify the following as much as possible—being sure that your answer involves no negative powers and all variables appear only once:

(a) $xy\sqrt{x^4y^5}$

(b) $\dfrac{\sqrt{x^{10}y^5}}{\sqrt{x^2y^3}}$

(c) $\sqrt[3]{x^{12}y^3z^{11}}$

Solution.

(a)
$$xy\sqrt{x^4y^5} = xy(x^{4/2}y^{5/2}) = xy(x^2y^{5/2}) = x^{1+2}y^{1+5/2} = x^3y^{7/2} = x^3\sqrt{y^7}$$

(b)
$$\frac{\sqrt{x^{10}y^5}}{\sqrt{x^2y^3}} = \frac{x^{10/2}y^{5/2}}{x^{2/2}y^{3/2}} = \frac{x^5y^{5/2}}{x^1y^{3/2}} = x^{5-1}y^{5/2-3/2} = x^4y^{2/2} = x^4y$$

(c)
$$\sqrt[3]{x^{12}y^3z^{11}} = x^{12/3}y^{3/3}z^{11/3} = x^4y^1z^{11/3} = x^4y\sqrt[3]{z^{11}}$$

Problem 5. (10pt) Showing all your work, simplify the following as much as possible—being sure that your answer involves no negative powers and all variables appear only once:

(a) $(y\sqrt{x})^4 \cdot (x^{-3}y^2)^{-1}$

(b) $(x\sqrt{y}) \cdot (y\sqrt[3]{x})$

(c) $\left(\dfrac{x\sqrt{y^5}}{y^2\sqrt{x^6}}\right)^{-1/2}$

Solution.

(a)

$$(y\sqrt{x})^4 \cdot (x^{-3}y^2)^{-1} = (yx^{1/2})^4 \cdot (x^{-3}y^2)^{-1} = (y^4x^{4/2}) \cdot (x^3y^{-2}) = x^{2+3}y^{4-2} = x^5y^2$$

(b)

$$(x\sqrt{y}) \cdot (y\sqrt[3]{x}) = (xy^{1/2}) \cdot (yx^{1/3}) = x^{1+1/3}y^{1/2+1} = x^{4/3}y^{3/2}$$

(c)

$$\left(\frac{x\sqrt{y^5}}{y^2\sqrt{x^6}}\right)^{-1/2} = \left(\frac{y^2\sqrt{x^6}}{x\sqrt{y^5}}\right)^{1/2} = \left(\frac{y^2x^{6/2}}{xy^{5/2}}\right)^{1/2} = \frac{y^{2/2}x^{6/4}}{x^{1/2}y^{5/4}} = \frac{yx^{3/2}}{x^{1/2}y^{5/4}} = x^{3/2-1/2}y^{1-5/4} = xy^{-1/4} = \frac{x}{\sqrt[4]{y}}$$

Problem 6. (10pt) Showing all your work, simplify the following as much as possible—being sure that your answer involves no negative powers and all variables appear only once:

(a) $\sqrt[3]{x^3y^2} \cdot \sqrt{xy^3}$

(b) $\left(\dfrac{x^5}{\sqrt{x}}\right)^{2/3}$

(c) $xy\left(\sqrt{\dfrac{x^2}{y^3}}\right)^{-4}$

Solution.

(a)
$$\sqrt[3]{x^3y^2} \cdot \sqrt{xy^3} = (x^{3/3}y^{2/3}) \cdot (x^{1/2}y^{3/2}) = x^{1+1/2}y^{2/3+3/2} = x^{3/2}y^{13/6}$$

(b)
$$\left(\frac{x^5}{\sqrt{x}}\right)^{2/3} = \left(\frac{x^5}{x^{1/2}}\right)^{2/3} = (x^{5-1/2})^{2/3} = (x^{9/2})^{2/3} = x^3$$

(c)
$$xy\left(\sqrt{\frac{x^2}{y^3}}\right)^{-4} = xy\left(\left(\frac{x^2}{y^3}\right)^{1/2}\right)^{-4} = xy\left(\frac{x^2}{y^3}\right)^{-2} = xy\left(\frac{y^3}{x^2}\right)^{2} = xy \cdot \frac{y^6}{x^4} = \frac{y^7}{x^3}$$