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MATH 100
Fall 2021
HW 9: Due 10/27

"Laziness is nothing more than the habit of resting before you get tired."

–Jules Renard

Problem 1. (10pt) Find the vertex form of the quadratic function $y = x^2 + 4x + 6$.

Solution. The x-coefficient is 4. We have $(\frac{1}{2} \cdot 4)^2 = 2^2 = 4$. Then we have...

$$y = x^2 + 4x + 6$$

$$y = x^2 + 4x + (4 - 4) + 6$$

$$y = (x^2 + 4x + 4) - 4 + 6$$

$$y = (x + 2)^2 + 2$$

Problem 2. (10pt) Find the vertex form of the quadratic function $y = x^2 + 4x - 5$.

Solution. The x-coefficient is 4. We have $(\frac{1}{2} \cdot 4)^2 = 2^2 = 4$. Then we have...

$$y = x^2 + 4x - 5$$

$$y = x^2 + 4x + (4 - 4) - 5$$

$$y = (x^2 + 4x + 4) - 4 - 5$$

$$y = (x + 2)^2 - 9$$

Problem 3. (10pt) Find the vertex form of the quadratic function $y = 2x^2 - 4x + 8$.

Solution. We factor out the 2. This gives us $y = 2(x^2 - 2x + 4)$. The x-coefficient is -2. We have $(\frac{1}{2} \cdot -2)^2 = (-1)^2 = 1$. Then we have...

$$y = 2(x^2 - 2x + 4)$$

$$y = 2(x^2 - 2x + (1 - 1) + 4)$$

$$y = 2\big((x^2 - 2x + 1) - 1 + 4\big)$$

$$y = 2\big((x - 1)^2 + 3\big)$$

$$y = 2(x - 1)^2 + 6$$

Problem 4. (10pt) Consider the quadratic function $f(x) = x^2 - 8x + 12$.

(a) Determine if the parabola opens upwards or downwards.

(b) Is the parabola convex or concave?

(c) Does the parabola have a maximum or minimum?

(d) Find the vertex and axis of symmetry.

(e) Find the maximum/minimum value of $f(x)$.

Solution.

(a) Because $a = 1 > 0$, the parabola opens upwards, i.e. the parabola is convex.

(b) Because the parabola opens downwards, it is convex.

(c) Because the parabola opens upwards, the vertex is a minimum.

(d) The vertex occurs when $x = -\frac{b}{2a} = -\frac{-8}{2(1)} = \frac{8}{2} = 4$. But then the axis of symmetry is $x = 4$. We have

$$y(4) = 4^2 - 8(4) + 12 = 16 - 32 + 12 = -4$$

Therefore, the vertex is $(4, -4)$. Alternatively, putting the parabola in vertex form:

$$y = x^2 - 8x + 12$$

$$y = x^2 - 8x + 16 - 16 + 12$$

$$y = (x-4)^2 - 4$$

we can easily see that the vertex is $(4, -4)$ and that the axis of symmetry is $x = 4$.

(e) Because the parabola opens upwards, the parabola has a minimum. The minimum occurs at the vertex. The vertex is $(4, -4)$. Therefore, the maximum value is -4.

Problem 5. (10pt) Consider the quadratic function $f(x) = -2x^2 - 4x + 4$.

(a) Determine if the parabola opens upwards or downwards.

(b) Is the parabola convex or concave?

(c) Does the parabola have a maximum or minimum?

(d) Find the vertex and axis of symmetry.

(e) Find the maximum/minimum value of $f(x)$.

Solution.

(a) Because $a = -2 < 0$, the parabola opens downwards, i.e. the parabola is concave.

(b) Because the parabola opens downwards, it is concave.

(c) Because the parabola opens downwards, the vertex is a maximum.

(d) The vertex occurs when $x = -\frac{b}{2a} = -\frac{-4}{2(-2)} = -\frac{4}{4} = -1$. But then the axis of symmetry is $x = -1$. We have

$$y(-1) = -2(-1)^2 - 4(-1) + 4 = -2 + 4 + 4 = 6$$

Therefore, the vertex is $(-1, 6)$. Alternatively, putting the parabola in vertex form:

$$\begin{aligned}
y &= -2x^2 - 4x + 4 \\
y &= -2(x^2 + 2x - 2) \\
y &= -2(x^2 + 2x + 1 - 1 - 2) \\
y &= -2\big((x+1)^2 - 3\big) \\
y &= -2(x+1)^2 + 6
\end{aligned}$$

we can easily see that the vertex is $(-1, 6)$ and that the axis of symmetry is $x = -1$.

(e) Because the parabola opens downwards, the parabola has a maximum. The maximum occurs at the vertex. The vertex is $(-1, 6)$. Therefore, the maximum value is 6.