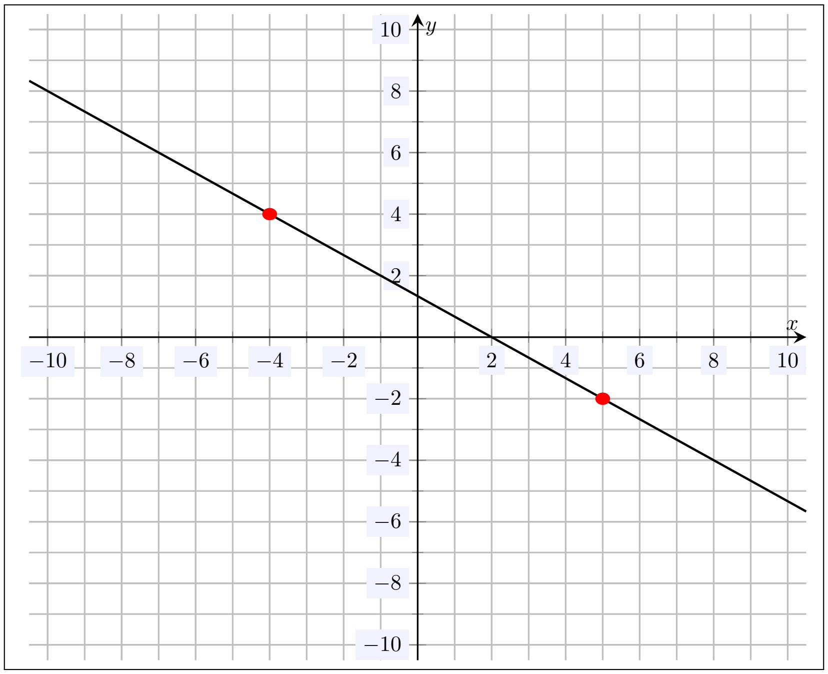
Name: *Caleb McWhorter — Solutions*
MATH 101
Fall 2023
HW 11: Due 11/06

"Learning is not attained by chance; it must be sought for with ardor and attended to with diligence."
–Abigail Adams

Problem 1. (10pt) Find the equation of the line plotted below.



Solution. Because this line is a linear function, we know that the line has the form $y = mx + b$. From the plot above, we can see that the points $(-4, 4)$ and $(5, -2)$ are on the line. We can then determine the slope of the line:

$$m = \frac{\Delta y}{\Delta x} = \frac{4 - (-2)}{-4 - 5} = \frac{4 + 2}{-4 + -5} = \frac{6}{-9} = -\frac{2}{3}$$

But then $y = -\frac{2}{3}\,x + b$. Because the line contains the point $(-4, 4)$, the point satisfies the equation for the line. But then...

$$\begin{aligned} y &= -\frac{2}{3}\,x + b \\ 4 &= -\frac{2}{3} \cdot -4 + b \\ 4 &= \frac{8}{3} + b \\ b &= \frac{4}{3} \end{aligned}$$

Alternatively, we can use the point-slope form of a linear function:

$$y = y_0 + m(x - x_0) = 4 + \frac{-2}{3}\big(x - (-4)\big) = 4 - \frac{2}{3}(x + 4) = 4 - \frac{2}{3}\,x - \frac{8}{3} = -\frac{2}{3}\,x + \frac{4}{3}$$

Therefore, the equation of the line is $y = -\frac{2}{3}\,x + \frac{4}{3}$.

Problem 2. (10pt) Find the equation of the line containing the point $(-5, 6)$ with slope $-\frac{1}{3}$.

Solution. Clearly, the line is not vertical. Therefore, the line must have the form $y = mx + b$. Because the line has slope $-\frac{1}{3}$, we have $m = -\frac{1}{3}$. Then we know $y = -\frac{1}{3}\,x + b$. Because the line contains the point $(-5, 6)$, it satisfies the equation of the line. But then...

$$\begin{aligned} y &= -\frac{1}{3}\,x + b \\ 6 &= -\frac{1}{3} \cdot -5 + b \\ 6 &= \frac{5}{3} + b \\ b &= \frac{13}{3} \end{aligned}$$

Alternatively, we can use the point-slope form of the line:

$$y = y_0 + m(x - x_0) = 6 + \frac{-1}{3}\big(x - (-5)\big) = 6 - \frac{1}{3}(x + 5) = 6 - \frac{1}{3}\,x - \frac{5}{3} = -\frac{1}{3}\,x + \frac{13}{3}$$

Therefore, the line is $y = -\frac{1}{3}\,x + \frac{13}{3}$.

Problem 3. (10pt) Find the equation of the line with x-intercept 5 and y-intercept -6.

Solution. Clearly, the line is not vertical. Therefore, the line has the form $y = mx + b$. Because the line contains the x and y-intercept, the line contain the point $(5, 0)$ and $(0, -6)$, respectively. But then we can compute the slope of the line:

$$m = \frac{\Delta y}{\Delta x} = \frac{0 - (-6)}{5 - 0} = \frac{0 + 6}{5 - 0} = \frac{6}{5}$$

But then $y = \frac{6}{5}\,x + b$. Because the line contains the point $(0, -6)$, the point satisfies the equation for the line. But then...

$$\begin{aligned} y &= \frac{6}{5}\,x + b \\ -6 &= \frac{6}{5} \cdot 0 + b \\ b &= -6 \end{aligned}$$

Of course, we could note that b is the y-intercept so that $b = -6$. Alternatively, we can use the point-slope form of a linear function:

$$y = y_0 + m(x - x_0) = -6 + \frac{6}{5}(x - 0) = \frac{6}{5}\,x - 6$$

Therefore, the equation of the line is $y = \frac{6}{5}\,x - 6$.

Problem 4. (10pt) Find the equation of the line parallel to the line $\ell(x) = \frac{4-x}{6}$ whose x-intercept is $(9, 0)$.

Solution. Because the line $\ell(x)$ is not vertical, it must be that the line in question is not vertical. Therefore, the line has the form $y = mx + b$. Because the line is parallel to the line $\ell(x)$, it must have the same slope as $\ell(x)$. The line $\ell(x) = \frac{4-x}{6} = \frac{4}{6} - \frac{1}{6}\,x$ has slope $-\frac{1}{6}$. Therefore, we must have $m = -\frac{1}{6}$. Then we know $y = -\frac{1}{6}\,x + b$. Because the line contains the point $(9, 0)$, it must satisfy the equation of the line. But then…

$$\begin{aligned} y &= -\frac{1}{6}\,x + b \\ 0 &= -\frac{1}{6} \cdot 9 + b \\ 0 &= -\frac{3}{2} + b \\ b &= \frac{3}{2} \end{aligned}$$

Alternatively, we can use the point-slope form of the line:

$$y = y_0 + m(x - x_0) = 0 + \frac{-1}{6}(x - 9) = -\frac{1}{6}\,x + \frac{3}{2}$$

Therefore, the line is $y = -\frac{1}{6}\,x + \frac{3}{2} = \frac{9-x}{6}$.