Quiz 1. *True/False*: If you had a bill of \$25.77 and were going to pay a tip of 20%, the total amount you would pay could be computed by finding $25.77(1.20)$.

Solution. The statement is *true*. Recall to calculate a percentage of a number N, we compute $N \cdot \%$, where N is the number and $\%$ is the percentage (written as a decimal). For instance, to compute 57% of 23, we compute $23(0.57) = 13.11$. To compute 172% of 150, we compute $150(1.72) = 258$. However, to compute a $\%$ percent increase or decrease of a number N, we compute $N(1 \pm \%)$, where N is the number, $\%$ is the percentage as a decimal, and we choose plus for increase and negative for decrease. For instance, to compute a 75% decrease of 13, we compute $13(1 - 0.75) = 13(0.25) = 3.25$. To compute a 115% increase of 120, we compute $120(1+1.15) = 120(2.15) = 258$. Here, we are increasing 25.77 by 20%, so we compute $25.77(1 + 0.20) = 25.77(1.20)$.

Quiz 2. *True/False*: The amount of concrete in tons, C, used to repair r roads remaining in a storage facility is given by $C(r) = 450.7 - 16.3r$. Because this function is linear, we can interpret the slope of $C(r)$ as saying that each road uses approximately 16.3 tons of concrete to repair.

Solution. The statement is *true*. The slope of the linear function $C(r) = 450.7 - 16.3r$ is...

$$m = -16.3 = -\frac{16.3}{1} = \frac{-16.3}{1}$$

Thinking of this slope as $\frac{\Delta \text{output}}{\Delta \text{input}}$, we can see that for each one increase in r, i.e. one additional road, there is a decrease by 16.3 tons in the amount of concrete remaining. Therefore, we can summarize this as that each road requires approximately 16.3 tons of concrete to repair.

Quiz 3. *True/False*: A company sells a product for $\$5.75$ per item. Each item costs approximately $\$1.37$ to manufacture and is produced in a machine that costs $\$87.50$ to operate. Given this data, we have $R(x) = 5.75$ and $C(x) = (1.37 + 87.50)x = 88.88x$.

Solution. The statement is *false*. If one sells x items, the revenue is $R(x) = 5.75 \cdot 7 = 5.75x$. Therefore, $R(x)$ is correct. However, we know that $C(x) = \text{VC} + \text{FC}$. The fixed costs are the machine operation costs, i.e. $\text{FC} = \$87.50$. The variable costs are the $\$1.37$ cost per item. If x items are produced, then the manufacture costs are $\text{VC} = 1.37 \cdot x = 1.37x$. Therefore, $C(x) = \text{VC} + \text{FC} = 1.37x + 87.50$.

Quiz 4. *True/False*: If the following matrix represents an augmented matrix in RREF, then the corresponding system has solution $x_1 = 5$, $x_2 = -3$, and $x_3 = 7$.

$$\begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Solution. The statement is *false*. Examining the equation corresponding to the last row, we see that $0 = 1$, which is impossible. Therefore, the original system of equations was inconsistent. But then the original system of equations has no solution.

Quiz 5. *True/False*: You can perform the following multiplication:

$$\begin{pmatrix} 1 & -1 & 0 & 5 & 3 \\ 0 & 4 & -2 & 6 & 1 \end{pmatrix} \begin{pmatrix} 3 & -2 \\ 3 & 8 \\ 4 & 0 \\ 2 & -1 \\ 0 & 5 \end{pmatrix}$$

Solution. The statement is *true*. Recall that you can multiply a $m \times n$ matrix with a $p \times q$ matrix if $n = p$. If so, you obtain a $m \times q$ matrix. The first matrix is 2×5 while the second matrix is 5×2. But because $5 = 5$, we can multiply these matrix to obtain a 2×2 matrix. One can check that the product is...

$$\begin{pmatrix} 10 & 0 \\ 16 & 31 \end{pmatrix}$$

Quiz 6. *True/False*: The matrix $\begin{pmatrix} -2 & 8 \\ -2 & 6 \end{pmatrix}$ has an inverse.

Solution. The statement is *true*. Recall that a matrix has an inverse if and only if the determinant of the matrix is *not* zero. We have...

$$\begin{pmatrix} -2 & 8 \\ -2 & 6 \end{pmatrix} = -2(6) - 8(-2) = -12 + 16 = 4 \neq 0$$

Therefore, the matrix is invertible. Recalling that if A is a 2×2 matrix (given below) that is invertible, we have...

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$A^{-1} = \frac{1}{\det A} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Therefore,

$$\begin{pmatrix} -2 & 8 \\ -2 & 6 \end{pmatrix}^{-1} = \frac{1}{4} \begin{pmatrix} 6 & -8 \\ 2 & -2 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} & -2 \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

Quiz 7. *True/False*: The point $(1, -3)$ satisfies the following system of inequalities:

$$\begin{aligned} x + y &\leq 0 \\ x - 2y &\leq 5 \end{aligned}$$

Solution. The statement is *false*. If a point satisfies a system of inequalities, it satisfies each of the inequalities individually—which we can check:

$$\begin{aligned} x + y &\leq 0 & \qquad x - 2y &\leq 5 \\ 1 + (-3) &\stackrel{?}{\leq} 0 & \qquad 1 - 2(-3) &\stackrel{?}{\leq} 5 \\ -2 &\leq 0 \ \checkmark & \qquad 1 + 6 &\stackrel{?}{\leq} 5 \\ & & \qquad 7 &\not\leq 5 \ \boldsymbol{\times} \end{aligned}$$

Because $(1, -3)$ does not satisfy all the inequalities, $(1, -3)$ does not satisfy the system of inequalities.