Name: _Caleb McWhorter — Solutions_

MATH 101

Fall 2022

HW 9: Due 10/24

Problem 1. (10pt) Let $f(x)$ be a function such that $f^{-1}(x)$ exists. A partial table of values for $f(x)$ is given below:

x	1	2	3	4	5
$f(x)$	5	7	0	9	3

Based on the table above (or your knowledge of functions and inverses), find the following:

(a) $f(3) = 0$

(b) $f^{-1}(3) = 5$

(c) $f(4) = 9$

(d) $f^{-1}(9) = 4$

(e) $f(f^{-1}(5)) = f(1) = 5$

(f) $f^{-1}(f(2)) = f^{-1}(7) = 2$

(g) $f^{-1}(f(-8)) = -8$

(h) $f(f^{-1}(10)) = 10$

Problem 2. (10pt) Let $f(x) = \frac{1}{4}(x - 3)$. Assume that $f^{-1}(x)$ exists.

(a) Find $f(15)$.

(b) Use (a) to explain why $f^{-1}(3) = 15$.

(c) Solve the equation given by $f(x) = 11$.

(d) Use (c) to explain why $f^{-1}(11) = 47$.

Solution.

(a) We have...

$$f(15) = \frac{1}{4}(15 - 3) = \frac{1}{4} \cdot 12 = 3$$

(b) We know from (a) that $f(15) = 3$. But then we must have $f^{-1}(3) = 15$.
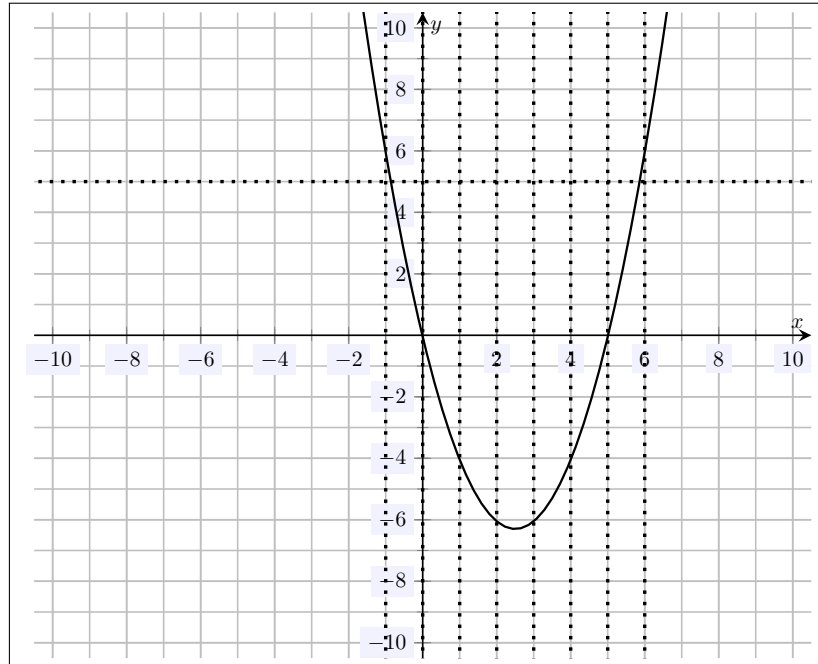
(c) We have...

$$f(x) = 11$$

$$\frac{1}{4}(x - 3) = 11$$

$$x - 3 = 44$$

$$x = 47$$

(d) We know from (c) that if $f(x) = 11$, then $x = 47$. But then $f(47) = 11$. This shows that $f^{-1}(11) = 47$.

Problem 3. (10pt) A graph of a relation $f(x)$ is shown below:

Using the graph above, answer the following:

(a) Is the relation $f(x)$ a function? Explain.

(b) Does the relation $f(x)$ have an inverse function? Explain.

Solution.

(a) Yes, the relation $f(x)$ is a function because it passes the vertical line test.

(b) No, the relation $f(x)$ does not have an inverse function because it fails the horizontal line test, e.g. at $y = 5$.