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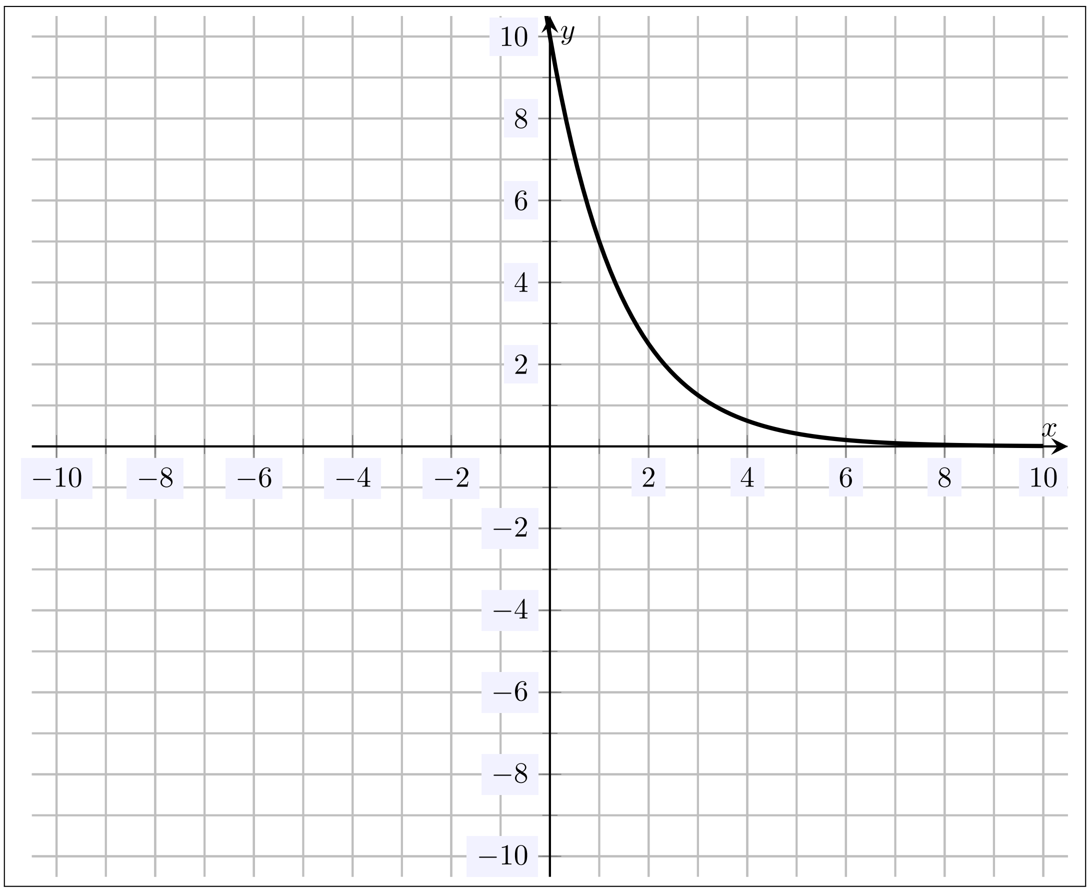
MATH 101
Winter 2021
HW 10: Due 01/20

"I'm fast. To give you a reference point, I'm somewhere between a snake and a mongoose. . . and a panther."
–Dwight Schrute, The Office

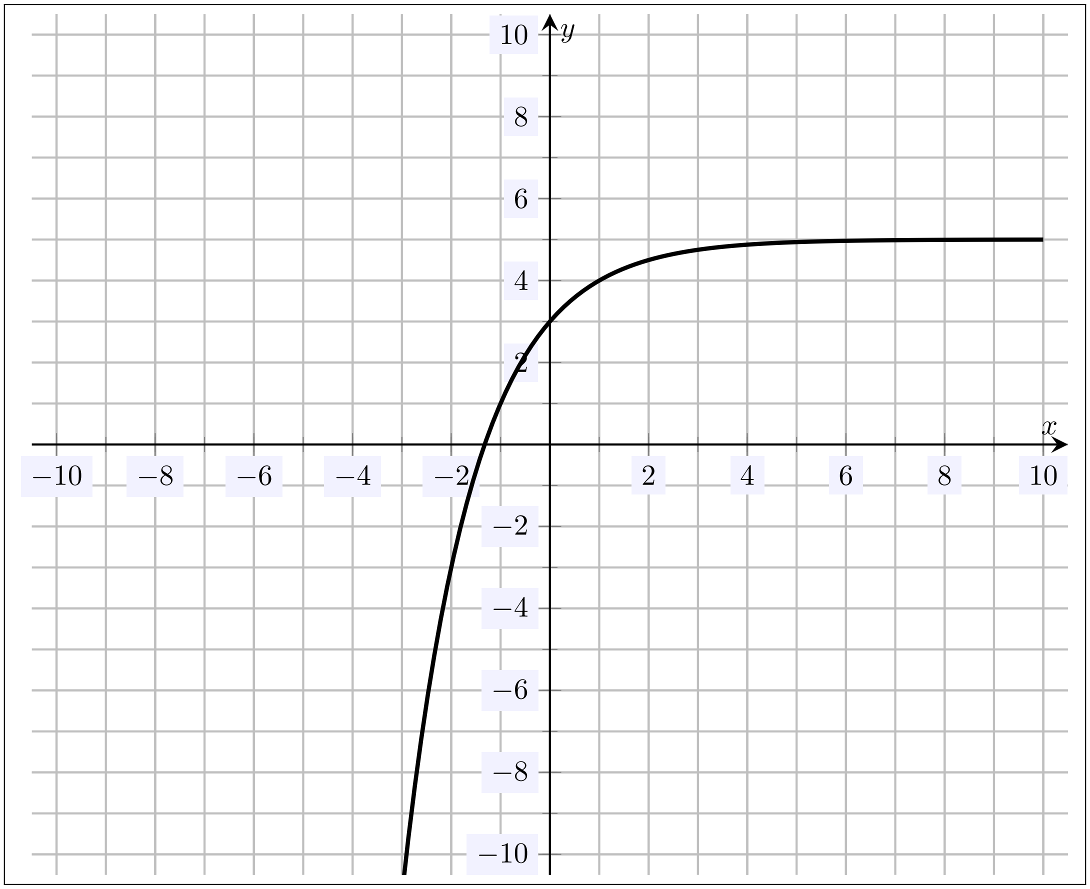
Problem 1. (10pt) Sketch the function $y = 10\left(\frac{1}{2}\right)^x$.



We know that $y(0) = 10\left(\frac{1}{2}\right)^0 = 10 \cdot 1 = 10$, so that the y-intercept is $(0, 10)$. We know also that the function having the form $y = Ab^{cx}$ with $A = 10 > 0$, $b = \frac{1}{2} < 1$, and $c = 1 > 0$, that the function has exponential decay, i.e. is decreasing. This gives the sketch above. Alternatively, we have. . .

x	-1	0	1	2	3	4	5	6	7	8	9	10
$f(x)$	20	10	5	$\frac{5}{2}$	$\frac{5}{4}$	$\frac{5}{8}$	$\frac{5}{16}$	$\frac{5}{32}$	$\frac{5}{64}$	$\frac{5}{128}$	$\frac{5}{256}$	$\frac{5}{512}$

Problem 2. (10pt) Sketch the function $y = 5 - 2^{1-x}$.



We know that $y(0) = 5 - 2^{1-0} = 5 - 2 = 3$, so that the y-intercept is $(0, 3)$. Rewriting y, we find...

$$y = 5 - 2^{1-x} = 5 + (-1) \cdot 2^{1-x} = 5 + (-1) \cdot 2^1 \cdot 2^{-x} = 5 + (-2)2^{-x}$$

The exponential part of this function has the form $y = Ab^{cx}$ with $A = -2 < 0$, $b = 2 > 1$, and $c = -1 < 0$, that the function is increasing. This gives the sketch above. Alternatively, we have...

x	-4	-3	-2	-1	0	1	2	3	4	5	6	7
$f(x)$	-27	-11	-3	1	3	4	$\frac{9}{2}$	$\frac{19}{4}$	$\frac{39}{8}$	$\frac{79}{16}$	$\frac{159}{32}$	$\frac{319}{64}$

Problem 3. (10pt) Write function $f(x) = 2\left(\frac{1}{3}\right)^{2-x}$ in the form $f(x) = Ab^x$, identifying A and b, and determine whether the function $f(x)$ is increasing or decreasing.

Solution. We have...

$$\begin{aligned} f(x) &= 2\left(\frac{1}{3}\right)^{2-x} \\ &= 2 \cdot \left(\frac{1}{3}\right)^{2} \cdot \left(\frac{1}{3}\right)^{-x} \\ &= 2 \cdot \frac{1}{9} \cdot \left(\frac{1}{3}\right)^{-x} \\ &= \frac{2}{9} \cdot \left(\left(\frac{1}{3}\right)^{-1}\right)^{x} \\ &= \frac{2}{9} \cdot 3^x \end{aligned}$$

Then $f(x) = \frac{2}{9} \cdot 3^x$ has the form $f(x) = Ab^x$ with $A = \frac{2}{9}$ and $b = 3$. Because this is a general exponential function Ab^{cx} with $A = \frac{2}{9} > 0$, $b = 3 > 1$, and $c = 1 > 0$, we know that $f(x)$ is increasing.

Problem 4. (10pt) Write function $f(x) = -5(2^{3x})$ in the form $f(x) = Ab^x$, identifying A and b, and determine whether the function $f(x)$ is increasing or decreasing.

Solution. We have...

$$\begin{aligned} f(x) &= -5(2^{3x}) \\ &= -5 \cdot (2^3)^x \\ &= -5(8^x) \end{aligned}$$

Then $f(x) = -5(8^x)$ has the form $f(x) = Ab^x$ with $A = -5$ and $b = 8$. Because this is a general exponential function Ab^{cx} with $A = -5 < 0$, $b = 8 > 1$, and $c = 1 > 0$, we know that $f(x)$ is decreasing.

Problem 5. (10pt) Write function $f(x) = 6 - 2^{1-2x}$ in the form $f(x) = Ab^x + C$, identifying A, b, and C, and determine whether the function $f(x)$ is increasing or decreasing.

Solution. We have...

$$f(x) = 6 - 2^{1-2x} = 6 - 2 \cdot 2^{-2x} = 6 - 2 \cdot (2^{-2})^x = 6 - 2 \cdot \left(\frac{1}{2^2}\right)^x = 6 - 2\left(\frac{1}{4}\right)^x = -2\left(\frac{1}{4}\right)^x + 6$$

Therefore, the function $f(x)$ has the form $f(x) = Ab^x + C$ with $A = -2$, $b = \frac{1}{4}$, and $C = 6$. Because the exponential part of this function has the form $y = Ab^{cx}$ with $A = -2 < 0$, $b = \frac{1}{4} < 1$, and $c = 1 > 0$, we know that $f(x)$ is increasing.

Problem 6. (10pt) Consider the function $y = -25(5^{-3x})$.

(a) Is the function increasing or decreasing? Explain.

(b) Find the y-intercept of this function.

(c) What are the x-intercepts and zeros for this function?

(d) Find $y(-1)$.

Solution.

(a) We have...

$$y = -25(5^{-3x}) = -25 \cdot (5^{-3})^x = -25\left(\frac{1}{5^3}\right)^x = -25\left(\frac{1}{125}\right)^x$$

Because this function has the form $f(x) = Ab^{cx}$ with $A = -25 < 0$, $b = \frac{1}{125} < 1$, and $c = 1 > 0$, we know that y is an increasing function.

(b) The y-intercept occurs when $x = 0$, but then...

$$y(0) = -25(5^{-3 \cdot 0}) = -25(5^0) = -25 \cdot 1 = -25$$

Therefore, the y-intercept is $(0, -25)$.

(c) The x-intercepts occur at zeros for $y(x)$. The zeros are the x-values such that $y(x) = 0$. But then we have...

$$\begin{aligned} -25(5^{-3x}) &= 0 \\ 5^{-3x} &= 0 \end{aligned}$$

But because $5^{-3x} > 0$ for all x, we know that $5^{-3x} \neq 0$. Therefore, $y(x)$ has no zeros. But then $y(x)$ also has no x-intercepts.

(d) We have...

$$y(-1) = -25(5^{-3 \cdot -1}) = -25(5^3) = -25(125) = -3125$$

Problem 7. (10pt) Showing all your work, solve the following equation:

$$3^{1-x} = 27$$

Solution. We have...

$$3^{1-x} = 27$$

$$3^{1-x} = 3^3$$

Comparing powers, this implies that $1 - x = 3$. But then $x = 1 - 3 = -2$.

Problem 8. (10pt) Showing all your work, solve the following equation:

$$64^x = \frac{1}{2}$$

Solution. We have...

$$\begin{aligned}
64^x &= \frac{1}{2} \\
64^x &= 2^{-1} \\
(2^6)^x &= 2^{-1} \\
2^{6x} &= 2^{-1}
\end{aligned}$$

Comparing powers, we have $6x = -1$, which implies $x = -\frac{1}{6}$.

Problem 9. (10pt) Showing all your work, solve the following equation:

$$2\left(\frac{1}{3}\right)^{-x} - 59 = -5$$

Solution. We have...

$$\begin{aligned}
2\left(\frac{1}{3}\right)^{-x} - 59 &= -5 \\
2\left(\frac{1}{3}\right)^{-x} &= 54 \\
\left(\frac{1}{3}\right)^{-x} &= 27 \\
(3^{-1})^{-x} &= 27 \\
3^x &= 3^3
\end{aligned}$$

Comparing powers, we see that $x = 3$.

Problem 10. (10pt) Showing all your work, solve the following equation:

$$2^{3x} - 7 = 9$$

Solution. We have…

$$\begin{aligned} 2^{3x} - 7 &= 9 \\ 2^{3x} &= 16 \\ 2^{3x} &= 2^4 \end{aligned}$$

Comparing powers, we find that $3x = 4$, which implies that $x = \frac{4}{3}$.