Name: _____

MATH 308
Fall 2022
HW 18: Due 12/06

"Combinatorialists use recurrence, generating functions, and such transformations as the Vandermonde convolution; others, to my horror, use contour integrals, differential equations, and other resources of mathematical analysis."

–John Riordan

Problem 1. (10pt) By counting functions ('ordinary' functions, injections, or surjections), showing all your work and fully explaining your reasoning, answer the following:

(a) How many ways 5 people can be assigned to 8 tasks, where each person can only be assigned to a single task but a task may have more than one person assigned to it. [Ans: 32,768]

(b) How many ways 5 people can be assigned to 8 tasks, where each person can only be assigned to a single task and each task may only have one person assigned to it. [Ans: 6,720]

(c) How many ways can 5 people be assigned to 3 tasks, where each task must have at least one person assigned to it? [Ans: 150]

Problem 2. (10pt) Using the principle of inclusion-exclusion, how many integers between 1 and 1000, inclusive, are...

(a) Divisible by at least one of 2, 3, 5? [Ans: 734]

(b) Divisible by 2 and 3 but not by 5? [Ans: 133]

(c) Divisible by 5 but not 2 nor 3? [Ans: 67]

(d) Divisible by 2, 3, and 5? [Ans: 33]

Problem 3. (10pt) Showing all your work and fully explaining your reasoning, use the (general) binomial theorem to answer the following:

(a) What is the coefficient of x^4y^{10} in $(x+y)^{14}$? [Ans: 1001]

(b) What is the coefficient of x^6y^5 in $(2x-3y)^{11}$? [Ans: $-7{,}185{,}024$]

(c) What is the coefficient of $x^{17}yz^2$ in $(x+y+z)^{20}$? [Ans: 3,420]

Problem 4. (10pt) Using the theory of dearrangements, showing all your work, and fully explaining your reasoning, answer the following:

(a) Find all the dearrangements of the set $S = \{1, 2, 3\}$.

(b) How many dearrangements are there for a set with four elements? [Ans: 9]

(c) Approximate how many dearrangements there are for a set with 10 elements. [Ans: 1,334,961]