

Name: \_\_\_\_\_

MATH 308

Fall 2023

HW 16: Due 12/12

*“Life is a school of probability.”*

*– Walter Bagehot*

**Problem 1.** (10pt) Let  $(\Omega, P(\Omega), \mathcal{P})$  be a finite probability space and let  $A, B, C, D \in \mathcal{P}(\Omega)$ .

$$P(A) = 0.56 \qquad P(D) = 0.28$$

$$P(B) = 0.71 \qquad P(B \cap C) = 0.09$$

$$P(C) = 0.30 \qquad P(A \cap C) = 0.12$$

- (a) Assuming  $C$  and  $D$  are independent, compute  $P(C \cup D)$ .
- (b) Are  $A$  and  $B$  disjoint? Explain.
- (c) Are  $B$  and  $C$  independent? Explain.
- (d) Compute  $P(A \mid C)$ .
- (e) If  $E_1, E_2 \in \mathcal{P}(\Omega)$ , is it possible that  $P(E_1 \cap E_2) \geq \min(P(E_1), P(E_2))$ ? Explain.

**Problem 2.** (10pt) You advising administrators for a local elementary school about suggested educational changes they can make to improve student performance. Below is some of the data on their students' performance on state exams last year, broken down by grade.

	1st	2nd	3rd	4th
Excellent	34	45	39	55
Satisfactory	67	55	58	71
Unsatisfactory	56	41	34	31

- (a) What percentage of these students are in 4th grade?
- (b) What percentage of these students were rated Excellent or Satisfactory?
- (c) What percentage of 4th graders were rated Satisfactory?
- (d) What percentage of Satisfactory students were 4th graders?
- (e) Does grade level and underperformance on state exams, i.e. being rated unsatisfactory, appear to be independent events? Explain.

**Problem 3.** (10pt) A community college is analyzing the midterm grades of their first semester freshmen. Out of 525 freshmen, the registrar reports that 121 students have failed a class within their major, 139 have failed a class outside their major, and that 72 students failed at least one class in and outside of their major.

- (a) Find the probability that a randomly selected freshman failed a class.
- (b) Find the probability that a randomly selected freshman did not fail a class.
- (c) Find the probability that a randomly selected freshman failed a class within their major if they failed a class outside their major.
- (d) Find the probability that a randomly selected freshman failed a class outside their major but did not fail a class within their major.

**Problem 4.** (10pt) You have been hired to do analysis for a marketing firm. From the company records, when consumers see an advertisement for their products, there is a 8% chance that the consumer will visit their website. Of those consumers that visit their website, 34% will make a purchase. Of those people that made a purchase, 62% will spend between \$1 and \$100, 27% will spend between between \$101 and \$300, and the rest will spend at least \$301.

- (a) What percentage of people that are advertised to will spend at least \$101 on the site?
- (b) What percentage of people that are advertised to will spend less than \$101 on the site?
- (c) What percentage of people that are advertised to will spend any money on the site?
- (d) What is the minimum expected revenue the company should expect to make (on average) if they advertise to 20,000 individuals?