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MATH 308
Fall 2022
HW 17: Due 12/06

"Some mathematicians feel that combinatorial analysis is not a branch of mathematics but rather a collection of clever but unrelated tricks."
–Frank Harary

Problem 1. (10pt) Suppose there are 8 appetizers, 15 entrées, and 6 desserts available at a restaurant. Using the multiplication principle or the addition principle, answer the following:

(a) How many ways can you order either an appetizer, entrées, or dessert? [Ans: 29]

(b) How many ways can you order a meal, i.e. appetizer, entrées, and dessert. [Ans: 720]

(c) How many ways can you order either an appetizer and an entrée, or an appetizer and a dessert? [Ans: 168]

Problem 2. (10pt) Using the theory of permutations, showing all your work, and fully justifying your reasoning, compute the following:

(a) The number of possible ways 15 people can finish a race, assuming that ties are not possible. [Ans: 1,307,674,368,000]

(b) The number of possible president, vice president, secretary, and treasurer that can be elected from 486 people, assuming no individual can hold more than one role. [Ans: 55,102,398,120]

(c) The number of possible passwords using 12 characters, assuming a character can be a digit or uppercase/lowercase letter. [Ans: 3,226,266,762,397,899,821,056]

(d) The number of distinct possible arrangements of the letters of the word 'syzygy.' [Ans: 120]

Problem 3. (10pt) Using the theory of combinations, showing all your work, and fully justifying your reasoning, compute the following:

(a) How many ways are there to choose a committee of 6 people from a collection of 20 people? [Ans: 38,760]

(b) How many 6 card hands can be dealt from a deck of 52 cards? [Ans: 20,358,520]

(c) How many ways are there to select any four bills from a jar containing a large number of \$1, \$5, \$10, \$20, \$50, and \$100 bills? [Ans: 70]

(d) How many nonnegative solutions $(x_1, x_2, x_3, x_4, x_5)$ are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 100$? [Ans: 4,598,126]

Problem 4. (10pt) Showing all your work and fully justifying your reasoning, answer the following:

(a) How many ways can the word 'frustrating' be arranged so that there are 4 letters between the 'u" and 'g'? [Ans: 1,088,640]

(b) How many ways can you choose a committee of 10 people with a designated representative for the committee from a collection of 50 people? [Ans: 102,722,781,700]

(c) If 50 people are broken up into groups by assigning to them to 4 rooms with 35 seats each, does one of the rooms have to have at least 13 people? Does one of the rooms have to have at least 23 empty seats?

(d) If you have a jury pool consisting of 14 men and 16 women, how many juries can be formed consisting of 5 men and 7 women? [Ans: 22,902,880]