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**MATH 101**
**Winter 2021**
**HW 3: Due 01/06**

*"Who's the one who didn't bring lice into the office? Meredith. Sure, I gave everybody pink eye once, and my ex keyed a few of their cars, and yeah, I BM'ed in the shredder on New Years. But I didn't bring the lice in. That was all Pam."*

*–Meredith Palmer, The Office*

**Problem 1.** (10pt) Compute the following:

(a) 60% of 77

(b) 32% of 1230

(c) 89% of 151

(d) 140% of 290

(e) 225% of 45

**Solution.**

(a)
$$77(0.60) = 46.2$$

(b)
$$1230(0.32) = 393.6$$

(c)
$$151(0.89) = 134.39$$

(d)
$$290(1.40) = 406$$

(e)
$$45(2.25) = 101.25$$

**Problem 2.** (10pt) Compute the following:

(a) 57 increased by 15%

(b) 630 decreased by 40%

(c) 485 decreased by 96%

(d) 110 increased by 120%

(e) 78 increased by 230%

**Solution.**

(a)

$$57(1 + 0.15) = 57(1.15) = 65.55$$

(b)

$$630(1 - 0.40) = 630(0.60) = 378$$

(c)

$$485(1 - 0.96) = 485(0.04) = 19.4$$

(d)

$$110(1 + 1.20) = 110(2.20) = 242$$

(e)

$$78(1 + 2.30) = 78(3.30) = 257.4$$

**Problem 3.** (10pt) Suppose you invest \$5,600 in an account that earns 6% annual interest compounded quarterly.

(a) Write a function which gives the amount of money in the account after $t$ years.

(b) Find the amount of money in the account after 7 years.

(c) Find the amount of money in the account after 27 months.

**Solution.**

(a) Let $M(t)$ be the amount of money in the account after $t$ years. Because the interest is compounded discretely, we know that $M(t) = P(1+\frac{r}{k})^{kt}$, where $P$ is the principal, $r$ is the yearly interest (as a decimal), and $k$ is the compounding rate per year. We know also that $P = 5600$, $r = 0.06$, and $k = 4$ (because the compounding is quarterly, i.e. 4 times per year). Therefore,

$$M(t) = P\left(1+\frac{r}{k}\right)^{kt} = 5600\left(1+\frac{0.06}{4}\right)^{4t} = 5600(1.015)^{4t}$$

(b) Because we are looking for the amount of money after 7 years, we know that $t = 7$.

$$M(7) = 5600(1.015)^{4\cdot 7} = 5600(1.015)^{28} = 5600(1.51722) = \$8496.44$$

(c) We need the time, $t$, to be in years. We know that 27 months is $\frac{27}{12} = 2.25$ years. But then we have...

$$M(2.25) = 5600(1.015)^{4\cdot 2.25} = 5600(1.015)^{9} = 5600(1.14339) = \$6402.98$$

**Problem 4.** (10pt) Suppose you invested money in an account which compounds interest discretely. The amount of money in the account after $t$ years is given by $M(t) = 683(1.0175)^{2t}$.

(a) How much was initially invested in the account?

(b) How often is the interest compounded?

(c) What is the interest rate on the account?

**Solution.**

(a) Because the interest is compounded discretely, we know that $M(t) = P(1 + \frac{r}{k})^{kt}$, where $P$ is the principal, $r$ is the yearly interest (as a decimal), and $k$ is the compounding rate per year. Because we know that $M(t) = 683(1.0175)^{2t}$, we must have...

$$\begin{aligned} M(t) = P\left(1 + \frac{r}{k}\right)^{kt} &= 683(1.0175)^{2t} \\ P &= 683 \\ 1 + \frac{r}{k} &= 1.0175 \\ kt &= 2t \end{aligned}$$

So we immediately know that $P = 683$, i.e. the initial amount invested in the account was \$683. From the last equation, we know that $kt = 2t$ for all $t$, i.e. $k = 2$. Therefore, the interest is being compounded twice per year, i.e. semiannually. But then we have...

$$\begin{aligned} 1 + \frac{r}{k} &= 1.0175 \\ 1 + \frac{r}{2} &= 1.0175 \\ \frac{r}{2} &= 0.0175 \\ r &= 0.0175 \cdot 2 \\ r &= 0.035 \end{aligned}$$

Therefore, the yearly interest rate is 3.5%.

(b) From the work in part (a), we found that the interest is compounded twice per year, i.e. semiannually.

(c) From the work in part (a), we know that the interest rate is 3.5%.

**Problem 5.** (10pt) A carton containing a dozen eggs costs \$3.26.

(a) What is the cost per egg?

(b) Approximately how much should 75 eggs cost?

(c) How many eggs could one purchase for \$27.30?

**Solution.**

(a)

$$\frac{\$3.26}{12 \text{ eggs}} = \frac{3.26}{12} \text{ \$/egg} = 0.271667 \text{ \$/egg}$$

(b)

$$\frac{0.271667 \text{ \$}}{1 \text{ egg}} = \frac{x}{75 \text{ eggs}}$$

$$x = 75 \text{ eggs} \cdot (0.271667 \text{ \$/egg })$$

$$x = \$20.375 \approx \$20.38$$

**OR**

$$0.271667 \text{ \$/egg} \cdot 75 \text{ eggs} = \$20.375 \approx \$20.38$$

(c)

$$\frac{0.271667 \text{ \$}}{1 \text{ egg}} = \frac{\$27.30}{x \text{ eggs}}$$

$$0.271667 \text{ \$/egg } x = \$27.30$$

$$x = \frac{\$27.30}{0.271667 \text{ \$/egg}}$$

$$x = 100.491 \text{ eggs} \approx 100 \text{ eggs}$$

**OR**

$$\frac{\$27.30}{0.271667 \text{ \$/egg}} = 100.491 \text{ eggs} \approx 100 \text{ eggs}$$

**Problem 6.** (10pt) Assume you have been driving on a highway for 3 hours and have traveled 191 miles.

(a) What is your average rate of speed?

(b) Assuming you continue at this rate of speed, how far will you have traveled 4.5 hours from now?

(c) Continuing at this speed, how long would it take to travel an additional 500 miles?

**Solution.**

(a)

$$\frac{191 \text{ miles}}{3 \text{ hours}} = \frac{191}{3} \text{ mph} = 63.67 \text{ mph}$$

(b)

$$d = vt$$

$$d = 63.67 \text{ mph} \cdot 4.5 \text{ hours}$$

$$d = 286.515 \text{ miles}$$

(c)

$$d = vt$$

$$500 \text{ miles} = 63.67 \text{ mph} \cdot t$$

$$t = \frac{500 \text{ miles}}{63.67 \text{ mph}}$$

$$t = 7.85 \text{ hours}$$

**Problem 7.** (10pt) The bones on a certain species of bird are approximately proportional to its wingspan. For one of its subspecies, a bird with a wing bone length of 1.5 ft has a wing span of 8.3 ft.

(a) If you find the remains of another bird of this subspecies with a bone length of 2.2 ft, how much would you estimate its wingspan was?

(b) If a bird of this subspecies has a wingspan of 7.9 ft, how long would you estimate the bone in its wing to be?

(c) Suppose you find a different species of bird with a wingspan of 3.8 ft and bone length of 0.79 ft. Do these two species of birds have approximately the same proportion of wingspan to bone length?

**Solution.**

(a)

$$\begin{aligned}\frac{1.5 \text{ ft}}{8.3 \text{ ft}} &= \frac{2.2 \text{ ft}}{x}\\ 1.5x &= 8.3 \cdot 2.2\\ 1.5x &= 18.26\\ x &= 12.1733 \text{ ft}\end{aligned}$$

(b)

$$\begin{aligned}\frac{1.5 \text{ ft}}{8.3 \text{ ft}} &= \frac{x}{7.9 \text{ ft}}\\ x &= 7.9 \cdot \frac{1.5}{8.3}\\ x &= 7.9 \cdot 0.180723\\ x &= 1.42771 \text{ ft}\end{aligned}$$

(c)

$$\frac{1.5 \text{ ft}}{8.3 \text{ ft}} \stackrel{?}{\approx} \frac{0.79 \text{ ft}}{3.8 \text{ ft}}$$

$$0.180723 \stackrel{?}{\approx} 0.207895$$

Because these are not ‘approximately’ equal (0.207895 is an approximately 15% increase from the 0.180723), these birds to not have ‘approximately’ the same proportions.

**Problem 8.** (10pt) Convert the following:

(a) 15 ft to m [1 ft = $0.3048$ m]

(b) 15 ft to km [1000 m = 1 km]

(c) 6400 ft to km

**Solution.**

(a)

$$\begin{array}{r|r} 15\text{ ft} & 0.3048\text{ m} \\ \hline & 1\text{ ft} \end{array} = 4.572\text{ m}$$

(b)

$$\begin{array}{r|r|r} 15\text{ ft} & 0.3048\text{ m} & 1\text{ km} \\ \hline & 1\text{ ft} & 1000\text{ m} \end{array} = 0.004572\text{ km}$$

(c)

$$\begin{array}{r|r|r} 6400\text{ ft} & 0.3048\text{ m} & 1\text{ km} \\ \hline & 1\text{ ft} & 1000\text{ m} \end{array} = 1.95072\text{ km}$$

**Problem 9.** (10pt) Convert the following:

(a) 17 hours to days [24 hours = 1 day]

(b) 27 oz to tons [16 oz = 1 lb; 2000 lb = 1 ton]

(c) 2.3 mi to in [1 mi = 5280 ft; 1 ft = 12 in]

**Solution.**

(a)

$$\begin{array}{r|r} \text{17 hours} & \text{1 day} \\ \hline & \text{24 hours} \end{array} = 0.708333 \text{ days}$$

(b)

$$\begin{array}{r|r|r} \text{27 oz} & \text{1 lb} & \text{1 ton} \\ \hline & \text{16 oz} & \text{2000 lb} \end{array} = 0.00084375 \text{ tons} = 8.4375 \cdot 10^{-4} \text{ tons}$$

(c)

$$\begin{array}{r|r|r} \text{2.3 mi} & \text{5280 ft} & \text{12 in} \\ \hline & \text{1 mi} & \text{1 ft} \end{array} = 145728 \text{ in}$$

**Problem 10.** (10pt) Convert the following:

(a) $400\ \text{ft}^2$ to $\text{in}^2$ [12 in = 1 ft]

(b) 60 mph to ft per second [1 mi = 5280 ft; 1 hr = 3600 s]

(c) $9.8\ \text{m/s}^2$ to $\text{ft/hr}^2$ [1 m = 3.28084 ft; 3600 s = 1 hr]

**Solution.**

(a)

$$\begin{array}{r|r|r} 400\ \text{ft}^2 & 12\ \text{in} & 12\ \text{in} \\ \hline & 1\ \text{ft} & 1\ \text{ft} \end{array} = 57600\ \text{in}^2$$

(b)

$$\begin{array}{r|r|r} 60\ \text{mi} & 5280\ \text{ft} & 1\ \text{hr} \\ \hline 1\ \text{hr} & 1\ \text{mi} & 3600\ \text{s} \end{array} = 88\ \text{ft/s}$$

(c)

$$\begin{array}{r|r|r|r} 9.8\ \text{m} & 3.28084\ \text{ft} & 3600\ \text{s} & 3600\ \text{s} \\ \hline 1\ \text{s}^2 & 1\ \text{m} & 1\ \text{hr} & 1\ \text{hr} \end{array} = 416692926.72\ \text{ft/s}^2 \approx 4.16693 \cdot 10^8\ \text{ft/s}^2$$