MAT 101: Exam 1
Fall – 2021
10/15/2021
85 Minutes

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Write your name on the appropriate line on the exam cover sheet. This exam contains 13 pages (including this cover page) and 15 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work.

Question	Points	Score
1	8	
2	8	
3	8	
4	6	
5	6	
6	4	
7	4	
8	6	
9	8	
10	6	
11	6	
12	4	
13	10	
14	8	
15	8	
Total:	100	

1. (8 points) Compute the following:

   (a) $50 + 50 - 25 \cdot 0 + 2 + 2 = 50 + 50 - 0 + 2 + 2 = 104$

   (b) $2 - 20/5 \cdot 3 = 2 - 4 \cdot 3 = 2 - 12 = -10$

   (c) $10 + 5 \cdot 2^2 - 3 = 10 + 5 \cdot 4 - 3 = 10 + 20 - 3 = 27$

   (d) $24/8 - 5(10 - 11)^3 = 3 - 5(-1)^3 = 3 - 5(-1) = 3 + 5 = 8$

2. (8 points) Simplify the following, being sure to have no negative exponents in your expression.

(a) $\dfrac{x^{-1}}{x^{-8}} = x^7$

(b) $\dfrac{xy^7}{x^5y^2} = \dfrac{y^5}{x^4}$

(c) $\dfrac{3x^2y^{-3}}{12x^6y^3} = \dfrac{3x^2}{12x^6y^3y^3} = \dfrac{3x^2}{12x^6y^6} = \dfrac{1}{4x^4y^6}$

(d) $\left(\dfrac{x^3}{y^{-2}}\right)^2 = \dfrac{x^6}{y^{-4}} = x^6y^4$

3. (8 points) Compute the following, being sure to simplify your answer completely:

(a) $2 - \frac{1}{3} = \frac{6}{3} - \frac{1}{3} = \frac{6-1}{3} = \frac{5}{3}$

(b) $\frac{5}{6} - \frac{3}{4} = \frac{10}{12} - \frac{9}{12} = \frac{10-9}{12} = \frac{1}{12}$

(c) $-\frac{14}{15} \cdot \frac{10}{21} = -\frac{2}{3} \cdot \frac{2}{3} = -\frac{4}{9}$

(d) $\frac{\frac{8}{9}}{\frac{4}{3}} = \frac{8}{9} \cdot \frac{3}{4} = \frac{2}{3}$

4. (6 points) Write a mathematical expression that computes the following:

   (a) 26% of 37 $37(0.26)$

   (b) 95% of 10 $10(0.95)$

   (c) 140% of 16 $16(1.40)$

5. (6 points) Write a mathematical expression that computes the following:

   (a) 40 increased by 60% $40(1 + 0.60) = 40(1.6)$

   (b) 71 decreased by 90% $71(1 - 0.90) = 71(0.10)$

   (c) 65 increased by 120% $65(1 + 1.20) = 65(2.20)$

6. (4 points) Convert the following decimal numbers to scientific notation:

   (a) $0.00125 = 1.25 \cdot 10^{-3}$

   (b) $796000 = 7.96 \cdot 10^{5}$

7. (4 points) Convert the following numbers in scientific notation to decimal notation:

   (a) $2.3 \cdot 10^{5} = 230\,000$

   (b) $5.7 \cdot 10^{-2} = 0.057$

8. (6 points) Find the prime factorizations of the following integers:

   (a) $120 = 2^3 \cdot 3 \cdot 5$

   (b) $84 = 2^2 \cdot 3 \cdot 7$

   (c) $76 = 2^2 \cdot 19$

9. (8 points) Compute the following:

   (a) $\gcd(6, 15) = 3$

   (b) $\gcd(574\,938, 1\,815\,000) = \gcd(2^1 \cdot 3^5 \cdot 7^1 \cdot 13^2, 2^3 \cdot 3^1 \cdot 5^4 \cdot 11^2) = 2^1 \cdot 3^1 = 6$

   (c) $\mathrm{lcm}(8, 10) = 40$

   (d) $\mathrm{lcm}(574\,938, 1\,815\,000) = \mathrm{lcm}(2^1 \cdot 3^5 \cdot 7^1 \cdot 13^2, 2^3 \cdot 3^1 \cdot 5^4 \cdot 11^2) = 2^3 \cdot 3^5 \cdot 5^4 \cdot 7^1 \cdot 11^2 \cdot 13^2 = 173\,918\,745\,000$

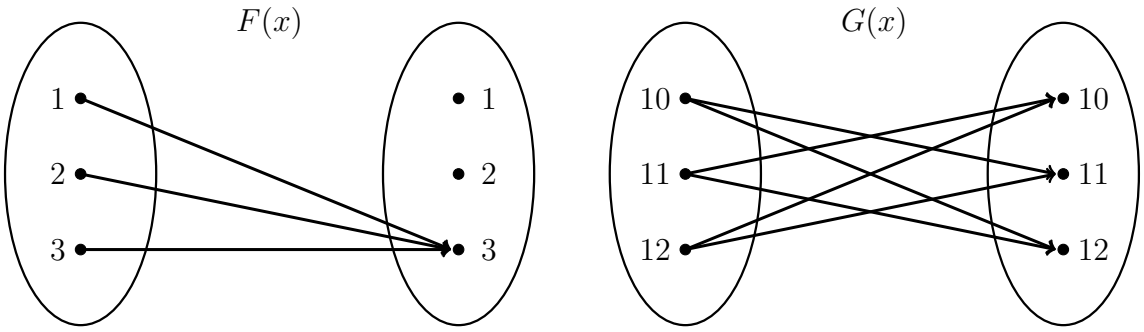
10. (6 points) Simplify the following as much as possible:

(a) $\sqrt{48} = \sqrt{16 \cdot 3} = 4\sqrt{3}$

(b) $\sqrt{90} = \sqrt{9 \cdot 10} = 3\sqrt{10}$

(c) $\sqrt[3]{2^6 \cdot 3^4 \cdot 5} = 2^2 \cdot 3^1 \sqrt[3]{3 \cdot 5} = 12\sqrt[3]{15}$

11. (6 points) Consider the following relations below:



x	$H(x)$
1	1
2	1
3	2
4	2
5	4

x	$J(x)$
5	-2
6	-1
8	0
9	1
5	2

$$K(x) := 14x - 9$$

$$L(x) := 5x(1 - x^3)$$

Determine if each of the relations given above is a function. If the relation is a function, write 'T' (True) and if the relation is *not* a function, write 'F' (False):

(a) ___T___: $F(x)$

(b) ___F___: $G(x)$

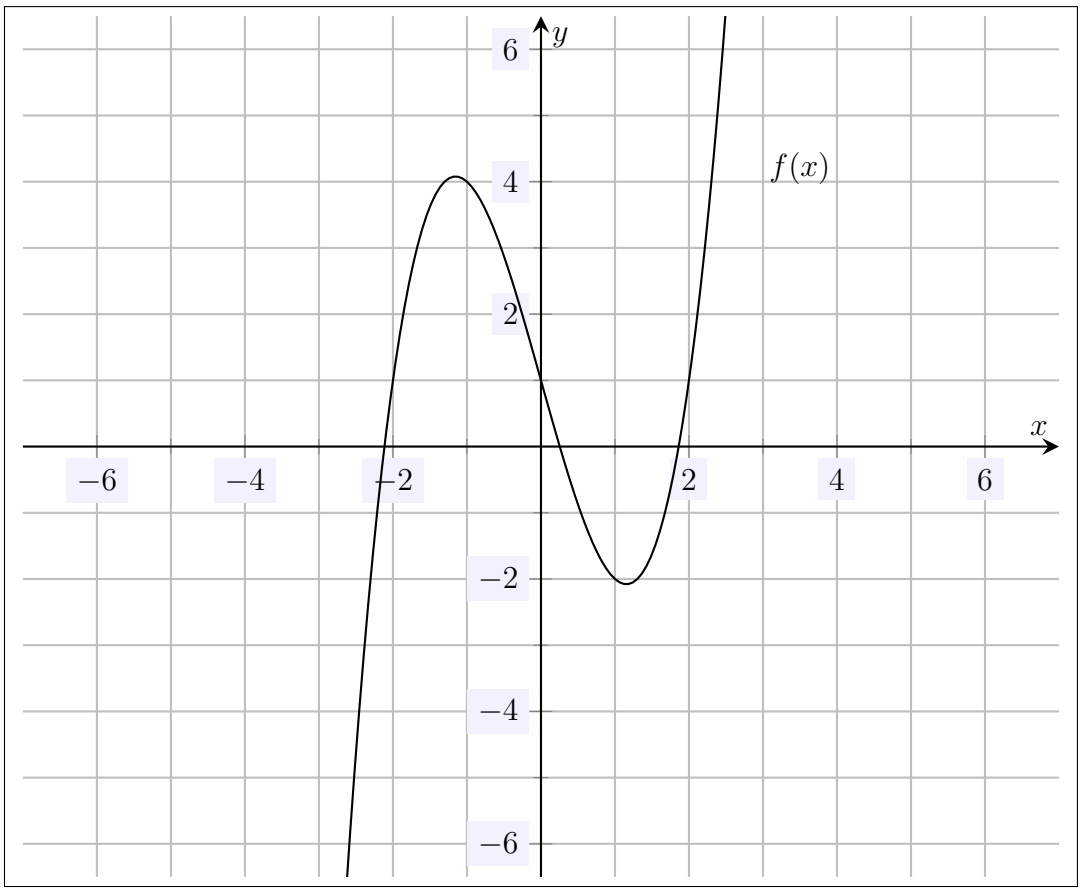
(c) ___T___: $H(x)$

(d) ___F___: $J(x)$

(e) ___T___: $K(x)$

(f) ___T___: $L(x)$

12. (4 points) Consider the relation plotted below.



(a) Is the relation plotted above a function? Explain.

The relation $f(x)$ above is a function because it passes the vertical line test.

(b) Does the relation above have an inverse function? Explain.

The relation $f(x)$ above does not have an inverse function because it fails the horizontal line test.

13. (10 points) Consider the functions given in the table below.

x	-2	-1	0	1	2
$f(x)$	3	-2	1	6	0
$g(x)$	1	2	-1	-2	-6

Compute the following:

(a) $f(1) = 6$

(b) $g(-1) - f(-2) = 2 - 3 = -1$

(c) $f(-1)g(0) = (-2)(-1) = 2$

(d) $(f - g)(1) = f(1) - g(1) = 6 - (-2) = 6 + 2 = 8$

(e) $(f \circ g)(-2) = f(g(-2)) = f(1) = 6$

(f) $(g \circ f)(2) = g(f(2)) = g(0) = -1$

(g) y-intercept of $g(x)$: $(0, -1)$

(h) x-intercept of $f(x)$: $(2, 0)$

14. (8 points) Find the equation of the line through $(-4, 12)$ and $(2, 3)$.

First, we calculate the slope...

$$m = \frac{12 - 3}{-4 - 2} = \frac{9}{-6} = -\frac{3}{2}$$

Now we use the fact that the line passes through the point $(2, 3)$*, i.e. when* $x = 2$ *then* $3 = 3$*...*

$$\begin{aligned} y &= mx + b \\ y &= -\frac{3}{2}x + b \\ 3 &= -\frac{3}{2} \cdot 2 + b \\ 3 &= -3 + b \\ b &= 6 \end{aligned}$$

Therefore, the equation of the line is $y = -\frac{3}{2}x + 6$.

15. (8 points) Find the equation of the line that is perpendicular to $y = 6$ that passes through the x-intercept of $y = x - 3$.

The line $y = 6$ is a vertical line. Because our line is perpendicular to this line, our line must be vertical, i.e. of the form $x = \#$. We know our line contains the x-intercept of the line $y = x - 3$. An x-intercept of a curve is where the curve passes through the x-axis, i.e. where $y = 0$. But then

$$\begin{aligned} y &= x - 3 \\ 0 &= x - 3 \\ x &= 3 \end{aligned}$$

Therefore, the x-intercept of the line $y = x - 3$ is the point $(3, 0)$. But then the equation of our line must be $x = 3$.