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**MATH 101**
**Spring 2024**
**HW 19: Due 04/22**

*"A million dollars isn't cool. You know what's cool? A billion dollars."*
*— Sean Parker, The Social Network*

**Problem 1.** (10pts) Showing all your work, factor the following quadratic expression:

$$2x^2 - 5x - 3$$

**Solution.** We find factors of $2 \cdot 3 = 6$ that add to $-5$. Because $-3 < 0$, the factors must have opposite signs. But then we have...

$$\begin{array}{rr} \multicolumn{2}{c}{\underline{\mathbf{6}}} \\ \boxed{1 \cdot -6\colon \quad -5} & \\ -1 \cdot 6\colon & 5 \\ 2 \cdot -3\colon & -1 \\ -2 \cdot 3\colon & 1 \end{array}$$

$$2x^2 - 5x - 3 = 2x^2 + x - 6x - 3 = (2x^2 + x) + (-6x - 3) = x(2x + 1) - 3(2x + 1) = (2x + 1)(x - 3)$$

**Problem 2.** (10pts) Use the quadratic formula to factor the following polynomial:

$$253x^2 - 7x - 98$$

**Solution.** If the roots of $f(x) = ax^2 + bx + c$ are $r_0, r_1$, then we know that $f(x) = a(x - r_0)(x - r_1)$. So we need to find the roots of the given quadratic function. The polynomial $253x^2 - 7x - 98$ has $a = 253$, $b = -7$, and $c = -98$. But then...

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(253)(-98)}}{2(253)} \\ &= \frac{7 \pm \sqrt{49 + 99176}}{506} \\ &= \frac{7 \pm \sqrt{99225}}{506} \\ &= \frac{7 \pm 315}{506} \end{aligned}$$

Therefore, the roots are $x = \frac{7-315}{506} = \frac{-308}{506} = -\frac{14}{23}$ and $x = \frac{7+315}{506} = \frac{322}{506} = \frac{7}{11}$. Therefore, we have...

$$253x^2 - 7x - 98 = 253\left(x - \frac{-14}{23}\right)\left(x - \frac{7}{11}\right) = 23\left(x + \frac{14}{23}\right) \cdot 11\left(x - \frac{7}{11}\right) = (23x + 14)(11x - 7)$$

**Problem 3.** (10pts) Find all the real zeros of the following polynomial:

$$x^5 - 9x$$

**Solution.** We make use of the difference of perfect squares, i.e. $x^2 - y^2 = (x-y)(x+y)$. We have...

$$x^5 - 9x = 0$$
$$x(x^4 - 9) = 0$$
$$x(x^2 - 3)(x^2 + 3) = 0$$

But then either $x = 0$, or $x^2 - 3 = 0$, or $x^2 + 3 = 0$. The fact case clearly implies $x = 0$. In the second case, we know that $x^2 = 3$, so that $x = \pm\sqrt{3}$. In the last case, we would have $x^2 = -3$, so that there are no real solutions. [The solutions to $x^2 = -3$ are $\pm i\sqrt{3}$.] Therefore, the real zeros of $x^5 - 9x$ are $-\sqrt{3}, 0, \sqrt{3}$.

**Problem 4.** (10pts) Showing all your work, solve the following equation:

$$\frac{x+1}{x-2} = \frac{6x}{x-4}$$

**Solution.** Cross multiplying, we have...

$$\begin{aligned} \frac{x+1}{x-2} &= \frac{x}{x-4} \\ (x+1)(x-4) &= 6x(x-2) \\ x^2 - 4x + x - 4 &= 6x^2 - 12x \\ x^2 - 3x - 4 &= 6x^2 - 12x \\ 0 &= 5x^2 - 9x + 4 \end{aligned}$$

We now factor $5x^2 - 9x + 4$. We find factors of $5 \cdot 4 = 20$ that add to $-9$. Because $4 > 0$, the factors must have the same signs. But then we have...

$$\begin{array}{rr} \multicolumn{2}{c}{\underline{\mathbf{20}}} \\ 1 \cdot 20: & 21 \\ -1 \cdot -20: & -21 \\ 2 \cdot 10: & 12 \\ -2 \cdot -10: & -12 \\ 4 \cdot 5: & 9 \\ \hline \multicolumn{1}{|r}{-4 \cdot -5:} & \multicolumn{1}{r|}{-9} \\ \hline \end{array}$$

$$5x^2 - 9x + 4 = 5x^2 - 4x - 5x + 4 = (5x^2 - 4x) + (-5x + 4) = x(5x - 4) - (5x - 4) = (5x - 4)(x - 1)$$

But then we know that $0 = (5x-4)(x-1)$. This implies that either $5x - 4 = 0$, which implies that $x = \frac{4}{5}$, or $x - 1 = 0$, which implies that $x = 1$. Therefore, the solutions are $x = \frac{4}{5}, 1$.