Quiz 1. *True/False*: If P is the proposition $6 < 5$ and Q is the proposition, “Earth is a planet,” then the logical statement $P \to Q$ is false.

Solution. The statement is *false*. Recall that the truth table for $P \to Q$ is as follows:

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

Here, P is the proposition $P : 6 < 5$ and Q is the proposition Q: “Earth is a planet.” It is clear that P is false and Q is true. But then examining the logic table above, we can see that $P \to Q$ is true.

Quiz 2. *True/False*: $\neg(P \to \neg Q) \equiv P \wedge Q$

Solution. The statement is *true*. To determine if two propositions are logically equivalent, one can either examine the truth table or apply logical rules to obtain one logical expression from the other. If we construct a truth table, we have…

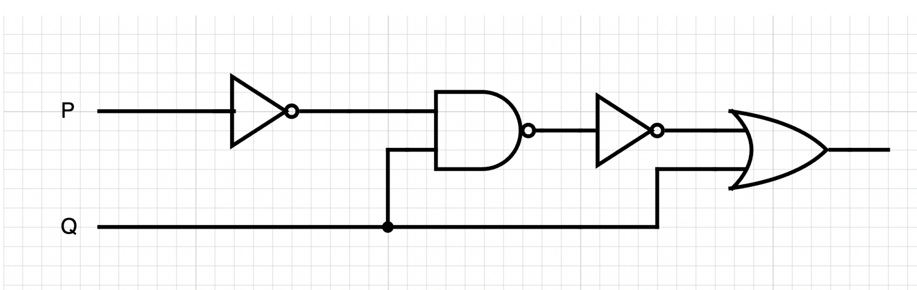
P	Q	$\neg Q$	$P \to \neg Q$	$\neg(P \to \neg Q)$	$P \wedge Q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	T	F	F

Because for each possible pair of choices for P and Q the outputs for $\neg(P \to \neg Q)$ and $P \wedge Q$ match, $\neg(P \to \neg Q) \equiv P \wedge Q$. Alternatively, we can transform one into the other by applying logical equivalences (recall $P \to Q \equiv \neg P \vee Q$ or $\neg(P \to Q) \equiv P \wedge \neg Q$):

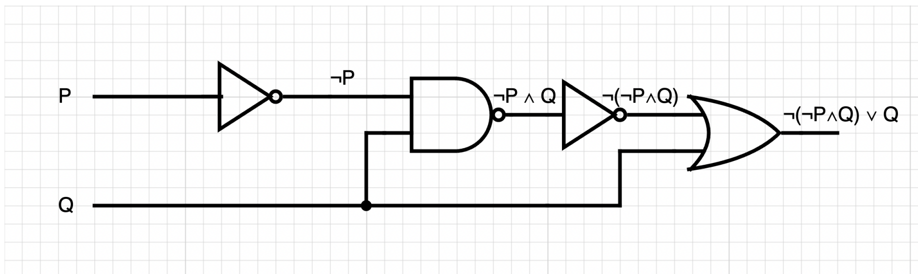
$$\neg(P \to \neg Q) \equiv \neg(\neg P \vee \neg Q) \equiv \neg(\neg P) \wedge \neg(\neg Q) \equiv P \wedge Q.$$

Quiz 3. *True/False*: The logic corresponding to the circuit shown below is the proposition:

$$(\neg P \wedge Q) \vee \neg Q.$$



Solution. The statement is *false*. We can trace through the circuit. We see that the current from P passes through a NOT gate and we obtain $\neg P$. This then feeds into an AND gate along with Q so that we obtain $\neg P \wedge Q$. The resulting current is then passed through a NOT gate, obtaining $\neg(\neg P \wedge Q)$. This finally reaches an OR gate—along with Q—to obtain $\neg(\neg P \wedge Q) \vee Q$. We can see a diagrammatic explanation below.



Quiz 4. *True/False*: Let the universe $\mathcal{U}$ be the set of real numbers and define $P(x)$ to be the predicate $P(x) : x^2 + x - 4 \geq 0$. Then $(\forall x)\big(\neg P(x)\big)$ is true.

Solution. The statement is *false*. If $P(x) : x^2 + x - 4 \geq 0$, then $\neg P(x) : x^2 + x - 4 < 0$. But then $(\forall x)\big(\neg P(x)\big)$ is the statement, "For all x, $x^2 + x - 4 < 0$." Now if $x = 1$, we have $\neg P(1) \colon 1^2 + 1 - 4 < 0$, i.e. $-2 < 0$, which is true. If $x = 0$, we have $\neg P(0) \colon 0^2 + 0 - 4 < 0$, i.e. $-4 < 0$, which is true. However, while $(\forall x)\big(\neg P(x)\big)$ is clearly true for *some* (we found at least two), it is not true *for all* x. As a counterexample, let $x = 10$. Then $\neg P(10) \colon 10^2 + 10 - 4 < 0$, which is $104 < 0$—clearly false. Therefore, $\neg P(x)$ is not true for all x. But then $(\forall x)\big(\neg P(x)\big)$ is false.