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MATH 100
Fall 2023
HW 2: Due 09/13

"I wasn't a failed DJ. I was pre-successful."
–Jason Mendoza, The Good Place

Problem 1. (10pt) Your history class course grade is determined by the following components:

Homeworks	25%
Paper I	10%
Paper II	10%
Paper III	10%
Project	30%
Quizzes	15%

Suppose that your homework average was 82%, your grades on the papers were 68%, 84%, and 76%, respectively, your project grade was 88%, and your quiz average was 94%.

(a) Compute your course average.

(b) If the project was a final project that had not yet occurred, i.e. you had not yet received the 88%, but all the other course grades were as listed above, then what is your current course average?

Solution.

(a) The course average will be a weighted average where each course component grade is weighted by the amount of the course grade that component is worth. But then we have...

$$\begin{aligned}\text{Course Grade} &= \sum \text{Course Worth} \cdot \text{Component Grade} \\ &= 0.25(82\%) + 0.10(68\%) + 0.10(84\%) + 0.10(76\%) + 0.30(88\%) + 0.15(94\%) \\ &= 20.5\% + 6.8\% + 8.4\% + 7.6\% + 26.4\% + 14.1\% \\ &= 83.8\%\end{aligned}$$

(b) We can compute the percentage earned towards the course average thus far the same way we computed the course average in (a):

$$\begin{aligned}\text{Course Grade} &= \sum \text{Course Worth} \cdot \text{Component Grade} \\ &= 0.25(82\%) + 0.10(68\%) + 0.10(84\%) + 0.10(76\%) + 0.15(94\%) \\ &= 20.5\% + 6.8\% + 8.4\% + 7.6\% + 14.1\% \\ &= 57.4\%\end{aligned}$$

The percentage of the course grade counted thus far is $25\%+10\%+10\%+10\%+15\% = 70\%$. But then the work above shows that the student has earned 57.4% from the 70% of the course grade counted thus far. But then the students course average is...

$$\text{Course Average} = \frac{\text{Percentage Earned}}{\text{Percentage Counted}} = \frac{0.574}{0.70} = 0.82 \rightsquigarrow 82\%$$

Note. This was implicit in (a), where you have earned 83.8% of the 100% of the course grade counted thus far: Course Average $= \frac{\textit{Percentage Earned}}{\textit{Percentage Counted}} = \frac{0.838}{1.00} = 0.838 \rightsquigarrow 83.8\%$*. Note also it is also possible to compute the average directly by treating this as a 'true' weighted average:* $\frac{\sum \textit{weight} \cdot \textit{value}}{\sum \textit{weight}}$*, which in our case would yield* $\frac{57.4\%}{70\%} = 0.82 \rightsquigarrow 82\%$.

Problem 2. (10pt) Suppose you received the following grades this semester:

Course	Credits	Grade
BIO 101: Human Biology	3	B
MATH 104: Precalculus	4	B+
RELS 102: Religion and Society	3	A–
ENG 207: Writing about World Mythology	3	B–
ECON 102: Principles of Microeconomics	3	C+

Given the following grade values,

Grade	Values	Grade	Values
A	4.0	C+	2.3
A–	3.7	C	2.0
B+	3.3	C–	1.7
B	3.0	D	1.0
B–	2.7	F	0

(a) Compute your semester GPA.

(b) If your previous GPA based on 67 credits was 3.308, what is your current overall GPA?

Solution.

(a) GPA is a weighted average where each letter grade earned is weighted by the number of credits for the corresponding course. But then we have...

$$\begin{aligned}
\text{GPA} &= \frac{\sum \text{Credits} \cdot \text{Letter Grade}}{\sum \text{Credits}} \\
&= \frac{3(3.0) + 4(3.3) + 3(3.7) + 3(2.7) + 3(2.3)}{3 + 4 + 3 + 3 + 3} \\
&= \frac{9 + 13.2 + 11.1 + 8.1 + 6.9}{3 + 4 + 3 + 3 + 3} \\
&= \frac{48.3}{16} \\
&\approx 3.019
\end{aligned}$$

(b) The new GPA will be the weighted average of the previous GPA with the current semester GPA, where each is weighted by the number of credits. Therefore, we have...

$$\begin{aligned}
\text{New GPA} &= \frac{\text{Previous Credits} \cdot \text{Previous GPA} + \text{Current Credits} \cdot \text{Current GPA}}{\text{Total Credits}} \\
&= \frac{67 \cdot 3.308 + 16 \cdot 3.019}{67 + 16} \\
&= \frac{221.636 + 48.304}{67 + 16} \\
&= \frac{269.94}{83} \\
&\approx 3.252
\end{aligned}$$

Problem 3. (10pt) Showing all your work, convert the following:

(a) 0.4 megagrams to centigrams

(b) 87 oz to stones [1 oz = 28.35 g, 1 stone = 6.35 kg]

(c) $98 per hour to MXP per minute [$0.057 = 1 MXP]

(d) 6,000 ft^2 to mi^2 [5280 ft = 1 mi.]

(e) 0.05 meters per square second to feet per square hour [0.3048 m = 1 ft]

Solution.

(a) We have...

$$\frac{0.4\text{ Mg} \left\| \begin{array}{c|c} 1000000\text{ g} & 100\text{ cg} \\ \hline 1\text{ Mg} & 1\text{ g} \end{array} \right.}{} = 40{,}000{,}000\text{ cg}$$

(b) We have...

$$\frac{87\text{ oz} \left\| \begin{array}{c|c|c} 28.35\text{ g} & 1\text{ kg} & 1\text{ stone} \\ \hline 1\text{ oz} & 1000\text{ g} & 66.35\text{ kg} \end{array} \right.}{} = 0.0371733233\text{ stone}$$

(c) We have...

$$\frac{\$98 \left\| \begin{array}{c|c} 1\text{ MXP} & 1\text{ hr} \\ \hline \$0.057 & 60\text{ min} \end{array} \right.}{1\text{ hr}} = \$28.655\text{ MXP per minute}$$

(d) We have...

$$\frac{6000\text{ ft}^2 \left\| \begin{array}{c|c} 1\text{ mi} & 1\text{ mi} \\ \hline 5280\text{ ft} & 5280\text{ ft} \end{array} \right.}{} = 0.00021522\text{ mi}^2$$

(e) We have...

$$\frac{0.05\text{ m} \left\| \begin{array}{c|c|c|c|c} 1\text{ ft} & 60\text{ s} & 60\text{ s} & 60\text{ min} & 60\text{ min} \\ \hline 0.3048\text{ m} & 1\text{ min} & 1\text{ min} & 1\text{ hr} & 1\text{ hr} \end{array} \right.}{1\text{ s}^2} \approx 2{,}125{,}984.25\text{ ft/hr}^2$$

Problem 4. (10pt) Suppose you are talking with your friend who has moved to Germany. The conversation has drifted to NYC housing. Currently, the cost of space in NYC is approximately $1,600 per square foot.

(a) For your friend, convert this to Euros per square meter. [€1= $1.07; 1 ft = 0.3048 m]

(b) Using (a), find the conversion factor from dollars per square foot to Euros per square meter.

(c) Use your answer from (b) to convert $1,845 per square foot to Euros per square meter.

Solution.

(a) We have...

$$\frac{\$1600}{1\text{ ft}^2} \left\| \frac{€1}{\$1.07} \right| \frac{1\text{ ft}}{0.3048\text{ m}} \left| \frac{1\text{ ft}}{0.3048\text{ m}} = €16{,}095.57 \text{ per square meter}$$

(b) From the work above, we can see that the conversion factor is...

$$\frac{1}{1.07} \cdot \frac{1}{0.3048} \cdot \frac{1}{0.3048} = 10.059729361$$

(c) We have...

$$\$1845 \text{ per square foot } \cdot 10.059729361 \approx €18,560.20 \text{ per square meter}$$

We can also check this directly:

$$\frac{\$1845}{1\text{ ft}^2} \left\| \frac{€1}{\$1.07} \right| \frac{1\text{ ft}}{0.3048\text{ m}} \left| \frac{1\text{ ft}}{0.3048\text{ m}} = €18{,}560.2 \text{ per square meter}$$