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**MATH 100**
**Fall 2021**
**HW 13: Due 11/15**

*"If at first you don't succeed, find out if the loser gets anything."*
*– William Lyon Phelps*

**Problem 1.** (10pt) Compute the following, being sure to show all your work and simplifying as much as possible:

$$\frac{3}{x+1} - \frac{2}{x-3}$$

**Solution.**

$$\begin{aligned}\frac{3}{x+1} - \frac{2}{x-3} &= \frac{3(x-3)-2(x+1)}{(x+1)(x-3)} \\ &= \frac{3x-9-2x-2}{(x+1)(x-3)} \\ &= \frac{x-11}{(x+1)(x-3)}\end{aligned}$$

**Problem 2.** (10pt) Compute the following, being sure to show all your work and simplifying as much as possible:

$$\frac{1}{x-1}+\frac{3}{x^2+3x-4}$$

**Solution.**

$$\begin{aligned}\frac{1}{x-1}+\frac{3}{x^2+3x-4} &= \frac{1}{x-1}+\frac{3}{(x-1)(x+4)}\\ &= \frac{1(x+4)}{(x-1)(x+4)}+\frac{3}{(x-1)(x+4)}\\ &= \frac{x+4+3}{(x-1)(x+4)}\\ &= \frac{x+7}{(x-1)(x+4)}\end{aligned}$$

**Problem 3.** (10pt) Compute the following, being sure to show all your work and simplifying as much as possible:

$$\frac{x}{x-5}+\frac{x+1}{x+2}$$

**Solution.**

$$\begin{aligned}\frac{x}{x-5}+\frac{x+1}{x+2} &= \frac{x(x+2)}{(x-5)(x+2)}+\frac{(x+1)(x-5)}{(x-5)(x+2)}\\ &= \frac{x(x+2)+(x+1)(x-5)}{(x-5)(x+2)}\\ &= \frac{(x^2+2x)+(x^2-5x+x-5)}{(x-5)(x+2)}\\ &= \frac{2x^2-2x-5}{(x-5)(x+2)}\end{aligned}$$

**Problem 4.** (10pt) Compute the following, being sure to show all your work and simplifying as much as possible:

$$\frac{x-2}{x^2+10x+16} - \frac{x}{x^2+7x-8}$$

**Solution.**

$$\begin{aligned}
\frac{x-2}{x^2+10x+16} - \frac{x}{x^2+7x-8} &= \frac{x-2}{(x+2)(x+8)} - \frac{x}{(x+8)(x-1)} \\
&= \frac{(x-2)(x-1)}{(x+2)(x+8)(x-1)} - \frac{x(x+2)}{(x+8)(x-1)(x+2)} \\
&= \frac{(x-2)(x-1) - x(x+2)}{(x+2)(x+8)(x-1)} \\
&= \frac{(x^2-x-2x+2) + (-x^2-2x)}{(x+2)(x+8)(x-1)} \\
&= \frac{-5x+2}{(x+2)(x+8)(x-1)}
\end{aligned}$$

**Problem 5.** (10pt) Compute the following, being sure to show all your work and simplifying as much as possible:

$$\frac{8}{x+1} \cdot \frac{x^2-1}{4x+4}$$

**Solution.**

$$\frac{8}{x+1} \cdot \frac{x^2-1}{4x+4} = \frac{8}{x+1} \cdot \frac{(x-1)(x+1)}{4(x+1)}$$

$$= \frac{{}^{2}\cancel{8}}{\cancel{x+1}} \cdot \frac{(x-1)\cancel{(x+1)}}{{}_{1}\cancel{4}(x+1)}$$

$$= \frac{2(x-1)}{x+1}$$

**Problem 6.** (10pt) Compute the following, being sure to show all your work and simplifying as much as possible:

$$\frac{x+6}{x^2-2x-24} \cdot \frac{x^2+5x+4}{2x+12}$$

**Solution.**

$$\begin{aligned}
\frac{x+6}{x^2-2x-24} \cdot \frac{x^2+5x+4}{2x+12} &= \frac{x+6}{(x-6)(x+4)} \cdot \frac{(x+1)(x+4)}{2(x+6)} \\
&= \frac{\cancel{x+6}}{(x-6)\cancel{(x+4)}} \cdot \frac{(x+1)\cancel{(x+4)}}{2\cancel{(x+6)}} \\
&= \frac{x+1}{2(x-6)}
\end{aligned}$$

**Problem 7.** (10pt) Compute the following, being sure to show all your work and simplifying as much as possible:

$$\frac{\dfrac{x+1}{x}}{\dfrac{x^2-1}{x^2+9x}}$$

**Solution.**

$$\frac{\dfrac{x+1}{x}}{\dfrac{x^2-1}{x^2+9x}} = \frac{x+1}{x} \cdot \frac{x^2+9x}{x^2-1}$$

$$= \frac{x+1}{x} \cdot \frac{x(x+9)}{(x-1)(x+1)}$$

$$= \frac{\cancel{x+1}}{\cancel{x}} \cdot \frac{\cancel{x}(x+9)}{(x-1)\cancel{(x+1)}}$$

$$= \frac{x+9}{x-1}$$

**Problem 8.** (10pt) Compute the following, being sure to show all your work and simplifying as much as possible:

$$\frac{\dfrac{x^2+x-2}{x^2+8x+15}}{\dfrac{x^2-2x-8}{x^2-9}}$$

**Solution.**

$$\frac{\dfrac{x^2+x-2}{x^2+8x+15}}{\dfrac{x^2-2x-8}{x^2-9}} = \frac{x^2+x-2}{x^2+8x+15} \cdot \frac{x^2-9}{x^2-2x-8}$$

$$= \frac{(x+2)(x-1)}{(x+3)(x+5)} \cdot \frac{(x-3)(x+3)}{(x-4)(x+2)}$$

$$= \frac{\cancel{(x+2)}(x-1)}{\cancel{(x+3)}(x+5)} \cdot \frac{(x-3)\cancel{(x+3)}}{(x-4)\cancel{(x+2)}}$$

$$= \frac{(x-1)(x-3)}{(x+5)(x-4)}$$