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MATH 100
Fall 2022
HW 1: Due 09/14

"I can be just as non-competitive as anybody. Matter of fact, I'm the most non-competitive, so I win."
–Peter Griffin, Family Guy

Problem 1. (10pt) Showing all the steps according to order of operations, compute the following:

(a) $10 + 10 - 16 \cdot 0 + 2 + 2$

(b) $(-1)^3 - 1 + 4^2/2$

(c) $15 - (6 - 10) + 3^2$

(d) $\dfrac{-4 - (2 - 4)^2}{3^2 - 1}$

Solution. Following the order of operations (PEMDAS), being sure to work from left to right, we have...

(a)

$$10 + 10 - 16 \cdot 0 + 2 + 2 = 10 + 10 - 0 + 2 + 2 = 20 - 0 + 2 + 2 = 20 + 2 + 2 = 22 + 2 = 24$$

(b)

$$(-1)^3 - 1 + 4^2/2 = -1 - 1 + 16/2 = -1 - 1 + 8 = -2 + 8 = 6$$

(c)

$$15 - (6 - 10) + 3^2 = 15 - (-4) + 3^2 = 15 - (-4) + 9 = 19 + 9 = 28$$

(d)

$$\frac{-4 - (2 - 4)^2}{3^2 - 1} = \frac{-4 - (-2)^2}{3^2 - 1} = \frac{-4 - 4}{9 - 1} = \frac{-8}{8} = -1$$

Problem 2. (10pt) Define the following sets:

$$\begin{aligned} A &= \{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10\} \\ B &= \{2,\ 4,\ 6,\ 8,\ 10\} \\ C &= \{1,\ 3,\ 5,\ 7,\ 9\} \\ D &= \{2,\ 3,\ 5,\ 7\} \\ E &= \{2,\ 3,\ 4,\ 6,\ 8,\ 9\} \end{aligned}$$

Consider all these sets as subsets of A. Compute the following:

(a) B^c

(b) $B \cup D$

(c) $E \setminus D$

(d) $C \cap E$

(e) $|A|$

Solution.

(a) The complement of a set A that is a subset of some larger set, say S, is denoted A^c and is the set of things that are in S that are *not* in A. We want to compute B^c, i.e. the things that are in A that are not in B (because we are considering B as a subset of A). For instance, we know that $5 \in B^c$ because $5 \in A$ but $5 \notin B$. We know that $2 \notin B^c$ because $2 \in B$. Continuing this process gives us...

$$B^c = \{1,\ 3,\ 5,\ 7,\ 9\}$$

(b) The union of two sets A and B, denoted $A \cup B$, is the set of things that are in A or that are in B (it could also be in both). We want to compute $B \cup D$. For instance, we know that $2, 4, 6, 8, 10 \in B \cup D$ because they are in B. We know that $2, 3, 5, 7 \in B \cup D$ because they are in D. Collecting these elements (eliminating redundant repeats) and writing them in order (for 'readability'), we have...

$$B \cup D = \{2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 10\}$$

(c) The set difference of sets A and B, denoted $A \setminus B$ or $A - B$, is the set of things that are in A but *not* in B; that is, $A \setminus B$ is the set A with anything that is in A that can be found in B removed. We want to compute $E \setminus D$. For instance, we know that $6 \in E \setminus D$ because $6 \in E$ but $6 \notin D$. We know also that $2 \notin E \setminus D$ because $2 \in D$. Continuing this process gives us...

$$E \setminus D = \{4,\ 6,\ 8,\ 9\}$$

(d) The intersection of two sets A and B, denoted $A \cap B$, is the set of things that are in A and B. We want to compute $C \cap E$. For instance, we know that $3 \in C \cap E$ because 3 is in C and E. We know that $8 \notin C \cap E$ because 8 is not in both of the sets C and E. Continuing this process, we have...

$$C \cap E = \{3, 9\}$$

(e) The cardinality (or size) of a finite set S, denoted $|S|$, is the number of elements in S. We want to compute $|A|$. Because A contains ten elements (the numbers 1–10), we have...

$$|A| = 10$$

Problem 3. (10pt) Define the following sets:

$A =$ All males over 40 years old. $C =$ All US Presidents, alive or dead.

$B =$ All people that have acted in a movie. $D =$ All persons under 6 ft tall.

Consider all of these sets as subsets of the set of all people alive. Being sure to completely justify your response, answer the following:

(a) Find an element of $A \cap B$.

(b) Is Jeff Bezos $\in A \cup C$? Is Jeff Bezos $\in C \cup D$?

(c) Is George Washington $\in C - B$?

(d) Is Danny DeVito $\in D^c$?

(e) Are sets B and C disjoint? [Hint: Consider US Presidents from the last 50 years.]

(a) An element of an intersection of two sets is something that is in *both* the sets. Elements of A are males over 40 years old while the elements of B are people that have acted in a movie. Therefore, the elements of $A \cap B$ will be those males over 40 years old that have acted in a movie. For instance, Colin Firth, Leonardo DiCaprio, Robert Downey Jr., Matt Damon, Robert De Niro , and Joe Pesci are all elements of $A \cap B$.

(b) An element of a union of two sets is something that is in either of the two sets. Elements of $A \cup C$ are then males over 40 years old or any US president, alive or dead, while the elements of $C \cup D$ are any US Presidents, dead or alive, or any person under 6 ft tall. Because Jeff Bezos is a male over 40 years old, he is in the set $A \cup C$—whether or not he is an alive or dead US President. Because Jeff Bezos is not over 6 ft tall (he is is 5'7), he is in the set $C \cup D$—even though he is not a US President, alive or dead.

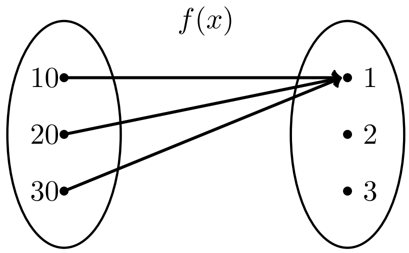
(c) An element of a difference of two sets is an element of the first set that *cannot* be found in the second set. Elements of $C - B$ are then US Presidents, alive or dead, that have *not* acted in a movie. Because George Washington was a US President (who happens to be dead) and never acted in a movie (as the first 'movie' was not created until the late 1880s). Therefore, George Washington is an elements of the set $C - B$.

(d) An element of the complement of a set is an element which is *not* in the given set. Elements of D^c are then people that are not under 6 ft tall. Because Danny DeVito is 4'10, he is under 6 ft tall. But then he is not one of the people that are not under 6 ft tall. Therefore, Danny DeVito is not an element of the set D^c.

(e) The sets B and C are disjoint if their intersection is empty, i.e. $B \cap C = \varnothing$; that is, two sets are disjoint if they have no element in common. The set B is the set of people that have acted in a movie and the set C is the set of US presidents, dead or alive. Elements of $B \cap C$ are then US presidents, dead or alive, that have acted in a movie. Because Ronald Reagan, Bill Clinton, and Donald Trump have all appeared in movie, $B \cap C$ is nonempty. Therefore, B and C are *not* disjoint.

Problem 4. (10pt) Determine whether the following relations are functions, being sure to justify your answer. If the relation is a function, determine its domain, codomain, and range. [For this problem, in determining a functions domain, codomain, and range, you may invoke the use/description of a graph.]

(a)



(b)

x	$g(x)$
1.0	1.0
1.5	4.3
3.0	-6.1
4.4	2.2
6.8	1.0

(c) $h(x, y) = x + y^4$.

(d) $j(x) =$ the multiple of two closest to x.

Solution. Recall that a relation is a function if for each input, there is only one possible output—not necessarily distinct from the outputs from other inputs.

(a) This relation is a function because for each input, there is a single output. We have $f(10) = 1$, $f(20) = 1$, and $f(30) = 1$. The domain of this function is $\{10, 20, 30\}$, the codomain is $\{1, 2, 3\}$, and the range is $\{1\}$.

(b) This relation is a function because for each input, there is a single output. We have $g(1.0) = 1.0$, $g(1.5) = 4.3$, $g(3.0) = -6.1$, $g(4.4) = 2.2$, and $g(6.8) = 1.0$. The domain of this function is $\{1.0, 1.5, 3.0, 4.4, 6.8\}$, the codomain is likely the set of real numbers or the set $\{1.0, 4.3, -6.1, 2.2, 1.0\}$, and the range is $\{1.0, 4.3, -6.1, 2.2, 1.0\}$.

(c) The relation is a function because for each set of inputs x, y, there is a single output—namely, the one obtained by evaluating $h(x, y)$ and following order of operations. The domain is the set of points (x, y) in the plane, the codomain and range is the set of real numbers. [Notice if we choose $y = 0$ and $x = r$, we have $h(r, 0) = r$ so that every real number output is possible.]

(d) The relation is *not* a function. For instance, $j(3)$ could be 2 or 4 because both are equally close to 3 so that the relation $j(x)$ is not well defined.

Problem 5. (10pt) Suppose that $f(x,y)$ is the function given by the following table:

$x\backslash y$	1	2	3	4
1	-2	7	4	-4
2	0	3	-1	1
3	5	-6	7	6
4	1	0	4	0

Showing all your work, compute the following:

(a) $f(3,2)$

(b) $f(3-1,2^2)$

(c) $5f(3,1)-8$

(d) $\dfrac{4-f\big(3^2+(-2)^3,1\big)}{2f(1,3)}$

Solution.

(a)

$$f(3,2)=-6$$

(b)

$$f(3-1,2^2)=f(3-1,4)=f(2,4)=1$$

(c)

$$5f(3,1)-8=5(5)-8=25-8=23$$

(d)

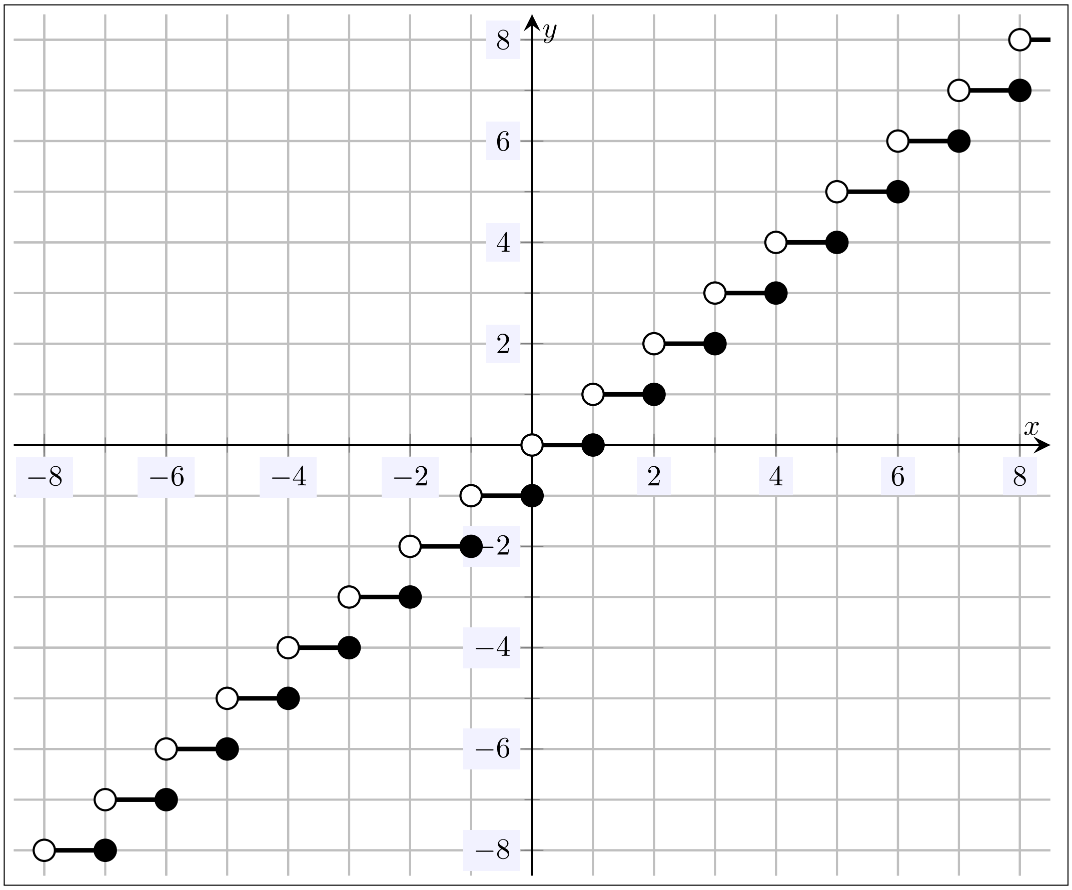
$$\frac{4-f(3^2+(-2)^3,1)}{2f(1,3)}=\frac{4-f(9+(-8),1)}{2f(1,3)}=\frac{4-f(1,1)}{2f(1,3)}=\frac{4-(-2)}{2(4)}=\frac{6}{8}=\frac{3}{4}$$

Problem 6. (10pt) Let rdwn(x) denote the largest integer that is *less than* x.

(a) Find rdwn(x) for $x = 0.5,\ 2.2,\ 5.9,\ 6.0,\ -1.5,\ -4.9,\ -7$.

(b) Explain why rdwn(x) is a function.

(c) Being as accurate as possible, sketch a graph of rdwn(x) on the plot below.



(a) We have rdwn$(0.5) = 0$, rdwn$(2.2) = 2$, rdwn$(5.9) = 5$, rdwn$(6.0) = 5$, rdwn$(-1.5) = -2$, rdwn$(-4.9) = -5$, and rdwn$(-7) = -8$.

(b) For any given input x, there is only one largest integer that is less than x.

(c) See the plot above.