

Name: Caleb McWhorter — Solutions

MATH 308

Fall 2022

HW 18: Due 12/06

*“Combinatorialists use recurrence, generating functions, and such transformations as the Vandermonde convolution; others, to my horror, use contour integrals, differential equations, and other resources of mathematical analysis.”*

*—John Riordan*

**Problem 1.** (10pt) By counting functions (‘ordinary’ functions, injections, or surjections), showing all your work and fully explaining your reasoning, answer the following:

- (a) How many ways 5 people can be assigned to 8 tasks, where each person can only be assigned to a single task but a task may have more than one person assigned to it. [Ans: 32,768]
- (b) How many ways 5 people can be assigned to 8 tasks, where each person can only be assigned to a single task and each task may only have one person assigned to it. [Ans: 6,720]
- (c) How many ways can 5 people be assigned to 3 tasks, where each task must have at least one person assigned to it? [Ans: 150]

**Problem 2.** (10pt) Using the principle of inclusion-exclusion, how many integers between 1 and 1000, inclusive, are...

- (a) Divisible by at least one of 2, 3, 5? [Ans: 734]
- (b) Divisible by 2 and 3 but not by 5? [Ans: 133]
- (c) Divisible by 5 but not 2 nor 3? [Ans: 67]
- (d) Divisible by 2, 3, and 5? [Ans: 33]

**Problem 3.** (10pt) Showing all your work and fully explaining your reasoning, use the (general) binomial theorem to answer the following:

- (a) What is the coefficient of  $x^4y^{10}$  in  $(x + y)^{14}$ ? [Ans: 1001]
- (b) What is the coefficient of  $x^6y^5$  in  $(2x - 3y)^{11}$ ? [Ans:  $-7,185,024$ ]
- (c) What is the coefficient of  $x^{17}yz^2$  in  $(x + y + z)^{20}$ ? [Ans: 3,420]

**Problem 4.** (10pt) Using the theory of dearrangements, showing all your work, and fully explaining your reasoning, answer the following:

- (a) Find all the dearrangements of the set  $S = \{1, 2, 3\}$ .
- (b) How many dearrangements are there for a set with four elements? [Ans: 9]
- (c) Approximate how many dearrangements there are for a set with 10 elements. [Ans: 1,334,961]