MAT 101: Exam 4
Summer – 2022
06/16/2022
85 Minutes

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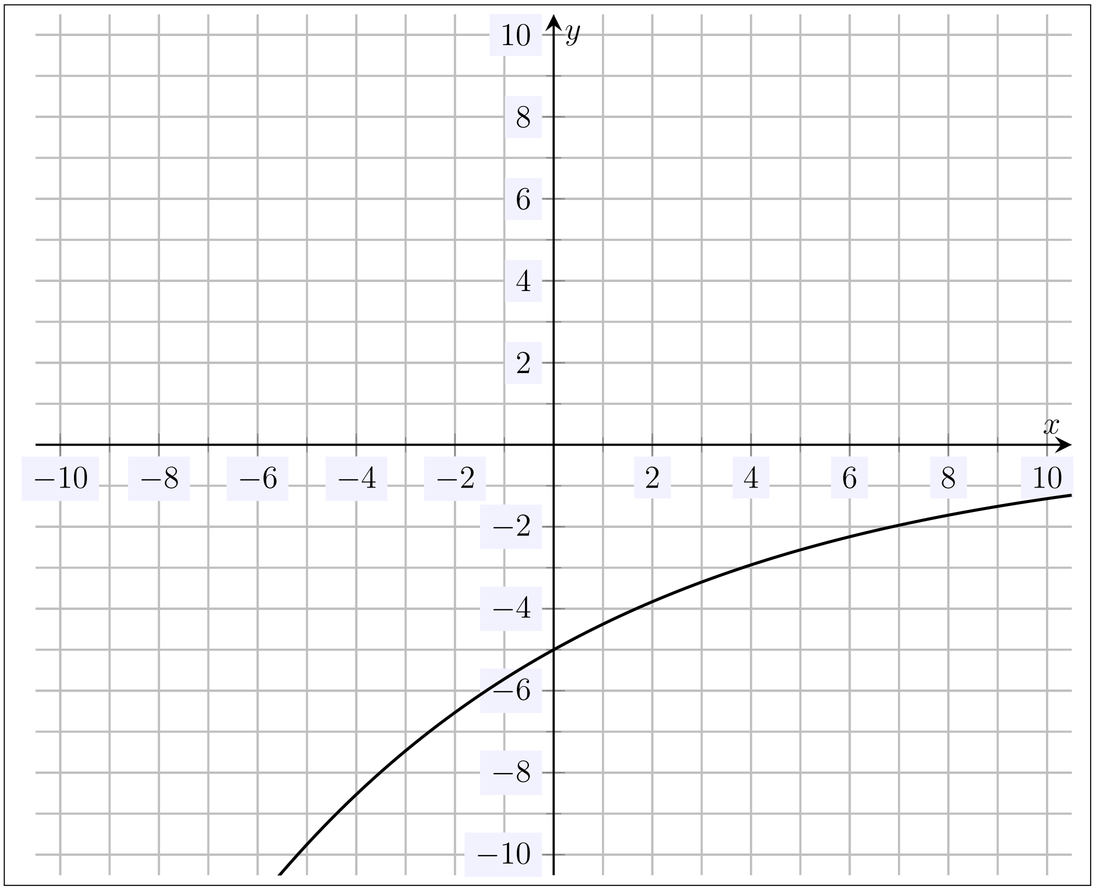
Write your name on the appropriate line on the exam cover sheet. This exam contains 21 pages (including this cover page) and 20 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
14	10	
15	10	
16	10	
17	10	
18	10	
19	10	
20	10	
Total:	200	

1. (10 points) Plot the quadratic function $f(x) = -5\left(\frac{7}{8}\right)^x$ as accurately as possible.



The function $f(x) = -5\left(\frac{7}{8}\right)^x$ *is in the form* Ab^{cx}. *Because* $b = \frac{7}{8} < 1$, $c = 1 > 0$, *and* $a = -5 < 0$, *we know that the function* $f(x)$ *is increasing. Because* $a < 0$, *we know that* $f(x)$ *is always negative. We know also that the* y*-intercept is given by* $f(0) = -5\left(\frac{7}{8}\right)^0 = -5 \cdot 1 = -5$, *so that the* y*-intercept is* $(0, -5)$. *Putting this information gives the sketch above.*

2. (10 points) Showing all your work, write the exponential function below in the form $y = Ab^x$.

$$-3\left(\frac{2}{7}\right)^{2x-1}$$

$$\begin{aligned}
-3\left(\frac{2}{7}\right)^{2x-1} &= -3 \cdot \left(\frac{2}{7}\right)^{2x} \cdot \left(\frac{2}{7}\right)^{-1} \\
&= -3 \cdot \left(\left(\frac{2}{7}\right)^{2}\right)^{x} \cdot \frac{7}{2} \\
&= -\frac{21}{2}\left(\frac{4}{49}\right)^{x}
\end{aligned}$$

3. (10 points) Fully justifying your answer, determine whether each of the following functions exhibit exponential growth or exponential decay:

   (a) $-7\left(\dfrac{12}{11}\right)^x$

   (b) $5\left(\dfrac{2}{3}\right)^{-x}$

   (c) $4(2^{1-x})$

Solution.

(a) The function $-7\left(\dfrac{12}{11}\right)^x$ *is in the form* Ab^{cx}*. Because* $b = \frac{12}{11} > 1$*,* $c = 1 > 0$*, and* $a = -7 < 0$*, we know that the function is decreasing.*

(b) The function $5\left(\dfrac{2}{3}\right)^{-x}$ *is in the form* Ab^{cx}*. Because* $b = \frac{2}{3} < 1$*,* $c = -1 < 0$*, and* $a = 5 > 0$*, we know that the function is increasing.*

(c) The function $4(2^{1-x}) = 4(2^1 2^{-x}) = 8(2^{-x})$ *is in the form* Ab^{cx}*. Because* $b = 2 > 1$*,* $c = -1 < 0$*, and* $a = 8 > 0$*, we know that the function is decreasing.*

4. (10 points) Showing all your work, rewrite each of the following exponential equations as a logarithmic equation or logarithmic equation as an exponential equation:

   (a) $\log_4(x) = y$

   (b) $e^{3y} = x$

   (c) $\log_x(5) = y$

Solution.

(a) $\log_4(x) = y \Longleftrightarrow 4^y = x$

(b) $e^{3y} = x \Longleftrightarrow \ln(x) = 3y$

(c) $\log_x(5) = y \Longleftrightarrow x^y = 5$

5. (10 points) Showing all your work, compute each of the following:

   (a) $\log_3(81)$

   (b) $\ln(1)$

   (c) $\log_5(5^{6741})$

   (d) $\ln(\sqrt[3]{e^2})$

   (e) $\log_{1/2}\left(\frac{1}{\sqrt{2}}\right)$

Solution.

(a) $\log_3(81) = \log_3(3^4) = 4$

(b) $\ln(1) = 0$

(c) $\log_5(5^{6741}) = 6741$

(d) $\ln(\sqrt[3]{e^2}) = \ln(e^{2/3}) = \dfrac{2}{3}$

(e) $\log_{1/2}\left(\dfrac{1}{\sqrt{2}}\right) = \log_{1/2}\left(\left(\dfrac{1}{2}\right)^{1/2}\right) = \dfrac{1}{2}$

6. (10 points) Showing all your work, write the following as a single logarithm:

$$\frac{1}{3}\big(6\ln(x) - 2\ln(y) + \ln(z)\big)$$

$$\frac{1}{3}\big(6\ln(x) - 2\ln(y) + \ln(z)\big)$$

$$2\ln(x) - \frac{2}{3}\ln(y) + \frac{1}{3}\ln(z)$$

$$\ln(x^2) - \ln(y^{2/3}) + \ln(z^{1/3})$$

$$\ln(x^2) - \ln(\sqrt[3]{y^2}) + \ln(\sqrt[3]{z})$$

$$\ln\left(\frac{x^2\sqrt[3]{z}}{\sqrt[3]{y^2}}\right)$$

$$\ln\left(x^2\sqrt[3]{\frac{z}{y^2}}\right)$$

7. (10 points) Assume $\log_4(x) = -3$, $\log_4(y) = 5$, and $\log_4(z) = -2$. Showing all your work, compute the following:

$$\log_4\left(\frac{x^3\sqrt{y}}{z^5}\right)$$

$$\log_4\left(\frac{x^3\sqrt{y}}{z^5}\right)$$

$$\log_4(x^3\sqrt{y}) - \log_4(z^5)$$

$$\log_4(x^3) + \log_4(\sqrt{y}) - \log_4(z^5)$$

$$3\log_4(x) + \frac{1}{2}\log_4(y) - 5\log_4(z)$$

$$3 \cdot -3 + \frac{1}{2} \cdot 5 - 5(-2)$$

$$-9 + \frac{5}{2} + 10$$

$$1 + \frac{5}{2}$$

$$\frac{7}{2}$$

8. (10 points) Showing all your work and simplifying as much as possible, express the following as a single logarithm in base 2:

$$\log_2(x) - \log_4(x) + \log_8(x)$$

$$\log_2(x) - \log_4(x) + \log_8(x)$$

$$\log_2(x) - \frac{\log_2(x)}{\log_2(4)} + \frac{\log_2(x)}{\log_2(8)}$$

$$\log_2(x) - \frac{\log_2(x)}{2} + \frac{\log_2(x)}{3}$$

$$\left(1 - \frac{1}{2} + \frac{1}{3}\right)\log_2(x)$$

$$\left(\frac{6}{6} - \frac{3}{6} + \frac{2}{6}\right)\log_2(x)$$

$$\frac{5\log_2(x)}{6}$$

9. (10 points) Showing all your work, determine how many digits are in 1728^{1729} when expressed in base 10.

10. (10 points) Showing all your work, determine how many digits are in 1728^{1729} when expressed in base 2.

11. (10 points) Showing all your work, determine if there is an integer k such that 3^k has 2022 digits when expressed in base 10. If there is such an integer, find them all. If not, explain why.

12. (10 points) Gerald meets with the banker Vivildi and decides to invest \$212 with him. Vivildi promises a return of 2.7% annual interest, compounded quarterly. How much interest will Gerald have earned after 3 years?

13. (10 points) Filipa takes out a loan of \$1000 at 9.4% annual interest, compounded continuously. How long until Filipa's amount owed has doubled?

14. (10 points) Cirri deposts \$162 in an account which earns 6.6% annual interest, compounded monthly. How long until Cirri's account has \$4000?

15. (10 points) Showing all your work, solve the following equation:

$$15 - 4(3^{1-x}) = 10$$

16. (10 points) Showing all your work, solve the following equation:

$$3\big(\log_7(2x+1)-4\big)=12$$

17. (10 points) Showing all your work, solve the following equation:

$$4e^{x^2} + 20 = 54$$

18. (10 points) Showing all your work, solve the following equation:

$$\log_2(3 - x) - 1 = 1 - \log_2(x + 3)$$

19. (10 points) Showing all your work, solve the following equation:

$$2(7^{6-x}) + 4 = 24$$

20. (10 points) Showing all your work, solve the following equation:

$$\log_2(x-2) = 2 + \log_2\left(\frac{1}{x+1}\right)$$