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MATH 108
Fall 2023
HW 13: Due 12/12

"You can't learn too much linear algebra."
–Benedict Gross

Problem 1. (10pt) Define the following:

$$\mathbf{u} = \begin{pmatrix} 1 \\ -1 \\ 5 \\ 7 \end{pmatrix}, \qquad \mathbf{v} = \begin{pmatrix} 0 \\ 2 \\ 4 \\ -6 \end{pmatrix}, \qquad \mathbf{w} = \begin{pmatrix} 8 \\ 1 \\ 0 \\ 5 \end{pmatrix}$$

Showing all your work, compute the following:

(a) $-3\mathbf{v}$

(b) $\mathbf{w} - \mathbf{u}$

(c) $\mathbf{v} \cdot \mathbf{w}$

Solution.

(a)
$$-3\mathbf{v} = -3\begin{pmatrix} 0 \\ 2 \\ 4 \\ -6 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -12 \\ 18 \end{pmatrix}$$

(b)
$$\begin{pmatrix} 8 \\ 1 \\ 0 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 5 \\ 7 \end{pmatrix} = \begin{pmatrix} 7 \\ 2 \\ -5 \\ -2 \end{pmatrix}$$

(c)
$$\begin{pmatrix} 0 \\ 2 \\ 4 \\ -6 \end{pmatrix} \cdot \begin{pmatrix} 8 \\ 1 \\ 0 \\ 5 \end{pmatrix} = 0(8) + 2(1) + 4(0) + (-6)5 = 0 + 2 + 0 - 30 = -28$$

Problem 2. (10pt) Define the following:

$$A = \begin{pmatrix} 1 & 3 & 0 \\ -2 & 5 & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} 8 & 4 & -1 \\ 2 & 0 & 6 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & 7 & 3 \\ -2 & 6 & 0 \end{pmatrix}$$

Showing all your work, compute the following:

(a) $-4B$

(b) $C - A$

(c) AB^T

Solution.

(a)

$$-4\begin{pmatrix} 8 & 4 & -1 \\ 2 & 0 & 6 \end{pmatrix} = \begin{pmatrix} -32 & -16 & 4 \\ -8 & 0 & -24 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 1 & 7 & 3 \\ -2 & 6 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 3 & 0 \\ -2 & 5 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 4 & 3 \\ 0 & 1 & -2 \end{pmatrix}$$

(c)

$$\begin{aligned}
\begin{pmatrix} 1 & 3 & 0 \\ -2 & 5 & 2 \end{pmatrix}\begin{pmatrix} 8 & 4 & -1 \\ 2 & 0 & 6 \end{pmatrix}^T &= \begin{pmatrix} 1 & 3 & 0 \\ -2 & 5 & 2 \end{pmatrix}\begin{pmatrix} 8 & 2 \\ 4 & 0 \\ -1 & 6 \end{pmatrix} \\
&= \begin{pmatrix} 1(8)+3(4)+0(-1) & 1(2)+3(0)+2(6) \\ -2(8)+5(4)+2(-1) & -2(2)+5(0)+2(6) \end{pmatrix} \\
&= \begin{pmatrix} 8+12+0 & 2+0+12 \\ -16+20-2 & -4+0+12 \end{pmatrix} \\
&= \begin{pmatrix} 20 & 14 \\ 2 & 8 \end{pmatrix}
\end{aligned}$$

Problem 3. (10pt) Define the following:

$$A = \begin{pmatrix} 4 & 6 & 1 & 0 & 5 \\ -1 & 2 & -3 & 0 & 4 \end{pmatrix}, \qquad \mathbf{u} = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \\ -3 \end{pmatrix}$$

(a) Can one compute $A\mathbf{u}$? If so, compute it. If not, explain why.

(b) Can one compute $A^T\mathbf{u}$? If so, compute it. If not, explain why.

Solution.

(a)

$$\begin{pmatrix} 4 & 6 & 1 & 0 & 5 \\ -1 & 2 & -3 & 0 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \\ -3 \end{pmatrix} = \begin{pmatrix} 4(1) + 6(0) + 1(2) + 0(0) + 5(-3) \\ -1(1) + 2(0) + (-3)2 + 0(0) + 4(-3) \end{pmatrix} = \begin{pmatrix} 4 + 0 + 2 + 0 - 15 \\ -1 + 0 - 6 + 0 - 12 \end{pmatrix} = \begin{pmatrix} -9 \\ -19 \end{pmatrix}$$

(b) The matrix A has dimension 2×5. Because the transpose interchanges rows and columns, A^T has dimension 5×2, i.e. 5 rows and 2 columns. The vector $\mathbf{u}$ has dimension 5×1. Because the number of columns of A^T (two) does not match the number of rows of $\mathbf{u}$ (five), one cannot form $A^T\mathbf{u}$.