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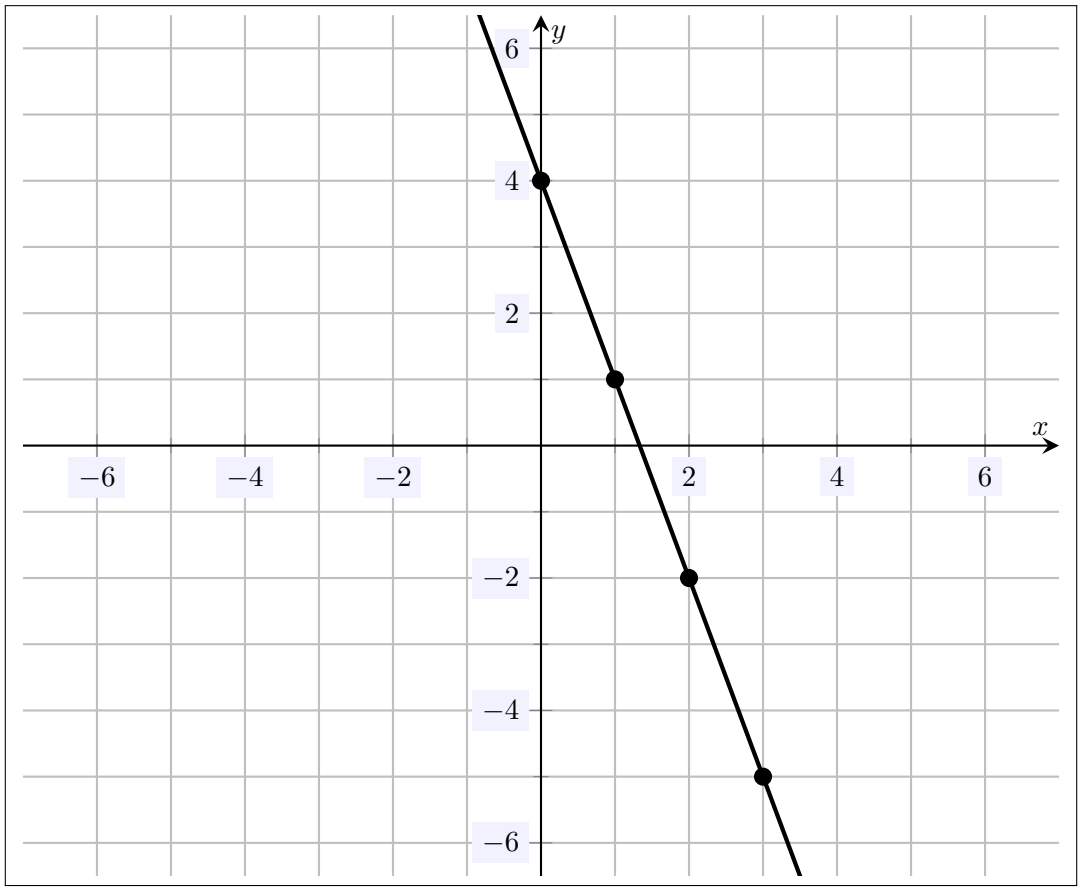
MATH 101
Fall 2021
HW 6: Due 10/08

"I'm fine. It's just that life is pointless and nothing matters and I'm always tired."
–Andy Dwyer, Parks and Recreation

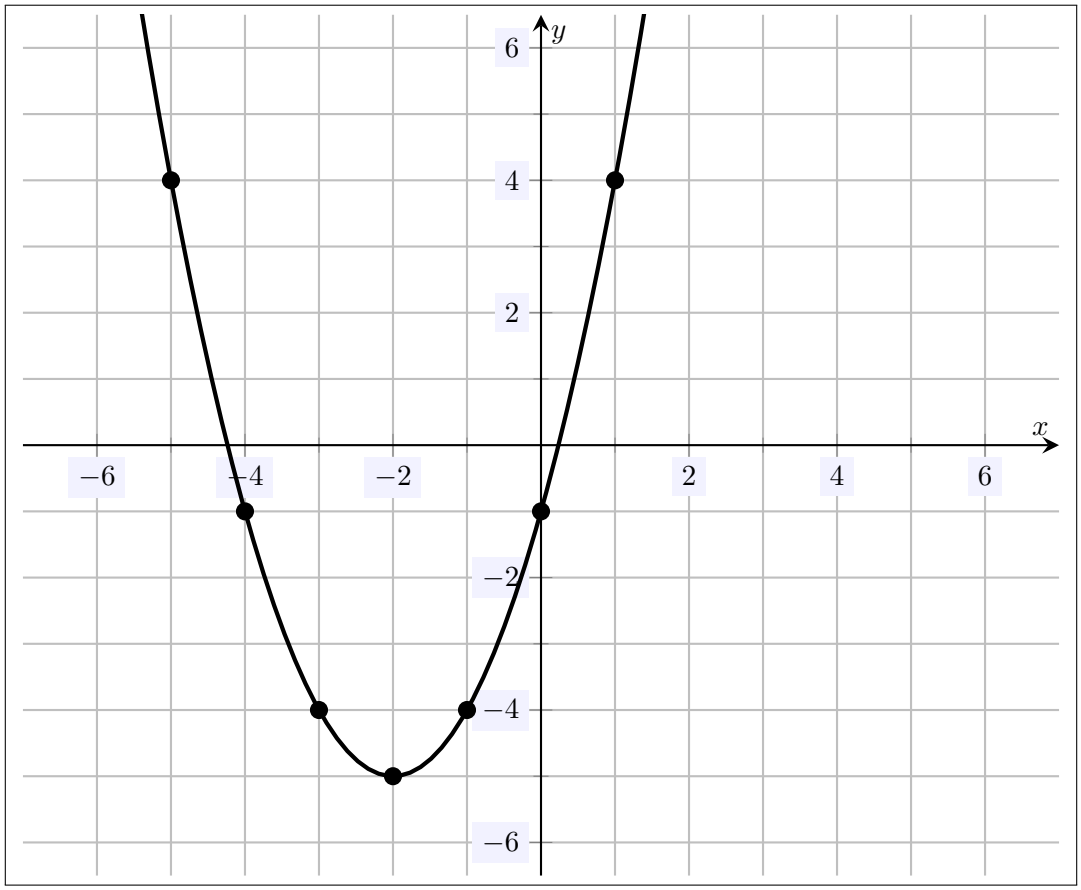
Problem 1. (10pt) Plot the function $f(x) := 4 - 3x$, being as accurate as possible.



Solution. To sketch the line, we should find around 8–12 equally spaces points on the curve and connect them "smoothly."

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	16	13	10	7	4	1	-2	-5	-8

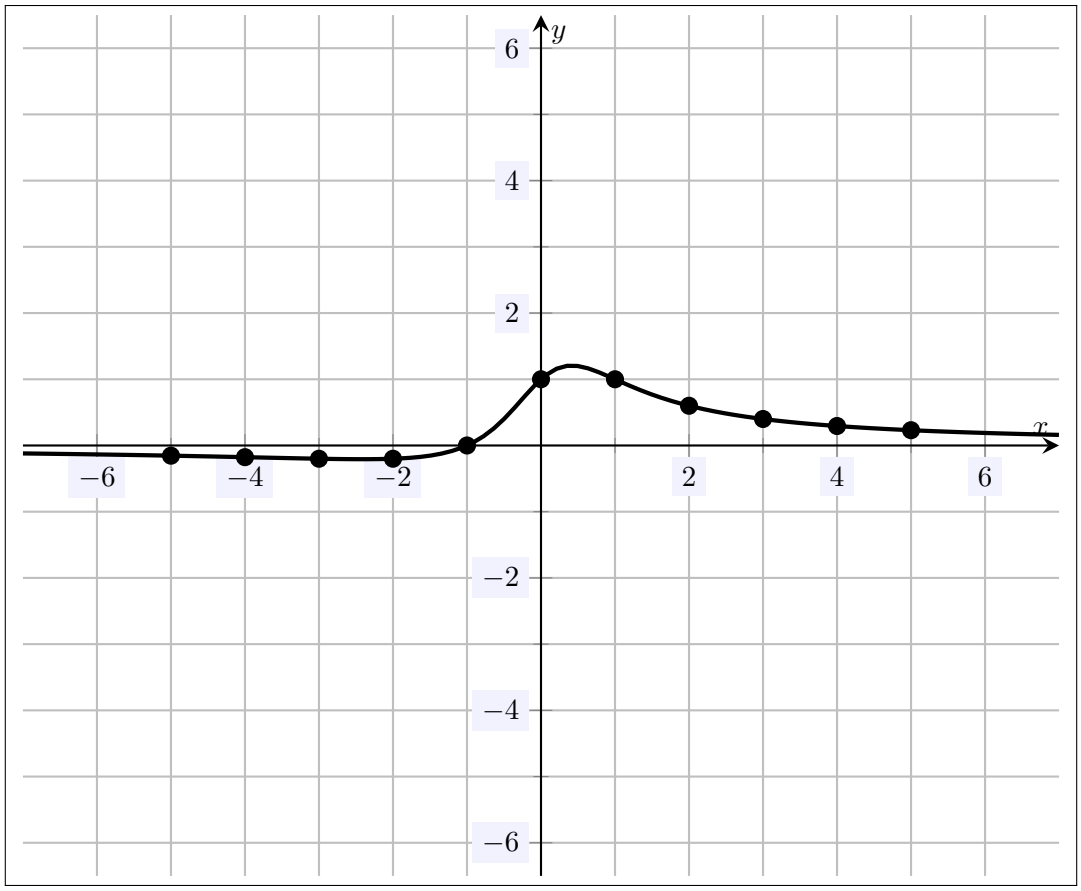
Problem 2. (10pt) Plot the function $f(x) := x^2 + 4x - 1$, being as accurate as possible.



Solution. To sketch the line, we should find around 8–12 equally spaces points on the curve and connect them "smoothly."

x	-5	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	4	-1	-4	-5	-4	-1	4	11	20	41

Problem 3. (10pt) Plot the function $f(x) := \dfrac{x+1}{x^2+1}$, being as accurate as possible.



Solution. To sketch the line, we should find around 8–12 equally spaces points on the curve and connect them "smoothly."

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f(x)$	$-\frac{2}{13}$	$-\frac{3}{17}$	$-\frac{1}{5}$	$-\frac{1}{5}$	0	1	1	$\frac{3}{5}$	$\frac{2}{5}$	$\frac{5}{17}$	$\frac{3}{13}$

Problem 4. (10pt) Let $f(x) := 5x - 3$.

(a) Find $f(1)$.

$$f(1) = 5(1) - 3 = 5 - 3 = 2$$

(b) What value(s) for x make the output of $f(x)$ twice the output from (a)?

The output from (a) was 2. So twice this would be 4. We want x so that $f(x) = 4$. But then

$$\begin{aligned} 5x - 3 &= 4 \\ 5x &= 7 \\ x &= \frac{7}{3} \end{aligned}$$

(c) Is $(1, 2)$ on the graph of $f(x)$? Explain.

We have $f(1) = 5(1) - 3 = 5 - 3 = 2$ so that $(1, 2)$ is a point on the graph. But this is exactly the given point. Alternatively, because $y = 5x - 3$, we check $2 \stackrel{?}{=} 5(1) - 3 = 2$ so that $(1, 2)$ is on the graph of $f(x)$.

(d) Is $(3, 5)$ on the graph of $f(x)$? Explain.

We have $f(3) = 5(3) - 3 = 15 - 3 = 12$ so that $(3, 12)$ is a point on the graph. But this is not the point $(3, 5)$. Therefore, $(3, 5)$ is not a point on the graph of $f(x)$. Alternatively, because $y = 5x - 3$, we check $5 \stackrel{?}{=} 5(3) - 3 = 12$, which is false so that $(3, 5)$ is not on the graph of $f(x)$.

Problem 5. (10pt) Define the following functions:

$$f(x) := x^3 - x$$
$$g(x) := x^2 - 2x + 3$$
$$h(x) := x^4 + x^2$$

Determine if the functions $f(x)$, $g(x)$ and $h(x)$ are even functions, odd functions, or neither. Be sure to justify your answer.

Solution. *First, observe that...*

$$f(-x) = (-x)^3 - (-x) = -x^3 + x = -(x^3 - x) = -f(x)$$

Therefore, $f(x)$ is odd because $f(-x) = -f(x)$. [Hence, $f(x)$ will be symmetric about the origin.] Next, observe that...

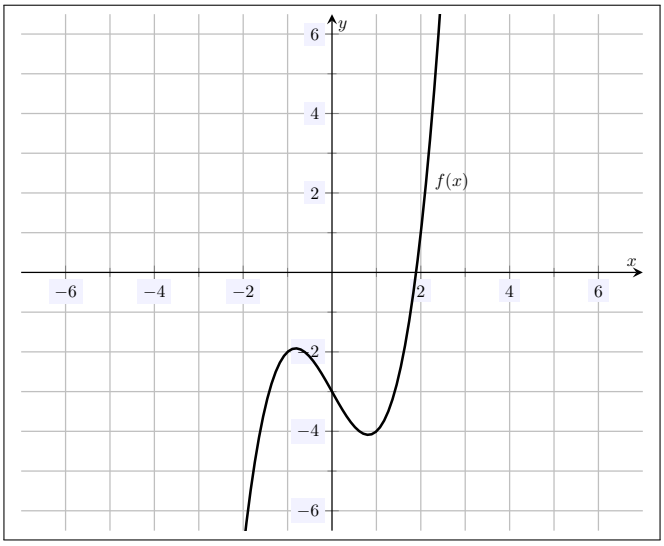
$$g(-x) = (-x)^2 - 2(-x) + 3 = x^2 + 2x + 3$$

But then $g(-x) \neq -g(x)$ and $g(-x) \neq g(x)$; therefore, $g(x)$ is neither odd nor even. Finally, observe that...

$$h(-x) = (-x)^4 + (-x)^2 = x^4 + x^2 = h(x)$$

Therefore, $h(x)$ is even because $h(-x) = h(x)$. [Hence, $h(x)$ will be symmetric about the y-axis.]

Problem 6. (10pt) Consider the function $f(x)$ plotted below.



(a) What is $f(1)$?

From the graph, we see that $f(1) = -4$.

(b) Is the point $(2, 1)$ on the graph of $f(x)$? Explain.

Because $f(x)$ *passes through the point* $(2, 1)$*, the point* $(2, 1)$ *is on the graph of* $f(x)$.

(c) Is the point $(-2, -2)$ on the graph of $f(x)$? Explain.

We can see that $f(x)$ *does not pass through the point* $(-2, -2)$*. Therefore,* $(-2, -2)$ *is not on the graph of* $f(x)$.

(d) Is the function $f(x)$ even, odd, or neither. Explain.

Because $f(x)$ *is not symmetric across the* y*-axis and also not symmetric through the origin,* $f(x)$ *can be neither even nor odd.*