Quiz 1. *True/False*: If you had a bill of \$25.77 and were going to pay a tip of 20%, the total amount you would pay could be computed by finding $25.77(1.20)$.

Solution. The statement is *true*. Recall to calculate a percentage of a number N, we compute $N \cdot \%$, where N is the number and $\%$ is the percentage (written as a decimal). For instance, to compute 57% of 23, we compute $23(0.57) = 13.11$. To compute 172% of 150, we compute $150(1.72) = 258$. However, to compute a $\%$ percent increase or decrease of a number N, we compute $N(1 \pm \%)$, where N is the number, $\%$ is the percentage as a decimal, and we choose plus for increase and negative for decrease. For instance, to compute a 75% decrease of 13, we compute $13(1 - 0.75) = 13(0.25) = 3.25$. To compute a 115% increase of 120, we compute $120(1+1.15) = 120(2.15) = 258$. Here, we are increasing 25.77 by 20%, so we compute $25.77(1 + 0.20) = 25.77(1.20)$.

Quiz 2. *True/False*: The amount of concrete in tons, C, used to repair r roads remaining in a storage facility is given by $C(r) = 450.7 - 16.3r$. Because this function is linear, we can interpret the slope of $C(r)$ as saying that each road uses approximately 16.3 tons of concrete to repair.

Solution. The statement is *true*. The slope of the linear function $C(r) = 450.7 - 16.3r$ is. . .

$$m = -16.3 = -\frac{16.3}{1} = \frac{-16.3}{1}$$

Thinking of this slope as $\frac{\Delta\text{output}}{\Delta\text{input}}$, we can see that for each one increase in r, i.e. one additional road, there is a decrease by 16.3 tons in the amount of concrete remaining. Therefore, we can summarize this as that each road requires approximately 16.3 tons of concrete to repair.

Quiz 3. *True/False*: A company sells a product for $\$5.75$ per item. Each item costs approximately $\$1.37$ to manufacture and is produced in a machine that costs $\$87.50$ to operate. Given this data, we have $R(x) = 5.75$ and $C(x) = (1.37 + 87.50)x = 88.88x$.

Solution. The statement is *false*. If one sells x items, the revenue is $R(x) = 5.75 \cdot 7 = 5.75x$. Therefore, $R(x)$ is correct. However, we know that $C(x) = \text{VC} + \text{FC}$. The fixed costs are the machine operation costs, i.e. $\text{FC} = \$87.50$. The variable costs are the $\$1.37$ cost per item. If x items are produced, then the manufacture costs are $\text{VC} = 1.37 \cdot x = 1.37x$. Therefore, $C(x) = \text{VC} + \text{FC} = 1.37x + 87.50$.

Quiz 4. *True/False*: If the following matrix represents an augmented matrix in RREF, then the corresponding system has solution $x_1 = 5$, $x_2 = -3$, and $x_3 = 7$.

$$\begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Solution. The statement is *false*. Examining the equation corresponding to the last row, we see that $0 = 1$, which is impossible. Therefore, the original system of equations was inconsistent. But then the original system of equations has no solution.

Quiz 5. *True/False*: You can perform the following multiplication:

$$\begin{pmatrix} 1 & -1 & 0 & 5 & 3 \\ 0 & 4 & -2 & 6 & 1 \end{pmatrix} \begin{pmatrix} 3 & -2 \\ 3 & 8 \\ 4 & 0 \\ 2 & -1 \\ 0 & 5 \end{pmatrix}$$

Solution. The statement is *true*. Recall that you can multiply a $m \times n$ matrix with a $p \times q$ matrix if $n = p$. If so, you obtain a $m \times q$ matrix. The first matrix is 2×5 while the second matrix is 5×2. But because $5 = 5$, we can multiply these matrix to obtain a 2×2 matrix. One can check that the product is...

$$\begin{pmatrix} 10 & 0 \\ 16 & 31 \end{pmatrix}$$

Quiz 6. *True/False*: The matrix $\begin{pmatrix} -2 & 8 \\ -2 & 6 \end{pmatrix}$ has an inverse.

Solution. The statement is *true*. Recall that a matrix has an inverse if and only if the determinant of the matrix is *not* zero. We have...

$$\begin{pmatrix} -2 & 8 \\ -2 & 6 \end{pmatrix} = -2(6) - 8(-2) = -12 + 16 = 4 \neq 0$$

Therefore, the matrix is invertible. Recalling that if A is a 2×2 matrix (given below) that is invertible, we have...

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$
$$A^{-1} = \frac{1}{\det A} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Therefore,

$$\begin{pmatrix} -2 & 8 \\ -2 & 6 \end{pmatrix}^{-1} = \frac{1}{4} \begin{pmatrix} 6 & -8 \\ 2 & -2 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} & -2 \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

Quiz 7. *True/False*: The point $(1,-3)$ satisfies the following system of inequalities:

$$\begin{aligned} x+y &\leq 0 \\ x-2y &\leq 5 \end{aligned}$$

Solution. The statement is *false*. If a point satisfies a system of inequalities, it satisfies each of the inequalities individually—which we can check:

$$\begin{aligned} x+y &\leq 0 \\ 1+(-3) &\stackrel{?}{\leq} 0 \\ -2 &\leq 0 \checkmark \end{aligned} \qquad\qquad \begin{aligned} x-2y &\stackrel{?}{\leq} 5 \\ 1-2(-3) &\stackrel{?}{\leq} 5 \\ 1+6 &\stackrel{?}{\leq} 5 \\ 7 &\not\leq 5 \text{ ✗} \end{aligned}$$

Because $(1,-3)$ does not satisfy all the inequalities, $(1,-3)$ does not satisfy the system of inequalities.

Quiz 8. *True/False*: To maximize $z = 5x+6y$ subject to $2x+3y \leq 6$, $-6x+y \leq 20$, and $x, y \geq 0$, the initial simplex tableau is...

$$\begin{array}{rrrr|r} 2 & 3 & 1 & 0 & 6 \\ -6 & 1 & 0 & 1 & 20 \\ \hline -5 & -6 & 0 & 0 & 0 \end{array}$$

Solution. The statement is *true*. First, note that the problem is in standard form. For each inequality, we introduce a slack variable so that we have...

$$\begin{aligned} 2x+3y+s_1 \qquad &= 6 \\ -6x+y \qquad +s_2 &= 20 \end{aligned}$$

Moving all the variables to the left side in $z = 5x+6y$, we have $z-5x-6y = 0$. Aligning the equations, we have...

$$\begin{aligned} 2x+3y+s_1 \qquad &= 6 \\ -6x+y \qquad +s_2 &= 20 \\ z-5x-6y \qquad\qquad &= 0 \end{aligned}$$

This gives us the initial simplex tableau...

$$\begin{array}{rrrr|r} 2 & 3 & 1 & 0 & 6 \\ -6 & 1 & 0 & 1 & 20 \\ \hline -5 & -6 & 0 & 0 & 0 \end{array}$$

Quiz 9. *True/False*: Given the following minimization problem:

$$\begin{aligned} \min w = x_1 + 2x_2 + 3x_3 \\ x_1 + x_2 + x_3 \geq 4 \\ x_1 - x_2 + x_3 \geq 6 \\ -x_1 + x_2 - x_3 \geq 8 \end{aligned}$$

the dual problem is...

$$\begin{pmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 2 \\ 1 & 1 & -1 & 3 \\ 4 & 6 & 8 & 0 \end{pmatrix}$$

Solution. The statement is *false*. First, observe that the problem is in standard form. Given a minimization problem in standard form, we first align all the inequalities with the function as the bottom equation:

$$\begin{aligned} x_1 + x_2 + x_3 \geq 4 \\ x_1 - x_2 + x_3 \geq 6 \\ -x_1 + x_2 - x_3 \geq 8 \\ x_1 + 2x_3 + 3x_3 = 0 \end{aligned}$$

From this, we form the matrix...

$$\begin{pmatrix} 1 & 1 & 1 & 4 \\ 1 & -1 & 1 & 6 \\ -1 & 1 & -1 & 8 \\ 1 & 2 & 3 & 0 \end{pmatrix}$$

We then take the transpose of this matrix...

$$\begin{pmatrix} 1 & 1 & 1 & 4 \\ 1 & -1 & 1 & 6 \\ -1 & 1 & -1 & 8 \\ 1 & 2 & 3 & 0 \end{pmatrix}^T = \begin{pmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 2 \\ 1 & 1 & -1 & 3 \\ 4 & 6 & 8 & 0 \end{pmatrix}$$

But then the corresponding maximization problem (the dual problem) is...

$$\begin{aligned} \max z = 4y_1 + 6y_2 + 8y_3 \\ y_1 + y_2 - y_3 \leq 1 \\ y_1 - y_2 + y_3 \leq 2 \\ y_1 + y_2 - y_3 \leq 3 \end{aligned}$$

Quiz 10. *True/False*: If \$4,000 is placed into an account that earns 5% interest, compounded quarterly, then the amount of money in the account after 8 years is...

$$4000\left(1+\frac{0.05}{4}\right)^{32}$$

Solution. The statement is *true*. We know that if P dollars is placed into an account earning an annual interest rate r, compounded k times per year, then the amount of money in the account after t years, F, is given by...

$$F = P\left(1+\frac{r}{k}\right)^{kt}$$

We have $P = 4000$, $r = 0.05$, $k = 4$, and $t = 8$ so that we have...

$$F = 4000\left(1+\frac{0.05}{4}\right)^{4\cdot 8} = 4000\left(1+\frac{0.05}{4}\right)^{32}$$

Quiz 11. *True/False*: An ordinary annuity is a series of equal payments, paid at equal intervals of time with payments occurring at the start of the payment period.

Solution. The statement is *false*. In an ordinary (simple) annuity, payments are made at the *end* of each payment period and the payment periods are the same as the interest periods (otherwise, it is a general ordinary annuity). If the payments are made at the *start* of each payment period and the payment periods are the same as the interest periods, then it is a (simple) annuity due (otherwise, it is a general annuity due). We can summarize this in the following chart:

	Payment at Start of Period	Payment at End of Period
Payment Period = Compounding Period	Ordinary Annuity Due	Ordinary (Simple) Annuity
Payment Period ≠ Compounding Period	General Annuity Due	General Annuity

Quiz 12. *True/False*: If you have a \$5,000 loan at 5.4% annual interest, compounded monthly, that you pay over 5 years using a series of monthly payments of \$95.28, then the amount you still owe on the loan after 2 years is...

$$P = 5000\, a_{\overline{36}|0.0045} = 3160.11$$

Solution. The statement is *false*. We know that the amount due on an (ordinary) amortized loan is given by $P = R\, a_{\overline{n-m}|i}$. We have $R = 95.28$, $i_p = r/k = 0.054/12 = 0.0045$, $n = kt = 12 \cdot 5 = 60$, and $m = kt_0 = 12 \cdot 2 = 24$. Then we have...

$$P = 95.28\, a_{\overline{36}|0.0045} = 3160.11$$