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MATH 108
Spring 2023
HW 13: Due 05/01

"There is hardly any theory which is more elementary [than linear algebra], in spite of the fact that generations of professors and textbook writers have obscured its simplicity by preposterous calculations with matrices."

–Jean Dieudonné

Problem 1. (10pt) Define $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ to be the vectors given below:

$$\mathbf{u} = \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix}, \qquad \mathbf{v} = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}, \qquad \mathbf{w} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$$

Compute the following:

(a) $-3\mathbf{v}$

(b) $\mathbf{w} - \mathbf{u}$

(c) $2\mathbf{u} + \mathbf{v}$

(d) $\mathbf{v} \cdot \mathbf{w}$

Solution.

(a)

$$-3\mathbf{v} = -3 \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ 9 \end{pmatrix}$$

(b)

$$\mathbf{w} - \mathbf{u} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 2-1 \\ -1-(-3) \\ 4-5 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

(c)

$$2\mathbf{u} + \mathbf{v} = 2 \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 2 \\ -6 \\ 10 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 2+0 \\ -6+2 \\ 10+(-3) \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 7 \end{pmatrix}$$

(d)

$$\mathbf{v} \cdot \mathbf{w} = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} = 0(2) + 2(-1) + (-3)4 = 0 - 2 - 12 = -14$$

Problem 2. (10pt) Define the following:

$$A = \begin{pmatrix} 1 & 0 & -3 \\ 2 & -1 & 1 \\ 0 & 5 & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 3 & 0 \\ -1 & 4 & -2 \end{pmatrix}, \qquad \mathbf{u} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

Compute the following:

(a) $2B$

(b) AB

(c) BA

(d) $A\mathbf{u}$

Solution.

(a)

$$2B = 2\begin{pmatrix} 2 & 1 & 1 \\ 0 & 3 & 0 \\ -1 & 4 & -2 \end{pmatrix} = \begin{pmatrix} 4 & 2 & 2 \\ 0 & 6 & 0 \\ -2 & 8 & -4 \end{pmatrix}$$

(b)

$$\begin{aligned} AB &= \begin{pmatrix} 1 & 0 & -3 \\ 2 & -1 & 1 \\ 0 & 5 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 & 1 \\ 0 & 3 & 0 \\ -1 & 4 & -2 \end{pmatrix} \\ &= \begin{pmatrix} 1(2) + 0(0) + (-3)(-1) & 1(1) + 0(3) + (-3)4 & 1(1) + 0(1) + (-3)(-2) \\ 2(2) + (-1)0 + 1(-1) & 2(1) + (-1)3 + 1(4) & 2(1) + (-1)0 + 1(-2) \\ 0(2) + 5(0) + 2(-1) & 0(1) + 5(3) + 2(4) & 0(1) + 0(0) + 2(-2) \end{pmatrix} \\ &= \begin{pmatrix} 2+0+3 & 1+0-12 & 1+0+6 \\ 4+0-1 & 2-3+4 & 2+0-2 \\ 0+0-2 & 0+15+8 & 0+0-4 \end{pmatrix} \\ &= \begin{pmatrix} 5 & -11 & 7 \\ 3 & 3 & 0 \\ -2 & 23 & -4 \end{pmatrix} \end{aligned}$$

(c)

$$BA = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 3 & 0 \\ -1 & 4 & -2 \end{pmatrix} \begin{pmatrix} 1 & 0 & -3 \\ 2 & -1 & 1 \\ 0 & 5 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 2(1)+1(2)+1(0) & 2(0)+1(-1)+1(5) & 2(-3)+1(1)+1(2) \\ 0(1)+3(2)+(-1)0 & 0(0)+3(-1)+0(5) & 0(-3)+3(1)+0(2) \\ (-1)1+4(2)+(-2)0 & -1(0)+4(-1)+(-2)5 & -1(-3)+4(1)+(-2)2 \end{pmatrix}$$

$$= \begin{pmatrix} 2+2+0 & 0-1+5 & -6+1+2 \\ 0+6+0 & 0-3+0 & 0+3+0 \\ -1+8+0 & 0-4-10 & 3+4-4 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 4 & -3 \\ 6 & -3 & 3 \\ 7 & -14 & 3 \end{pmatrix}$$

(d)

$$A\mathbf{u} = \begin{pmatrix} 1 & 0 & -3 \\ 2 & -1 & 1 \\ 0 & 5 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1(1)+0(-1)+(-3)1 \\ 2(1)+(-1)(-1)+1(1) \\ 0(1)+5(-1)+2(1) \end{pmatrix} = \begin{pmatrix} 1+0-3 \\ 2+1+1 \\ 0-5+2 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ -3 \end{pmatrix}$$