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MATH 307
Spring 2023
HW 6: Due 02/27 (28)

"The laws of probability, so true in general, so fallacious in particular."
–Edward Gibbon

Problem 1. (10pt) The probabilities of several events in a finite probability space are given below:

$$\begin{array}{ll} P(A) = 0.45 & P(A \mid C) = 0.10 \\ P(B) = 0.50 & P(B \cap C) = 0.25 \\ P(C) = 0.30 & P(A \cap B) = 0.15 \end{array}$$

(a) Find $P(A \cup B)$.

(b) Find $P(B \mid C)$.

(c) Find $P(A \cap C)$.

(d) If A and C were independent, find $P(A \cap C)$.

(e) Using (c) and (d), determine if A and C are independent.

Solution.

(a) We have...

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.45 + 0.50 - 0.15 \approx 0.80$$

(b) We have...

$$P(B \mid C) = \frac{P(B \cap C)}{P(C)} = \frac{0.25}{0.30} \approx 0.8333$$

(c) We have...

$$P(A \cap C) = P(C)P(A \mid C) = 0.30 \cdot 0.10 \approx 0.03$$

(d) If A and C were independent, we would have...

$$P(A \cap C) = P(A) \cdot P(C) = 0.45 \cdot 0.30 \approx 0.135$$

(e) If A and C were independent, then $P(A \cap C) = P(A) \cdot P(C)$. From (c), we know that $P(A \cap C) = 0.03$. But in (d), we found $P(A) \cdot P(C) = 0.135$. But then $P(A \cap C) \neq P(A) \cdot P(C)$. Therefore, A and C are not independent.

Problem 2. (10pt) Dr. Graham's 5th grade class has created a table of the number of times a school in the district has delayed or cancelled school in the last decade based on the amount of snow they received the night before. Their results are summarized below.

	0" – 0.9"	1" – 2.9"	2.9" – 4.9"	≥ 5"	Total
On-Time	1200	800	350	5	2355
Delayed	26	54	875	1022	1977
Canceled	0	3	55	410	468
Total	1226	857	1280	1437	4800

(a) What percentage of the time did schools cancel when it snowed?

(b) What percentage of the time that it snowed did they receive 1" – 2.9"?

(c) On days when it snowed more than 5", what percentage of schools cancelled school?

(d) On days when schools delayed, what percentage of those days did it snow between 0" and 0.9"?

Solution.

(a) We have...

$$P(\text{cancel and snowed}) = \frac{468}{4800} \approx 0.0975$$

Therefore, schools cancelled 9.75% of the time when it snowed.

(b) We have...

$$P(\text{receive 1" – 2.9"}) = \frac{857}{4800} \approx 0.1785$$

Therefore, the district receives 1" to 2.9" of snow 17.85% of the time.

(c) We have...

$$P(\text{cancel} \mid \text{more than 5"}) = \frac{P(\text{cancel with more than 5"})}{P(\text{more than 5"})} = \frac{410}{1437} \approx 0.2853$$

Therefore, on days when they received more than 5" of snow, schools cancelled 28.53% of the time.

(d) We have...

$$P(\text{between 0" and 0.9"} \mid \text{delayed}) = \frac{P(\text{delayed and between 0" and 0.9"})}{P(\text{delayed})} = \frac{26}{1977} \approx 0.0132$$

Therefore, on days when schools delayed, it was when there was only 0" to 0.9" of snow 1.32% of the time.

Problem 3. (10pt) Mr. Bobbert has a 4th grade class with 30 students. In this class, there are 18 students that like sports, 9 students that like video games, and 8 students that like both.

(a) Find the probability that a randomly selected student like sports or video games.

(b) Find the probability that a randomly selected student only likes sports.

(c) Find the probability that a randomly selected student does not like sports nor video games.

(d) If a student enjoys video games, find the probability that they also enjoy sports.

Solution. We know that there are 18 students that like sports, including the 8 that like sports and video games. Therefore, $18 - 8 = 10$ students like only sports. Similarly, $9 - 8$ students like only video games. Then the total number of students like that sports or video games is $10 + 8 + 1 = 19$. Therefore, $30 - 19 = 11$ students that like neither. Then...

(a) We have...

$$P(\text{sports or video games}) = \frac{10+8+1}{30} = \frac{19}{30} \approx 0.6333$$

(b) We have...

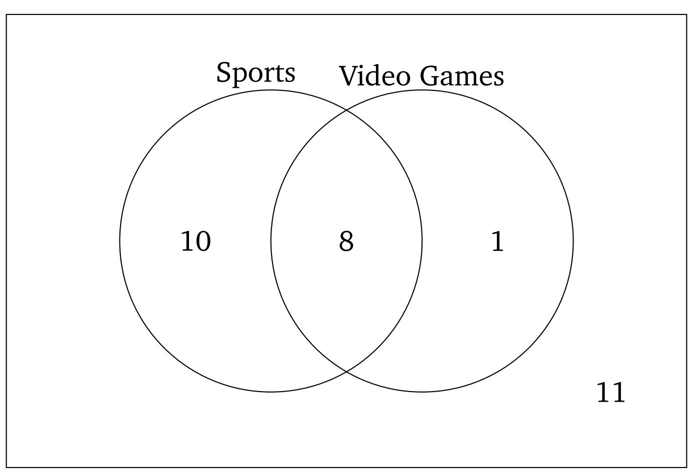
$$P(\text{only sports}) = \frac{10}{30} \approx 0.3333$$

(c) We have...

$$P(\text{neither}) = \frac{11}{30} \approx 0.3667$$

(d) We have...

$$P(\text{sports} \mid \text{video games}) = \frac{P(\text{sports and video games})}{P(\text{video games})} = \frac{8}{8+1} = \frac{8}{9} \approx 0.8889$$



Problem 4. (10pt) Ms. Streikert has a 6th grade English class. She finds that there is a 75% chance that a student studies for an exam. If a student studies for an exam, there is an 90% chance that they pass the exam. If a student does not study for an exam, there is a 85% chance that they fail the exam.

(a) Find the probability that a randomly selected student fails the exam.

(b) Find the probability that a randomly selected student studies or passes the exam.

(c) Find the probability that a randomly selected student both studies and fails the exam.

(d) If a student fails the exam, find the probability that they did not study.

Solution.

(a) We have...

$$P(\text{fails}) = 0.0750 + 0.2125 = 0.2875$$

(b) We have...

$$P(\text{studies or passes}) = 0.6750 + 0.0750 + 0.0375 = 0.7875$$

(c) We have...

$$P(\text{studies and fails}) = 0.0750$$

(d) We have...

$$P(\text{not study} \mid \text{fails}) = \frac{P(\text{not study and fails})}{P(\text{fails})} = \frac{0.2125}{0.0750 + 0.2125} = \frac{0.2125}{0.2875} \approx 0.73913$$