

Name: _____

MATH 308

Fall 2023

HW 15: Due 11/14

“One reason why Combinatorics has been slow to become accepted as part of mainstream Mathematics is the common belief that it consists of a bag of isolated tricks, a number of areas. . . with little or no connexion between them. We shall see that they have numerous threads weaving them together into a beautifully patterned tapestry.”

–Richard K. Guy

Problem 1. (10pt) Showing all your work, complete the following:

- (a) Writing $(3x^2 - \frac{1}{x})^{12}$ in descending power of x , find the ‘middle’ term.
- (b) Find the coefficient of x^3y^2 in $(2x - 3y)^5$.
- (c) Find the coefficient of x^2yz^3 in $(2x - y + 3z)^6$

Problem 2. (10pt) If you distribute 27 tasks amongst 6 people, what is the maximum number of tasks a person can be assigned? What is the minimum number of tasks someone can be assigned to? Does there have to be a person with at least 5 tasks assigned to them? Explain. Does there have to be a person with at most 4 tasks assigned to them? Explain.

Problem 3. (10pt) Consider the set of positive integers less than 10,000. How many of these numbers are...

- (a) ...divisible by 5 or 7?
- (b) ...divisible by 2 and 5 but not 20?
- (c) ...divisible by 2, 3, or 5?