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MATH 100
Fall 2022
HW 14: Due 11/07

"A business that makes nothing but money is a poor business."
–Henry Ford

Problem 1. (10pt) If you place \$620 in a savings account that earns 1.3% annual interest, compounded monthly, find the amount that you have after 8 years.

Solution. We know that if P dollars accumulating interest at an annual interest rate of r, compounded k times year, then the amount after t years is $P\left(1+\frac{r}{k}\right)^{kt}$. The initial amount of money is $P = \$620$. The annual interest rate is $r = 0.013$, compounded each month, i.e. $k = 12$ times per year. Then after $t = 8$ years, we have...

$$\$620\left(1+\frac{0.013}{12}\right)^{12\cdot 8} = \$620(1.0010833)^{96} = \$620(1.1095344) \approx \$687.91$$

Therefore, the amount in the account after 8 years is \$687.91.

Problem 2. (10pt) Suppose that you take out a loan for \$1,500 at 7.1% annual interest, compounded daily, for a period of 2 years. Find the amount of interest that you pay on the loan.

Solution. We know that if P dollars accumulating interest at an annual interest rate of r, compounded k times year, then the amount after t years is $P\left(1+\frac{r}{k}\right)^{kt}$. The initial amount of money is $P = \$1500$. The annual interest rate is $r = 0.071$, compounded each day, i.e. $k = 365$ times per year. Then after $t = 2$ years, we have...

$$\$1500\left(1+\frac{0.071}{365}\right)^{365\cdot 2} = \$1500(1.00019452)^{730} = \$1500(1.15256) \approx \$1728.84$$

Therefore, the owed on the loan after 4 years is \$1,728.84. But because the original amount of the loan was \$1,500, the rest must be interest. Therefore, one pays $\$1,728.84 - \$1,500 = \$228.84$ in interest.

Problem 3. (10pt) Suppose that you plan on saving \$3,000 to put down on a car. You place \$2,600 into an account which earns 2% annual interest, compounded quarterly. How long until you have enough money in the account to put down for the car?

Solution. We know that if P dollars accumulating interest at an annual interest rate of r, compounded k times year, then the amount of years, t, required to reach F dollars is $t = \frac{\ln(F/P)}{k\ln(1+r/k)}$. The initial amount of money is $P = \$2600$. The annual interest rate is $r = 0.02$, compounded each quarter, i.e. $k = 4$ times per year. The amount desired is $F = \$3000$. We then have...

$$\frac{\ln(\$3000/\$2600)}{4\ln(1+0.02/4)} = \frac{\ln(1.15385)}{4\ln(1.005)} = \frac{\ln(1.15385)}{4(0.00498754)} = \frac{0.143104}{0.0199502} = 7.173$$

Therefore, the amount of time required to save \$2,600 is 7.173 years, i.e. 7 years, 2 months, and 2.31 days.