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MATH 101
Spring 2022
HW 15: Due 05/10

"If you think you can do a thing or think you can't do a thing, you're right."
–Henry Ford

Problem 1. (10pt) Write the following functions in the form $y = Ab^x$ and determine whether the function is increasing or decreasing:

(a) $f(x) = -6(7^{-2x+1})$

(b) $g(x) = 8\left(\dfrac{16}{9}\right)^{x/2}$

(c) $h(x) = -15\left(\dfrac{1}{2}\right)^{1-x}$

Solution.

(a) We have...

$$f(x) = -6(7^{-2x+1}) = -6(7^{-2x} \cdot 7^1) = -42((7^{-2})^x) = -42\left(\frac{1}{49}\right)^x$$

Because $A = -42 < 0$, $b = \frac{1}{49} < 1$, and $c = 1 > 0$, $f(x)$ is increasing.

(b) We have...

$$g(x) = 8\left(\frac{16}{9}\right)^{x/2} = 8\left(\left(\frac{16}{9}\right)^{1/2}\right)^x = 8\left(\frac{4}{3}\right)^x$$

Because $A = 8 > 0$, $b = \frac{4}{3} > 1$, and $c = 1 > 0$, $g(x)$ is increasing.

(c) We have...

$$h(x) = -15\left(\frac{1}{2}\right)^{1-x} = -15\left(\left(\frac{1}{2}\right)^1 \cdot \left(\frac{1}{2}\right)^{-x}\right) = -\frac{15}{2}\left(\left(\frac{1}{2}\right)^{-1}\right)^x = -\frac{15}{2}\left(2^x\right)$$

Because $A = -\frac{15}{2} < 0$, $b = 2 > 1$, and $c = 1 > 0$, $h(x)$ is decreasing.

Problem 2. (10pt) Rewrite the following logarithm in terms of $\log_2 x$, $\log_2 y$, and constants:

$$\log_2\left(\frac{x^7}{4y^8}\right)$$

Solution.

$$\begin{aligned}
\log_2\left(\frac{x^7}{4y^8}\right) &= \log_2(x^7) - \log_2(4y^8) \\
&= \log_2(x^7) - \big(\log_2(4) + \log_2(y^8)\big) \\
&= \log_2(x^7) - \log_2(4) - \log_2(y^8) \\
&= 7\log_2(x) - 2 - 8\log_2(y) \\
&= 7\log_2(x) - 8\log_2(y) - 2
\end{aligned}$$

Problem 3. (10pt) Solve the following equation:

$$\log_3(5 - x) + 4 = 36$$

Solution.

$$\log_3(5 - x) + 4 = 36$$

$$\log_3(5 - x) = 32$$

$$3^{\log_3(5-x)} = 3^{32}$$

$$5 - x = 3^{32}$$

$$x = 5 - 3^{32}$$

Problem 4. (10pt) Solve the following equation:

$$e^{2x-1} = 17$$

Solution.

$$\begin{aligned} e^{2x-1} &= 17 \\ \ln e^{2x-1} &= \ln(17) \\ 2x - 1 &= \ln(17) \\ 2x &= \ln(17) + 1 \\ x &= \frac{\ln(17) + 1}{2} \end{aligned}$$

Problem 5. (10pt) Suppose you invest \$500 in an account that earns 4.2% annual interest, compounded quarterly. How long until you have \$800 saved?

Solution.

$$\begin{aligned}
F &= P\left(1+\frac{r}{k}\right)^{kt} \\
800 &= 500\left(1+\frac{0.042}{4}\right)^{4t} \\
800 &= 500(1.0105)^{4t} \\
(1.0105)^{4t} &= 1.6 \\
\ln(1.0105)^{4t} &= \ln(1.6) \\
4t\ln(1.0105) &= \ln(1.6) \\
t &= \frac{\ln(1.6)}{4\ln(1.0105)} \\
t &\approx 11.25 \text{ years}
\end{aligned}$$

Problem 6. (10pt) If you take out a \$10,000 loan at a 7% annual interest rate, compounded continuously, how long until the loan amount has doubled?

Solution.

$$
\begin{aligned}
F &= Pe^{rt} \\
20000 &= 10000e^{0.07t} \\
e^{0.07t} &= 2 \\
\ln e^{0.07t} &= \ln(2) \\
0.07t &= \ln(2) \\
t &= \frac{\ln(2)}{0.07} \\
t &\approx 9.9 \text{ years}
\end{aligned}
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