Name: *Caleb McWhorter — Solutions*
MATH 101
Fall 2023
HW 5: Due 09/25

"I turned myself into a pickle, Morty! I'm Pickle Rick!"
–Rich Sanchez, Rick & Morty

Problem 1. (10pt) Express each of the following decimal numbers as a rational number in simplest form and express each of the rational numbers as a decimal number:

(a) 0.85

(b) $\frac{5}{12}$

(c) 1.12

(d) $\frac{11}{6}$

Problem 2. (10pt) Showing all your work, express the number $0.\overline{2023}$ as a rational number.

Problem 3. (10pt) Perform the following operations in $\mathbb{C}$:

(a) $(\frac{2}{3} + 5i) + (\frac{1}{2} - \frac{3}{4}i)$

(b) $(15 + 6i) - (9 - 4i)$

(c) $(6 - 3i)(8 + 5i)$

(d) $\dfrac{5 - 7i}{4 + 3i}$

(e) $(1 + 2i)(\overline{1 + 2i})$

Problem 4. (10pt) Every quadratic equation $ax^2 + bx + c = 0$ has exactly two (not necessarily distinct) solutions when the solutions are allowed to be complex numbers. Without explicitly solving the equation, verify that the two solutions to $x^2 - 2x + 5 = 0$ are $x_0 = 1 \pm 2i$; that is, substitute both $x = 1 + 2i$ and $x = 1 - 2i$ into $x^2 - 2x + 5$ and show that one obtains a zero for this function in each case.