Name: ______________________________

MATH 308

Fall 2022

HW 13: Due 11/10

"... there is no apparent reason why one number is prime and another not. To the contrary, upon looking at these numbers one has the feeling of being in the presence of one of the inexplicable secrets of creation."
–Don Zagier

Problem 1. (10pt) Showing all your work and fully justifying your reasoning, complete the following:

(a) Using the definition of even, show that -484 is even.

(b) Using the definition of odd, show that 151 is odd.

(c) Find the prime factorization of 360.

(d) Find all the prime divisors of 45!.

(e) Can an integer of the form $n^4 - 9$, where $n \in \mathbb{Z}$, be prime?

Problem 2. (10pt) Showing all your work and fully justifying your reasoning, complete the following:

(a) List at least ten multiples of 17.

(b) List the divisors of 120.

(c) What are the prime divisors of 120?

Problem 3. (10pt) Showing all your work and justifying your reasoning, complete the following:

(a) By enumerating the divisors of 40 and 100, compute $\gcd(40, 100)$.

(b) By enumerating sufficient multiples of 25 and 60, compute $\operatorname{lcm}(25, 60)$.

(c) Compute $\gcd(2^{100} \cdot 3^{200} \cdot 5^{600} \cdot 11^{100},\ 2^{300} \cdot 3^{100} \cdot 5^{600} \cdot 7^{800})$.

(d) Compute $\operatorname{lcm}(2^{100} \cdot 3^{200} \cdot 5^{600} \cdot 11^{100},\ 2^{300} \cdot 3^{100} \cdot 5^{600} \cdot 7^{800})$.

Problem 4. (10pt) Showing all your work and justifying your reasoning, complete the following:

(a) Prove or disprove: if p is prime, then $p^2 + 1$ is also prime.

(b) Using the fact that $ab = \operatorname{lcm}(a, b) \cdot \gcd(a, b)$ and $\gcd(196, 1320) = 4$, compute $\operatorname{lcm}(196, 1320)$.

(c) If $a, b \in \mathbb{Z}$ such that $\gcd(a, b) = p$, where p is prime, what are the possible values for $\gcd(a^2, b)$, $\gcd(a, b^2)$, $\gcd(a^2, b^2)$, and $\gcd(a^2, b^3)$?