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MATH 101
Fall 2021
HW 12: Due 11/05

"Science and everyday life cannot and should not be separated."
–Rosalind Franklin

Problem 1. (10pt) Use the quadratic formula to factor $x^2 - 6x - 3$. Show all your work.

Solution. We have...

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
x &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-3)}}{2(1)} \\
x &= \frac{6 \pm \sqrt{36 + 12}}{2} \\
x &= \frac{6 \pm \sqrt{48}}{2} \\
x &= \frac{6 \pm \sqrt{16 \cdot 3}}{2} \\
x &= \frac{6 \pm 4\sqrt{3}}{2} \\
x &= 3 \pm 2\sqrt{3}
\end{aligned}$$

Observe that $a = 1$. Therefore, the factorization is...

$$x^2 - 6x - 3 = 1 \cdot \big(x - (3 + 2\sqrt{3})\big)\big(x - (3 - 2\sqrt{3})\big) = \big(x - (3 + 2\sqrt{3})\big)\big(x - (3 - 2\sqrt{3})\big)$$

Problem 2. (10pt) Use the quadratic formula to factor $x^2 - 6x + 10$. Show all your work.

Solution. We have...

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 40}}{2}$$

$$x = \frac{6 \pm \sqrt{-4}}{2}$$

$$x = \frac{6 \pm 2i}{2}$$

$$x = 3 \pm i$$

Observe that $a = 1$. Therefore, the factorization is...

$$x^2 - 6x + 10 = 1 \cdot \big(x - (3 + i)\big)\big(x - (3 - i)\big) = \big(x - (3 + i)\big)\big(x - (3 - i)\big)$$

Problem 3. (10pt) Use the quadratic formula to factor $4x^2 - 8x + 3$. Show all your work.

Solution. We have...

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(3)}}{2(4)}$$

$$x = \frac{8 \pm \sqrt{64 - 48}}{8}$$

$$x = \frac{8 \pm \sqrt{16}}{8}$$

$$x = \frac{8 \pm 4}{8}$$

$$x = \frac{8+4}{8}, \quad \frac{8-4}{8}$$

$$x = \frac{12}{8}, \quad \frac{4}{8}$$

$$x = \frac{3}{2}, \quad \frac{1}{2}$$

Observe that $a = 4$. Therefore, the factorization is...

$$4x^2 - 8x + 3 = 4\left(x - \frac{3}{2}\right)\left(x - \frac{1}{2}\right)$$

Note that this is the same as the normal factorization:

$$\begin{aligned} 4x^2 - 8x + 3 &= 4\left(x - \frac{3}{2}\right)\left(x - \frac{1}{2}\right) \\ &= 2 \cdot 2 \cdot \left(x - \frac{3}{2}\right)\left(x - \frac{1}{2}\right) \\ &= 2\left(x - \frac{3}{2}\right) \cdot 2\left(x - \frac{1}{2}\right) \\ &= (2x-3)(2x-1) \end{aligned}$$

Problem 4. (10pt) Find the x-intercepts of the quadratic function $y = x^2 - 11x + 30$. Show all your work.

Solution. The x-intercepts correspond to the points where $y = 0$. But then

$$x^2 - 11x + 30 = 0$$

$$(x - 5)(x - 6) = 0$$

Then either $x - 5 = 0$, i.e. $x = 5$, or $x - 6 = 0$, i.e. $x = 6$. Therefore, the x-intercepts are $(5, 0)$ and $(6, 0)$.

Problem 5. (10pt) Find the x-intercepts of the quadratic function $y = x^2 - 6x + 9$. Show all your work.

Solution. The x-intercepts correspond to the points where $y = 0$. But then

$$x^2 - 6x + 9 = 0$$

$$(x-3)^2 = 0$$

But then $x - 3 = 0$, i.e. $x = 3$. Therefore, the only x-intercept is $(3, 0)$.

Problem 6. (10pt) Find the x-intercepts of the quadratic function $y = 5x^2 - 19x + 12$. Show all your work.

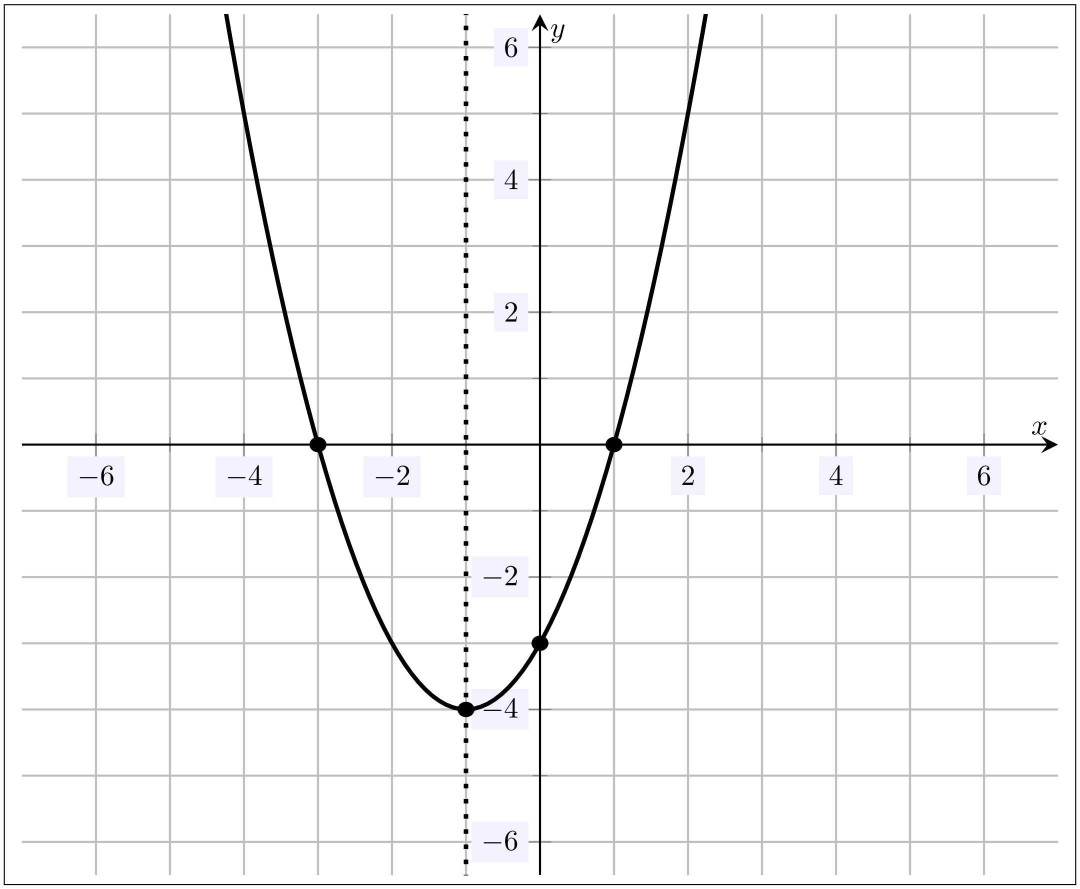
Solution. The x-intercepts correspond to the points where $y = 0$. But then

$$5x^2 - 19x + 12 = 0$$

$$(5x - 4)(x - 3) = 0$$

Then either $5x - 4 = 0$, i.e. $x = \frac{4}{5}$, or $x - 3 = 0$, i.e. $x = 3$. Therefore, the x-intercepts are $(\frac{4}{5}, 0)$ and $(3, 0)$.

Problem 7. (10pt) Plot the function $y = x^2 + 2x - 3$. Your plot should include the vertex, axis of symmetry, y-intercept, and x-intercepts. Show all your work.



The x-coordinate of the vertex is $x = -\frac{b}{2a} = -\frac{2}{2(1)} = -1$. The y-coordinate is then $y(-1) = (-1)^2 + 2(-1) - 3 = 1 - 2 - 3 = -4$. Therefore, the vertex is $(-1, -4)$. This also means the axis of symmetry is $x = -1$. To find the x-intercepts, we solve $x^2 + 2x - 3 = 0$. But this implies $(x + 3)(x - 1) = 0$ so that either $x + 3 = 0$, i.e. $x = -3$, or $x - 1 = 0$, i.e. $x = 1$. Therefore, the x-intercepts are $(-3, 0)$ and $(1, 0)$. The y-intercept corresponds to the value when $x = 0$. But then $y = 0^2 + 2(0) - 3 = -3$. Therefore, the y-intercept is $(0, -3)$.