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MATH 108
Fall 2023
HW 14: Due 12/12

"It is a curious historical fact that modern quantum mechanics began with two quite different mathematical formulations: the differential equation of Schrödinger and the matrix algebra of Heisenberg. The two apparently dissimilar approaches were proved to be mathematically equivalent."

–Richard Feynman

Problem 1. (10pt) Find the augmented matrix to the corresponding system of equations:

$$
\begin{gathered}
x - 2y + 3z - w = 10 \\
x + 4y - 26w = 19 \\
-6x + 19z + w = 25
\end{gathered}
$$

Solution. First, we order the variables as x, y, z, and then w. We also make sure each equality has all variables present. This gives us the following system of equations:

$$
\begin{gathered}
x - 2y + 3z - w = 10 \\
x + 4y + 0z - 26w = 19 \\
-6x + 0y + 19z + w = 25
\end{gathered}
$$

Therefore, the augmented matrix is...

$$
\begin{pmatrix}
1 & -2 & 3 & -1 & 10 \\
1 & 4 & 0 & -26 & 19 \\
-6 & 0 & 19 & 1 & 25
\end{pmatrix}
$$

Problem 2. (10pt) The matrix below is the initial augmented matrix for a system of linear equations. Find the system of linear equations.

$$\begin{pmatrix} 5 & -3 & 1 & 8 \\ 1 & 0 & -1 & 5 \\ -6 & 2 & 9 & 1 \\ 5 & 6 & 7 & 12 \end{pmatrix}$$

Solution. Each column corresponds to a variable in the system—except the last column that corresponds to the 'other side' of the equalities. Therefore, there are $4 - 1 = 3$ variables in the system, which we will label x, y, z. Therefore, the system of equations must be...

$$\begin{aligned} 5x - 3y + z &= 8 \\ x - z &= 5 \\ -6x + 2y + 9z &= 1 \\ 5x + 6y + 7z &= 12 \end{aligned}$$

Problem 3. (10pt) The following matrix is the RREF of an augmented matrix coming from a system of equations. Did this system of equations have a solution? If the system of equations had a solution, find all the possible solutions. If the system did not have a solution, explain why.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -5 \\ 0 & 0 & 0 & 1 & -2 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Solution. Each of the columns of the matrix corresponds to a variable—except for the last column which corresponds to the 'other' side of the equalities. There are then $6-1=5$ variables. We mark the pivot columns of the matrix:

$$\begin{pmatrix} \boxed{1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \boxed{1} & 0 & 0 & 0 & -5 \\ 0 & 0 & 0 & \boxed{1} & -2 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Therefore, x_1, x_2, x_4 will be 'fixed.' We then take x_3, x_5 to be free variables. The first row tells us that $x_1 = 0$. The second row tells us that $x_2 = -5$. The third row tells us that $x_4 - 2x_5 = 7$, which implies that $x_4 = 2x_5 + 7$. Therefore, the solution is...

$$\begin{cases} x_1 = 0 \\ x_2 = -5 \\ x_3 \colon \text{ free} \\ x_4 = 2x_5 + 7 \\ x_5 \colon \text{ free} \end{cases}$$

Problem 4. (10pt) The following matrix is the RREF of an augmented matrix coming from a system of equations. Did this system of equations have a solution? If the system of equations had a solution, find all the possible solutions. If the system did not have a solution, explain why.

$$\begin{pmatrix} 1 & 0 & -9 \\ 0 & 1 & 0 \end{pmatrix}$$

Solution. Each of the columns of the matrix corresponds to a variable—except for the last column which corresponds to the 'other' side of the equalities. There are then $3 - 1 = 2$ variables. From the first row, we see that $x_1 = -9$. From the second row, we see that $x_2 = 0$. Therefore, the unique solution is $(x_1, x_2) = (0, 0)$, i.e. ...

$$\begin{cases} x_1 = 0 \\ x_2 = 0 \end{cases}$$

Problem 5. (10pt) The following matrix is the 'RREF' of an augmented matrix coming from a system of equations. Did this system of equations have a solution? If the system of equations had a solution, find all the possible solutions. If the system did not have a solution, explain why.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 5 \end{pmatrix}$$

Solution. The last row of the matrix implies that $0 = 5$. Therefore, there is no solution to the system of equations.