Name: ______________________________
MATH 308
Fall 2021
HW 11: Due 11/05

"Before software should be reusable, it should be usable."
–Ralph Johnson

Problem 1. (10pt) Show that the following sets have the same cardinality by finding a bijection between them (you need not prove that your function is bijective):

(a) $A = (-2, 2)$, $B = (5, 6)$

(b) $A = \{0, 1\} \times \mathbb{N}$, $B = \mathbb{N}$

(c) $A = [0, 1]$, $B = (0, 1)$

Problem 2. (10pt) Show that $\mathbb{N}$ and $\mathbb{Z} \times \mathbb{Z}$ have the same cardinality using the Schröder-Cantor-Bernstein Theorem.

Problem 3. (10pt) In class, we discussed that given a set S, the cardinality of the power set of S, $\mathcal{P}(S)$, is strictly larger. Thus, we can construct sets with arbitrarily large cardinality. Prove this fact, i.e. prove the following:

(a) $|S| \leq |\mathcal{P}(S)|$, i.e. find an injection $f : S \to \mathcal{P}(S)$.

(b) there does not exist a bijection $\phi : S \to \mathcal{P}(S)$. [Hint: Show there is not a surjection by considering the set $A := \{s \in S : s \notin \phi(s)\} \subseteq S$.]

(c) Explain why this implies there can be no 'set of all sets.'

Problem 4. (10pt) Determine if the following sets are countable or uncountable (give a brief explanation; however, a formal proof is not necessary):

(a) $A = \{\log n \colon n \in \mathbb{N}\}$.

(b) $B =$ set of perfect squares.

(c) $C = \{(m, n) \in \mathbb{N} \times \mathbb{N} \colon 2 \leq m \leq n^2\}$.

(d) $D =$ set of all irrational numbers.

(e) $E =$ set of linear functions $f : \mathbb{R} \to \mathbb{R}$.

(f) $F =$ set of all finite binary strings.

(g) $G =$ set of *all* binary strings.

(h) $H =$ set of all functions $f : \{0, 1\} \to \mathbb{N}$.

(i) $I =$ set of all functions $f : \mathbb{N} \to \{0, 1\}$.

(j) $J =$ set of all possible dictionary 'words.'

(k) $K =$ set of all subsets of $\mathbb{N}$.

Problem 5. (10pt) Mimic Cantor's proof that the set $\mathbb{R}$ is uncountable to prove that the set of all real numbers without a 7 in their decimal expansion is uncountable.