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MATH 108
Fall 2022
HW 7: Due 10/13

"The only function of economic forecasting is to make astrology look respectable."
–John Galbraith

Problem 1. (10pt) As accurately as possible and showing all your work, find the least square regression line, along with the r and r^2 value, for the dataset $\{(1,0),(0,1),(1,1),(2,6)\}$. Show all your work.

Solution. We have 4 points so that $n = 4$. First, we compute the x and y averages—$\overline{x}$ and $\overline{y}$, respectively.

$$\overline{x} = \frac{\sum x_i}{n} = \frac{1+0+1+2}{4} = \frac{4}{4} = 1$$
$$\overline{y} = \frac{\sum y_i}{n} = \frac{0+1+1+6}{4} = \frac{8}{4} = 2$$

Now we compute s_x, s_y, r: Then we have

x	y	$x_i - \overline{x}$	$(x_i - \overline{x})^2$	$y_i - \overline{y}$	$(y_i - \overline{y})^2$
1	0	0	0	-2	4
0	1	-1	1	-1	1
1	1	0	0	-1	1
2	6	1	1	4	16
	Total:		2		22

$$s_x^2 = \frac{1}{n-1}\sum(x_i - \overline{x})^2 = \frac{1}{4-1}\cdot 2 = 0.6667 \Longrightarrow s_x = \sqrt{0.6667} = 0.8165$$
$$s_y^2 = \frac{1}{n-1}\sum(y_i - \overline{y})^2 = \frac{1}{4-1}\cdot 22 = 7.3333 \Longrightarrow s_y = \sqrt{7.3333} = 2.7080$$

Now we also compute the r value:

x	y	$x_i - \overline{x}$	$y_i - \overline{y}$	$(x_i - \overline{x})(y_i - \overline{y})$
1	0	0	-2	0
0	1	-1	-1	1
1	1	0	-1	0
2	6	1	4	4
			Total:	5

$$r = \frac{1}{n-1}\frac{1}{s_x s_y}\sum(x_i - \overline{x})(y_i - \overline{y}) = \frac{1}{4-1}\cdot\frac{1}{0.8165\cdot 2.7080}\cdot 5 = 0.7537788$$

Therefore, $r^2 = 0.5682$. Finally, we can compute our regression coefficients:

$$b_1 = r\,\frac{s_y}{s_x} = 0.7537788\cdot\frac{2.7080}{0.8165} = 2.50 \quad\text{and}\quad b_0 = \overline{y} - b_1\overline{x} = 2 - 2.50\cdot 1 = -0.5$$

Therefore, as $\widehat{y} = b_1 x + b_0$, we know $\widehat{y} = 2.50x - 0.5$.

Problem 2. (10pt) Given the following information below, find the least square regression line. Show all your work.

$$\begin{aligned} n &= 10 & R &= -0.0023 \\ \overline{x} &= 0.97 & s_x^2 &= 30.32 \\ \overline{y} &= -1.33 & s_y^2 &= 36.54 \end{aligned}$$

Solution. We are given s_x^2 and s_y^2 so that we have...

$$s_x = \sqrt{s_x^2} = \sqrt{30.32} \approx 5.50636$$

$$s_y = \sqrt{s_y^2} = \sqrt{36.54} \approx 6.04483$$

But then we have...

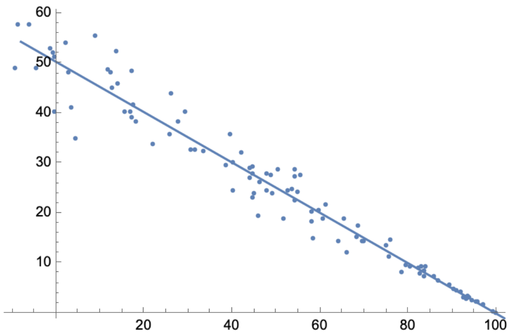
$$b_1 = r\,\frac{s_y}{s_x} = -0.0023 \cdot \frac{6.04483}{5.50636} \approx -0.00252492$$

$$b_0 = \overline{y} - b_1\overline{x} = -1.33 - (-0.00252492) \cdot 0.97 = -1.33 - (-0.00244917) = -1.32755083$$

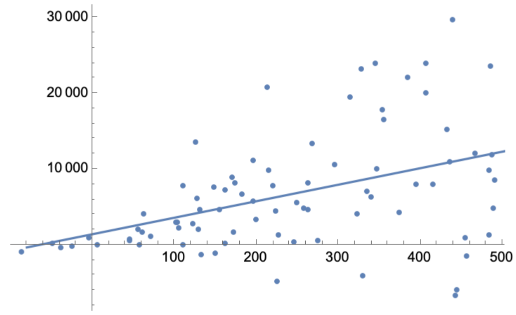
Therefore, the model is...

$$\widehat{y} = -0.00252492x - 1.32755083$$

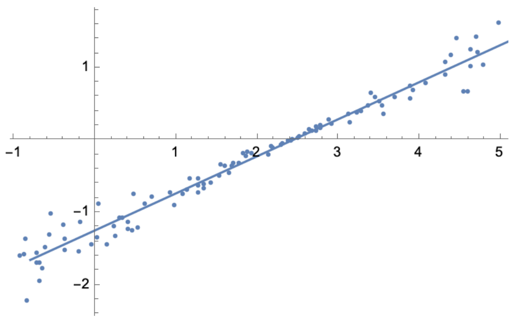
Problem 3. (10pt) Match each regression coefficient to its corresponding graph.



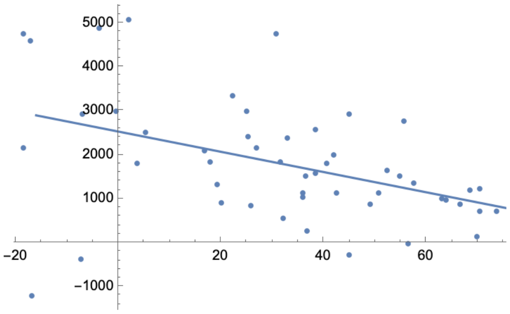
(a)



(b)



(c)



(d)

(i) ___(a)___: $R = -0.9197$

(ii) ___(d)___: $R = -0.6023$

(iii) ___(b)___: $R = 0.2527$

(iv) ___(c)___: $R = 0.9616$

Problem 4. (10pt) A researcher is predicting penguin weights given their final adult height. They create a linear regression model for the weight of the penguin (in lbs), W, given its heigh in cm, h. Their model is $W(h) = 0.8h - 56.2$.

(a) What are b_0 and b_1 for this linear regression?

(b) How much does a penguin's weight increase per centimeter taller that it is, according to this model?

(c) Does the y-intercept for this model hold any meaning? Explain.

(d) Predict a penguin's weight if its height is 125 cm. Suppose one of the penguins in their dataset has a height of 125 cm and weight of 48.6 lbs. Find the residual for this datapoint.

(e) The researcher finds an R^2 value of 0.4329. Is this linear model a good predictor of a penguin's weight given its height? Explain.

Solution.

(a) We know that b_1 is the slope of the linear model and b_0 is the y-intercept of the linear model. Because $W(h) = 0.8h - 56.2$ has slope 0.8 and y-intercept -56.2, we know that $b_1 = 0.8$ and $b_0 = -56.2$.

(b) This is the rate of change of the penguin's weight with respect to their height. But this is precisely the slope of the linear model. Because we have $b_1 = 0.8 = \dfrac{0.8}{1} \leftrightarrow \dfrac{\Delta W}{\Delta h}$. Treating this as $\Delta W = 0.8$ and $\Delta h = 1$, we see that for every additional 1 cm the penguin is in height, the model predicts the penguin's weight increases by 0.8 lbs.

(c) We know that the y-intercept occurs when the input is 0. But then we have $W(0) = 0.8(0) - 56.2 = 0 - 56.2 = -56.2$. Therefore, the y-intercept is $(0, -56.2)$. This says that a penguin that is 0 cm tall weighs -56.2 lbs. But what is a 0 cm tall penguin? Moreover, what does negative weight mean? Therefore, it is unlikely that the y-intercept for this model has an interpretation in the context of the problem.

(d) We have...

$$W(125) = 0.8(125) - 56.2 = 100.0 - 56.2 = 43.8 \text{ lb}$$

Therefore, the model predicts that a penguin that is 125 cm tall weighs 43.8 lb. But then the residual, e_i, for a penguin that is 125 cm that actually weighs 48.6 lb is $e_i = y - \widehat{y} = 48.6 \text{ lb} - 43.8 \text{ lb} = 4.8 \text{ lb}$.

(e) We know the closer R^2 is to 1, the better the model. If $R^2 = 1$, then the data is perfectly linear. By 'most' standards, $R^2 = 0.4329$ is not a good indication that this is a good model. This says that only 43.29% of the variation in the penguin weight due to height is explained by the model. Typically, we would desire R^2 to be greater than 0.60, 0.70, 0.85, 0.95, or 0.99.