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MATH 101
Summer 2022
HW 2: Due 05/25

"Sometimes the questions are complicated and the answers are simple."
–Dr. Seuss (Theodor Seuss Geisel)

Problem 1. (10pt) Is the following statement true or false, explain: for a nonzero real number x and integer n, $x^n = \dfrac{1}{x^{-n}}$.

Solution. The statement is true. Observe that if $x \neq 0$ is a real number and n is an integer, then we have...

$$\frac{1}{x^{-n}} = \frac{1}{1/x^n} = \frac{1}{1} \cdot \frac{x^n}{1} = x^n$$

Problem 2. (10pt) Write the following complex numbers in the form $a + bi$:

(a) $2(1 - 3i) - (6 - 4i)$

(b) $(3 + 4i)(2 - i)$

(c) $(2i)^5$

(d) $\dfrac{5 - i}{1 + 2i}$

Solution.

(a)

$$2(1 - 3i) - (6 - 4i) = (2 - 6i) - (6 - 4i) = 2 - 6i - 6 + 4i = -4 - 2i$$

(b)

$$(3 + 4i)(2 - i) = 6 - 3i + 8i - 4i^2 = 6 - 3i + 8i - 4(-1) = 6 - 3i + 8i + 4 = 10 + 5i$$

(c)

$$(2i)^5 = 2^5 \cdot i^5 = 2^5 \cdot (i^4 \cdot i) = 32 \cdot (1 \cdot i) = 32i$$

(d)

$$\frac{5 - i}{1 + 2i} = \frac{5 - i}{1 + 2i} \cdot \frac{1 - 2i}{1 - 2i} = \frac{5 - 10i - i + 2i^2}{1 - 2i + 2i - 4i^2} = \frac{5 - 11i - 2}{1 - 4(-1)} = \frac{3 - 11i}{5} = \tfrac{3}{5} - \tfrac{11}{5}\, i$$

Problem 3. (10pt) Write the following numbers in scientific notation as a decimal value:

(a) $5.77 \cdot 10^5$

(b) $-1.41 \cdot 10^{-2}$

(c) $8.9 \cdot 10^0$

(d) $4.5 \cdot 10^{-6}$

Solution.

(a) $5.77 \cdot 10^5 = 577\,000$

(b) $-1.41 \cdot 10^{-2} = -0.0141$

(c) $8.9 \cdot 10^0 = 8.9$

(d) $4.5 \cdot 10^{-6} = 0.0000045$

Problem 4. (10pt) Write the following decimal numbers in scientific notation:

(a) 34100000

(b) 7.7

(c) 0.6631

(d) 0.0000004

Solution.

(a) $34100000 = 3.41 \cdot 10^7$

(b) $7.7 = 7.7 \cdot 10^0$

(c) $0.6631 = 6.631 \cdot 10^{-1}$

(d) $0.0000004 = 4.0 \cdot 10^{-7}$

Problem 5. (10pt) Using no negative powers and showing all your work, simplify the following:

(a) $(x^5y^3)^2(x^{10}y^2)^0$

(b) $x^3(x^2y)^2y^3$

(c) $x^3y^8(x^5y^2)^{-2}$

(d) $(xy^4)^{-3}(x^5y^{-3})^{-5}$

Solution.

(a)
$$(x^5y^3)^2(x^{10}y^2)^0 = (x^{5\cdot 2}y^{3\cdot 2}) \cdot 1 = x^{10}y^6$$

(b)
$$x^3(x^2y)^2y^3 = x^3 \cdot x^4y^2 \cdot y^3 = x^{3+4}y^{2+3} = x^7y^5$$

(c)
$$x^3y^8(x^5y^2)^{-2} = \frac{x^3y^8}{(x^5y^2)^2} = \frac{x^3y^8}{x^{10}y^4} = \frac{\cancel{x^3}y^{\cancel{8}^{4}}}{\cancel{x^{10}}^{7}\cancel{y^4}} = \frac{y^4}{x^7}$$

(d)
$$(xy^4)^{-3}(x^5y^{-3})^{-5} = x^{-3}y^{-12} \cdot x^{-25}y^{15} = x^{-3-25}y^{-12+15} = x^{-28}y^3 = \frac{y^3}{x^{28}}$$

Problem 6. (10pt) Using no negative powers and showing all your work, simplify the following:

(a) $\dfrac{x^5y^7}{x^4y^{12}}$

(b) $\dfrac{x^8y^{-3}}{x^{12}y^{-6}}$

(c) $\dfrac{(xy^3)^{-2}}{x^{-5}y^2}$

(d) $\dfrac{x^3(x^2y)^0}{y^5(xy^3)^{-2}}$

Solution.

(a)
$$\frac{x^5y^7}{x^4y^{12}} = x^{5-4}y^{7-12} = x^1y^{-5} = \frac{x}{y^5}$$

(b)
$$\frac{x^8y^{-3}}{x^{12}y^{-6}} = \frac{x^8y^6}{x^{12}y^3} = \frac{y^3}{x^4}$$

(c)
$$\frac{(xy^3)^{-2}}{x^{-5}y^2} = \frac{x^5}{(xy^3)^2y^2} = \frac{x^5}{x^2y^6y^2} = \frac{x^5}{x^2y^8} = \frac{x^3}{y^8}$$

(d)
$$\frac{x^3(x^2y)^0}{y^5(xy^3)^{-2}} = \frac{x^3 \cdot 1 \cdot (xy^3)^2}{y^5} = \frac{x^3 \cdot x^2y^6}{y^5} = \frac{x^5y^6}{y^5} = x^5y$$

Problem 7. (10pt) Using no negative powers, being sure each variable appears only once, and showing all your work, simplify the following:

(a) $xy\sqrt{x^5y^3}$

(b) $\dfrac{y\sqrt{x^5}}{\sqrt{x^3y}}$

(c) $\dfrac{(x^2y^5)^{1/2}}{(x^6y^2)^{-1/3}}$

(d) $(x^{10}y^{-4})^{-1/4}\sqrt{xy}$

Solution.

(a)
$$xy\sqrt{x^5y^3} = xy(x^5y^3)^{1/2} = xy \cdot x^{5/2}y^{3/2} = x^{1+5/2}y^{1+3/2} = x^{7/2}y^{5/2} = \sqrt{x^7y^5}$$

(b)
$$\frac{y\sqrt{x^5}}{\sqrt{x^3y}} = \frac{yx^{5/2}}{(x^3y)^{1/2}} = \frac{yx^{5/2}}{x^{3/2}y^{1/2}} = x^{5/2-3/2}y^{1-1/2} = xy^{1/2} = x\sqrt{y}$$

(c)
$$\frac{(x^2y^5)^{1/2}}{(x^6y^2)^{-1/3}} = (x^{2/2}y^{5/2}) \cdot (x^6y^2)^{1/3} = xy^{5/2} \cdot x^2y^{2/3} = x^{1+2}y^{5/2+2/3} = x^3y^{19/6} = x^3\sqrt[6]{y^{19}}$$

(d)
$$(x^{10}y^{-4})^{-1/4}\sqrt{xy} = x^{-10/4}y^{4/4} \cdot (xy)^{1/2} = x^{-5/2}y \cdot x^{1/2}y^{1/2} = x^{-5/2+1/2}y^{1+1/2} = x^{-2}y^{3/2} = \frac{\sqrt{y^3}}{x^2}$$

Problem 8. (10pt) Showing all your work, simplify the following as much as possible:

(a) $\sqrt{75}$

(b) $\sqrt{250}$

(c) $\sqrt{360}$

(d) $\sqrt{2^7 \cdot 3^3 \cdot 5^2 \cdot 11}$

Solution.

(a)
$$\sqrt{75} = \sqrt{3 \cdot 5^2} = 5\sqrt{3}$$

(b)
$$\sqrt{250} = \sqrt{2 \cdot 5^3} = \sqrt{2 \cdot (5^2 \cdot 5)} = 5\sqrt{2 \cdot 5} = 5\sqrt{10}$$

(c)
$$\sqrt{360} = \sqrt{2^3 \cdot 3^2 \cdot 5} = \sqrt{(2^2 \cdot 2) \cdot 3^2 \cdot 5} = 2 \cdot 3\sqrt{2 \cdot 5} = 6\sqrt{10}$$

(d)
$$\sqrt{2^7 \cdot 3^3 \cdot 5^2 \cdot 11} = \sqrt{(2^6 \cdot 2) \cdot (3^2 \cdot 3) \cdot 5^2 \cdot 11} = 2^3 \cdot 3 \cdot 5\sqrt{2 \cdot 3 \cdot 11} = 120\sqrt{66}$$

Problem 9. (10pt) Showing all your work, simplify the following as much as possible:

(a) $\sqrt[3]{40}$

(b) $\sqrt[4]{80}$

(c) $\sqrt[3]{2^8 \cdot 3^2 \cdot 5 \cdot 7^6}$

(d) $\sqrt[5]{2^4 \cdot 5^{10} \cdot 7^5}$

Solution.

(a)
$$\sqrt[3]{40} = \sqrt[3]{2^3 \cdot 5} = 2\sqrt[3]{5}$$

(b)
$$\sqrt[4]{80} = \sqrt[4]{2^4 \cdot 5} = 2\sqrt[4]{5}$$

(c)
$$\sqrt[3]{2^8 \cdot 3^2 \cdot 5 \cdot 7^6} = \sqrt[3]{(2^6 \cdot 2^2) \cdot 3^2 \cdot 5 \cdot 7^6} = 2^2 \cdot 7^2 \sqrt[3]{2^2 \cdot 3^2 \cdot 5} = 196\sqrt[3]{180}$$

(d)
$$\sqrt[5]{2^4 \cdot 5^{10} \cdot 7^5} = 5^2 \cdot 7^1 \sqrt[5]{2^4} = 175\sqrt[5]{16}$$

Problem 10. (10pt) Rationalize the following fractions:

(a) $\dfrac{1}{\sqrt{11}}$

(b) $\dfrac{4}{\sqrt{13}}$

(c) $\dfrac{5}{3+\sqrt{7}}$

(d) $\dfrac{1}{\sqrt[5]{3}}$

Solution.

(a)
$$\frac{1}{\sqrt{11}} = \frac{1}{\sqrt{11}} \cdot \frac{\sqrt{11}}{\sqrt{11}} = \frac{\sqrt{11}}{11}$$

(b)
$$\frac{4}{\sqrt{13}} = \frac{4}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{4\sqrt{13}}{13}$$

(c)
$$\frac{5}{3+\sqrt{7}} = \frac{5}{3+\sqrt{7}} \cdot \frac{3-\sqrt{7}}{3-\sqrt{7}} = \frac{15-5\sqrt{7}}{9-3\sqrt{7}+3\sqrt{7}-7} = \frac{15-5\sqrt{7}}{2}$$

(d)
$$\frac{1}{\sqrt[5]{3}} = \frac{1}{3^{1/5}} = \frac{1}{3^{1/5}} \cdot \frac{3^{4/5}}{3^{4/5}} = \frac{3^{4/5}}{3^1} = \frac{\sqrt[5]{3^4}}{3} = \frac{\sqrt[5]{81}}{3}$$