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MATH 101
Winter 2021
HW 11: Due 01/21

"Yeah, I'm not a temp anymore. I got Jim's old job. Which means at my 10-year high school reunion, it will not say 'Ryan Howard is a temp.' It will say, 'Ryan Howard is a junior sales associate at a mid-range paper supply firm.' That'll show 'em."

–Ryan Howard, The Office

Problem 1. (10pt) Showing all your work, compute the following:

(a) $\log_3(27) - \log_3(3) + \log_3(1)$

(b) $\log_6\left(\frac{1}{36}\right)$

(c) $\log_{12}(12^{1/5})$

Solution.

(a)

$$\log_3(27) - \log_3(3) + \log_3(1) = \log_3(3^3) - \log_3(3^1) + \log_3(3^0) = 3 - 1 + 0 = 2$$

(b)

$$\log_6\left(\frac{1}{36}\right) = \log_6(36^{-1}) = \log_6\left((6^2)^{-1}\right) = \log_6(6^{-2}) = -2$$

(c)

$$\log_{12}(12^{1/5}) = \frac{1}{5}$$

Problem 2. (10pt) Showing all your work, compute the following:

(a) $\ln(e^2) + 3\ln(1)$

(b) $\ln(\sqrt[3]{e})$

(c) $\ln(e^{4/3})$

Solution.

(a)

$$\ln(e^2) + 3\ln(1) = \ln(e^2) + 3\ln(e^0) = 2 + 3(0) = 2 + 0 = 2$$

(b)

$$\ln(\sqrt[3]{e}) = \ln(e^{1/3}) = \frac{1}{3}$$

(c)

$$\ln(e^{4/3}) = \frac{4}{3}$$

Problem 3. (10pt) Showing all your work, write the following in terms of $\log x$ and $\log y$.

$$\log_6\left(\frac{36x^5}{\sqrt{y}}\right)$$

Solution.

$$\begin{aligned}
\log_6\left(\frac{36x^5}{\sqrt{y}}\right) &= \log_6(36x^5) - \log_6(\sqrt{y}) \\
&= \log_6(36) + \log_6(x^5) - \log_6(\sqrt{y}) \\
&= \log_6(6^2) + \log_6(x^5) - \log_6(y^{1/2}) \\
&= 2 + 5\log_6(x) - \frac{1}{2}\log_6(y) \\
&= 5\log_6(x) - \frac{1}{2}\log_6(y) + 2
\end{aligned}$$

Problem 4. (10pt) Showing all your work, write the following in terms of $\log x$, $\log y$, and $\log z$.

$$\ln\left(\frac{z^6\sqrt[3]{x^2}}{y^5}\right)$$

Solution.

$$\begin{aligned}\ln\left(\frac{z^6\sqrt[3]{x^2}}{y^5}\right) &= \ln(z^6\sqrt[3]{x^2}) - \ln(y^5)\\ &= \ln(z^6) + \ln(\sqrt[3]{x^2}) - \ln(y^5)\\ &= \ln(z^6) + \ln(x^{2/3}) - \ln(y^5)\\ &= 6\ln(z) + \frac{2}{3}\ln(x) - 5\ln(y)\end{aligned}$$

Problem 5. (10pt) Without using negative powers, write the following as a single logarithm:

$$-6\log_2(x) + \frac{3}{2}\log_2(y) - 8$$

Solution.

$$\begin{aligned}
-6\log_2(x) + \frac{3}{2}\log_2(y) - 8 &= -6\log_2(x) + \frac{3}{2}\log_2(y) - \log_2(2^8) \\
&= \log_2(x^{-6}) + \log_2(y^{3/2}) - \log_2(2^8) \\
&= \log_2(x^{-6}y^{3/2}) - \log_2(2^8) \\
&= \log_2\left(\frac{x^{-6}y^{3/2}}{2^8}\right) \\
&= \log_2\left(\frac{y^{3/2}}{2^8x^6}\right) \\
&= \log_2\left(\frac{\sqrt{y^3}}{256x^6}\right)
\end{aligned}$$

Problem 6. (10pt) Without using negative powers, write the following as a single logarithm:

$$\frac{6\ln x - 2\ln y + \ln z}{2}$$

Solution.

$$\begin{aligned}
\frac{6\ln x - 2\ln y + \ln z}{2} &= 3\ln(x) - \ln(y) + \frac{1}{2}\ln(z) \\
&= \ln(x^3) - \ln(y) + \ln(z^{1/2}) \\
&= \ln\left(\frac{x^3}{y}\right) + \ln(z^{1/2}) \\
&= \ln\left(\frac{x^3 z^{1/2}}{y}\right) \\
&= \ln\left(\frac{x^3\sqrt{z}}{y}\right)
\end{aligned}$$

Problem 7. (10pt) Showing all your work, solve the following equation:

$$\log_5(2x-3)+8=10$$

Solution.

$$\log_5(2x-3)+8=10$$

$$\log_5(2x-3)=2$$

$$5^{\log_5(2x-3)}=5^2$$

$$2x-3=25$$

$$2x=28$$

$$x=14$$

Problem 8. (10pt) Showing all your work, solve the following equation:

$$\ln(1-x) = \frac{2}{3}$$

Solution.

$$\ln(1-x) = \frac{2}{3}$$

$$e^{\ln(1-x)} = e^{2/3}$$

$$1 - x = e^{2/3}$$

$$x = 1 - e^{2/3}$$

Problem 9. (10pt) Showing all your work, solve the following equation:

$$11^{-x} - 12 = 20$$

Solution.

$$11^{-x} - 12 = 20$$

$$11^{-x} = 32$$

$$\log_{11}(11^{-x}) = \log_{11}(32)$$

$$-x = \log_{11}(32)$$

$$x = -\log_{11}(32)$$

Note: We can also write this as $x = -\log_{11}(32) = \log_{11}(32^{-1} = \log_{11}\left(\frac{1}{32}\right)$.

OR

$$11^{-x} - 12 = 20$$

$$11^{-x} = 32$$

$$\ln(11^{-x}) = \ln(32)$$

$$-x\ln(11) = \ln(32)$$

$$x = -\frac{\ln(32)}{\ln(11)}$$

Note: By the change of base equation, we know that $-\frac{\ln(32)}{\ln(11)} = -\log_{11}(32)$.

Problem 10. (10pt) Showing all your work, solve the following equation:

$$2\ln(x) - 4 = 6 - \ln(x)$$

Solution.

$$2\ln(x) - 4 = 6 - \ln(x)$$

$$3\ln(x) = 10$$

$$\ln(x) = \frac{10}{3}$$

$$e^{\ln(x)} = e^{10/3}$$

$$x = \sqrt[3]{e^{10}}$$

OR

$$2\ln(x) - 4 = 6 - \ln(x)$$

$$3\ln(x) = 10$$

$$\ln(x^3) = 10$$

$$e^{\ln(x^3)} = e^{10}$$

$$x^3 = e^{10}$$

$$x = e^{10/3}$$