

Quiz 1. *True/False:* Both $12 = 3 \cdot 4$ and $12 = 2^2 \cdot 3$ are prime factorizations of 12.

Solution. The statement is *false*. A factorization of an integer n is a product of integers that yields n . For instance, if $n = 100$, then $n = 1 \cdot 100, 10 \cdot 10, 5 \cdot 20, \dots$ are all factorizations of 100. A prime factorization is a factorization where all the numbers in the product are primes or powers of primes. [If n is prime, we allow $n = n$ to be the prime factorization, i.e. the ‘empty’ product.] Then in the instance of $n = 100$, the factorizations $5 \cdot 20$ cannot be a prime factorization because 20 is not prime. In the given problem, $12 = 3 \cdot 4$ is *not* a prime factorization because 4 is not prime ($4 = 2 \cdot 2$), while $12 = 2^2 \cdot 3$ is a prime factorization because we have written 12 as a product of (powers of) primes. By the Fundamental Theorem of Arithmetic, every integer greater than 1 is either prime or can be written uniquely (up to order, e.g. $6 = 2 \cdot 3 = 3 \cdot 2$) as a product of primes.

Quiz 2. *True/False:* $\gcd(2^{50} \cdot 3^{60} \cdot 7^{40}, 2^{30} \cdot 3^{70} \cdot 5^{90}) = 2^{30} \cdot 3^{60} \cdot 5^{90} \cdot 7^{40}$

Solution. The statement is *false*. If one wishes to compute $\gcd(a, b)$, one can compute the prime factorizations of a, b and find the product of the primes appearing in *both* prime factorizations of a, b , each to the smaller of the prime powers involved in the factorizations of a, b . For instance, if we wanted to compute $\gcd(2520, 74844) = \gcd(2^3 \cdot 3^2 \cdot 5^1 \cdot 7, 2^2 \cdot 3^5 \cdot 7 \cdot 11)$, observe that the primes occurring both are 2, 3, 7. The smallest power for each is 2, 2, 1, respectively. Therefore, $\gcd(2520, 74844) = \gcd(2^3 \cdot 3^2 \cdot 5^1 \cdot 7, 2^2 \cdot 3^5 \cdot 7 \cdot 11) = 2^2 \cdot 3^2 \cdot 7^1 = 252$. In the given problem, while the smallest power of each prime was chosen, *every* prime was used rather than just the primes both factorizations have in common.