Name: *Caleb McWhorter — Solutions*
MATH 108
Fall 2021
HW 8: Due 11/09

"I did not attend his funeral, but I sent a nice letter saying I approved of it."
–Mark Twain

Problem 1. (10pt) Find the least square regression line for the points: $(1,1),(1,0),(2,3),(3,4)$. Show all your work.

Solution. We have 4 points so that $n = 4$. First, we compute the x and y averages—$\overline{x}$ and $\overline{y}$, respectively.

$$\overline{x} = \frac{\sum x_i}{n} = \frac{1+1+2+3}{4} = \frac{7}{4} \approx 1.75$$
$$\overline{y} = \frac{\sum y_i}{n} = \frac{1+0+3+4}{4} = \frac{8}{4} \approx 2.00$$

Now we compute s_x, s_y, r: Then we have

x	y	$x_i - \overline{x}$	$(x_i - \overline{x})^2$	$y_i - \overline{y}$	$(y_i - \overline{y})^2$
1	1	-0.75	0.5625	-1	1
1	0	-0.75	0.5625	-2	4
2	3	0.25	0.0625	1	1
3	4	1.25	1.5625	2	4
	Total:		2.75		10

$$s_x^2 = \frac{1}{n-1} \sum (x_i - \overline{x})^2 = \frac{1}{4-1} \cdot 2.75 = 0.9167 \Longrightarrow s_x = \sqrt{0.9167} = 0.9574$$
$$s_y^2 = \frac{1}{n-1} \sum (y_i - \overline{y})^2 = \frac{1}{4-1} \cdot 10 = 3.3333 \Longrightarrow s_y = \sqrt{3.3333} = 1.8257$$

Now we also compute the r value:

x	y	$x_i - \overline{x}$	$\frac{x_i - \overline{x}}{s_x}$	$y_i - \overline{y}$	$\frac{y_i - \overline{y}}{s_y}$	$\frac{x_i - \overline{x}}{s_x} \cdot \frac{y_i - \overline{y}}{s_y}$
1	1	-0.75	-0.7833	-1	-0.5477	0.4291
1	0	-0.75	-0.7833	-2	-1.0954	0.8581
2	3	0.25	0.2611	1	0.5477	0.1430
3	4	1.25	1.3056	2	1.0954	1.4302
	Total:					2.8604

$$r = \frac{1}{n-1} \sum \left(\frac{x_i - \overline{x}}{s_x} \right) \left(\frac{y_i - \overline{y}}{s_y} \right) = \frac{1}{4-1} \cdot 2.8604 = 0.9534$$

Therefore, $r^2 = 0.9090$. Finally, we can compute our regression coefficients:

$$b_1 = r \, \frac{s_y}{s_x} = 0.9534 \cdot \frac{1.8257}{0.9574} = 1.818 \quad \text{and} \quad b_0 = \overline{y} - b_1 \overline{x} = 2.00 - 1.818 \cdot 1.75 = -1.181$$

Therefore, as $\widehat{y} = b_1 x + b_0$, we know $\widehat{y} = 1.818x - 1.181$.

Problem 2. (10pt) Given the following information below, find the least square regression line. Show all your work.

$$\begin{aligned} n &= 11 \\ \overline{x} &= 3.45, \quad \sigma_x^2 &= 7.073 \\ \overline{y} &= 6.81, \quad \sigma_y^2 &= 5.371 \\ R &= 0.802 \end{aligned}$$

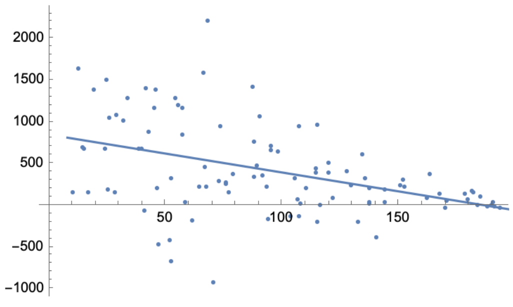
Solution. Because $\sigma_x^2 = 7.073$ and $\sigma_y^2 = 5.371$, we know that $\sigma_x = \sqrt{7.073} = 2.6595$ and $\sigma_y = \sqrt{5.371} = 2.3175$. But then...

$$b_1 = R\,\frac{\sigma_y^2}{\sigma_x^2} = 0.802\,\frac{2.3175}{2.6595} = 0.699$$

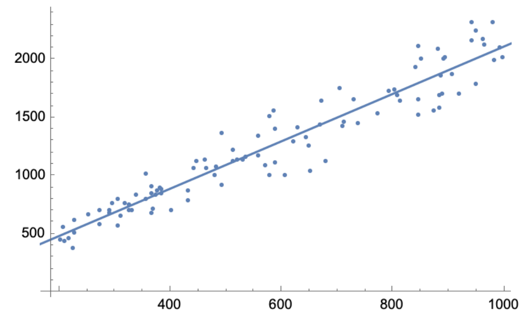
$$b_1 = \overline{y} - b_1\overline{x} = 6.81 - 0.699 \cdot 3.45 = 4.398$$

Therefore, as $\widehat{y} = b_1 x + b_0$, we know that $\widehat{y} = 0.699x + 4.398$.

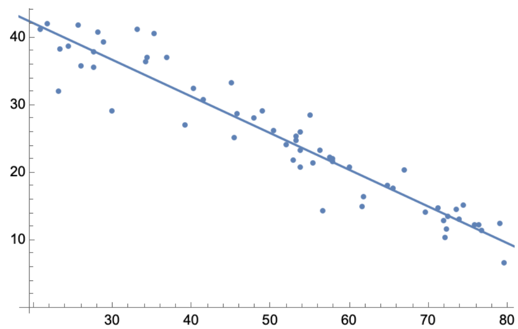
Problem 3. (10pt) Match each regression coefficient to its corresponding graph.



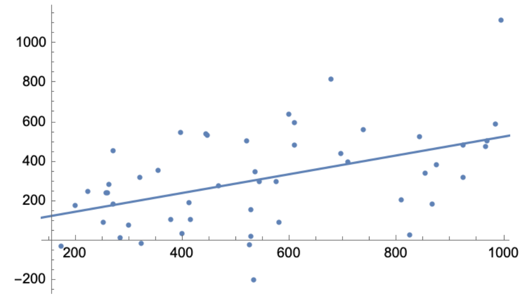
(a)



(b)



(c)



(d)

(i) ___*(c)*___: $R = -0.9529$

(ii) ___*(a)*___: $R = -0.4354$

(iii) ___*(d)*___: $R = 0.4759$

(iv) ___*(b)*___: $R = 0.9573$

Problem 4. (10pt) The lengths (in cm) of twenty snakes are taken 6 months after hatching and 2 years after hatching. The data is given below.

$$(41.2, 163.6), (18.1, 68.9), (42.3, 151.6), (13.2, 43.9), (45.8, 189.5),$$
$$(42.7, 180.5), (24.4, 92.8), (49.0, 166.), (24.6, 101.1), (18.9, 77.5),$$
$$(16.3, 63.6), (36.3, 142.2), (32.2, 124.3), (36.3, 121.), (24.7, 77.8),$$
$$(40.1, 139.7), (22.3, 72.8), (42.4, 182.2), (21.4, 73.), (12.3, 53.1)$$

A linear regression for this data was found to be $\widehat{y} = 3.9x - 3.1$ with $R = 0.9381$.

(a) Was the linear regression a good fit for the data? Explain.

(b) Find the residual for the data point $(41.2, 163.6)$. Was the model under or over prediction for the length of the snake? Explain.

(c) Given this data and model, predict the length of a snake after 2 years that measures 32.7 cm 6 months after hatching.

(d) Should this model be used to predict the length of a snake which is 65 cm six months after hatching? Explain.

Solution.

(a) Observe that $R = 0.9381$ and that $R^2 = 0.8800$, both of which (being 'close' to 1) are good both indicators that this linear regression is a 'good' fit for the data.

(b) We have $\widehat{y}(41.2) = 3.9(41.2) - 3.1 = 160.68 - 3.1 = 157.58$ cm. Therefore, the residual is $e = y_i - \widehat{y} = 163.6 - 157.58 = 6.02$ cm. But then the model has under predicted the length of the snake.

(c) We predict the snake has length $\widehat{y}(32.7) = 3.9(32.7) - 3.1 = 127.53 - 3.1 = 124.43$ cm.

(d) No, it would *not* be appropriate to use this model to make a prediction for a snake this long so soon after hatching. The model was built using only snakes that were length 12.3 cm to 49.0 cm in length. The length 65 cm is 'far' above our 49.0 cm. Therefore, it would be inappropriate to extrapolate that far using our model.