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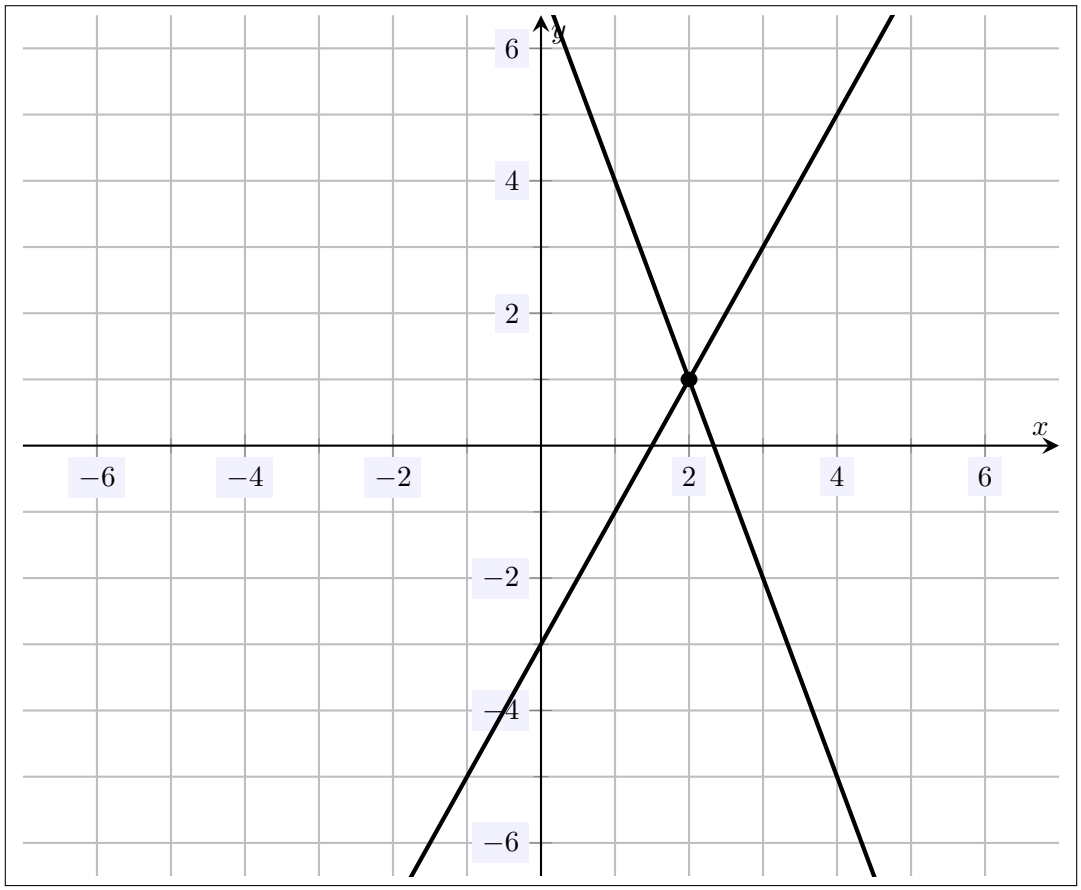
MATH 100
Fall 2021
HW 15: Due 12/08

"Nothing goes over my head. My reflexes are too fast. I would catch it."
–Drax, Guardians of the Galaxy

Problem 1. (10pt) Find and plot the solution to the system of equations given in the graph below:



Examining the graph, we see that the solution to the simultaneous linear system is $(2, 1)$, which we plot on the graph.

Problem 2. (10pt) Show that $(1, -2)$ is the solution to the system of equations below.

$$-5x + 3y = -11$$
$$3x - 5y = 13$$

Solution. If $(1, -2)$, i.e. $x = 1$ and $y = -2$, is the solution to the system, then the point satisfies each of the equations. We check this:

$$-5x + 3y = -5(1) + 3(-2) = -5 - 6 = -11 \quad ✓$$
$$3x - 5y = 3(1) - 5(-2) = 3 + 10 = 13 \quad ✓$$

Therefore, $(1, -2)$ is a solution to the system of equalities.

Problem 3. (10pt) Determine if the system of equations below has a solution. If not, explain why; if so, find the solution.

$$\begin{aligned} 4x - 3y &= -7 \\ 2x + 4y &= 13 \end{aligned}$$

Solution. This a system of linear equations. The system will have a solution if and only if the lines intersect. But this will only happen if they are not parallel. So we find the slopes of each line.

$$\begin{aligned} 4x - 3y &= -7 & \qquad 2x + 4y &= 13 \\ -3y &= -4x - 7 & \qquad 4y &= -2x + 13 \\ y &= \frac{4}{3}x + \frac{7}{3} & \qquad y &= -\frac{1}{2}x + \frac{13}{4} \end{aligned}$$

The slope of the first line is $m_1 = \frac{4}{3}$ while the slope of the second line is $m_2 = -\frac{1}{2}$. Because $m_1 \neq m_2$, the lines are not parallel. But then the lines intersect so that there is a solution to the system of equations. Now we find the solution by using both substitution and elimination.
If we use substitution, we can solve for y in the first equation. This yields $y = \frac{4}{3}x + \frac{7}{3}$. Using this in the second equation, we have...

$$\begin{aligned} 2x + 4y &= 13 \\ 2x + 4\left(\frac{4}{3}x + \frac{7}{3}\right) &= 13 \\ 2x + \frac{16}{3}x + \frac{28}{3} &= 13 \\ 3\left(2x + \frac{16}{3}x + \frac{28}{3}\right) &= 13 \cdot 3 \\ 6x + 16x + 28 &= 39 \\ 22x + 28 &= 39 \\ 22x &= 11 \\ x &= \frac{1}{2} \end{aligned}$$

But then we have $y = \frac{4}{3} \cdot \frac{1}{2} + \frac{7}{3} = \frac{2}{3} + \frac{7}{3} = \frac{9}{3} = 3$. Therefore, the solution is $(\frac{1}{2}, 3)$.

Using elimination, suppose we eliminate x. Then we multiply the first equation by 1 and the second equation by -2 and add them. This gives us...

$$\begin{aligned} 4x - 3y &= -7 \\ -4x - 8y &= -26 \\ \hline -11y &= -33 \\ y &= 3 \end{aligned}$$

Using this in the first equation, we find

$$\begin{aligned} 4x - 3y &= -7 \\ 4x - 3(3) &= -7 \\ 4x - 9 &= -7 \\ 4x &= 2 \\ x &= 1/2 \end{aligned}$$

Therefore, the solution is $(\frac{1}{2}, 3)$.

Problem 4. (10pt) Determine if the system of equations below has a solution. If not, explain why; if so, find the solution.

$$10x - 4y = 12$$
$$-5x + 2y = 4$$

Solution. This a system of linear equations. The system will have a solution if and only if the lines intersect. But this will only happen if they are not parallel. So we find the slopes of each line.

$$\begin{aligned} 10x - 4y &= 12 \\ -4y &= -10x + 12 \\ y &= \frac{5}{2}x + 3 \end{aligned} \qquad \begin{aligned} -5x + 2y &= 4 \\ 2y &= 5x + 4 \\ y &= \frac{5}{2}x + 2 \end{aligned}$$

The slope of the first line is $m_1 = \frac{5}{2}$ while the slope of the second line is $m_2 = -\frac{5}{2}$. Because $m_1 = m_2$, the lines are parallel. But then the lines do not intersect so that there is no solution to the system of equations.