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MATH 101
Fall 2023
HW 9: Due 10/30

"Mathematics is the most beautiful and most powerful creation of human spirit."
–Stefan Banach

Problem 1. (10pt) Values for several functions are given in the table below.

x	-3	-2	-1	0	1	2	3
$f(x)$	4	8	-1	5	-3	0	-2
$g(x)$	1	6	0	-6	-7	-3	1
$h(x)$	-4	0	3	5	10	3	9

Given the data above, compute the following:

(a) $(h+g)(-2) = h(-2) + g(-2) = 0 + 6 = 6$

(b) $(f-g)(0) = f(0) - g(0) = 5 - (-6) = 5 + 6 = 11$

(c) $(5h)(1) = 5h(1) = 5 \cdot 10 = 50$

(d) $\left(\dfrac{h}{f}\right)(1) = \dfrac{h(1)}{f(1)} = \dfrac{10}{-3} = -\dfrac{10}{3}$

(e) $g(-3)\,h(3) = 1 \cdot 9 = 9$

(f) $g\big(-1 - f(3)\big) = g\big(-1 - (-2)\big) = g(-1+2) = g(1) = -7$

(g) $(h \circ g)(2) = h\big(g(2)\big) = h(-3) = -4$

(h) $(g \circ h)(2) = g\big(h(2)\big) = g(3) = 1$

(i) $(f \circ g)(-1) = f\big(g(-1)\big) = f(0) = 5$

(j) $(h \circ g \circ f)(1) = h\Big(g\big(f(1)\big)\Big) = h\big(g(-3)\big) = h(1) = 10$

Problem 2. (10pt) Suppose $f(x)$ and $g(x)$ are the functions given below.

$$f(x) = 2x - 3$$

$$g(x) = x^2 + 2x - 1$$

Compute the following:

(a) $f(5) = 2(5) - 3 = 10 - 3 = 7$

(b) $g(-2) = (-2)^2 + 2(-2) - 1 = 4 - 4 - 1 = -1$

(c) $f(0) - 3g(2) = (2 \cdot 0 - 3) - 3\big(2^2 + 2(2) - 1\big) = -3 - 3(7) = -3 - 21 = -24$

(d) $(f - g)(x) = f(x) - g(x) = (2x - 3) - (x^2 + 2x - 1) = 2x - 3 - x^2 - 2x + 1 = -x^2 - 2$

(e) $(fg)(x) = f(x)\, g(x) = (2x-3)(x^2+2x-1) = 2x^3+4x^2-2x-3x^2-6x+3 = 2x^3+x^2-8x+3$

(f) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{2x - 3}{x^2 + 2x - 1}$

(g) $(f \circ g)(0) = f\big(g(0)\big) = f(0^2 + 2(0) - 1) = f(0 + 0 - 1) = f(-1) = 2(-1) - 3 = -2 - 3 = -5$

(h) $(g \circ f)(0) = g\big(f(0)\big) = g(2(0) - 3) = g(0 - 3) = g(-3) = (-3)^2 + 2(-3) - 1 = 9 - 6 - 1 = 2$

(i) $(f \circ g)(x) = f\big(g(x)\big) = f(x^2 + 2x - 1) = 2(x^2 + 2x - 1) - 3 = 2x^2 + 4x - 2 - 3 = 2x^2 + 4x - 5$

(j) $(g \circ f)(x) = g\big(f(x)\big) = g(2x-3) = (2x-3)^2+2(2x-3)-1 = (4x^2-12x+9)+(4x-6)-1 = 4x^2 - 8x + 2$

Problem 3. (10pt) Let $f(x)$ be the function given by $f(x) = 3x - 7$.

(a) Find a value in the range of f. Be sure to justify why the value is in the range.

(b) Compute $f(4)$. Is $(4, 1)$ on the graph of f? Explain.

(c) Is there an x such that $f(x) = 11$? Explain.

(d) Is $1 \in f^{-1}(3)$? Explain.

(e) Assuming f^{-1} exists, what is $f(f^{-1}(\pi))$ and $f^{-1}(f(\sqrt{2}))$?

Solution.

(a) We know that the range of f is the set of outputs of f. Therefore, we can obtain an output by evaluating f at any value in its domain. For example, $f(0) = 3(0) - 7 = -7$, $f(10) = 3(10) - 7 = 23$, and $f(-5) = 3(-5) - 7 = -22$ are all values in the range of f.

(b) We have $f(4) = 3(4) - 7 = 12 - 7 = 5$. This implies that $(4, 5)$ is a point on the graph. Therefore, $(4, 1)$ cannot be on the graph of f. If it were on the graph, then we would know that $f(4) = 1$. But we know $f(4) = 5 \neq 1$.

(c) If there were x such that $f(x) = 11$, then...

$$\begin{aligned} f(x) &= 11 \\ 3x - 7 &= 11 \\ 3x &= 18 \\ x &= 6 \end{aligned}$$

Of course, this assumes there is an x such that $f(x) = 11$; that is, we have shown that $x = 6$ is the only *possible* value. We can verify this possible solution: $f(6) = 3(6) - 7 = 18 - 7 = 11$. Therefore, there is such an x-value—namely, $x = 6$.

(d) If $1 \in f^{-1}(3)$, then $f(1) = 3$. We have $f(1) = 3(1) - 7 = 3 - 7 = -4$. Therefore, $1 \notin f^{-1}(3)$.

(e) If f^{-1} exists, then we know that $(f \circ f^{-1})(x) = f\big(f^{-1}(x)\big)$ and $(f^{-1} \circ f)(x) = f^{-1}\big(f(x)\big)$ for all x. But then we would have $f(f^{-1}(\pi)) = \pi$ and $f^{-1}(f(\sqrt{2})) = \sqrt{2}$.