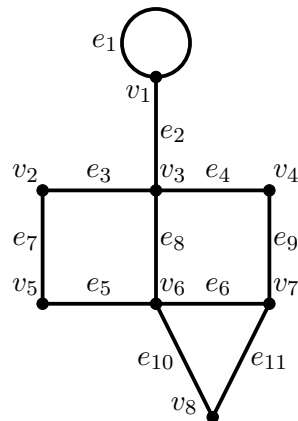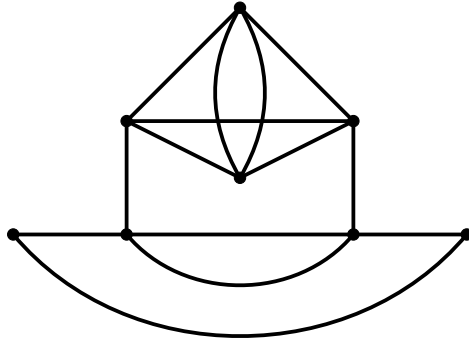*"All paths are not equal; if they were, they wouldn't be paths but rather the points at each end."*

−H.E. Huntley

Problem 1. (10pt) Does the graph G below have an Euler circuit or Euler trail? If it has an Euler circuit or Euler trail, find it. If it does not have an Euler circuit or Euler trail, explain why not.

Problem 2. (10pt) Let G be the graph given below.

(a) Does G have an Euler trail? If it does, find it. If it does not, explain why.

(b) Does G have a Hamiltonian circuit? If it does, find it. If it does not, explain why.

Problem 3. (10pt) Suppose G is the graph given below on the left and that H is a directed graph with adjacency matrix A. Showing all your work and fully justifying your responses, answer the questions below.
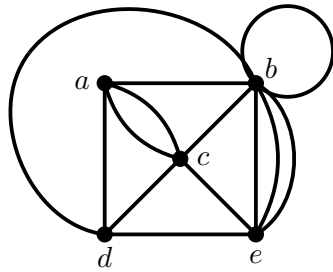
$$A^8 = \begin{pmatrix} 408628 & 1456983 & 1201872 & 1045608 \\ 217055 & 774044 & 638429 & 555540 \\ 442957 & 1577690 & 1299626 & 1131303 \\ 280444 & 1001303 & 825067 & 716683 \end{pmatrix}$$

(a) How many walks are there of length 1 from b to e? What about from d to b?

(b) How many walks are there of length 2 from c to itself? What about from e to a?

(c) How many walks are there of length 4 from a to c? What about from b to itself?

(d) How many connected components does G have?

(e) How many walks are there of length 8 from v_1 to v_3 in H? What about v_4 to v_2?

Problem 4. (10pt) Suppose that G is an undirected graph with adjacency matrix, A, given below.

$$A = \begin{pmatrix} 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Using only this adjacency matrix, showing all your work, fully justifying your responses, answer the following:

(a) Is G a simple graph?

(b) Is G a multigraph?

(c) How many connected components does G have?

(d) Find the degrees of vertices v_1, v_8, and v_5.

(e) What is the degree of G?