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MATH 101

Spring 2022

HW 3: Due 02/15

“Stop touching things.”

–Mandalorian, The Mandalorian

Problem 1. (10pt) Is the following statement true or false, explain: Any number to the zero power is 1, i.e. $x^0 = 1$ for all real numbers x .

Solution. The statement is false. We know that if $x \neq 0$, then $x^0 = 1$, e.g. $1^0 = 1$, $\pi^0 = 1$, $(\frac{2}{3})^0 = 1$, etc. However, 0^0 is undefined.

Problem 2. (10pt) Is the following statement true or false, explain: $\frac{1}{x^{-3}} = \frac{1}{x \cdot x \cdot x}$

Solution. The statement is false. We have...

$$\frac{1}{x^{-3}} = \frac{1}{\frac{1}{x^3}} = x^3$$

Problem 3. (10pt) Is the following statement true or false, explain using words and a calculator computation: $\sqrt[3]{2}$ is the number that when cubed yields 2.

Solution. The statement is true. Recall that $\sqrt[n]{r}$ stands for the number that when we multiply it by itself n times, we obtain r . Therefore, $\sqrt[3]{2}$ stands for the number that when we cube it, we obtain 2. We have

$$(\sqrt[3]{2})^3 = (2^{1/3})^3 = 2^1 = 2$$

Numerically, we have

$$\begin{aligned}(\sqrt[3]{2})^3 &= \sqrt[3]{2} \cdot \sqrt[3]{2} \cdot \sqrt[3]{2} \\&\approx 1.25992104989487 \cdot 1.25992104989487 \cdot 1.25992104989487 \\&\approx 1.587401051968192 \cdot 1.25992104989487 \\&\approx 2\end{aligned}$$

Problem 4. (10pt) Showing all your work, simplify the following as much as possible:

(a) $(x^{-2}y^5)^3$

(b) $\frac{x^{-3}y^4}{x^3y^5}$

(c) $x(x^5y)^2y^{-6}$

Solution.

(a)

$$(x^{-2}y^5)^3 = x^{-6}y^{15} = \frac{y^{15}}{x^6}$$

(b)

$$\frac{x^{-3}y^4}{x^3y^5} = \frac{y^4}{x^3x^3y^5} = \frac{1}{x^6y}$$

(c)

$$x(x^5y)^2y^{-6} = xx^{10}y^2y^{-6} = \frac{xx^{10}y^2}{y^6} = \frac{x^{11}}{y^4}$$

Problem 5. (10pt) Showing all your work, simplify the following as much as possible:

(a) $\left(\frac{x^3}{y^{-1}}\right)^{-1}$

(b) $\frac{(x^2y)^0x^4}{(y^3)^2}$

(c) $\frac{(x^{-3}y^4)^{-5}x^2y}{x^{-2}y^0}$

Solution.

(a)

$$\left(\frac{x^3}{y^{-1}}\right)^{-1} = \frac{y^{-1}}{x^3} = \frac{1}{x^3y}$$

(b)

$$\frac{(x^2y)^0x^4}{(y^3)^2} = \frac{1 \cdot x^4}{y^6} = \frac{x^4}{y^6}$$

(c)

$$\frac{(x^{-3}y^4)^{-5}x^2y}{x^{-2}y^0} = \frac{x^{15}y^{-20}x^2y}{x^{-2}y^0} = \frac{x^{15}x^4y}{y^{20}y^0} = \frac{x^{19}y}{y^{20}} = \frac{x^{19}}{y^{19}}$$

Problem 6. (10pt) Showing all your work, simplify the following as much as possible:

(a) $(x^7y^8)^{1/2}$

(b) $\left(\frac{\sqrt{x^5}}{\sqrt[3]{y^2}}\right)^4$

(c) $\frac{x(x^{3/2}y^{2/3})^2}{(x^6y)^{1/3}}$

Solution.

(a)

$$(x^7y^8)^{1/2} = x^{7/2}y^4$$

(b)

$$\left(\frac{\sqrt{x^5}}{\sqrt[3]{y^2}}\right)^4 = \left(\frac{x^{5/2}}{y^{2/3}}\right)^4 = \frac{x^{10}}{y^{8/3}}$$

(c)

$$\frac{x(x^{3/2}y^{2/3})^2}{(x^6y)^{1/3}} = \frac{xx^3y^{4/3}}{x^2y^{1/3}} = \frac{x^4y^{4/3}}{x^2y^{1/3}} = x^2y$$

Problem 7. (10pt) Showing all your work, simplify the following as much as possible:

(a) $\frac{(y\sqrt{x})^4}{\sqrt{y}x^{-3/2}}$

(b) $(\sqrt[3]{yx^2})^2(yx^2)^{1/3}$

(c) $\left(\frac{x^4}{y^7}\right)^{-2/3}$

Solution.

(a)

$$\frac{(y\sqrt{x})^4}{\sqrt{y}x^{-3/2}} = \frac{(yx^{1/2})^4}{y^{1/2}x^{-3/2}} = \frac{y^4x^2}{y^{1/2}x^{-3/2}} = \frac{y^4x^2x^{3/2}}{y^{1/2}} = x^{7/2}y^{7/2}$$

(b)

$$(\sqrt[3]{yx^2})^2(yx^2)^{1/3} = (y^{1/3}x^{2/3})^2(yx^2)^{1/3} = y^{2/3}x^{4/3} \cdot y^{1/3}x^{2/3} = x^2y$$

(c)

$$\left(\frac{x^4}{y^7}\right)^{-2/3} = \left(\frac{y^7}{x^4}\right)^{2/3} = \frac{y^{14/3}}{x^{8/3}}$$

Problem 8. (10pt) Showing all your work, simplify the following as much as possible:

(a) $\sqrt{72}$

(b) $\sqrt{180}$

(c) $\sqrt{500}$

Solution.

(a)

$$\sqrt{72} = \sqrt{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3} = 2 \cdot 3\sqrt{2} = 6\sqrt{2}$$

(b)

$$\sqrt{180} = \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5} = 2 \cdot 3\sqrt{5} = 6\sqrt{5}$$

(c)

$$\sqrt{500} = \sqrt{2 \cdot 2 \cdot 5 \cdot 5 \cdot 5} = 2 \cdot 5\sqrt{5} = 10\sqrt{5}$$

Problem 9. (10pt) Showing all your work, simplify the following as much as possible:

(a) $\frac{\sqrt{60}}{3}$

(b) $\sqrt[3]{80}$

(c) $\sqrt[4]{2^{12} \cdot 3^3 \cdot 5^9 \cdot 7^4}$

Solution.

(a)

$$\frac{\sqrt{60}}{3} = \frac{\sqrt{2 \cdot 2 \cdot 3 \cdot 5}}{3} = \frac{2\sqrt{3 \cdot 5}}{3} = \frac{2\sqrt{15}}{3}$$

(b)

$$\sqrt[3]{80} = \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 5} = 2\sqrt[3]{2 \cdot 5} = 2\sqrt[3]{10}$$

(c)

$$\sqrt[4]{2^{12} \cdot 3^3 \cdot 5^9 \cdot 7^4} = 2^3 \cdot 5^2 \cdot 7\sqrt[4]{3^3 \cdot 5} = 1400\sqrt[4]{135}$$

Problem 10. (10pt) Rationalize the following fractions:

(a) $\frac{1}{\sqrt{5}}$

(b) $\frac{2}{\sqrt{3}}$

(c) $\frac{4}{1 - \sqrt{2}}$

(d) $\frac{6}{3 + \sqrt{6}}$

(e) $\frac{1}{\sqrt[3]{4}}$

Solution.

(a)

$$\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

(b)

$$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

(c)

$$\frac{4}{1 - \sqrt{2}} = \frac{4}{1 - \sqrt{2}} \cdot \frac{1 + \sqrt{2}}{1 + \sqrt{2}} = \frac{4(1 + \sqrt{2})}{1 + \sqrt{2} - \sqrt{2} - 2} = \frac{4(1 + \sqrt{2})}{-1} = -4(1 + \sqrt{2})$$

(d)

$$\frac{6}{3 + \sqrt{6}} = \frac{6}{3 + \sqrt{6}} \cdot \frac{3 - \sqrt{6}}{3 - \sqrt{6}} = \frac{6(3 - \sqrt{6})}{9 - 3\sqrt{6} + 3\sqrt{6} - 6} = \frac{6(3 - \sqrt{6})}{3} = 2(3 - \sqrt{6})$$

(e)

$$\frac{1}{\sqrt[3]{4}} = \frac{1}{4^{1/3}} = \frac{1}{4^{1/3}} \cdot \frac{4^{2/3}}{4^{2/3}} = \frac{4^{2/3}}{4^1} = \frac{\sqrt[3]{4^2}}{4} = \frac{\sqrt[3]{4 \cdot 4}}{4} = \frac{\sqrt[3]{2 \cdot 2 \cdot 2 \cdot 2}}{4} = \frac{2\sqrt[3]{2}}{4} = \frac{\sqrt[3]{2}}{2}$$