Name: _____
MATH 308
Fall 2021
HW 13: Due 11/22

"All generalizations are false, including this one."

–Mark Twain

Problem 1. (10pt) Prove that the product of two even integers is even and that the product of an even integer with an odd integer is even.

Problem 2. (10pt) Prove that if the square of an integer is even, then the integer is even. Use this to prove that if $n^2 + 1$ is a prime greater than 5, then the digit in the 1's place of n is 0, 4, or 6.

Problem 3. (10pt) Use the division algorithm to write $180 = 7q + r$, where $q, r \in \mathbb{Z}$ and $0 \leq r < 7$.

Problem 4. (10pt) Use the division algorithm to prove that the 1's digit of a perfect square is never 2, 3, 7, or 8.

Problem 5. (10pt) Prove or disprove: Let $x, a, b \in \mathbb{Z}$. If x does not divide a and x does not divide b, then x does not divide ab.

Problem 6. (10pt) Prove that if n is composite, then n has a prime factor p with $p \leq \sqrt{n}$. Use this to show that 1321 is prime.