Name: _____

MATH 308

Fall 2023

HW 12: Due 11/10

"No part of mathematics is ever, in the long run, "useless." Most of number theory has very few "practical" applications. That does not reduce its importance, and if anything it enhances its fascination. No one can predict when what seems to be a most obscure theorem may suddenly be called upon to play some vital and hitherto unsuspected role."

−C. Stanley

Problem 1. (10pt) Showing all your work and fully justifying your answer, complete the following:

(a) From the definition, determine whether 79 is odd or even.

(b) From the definition, determine whether or not -343 a perfect cube.

(c) Find the prime factorization of 840. Find all the divisors of 840.

(d) Can an integer of the form $n^2 + 7n + 6$, where $n \in \mathbb{N}$, be prime? Explain.

Problem 2. (10pt) Using the given a, b, express the division $\frac{b}{a}$ using the division algorithm. Be sure to show all your work.

(a) $a = 16$, $b = 2797$

(b) $a = -29$, $b = -7015$

(c) $a = 56$, $b = 55664$

Problem 3. (10pt) Showing all your work and fully justifying your reasoning, complete the following:

(a) Use the Euclidean Algorithm to find $\gcd(459, 303)$.

(b) Use the extended Euclidean algorithm, express $\gcd(459, 303)$ as a linear combination of 459 and 303.

(c) Is it possible to find integers x, y such that $459x + 301y = 5$? If not, explain why. If so, find them.

(d) Is it possible to find integers x, y such that $459x + 301y = 6$? If not, explain why. If so, find them.

Problem 4. (10pt) Showing all your work, convert the given base-10 number, binary number, or hexadecimal to the given base b:

(a) 7187; $b = 16$

(b) 119; $b = 2$

(c) 11101_2; $b = 10$

(d) $801c$; $b = 10$