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MATH 101

Fall 2023

HW 3: Due 09/18

"I saw a lot of my body... and I didn't like it."

—David Rose, *Schitt's Creek*

Problem 1. (10pt) Showing all your work, simplify the following as much as possible (express any denominators using negative powers):

(a) $ab(a^3b^2)^0$

(b) $x^4y^9x^{22}y^5$

(c) $\frac{r^0s^5}{r^4s^3}$

(d) $(x^8y^{10})\left(\frac{x^3}{y^8}\right)$

(e) $\frac{r^{12}s^4t^5}{s^3r^{20}t^5}$

Solution.

(a)

$$ab(a^3b^2)^0 = ab \cdot 1 = ab$$

(b)

$$x^4y^9x^{22}y^5 = x^4x^{22} \cdot y^9y^5 = x^{4+22}y^{9+5} = x^{26}y^{14}$$

(c)

$$\frac{r^0s^5}{r^4s^3} = \frac{1 \cdot s^5}{r^4s^3} = \frac{s^5}{r^4s^3} = \frac{\cancel{s^3}^2}{r^4\cancel{s}^3} = \frac{s^2}{r^4} = s^2r^{-4}$$

(d)

$$(x^8y^{10})\left(\frac{x^3}{y^8}\right) = \frac{x^8x^3y^{10}}{y^8} = \frac{x^{8+3}y^{10}}{y^8} = \frac{x^{11}y^{10}}{y^8} = \frac{x^{11}y^{\cancel{10}^2}}{\cancel{y}^8} = x^{11}y^2$$

(e)

$$\frac{r^{12}s^4t^5}{s^3r^{20}t^5} = \frac{\cancel{r}^{12}s^4\cancel{t}^5}{\cancel{s}^3r^{\cancel{20}^8}\cancel{t}^5} = \frac{s}{r^8} = sr^{-8}$$

Problem 2. (10pt) Showing all your work, simplify the following as much as possible (do not express your answer using any negative powers):

(a) $\frac{a^6 b^3}{a^{18} b^5}$

(b) $\frac{x^6 y^9}{x^{-6} y^{16}}$

(c) $\frac{r^{18} s^7 r^{-3} s^{-2}}{r^{11} s^5}$

(d) $\frac{a^0 b^{-5}}{a^{-8} b^7} \cdot \frac{b^3}{a^6}$

(e) $\frac{x}{y} \left(\frac{x^3 y^{-11}}{x^{-5} y^{12}} \cdot \frac{x^{-4} y^9}{x^4 y^{-3}} \right)^0$

Solution.

(a)

$$\frac{a^6 b^3}{a^{18} b^5} = \frac{\cancel{a^6} \cancel{b^3}}{\cancel{a^{18}}^{12} \cancel{b^3}^2} = \frac{1}{a^{12} b^2}$$

(b)

$$\frac{x^6 y^9}{x^{-6} y^{16}} = \frac{x^6 x^6 y^9}{y^{16}} = \frac{x^{6+6} y^9}{y^{16}} = \frac{x^{12} y^9}{y^{16}} = \frac{x^{12} \cancel{y^9}}{\cancel{y^9}^7} = \frac{x^{12}}{y^7}$$

(c)

$$\frac{r^{18} s^7 r^{-3} s^{-2}}{r^{11} s^5} = \frac{r^{18} r^{-3} s^7 s^{-2}}{r^{11} s^5} = \frac{r^{18-3} s^{7-2}}{r^{11} s^5} = \frac{r^{15} s^5}{r^{11} s^5} = \frac{\cancel{r^{15}}^4 \cancel{s^5}}{\cancel{r^{11}}^4 \cancel{s^5}} = r^4$$

(d)

$$\frac{a^0 b^{-5}}{a^{-8} b^7} \cdot \frac{b^3}{a^6} = \frac{b^{-5}}{a^{-8} b^7} \cdot \frac{b^3}{a^6} = \frac{b^{-5} b^3}{a^{-8} a^6 b^7} = \frac{b^{-5+3}}{a^{-8+6} b^7} = \frac{b^{-2}}{a^{-2} b^7} = \frac{a^2}{b^2 b^7} = \frac{a^2}{b^{2+7}} = \frac{a^2}{b^9}$$

(e)

$$\frac{x}{y} \left(\frac{x^3 y^{-11}}{x^{-5} y^{12}} \cdot \frac{x^{-4} y^9}{x^4 y^{-3}} \right)^0 = \frac{x}{y} \cdot 1 = \frac{x}{y}$$

Problem 3. (10pt) Showing all your work, simplify the following as much as possible (do not express your answer using any negative powers):

(a) $((x^2y^3)^3)^3$

(b) $(r^3s)^6(r^2s^9)^4$

(c) $(x^{-4}y^6)^{-2}(x^2y^5)$

(d) $b^6 \left(\frac{a^6b^3}{a^{-3}} \right)^{-4}$

(e) $(xy^2)^{-3} \left(\frac{(xy)^2}{xy^{-1}} \right)^{-1}$

Solution.

(a)

$$((x^2y^3)^3)^3 = (x^2y^3)^9 = x^{18}y^{27}$$

(b)

$$(r^3s)^6(r^2s^9)^4 = (r^{18}s^6)(r^8s^{36}) = r^{18}r^8s^6s^{36} = r^{18+8}s^{6+36} = r^{26}s^{42}$$

(c)

$$(x^{-4}y^6)^{-2}(x^2y^5) = (x^8y^{-12})(x^2y^5) = x^8x^2y^{-12}y^5 = x^{8+2}y^{-12+5} = x^{10}y^{-7} = \frac{x^{10}}{y^7}$$

(d)

$$b^6 \left(\frac{a^6b^3}{a^{-3}} \right)^{-4} = b^6 (a^6a^3b^3)^{-4} = b^6(a^9b^3)^{-4} = b^6a^{-36}b^{-12} = a^{-36}b^6b^{-12} = a^{-36}b^{-6} = \frac{1}{a^{36}b^6}$$

(e)

$$(xy^2)^{-3} \left(\frac{(xy)^2}{xy^{-1}} \right)^{-1} = (x^{-3}y^{-6}) \cdot \frac{xy^{-1}}{(xy)^2} = x^{-3}y^{-6} \cdot \frac{xy^{-1}}{x^2y^2} = \frac{x^{-3}xy^{-6}y^{-1}}{x^2y^2} = \frac{x^{-2}y^{-7}}{x^2y^2} = \frac{1}{x^2x^2y^2y^7} = \frac{1}{x^4y^9}$$

Problem 4. (10pt) Showing all your work, simplify the following as much as possible:

(a) $\frac{(4x^2)^3(3x^4)}{(6x^3)^4}$

(b) $\left(-\frac{3x^{-2}y^7}{2x^3y^5}\right)^{-2}$

(c) $\left(\frac{x^6y^3 \cdot (x^5y)^2}{x^{-4}y^{12}}\right)^3$

(d) $\left(\frac{x^{r+s}}{x^{2r+5}}\right)^4$

(e) $(xy)^{-n} \cdot \frac{x^{n-1}y^{m-1}}{x^{2n}y^n}$

Solution.

(a)
$$\frac{(4x^2)^3(3x^4)}{(6x^3)^4} = \frac{4^3x^6 \cdot 3x^4}{6^4x^{12}} = \frac{(2^2)^3 \cdot 3 \cdot x^6x^4}{(2 \cdot 3)^4x^{12}} = \frac{2^6 \cdot 3 \cdot x^{10}}{2^4 \cdot 3^4x^{12}} = \frac{2^2 \cdot 1 \cdot 1}{1 \cdot 3^3 \cdot x^2} = \frac{4}{27x^2}$$

(b)
$$\left(-\frac{3x^{-2}y^7}{2x^3y^5}\right)^{-2} = \left(-\frac{2x^3y^5}{3x^{-2}y^7}\right)^2 = \frac{4x^6y^{10}}{9x^{-4}y^{14}} = \frac{4x^{10}}{9y^4}$$

(c)
$$\left(\frac{x^6y^3 \cdot (x^5y)^2}{x^{-4}y^{12}}\right)^3 = \frac{x^{18}y^9 \cdot (x^5y)^6}{x^{-12}y^{36}} = \frac{x^{18}y^9 \cdot x^{30}y^6}{x^{-12}y^{36}} = \frac{x^{48}y^{15}}{x^{-12}y^{36}} = x^{48-(-12)}y^{15-36} = x^{60}y^{-21} = \frac{x^{60}}{y^{21}}$$

(d)
$$\left(\frac{x^{r+s}}{x^{2r+5}}\right)^4 = \frac{x^{4r+4s}}{x^{8r+20}} = x^{(4r+4s)-(8r+20)} = x^{4s+4s-8r-20} = x^{4s-4r-20}$$

(e)

$$(xy)^{-n} \cdot \frac{x^{n-1}y^{m-1}}{x^{2n}y^n} = x^{-n}y^{-n} \cdot \frac{x^{n-1}y^{m-1}}{x^{2n}y^n} = x^{-n}y^{-n} \cdot x^{n-1}y^{m-1} \cdot x^{-2n}y^{-n} = x^{-n+(n-1)-2n}y^{-n+(m-1)-n} = x^{-2n-1}y^{m-2n-1}$$