Name: _____

MATH 308
Fall 2022
HW 22: Due 12/15

"The art of simplicity is a puzzle of complexity."
–Douglas Horton

Problem 1. (10pt) Showing all your work and fully justifying your reasoning, answer the following:

(a) Show that $f(x) = x^5 + 3x^4 - x^2 + 6$ is $\Omega(x^5)$. Does this imply that $f(x)$ is $\Omega(x^4)$?

(b) Show that $g(x) = x^4 - 3x^2 + 6x^2 - 8$ is $O(x^5)$. Does this imply that $g(x)$ is $O(x^6)$?

(c) Show that $h(x) = x^3 - x + 7$ is $\Theta(x^3)$. For $n \neq 3$, can $h(x)$ be $\Theta(x^n)$? Explain.

Problem 2. (10pt) Show that $\sum_{i=0}^{n}(3i+2)$ is $\Theta(n^2)$.

Problem 3. (10pt) Define $f(x) = 2x + \log x$ and $h(x) = x^2 + 2^x + 5$. Show that $f(x)$ is $\Theta(x)$ and $h(x)$ is $\Theta(2^x)$.

Problem 4. (10pt) Assume that each addition, subtraction, multiplication, division, and `print` 'costs' one flop while defining/redefining variables 'cost' no flops. Suppose you have an algorithm, whose pseudocode is given below.

```
for i := 1 to n
   for j := 1 to n
      a := i^3*j + i - 2 + n
      print(a)
   next j
next i
```

(a) Find the outputs for this algorithm for $n = 2$.

(b) Find the total number of flops performing this algorithm. What is Θ for this algorithm?