MAT 101: Exam 1
Summer – 2022
05/26/2022
85 Minutes

Name: *Caleb M^{c}Whorter — Solutions*

Write your name on the appropriate line on the exam cover sheet. This exam contains 21 pages (including this cover page) and 20 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
14	10	
15	10	
16	10	
17	10	
18	10	
19	10	
20	10	
Total:	200	

1. (10 points) For each of the following numbers, indicate with a checkmark whether each of the numbers is a natural, integer, rational, irrational, real, or complex number.

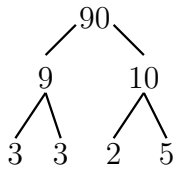
	Natural	Integer	Rational	Irrational	Real	Complex
13	✓					
0						
-7						
$34/4$						
$0.\overline{13}$						
$5/7$						
-5.78						
$\sqrt{49}$						
π						
$\sqrt{-4}$						

2. (10 points) Find the prime factorizations for each of the following integers:

   (a) 29

   (b) 84

   (c) 495



3. (10 points) Using an argument involving the square root, show that the integer 109 is prime.

4. (10 points) Showing all your work, compute the following:

   (a) $\gcd(50, 66)$

   (b) $\mathrm{lcm}(14, 22)$

   (c) $\gcd(2^{12} \cdot 3^{40} \cdot 7^{12} \cdot 13^{70}, 2^{30} \cdot 3^{28} \cdot 5^{46} \cdot 11^{90})$

   (d) $\mathrm{lcm}(2^{12} \cdot 3^{40} \cdot 7^{12} \cdot 13^{70}, 2^{30} \cdot 3^{28} \cdot 5^{46} \cdot 11^{90})$

5. (10 points) Showing all your work, reduce the following rational numbers completely:

   (a) $\frac{21}{45}$

   (b) $\frac{7}{22}$

   (c) $\frac{132}{30}$

6. (10 points) Showing all your work and being sure to simplify as much as possible, compute the following:

   (a) $\frac{5}{7} - \frac{8}{21}$

   (b) $\frac{9}{8} + \frac{7}{12}$

   (c) $\frac{20}{99} \cdot \frac{21}{10}$

   (d) $\dfrac{\frac{26}{15}}{\frac{20}{9}}$

7. (10 points) Convert the following decimal numbers to a rational number, being sure to simplify as much as possible:

   (a) 0.240

   (b) $0.\overline{18}$

$$\begin{array}{rrcl} & 100N & = & 12.1212121\overline{12} \\ - & N & = & 0.1212121\overline{12} \\ \hline & 99N & = & 12 \\ & N & = & \frac{12}{99} \\ & N & = & \frac{4}{33} \end{array}$$

$$0.\overline{12} = \frac{4}{33}$$

8. (10 points) Write the following numbers in scientific notation:

   (a) 178.3

   (b) 4.64

   (c) 0.0000091

   (d) 167000

9. (10 points) Write the following scientific numbers in ordinary decimal notation:

   (a) $1.45 \cdot 10^{5}$

   (b) $6.92 \cdot 10^{-3}$

   (c) $-8.3 \cdot 10^{0}$

10. (10 points) Simplify the follow expressions, showing all your work and using no negative powers:

    (a) $x^5y^2(xy^3)^0(x^3y)^4$

    (b) $z^4(x^5z^3)^{-2}(x^3y^2)^2$

    (c) $\dfrac{x^{-5}y^6}{(x^3y^{-4})^{-2}}$

    (d) $\dfrac{(x^8y^{12})^{1/4}}{\sqrt{x^4y^5}}$

11. (10 points) Showing all your work, simplify the following as much as possible:

    (a) $\sqrt{68}$

    (b) $\sqrt{2^8 \cdot 3 \cdot 5^3}$

    (c) $\sqrt[5]{2^{10} \cdot 3^5 \cdot 5^2 \cdot 7^6}$

12. (10 points) Showing all your work and simplifying as much as possible, rationalize the following:

    (a) $\dfrac{10}{\sqrt{5}}$

    (b) $\dfrac{14}{3-\sqrt{2}}$

13. (10 points) Compute each of the following and write the result in the form $a + bi$, being sure to simplify as much as possible:

    (a) $(10 - 7i) - (5 - 11i)$

    (b) $(4 - 3i)(11 + 6i)$

    (c) $\dfrac{1 + i}{1 - 3i}$

14. (10 points) Showing all your work, compute each of the following:

    (a) 18% of 940

    (b) 88% of 7

    (c) 132% of 65

15. (10 points) Showing all your work, compute each of the following:

    (a) 900 decreased by 36%

    (b) 66 increased by 45%

    (c) 19 increased by 160%

16. (10 points) A student is taking a course with the following grade components:

Presentation	5%
Homework	25%
Quizzes	15%
Midterm	25%
Final	30%

Suppose that the student received a 94% on the presentation, 78% on the midterm, and 84% on the final. If the student had a 89% quiz average and 92% homework average, find the student's course average.

17. (10 points) A local college uses the following credit weights in their GPA calculations:

A	4.0	C+	2.3
A−	3.7	C	2.0
B+	3.3	C−	1.7
B	3.0	D	1.0
B−	2.7	F	0.0

Suppose that student received the following grades this semester:

Course	Credits	Grade
German I	3	A−
Freshman Seminar	1	A
Calculus III	4	A−
Physics I	4	B−
Cultural Anthropology	3	B
The Soviet Empire	3	C+

Showing all your work, compute this student's semester GPA.

18. (10 points) A certain drug is to be administered such that the patient gets 45 mg for every 3 pounds (lb) that a patient weighs. How many milligrams (mg) should a 165 lb patient receive?

19. (10 points) Mankind has met an alien civilization. They use a unit of distance known as a 'kellikarn' (kn) and a unit of time called a 'starpoint' (sp). It is known 15 kn is 3.2 ft and 17 sp is 1.4 min. What is 26.1 kn/sp in miles per hour? Note that there are 5280 ft in one mile.

20. (10 points) Suppose that the average cost of land in Montana is approximately $2,000 per acre. If one acre is 4840 yd^2 and 1 yard (yd) is 3 feet (ft), what is the average cost of land in Montana per square foot?