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MATH 101

Winter 2021

HW 2: Due 01/05

*"I don't talk trash, I talk smack. They're totally different. Trash talk is hypothetical, like: your mom is so fat she can eat the internet. But smack talk is happening like right now. Like: you're ugly and I know it for a fact 'cause I got the evidence right there."*

*–Kelly Kapoor, The Office*

**Problem 1.** (10pt) Showing all your work, simplify the following as much as possible:

(a)  $(x^3y^{-1})^4$

(b)  $\frac{x^5y^{-6}}{x^3y^2}$

(c)  $\left(\frac{x^{-2}}{y^4}\right)^{-1}$

(d)  $\frac{(xy)^0x^{-3}}{(y^2)^3}$

(e)  $\frac{(x^{-2}y^3)^{-5}xy^6}{x^0y^{-2}}$

**Solution.**

(a)

$$(x^3y^{-1})^4 = x^{12}y^{-4} = \frac{x^{12}}{y^4}$$

(b)

$$\frac{x^5y^{-6}}{x^3y^2} = \frac{x^5}{x^3y^2y^6} = \frac{x^5}{x^3y^8} = \frac{x^2}{y^8}$$

(c)

$$\left(\frac{x^{-2}}{y^4}\right)^{-1} = \frac{y^4}{x^{-2}} = x^2y^4$$

(d)

$$\frac{(xy)^0x^{-3}}{(y^2)^3} = \frac{1 \cdot x^{-3}}{y^6} = \frac{1}{x^3y^6}$$

(e)

$$\frac{(x^{-2}y^3)^{-5}xy^6}{x^0y^{-2}} = \frac{x^{10}y^{-15}xy^6}{1 \cdot y^{-2}} = \frac{x^{11}y^6y^2}{y^{15}} = \frac{x^{11}y^8}{y^{15}} = \frac{x^{11}}{y^7}$$

**Problem 2.** (10pt) Showing all your work, simplify the following as much as possible:

(a)  $(x^4y^5)^{1/2}$

(b)  $\left(\frac{\sqrt{x}}{\sqrt[3]{y^2}}\right)^3$

(c)  $\frac{(x\sqrt{y})^3}{\sqrt{x}y^{-3/2}}$

(d)  $(\sqrt[3]{xy^2})^2(xy^2)^{1/3}$

(e)  $\left(\frac{x^6}{y^5}\right)^{-2/3}$

**Solution.**

(a)

$$(x^4y^5)^{1/2} = x^{4/2}y^{5/2} = x^2y^{5/2} = x^2\sqrt{y^5}$$

(b)

$$\left(\frac{\sqrt{x}}{\sqrt[3]{y^2}}\right)^3 = \left(\frac{x^{1/2}}{y^{2/3}}\right)^3 = \frac{x^{3/2}}{y^2} = \frac{\sqrt[3]{x^2}}{y^2}$$

(c)

$$\frac{(x\sqrt{y})^3}{\sqrt{x}y^{-3/2}} = \frac{(xy^{1/2})^3}{x^{1/2}y^{-3/2}} = \frac{x^3y^{3/2} \cdot y^{3/2}}{x^{1/2}} = \frac{x^3y^3}{x^{1/2}} = x^{5/2}y^3 = y^3\sqrt{x^5}$$

(d)

$$(\sqrt[3]{xy^2})^2(xy^2)^{1/3} = (x^{1/3}y^{2/3})^2(xy^2)^{1/3} = x^{2/3}y^{4/3}x^{1/3}y^{2/3} = x^1y^{6/3} = xy^2$$

(e)

$$\left(\frac{x^6}{y^5}\right)^{-2/3} = \left(\frac{y^5}{x^6}\right)^{2/3} = \frac{y^{10/3}}{x^{12/3}} = \frac{y^{10/3}}{x^4} = \frac{\sqrt[3]{y^{10}}}{x^4}$$

**Problem 3.** (10pt) Showing all your work, simplify the following as much as possible:

(a)  $\sqrt{28}$

(b)  $\sqrt{120}$

(c)  $\frac{\sqrt{90}}{3}$

(d)  $\sqrt[3]{360}$

(e)  $\sqrt[4]{2^9 \cdot 3^5 \cdot 5^2 \cdot 7^4}$

**Solution.**

(a)

$$\sqrt{28} = \sqrt{2^2 \cdot 7} = 2\sqrt{7}$$

(b)

$$\sqrt{120} = \sqrt{2^3 \cdot 3 \cdot 5} = 2\sqrt{2 \cdot 3 \cdot 5} = 2\sqrt{30}$$

(c)

$$\frac{\sqrt{90}}{3} = \frac{\sqrt{2 \cdot 3^2 \cdot 5}}{3} = \frac{3\sqrt{2 \cdot 5}}{3} = \sqrt{10}$$

(d)

$$\sqrt[3]{360} = \sqrt[3]{2^3 \cdot 3^2 \cdot 5} = 2\sqrt[3]{3^2 \cdot 5} = 2\sqrt[3]{45}$$

(e)

$$\sqrt[4]{2^9 \cdot 3^5 \cdot 5^2 \cdot 7^4} = 2^2 \cdot 3^1 \cdot 7 \sqrt[4]{2 \cdot 3 \cdot 5^2} = 84\sqrt[4]{150}$$

**Problem 4.** (10pt) Rationalize the following fractions:

(a)  $\frac{1}{\sqrt{3}}$

(b)  $\frac{6}{\sqrt{5}}$

(c)  $\frac{4}{1 + \sqrt{6}}$

(d)  $\frac{6}{3 - \sqrt{7}}$

(e)  $\frac{1}{\sqrt[3]{12}}$

**Solution.**

(a)

$$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

(b)

$$\frac{6}{\sqrt{5}} = \frac{6}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{6\sqrt{5}}{5}$$

(c)

$$\frac{4}{1 + \sqrt{6}} = \frac{4}{1 + \sqrt{6}} \cdot \frac{1 - \sqrt{6}}{1 - \sqrt{6}} = \frac{4 - 4\sqrt{6}}{1 - \sqrt{6} + \sqrt{6} - 6} = \frac{4 - 4\sqrt{6}}{-5} = \frac{4\sqrt{6} - 4}{5}$$

(d)

$$\frac{6}{3 - \sqrt{7}} = \frac{6}{3 - \sqrt{7}} \cdot \frac{3 + \sqrt{7}}{3 + \sqrt{7}} = \frac{18 + 6\sqrt{7}}{9 + 3\sqrt{7} - 3\sqrt{7} - 7} = \frac{18 + 6\sqrt{7}}{2} = 9 + 3\sqrt{7}$$

(e)

$$\frac{1}{\sqrt[3]{12}} = \frac{1}{(12)^{1/3}} \cdot \frac{(12)^{2/3}}{(12)^{2/3}} = \frac{(12)^{2/3}}{12} = \frac{\sqrt[3]{12^2}}{12} = \frac{\sqrt[3]{144}}{12} = \frac{\sqrt[3]{8 \cdot 18}}{12} = \frac{2\sqrt[3]{18}}{12} = \frac{\sqrt[3]{18}}{6}$$

**Problem 5.** (10pt) Convert the following numbers from scientific to decimal notation:

(a)  $1.5 \cdot 10^4$

(b)  $3.19 \cdot 10^{-3}$

(c)  $-4.33 \cdot 10^0$

(d)  $1.574 \cdot 10^2$

(e)  $8.48 \cdot 10^{-6}$

**Solution.**

(a)  $1.5 \cdot 10^4 = 15000$

(b)  $3.19 \cdot 10^{-3} = 0.00319$

(c)  $-4.33 \cdot 10^0 = -4.33$

(d)  $1.574 \cdot 10^2 = 157.4$

(e)  $8.48 \cdot 10^{-6} = 0.00000848$

**Problem 6.** (10pt) Convert the following numbers from decimal to scientific notation:

(a) 14500000

(b) 0.004

(c) 878410

(d) 0.0000077

(e) 1.55

**Solution.**

(a)  $14500000 = 1.45 \cdot 10^7$

(b)  $0.004 = 4.0 \cdot 10^{-3}$

(c)  $878410 = 8.78410 \cdot 10^5$

(d)  $0.0000077 = 7.7 \cdot 10^{-6}$

(e)  $1.55 = 1.55 \cdot 10^0$

**Problem 7.** (10pt) Express the following rational numbers as a decimal:

(a)  $\frac{3}{8}$

(b)  $\frac{13}{4}$

(c)  $\frac{4}{5}$

(d)  $\frac{1}{9}$

(e)  $\frac{4}{33}$

**Solution.**

(a)  $\frac{3}{8} : 8 \overline{) 3.000} . \text{ Then } \frac{3}{8} = 0.375.$

$$\begin{array}{r} 0.375 \\ 8 \overline{) 3.000} \\ \underline{24} \phantom{00} \\ 60 \phantom{0} \\ \underline{56} \phantom{0} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

(b)  $\frac{13}{4} : 4 \overline{) 13.00} . \text{ Then } \frac{13}{4} = 3.25.$

$$\begin{array}{r} 3.25 \\ 4 \overline{) 13.00} \\ \underline{12} \phantom{00} \\ 10 \phantom{0} \\ \underline{8} \phantom{0} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

(c)  $\frac{4}{5} : 5 \overline{) 4.0} . \text{ Then } \frac{4}{5} = 0.8.$

$$\begin{array}{r} 0.8 \\ 5 \overline{) 4.0} \\ \underline{40} \\ 0 \end{array}$$

(d)  $\frac{1}{9} : 9 \overline{) 1.0} . \text{ Then } \frac{1}{9} = 0.\overline{1}.$

$$\begin{array}{r} 0.\overline{1} \\ 9 \overline{) 1.0} \\ \underline{9} \phantom{0} \\ 10 \\ \underline{9} \\ 1 \end{array}$$

(e)  $\frac{4}{33} : 33 \overline{) 4.00} . \text{ Then } \frac{4}{33} = 0.\overline{12}.$

$$\begin{array}{r} 0.\overline{12} \\ 33 \overline{) 4.00} \\ \underline{33} \phantom{00} \\ 60 \phantom{0} \\ \underline{66} \phantom{0} \\ 70 \\ \underline{66} \\ 4 \end{array}$$

**Problem 8.** (10pt) Express the following decimal numbers as rational numbers:

(a)  $-6$

(b)  $1.4$

(c)  $0.54$

(d)  $0.2222\overline{2}$

(e)  $0.1010\overline{10}$

**Solution.**

(a)

$$-6 = \frac{-6}{1}$$

(b)

$$1.4 = \frac{14}{10} = \frac{7}{5}$$

(c)

$$0.54 = \frac{54}{100} = \frac{27}{50}$$

(d)

$$\begin{array}{rcl} 10N & = & 2.2222\overline{2} \\ N & = & 0.2222\overline{2} \\ \hline 9N & = & 2 \\ N & = & \frac{2}{9} \end{array}$$

(e)

$$\begin{array}{rcl} 100N & = & 10.1010\overline{10} \\ N & = & 0.1010\overline{10} \\ \hline 99N & = & 10 \\ N & = & \frac{10}{99} \end{array}$$



**Problem 9.** (10pt) Showing all your work, compute the following:

(a)  $(1 - 5i) + (6 + 8i)$

(b)  $(5 + 6i) - 2(4 - i)$

(c)  $(3 + i)(4 + 2i)$

(d)  $\frac{1 + i}{8 + i}$

(e)  $(5 - 3i)^2$

**Solution.**

(a)

$$(1 - 5i) + (6 + 8i) = (1 + 6) + (-5i + 8i) = 7 + 3i$$

(b)

$$(5 + 6i) - 2(4 - i) = (5 + 6i) + (-8 + 2i) = (5 - 8) + (6i + 2i) = -3 + 8i$$

(c)

$$(3 + i)(4 + 2i) = 12 + 6i + 4i + 2i^2 = 12 + 10i - 2 = 10 + 10i$$

(d)

$$\frac{1 + i}{8 + i} = \frac{1 + i}{8 + i} \cdot \frac{8 - i}{8 - i} = \frac{(1 + i)(8 - i)}{(8 + i)(8 - i)} = \frac{8 - i + 8i - i^2}{64 - 8i + 8i - i^2} = \frac{8 + 7i + 1}{64 + 1} = \frac{9 + 7i}{65} = \frac{9}{65} + \frac{7}{65}i$$

(e)

$$(5 - 3i)^2 = (5 - 3i)(5 - 3i) = 25 - 15i - 15i + 9i^2 = 25 - 30i - 9 = 16 - 30i$$

**Problem 10.** (10pt) Simplifying as much as possible, express the following as a single complex number of the form  $a + bi$ :

(a) 7

(b)  $\sqrt{-4}$

(c)  $6 - \sqrt{-18}$

(d)  $(2i)^3$

(e)  $\frac{1 + \sqrt{-9}}{3}$

**Solution.**

(a)

$$7 = 7 + 0i$$

(b)

$$\sqrt{-4} = \sqrt{4}i = 2i = 0 + 2i$$

(c)

$$6 - \sqrt{-18} = 6 - \sqrt{18}i = 6 - \sqrt{9 \cdot 2}i = 6 - 3\sqrt{2}i$$

(d)

$$(2i)^3 = 2^3 i^3 = 8(-i) = -8i = 0 - 8i$$

(e)

$$\frac{1 + \sqrt{-9}}{3} = \frac{1 + \sqrt{9}i}{3} = \frac{1 + 3i}{3} = \frac{1}{3} + i$$