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MATH 101
Spring 2024
HW 2: Due 01/29

"All opinions are not equal. Some are a very great deal more robust, sophisticated and well supported in logic and argument than others."
— Douglas Adams

Problem 1. (10pts) Showing all your work, reduce the following rational numbers:

(a) $\frac{36}{20}$

(b) $\frac{165}{44}$

(c) $\frac{23}{5}$

(d) $\frac{16}{80}$

(e) $\frac{70}{105}$

Solution.

(a)
$$\frac{36}{20} = \frac{9 \cdot 4}{5 \cdot 4} = \frac{9 \cdot \cancel{4}}{5 \cdot \cancel{4}} = \frac{9}{5}$$

(b)
$$\frac{165}{44} = \frac{11 \cdot 15}{11 \cdot 4} = \frac{\cancel{11} \cdot 15}{\cancel{11} \cdot 4} = \frac{15}{4}$$

(c)
$$\frac{23}{5} = \frac{23}{5}$$

(d)
$$\frac{16}{80} = \frac{16}{5 \cdot 16} = \frac{\cancel{16}}{5 \cdot \cancel{16}} = \frac{1}{5}$$

(e)
$$\frac{70}{105} = \frac{35 \cdot 2}{35 \cdot 3} = \frac{\cancel{35} \cdot 2}{\cancel{35} \cdot 3} = \frac{2}{3}$$

Problem 2. (10pts) Showing all your work and simplifying as much as possible, compute the following:

(a) $\frac{3}{7} + \frac{5}{2}$

(b) $\frac{11}{3} - \frac{5}{33}$

(c) $\frac{12}{25} + \frac{7}{10}$

(d) $\frac{18}{5} - \frac{10}{11}$

(e) $\frac{5}{3} + \frac{11}{2} - \frac{1}{7}$

Solution.

(a)
$$\frac{3}{7} + \frac{5}{2} = \frac{3}{7} \cdot \frac{2}{2} + \frac{5}{2} \cdot \frac{7}{7} = \frac{6}{14} + \frac{35}{14} = \frac{6+35}{14} = \frac{41}{14}$$

(b)
$$\frac{11}{3} - \frac{5}{33} = \frac{11}{3} \cdot \frac{11}{11} - \frac{5}{33} = \frac{121}{33} - \frac{5}{33} = \frac{121-5}{33} = \frac{116}{33}$$

(c)
$$\frac{12}{25} + \frac{7}{10} = \frac{12}{25} \cdot \frac{2}{2} + \frac{7}{10} \cdot \frac{5}{5} = \frac{24}{50} + \frac{35}{50} = \frac{24+35}{50} = \frac{59}{50}$$

(d)
$$\frac{18}{5} - \frac{10}{11} = \frac{18}{5} \cdot \frac{11}{11} - \frac{10}{11} \cdot \frac{5}{5} = \frac{198}{55} - \frac{50}{55} = \frac{198-50}{55} = \frac{148}{55}$$

(e)
$$\frac{5}{3} + \frac{11}{2} - \frac{1}{7} = \frac{5}{3} \cdot \frac{14}{14} + \frac{11}{2} \cdot \frac{21}{21} - \frac{1}{7} \cdot \frac{6}{6} = \frac{70}{42} + \frac{231}{42} - \frac{6}{42} = \frac{70+231-6}{42} = \frac{295}{42}$$

Problem 3. (10pts) Showing all your work and simplifying as much as possible, compute the following:

(a) $\frac{6}{55} \cdot \frac{44}{21}$

(b) $\frac{\frac{49}{12}}{\frac{7}{20}}$

(c) $\frac{\frac{11}{5}}{\frac{3}{26}}$

(d) $\frac{30}{18} \cdot \frac{27}{70}$

(e) $\frac{\frac{180}{175}}{\frac{30}{98}}$

Solution.

(a)
$$\frac{6}{55} \cdot \frac{44}{21} = \frac{\cancel{6}^{\,2}}{\cancel{55}^{\,5}} \cdot \frac{\cancel{44}^{\,4}}{\cancel{21}^{\,7}} = \frac{2}{5} \cdot \frac{4}{7} = \frac{8}{35}$$

(b)
$$\frac{\frac{49}{12}}{\frac{7}{20}} = \frac{49}{12} \cdot \frac{20}{7} = \frac{\cancel{49}^{\,7}}{\cancel{12}^{\,3}} \cdot \frac{\cancel{20}^{\,5}}{\cancel{7}^{\,1}} = \frac{7}{3} \cdot \frac{5}{1} = \frac{35}{3}$$

(c)
$$\frac{\frac{11}{5}}{\frac{3}{26}} = \frac{11}{5} \cdot \frac{26}{3} = \frac{286}{15}$$

(d)
$$\frac{30}{18} \cdot \frac{27}{70} = \frac{\cancel{30}^{\,3}}{\cancel{18}^{\,2}} \cdot \frac{\cancel{27}^{\,3}}{\cancel{70}^{\,7}} = \frac{3}{2} \cdot \frac{3}{7} = \frac{9}{14}$$

(e)
$$\frac{\frac{180}{175}}{\frac{30}{98}} = \frac{180}{175} \cdot \frac{98}{30} = \frac{\cancel{180}^{\,6}}{\cancel{175}^{\,25}} \cdot \frac{\cancel{98}^{\,14}}{\cancel{30}^{\,1}} = \frac{6}{25} \cdot \frac{14}{1} = \frac{84}{25}$$

Problem 4. (10pts) Explain whether the following statements are true or false:

(a) All real numbers are rational.

(b) All rational numbers have a decimal expansion which terminates.

(c) There is only one way to express a rational number.

(d) All rational numbers are numbers between 0 and 1.

Solution.

(a) The statement is *false*. For instance, the real numbers $\sqrt{2}$, π, e, $0.12345678910111213 14\cdots$ are all irrational.

(b) The statement is *false*. A rational number is a real number that can be written in the form $\frac{a}{b}$, where a, b are integers. A real number is rational if and only if its decimal expansion terminates or repeats. We know that $\frac{1}{3}$ is rational and $\frac{1}{3} = 0.\overline{3}$ has a decimal expansion that does not terminate.

(c) The statement is *false*. For instance, $\frac{5}{10} = \frac{1}{2} = 0.5$ has infinitely many representations.

(d) The statement is *false*. For instance, $\frac{13}{5}$ is rational and $\frac{13}{5} = 2.6 > 1$.