

Name: \_\_\_\_\_

MATH 308

Fall 2021

HW 4: Due 10/08

*"I see the muscle shirt came today. Muscles coming tomorrow?"*

*– Wayne, Letterkenny*

**Problem 1.** (10pt) Read Keith Conrad's "[Advice on Mathematical Writing](#)." What are some things that you learned about good mathematical exposition that you may have otherwise thought?

**Problem 2.** (10pt) Watch 3Blue1Brown's "The unexpectedly hard windmill question (2011 IMO, Q2)" and "The hardest problem on the hardest test." What proof strategies do you believe these videos exhibit?

**Problem 3.** (10pt) Prove that if  $n^2 + (n + 1)^2 = (n + 2)^2$ , then  $n = 3$ .

**Problem 4.** (10pt) Recall that an integer  $n$  is called *even* if there is an integer  $k$  such that  $n = 2k$  and called *odd* if there is an integer  $k$  such that  $n = 2k + 1$ . Prove that the product of two odd integers is odd.

**Problem 5.** (10pt) Rewrite the proof below to be shorter using either “without loss of generality” or “mutatis mutandis”:

**Theorem.** For all  $a, b \in \mathbb{R}$ ,  $|ab| = |a| |b|$ .

*Proof.*

*Case 1* ( $a, b \geq 0$ ): Here  $|a| = a$ ,  $|b| = b$ , and  $ab \geq 0$ . But then  $|ab| = ab = |a| |b|$ .

*Case 2* ( $a < 0, b \geq 0$ ): Here  $|a| = -a$  and  $|b| = b$ . If  $b = 0$ , then  $|b| = 0$  and  $ab = 0$ . But then  $|ab| = |0| = 0 = -ab = |a| |b|$ . Otherwise,  $b > 0$  and then  $ab < 0$ . Then  $|ab| = -ab = |a| |b|$ .

*Case 3* ( $a \geq 0, b < 0$ ): Here  $|a| = a$  and  $|b| = -b$ . If  $a = 0$ , then  $|a| = 0$  and  $ab = 0$ . But then  $|ab| = |0| = 0 = -ab = |a| |b|$ . Otherwise,  $a > 0$  and then  $ab < 0$ . Then  $|ab| = -ab = |a| |b|$ .

*Case 4* ( $a, b < 0$ ): Here  $|a| = -a$ ,  $|b| = -b$ , and  $ab > 0$ . Then  $|ab| = ab = (-a)(-b) = |a| |b|$ . □

**Problem 6.** (10pt) By mimicking the proof that  $\sqrt{2}$  is irrational, prove that  $\sqrt{p}$  is irrational for any prime  $p$ .

**Problem 7.** (10pt) Consider the checkerboard below that has two squares from each corner removed from the board. Prove that this board cannot be covered with the ‘T-shapes’ (or its rotations) shown on the right.

