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**MATH 101**
**Spring 2022**
**HW 13: Due 05/03**

*"You have no idea, how much poetry there is in the calculation of a table of logarithms!"*
*–Carl Friedrich Gauss*

**Problem 1.** (10pt) Compute the following:

(a) $\log_7(1)$

(b) $\log_2(128)$

(c) $\log_4\left(\dfrac{1}{16}\right)$

(d) $\ln(e)$

(e) $\ln(e^{2/3})$

**Solution.**

(a) $\log_7(1) = 0$

(b) $\log_2(128) = \log_2(2^7) = 7$

(c) $\log_4\left(\dfrac{1}{16}\right) = \log_4\left(\dfrac{1}{4^2}\right) = \log_4(4^{-2}) = -2$

(d) $\ln(e) = \log_e(e^1) = 1$

(e) $\ln(e^{2/3}) = \log_e(e^{2/3}) = \frac{2}{3}$

**Problem 2.** (10pt) Write the following in terms of $\ln x$, $\ln y$, and $\ln z$:

$$\ln\left(\frac{x^2y}{z^6}\right)$$

**Solution.**

$$\begin{aligned}\ln\left(\frac{x^2y}{z^6}\right) &= \ln(x^2y) - \ln(z^6)\\ &= \ln(x^2) + \ln(y) - \ln(z^6)\\ &= 2\ln(x) + \ln(y) - 6\ln(z)\end{aligned}$$

**Problem 3.** (10pt) Write the following as a single logarithm involving no negative powers:

$$5\log_2(x) - 2\log_2\left(\frac{1}{y^2}\right) - 3\log_2(z) + 3$$

**Solution.**

$$\begin{aligned}
5\log_2(x) - 2\log_2\left(\frac{1}{y^2}\right) - 3\log_2(z) + 3 &= 5\log_2(x) - 2\log_2\left(\frac{1}{y^2}\right) - 3\log_2(z) + \log_2(2^3) \\
&= 5\log_2(x) - 2\log_2\left(\frac{1}{y^2}\right) - 3\log_2(z) + \log_2(8) \\
&= \log_2(x^5) + \log_2\left(\frac{1}{y^{-4}}\right) + \log_2(z^{-3}) + \log_2(8) \\
&= \log_2\left(\frac{x^5 \cdot z^{-3} \cdot 8}{y^{-4}}\right) \\
&= \log_2\left(\frac{8x^5y^4}{z^3}\right)
\end{aligned}$$

**Problem 4.** (10pt) Solve the following equations:

(a) $15\left(\frac{1}{2}\right)^x = 45$

(b) $3^{2-x} + 5 = 15$

(c) $e^{x/3} - 12 = 28$

**Solution.**

(a)

$$\begin{aligned} 15\left(\frac{1}{2}\right)^x &= 45 \\ \left(\frac{1}{2}\right)^x &= 3 \\ \log_2\left(\left(\frac{1}{2}\right)^x\right) &= \log_{1/2}(3) \\ x &= \log_{1/2}(3) \end{aligned}$$

(b)

$$\begin{aligned} 3^{2-x} + 5 &= 15 \\ 3^{2-x} &= 10 \\ \log_3(3^{2-x}) &= \log_3(10) \\ 2 - x &= \log_3(10) \\ x &= 2 - \log_3(10) \end{aligned}$$

(c)

$$\begin{aligned} e^{x/3} - 12 &= 28 \\ e^{x/3} &= 40 \\ \ln(e^{x/3}) &= \ln(40) \\ \frac{x}{3} &= \ln(40) \\ x &= 3\ln(40) \end{aligned}$$