MAT 100: Exam 3
Fall – 2023
12/13/2023
85 Minutes

Name: *Caleb M^{c}Whorter — Solutions*

Write your name on the appropriate line on the exam cover sheet. This exam contains 9 pages (including this cover page) and 8 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
Total:	80	

1. (10 points) Indicate whether the underlined measurement represents a quantitative or categorical variable.

   (a) I would describe their relationship as more of a situationship.

   (b) Dr. Draye entered the trial participants as 0 for placebo and 1 for active drug.

   (c) Troye easily spend nineteen hours this week watching *Jingle All the Way*.

   (d) My venti, seven pump pumpkin spice latte with eight shots of espresso and one pump of maple pecan sauce cost me $27.85.

Solution.

(a) Categorical

(b) Categorical

(c) Quantitative

(d) Quantitative

2. (10 points) Indicate whether the described experiment represents a convenience, systematic, stratified, or cluster sampling.

   (a) Saint Tom Aquiner College surveyed fifty people from each dorm room to determine whether their students had holiday spirit.

   (b) Ben surveyed every AI programmer from twenty of the top fifty AI companies about their views on AI safety.

   (c) Tequila Mockingbird liquor store surveyed every twentieth customer purchasing nog about their customer satisfaction.

   (d) You ask every member of your fam whether or not you got the drip.

Solution.

(a) Stratified

(b) Cluster

(c) Systematic

(d) Convenience

3. (10 points) Indicate whether the underlined measurement represents a nominal, ordinal, interval, or ratio level measurement.

   (a) The eighth Final Fantasy game is the best one.

   (b) The weather outside is frightful.

   (c) Despite it being the Christmas season, it is 43°F outside.

   (d) There are 57 people in line to try fish sticks at the new restaurant *Frying Nemo*.

Solution.

(a) Ordinal

(b) Nominal

(c) Interval

(d) Ratio

4. (10 points) A professor surveyed students on the number of breakdowns they had during a single math homework assignment. The results are found below.

$$1, \quad 10, \quad 17, \quad 21, \quad 23, \quad 24, \quad 24, \quad 25, \quad 25, \quad 27, \quad 30, \quad 34, \quad 39$$

(a) Compute the median for this data set.

(b) Compute the 5-number summary for this data set.

(c) Find the IQR for this dataset.

(d) Sketch a box plot for this dataset.

Solution. Observe that the data has already been placed in ascending order.

(a) The median for this dataset will be the $\frac{13}{2} = 6.5 \rightsquigarrow 7$th value in this dataset. Therefore, the median is 24.

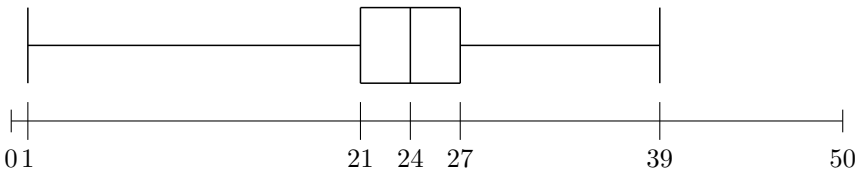
(b) The five-number summary for a dataset consists of the min, Q_1, median, Q_3, and max. Recall that Q_1 is the 25th percentile and Q_3 is the 75th percentile. Therefore, Q_1 is the $0.25 \cdot 13 = 3.25 \rightsquigarrow 4$th value in the dataset, which is 21, and Q_3 is the $0.75 \cdot 13 = 9.75 \rightsquigarrow 10$th value in the dataset, which is 27. Therefore, the five-number summary is...

Min	Q_1	Median	Q_3	Max
1	21	24	27	39

(c) We have...

$$\text{I.Q.R.} = Q_3 - Q_1 = 27 - 21 = 6$$

(d)



5. (10 points) The number of minutes four students spent studying for a math final exam are found below.

$$1, \quad 6, \quad 6, \quad 12$$

(a) Compute the mean for this data set.

(b) Compute the standard deviation for this data set.

(c) Is the median or mean a more robust measure of center for this data set? Explain.

Solution.

(a) The mean is...

$$\overline{x} = \frac{\sum x_i}{n} = \frac{1+6+6+12}{4} = \frac{25}{4} = 6.25$$

(b) We compute all the individual terms to compute the standard deviation:

x_i	$x_i - \overline{x}$	$(x_i - \overline{x})^2$
1	-5.25	27.5625
6	-0.25	0.0625
6	-0.25	0.0625
12	5.75	33.0625
		Total: 60.75

But then the standard deviation is...

$$\sigma = \sqrt{\frac{1}{n-1}\sum (x_i - \overline{x})^2} = \sqrt{\frac{1}{3} \cdot 60.72} = \sqrt{20.25} = 4.5$$

(c) The median is always a more robust measure of center than the mean as it is resistant to outliers whereas the mean is not resistant to outliers. The median of this dataset is clearly 6.

6. (10 points) Though one's man may not walk 500 miles for you, the number of miles a man is willing to walk is found to be normally distributed with mean 42.1 miles and standard deviation 8.6 miles

   (a) Find the percent of men that will walk less than 40 miles for you.

   (b) Find the percent of men that will walk more than 40 miles for you.

   (c) Find the number of miles a man needs to walk to be in the top 1% of men that will walk for you.

Solution.

(a)
$$z_{40} = \frac{40 - 42.1}{8.6} = \frac{-2.1}{8.6} = -0.24 \rightsquigarrow 0.4052$$
Therefore, $P(x \leq 40) = 0.4052$.

(b) We have...
$$P(x \geq 40) = 1 - P(x \leq 40) = 1 - 0.4052 = 0.5948$$

(c) If a man walked the minimal number of miles to be in the top 1% of men that will walk for you, then this man has walked more miles than 99% of the men have walked for you. But then if x were this minimal number of miles, we know that $z_x \rightsquigarrow 0.99$. Examining a z-score chart, it is clear that $z_x \approx 2.33$. But then...
$$z_x = \frac{x - \mu}{\sigma}$$
$$2.33 = \frac{x - 42.1}{8.6}$$
$$20.038 = x - 42.1$$
$$x = 62.038$$
Therefore, a man needs to walk at least 62.038 miles to be in the top 1% of men that would walk for you.

7. (10 points) Scientists have put their top minds on the problem of what the average 'rizz' rating is for current celebrities. Participants were asked to rate the 'rizz' of various Hollywoo stars and celebrities[1] on a scale of 0 to 100. The study examined 37 individuals and found an average 'rizz' rating of 72.4. The standard deviation for 'rizz' ratings for the population was known to be 13.9. Find a 98% confidence interval for the average 'rizz' rating of Hollywoo stars and celebrities.

Solution. We assume that the sample was a simple random sample. The study examined $n = 37 > 30$ individuals. Therefore, this sample is 'sufficiently large' for the Central Limit Theorem to apply. If the confidence interval contains 98% of the values, there are 2% of values outside of this interval. Because the normal distribution is symmetric, this leaves 1% of values above/below the confidence interval. But then the upper value in this confidence interval is greater than 98% + 1%= 99% of values in the distribution. If x were this upper value, then $z_x \rightsquigarrow 0.99$. Examining a z-score chart, we have $z_x \approx 2.33$. But then the confidence interval is given by...

$$\begin{aligned} \overline{x} \quad &\pm \quad z^* \frac{\sigma}{\sqrt{n}} \\ 72.4 \quad &\pm \quad 2.33 \cdot \frac{13.9}{\sqrt{37}} \\ 72.4 \quad &\pm \quad 2.33 \cdot 2.28515 \\ 72.4 \quad &\pm \quad 5.32 \end{aligned}$$

Therefore, the confidence interval is $(67.08, 77.72)$, i.e. "there is a 98% chance that the average 'rizz' rating for Hollywoo stars and celebrities is between 67.08% and 77.72%."[2]

[1] What do they know? Do they know things? Let's find out.

[2] Strictly speaking, a 98% chance that this interval contains the true mean.

8. (10 points) Visitors to a winter wonderland festival at a local college were asked what their favorite Christmas movie was. The results of the survey, broken down by age, are given below.

	18 – 21	22 – 35	35 – 50	50+	Total
Elf	40	18	30	32	120
Home Alone	20	38	42	14	114
Muppet Christmas Carol	24	42	39	33	138
A Christmas Story	2	7	23	50	82
Gremlins	0	0	1	3	4
Total	86	105	135	132	458

(a) What is the probability that an individual surveyed was 35 – 50 or preferred *Muppet Christmas Carol*?

(b) What is the probability that an individual surveyed was 22 – 35 and preferred *Elf*?

(c) What is the probability that an individual that was 51+ preferred *A Christmas Story*?

(d) Are preferring *Gremlins* and being 18 – 21 disjoint events? Can these events be independent?

Solution.

(a)
$$P(\text{35 – 50 or } \textit{Muppet Christmas Carol}) = \frac{135 + 138 - 39}{458} = \frac{234}{458} \approx 0.5109$$

(b)
$$P(\text{22 – 35 and } \textit{Elf}) = \frac{18}{458} \approx 0.0393$$

(c)
$$P(\textit{A Christmas Story} \mid 51+) = \frac{50}{132} \approx 0.3788$$

(d) Yes, there were no people that preferred *Gremlins* and were 18 – 21. But then these events cannot be independent because they are disjoint. [Disjoint events are *never* independent.]