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MATH 308
Fall 2022
HW 16: Due 12/06

"Algebra is the metaphysics of arithmetic."

–John Ray

Problem 1. (10pt) Let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^4$ be defined by $\mathbf{u} = \begin{pmatrix} 1 \\ 0 \\ -3 \\ 2 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 4 \\ -1 \\ 6 \\ -5 \end{pmatrix}$. Showing all your work, compute the following:

(a) $-6\mathbf{u}$

(b) $\mathbf{v} - \mathbf{u}$

(c) $\mathbf{u} + 2\mathbf{v}$

(d) $\mathbf{u} \cdot \mathbf{v}$

Solution.

(a)

$$-6\mathbf{u} = -6 \begin{pmatrix} 1 \\ 0 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -6 \\ 0 \\ 18 \\ -12 \end{pmatrix}$$

(b)

$$\mathbf{v} - \mathbf{u} = \begin{pmatrix} 4 \\ -1 \\ 6 \\ -5 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 9 \\ -7 \end{pmatrix}$$

(c)

$$\mathbf{u} + 2\mathbf{v} = \begin{pmatrix} 1 \\ 0 \\ -3 \\ 2 \end{pmatrix} + 2 \begin{pmatrix} 4 \\ -1 \\ 6 \\ -5 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -3 \\ 2 \end{pmatrix} + \begin{pmatrix} 8 \\ -2 \\ 12 \\ -10 \end{pmatrix} = \begin{pmatrix} 9 \\ -2 \\ 9 \\ -8 \end{pmatrix}$$

(d)

$$\mathbf{u} \cdot \mathbf{v} = \begin{pmatrix} 1 \\ 0 \\ -3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -1 \\ 6 \\ -5 \end{pmatrix} = 1(4) + 0(-1) + (-3)6 + 2(-5) = 4 + 0 - 18 - 10 = -24$$

Problem 2. (10pt) Define matrices A, B, C as follows:

$$A = \begin{pmatrix} 1 & 0 & -4 \\ -2 & 3 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 2 & -2 \\ 5 & 1 & 4 \end{pmatrix}, \qquad C = \begin{pmatrix} 2 & 0 \\ -1 & 6 \\ 5 & 3 \end{pmatrix}$$

Showing all your work, compute the following:

(a) $4A$

(b) $A - B$

(c) $3A + B$

(d) AC

(e) B^T

Solution.

(a)

$$4\begin{pmatrix} 1 & 0 & -4 \\ -2 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 0 & -16 \\ -8 & 12 & 4 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 1 & 0 & -4 \\ -2 & 3 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 2 & -2 \\ 5 & 1 & 4 \end{pmatrix} = \begin{pmatrix} 1 & -2 & -2 \\ -7 & 2 & -3 \end{pmatrix}$$

(c)

$$3\begin{pmatrix} 1 & 0 & -4 \\ -2 & 3 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 2 & -2 \\ 5 & 1 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 0 & -12 \\ -6 & 9 & 3 \end{pmatrix} + \begin{pmatrix} 0 & 2 & -2 \\ 5 & 1 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 2 & -14 \\ -1 & -8 & 7 \end{pmatrix}$$

(d)

$$\begin{pmatrix} 1 & 0 & -4 \\ -2 & 3 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ -1 & 6 \\ 5 & 3 \end{pmatrix} = \begin{pmatrix} 1(2) + 0(-1) + (-4)5 & 1(0) + 0(6) + (-4)3 \\ -2(2) + 3(-1) + 1(5) & -2(0) + 3(6) + 1(3) \end{pmatrix} = \begin{pmatrix} -18 & -12 \\ -2 & 21 \end{pmatrix}$$

(e)

$$\begin{pmatrix} 0 & 2 & -2 \\ 5 & 1 & 4 \end{pmatrix}^T = \begin{pmatrix} 0 & 5 \\ 2 & 1 \\ -2 & 4 \end{pmatrix}$$

Problem 3. (10pt) Define matrices A, B, C as follows:

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 3 \end{pmatrix}, \qquad B = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}, \qquad C = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Showing all your work and explaining your reasoning, answer the following:

(a) What is B^2?

(b) If CA is defined, compute it. If not, explain why.

(c) What is a_{23}? What is b_{21}?

(d) If $M = AC$, without explicitly computing AC, what is m_{23}?

Solution.

(a) We have...

$$B^2 = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}^2 = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 2(2) + (-1)0 & 2(-1) + (-1)3 \\ 0(2) + 3(0) & 0(-1) + 3(3) \end{pmatrix} = \begin{pmatrix} 4 & -5 \\ 0 & 9 \end{pmatrix}$$

(b) The product CA is not defined. The product of two matrices, say AB, where A is a $m \times n$ matrix and B is a $r \times s$ matrix, is defined if and only if $n = r$. Here, we have $m = n = 3$ and $r = 2$ and $s = 3$. Because $n = 3 \neq 2 = r$, the product is not defined.

(c) We have $a_{23} = 3$ and $b_{21} = 1$.

(d) We know m_{23} is the dot product of the second row of A with the third column of C. But then we have...

$$m_{23} = 0(0) + (-1)0 + 3(3) = 0 + 0 + 9 = 9$$

Problem 4. (10pt) If A, B are matrices, is it true $(A+B)^2 = A^2 + 2AB + B^2$? If so, explain why. If not, explain why not.

Solution. The statement is false. Matrix multiplication is not commutative.