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MATH 108

Spring 2023

HW 13: Due 05/01

“There is hardly any theory which is more elementary [than linear algebra], in spite of the fact that generations of professors and textbook writers have obscured its simplicity by preposterous calculations with matrices.”

–Jean Dieudonné

Problem 1. (10pt) Define \mathbf{u} , \mathbf{v} , and \mathbf{w} to be the vectors given below:

$$\mathbf{u} = \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}, \quad \mathbf{w} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$$

Compute the following:

- (a) $-3\mathbf{v}$
- (b) $\mathbf{w} - \mathbf{u}$
- (c) $2\mathbf{u} + \mathbf{v}$
- (d) $\mathbf{v} \cdot \mathbf{w}$

Solution.

(a)

$$-3\mathbf{v} = -3 \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ 9 \end{pmatrix}$$

(b)

$$\mathbf{w} - \mathbf{u} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 2-1 \\ -1-(-3) \\ 4-5 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

(c)

$$2\mathbf{u} + \mathbf{v} = 2 \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 2 \\ -6 \\ 10 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 2+0 \\ -6+2 \\ 10+(-3) \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 7 \end{pmatrix}$$

(d)

$$\mathbf{v} \cdot \mathbf{w} = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} = 0(2) + 2(-1) + (-3)4 = 0 - 2 - 12 = -14$$

Problem 2. (10pt) Define the following:

$$A = \begin{pmatrix} 1 & 0 & -3 \\ 2 & -1 & 1 \\ 0 & 5 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 3 & 0 \\ -1 & 4 & -2 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

Compute the following:

(a) $2B$

(b) AB

(c) BA

(d) $A\mathbf{u}$

Solution.

(a)

$$2B = 2 \begin{pmatrix} 2 & 1 & 1 \\ 0 & 3 & 0 \\ -1 & 4 & -2 \end{pmatrix} = \begin{pmatrix} 4 & 2 & 2 \\ 0 & 6 & 0 \\ -2 & 8 & -4 \end{pmatrix}$$

(b)

$$\begin{aligned} AB &= \begin{pmatrix} 1 & 0 & -3 \\ 2 & -1 & 1 \\ 0 & 5 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 & 1 \\ 0 & 3 & 0 \\ -1 & 4 & -2 \end{pmatrix} \\ &= \begin{pmatrix} 1(2) + 0(0) + (-3)(-1) & 1(1) + 0(3) + (-3)4 & 1(1) + 0(1) + (-3)(-2) \\ 2(2) + (-1)0 + 1(-1) & 2(1) + (-1)3 + 1(4) & 2(1) + (-1)0 + 1(-2) \\ 0(2) + 5(0) + 2(-1) & 0(1) + 5(3) + 2(4) & 0(1) + 0(0) + 2(-2) \end{pmatrix} \\ &= \begin{pmatrix} 2 + 0 + 3 & 1 + 0 - 12 & 1 + 0 + 6 \\ 4 + 0 - 1 & 2 - 3 + 4 & 2 + 0 - 2 \\ 0 + 0 - 2 & 0 + 15 + 8 & 0 + 0 - 4 \end{pmatrix} \\ &= \begin{pmatrix} 5 & -11 & 7 \\ 3 & 3 & 0 \\ -2 & 23 & -4 \end{pmatrix} \end{aligned}$$

(c)

$$\begin{aligned}BA &= \begin{pmatrix} 2 & 1 & 1 \\ 0 & 3 & 0 \\ -1 & 4 & -2 \end{pmatrix} \begin{pmatrix} 1 & 0 & -3 \\ 2 & -1 & 1 \\ 0 & 5 & 2 \end{pmatrix} \\&= \begin{pmatrix} 2(1) + 1(2) + 1(0) & 2(0) + 1(-1) + 1(5) & 2(-3) + 1(1) + 1(2) \\ 0(1) + 3(2) + (-1)0 & 0(0) + 3(-1) + 0(5) & 0(-3) + 3(1) + 0(2) \\ (-1)1 + 4(2) + (-2)0 & -1(0) + 4(-1) + (-2)5 & -1(-3) + 4(1) + (-2)2 \end{pmatrix} \\&= \begin{pmatrix} 2 + 2 + 0 & 0 - 1 + 5 & -6 + 1 + 2 \\ 0 + 6 + 0 & 0 - 3 + 0 & 0 + 3 + 0 \\ -1 + 8 + 0 & 0 - 4 - 10 & 3 + 4 - 4 \end{pmatrix} \\&= \begin{pmatrix} 2 & 4 & -3 \\ 6 & -3 & 3 \\ 7 & -14 & 3 \end{pmatrix}\end{aligned}$$

(d)

$$A\mathbf{u} = \begin{pmatrix} 1 & 0 & -3 \\ 2 & -1 & 1 \\ 0 & 5 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1(1) + 0(-1) + (-3)1 \\ 2(1) + (-1)(-1) + 1(1) \\ 0(1) + 5(-1) + 2(1) \end{pmatrix} = \begin{pmatrix} 1 + 0 - 3 \\ 2 + 1 + 1 \\ 0 - 5 + 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ -3 \end{pmatrix}$$