Name: _____

MATH 308
Fall 2023
HW 11: Due 11/10

"Mathematics is the art of giving the same name to different things."
–Henri Poincaré

Problem 1. (10pt) Consider the relation $\sim$ on $\mathbb{Z}\times\mathbb{Z}$ given by $(a,b) \sim (c,d)$ if and only if $a+c = b+d$.

(a) What assumptions does this relation need to satisfy to be an equivalence relation?

(b) Is $(1,0) \sim (3,4)$? Explain.

(c) Is $(-2,1) \sim (1,1)$? Explain.

(d) Is this relation symmetric? Explain.

(e) Is this relation reflective? Explain.

Problem 2. (10pt) Showing all your work, compute the following:

(a) $\displaystyle\sum_{k=-3}^{100} 5$

(b) $\displaystyle\sum_{k=0}^{200} k^2$

(c) $\displaystyle\sum_{k=100}^{200} k$

(d) $\displaystyle\sum_{k=0}^{150} (2k-3)$

Problem 3. (10pt) Showing all your work, find a closed-form expression for the following sum:

$$\sum_{k=2}^{n}(2k^2 - k + 4n)$$