

Quiz 1. True/False: The integer 45 has prime factorization $45 = 3 \cdot 15$, which shows that 3 and 15 are divisors of 45. Furthermore, we know that 1 is a multiple of 45.

Solution. The statement is *false*. While it is true that $45 = 3 \cdot 15$ is a *factorization* of 45, it is not a *prime factorization* of 45 because $15 = 3 \cdot 5$. The prime factorization of 45 is $45 = 3^2 \cdot 5$. It is true that if $45 = 3 \cdot 15$, then 3 and 15 are divisors of 45. Finally, while 1 is a divisor of 45 because $45 = 45 \cdot 1$, 1 is not a multiple of 45 because there is not an integer k such that $1 = 45k$.

Quiz 2. True/False: $\frac{\frac{2a}{b}}{\frac{4a}{bc}} = 8c$

Solution. The statement is *false*. We have...

$$\frac{\frac{2a}{b}}{\frac{4a}{bc}} = \frac{2a}{b} \cdot \frac{bc}{4a} = \frac{\cancel{2}a}{\cancel{b}} \cdot \frac{\cancel{b}c}{\cancel{4}^2a} = \frac{c}{2}$$

Quiz 3. True/False: The expression $\frac{(xy^3)^{-2}}{(x^{-3}y^8)^2}$ when fully simplified is $\frac{x^4}{y^{22}}$.

Solution. The statement is *true*. We have...

$$\frac{(xy^3)^{-2}}{(x^{-3}y^8)^2} = \frac{x^{-2}y^{-6}}{x^{-6}y^{16}} = \frac{x^6}{x^2y^6y^{16}} = \frac{x^6}{x^2y^{22}} = \frac{x^4}{y^{22}}$$

Quiz 4. True/False: $\left(\frac{(x^2y^3)^4}{x^{-3}y^8}\right)^{-1/2} = \frac{1}{y^2 \sqrt[11]{x^2}}$

Solution. The statement is *true*. We have...

$$\left(\frac{(x^2y^3)^4}{x^{-3}y^8}\right)^{-1/2} = \left(\frac{x^{-3}y^8}{(x^2y^3)^4}\right)^{1/2} = \left(\frac{x^{-3}y^8}{x^8y^{12}}\right)^{1/2} = \left(\frac{y^8}{x^{11}y^{12}}\right)^{1/2} = \left(\frac{1}{x^{11}y^4}\right)^{1/2} = \frac{1}{x^{11/2}y^{4/2}} = \frac{1}{y^2\sqrt{x^{11}}}$$

Therefore, the quiz statement is false. The quiz statement has $\sqrt[11]{x^2} = x^{2/11}$ instead of $\sqrt{x^{11}} = x^{11/2}$.

Quiz 5. *True/False:* The real number $0.123412341234\dots$ is a rational number; therefore, one can find integers a, b such that $\frac{a}{b} = 0.123412341234\dots$

Solution. The statement is *true*. A rational number is a real number of the form $\frac{a}{b}$, where a, b are integers and $b \neq 0$. Equivalently, a rational number is a real number whose decimal expansion either terminates or repeats. Because the decimal expansion of $0.123412341234\dots$ repeats, it must be that $0.123412341234\dots$ is rational. Therefore, there must be integers a, b such that $\frac{a}{b} = 0.123412341234\dots$. In fact, if $N = 0.123412341234\dots$, we have...

$$\begin{array}{rcl} 10000N & = & 1234.123412341234\overline{1234} \\ - N & = & 0.123412341234\overline{1234} \\ \hline 9999N & = & 1234 \\ N & = & \frac{1234}{9999} \end{array}$$

Quiz 6. *True/False:* Suppose a course has grade components of homework (50%), quizzes (10%), a midterm (20%), and a final (20%). If you had a 80% homework average, 75% quiz average, and received a 60% on the midterm, then your average is...

$$0.50(80\%) + 0.10(75\%) + 0.20(60\%) = 40\% + 7.5\% + 12\% = 59.5\%$$

Solution. The statement is *false*. One's course average is a weighted average where each percentage earned is weighted by the components worth. But then...

$$\text{Course Average} = \frac{\sum w_i x_i}{\sum w_i} = \frac{0.50 \cdot 0.80 + 0.10 \cdot 0.75 + 0.20 \cdot 0.60}{0.50 + 0.10 + 0.20} = \frac{0.40 + 0.075 + 0.12}{0.80} = \frac{0.595}{0.80} = 0.74375$$