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MATH 100

Fall 2021

HW 7: Due 10/13

*“Be nice to your sister, Bill. Some day you’ll be sleeping on her couch
after your first divorce”*

–Frank Murphy, F is for Family

Problem 1. (10pt) Find the equation of the line through the point $(3, -1)$ with slope -2 .

Solution. Because the line is not vertical, it must be of the form $y = mx + b$. Because the slope is -2 , we have $y = -2x + b$. Using the point $(3, -1)$, i.e. $x = 3$ and $y = -1$, we have

$$y = -2x + b$$

$$-1 = -2(3) + b$$

$$-1 = -6 + b$$

$$b = 5$$

Therefore, the equation of the line is $y = -2x + 5 = 5 - 2x$.

Problem 2. (10pt) Find the equation of the line through $(-2, 2)$ and $(3, 4)$.

Solution. Because these points are not vertically aligned, we know the line is not vertical. Therefore, the line has the form $y = mx + b$. First, we compute the slope:

$$m = \frac{2 - 4}{-2 - 3} = \frac{-2}{-5} = \frac{2}{5}$$

Then $y = \frac{2}{5}x + b$. Now we use the point $(3, 4)$, i.e. $x = 3$ and $y = 4$, and find that...

$$\begin{aligned}y &= \frac{2}{5}x + b \\4 &= \frac{2}{5} \cdot 3 + b \\4 &= \frac{6}{5} + b \\b &= 4 - \frac{6}{5} \\b &= \frac{20}{5} - \frac{6}{5} \\b &= \frac{20 - 6}{5} \\b &= \frac{14}{5}\end{aligned}$$

Therefore, the equation of the line is $y = \frac{2}{5}x + \frac{14}{5} = \frac{2x + 14}{5}$.

Problem 3. (10pt) Find the equation of the horizontal line through $(0, -2)$.

Solution. Because the line is not vertical, the line has the form $y = mx + b$. But because the line is horizontal, we know the line has slope 0, i.e. $m = 0$. Then we have $y = b$. We use the point $(0, -2)$, i.e. $x = 0$ and $y = -2$. But then $-2 = b$. Therefore, $y = -2$.

Problem 4. (10pt) Find the equation of the line parallel to the line $y = 6 - x$ containing the point $(-6, 1)$.

Solution. The line $y = 6 - x$ is not vertical. Therefore, a line parallel to it is not vertical. Therefore, our line has the form $y = mx + b$. Because this line is parallel to the line $y = 6 - x$, it must have slope equal to the slope of the line $y = 6 - x$. The slope of $y = 6 - x$ is -1 . Therefore, we know that $m = -1$, i.e. $y = -x + b$. Now we use the point $(-6, 1)$, i.e. $x = -6$ and $y = 1$. But then...

$$y = -x + b$$

$$1 = -(-6) + b$$

$$1 = 6 + b$$

$$b = -5$$

Therefore, the line is $y = -x - 5$.

Problem 5. (10pt) Find the equation of the line perpendicular to the line $y = \frac{5}{3}x + 1$ passing through the point $(10, -13)$.

Solution. Because the line $y = \frac{5}{3}x + 1$ is not horizontal, a line perpendicular to it is not vertical. Then our line is not vertical so that it has the form $y = mx + b$. Because our line is perpendicular to the line $y = \frac{5}{3}x + 1$, the slope of our line is the negative reciprocal of the slope of the line $y = \frac{5}{3}x + 1$. The slope of the line $y = \frac{5}{3}x + 1$ is $\frac{5}{3}$. Therefore, the slope of our line is $-\frac{3}{5}$, i.e. $m = -\frac{3}{5}$. Therefore, $y = -\frac{3}{5}x + b$. Using the point $(10, -13)$, i.e. $x = 10$ and $y = -13$, we have...

$$\begin{aligned}y &= -\frac{3}{5}x + b \\-13 &= -\frac{3}{5} \cdot 10 + b \\-13 &= -6 + b \\b &= -7\end{aligned}$$

Therefore, the equation of the line $y = -\frac{3}{5}x - 7$.

Problem 6. (10pt) Find the equation of the line perpendicular to the line $y = 6$ that contains the point $(-3, 9)$.

Solution. The line $y = 6$ is horizontal. Therefore, a line perpendicular to the line $y = 6$ must be vertical, i.e. of the form $x = \#$. Because our line contains the point $(-3, 9)$, we know that the line must be $x = -3$.

Problem 7. (10pt) Find the equation of the line that is perpendicular to the line $y = 7x - 1$ that passes through the x -intercept of the line $y = 4x - 8$.

Solution. The line $y = 7x - 1$ is not horizontal. Therefore, a line perpendicular to it is not vertical. Then our line must have the form $y = mx + b$. The line $y = 7x - 1$ has slope 7. Because our line is perpendicular to the line $y = 7x - 1$, the slope must be the negative reciprocal of the slope of the line $y = 7x - 1$. Therefore, we know that $m = -\frac{1}{7}$. Then we know that $y = -\frac{1}{7}x + b$. We know that the line contains the x -intercept of the line $y = 4x - 8$. The x -intercept of a curve is where the curve passes through the x -axis, i.e. where $y = 0$. But then

$$y = 4x - 8$$

$$0 = 4x - 8$$

$$4x = 8$$

$$x = 2$$

Therefore, the x -intercept of $y = 4x - 8$ is the point $(2, 0)$. Then our line contains $(2, 0)$, i.e. $x = 2$ and $y = 0$. But then...

$$y = -\frac{1}{7}x + b$$

$$0 = -\frac{1}{7} \cdot 2 + b$$

$$0 = -\frac{2}{7} + b$$

$$b = \frac{2}{7}$$

Therefore, the equation of the line is $y = -\frac{1}{7}x + \frac{2}{7} = \frac{-x + 2}{7} = \frac{2 - x}{7}$.