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MATH 307

Spring 2023

HW 11: Due 04/14

*“The power of mathematics is often to change one thing into another;  
to change geometry into language.”*

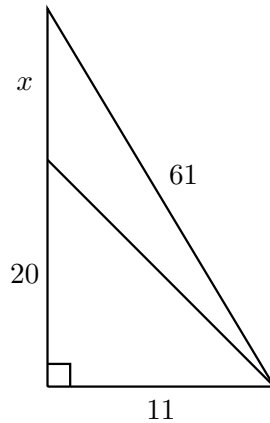
*–Marcus du Sautoy*

**Problem 1.** (10pt) The speed of an airplane is 821.3 ft/s. Find this speed in miles per hour (mph).  
[Note: 5280 ft = 1 mi]

**Solution.**

$$\frac{821.3 \text{ ft}}{1 \text{ s}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \cdot \frac{60 \text{ s}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = 559.977 \text{ mph}$$

**Problem 2.** (10pt) Find  $x$  in the diagram below:



**Solution.** The large triangle is a right triangle. Therefore, by the Pythagorean Theorem,  $a^2 + b^2 = c^2$ , where  $a$  and  $b$  are the legs of the triangle and  $c$  is the hypotenuse. The larger triangle has legs  $11$  and  $x + 20$  and hypotenuse  $61$ . But then we have...

$$(x + 20)^2 + 11^2 = 61^2$$

$$(x + 20)^2 + 121 = 3721$$

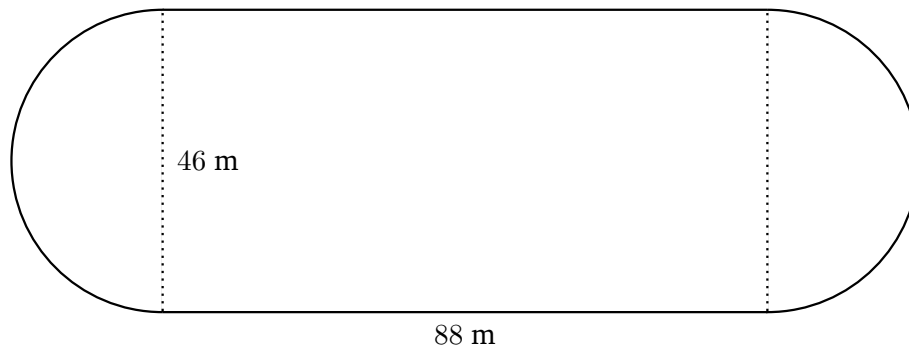
$$(x + 20)^2 = 3600$$

$$\sqrt{(x + 20)^2} = \sqrt{3600}$$

$$x + 20 = 60$$

$$x = 40$$

**Problem 3.** (10pt) A ‘track’ consists of two semicircles adjoined to the sides of a rectangle. Find the perimeter and the area of the ‘track’ below.



**Solution.** The region above consists of two half circles and a rectangle. The perimeter is the distance around the figure. The two circle halves can be combined into one circle with diameter 46 m, i.e. radius 23 m. The perimeter of a circle is  $2\pi r$ . Only the upper and lower sides of the rectangle are included in the perimeter. Therefore, the perimeter is...

$$P = 2\pi r + 2h = 2\pi \cdot 23 + 2 \cdot 88 = 46\pi + 176 \approx 320.513 \text{ m}$$

The area of the above figure can be broken into the area from the rectangle and the area from two half circles, i.e. one whole circle. The area of a rectangle is  $bh$  and the area of a circle is  $\pi r^2$ . Therefore, we have...

$$A = \pi r^2 + bh = \pi \cdot 23^2 + 88(46) = 529\pi + 4048 \approx 5709.9 \text{ m}^2$$