Name: _____
MATH 308
Fall 2021
HW 1: Due 09/24

"I'm not superstitious, but I am a little stitious."
–Michael Scott, The Office

Problem 1. (10pt) Determine if the following sentences are propositions. If the sentence is a proposition, mark it 'T'; otherwise, mark the sentence 'F.'

(a) _____: $|3 - 5| > 10$

(b) _____: I just started watching *'The Chair.'*

(c) _____: The universe is infinite.

(d) _____: $n + 1$ is odd.

(e) _____: Why are you doing this homework?

Problem 2. (10pt) Give an original example of a proposition.

Problem 3. (10pt) Give an original non-example of a proposition.

Problem 4. (10pt) Determine if the following propositions are true (T) or false (F).

(a) _____: If n is an integer, then $2n$ is even.

(b) _____: Every prime number is odd.

(c) _____: $x^2 + 1 > 0$

(d) _____: It will either rain tomorrow or not.

(e) _____: If $x^2 = 9$, then $x = 3$,

Problem 5. (10pt) Negate the following sentences:

(a) $2 \cdot 2 = 4$ or $3 \cdot 3 = 6$

(b) Everyone in the room has taken a mathematics course.

(c) She speaks German and English.

(d) $x > 1$ and x is an integer.

(e) If you study for the exam, then you will pass.

Problem 6. (10pt) Negate each of the following propositional formulas P by finding a formula logically equivalent P in which the negation applies only to individual atoms.

(a) $P \vee (\neg Q)$

(b) $\neg Q \to \neg P$

(c) $(P \vee Q) \wedge (\neg P \vee \neg Q)$

(d) $P \wedge Q \to P \vee Q$

(e) $P \vee (Q \Leftrightarrow R)$

Problem 7. (10pt) Express the proposition "P unless Q" in terms of the propositions P and Q and the logical symbols $\neg, \wedge, \vee, \rightarrow$. [Unless can mean many things, here it means "if not."]

Problem 8. (10pt) Recall that the 'exclusive or', denoted $\veebar$, was defined by $P \veebar Q \Leftrightarrow (P \vee Q) \wedge \neg(P \wedge Q)$. Show that $P \veebar Q$ is logically equivalent to $P \leftrightarrow \neg\, Q$.

Problem 9. (10pt) Compute the truth tables for the following compound propositions. In each case, indicate whether the propositional formula is a tautology, contradiction, or neither.

(a) $(P \wedge Q) \wedge (R \wedge \neg Q)$

(b) $(P \leftrightarrow Q) \leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$

(c) $(P \rightarrow T_0) \wedge (F_0 \rightarrow Q)$

Problem 10. (10pt) Determine if the logical symbol $\rightarrow$ is associative. Be sure to fully justify your answer.

Problem 11. (10pt) Give the converse and contrapositive of the following statements.

(a) $P \to Q$

(b) If it is snowing outside, then it is cold.

Problem 12. (10pt) Determine if the following argument is logical. Explain.

$$\begin{array}{c} P \to R \\ \neg P \to Q \\ Q \to S \\ \hline \therefore \neg R \to S \end{array}$$