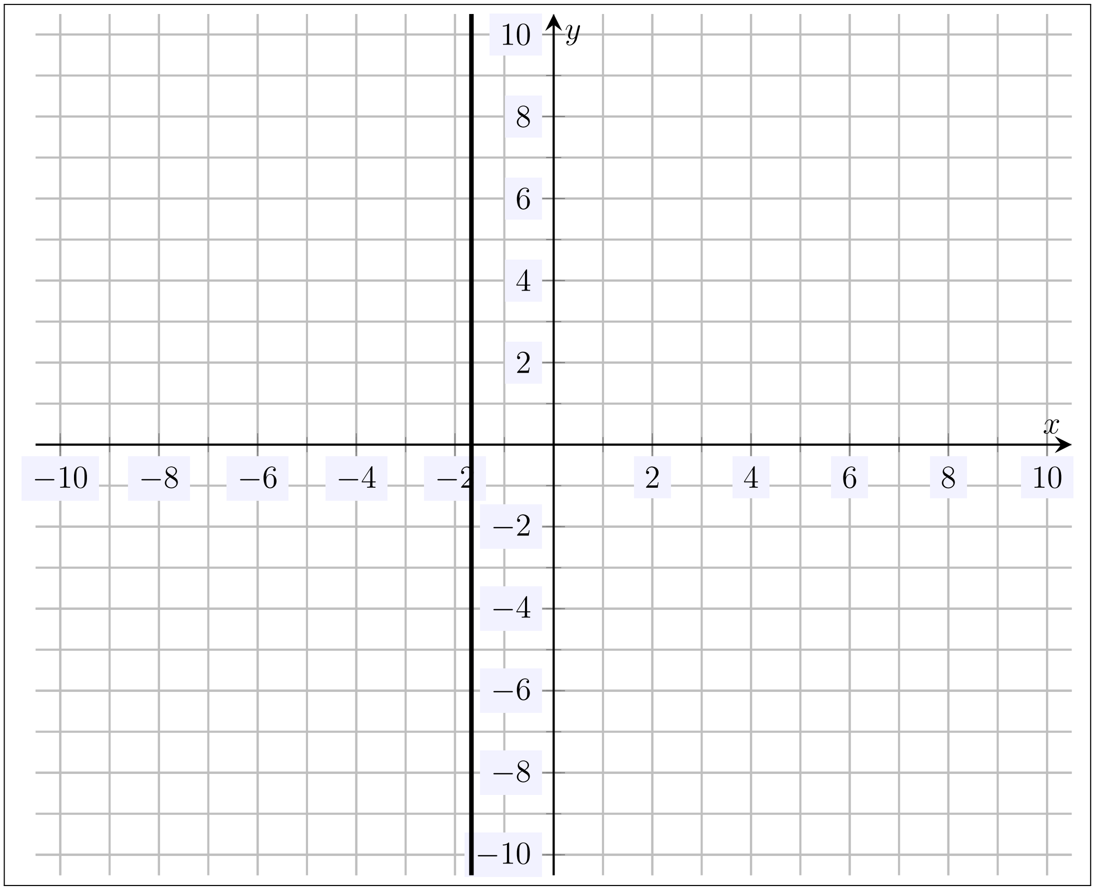
MAT 101: Exam 2 **Name:** *Caleb McWhorter — Solutions*
Fall – 2022
11/21/2022
85 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 17 pages (including this cover page) and 15 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work.

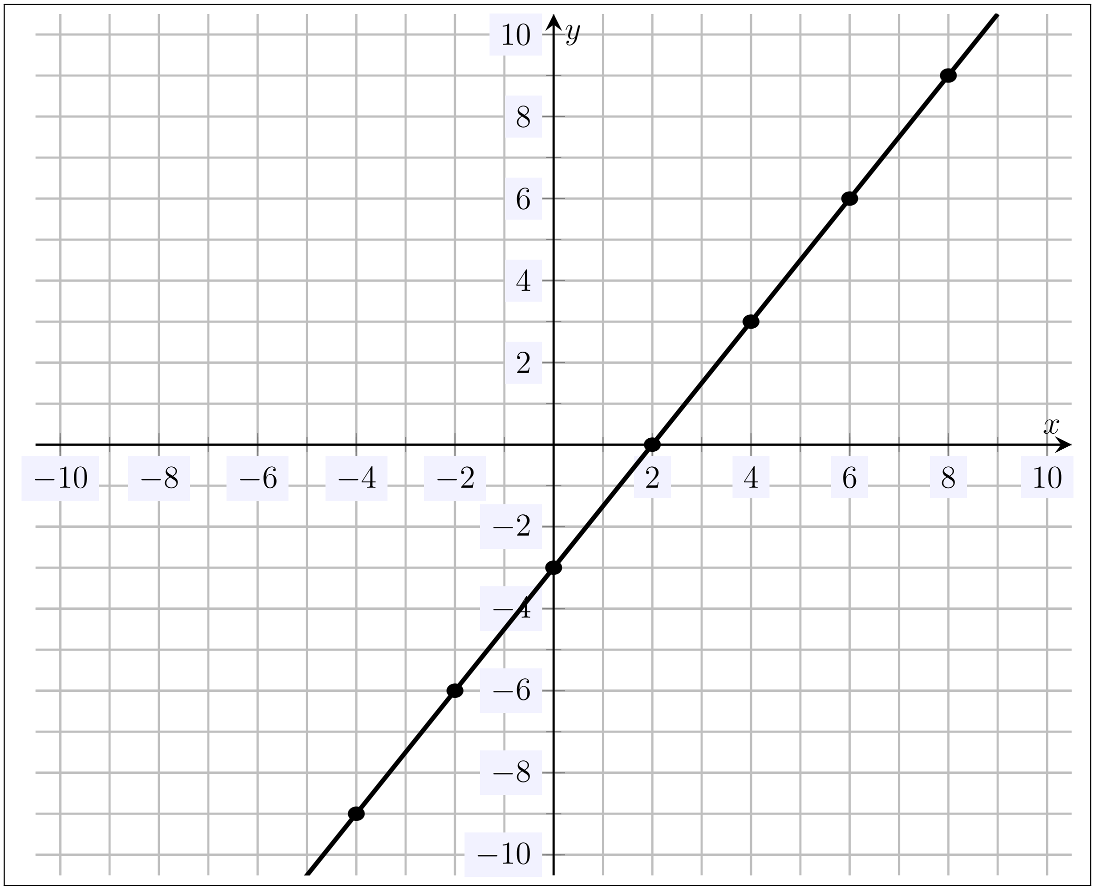
Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
14	10	
15	10	
Total:	150	

1. (10 points) As accurately as possible, plot the line $x = -\frac{5}{3}$ on the graph below.



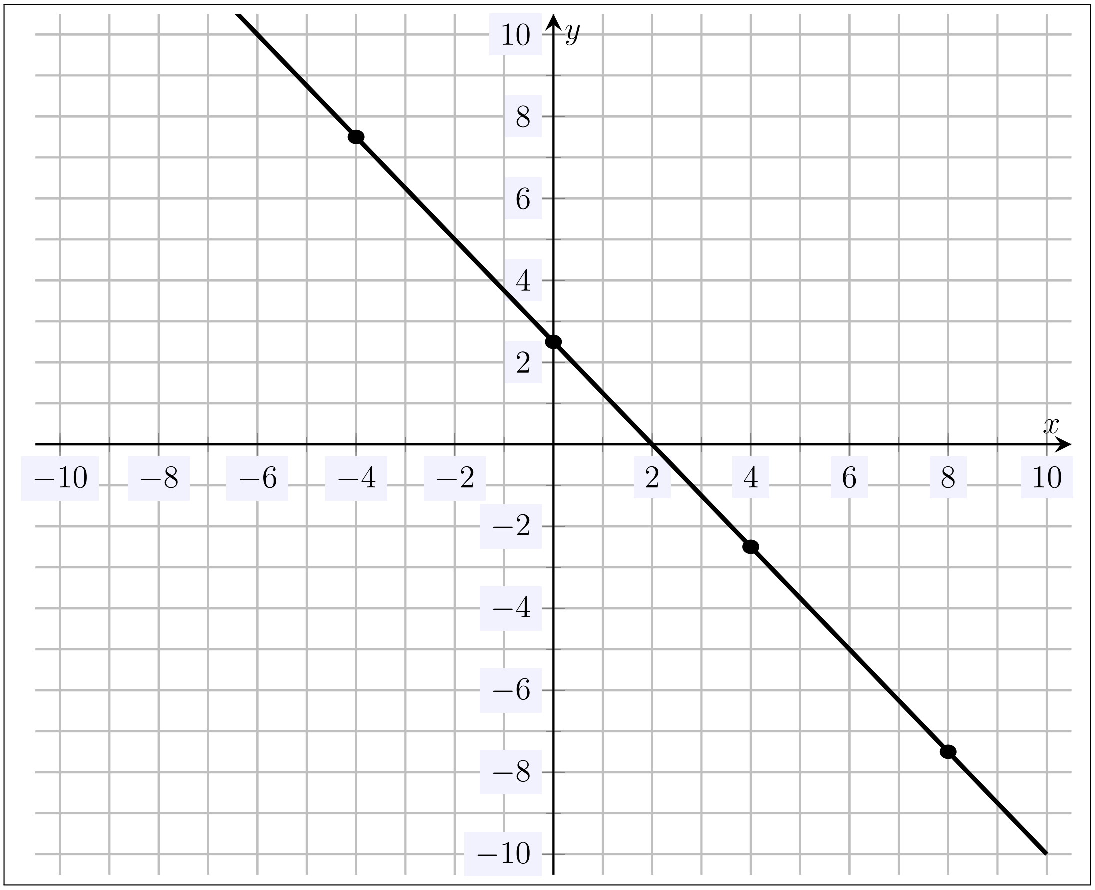
This is a vertical line located at $x = -\frac{5}{3}$.

2. (10 points) As accurately as possible, plot the line $y = \frac{3}{2}x - 3$ on the graph below.



The line $y = \frac{3}{2}x - 3$ has y-intercept $(0, -3)$. The slope of the line is $m = \frac{3}{2}$. Interpreting $m = \frac{3}{2}$ as $\frac{\Delta y}{\Delta x}$, we see that for each increase of 2 in x, there is a corresponding increase of 3 in y. Alternatively, writing $m = \frac{3}{2} = \frac{-3}{-2}$ and interpreting this as $\frac{\Delta y}{\Delta x}$, we see that for each decrease of 2 in x, there is a decrease of 3 in y. Using the y-intercept and these interpretations of the slope to sketch points on the line (shown above), we obtain the sketch of the line.

3. (10 points) As accurately as possible, plot the line $5x + 4y = 10$ on the graph below.



First, observe that we have...

$$\begin{aligned} 5x + 4y &= 10 \\ 4y &= -5x + 10 \\ y &= -\frac{5}{4}x + \frac{10}{4} \\ y &= -\frac{5}{4}x + \frac{5}{2} \end{aligned}$$

The line $y = -\frac{5}{4}x - 3$ has y-intercept $\left(0, \frac{5}{2}\right)$. The slope of the line is $m = -\frac{5}{4} = \frac{-5}{4}$. Interpreting $m = \frac{-5}{4}$ as $\frac{\Delta y}{\Delta x}$, we see that for each increase of 4 in x, there is a corresponding decrease of 5 in y. Alternatively, writing $m = -\frac{5}{4} = \frac{5}{-4}$ and interpreting this as $\frac{\Delta y}{\Delta x}$, we see that for each decrease of 4 in x, there is an increase of 5 in y. Using the y-intercept and these interpretations of the slope to sketch points on the line (shown above), we obtain the sketch of the line.

Alternatively, considering $5x + 4y = 10$, if $y = 0$, we have $5x = 10$, so that $x = 2$. But then $(2, 0)$ is a point on the line. If $x = 0$, we have $4y = 10$, so that $y = \frac{10}{4} = \frac{5}{2}$. But then $\left(0, \frac{5}{2}\right)$ is on the line. Connecting these points via a straight line, we have the line shown above.

4. (10 points) Consider the line given by $5x - 4y = 7$.

   (a) Showing all your work and without referencing a graph, determine if $(-3, -2)$ is on the line.

   (b) Showing all your work and without referencing a graph, determine if $\left(1, -\frac{1}{2}\right)$ is on the line.

Solution.

(a) We have...

$$\begin{aligned} 5x - 4y &= 7 \\ 5(-3) - 4(-2) &\stackrel{?}{=} 7 \\ -15 + 8 &\stackrel{?}{=} 7 \\ -7 &\neq 7 \\ &\text{✗} \end{aligned}$$

(b) We have...

$$\begin{aligned} 5x - 4y &= 7 \\ 5(1) - 4\left(-\frac{1}{2}\right) &\stackrel{?}{=} 7 \\ 5 + 2 &\stackrel{?}{=} 7 \\ 7 &= 7 \\ &\checkmark \end{aligned}$$

5. (10 points) Showing all your work and being sure to list the points, find at least three distinct points on the line $-7x + 3y = 10$.

Solution. *To find points on the line, we can simple plug-in any value for* x, y *and solve for the other. For instance, if...*

$x = 0$*:*

$$\begin{aligned} -7x + 3y &= 10 \\ 0 + 3y &= 10 \\ 3y &= 10 \\ y &= \frac{10}{3} \end{aligned}$$

Therefore, $\left(0, \frac{10}{3}\right)$ *is on the line.*

$y = 0$*:*

$$\begin{aligned} -7x + 3y &= 10 \\ -7x + 0 &= 10 \\ -7x &= 10 \\ x &= -\frac{10}{7} \end{aligned}$$

Therefore, $\left(-\frac{10}{7}, 0\right)$ *is on the line.*

$x = 1$*:*

$$\begin{aligned} -7x + 3y &= 10 \\ -7 + 3y &= 10 \\ 3y &= 17 \\ y &= \frac{17}{3} \end{aligned}$$

Therefore, $\left(1, \frac{17}{3}\right)$ *is on the line.*

$y = 1$*:*

$$\begin{aligned} -7x + 3y &= 10 \\ -7x + 3 &= 10 \\ -7x &= 7 \\ x &= -1 \end{aligned}$$

Therefore, $(-1, 1)$ *is on the line.*

6. (10 points) Consider the linear function $f(x) = 7 - \frac{6}{5}x$. Showing all your work, answer the following:

   (a) Find the rate of change of $f(x)$.

   (b) Interpret the rate of change of $f(x)$.

   (c) Determine whether $f(x)$ is an increasing or decreasing function.

   (d) Determine the y-intercept of $f(x)$.

   (e) Find the exact value of $f\left(\frac{2}{3}\right)$.

Solution.

(a) The rate of change of $f(x)$ is the slope. Because $f(x)$ has the form $y = mx + b$, we see that the slope of $f(x)$ is $m = -\frac{6}{5}$.

(b) We have $m = -\frac{6}{5}$. Writing $m = -\frac{6}{5} = \frac{-6}{5}$ and interpreting this as $\frac{\Delta y}{\Delta x}$, we see that each increase of 5 in x results in a decrease of 6 in y. Alternatively, writing $m = -\frac{6}{5} = \frac{6}{-5}$ and interpreting this as $\frac{\Delta y}{\Delta x}$, we see that each decrease of 5 in x results in an increase of 6 in y.

(c) Because $m = -\frac{6}{5} < 0$, we see that $f(x)$ is decreasing.

(d) Because $f(x) = 7 - \frac{6}{5}x$ has the form $y = mx + b$, we see that the y-intercept is 7, i.e. $(0, 7)$.

(e) We have...

$$f\left(\frac{2}{3}\right) = 7 - \frac{6}{5} \cdot \frac{2}{3} = 7 - \frac{2}{5} \cdot \frac{2}{1} = 7 - \frac{4}{5} = \frac{35}{5} - \frac{4}{5} = \frac{31}{5}$$

7. (10 points) Consider the linear function $f(x) = -3(2x - 7)$. Showing all your work, answer the following:

   (a) Find the rate of change of $f(x)$.

   (b) Interpret the rate of change of $f(x)$.

   (c) Determine whether $f(x)$ is an increasing or decreasing function.

   (d) Determine the x-intercept of $f(x)$.

   (e) Find an x-value such that $f(x) = 9$.

Solution. *First, we write* $f(x) = -3(2x - 7) = -6x + 21$.

(a) The rate of change of $f(x)$ *is the slope. Because* $f(x) = -6x + 21$ *has the form* $y = mx + b$*, we see that the slope of* $f(x)$ *is* $m = -6$.

(b) We have $m = -6$. *Writing* $m = -6 = \frac{-6}{1}$ *and interpreting this as* $\frac{\Delta y}{\Delta x}$*, we see that each increase of 1 in* x *results in a decrease of 6 in* y*. Alternatively, writing* $m = -6 = -\frac{6}{1} = \frac{6}{-1}$ *and interpreting this as* $\frac{\Delta y}{\Delta x}$*, we see that each decrease of 1 in* x *results in an increase of 6 in* y.

(c) Because $m = -6 < 0$*, we see that* $f(x)$ *is decreasing.*

(d) The x*-intercept occurs when* $f(x) = 0$*. But then…*

$$\begin{aligned} f(x) &= 0 \\ -3(2x-7) &= 0 \\ 2x - 7 &= 0 \\ 2x &= 7 \\ x &= \frac{7}{2} \end{aligned}$$

Therefore, the x*-intercept is* $\frac{7}{2}$*, i.e.* $\left(\frac{7}{2}, 0\right)$.

(e) If $f(x) = 9$*, we have…*

$$\begin{aligned} f(x) &= 9 \\ -6x + 21 &= 9 \\ -6x &= -12 \\ x &= 2 \end{aligned}$$

We can check: $f(2) = -6(2) + 21 = -12 + 21 = 9$.

8. (10 points) Find the equation of the line parallel to the line $y = 6 - x$ that has x-intercept -12.

Solution. *Because the line is parallel to $y = 6 - x$, the line has the form $y = mx + b$. Because the line is parallel to $y = 6 - x$, it must have the same slope. The slope of the line $y = 6 - x$ has slope -1; therefore, the line has slope $m = -1$. Because the line has x-intercept -12, we know that $(-12, 0)$ is on the line. But then we have. . .*

$$y = mx + b$$

$$y = -x + b$$

$$0 = -(-12) + b$$

$$0 = 12 + b$$

$$b = -12$$

Therefore, the line is $y = -x - 12$.

9. (10 points) Find the equation of the line perpendicular to the line $y = -\frac{\pi}{2}$ that contains the x-intercept of the line $-7x + 5y = -3$.

Solution. *Because the line is perpendicular to $y = -\frac{\pi}{2}$, which is horizontal, our line must be vertical. Therefore, the line has the form $x = c$ for some c. The x-intercept of the line $-7x + 5y = -3$ occurs when $y = 0$. But then we have $-7x = -3$. But then $x = \frac{3}{7}$. Therefore, the line contains the point $\left(\frac{3}{7}, 0\right)$. But then the line must be $x = \frac{3}{7}$.*

10. (10 points) Find the equation of the line that contains the y-intercept of $y = \frac{7}{11}\,x - 5$ and the point of intersection of $y = 4 - 3x$ and $y = \frac{1}{2}\,x + 18$.

Solution. *Because the line* $y = \frac{7}{11}\,x - 5$ *has the form* $y = mx + b$*, the its* y*-intercept is* $(0, -5)$*. We now find the point of intersection of* $y = 4 - 3x$ *and* $y = \frac{1}{2}\,x + 18$*:*

$$\begin{aligned} 4 - 3x &= \frac{1}{2}\,x + 18 \\ 2(4 - 3x) &= 2\left(\frac{1}{2}\,x + 18\right) \\ 8 - 6x &= x + 36 \\ 8 - 7x &= 36 \\ -7x &= 28 \\ x &= -4 \end{aligned}$$

Using this in the line $y = 4 - 3x$*. We have* $y = 4 - 3(-4) = 4 + 12 = 16$*. Therefore, the line contains the point* $(-4, 16)$*. Because the line contains the point* $(0, -5)$ *and* $(-4, 16)$*, we have...*

$$m = \frac{\Delta y}{\Delta x} = \frac{16 - (-5)}{-4 - 0} = \frac{16 + 5}{-4 - 0} = \frac{21}{-4} = -\frac{21}{4}$$

But the line contains $(0, -5)$*, so that we have...*

$$\begin{aligned} y &= mx + b \\ -5 &= -\frac{21}{4} \cdot 0 + b \\ -5 &= 0 + b \\ b &= -5 \end{aligned}$$

Therefore, the line $y = -\frac{21}{4}\,x - 5$*.*

11. (10 points) Showing all your work, find the solution to the following:

$$2(x-9) = \frac{x}{3} + 7$$

Solution. *We have. . .*

$$\begin{aligned} 2(x-9) &= \frac{x}{3} + 7 \\ 2x - 18 &= \frac{x}{3} + 7 \\ 3(2x-18) &= 3\left(\frac{x}{3} + 7\right) \\ 6x - 54 &= x + 21 \\ 5x - 54 &= 21 \\ 5x &= 75 \\ x &= 15 \end{aligned}$$

12. (10 points) Showing all your work, find the solution to the following:

$$\frac{5x-3}{1-x} = 13$$

Solution. *We have. . .*

$$\begin{aligned}
\frac{5x-3}{1-x} &= 13 \\
5x-3 &= 13(1-x) \\
5x-3 &= 13-13x \\
18x-3 &= 13 \\
18x &= 16 \\
x &= \frac{16}{18} \\
x &= \frac{8}{9}
\end{aligned}$$

13. (10 points) Without explicitly finding the intersection, explain why the following lines $y = 2(7 - 5x)$ and $y = 5x + 2$ intersect. Showing all your work, find the intersection of the lines.

Solution. *The line* $y = 2(7 - 5x) = 14 - 10x$ *has some* $m = -10$. *The line* $y = 5x + 2$ *has slope 5. Because* $-10 \neq 5$*, the lines cannot be parallel. But because the lines are not parallel, the lines must intersect.*

14. (10 points) Thai Tanic is a new restaurant chain that has been expanding across the Northwest. It is projected that next year, it will have \$800,000 in profits and the following year it will have \$1.2 million in profits.

   (a) Under what assumptions is a linear model to predict the growth rate of this business appropriate?

   (b) Find a linear model for the profit of this company t years from today.

   (c) Interpret the slope of your linear model in (b).

   (d) Interpret the y-intercept of your linear model in (b), if possible.

   (e) How long until the company has a profit of \$5 million?

Solution. *We will let the profits, P, at a time t years from now be denoted by $P(t)$.*

(a) The rate of change of the profits must be constant.

(b) Because $P(t)$ is a linear function, we know $P(t) = mt + b$ for some m, b. We know that in a year, the company will have \$800,000 in profits, i.e. $(1, 800000)$ is a point on the graph of $P(t)$. Similarly, because the company makes \$1,200,000 two years from now, $(2, 1200000)$ is on the graph of $P(t)$. But then we have...

$$m = \frac{1200000 - 800000}{2 - 1} = \frac{400000}{1} = 400000$$

Because $(1, 800000)$ is on the line, we have...

$$\begin{aligned} P(t) &= mt + b \\ P(t) &= 400000t + b \\ 800000 &= 400000(1) + b \\ 800000 &= 400000 + b \\ b &= 400000 \end{aligned}$$

Therefore, $P(t) = 400000t + 400000 = 400000(t + 1)$.

(c) From (b), we know that $m = 400000$. Interpreting $m = 400000 = \frac{400000}{1}$ as $\frac{\Delta P}{\Delta t}$, we see that every additional year, the company makes an additional \$400,000 in profit, i.e. the company makes an additional \$400,000 in profit per year.

(d) From (b), the y-intercept is 400,000. The y-intercept occurs when $t = 0$, i.e. the current year. Therefore, the company is currently making \$400,000 in profit.

(e) *If the profit is \$5 million, then* $P(t) = 5000000$. *But then...*

$$\begin{aligned} P(t) &= 5000000 \\ 400000t + 400000 &= 5000000 \\ 400000t &= 4600000 \\ t &= 11.5 \end{aligned}$$

Therefore, the company will have a profit of \$5 million in 11.5 years.

15. (10 points) Sia Gogh is driving down the highway at 65 mph. She has been driving for 2 hours. Sia determines her total distance traveled t hours from now is approximately $D(t) = 65t + 130$.

(a) Explain why her distance traveled is approximately linear.

(b) Interpret the slope of $D(t)$.

(c) Interpret the y-intercept of $D(t)$.

(d) How far has she traveled after a total of 10 hours?

Solution.

(a) Because $D(t) = 65t + 130$ has the form $y = mx + b$ with $m = 65$, $x = t$, and $b = 130$, we know that $D(t)$ is linear.

(b) The slope of $D(t) = 65t + 130$ is $m = 65$. Interpreting the slope $m = 65 = \frac{65}{1}$ as $\frac{\Delta D}{\Delta t}$, we see that every additional hour from now sees her distance driven increasing by 65 miles, i.e. she is currently traveling at an average rate of 65 mph.

(c) The y-intercept of $D(t) = 65t + 130$ is 130. The y-intercept occurs when $t = 0$, i.e. 0 hours from now. But then currently, her total distance traveled is 130 miles. [Note: She has already driven for 2 hours.]

(d) She has already driven 2 hours. Therefore, she would need to drive an additional 8 hours from now to drive for a total of 10 hours. But then...

$$D(8) = 65(8) + 130 = 520 + 130 = 650$$

Therefore, after a total of 10 hours, she has driven 650 miles.