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MATH 101
Fall 2022
HW 10: Due 10/24

"VIP is always better, Vivian."
–Anna Delvey (Sorokin), Inventing Anna

Problem 1. (10pt) A function $f(x)$ has a table of values given below. Using this table, explain why $f^{-1}(x)$ cannot exist.

x	1	2	3	4	5
$f(x)$	6	3	9	6	1

Solution. Because $f(1) = 6$, we know that $f^{-1}(6) = 1$. But we also have $f(4) = 6$, so that $f^{-1}(6) = 4$. But we cannot have both $f^{-1}(6) = 1$ and $f^{-1}(6) = 4$. Therefore, $f^{-1}(6)$ is not well defined so that $f^{-1}(x)$ does not exist.

Problem 2. (10pt) Let $f(x) = 4x + 3$ and $g(x) = \frac{1}{4}(x - 3)$. Show that $g(x)$ is the inverse of $f(x)$ by showing that $(f \circ g)(x) = f(g(x)) = x$ and $(g \circ f)(x) = x$.

Solution. We have...

$$\begin{aligned}
(f \circ g)(x) &= f\big(g(x)\big) \\
&= f\left(\frac{1}{4}(x-3)\right) \\
&= 4 \cdot \frac{1}{4}(x-3) + 3 \\
&= (x-3) + 3 \\
&= x
\end{aligned}
\qquad\qquad
\begin{aligned}
(g \circ f)(x) &= g\big(f(x)\big) \\
&= g(4x+3) \\
&= \frac{1}{4}\big((4x+3) - 3\big) \\
&= \frac{1}{4} \cdot 4x \\
&= x
\end{aligned}$$

Therefore, because $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$, we know that $g(x) = f^{-1}(x)$.

Problem 3. (10pt) Let $y = \frac{1}{3}\,x + 5$.

(a) By interchanging the roles of y and x, find the inverse to the function $f(x) = \frac{1}{3}\,x + 5$.

(b) Use the answer from (a) to find $f^{-1}(-2)$.

Solution.

(a) We can write $f(x) = \frac{1}{3}\,x + 5$ as $y = \frac{1}{3}\,x + 5$. Interchanging the roles of x and y, we have $x = \frac{1}{3}\,y + 5$. But then…

$$\begin{aligned} x &= \frac{1}{3}\,y + 5 \\ x - 5 &= \frac{1}{3}\,y \\ y &= 3(x - 5) \end{aligned}$$

But then we have $f^{-1}(x) = 3(x - 5)$.

(b) We have…

$$f^{-1}(-2) = 3(-2 - 5) = 3(-7) = -21$$