Name: _____

MATH 308
Fall 2023
HW 6: Due 10/05

"I know that the great Hilbert said, 'We will not be driven out of the paradise Cantor has created for us,' and I reply, 'I see no reason for walking in!' "

–Richard Hamming

Problem 1. (10pt) Let A and B be sets. For each of the following sets, compute the *complement* of the given set. Be sure to show all your work and simplify your set expression as much as possible.

(a) $(A \Delta B) \cup B^c$

(b) $(A \cup B^c) \cap (A \cap B)^c$

(c) $A - (A - B)$

Problem 2. (10pt) Let $X = \{a, \{b\}, \{a, b\}\}$.

(a) Compute $\mathcal{P}(X)$. What is the cardinality of this set?

(b) Determine whether the following are true or false—no justification is necessary:

(i) $\varnothing \in X$

(ii) $\varnothing \subseteq X$

(iii) $a \in X$

(iv) $\{a\} \in X$

(v) $\{a\} \subseteq X$

(vi) $\varnothing \in \mathcal{P}(X)$

(vii) $\mathcal{P}(X) \subseteq \mathcal{P}(X)$

(viii) $\{a, b\} \in \mathcal{P}(X)$

(ix) $\{a, b\} \subseteq \mathcal{P}(X)$

(x) $\{\{a, b\}\} \subseteq \mathcal{P}(X)$

Problem 3. (10pt) For integers n, let $X_n = (n, n+1)$, and for natural numbers m, let $Y_m = \left[\frac{1}{m}, m\right)$. Compute the following:

(a) $\bigcup_{i=-1}^{2} X_i$

(b) $\bigcap_{k=2}^{5} Y_k$

(c) $\bigcup_{n\in\mathbb{Z}} X_n$

(d) $\bigcup_{m\in\mathbb{N}} Y_m$

(e) $\left(\bigcup_{m\in\mathbb{N}} Y_m\right)^c$

Problem 4. (10pt) Let $A = \{-1, 0, 1\}$, $B = \{a, b\}$, and $C = \{\sqrt{2}, \pi\}$.

(a) Compute $A \times B$.

(b) Is $A \times B = B \times A$? Explain.

(c) Compute $\mathcal{P}(B \times C)$.

(d) If $X = \varnothing$, what is $X \times Y$ for any set Y?

(e) If X, Y are sets and $X \times Y = Y \times X$, is it necessarily true that $X = Y$? Explain. [Hint: Use part (d).]