Name: *Caleb McWhorter — Solutions*
MATH 307
Spring 2023
HW 7: Due 03/01 (02)

"Take chances, make mistakes. That's how you grow. Pain nourishes your courage. You have to fail in order to practice being brave."
–Mary Tyler Moore

Problem 1. (10pt) Being as detailed as possible, answer the following:

(a) What are two measures of 'center' for a dataset?

(b) What are two measures of 'variability' for a dataset?

(c) Name a measure of center which is robust and a measure of center which is not robust.

Solution.

(a) There are many notions of 'center.' Among the more common measures of center are mean, median, midrange, etc.

(b) There are many notions of 'variability' for a data set. Among the more common measures of variability are standard deviation, IQR, and range.

(c) The median is a measure of center which is robust while the mean is a measure of center which is not robust. For instance, given the dataset 1, 2, 3, both the median and mean are 2. However, if the outlier 100 is added, the median changes to 2.5 while the mean changes to 53.

Problem 2. (10pt) Stewart is tracking snowfall received over the past few days in his area. He finds the following number of inches of snow has fallen over the past few days:

$$1.5” \qquad 4.2” \qquad 0” \qquad 0.8” \qquad 5.1”$$

(a) Find the mean inches of snow that fell this week.

(b) Find the standard deviation for the this week's snowfall.

(c) If Stewart underestimated the inches that fell each day by 0.5", what would be the average snowfall for the week?

Solution.

(a) We have...

$$\overline{x} = \frac{1.5” + 4.2” + 0” + 0.8” + 5.1”}{5} = \frac{11.6”}{5} = 2.32”$$

(b) We have...

x_i	$x_i - \overline{x}$	$(x_i - \overline{x})^2$
1.5	-0.82	0.6724
4.2	1.88	3.5344
0	-2.32	5.3824
0.8	-1.52	2.3104
5.1	2.78	7.7284
	Total:	19.6280

Therefore, we have...

$$s^2 = \frac{1}{n-1}\sum(x_i - \overline{x})^2 = \frac{1}{5-1} \cdot 19.6280 = \frac{19.6280}{4} = 4.907$$

But then we have $s = \sqrt{s^2} = \sqrt{4.907} \approx 2.215”$.

(c) Because each day's snowfall is 0.5" more than what Stewart measured, we anticipate the average snowfall is 0.5" more than what was computed in (a). But then we have $\overline{x} = 2.32” + 0.5” = 2.82”$. We can confirm this with a direct computation:

$$\overline{x} = \frac{2.0” + 4.7” + 0.5” + 1.3” + 5.6”}{5} = \frac{14.1”}{5} = 2.82”$$

Problem 3. (10pt) Charlie is at a town hall meeting. He goes around the room and asks each person their age. Charlie finds the following responses for the ages of the people at the meeting:

$$66 \quad 50 \quad 67 \quad 40 \quad 70 \quad 56 \quad 69 \quad 57 \quad 40 \quad 44$$

(a) Find the median for this dataset.

(b) Find the IQR for this dataset.

(c) Sketch a box and whisker plot for this dataset.

Solution.

(a) Ordering the data, we have...

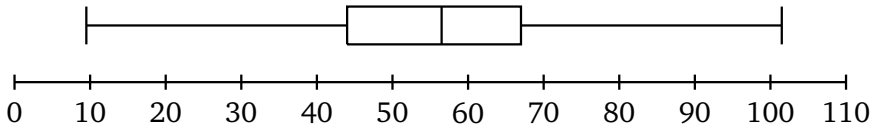
$$40 \quad 40 \quad 44 \quad 50 \quad 56 \quad 57 \quad 66 \quad 67 \quad 69 \quad 70$$

There are 10 numbers. Therefore, because $\frac{10}{2} = 5$, the median is the average of the 5th and 6th number. Then the median is $\frac{56+57}{2} = \frac{113}{2} = 56.5$.

(b) The numbers less than this median are 40, 40, 44, 50, and 56. The median of these numbers is 44. But then $Q_1 = 44$. The numbers greater than 56.5 are 57, 66, 67, 69, and 70. The median of these numbers is 67. But then $Q_3 = 67$. Therefore, we have...

$$\text{IQR} = Q_3 - Q_1 = 67 - 44 = 23$$

(c) Observe that $1.5\text{IQR} = 1.5 \cdot 23 = 34.5$. We now have $Q_1 - 1.5\text{IQR} = 44 - 34.5 = 9.5$ and $Q_3 + 1.5\text{IQR} = 67 + 34.5 = 101.5$. The box and whisker plot is then...



Problem 4. (10pt) Lorelei is finding the mean of the numbers 17, 18, 26, 19. She finds that the mean is 40. By comparing Lorelei's mean as a measure of 'center' to the size of the numbers, how would you explain to her why this cannot be correct. What has Lorelei done incorrectly? How would you explain to her how to fix her work?

Solution. If the mean is a measure of 'center', then it should be in the 'center' of the numbers. However, 40 is much larger than all the listed numbers. Therefore, it should not be possible for 40 to be the 'center' of these numbers. We know that...

$$\overline{x} = \frac{\text{sum numbers}}{\text{amount of numbers}} = \frac{17+18+26+19}{4} = \frac{80}{4} = 20$$

Therefore, the correct mean is 20. We can see from this calculation that Lorelei likely did the following:

$$\overline{x} = \frac{\text{sum numbers}}{2} = \frac{17+18+26+19}{2} = \frac{80}{2} = 40$$

Likely, this is a result of computing the mean of only two numbers many times and internalizing that to find the mean you find the sum of the numbers and divide by 2.