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MATH 101
Winter 2021
HW 9: Due 01/19

"When Pam gets Michael's old chair, I get Pam's old chair. Then I'll have two chairs. Only one to go."
–Creed Bratton, The Office

Problem 1. (10pt) Explain why the following system of equations does or does not have a solution:

$$\begin{cases} -2x + 3y = -15 \\ \ \ 4x + 6y = 6 \end{cases}$$

Solution. We solve for y in each of the equations:

$$\begin{aligned} -2x + 3y &= -15 & \qquad 4x + 6y &= 6 \\ 3y &= 2x - 15 & 6y &= -4x + 6 \\ y &= \frac{2}{3}\,x - 5 & y &= -\frac{2}{3}\,x + 1 \end{aligned}$$

The slope of the first line is $m_1 = \frac{2}{3}$ and the slope of the second line is $m_2 = -\frac{2}{3}$. Because $m_1 \neq m_2$, the lines are not parallel. Therefore, the lines must intersect. [Notice that the y-intercepts are different, so that the lines are distinct.] But then the system of equations has a solution.

Problem 2. (10pt) Determine if the point $(-2,-3)$ is a solution to the following system of equations:

$$\begin{aligned} -5x + 3y &= 1 \\ 6x - 7y &= -33 \end{aligned}$$

Solution. If $(-2,-3)$ is a solution to the system of equations, it lies on both the given lines. But then the point satisfies both of the given equations. We check this:

$$\begin{aligned} -5x + 3y &= 1 \\ -5(-2) + 3(-3) &\stackrel{?}{=} 1 \\ 10 - 9 &\stackrel{?}{=} 1 \\ 1 &\stackrel{\checkmark}{=} 1 \end{aligned}$$

so that $(-2,-3)$ lies along the first line but...

$$\begin{aligned} 6x - 7y &= -33 \\ 6(-2) - 7(-3) &\stackrel{?}{=} -33 \\ -12 + 21 &\stackrel{?}{=} -33 \\ 9 &\stackrel{\times}{=} -33 \end{aligned}$$

so that $(-2,-3)$ does not lie along the second line. Therefore, $(-2,-3)$ is not a solution to the system of equations.

Problem 3. (10pt) Showing all your work, solve the following system of equations:

$$\begin{aligned} 4x - y &= -11 \\ x + 5y &= 13 \end{aligned}$$

Solution. Using substitution, we solve for y in the first equation:

$$\begin{aligned} 4x - y &= -11 \\ -y &= -4x - 11 \\ y &= 4x + 11 \end{aligned}$$

Using this in the second equation, we find...

$$\begin{aligned} x + 5y &= 13 \\ x + 5(4x + 11) &= 13 \\ x + 20x + 55 &= 13 \\ 21x + 55 &= 13 \\ 21x &= -42 \\ x &= -2 \end{aligned}$$

But then we know that $y = 4x + 11 = 4(-2) + 11 = -8 + 11 = 3$. Therefore, the solution is $(x, y) = (-2, 3)$.

OR

Using elimination, we eliminate x by multiplying the second equation by -4 and adding the equations:

$$\begin{array}{r} 4x - y = -11 \\ \underline{-4x - 20y = -52} \\ -21y = -63 \\ y = 3 \end{array}$$

Using the first equation, we have...

$$\begin{aligned} 4x - y &= -11 \\ 4x - 3 &= -11 \\ 4x &= -8 \\ x &= -2 \end{aligned}$$

Therefore, the solution to the system of equations is $(x, y) = (-2, 3)$.

Problem 4. (10pt) Showing all your work, solve the following system of equations:

$$\begin{aligned} 4x - 5y &= -6 \\ 6x + 3y &= 12 \end{aligned}$$

Solution. Using substitution, we solve for y in the first equation:

$$\begin{aligned} 4x - 5y &= -6 \\ -5y &= -4x - 6 \\ y &= \frac{4}{5}\,x + \frac{6}{5} \end{aligned}$$

Using this in the second equation, we find...

$$\begin{aligned} 6x + 3y &= 12 \\ 6x + 3\left(\frac{4}{5}\,x + \frac{6}{5}\right) &= 12 \\ 6x + \frac{12}{5}\,x + \frac{18}{5} &= 12 \\ 5\left(6x + \frac{12}{5}\,x + \frac{18}{5}\right) &= 12 \cdot 5 \\ 30x + 12x + 18 &= 60 \\ 42x + 18 &= 60 \\ 42x &= 42 \\ x &= 1 \end{aligned}$$

But then we know that $y = \frac{4}{5} \cdot 1 + \frac{6}{5} = \frac{4}{5} + \frac{6}{5} = \frac{10}{5} = 2$. Therefore, the solution is $(x, y) = (1, 2)$.

OR

Using elimination, we eliminate x by multiplying the first equation by 3 and the second equation by -2 and adding the equations:

$$\begin{aligned} 12x - 15y &= -18 \\ \underline{-12x - 6y} &\underline{\;= -24} \\ -21y &= -42 \\ y &= 2 \end{aligned}$$

Using the first equation, we have...

$$\begin{aligned} 4x - 5y &= -6 \\ 4x - 10 &= -6 \\ 4x &= 4 \\ x &= 1 \end{aligned}$$

Therefore, the solution to the system of equations is $(x, y) = (1, 2)$.

Problem 5. (10pt) Showing all your work, solve the following system of equations:

$$\begin{aligned} 3x - 2y &= 7 \\ -6x + 3y &= -11 \end{aligned}$$

Solution. Using substitution, we solve for y in the first equation:

$$\begin{aligned} 3x - 2y &= 7 \\ -2y &= -3x + 7 \\ y &= \frac{3}{2}x - \frac{7}{2} \end{aligned}$$

Using this in the second equation, we find...

$$\begin{aligned} -6x + 3y &= -11 \\ -6x + 3\left(\frac{3}{2}x - \frac{7}{2}\right) &= -11 \\ -6x + \frac{9}{2}x - \frac{21}{2} &= -11 \\ 2\left(-6x + \frac{9}{2}x - \frac{21}{2}\right) &= -11 \cdot 2 \\ -12x + 9x - 21 &= -22 \\ -3x - 21 &= -22 \\ -3x &= -1 \\ x &= \frac{1}{3} \end{aligned}$$

But then we know that $y = \frac{3}{2} \cdot \frac{1}{3} - \frac{7}{2} = \frac{1}{2} - \frac{-6}{2} = -3$. Therefore, the solution is $(x, y) = (\frac{1}{3}, -3)$.

OR

Using elimination, we eliminate x by multiplying the first equation by 2 and adding the equations:

$$\begin{array}{r} 6x - 4y = 14 \\ \underline{-6x + 3y = -11} \\ -y = 3 \\ y = -3 \end{array}$$

Using the first equation, we have...

$$\begin{aligned} 3x - 2y &= 7 \\ 3x + 6 &= 7 \\ 3x &= 1 \\ x &= \frac{1}{3} \end{aligned}$$

Therefore, the solution to the system of equations is $(x, y) = (\frac{1}{3}, -3)$.

Problem 6. (10pt) Compute the following, simplifying as much as possible:

$$\frac{x}{x-1}+\frac{x+1}{x^2+4x-5}$$

Solution.

$$\begin{aligned}\frac{x}{x-1}+\frac{x+1}{x^2+4x-5} &= \frac{x}{x-1}+\frac{x+1}{(x-1)(x+5)}\\ &= \frac{x(x+5)}{(x-1)(x+5)}+\frac{x+1}{(x-1)(x+5)}\\ &= \frac{x^2+5x}{(x-1)(x+5)}+\frac{x+1}{(x-1)(x+5)}\\ &= \frac{x^2+6x+1}{(x-1)(x+5)}\end{aligned}$$

Problem 7. (10pt) Compute the following, simplifying as much as possible:

$$\frac{3-x}{x^2-4} - \frac{5x}{x^2+5x-14}$$

Solution.

$$\begin{aligned}
\frac{3-x}{x^2-4} - \frac{5x}{x^2+5x-14} &= \frac{3-x}{(x-2)(x+2)} - \frac{5x}{(x-2)(x+7)} \\
&= \frac{(3-x)(x+7)}{(x-2)(x+2)(x+7)} - \frac{5x(x+2)}{(x-2)(x+2)(x+7)} \\
&= \frac{3x+21-x^2-7x}{(x-2)(x+2)(x+7)} - \frac{5x^2+10x}{(x-2)(x+2)(x+7)} \\
&= \frac{-x^2-4x+21}{(x-2)(x+2)(x+7)} - \frac{5x^2+10x}{(x-2)(x+2)(x+7)} \\
&= \frac{(-x^2-4x+21)-(5x^2+10x)}{(x-2)(x+2)(x+7)} \\
&= \frac{-x^2-4x+21-5x^2-10x}{(x-2)(x+2)(x+7)} \\
&= \frac{-6x^2-14x+21}{(x-2)(x+2)(x+7)}
\end{aligned}$$

Problem 8. (10pt) Compute the following, simplifying as much as possible:

$$\frac{x^2+5x-6}{x^2-5x+24} \cdot \frac{x^2-9}{x^2+8x-9}$$

Solution.

$$\begin{aligned}
\frac{x^2+5x-6}{x^2-5x+24} \cdot \frac{x^2-9}{x^2+8x-9} &= \frac{(x-1)(x+6)}{(x-8)(x+3)} \cdot \frac{(x-3)(x+3)}{(x-1)(x+9)} \\
&= \frac{\cancel{(x-1)}(x+6)}{(x-8)\cancel{(x+3)}} \cdot \frac{(x-3)\cancel{(x+3)}}{\cancel{(x-1)}(x+9)} \\
&= \frac{(x-3)(x+6)}{(x-8)(x+9)}
\end{aligned}$$

Problem 9. (10pt) Compute the following, simplifying as much as possible:

$$\frac{\frac{4x^2-9}{x^2+5x+4}}{\frac{2x^2-x-6}{x^2-4x-32}}$$

Solution.

$$\frac{\frac{4x^2-9}{x^2+5x+4}}{\frac{2x^2-x-6}{x^2-4x-32}} = \frac{4x^2-9}{x^2+5x+4} \cdot \frac{x^2-4x-32}{2x^2-x-6}$$

$$= \frac{(2x-3)(2x+3)}{(x+1)(x+4)} \cdot \frac{(x-8)(x+4)}{(x-2)(2x+3)}$$

$$= \frac{(2x-3)\cancel{(2x+3)}}{(x+1)\cancel{(x+4)}} \cdot \frac{(x-8)\cancel{(x+4)}}{(x-2)\cancel{(2x+3)}}$$

$$= \frac{(x-8)(2x-3)}{(x-2)(x+1)}$$

Problem 10. (10pt) Compute the following, simplifying as much as possible:

$$\frac{4x+3}{x-10} - \frac{\frac{x+6}{x-7}}{\frac{x^2-4x-60}{x^2-6x-7}}$$

Solution.

$$\begin{aligned}
\frac{4x+3}{x-10} - \frac{\frac{x+6}{x-7}}{\frac{x^2-4x-60}{x^2-6x-7}} &= \frac{4x+3}{x-10} - \frac{x+6}{x-7} \cdot \frac{x^2-6x-7}{x^2-4x-60} \\
&= \frac{4x+3}{x-10} - \frac{x+6}{x-7} \cdot \frac{(x-7)(x+1)}{(x-10)(x+6)} \\
&= \frac{4x+3}{x-10} - \frac{\cancel{x+6}}{\cancel{x-7}} \cdot \frac{\cancel{(x-7)}(x+1)}{(x-10)\cancel{(x+6)}} \\
&= \frac{4x+3}{x-10} - \frac{x+1}{x-10} \\
&= \frac{4x+3-(x+1)}{x-10} \\
&= \frac{4x+3-x-1}{x-10} \\
&= \frac{3x-2}{x-10}
\end{aligned}$$