Name: _____
MATH 308
Fall 2023
HW 1: Due 09/07

"And I knew exactly what to do. . . but in a much more real sense, I had no idea what to do."
–Michael Scott, The Office

Problem 1. (10pt) Determine if each of the following are propositions. If the example is a proposition, state its truth value with a brief justification. If the example is *not* a proposition, briefly explain why:

(a) $3^2 - 15 = 6$

(b) The statement in (c) is false.

(c) George Orwell wrote *A Remembrance of Things Past*.

(d) There is intelligent life in the universe.

(e) $x - 3 \leq 10$

Problem 2. (10pt) For each of the following, either define appropriate primitive propositions (using P, Q, R, etc.) and write the 'statement' using logical connectives, or give an English sentence for the given primitives and 'translate' the logical 'sentence' into an English sentence:

(a) $P \to (\neg Q \vee R)$

(b) You will succeed, if you believe and work hard.

(c) $Q \wedge (\neg P \vee Q)$

(d) I pay rent, or I lose my job and starve.

Problem 3. (10pt) Consider the following compound statement: $\neg(P \to \neg Q) \wedge \neg Q$

(a) Determine whether the given compound statement is a tautology, contradiction, or neither. Be sure to justify your response.

(b) Using a truth table, show that the first part of the given compound statement, i.e. $\neg(P \to \neg Q)$, is logically equivalent to $P \wedge Q$.

(c) By 'simplifying' the expression $\neg\big(P \vee \neg(P \wedge Q)\big)$, show that this compound statement is logically equivalent to the compound statement given at the start of the problem.

Problem 4. (10pt) Fix a real number x. Consider the statement, "if $x^2 > 4$, then $x > 2$'

(a) Determine the truth value of this statement with an explanation.

(b) Rewrite the given statement by defining appropriate primitive propositions and logical connectives.

(c) Find the negation, converse, and contrapositive of your result from (b).

(d) Rewrite your answers from (c) as English sentences. Then determine the truth value, with explanation, of each of the statements.