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MATH 108
Spring 2024
HW 17: Due 04/15

*“Why would you sow secretly? Why would you
prune privately? Why would you cull
clandestinely? Hidden hoe-ing and whatnot?”*
— Barbara Howard, Abbott Elementary

Problem 1. (10pts) Find the augmented matrix to the corresponding system of equations:

$$x + y + z - w = 15$$

$$2x - 3y + 14w = -12$$

$$-x + 5y + 8z = 27$$

$$x - y + z - w = 8$$

Solution. First, we order the variables as x , y , z , and then w . We also make sure each equality has all variables present. This gives us the following system of equations:

$$x + y + z - w = 15$$

$$2x - 3y + 0z + 14w = -12$$

$$-x + 5y + 8z + 0w = 27$$

$$x - y + z - w = 8$$

Therefore, the augmented matrix is...

$$\left(\begin{array}{ccccc} 1 & 1 & 1 & -1 & 15 \\ 2 & -3 & 0 & 14 & -12 \\ -1 & 5 & 8 & 0 & 27 \\ 1 & -1 & 1 & -1 & 8 \end{array} \right)$$

Problem 2. (10pts) The matrix below is the initial augmented matrix for a system of linear equations. Find the system of linear equations.

$$\begin{pmatrix} -4 & 1 & 12 \\ 3 & 7 & -11 \end{pmatrix}$$

Solution. Each column corresponds to a variable in the system—except the last column that corresponds to the ‘other side’ of the equalities. Therefore, there are $3 - 1 = 2$ variables in the system, which we will label x and y . Therefore, the system of equations must be...

$$\begin{aligned} -4x + y &= 12 \\ 3x + 7y &= -11 \end{aligned}$$

Problem 3. (10pts) The following matrix is the RREF of an augmented matrix coming from a system of equations. Did this system of equations have a solution? If the system of equations had a solution, find all the possible solutions. If the system did not have a solution, explain why.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 12 \end{pmatrix}$$

Solution. Each of the columns of the matrix corresponds to a variable—except for the last column which corresponds to the ‘other’ side of the equalities. There are then $4 - 1 = 3$ variables. From the first row, we see that $x_1 = 0$. From the second row, we see that $x_2 = -5$. From the third row, we see that $x_3 = 12$. Therefore, the unique solution is $(x_1, x_2, x_3) = (0, -5, 12)$, i.e. ...

$$\begin{cases} x_1 = 0 \\ x_2 = -5 \\ x_3 = 12 \end{cases}$$

Problem 4. (10pts) The following matrix is the RREF of an augmented matrix coming from a system of equations. Did this system of equations have a solution? If the system of equations had a solution, find all the possible solutions. If the system did not have a solution, explain why.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & -7 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

Solution. The last row of the matrix implies that $0 = -1$. Therefore, there is no solution to the system of equations.