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MATH 100
Fall 2023
HW 8: Due 10/30

"I'm proud to pay taxes in the United States; the only thing is, I could be just as proud for half the money."
–Arthur Godfrey

Problem 1. (10pt) The CPI this year is 304.3. Last year, the CPI was 292.7.

(a) What was the inflation rate from last year to this year?

(b) Predict the current cost of a good that cost \$459.99 last year.

(c) If the inflation rate in (a) was constant every year, what will be the percentage increase in the cost of goods in 100 years compared to this year?

(d) Suppose that the current cost of a cup of coffee is \$3.50. If the inflation rate for the past 50 years was the rate computed in (a), what was the cost of a cup of coffee 50 years ago?

Solution.

(a) We know that New CPI $=$ Old CPI $\cdot$ $(1 +$ Inf. Rate$)$. But then $1 +$ Inf. Rate $= \frac{\text{New CPI}}{\text{Old CPI}}$. But then...

$$1 + \text{Inf. Rate} = \frac{\text{New CPI}}{\text{Old CPI}} = \frac{304.3}{292.7} = 1.0396310215237445 \approx 1.0396$$

Therefore, the inflation rate was 3.96%.

(b) If the inflation represents the true percentage increase in price, then we know New Price $=$ Old Price$(1 +$ Inf. Rate$)$. But then...

$$\text{New Price} = \$459.99(1 + 0.0396310215237445) = \$459.99(1.0396310215237445) \approx \$478.22$$

(c) If we increase a number # by $\%_d$ a total of n times, the result is $\#(1 + \%_d)^n$, where $\%_d$ is the percentage increase written as a decimal. Then if a good costs P now, the price after 100 years of 3.96% annual inflation will be...

$$P(1+0.0396310215237445)^{100} = P(1.0396310215237445)^{100} = P(48.74420509385) = P(1+47.74420509385)$$

We can recognize this as a 4,774.42% increase in price.

(d) Let the price of a cup of coffee 50 years ago be P. Using the logic in (c), we know that the price after 50 years of 3.96% inflation will be $P(1+0.0396310215237445)^{50} = P(1.0396310215237445)^{50}$. But we know this final price must be \$3.50. So we have...

$$\$3.50 = P(1.0396310215237445)^{50}$$
$$\$3.50 = 6.981705027702761P$$
$$P = \frac{\$3.50}{6.981705027702761}$$
$$P \approx \$0.50$$

Problem 2. (10pt) The 2023 federal tax brackets are given below. The standard deduction for single filers for the year 2023 is \$13,850. Compute the federal taxes in 2023 for a single filer taking the standard deduction that made \$135,000 that year.

Tax Rate	Taxable Income
10%	Up to \$11,000
12%	\$11,001 – \$44,725
22%	\$44,726 – \$95,375
24%	\$95,376 – \$182,100
32%	\$182,101 – \$231,250
35%	\$231,251 – \$578,125
37%	$\geq$ \$578,126

Solution. We know the taxable income is their net income minus their deduction. Therefore, this person's taxable income is $\$135{,}000 - \$13{,}850 = \$121{,}150$. This individual's federal tax is then the tax rate applied to the amount of income in each tax bracket. Therefore, this individual's federal tax liability is...

$$\begin{aligned}\text{Fed. Tax} &= 0.10(\$11{,}000 - \$0) + 0.12(44{,}725 - \$11{,}000) + 0.22(\$95{,}375 - \$44{,}725) + 0.24(\$121{,}150 - \$95{,}375) \\ &= 0.10(\$11{,}000) + 0.12(\$33{,}725) + 0.22(\$50{,}650) + 0.24(\$25{,}775) \\ &= \$1{,}100 + \$4{,}047 + \$11{,}143 + \$6{,}186 \\ &= \$22{,}476\end{aligned}$$

Problem 3. (10pt) You invest \$5,000 into an account that earns 11.3% simple annual interest.

(a) How much will the account have after 5 years?

(b) How much should you have invested to have the \$8,000 after 5 years?

Solution.

(a) We know that $F = P(1 + rt)$, where F is the future value, P is the present value (principal), r is the annual interest rate, and t is the number of years. But then we have...

$$F = P(1 + rt) = \$5{,}000(1 + 0.113 \cdot 5) = \$5{,}000(1 + 0.565) = \$5{,}000(1.565) \approx \$2{,}825$$

Therefore, the account will have \$2,825 after 5 years.

(b) We know $P = \frac{F}{1+rt}$, where P is the present value (principal), F is the future value, r is the annual interest rate, and t is the number of years. But then we have...

$$P = \frac{F}{1 + rt} = \frac{\$8{,}000}{1 + 0.113 \cdot 5} = \frac{\$8{,}000}{1 + 0.565} = \frac{\$8{,}000}{1.565} \approx \$5{,}111.82$$

Therefore, you should have invested \$5,111.82 to have \$8,000 after 5 years.