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MATH 101
Winter 2021
HW 5: Due 01/11

"I just want to lie on the beach and eat hot dogs. That's all I've ever wanted."
–Kevin Malone, The Office

Problem 1. (10pt) Determine if the following function is linear. Explain why or why not.

x	$f(x)$
1.2	7.16
2.8	12.39
4.4	16.12
6.0	22.13
7.6	25.08

Solution. We compute the slopes:

$$m = \frac{12.39 - 7.16}{2.8 - 1.2} = \frac{5.23}{1.6} = 3.26875$$

$$m = \frac{16.12 - 12.39}{4.4 - 2.8} = \frac{3.73}{1.6} = 2.33125$$

$$m = \frac{22.13 - 16.12}{6.0 - 4.4} = \frac{6.01}{1.6} = 3.75625$$

$$m = \frac{25.08 - 22.13}{7.6 - 6.0} = \frac{2.95}{1.6} = 1.84375$$

Because the slope is not constant, this function cannot be linear.

Problem 2. (10pt) A linear function has a table whose values are given below. Find the equation of the linear function. Be sure to specify the slope and y-intercept.

x	$f(x)$
2	20
7	-5
12	-30
17	-55

Solution. Because this line is clearly not vertical, we know that $y = mx + b$. First, we compute the slope:

$$m = \frac{-5 - 20}{7 - 2} = \frac{-25}{5} = -5$$

Now we use the fact that $(2, 20)$ is on the line:

$$y = mx + b$$

$$y = -5x + b$$

$$20 = -5(2) + b$$

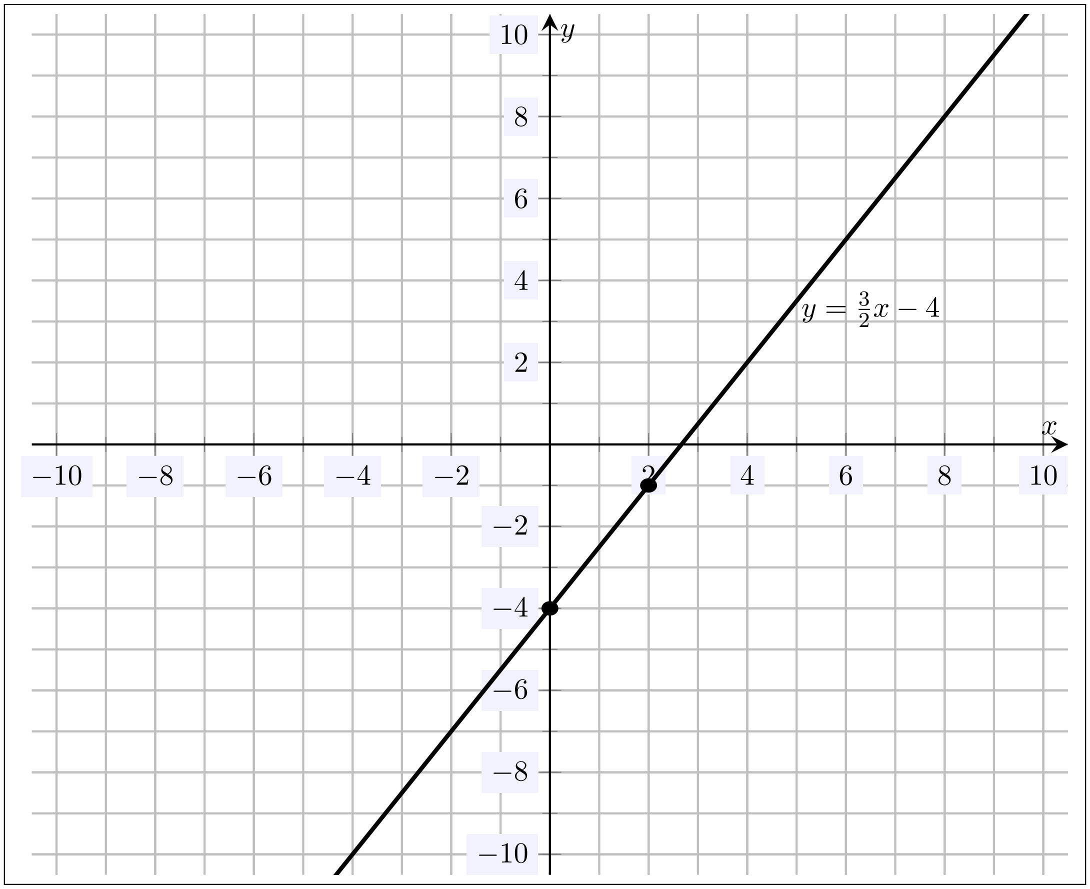
$$20 = -10 + b$$

$$b = 30$$

Therefore, we have...

$$\boxed{y = -5x + 30}$$

Problem 3. (10pt) Plot the linear function $y = \frac{3}{2}x - 4$ using the "two-point" method.

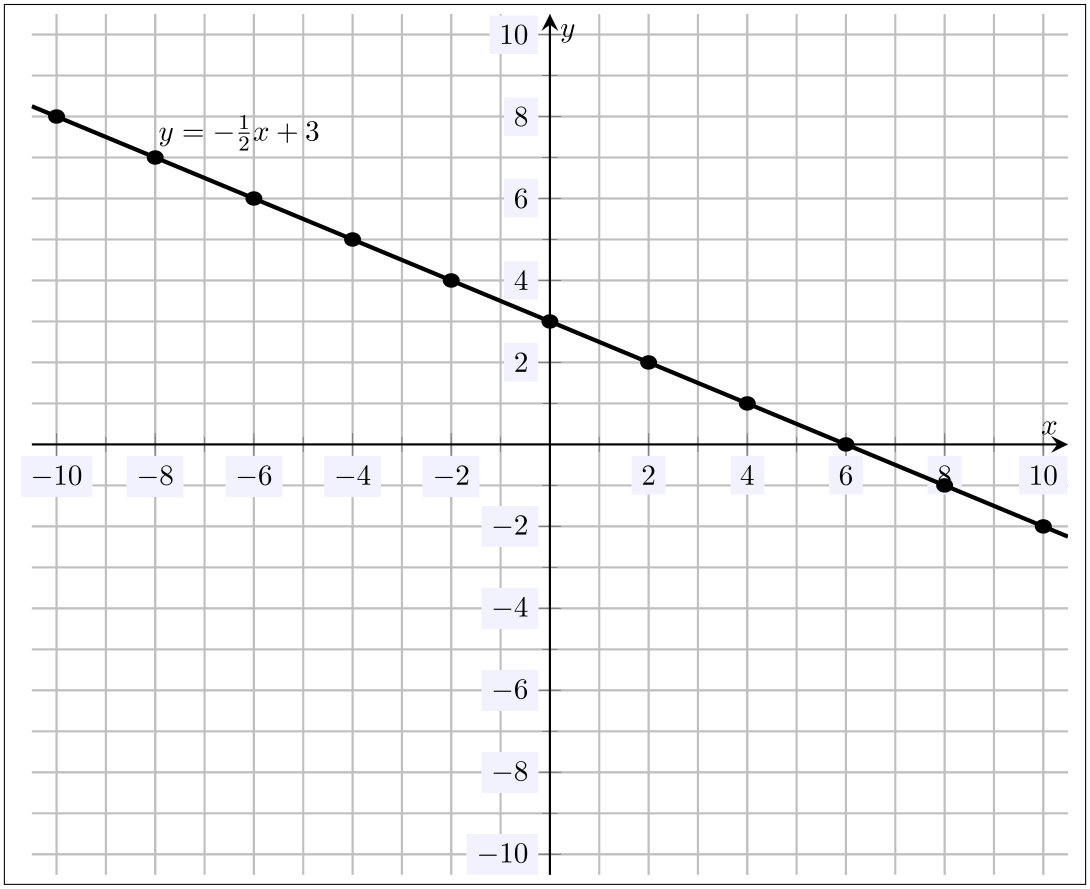


We compute any two points on the curve. For instance, we have...

$$y(0) = \frac{3}{2} \cdot 0 - 4 = 0 - 4 = -4 \rightsquigarrow (0, -4)$$
$$y(2) = \frac{3}{2} \cdot 2 - 4 = 3 - 4 = -1 \rightsquigarrow (2, -1)$$

We can then plot these point and connect them linearly.

Problem 4. (10pt) Plot the linear function $y = -\frac{1}{2}x + 3$ using the "slope" method.



We find a point on the line. For instance, we know $y(0) = -\frac{1}{2} \cdot 0 + 3 = 0 + 3 = 3$, which gives the point $(0, 3)$. We know the slope is $m = -\frac{1}{2}$, which we can interpret as...

$$\frac{\Delta y}{\Delta x} = \frac{-1}{2} = \frac{1}{-2}$$

That is, for every increase of 2 in x, y decreases by 1 or for every decrease in 2 in x there is a corresponding increase of 1 in y. This yields the plot above.

Problem 5. (10pt) Suppose water is draining from a tank. The number of gallons of water in the tank t hours from now is given by $W(t) = 567.8 - 24.1t$.

(a) Is $W(t)$ linear? Explain.

(b) What is the slope of $W(t)$? Interpret the slope.

(c) Explain how we can know that water is draining from the tank using (b).

(d) What is the y-intercept for $W(t)$? Interpret this intercept.

(e) Sketch a plot of $W(t)$ and estimate when the tank will be completely empty.

Solution.

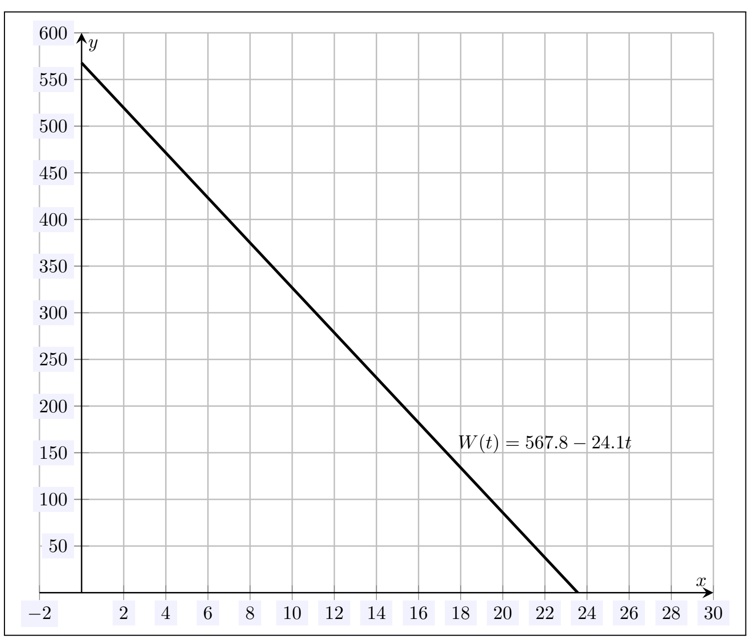
(a) Yes. Writing $W(t) = -24.1t + 567.8$, we see that $W(t)$ has the form $mx + b$, where $x = t$, $m = -24.1$, and $b = 567.8$. Moreover, the water is draining from the tank at a constant rate so that $W(t)$ must be linear.

(b) From (a), we see that $m = -24.1$. Thinking of -24.1 as $\frac{-24.1}{1}$ and interpreting it as $\dfrac{\Delta y}{\Delta x}$, we see that for each increase of 1 in x, we see a corresponding decrease of 24.1 in y. In context, this slope means that each hour 24.1 gallons of water drains from the tank.

(c) Because $m < 0$, we know that $W(t)$ is decreasing. Because this represents the amount of water in the tank, we know that the amount of water in the tank is decreasing.

(d) We have $W(0) = 567.8 - 24.1(0) = 567.8 - 0 = 567.8$. Therefore, the y-intercept is $(0, 567.8)$. This means when $t = 0$, then $W = 567.8$. We know $t = 0$ is 0 hours from now, i.e. the initial time, and that at this moment there was 567.8 gallons of water in the tank. Therefore, we can interpret the y-intercept as the information that the initial amount of water in the tank was 567.8 gallons.

(e) The tank is empty when $W(t) = 0$. From the plot below, we can see that the tank completely empties sometime between 22 hours and 24 hours from now. We can approximate this as 23.5 hours from now. Computing this exactly, we find the tank empties in 23.5602 hours.



Problem 6. (10pt) Consider the linear equation $12x - 2y = 56$.

(a) Solve the linear equation for y.

(b) Determine the slope and y-intercept for the corresponding line.

(c) Interpret the slope in at least two different ways.

Solution.

(a)

$$\begin{aligned} 12x - 2y &= 56 \\ -2y &= -12x + 56 \\ y &= 6x - 28 \end{aligned}$$

(b) Because $y = 6x - 28$ is in the form $y = mx + b$, we know that $m = 6$, i.e. the slope is 6, and that $b = -28$, which implies that the y-intercept is $(0, -28)$.

(c) We know that the slope is $m = 6 = \frac{6}{1}$. Thinking of the slope as $\frac{6}{1} = \dfrac{\Delta y}{\Delta x}$, we can interpret this as $\Delta x = 1$ and $\Delta y = 6$. But then for every increase of 1 in x, we see a corresponding increase of 6 in y. Alternatively, thinking of the slope as $\frac{6}{1} = \frac{-6}{-1} = \dfrac{\Delta y}{\Delta x}$, we can interpret this as $\Delta x = -1$ and $\Delta y = -6$. But then for every decrease of 1 in x, we see a corresponding decrease of 6 in y.

Problem 7. (10pt) Consider the linear equation $7.6x + 14.9y = 429.1$.

(a) Solve the linear equation for y.

(b) Determine the slope and y-intercept for the corresponding line.

(c) Interpret the slope in at least two different ways.

Solution.

(a)

$$\begin{aligned} 7.6x + 14.9y &= 429.1 \\ 14.9y &= -7.6x + 429.1 \\ y &= -0.510067x + 28.7987 \end{aligned}$$

(b) Because $y = -0.510067x + 28.7987$ is in the form $y = mx + b$, we know that $m = -0.510067$, i.e. the slope is -0.510067, and that $b = 28.7987$, which implies that the y-intercept is $(0, 28.7987)$.

(c) We know that the slope is $m = -0.510067 = \frac{-0.510067}{1}$. Thinking of the slope as $\frac{-0.510067}{1} = \dfrac{\Delta y}{\Delta x}$, we can interpret this as $\Delta x = 1$ and $\Delta y = -0.510067$. But then for every increase of 1 in x, we see a corresponding decrease of 0.510067 in y. Alternatively, thinking of the slope as $\frac{-0.510067}{1} = \frac{0.510067}{-1} = \dfrac{\Delta y}{\Delta x}$, we can interpret this as $\Delta x = -1$ and $\Delta y = 0.510067$. But then for every decrease of 1 in x, we see a corresponding increase of 0.510067 in y.

Problem 8. (10pt) Consider the line given by $y = -\frac{7}{6}x + 5$.

(a) Put the line in standard form.

(b) Is the point $(-6, 10)$ on the line? Explain.

(c) Is the point $(12, -9)$ on the line? Explain.

Solution.

(a)

$$\begin{aligned} y &= -\frac{7}{6}x + 5 \\ 6y &= -7x + 30 \\ 7x + 6y &= 30 \end{aligned}$$

(b) If $(-6, 10)$ is on the line, it satisfies the equation of the line. We check this...

$$\begin{aligned} 7x + 6y &= 30 \\ 7(-6) + 6(10) &\stackrel{?}{=} 30 \\ -42 + 60 &\stackrel{?}{=} 30 \\ 18 &\neq 30 \end{aligned}$$

Therefore, $(-6, 10)$ is *not* on the line. Alternatively, because this is a linear *function*, if $(-6, 10)$ is on the line then we must have $f(-6) = 10$. We check this...

$$f(-6) = -\frac{7}{6} \cdot -6 + 5 = 7 + 5 = 12$$

Therefore, $(-6, 10)$ is *not* on the line. However, the point $(-6, 12)$ is on the line.

(c) If $(12, -9)$ is on the line, it satisfies the equation of the line. We check this...

$$\begin{aligned} 7x + 6y &= 30 \\ 7(12) + 6(-9) &\stackrel{?}{=} 30 \\ 84 - 54 &\stackrel{?}{=} 30 \\ 30 &= 30 \end{aligned}$$

Therefore, $(12, -9)$ is on the line. Alternatively, because this is a linear *function*, if $(12, -9)$ is on the line then we must have $f(12) = -9$. We check this...

$$f(12) = -\frac{7}{6} \cdot 12 + 5 = -7(2) + 5 = -14 + 5 = -9$$

Therefore, $(12, -9)$ is on the line.

Problem 9. (10pt) Find the equation of the line with slope $-\frac{15}{4}$ and y-intercept $(0, -8)$.

Solution. Because this line is not vertical, we know that the line has the form $y = mx + b$. Because the slope is $-\frac{15}{4}$, we know that $m = -\frac{15}{4}$. Therefore, we know that $y = -\frac{15}{4}x + b$. We know that the line contains the point $(0, -8)$. But then $(0, -8)$ satisfies the equation of the line. Then...

$$y = -\frac{15}{4}x + b$$

$$-8 = -\frac{15}{4} \cdot 0 + b$$

$$-8 = 0 + b$$

$$b = -8$$

Therefore, $y = -\frac{15}{4}x - 8$. Alternatively, once we know that $y = -\frac{15}{4}x + b$, because the y-intercept is $(0, -8)$, we know that $b = -8$. Therefore, $y = -\frac{15}{4}x - 8$. In either case, the equation of the line is...

$$\boxed{y = -\frac{15}{4}x - 8}$$

Problem 10. (10pt) Find the equation of the line with slope 5 passing through the point $(-3, 10)$.

Solution. Because the line is not vertical, we know that $y = mx + b$. We also know that the line has slope 5, i.e. $m = 5$. Therefore, we know $y = 5x + b$. Using the fact that the line contains $(-3, 10)$, we know...

$$y = 5x + b$$

$$10 = 5(-3) + b$$

$$10 = -15 + b$$

$$25 = b$$

Therefore, the equation of the line is...

$$\boxed{y = 5x + 25}$$