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MATH 101
Fall 2022
HW 23: Due 12/12

"A man only learns in two ways: one by reading, and the other by association with smarter people."
–Will Rogers

Problem 1. (10pt) Solve the following equations:

(a) $(x+6)(x-12)=0$

(b) $x^2+15x+56=0$

(c) $81-x^2=0$

(d) $9x=x^2-36$

(e) $6x^2=x+2$

Solution.

(a) We know $(x+6)(x-12)=0$ implies that either $x+6=0$, which means $x=-6$, or $x-12=0$, which implies that $x=12$. Therefore, the solutions are $x=-6,12$.

(b) We have...

$$\begin{aligned} x^2+15x+56&=0 \\ (x+8)(x+7)&=0 \end{aligned}$$

But this implies that either $x+8=0$, which implies $x=-8$, or $x+7=0$, which implies $x=-7$. Therefore, the solutions are $x=-8,-7$.

(c) We have...

$$\begin{aligned} 81-x^2&=0 \\ (8-x)(8+x)&=0 \end{aligned}$$

But this implies that either $8-x=0$, which implies $x=8$, or $8+x=0$, which implies $x=-8$. Therefore, the solutions are $x=-8,8$.

(d) We have...

$$\begin{aligned} 9x&=x^2-36 \\ x^2-9x+36&=0 \\ (x-12)(x+3)&=0 \end{aligned}$$

But this implies that either $x-12=0$, which implies $x=12$, or $x+3=0$, which implies $x=-3$. Therefore, the solutions are $x=-3,12$.

(e) We have...

$$\begin{aligned} 6x^2 &= x + 2 \\ 6x^2 - x - 2 &= 0 \\ (2x+1)(3x-2) &= 0 \end{aligned}$$

But this implies that either $2x + 1 = 0$, which implies $2x = -1$ so that $x = -\frac{1}{2}$, or $3x - 2 = 0$, which implies $3x = 2$ so that $x = \frac{2}{3}$. Therefore, the solutions are $x = -\frac{1}{2}, \frac{2}{3}$.

Problem 2. (10pt) Showing all your work, factor the following polynomial: $8x^2 + 34x - 30$

Solution. We know $f(x) = 8x^2 + 34x - 30$ is a quadratic function of the form $ax^2 + bx + c$ with $a = 8$, $b = 34$, and $c = -30$. If $f(x)$ has roots r_1, r_2, then $f(x)$ factors as $a(x - r_1)(x - r_2)$. We find the roots of $f(x) = 8x^2 + 34x - 30$, i.e. the solutions to $8x^2 + 34x - 30 = 0$, using the quadratic formula:

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-34 \pm \sqrt{34^2 - 4(8)(-30)}}{2(8)} \\ &= \frac{-34 \pm \sqrt{1156 + 960}}{16} \\ &= \frac{-34 \pm \sqrt{2116}}{16} \\ &= \frac{-34 \pm 46}{16} \end{aligned}$$

Therefore, the roots are $x = \frac{-34-46}{16} = \frac{-80}{16} = -5$ and $x = \frac{-34+46}{16} = \frac{12}{16} = \frac{3}{4}$. But then the polynomial $8x^2 + 34x - 30$ factors as...

$$\begin{aligned} 8x^2 + 34x - 30 &= a(x - r_1)(x - r_2) \\ &= 8\big(x - (-5)\big)\left(x - \frac{3}{4}\right) \\ &= 8(x + 5)\left(x - \frac{3}{4}\right) \\ &= 2(x + 5) \cdot 4\left(x - \frac{3}{4}\right) \\ &= 2(x + 5)(4x - 3) \end{aligned}$$

Problem 3. (10pt) Use the quadratic equation to factor the following polynomial: $120x^2 + 234x - 165$

Solution. We know $f(x) = 120x^2 + 234x - 165$ is a quadratic function of the form $ax^2 + bx + c$ with $a = 120$, $b = 234$, and $c = -165$. If $f(x)$ has roots r_1, r_2, then $f(x)$ factors as $a(x - r_1)(x - r_2)$. We find the roots of $f(x) = 120x^2 + 234x - 165$, i.e. the solutions to $120x^2 + 234x - 165 = 0$, using the quadratic formula:

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-234 \pm \sqrt{234^2 - 4(120)(-165)}}{2(120)} \\ &= \frac{-234 \pm \sqrt{54756 + 79200}}{240} \\ &= \frac{-234 \pm \sqrt{133956}}{240} \\ &= \frac{-234 \pm 366}{240} \end{aligned}$$

Therefore, the roots are $x = \frac{-234-366}{240} = \frac{-600}{240} = -\frac{5}{2}$ and $x = \frac{-234+366}{240} = \frac{132}{240} = \frac{11}{20}$. But then the polynomial $120x^2 + 234x - 165$ factors as...

$$\begin{aligned} 120x^2 + 234x - 165 &= a(x - r_1)(x - r_2) \\ &= 120\left(x - \frac{-5}{2}\right)\left(x - \frac{11}{20}\right) \\ &= 120\left(x + \frac{5}{2}\right)\left(x - \frac{11}{20}\right) \\ &= 3 \cdot 2\left(x + \frac{5}{2}\right) \cdot 20\left(x - \frac{11}{20}\right) \\ &= 3(2x + 5)(20x - 11) \end{aligned}$$