**MATH 108**
**Fall 2021**
**HW 4: Due 10/05**

*"There are three choices in this life: be good, get good, or give up."*
*−Dr. Gregory House, House*

**Problem 1.** (10pt) Find the matrix associated with the system of equations below.

$$2x_1 - x_2 + 5x_3 = 5$$
$$x_1 \qquad + x_3 = -1$$
$$3x_2 - 6x_3 = 4$$

**Solution.**

$$\begin{pmatrix} 2 & -1 & 5 & 5 \\ 1 & 0 & 1 & -1 \\ 0 & 3 & -6 & 4 \end{pmatrix}$$

**<u>OR</u>**

$$\left(\begin{array}{ccc|c} 2 & -1 & 5 & 5 \\ 1 & 0 & 1 & -1 \\ 0 & 3 & -6 & 4 \end{array}\right)$$

**Problem 2.** (10pt) Write the system of equations associated to the matrix below.

$$\begin{pmatrix} 1 & -1 & 3 & 5 & 6 \\ 0 & 1 & 4 & 9 & -2 \\ 1 & 2 & 0 & -6 & 3 \\ 2 & -1 & 4 & 1 & 7 \end{pmatrix}$$

**Solution.**

$$
\begin{aligned}
x_1 &- x_2 + 3x_3 + 5x_4 &= 6 \\
&\phantom{-} x_2 + 4x_3 + 9x_4 &= -2 \\
x_1 &+ 2x_2 \phantom{{}+ 4x_3} - 6x_4 &= 3 \\
2x_1 &- x_2 + 4x_3 + x_4 &= 7
\end{aligned}
$$

**Problem 3.** (10pt) Use matrix methods to solve the system of equations below. Show all your work.

$$6x_1 - x_2 = 13$$
$$2x_1 + 3x_2 = 1$$

**Solution.**

$$\left( \begin{array}{cc|c} 6 & -1 & 13 \\ 2 & 3 & 1 \end{array} \right) \qquad\qquad R_1 + -3R_2 \to R_2$$

$$\left( \begin{array}{cc|c} 6 & -1 & 13 \\ 0 & -10 & 10 \end{array} \right) \qquad\qquad -\frac{1}{10}R_2 \to R_2$$

$$\left( \begin{array}{cc|c} 6 & -1 & 13 \\ 0 & 1 & -1 \end{array} \right) \qquad\qquad R_2 + R_1 \to R_1$$

$$\left( \begin{array}{cc|c} 6 & 0 & 12 \\ 0 & 1 & -1 \end{array} \right) \qquad\qquad \frac{1}{6}R_1 \to R_1$$

$$\left( \begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \end{array} \right)$$

Therefore, the solution is $(x_1, x_2) = (2, -1)$, i.e. $x_1 = 2$ and $x_2 = -1$:

$$\begin{cases} x_1 = 2 \\ x_2 = -1 \end{cases}$$

**Problem 4.** (10pt) Use matrix methods to solve the system of equations below. Show all your work.

$$x_1 - x_2 + x_3 = 2$$
$$2x_1 + 2x_2 - x_3 = 9$$
$$x_2 - 3x_3 = 8$$

**Solution.**

$$\left( \begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 2 & 2 & -1 & 9 \\ 0 & 1 & -3 & 8 \end{array} \right) \qquad -2R_1 + R_2 \to R_2$$

$$\left( \begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 4 & -3 & 5 \\ 0 & 1 & -3 & 8 \end{array} \right) \qquad R_2 - 4R_3 \to R_3$$

$$\left( \begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 4 & -3 & 5 \\ 0 & 0 & 9 & -27 \end{array} \right) \qquad \frac{1}{9}R_3 \to R_3$$

$$\left( \begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 4 & -3 & 5 \\ 0 & 0 & 1 & -3 \end{array} \right) \qquad 3R_3 + R_2 \to R_2$$

$$\left( \begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 4 & 0 & -4 \\ 0 & 0 & 1 & -3 \end{array} \right) \qquad -R_3 + R_1 \to R_1$$

$$\left( \begin{array}{ccc|c} 1 & -1 & 0 & 5 \\ 0 & 4 & 0 & -4 \\ 0 & 0 & 1 & -3 \end{array} \right) \qquad \frac{1}{4}R_2 \to R_2$$

$$\left( \begin{array}{ccc|c} 1 & -1 & 0 & 5 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -3 \end{array} \right) \qquad R_2 + R_1 \to R_1$$

$$\left( \begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -3 \end{array} \right)$$

Therefore, the solution is $(x_1, x_2, x_3) = (4, -1, -3)$, i.e. $x_1 = 4$, $x_2 = -1$, and $x_3 = -3$,

$$\begin{cases} x_1 = 4 \\ x_2 = -1 \\ x_3 = -3 \end{cases}$$

**Problem 5.** (10pt) Use WolframAlpha's `RowReduce` to find the solution to the following system of equations:

$$-6x_1 - x_2 + 7x_3 - 4x_4 + 3x_5 = 83$$

$$2x_1 + 5x_2 + 2x_3 + 5x_4 + 7x_5 = \frac{67}{3}$$

$$-8x_1 - x_2 - 9x_3 - 10x_4 = -111$$

$$7x_1 - 6x_2 + 3x_3 - 5x_4 + 9x_5 = \frac{97}{2}$$

$$12x_1 - 4x_2 - x_3 + 5x_4 + 6x_5 = 0$$

**Solution.** The associated matrix is...

$$\begin{pmatrix} -6 & -1 & 7 & -4 & 3 & | & 83 \\ 2 & 5 & 2 & 5 & 7 & | & 67/3 \\ -8 & -1 & -9 & -10 & 0 & | & -111 \\ 7 & -6 & 3 & -5 & 9 & | & 97/2 \\ 12 & -4 & -1 & 5 & 6 & | & 0 \end{pmatrix}$$

Using WolframAlpha's `RowReduce` function, we obtain...

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & | & 1/2 \\ 0 & 1 & 0 & 0 & 0 & | & -1 \\ 0 & 0 & 1 & 0 & 0 & | & 12 \\ 0 & 0 & 0 & 1 & 0 & | & 0 \\ 0 & 0 & 0 & 0 & 1 & | & 1/3 \end{pmatrix}$$

Therefore, the solution is $(x_1, x_2, x_3, x_4, x_5) = (1/2, -1, 12, 0, 1/3)$, i.e. $x_1 = \frac{1}{2}$, $x_2 = -1$, $x_3 = 12$, $x_4 = 0$, and $x_5 = \frac{1}{3}$:

$$\begin{cases} x_1 = \frac{1}{2} \\ x_2 = -1 \\ x_3 = 12 \\ x_4 = 0 \\ x_5 = \frac{1}{3} \end{cases}$$