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MATH 101

Fall 2022

HW 13: Due 11/07

*"Above all, don't fear difficult moments.
The best comes from them."*

—Rita Levi-Montalcini

Problem 1. (10pt) Consider the linear function $f(x) = 7 - \frac{6}{7}x$.

- (a) Find the rate of change of $f(x)$.
- (b) Is $f(x)$ increasing or decreasing? Explain.
- (c) Find the y -intercept of $f(x)$.
- (d) Find $f(-3)$.

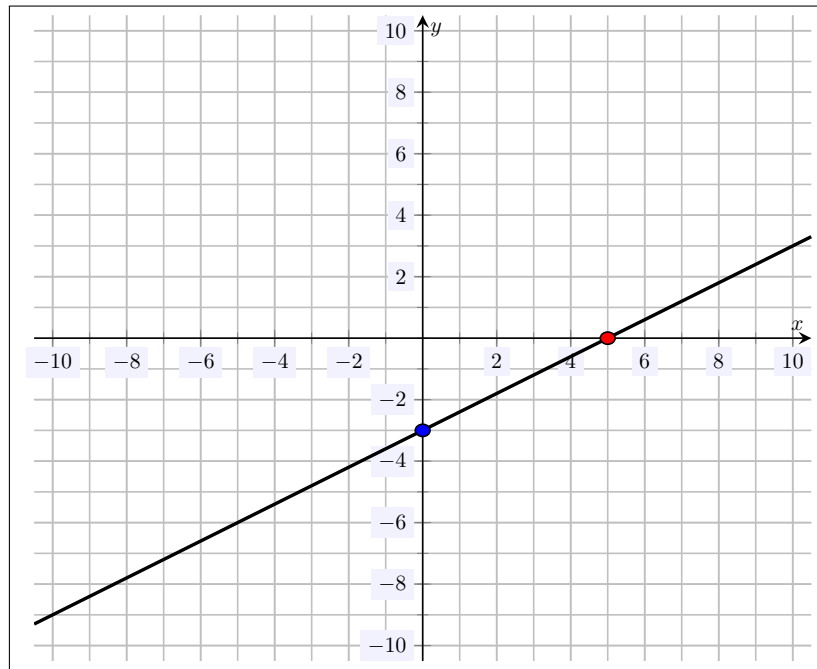
Solution.

- (a) The rate of change of a linear function is its slope. Because $f(x) = 7 - \frac{6}{7}x$ has the form $y = mx + b$ with $m = -\frac{6}{7}$ and $b = 7$, we know that the slope of $f(x)$ is $-\frac{6}{7}$. Therefore, the rate of change of $f(x)$ is $-\frac{6}{7}$.
- (b) Because the rate of change, i.e. slope, of $f(x)$ is negative, we know that $f(x)$ is a decreasing function.
- (c) Because $f(x) = 7 - \frac{6}{7}x$ has the form $y = mx + b$ with $m = -\frac{6}{7}$ and $b = 7$, we know that the y -intercept of $f(x)$ is 7, i.e. $(0, 7)$.
- (d) We have...

$$f(3) = 7 - \frac{6}{7} \cdot -3 = 7 + \frac{18}{7} = \frac{49}{7} + \frac{18}{7} = \frac{67}{7}$$

Problem 2. (10pt) Using the plot of the linear function $f(x)$ below, answer the following:

- (a) Find the slope of the given line.
- (b) Find the y -intercept of the given line.
- (c) Find $f(x)$.
- (d) Find the x -intercept of $f(x)$.



Solution. We know that the line has the form $y = mx + b$. Observe that we have the point $(0, -3)$ on the line. For each increase of 5 in x , there is a corresponding 3 increase in y . Therefore, we have $m = \frac{\Delta y}{\Delta x} = \frac{3}{5}$. But then we know that $y = \frac{3}{5}x + b$. Using the fact that $(0, -3)$ is on the line, we have $-3 = \frac{3}{5} \cdot 0 + b$ so that $b = -3$. Therefore, the line is $y = \frac{3}{5}x - 3$. But then $f(x) = \frac{3}{5}x - 3$.

- (a) Because $f(x) = \frac{3}{5}x - 3$ has the form $y = mx + b$ with $m = \frac{3}{5}$ and $b = -3$, we know that the slope of $f(x)$ is $\frac{3}{5}$.
- (b) Because $f(x) = \frac{3}{5}x - 3$ has the form $y = mx + b$ with $m = \frac{3}{5}$ and $b = -3$, we know that the y -intercept of $f(x)$ is -3 , i.e. $(0, -3)$. Note we can also see the y -intercept as the marked blue point, $(0, -3)$, on the graph.
- (c) From the work above, we know that $f(x) = \frac{3}{5}x - 3$.

(d) The x -intercept occurs when $y = 0$. But then we have...

$$f(x) = \frac{3}{5}x - 3$$

$$0 = \frac{3}{5}x - 3$$

$$\frac{3}{5}x = 3$$

$$\frac{5}{3} \cdot \frac{3}{5}x = 3 \cdot \frac{5}{3}$$

$$x = 5$$

Therefore, the x -intercept is $x = 5$, i.e. $(5, 0)$. We can also see the x -intercept as the red point, $(5, 0)$, marked on the graph.