Name: *Caleb McWhorter — Solutions*
MATH 101
Fall 2022
HW 5: Due 10/03

"Between two evils, I always pick the one I never tried before."
–Mae West

Problem 1. (10pt) Showing all your work, convert the following indicated units to the units indicated in brackets using the given information:

(a) 518.3 m to cm

(b) 5,100 ft to km [1 ft = 0.3048 m]

(c) 60 mph to km per hour [1 mi = 5280 ft, 1 m = 3.28084 ft]

(d) 45 mph to ft per second [5280 ft = 1 mi]

(e) 2000 ft^3 to cm^3 [5 m = 16.4042 ft]

Solution.

(a)

$$\begin{array}{r|r} 518.3\text{ m} & 100\text{ cm} \\ \hline & 1\text{ m} \end{array} = 51830\text{ cm}$$

(b)

$$\begin{array}{r|r|r} 5100\text{ ft} & 0.3048\text{ m} & 1\text{ km} \\ \hline & 1\text{ ft} & 1000\text{ m} \end{array} = 1.55448\text{ km}$$

(c)

$$\begin{array}{r|r|r|r} 60\text{ mi} & 5280\text{ ft} & 1\text{ m} & 1\text{ km} \\ \hline 1\text{ hr} & 1\text{ mi} & 3.28084\text{ ft} & 1000\text{ m} \end{array} = 96.56063\text{ km per hour}$$

(d)

$$\begin{array}{r|r|r|r} 45\text{ mi} & 5280\text{ ft} & 1\text{ hr} & 1\text{ min} \\ \hline 1\text{ hr} & 1\text{ mi} & 60\text{ min} & 60\text{ s} \end{array} = 66\text{ ft per second}$$

(e)

$$\begin{array}{r|r|r|r|r|r|r} 2000\text{ ft}^3 & 5\text{ m} & 5\text{ m} & 5\text{ m} & 100\text{ cm} & 100\text{ cm} & 100\text{ cm} \\ \hline & 16.4042\text{ ft} & 16.4042\text{ ft} & 16.4042\text{ ft.} & 1\text{ m} & 1\text{ m} & 1\text{ m} \end{array}$$

$$= 56633687.7472\text{ cm}^3$$

Problem 2. (10pt) Avery's husband has to order a special paint they want for their patio. The paint has to be shipped from Europe. He wants to be sure that he orders enough paint to paint the patio, which is 240 ft^2. Each can of paint costs €29 and claims it can cover 2.6 m^2.

(a) How many square feet can the paint cover? [3.28084 ft = 1 m]

(b) How many cans should he order?

(c) What is the cost of the paint in Euros per meter squared?

(d) What is the cost of the paint in USD per square foot? [\$1 = € 0.98]

Solution.

(a) We convert the 2.6 m^2 that each can of paint can cover to square feet:

$$\begin{array}{r|r|r} 2.6\text{ m}^2 & 3.28084\text{ ft} & 3.28084\text{ ft} \\ \hline & 1\text{ m} & 1\text{ m} \end{array} = 27.9861\text{ ft}^2$$

(b) If each can of paint can cover 27.9861 ft^2 of area and there is 240 ft^2 to cover, then the minimum number of cans of paint required is 240 ft$^2/(27.9861$ ft$^2/$can$) \approx 8.57569$ cans. But then a minimum of 9 cans of paint is required to cover the entire wall. [Notice ≈ 8.6 is the minimum required, but you can only bring whole cans of paint and 8 cans is too few.]

(c) The cost of the pain in Euros per meter squared is $\text{€}29/2.6$ m^2 $\approx \text{€}11.15385/$m^2.

(d) We can either convert the cost in Euro per meter squared to USD per square foot:

$$\begin{array}{r|r|r|r} \text{€}29 & \$1 & 1\text{ m} & 1\text{ m} \\ \hline 2.6\text{ m}^2 & \text{€}0.98 & 3.28084\text{ ft} & 3.28084\text{ ft} \end{array} = \$1.0574/\text{ft}^2$$

which can also be computed...

$$\begin{array}{r|r|r|r} \text{€}11.15385 & \$1 & 1\text{ m} & 1\text{ m} \\ \hline 1\text{ m}^2 & \text{€}0.98 & 3.28084\text{ ft} & 3.28084\text{ ft} \end{array} = \$1.0574/\text{ft}^2$$

We can also convert the cost from Euro to USD, $\text{€}29 \cdot \$1/\text{€}0.98 = \$29.5918$, and we know from (a) the square foot that the paint covers, namely 27.9861 ft^2. But then the cost in USD per square foot is $\$29.5918/27.9861$ ft^2 $\approx \$1.0574/$ft^2.

Problem 3. (10pt) Aliens arrive on Earth and try to communicate with humans. Being intelligent beings, they first try to understand our mathematical systems. Aliens measure speed in blips per flarg. They claim to have traveled to Earth at 587 blips per flarg. We discover that in their units, 1 blips is 800 bloop and 465 bloop is 1,000 miles. We discover also 1 flarg is 8.2 s. What speed (in miles per second) did they travel to Earth? If the speed of light is 186,282 miles per second, what percent of the speed of light did they travel?

Solution. We merely need convert from blips per flarg to miles per second:

$$\begin{array}{r|r|r|r} 587 \text{ blip} & 800 \text{ bloop} & 1000 \text{ mi} & 1 \text{ flarg} \\ \hline 1 \text{ flarg} & 1 \text{ blips} & 465 \text{ bloop} & 8.2 \text{ s} \end{array} = 123{,}157.6187 \text{ mi per second (mips)}$$

As a percentage of the speed of light, this is 123157.6187 mips/186282 mips ≈ 0.6611, i.e. 66.11% of the speed of light.

Problem 4. (10pt) Alden drives to visit his family. On the outgoing trip, he runs into no traffic and is able to drive at 55 mph the entire way, completing the trip in only a few hours. However, on the return trip he runs into construction on the highway and is only able to drive 35 mph. It takes him 2 hours longer on the return trip than on the original trip. How many miles is his home from his family's home?

Solution. We do not know the distance from Alden to his family's home. Let d be this distance. We know that he makes the trip in some amount of hours; let's call this amount of hours t_1. Because $d = vt$, we know that $d = 55t_1$. On the return trip, he takes 2 hours longer. Therefore, he took $t_2 := t_1 + 2$ hours to drive back. Again, because $d = vt$ and the fact that he drives 35 mph on the return trip, he traveled a total distance of $35t_2 = 35(t_1 + 2)$ miles on the return trip. But because this was the same distance d, we know that $d = 35(t_1 + 2)$. But then we have. . .

$$
\begin{aligned}
d &= d \\
55t_1 &= 35t_2 \\
55t_1 &= 35(t_1 + 2) \\
55t_1 &= 35t_1 + 70 \\
20t_1 &= 70 \\
t_1 &= 3.5 \text{ hrs}
\end{aligned}
$$

Therefore, he originally drove for 3.5 hours. But then he traveled a total distance of $d = 55$ mph $\cdot$ 3.5 hrs $= 192.5$ mi. Therefore, his family's home is 192.5 miles from his home.

Problem 5. (10pt) Water is flowing into a large vat that can contain 587 ft^3 of water. Suppose that water is flowing it at a rate of 43.7 gallons per minute.

(a) Find the rate at which the water is flowing in ft^3 per minute. [1 gallon = 0.134 ft^3]

(b) How long does it take to fill the whole tank?

(c) Assuming the tank begins empty, how much of the tank is unfilled after 10 minutes?

(d) If the tank started with 250 ft^3 of water, what volume remains unfilled in the tank one hour after the water begins filling the tank?

Solution.

(a) We simply convert 43.7 gallons per minute to ft^3 per minute:

$$\begin{array}{r|l} 43.7 \text{ gallons} & 0.135 \text{ ft}^3 \\ \hline 1 \text{ min} & 1 \text{ gallon} \end{array} = 5.8995 \text{ ft}^3/\text{min}$$

(b) We know that $C = rt$, where C is the change, r is the rate, and t is time. Because the water is flowing in at a rate of 5.8995 ft^3/min and the change required to fill the tank is 587 ft^3, i.e. the tank holds 587 ft^3 of liquid, we know that 587 ft^3 $= 5.8995$ $\text{ft}^3/\text{min} \cdot t$. But then $t = 587$ $\text{ft}^3/(5.8995$ $\text{ft}^3/\text{min}) = 99.5$ min, i.e. 1 hour, 39 minutes, and 30 seconds.

(c) We know that $C = rt$, where C is the change, r is the rate, and t is time. Because the water is flowing in at a rate of 5.8995 ft^3/min and the change required to fill the tank is 587 ft^3, i.e. the tank holds 587 ft^3 of liquid, we know that $C = 5.8995$ $\text{ft}^3/\text{min} \cdot 10$ min $= 58.995$ ft^3. But this is the amount of water in the tank. The amount of unfilled space is then 587 $\text{ft}^3 - 58.995$ ft^3 $=$ 528.005 ft^3.

(d) We know that 1 hour is 60 minutes and that $C = rt$. The amount of water added is then $C = 5.8995 \cdot 60 = 353.97$ ft^3. The tank began with 250 ft^3 of water. But then the total amount of water in the tank is 250 $\text{ft}^3 + 353.96$ $\text{ft}^3 = 603.96$ ft^3. Because this is greater than the total amount the tank can hold, there is no volume that is unfilled, i.e. the tank is overflowing. [In fact, the tank has been overflowing for the past 2 minutes and 52 seconds.]

Problem 6. (10pt) Ann Velope is stuffing envelopes for an upcoming charity event. Counting, she has been able to stuff 116 envelopes in the last 20 minutes.

(a) What is her rate in envelopes per hour?

(b) How long will it take for her to fill 1,200 envelopes?

(c) If her coworker helps her and he can stuff 250 envelopes per hour, how long would it take both of them to stuff 2,000 envelopes?

(d) Suppose instead that Ann can stuff some large number of envelopes in 4 hours, while her coworker can do the same task in 6 hours. Suppose that the coworker starts stuffing envelopes, then an hour later Ann joins them to help speed things up. Assuming they work at their usual speeds, how long will it take them to stuff all the envelopes?

Solution.

(a) We simply convert her rate of 116 envelopes per 20 minutes to envelopes per hour:

$$\begin{array}{r|l} \text{116 envelopes} & \text{60 min} \\ \hline \text{20 minutes} & \text{1 hour} \end{array} = \text{348 envelopes/hour}$$

(b) Because we know that $C = rt$, where C is the change, r is the rate, and t is the time, and she needs a change of 1,2000 envelopes—stuffing at a rate of 348 envelopes/hour, it takes her...

$$C = rt$$

$$1200 \text{ envelopes} = 348 \text{ envelopes/hour} \cdot t$$

$$t = 3.44828 \text{ hours}$$

That is, it takes her 3 hours, 26 minutes, and 53.8 seconds.

(c) Combined, they can stuff 348 envelopes $+ 250$ envelopes $= 598$ envelopes each hour. But then using the method from (b), we have...

$$C = rt$$

$$2000 \text{ envelopes} = 598 \text{ envelopes/hour} \cdot t$$

$$t = 3.34448 \text{ hours}$$

That is, it will take them 3 hours, 20 minutes, and 40.1 seconds.

(d) We know that stuffing envelopes at a rate of r envelopes per hour for t hours, a total of $C = rt$ envelopes have been stuffed. Let A be the rate at which Ann stuffs envelopes. Then we know that $C = rt = 4A$. Let W be the rate at which her coworker stuffs envelopes. Because he does this same task in 6 hours, we have $C = rt = 6W$. But then we know that $4A = C = 6W$

so that $A = \frac{4}{6}W = \frac{2}{3}W$; equivalently, $W = \frac{3}{2}A$. Now if they combine their efforts, they stuff envelopes at a rate of $A + W$ envelopes per hour. But then we have...

$$\begin{aligned}
C &= rt \\
C &= (A+W)t \\
C &= \left(A + \frac{2}{3}A\right)t \\
C &= \frac{5}{3}A \cdot t \\
4A &= \frac{5}{3}A \cdot t \\
t &= 4A \cdot \frac{3}{5} \cdot \frac{1}{A} \\
t &= \frac{12}{5} \\
t &= 2.4 \text{ hours}
\end{aligned}$$

Therefore, they take a combined 2.4 hours, i.e. 2 hours and 24 minutes, to stuff the envelopes.