

Name: \_\_\_\_\_

MATH 308

Fall 2022

HW 5: Due 09/22

*“To choose one sock from each of infinitely many pairs of socks requires the Axiom of Choice, but for shoes the Axiom is not needed.”*

*– Bertrand Russell*

**Problem 1.** (10pt) For each of the sets described below, either give the set by enumerating all its elements (if possible) or give the set using set-builder notation. Also for each set, give an element and non-element of the set.

- (a) The set of integer multiples of 8.
- (b) The set of negative solutions to  $(x - 4)(x + 1)(x + 6) = 0$ .
- (c) The set of nonnegative rational numbers less than 1.
- (d) The set of real numbers with a real-valued square root.
- (e) The set of integer cubes with absolute value less than 100.

**Problem 2.** (10pt) For each of the sets given below, describe the sets in words. Also for each set, give an example of an element and non-element of the set.

(a)  $\{2, 3, 5, 7, 11, 13, \dots\}$

(b)  $\{\dots, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, 16, \dots\}$

(c)  $\{n \in \mathbb{N} : n^2 = 30 - n\}$

(d)  $\{k \in \mathbb{Z} : (3k + 1)/5 \in \mathbb{Z}\}$

(e)  $\{n \in \mathbb{N} : (\exists k \in \mathbb{N})(n = 3k + 1)\}$

**Problem 3.** (10pt) Define the following sets:

$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$B = \{2, 4, 6, 8, 10\}$$

$$C = \{1, 3, 5, 7, 9\}$$

$$D = \{2, 3, 5, 7\}$$

$$E = \{1, 2, 4, 8, 10\}$$

$$F = \{3, 5, 8, 9, 10\}$$

Consider each of the sets above as coming from the universal set  $\mathcal{U} := A$ . Compute the following:

(a)  $D^c$

(d)  $E \setminus F$

(b)  $B \cup C$

(e)  $E \Delta F$

(c)  $C \cup (B \cap D)$

(f)  $(B \cup C)^c$

**Problem 4.** (10pt) Let the universal set of discourse be the set of integers. Define the following sets:

$A$  = set of even integers

$B$  = set of odd integers

$C$  = set of prime integers

$D$  = set of square integers

$E$  = set of nonnegative integers

$F$  = set of positive integers

$G$  = set of integers strictly between 0 and 20

$H$  = set of integers that are a multiple of 5

Compute the sets below. When giving your solution, either enumerate all the elements of the resulting set (if possible), give the set using set-builder notation, or give the set using some 'standard' notation.

(a)  $B^c$

(f)  $E \Delta F$

(b)  $A \cup B$

(g)  $C \cap H$

(c)  $A \cap C$

(h)  $D \cap E^c$

(d)  $B \cap C$

(i)  $D^c$

(e)  $G - D$