Name: *Caleb McWhorter — Solutions*
MATH 101
Fall 2022
HW 1: Due 09/014

"Blackmail is such an ugly word. I prefer 'extortion.' The 'x' makes it sound cool."
–Bender Rodriguez, Futurama

Problem 1. (10pt) Showing all your work, find the prime factorizations of the following:

(a) 45

(b) 30

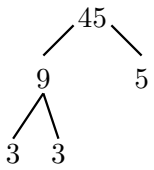
(c) 17

(d) 44

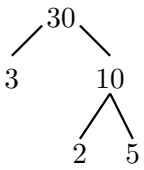
(e) 220

Solution.

(a) $45 = 3^2 \cdot 5$

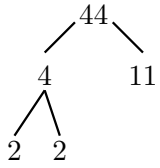


(b) $30 = 2 \cdot 3 \cdot 5$

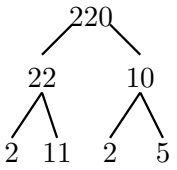


(c) $17 = 17^1$

(d) $44 = 2^2 \cdot 11$



(e) $220 = 2^2 \cdot 5 \cdot 11$



Problem 2. (10pt) Without using a calculator, answer the following:

(a) Does 2 divide 6701? Explain.

(b) Does 3 divide 3801437? Explain.

(c) Does 4 divide 19154300? Explain.

(d) Does 5 divide 648520? Explain.

(e) Does 9 divide 94321836? Explain.

Solution.

(a) 2 divides an integer if and only if the integer is even. Because 6701 is not even, 6701 is not divisible by 2.

(b) 3 divides an integer if and only if the sum of its digits is divisible by 3. Because $3+8+0+1+4+3+7=26$ is not divisible by 3, 3801437 is not divisible by 3.

(c) 4 divides an integer if and only if the last two digits form an integer divisible by 4. Because $00=0$ is divisible by 4, 19154300 is divisible by 4.

(d) 5 divides an integer if and only if it ends in 0 or 5. Because 648520 ends in 0, 648520 is divisible by 5.

(e) 9 divides an integer if and only if the sum of its digits is divisible by 9. Because $9+4+3+2+1+8+3+6=36$ is divisible by 9, 94321836 is divisible by 9.

Problem 3. (10pt) Showing all your work, compute the following:

(a) $\gcd(10, 14)$

(b) $\operatorname{lcm}(10, 14)$

(c) $\gcd(2^{100} \cdot 3^{40} \cdot 7^{11} \cdot 11^{30},\ 2^{20} \cdot 5^{80} \cdot 7^{60} \cdot 11^{10})$

(d) $\operatorname{lcm}(2^{100} \cdot 3^{40} \cdot 7^{11} \cdot 11^{30},\ 2^{20} \cdot 5^{80} \cdot 7^{60} \cdot 11^{10})$

Solution.

(a)

$$\gcd(10, 14) = \gcd(2 \cdot 5, 2 \cdot 7) = 2$$

(b)

$$\operatorname{lcm}(10, 14) = \operatorname{lcm}(2 \cdot 5, 2 \cdot 7) = 2 \cdot 5 \cdot 7 = 70$$

(c)

$$\gcd(2^{100} \cdot 3^{40} \cdot 7^{11} \cdot 11^{30},\ 2^{20} \cdot 5^{80} \cdot 7^{60} \cdot 11^{10}) = 2^{20} \cdot 7^{11} \cdot 11^{10} = 53\,778\,069\,122\,557\,835\,522\,080\,768$$

(d)

$$\operatorname{lcm}(2^{100} \cdot 3^{40} \cdot 7^{11} \cdot 11^{30},\ 2^{20} \cdot 5^{80} \cdot 7^{60} \cdot 11^{10}) = 2^{100} \cdot 3^{40} \cdot 5^{80} \cdot 7^{60} \cdot 11^{30}$$

Problem 4. (10pt) Showing all your work and being sure to simplify as much as possible, compute the following:

(a) $\dfrac{5}{12} - \dfrac{7}{22}$

(b) $\dfrac{3}{4} + \frac{7}{8} - \frac{1}{6}$

(c) $\dfrac{3}{4} \cdot \dfrac{16}{27} \cdot \dfrac{9}{4}$

(d) $\dfrac{\frac{15}{21}}{\frac{5}{2}}$

(e) $\dfrac{\frac{13}{15}}{\frac{39}{40}}$

Solution.

(a)
$$\frac{5}{12} - \frac{7}{22} = \frac{55}{132} - \frac{42}{132} = \frac{13}{132}$$

(b)
$$\frac{3}{4} + \frac{7}{8} - \frac{1}{6} = \frac{18}{24} + \frac{21}{24} - \frac{4}{24} = \frac{35}{24}$$

(c)
$$\frac{3}{4} \cdot \frac{16}{27} \cdot \frac{9}{4} = \frac{\cancel{3}^{1}}{\cancel{4}^{1}} \cdot \frac{\cancel{16}^{\cancel{4}^{1}}}{\cancel{27}^{\cancel{9}^{1}}} \cdot \frac{\cancel{9}^{1}}{\cancel{4}^{1}} = 1$$

(d)
$$\frac{\frac{15}{21}}{\frac{5}{2}} = \frac{15}{21} \cdot \frac{2}{5} = \frac{\cancel{15}^{\cancel{3}^{1}}}{\cancel{21}^{7}} \cdot \frac{2}{\cancel{5}^{1}} = \frac{2}{7}$$

(e)
$$\frac{\frac{13}{15}}{\frac{39}{40}} = \frac{13}{15} \cdot \frac{40}{39} = \frac{\cancel{13}^{1}}{\cancel{15}^{3}} \cdot \frac{\cancel{40}^{8}}{\cancel{39}^{3}} = \frac{8}{9}$$

Problem 5. (10pt) For each of the following, either convert the given improper fraction to a mixed number or the given mixed number to an improper fraction:

(a) $5\frac{7}{8}$

(b) $\frac{90}{7}$

(c) $\frac{27}{5}$

(d) $-6\frac{3}{4}$

Solution.

(a)

$$5\frac{7}{8} = \frac{5(8)+7}{8} = \frac{40+7}{8} = \frac{47}{8}$$

(b)

$$\frac{90}{7} = \frac{84+6}{7} = \frac{12(7)+6}{7} = 12\tfrac{6}{7}$$

(c)

$$\frac{27}{5} = \frac{25+2}{5} = \frac{5(5)+2}{5} = 5\tfrac{2}{5}$$

(d)

$$-6\frac{3}{4} = -\left(6\frac{3}{4}\right) = -\left(\frac{6(4)+3}{4}\right) = -\left(\frac{24+3}{4}\right) = -\frac{27}{4}$$

Problem 6. (10pt) For each of the following, either convert the given fraction to a decimal or the given decimal to a fraction:

(a) $\frac{3}{8}$

(b) 0.35

(c) $\frac{30}{7}$

(d) $0.\overline{75}$

Solution.

(a)

$$\begin{array}{r} 0.375 \\ 8\overline{)\,3.000} \\ \underline{2.4} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

$$\frac{3}{8} = 0.375$$

(b)

$$0.35 = \frac{35}{100} = \frac{\cancel{35}^{7}}{\cancel{100}^{20}} = \frac{7}{20}$$

(c)

$$\begin{array}{r} 4.\overline{285714} \\ 7\overline{)\,30.000000} \\ \underline{28} \\ 2.0 \\ \underline{1.4} \\ 60 \\ \underline{56} \\ 40 \\ \underline{35} \\ 50 \\ \underline{49} \\ 10 \\ \underline{7} \\ 30 \\ \underline{28} \\ 2 \end{array}$$

$$\frac{30}{7} = 4.\overline{285714}$$

(d)

$$\begin{array}{rrcc} & 100N & = & 75.757575757\overline{75} \\ - & N & = & 0.757575757\overline{75} \\ \hline & 99N & = & 75 \\ & N & = & \frac{75}{99} \\ & N & = & \frac{25}{33} \end{array}$$