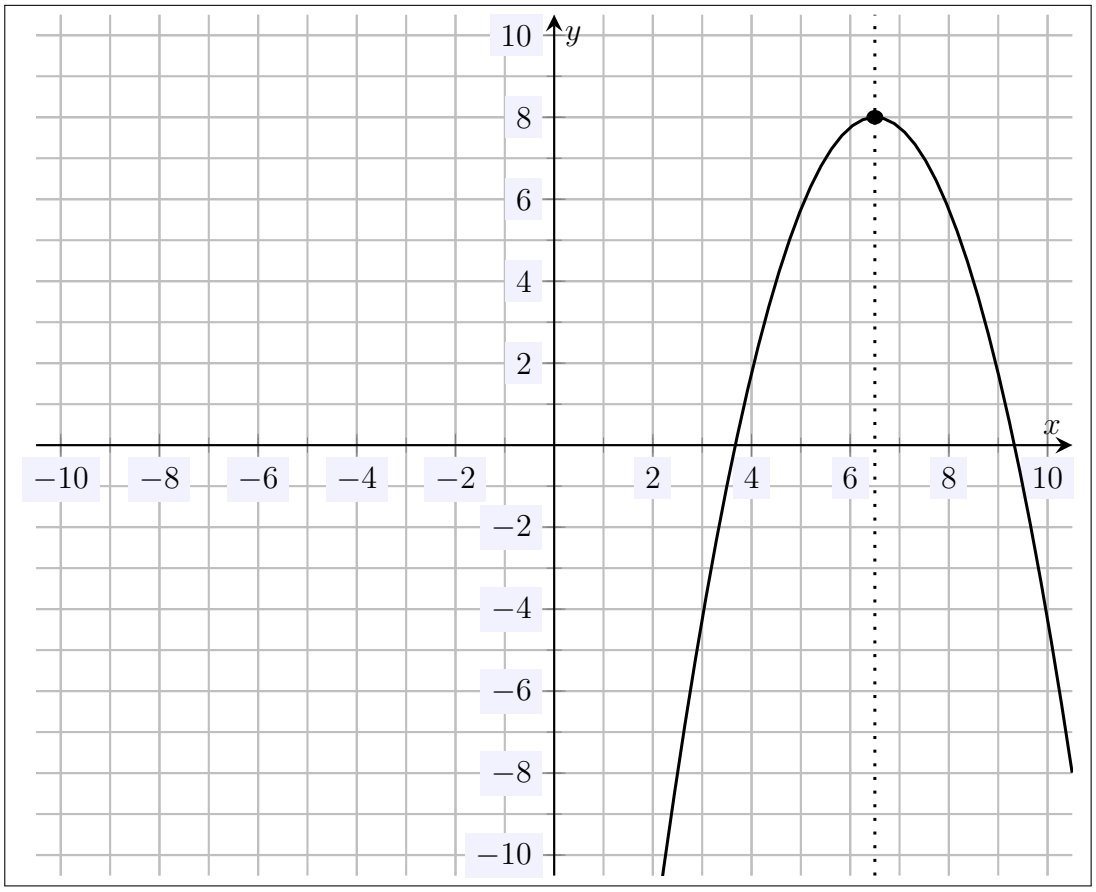
MAT 101: Exam 2 **Name:** _____
Spring – 2022
04/14/2022
85 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 21 pages (including this cover page) and 20 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
14	10	
15	10	
16	10	
17	10	
18	10	
19	10	
20	20	
Total:	210	

1. (10 points) Sketch the quadratic function $f(x) = 8 - \left(x - \dfrac{13}{2}\right)^2$ on the plot below. Your sketch should include the vertex and axis of symmetry—being placed as accurately as possible.



We have $f(x) = 8 - \left(x - \dfrac{13}{2}\right)^2 = -\left(x - \dfrac{13}{2}\right)^2 + 8$. *Therefore,* $f(x)$ *is in vertex form, i.e. the form* $f(x) = a(x-p)^2 + q$, *where* (p, q) *is the vertex. We then know that the vertex of* $f(x)$ *is* $(-\frac{13}{2}, 8)$ *so that the axis of symmetry is* $x = \frac{13}{2}$. *Because* $a = -1 < 0$, *the parabola opens downwards. This gives the sketch above.*

2. Consider the quadratic function $y = (x+5)^2 + 6$.

   (a) (2 points) Identify a, b, and c for this quadratic function.

   $$y = (x+5)^2 + 6 = (x^2 + 10x + 25) + 6 = x^2 + 10x + 31$$

   Therefore, $a = 1$, $b = 10$, $c = 31$.

   (b) (2 points) Does this quadratic function open upwards or downwards?

   Because $a = 1 > 0$*, the quadratic function opens upwards.*

   (c) (2 points) Is this quadratic function convex or concave?

   Because $a = 1 > 0$*, the quadratic function is convex*

   (d) (2 points) What is the vertex of this quadratic function? What is the axis of symmetry?

   The quadratic function $y = (x+5)^2 + 6$ *is in vertex form, i.e.* $y = a(x-p)^2 + q$*, where* (p, q) *is the vertex. Therefore, the vertex is* $(-5, 6)$ *and the axis of symmetry is* $x = -5$*.*

   (e) (2 points) Find the maximum and minimum values for y.

   Because the parabola opens upwards, there is no maximum value, i.e. it does not exist. The parabola has a minimum. Because the vertex is $(-5, 6)$*, we know the minimum value is 6—the* y*-coordinate of the vertex.*

3. (10 points) Find the vertex form of the quadratic function $f(x) = -3x^2 + 12x - 7$.

Completing the square, we have...

$$\begin{aligned} f(x) &= -3x^2 + 12x - 7 \\ &= -3\left(x^2 - 4x + \frac{7}{3}\right) \\ &= -3\left(x^2 - 4x + 4 - 4 + \frac{7}{3}\right) \\ &= -3\left((x^2 - 4x + 4) - \frac{12}{3} + \frac{7}{3}\right) \\ &= -3\left((x-2)^2 - \frac{5}{3}\right) \\ &= -3(x-2)^2 + 5 \end{aligned}$$

Alternatively, we can use the 'evaluation method':

$$\begin{aligned} a &= -3 \\ x &= -\frac{b}{2a} = -\frac{12}{2(-3)} = -\frac{12}{-6} = -(-2) = 2 \\ f(2) &= -3(2^2) + 12(2) - 7 = -3(4) + 12(2) - 7 = -12 + 24 - 7 = 5 \end{aligned}$$

Therefore, the vertex form is $f(x) = -3(x-2)^2 + 5$.

4. (10 points) Factor $x^2 + 16x - 80$ completely.

80	
$1 \cdot -80$	-79
$-1 \cdot 80$	79
$2 \cdot -40$	-38
$-2 \cdot 40$	38
$4 \cdot -20$	-16
$\boxed{-4 \cdot 20}$	$\boxed{16}$
$5 \cdot -16$	-11
$-5 \cdot 16$	11
$8 \cdot -10$	-2
$-8 \cdot 10$	2

Therefore,

$$x^2 + 16x - 80 = (x - 4)(x + 20)$$

5. (10 points) Factor $3x^2 - 9x - 120$ completely.

First, observe that $3x^2 - 9x - 120 = 3(x^2 - 3x - 40)$. *Then...*

$\underline{\mathbf{40}}$	
$1 \cdot -40$	-39
$-1 \cdot 40$	39
$2 \cdot -20$	-18
$-2 \cdot 20$	18
$4 \cdot -10$	-6
$-4 \cdot 10$	6
$\boxed{5 \cdot -8}$	$\boxed{-3}$
$-5 \cdot 8$	3

Therefore,

$$3x^2 - 9x - 120 = 3(x^2 - 3x - 40) = 3(x + 5)(x - 8)$$

6. Factor the following completely:

(a) (5 points) $16x - 20x^2$

Observe, we can factor out $4x$*:*

$$16x - 20x^2 = 4x(4 - 5x)$$

(b) (5 points) $49 - x^2$

Observe that this is a difference of perfect squares:

$$49 - x^2 = (7 - x)(7 + x)$$

7. (10 points) Factor $10x^2 + 43x - 35$ completely.

8. (10 points) Solve the following:

$$5(6-x) = \frac{4}{3}x + 30$$

$$5(6-x) = \frac{4}{3}x + 30$$

$$30 - 5x = \frac{4}{3}x + 30$$

$$-5x = \frac{4}{3}x$$

$$-15x = 4x$$

$$-19x = 0$$

$$x = 0$$

9. (10 points) Solve the following:

$$9 = x(10 - x)$$

$$\begin{aligned} 9 &= x(10 - x) \\ 9 &= 10x - x^2 \\ x^2 - 10x + 9 &= 0 \\ (x - 1)(x - 9) &= 0 \end{aligned}$$

But then either $x - 1 = 0$*, so that* $x = 1$*, or* $x - 9 = 0$*, so that* $x = 9$*.*

10. (10 points) Solve the following:

$$6 - x = 12x + 7$$

$$6 - x = 12x + 7$$

$$-1 = 13x$$

$$x = -\frac{1}{13}$$

11. (10 points) Solve the following:

$$x(3x-1) = x(x+5)$$

$$\begin{aligned} x(3x-1) &= x(x+5) \\ 3x^2 - x &= x^2 + 5x \\ 2x^2 - 6x &= 0 \\ 2x(x-3) &= 0 \end{aligned}$$

But then either $2x = 0$*, so that* $x = 0$*, or* $x - 3 = 0$*, so that* $x = 3$*.*

12. (10 points) Solve the following:

$$5(x+5) = 4x^2 + 5x$$

$$\begin{aligned} 5(x+5) &= 4x^2 + 5x \\ 5x + 25 &= 4x^2 + 5x \\ 25 &= 4x^2 \\ 0 &= 4x^2 - 25 \\ 0 &= (2x-5)(2x+5) \end{aligned}$$

But then either $2x - 5 = 0$*, which implies* $x = \dfrac{5}{2}$*, or* $2x + 5 = 0$*, which implies that* $x = -\dfrac{5}{2}$.

13. (10 points) Use the quadratic formula to solve the following:

$$6 - 5x^2 = 4x(1 - x)$$

14. (10 points) Use the quadratic formula to factor $x^2 - 10x + 23$.

15. (10 points) Use the discriminant to determine whether the function $384x^2+232x-175$ factors 'nicely', i.e. over the integers. If not, determine whether it even factors over the real numbers or requires complex numbers to factor.

$$\begin{aligned} D &= b^2 - 4ac \\ &= 232^2 - 4(384)(-175) \\ &= 53824 + 268800 \\ &= 322624 \\ &= 568^2 \end{aligned}$$

Because the discriminant is a perfect square, the function $384x^2 + 232x - 175$ *factors 'nicely', i.e. over the integers. Therefore,* $384x^2 + 232x - 175$ *also factors 'nicely' over the rational numbers, real numbers, and complex numbers.*

16. (10 points) Showing all your work, determine if the point $(5, -4)$ is a solution to the following system of equations:

$$2x + y = 6$$

$$-5x - 6y = -49$$

The point $(x, y) = (5, -4)$ is a solution to the system of equations if and only if it satisfies both of the equations. We check this:

$$2x + y = 6$$

$$2(5) + (-4) \stackrel{?}{=} 6$$

$$10 - 4 \stackrel{?}{=} 6$$

$$6 = 6$$

✓

and

$$-5x - 6y = -49$$

$$-5(5) - 6(-4) \stackrel{?}{=} -49$$

$$-25 + 24 \stackrel{?}{=} -49$$

$$-1 \neq -49$$

✗

Because $(5, -4)$ does not satisfy both of the equations, $(x, y) = (5, -4)$ is not a solution to the system of equations. [Indeed, there is no solution to this system of equations because the given lines are parallel.]

17. (10 points) Showing all your work, determine whether the following system of equations has a solution. If it has a solution, you do not need to find the solution.

$$10x - 4y = -24$$

$$-5x + 2y = -2$$

18. (10 points) Solve the following system of equations:

$$3x + 5y = 16$$

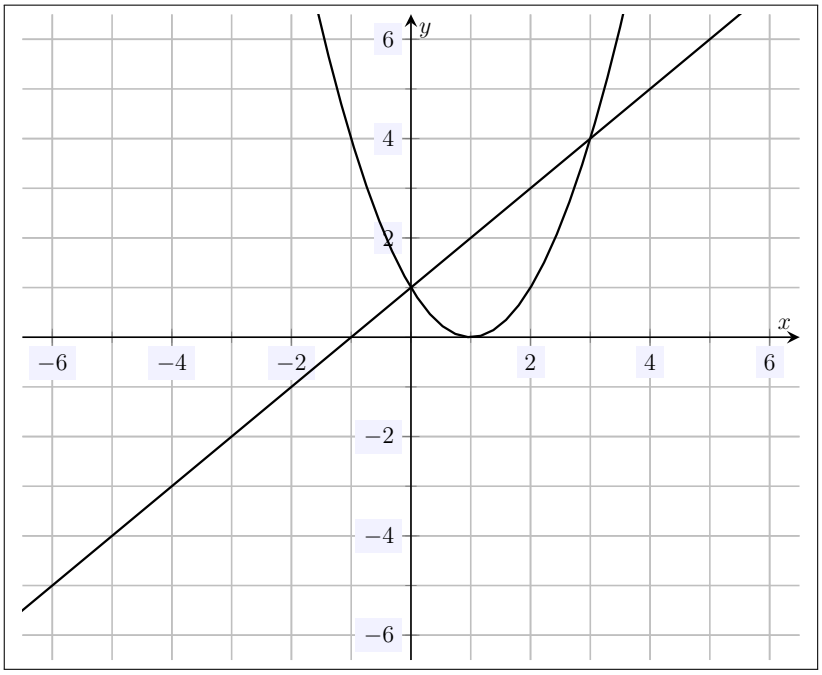
$$x + 6y = 1$$

19. (10 points) Solve the following system of equations:

$$4x + 7y = -8$$

$$2x - 5y = -4$$

20. (10 points) A quadratic function $y = x^2 - 2x + 1$ and a linear function $y = x + 1$ are plotted below.



(a) (5 points) Using the plot above, solve the following system of equations:

$$-x + y = 1$$
$$2x + y = x^2 + 1$$

(b) (5 points) Setting the functions equal, verify the solution(s) to the system of equations from (a).