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MATH 100
Fall 2022
HW 2: Due 09/19

"This is not yours to fix alone. You act like you're all alone out there in the world, but you're not. You're not alone"
–Joyce Byers, Stranger Things

Problem 1. (10pt) Showing all your work, compute the following:

(a) 55% of 143

(b) 1% of 3.6

(c) 49% of 49

(d) 121% of 4000

Solution. Recall to find a percent, $\%$, of a number N, we compute $N \cdot \%_d$, where $\%_d$ is the percentage written as a decimal. We then have...

(a)
$$143(0.55) = 78.65$$

(b)
$$3.6(0.01) = 0.036$$

(c)
$$49(0.49) = 24.01$$

(d)
$$4000(1.21) = 4840$$

Problem 2. (10pt) Showing all your work, compute the following:

(a) 78 increased by 40%

(b) 94 decreased by 65%

(c) 166 decreased by 2%

(d) 1820 increased by 163%

Solution. To find a percentage, $\%$, increase or decrease of a number N, we compute $N(1 \pm \%_d)$, where we choose '$+$' if it is a percentage increase, '$-$' if it is a percentage decrease, and $\%_d$ is the percentage written as a decimal. We then have...

(a)

$$78(1 + 0.40) = 78(1.40) = 109.2$$

(b)

$$94(1 - 0.65) = 94(0.35) = 32.9$$

(c)

$$166(1 - 0.02) = 166(0.98) = 162.68$$

(d)

$$1820(1 + 1.63) = 1820(2.63) = 4786.6$$

Problem 3. (10pt) Why is 80% of 485 the same value as the value obtained by reducing 485 by 20%? Be sure to give an explanation that does not simply involve computing both. Then compute both values as stated.

Solution. If we find 80% of 485, then 485 has lost 20% of its value. Vice versa, if we reduce 20% of its value, then the resulting number has only 80% of its original value. Therefore, 80% of 485 must have the same value as reducing 485 by 20%. Computing both, we have...

$$\text{80\% of 485: } 485(0.80) = 388$$

$$\text{485 decreased by 20\%: } 485(1 - 0.20) = 485(0.80) = 388$$

Problem 4. (10pt) Sally Forth is looking to book a vacation trip. She does not want to spend more than $3,500 on the trip. The prices listed on the travel website she is using to book the trip do not include 7% sales tax or a $50 booking surcharge that the website charges (applied to the cost of the trip *before* the tax). What is the highest advertised price on the website that she can book and actually afford?

Solution. We do not know the largest possible cost of a trip that Sally can afford. Let P denote the highest advertised price trip that she can afford. If P is the advertised cost, then once you book the trip you are charged the $50 surcharge, followed by a 7% tax on the combined cost of the trip and surcharge. The cost of the trip and the surcharge is $P + \$50$. Once the 7% tax is added to this, i.e. a 7% increase on this price, we have a total cost of...

$$(P + \$50) \cdot (1 + 0.07) = (P + \$50)(1.07).$$

The maximum that this final price can be is $3,500. But then $3,500 is the most that the total cost, given above, can be. Therefore, we have...

$$\begin{aligned}
(P + \$50)(1.07) &= \$3500 \\
\frac{(P + \$50)\cancel{(1.07)}}{\cancel{1.07}} &= \frac{\$3500}{1.07} \\
P + \$50 &= \$3271.03 \\
P + \$50 - \$50 &= \$3271.03 - \$50 \\
P &= \$3221.03.
\end{aligned}$$

Therefore, the most expensive advertised price for a trip that Sally can afford to book is $3,221.03.

Problem 5. (10pt) Ophelia Pane is taking a Mathematics course. For this Mathematics course, the grading scheme is as follows:

Participation	5%	Quizzes	10%
Activities	5%	Exams	30%
Project	10%	Homework	40%

Suppose she had a 90% participation average, 100% activities average, 85% project average, 75% quiz average, and 79% homework average. She received a 72% on exam 1, 89% on exam 2, and 84% on exam 3, which gave her a 81.67 exam average (because they were weighted equally). What was her finally average in the course?

Solution. This is a weighted average. Recall that a weighted average is given by...

$$\sum \text{value} \cdot \text{weight},$$

where $\sum$ stands for the sum of the values value $\cdot$ weight. For instance, if one value is the 10% project grade and one has a 100% on the project, one receives $10\%(1.00) = 10\%$ on that portion of the course grade. If one receives a 50% on the project, one receives $10\%(0.50) = 5\%$ on that portion of the course grade. One finds each of these values for each portion of the course grade and adds them up. We compute this value:

$$\begin{aligned}
\text{Course Average} &= \sum \text{value} \cdot \text{weight} \\
&= 5\%(0.90) + 5\%(1.00) + 10\%(0.85) + 10\%(0.75) + 30\%(0.8167) + 40\%(0.79) \\
&= 4.5\% + 5.0\% + 8.5\% + 7.5\% + 24.501\% + 31.6\% \\
&= 81.601\% \\
&\approx 81.6\%
\end{aligned}$$

Therefore, Ophelia has an 81.6% course average.

Problem 6. (10pt) Suppose that Jim Nasium is in his first semester at college. His transcript at the end of the first semester was as follows:

Course	Letter Grade	Credits
First-Year Seminar	A	1
Calculus	A−	4
Introductory Philosophy	C+	3
German I	B−	3
Writing I	B+	3
American Poets	D	3

A	4.0	C+	2.3
A–	3.7	C	2.0
B+	3.3	C–	1.7
B	3.0	D	1.0
B–	2.7	F	0.0

What was Jim's GPA for his first semester? [The college's letter grade scheme is show above on the right.]

Solution. This is a weighted average. Recall that a weighted average is given by...

$$\sum \text{value} \cdot \text{weight},$$

where $\sum$ stands for the sum of the values value $\cdot$ weight. Because all the weights in the case of GPA are measured out of the total credits, we can write this simply as:

$$\frac{\sum \text{value} \cdot \text{credits}}{\text{total credits}}.$$

For instance, if one receives an 'A' (which has value 4.0) in a course, one then multiplies by its credits (for instance 3 credits) for a total of $4.0 \cdot 3 = 12.0$ contribution towards the GPA. We can then compute Jim's GPA:

$$\begin{aligned}
\text{GPA} &= \frac{\sum \text{value} \cdot \text{credits}}{\text{total credits}} \\
&= \frac{4.0(1) + 3.7(4) + 2.3(3) + 2.7(3) + 3.3(3) + 1.0(3)}{1 + 4 + 3 + 3 + 3 + 3} \\
&= \frac{4.0 + 14.8 + 6.9 + 8.1 + 9.9 + 3}{17} \\
&= \frac{46.7}{17} \\
&= 2.74706 \\
&\approx 2.747
\end{aligned}$$

Therefore, Jim has a 2.747 GPA.