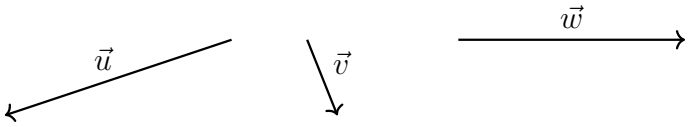
MAT 107: Exam 2 **Name:** _____
Winter – 2022
01/16/2023
Time Limit: '∞'

Write your name on the appropriate line on the exam cover sheet. This exam contains 11 pages (including this cover page) and 10 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total:	100	

1. (10 points) Define the following vectors:



As accurately as possible, sketch the following:

(a) $\vec{u} + \vec{w}$

(b) $\vec{v} - \vec{u}$

(c) $-\frac{1}{2}\vec{w}$

(d) $\vec{u} + 2\vec{v}$

2. (10 points) Let $\vec{u} = \langle 2, -1, 0 \rangle$ and $\vec{v} = \hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}$. Find the following:

   (a) $-3\vec{u}$

   (b) $\vec{v} - \vec{u}$

   (c) $3\vec{u} + 2\vec{v}$

   (d) $\vec{u} \cdot \vec{v}$

   (e) The angle between $\vec{u}$ and $\vec{v}$.

3. (10 points) Suppose you are a sprite in a 2D video game. Currently, you are at $\vec{p} = \langle 2.4, 3.7 \rangle$. You are moving in the direction given by $\langle -2, 1 \rangle$ at speed 1.6. Find your position one game 'tick' from now.

4. (10 points) Define the following:

$$A = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 4 & 1 \\ -6 & 1 & 3 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & -1 & 5 \\ 3 & 2 & 1 \\ -2 & 1 & 0 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & 4 & 0 \\ 2 & -1 & 5 \end{pmatrix}$$

Find the following:

(a) $4C$

(b) $A - B$

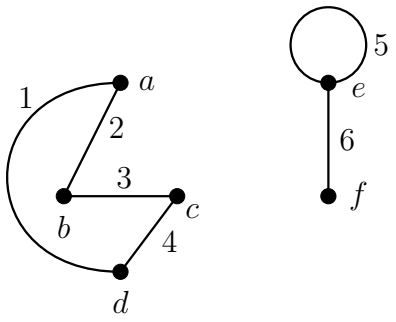
(c) CA

5. (10 points) Define $A = \begin{pmatrix} -4 & 1 \\ 2 & 6 \end{pmatrix}$ and $\vec{u} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

   (a) Compute $A\vec{u}$.

   (b) Explain why you cannot compute $\vec{u}A$.

6. (10 points) Let G be the graph given below:



(a) What is adjacent to a?

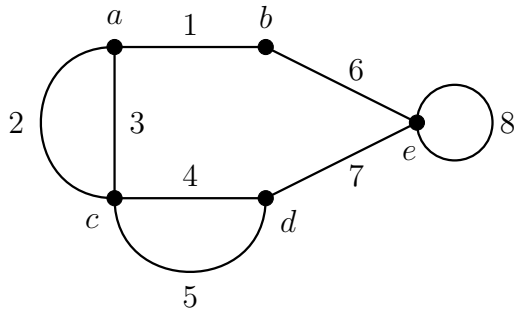
(b) What is adjacent to 3?

(c) Are there parallel edges? Explain.

(d) Is the graph connected? Explain.

(e) Is the graph simple? Explain.

7. (10 points) Let G be the graph given below:



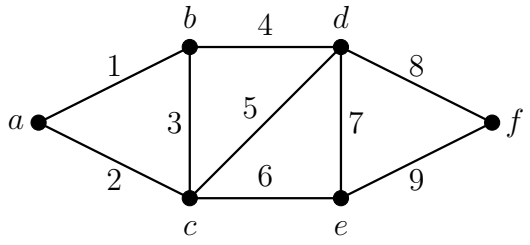
(a) Find the degree of the vertex c.

(b) Find the degree of the vertex e.

(c) Find the degree of the graph.

(d) Give an example of a trail from a to c that is not a path.

8. (10 points) Consider the graph below:

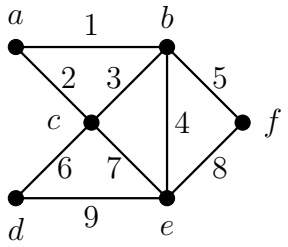


(a) Explain why the graph does not have an Euler circuit.

(b) Explain why the graph has an Euler trail.

(c) Find an Euler trial for this graph.

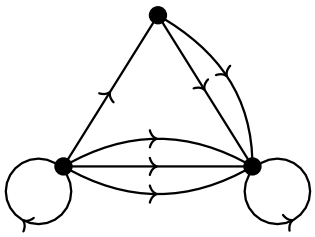
9. (10 points) Consider the graph below:



(a) Find an Euler circuit for this graph.

(b) Find a Hamiltonian circuit for this graph.

10. (10 points) Let G be the graph below:



(a) Find the adjacency matrix for G.

(b) Find the number of walks from 1 to 3 of length 2.