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MATH 308
Fall 2022
HW 19: Due 12/06

"It is a part of probability that many improbable things will happen."
–Aristotle

Problem 1. (10pt) The probabilities of several events in a finite probability space are given below:

$$\begin{aligned} P(A) &= 0.40 \qquad P(A \cap B) = 0.10 \\ P(B) &= 0.25 \qquad P(A \cap C) = 0.05 \\ P(C) &= 0.70 \qquad P(B \cap C) = 0.00 \\ P(D) &= 0.65 \qquad P(B \cup D) = 0.75 \end{aligned}$$

(a) Find $P(A \cup B)$.

(b) Assuming C and D are independent, find $P(C \cup D)$.

(c) Assuming A and D are independent events, find $P(A \cap D)$.

(d) Find $P(A \mid B)$.

(e) Find $P(B \cap D)$.

(f) Are C and D disjoint? Explain.

(g) Are A and C independent? Explain.

(h) Are B and C independent? Explain.

Solution.

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.40 + 0.25 - 0.10 = 0.55$

(b) Because C and D are independent, $P(C \cap D) = P(C) \cdot P(D)$. But then $P(C \cup D) = P(C) + P(D) - P(C \cap D) = 0.70 + 0.65 - 0.70(0.65) = 0.895$

(c) Because A and D are independent, we know that $P(A \cap D) = P(A) \cdot P(D) = 0.40 \cdot 0.65 = 0.26$.

(d) We have $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.10}{0.25} = 0.40$

(e) We know that $P(B \cup D) = P(B) + P(D) - P(B \cap D)$. But then $0.75 = 0.25 + 0.65 - P(B \cap D)$ so that $P(B \cap D) = 0.15$

(f) Events C and D cannot be disjoint. If there were, then we would have $P(C \cup D) = P(C) + P(D) = 0.70 + 0.65 = 1.35 > 1$, which is impossible.

(g) If A and C were independent, then $P(A \cap C) = P(A) \cdot P(C) = 0.40 \cdot 0.70 = 0.28$. But we know $P(A \cap C) = 0.05$. Therefore, A and C are not independent.

(h) Because this is a finite probability space, $P(B \cap C) = 0.00$ implies that B and C are disjoint. But disjoint events cannot be independent. Therefore, B and C are not independent.

Problem 2. (10pt) Below are two charts giving information on survival/death for the April 15, 1912 sinking of the Titanic.

	Men	Women	Boys	Girls	Total
Survived	338	316	29	27	710
Died	1352	109	35	18	1514
Total	1690	425	64	45	2224

	Survived	Died	Total
First Class Men	57	118	175
First Class Women	140	4	144
First Class Children	5	1	6
Second Class Men	14	154	168
Second Class Women	80	13	93
Second Class Children	24	0	24
Third Class Men	75	387	462
Third Class Women	76	89	165
Third Class Children	27	52	79
Crew (Men)	192	693	885
Crew (Women)	20	3	23
Total	710	1514	2224

Let S denote the set of people that survived, D denote the people that died, W denote the set of women, M denote the set of men, C denote the set of children, F denote the set of first class passengers, N denote the set of second class passengers, T denote the set of third class passengers, and R denote the set of crew members. State each of the following probabilities as complete sentence and find its probability.

(a) $P(M \cap S)$

(b) $P(W \cup F)$

(c) $P(F \mid S)$

(d) $P(C \cap T \cap D)$

(e) $P\big((N \cap S) \cup (R \cap D)\big)$

Problem 3. (10pt) Suppose a computer programmer has a choice between two algorithms to solve a problem. The first algorithm will run in under 5 hours 60% of the time. The other algorithm will finish in under 5 hours 80% of the time. However, there is a 5% chance that the second algorithm will simply crash.

(a) If you randomly select an algorithm, what is the probability that you will have a solution in under 5 hours.

(b) If you randomly select an algorithm, what is the probability it crashes or does not produce a solution in under 5 hours?

(c) If you randomly select an algorithm, what is the probability that was the first algorithm and it produced a solution in under 5 hours?

(d) If you randomly select an algorithm and it produces a solution in more than 5 hours, what is the probability that it was the second algorithm?

Problem 4. (10pt) Below is a table giving a probability space along with a random variable X. Find EX.

S	1	2	3	4	5
$P(S)$	0.20	0.15	0.16	0.46	0.03
X	5	-3	4	6	0