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MATH 101
Winter 2021
HW 8: Due 01/14

"Do I need to be liked? Absolutely not. I like to be liked. I enjoy being liked. I have to be liked, but it's not like this compulsive need to be liked, like my need to be praised."

–Michael Scott, The Office

Problem 1. (10pt) Showing all your work, factor $x^2 + 23x + 120$.

Solution.

$\underline{\mathbf{120}}$	
$1 \cdot 120$	121
$-1 \cdot -120$	-121
$2 \cdot 60$	62
$-2 \cdot -60$	-62
$3 \cdot 40$	43
$-3 \cdot -40$	-43
$4 \cdot 30$	34
$-4 \cdot -30$	-34
$5 \cdot 24$	29
$-5 \cdot -24$	-29
$6 \cdot 20$	26
$-6 \cdot -20$	-26
$\boxed{8 \cdot 15}$	$\boxed{23}$
$-8 \cdot -15$	-23
$10 \cdot 12$	22
$-10 \cdot -12$	-22

Therefore,

$$x^2 + 23x + 120 = (x + 8)(x + 15)$$

Problem 2. (10pt) Showing all your work, factor $-x^2 - 6x + 72$.

Solution. First, we rewrite this as $-(x^2 + 6x - 72)$. Then...

	56
$1 \cdot -72$	-71
$-1 \cdot 72$	71
$2 \cdot -36$	-34
$-2 \cdot 36$	34
$3 \cdot -26$	-23
$-3 \cdot 26$	23
$4 \cdot -18$	-14
$-4 \cdot 18$	14
$6 \cdot -12$	-6
$\boxed{-6 \cdot 12}$	$\boxed{6}$
$8 \cdot -9$	-1
$-8 \cdot 9$	1

Therefore,

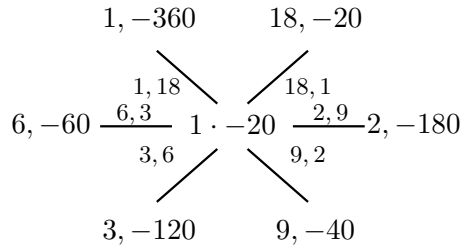
$$-x^2 - 6x + 72 = -(x^2 + 6x - 72) = -(x - 6)(x + 12)$$

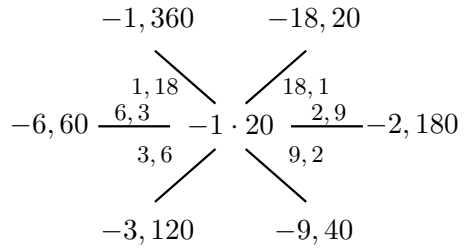
Problem 3. (10pt) Showing all your work, factor $18x^2 + 67x - 20$.

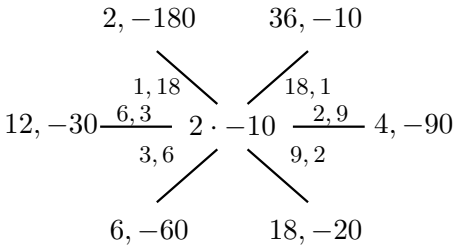
Solution.

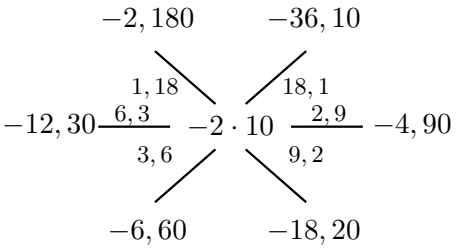
$$\underline{\mathbf{20}}$$
$$1 \cdot -20$$
$$-1 \cdot 20$$
$$2 \cdot -10$$
$$-2 \cdot 10$$
$$4 \cdot -5$$
$$-4 \cdot 5$$

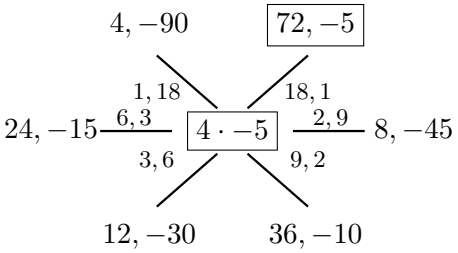
Then as $18 = 1 \cdot 18 = 2 \cdot 9 = 3 \cdot 6$, we have...

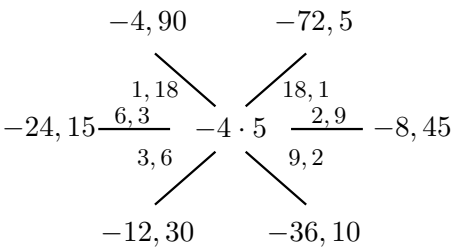












Therefore,

$$18x^2 + 67x - 20 = (18x - 5)(x + 4)$$

Problem 4. (10pt) Use the quadratic equation to factor $100x^2 - 225x + 126$.

Solution. We have...

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-225) \pm \sqrt{(-225)^2 - 4(100)(126)}}{2(100)}$$

$$x = \frac{225 \pm \sqrt{50625 - 50400}}{200}$$

$$x = \frac{225 \pm \sqrt{225}}{200}$$

$$x = \frac{225 \pm 15}{200}$$

But then the roots of $100x^2 - 225x + 126$ are $x = \frac{225+15}{200} = \frac{240}{200} = \frac{6}{5}$ or $x = \frac{225-15}{200} = \frac{21}{20}$. Observe that $a = 1$. Therefore, the factorization is...

$$100x^2 - 225x + 126 = 100 \cdot \left(x - \frac{6}{5}\right)\left(x - \frac{21}{20}\right) = 5 \cdot \left(x - \frac{6}{5}\right) \cdot 20\left(x - \frac{21}{20}\right) = (5x - 6)(20x - 21)$$

Problem 5. (10pt) Use the quadratic equation to factor $x^2 - 6x + 1$.

Solution. We have...

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 4}}{2}$$

$$x = \frac{6 \pm \sqrt{32}}{2}$$

$$x = \frac{6 \pm \sqrt{16 \cdot 2}}{2}$$

$$x = \frac{6 \pm 4\sqrt{2}}{2}$$

$$x = 3 \pm 2\sqrt{2}$$

Then the roots of $x^2 - 6x + 1$ are $x = 3 + 2\sqrt{2}$ and $x = 3 - 2\sqrt{2}$. Observe that $a = 1$. Therefore, the factorization is...

$$x^2 - 6x + 1 = 1 \cdot \big(x - (3 + 2\sqrt{2})\big)\big(x - (3 - 2\sqrt{2})\big) = \big(x - (3 + 2\sqrt{2})\big)\big(x - (3 - 2\sqrt{2})\big)$$

Problem 6. (10pt) Showing all your work, solve the equation $x^2 - 9x + 14 = 0$.

Solution.

$$x^2 - 9x + 14 = 0$$

$$(x - 2)(x - 7) = 0$$

But then either $x - 2 = 0$, which implies $x = 2$, or $x - 7 = 0$, which implies $x = 7$. Therefore, $x = 2$ or $x = 7$.

Problem 7. (10pt) Showing all your work, solve the equation $x^2 + 2x = 48$.

Solution. We have...

$$x^2 + 2x = 48$$

$$x^2 + 2x - 48 = 0$$

$$(x + 8)(x - 6) = 0$$

But then either $x + 8 = 0$, which implies $x = -8$, or $x - 6 = 0$, which implies $x = 6$. Therefore, $x = -8$ or $x = 6$.

Problem 8. (10pt) Showing all your work, solve the equation $x = 15 - 2x^2$.

Solution. We have...

$$x = 15 - 2x^2$$

$$2x^2 + x - 15 = 0$$

$$(2x - 5)(x + 3) = 0$$

But then either $2x - 5 = 0$, which implies that $2x = 5$ so that $x = \frac{5}{2}$, or $x + 3 = 0$, which implies that $x = -3$. Therefore, $x = -3$ or $x = \frac{5}{2}$.

Problem 9. (10pt) Use the quadratic equation to solve the equation $x^2 = 2x + 13$.

Solution. First, we write the equation as...

$$x^2 = 2x + 13$$
$$x^2 - 2x - 13 = 0$$

Then we have...

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-13)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 + 52}}{2}$$

$$x = \frac{2 \pm \sqrt{56}}{2}$$

$$x = \frac{2 \pm \sqrt{4 \cdot 14}}{2}$$

$$x = \frac{2 \pm 2\sqrt{14}}{2}$$

$$x = 1 \pm \sqrt{14}$$

Then either $x = 1 + \sqrt{14}$ or $x = 1 - \sqrt{14}$. Therefore, $x = 1 - \sqrt{14}, 1 + \sqrt{14}$.

Problem 10. (10pt) Use the quadratic equation to solve the equation $x^2 + 29 = 10x$.

Solution. First, we write the equation as...

$$x^2 + 29 = 10x$$
$$x^2 - 10x + 29 = 0$$

Then we have...

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(29)}}{2(1)}$$

$$x = \frac{10 \pm \sqrt{100 - 116}}{2}$$

$$x = \frac{10 \pm \sqrt{-16}}{2}$$

$$x = \frac{10 \pm \sqrt{16}i}{2}$$

$$x = \frac{10 \pm 4i}{2}$$

$$x = 5 \pm 2i$$

Then $x = 5 + 2i$ or $x = 5 - 2i$. Therefore, $x = 5 - 2i, 5 + 2i$.