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MATH 101
Summer 2022
HW 1: Due 05/24

"It is easy to forget now, how effervescent and free we all felt that summer."

–Anna Godbersen

Problem 1. (10pt) Showing all your work and simplifying as much as possible, compute each of the following:

(a) $15/5(1+2)$

(b) $20/(4(2+3))$

(c) $\dfrac{4^2/2-8+3\cdot -2}{(12-4)(4-5)}$

(d) $\dfrac{8-12}{-2^2}+7\cdot 12/2$

(e) $4(-1)^3-2(-1)^3+6\cdot 5/4$

Solution.

(a)
$$15/5(1+2)=15/5(3)=3(3)=9$$

(b)
$$20/(4(2+3))=20/(4(5))=20/20=1$$

(c)
$$\frac{4^2/2-8+3\cdot -2}{(12-4)(4-5)}=\frac{4^2/2-8+3\cdot -2}{8\cdot -1}=\frac{16/2-8-6}{-8}=\frac{8-8-6}{-8}=\frac{-6}{-8}=\frac{6}{8}=\frac{3}{4}$$

(d)
$$\frac{8-12}{-2^2}+7\cdot 12/2=\frac{-4}{-4}+7\cdot 6=1+42=43$$

(e)
$$4(-1)^3-2(-1)^3+6\cdot 5/4=4(-1)-2(-1)+30/4=-4+2+\frac{15}{2}=-2+\frac{15}{2}=\frac{-4}{2}+\frac{15}{2}=\frac{11}{2}$$

Problem 2. (10pt) Showing all your work, find the prime factorizations of the following integers:

(a) 90

(b) 141

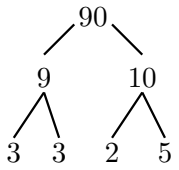
(c) 149

(d) 27

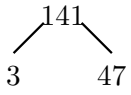
(e) 185

Solution.

(a) $90 = 2 \cdot 3^2 \cdot 5$

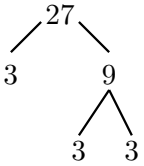


(b) $141 = 3 \cdot 47$

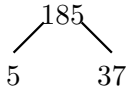


(c) $149 = 149^1$

(d) $27 = 3^3$



(e) $185 = 5 \cdot 37$



Problem 3. (10pt) Compute each of the following by finding the divisors/multiples of the given integers:

(a) $\gcd(18, 24)$

(b) $\gcd(60, 125)$

(c) $\mathrm{lcm}(14, 20)$

(d) $\mathrm{lcm}(10, 21)$

Solution.

(a)

18: 1, 2, 3, **6**, 9, 18
24: 1, 2, 3, 4, **6**, 8, 12, 24

Therefore, $\gcd(18, 24) = 6$.

(b)

60: 1, 2, 3, 4, **5**, 6, 10, 12, 15, 20, 30, 60
125: 1, **5**, 25, 125

Therefore, $\gcd(60, 125) = 5$.

(c)

14: 14, 28, 42, 56, 70, 84, 98, 112, 126, **140**
20: 20, 40, 60, 80, 100, 120, **140**, 160, 180, 200

Therefore, $\mathrm{lcm}(14, 20) = 140$.

(d)

10: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140, 150, 160, 170, 180, 190, 200, **210**
21: 21, 42, 63, 84, 105, 126, 147, 168, 189, **210**

Therefore, $\mathrm{lcm}(10, 21) = 210$.

Problem 4. (10pt) Use the prime factorizations of the given integers to compute each of the following:

(a) $\gcd(142, 200)$

(b) $\operatorname{lcm}(72, 204)$

(c) $\gcd(2^{11} \cdot 3^8 \cdot 7^2 \cdot 17^4, 2^5 \cdot 3^2 \cdot 5^6 \cdot 11^{30})$

(d) $\operatorname{lcm}(2^{11} \cdot 3^8 \cdot 7^2 \cdot 17^4, 2^5 \cdot 3^2 \cdot 5^6 \cdot 11^{30})$

Solution.

(a)

$$\gcd(142, 200) = \gcd(2 \cdot 71, 2^3 \cdot 5^2) = 2$$

(b)

$$\operatorname{lcm}(72, 204) = \operatorname{lcm}(2^3 \cdot 3^2, 2^2 \cdot 3 \cdot 17) = 2^3 \cdot 3^2 \cdot 17 = 1224$$

(c)

$$\gcd(2^{11} \cdot 3^8 \cdot 7^2 \cdot 17^4, 2^5 \cdot 3^2 \cdot 5^6 \cdot 11^{30}) = 2^5 \cdot 3^2 = 288$$

(d)

$$\begin{aligned}\operatorname{lcm}(2^{11} \cdot 3^8 \cdot 7^2 \cdot 17^4, 2^5 \cdot 3^2 \cdot 5^6 \cdot 11^{30}) &= 2^{11} \cdot 3^8 \cdot 5^6 \cdot 7^2 \cdot 11^{30} \cdot 17^4 \\ &= 14993131026926250123829433260382033579359008000000\end{aligned}$$

Problem 5. (10pt) For each of the following, either convert the rational number from an improper fraction to a proper fraction or vice versa:

(a) $5\frac{6}{7}$

(b) $\frac{35}{3}$

(c) $-9\frac{3}{4}$

(d) $-\frac{26}{5}$

Solution.

(a)

$$5\tfrac{6}{7} = 5 + \frac{6}{7} = \frac{35}{7} + \frac{6}{7} = \frac{41}{7}$$

(b)

$$\begin{array}{r} 11.\overline{6} \\ 3\overline{)\,35.0} \\ \underline{3} \\ 05 \\ \underline{3} \\ 2.0 \\ \underline{1.8} \\ 2 \end{array}$$

Then we have $35 - 3(11) = 35 - 33 = 2$ so that $\frac{35}{3} = 11\frac{2}{3}$.

(c)

$$-9\tfrac{3}{4} = -\left(9 + \frac{3}{4}\right) = -\left(\frac{36}{4} + \frac{3}{4}\right) = -\frac{39}{4}$$

(d)

$$\begin{array}{r} 5.2 \\ 5\overline{)\,26.0} \\ \underline{25} \\ 1.0 \\ \underline{1.0} \\ 0 \end{array}$$

Then we have $26 - 5(5) = 26 - 25 = 1$ so that $-\frac{26}{5} = -5\frac{1}{5}$.

Problem 6. (10pt) Completely reduce the following rational numbers, showing all your work:

(a) $\frac{15}{33}$

(b) $-\frac{140}{90}$

(c) $\frac{210}{308}$

(d) $\frac{10}{21}$

Solution.

(a)

$$\frac{15}{33} = \frac{\cancel{3} \cdot 5}{\cancel{3} \cdot 11} = \frac{5}{11}$$

(b)

$$-\frac{140}{90} = -\frac{14 \cdot \cancel{10}}{9 \cdot \cancel{10}} = -\frac{14}{9}$$

(c)

$$\frac{210}{308} = \frac{21 \cdot 10}{4 \cdot 77} = \frac{3 \cdot \cancel{7} \cdot 10}{4 \cdot \cancel{7} \cdot 11} = \frac{3 \cdot \cancel{10}^{5}}{\cancel{4}^{2} \cdot 11} = \frac{3 \cdot 5}{2 \cdot 11} = \frac{15}{22}$$

(d)

$$\frac{10}{21} = \frac{2 \cdot 5}{3 \cdot 7} = \frac{10}{21}$$

Problem 7. (10pt) Simplifying as much as possible and showing all your work, compute the following:

(a) $\frac{12}{15} - \frac{5}{9}$

(b) $\frac{1}{6} + \frac{7}{12}$

(c) $-\frac{5}{12} + \frac{7}{18}$

(d) $2 + \frac{1}{3} - \frac{5}{2}$

Solution.

(a)
$$\frac{12}{15} - \frac{5}{9} = \frac{36}{45} - \frac{25}{45} = \frac{36-25}{45} = \frac{11}{45}$$

(b)
$$\frac{1}{6} + \frac{7}{12} = \frac{2}{12} + \frac{7}{12} = \frac{2+7}{12} = \frac{9}{12} = \frac{3}{4}$$

(c)
$$-\frac{5}{12} + \frac{7}{18} = -\frac{15}{36} + \frac{14}{36} = \frac{-15+14}{36} = -\frac{1}{36}$$

(d)
$$2 + \frac{1}{3} - \frac{5}{2} = \frac{12}{6} + \frac{2}{6} - \frac{15}{6} = \frac{12+2-15}{6} = -\frac{1}{6}$$

Problem 8. (10pt) Simplifying as much as possible and showing all your work, compute the following:

(a) $\frac{15}{14} \cdot \frac{7}{33}$

(b) $\dfrac{\frac{5}{6}}{\frac{7}{15}}$

(c) $\frac{19}{4} \cdot -\frac{10}{9}$

(d) $\dfrac{\frac{2}{45}}{\frac{20}{21}}$

Solution.

(a)
$$\frac{15}{14} \cdot \frac{7}{33} = \frac{\cancel{3} \cdot 5}{2 \cdot \cancel{7}} \cdot \frac{\cancel{7}}{\cancel{3} \cdot 11} = \frac{5}{22}$$

(b)
$$\frac{\frac{5}{6}}{\frac{7}{15}} = \frac{5}{6} \cdot \frac{15}{7} = \frac{5}{\cancel{6}^{2}} \cdot \frac{\cancel{15}^{5}}{7} = \frac{25}{14}$$

(c)
$$\frac{19}{4} \cdot -\frac{10}{9} = \frac{19}{\cancel{4}^{2}} \cdot -\frac{\cancel{10}^{5}}{9} = -\frac{95}{18}$$

(d)
$$\frac{\frac{2}{45}}{\frac{20}{21}} = \frac{2}{45} \cdot \frac{21}{20} = \frac{\cancel{2}}{\cancel{3} \cdot 3 \cdot 5} \cdot \frac{\cancel{3} \cdot 7}{\cancel{2} \cdot 2 \cdot 5} = \frac{7}{150}$$

Problem 9. (10pt) Showing all your work, convert the following rational numbers to decimals:

(a) $\frac{4}{9}$

(b) $\frac{7}{20}$

(c) $\frac{2}{11}$

Solution.

(a)

$$\begin{array}{r} 0.\overline{4} \\ 9\overline{)\,4.0} \\ \underline{3.6} \\ 4 \end{array}$$

(b)

$$\begin{array}{r} 0.35 \\ 20\overline{)\,7.00} \\ \underline{6.0} \\ 1.00 \\ \underline{1.00} \\ 0 \end{array}$$

(c)

$$\begin{array}{r} 0.\overline{18} \\ 11\overline{)\,2.00} \\ \underline{1.1} \\ 90 \\ \underline{88} \\ 2 \end{array}$$

Problem 10. (10pt) Showing all your work, convert the following decimals to rational numbers:

(a) 0.7

(b) 0.125

(c) $0.12121212\overline{12}$

Solution.

(a)

$$0.7 = \frac{7}{10}$$

(b)

$$0.125 = \frac{125}{1000} = \frac{3}{40}$$

(c)

$$\begin{array}{rrcc} & 100N & = & 12.12121212\overline{12} \\ - & N & = & 0.1212121212\overline{12} \\ \hline & 99N & = & 12 \\ & N & = & \frac{12}{99} \\ & N & = & \frac{4}{33} \end{array}$$

$$0.\overline{12} = \frac{4}{33}$$