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MATH 108
Fall 2021
HW 4: Due 10/05

"There are three choices in this life: be good, get good, or give up."
–Dr. Gregory House, House

Problem 1. (10pt) Find the matrix associated with the system of equations below.

$$\begin{aligned} 2x_1 - x_2 + 5x_3 &= 5 \\ x_1 \qquad\quad + x_3 &= -1 \\ 3x_2 - 6x_3 &= 4 \end{aligned}$$

Solution.

$$\begin{pmatrix} 2 & -1 & 5 & 5 \\ 1 & 0 & 1 & -1 \\ 0 & 3 & -6 & 4 \end{pmatrix}$$

<u>OR</u>

$$\left(\begin{array}{ccc|c} 2 & -1 & 5 & 5 \\ 1 & 0 & 1 & -1 \\ 0 & 3 & -6 & 4 \end{array}\right)$$

Problem 2. (10pt) Write the system of equations associated to the matrix below.

$$\begin{pmatrix} 1 & -1 & 3 & 5 & 6 \\ 0 & 1 & 4 & 9 & -2 \\ 1 & 2 & 0 & -6 & 3 \\ 2 & -1 & 4 & 1 & 7 \end{pmatrix}$$

Solution.

$$\begin{array}{rcrcrcrcr} x_1 & - & x_2 & + & 3x_3 & + & 5x_4 & = & 6 \\ & & x_2 & + & 4x_3 & + & 9x_4 & = & -2 \\ x_1 & + & 2x_2 & & & - & 6x_4 & = & 3 \\ 2x_1 & - & x_2 & + & 4x_3 & + & x_4 & = & 7 \end{array}$$

Problem 3. (10pt) Use matrix methods to solve the system of equations below. Show all your work.

$$6x_1 - x_2 = 13$$
$$2x_1 + 3x_2 = 1$$

Solution.

$$\left(\begin{array}{cc:c} 6 & -1 & 13 \\ 2 & 3 & 1 \end{array}\right) \qquad R_1 + -3R_2 \to R_2$$

$$\left(\begin{array}{cc:c} 6 & -1 & 13 \\ 0 & -10 & 10 \end{array}\right) \qquad -\frac{1}{10}R_2 \to R_2$$

$$\left(\begin{array}{cc:c} 6 & -1 & 13 \\ 0 & 1 & -1 \end{array}\right) \qquad R_2 + R_1 \to R_1$$

$$\left(\begin{array}{cc:c} 6 & 0 & 12 \\ 0 & 1 & -1 \end{array}\right) \qquad \frac{1}{6}R_1 \to R_1$$

$$\left(\begin{array}{cc:c} 1 & 0 & 2 \\ 0 & 1 & -1 \end{array}\right)$$

Therefore, the solution is $(x_1, x_2) = (2, -1)$, i.e. $x_1 = 2$ and $x_2 = -1$:

$$\begin{cases} x_1 = 2 \\ x_2 = -1 \end{cases}$$

Problem 4. (10pt) Use matrix methods to solve the system of equations below. Show all your work.

$$\begin{aligned} x_1 - x_2 + x_3 &= 2 \\ 2x_1 + 2x_2 - x_3 &= 9 \\ x_2 - 3x_3 &= 8 \end{aligned}$$

Solution.

$$\left(\begin{array}{rrr|r} 1 & -1 & 1 & 2 \\ 2 & 2 & -1 & 9 \\ 0 & 1 & -3 & 8 \end{array}\right) \qquad -2R_1 + R_2 \to R_2$$

$$\left(\begin{array}{rrr|r} 1 & -1 & 1 & 2 \\ 0 & 4 & -3 & 5 \\ 0 & 1 & -3 & 8 \end{array}\right) \qquad R_2 - 4R_3 \to R_3$$

$$\left(\begin{array}{rrr|r} 1 & -1 & 1 & 2 \\ 0 & 4 & -3 & 5 \\ 0 & 0 & 9 & -27 \end{array}\right) \qquad \frac{1}{9}R_3 \to R_3$$

$$\left(\begin{array}{rrr|r} 1 & -1 & 1 & 2 \\ 0 & 4 & -3 & 5 \\ 0 & 0 & 1 & -3 \end{array}\right) \qquad 3R_3 + R_2 \to R_2$$

$$\left(\begin{array}{rrr|r} 1 & -1 & 1 & 2 \\ 0 & 4 & 0 & -4 \\ 0 & 0 & 1 & -3 \end{array}\right) \qquad -R_3 + R_1 \to R_1$$

$$\left(\begin{array}{rrr|r} 1 & -1 & 0 & 5 \\ 0 & 4 & 0 & -4 \\ 0 & 0 & 1 & -3 \end{array}\right) \qquad \frac{1}{4}R_2 \to R_2$$

$$\left(\begin{array}{rrr|r} 1 & -1 & 0 & 5 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -3 \end{array}\right) \qquad R_2 + R_1 \to R_1$$

$$\left(\begin{array}{rrr|r} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -3 \end{array}\right)$$

Therefore, the solution is $(x_1, x_2, x_3) = (4, -1, -3)$, i.e. $x_1 = 4$, $x_2 = -1$, and $x_3 = -3$,

$$\begin{cases} x_1 = 4 \\ x_2 = -1 \\ x_3 = -3 \end{cases}$$

Problem 5. (10pt) Use WolframAlpha's `RowReduce` to find the solution to the following system of equations:

$$\begin{aligned} -6x_1 - x_2 + 7x_3 - 4x_4 + 3x_5 &= 83 \\ 2x_1 + 5x_2 + 2x_3 + 5x_4 + 7x_5 &= \frac{67}{3} \\ -8x_1 - x_2 - 9x_3 - 10x_4 &= -111 \\ 7x_1 - 6x_2 + 3x_3 - 5x_4 + 9x_5 &= \frac{97}{2} \\ 12x_1 - 4x_2 - x_3 + 5x_4 + 6x_5 &= 0 \end{aligned}$$

Solution. The associated matrix is...

$$\left(\begin{array}{rrrrr:r} -6 & -1 & 7 & -4 & 3 & 83 \\ 2 & 5 & 2 & 5 & 7 & 67/3 \\ -8 & -1 & -9 & -10 & 0 & -111 \\ 7 & -6 & 3 & -5 & 9 & 97/2 \\ 12 & -4 & -1 & 5 & 6 & 0 \end{array}\right)$$

Using WolframAlpha's `RowReduce` function, we obtain...

$$\left(\begin{array}{ccccc:c} 1 & 0 & 0 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 12 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1/3 \end{array}\right)$$

Therefore, the solution is $(x_1, x_2, x_3, x_4, x_5) = (1/2, -1, 12, 0, 1/3)$, i.e. $x_1 = \frac{1}{2}$, $x_2 = -1$, $x_3 = 12$, $x_4 = 0$, and $x_5 = \frac{1}{3}$:

$$\begin{cases} x_1 = \frac{1}{2} \\ x_2 = -1 \\ x_3 = 12 \\ x_4 = 0 \\ x_5 = \frac{1}{3} \end{cases}$$