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MATH 108
Spring 2022
Written HW 9: Due 04/11

"Economics has never been a science—and it is even less now than a few years ago."
–Paul A. Samuelson

Problem 1. (10pt) Bennie Factor is setting up a college fund for his children. He will deposit $1,500 every 6 months into an account that earns 4.6% annual interest, compounded semiannually. How much is in the account after 18 years?

Solution. This is an annuity. We have six-month payments of $R = 1500$, an annual interest rate of $r = 0.046$ (so $i_p = r/k = 0.023$), and number of compounds $n = 18 \cdot 2 = 36$. We then have...

$$s_{\overline{n}|\, i_p} = s_{\overline{36}|\, 0.023} = \frac{(1 + 0.023)^{36} - 1}{0.023} = 55.1031$$

so that we have...

$$F = R\, s_{\overline{36}|\, 0.023} = 1500(55.1031) = 82654.7074 \approx \$82,654.71$$

Problem 2. (10pt) Holly Wood is saving money for her senior project in film school. To produce her film, she needs \$6,000 in 18 months. If she makes equal monthly payments into an account earning 2.3% annual interest, compounded monthly, how much money does she need to deposit into the account each month?

Solution. This is an annuity. We have future value $F = 6000$, annual interest $r = 0.023$ (so that $i_p = r/k = 0.00191667$), and number of payments $n = 18$. We then have...

$$s_{\overline{n}|i_p} = s_{\overline{18}|0.00191667} = \frac{(1 + 0.00191667)^{18} - 1}{0.00191667} = 18.2963$$

so that we have...

$$R = \frac{F}{s_{\overline{18}|0.00191667}} = \frac{6000}{18.2963} = 327.936 \approx \$327.94$$

Problem 3. (10pt) Jack Pott wins a $125,000 lottery. To make the money last as long as possible, he deposits this money into an account that earns 8% annual interest, compounded quarterly. If he wants to withdraw money from this account monthly so that the money lasts 10 years, what should he withdraw from the account each quarter?

Solution. This is an annuity. We have present value $P = 125000$, annual interest $r = 0.08$ (so that $i_p = r/k = 0.08/4 = 0.02$), and number of compounds $n = 4 \cdot 10 = 40$. We then have...

$$a_{\overline{n}|i_p} = a_{\overline{40}|0.02} = \frac{1-(1+0.02)^{-40}}{0.02} = 27.3554$$

so that we have...

$$R = \frac{P}{a_{\overline{120}|0.02}} = \frac{125000}{27.3554} = 4569.568 \approx \$4,569.47$$

Problem 4. (10pt) To purchase a new car, Horace Cope takes out a loan for \$9,000 at a rate of 8.5% annual interest, compounded monthly. He will make equal monthly payments on this loan for 2 years. What are the monthly payments?

Solution. This is an amortization. We have principal $P = 9000$, annual interest $r = 0.085$ (so that $i_p = r/k = 0.00708333$), and number of compounds $n = 2 \cdot 12 = 24$. We then have...

$$a_{\overline{n}|i_p} = a_{\overline{24}|0.00708333} = \frac{1 - (1 + 0.00708333)^{-24}}{0.00708333} = 21.9995$$

so that we have...

$$R = \frac{P}{a_{\overline{24}|0.00708333}} = \frac{9000}{21.9995} = 409.101 \approx \$409.10$$