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MATH 108
Spring 2024
HW 15: Due 04/08

"Yeah. Yeah, I... I can see this. I mean, it's not for me, but people will like it. It's Starbucks. It's what American wants."
— Matthew MacDell, Big Mouth

Problem 1. (10pts) Define the following:

$$\mathbf{u} = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 4 \end{pmatrix}, \qquad \mathbf{v} = \begin{pmatrix} 1 \\ -3 \\ 8 \\ 2 \end{pmatrix}, \qquad \mathbf{w} = \begin{pmatrix} 6 \\ -2 \\ -1 \\ 0 \end{pmatrix}$$

Showing all your work, compute the following:

(a) $-3\mathbf{w}$

(b) $\mathbf{v} - \mathbf{u}$

(c) $\mathbf{u} \cdot \mathbf{w}$

Solution.

(a)

$$-3\mathbf{w} = -3 \begin{pmatrix} 6 \\ -2 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -18 \\ 6 \\ 3 \\ 0 \end{pmatrix}$$

(b)

$$\mathbf{v} - \mathbf{u} = \begin{pmatrix} 1 \\ -3 \\ 8 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 1-1 \\ -3-0 \\ 8-(-1) \\ 2-4 \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \\ 9 \\ -2 \end{pmatrix}$$

(c)

$$\mathbf{u} \cdot \mathbf{w} = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -2 \\ -1 \\ 0 \end{pmatrix} = 1(6) + 0(-2) + (-1)(-1) + 4(0) = 6 + 0 + 1 + 0 = 7$$

Problem 2. (10pts) Define the following:

$$A = \begin{pmatrix} -1 & 2 & 0 \\ 0 & 6 & -2 \end{pmatrix}, \qquad B = \begin{pmatrix} 6 & -3 & -1 \\ 1 & 1 & 0 \end{pmatrix}, \qquad C = \begin{pmatrix} 0 & 2 & -5 \\ 6 & 0 & 4 \end{pmatrix}$$

Showing all your work, compute the following:

(a) $3A$

(b) $B - A$

(c) CA^T

Solution.

(a)

$$3A = 3\begin{pmatrix} -1 & 2 & 0 \\ 0 & 6 & -2 \end{pmatrix} = \begin{pmatrix} -3 & 6 & 0 \\ 0 & 18 & -6 \end{pmatrix}$$

(b)

$$B - A = \begin{pmatrix} 6 & -3 & -1 \\ 1 & 1 & 0 \end{pmatrix} - \begin{pmatrix} -1 & 2 & 0 \\ 0 & 6 & -2 \end{pmatrix} = \begin{pmatrix} 6-(-1) & -3-2 & -1-0 \\ 1-0 & 1-6 & 0-(-2) \end{pmatrix} = \begin{pmatrix} 7 & -5 & -1 \\ 1 & -5 & 2 \end{pmatrix}$$

(c)

$$\begin{aligned}
CA^T &= \begin{pmatrix} 0 & 2 & -5 \\ 6 & 0 & 4 \end{pmatrix} \begin{pmatrix} -1 & 2 & 0 \\ 0 & 6 & -2 \end{pmatrix}^T \\
&= \begin{pmatrix} 0 & 2 & -5 \\ 6 & 0 & 4 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 2 & 6 \\ 0 & -2 \end{pmatrix} \\
&= \begin{pmatrix} 0(-1) + 2(2) + (-5)0 & 0(0) + 2(6) + (-5)(-2) \\ 6(-1) + 0(2) + 4(0) & 6(0) + 0(6) + 4(-2) \end{pmatrix} \\
&= \begin{pmatrix} 0+4+0 & 0+12+10 \\ -6+0+0 & 0+0-8 \end{pmatrix} \\
&= \begin{pmatrix} 4 & 22 \\ -6 & -8 \end{pmatrix}
\end{aligned}$$

Problem 3. (10pts) Define the following:

$$A = \begin{pmatrix} 1 & -1 \\ 0 & 3 \\ -4 & 2 \\ 0 & 6 \end{pmatrix}, \qquad \mathbf{u} = \begin{pmatrix} 4 \\ -2 \\ 0 \\ 1 \end{pmatrix}$$

(a) Can one compute $A\mathbf{u}$? If so, compute it. If not, explain why.

(b) Can one compute $A^T\mathbf{u}$? If so, compute it. If not, explain why.

Solution.

(a) No, we cannot compute $A\mathbf{u}$. To multiply a $m \times n$ matrix with a $r \times s$ matrix, it must be that $n = r$, i.e. the number of columns of the first must be the number of rows of the second. The matrix A is 4×2 and the matrix/vector $\mathbf{u}$ is 4×1. Because $n = 2 \neq 4 = r$, we cannot form $A\mathbf{u}$.

(b) The matrix A^T switches the rows and columns of A. But then A^T is a 2×4 matrix. But then one can form $A^T\mathbf{u}$. We have...

$$\begin{aligned}
A^T\mathbf{u} &= \begin{pmatrix} 1 & -1 \\ 0 & 3 \\ -4 & 2 \\ 0 & 6 \end{pmatrix}^T \begin{pmatrix} 4 \\ -2 \\ 0 \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 & -4 & 0 \\ -1 & 3 & 2 & 6 \end{pmatrix} \begin{pmatrix} 4 \\ -2 \\ 0 \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} 1(4) + 0(-2) + (-4)0 + 0(1) \\ -1(4) + 3(-2) + 2(0) + 6(1) \end{pmatrix} \\
&= \begin{pmatrix} 4 + 0 + 0 + 0 \\ -4 - 6 + 0 + 6 \end{pmatrix} \\
&= \begin{pmatrix} 4 \\ -4 \end{pmatrix}
\end{aligned}$$