

**Check-In 08/22.** (*True/False*) Let  $f(x)$  be a relation with  $f(2) = 7$  and  $f(-3) = 7$ . Because  $f(2)$  and  $f(-3)$  are both 7,  $f$  cannot be a function.

**Solution.** The statement is *false*. A relation is a function if there is only one possible output for a given input, i.e. given an input, one knows with certainty what the output is. We know that  $f(2) = 7$  and  $f(-3) = 7$ ; that is, given the inputs of  $x = 2$  or  $x = -3$ , we know the output. The fact that the outputs are the same is irrelevant. There are many functions with the property that  $f(2) = 7$  and  $f(-3) = 7$ . For instance, there must be a linear function through these two points, i.e.  $y = 7$ . An example of a quadratic function through these points is  $y = \frac{7x(x+1)}{6}$ .

**Check-In 08/27.** (*True/False*) If  $S(t) = 0.008t + 57.81$  represents the stock price for a company  $t$  minutes after opening, then the rate of change of the stock value is 0.008, i.e. the stock is gaining \$0.008 per minute in value, and the opening price of the stock was \$57.81.

**Solution.** The statement is *false*. The stock price at opening would be the stock price at  $t = 0$ . But  $S(0) = 0.008(0) + 57.81 = 57.81$ . Therefore, the opening stock price was \$57.81. Observe that  $S(t)$  is a linear function, i.e. a function of the form  $y = mx + b$  with  $y = S$ ,  $x = t$ ,  $m = 0.008$ , and  $b = 57.81$ . We know the rate of change of a linear function is its slope. But then the rate of change of  $S(t)$  is  $m = 0.008$ , i.e. there is an increase of \$0.008 per minute in the value of the stock.

**Check-In 08/29.** (*True/False*) If the production cost of a certain item is constant, then the cost to produce  $q$  items,  $C(q)$  is linear. Furthermore, the slope of  $C(q)$  is the marginal cost and  $C(0)$  is the fixed cost.

**Solution.** The statement is *true*. If the cost of production for the item is constant, then the production cost has a constant rate of change. But then the cost function to produce  $q$  items,  $C(q)$ , must be linear. We know the marginal cost for a linear cost function is its slope. Furthermore,  $C(0)$  is the fixed costs. But the  $y$ -intercept of  $C(q)$  is precisely  $C(0)$ .

**Check-In 09/03.** (*True/False*) Let  $f(x) = 17(0.93)^x$ . Because  $f(x)$  the form  $Ab^x$  with  $A = 17$  and  $b = 0.93$ , it is exponential. Furthermore,  $A = 17$  represents the  $y$ -intercept of 17, i.e. an initial value of 17, and  $b = 0.93$  can be interpreted as a 93% decrease of the initial value of 17 a total of  $x$ -times.

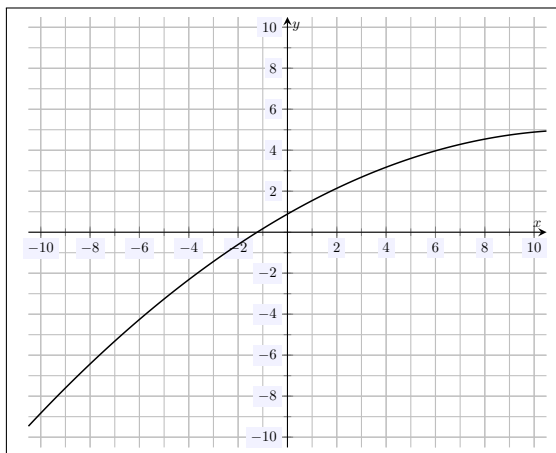
**Solution.** The statement is *false*. An exponential function is a function of the form  $Ab^x$ . Therefore,  $f(x) = 17(0.93)^x$  is an exponential function with  $A = 17$  and  $b = 0.93$ . We know that  $A = 17$  is the  $y$ -intercept because  $f(0) = 17(0.93)^0 = 17(1) = 17$ . We know that for any exponential function, we can interpret  $b$  as a percentage increase/decrease. We know that  $0 < b < 1$ . Therefore, we know that  $f(x)$  is exponentially decreasing. We have  $b = 0.93 = 1 - 0.07$ . Therefore, we can interpret  $f(x)$  as a 7% decrease of the initial value of 17 a total of  $x$ -times.

**Check-In 09/05.** (True/False) Because multiplication is commutative,  $(f \circ g)(x) = (g \circ f)(x)$ .

**Solution.** The statement is *false*. It is true that multiplication is commutative. However,  $f \circ g$  does not denote multiplication but rather function composition. We know that  $(f \circ g)(x) = f(g(x))$ . There is no need for  $(f \circ g)(x) = (g \circ f)(x)$ . Although it can happen, it is certainly (typically) false. For instance, if  $f(x) = 0$  and  $g(x) = 1$ . Then  $(f \circ g)(x) = f(g(x)) = f(1) = 0$  and  $(g \circ f)(x) = g(f(x)) = g(0) = 1$ .

**Check-In 09/19.** (True/False) If  $f(x)$  is a function which is twice differentiable and  $f'(x) > 0$ , then  $f''(x) > 0$ .

**Solution.** The statement is *false*. Recall that if  $f'(x) > 0$ , the function  $f(x)$  is increasing at that  $x$ -value, and if  $f'(x) < 0$ , the function  $f(x)$  is decreasing at that  $x$ -value. Furthermore, recall that if  $f''(x) > 0$ , the function  $f(x)$  is concave up at that  $x$ -value, and if  $f''(x) < 0$ , the function  $f(x)$  is concave down at that  $x$ -value. Therefore, the question is asking if a function is increasing, does it have to be concave up. This is certainly not the case. For instance, consider the function  $f(x)$  shown below.



This function is clearly everywhere increasing, so that  $f'(x) > 0$ . However, observe that the function is concave down, so that  $f''(x) < 0$ . The sign of  $f'$  and  $f''$  do indeed give you information about  $f(x)$ . However, the signs of  $f$ ,  $f'$ , and  $f''$  do not need to be the same.