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MATH 111-I
Spring 2025
Quiz 7

Problem 1: Find the domain of the following functions:

(a) $f(x) = x^3 - 4x + 8$

(b) $g(x) = \dfrac{6}{x+9}$

(c) $h(x) = \sqrt{2x-5}$

(a) The domain of any polynomial is all real numbers. Therefore, the domain of $f(x)$ is all real numbers, i.e. $\mathbb{R}$ or $(-\infty, \infty)$.

(b) We know a 'fraction' is defined so long as the denominator is not 0. We have $x+9=0$, then $x=-9$. Therefore, the domain of $g(x)$ is all real numbers except for $x=-9$, i.e. $(-\infty,-9)\cup(-9,\infty)$ or $\mathbb{R}\setminus\{-9\}$.

(c) The domain of an even root is all inputs that are nonnegative, i.e. greater than or equal to 0. So, we need $2x-5\geq 0$. This implies that $2x\geq 5$ so that $x\geq\frac{5}{2}$. Therefore, the domain of $h(x)$ is the set of real numbers with $x\geq\frac{5}{2}$, i.e. $[\frac{5}{2},\infty)$.

Problem 2: Find the domain and range of the function $f(x) = 4-(x+1)^2$.

The function $f(x)$ is a quadratic function. The domain of a quadratic function—indeed, any polynomial—is the set of all real numbers, i.e. $\mathbb{R}$ or $(-\infty,\infty)$.

We can see that $a=-1<0$. [This is because $f(x)$ is in vertex form. Alternatively, we can expand $f(x)$: $4-(x+1)^2=4-(x+1)(x+1)=4-(x^2+x+x+1)=4-(x^2+2x+1)=-x^2-2x-3$, where we can see that $a=-1$.] But then $f(x)$ opens downwards, i.e. $f(x)$ has a maximum value but no minimum value. We can see from $f(x)$ that $f(x)$ has vertex $(-1,4)$. The vertex is the maximum y-value. But then the range must be $(-\infty,4]$.