The Mathical Adventures of Robbie the Red
Braking, Circuits, and Linear Equations

To receive full or partial credit, you must show all work on your own paper.

Robbie the Red Robot has attached his wheels this morning so that he can go tear up the pavement in a local parking lot. As Robbie is making his warm-up laps around the lot, a deer has the audacity to jump out in front of him and he must quickly hit the brakes! The following table shows Robbie's velocity (measured in meters per second) at two points in time since he applied the brakes.

Time t	Velocity $v(t)$
2 seconds	35.65 m/sec
4.5 seconds	15.15 m/sec

Robbie's velocity can be modeled by the equation $v(t) = at + v_0$ where a is Robbie's acceleration (measured in m/sec^2) and v_0 is his initial velocity (i.e. his velocity at the moment he applied the brakes). Ah, but the equation for $v(t)$ should remind us of something in math! Indeed, compare

$$v(t) = at + v_0 \qquad \text{to} \qquad y = mx + b, \qquad \text{the slope-intercept form of a line!}$$

Problem 1. *(6 points)* Noting that a corresponds to slope of a line and v_0 corresponds to its y-intercept,

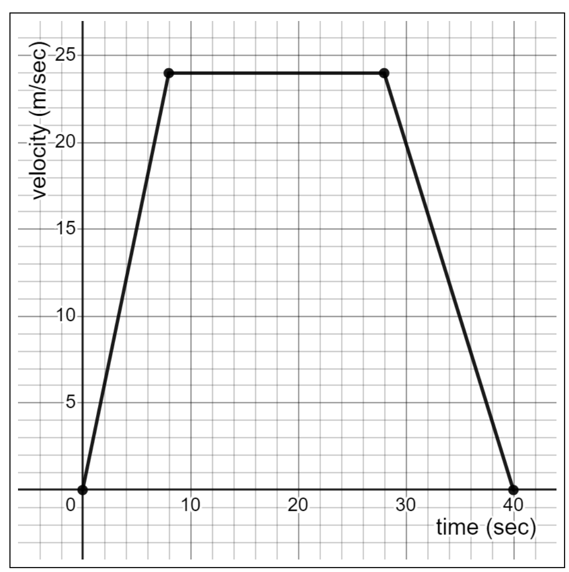
(a) Find the values of a and v_0 to fill out the equation for Robbie's velocity.

(b) Sketch a graph of the line described by $v(t)$.

(c) Identify/label on your graph the initial velocity, the acceleration, and the time it took Robbie to stop.

Through a stroke of luck, Robbie stops just short of hitting the deer, and Robbie decides to take a more measured approach to his driving for the rest of the day to avoid any more heart-pounding sudden stops.

The graph at right shows Robbie's velocity (in meters per second) as he nervously makes his next lap around the parking lot. (Some points have been marked on the graph for your convenience.)



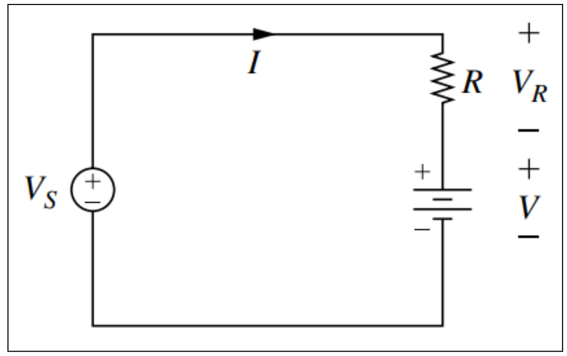
Explore. Using the graph at right,

(a) Describe in your own words how Robbie's velocity was changing over the 40-second interval.

(b) Write a piecewise-defined function $v(t)$ that describes Robbie's velocity in accordance with the graph.

As Robbie is returning home in the evening, his wheels sputter out and die. On running a self-diagnostic, Robbie discovers that one of his closed circuits (pictured below at right) has somehow stopped obeying Kirchhoff's voltage law (KVL)!

A quick refresher on Ohm's law and KVL is in order:



- **Ohm's law** states that the voltage drop V_R across a resistor (measured in volts) is equal to the current (measured in amperes) flowing through the resistor times the resistance R (measured in Ohms). In symbols, $V_R = IR$.

- **Kirchhoff's voltage law** states that the "sum of voltage rises" is equal to the "sum of voltage drops". (Voltage rise occurs when the current *flows out of* a "+" symbol, and voltage drop occurs when the current *flows into* a "+".)

In the case of Robbie's closed circuit, the current "flows out" of V_s and "flows into" R and then V. With this observation, KVL gives the following equation:

$$V_s = V_R + V, \quad \text{and Ohm's law transforms this into} \quad V_s = IR + V.$$

The resistance R and the "battery" V are constants here, so we'll treat V_s and I as the dependent and independent variable, respectively. Notice then that $V_s = RI + V$ bears a striking resemblance to $y = mx + b$.

Suppose that the source voltage V_s and the current I have value pairs as in the table below:

Current I	Voltage V_s
0.75 amperes	12 volts
1.5 amperes	18 volts

Problem 2. *(6 points)* If Robbie's rogue circuit were obeying Kirchhoff's voltage law,

(a) Use the table above to find values for R and V that fill out the equation $V_s = RI + V$. Then sketch a graph of this linear equation.

(b) The equation $V_s = RI + V$ expresses voltage as a function of current, but Robbie would like to interchange the roles of the dependent and independent variables. Assist him by rewriting this equation so that it expresses current as a function of voltage, and then sketch a graph of the new equation.