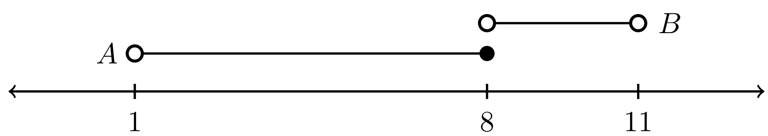
Check-In 08/22. *(True/False) True/False*: If $A = (1, 8]$ and $B = (8, 11)$, then $A \cap B = \{8\}$.

Solution. The statement is *false*. The set A is the set of numbers greater than 1 but at most 8, i.e. the numbers less than 8—including 8. Whereas B is the set of numbers greater than 8 but less than 11. The set $A \cap B$ is the set of elements in *both* A and B. The only real number that *could* be in both A and B is 8. However, 8 is in B but not in A. Therefore, $8 \notin A \cap B$. But then $A \cap B$ is empty, i.e. $A \cap B = \varnothing$. We can see this by sketching the intervals and seeing that there is no 'overlap.'



Check-In 08/26. *(True/False) True/False*: $\left(\dfrac{y^{-2}}{x^4x^2}\right)^{-3} = \dfrac{x^{24}}{y^5}$

Solution. The statement is *false*. Recall that $x^ax^b = x^{a+b}$, $(x^a)^b = x^{ab}$, and $x^{-1} = \frac{1}{x}$. Then we have...

$$\left(\frac{y^{-2}}{x^4x^2}\right)^{-3} = \left(\frac{y^{-2}}{x^6}\right)^{-3} = \frac{y^6}{x^{-18}} = x^{18}y^6$$

Check-In 08/27. *(True/False) True/False*: It is possible to have a right triangle with sides of length $4, 7, 12$.

Solution. The statement is *false*. We know a triangle with sides a, b, c is a right triangle if and only if $a^2 + b^2 = c^2$. We have...

$$\begin{gathered} a^2 + b^2 \stackrel{?}{=} c^2 \\ 4^2 + 7^2 \stackrel{?}{=} 12^2 \\ 16 + 49 \stackrel{?}{=} 144 \\ 65 \neq 144 \end{gathered}$$

Therefore, there exists no such right triangle. In fact, there does not even exist a triangle with such sides. A triangle with sides a, b, c exists if and only if the length of any pairs of sides of the triangle add up to greater than the third. [This is called the triangle inequality.] But $4 + 7 = 11 \not> 12$. Therefore, there does not exist a triangle with sides 4, 7, and 12.

Check-In 08/28. *(True/False) True/False*: Let $f(x)$ be a relation with $f(2) = 7$ and $f(-3) = 7$. Because $f(2)$ and $f(-3)$ are both 7, f cannot be a function.

Solution. The statement is *false*. A relation is a function if there is only one possible output for a given input, i.e. given an input, one knows with certainty what the output is. We know that $f(2) = 7$

and $f(-3) = 7$; that is, given the inputs of $x = 2$ or $x = -3$, we know the output. The fact that the outputs are the same is irrelevant. There are many functions with the property that $f(2) = 7$ and $f(-3) = 7$. For instance, there must be a linear function through these two points, i.e. $y = 7$. An example of a quadratic function through these points is $y = \frac{7x(x+1)}{6}$.

Check-In 08/29. *(True/False) True/False*: Let $f(x)$ be a function. The y-intercept of $f(x)$ is $f(0)$ and the x-intercept(s) of $f(x)$ are the x-values where $f(x) = 0$.

Solution. The statement is *true*. The y-intercept is where the graph of the function intersects the y-axis, which is the line $x = 0$. But then the y-intercept must be the function value at $x = 0$, i.e. $f(0)$. An x-intercept is where the graph of the function intersects the x-axis, which is the line $y = 0$. But this means the output of the function is zero. Therefore, an x-intercept is an x-value such that $f(x) = 0$.

Check-In 09/03. *(True/False)* If you are walking straight towards a building at a constant speed, then your distance from the building is given by a linear function. The y-intercept would represent your initial distance from the building and the slope would represent your walking speed. Furthermore, the x-intercept would represent the time you arrive at the building.

Solution. The statement is *true*. Let $D(t)$ denote your distance from the building. Because you are walking towards the building at a constant speed, the distance between you and the building is decreasing at a constant rate. But functions with a constant rate of change are linear. Therefore, $D(t)$ must be linear. We know that the y-intercept occurs when $t = 0$, i.e. the y-intercept is $D(0)$. But $D(0)$ is the distance you are at $t = 0$, i.e. the start. Therefore, $D(0)$ must denote your initial distance from the building. The slope of $D(t)$ represents the rate of change of your distance from the building. But this distance is only changing because you are walking towards the building. Therefore, the (absolute value of) the slope of $D(t)$ is your walking speed. Finally, an x-intercept is an input such that $D(t) = 0$. But $D(t) = 0$ implies that at the given time, your distance from the building is zero, i.e. you have arrived at the building. Therefore, the x-intercept of $D(t)$ represents the time you arrive at the building.

Check-In 09/04. *(True/False)* Because multiplication is commutative, $(f \circ g)(x) = (g \circ f)(x)$.

Solution. The statement is *false*. While *multiplication* is commutative, $f \circ g$ *does not* represent multiplication. Recall that $f \circ g$ represents function composition, i.e. $(f \circ g)(x) = f\big(g(x)\big)$. Function composition is *not* commutative. For instance, suppose that $f(x) = 5$ and $g(x) = -6$. Then for any x, $(f \circ g)(x) = f\big(g(x)\big) = f(-6) = 5$ and $(g \circ f)(x) = g\big(f(x)\big) = g(5) = -6$. But then $(f \circ g)(x) \neq (g \circ f)(x)$ for any x.

Check-In 09/05. *(True/False)* If $f(x)$ is a function, then $\dfrac{f(-5+h)-f(-5)}{h}$ represents the average rate of change for $f(x)$ on the interval containing -5 and $-5+h$. If $f(x)$ is linear, this is the slope of $f(x)$.

Solution. The statement is *true*. Recall the average rate of change for a function $f(x)$ on the interval $[a,b]$ is...

$$\text{Avg. ROC}_{[a,b]}\, f(x) = \frac{f(b)-f(a)}{b-a} = \frac{f(a)-f(b)}{a-b}$$

That is, the average rate of change for a function on the interval $[a,b]$ is the slope of the line segment through $\big(a, f(a)\big)$ and $\big(b, f(b)\big)$. Because we want the interval containing -5 and $-5+h$, i.e. $[-5,-5+h]$ or $[-5+h,-5]$ (depending on whether $h>0$ or not), the average rate of change on this interval is...

$$\frac{f(-5+h)-f(-5)}{(-5+h)-(-5)} = \frac{f(-5+h)-f(-5)}{-5+h+5} = \frac{f(-5+h)-f(-5)}{h}$$

Check-In 09/11. *(True/False)* The function $f(x)=6-(x+2)^2$ is quadratic. Furthermore, it is convex and has vertex $(2,6)$.

Solution. The statement is *false*. Observe that $f(x)=6-(x+2)^2=6-(x^2+4x+4)=6-x^2-4x-4=-x^2-4x+2$. Therefore, $f(x)$ is quadratic with $a=-1$, $b=-4$, and $c=2$. We can also see that $f(x)=6-(x+2)^2=-(x+2)^2+6$ is in vertex form—meaning $f(x)$ must be quadratic. Because $a=-1<0$, we know that $f(x)$ is concave—not convex. We know that if $a>0$, a quadratic is convex, and if $a<0$, a quadratic is concave. Furthermore, because $f(x)=-(x+2)^2+6=-\big(x-(-2)\big)^2+6$, the vertex of $f(x)$ is $(-2,6)$—not $(2,6)$. Recall, the x-coordinate of the vertex makes the 'square term' of the vertex form zero and the y-coordinate of the vertex is what remains after the square term is zero.

Check-In 09/12. *(True/False)* A factorization of $12x^2-77x+120$ is $(3x-8)(4x-15)$.

Solution. The statement is *true*. Finding the factorization of $12x^2-77x+120$ may be difficult. However, it is routine to verify that a proposed factorization is correct—this always allows us to easily check whether we have obtained a correct factorization of a polynomial:

$$(3x-8)(4x-15)=12x^2-45x-32x+120=12x^2-77x+120$$

Check-In 09/16. *(True/False)* $9-4x^2=(3-2x)(3+2x)$

Solution. The statement is *true*. This is a special type of factorization—the difference of perfect squares: $a^2-b^2=(a-b)(a+b)$. Here, we have $a=3$ and $b=2x$ because $a^2=9$ and $b^2=(2x)^2=4x^2$. But then $9-4x^2=(3-2x)(3+2x)$. While the *difference* of perfect squares factors, the *sum* of perfect squares, i.e. a^2+b^2, *never* factors over the real numbers.

Check-In 09/17. *(True/False)* If $f(x) = 2x^3 - 3x^2 + 4x - 5$, then the value of $f(-4)$ is the remainder of $f(x)$ when divided by $x + 4$.

Solution. The statement is *true*. Recall that if we divide a polynomial $f(a)$ by $x - a$, then $f(a)$ is the remainder when we divide $f(x)$ by $x - a$. Here we have $x + 4 = x - (-4)$. Therefore, $f(-4)$ is the remainder when we divide $f(x)$ by $x + 4$. We can also check this directly: $f(-4) = 2(-4)^3 - 3(-4)^2 + 4(-4) - 5 = -128 - 48 - 16 - 5 = -197$ and

$$
\begin{array}{rrrrr}
 & & 2x^2 & -11x & +48 \\ \cline{2-5}
x+4\,\big) & 2x^3 & -3x^2 & +4x & -5 \\
 & -2x^3 & -8x^2 & & \\ \cline{2-3}
 & & -11x^2 & +4x & \\
 & & 11x^2 & +44x & \\ \cline{3-4}
 & & & 48x & -5 \\
 & & & -48x & -192 \\ \cline{4-5}
 & & & & -197
\end{array}
$$

Alternatively, we could compute the division using synthetic division (because $x + 4$ is linear):

$$
\begin{array}{r|rrrr}
-4 & 2 & -3 & 4 & -5 \\
 & & -8 & 44 & -192 \\ \hline
 & 2 & -11 & 48 & -197
\end{array}
$$

Check-In 09/18. *(True/False)* The domain of $f(x) = \dfrac{x^2 - 1}{x^2 + 4x + 3}$ is $x \neq -3, \pm 1$.

Solution. The statement is *true*. The domain of a rational function, $f(x) = \frac{a(x)}{b(x)}$, are the values where $b(x) \neq 0$. We have $a(x) = x^2 - 1$ and $b(x) = x^2 + 4x + 3$. We have…

$$
\begin{aligned}
b(x) &= 0 \\
x^2 + 4x + 3 &= 0 \\
(x + 1)(x + 3) &= 0
\end{aligned}
$$

But then either $x + 1 = 0$, which implies $x = -1$, or $x + 3 = 0$, which implies $x = -3$. Therefore, the domain of $f(x)$ is all real numbers except $x = -3, -1$. The mistake made here is excluding the values where the numerator, $a(x)$, is also zero: if $a(x) = 0$, then $x^2 - 1 = 0$. But then $(x - 1)(x + 1) = 0$, which implies that $x - 1 = 0$, so that $x = 1$, or $x + 1 = 0$, so that $x = -1$. These values are zeros unless they are zeros shared by the denominator. Therefore, $x = 1$ is a zero of $f(x)$. However, $x = -1$ is not a zero of $f(x)$—it is not in the domain. Observe that if $x \neq -1$, then…

$$
f(x) = \frac{x^2 - 1}{x^2 + 4x + 3} = \frac{(x - 1)\cancel{(x + 1)}}{\cancel{(x + 1)}(x + 3)} = \frac{x - 1}{x + 3}
$$

Therefore, $x = -1$ corresponds to a hole of $f(x)$: $\frac{x-1}{x+3}\Big|_{x=-1} = \frac{-1-1}{-1+3} = \frac{-2}{2} = -1$ so that the hole is $(-1, -1)$.

Check-In 09/19. *(True/False)* The function $f(x) = 4\left(\frac{1}{3}\right)^{-x}$ grows exponentially and has y-intercept 4.

Solution. The statement is *true*. We rewrite $f(x)$:

$$f(x) = 4\left(\frac{1}{3}\right)^{-x} = 4\left(\left(\frac{1}{3}\right)^{-1}\right)^{x} = 4(3^x)$$

Therefore, $f(x)$ is exponential because it has the form Ab^x with $A = 4$ and $b = 3$. We know that $f(x)$ is growing exponentially because $b = 3 > 1$. The y-intercept of Ab^x is A. Therefore, because $A = 4$, $f(x)$ has y-intercept 4.