**Check-In 08/21.** (*True/False*) The integral  $\int x \sqrt[3]{x-2} dx$  can be treated as a 'shifting integral' by using the *u*-substitution u=x-2.

**Solution.** The statement is *true*. We 'want' to be able to distribute the x across the cube-root but we cannot—this is not a valid operation. However, if we make the u-substitution u=x-2, then we will be able to distribute in a way that makes this integral 'routine.' So, let u=x-2, then du=dx. Moreover, because u=x-2, we know that x=u+2. But then...

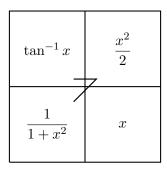
$$\int x \sqrt[3]{x-2} \ dx = \int (u+2) \sqrt[3]{u} \ du = \int \left( u^{4/3} + 2u^{1/3} \right) \ du = \frac{3}{7} u^{7/3} + \frac{3}{4} \cdot 2u^{4/3} + C = \frac{3}{7} (x-2)^{7/3} + \frac{3}{2} (x-2)^{4/3} + C$$

Note that a computer algebra system may write the answer (though you will *not* be expected to) like this:

$$\tfrac{3}{7}(x-2)^{7/3} + \tfrac{3}{2}(x-2)^{4/3} + C = (x-2)^{4/3} \left( \tfrac{3}{7}(x-2) + \tfrac{3}{2} \right) + C = (x-2)^{4/3} \left( \tfrac{3}{7}x + \tfrac{9}{14} \right) + C = \tfrac{3}{14}(x-2)^{4/3} \left( 2x + 3 \right) + C$$

**Check-In 08/26.** (*True/False*) Using integration-by-parts to evaluate  $\int x \tan^{-1}(x) dx$ , one chooses  $u = \tan^{-1} x$  and dv = x.

**Solution.** The statement is *true*. Using LIATE, the first term that appears is 'I' for inverse trig. Therefore, we choose  $u = \tan^{-1} x$ . But then dv = x. We then fill in our box:



Using the 'rule of 7', we have...

$$\int x \tan^{-1} x \, dx = \frac{1}{2} x^2 \tan^{-1} x - \frac{1}{2} \int \frac{x^2}{1 + x^2} \, dx$$

We now need only evaluate the integral on the right. Dividing  $1+x^2$  into  $x^2$ , we have a remainder of -1, i.e.  $\frac{x^2}{1+x^2}=1+\frac{-1}{1+x^2}$ . Therefore, we have...

$$\frac{1}{2} \int \frac{x^2}{1+x^2} dx = \frac{1}{2} \int \left(1 + \frac{-1}{1+x^2}\right) dx = \frac{1}{2} \left(x - \tan^{-1} x\right) + C$$

But then...

$$\int x \tan^{-1} x \, dx = \frac{1}{2} x^2 \tan^{-1} x - \frac{1}{2} \int \frac{x^2}{1+x^2} \, dx$$

$$= \frac{1}{2} x^2 \tan^{-1} x - \frac{1}{2} (x - \tan^{-1} x) + C$$

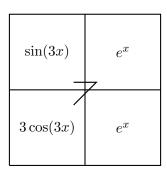
$$= \frac{1}{2} x^2 \tan^{-1} x - \frac{1}{2} x + \frac{1}{2} \tan^{-1} x + C$$

$$= \frac{x^2 \tan^{-1} x - x + \tan^{-1} x}{2} + C$$

$$= \frac{(x^2 + 1) \tan^{-1} x - x}{2} + C$$

**Check-In 08/28.** (*True/False*) The integral  $\int e^x \sin(3x) dx$  can be treated as an integration-by-parts 'looping' integral.

**Solution.** The statement is *true*. Using integration-by-parts for  $\int e^x \sin(3x) \, dx$  would result in an integral that would 'loop' back to itself. Generally, an integrand of the form exponential  $\cdot$  (sin or cos) or trig  $\cdot$  trig will have this property. Using traditional integration-by-parts, by LIATE, we choose  $u = \sin(3x)$  and  $dv = e^x$ . Filling out our box, we have...

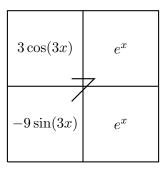


Using the 'rule of seven', we then have...

$$\int e^x \sin(3x) \ dx = e^x \sin(3x) - \int 3e^x \cos(3x) \ dx$$

To integrate  $\int 3e^x \cos(3x) dx$ , we again use integration-by-parts. Using LIATE, we choose u =

 $3\cos(3x)$  and  $dv=e^x$ . Filling out the box, we have...



Using the 'rule of seven', we then have

$$\int 3e^x \cos(3x) \ dx = 3e^x \cos(3x) - \int -9e^x \sin(3x) \ dx = 3e^x \cos(3x) + 9 \int e^x \sin(3x) \ dx$$

But then we have...

$$\int e^x \sin(3x) \ dx = e^x \sin(3x) - \int 3e^x \cos(3x) \ dx = \int e^x \sin(3x) \ dx = e^x \sin(3x) - \left(3e^x \cos(3x) + 9 \int e^x \sin(3x) \ dx\right) = \int e^x \sin(3x) \ dx = \int e^x$$

Therefore, we have...

$$\int e^x \sin(3x) \, dx = e^x \sin(3x) - \left(3e^x \cos(3x) + 9 \int e^x \sin(3x) \, dx\right)$$

$$\int e^x \sin(3x) \, dx = e^x \sin(3x) - 3e^x \cos(3x) - 9 \int e^x \sin(3x) \, dx$$

$$10 \int e^x \sin(3x) \, dx = e^x \sin(3x) - 3e^x \cos(3x)$$

$$\int e^x \sin(3x) \, dx = \frac{e^x \sin(3x) - 3e^x \cos(3x)}{10} + C$$

$$\int e^x \sin(3x) \, dx = \frac{e^x}{10} \left(\sin(3x) - 3\cos(3x)\right) + C$$

Alternatively, we can use an alternation of the tabular method of integration-by-parts. We choose  $u = \sin(3x)$  and  $dv = e^x$ . We then have...

$$\begin{array}{c|c}
u & dv \\
\hline
\sin(3x) & + e^x \\
3\cos(3x) & - e^x \\
-9\sin(3x) & + e^x
\end{array}$$

Therefore, we have...

$$\int e^x \sin(3x) \, dx = e^x \sin(3x) - 3\cos(3x)e^x - 9 \int e^x \sin(3x) \, dx$$

Solving for our integral, we have...

$$\int e^x \sin(3x) \, dx = e^x \sin(3x) - 3\cos(3x)e^x - 9 \int e^x \sin(3x) \, dx$$

$$10 \int e^x \sin(3x) \, dx = e^x \sin(3x) - 3e^x \cos(3x)$$

$$\int e^x \sin(3x) \, dx = \frac{e^x \sin(3x) - 3e^x \cos(3x)}{10} + C$$

$$\int e^x \sin(3x) \, dx = \frac{e^x}{10} \left( \sin(3x) - 3\cos(3x) \right) + C$$