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MATH 111-I
Spring 2025
Quiz 3

Problem 1: Compute the average rate of change for $f(x) = 1 - 3x$ on the interval $[-1, 1]$. Show all your work.

The average rate of change for $f(x)$ on an interval $[a, b]$ is the slope of the line through the endpoints, i.e. $\frac{f(b)-f(a)}{b-a}$.

$$f(1) = 1 - 3(1) = 1 - 3 = -2$$
$$f(-1) = 1 - 3(-1) = 1 + 3 = 4$$
$$m = \frac{f(b) - f(a)}{b - a} = \frac{f(1) - f(-1)}{1 - (-1)} = \frac{-2 - 4}{1 + 1} = \frac{-6}{2} = -3$$

Therefore, the average rate of change is -3. Alternatively, observe that $f(x) = 1 - 3x$ is linear because it has the form $y = mx + b$ with $y = f(x)$, $x = x$, $m = -3$, and $b = 1$. We know the average rate of change of a line is its slope. The slope of the line $f(x)$ is $m = -3$; therefore, the average rate of change must be -3.

Problem 2: A physicist is tracking the temperature of a metal rod as a heat pulse is 'injected' into the rod. The physicist observes that the rate of change in the temperature in the rod is constant. They will build a model for the temperature of the rod (in Kelvin), $K(t)$, t minutes from now.

(a) Explain why $K(t)$ is linear.

We know the rate of change in the temperature in the rod is constant. But functions with a constant rate of change are linear. Therefore, it must be that $K(t)$ is linear.

(b) Suppose that $K(t) = 0.9t + 297$. Find and interpret the slope of $K(t)$.

We see that $K(t)$ has the form $y = mx + b$ with $y = K(t)$, $x = t$, $m = 0.9$, and $b = 297$. Therefore, the slope is $m = 0.9$. We know that $m = \frac{\Delta output}{\Delta input} = \frac{\Delta temperature}{\Delta time} = \frac{0.9}{1}$. Therefore, every 1 increase in minutes results in an increase of 0.9 K in temperature, i.e. the rod's temperature is increasing by 0.9 K every minute.

(c) Still assuming $K(t) = 0.9t + 297$, find and interpret the y-intercept of $K(t)$.

We see that $K(t)$ has the form $y = mx + b$ with $y = K(t)$, $x = t$, $m = 0.9$, and $b = 297$. Therefore, the slope is $m = 0.9$. We know that $b = 297$ is the y-intercept, i.e. $K(0) = 297$. But then the temperature $t = 0$ minutes from now is 297, i.e. the initial temperature of the rod is 297 K.

(d) Assuming $K(t)$ is given as above, compute $K(10)$. Explain what $K(10)$ represents.

We have $K(10) = 0.9(10)+297 = 9+297 = 306$. But then the temperature of the rod $t = 6$ minutes from now is 306 K, i.e. the temperature of the rod in 6 minutes will be 306 K.