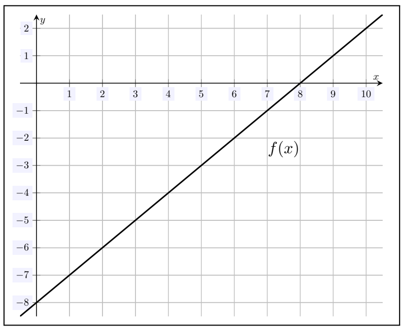
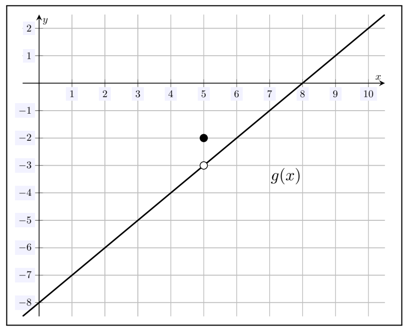
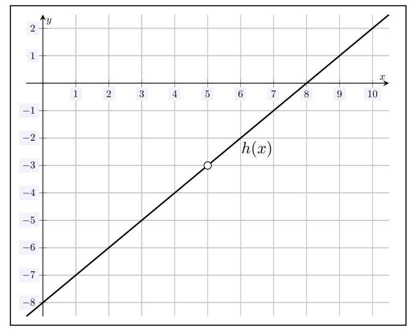
Check-In 08/22. *(True/False)* If $\lim\limits_{x\to 5} f(x) = -3$, then $f(5) = -3$.

Solution. The statement is *false*. A function's limit (if it even exists) *does not* have to be the same as the function's value at that limiting value—the function does not even have to be defined there! Consider the three examples below.







For the graph of $f(x)$ on the left, $\lim\limits_{x\to 5} f(x) = -3$, so they are equal. However, observe that for $g(x)$ (the middle graph), we have $\lim\limits_{x\to 5} g(x) = -3$ but $g(5) = -2$, so that $\lim\limits_{x\to 5} g(x) \neq g(5)$. Similarly, in the graph of $h(x)$ on the right, $\lim\limits_{x\to 5} h(x) = -3$ but $h(-3)$ is not defined, so that $\lim\limits_{x\to 5} h(x) \neq h(-3)$. A function's value (if even defined) need not be related to its limit (if the limit even exists).

Check-In 08/25. *(True/False)* The limit $\lim\limits_{x\to 0} \dfrac{\sin x}{x} = \text{DNE}$ because $\frac{\sin x}{x}$ becomes $\frac{0}{0}$ when one 'plugs-in' $x = 0$ and $\frac{0}{0}$ is undefined.

Solution. The statement is *false*. In fact, $\lim\limits_{x\to 0} \dfrac{\sin x}{x} = 1$. Yes, $\frac{0}{0}$, $\pm\frac{\infty}{\infty}$, $0 \cdot \infty$, $\infty - \infty$, 0^0, 1^∞, and ∞^0 are indeterminant/undefined expressions. However, that does not mean that the limit they arise from does not exist. The limit could exist or not—just like *any* limit. 'Running into' one of these expressions when evaluating a limit at its limiting value only means that one needs to try a different approach to determine if the limit exists or not.

Check-In 08/27. *(True/False)* $\lim\limits_{x\to 0} \dfrac{\tan(5x)}{5x} = 1$

Solution. The statement is *true*. Recall that $\lim\limits_{x\to 0} \dfrac{\sin(x)}{x} = 1$. We should think of this as $\lim\limits_{\square\to 0} \dfrac{\sin \square}{\square} = 1$. But then. . .

$$\lim_{x\to 0} \frac{\tan(5x)}{5x} = \lim_{x\to 0} \frac{\frac{\sin(5x)}{\cos(5x)}}{5x} = \lim_{x\to 0} \frac{\sin(5x)}{5x\cos x} = \lim_{x\to 0} \left(\frac{\sin(5x)}{5x} \cdot \frac{1}{\cos(5x)} \right)$$

Using the fact that $\lim\limits_{\square\to 0} \dfrac{\sin \square}{\square} = 1$, we then have. . .

$$\lim_{x\to 0} \frac{\tan(5x)}{5x} = \lim_{x\to 0} \left(\frac{\sin(5x)}{5x} \cdot \frac{1}{\cos(5x)} \right) = 1 \cdot \frac{1}{\cos(0)} = 1 \cdot \frac{1}{1} = 1$$

Check-In 08/29. *(True/False)* $\lim\limits_{x \to 2^+} \dfrac{x-3}{x-2} = \infty$

Solution. The statement is *false*. Observe that plugging-in $x = 2$, we obtain $\frac{-1}{0}$, which is undefined. However, this indicates that this is a 'thinking limit'. Because in the limit, x is 'close' to 2, $x - 3$ is negative. As we approach 2 from the right, $x > 2$ so that $x - 2$ is positive. But then...

$$\lim_{x \to 2^+} \frac{\overbrace{x-3}^{-}}{\underbrace{x-2}_{+}} = -\infty$$