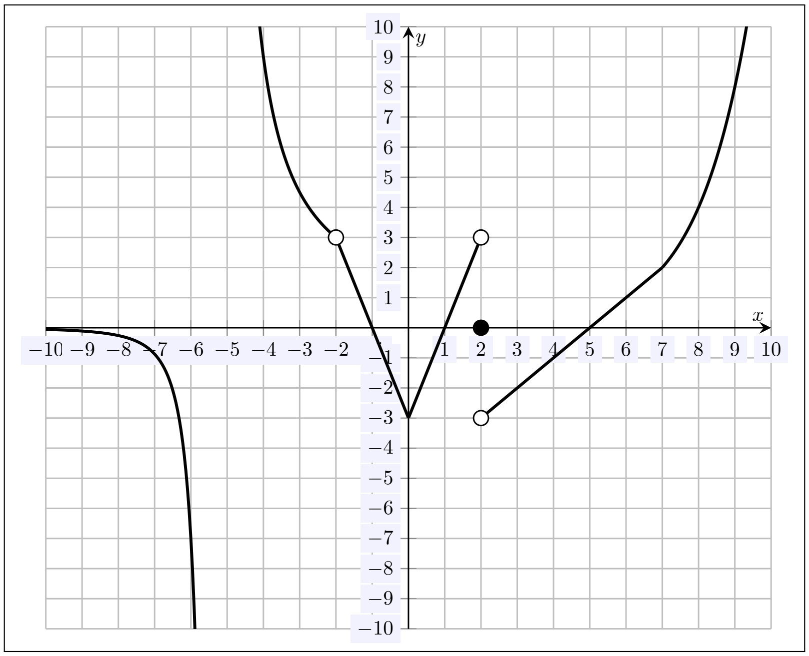
MATH 141 — Fall 2024
Exam 1 Review

Problem 1. Use the plot of the function $f(x)$ below to answer the following questions:



(a) $f(2)$

(b) $\lim\limits_{x\to 2^-} f(x)$

(c) $\lim\limits_{x\to 2^+} f(x)$

(d) $\lim\limits_{x\to 2} f(x)$

(e) $\lim\limits_{x\to -2^-} f(x)$

(f) $\lim\limits_{x\to -2^+} f(x)$

(g) $\lim\limits_{x\to -2} f(x)$

(h) $\lim\limits_{x\to -\infty} f(x)$

(i) $\lim\limits_{x\to \infty} f(x)$

(j) What is the y-intercept of $f(x)$?

(k) What are the zeros of $f(x)$?

(l) If $f(x)$ has any vertical asymptotes, give their equation.

(m) Where is $f(x)$ continuous?

(n) List at least 4 values for x at which $f(x)$ is not differentiable.

Problem 2. Showing all your work, compute the following limits:

(a) $\lim\limits_{h\to 0} \dfrac{(9+h)^2 - 81}{h}$

(b) $\lim\limits_{w\to \infty} \dfrac{\pi w^2 - w + 6}{2w^2 - 4}$

(c) $\lim\limits_{x\to 4} \dfrac{\sqrt{x+5} - 3}{x - 4}$

(d) $\lim\limits_{b\to -\infty} \dfrac{x^7 + x^2 - 6}{x^2 + 5x - 3}$

(e) $\lim\limits_{x\to 0} \dfrac{3x}{\sin x}$

(f) $\lim\limits_{x\to \infty} \left(1 + \dfrac{2}{x}\right)^x$

Problem 3. Showing all your work, compute the following limits:

(a) $\displaystyle\lim_{c\to 1} \frac{x^2}{\pi x^2 - \sqrt[3]{x} + 8}$

(b) $\displaystyle\lim_{x\to 0^-} \frac{(x+3)^2 - 9}{x}$

(c) $\displaystyle\lim_{x\to 0} \frac{\sin x}{\sqrt[3]{x}}$

(d) $\displaystyle\lim_{x\to 3^+} \frac{x^2+5x+6}{x^2+2x+1}$

(e) $\displaystyle\lim_{a\to\infty} \left(1+\frac{1}{a}\right)^{3a}$

(f) $\displaystyle\lim_{x\to 0} \frac{x^2 - x\cos x}{x}$

(g) $\displaystyle\lim_{x\to 0} \frac{5x}{\sqrt{x+7}-\sqrt{7}}$

(h) $\displaystyle\lim_{x\to\infty} \arctan(x)$

(i) $\displaystyle\lim_{x\to 4} \cos^2\left(\frac{5\pi}{x}\right)$

(j) $\displaystyle\lim_{x\to\infty} x^3 \sin\left(\frac{1}{x^3}\right)$

(k) $\displaystyle\lim_{x\to\infty} \frac{1-x^2}{x^4-1}$

(l) $\displaystyle\lim_{b\to 0} \frac{\frac{1}{2+b}-\frac{1}{b}}{b}$

(m) $\displaystyle\lim_{x\to\infty} \frac{x^6}{(2x^2-1)^3}$

(n) $\displaystyle\lim_{x\to\infty} \frac{x^2-5x+6}{x^3-8x+9}$

(o) $\displaystyle\lim_{x\to-\infty} 7^x$

(p) $\displaystyle\lim_{r\to 0} r^8 \sin\left(\frac{e^r}{r^8}\right)$

(q) $\displaystyle\lim_{x\to 1^-} \frac{5-2x}{x-1}$

(r) $\displaystyle\lim_{x\to 0} x\sin x$

(s) $\displaystyle\lim_{h\to -4^+} \frac{|h+4|}{5-h}$

(t) $\displaystyle\lim_{h\to -3^-} \frac{h+10}{|h+3|}$

(u) $\displaystyle\lim_{x\to\infty} \frac{(x+1)(x-3)(x^2-5)}{x^2-6}$

(v) $\displaystyle\lim_{x\to 0^-} \left(1+\frac{1}{e^x}\right)^x$

(w) $\displaystyle\lim_{x\to\pi^+} \left(x^2-3^x\right)$

Problem 4. Decide whether the following statements are true or false. Be sure to justify your answer.

(a) If $f(x)$ and $g(x)$ are continuous at $x=a$, then $(f+g)(x)$ is continuous at $x=a$.

(b) If $f(x)$ is continuous at $x=a$, then $(f(x))^2$ is continuous at $x=a$.

(c) If $f(x)$ and $g(x)$ are everywhere continuous, then $(fg)(x)$ is everywhere continuous.

(d) If $f(x)$ and $g(x)$ are continuous at $x=a$, then $(f\circ g)(x)$ is continuous at $x=a$.

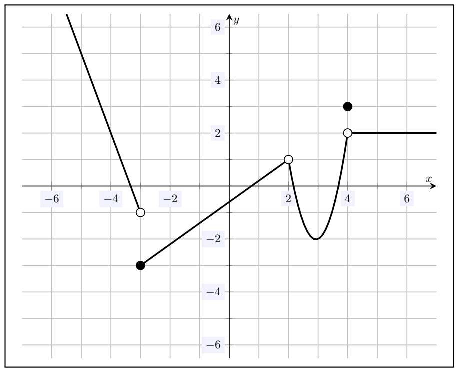
(e) If $f(x)$ is continuous at $x=a$, then $|f(x)|$ is continuous at $x=a$.

(f) If $|f(x)|$ is continuous at $x=a$, then $f(x)$ is continuous at $x=a$.

(g) If $f(x)$ and $g(x)$ are discontinuous at $x=a$, then $(f+g)(x)$ is discontinuous at $x=a$.

(h) If $f(x) < g(x)$ for $x>0$ and $\displaystyle\lim_{x\to\infty} f(x)$ and $\displaystyle\lim_{x\to\infty} g(x)$ exist, then $\displaystyle\lim_{x\to\infty} f(x) < \lim_{x\to\infty} g(x)$.

Problem 5. For the function $f(x)$, whose graph is shown in the figure below, compute the following limits. If the limit does not exist, write 'DNE.'



(a) $\lim_{x \to -3^-} f(x)$

(b) $\lim_{x \to -3^+} f(x)$

(c) $\lim_{x \to -3} f(x)$

(d) $\lim_{x \to 2^-} f(x)$

(e) $\lim_{x \to 2^+} f(x)$

(f) $\lim_{x \to 2} f(x)$

(g) $\lim_{x \to -\infty} f(x)$

(h) $\lim_{x \to \infty} f(x)$

Also, determine if $f(x)$ continuous at $x = 4$. Be sure to justify your answer using the definition of continuity.

Problem 6. Showing all your work, compute the following limits:

(a) $\lim_{x \to 8} \sin\left(\frac{\pi x}{2}\right)$

(b) $\lim_{h \to -2} \frac{h^2 - h - 2}{h - 2}$

(c) $\lim_{x \to 0} \frac{\frac{1}{x+5} - \frac{1}{5}}{x}$

(d) $\lim_{x \to 4} \left| \frac{x}{x+4} \right|$

(e) $\lim_{x \to 9} \frac{3 - \sqrt{x}}{x - 9}$

(f) $\lim_{x \to 4} \csc\left(\frac{11\pi}{x}\right)$

(g) $\lim_{x \to \infty} \frac{6 - 5x^2}{3x^2 + x - 9}$

(h) $\lim_{\theta \to 0} \frac{\cos\theta \tan\theta}{\theta}$

(i) $\lim_{x \to \infty} x \sin\left(\frac{1}{x}\right)$

(j) $\lim_{x \to \infty} \tan^{-1}(x - 2^x)$

(k) $\lim_{x \to 0} \left(1 + \frac{x}{5}\right)^{1/x}$

(l) $\lim_{x \to -4^-} \frac{3x + 5}{x + 4}$

(m) $\lim_{a \to 0} \frac{16 - (4 - a)^2}{a}$

(n) $\lim_{x \to -1^-} \frac{x^2}{x + 1}$

(o) $\lim_{j \to -7} \frac{|j + 1|}{j}$

(p) $\lim_{x \to \infty} \frac{x^5}{100x^4 + 6x^3 + 5x^2 - x + 9}$

(q) $\lim_{x \to 6} \frac{\sqrt{10 - x} - 2}{x - 6}$

Problem 7. Decide whether the following statements are true or false. Be sure to justify your answer.

(a) If neither $\lim_{x\to 2} f(x)$ nor $\lim_{x\to 2} g(x)$ exist, then $\lim_{x\to 2}(f(x)+g(x))$ does not exist.

(b) If neither $\lim_{x\to 2} f(x)$ nor $\lim_{x\to 2} g(x)$ exist, then $\lim_{x\to 2}(f(x)\cdot g(x))$ does not exist.

(c) If $\lim_{x\to\infty} f(x)=\infty$, then $\lim_{x\to\infty}(f(x)+g(x))=\infty$.

(d) If $\lim_{x\to 7} f(x)$ and $\lim_{x\to 7} g(x)$ exist, then $\lim_{x\to 7}(f(x)-g(x))$ must exist.

(e) If $\lim_{x\to 1} f(x)$ and $\lim_{x\to 1} g(x)$ exist, then $\lim_{x\to 1}(f(x)+g(x))$ must exist.

(f) If $\lim_{x\to 0} f(x)$ exists, then $f(0)$ exists.

(g) If $f(x)<g(x)$ for all x, then $\lim_{x\to 0} f(x)<\lim_{x\to 0} g(x)$.

(h) If $\lim_{x\to 9} f(x)=0$, then $\lim_{x\to 9} f(x)g(x)=0$.

(i) If $\lim_{x\to 7} f(x)=-9$, then $\lim_{x\to 7}|f(x)|=9$.

(j) If $\lim_{x\to 3^-} f(x)$ exists, then $\lim_{x\to 3^+} f(x)$ exists.

(k) If $\lim_{x\to 5} f(x)$ exists, then $\lim_{x\to 5} f(x)=f(5)$.

(l) If $\lim_{x\to 4}|f(x)|=L$, then $\lim_{x\to 4} f(x)=L$.

(m) If $\lim_{x\to -4^-} f(x)$ and $\lim_{x\to -4^+} f(x)$ exist, then $\lim_{x\to -4} f(x)$ exists.

(n) If $\lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)$, then $\lim_{x\to 0} f(x)$ exists.

(o) If $\lim_{x\to \pi} f(x)$ exists, then $\lim_{x\to \pi^+} f(x)$ exists.

(p) If $\lim_{x\to 1^-} f(x)$ exists, then $\lim_{x\to 1} f(x)$ exists.

(q) If $f(x)$ is continuous at $x=a$, then $\lim_{x\to a^-} f(x)=f(a)$.

(r) If $f(x)$ is everywhere continuous, then $\lim_{x\to -3} f(x^2)=f(9)$.

(s) *Any* continuous function is differentiable.

(t) *Any* differentiable function is continuous.

(u) If a function is differentiable, then it is differentiable everywhere.

(v) If $g(2)=0$, then $\frac{f(x)}{g(x)}$ has a vertical asymptote at $x=2$.

Problem 8. Showing all your work, compute the following limits:

(a) $\lim_{x\to 0^-} \dfrac{\sqrt{2x+5}-4}{x^2+1}$

(b) $\lim_{u\to 5} \dfrac{u+5}{u^2-2u+3}$

(c) $\lim_{x\to -\infty} e^x$

(d) $\lim_{a\to 0} \dfrac{a}{\sqrt{a+9}-3}$

(e) $\lim_{j\to -5} \dfrac{j^2+4j-5}{2j^2+13j+15}$

(f) $\lim_{x\to \infty} \dfrac{2^x}{6^x}$

(g) $\lim_{h\to 0} \dfrac{\sqrt{2+h}-\sqrt{2}}{h}$

(h) $\lim_{b\to 5} \cos\left(\pi - \dfrac{\pi b}{4}\right)$

(i) $\lim_{k\to 8} \dfrac{k^2-5k-24}{k^2-13k+40}$

(j) $\lim_{x\to \frac{3}{2}} \dfrac{2x^2+7x-15}{2x^2-15x+18}$

(k) $\lim_{x\to 0^-} \dfrac{x}{x^2-7x+9}$

(l) $\lim_{t\to 3} \dfrac{t-3}{3-\sqrt{12-t}}$

(m) $\lim_{c\to 0} \dfrac{5c}{\sin(7c)}$

(n) $\lim_{b\to -5^+} \dfrac{b+4}{|b+5|}$

(o) $\lim_{x\to \infty} \arctan\left(e^x\right)$

(p) $\lim_{x\to \infty} \left(1+\dfrac{\pi}{x}\right)^{\pi x}$

(q) $\lim_{x\to \infty} \arctan(1-x)$

(r) $\lim_{x\to 0} \dfrac{1-\cos(\sin x)}{\sin x}$

(s) $\lim_{x\to \infty} e^{2x}\sin\left(e^{-x}\right)$

(t) $\lim_{x\to 0} \dfrac{\sin(2x)}{\sin(\pi x)}$

(u) $\lim_{u\to 1} u\sec\left(\dfrac{u^2}{4}\right)$

(v) $\lim_{x\to \infty} \dfrac{1+8^x}{9^x}$

(w) $\lim_{u\to \infty} \dfrac{(2u-6)(3u-1)(5u+2)}{(u-1)(u+1)(7u-8)}$

(x) $\lim_{x\to -3} \dfrac{\frac{1}{6}+\frac{1}{x-3}}{x+3}$

(y) $\lim_{x\to -\infty} \dfrac{5-e^x}{9^x}$

(z) $\lim_{u\to 0} \dfrac{\tan(4u)}{\tan(8u)}$

Problem 9. Showing all your work, compute the following limits:

(a) $\lim_{x\to -\infty} \dfrac{x^5+4x^2+1}{x^2+6}$

(b) $\lim_{x\to 0^+} \dfrac{5x}{x^2-6x}$

(c) $\lim_{x\to 0^+} \tan^{-1}\left(\dfrac{1}{x}\right)$

(d) $\lim_{x\to -\infty} \dfrac{x^3+x-6}{x+6}$

(e) $\lim_{x\to \infty} 5^x$

(f) $\lim_{r\to 0} \dfrac{\tan^2 r}{r}$

(g) $\lim_{\ell\to -3} \dfrac{x^2+x-6}{x^2-9}$

(h) $\lim_{a\to \pi} a\sec a$

(i) $\lim_{y\to 0} \dfrac{y}{y^2-3y+4}$

(j) $\lim_{x\to 0^+} (x^2-5x+8)$

(k) $\lim_{y\to 1^-} ye^{\pi y}$

Problem 10. Decide whether the following statements are true or false. Be sure to justify your answer.

(a) If f, g are differentiable, then $\frac{d}{dx}(fg) = f'g'$.

(b) If a function is continuous, then it is differentiable.

(c) If a function is differentiable, then it is continuous.

(d) If $\lim_{x\to a} f(x)$ exists, then $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$ exist.

(e) If $\lim_{x\to a} f(x)$ exists, then $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)$.

(f) If $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$ exist, then $\lim_{x\to a} f(x)$ exists.

(g) If $\lim_{x\to a} f(x) = M$ and $\lim_{x\to a} g(x) = N$, then $\lim_{x\to a} \left(\frac{f(x)}{g(x)}\right) = \frac{M}{N}$.

(h) Polynomials are everywhere continuous.

(i) Rational functions are continuous everywhere.

(j) A tangent line to a function $f(x)$ intersects the function only once.

(k) A tangent line to a function $f(x)$ cannot intersect the function infinitely many times.

(l) $\lim_{x\to a} (f(x) + g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$.

(m) All continuous functions have at least one x-value at which they are differentiable.

(n) All functions on $\mathbb{R}$ have a limit at some x-value in their domain.

(o) If f, g are differentiable, then $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)}{g'(x)}$.

(p) A tangent line to a function $f(x)$ at $x = a$ has the same value of $f(x)$ at $x = a$.

(q) Every function has a tangent line wherever it is defined.

(r) If $g(x) < f(x)$ on (a, b), then $g'(x) < f'(x)$ on (a, b).

(s) If $\lim_{x\to\infty} f(x) = L$, i.e. x has a horizontal asymptote, then $\lim_{x\to-\infty} f(x) = L$.

(t) There is a function with a zero at $x = 0$ and a y-intercept of 6.

(u) If $f(x)$ is differentiable and decreasing on (a, b), then $f'(x) < 0$ on (a, b).

(v) If $f'(x) > 0$ on (a, b), then $f(x)$ is increasing on (a, b).

(w) If $f'(x) > 0$ on (a, b), then $f(x) > 0$ on (a, b).

(x) If $f(x)$ is continuous at $x = a$, then $\lim_{x\to a} f(x)$ exists.

(y) If $f(x), g(x)$ are continuous, then $\frac{f(x)}{g(x)}$ is continuous whenever it is defined.

Problem 11. Showing all your work, compute the following limits:

(a) $\lim_{\theta\to\frac{\pi}{2}} \sec\theta$

(b) $\lim_{h\to-\infty} \frac{h^5+6h^2+1}{h^8}$

(c) $\lim_{u\to3^-} |2u-8|$

(d) $\lim_{t\to3} \frac{3x}{\sqrt{x+6}}$

(e) $\lim_{y\to3^+} \frac{3-y}{y^2-9}$

(f) $\lim_{x\to0} \left(3x^2-e^x\right)$

(g) $\lim_{x\to\infty} \frac{1-x^6}{10x^2+3x-5}$

(h) $\lim_{r\to1} \frac{\sqrt{2r-1}-1}{r-1}$

(i) $\lim_{x\to0} \arctan\left(\frac{\sin x}{x}\right)$

(j) $\lim_{x\to\infty} \frac{2^x+3^x}{5^x}$

(k) $\lim_{x\to0^-} \frac{4-(2-x)^2}{x}$

(l) $\lim_{x\to\infty} \left(8+4^{4-x}\right)$

(m) $\lim_{x\to-\infty} \frac{x+6}{1-x^2}$

(n) $\lim_{a\to-\infty} \frac{a^6-a^5+a^4-a^3+a^2-a+1}{4a^6-100a^3}$

(o) $\lim_{x\to\infty} \cos\left(\frac{1}{x}\right)$

(p) $\lim_{a\to\infty} \frac{a^7-15a^4+3a^2-5}{a^{10}+a^2}$

(q) $\lim_{x\to-\infty} \frac{e^x}{\pi^x}$

(r) $\lim_{x\to-\infty} \frac{e^{2x}}{\pi^x}$

(s) $\lim_{y\to0} \sin\left(\frac{1}{y^2}\right)$

Problem 12. Showing all your work, compute the following limits:

(a) $\lim_{q\to0} \frac{\sin(w^2)}{w}$

(b) $\lim_{x\to1} \tan\left(\frac{\pi x}{2}\right)$

(c) $\lim_{x\to0} \frac{\sqrt{x+3}-\sqrt{3}}{x(\sqrt{x+1}+5)}$

(d) $\lim_{k\to1^-} \csc(8\pi k)$

(e) $\lim_{u\to1} \frac{3u^2+2u-5}{u-1}$

(f) $\lim_{x\to\sqrt{2}} (x^4-\cos^2 x)$

(g) $\lim_{x\to-1} \frac{x+6}{x+1}$

(h) $\lim_{x\to\infty} (4-e^x)$

(i) $\lim_{\phi\to0} (1+\phi)^{1/\phi}$

(j) $\lim_{v\to6} \frac{v}{|v-6|}$

(k) $\lim_{u\to1} \frac{|u-5|}{u+6}$

(l) $\lim_{t\to8^-} \frac{|t-8|}{t-8}$

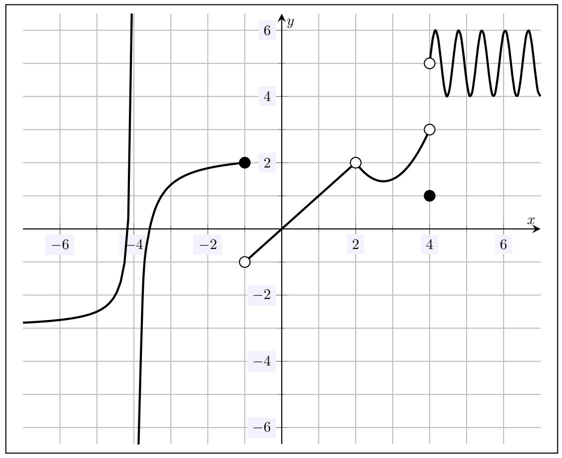
(m) $\lim_{x\to\infty} \frac{x^2+x-5}{(x-3)(x+5)}$

(n) $\lim_{x\to\infty} \sin\left(\frac{x-1}{x^2+2x+5}\right)$

(o) $\lim_{x\to\infty} \left(\frac{2x-1}{5x+9}\right)^4$

(p) $\lim_{x\to12} \tan^3(4x)$

Problem 13. For the function $f(x)$, whose graph is shown in the figure below, compute the following limits. If the limit does not exist, write 'DNE.'



(a) $\lim_{x\to -4^-} f(x)$

(b) $\lim_{x\to -4^+} f(x)$

(c) $\lim_{x\to -4} f(x)$

(d) $f(-4)$

(e) $\lim_{x\to -1^-} f(x)$

(f) $\lim_{x\to -1^+} f(x)$

(g) $\lim_{x\to -1} f(x)$

(h) $f(-1)$

(i) $\lim_{x\to 2^-} f(x)$

(j) $\lim_{x\to 2^+} f(x)$

(k) $\lim_{x\to 2} f(x)$

(l) $f(2)$

(m) $\lim_{x\to 4^-} f(x)$

(n) $\lim_{x\to 4^+} f(x)$

(o) $\lim_{x\to 4} f(x)$

(p) $f(4)$

(q) $\lim_{x\to -\infty} f(x)$

(r) $\lim_{x\to \infty} f(x)$

Problem 14. Showing all your work, compute the following limits:

(a) $\lim_{v\to 0} \frac{\tan v}{v}$

(b) $\lim_{x\to -\infty} \frac{1-x}{x+1}$

(c) $\lim_{x\to \frac{\pi}{2}} \frac{\cos x}{\cot x}$

(d) $\lim_{x\to \infty} 3x \sin\left(\frac{4}{x}\right)$

(e) $\lim_{x\to \infty} \frac{1-x^3}{x+5}$

(f) $\lim_{u\to 0^+} \frac{\sqrt{u+1}-1}{u}$

(g) $\lim_{k\to 2} \frac{|k^2-4|}{2k}$

(h) $\lim_{x\to \infty} e^x \sin\left(e^{-x}\right)$

(i) $\lim_{y\to 0} \frac{\cos y - 1}{2y^2}$

(j) $\lim_{x\to 5} \frac{x}{x-5}$

(k) $\lim_{y\to 6} \tan\left(\frac{6\pi}{y}\right)$

(l) $\lim_{b\to 5} \cot^2(\pi x)$

(m) $\lim_{x\to\infty} \arctan\left(\frac{1}{x}\right)$

(n) $\lim_{\psi\to\pi/4} \frac{1-\tan\psi}{\sin\psi-\cos\psi}$

(o) $\lim_{x\to 6} |x+6|$

(p) $\lim_{w\to\infty} \left(1+\frac{3}{4w}\right)^{9w}$

(q) $\lim_{x\to 0} x\sin\left(\frac{1}{x}\right)$

(r) $\lim_{x\to 0} x^3\cos\left(\frac{1}{x^2}\right)$

Problem 15. Define the following functions:

$$f(x) = \begin{cases} 2x-5, & x<0 \\ x^2+5x-1, & x\geq 0 \end{cases} \qquad g(x) = \begin{cases} 2e^x-1, & x\leq 0 \\ \dfrac{\sin x}{x}, & x>0 \end{cases} \qquad h(x) = \begin{cases} 1-x^2, & x<-2 \\ \cos x, & -2<x\leq 5 \\ \ln|1-x|, & x>5 \end{cases}$$

Showing all your work, compute the following limits:

(a) $\lim_{x\to 0^+} f(x)$

(b) $\lim_{x\to 0^-} f(x)$

(c) $\lim_{x\to 0} f(x)$

(d) $\lim_{x\to 15} f(x)$

(e) $\lim_{x\to -\pi} f(x)$

(f) $\lim_{x\to 0^-} g(x)$

(g) $\lim_{x\to 0^+} g(x)$

(h) $\lim_{x\to 0} g(x)$

(i) $\lim_{x\to \pi} g(x)$

(j) $\lim_{x\to -20} g(x)$

(k) $\lim_{x\to -2^-} h(x)$

(l) $\lim_{x\to -2^+} h(x)$

(m) $\lim_{x\to -2} h(x)$

(n) $\lim_{x\to 5^-} h(x)$

(o) $\lim_{x\to 5^+} h(x)$

(p) $\lim_{x\to 5} h(x)$

(q) $\lim_{x\to 10} h(x)$

(r) $\lim_{x\to -3} h(x)$

(s) $\lim_{x\to 0} h(x)$

Problem 16. Assume that $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, where $L, M \neq 0$. Showing all your work, compute the following limits:

(a) $\lim_{x\to a} (f(x) - 5g(x))$

(b) $\lim_{x\to a} \left(\frac{4-f(x)}{[g(x)]^2}\right)$

(c) $\lim_{x\to a} \left(g(x)\sqrt[3]{f(x)}\right)$

(d) $\lim_{x\to a} (|f(x)| + \sin(g(x)))$

Problem 17. Showing all your work, compute the following limits:

(a) $\displaystyle\lim_{x\to\infty}\left(1+\frac{5}{x}\right)^{6x}$

(b) $\displaystyle\lim_{r\to 2}|r^2-1|$

(c) $\displaystyle\lim_{h\to 0}\frac{(1-\cos h)^2}{h}$

(d) $\displaystyle\lim_{x\to 0}\sin\left(\frac{1}{x}\right)$

(e) $\displaystyle\lim_{k\to-\infty}\frac{k^3-5k^2+15}{(k^2-1)(k^2+8)}$

(f) $\displaystyle\lim_{x\to 0}\cos\left(\frac{1}{x}\right)$

(g) $\displaystyle\lim_{x\to-\infty}\frac{5x^3-x+7}{10x^3-x^2+4}$

(h) $\displaystyle\lim_{x\to 0}\frac{1-\cos^2 x}{x}$

(i) $\displaystyle\lim_{x\to-\infty}\cos\left(\frac{x^2}{x^3+1}\right)$

(j) $\displaystyle\lim_{x\to 6}\sec\left(3\pi-\frac{\pi}{x}\right)$

(k) $\displaystyle\lim_{b\to 0}\frac{b}{2\cos b-2b\cos^2 b}$

(l) $\displaystyle\lim_{y\to\infty}\frac{(1-y)(y^2+1)(y+7)}{y^2+2y+12}$

(m) $\displaystyle\lim_{x\to 2}\frac{x^5-32}{x-2}$

(n) $\displaystyle\lim_{x\to\pi}\left(\frac{7}{3}-\cot^2\left(\frac{x}{3}\right)\right)$

(o) $\displaystyle\lim_{y\to 0}\frac{8}{\sec(5y)}$

(p) $\displaystyle\lim_{r\to\infty}\frac{1000r+7}{r^2-12}$

(q) $\displaystyle\lim_{y\to-\infty}\frac{\pi y^2-\sqrt[3]{5}y}{\pi y^4+1}$

(r) $\displaystyle\lim x\to 1^{+}\frac{x+1}{|x-1|}$

(s) $\displaystyle\lim_{r\to\infty}\frac{1-r^5}{r^5+6}$

(t) $\displaystyle\lim_{a\to 4}\frac{a-4}{|a-4|}$

Problem 18. Give an example of...

(a) a continuous function (algebraically and graphically).

(b) a differentiable function.

(c) a function which is not differentiable (algebraically and graphically).

(d) a function whose limit exists (algebraically and graphically).

(e) a function whose limit does not exist (algebraically and graphically).

(f) a function whose left and right limits exist but whose limit does not exist.

(g) a function whose left and right limits are equal but whose limit does not exist.

(h) a function with a vertical asymptote.

(i) a function with a horizontal asymptote.

(j) a function with a zero.

(k) a function with no zeros.

(l) a function with no y-intercept.

(m) a function with a jump discontinuity.

(n) a function with an infinite discontinuity.

(o) a function with a removable discontinuity.

(p) a function with an infinite amount of zeros.

(q) a function with infinitely many infinite discontinuities.

(r) a function with infinitely many removable discontinuities.

(s) a polynomial with roots $x = -1, 2, 3$.

(t) a polynomial with roots $(-6, 0), (2, 0)$ and y-intercept $(0, 5)$.

(u) a function with y-intercept -2 and a zero at $x = 4$.

(v) a graph which is not the plot of a function.

(w) a graph which is not a function of x or y but is a function of some variable.

Problem 19. Assume that $\lim_{x\to 0} f(x) = L$ and $\lim_{x\to 0} g(x) = M$, where $L, M \neq 0$. Showing all your work, compute the following limits:

(a) $\lim_{x\to 0} (f(x) - g(x))$

(b) $\lim_{x\to 0} \left(f(x)g(x) + f(x^2)\right)$

(c) $\lim_{x\to 0} \left(\dfrac{f(\sin(x))}{5g(x)}\right)$

(d) $\lim_{x\to 0} \sin(xf(x))$

(e) $\lim_{x\to 0} \left(g(x)e^{L-f(x)}\right)$

(f) $\lim_{x\to 0} \left(\cos(g(x)) \cdot \dfrac{\sin(L - f(x))}{L - f(x)}\right)$

Problem 20. Fully justifying your answer, find the largest subset of the real numbers over which the following functions are continuous:

(a) $x^3 - 5x + 9$

(b) $xe^x + \sin x$

(c) $\ln\left|\cos\left(4^{x^2}\right)\right|$

(d) $\dfrac{2x-1}{4-7x}$

(e) $\dfrac{x+5}{\sqrt[3]{4-x^2}}$

(f) $\dfrac{x^2-1}{x+1}$

(g) $\dfrac{(x-6)(x+9)}{x^2-x-12}$

(h) $x^3 \sin\left(\dfrac{1}{x}\right)$

(i) $4x^5 + \tan x$

(j) $\sqrt[4]{x^2+6x+9}$

(k) $\dfrac{\arctan(3^x)}{x^2-2}$

Problem 21. Fully justifying your answer, find the largest subset of the real numbers over which the following functions are continuous:

(a) $5x^5 - 4x^3 + x - 17$

(b) $\dfrac{x-6}{5-x}$

(c) $\dfrac{x^3 - x + 3}{e^x + 1}$

(d) $\dfrac{7x-5}{x^2+8x+15}$

(e) $\dfrac{\cos(\ln x)}{\sin x}$

(f) $\sqrt{x^2+5}$

(g) $\cos\left|\dfrac{5-e^x}{x^2+1}\right|$

(h) $\sqrt[3]{x^3-6x+8}$

(i) $\sqrt[4]{5-x}$

(j) $\sqrt{\dfrac{x+5}{x-4}}$

(k) $x5^x \sec(x)$

Problem 22. Fully justifying your answer, find the largest subset of the real numbers over which the function $f(x) = \dfrac{x^3 - xe^x + \sin(x) - \sqrt{16-x^2}}{x^3 - 3x^2 - 4x}$ is continuous.

Problem 23. Fully justifying your answer, find the largest subset of the real numbers over which the function $g(x) = \dfrac{x6^x - \ln(x) + \cos(4-x)}{(x^2+3x-18)\sqrt{x+4}\sqrt[3]{x-10}}$ is continuous.

Problem 24. Find and classify any discontinuities for the following functions—if the discontinuity involves a hole, find the location of the hole:

(a) $\dfrac{x^2-1}{x+1}$

(b) $\dfrac{x^2+5x-1}{x^2+5x+6}$

(c) $\dfrac{\sin(2x)}{x}$

(d) $\dfrac{1-\cos(3x)}{x}$

(e) $\dfrac{x^2+5x-6}{x^3+11x^2-12x}$

(f) x^2+6x+8

(g) $\dfrac{(x+6)(x-7)(x-1)}{(x-1)(x^2-49)}$

(h) $x\sin\left(\dfrac{1}{x}\right)$

(i) $\dfrac{4-x}{x^2+4}$

(j) $\dfrac{\sin^2(x)-1}{1+\sin x}$

Problem 25. Find a value c which makes the following function everywhere continuous on the real line. Be sure to fully justify why the function is everywhere continuous.

$$f(x) = \begin{cases} \dfrac{\sin(3x)}{x}, & x \le 0 \\ c - 5x, & x > 0 \end{cases}$$

Problem 26. Find a value c which makes the following function everywhere continuous on the real line. Be sure to fully justify why the function is everywhere continuous.

$$f(x) = \begin{cases} \dfrac{x^2-2x-15}{x^2+5x+6}, & x \neq -3 \\ c, & x = -3 \end{cases}$$

Problem 27. Find values m, b which makes the following function everywhere continuous on the real line. Be sure to fully justify why the function is everywhere continuous.

$$f(x) = \begin{cases} x^2-1, & x \leq -2 \\ mx+b, & -2 < x \leq 3 \\ x-2^x, & x > 3 \end{cases}$$

Problem 28. Find any vertical, horizontal, and slant asymptotes for the following functions:

(a) $\dfrac{3x-5}{x+4}$

(b) $\dfrac{x^2-x+4}{x+1}$

(c) $\dfrac{5x-2}{x^2+9}$

(d) $\dfrac{x^2-7x+4}{x^3-x^2}$

(e) $\dfrac{(x-1)(x+1)(x-5)(x-9)}{(x+1)(x+4)(x^5-9)}$

(f) $\dfrac{4-x^2}{2x^2+9}$

(g) $\dfrac{x^2-4}{x^2+6x+8}$

(h) $\dfrac{x^3+4x^2-x+8}{x^2-x-6}$

Problem 29. Use the definition of the derivative to find the value of the derivative of the given function at the indicated value.

(a) $f(x) = x^2 - x + 4$, $a = 1$

(b) $g(x) = 2x^2 - 3x + 5$, $a = -2$

(c) $h(x) = \sin x$, $a = \frac{\pi}{6}$

(d) $j(x) = \cos x$, $a = \pi$

(e) $k(x) = \dfrac{x+1}{x-2}$, $a = 0$

(f) $\ell(x) = \sqrt{x}$, $a = 25$

(g) $m(x) = \sqrt{4-x}$, $a = -12$

(h) $n(x) = \dfrac{1}{\sqrt{x}}$, $a = 9$

(i) $p(x) = 4 - 7x^2$, $a = -1$

(j) $q(x) = e^x$, $a = 0$

(k) $r(x) = \dfrac{4}{x+1}$, $a = 3$

Problem 30. Use the definition of the derivative to find the derivative of the following functions:

(a) $f(x) = x^2 - 7$

(b) $g(x) = x^2 - 2x + 5$

(c) $h(x) = 3x^2 - x + 3$

(d) $j(x) = \sqrt{x}$

(e) $k(x) = (x+1)^3$

(f) $\ell(x) = (1-4x)^2$

(g) $m(x) = \sqrt{6-x}$

(h) $n(x) = (2x-1)^3$

(i) $p(x) = \dfrac{x-1}{x+5}$

(j) $q(x) = \dfrac{1}{\sqrt{x}}$

(k) $r(x) = \dfrac{3x+1}{x-1}$

(l) $s(x) = \dfrac{1}{x^2}$

(m) $t(x) = \sin x$

(n) $u(x) = \cos x$

(o) $v(x) = e^x$

(p) $w(x) = 2^x$

(q) $y(x) = \csc x$

Problem 31. Use the definition of the derivative to find the equation of the tangent line to the function at the given point.

(a) $x^2 + 4x - 1$, $a = 2$

(b) $1 - x^2$, $a = -3$

(c) $3x^2 - x + 1$, $a = 1$

(d) $\sqrt{x}$, $a = 25$

(e) $\dfrac{1}{x}$, $a = -3$

(f) $\dfrac{x+3}{x}$, $a = 3$

(g) $\dfrac{x+1}{x-2}$, $a = -1$

(h) $\sqrt{20-x}$, $a = 4$

(i) $\dfrac{1}{\sqrt{x}}$, $a = 4$

(j) $(2x-7)$, $a = 1$

(k) $\sin x$, $a = \frac{\pi}{4}$

(l) $\cos x$, $a = \frac{5\pi}{3}$

Problem 32. Showing all your work, find the second derivatives of the following functions:

(a) $11 - x$

(b) $x^2 + 8x - 2$

(c) $4x^2 + 5x - 9$

(d) $x^4 - 3x^2 + 5x - 8$

(e) $x^5 - x^2 + 9$

(f) e^x

(g) 2^x

(h) $\dfrac{x^3 - 5x^2 + 9x - 1}{x^2}$

(i) $\dfrac{x+1}{x-1}$

(j) $\sin(3x)$

(k) $\dfrac{x^2 - x + 7 - \sqrt[3]{x}}{\sqrt{x}}$

(l) $\dfrac{x^2-5}{9-x^2}$

(m) $\sin(x)\sin(2x)$

(n) $x^4 e^x$

(o) $x\sin(x^2)$

(p) $\csc x \tan x$

(q) $\arccos(1-x)$

(r) $\arctan(5^x)$

(s) $6^x \sec(e^x)\ln x$

(t) $\dfrac{x4^x - 4^{2x}}{x\arccos x}$

(u) $3x^5 \tan x\sqrt{5-x}$

(v) $\dfrac{5x-1}{\sqrt[3]{x^2+9}}$

Problem 33. Showing all your work, find the derivatives of the following functions:

(a) $9 - 5x$

(b) $x^2 + 3x - 8$

(c) $x^4 + 4x^2 - x + 9$

(d) $6 - \sqrt{x}$

(e) $\dfrac{1}{\sqrt[3]{x}}$

(f) $\sqrt[5]{x^7}$

(g) $\dfrac{x-1}{x^2}$

(h) $\log_7(x)$

(i) $\log_\pi x$

(j) 5^{1-x}

(k) $\dfrac{6-x}{x+4}$

(l) $x^4 4^x$

(m) $\sin^4(-x)$

(n) π^x

(o) $\sqrt[3]{7}\pi^{3/5}e^{1-\pi}$

(p) $\sec(5x)$

(q) $\arcsin(2x)$

(r) $\arccos(\ln x)$

(s) $\tan^4(x)$

(t) $x5^x \log_9 x$

(u) x^x

(v) x^{2x}

Problem 34. Showing all your work, find the derivatives of the following functions:

(a) $x^4 e^{2x} \tan x$

(b) $\csc(x)\cot(-x)$

(c) $\dfrac{x - 6^x}{\ln x}$

(d) $(\sin x - e^x)^{100}$

(e) $(2x-5)^{12}(4-x)^{10}$

(f) $\dfrac{\sec(2x)}{5^x}$

(g) $\dfrac{x^2 - \cot(2x)}{x - e^{-x}}$

(h) $(2x)^{\cos x}$

(i) $\dfrac{\arctan(4x)}{1-x}$

(j) $\dfrac{x3^x \arctan(1-x)}{5x \log_2 x}$

(k) $\sec x \tan x \operatorname{arccsc} x$

(l) $\sec\left(e^{\log_5(1-x^2)}\right)$

(m) $(1-x)^{x-1}$

(n) $(\sin x)^x$

(o) $\sqrt{x}^{\,\tan x}$

Problem 35. Showing all your work, find the derivatives of the following functions:

(a) $\ln\big(\sin\big(\csc(2x)\big)\big)$

(b) $\dfrac{x5^x + \sec(6 - \sqrt{x})}{(x^2-5)^9}$

(c) $\dfrac{5^{-x} + \sin^2(2^x)}{\operatorname{arccot} x - e^{x^2}}$

(d) $(1-x)^4 8^{-x} \operatorname{arcsec}(x^2 e^x)\tan^2\big(1 - \ln(4x)\big)$

(e) $\sqrt[5]{\sin^5(x^2 + 5\sqrt{x})\big)^8}$

(f) $\left(2^{-\operatorname{arccot} x} + \log_6(\sqrt[10]{x})\right)^5$

Problem 36. Showing all your work, find the following limits:

(a) $\lim_{x\to\infty} (\ln(3x) - \ln(5x))$

(b) $\lim_{x\to\infty} \left(\sqrt{9x^2+1} - 3x\right)$

(c) $\lim_{x\to\infty} \ln(5x+1) - \ln(3x+2)$

(d) $\lim_{x\to\infty} \sqrt{x^6+2x^3} - x^3$

(e) $\lim_{x\to\infty} \left(\sqrt{x^2+3x+1} - 2x\right)$

(f) $\lim_{x\to\infty} \sqrt{x+2} - \sqrt{x-1}$

(g) $\lim_{x\to\infty} \ln(3x^2-4) - \ln(2x^2+1)$

(h) $\lim_{x\to\infty} \dfrac{x^{4/3} + x + \sqrt[3]{x}}{(2x^{2/3}+5)^2}$

(i) $\lim_{x\to\infty} \dfrac{10x + \sqrt{x+3}}{5x-1}$

(j) $\lim_{x\to\infty} \sqrt{x^2+1} - \sqrt{x^2-1}$

(k) $\lim_{x\to\infty} \left(x - x\cos\left(\dfrac{1}{x}\right)\right)$

(l) $\lim_{x\to\infty} \sqrt{x^2+6x+1} - x$

(m) $\lim_{x\to\infty} \left(\sqrt{9x^2+1} - 3x\right)$

(n) $\lim_{x\to\infty} \sqrt{2x^2+4x-1} - \sqrt{2x^2+8x+7}$

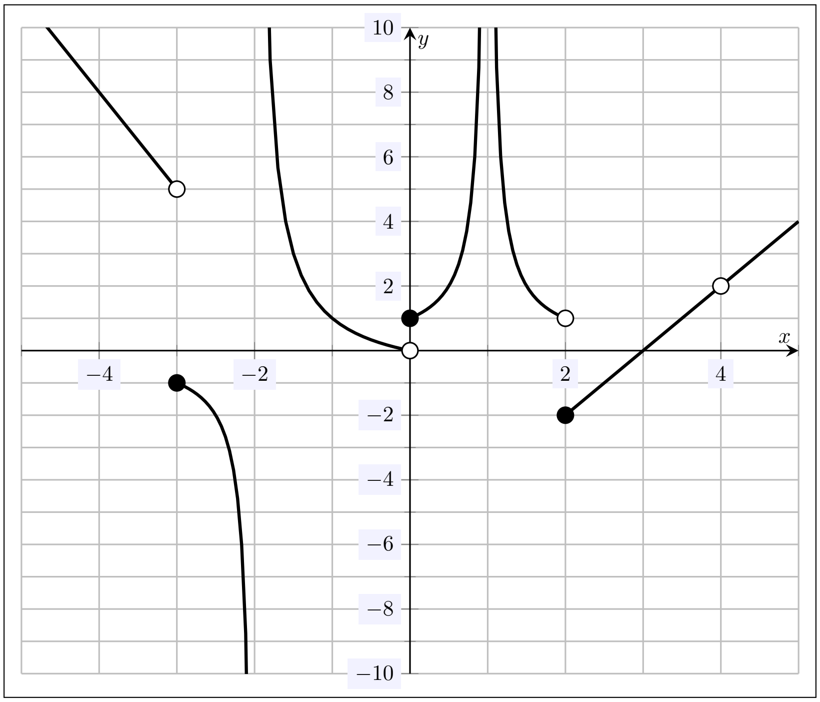
Problem 37. Find the values of x at which the following function is continuous. Explain your reasoning.

$$f(x) = \begin{cases} -2-x, & -1 \le x \\ -1, & -1 < x \le 0 \\ \sqrt{x}, & 0 < x < 1 \\ 2-x, & 1 \le x < 2 \\ (x-2)^2, & 2 \le x \end{cases}$$

Problem 38. Show that the following function is everywhere continuous.

$$f(x) = \begin{cases} \dfrac{\sin(x-3)}{x-3}, & x \neq 3 \\ 1, & x = 3 \end{cases}$$

Problem 39. For the following plot, find the values of x for which the function is discontinuous and identify the type of discontinuity.



Problem 40. Explain why the following functions are discontinuous:

(a) $f(x) = \sin(1/x)$

(b) $h(x) = \dfrac{1}{2-x}$

(c) $r(x) = \begin{cases} 2x+3, & x < 1 \\ x-7, & x \geq 1 \end{cases}$

(d) $s(x) = \begin{cases} -2x, & x < 0 \\ 4x, & x > 0 \end{cases}$

Problem 41. Find the intervals on which the following functions are continuous:

(a) $f(x) = 2x + 3$

(b) $g(x) = \dfrac{1}{6-5x}$

(c) $h(x) = \dfrac{x-7}{x+6}$

(d) $r(x) = \dfrac{\sin x}{x^2+2x+3}$

(e) $s(x) = \sin(\cos(x^2+1))$

(f) $t(x) = \dfrac{x\sin(1-x)}{\sqrt{x^2+2}}$

Problem 42. If $p(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$ is a polynomial, does there exist a positive integer n such that $p^{(n)}(x) = 0$? Explain.

Problem 43. Does there exist a function $f(x)$ such that $f^{(n)}(x)$ exists for all positive integers n but $f^{(n)}(x) \neq 0$ for all positive integers n? Explain.

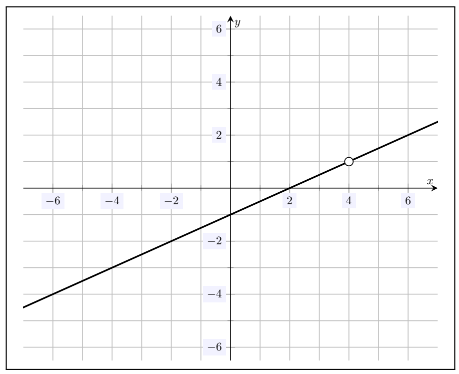
Problem 44. Does there exist a function $f(x)$ such that $f^{(n)}(x)$ exists for all positive integers n and $f^{(n)}(x) > 0$ for all positive integers n? Explain.

Problem 45. Does there exist a function $f(x)$ such that $f^{(n)}(x)$ exists for all positive integers n and $f^{(n)}(x) < 0$ for all positive integers n? Explain.

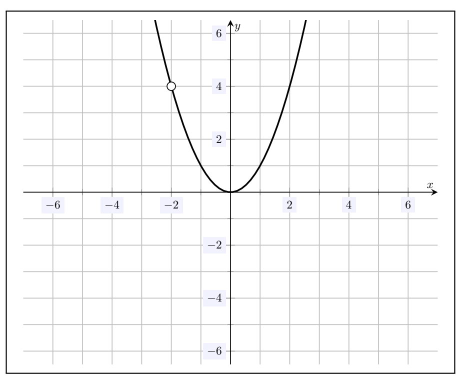
Problem 46. Does there exist a function $f(x)$ such that $f^{(n)}(x)$ exists for all positive integers n and that $f^{(n)}(x)$ changes sign infinitely many times for each such n? Explain.

Problem 47. Does there exist a function which is differentiable but its derivative is not? Explain.

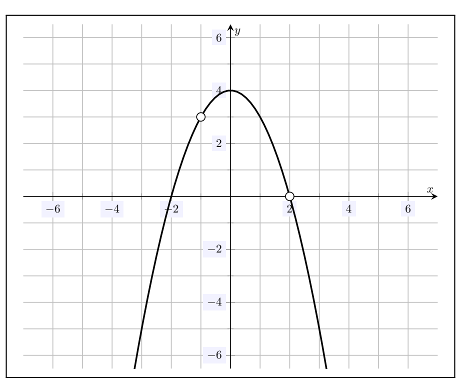
Problem 48. Find a function whose graph could be given below.



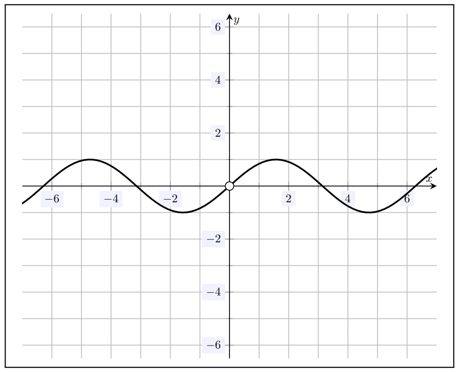
Problem 49. Find a function whose graph could be given below.



Problem 50. Find a function whose graph could be given below.



Problem 51. Find a function whose graph could be given below.



Problem 52. Define $f(x)$ to be the following function:

$$f(x) = \begin{cases} x^2 2^{-x}, & x \geq 0 \\ 4 - x, & x < 0 \end{cases}$$

Use the definition of $f(x)$ to find the following:

(a) $f(0)$

(b) $\lim_{x \to 2} f(x)$

(c) y-intercepts

(d) $\lim_{x \to 0^-} f(x)$

(e) x-intercepts

(f) $\lim_{x \to 0^+} f(x)$

(g) Classify any discontinuities for $f(x)$

(h) $\lim_{x \to 0} f(x)$

Problem 53. Define $f(x)$ to be the following function:

$$f(x) = \frac{(x+1)(2x-3)(x+2)}{(3x-7)(x+2)(x+3)}$$

(a) What is the y-intercept of $f(x)$?

(b) What are the x-intercepts of $f(x)$?

(c) What are the vertical asymptotes for $f(x)$?

(d) Where is $f(x)$ continuous?

(e) If $f(x)$ has any discontinuities, classify them.

(f) Identify any horizontal asymptotes $f(x)$ might have.

Problem 54. Evaluate the following limits:

(a) $\lim_{x\to 0^+} \ln x$

(b) $\lim_{x\to 2^+} \dfrac{x+6}{x-2}$

(c) $\lim_{x\to 1^-} \dfrac{x-4}{x+1}$

(d) $\lim_{x\to -2} \dfrac{2x+4}{x+2}$

(e) $\lim_{x\to 0} \dfrac{\cos x}{x}$

Problem 55. Calculate the following limits:

(a) $\lim_{x\to 0} \csc x - \cot x$

(b) $\lim_{x\to 0} \dfrac{3x}{\sin 5x}$

(c) $\lim_{x\to 0} \dfrac{\csc 7x}{\csc 5x}$

(d) $\lim_{x\to 0} \sin^2 3x$

(e) $\lim_{x\to 0} \dfrac{\tan x}{\sin x}$

(f) $\lim_{x\to 0} \dfrac{\sin^2(3x)}{x}$

(g) $\lim_{x\to 0} \dfrac{\tan x}{x}$

Problem 56. Use Squeeze Theorem to evaluate the following limits:

(a) $\lim_{x\to 0} x\sin(1/x)$

(b) $\lim_{x\to 0} x^2\cos(1/x)$

(c) $\lim_{x\to 0} |x|\cos^2(1/x)$

(d) $\lim_{x\to 0} x^3 e^{\sin(1/x)}$

(e) $\lim_{x\to \infty} \dfrac{x^x}{(2x)!}$

(f) $\lim_{x\to \infty} (x!)^{1/x^2}$

Problem 57. Use the Intermediate Value Theorem to show there is a solution to the following equations over the given interval:

(a) $4^x = x^2 + 1$ over $[-2, 1]$

(b) $x^3 + \cos x = 2$ over $[0, 10]$

(c) $e^{-x^2} - x = 0$ over $[0, 1]$

(d) $x^3 + x + 1 = 0$ over $[-1, 0]$

(e) $\pi^{13.475}x^{15} - \sqrt{e^3}x^12 - x^9 + 1478x + 14.2345 = e^\pi x^{13} - \sqrt{1 + \sqrt{2 + \sqrt{3}}}x^{10} - 99.99x^2 + 2^{4^{6^8}}$

Problem 58. In Special Relativity, the energy of a particle moving at a velocity v is given by

$$E(v) = \frac{mc^2}{\sqrt{1 - v^2/c^2}},$$

where c is the speed of light and m is the mass of the particle. What happens if $v = 0$? What happens as v approaches c? What does this limit imply? Is this something you already knew?

Problem 59. Let $f(x) = [\![x]\!]$ denote the largest integer n such that $n \le x$. For example, $[\![1.5]\!] = 1$, $[\![2]\!] = 2$, $[\![-1]\!] = -1$, $[\![-2.2]\!] = -3$, and $[\![0]\!] = 0$. This function is used in Computer Science since $[\![x]\!]$ gives the 'integer part' of x.

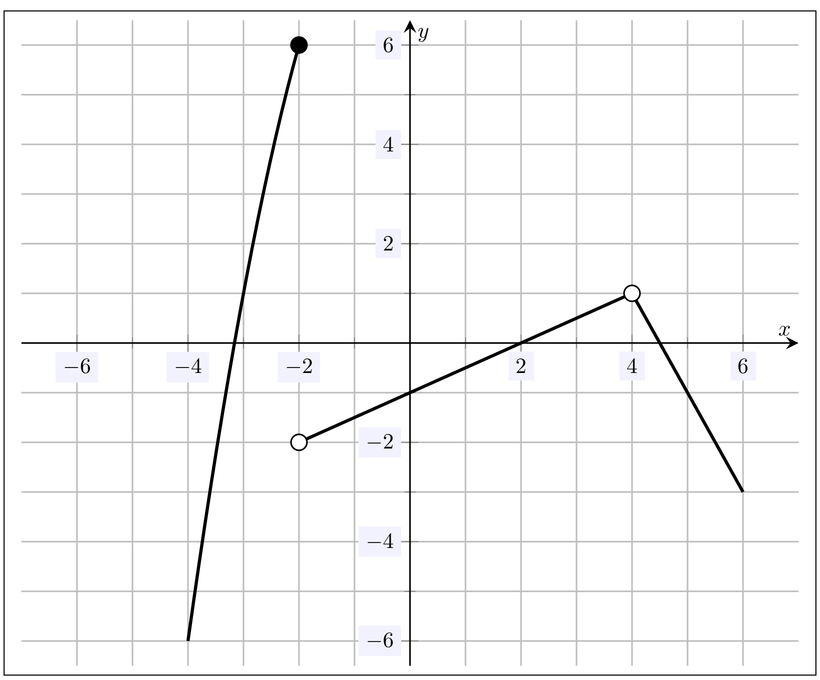
(a) Graph the function $f(x) = [\![x]\!]$

(b) Determine $\lim_{x\to 3.2^+} f(x)$, $\lim_{x\to 3.2^-} f(x)$, and $\lim_{x\to 3.2} f(x)$.

(c) Determine $\lim_{x\to 5^+} f(x)$, $\lim_{x\to 5^-} f(x)$, and $\lim_{x\to 5} f(x)$.

(d) Using the previous parts for what values a does $\lim_{x\to a} f(x)$ exist?

Problem 60. Use the graph of $f(x)$ below to evaluate the following:



(a) $\lim_{x\to 2^+} f(x)$

(b) $\lim_{x\to 2^-} f(x)$

(c) $\lim_{x\to 2} f(x)$

(d) $f(2)$

(e) $\lim_{x\to -2^-} f(x)$

(f) $\lim_{x\to -2^+} f(x)$

(g) $\lim_{x\to -2} f(x)$

(h) $f(-2)$

(i) $\lim_{x\to 4^-} f(x)$

(j) $\lim_{x\to 4^+} f(x)$

(k) $\lim_{x\to 4} f(x)$

(l) $f(4)$

Problem 61. Find the x-intercepts, y-intercepts, vertical asymptotes, and horizontal asymptotes of the following function. If there are discontinuities, identify them. If there are removable discontinuities, identify the point.

$$\frac{(x+2)(x-3)(x+7)(2x-3)}{(x-3)(2x+1)(x-2)(x-7)}$$

Problem 62. Let $f(x) = x^2 + 5x - 1$. Find the average velocity of $f(x)$ on $[-1, 2]$. Use the definition of the derivative to find the instantaneous velocity of $f(x)$ at $x = 1$.

Problem 63. Define $f(x)$ to be the following function:

$$f(x) = \begin{cases} 1 - x, & x \leq 1 \\ x^2 + ax + b, & x > 1 \end{cases}$$

Find values a, b so that $f(x)$ is everywhere continuous and differentiable.

Problem 64. If $f(x)$ is a function defined around $x = a$, explain why one can define $f'(a) := \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$. Use this definition of the derivative to find $f'(x)$, where $f(x) = x^2 + 3x - 6$.

Problem 65. State the Sandwich Theorem. Give an example of a limit that it can be used to compute.

Problem 66. Find the following limits (if they exist):

(a) $\lim_{x\to\infty} \frac{6}{x^2+4}$

(b) $\lim_{x\to\infty} 2^{-x}$

(c) $\lim_{x\to\infty} \ln(x+6)$

(d) $\lim_{x\to\infty} \ln\left(\frac{2x+1}{3x-2}\right)$

(e) $\lim_{x\to\infty} \cos(1/x)$

(f) $\lim_{x\to\infty} x\sin(1/x)$

Problem 67. For each part below, give an example of a function with given properties. If no such function exists, explain why.

(a) A hole at $x = 5$.

(b) Holes at $x = -3, 0$.

(c) A hole at the point $(1, -6)$.

(d) Horizontal asymptote $y = 7$.

(e) Vertical asymptote at $x = \pi$.

(f) Horizontal asymptote $y = 0$ and a hole at $x = -2$.

(g) A function which crosses a horizontal asymptote an infinite number of times.

(h) A function which is not defined at $x = 7$ and $x = 10$.

(i) Vertical asymptote at $x = 0$, a horizontal of $y = -5$, and a hole at $x = 2$.

(j) A function with y-intercepts 4 and -5.

(k) A function with x-intercepts -6 and 7.

(l) A function with y-intercept 6 and x-intercepts $-4, 9$.

(m) A continuous function that is not differentiable at $x = 6$.

(n) A continuous function that is not differentiable at $x = -3, 4$.

(o) A differentiable function that is not continuous.

Problem 68. Evaluate the following limits:

(a) $\displaystyle\lim_{w\to 0} \frac{w}{|w|}$

(b) $\displaystyle\lim_{w\to -2} \frac{2w+4}{|w+2|}$

(c) $\displaystyle\lim_{w\to 6} \frac{|w-5|-1}{w-6}$

(d) $\displaystyle\lim_{w\to 3} \frac{w^2+w-12}{|w-3|}$

(e) $\displaystyle\lim_{w\to 2} \left(3w^3 - |w-2|\right)$

Problem 69. For each part below, give an example of a function with given properties. If no such function exists, explain why.

(a) A function with infinitely many y-intercepts.

(b) A function with infinitely many vertical asymptotes.

(c) A function with infinitely many zeros.

(d) A function with infinitely many zeros but $\lim_{x\to\infty} f(x) = \infty$.

(e) A function with infinitely many zeros that is unbounded.

(f) A function with a jump discontinuity at $x = 4$.

(g) A function where $\lim_{x\to 5} f(x) = 9$.

(h) A function where $\lim_{x\to 1} f(x) = 0$

(i) A function where $\lim_{x\to 0} f(x) = 2$ but $\lim_{x\to 0^+} f(x) = 1$.

(j) A function with a removable discontinuity at $x = 0$.

(k) A function with an infinite discontinuity at $x = 5$ and a jump discontinuity at $x = 2$.

(l) A function where $\lim_{x\to 1^+} f(x) \neq \lim_{x\to 1^-} f(x)$.

(m) A function which is everywhere differentiable but has a jump discontinuity at $x = 5$.

(n) A function where $f'(x)$ exists but $f''(x)$ does not exist.

(o) A function that is infinitely differentiable, i.e. $f^{(n)}(x)$ exists for all positive integers n.

(p) A function where $\lim_{x\to 7^+} f(x) = 4$ but $\lim_{x\to 7^-} f(x) = -1$.

(q) A function that is nowhere differentiable.

Problem 70. Use the Squeeze Theorem to prove the following:

(a) $\lim_{x\to 0} x^2 \sin^2\left(\frac{1}{x}\right) = 0$

(b) $\lim_{x\to 0} x^2 \cos\left(\frac{1}{x^2}\right) = 0$

(c) $\lim_{x\to 0} x^2 e^{\sin 1/x} = 0$

(d) $\lim_{x\to\infty} \frac{2+\sin x}{x-3} = 0$

Problem 71. The following represents the derivative of some function f at some value a. Find such an f and a:

$$\lim_{h\to 0} \frac{(2+h)^3 - 8}{h}$$

Problem 72. The following represents the derivative of some function f at some value a. Find such an f and a:

$$\lim_{h\to 0} \frac{\sqrt{9+h} - 3}{h}$$

Problem 73. The following represents the derivative of some function f at some value a. Find such an f and a:

$$\lim_{h\to 0} \frac{\frac{1}{(h-3)^2} - \frac{1}{9}}{h}$$

Problem 74. Use the Intermediate Value Theorem to show that there is a solution to the given equation.

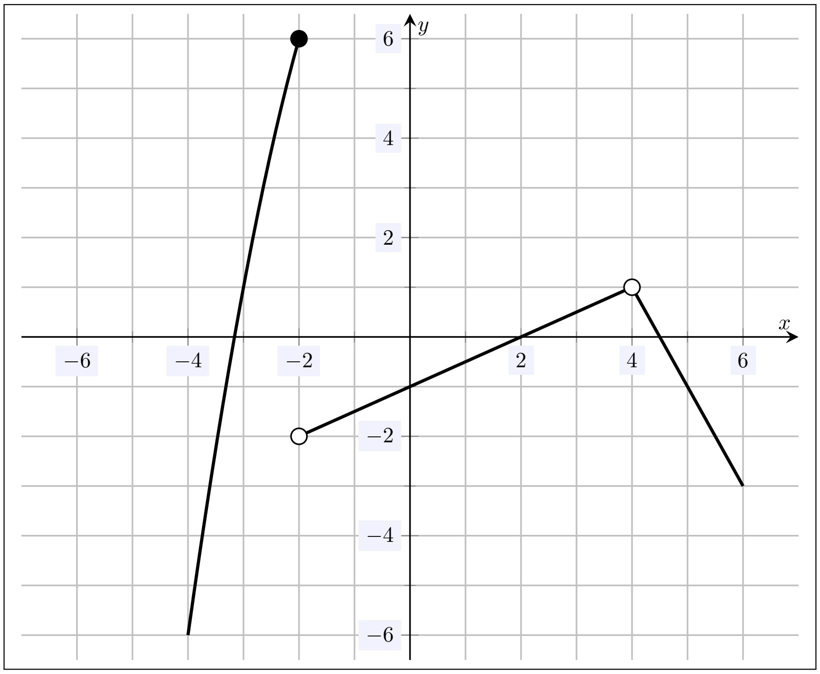
(a) $\sin x = x$

(b) $4x^2 - 4 = 2x$

(c) $e^x = 10 - \sqrt{x}$

(d) $\pi x^{15} + e^2 x^{13} - 5x^4 + \sqrt[3]{2} = e^{\pi} x^{12} + \pi^e x^3 + 6x - 1729$

Problem 75. Use the graph of $f(x)$ below to evaluate the following:



(a) $\lim_{x \to 2^+} f(x)$

(b) $\lim_{x \to 2^-} f(x)$

(c) $\lim_{x \to 2} f(x)$

(d) $f(2)$

(e) $\lim_{x \to -2^-} f(x)$

(f) $\lim_{x \to -2^+} f(x)$

(g) $\lim_{x \to -2} f(x)$

(h) $f(-2)$

(i) $\lim_{x \to 4^-} f(x)$

(j) $\lim_{x \to 4^+} f(x)$

(k) $\lim_{x \to 4} f(x)$

(l) $f(4)$