MATH 141 — Class Learning Outcomes — Fall 2025

Here are some of the learning outcomes for each lecture. That is, after lecture, the homework, and studying, what students should know from each lecture. Though this list is not necessarily *completely* comprehensive, the most important ideas/concepts are contained in each list. Students should be sure to feel comfortable with each bullet point before an exam. These learning goals are broken down by class date—with the class topic given. The classes are given in reverse-chronological order for ease of access to the most recent class. You may also click any of the hyperlinks below to jump to that date.

09/08, 09/10, 09/12, Monday, Wednesday, Friday: Derivative Rules and Chain, Product, & Quotient Rule

- Know the 'essential' derivatives:
  - $\dfrac{d}{dx}\,(\text{constant}) = 0$
  - $\dfrac{d}{dx}\,\square^n = n\,\square^{n-1}$
  - $\dfrac{d}{dx}\,\#^{\square} = \#^{\square}\ln\#$
  - $\dfrac{d}{dx}\,e^{\square} = e^{\square}$
  - $\dfrac{d}{dx}\,\log_b(\,\square\,) = \dfrac{1}{(\,\square\,)\ln b}$
  - $\dfrac{d}{dx}\,\ln(\,\square\,) = \dfrac{1}{\square}$
  - $\dfrac{d}{dx}\,\sin(\,\square\,) = \cos(\,\square\,)$
  - $\dfrac{d}{dx}\,\cos(\,\square\,) = -\sin(\,\square\,)$
  - $\dfrac{d}{dx}\,\tan(\,\square\,) = \sec^2(\,\square\,)$
  - $\dfrac{d}{dx}\,\csc(\,\square\,) = -\csc(\,\square\,)\cot(\,\square\,)$
  - $\dfrac{d}{dx}\,\sec(\,\square\,) = \sec(\,\square\,)\tan(\,\square\,)$
  - $\dfrac{d}{dx}\,\cot(\,\square\,) = -\csc^2(\,\square\,)$
  - $\dfrac{d}{dx}\,\arcsin(\,\square\,) = \dfrac{1}{\sqrt{1-(\,\square\,)^2}}$
  - $\dfrac{d}{dx}\,\arccos(\,\square\,) = \dfrac{-1}{\sqrt{1-(\,\square\,)^2}}$
  - $\dfrac{d}{dx}\,\arctan(\,\square\,) = \dfrac{1}{1+(\,\square\,)^2}$
  - $\dfrac{d}{dx}\,\operatorname{arccsc}(\,\square\,) = \dfrac{-1}{|\square|\sqrt{(\,\square\,)^2-1}}$
  - $\dfrac{d}{dx}\,\operatorname{arcsec}(\,\square\,) = \dfrac{1}{|\square|\sqrt{(\,\square\,)^2-1}}$
  - $\dfrac{d}{dx}\,\operatorname{arccot}(\,\square\,) = \dfrac{-1}{1+(\,\square\,)^2}$

- Know that if $f(x), g(x)$ are differentiable and c is a constant that $\dfrac{d}{dx}\big(cf(x)\big) = cf'(x)$ and $\dfrac{d}{dx}\big(f(x) \pm g(x)\big) = f'(x) \pm g'(x)$ and be able to use this in practice.

- Know the chain rule: if $f(x), g(x)$ are differentiable, then $\dfrac{d}{dx} f\big(g(x)\big) = f'\big(g(x)\big) \cdot g'(x)$ and be able to use this to compute the derivative of a function.

- Know the product rule: if $f(x), g(x)$ are differentiable, then $\dfrac{d}{dx}\big(f(x)g(x)\big) = f'(x)g(x) + f(x)g'(x)$ and be able to use this to compute the derivative of a function.

- Know the quotient rule: if $f(x), g(x)$ are differentiable, then $\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right) = \dfrac{f'(x)g(x) - g'(x)f(x)}{\big(g(x)\big)^2}$ and be able to use this to compute the derivative of a function.

- Be able to compute the derivative of functions which use the chain, product, or quotient rule at the same time.

- Be able to compute the tangent line of a function at a point: the tangent line to $f(x)$ at $x = a$ is $y = f(a) + f'(a)\big(x - a\big)$.

09/05, Friday: Derivative Definition

- Be able to articulate the motivation for the derivative, especially the fact that the derivative generalizes the notion of rate of change/slope of a line to arbitrary functions.
- Be able to state the definition of the derivative of $f(x)$ at $x = a$: $f(x)$ is differentiable at $x = a$ if $\lim_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$ exists; if so, we write this value as $f'(a)$ or $\frac{df}{dx}(a)$.
- Be able to state the definition of the derivative of $f(x)$: $f(x)$ is differentiable if $\lim_{h\to 0} \dfrac{f(a+h)-f(x)}{h}$ exists; if so, we write this function as $f'(x)$ or $\frac{df}{dx}$.
- Understand that the value of the derivative is the rate of change of the function.
- Know that value of the derivative at a point is the slope of the tangent line at that point.
- Know and be able to explain why that if $f(x)$ is differentiable at $x = a$ and Δx is 'small' that $f(a+\Delta x) - f(a) \approx f'(a) \cdot \Delta x$.
- Know and be able to use the notation for higher order derivatives: $f''(x)$, $f'''(x)$, $f^{(4)}(x)$, etc. and $\dfrac{d^2 f}{dx^2}$, $\dfrac{d^3 f}{dx^3}$, etc.
- Know that if $f(x)$ represents position, then $f'(x)$ represents velocity, $f''(x)$ represents acceleration, $f'''(x)$ represents the jerk (or jolt), $f^{(4)}(x)$ represents the snap (or jounce), $f^{(5)}(x)$ represents the crackle, and $f^{(6)}(x)$ represents the pop.
- Be able to compute the derivative of a function $f(x)$ at $x = a$ using the definition of the derivative, e.g. $f'(-1)$ where $f(x) = x - x^2$:

$$\begin{aligned}
f'(-1) &= \lim_{h\to 0} \frac{f(-1+h) - f(-1)}{h} \\
&= \lim_{h\to 0} \frac{\big((-1+h) - (-1+h)^2\big) - (-1-(-1)^2)}{h} \\
&= \lim_{h\to 0} \frac{(-h^2+3h-2) - (-2)}{h} \\
&= \lim_{h\to 0} \frac{-h^2+3h}{h} \\
&= \lim_{h\to 0} (-h+3) \\
&= 3
\end{aligned}$$

- Be able to compute *the derivative* of a function $f(x)$ using the definition of the derivative, e.g.

$f'(x)$ where $f(x) = x - x^2$:

$$
\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{\big((x+h) - (x+h)^2\big) - (x - x^2)}{h} \\
&= \lim_{h \to 0} \frac{\big(-x^2 - 2hx + x + h - h^2\big) - (x - x^2)}{h} \\
&= \lim_{h \to 0} \frac{-2hx + h - h^2}{h} \\
&= \lim_{h \to 0} (-2x + 1 - h) \\
&= -2x + 1
\end{aligned}
$$

- Know that if $f(x)$ is differentiable at $x = a$, then $f(x)$ is continuous at $x = a$. So, if $f(x)$ is not continuous at $x = a$, then $f(x)$ cannot be differentiable at $x = a$.
- Know that a continuous function may not be differentiable and be able to give an example, e.g. $f(x) = |x|$ at $x = 0$.
- Know (with an example) the definition of corner point and cusp/singularity, e.g. $f(x) = |x|$ at $x = 0$ or $f(x) = x^{1/3}$ at $x = 0$.
- Know that there are continuous functions which are nowhere differentiable, e.g. the Weierstrass function.

09/03, Wednesday: Continuity

- Understand that continuous functions $f(x)$ are functions that one can draw 'without lifting up one's pen.'
- Be able to state and understand the definition of continuity: a function $f(x)$ is continuous at $x = r$ if $f(r) = \lim_{x \to r} f(x)$. Otherwise, we say that $f(x)$ is not continuous at $x = r$ or that $f(x)$ has a discontinuity at $x = r$ and say $f(x)$ is discontinuous. The function is continuous on an interval I if it is continuous for every x-value in that interval. Otherwise, we say that $f(x)$ is not continuous on I or that $f(x)$ is discontinuous on I.
- Understand that the continuity of $f(x)$ at $x = r$ implies: $f(r)$ is defined, $\lim_{x \to r^-} f(x)$ exists, $\lim_{x \to r^+} f(x)$ exists, $\lim_{x \to r^-} f(x) = \lim_{x \to r^+} f(x)$, and $\lim_{x \to r} f(x)$.
- Be able draw examples of each of the common types of discontinuity: removable discontinuity, jump discontinuity, infinite discontinuity, and essential discontinuity.
- Be able to state the limit definition of each of the common types of discontinuity: removable discontinuity, jump discontinuity, infinite discontinuity, and essential discontinuity, e.g. a removable discontinuity for $f(x)$ is an x-value such that $f(r)$ is not defined but $\lim_{x \to r} f(x)$ exists and is finite.
- Understand how the assumption $f(r) = \lim_{x \to r} f(x)$ excludes each of the different types of discontinuity.
- Know common types of continuous functions: polynomials (everywhere), exponential functions (everywhere), logarithmic functions (whenever defined), rational functions (wherever defined), $\sin x$ and $\cos x$ (everywhere), and power functions (whenever defined).
- Be able to determine whether a given function $f(x)$ is continuous at $x = r$ and if not what type of discontinuity $f(x)$ has.
- Know and be able to use the common continuity theorems: if $f(x), g(x)$ are continuous at c, d are real numbers, then…
  - $cf(x) \pm d$ is continuous, i.e. scaling and shifting a continuous function gives a continuous function
  - $f(x) \pm g(x)$ is continuous, i.e. sums/differences of continuous functions are continuous
  - $f(x)g(x)$ is continuous, i.e. products of continuous functions are continuous
  - $\dfrac{f(x)}{g(x)}$ is continuous whenever $g(x) \neq 0$, i.e. quotients of continuous functions are continuous
  - $f\big(g(x)\big)$ is continuous (whenever defined), i.e. composition of continuous functions are continuous
  - $f^{c/d}(x)$ is continuous (whenever defined), i.e. powers of continuous functions are continuous
- Be able to use the continuity theorems to explain why a given function is continuous

- Be able to determine whether a function is continuous on a given interval.
- Be able to determine the largest interval on which a given function is continuous.
- Be able to sketch examples of continuous functions on a given interval or sketch functions with a specified type of discontinuity on a given interval.
- Be able to give algebraic examples of a function with a specified type of discontinuity at a specified x-value.

08/22, 08/25, 08/27, 08/29, Friday, Monday, Wednesday, Friday: Limit Techniques

- Know that $\frac{0}{0}$, $\pm\frac{\infty}{\infty}$, $0 \cdot \infty$, $\infty - \infty$, 0^0, 1^∞, and ∞^0 are all *indeterminant forms*, i.e. expressions whose value cannot be a priori determined. Know also that $\pm\frac{1}{0}$ is undefined but that obtaining this 'value' means that the limit will be ∞, $-\infty$, or DNE—depending on the limit.

- Know the common limit techniques: 'Plug 'n Chug', Algebra, Conjugation, "Special Limits", Piecewise limits, "Thinking" Limits, "Rational" Limits, and Squeeze Theorem. For example, …

$$\lim_{x \to \frac{1}{2}} (\sin(\pi x) + 2x) = \sin\left(\tfrac{\pi}{2}\right) + 2 \cdot \tfrac{1}{2} = 1 + 1 = 2$$

$$\lim_{x \to 0} \frac{\sin(4x)}{5x} = \lim_{x \to 0} \left(\frac{1}{5} \cdot \frac{\sin(4x)}{4x} \cdot 4 \right) = \frac{1}{5} \cdot 1 \cdot 4 = \frac{4}{5}$$

$$\lim_{x \to 0} \frac{x - x\cos x}{x^2} = \lim_{x \to 0} \left(x \cdot \frac{1 - \cos x}{x} \right) = 0 \cdot 0 = 0$$

$$\lim_{x \to \infty} \left(1 + \frac{3}{4x} \right)^{2x} = \lim_{x \to \infty} \left(1 + \frac{1}{\frac{4x}{3}} \right)^{2x} = \lim_{x \to \infty} \left(1 + \frac{1}{\frac{4x}{3}} \right)^{4x/3 \cdot (2 \cdot 3/4)} = e^{2 \cdot 3/4} = e^{3/2}$$

$$\lim_{x \to 3^+} \frac{|3 - x|}{x - 3} = \lim_{x \to 3^+} \frac{-(3 - x)}{x - 3} = \lim_{x \to 3^+} \frac{x - 3}{x - 3} = 1$$

$$\lim_{x \to -2^-} \frac{x + 4}{x + 2} \overset{\frac{1}{0}}{=} \lim_{x \to -2^-} \frac{\overbrace{x + 4}^{+}}{\underbrace{x + 2}_{-}} = -\infty$$

$$\lim_{x \to \infty} \frac{x + 4}{x^2 - 1} = \lim_{x \to \infty} \frac{x + 4}{x^2 - 1} \cdot \frac{1/x^2}{1/x^2} = \lim_{x \to \infty} \frac{\frac{x}{x^2} + \frac{4}{x^2}}{\frac{x^2}{x^2} - \frac{1}{x^2}} = \lim_{x \to \infty} \frac{\frac{1}{x} + \frac{4}{x^2}}{1 - \frac{1}{x^2}} = \frac{0 + 0}{1 - 0} = 0$$

- Know that…

$$\lim_{\square \to 0} \frac{\sin \square}{\square} = 1 \qquad \lim_{\square \to 0} \frac{\square}{\sin \square} = 1 \qquad \lim_{\square \to 0} \frac{1 - \cos \square}{\square} = 0 \qquad \lim_{x \to \infty} \left(1 + \frac{1}{x} \right)^x = e$$

- Recall that if $p(x), q(x)$ are polynomials, then…

$$\lim_{x \to \pm\infty} \frac{p(x)}{q(x)} = \begin{cases} 0, & \deg q > \deg p \\ \pm\infty(\text{depends on signs}), & \deg p > \deg q \\ \text{ratio leading coefficients}, & \deg p = \deg q \end{cases}$$

- Know and be able to compute limits which are approximately rational and use the same idea of $\cdot \dfrac{1/\text{den. dom. term}}{1/\text{den. dom. term}}$, i.e. dividing the numerator and denominator by the dominating term in the denominator, e.g.

$$\lim_{x\to\infty} \frac{3x-1}{\sqrt{5x^2+1}} = \lim_{x\to\infty} \frac{3x-1}{\sqrt{5x^2+1}} \cdot \frac{1/\sqrt{x^2}}{1/\sqrt{x^2}} = \lim_{x\to\infty} \frac{\frac{3x}{x} - \frac{1}{x}}{\sqrt{\frac{5x^2+1}{x^2}}} = \lim_{x\to\infty} \frac{3-\frac{1}{x}}{\sqrt{5+\frac{1}{x^2}}} = \frac{3-0}{\sqrt{5+0}} = \frac{3}{\sqrt{5}}$$

$$\lim_{x\to\infty} \frac{2^x}{3^x+4^x} = \lim_{x\to\infty} \frac{2^x}{3^x+4^x} \cdot \frac{1/4^x}{1/4^x} = \lim_{x\to\infty} \frac{\left(\frac{1}{2}\right)^x}{\left(\frac{3}{4}\right)^x + 1^x} = \frac{0}{0+1} = 0$$

- Know and be able to use the 'trick' for conjugation-like limits at $\pm\infty$, e.g.

$$\lim_{x\to\infty} (x - \sqrt{x^2+x}) = \lim_{x\to\infty} (x-\sqrt{x^2+x}) \cdot \frac{x+\sqrt{x^2+x}}{x+\sqrt{x^2+x}} == \lim_{x\to\infty} \frac{x^2-(x^2+x)}{x+\sqrt{x^2+x}} = \lim_{x\to\infty} \frac{-x}{x+\sqrt{x^2+x}} = \cdots = -\frac{1}{2}$$

  where '$\cdots$' is simply using the extension of rational limits we saw above.

- Know and be able to state/apply the Squeeze Theorem: if $g(x) \leq f(x) \leq h(x)$ and $\lim\limits_{x\to a} g(x) = L = \lim\limits_{x\to a} h(x)$, then $\lim\limits_{x\to a} f(x) = L$. For example, we know that $-4x \leq 4x \sin\left(\frac{1}{x}\right) \leq 4x$ because $-1 \leq \sin x \leq 1$ for all x. We know $\lim\limits_{x\to 0}(-4x) = 0 = \lim\limits_{x\to 0} 4x$. Therefore, by Squeeze Theorem, $\lim\limits_{x\to 0} 4x \sin\left(\frac{1}{x}\right) = 0$.

- Be able to explain the Squeeze Theorem graphically.

- Know that one always tries 'Plug 'n Chug' first. If one can obtain a value and the function is *not* piecewise, then the obtained value is likely the limit.

- Be able to compute limits algebraically using the common limit techniques.

- Be able to articulate the difference algebraically and graphically between functions such as $\dfrac{(x+2)(x-6)}{x+2}$ and $x-6$.

- Know that one never writes $\lim\limits_{x\to a} =$ to mean "the limit equals"—limits must always be followed by a function of some kind.

- Know never to drop $\lim\limits_{x\to a}$ until one actually evaluates the expression at $x = a$.

- Know that functions can be undefined and limits can be DNE, but not the other way around. That is, we can say 'the function is undefined' but we would never say 'the function is DNE', and we can say 'the limit DNE' but would never say 'the limit is undefined.'

08/20, Wednesday: Graphical Limits

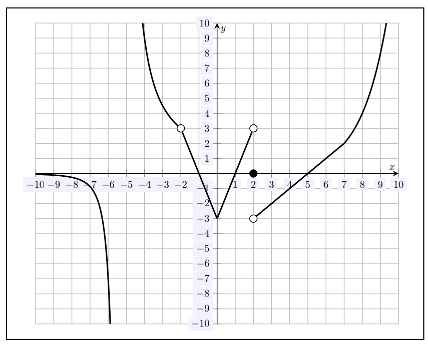
- Know that Calculus is the study of change and the underlying tool throughout most of Calculus is limits.

- Know the definition and intuitive definition of a limit: L is the limit of the function $f(x)$ as x 'approaches' a if the outputs of $f(x)$ get 'arbitrarily close' to L as x gets 'arbitrarily close' to a. If so, we write $\lim\limits_{x\to a} f(x) = L$. Otherwise, we say that the limit does not exist.

- Be able to properly and correctly read and write using limit notation.

- Understand and compute limits numerically by using or constructing tables of values.

- Be able to understand and compute limits graphically, e.g. compute $\lim\limits_{x\to -2} f(x)$ or $\lim\limits_{x\to 2} f(x)$ for the function $f(x)$ given below.

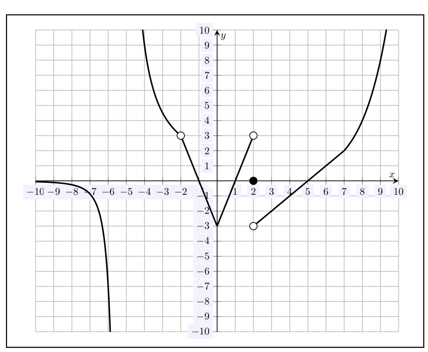


- Be able to understand and articulate the ways a limit can not exist using a graph.

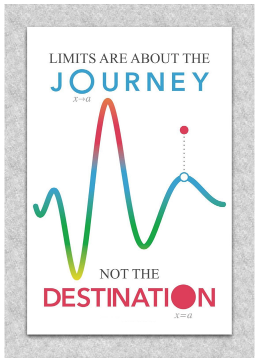
- Be able to construct examples of functions graphically with a specific limit or that do not exist in specified ways, e.g. construct a function $f(x)$ such that $\lim\limits_{x\to 1} f(x) = 4$ but $f(1) = -3$.

- Know the definition and intuitive definition of the left- and right-hand limit, e.g. the left-hand limit is L if the outputs of $f(x)$ get arbitrarily 'close' to L as the inputs x get arbitrarily 'close' to a 'on the left', i.e. for all x-values $x < a$ but arbitrarily 'close' to a; if so, we write $\lim\limits_{x\to a^-} f(x) = L$ and otherwise say that the limit does not exist.

- Be able to understand and compute left- and right-hand limits graphically, e.g. compute $\lim\limits_{x\to 2^-} f(x)$, $\lim\limits_{x\to 2^+} f(x)$, $\lim\limits_{x\to -6^-} f(x)$, etc., where $f(x)$ is the function given below



- Be able to construct examples of functions graphically with specified values, left/right limits, and limits, e.g. draw a function with $f(1) = 2$, $\lim\limits_{x\to 1^-} f(x) = 2$, but $\lim\limits_{x\to 1} f(x)$ does not exist.
- Understand that $f(x)$ does not need to be defined at $x = a$ to discuss limits at $x = a$.
- Understand that limits are what happens 'near' $x = a$, not what happens at $x = a$, i.e. "limits are about the journey, not the destination."



- Know that $\lim\limits_{x\to a} f(x)$ exists if and only if $\lim\limits_{x\to a^-} f(x)$ exists, $\lim\limits_{x\to a^+} f(x)$ exits, and $\lim\limits_{x\to a^-} f(x) = \lim\limits_{x\to a^+} f(x)$.
- Know the definition and intuitive definition of a limit at $\pm\infty$, e.g. we say that $\lim\limits_{x\to\infty} f(x) = L$ if the outputs of $f(x)$ arbitrarily 'close' to L as x gets 'larger and larger'.
- Be able to compute limits at infinity, i.e. $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$, graphically.
- Be able to define what a horizontal asymptote and a vertical asymptote is: a horizontal asymptote for $f(x)$ is a line $y = A$ such that $\lim\limits_{x\to\infty} f(x) = A$ or $\lim\limits_{x\to-\infty} f(x) = A$, and a vertical asymptote for $f(x)$ is a line $x = A$ such that $\lim\limits_{x\to A^-} f(x) = \pm\infty$ or $\lim\limits_{x\to A^+} f(x) = \pm\infty$.
- Be able to find and identify horizontal asymptotes and vertical asymptotes for a function graphically.
- Be able to find and identify horizontal asymptotes and vertical asymptotes for a function algebraically.
- Understand and be able to use all the limit theorems: if $\lim f = L$ and $\lim g = M$ (and L, M are finite) and c is a real number, then...
  - $\lim(cf) = c\lim f = cL$
  - $\lim(f \pm g) = \lim f \pm \lim g = L \pm M$
  - $\lim(fg) = \lim f \cdot \lim g = LM$
  - $\lim\left(\frac{f}{g}\right) = \frac{\lim f}{\lim g} = \frac{L}{M}$, if $M \neq 0$
- (Optional) Know and be able to explain the ϵ-δ definition of a limit: the limit of $f(x)$ as x approaches a is L, denoted $\lim\limits_{x\to a} f(x) = L$, if for all $\epsilon > 0$, there exists $\delta > 0$ such that $|f(x) - L| < \epsilon$ whenever $|x - a| < \delta$.