MATH 141 — Fall 2024
Exam 2 — Extra Problems

Derivatives

Problem 1. Let $f(x) = x^3 - 12x + 5$. Find the intervals of increase and decrease, intervals of concavity, inflection points, and local minima and maxima of $f(x)$.

Optimization

Problem 2. A box with a square base and open top must have a volume of 32 cubic centimeters. Find the dimensions of the box that minimize the amount of material used.

Problem 3. There is 300 square cm of material available to make a box with a square base and an open top. Find the dimensions of the box with the largest possible volume.

Problem 4. A cylindrical can without a top is made to contain 1000 cm^3 of liquid. Find the dimensions that will minimize the surface area of the metal to make the can.

Problem 5. An open rectangular box that holds 36,000 cubic inches is to have a rectangular base that is twice as long as it is wide. What dimensions for the box will require the least material?

Related Rates

Problem 6. A kite 100 feet above the ground moves horizontally at a speed of 10 feet per second. At what rate is the angle between the string and the horizontal decreasing when 200 feet of string is let out?

Problem 7. A ladder 10 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 3 ft/sec., how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 feet from the wall?

Problem 8. During a game of baseball, a batter hits the ball and runs towards first base at a speed of 25 ft per second? At what rate is their distance from second base decreasing when they are one quarter of the way from first base, i.e. 67.5 ft from first base?

Problem 9. Water is being poured into a conical reservoir at the rate of π cubic feet per second. The reservoir has a radius of 6 ft across the top and a height of 12 ft. At what rate is the depth of the water increasing when the depth is 6 ft?

Problem 10. Car A is leaving at a right-angled intersection and heading south at 60 mph. Car B is leaving the intersection and heading east at 70 mph. When Car A is 6 mi away from the intersection and Car B is 8 mi to the east, how fast is the distance between the cars changing?

Problem 11. A spotlight on the ground shines on a wall 40 ft away. A person 6 ft tall walks from the spotlight toward the wall at a speed of 6 ft/sec. How fast is the length of their shadow on the wall decreasing when they are 15 ft away from the spotlight?

IVT

Problem 12. Show that the polynomial $2x^9 - 5x^4 + 7x - 3$ has a root for some x with $-1 \leq x \leq 1$.

Problem 13. Show that the equation $2^x = 5x - 2$ has a solution on the interval $[0, 1]$.

Problem 14. Let $f(x) = x^3 - x^2 + x - 1$. Show there exists a real number a such that $f(x) = -2$. [Hint. Consider the interval $[-1, 0]$.]

Problem 15. Suppose that $f(x)$ is continuous on $[0, 1]$ and $-1 \leq f(x) \leq 1$ for $x \in [0, 1]$. Show there is a $c \in [0, 1]$ such that $f(c)^2 = c$.

MVT

Problem 16. Find the values c that satisfy the MVT for $f(x) = x^3 + 5x^2 - 7x + 5$ on $[-1, 2]$.

Problem 17. Show that $3x + 2\sin x = 5$ has exactly one solution.

Problem 18. Prove that if two runners race each other and end in a tie, they must have run at the same speed at some point during the race.

Problem 19. If $f(x)$ is a differentiable function on $[2, 9]$ with $f(2) = -3$ and $-1 \leq f'(x) \leq 5$ for $x \in [2, 9]$, what are the smallest and largest possible values for $f(9)$?

IVT & MVT

Problem 20. Show that every positive real number greater than one has a unique positive, real square root.