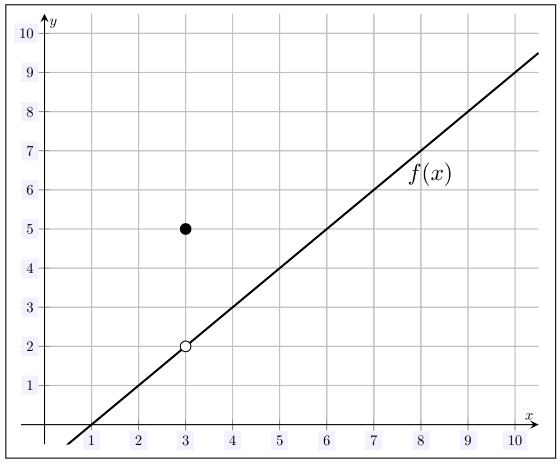
Check-In 01/15. *(True/False) True/False*: If $f(3) = 5$, then $\lim_{x\to 3} f(x) = 5$.

Solution. The statement is *false*. Recall that the limit of a function (if it exists) is what the output gets 'close' to as the input gets 'close' to its limiting value. The fact that $f(3) = 5$ does not mean the outputs are all 'close' to 5 when x is 'close' to 3. For instance, consider the function $f(x)$ plotted below.



Despite the fact that $f(3) = 5$, $\lim_{x\to 3} f(x) = 2$ because all the outputs are 'close' to 2 when the inputs are 'close' to 3.

Check-In 01/17. *(True/False) True/False*: Let $f(x)$ be a function defined on all real numbers such that $\lim_{x\to\pi} f(x) = 10$. Then it must be that $\lim_{x\to\pi^+} f(x) = 10$.

Solution. The statement is *true*. Recall that the limit (if it exists) is what the output gets 'close' to as the input gets 'close' to its limiting value. Because $\lim_{x\to\pi} f(x) = 10$, the outputs of $f(x)$ are all 'close' to 10 whenever x is 'close' to π—no matter how x is 'close' to π. The right-hand limit $\lim_{x\to\pi^+} f(x)$ asks what the outputs are 'close' to if x is 'close' to π—but bigger than π. But we already know that the outputs are 'close' to 10. Therefore, it must be that $\lim_{x\to\pi^+} f(x) = 10$. Recall that $\lim_{x\to a} f(x) = L$ if and only if $\lim_{x\to a^-} f(x) = L$ and $\lim_{x\to a^+} f(x) = L$.

Check-In 01/22. *(True/False)* $\lim_{x\to\infty}\left(1+\dfrac{1}{3x}\right)^x = e^3$

Solution. The statement is *false*. Recall that $\lim_{x\to\infty}\left(1+\dfrac{1}{x}\right)^x = e$. But then...

$$\lim_{x\to\infty}\left(1+\frac{1}{3x}\right)^x = \lim_{x\to\infty}\left(1+\frac{1}{3x}\right)^{x\cdot 3/3} = \lim_{x\to\infty}\left[\left(1+\frac{1}{3x}\right)^{3x}\right]^{1/3} = e^{1/3} = \sqrt[3]{e}$$

Check-In 01/24. *(True/False)* $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = (1+0)^\infty = 1^\infty = 1$

Solution. The statement is *false*. One does obtain 1^∞ after naïvely plugging in $x=\infty$. However, ∞ is not a number; moreover, although one might feel otherwise, it is simply need not be the case that $1^\infty = 1$. Indeed, 1^∞ is an indeterminant form. One could correctly recall that...

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$$