Check-In 08/21. *(True/False)* The integral $\int x\sqrt[3]{x-2}\,dx$ can be treated as a 'shifting integral' by using the u-substitution $u = x - 2$.

Solution. The statement is *true*. We 'want' to be able to distribute the x across the cube-root but we cannot—this is not a valid operation. However, if we make the u-substitution $u = x - 2$, then we will be able to distribute in a way that makes this integral 'routine.' So, let $u = x - 2$, then $du = dx$. Moreover, because $u = x - 2$, we know that $x = u + 2$. But then…

$$\int x\sqrt[3]{x-2}\,dx = \int (u+2)\sqrt[3]{u}\,du = \int \left(u^{4/3} + 2u^{1/3}\right)\,du = \tfrac{3}{7}\,u^{7/3} + \tfrac{3}{4}\cdot 2u^{4/3} + C = \tfrac{3}{7}(x-2)^{7/3} + \tfrac{3}{2}(x-2)^{4/3} + C$$

Note that a computer algebra system may write the answer (though you will *not* be expected to) like this:

$$\tfrac{3}{7}(x-2)^{7/3} + \tfrac{3}{2}(x-2)^{4/3} + C = (x-2)^{4/3}\left(\tfrac{3}{7}(x-2) + \tfrac{3}{2}\right) + C = (x-2)^{4/3}\left(\tfrac{3}{7}x + \tfrac{9}{14}\right) + C = \tfrac{3}{14}(x-2)^{4/3}\,(2x+3) + C$$

Check-In 08/26. *(True/False)* Using integration-by-parts to evaluate $\int x\tan^{-1}(x)\,dx$, one chooses $u = \tan^{-1} x$ and $dv = x$.

Solution. The statement is *true*. Using LIATE, the first term that appears is 'I' for inverse trig. Therefore, we choose $u = \tan^{-1} x$. But then $dv = x$. We then fill in our box:

$\tan^{-1} x$	$\dfrac{x^2}{2}$
$\dfrac{1}{1+x^2}$	x

Using the 'rule of 7', we have…

$$\int x\tan^{-1} x\,dx = \frac{1}{2}\,x^2\tan^{-1} x - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx$$

We now need only evaluate the integral on the right. Dividing $1 + x^2$ into x^2, we have a remainder of -1, i.e. $\dfrac{x^2}{1+x^2} = 1 + \dfrac{-1}{1+x^2}$. Therefore, we have…

$$\frac{1}{2}\int \frac{x^2}{1+x^2}\,dx = \frac{1}{2}\int \left(1 + \frac{-1}{1+x^2}\right)\,dx = \frac{1}{2}\left(x - \tan^{-1} x\right) + C$$

But then...

$$\begin{aligned}
\int x\tan^{-1}x\;dx &= \frac{1}{2}\,x^2\tan^{-1}x-\frac{1}{2}\int\frac{x^2}{1+x^2}\;dx\\
&= \frac{1}{2}\,x^2\tan^{-1}x-\frac{1}{2}\left(x-\tan^{-1}x\right)+C\\
&= \frac{1}{2}\,x^2\tan^{-1}x-\frac{1}{2}\,x+\frac{1}{2}\,\tan^{-1}x+C\\
&= \frac{x^2\tan^{-1}x-x+\tan^{-1}x}{2}+C\\
&= \frac{(x^2+1)\tan^{-1}x-x}{2}+C
\end{aligned}$$

Check-In 08/28. *(True/False)* The integral $\displaystyle\int e^x\sin(3x)\;dx$ can be treated as an integration-by-parts 'looping' integral.

Solution. The statement is *true*. Using integration-by-parts for $\displaystyle\int e^x\sin(3x)\;dx$ would result in an integral that would 'loop' back to itself. Generally, an integrand of the form exponential $\cdot$ (sin or cos) or trig $\cdot$ trig will have this property. Using traditional integration-by-parts, by LIATE, we choose $u=\sin(3x)$ and $dv=e^x$. Filling out our box, we have...

$\sin(3x)$	e^x
$3\cos(3x)$	e^x

Using the 'rule of seven', we then have...

$$\int e^x\sin(3x)\;dx=e^x\sin(3x)-\int 3e^x\cos(3x)\;dx$$

To integrate $\displaystyle\int 3e^x\cos(3x)\;dx$, we again use integration-by-parts. Using LIATE, we choose $u=$

$3\cos(3x)$ and $dv = e^x$. Filling out the box, we have...

$3\cos(3x)$	e^x
$-9\sin(3x)$	e^x

Using the 'rule of seven', we then have

$$\int 3e^x \cos(3x)\ dx = 3e^x \cos(3x) - \int -9e^x \sin(3x)\ dx = 3e^x \cos(3x) + 9 \int e^x \sin(3x)\ dx$$

But then we have...

$$\int e^x \sin(3x)\ dx = e^x \sin(3x) - \int 3e^x \cos(3x)\ dx = \int e^x \sin(3x)\ dx = e^x \sin(3x) - \left(3e^x \cos(3x) + 9 \int e^x \sin(3x)\ dx\right)$$

Therefore, we have...

$$\int e^x \sin(3x)\ dx = e^x \sin(3x) - \left(3e^x \cos(3x) + 9 \int e^x \sin(3x)\ dx\right)$$

$$\int e^x \sin(3x)\ dx = e^x \sin(3x) - 3e^x \cos(3x) - 9 \int e^x \sin(3x)\ dx$$

$$10 \int e^x \sin(3x)\ dx = e^x \sin(3x) - 3e^x \cos(3x)$$

$$\int e^x \sin(3x)\ dx = \frac{e^x \sin(3x) - 3e^x \cos(3x)}{10} + C$$

$$\int e^x \sin(3x)\ dx = \frac{e^x}{10} \big(\sin(3x) - 3\cos(3x)\big) + C$$

Alternatively, we can use an alternation of the tabular method of integration-by-parts. We choose $u = \sin(3x)$ and $dv = e^x$. We then have...

$$\begin{array}{cc} u & dv \\ \hline \sin(3x) & e^x \\ & {\scriptstyle +} \\ 3\cos(3x) & e^x \\ & {\scriptstyle -} \\ -9\sin(3x) & \xrightarrow{+} e^x \end{array}$$

Therefore, we have...

$$\int e^x \sin(3x)\ dx = e^x \sin(3x) - 3\cos(3x)e^x - 9 \int e^x \sin(3x)\ dx$$

Solving for our integral, we have...

$$\int e^x \sin(3x)\ dx = e^x \sin(3x) - 3\cos(3x)e^x - 9\int e^x \sin(3x)\ dx$$

$$10\int e^x \sin(3x)\ dx = e^x \sin(3x) - 3e^x \cos(3x)$$

$$\int e^x \sin(3x)\ dx = \frac{e^x \sin(3x) - 3e^x \cos(3x)}{10} + C$$

$$\int e^x \sin(3x)\ dx = \frac{e^x}{10}\big(\sin(3x) - 3\cos(3x)\big) + C$$