

MATH 142: Exam 3

Name: _____

Fall —₁ 2025

11/20/2025

75 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 8 pages (including this cover page) and 6 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	18	
2	16	
3	14	
4	16	
5	18	
6	18	
Total:	100	

1. (18 points) Fill out the table of Maclaurin series below.

Function	First Two Nonzero Terms	Series	Interval of Convergence
$\frac{1}{1-x}$	_____	_____	_____
e^x	_____	_____	_____
$\sin(x)$	_____	_____	_____
$\cos(x)$	_____	_____	_____
$\ln(1+x)$	_____	_____	_____
$\arctan(x)$	_____	_____	_____

2. (16 points) Showing all your work, find the Taylor series for $f(x) = \frac{3}{x^2}$ centered at $x = -1$. You may *not* use any known Taylor series to find this Taylor series. It must be derived “from scratch.” You *do not* need to find the interval of convergence for this Taylor series but you should simplify the resulting series.

3. (14 points) Showing all your work and fully justifying your reasoning, find the center, radius of convergence, and interval of convergence for each following power series:

(a) $\sum_{n=0}^{\infty} n! (5x + 2)^n$

(b) $\sum_{n=0}^{\infty} (-1)^n \frac{x^n}{(2n)!}$

4. (16 points) Showing all your work, complete the following:

(a) Find the Maclaurin series for $x \cos(x^3)$.

(b) Compute $\sum_{n=0}^{\infty} n \left(-\frac{1}{2}\right)^{n-1}$ [Hint. Differentiate $\sum_{n=0}^{\infty} x^n$.]

(c) Compute $\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \frac{1}{7} + \dots$

(d) Replacing $\sin x$ with a Taylor series, compute $\lim_{x \rightarrow 0} \frac{x - \sin x}{3x^3}$. [You will receive no credit for using l'Hôpital's rule. However, you may check your answer with it.]

5. (18 points) Showing all your work and fully justifying your reasoning, find the center, radius of convergence, and interval of convergence for the following power series:

$$\sum_{n=1}^{\infty} \frac{(x - 59)^n}{8^n \sqrt{n}}$$

6. Consider the function $f(x) = \frac{1}{x}$. Showing all your work and fully justifying your reasoning, complete the following:
- (6 points) Find the second degree Taylor polynomial, $T_2(x)$, for $f(x)$ centered at $x = 2$.
 - (6 points) Find the maximum error approximating $f(2.1)$ using $T_2(2.1)$.
 - (6 points) Find the maximum error approximating the values of $f(x)$ using $T_2(x)$ on $[1, 3]$.