MATH 115 (08/28): Sets, Geometry, & Function Problems

Problem 1. Consider the intervals $A = [-3, 3]$, $B = (-2, 1]$, $C = [0, 5]$, $D = \{0\}$, $E = (-2, -1)$, and $F = (1, 4)$. Expressing your answer as simply as possible, compute the following:

(a) $A \cap B$

(b) $A \cup C$

(c) $B \cap D$

(d) $A \cup F$

(e) $C \cap F$

(f) $A \cap (E \cup F)$

Problem 2. Define the points $A = (-1, 3)$, $B = (1, 1)$, and $C = (2, 6)$.

(a) Find the distance from A to the y-axis.

(b) Find the distance from C to the x-axis.

(c) Find the slope of the segment connecting A and B.

(d) Find the midpoints of AC.

(e) Find the distances between each pair of points.

(f) Is the triangle formed by ABC a right triangle? Explain.

Problem 3. Consider the relation given by $x = y^2 + y - 12$.

(a) Find a point on the graph given by this equation.

(b) Is $(-18, -2)$ on the graph given by this equation? Explain.

(c) Is the given relation a function of x? Explain.

(d) Is the relation a function of y? Explain.

(e) Find the x-intercepts for the graph of this relation.

(f) Find the y-intercepts for the graph of this relation.

Problem 4. A rope is attached to the tip of a boat. The rope is pulled taunt over a pulley suspended 8 ft above the ground at the edge of the water. The boat is 30 ft from the shore and rope is being pulled over the pulley at a rate of 2 ft per second. Find the distance the tip of the boat is from the shore after t seconds.

Problem 5. Simplify the following as much as possible:

(a) $\dfrac{x^4(x^5y^{-2})^{-3}}{x^{-1}(xy)^5(x^0y^{-8})}$

(b) $\left(\dfrac{x^{-2}}{y^4}\right)^{-5}\sqrt{\dfrac{x^4}{y^2}}$

(c) $\left(\left(\dfrac{(x^0y^5)^{-2}y^4x^{-2}}{x^{-10}y^5}\right)^2\right)^{-1}$

Problem 6. Consider the relation given by $f(x) = \dfrac{x+3}{x}$.

(a) Is $f(x)$ a function? Explain.

(b) Find a point on the graph given by $f(x)$? Explain.

(c) Is $(-1,-2)$ on the graph given by $f(x)$? Explain.

(d) Is π in the domain of $f(x)$? Explain.

(e) Is 0 in the domain of $f(x)$? Explain.

(f) Find something in the range/image of $f(x)$. Justify your answer.

(g) Is $5 \in$ im f, i.e. $5 \in$ range(f)? Justify your answer.

(h) Do the graphs of $f(x)$ and $g(x) = 2(3-x)$ intersect? Explain.

(i) Find and simplify $f(1+a)$.

Problem 7. Complete the following:

(a) Find an expression 'describing' the set of all points a distance 5 from the point $(4,-3)$.

(b) Find the equation of a circle with center $(1,7)$ that intersects the x-axis only once.

(c) Sketch a relation whose domain is the positive real numbers and passes the vertical line everywhere except at $x = 5$.

(d) Let $\mathcal{O}$ be the origin and P be the point in Quadrant II where the graph of $f(x) = 6 - x^2$ intersects the line $y = 4$. Find the area of the rectangle that has $\mathcal{O}P$ as one of its diagonals.

Problem 8. Let $A = \{0,1,2,3\}$ and $B = \{-1,0,1,5,9\}$.

(a) Give an example of a relation from A to B that is not a function.

(b) Give an example of a function from A to B.

(c) Give an example of a function from A to B that is not constant but is constant on the subset of A given by $\{0,2\}$.

(d) Give an example of a function from A to B that is increasing.

(e) Give an example of a function from A to B that is not increasing.

(f) Is it possible to define a function from A to B whose range and codomain are the same? Explain.

Problem 9. Let $F(x) = \dfrac{\sqrt{x+3}}{x-2}$.

(a) Is $F(x)$ a function of x? Explain.

(b) What is the domain of $F(x)$?

(c) What is $F(10)$? What is the corresponding point on the graph of $F(x)$?

(d) Find the x- and y-intercepts for $F(x)$.

(e) Is $-2 \in$ range $(F(x))$? Explain.