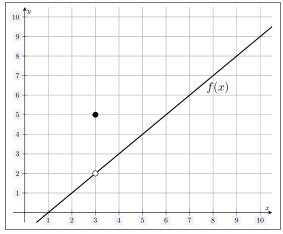
Check-In 01/15. (True/False) True/False: If f(3) = 5, then $\lim_{x \to 3} f(x) = 5$.

Solution. The statement is *false*. Recall that the limit of a function (if it exists) is what the output gets 'close' to as the input gets 'close' to its limiting value. The fact that f(3)=5 does not mean the outputs are all 'close' to 5 when x is 'close' to 3. For instance, consider the function f(x) plotted below.



Despite the fact that f(3)=5, $\lim_{x\to 3}f(x)=2$ because all the outputs are 'close' to 2 when the inputs are 'close' to 3.

Check-In 01/17. (*True/False*) *True/False*: Let f(x) be a function defined on all real numbers such that $\lim_{x\to\pi}f(x)=10$. Then it must be that $\lim_{x\to\pi^+}f(x)=10$.

Solution. The statement is *true*. Recall that the limit (if it exists) is what the output gets 'close' to as the input gets 'close' to its limiting value. Because $\lim_{x\to\pi}f(x)=10$, the outputs of f(x) are all 'close' to 10 whenever x is 'close' to π —no matter how x is 'close' to π . The right-hand limit $\lim_{x\to\pi^+}f(x)$ asks what the outputs are 'close' to if x is 'close' to π —but bigger than π . But we already know that the outputs are 'close' to 10. Therefore, it must be that $\lim_{x\to\pi^+}f(x)=10$. Recall that $\lim_{x\to a}f(x)=L$ if and only if $\lim_{x\to a^-}f(x)=L$ and $\lim_{x\to a^+}f(x)=L$.