MATH 142: Final Exam **Name:** ______________________________
Fall — 2025
12/09/2025
150 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 18 pages (including this cover page) and 15 questions. Check that you have every page of the exam. Answer the questions in the spaces provided. Be sure to follow instructions, answer every question completely, and show all your work.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
14	10	
15	10	
Total:	150	

'Twas the night before Christmas, all frozen and still,
Santa is working hard, but Math makes him quite ill.
The sleigh is loaded, and the reindeer are prepared,
But with problems unsolved, no gifts can be shared.

"These problems are hard!", Santa cried out in fright.
"It's clear all my derivatives and sums aren't right!"
This math needs to be done before Santa can take flight,
He'll need a Calculus ace to help Save Christmas tonight!

1. (10 points) *On the brink of his flight, Santa is overcome with dismay,*
For he'd forgotten his integrals, quick, he needs help right away!

Showing all your work, compute the following:

(a) $\displaystyle\int \frac{\ln x}{\sqrt{x}}\, dx$

(b) $\displaystyle\int xe^{2x}\, dx$

2. (10 points) *Asked if he knew integral methods, Santa sat in silence, thinking it through. He shouted "67!". The elves let out a groan, "Okay, so we're not passing you."*

Showing all your work, compute the following:

$$\int \frac{x^2 + 4x - 6}{x^2(x+3)}\, dx$$

3. (10 points) *Though Rudolph's red nose shone brightly, it dims at the integral ahead,*
Santa needs help! Quick, compute this integral or Christmas could be dead!

Showing all your work, determine whether the following integral converges or diverges:

$$\int_1^2 \frac{dx}{(2-x)^3}$$

4. (10 points) *For the naughtiest students, Krampus brought integrals to fear,*
"Solve these quickly or you'll be stuck in Calculus next year!"

Showing all your work, compute the following:

$$\int \frac{dx}{(16 - x^2)^{3/2}}$$

5. (10 points) *"These integrals!" cried Santa. "These problems must be rigged."*
"No one reminded me, I had to be good at trig!

Showing all your work, compute the following:

$$\int \sec^7(\theta)\tan(\theta)\ d\theta$$

6. (10 points) *All through the night, ole Santa worked—pressing on with cheer.*

   Suddenly, Santa froze and gasped. He'd forgotten his series this year!

   Showing all your work and completely justifying your logic, determine whether the following series converges or diverges:

$$\sum_{n=3}^{\infty} (-1)^n \frac{\ln n}{n}$$

7. (10 points) *What Santa knew about series wasn't quite true.*

   Turns out, he's gonna need some help getting through!

   Showing all your work and completely justifying your logic, determine whether the following series converges or diverges:

$$\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}$$

8. (10 points) *Though the elves did their best, their calculations gave Santa a fright,*
He ho'd, "Bring help—these series might ground us tonight!"

Showing all your work and completely justifying your logic, determine whether the following series diverges, converges conditionally, or converges absolutely.

$$\sum_{n=0}^{\infty} \frac{(-1)^{n-1}}{7+2n}$$

9. (10 points) *Santa's trajectory not correct, "Someone help me!"*

   No one told me I that couldn't use Chat-GPT!

   Showing all your work and completely justifying your logic, determine whether the series below converges or diverges. If the series converges, find its sum. If it diverges, explain why.

$$\sum_{n=0}^{\infty} \frac{3}{2^{n-1}}$$

10. (10 points) *Santa hurtled through the sky, the fate of Christmas so near,*

But he shook with fear, the series were too tricky to handle this year!

Showing all your work and completely justifying your logic, determine whether the following series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{5n}{n^3 + 4}$$

11. (10 points) *Santa puzzled through power series, each term driving him insane,*
But "At this rate, I'm as safe as a budget Boeing plane!"

Showing all your work and completely justifying your logic, determine the center, radius, and interval of convergence for the following power series:

$$\sum_{n=0}^{\infty} (-1)^n \frac{(x+6)^n}{n!}$$

12. (10 points) *As he plotted his course, Santa frowned at each point of divergence,,*
For he couldn't take flight till someone helped find his interval of convergence.

Showing all your work and completely justifying your logic, determine the center, radius, and interval of convergence for the following power series:

$$\sum_{n=1}^{\infty} \frac{x^n}{n\,3^n}$$

13. (10 points) *As he tallied the sum, Santa muttered, "These terms never cease!"*
"If this series keeps growing, I'll be stuck here till after next Christmas feast!"

(a) Use the Maclaurin series for $\cos(x)$ to find a Maclaurin series for $x\cos(x^2)$.

(b) Use (a) to compute the following sum:

$$\sum_{n=0}^{\infty}(-1)^n \frac{\pi^{\frac{4n+1}{2}}}{(2n)!}$$

14. (10 points) *Five golden rings, four calling birds, three French hens,*
Two turtle doves, and a partridge in a pear tree!
. . . but they were all of them deceived for another ring was made.

Define the function $f(x)$ to be $f(x) = xe^{-x^4}$.

(a) Using a known power series, express $f(x)$ as an infinite series.

(b) Use your answer from (a) to express the following integral as a series:

$$\int xe^{-x^4}\,dx$$

15. (10 points) *With presents delivered and Christmas nearly in the clear,*

Santa has one last task, and then you're done for this year!

Showing all your work, find the third degree Taylor polynomial, $T_3(x)$, centered at $x = 0$ for the function $f(x) = \sqrt{x+1}$.

The night was set right and magic of Christmas came true.
Santa let out a sigh, these students really came through.
Each integral untangled, each convergence made precise,
They restored all the magic that keeps Christmas nice.

And Santa called out as he rode through the night,
His voice full of cheer and his heart lifted light:
"Merry Christmas to all! But before I depart,
Never forget your Calculus—the math of the heart!"