Linear u-Substitution: u-Sub. in Your Head

Determine whether each of the following integrals is an 'obvious' 'linear u-sub.' If it is, compute the integral 'in your head.'

Problems

(1) $\displaystyle\int \sin(\pi x)\,dx$

(2) $\displaystyle\int e^{5x}\,dx$

(3) $\displaystyle\int \frac{dx}{x^2-10x+25}$

(4) $\displaystyle\int \tan(6s)\,ds$

(5) $\displaystyle\int \ln(2x)\,dx$

(6) $\displaystyle\int \frac{\left(1+\frac{1}{x}\right)^6}{x^2}\,dx$

(7) $\displaystyle\int (6x-7)^4\,dx$

(8) $\displaystyle\int \frac{\sin\theta}{\cos^2\theta+1}\,d\theta$

(9) $\displaystyle\int \sin(1-3a)\,da$

(10) $\displaystyle\int e^{9-x}\,dx$

(11) $\displaystyle\int \ln(9x)\,dx$

(12) $\displaystyle\int \sqrt[3]{1-5x}\,dx$

(13) $\displaystyle\int (1-t)^{11}\,dt$

(14) $\displaystyle\int \sin(\cos(x))\sin x\,dx$

(15) $\displaystyle\int \cos\left(\frac{x}{2}\right)\,dx$

(16) $\displaystyle\int \left(\frac{5}{7}\right)^{x/6}\,dx$

(17) $\displaystyle\int \frac{e^{\ln x}}{x}\,dx$

(18) $\displaystyle\int \frac{dt}{6t-13}$

(19) $\displaystyle\int 8^{1-x}\,dx$

(20) $\displaystyle\int \cos(5x)\,dx$

(21) $\displaystyle\int \cot(1-x)\,dx$

(22) $\displaystyle\int \frac{dx}{\sqrt{x}(\sqrt{x}+1)}$

(23) $\displaystyle\int \frac{1}{(5x+2)^4}\,dx$

(24) $\displaystyle\int \sin\left(\frac{x}{7}\right)\,dx$

(25) $\displaystyle\int 5^{6x}\,dx$

(26) $\displaystyle\int \frac{e^x+5}{e^x}\,dx$

(27) $\displaystyle\int \left(\frac{2x-1}{3}\right)^5\,dx$

(28) $\displaystyle\int \csc^2(9z)\,dz$

(29) $\displaystyle\int \frac{x}{\sqrt{x+1}}\,dx$

(30) $\displaystyle\int \cos(2w+1)\,dw$

(31) $\displaystyle\int e^{3m-4}\,dm$

(32) $\displaystyle\int \ln\left(\frac{x}{5}\right)\,dx$

(33) $\displaystyle\int (r+3)^8\,dr$

(34) $\displaystyle\int q^3\sqrt{q^2+1}\,dq$

(35) $\displaystyle\int (3x-7)^3\,dx$

(36) $\displaystyle\int \sec(4x)\tan(4x)\,dx$

(37) $\displaystyle\int \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$

(38) $\displaystyle\int \sqrt{3z+1}\,dz$

(39) $\displaystyle\int \log_8(4x)\,dx$

(40) $\displaystyle\int e^{n/7}\,dn$

(41) $\displaystyle\int \csc(2x)\cot(2x)\,dx$

(42) $\displaystyle\int \frac{\tan\sqrt[3]{x^2}}{\sqrt[3]{x}}\,dx$

(43) $\displaystyle\int e^{x/5}\,dx$

(44) $\displaystyle\int \tan\left(\frac{x}{13}\right)\,dx$

(45) $\displaystyle\int \log_2(3a+4)\,da$

(46) $\displaystyle\int \frac{dx}{(x+10)^3}$

(47) $\displaystyle\int x^4\sec^2(3x^5)\,dx$

(48) $\displaystyle\int (0.3)^{x/0.1}\,dx$

(49) $\displaystyle\int (6x+13)^{2/3}\,dx$

(50) $\displaystyle\int \frac{dx}{x\ln\sqrt{x}}$

(51) $\displaystyle\int \pi^{\pi x}\,dx$

(52) $\displaystyle\int \frac{dx}{3-2x}$

(53) $\displaystyle\int (4x)^{100}\,dx$

(54) $\displaystyle\int \frac{4\ln x}{x(\ln x+10)}\,dx$

(55) $\displaystyle\int (5b^2+15)(b^3+15b)^{11}\,db$

Solutions

Note. The solutions are not simplified—rather given as one would write 'from one's head.'

(1) $-\dfrac{\cos(\pi x)}{\pi} + C$

(2) $\dfrac{e^{5x}}{5} + C$

(3) Not an obvious linear u-sub.

(4) $\dfrac{\ln(\sec 6x)}{6} + C$

(5) $\dfrac{2x - 2x\ln(2x)}{2} + C$

(6) Not a linear u-sub.

(7) $\dfrac{(6x-7)^5}{5} \cdot \dfrac{1}{6} + C$

(8) Not a linear u-sub.

(9) $-\cos(1-3x) \cdot -\frac{1}{3} + C$

(10) $-e^{9-x} + C$

(11) $\dfrac{9x - 9x\ln(9x)}{9} + C$

(12) $\dfrac{3(1-5x)^{4/3}}{4} \cdot -\dfrac{1}{5} + C$

(13) $-\dfrac{(1-t)^{12}}{12} + C$

(14) Not a linear u-sub.

(15) $2\sin\left(\dfrac{x}{2}\right) + C$

(16) $\dfrac{\left(\dfrac{5}{7}\right)^{x/6}}{\ln(5/7)} \cdot 6 + C$

(17) Not a linear u-sub.

(18) $\dfrac{\ln|6t-13|}{6} + C$

(19) $-\dfrac{8^{1-x}}{\ln 8} + C$

(20) $\dfrac{\sin(5x)}{5} + C$

(21) $-\ln(\sin(1-x)) + C$

(22) Not a linear u-sub.

(23) $\dfrac{(5x+2)^{-3}}{-3} \cdot \dfrac{1}{5} + C$

(24) $-\cos\left(\dfrac{x}{7}\right) \cdot 7 + C$

(25) $\dfrac{5^{6x}}{\ln 5} \cdot \dfrac{1}{6} + C$

(26) Not a linear u-sub.

(27) $\left(\dfrac{2x-1}{3}\right)^6 \cdot \dfrac{1}{6} \cdot \dfrac{2}{3} + C$

(28) $\dfrac{-\cot(9z)}{9} + C$

(29) Not an obvious linear u-sub.

(30) $\dfrac{\sin(2w+1)}{2} + C$

(31) $\dfrac{e^{3m-4}}{3} + C$

(32) $\left(\dfrac{x}{5} - \dfrac{x}{5}\ln\left(\dfrac{x}{5}\right)\right) \cdot 5 + C$

(33) $\dfrac{(r+3)^9}{9} + C$

(34) Not an obvious (linear) u-sub.

(35) $\dfrac{(3x-7)^4}{4} \cdot \dfrac{1}{3} + C$

(36) $\dfrac{\sec(4x)}{4} + C$

(37) Not a linear u-sub.

(38) $\dfrac{2(3z+1)^{3/2}}{3} \cdot \dfrac{1}{3} + C$

(39) $\dfrac{4x - 4x\log_8(4x)}{\ln 8} \cdot \dfrac{1}{4} + C$

(40) $7e^{n/7} + C$

(41) $\dfrac{-\csc(2x)}{2} + C$

(42) Not a linear u-sub.

(43) $5e^{x/5} + C$

(44) $13\ln\left|\sec\left(\dfrac{x}{13}\right)\right| + C$

(45) $\dfrac{(3a+4) - (3a+4)\log_2(3a+4)}{\ln 2} \cdot \dfrac{1}{3} + C$

(46) $\dfrac{(x+10)^{-2}}{-2} + C$

(47) Not a linear u-sub.

(48) $\dfrac{(0.3)^{x/0.1}}{\ln(0.3)} \cdot 0.1 + C$

(49) $\dfrac{3(6x+13)^{5/3}}{5} \cdot \dfrac{1}{6} + C$

(50) Not a linear u-sub.

(51) $\dfrac{\pi^{\pi x}}{\ln \pi} \cdot \dfrac{1}{\pi} + C$

(52) $\ln|3-2x| \cdot \dfrac{1}{-2} + C$

(53) $\dfrac{(4x)^{101}}{101} \cdot \dfrac{1}{4} + C$

(54) Not a linear u-sub.

(55) Not a linear u-sub.