MATH 142: Exam 3 **Name:** _____
Spring — 2025
04/17/2025
75 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 8 pages (including this cover page) and 7 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	10	
2	20	
3	20	
4	15	
5	15	
6	10	
7	10	
Total:	100	

1. (10 points) Answer the following:

   (a) What is the Taylor series for $\sin x$ centered at $x = 0$? Be sure to give the series and its interval of convergence.

   (b) What is the Taylor series for $\cos x$ centered at $x = 0$? Be sure to give the series and its interval of convergence.

   (c) What is the Taylor series for $\dfrac{1}{1-x}$ centered at $x = 0$? Be sure to give the series and its interval of convergence.

2. (20 points) Find the first four nonzero terms of the Taylor series centered at $x = 1$ for the function $f(x) = 3x^2 + e^{1-x}$.

3. (20 points) Find the center, radius of convergence, and interval of convergence for the following power series:

$$\sum_{n=1}^{\infty} \frac{(x-4)^n}{n\,5^n}$$

4. (15 points) Showing all your work, complete the following parts:

   (a) Find the Maclaurin series for e^{-x^2}.

   (b) Use (a) to 'compute' the following:

$$\int_0^1 e^{-x^2}\, dx$$

5. (15 points) Find the center, radius of convergence, and interval of convergence for the following power series:

$$\sum_{n=1}^{\infty} \frac{n!\, x^n}{\sqrt{n}}$$

6. (10 points) Showing all your work, compute the following sum:

$$\sum_{n=0}^{\infty} \frac{3^n}{n!}$$

7. (10 points) Consider the function $f(x) = e^x$. Let $T_2(x)$ be the second degree Taylor polynomial centered at $x = 0$ for $f(x)$. If one uses $T_2(x)$ to approximate $f(x)$ on $(-1, 1)$, what is the largest possible error using this approximation? Be sure to fully justify your answer.