MATH 141: Exam 2
Fall —1 2025
10/24/2025
50 Minutes

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Write your name on the appropriate line on the exam cover sheet. This exam contains 7 pages (including this cover page) and 6 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	15	
2	15	
3	15	
4	15	
5	15	
6	15	
Total:	90	

1. (15 points) The approximation $\sqrt[3]{1+x} \approx 1 + \frac{x}{3}$ for "small" x is often used in Physics. Justify this approximation by finding the linearization of $f(x) = \sqrt[3]{1+x}$ when $x = 0$.

$$\begin{cases} F(x) = \sqrt[3]{1+x} = (1+x)^{1/3} \\ F(0) = \sqrt[3]{1+0} = 1 \end{cases}$$

$$\begin{cases} F'(x) = \frac{1}{3}(1+x)^{-2/3} \\ F'(0) = \frac{1}{3}(1+0)^{-2/3} = 1/3 \end{cases}$$

$$\boxed{\ell(x) = 1 + \frac{1}{3}(x-0)}$$

2. Showing all your work, answer the following:

(a) (5 points) Given that $(x+y)^5 = x - y^2$, find $\frac{dy}{dx}$.

$$\frac{d}{dx}\left((x+y)^5\right) = \frac{d}{dx}\left(x - y^2\right)$$

$$5(x+y)^4 \cdot (1+y') = 1 - 2yy'$$

$$5(x+y)^4 + 5(x+y)^4 y' = 1 - 2yy'$$

$$5(x+y)^4 y' + 2yy' = 1 - 5(x+y)^4$$

$$\left(5(x+y)^4 + 2y\right)y' = 1 - 5(x+y)^4$$

$$\boxed{\frac{dy}{dx} = \frac{1 - 5(x+y)^4}{5(x+y)^4 + 2y}}$$

(b) (5 points) Assuming that x depends on time but y *does not*, differentiate the following expression with respect to time, t: $x^2 + y^3 - \sin(xy)$

$$\frac{d}{dt}\left(x^2 + y^3 - \sin(xy)\right)$$

$$\boxed{2x\frac{dx}{dt} - \cos(xy) \cdot y\frac{dx}{dt}}$$

(c) (5 points) Assuming that *both* x and y depend on time, differentiate the following expression with respect to time, t: $x^2 + y^3 - \sin(xy)$

$$\frac{d}{dt}\left(x^2 + y^3 - \sin(xy)\right)$$

$$\boxed{2x\frac{dx}{dt} + 3y^2\frac{dy}{dt} - \cos(xy)\left(\frac{dx}{dt}y + \frac{dy}{dt}x\right)}$$

3. (15 points) Showing all your work, compute the following limits:

(a) $\lim_{x\to 0} \dfrac{x}{\arctan(x)}$ $\overset{LH}{=} \lim_{x\to 0} \dfrac{1}{\frac{1}{1+x^2}} = \lim_{x\to 0} (1+x^2) = \boxed{1}$

(b) $\lim_{x\to\infty} \dfrac{\ln x}{\sqrt[4]{x}}$ $\overset{LH}{=} \lim_{x\to\infty} \dfrac{\frac{1}{x}}{\frac{1}{4}x^{-3/4}} = \lim_{x\to\infty} \dfrac{4x^{3/4}}{x} = \lim_{x\to\infty} \dfrac{4}{x^{1/4}} = \boxed{0}$

(c) $\lim_{x\to 0}(\cos x)^{-4/x^2}$

$L := \lim_{x\to 0} (\cos x)^{-4/x^2}$

$\ln L = \lim_{x\to 0} \ln(\cos x)^{-4/x^2}$

$\ln L = \lim_{x\to 0} \dfrac{-4\ln(\cos x)}{x^2}$

$\ln L \overset{LH}{=} \lim_{x\to 0} \dfrac{-4\cdot\frac{1}{\cos x}\cdot -\sin x}{2x}$

$\ln L = \lim_{x\to 0} \dfrac{4\sin x}{2x\cos x}$

$\ln L \overset{LH}{=} \lim_{x\to 0} \dfrac{4\cos x}{2\cos x - 2x\sin x}$

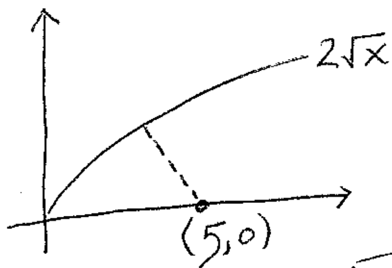
$\ln L = \dfrac{4}{2-0}$

$\ln L = 2$

$e^{\ln L} = e^2$

$\boxed{L = e^2}$

4. (15 points) Find the point on curve $y = 2\sqrt{x}$ closest to the point $(5, 0)$.



$$D = \sqrt{(x-5)^2 + (y-0)^2}$$

$$D = \sqrt{(x-5)^2 + (2\sqrt{x})^2}$$

$$D = \sqrt{x^2 - 10x + 25 + 4x}$$

$$D = \sqrt{x^2 - 6x + 25}$$

$$D^2 = \underbrace{x^2 - 6x + 25}_{f(x)}$$

$$F'(x) = 2x - 6$$

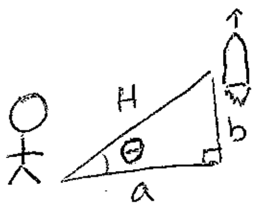
$$F' = 0$$

$$2x - 6 = 0$$

$$\boxed{x = 3}$$

$$\boxed{(3, 2\sqrt{3})}$$

5. (15 points) An amateur rocket builder stands 900 ft from the launch point of their rocket. Approximately two seconds after launch, they estimate that the rocket makes an angle of 60° angle with the ground and this angle is changing at a rate of 8° per second. Assuming the rocket rises vertically from the pad, accelerates to its maximum speed instantaneously, and travels at a constant speed, estimate the speed of the rocket.



Know

$a = 900$ Ft * Constant

$\theta = \frac{\pi}{3}$

$\theta' = \cancel{8}^{2}° \cdot \frac{\pi}{\cancel{180}_{45}} = \frac{2\pi}{45}$

Want

$\frac{db}{dt}$

$\tan\theta = \frac{b}{a}$

$\tan\theta = \frac{b}{900}$

$b = 900 \tan\theta$

$\frac{db}{dt} = 900 \sec^2\theta \cdot \frac{d\theta}{dt}$

$\frac{db}{dt} = 900 \left(\sec\frac{\pi}{3}\right)^2 \cdot \frac{2\pi}{45}$

$= \cancel{900}^{20} \cdot 2^2 \cdot \frac{2\pi}{\cancel{45}_1}$

$= 20 \cdot 4 \cdot 2\pi$

$= 160\pi$

$\boxed{160\pi}$

6. (15 points) A function and its derivatives are given below.

$$f(x) = \frac{x^2+4}{x}, \qquad f'(x) = \frac{x^2-4}{x^2}, \qquad f''(x) = \frac{8}{x^3} \Bigg\} \text{ Undef @ } x = 0$$

Showing all your work, use the information above to answer the questions below.

(a) Find and classify the critical values (if any) for $f(x)$ as local maxima, local minima, or neither.

$\frac{x^2-4}{x^2} = 0$

$x^2 - 4 = 0$

$(x-2)(x+2) = 0$

F': $+$ | -2 | $-$ | 0 | $-$ | 2 | $+$

$\boxed{\begin{array}{l} x = -2: \text{ local max} \\ x = 2: \text{ local min} \\ x = 0: \text{ neither, not in domain } F(x) \end{array}}$

(b) Find the intervals where $f(x)$ is increasing and find the intervals where $f(x)$ is decreasing, if any such intervals exist.

$\boxed{\begin{array}{l} \text{Inc: } (-\infty, -2) \cup (2, \infty) \\ \text{Dec: } (-2, 0) \;\&\; (0, 2) \end{array}}$

(c) Find the intervals where $f(x)$ is concave and find the intervals where $f(x)$ is convex, if any such intervals exist.

$\frac{8}{x^3} = 0$

$\cancel{8 = 0}$

Undef @ $x = 0$

f'': $-$ | 0 | $+$

$\boxed{\begin{array}{l} \text{Concave: } (-\infty, 0) \\ \text{Convex: } (0, \infty) \end{array}}$

(d) Find any inflection points for $f(x)$, if any.

$\boxed{\text{None,}}$ F undef. @ $x = 0$.