Check-In 01/16. *(True/False)* Given $\int_0^{\pi} e^{\sin x} \cos x \, dx$, the u-substitution $u = \sin x$ transforms this integral into $\int_0^{\pi} e^u \, du$.

Solution. The statement is *false*. If $u = \sin x$, then $du = \cos x \, dx$. So indeed, this u-substitution would transform the integral $\int e^{\sin x} \cos x \, dx$ into the integral $\int e^u \, du$. However with definite integrals, one needs to remember to transform the limits as well. If $x = 0$, then $u = \sin(0) = 0$. If $x = \pi$, then $u = \sin(\pi) = 0$. Therefore, the correct substitution is $\int_0^{\pi} e^{\sin x} \cos x \, dx = \int_0^0 e^u \, du = 0$.

Check-In 01/21. *(True/False)* To integrate $\int \operatorname{arccot} \theta \, d\theta$, one can use integration-by-parts by choosing $u = \operatorname{arccot} \theta$ and $dv = 1$.

Solution. The statement is *true*. Using LIATE, it is likely that the choice of $u = \operatorname{arccot} \theta$ will work. With 'nothing left' in the integrand, this means that $dv = 1$. We fill in our box as follows:

$$\begin{array}{|c|c|}\hline \operatorname{arccot}\theta & \\ \hline & 1 \\ \hline \end{array} \implies \begin{array}{|c|c|}\hline \operatorname{arccot}\theta & \theta \\ \hline \dfrac{-1}{1+\theta^2} & 1 \\ \hline \end{array}$$

Then using the 'Rule of 7', we find that...

$$\int \operatorname{arccot} \theta \, d\theta = \theta \operatorname{arccot} \theta - \int \frac{-\theta}{1+\theta^2} \, d\theta = \theta \operatorname{arccot} \theta + \int \frac{\theta}{1+\theta^2} \, d\theta$$

Using the u-substitution $u = 1 + \theta^2$, we see that $\int \frac{\theta}{1+\theta^2} \, d\theta = \frac{1}{2} \ln |1 + \theta^2| + C$. Therefore, we have...

$$\int \operatorname{arccot} \theta \, d\theta = \theta \operatorname{arccot} \theta + \frac{1}{2} \ln |1 + \theta^2| + C$$