

## Exam 2 Review

**Problem 1.** Identify  $a, b, c$  from the standard form and also find the vertex form of the following quadratic functions:

(a)  $x^2 + 6x + 1$

(c)  $x^2 - 2x + 5$

(b)  $2x^2 - 20x + 53$

(d)  $4x^2 - 4x - 5$

**Problem 2.** Find  $a, b, c$  from the standard form of the following quadratic functions. Also, find the vertex and axis of symmetry for each of them.

(a)  $7 - (x + 1)^2$

(b)  $x^2 - 8x + 26$

(c)  $(x + 10)^2$

(d)  $2(x - 7) - 15$

**Problem 3.** Find the vertex form of the following by using completing the square and then by using the evaluation method:

(a)  $x^2 + 10x + 15$

(b)  $-x^2 + 8x - 14$

(c)  $3x^2 + 6x - 4$

**Problem 4.** Consider the quadratic function  $f(x) = x^2 - 6x + 14$ .

(a) Find  $a, b, c$  for this quadratic function.

(b) Does  $f(x)$  open upwards or downwards? Explain.

(c) Is this quadratic function convex or concave? Explain.

(d) Find the vertex and axis of symmetry for  $f(x)$ .

(e) Find the minimum value of  $f(x)$ , if it exists. If it does not exist, explain why.

(f) Find the maximum value of  $f(x)$ , if it exists. If it does not exist, explain why.

**Problem 5.** Consider the quadratic function  $f(x) = 12 - (x + 3)^2$ .

(a) Find  $a, b, c$  for this quadratic function.

(b) Does  $f(x)$  open upwards or downwards? Explain.

(c) Is this quadratic function convex or concave? Explain.

- (d) Find the vertex and axis of symmetry for  $f(x)$ .
- (e) Find the minimum value of  $f(x)$ , if it exists. If it does not exist, explain why.
- (f) Find the maximum value of  $f(x)$ , if it exists. If it does not exist, explain why.

**Problem 6.** Consider the quadratic function  $f(x) = 2x^2 - 4x + 9$ .

- (a) Find  $a, b, c$  for this quadratic function.
- (b) Does  $f(x)$  open upwards or downwards? Explain.
- (c) Is this quadratic function convex or concave? Explain.
- (d) Find the vertex and axis of symmetry for  $f(x)$ .
- (e) Find the minimum value of  $f(x)$ , if it exists. If it does not exist, explain why.
- (f) Find the maximum value of  $f(x)$ , if it exists. If it does not exist, explain why.

**Problem 7.** Showing all your work, factor each of the following as much as possible:

- |                        |                         |
|------------------------|-------------------------|
| (a) $x^2 + 2x - 24$    | (f) $10x^2 + 10x - 300$ |
| (b) $x^2 - 4x + 4$     | (g) $x^2 + 18x + 56$    |
| (c) $3x^2 + 24x - 27$  | (h) $x^4 - 16$          |
| (d) $-2x^2 + 10x - 24$ | (i) $x^2 - 2x - 120$    |
| (e) $49 - x^2$         | (j) $x^2 + x - 132$     |

**Problem 8.** Solve the following quadratic equations by completing the square. Then solve the equations by using the quadratic formula. Verify your solution(s).

- (a)  $x^2 = 2(5x - 11)$
- (b)  $x(2 - x) = -224$
- (c)  $(x - 2)(x + 2) = 2x$

**Problem 9.** Showing all your work, solve the following quadratic equations and then verify your solution(s):

- |                          |                       |
|--------------------------|-----------------------|
| (a) $x^2 - 10x + 24 = 0$ | (e) $2x^2 + 22x = 84$ |
| (b) $15 = x(x - 2)$      | (f) $6x^2 = x + 1$    |
| (c) $x^2 - x = 6$        | (g) $88 - 4x^2 = 36x$ |
| (d) $x(6x + 1) = 15$     | (h) $7x - x^2 = 6$    |

(i)  $x^2 + 6x - 11 = 0$

(j)  $231x^2 = 353x - 60$

**Problem 10.** Showing all your work, solve the following equations and then verify your solution(s):

(a)  $x^4 + 2x^2 = 15$

(d)  $x^2 + \frac{4}{x^2} = 5$

(b)  $\frac{x+1}{x+5} = \frac{x}{2x+1}$

(e)  $\frac{3}{5x+2} = x$

(c)  $x - \sqrt{x} - 6 = 0$

(f)  $x^6 - 10x^3 = -25$

**Problem 11.** Showing all your work, find the domain of the following functions:

(a)  $5 - (x+6)^2$

(d)  $\sqrt{2x+10}$

(b)  $x^5 - 4x + 12$

(e)  $\sqrt{6-x}$

(c)  $\frac{1}{7-x}$

(f)  $\frac{\sqrt{x}}{x-10}$

**Problem 12.** Showing all your work, find the domain and range of the following functions:

(a)  $3(x+9)$

(d)  $\sqrt{x}$

(b)  $\frac{1}{x}$

(e)  $(x+5)^2 - 11$

(c)  $1 - 4x - x^2$

(f)  $6x + 17$

**Problem 13.** Without explicitly solving the following equations, determine whether there is a solution. Be sure to fully justify your reasoning.

(a)  $2x^2 + 5x = 3$

(b)  $x^2 + 7 = x$

(c)  $(3-x)(3+x) = 8x$

**Problem 14.** Without explicitly solving the following equations, determine whether the given quadratic can be factored. Be sure to fully justify your reasoning.

(a)  $x^2 + 4x - 45$

(b)  $x^2 + 3x - 1$

(c)  $6x^2 - 8x + 2$

**Problem 15.** Use the quadratic formula to factor the following:

(a)  $x^2 - 4x - 32$

(c)  $24x^2 - 26x - 15$

(b)  $24x^2 + 26 - 5$

(d)  $72x^2 + 71x - 120$

**Problem 16.** Values for several functions are given in the table below.

$x$	-3	-2	-1	0	1	2	3
$f(x)$	4	8	-1	5	-3	0	-2
$g(x)$	1	6	0	-6	-7	-3	1
$h(x)$	-4	0	3	5	10	3	9

Given the data above, compute the following:

(a)  $(h + g)(-2) =$

(f)  $g(-1 - f(3)) =$

(b)  $(f - g)(0) =$

(g)  $(h \circ g)(2) =$

(c)  $(5h)(1) =$

(h)  $(g \circ h)(2) =$

(d)  $\left(\frac{h}{f}\right)(1) =$

(i)  $(f \circ g)(-1) =$

(e)  $g(-3)h(3) =$

(j)  $(h \circ g \circ f)(1) =$

**Problem 17.** Write each of the following functions as a composition of functions  $f(g(x))$ .

(a)  $(x - 5)^2 + 8$

(d)  $\frac{1}{(3x + 5)^4}$

(b)  $\frac{1}{x + 8}$

(e)  $\sqrt{x^2 + 10}$

(c)  $(1 - x)^{11}$

(f)  $\frac{1}{2(x - 5)^3}$

**Problem 18.** Suppose  $f(x)$  and  $g(x)$  are the functions given below.

$$f(x) = 2x - 3$$

$$g(x) = x^2 + 2x - 1$$

Compute the following:

(a)  $f(5) =$

(d)  $(f - g)(x) =$

(b)  $g(-2) =$

(e)  $(fg)(x) =$

(c)  $f(0) - 3g(2) =$

$$(f) \left(\frac{f}{g}\right)(x) =$$

$$(h) (g \circ f)(0) =$$

$$(g) (f \circ g)(0) =$$

$$(i) (f \circ g)(x) =$$

$$(j) (g \circ f)(x) =$$

**Problem 19.** Let  $f(x)$  be the function given by  $f(x) = 3x - 7$ .

(a) Find a value in the range of  $f$ . Be sure to justify why the value is in the range.

(b) Compute  $f(4)$ . Is  $(4, 1)$  on the graph of  $f$ ? Explain.

(c) Is there an  $x$  such that  $f(x) = 11$ ? Explain.

(d) Is  $1 \in f^{-1}(3)$ ? Explain.

(e) Assuming  $f^{-1}$  exists, what is  $f(f^{-1}(\pi))$  and  $f^{-1}(f(\sqrt{2}))$ ?

**Problem 20.** Suppose  $f(x)$  and  $g(x)$  are the functions given below.

$x$	-3	-2	-1	0	1	2	3
$f(x)$	5	2	0	-1	-2	-4	-5
$g(x)$	1	1	5	2	-3	-3	4
$h(x)$	-6	7	1	-2	0	1	-1

Compute the following:

$$(a) (f + g)(3) =$$

$$(f) h(-1 - f(0)) =$$

$$(b) (f - g)(-1) =$$

$$(g) (g \circ f)(-2) =$$

$$(c) (5h)(1) =$$

$$(h) (h \circ g)(1) =$$

$$(d) \left(\frac{h}{g}\right)(-3) =$$

$$(i) (g \circ h)(1) =$$

$$(e) f(2)h(-2) =$$

$$(j) (g \circ f \circ h)(-1) =$$

**Problem 21.** Suppose  $f(x)$  and  $g(x)$  are the functions given below.

$$f(x) = 3x - 10$$

$$g(x) = 2x^2 - x + 5$$

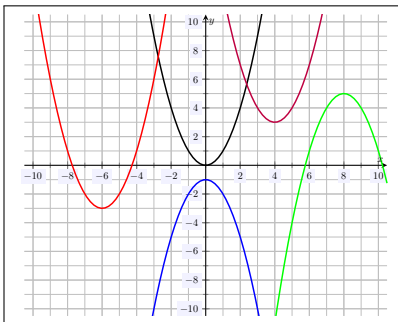
Compute the following:

- (a)  $f(3) =$
- (b)  $g(-2) =$
- (c)  $5f(6) - g(1) =$
- (d)  $f(x) - g(x) =$
- (e)  $f(x)g(x) =$
- (f)  $\left(\frac{f}{g}\right)(x) =$
- (g)  $(f \circ g)(0) =$
- (h)  $(g \circ f)(3) =$
- (i)  $(f \circ g)(x) =$
- (j)  $(g \circ f)(x) =$

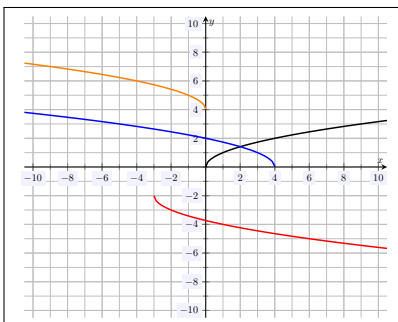
**Problem 22.** Let  $f(x) = x^2$ ,  $g(x) = \frac{1}{x}$ ,  $h(x) = \sqrt{x}$ , and  $k(x) = 3x + 1$ . For each of the given functions, determine the function described:

- (a) the function shifted two up and three to the right.
- (b) the function shifted 5 to the left and 4 down.
- (c) the function reflected through the  $x$  and  $y$ -axis.
- (d) the function compressed in the  $x$ -directed by a factor of 3.
- (e) the function scaled vertically by a factor of 2 and then shifted 5 downward.
- (f) the function shifted two upwards, reflected across the  $y$ -axis, and then shifted two right.
- (g) the function shifted 6 down, 5 to the left, and then reflected across the  $x$ -axis.

**Problem 23.** A function  $f(x)$  is plotted in black. Determine the other functions plotted in terms of  $f(x)$ .



**Problem 24.** A function  $f(x)$  is plotted in black. Determine the other functions plotted in terms of  $f(x)$ .



**Problem 25.** If  $f(x)$  is a function, describe the graph of the given function in terms of the graph of  $f(x)$ .

(a)  $f(x + 3) - 4$

(e)  $f(3x)$

(b)  $5 - f(x - 1)$

(f)  $-6f(x + 5)$

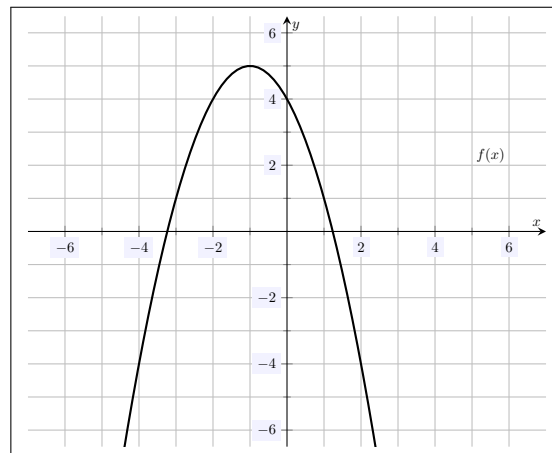
(c)  $-f(-x)$

(g)  $f(-x) + 7$

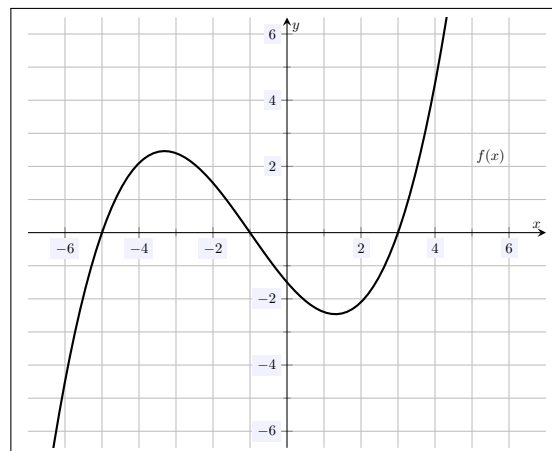
(d)  $f(-x + 3)$

(h)  $2f(1 - x) + 1$

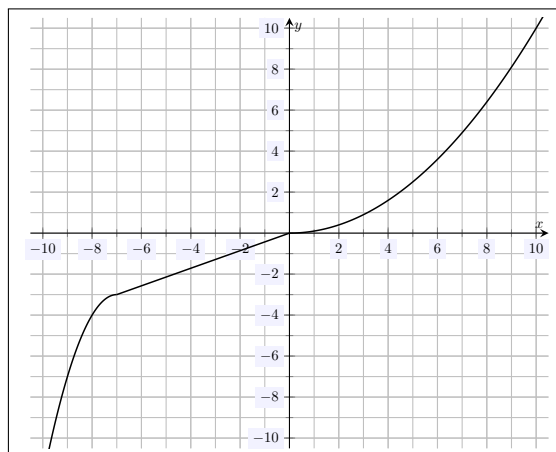
**Problem 26.** Determine if the relation below is a function or not. If it is a function, explain why. If it is not a function, explain why. Determine also whether the relation has an inverse function. If it has an inverse function, explain why and sketch it. If it does not have an inverse function, explain why not.



**Problem 27.** Determine if the relation below is a function or not. If it is a function, explain why. If it is not a function, explain why. Determine also whether the relation has an inverse function. If it has an inverse function, explain why and sketch it. If it does not have an inverse function, explain why not.



**Problem 28.** Determine if the relation below is a function or not. If it is a function, explain why. If it is not a function, explain why. Determine also whether the relation has an inverse function. If it has an inverse function, explain why and sketch it. If it does not have an inverse function, explain why not.



**Problem 29.** Find the inverse to the following functions. Also, verify that your inverse is correct:

(a)  $2x - 1$

(c)  $\frac{6x - 9}{-2}$

(b)  $\frac{1}{3}x + 5$

(d)  $8(3 - x)$

**Problem 30.** Determine whether the following statements are true or false. Be sure to justify your answer:

(a) A function has an inverse if it passes the HLT.

(b) A function can have more than one inverse.

(c) All functions have an inverse.

(d) The function  $x^2 + 8$  has an inverse.

(e) If  $f$  has an inverse, then  $f(f^{-1}(\pi)) = \pi$ .

(f) All lines have an inverse.