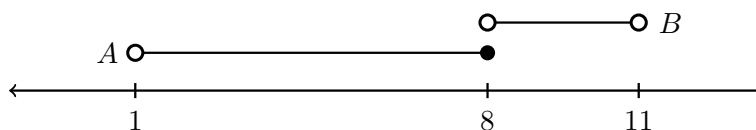


Check-In 08/22. (True/False) True/False: If $A = (1, 8]$ and $B = (8, 11)$, then $A \cap B = \{8\}$.

Solution. The statement is *false*. The set A is the set of numbers greater than 1 but at most 8, i.e. the numbers less than 8—including 8. Whereas B is the set of numbers greater than 8 but less than 11. The set $A \cap B$ is the set of elements in *both* A and B . The only real number that *could* be in both A and B is 8. However, 8 is in B but not in A . Therefore, $8 \notin A \cap B$. But then $A \cap B$ is empty, i.e. $A \cap B = \emptyset$. We can see this by sketching the intervals and seeing that there is no ‘overlap.’



Check-In 08/26. (True/False) True/False: $\left(\frac{y^{-2}}{x^4x^2}\right)^{-3} = \frac{x^{24}}{y^5}$

Solution. The statement is *false*. Recall that $x^a x^b = x^{a+b}$, $(x^a)^b = x^{ab}$, and $x^{-1} = \frac{1}{x}$. Then we have...

$$\left(\frac{y^{-2}}{x^4x^2}\right)^{-3} = \left(\frac{y^{-2}}{x^6}\right)^{-3} = \frac{y^6}{x^{-18}} = x^{18}y^6$$

Check-In 08/27. (True/False) True/False: It is possible to have a right triangle with sides of length 4, 7, 12.

Solution. The statement is *false*. We know a triangle with sides a, b, c is a right triangle if and only if $a^2 + b^2 = c^2$. We have...

$$\begin{aligned} a^2 + b^2 &\stackrel{?}{=} c^2 \\ 4^2 + 7^2 &\stackrel{?}{=} 12^2 \\ 16 + 49 &\stackrel{?}{=} 144 \\ 65 &\neq 144 \end{aligned}$$

Therefore, there exists no such right triangle. In fact, there does not even exist a triangle with such sides. A triangle with sides a, b, c exists if and only if the length of any pairs of sides of the triangle add up to greater than the third. [This is called the triangle inequality.] But $4 + 7 = 11 \not> 12$. Therefore, there does not exist a triangle with sides 4, 7, and 12.

Check-In 08/28. (True/False) True/False: Let $f(x)$ be a relation with $f(2) = 7$ and $f(-3) = 7$. Because $f(2)$ and $f(-3)$ are both 7, f cannot be a function.

Solution. The statement is *false*. A relation is a function if there is only one possible output for a given input, i.e. given an input, one knows with certainty what the output is. We know that $f(2) = 7$

and $f(-3) = 7$; that is, given the inputs of $x = 2$ or $x = -3$, we know the output. The fact that the outputs are the same is irrelevant. There are many functions with the property that $f(2) = 7$ and $f(-3) = 7$. For instance, there must be a linear function through these two points, i.e. $y = 7$. An example of a quadratic function through these points is $y = \frac{7x(x+1)}{6}$.

Check-In 08/29. (True/False) True/False: Let $f(x)$ be a function. The y -intercept of $f(x)$ is $f(0)$ and the x -intercept(s) of $f(x)$ are the x -values where $f(x) = 0$.

Solution. The statement is *true*. The y -intercept is where the graph of the function intersects the y -axis, which is the line $x = 0$. But then the y -intercept must be the function value at $x = 0$, i.e. $f(0)$. An x -intercept is where the graph of the function intersects the x -axis, which is the line $y = 0$. But this means the output of the function is zero. Therefore, an x -intercept is an x -value such that $f(x) = 0$.

Check-In 09/03. (True/False) If you are walking straight towards a building at a constant speed, then your distance from the building is given by a linear function. The y -intercept would represent your initial distance from the building and the slope would represent your walking speed. Furthermore, the x -intercept would represent the time you arrive at the building.

Solution. The statement is *true*. Let $D(t)$ denote your distance from the building. Because you are walking towards the building at a constant speed, the distance between you and the building is decreasing at a constant rate. But functions with a constant rate of change are linear. Therefore, $D(t)$ must be linear. We know that the y -intercept occurs when $t = 0$, i.e. the y -intercept is $D(0)$. But $D(0)$ is the distance you are at $t = 0$, i.e. the start. Therefore, $D(0)$ must denote your initial distance from the building. The slope of $D(t)$ represents the rate of change of your distance from the building. But this distance is only changing because you are walking towards the building. Therefore, the (absolute value of) the slope of $D(t)$ is your walking speed. Finally, an x -intercept is an input such that $D(t) = 0$. But $D(t) = 0$ implies that at the given time, your distance from the building is zero, i.e. you have arrived at the building. Therefore, the x -intercept of $D(t)$ represents the time you arrive at the building.

Check-In 09/04. (True/False) Because multiplication is commutative, $(f \circ g)(x) = (g \circ f)(x)$.

Solution. The statement is *false*. While *multiplication* is commutative, $f \circ g$ *does not* represent multiplication. Recall that $f \circ g$ represents function composition, i.e. $(f \circ g)(x) = f(g(x))$. Function composition is *not* commutative. For instance, suppose that $f(x) = 5$ and $g(x) = -6$. Then for any x , $(f \circ g)(x) = f(g(x)) = f(-6) = 5$ and $(g \circ f)(x) = g(f(x)) = g(5) = -6$. But then $(f \circ g)(x) \neq (g \circ f)(x)$ for any x .

Check-In 09/05. (True/False) If $f(x)$ is a function, then $\frac{f(-5+h) - f(-5)}{h}$ represents the average rate of change for $f(x)$ on the interval containing -5 and $-5+h$. If $f(x)$ is linear, this is the slope of $f(x)$.

Solution. The statement is *true*. Recall the average rate of change for a function $f(x)$ on the interval $[a, b]$ is...

$$\text{Avg. ROC}_{[a,b]} f(x) = \frac{f(b) - f(a)}{b - a} = \frac{f(a) - f(b)}{a - b}$$

That is, the average rate of change for a function on the interval $[a, b]$ is the slope of the line segment through $(a, f(a))$ and $(b, f(b))$. Because we want the interval containing -5 and $-5+h$, i.e. $[-5, -5+h]$ or $[-5+h, -5]$ (depending on whether $h > 0$ or not), the average rate of change on this interval is...

$$\frac{f(-5+h) - f(-5)}{(-5+h) - (-5)} = \frac{f(-5+h) - f(-5)}{-5+h+5} = \frac{f(-5+h) - f(-5)}{h}$$

Check-In 09/11. (True/False) The function $f(x) = 6 - (x+2)^2$ is quadratic. Furthermore, it is convex and has vertex $(2, 6)$.

Solution. The statement is *false*. Observe that $f(x) = 6 - (x+2)^2 = 6 - (x^2 + 4x + 4) = 6 - x^2 - 4x - 4 = -x^2 - 4x + 2$. Therefore, $f(x)$ is quadratic with $a = -1$, $b = -4$, and $c = 2$. We can also see that $f(x) = 6 - (x+2)^2 = -(x+2)^2 + 6$ is in vertex form—meaning $f(x)$ must be quadratic. Because $a = -1 < 0$, we know that $f(x)$ is concave—not convex. We know that if $a > 0$, a quadratic is convex, and if $a < 0$, a quadratic is concave. Furthermore, because $f(x) = -(x+2)^2 + 6 = -(x - (-2))^2 + 6$, the vertex of $f(x)$ is $(-2, 6)$ —not $(2, 6)$. Recall, the x -coordinate of the vertex makes the ‘square term’ of the vertex form zero and the y -coordinate of the vertex is what remains after the square term is zero.

Check-In 09/12. (True/False) A factorization of $12x^2 - 77x + 120$ is $(3x - 8)(4x - 15)$.

Solution. The statement is *true*. Finding the factorization of $12x^2 - 77x + 120$ may be difficult. However, it is routine to verify that a proposed factorization is correct—this always allows us to easily check whether we have obtained a correct factorization of a polynomial:

$$(3x - 8)(4x - 15) = 12x^2 - 45x - 32x + 120 = 12x^2 - 77x + 120$$

Check-In 09/16. (True/False) $9 - 4x^2 = (3 - 2x)(3 + 2x)$

Solution. The statement is *true*. This is a special type of factorization—the difference of perfect squares: $a^2 - b^2 = (a - b)(a + b)$. Here, we have $a = 3$ and $b = 2x$ because $a^2 = 9$ and $b^2 = (2x)^2 = 4x^2$. But then $9 - 4x^2 = (3 - 2x)(3 + 2x)$. While the *difference* of perfect squares factors, the *sum* of perfect squares, i.e. $a^2 + b^2$, *never* factors over the real numbers.

Check-In 09/17. (True/False) If $f(x) = 2x^3 - 3x^2 + 4x - 5$, then the value of $f(-4)$ is the remainder of $f(x)$ when divided by $x + 4$.

Solution. The statement is *true*. Recall that if we divide a polynomial $f(x)$ by $x - a$, then $f(a)$ is the remainder when we divide $f(x)$ by $x - a$. Here we have $x + 4 = x - (-4)$. Therefore, $f(-4)$ is the remainder when we divide $f(x)$ by $x + 4$. We can also check this directly: $f(-4) = 2(-4)^3 - 3(-4)^2 + 4(-4) - 5 = -128 - 48 - 16 - 5 = -197$ and

$$\begin{array}{r}
 2x^2 - 11x + 48 \\
 x + 4 \overline{) 2x^3 - 3x^2 + 4x - 5} \\
 \underline{- 2x^3 - 8x^2} \\
 -11x^2 + 4x \\
 \underline{11x^2 + 44x} \\
 48x - 5 \\
 \underline{- 48x - 192} \\
 -197
 \end{array}$$

Alternatively, we could compute the division using synthetic division (because $x + 4$ is linear):

$$\begin{array}{r|rrrr}
 -4 & 2 & -3 & 4 & -5 \\
 & & -8 & 44 & -192 \\
 \hline
 & 2 & -11 & 48 & -197
 \end{array}$$

Check-In 09/18. (True/False) The domain of $f(x) = \frac{x^2 - 1}{x^2 + 4x + 3}$ is $x \neq -3, \pm 1$.

Solution. The statement is *true*. The domain of a rational function, $f(x) = \frac{a(x)}{b(x)}$, are the values where $b(x) \neq 0$. We have $a(x) = x^2 - 1$ and $b(x) = x^2 + 4x + 3$. We have...

$$\begin{aligned}
 b(x) &= 0 \\
 x^2 + 4x + 3 &= 0 \\
 (x + 1)(x + 3) &= 0
 \end{aligned}$$

But then either $x + 1 = 0$, which implies $x = -1$, or $x + 3 = 0$, which implies $x = -3$. Therefore, the domain of $f(x)$ is all real numbers except $x = -3, -1$. The mistake made here is excluding the values where the numerator, $a(x)$, is also zero: if $a(x) = 0$, then $x^2 - 1 = 0$. But then $(x - 1)(x + 1) = 0$, which implies that $x - 1 = 0$, so that $x = 1$, or $x + 1 = 0$, so that $x = -1$. These values are zeros unless they are zeros shared by the denominator. Therefore, $x = 1$ is a zero of $f(x)$. However, $x = -1$ is not a zero of $f(x)$ —it is not in the domain. Observe that if $x \neq -1$, then...

$$f(x) = \frac{x^2 - 1}{x^2 + 4x + 3} = \frac{(x - 1)\cancel{(x + 1)}}{\cancel{(x + 1)}(x + 3)} = \frac{x - 1}{x + 3}$$

Therefore, $x = -1$ corresponds to a hole of $f(x)$: $\frac{x-1}{x+3} \Big|_{x=-1} = \frac{-1-1}{-1+3} = \frac{-2}{2} = -1$ so that the hole is $(-1, -1)$.

Check-In 09/19. (True/False) The function $f(x) = 4 \left(\frac{1}{3}\right)^{-x}$ grows exponentially and has y -intercept 4.

Solution. The statement is *true*. We rewrite $f(x)$:

$$f(x) = 4 \left(\frac{1}{3}\right)^{-x} = 4 \left(\left(\frac{1}{3}\right)^{-1}\right)^x = 4(3^x)$$

Therefore, $f(x)$ is exponential because it has the form Ab^x with $A = 4$ and $b = 3$. We know that $f(x)$ is growing exponentially because $b = 3 > 1$. The y -intercept of Ab^x is A . Therefore, because $A = 4$, $f(x)$ has y -intercept 4.

Check-In 09/23. (True/False) For any base $b > 0$, $\log_b(x + y) = \log_b x + \log_b y$.

Solution. The statement is *false*. For instance, consider the case where $x = y = 1$ and $b = 2$. On the left, we have $\log_2(1+1) = \log_2(2) = 1$. On the right, we have $\log_2(1) + \log_2(1) = 0 + 0 = 0$. Therefore, we see that $\log_b(x + y) \neq \log_b x + \log_b y$. However, it is the case that $\log_b(xy) = \log_b x + \log_b y$.

Check-In 09/24. (True/False) $\sqrt{x^{\log_b y}} = x^{\log_b \sqrt{y}}$

Solution. The statement is *true*. Recall that $(x^a)^b = x^{ab}$ and $r \log_b x = \log_b(x^r)$. But then...

$$\sqrt{x^{\log_b y}} = (x^{\log_b y})^{1/2} = x^{\frac{1}{2} \log_b y} = x^{\log_b y^{1/2}} = x^{\log_b \sqrt{y}}$$

Check-In 09/25. (True/False) Both $x = \log_2(10)$ and $2 \log_4(10)$ are solutions to $2^x = 10$.

Solution. The statement is *true*. We can check whether these are solutions:

$$\begin{aligned} 2^x &= 2^{\log_2 10} = 10 \\ 2^x &= 2^{2 \log_4 10} = (2^2)^{\log_4 10} = 4^{\log_4 10} = 10 \end{aligned}$$

Therefore, these are both solutions to the given equation. Alternatively, one can also convert from one log base to another using change of base: $\log_b x = \frac{\log_a x}{\log_a b}$. Using the fact that $4^{1/2} = \sqrt{4} = 2$, we have...

$$\log_2(10) = \frac{\log_4 10}{\log_4 2} = \frac{\log_4 10}{\frac{1}{2}} = 2 \log_4 10$$

Of course, one then still need show that at least one of these is actually a solution to the given equation—as above.

Check-In 10/02. (True/False) There is an integer k such that 34^k has 8,085 digits.

Solution. The statement is *true*. Recall that the number of digits in N in base- b is one more than $\lfloor \log_b N \rfloor$. But then if 34^k has 8,085 digits, it must be that $\lfloor \log_{10} 34^k \rfloor + 1 = 8085$. That is, we have $8084 \leq \log_{10} 34^k < 8085$. But then...

$$\begin{aligned} 8084 &\leq \log_{10} 34^k < 8085 \\ 8084 &\leq k \log_{10} 34 < 8085 \\ \frac{8084}{\log_{10} 34} &\leq k < \frac{8085}{\log_{10} 34} \quad 5278.56 \leq k < 5279.21 \end{aligned}$$

Therefore, choosing $k = 5279$, we see that 34^k has 8,085 digits. We can check this: the number of digits in 34^{5279} is...

$$\lfloor \log_{10} 34^{5279} \rfloor + 1 = \lfloor 5279 \log_{10} 34 \rfloor + 1 = \lfloor 5279(1.531478917) \rfloor + 1 = \lfloor 8084.6772 \rfloor + 1 = 8084 + 1 = 8085$$

Check-In 10/07. (True/False) If $|x + 1| = 4$, then $x = 3$.

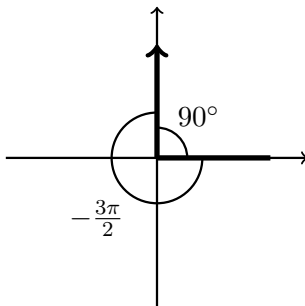
Solution. The statement is *false*. Observe that if $x = 3$, we have $|x + 1| = |3 + 1| = |4| = 4$. So $x = 3$ is a solution, but it is not the only solution. Depending on whether $x \geq -1$ or $x < -1$, we know that $|x + 1| = x + 1$ or $|x + 1| = -(x + 1)$, respectively. But then...

$$\begin{array}{ll} |x + 1| = 4 & |x + 1| = 4 \\ x + 1 = 4 & -(x + 1) = 4 \\ x = 3 & x + 1 = -4 \\ & x = -5 \end{array}$$

Therefore, if $|x + 1| = 4$, then $x = 3$ or $x = -5$. It is not necessarily the case that $x = 3$.

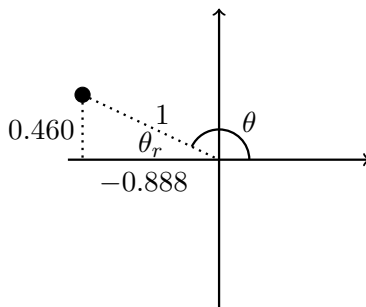
Check-In 10/08. (True/False) The angle $-\frac{3\pi}{2}$ radians is coterminal with the angle 90° .

Solution. The statement is *true*. Converting $\frac{3\pi}{2}$ radians to degrees, we have 90° . Therefore, $-\frac{3\pi}{2}$ radians is -270° . Plotting both these angles, we see that they have the same vertex, initial side, and terminal side. Therefore, these angles are coterminal.



Check-In 10/09. (*True/False*) The point $(-0.888, 0.460)$ is on the unit circle. If θ is the angle this point makes with the origin and positive x -axis, then $\sin \theta = -0.888$ and $\cos \theta = 0.460$.

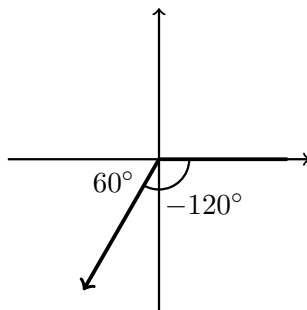
Solution. The statement is *false*. Recall the points on the unit circle are of the form $(\cos \theta, \sin \theta)$. But then if $(-0.888, 0.460)$ is on the unit circle, then $\cos \theta = -0.888$ and $\sin \theta = 0.460$. The given values are reversed. We can also do this using a sketch via reference angles.



But then $\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{0.460}{1} = 0.460$ and $\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{-0.888}{1} = -0.888$.

Check-In 10/10. (*True/False*) $\cos(-120^\circ) = -\frac{\sqrt{3}}{2}$

Solution. The statement is *false*. We plot the angle -120° . We can see that the reference angle for -120° is 60° . We can see that -120° is in Quadrant III, where $\cos \theta$ is negative.



Therefore, $\cos(-120^\circ) = -\cos(60^\circ) = -\frac{1}{2}$.

Check-In 10/14. (*True/False*) If $f(x)$ is a trigonometric function, θ is an angle, and θ_r is the reference angle for θ , then $f(\theta) = f(\theta_r)$.

Solution. The statement is *false*. Consider the angle $\theta = -30^\circ$ and the trigonometric function $f(x) = \sin x$. We know the reference angle for -30° is 30° , i.e. $\theta_r = 30^\circ$. Now...

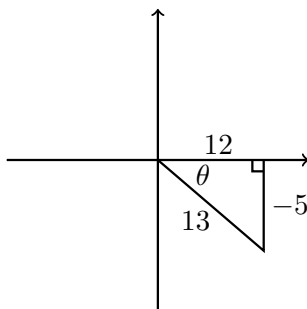
$$f(\theta) = f(-30^\circ) = \sin(-30^\circ) = \sin(270^\circ) = -\frac{1}{2}$$

$$f(\theta_r) = f(30^\circ) = \sin(30^\circ) = \frac{1}{2}$$

But then $f(\theta) = f(\theta_r)$. What is true is that $|f(\theta)| = |f(\theta_r)|$, i.e. $f(\theta)$ and $f(\theta_r)$ are the same value up to sign; that is, $f(\theta)$ is either $+f(\theta_r)$ or $-f(\theta_r)$.

Check-In 10/15. (True/False) If $\cos \theta = \frac{12}{13}$ and $\sin \theta < 0$, then $\tan \theta = \frac{5}{12}$.

Solution. The statement is *false*. Because $\cos \theta = \frac{12}{13} > 0$, we know that θ is an angle in QI or QIV. Because $\sin \theta < 0$, it must be that θ is an angle in QIII or QIV. Therefore, θ is an angle in QIV. But in QIV, $\tan \theta \leq 0$. Therefore, $\tan \theta \neq \frac{5}{12}$. If we want to find $\tan \theta$ directly, we use the fact that θ is in QIV and that $\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{12}{13}$; therefore, we can assume we have...



There the opposite side was found using the Pythagorean Theorem: $a^2 + 12^2 = 13^2$, so $a^2 = 13^2 - 12^2 = 169 - 144 = 25$, i.e. $a = \sqrt{25} = 5$. Therefore, $\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{-5}{12}$.

Check-In 10/16. (True/False) $\sin^2(50^\circ) + \cos^2(310^\circ) = 1$

Solution. The statement is *true*. Recall that $\sin^2 \theta + \cos^2 \theta = 1$ for any angle θ . Now the reference angle for 310° is 50° and we know that 310° is in QIV. But then $\cos(310^\circ) = \cos(50^\circ)$. But then...

$$\sin^2(50^\circ) + \cos^2(310^\circ) = \sin^2(50^\circ) + \cos^2(50^\circ) = 1$$

Check-In 10/21. (True/False) The amplitude of $y = -3 \sin(2x)$ is 3.

Solution. The statement is *true*. The amplitude of a sine or cosine wave is half the height from its maximum to minimum value, i.e. the distance from its midline to a maximum or minimum. We know that $y = \sin(bx)$ and $y = \cos(bx)$ have amplitude 1 for any b . We know that multiplication by a constant A stretches/shrinks a graph by a factor A , i.e. the graph of $y = Af(x)$ is the graph of $y = f(x)$ stretched/shrunk a graph by a factor A . In this case, we have $A = -3$ so that it must be that the amplitude is 3. Alternatively, we know the maximum/minimum value of sine and cosine is 1 or -1 , respectively. Therefore, the maximum of y is $-3 \cdot -1 = 3$ and the minimum value of y is $y = -3 \cdot 1 = -3$. Therefore, the amplitude is $\frac{\text{max} - \text{min}}{2} = \frac{3 - (-3)}{2} = \frac{6}{2} = 3$.

Check-In 10/22. (True/False) If one is to sketch a period of $y = \cos(4\pi x)$, then one first finds the period of $\frac{1}{2}$. The phase shift is 0, so the points used for the plot are $0, \frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}$.

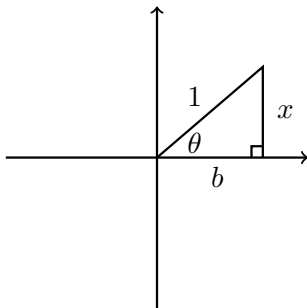
Solution. The statement is *true*. We know that if $y = A \cos(b(x - c))$, then the amplitude is A , the period is $\frac{2\pi}{b}$, and the phase shift is c . We first write y in this form: $y = \cos(4\pi(x - 0))$. Therefore, the phase shift is 0 and the period is $P := \frac{2\pi}{4\pi} = \frac{1}{2}$. Therefore, we sketch a period of y over the interval $[c, c + P] = [0, 0 + \frac{1}{2}] = [0, \frac{1}{2}]$. We break this interval into four equal length pieces, which must have length $\frac{P}{4} = \frac{1/2}{4} = \frac{1}{8}$. Therefore, the points used for the plot are $0, \frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}$.

Check-In 10/23. (True/False) $\arctan(-\sqrt{3}) = 300^\circ$

Solution. The statement is *false*. Recall that $\arctan y = \tan^{-1} y$ ‘asks the question,’ “What angle θ is required to obtain y .” That is, $\arctan y = \theta$ if and only if $\tan \theta = y$. However, while $\tan(300^\circ) = -\sqrt{3}$, to make $\arctan x$ a function, $\arctan x$ always returns an angle in the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$, i.e. $(-90^\circ, 90^\circ)$. Therefore, we need an angle coterminal with 300° that lies in this interval. But then $\arctan(-\sqrt{3}) = -60^\circ$, i.e. $\arctan(-\sqrt{3}) = -\frac{\pi}{3}$.

Check-In 10/24. (True/False) If $0 \leq x \leq 1$, then $\arcsin(x) = \arctan\left(\frac{x}{\sqrt{1-x^2}}\right)$.

Solution. The statement is *true*. Suppose that $\theta = \arcsin x$. Then $\sin \theta = x$. We can construct a right triangle such that $\sin \theta = \frac{\text{opp.}}{\text{hyp.}} = x = \frac{x}{1}$. For instance, the right triangle with ‘opposite’ side x and hypotenuse 1, drawn below.



But then by the Pythagorean Theorem, $a^2 + b^2 = c^2$, i.e. $x^2 + b^2 = 1^2$. This implies that $b^2 = 1 - x^2$, i.e. $b = \sqrt{1 - x^2}$. But then $\tan \theta = \frac{\text{opp.}}{\text{adj.}} = \frac{x}{\sqrt{1-x^2}}$. But then $\arctan\left(\frac{x}{\sqrt{1-x^2}}\right) = \theta = \arcsin(x)$.

Check-In 10/30. (True/False) For any real number y , $\sin(\arcsin(y)) = y$.

Solution. The statement is *true*. We know that if $\theta = \arcsin y$, then $\sin \theta = y$. But then we know that $\sin(\arcsin(y)) = \sin(\theta) = y$. However, the ‘reverse’ need not be true because of range restriction for the inverse trigonometric functions, i.e. it need not be the case that $\arcsin(\sin(y)) = y$. For instance, $\arcsin(\sin(0)) = \sin(0) = 0$. However, $\arcsin(\sin(2\pi)) = \arcsin(0) = 0 \neq 2\pi$.

Check-In 10/31. (True/False) $(\cos(15^\circ) + \sin(-15^\circ))(\cos(15^\circ) + \sin(15^\circ)) = \frac{\sqrt{3}}{2}$

Solution. The statement is *true*. Recall that $\sin(-\theta) = -\sin(\theta)$ and that $\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$. But then...

$$(\cos(15^\circ) + \sin(-15^\circ))(\cos(15^\circ) + \sin(15^\circ))$$

$$(\cos(15^\circ) - \sin(15^\circ))(\cos(15^\circ) + \sin(15^\circ))$$

$$\cos^2(15^\circ) - \sin^2(15^\circ)$$

$$\cos(2 \cdot 15^\circ)$$

$$\cos(30^\circ)$$

$$\frac{\sqrt{3}}{2}$$

Check-In 11/04. (True/False) $\sin^2 x = \frac{1 + \cos(2x)}{2}$

Solution. The statement is *false*. The actual identity is $\sin^2 x = \frac{1 - \cos(2x)}{2}$. We can tell this ‘identity’ is not true by choosing a special angle and testing it. For instance, choosing $x = \frac{\pi}{2}$, we have...

$$\sin^2 x \Big|_{x=\frac{\pi}{2}} = \sin^2 \left(\frac{\pi}{2} \right) = 1^2 = 1$$

$$\frac{1 + \cos(2x)}{2} \Big|_{x=\frac{\pi}{2}} = \frac{1 + \cos(\pi)}{2} = \frac{1 - 1}{2} = 0$$

Therefore, the ‘identity’ cannot be true.

Check-In 11/06. (True/False) $\sin(x - y) = \sin x \cos y - \sin y \cos x$

Solution. The statement is *true*. This is the sum angle identity for sine. We can use this to find certain sine values. For instance, we have...

$$\begin{aligned} \sin(15^\circ) &= \sin(45^\circ - 30^\circ) \\ &= \sin(45^\circ) \cos(30^\circ) - \sin(30^\circ) \cos(45^\circ) \\ &= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{1}{\sqrt{2}} \\ &= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} \\ &= \frac{\sqrt{3} - 1}{2\sqrt{2}} \approx 0.258819 \end{aligned}$$

Check-In 11/07. (True/False) If $\csc\left(\frac{\theta}{3}\right) = -2$, then it must be that $\theta = -\frac{\pi}{2} + 2\pi n$, where n is an integer.

Solution. The statement is *false*. First, observe that if $\csc\left(\frac{\theta}{3}\right) = -2$, then we must have $\sin\left(\frac{\theta}{3}\right) = -\frac{1}{2}$. But then we know that $\frac{\theta}{3} = \frac{7\pi}{6} + 2\pi n$ or $\frac{\theta}{3} = \frac{11\pi}{6} + 2\pi n$, where n is an integer. But then $\theta = \frac{7\pi}{2} + 6\pi n$ or $\theta = \frac{11\pi}{2} + 6\pi n$, where n is an integer. Note that the second type of solution (by subtracting a 6π -radians) could be written as $\theta = -\frac{\pi}{2} + 6\pi n$, where n is an integer. The given solution only finds one ‘type’ of the ‘two types’ of solutions.

Check-In 11/11. (True/False) There are infinitely many solutions to $4x \sin^2 x = 4(2 - x \cos^2 x)$.

Solution. The statement is *false*. While ‘many’ trigonometric equations have infinitely many solutions, this need not always be the case. Recall that $\sin^2 \theta + \cos^2 \theta = 1$ for all θ . But then...

$$4x \sin^2 x = 4(2 - x \cos^2 x)$$

$$4x \sin^2 x = 8 - 4x \cos^2 x$$

$$4x \sin^2 x + 4x \cos^2 x = 8$$

$$4x (\sin^2 x + \cos^2 x) = 8$$

$$4x \cdot 1 = 8$$

$$x = 2$$

Therefore, the given equation has only one solution.

Check-In 11/12. (True/False) The solutions to $\sin(2x) + \sin(x) = 0$ are $2\pi n$, where n is an integer.

Solution. The statement is *false*. Recall that $\sin(2x) = 2 \sin x \cos x$. But then...

$$\sin(2x) + \sin(x) = 0$$

$$2 \sin x \cos x + \sin x = 0$$

$$\sin x (2 \cos x + 1) = 0$$

Therefore, either $\sin x = 0$, which implies that $x = n\pi$, where n is an integer, or $2 \cos x + 1 = 0$. If $2 \cos x + 1 = 0$, then $\cos x = -\frac{1}{2}$. But this implies that either $x = \frac{5\pi}{6} + 2\pi n$ or $x = \frac{7\pi}{6} + 2\pi n$, where n is an integer. Therefore, the solution(s) presented in the statement are only one ‘type’ of solution to the given equation.

Check-In 11/13. (True/False) $\frac{\sin \theta - \sin^3 \theta}{\cos^2 \theta} = \sin \theta$

Solution. The statement is *true*. Recall that $\sin^2 \theta + \cos^2 \theta = 1$, which implies that $\cos^2 \theta = 1 - \sin^2 \theta$. But then...

$$\frac{\sin \theta - \sin^3 \theta}{\cos^2 \theta} = \frac{\sin \theta(1 - \sin^2 \theta)}{\cos^2 \theta} = \frac{\sin \theta \cdot \cos^2 \theta}{\cos^2 \theta} = \sin \theta$$

Check-In 11/14. (True/False) The law of cosine states that $c^2 = a^2 + b^2 - 2ab \cos C$ and in the case when c is the hypotenuse of a right triangle, then the law simplifies to the Pythagorean theorem.

Solution. The statement is *true*. The law of cosines states that for any triangle Δabc with corresponding angles A, B, C , that $c^2 = a^2 + b^2 - 2ab \cos C$, where C is the angle corresponding to side c . Consider the case of a right triangle with hypotenuse c . We know the angle corresponding to c is 90° . But then by the law of cosines would imply...

$$c^2 = a^2 + b^2 - 2ab \cos C = a^2 + b^2 - 2ab \cos(90^\circ) = a^2 + b^2 - 2ab(0) = a^2 + b^2$$

This is the Pythagorean Theorem. Therefore, we can view the law of cosines is a generalization of the pythagorean theorem to non-right triangles.