MATH 142: Final Exam **Name:** ______________________________
Spring — 2025
05/06/2025
150 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 21 pages (including this cover page) and 18 questions. Check that you have every page of the exam.

This exam consists of three parts—each corresponding to a previous course exam—with six questions. **Choose 5 of the 6 questions for each part and answer only these 5 questions.** That is, choose five of the six questions from Questions 1 through 6, choose five of the six questions from Questions 7 through 12, and choose five of the six questions from Questions 13 through 18 to answer and answer only these questions. Clearly indicate which questions you are choosing *not* to answer by crossing out their score box on the cover page. You are not required to answer the bonus questions and doing so will not adversely affect your score.

Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each chosen question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Exam 1			**Exam 2**			**Exam 3**		
Question	Points	Score	Question	Points	Score	Question	Points	Score
1	10		7	10		13	10	
2	10		8	10		14	10	
3	10		9	10		15	10	
4	10		10	10		16	10	
5	10		11	10		17	10	
6	10		12	10		18	10	
Total	50		**Total**	50		**Total**	50	

Exam Total:

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1. (10 points) Showing all your work, evaluate the following:

$$\int x^3 \ln x \; dx$$

2. (10 points) Showing all your work, compute the following:

$$\int \frac{x^2}{\sqrt{9-x^2}}\,dx$$

3. (10 points) Showing all your work, evaluate the following:

$$\int_0^{\pi} \sin^3\theta \cos^2\theta \; d\theta$$

4. (10 points) Showing all your work, evaluate the following integral:

$$\int \frac{x^2 + 11x + 3}{(x-1)^2(x+4)}\, dx$$

5. (10 points) Determine whether the following improper integral converges or diverges. If it converges, compute the value of the integral. Be sure to rigorously justify all your computations.

$$\int_2^\infty \frac{dx}{x\,(\ln x)^2}$$

6. (10 points) Showing all your work, find the following:

$$\int x^2 \sin(2x)\ dx$$

7. (10 points) Compute the arc length of the curve $y = 2 + 4x^{3/2}$, where $0 \leq x \leq 2$.

8. (10 points) Determine whether the following series is absolutely convergent, conditionally convergent, or divergent. Be sure to fully justify your reasoning with the appropriate series tests.

$$\sum_{n=0}^{\infty} (-1)^n \frac{n+1}{n^2+1}$$

$n=0$

9. (10 points) Completely justifying your answer, determine whether the following series converges or diverges. If the series converges, compute its sum.

$$\sum_{n=1}^{\infty} \frac{3(5^n)}{2^{3n}}$$

10. (10 points) Determine whether the following series is convergent or divergent. Be sure to fully justify your reasoning.

$$\sum_{n=1}^{\infty} \frac{n^2\,4^n}{n!}$$

11. (10 points) Determine whether the following series converges or diverges. Use appropriate series tests to justify your responses.

$$\sum_{n=1}^{\infty} \frac{2n-1}{n^2+1}$$

12. (10 points) Showing all your work, compute the sum of the series

$$\sum_{k=1}^{\infty} \frac{1}{(k+2)(k+3)}$$

13. (10 points) Find the center, radius of convergence, and interval of convergence for the following series:

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^2\,3^n}$$

14. (10 points) Find the first three nonzero terms of the Taylor series for $f(x) = 8x^{3/2}$ centered at $x = 1$.

15. (10 points) Find the Taylor series centered at $c = \frac{\pi}{2}$ for the function $f(x) = \sin(2x)$. You may not make use of any known Taylor series. Your answer must include an expression for the nth term of the Taylor series.

16. (10 points) Find the center, radius of convergence, and interval of convergence for the following series:

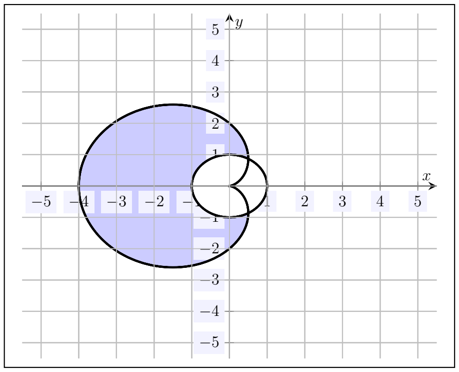
$$\sum_{n=0}^{\infty}(-1)^n \frac{x^n}{n!\,3^n}$$

17. (10 points) Give the first three nonzero terms of the Maclaurin series for $f(x) = \sin(x)$. If one uses this polynomial to approximate $f(x)$ on $(-1, 1)$, what is an upper bound for the maximum error for this approximation? Be sure to fully justify your upper bound.

18. (10 points) Showing all your work, find the sum of the series

$$\sum_{n=0}^{\infty}(-1)^n \frac{(\pi^2)^n}{(2n)!}$$

Bonus I. Find the area of the region bound by the curves $r = 1$ and $r = 2 - 2\cos\theta$, shown below.



Bonus II. Find a parametrization for the curves described below:

(a) A circle with radius 5 centered at $(-2, 3)$.

(b) The line $y = 2x + 3$.

(c) The portion of the parabola $x = y^2 - 3$ between $y = -1$ and $y = 2$.