MATH 344: Exam 1 **Name:** ______________________________

Spring —$_{2}$ 2026

02/13/2026

50 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 8 pages (including this cover page) and 6 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	15	
2	15	
3	15	
4	15	
5	15	
6	15	
Total:	90	

1. (15 points) Let $\mathbf{u} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ be vectors in $\mathbb{R}^3$.

   (a) Find a *unit* vector which is parallel to $\mathbf{u} - 2\mathbf{v}$.

   (b) Determine the *exact* angle between $\mathbf{u}$ and $\mathbf{v}$.

   (c) Compute $\mathrm{proj}_{\mathbf{v}}\mathbf{u}$.

2. (15 points) Consider the system of linear equations given below:

$$\begin{cases} x - 3y = 6 \\ 2x - 7y = 13 \end{cases}$$

(a) Write this system of equations in matrix-vector form, i.e. $A\mathbf{x} = \mathbf{b}$.

(b) Explain why the matrix A is invertible without explicitly finding its inverse.

(c) Find the matrix A^{-1}.

(d) Solve the system of equations using the inverse.

3. (15 points) Consider the matrix A and vector $\mathbf{x}$ given below.

$$A = \begin{pmatrix} 1 & 2 & 1 \\ -3 & 4 & 1 \\ 1 & 2 & -3 \end{pmatrix}, \qquad \mathbf{x} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$$

(a) Find A^T.

(b) Compute $A\mathbf{x}$ by treating the product as a linear combination of columns of A. [You may receive partial credit if you compute $A\mathbf{x}$ through another method.]

(c) Find the $(2,1)$-cofactor of A, C_{21}.

Now consider the matrices B, C, D given below.

$$B = \begin{pmatrix} 5 & -8 & 4 & 0 & 8 \\ 2 & -3 & 7 & 9 & 0 \\ 1 & -2 & 1 & 0 & 2 \\ 4 & 0 & 0 & -3 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 4 & -2 & 6 & 1 & 0 \\ 0 & 3 & 9 & 4 & 2 & -1 \\ 0 & 6 & 2 & -1 & 9 & 0 \\ 4 & 1 & 0 & -3 & 5 & 8 \\ 9 & 0 & 1 & 7 & -4 & 8 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

(d) Without explicitly computing BC, determine its size.

(d) Without explicitly computing BC, find the $(3, 2)$-entry of the product, i.e. $(BC)_{32}$.

(e) Circle the pivot positions in D and determine its rank and nullity.

4. (15 points) Suppose an augmented matrix $[A \;\; \mathbf{b}]$ coming from a linear system of equations was placed into REF using the following row operations:

$$\begin{gathered} R_1 \longleftrightarrow R_3 \\ -R_1 + 2R_2 \to R_2 \\ R_1 + R_3 \to R_3 \\ \tfrac{1}{3}R_2 \to R_2 \\ R_2 - R_3 \to R_3 \end{gathered}$$

After these row operations, the following augmented matrix was obtained:

$$\left[\begin{array}{ccc:c} 1 & 1 & -1 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 2 & -4 \end{array}\right]$$

(a) Labeling your variables $x_1, x_2, \ldots$, find the solution to the original system of linear equations using back-substitution.

(b) Compute the determinant of the original coefficient matrix A.

5. (15 points) Matrices M, N below are in RREF and represent augmented matrices coming from systems of linear equations. For each matrix, determine whether the corresponding system is consistent or inconsistent. If it is inconsistent, explain why. If it is consistent, either find the unique solution or parametrize the solution set (if infinitely many solutions exist). Use variables $x_1, x_2, \ldots$.

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \qquad N = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

(a) The System Corresponding to M:

(b) The System Corresponding to N:

6. (15 points) A system of linear equations with infinitely many solutions has an augmented matrix whose RREF form is given below.

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 3 \\ 0 & 1 & 2 & -1 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Find all of the possible solutions in vector form. Label the variables $x_1, x_2, \ldots$. Also, give a concrete example of one of the infinitely many possible solutions.