Check-In 08/21. *(True/False)* The integral $\int x\sqrt[3]{x-2}\,dx$ can be treated as a 'shifting integral' by using the u-substitution $u = x - 2$.

Solution. The statement is *true*. We 'want' to be able to distribute the x across the cube-root but we cannot—this is not a valid operation. However, if we make the u-substitution $u = x - 2$, then we will be able to distribute in a way that makes this integral 'routine.' So, let $u = x - 2$, then $du = dx$. Moreover, because $u = x - 2$, we know that $x = u + 2$. But then...

$$\int x\sqrt[3]{x-2}\,dx = \int (u+2)\sqrt[3]{u}\,du = \int \left(u^{4/3} + 2u^{1/3}\right)\,du = \tfrac{3}{7}\,u^{7/3} + \tfrac{3}{4}\cdot 2u^{4/3} + C = \tfrac{3}{7}(x-2)^{7/3} + \tfrac{3}{2}(x-2)^{4/3} + C$$

Note that a computer algebra system may write the answer (though you will *not* be expected to) like this:

$$\tfrac{3}{7}(x-2)^{7/3} + \tfrac{3}{2}(x-2)^{4/3} + C = (x-2)^{4/3}\left(\tfrac{3}{7}(x-2) + \tfrac{3}{2}\right) + C = (x-2)^{4/3}\left(\tfrac{3}{7}x + \tfrac{9}{14}\right) + C = \tfrac{3}{14}(x-2)^{4/3}\,(2x+3) + C$$