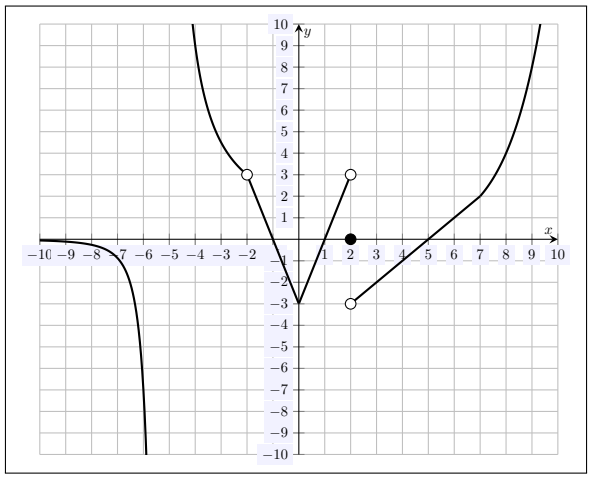
# Exam 1 Review

**Problem 1.** Use the plot of the function $f(x)$ below to answer the following questions:




(i) $f(2)$

(ii) $\lim_{x\to 2^-} f(x)$

(iii) $\lim_{x\to 2^+} f(x)$

(iv) $\lim_{x\to 2} f(x)$

(v) $\lim_{x\to -2^-} f(x)$

(vi) $\lim_{x\to -2^+} f(x)$

(vii) $\lim_{x\to -2} f(x)$

(viii) $\lim_{x\to -\infty} f(x)$

(ix) $\lim_{x\to \infty} f(x)$

(x) What is the $y$-intercept of $f(x)$?

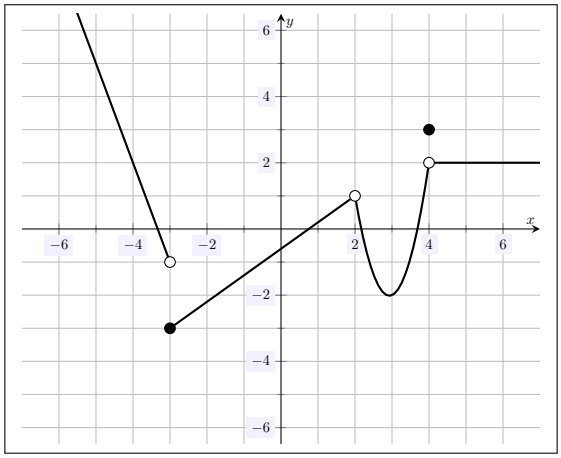
(xi) What are the zeros of $f(x)$?

(xii) If $f(x)$ has any vertical asymptotes, give their equation.

(xiii) Where is $f(x)$ continuous?

(xiv) List at least 4 values for $x$ at which $f(x)$ is not differentiable.

**Problem 2.** For the function $f(x)$, whose graph is shown in the figure below, compute the following limits. If the limit does not exist, write 'DNE.'



(i) $\lim_{x\to -3^-} f(x)$

(ii) $\lim_{x\to -3^+} f(x)$

(iii) $\lim_{x\to -3} f(x)$

(iv) $\lim_{x\to 2^-} f(x)$

(v) $\lim_{x\to 2^+} f(x)$

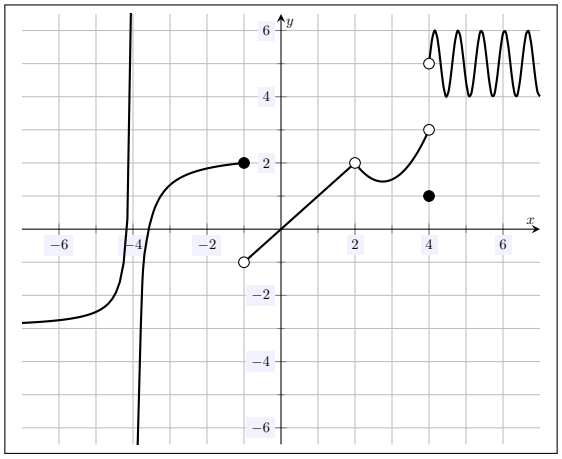
(vi) $\lim_{x\to 2} f(x)$

(vii) $\lim_{x\to -\infty} f(x)$

(viii) $\lim_{x\to \infty} f(x)$

Also, find and classify and discontinuities for $f(x)$. Determine any values where $f(x)$ is not differentiable. Finally, determine the value of the derivative at $x = -1$ and $x = 5$.

**Problem 3.** For the function $f(x)$, whose graph is shown in the figure below, compute the following limits. If the limit does not exist, write 'DNE.'




(i) $\lim_{x\to -4^-} f(x)$

(ii) $\lim_{x\to -4^+} f(x)$

(iii) $\lim_{x\to -4} f(x)$

(iv) $f(-4)$

(v) $\lim_{x\to -1^-} f(x)$

(vi) $\lim_{x\to -1^+} f(x)$

(vii) $\lim_{x\to -1} f(x)$

(viii) $f(-1)$

(ix) $\lim_{x\to 2^-} f(x)$

(x) $\lim_{x\to 2^+} f(x)$

(xi) $\lim_{x\to 2} f(x)$

(xii) $f(2)$

(xiii) $\lim_{x\to 4^-} f(x)$

(xiv) $\lim_{x\to 4^+} f(x)$

(xv) $\lim_{x\to 4} f(x)$

(xvi) $f(4)$

(xvii) $\lim_{x\to -\infty} f(x)$

(xviii) $\lim_{x\to \infty} f(x)$

**Problem 4.** Showing all your work, compute the following limits:

(i) $\lim_{a\to 0} \dfrac{\sin(5a)}{3a}$

(ii) $\lim_{x\to -\infty} \left(\dfrac{1}{x} - \arctan x\right)$

(iii) $\lim_{x\to 4} \dfrac{x^2-5x+4}{x^2-2x-8}$

(iv) $\lim_{r\to 0} \dfrac{\tan^2(r)}{r}$

(v) $\lim_{x\to 1} \dfrac{x+5}{|x+1|}$

(vi) $\lim_{x\to\infty} \dfrac{4x^2-3x+7}{6x^2+7}$

(vii) $\lim_{s\to\infty} \left(1+\dfrac{6}{s}\right)^{2s}$

(viii) $\lim_{s\to 3^+} \dfrac{1-s}{s-3}$

(ix) $\lim_{q\to 0^+} (1+5q)^{4/q}$

(x) $\lim_{x\to \frac{\pi}{2}} \dfrac{\cos x}{\cot x}$

(xi) $\lim_{x\to 5^-} \dfrac{5x-25}{|x-5|}$

(xii) $\lim_{x\to\infty} x\sin\left(\dfrac{1}{x}\right)$

(xiii) $\lim_{x\to -4} \dfrac{x+4}{x^2-16}$

(xiv) $\lim_{x\to 0} \dfrac{\cos x - 1}{2x^2}$

(xv) $\lim_{g\to\infty} \dfrac{4-g}{g^3+1}$

(xvi) $\lim_{h\to 0} 3\sqrt{h}\sin\left(\dfrac{5}{h}\right)$

(xvii) $\lim_{v\to -3^-} \dfrac{4v+1}{v+3}$

(xviii) $\lim_{y\to\infty} \left(1-\dfrac{1}{5y}\right)^{y}$

(xix) $\lim_{w\to 0} \dfrac{\sin(6w)}{\sin(2w)}$

(xx) $\lim_{u\to 0} \dfrac{\tan(\pi u)}{\sin(\pi u)}$

(xxi) $\lim_{t\to -\infty} \dfrac{1-t^3}{5t+6}$

(xxii) $\lim_{x\to 3} \left(\dfrac{2x}{x+3} - |x|\right)$

**Problem 5.** Determine whether the given function is continuous on the given interval $\mathcal{I}$:

(i) $f(x) = \dfrac{x\sin x}{x-5}$, $\mathcal{I} = (-4, 4)$

(ii) $g(x) = e^x\cos x - 3x^2$, $\mathcal{I} = (-\infty, \infty)$

(iii) $h(x) = \dfrac{4x}{x^2-x-6}$, $\mathcal{I} = (1, \infty)$

(iv) $j(x) = \sqrt[3]{x} - \log_6(x^2+1)$, $\mathcal{I} = (-\infty, \infty)$

**Problem 6.** Determine the largest interval where the following functions are continuous:

(i) $f(x) = (1-2x)^3$

(ii) $g(x) = \sin^2 x - \cos x + \sqrt{x}$

(iii) $h(x) = \dfrac{x}{x+3}$

(iv) $j(x) = e^x - \ln x$

(v) $k(x) = \dfrac{x^2-1}{x^3-4x^2-5x}$

(vi) $\ell(x) = \sin(1-x^2) + \arctan x + 5$

**Problem 7.** Find $a, b$ so that the following function is everywhere continuous. Be sure to use any necessary theorems and the definition of continuity to prove $f(x)$ is everywhere continuous.

$$f(x) = \begin{cases} 6 - x, & x \leq -1 \\ ax - b, & -1 < x \leq 3 \\ x^2 - 3x, & 3 < x \end{cases}$$

**Problem 8.** Using the definition of the derivative, find the derivative of the given function at the indicated value:

(i) $f(x) = x^2 - 5x + 2$, $a = 1$

(ii) $g(x) = \dfrac{1}{x}$, $a = -3$

(iii) $h(x) = \sqrt{x + 7}$, $a = 2$

(iv) $j(x) = |2x - 1|$, $a = 3$

**Problem 9.** Using the definition of the derivative, find the derivative of the given function:

(i) $f(x) = 2x^2 - 3x + 5$

(ii) $g(x) = \sqrt{1 - x}$

(iii) $h(x) = \dfrac{x}{x + 1}$

(iv) $j(x) = 2x^{3/2}$

**Problem 10.** Showing all your work, compute the following:

(i) $\dfrac{d}{dx}\left(5\sqrt[3]{x^2} - 4e^{2x}\right)$

(ii) $\dfrac{d}{dx}\cos^3(2^{-x})$

(iii) $\dfrac{d}{dx}\log_5(x)\arctan(5x)$

(iv) $\dfrac{d}{dx}\left(\dfrac{x - x^3}{5x - 6}\right)$

(v) $\dfrac{d^2}{dx^2}\left(-2\csc(e^x) - \operatorname{arcsec} x\right)$

(vi) $\dfrac{d}{dx}\left(\dfrac{8^{-x}\sin x}{(2x + 1)^2}\right)$

(vii) $\dfrac{d}{dx}e^{10x}\sin x \cos x$

(viii) $\dfrac{d^2}{dx^2}\left(\dfrac{1}{3^x} - \dfrac{1}{x^2 - x}\right)$

(ix) $\dfrac{d}{dx}\left(\dfrac{\sec x - x}{\sqrt{x}\ln x}\right)$

(x) $\dfrac{d}{dx}\left(1 - \dfrac{x}{x - \cot x}\right)^5$

(xi) $\dfrac{d}{dx}x^3\tan^2(3x)$

(xii) $\dfrac{d^2}{dx^2}(3x^2 - 5x + 9)^{12}$