Check-In 08/22. *(True/False)* Let $f(x)$ be a relation with $f(2) = 7$ and $f(-3) = 7$. Because $f(2)$ and $f(-3)$ are both 7, f cannot be a function.

Solution. The statement is *false*. A relation is a function if there is only one possible output for a given input, i.e. given an input, one knows with certainty what the output is. We know that $f(2) = 7$ and $f(-3) = 7$; that is, given the inputs of $x = 2$ or $x = -3$, we know the output. The fact that the outputs are the same is irrelevant. There are many functions with the property that $f(2) = 7$ and $f(-3) = 7$. For instance, there must be a linear function through these two points, i.e. $y = 7$. An example of a quadratic function through these points is $y = \frac{7x(x+1)}{6}$.

Check-In 08/27. *(True/False)* If $S(t) = 0.008t + 57.81$ represents the stock price for a company t minutes after opening, then the rate of change of the stock value is 0.008, i.e. the stock is gaining $\$0.008$ per minute in value, and the opening price of the stock was $\$57.81$.

Solution. The statement is *false*. The stock price at opening would be the stock price at $t = 0$. But $S(0) = 0.008(0) + 57.81 = 57.81$. Therefore, the opening stock price was \$57.81. Observe that $S(t)$ is a linear function, i.e. a function of the form $y = mx + b$ with $y = S$, $x = t$, $m = 0.008$, and $b = 57.81$. We know the rate of change of a linear function is its slope. But then the rate of change of $S(t)$ is $m = 0.008$, i.e. there is an increase of \$0.008 per minute in the value of the stock.

Check-In 08/29. *(True/False)* If the production cost of a certain item is constant, then the cost to produce q items, $C(q)$ is linear. Furthermore, the slope of $C(q)$ is the marginal cost and $C(0)$ is the fixed cost.

Solution. The statement is *true*. If the cost of production for the item is constant, then the production cost has a constant rate of change. But then the cost function to produce q items, $C(q)$, must be linear. We know the marginal cost for a linear cost function is its slope. Furthermore, $C(0)$ is the fixed costs. But the y-intercept of $C(q)$ is precisely $C(0)$.

Check-In 09/03. *(True/False)* Let $f(x) = 17(0.93)^x$. Because $f(x)$ the form Ab^x with $A = 17$ and $b = 0.93$, it is exponential. Furthermore, $A = 17$ represents the y-intercept of 17, i.e. an initial value of 17, and $b = 0.93$ can be interpreted as a 93% decrease of the initial value of 17 a total of x-times.

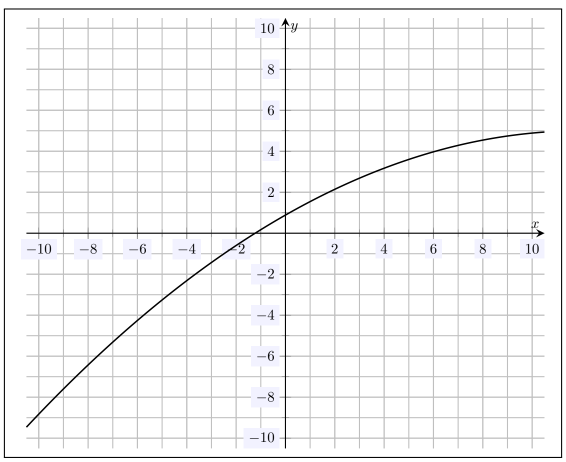
Solution. The statement is *false*. An exponential function is a function of the form Ab^x. Therefore, $f(x) = 17(0.93)^x$ is an exponential function with $A = 17$ and $b = 0.93$. We know that $A = 17$ is the y-intercept because $f(0) = 17(0.93)^0 = 17(1) = 17$. We know that for any exponential function, we can interpret b as a percentage increase/decrease. We know that $0 < b < 1$. Therefore, we know that $f(x)$ is exponentially decreasing. We have $b = 0.93 = 1 - 0.07$. Therefore, we can interpret $f(x)$ as a 7% decrease of the initial value of 17 a total of x-times.

Check-In 09/05. *(True/False)* Because multiplication is commutative, $(f \circ g)(x) = (g \circ f)(x)$.

Solution. The statement is *false*. It is true that multiplication is commutative. However, $f \circ g$ does not denote multiplication but rather function composition. We know that $(f \circ g)(x) = f\big(g(x)\big)$. There is no need for $(f \circ g)(x) = (g \circ f)(x)$. Although it can happen, it is certainly (typically) false. For instance, if $f(x) = 0$ and $g(x) = 1$. Then $(f \circ g)(x) = f\big(g(x)\big) = f(1) = 0$ and $(g \circ f)(x) = g\big(f(x)\big) = g(0) = 1$.

Check-In 09/19. *(True/False)* If $f(x)$ is a function which is twice differentiable and $f'(x) > 0$, then $f''(x) > 0$.

Solution. The statement is *false*. Recall that if $f'(x) > 0$, the function $f(x)$ is increasing at that x-value, and if $f'(x) < 0$, the function $f(x)$ is decreasing at that x-value. Furthermore, recall that if $f''(x) > 0$, the function $f(x)$ is concave up at that x-value, and if $f''(x) < 0$, the function $f(x)$ is concave down at that x-value. Therefore, the question is asking if a function is increasing, does it have to be concave up. This is certainly not the case. For instance, consider the function $f(x)$ shown below.



This function is clearly everywhere increasing, so that $f'(x) > 0$. However, observe that the function is concave down, so that $f''(x) < 0$. The sign of f' and f'' do indeed give you information about $f(x)$. However, the signs of f, f', and f'' *do not* need to be the same.

Check-In 09/24. *(True/False)* If $C(q)$ is a cost function, then $C'(q)$ is the marginal cost.

Solution. The statement is *true*. In the case of linear cost functions, we knew that the marginal cost was the slope of the line, i.e. its derivative. For general cost functions, we define the marginal cost to be the derivative of the cost function. Generally, we know the marginal cost is the cost of producing the next item. We know that $C'(q) := \frac{dC}{dq}$. But then the cost of the next item should approximately be $\frac{dC}{dq}\big((q+1) - q\big) = \frac{dC}{dq} \cdot 1 = \frac{dC}{dq}$. That is, the cost of the next additional item is approximately $C'(q)$.

Check-In 09/26. *(True/False)* $\frac{d}{dx}(7x^3 - e^x + \log_2 x) = 21x^2 - e^x + \frac{1}{x \ln 2}$

Solution. The statement is *true*. Recall that $\frac{d}{dx} x^n = nx^{n-1}$, $\frac{d}{dx} e^x = e^x$, and $\frac{d}{dx} \log_2 x = \frac{1}{x \ln 2}$. Therefore, we have...

$$\frac{d}{dx}(7x^3 - e^x + \log_2 x) = (7 \cdot 3x^2 - e^x + \frac{1}{x \ln 2}) = 21x^2 - e^x + \frac{1}{x \ln 2}$$

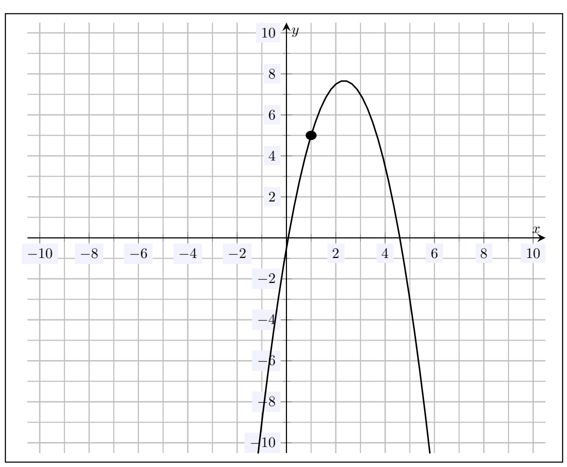
Check-In 10/03. *(True/False)* $\frac{d}{dx}\left(4x^2 - \frac{1}{x}\right)^5 = 5\left(8x - \frac{1}{x^2}\right)^4$

Solution. The statement is *false*. Recall that the chain rule states that $\frac{d}{dx} f\big(g(x)\big) = f'\big(g(x)\big) \cdot g'(x)$. We have here $f(x) = x^5$ and $g(x) = 4x^2 - \frac{1}{x}$. We know that $f'(x) = 5x^4$ and $g'(x) = 8x + \frac{1}{x^2}$. Therefore, we have...

$$\frac{d}{dx}\left(4x^2 - \frac{1}{x}\right)^5 = 5\left(4x^2 - \frac{1}{x}\right)^4 \cdot \left(8x + \frac{1}{x^2}\right)$$

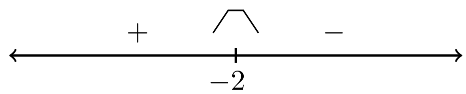
Check-In 10/08. *(True/False)* Suppose that $f(1) = 5$, $f'(1) = 4$, and $f''(1) = -3$. Because $f'(1) > 0$, $f(x)$ is increasing at $x = 1$. Furthermore, because $f''(1) < 0$, $x = 1$ must be a local maxima for $f(x)$.

Solution. The statement is *false*. It is true that if $f'(a) > 0$, then $f(x)$ is increasing at $x = a$. Therefore, because $f'(1) > 0$, $f(x)$ is increasing at $x = 1$. We know that if $f''(a) < 0$ *and* $f'(a) = 0$, then $f(x)$ has a local maxima at $x = a$—this is the second derivative test. Though $f''(1) < 0$, we know $f'(1) \neq 0$ because $f'(1) > 0$. Therefore, $x = 1$ need not be a local maxima. For instance, the function below has $f(1) = 5$, $f'(1) = 4$, and $f''(1) = -3$ but $x = 1$ is not a local maxima.



Check-In 10/10. *(True/False)* Suppose that $f(-2) = 10$ and $f'(-2) = 0$. If f' changes sign from positive to negative across $x = -2$, then $f(x)$ has a local maximum at $x = -2$.

Solution. The statement is *true*. This is the first derivative test. We know that if $f'(a) = 0$ and $f'(x)$ changes sign from positive to negative across $x = a$, then $x = a$ is a local maximum.



Check-In 10/24. *(True/False)* If $v(t)$ is a person's velocity at time t, then the total area between the graph of $v(t)$ and the x-axis (with area under the x-axis counted as negative) is the net change in the distance traveled.

Solution. The statement is *true*. We approximate this area using the sum of areas of many rectangles. [If we want the *exact* value, either we need to make a geometric argument or use an infinite amount of rectangles.] Picture the area of one of these rectangles: $A = bh$. The base of this rectangle is a change in the inputs—t, i.e. Δt. This has units of time. The height of this rectangle is $f(t)$ for some t, which has units of $v(t)$, e.g. m/s. But then the area of the rectangle has units of $A =$ time · distance/time = distance, where this distance can be negative (meaning the total distance is decreasing) if $v(t) < 0$. Adding these rectangles, we are summing distances. Therefore, the total area between the graph of $v(t)$ and the x-axis (with area under the x-axis counted as negative) is the net change in the distance traveled.

Check-In 10/29. *(True/False)* Because $x^2 + y^2 = 1$ is a circle with radius 1 centered at $(0, 0)$, $\int_{-1}^{1} \sqrt{1 - x^2}\, dx = \frac{\pi}{2}$.

Solution. The statement is *true*. Let $y = \sqrt{1 - x^2}$. We want to compute $\int_{-1}^{1} \sqrt{1 - x^2}\, dx = \int_{-1}^{1} y(x)\, dx$. But if $y = \sqrt{1 - x^2}$, then $y^2 = 1 - x^2$, which implies that $x^2 + y^2 = 1$. But this is the circle with radius 1 centered at the origin. This shows us that $y = \sqrt{1 - x^2}$ on $[-1, 1]$ is the 'upper half' of this unit circle. We know that $\int_{-1}^{1} \sqrt{1 - x^2}\, dx$ is the (signed) area between the curve and the x-axis. We know that the area of a circle is πr^2. For the whole unit circle, $A = \pi r^2 = \pi(1^2) = \pi$. But then the area of the half circle is $\frac{\pi}{2}$. Therefore, $\int_{-1}^{1} \sqrt{1 - x^2}\, dx = \frac{\pi}{2}$.

Check-In 10/31. *(True/False)* Estimating $\int_{1}^{9} f(x)\; dx$ using a right hand sum with sixteen equal width rectangles means you would use rectangles with width 0.5 and the first rectangle would have height $f(1)$.

Solution. The statement is *false*. We want to approximate $\int_a^b f(x)\, dx$ with $a = 1$ and $b = 9$ using $n = 16$ equal width rectangles. Because the rectangles have equal width, we can break the internal into n equal pieces. Then each rectangle will have width $\Delta x = \frac{b-a}{n} = \frac{9-1}{16} = \frac{8}{16} = \frac{1}{2} = 0.5$. Because the interval for this integral is $[1, 9]$ and we begin at $x = 1$, the intervals we use for the approximation are $[1, 1.5]$, $[1.5, 2]$, $[2, 2.5]$, $\ldots$, $[8.5, 9]$. If we use a right hand sum, then we use the value on the 'right' of each interval. But then the height of the first box would be the 'height' at $x = 1.5$, i.e. $f(1.5)$. Therefore, the statement is false as it uses $f(1)$ for the initial box height—-the one use for the *left* hand sum.

Check-In 11/07. *(True/False)* $\int \frac{1}{x}\, dx = \ln|x|$

Solution. The statement is *false*. Recall that $\int f(x)\, dx$ is the antiderivative of $f(x)$, i.e. a function $F(x)$ such that $F'(x) = f(x)$. Of course, we have $\frac{d}{dx} \ln|x| = \frac{1}{x}$. However, any constant, C, plus $\ln|x|$ would produce another function such that its derivative is $\frac{1}{x}$. Therefore, we should have $\int \frac{1}{x}\, dx = \ln|x| + C$, where C is a constant. One always needs this constant of integration for any indefinite integral.

Check-In 11/12. *(True/False)* $\int_0^1 (3x^2 - x)\, dx = \frac{1}{2}$

Solution. The statement is *true*. Recall the power rule for indefinite integrals: $\int x^n\, dx = \frac{1}{n+1}\, x^{n+1} + C$, if $n \neq -1$, i.e. $\int \frac{1}{x}\, dx = \ln|x| + C$. But then using the Fundamental Theorem of Calculus, we have...

$$\int_0^1 (3x^2 - x)\, dx = \left(\frac{3}{3}\, x^3 - \frac{1}{2}\, x^x \right) \Bigg|_0^1 = \left(x^3 - \frac{1}{2}\, x^x \right) \Bigg|_0^1 = \left(1 - \frac{1}{2} \right) - (0 - 0) = \frac{1}{2} - 0 = \frac{1}{2}$$

Check-In 11/14. *(True/False)* Using the u-substitution $u = \sqrt{x}$ transforms the integral $\int_0^4 2\sqrt{x}\, e^{\sqrt{x}}\, dx$ into the integral $\int_0^4 e^u\, du$.

Solution. The statement is *false*. If we have $u = \sqrt{x}$, then $du = \frac{1}{2}\, x^{-1/2}\, dx = \frac{1}{2\sqrt{x}}\, dx$. But then $dx = 2\sqrt{x}\, dx$. Finally, one needs to convert the limits of integration: if $x = 0$, then $u = \sqrt{0} = 0$, and if $x = 4$, then $u = \sqrt{4} = 2$. Therefore, the substituted integral becomes...

$$\int_0^4 2\sqrt{x}\, e^{\sqrt{x}}\, dx = \int_0^2 2\sqrt{x}\, e^u \cdot 2\sqrt{x}\, du = \int_0^2 4x\, e^u\, du$$

Not only does the given integral not have the correct bounds, certainly, the integrand above not simplify to the given integrand. [In fact, the correct substituted integral is $\int_0^2 4u^2e^u\, du$, which is beyond the scope of this course.]

Check-In 11/19. *(True/False)* To evaluate $\int \frac{x^3 - x + 5}{x}\, dx$, one 'should use' u-substitution.

Solution. The statement is *false*. The only 'useful' choices would be $u = x$ or $u = x^3 - x + 5$. If $u = x$, then one is simply literally rewriting the integral in u's instead of x's. If $u = x^3 - x + 5$, then $du = (3x^2 - 1)\, dx$, so that $dx = \frac{1}{3x^2 - 1}\, du$. But there is no $3x^2 - 1$ term in the numerator to cancel, so that the resulting integral will not be entirely in u's. Therefore, this would not be a valid u-substitution. The correct method is to apply algebra by dividing the x into each term in the numerator and apply basic integration rules:

$$\int \frac{x^3 - x + 5}{x}\, dx = \int \left(x^2 - 1 + \frac{5}{x} \right)\, dx = \frac{x^3}{3} - x + 5\ln|x| + C$$