π-Day
Polar Form & De Moivre's Theorem

Quick Facts

- $z = a + bi$ is a complex number. This form of z is called the *rectangular form/representation*.
- $|z|$ is the *absolute value* or *modulus* of z. Furthermore, $|z| = \sqrt{a^2 + b^2}$ and is the distance from the 'point' z to the origin. [Draw a right triangle!]
- The *polar form/representation* of z is $r(\cos\theta + i\sin\theta)$, where $r = |z|$ and θ is the 'angle plotting z makes.'
- We find the polar form of z by finding r, finding θ, and then writing out the polar form.
- De Moivre's (duh-mwah-vwurr) Theorem: $z^n = r^n\big(\cos(n\theta) + i\sin(n\theta)\big)$.
- We use De Moivre's Theorem to compute powers of z by finding the polar form of z, writing out the expression above, and then simplifying the expression.
- We use the following to compute roots of complex numbers:

$$\underbrace{\sqrt[n]{z}}_{\text{Or } z^{1/n}} = \sqrt[n]{r}\left(\cos\left(\frac{\theta + 2\pi k}{n}\right) + i\sin\left(\frac{\theta + 2\pi k}{n}\right)\right) \qquad \text{(radians)}$$

$$\underbrace{\sqrt[n]{z}}_{\text{Or } z^{1/n}} = \sqrt[n]{r}\left(\cos\left(\frac{\theta + 360^\circ k}{n}\right) + i\sin\left(\frac{\theta + 360^\circ k}{n}\right)\right) \qquad \text{(degrees)}$$

  where $k = 0, 1, \ldots, n-1$. These roots are equally spaced points (separated by an angle $\frac{360^\circ}{n}$, starting at angle $\frac{\theta}{n}$) on the circle at the origin with radius $\sqrt[n]{r}$.
- To find roots of complex numbers, we find the polar form of z, use the above expression (writing it out for each of $k = 1, 2, \ldots, n-1$, and then simplify each expression.