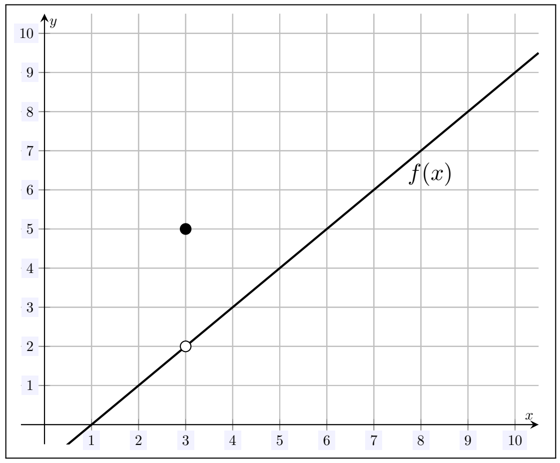
Check-In 01/15. *(True/False) True/False*: If $f(3) = 5$, then $\lim_{x \to 3} f(x) = 5$.

Solution. The statement is *false*. Recall that the limit of a function (if it exists) is what the output gets 'close' to as the input gets 'close' to its limiting value. The fact that $f(3) = 5$ does not mean the outputs are all 'close' to 5 when x is 'close' to 3. For instance, consider the function $f(x)$ plotted below.



Despite the fact that $f(3) = 5$, $\lim_{x \to 3} f(x) = 2$ because all the outputs are 'close' to 2 when the inputs are 'close' to 3.

Check-In 01/17. *(True/False) True/False*: Let $f(x)$ be a function defined on all real numbers such that $\lim_{x \to \pi} f(x) = 10$. Then it must be that $\lim_{x \to \pi^+} f(x) = 10$.

Solution. The statement is *true*. Recall that the limit (if it exists) is what the output gets 'close' to as the input gets 'close' to its limiting value. Because $\lim_{x \to \pi} f(x) = 10$, the outputs of $f(x)$ are all 'close' to 10 whenever x is 'close' to π—no matter how x is 'close' to π. The right-hand limit $\lim_{x \to \pi^+} f(x)$ asks what the outputs are 'close' to if x is 'close' to π—but bigger than π. But we already know that the outputs are 'close' to 10. Therefore, it must be that $\lim_{x \to \pi^+} f(x) = 10$. Recall that $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = L$.

Check-In 01/22. *(True/False)* $\lim_{x \to \infty} \left(1 + \frac{1}{3x}\right)^x = e^3$

Solution. The statement is *false*. Recall that $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$. But then...

$$\lim_{x \to \infty} \left(1 + \frac{1}{3x}\right)^x = \lim_{x \to \infty} \left(1 + \frac{1}{3x}\right)^{x \cdot 3/3} = \lim_{x \to \infty} \left[\left(1 + \frac{1}{3x}\right)^{3x}\right]^{1/3} = e^{1/3} = \sqrt[3]{e}$$

Check-In 01/24. *(True/False)* $\lim_{x\to\infty}\left(1+\dfrac{1}{x}\right)^x = (1+0)^\infty = 1^\infty = 1$

Solution. The statement is *false*. One does obtain 1^∞ after naïvely plugging in $x=\infty$. However, ∞ is not a number; moreover, although one might feel otherwise, it is simply need not be the case that $1^\infty = 1$. Indeed, 1^∞ is an indeterminant form. One could correctly recall that...

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$$

Check-In 01/27. *(True/False)* The function $f(x) = \dfrac{e^x \sin(\sqrt[3]{x})}{x^2+6x+9}$ is continuous on any interval which does not contain $x=-3$.

Solution. The statement is *true*. We know that e^x, $\sin x$, $\sqrt[3]{x}$, and x^2+6x+9 are everywhere continuous. But then $\sin(\sqrt[3]{x})$ is everywhere continuous, because it is a composition of continuous functions. This makes $e^x\sin(\sqrt[3]{x})$ continuous, because it is the product of continuous functions. But then $f(x) = \dfrac{e^x \sin(\sqrt[3]{x})}{x^2+6x+9}$ is continuous so long as $x^2+6x+9\neq 0$, because it would be a quotient of continuous functions. Observe that $x^2+6x+9 = (x+3)^2$. Therefore, if $x^2+6x+9=0$, then $(x+3)^2 = 0$ so that $x=-3$. Therefore, $f(x)$ is continuous on any interval not containing -3.

Check-In 01/29. *(True/False)* The limit $\lim_{x\to 0}\dfrac{\sqrt{x+9}-3}{x}$ represents $f'(9)$, where $f(x)=\sqrt{x}$.

Solution. The statement is *true*. The definition of the derivative at $x=a$ is $f'(a) = \lim_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$. Taking $f(x)=\sqrt{x}$ and $a=9$, we would have $f'(9) = \lim_{h\to 0}\dfrac{f(9+h)-f(9)}{h} = \lim_{h\to 0}\dfrac{\sqrt{9+h}-\sqrt{9}}{h} = \lim_{h\to 0}\dfrac{\sqrt{h+9}-3}{h}$. This is the same as the given limit with the role of h and x interchanged.

Check-In 01/31. *(True/False)* $\dfrac{d}{dx}\sin(\ln x) = \cos\left(\dfrac{1}{x}\right)$

Solution. The statement is *false*. We have a derivative of a composition of functions. This requires chain rule: $\frac{d}{dx} f\big(g(x)\big) = f'\big(g(x)\big)\cdot g'(x)$. Here, we have $f(x)=\sin x$ and $g(x)=\ln x$. The correct derivative should be...

$$\frac{d}{dx}\sin(\ln x) = \cos(\ln x)\cdot\frac{1}{x}$$

Here, the 'rule' $\frac{d}{dx} f\big(g(x)\big) = f'\big(g'(x)\big)$ has been applied, which is incorrect.