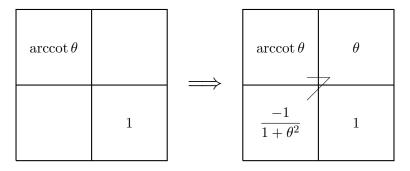
Check-In 01/16. (True/False) Given $\int_0^\pi e^{\sin x} \cos x \ dx$, the u-substitution $u = \sin x$ transforms this integral into $\int_0^\pi e^u \ du$.

Solution. The statement is *false*. If $u=\sin x$, then $du=\cos x\,dx$. So indeed, this u-substitution would transform the integral $\int e^{\sin x}\cos x\,dx$ into the integral $\int e^u\,du$. However with definite integrals, one needs to remember to transform the limits as well. If x=0, then $u=\sin(0)=0$. If $x=\pi$, then $u=\sin(\pi)=0$. Therefore, the correct substitution is $\int_0^\pi e^{\sin x}\cos x\,dx=\int_0^0 e^u\,du=0$.

Check-In 01/21. (True/False) To integrate $\int \operatorname{arccot} \theta \, d\theta$, one can use integration-by-parts by choosing $u = \operatorname{arccot} \theta$ and dv = 1.

Solution. The statement is *true*. Using LIATE, it is likely that the choice of $u = \operatorname{arccot} \theta$ will work. With 'nothing left' in the integrand, this means that dv = 1. We fill in our box as follows:



Then using the 'Rule of 7', we find that...

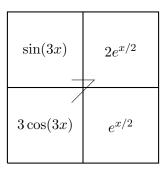
$$\int \operatorname{arccot} \theta \, d\theta = \theta \operatorname{arccot} \theta - \int \frac{-\theta}{1 + \theta^2} \, d\theta = \theta \operatorname{arccot} \theta + \int \frac{\theta}{1 + \theta^2} \, d\theta$$

Using the u-substitution $u=1+\theta^2$, we see that $\int \frac{\theta}{1+\theta^2} d\theta = \frac{1}{2} \ln|1+\theta^2| + C$. Therefore, we have... $\int \operatorname{arccot} \theta \, d\theta = \theta \operatorname{arccot} \theta + \frac{1}{2} \ln|1+\theta^2| + C$

Check-In 01/23. (*True/False*) The integral $\int e^{x/2} \sin(3x) dx$ is a 'looping' integral.

Solution. The statement is *true*. Recall that integrals whose integrand is the product of an exponential function with $\sin x$ or $\cos x$ 'loop.' We can see this directly: choose $u = \sin(3x)$. Using

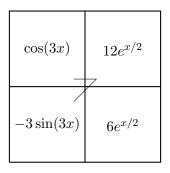
the 'box method', we have...



Therefore, we have...

$$\int e^{x/2}\sin(3x)\ dx = 2e^{x/2}\sin(3x) - \int 6e^{x/2}\cos(3x)\ dx$$

But this integral on the right also requires integration-by-parts: we choose $u = \cos(3x)$ and then...



So then we have...

$$\int e^{x/2} \sin(3x) \, dx = 2e^{x/2} \sin(3x) - \int 6e^{x/2} \cos(3x) \, dx$$

$$= 2e^{x/2} \sin(3x) - \left(12e^{x/2} \cos(3x) - \int -36e^{x/2} \sin(3x) \, dx\right)$$

$$= 2e^{x/2} \sin(3x) - 12e^{x/2} \cos(3x) + \int -36e^{x/2} \sin(3x) \, dx$$

$$= 2e^{x/2} \sin(3x) - 12e^{x/2} \cos(3x) - 36 \int e^{x/2} \sin(3x) \, dx$$

Observe that we have 'looped'—obtaining a multiple of the original integral on the right. Adding $36 \int e^{x/2} \sin(3x) \ dx$ to both sides, we have. . .

$$37 \int e^{x/2} \sin(3x) \ dx = 2e^{x/2} \sin(3x) - 12e^{x/2} \cos(3x)$$

Therefore, we have...

$$\int e^{x/2}\sin(3x)\ dx = \frac{2e^{x/2}\sin(3x) - 12e^{x/2}\cos(3x)}{37} + C$$

We can shortcut this work by adjusting tabular integration:

$$\begin{array}{c|c}
u & dv \\
\hline
sin(3x) & + e^{x/2} \\
3\cos(3x) & + 2e^{x/2} \\
-9\sin(3x) & + 4e^{x/2}
\end{array}$$

Therefore, we have...

$$\int e^{x/2} \sin(3x) \, dx = 2e^{x/2} - 12e^{x/2} \cos(3x) - 36 \int e^{x/2} \sin(3x) \, dx$$
$$37 \int e^{x/2} \sin(3x) \, dx = 2e^{x/2} - 12e^{x/2} \cos(3x)$$
$$\int e^{x/2} \sin(3x) \, dx = \frac{2e^{x/2} \sin(3x) - 12e^{x/2} \cos(3x)}{37} + C$$

Check-In 01/28. (*True/False*) To integrate $\int \cos^8 \theta \sin^5 \theta \ d\theta$, one should choose $u = \cos \theta$.

Solution. The statement is *true*. Observe that if we choose $u=\cos\theta$, then $\cos^8\theta$ becomes u^8 . We know du will then produce a $\sin\theta$ —specifically $-\sin\theta$. This 'uses' one of the $\sin\theta$'s in the integrand. This leaves $\sin^4\theta$ remaining in the integrand. But we can replace even powers of $\sin\theta$ in terms of $\cos\theta$. So we can carry out this substitution. Observe that if $u=\cos\theta$, then $du=-\sin\theta\ d\theta$. But then using the fact that $\sin^2\theta=1-\cos^2\theta$ (so that $\sin^4\theta=(\sin^2\theta)^2=(1-\cos^2\theta)^2$), we have...

$$\int \cos^8 \theta \sin^5 \theta \, d\theta = \int \cos^8 \theta \sin^4 \theta \cdot \sin \theta \, d\theta$$

$$= -\int \cos^8 \theta \sin^4 \theta \cdot -\sin \theta \, d\theta$$

$$= -\int \cos^8 \theta (1 - \cos^2 \theta)^2 \cdot -\sin \theta \, d\theta$$

$$= -\int u^8 (1 - u^2)^2 \, du$$

$$= -\int u^8 (1 - 2u^2 + u^4) \, du$$

$$= \int -u^8 + 2u^{10} - u^{12} \, du$$

$$= -\frac{u^9}{9} + \frac{2u^{11}}{11} - \frac{u^{13}}{13} + C$$

$$= -\frac{\cos^9 \theta}{9} + \frac{2\cos^{11} \theta}{11} - \frac{\cos^{13} \theta}{13} + C$$

Alternatively, observe that if we had chosen $u = \sin \theta$, then $\sin^5 \theta$ becomes u^5 . We know that du produces a $\cos \theta$. This 'uses' one of the $\cos \theta$'s in the integrand. This leaves $\cos^7 \theta$ remaining

in the integrand. However, we can only replace even powers of $\cos\theta$ in terms of $\sin\theta$ using $\cos^2\theta=1-\sin^2\theta$. So without using other identities or using other techniques, we cannot 'simply' choose $u=\sin\theta$ for this integral.