

Check-In 08/22. (True/False) Let $f(x)$ be a relation with $f(2) = 7$ and $f(-3) = 7$. Because $f(2)$ and $f(-3)$ are both 7, f cannot be a function.

Solution. The statement is *false*. A relation is a function if there is only one possible output for a given input, i.e. given an input, one knows with certainty what the output is. We know that $f(2) = 7$ and $f(-3) = 7$; that is, given the inputs of $x = 2$ or $x = -3$, we know the output. The fact that the outputs are the same is irrelevant. There are many functions with the property that $f(2) = 7$ and $f(-3) = 7$. For instance, there must be a linear function through these two points, i.e. $y = 7$. An example of a quadratic function through these points is $y = \frac{7x(x+1)}{6}$.

Check-In 08/27. (True/False) If $S(t) = 0.008t + 57.81$ represents the stock price for a company t minutes after opening, then the rate of change of the stock value is 0.008, i.e. the stock is gaining \$0.008 per minute in value, and the opening price of the stock was \$57.81.

Solution. The statement is *false*. The stock price at opening would be the stock price at $t = 0$. But $S(0) = 0.008(0) + 57.81 = 57.81$. Therefore, the opening stock price was \$57.81. Observe that $S(t)$ is a linear function, i.e. a function of the form $y = mx + b$ with $y = S$, $x = t$, $m = 0.008$, and $b = 57.81$. We know the rate of change of a linear function is its slope. But then the rate of change of $S(t)$ is $m = 0.008$, i.e. there is an increase of \$0.008 per minute in the value of the stock.

Check-In 08/29. (True/False) If the production cost of a certain item is constant, then the cost to produce q items, $C(q)$ is linear. Furthermore, the slope of $C(q)$ is the marginal cost and $C(0)$ is the fixed cost.

Solution. The statement is *true*. If the cost of production for the item is constant, then the production cost has a constant rate of change. But then the cost function to produce q items, $C(q)$, must be linear. We know the marginal cost for a linear cost function is its slope. Furthermore, $C(0)$ is the fixed costs. But the y -intercept of $C(q)$ is precisely $C(0)$.

Check-In 09/03. (True/False) Let $f(x) = 17(0.93)^x$. Because $f(x)$ the form Ab^x with $A = 17$ and $b = 0.93$, it is exponential. Furthermore, $A = 17$ represents the y -intercept of 17, i.e. an initial value of 17, and $b = 0.93$ can be interpreted as a 93% decrease of the initial value of 17 a total of x -times.

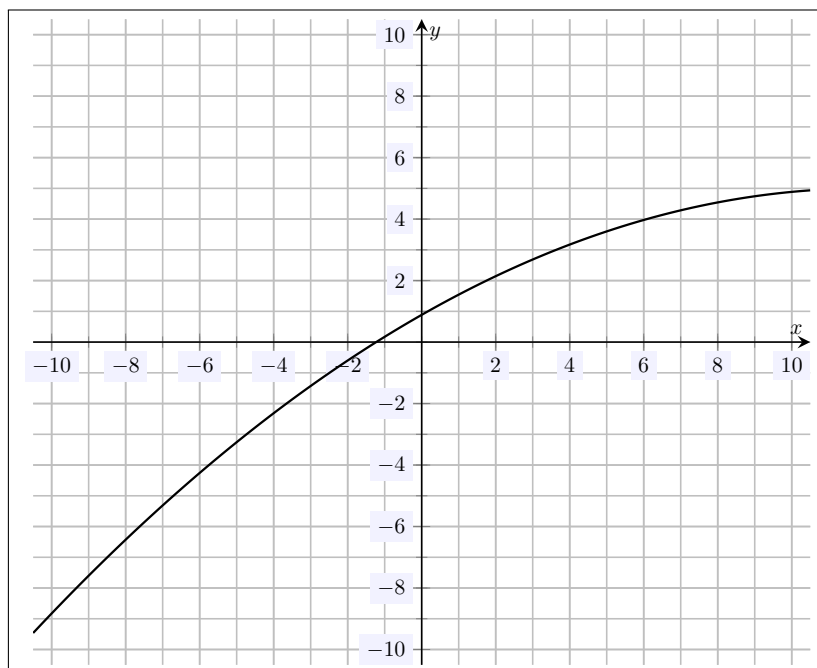
Solution. The statement is *false*. An exponential function is a function of the form Ab^x . Therefore, $f(x) = 17(0.93)^x$ is an exponential function with $A = 17$ and $b = 0.93$. We know that $A = 17$ is the y -intercept because $f(0) = 17(0.93)^0 = 17(1) = 17$. We know that for any exponential function, we can interpret b as a percentage increase/decrease. We know that $0 < b < 1$. Therefore, we know that $f(x)$ is exponentially decreasing. We have $b = 0.93 = 1 - 0.07$. Therefore, we can interpret $f(x)$ as a 7% decrease of the initial value of 17 a total of x -times.

Check-In 09/05. (True/False) Because multiplication is commutative, $(f \circ g)(x) = (g \circ f)(x)$.

Solution. The statement is *false*. It is true that multiplication is commutative. However, $f \circ g$ does not denote multiplication but rather function composition. We know that $(f \circ g)(x) = f(g(x))$. There is no need for $(f \circ g)(x) = (g \circ f)(x)$. Although it can happen, it is certainly (typically) false. For instance, if $f(x) = 0$ and $g(x) = 1$. Then $(f \circ g)(x) = f(g(x)) = f(1) = 0$ and $(g \circ f)(x) = g(f(x)) = g(0) = 1$.

Check-In 09/19. (True/False) If $f(x)$ is a function which is twice differentiable and $f'(x) > 0$, then $f''(x) > 0$.

Solution. The statement is *false*. Recall that if $f'(x) > 0$, the function $f(x)$ is increasing at that x -value, and if $f'(x) < 0$, the function $f(x)$ is decreasing at that x -value. Furthermore, recall that if $f''(x) > 0$, the function $f(x)$ is concave up at that x -value, and if $f''(x) < 0$, the function $f(x)$ is concave down at that x -value. Therefore, the question is asking if a function is increasing, does it have to be concave up. This is certainly not the case. For instance, consider the function $f(x)$ shown below.



This function is clearly everywhere increasing, so that $f'(x) > 0$. However, observe that the function is concave down, so that $f''(x) < 0$. The sign of f' and f'' do indeed give you information about $f(x)$. However, the signs of f , f' , and f'' do not need to be the same.