

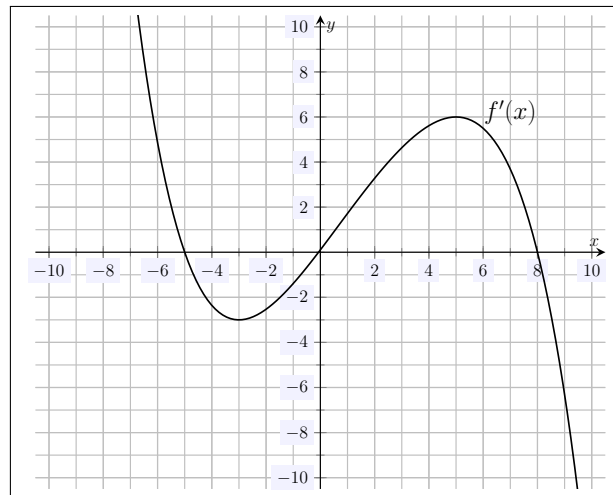
MATH 141: Exam 2
Spring — 2025
03/07/2025
50 Minutes

Name: _____

Write your name on the appropriate line on the exam cover sheet. This exam contains 8 pages (including this cover page) and 6 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	20	
2	16	
3	16	
4	16	
5	16	
6	16	
Total:	100	

1. (20 points) Let $f(x)$ be a twice continuously differentiable function whose *derivative*, $f'(x)$, is plotted below.



Based on the plot above, answer the following questions:

- (a) On what interval(s)—if any—is $f(x)$ increasing?
- (b) On what interval(s)—if any—is $f(x)$ decreasing?
- (c) On what interval(s)—if any—is $f(x)$ concave up?
- (d) On what interval(s)—if any—is $f(x)$ concave down?
- (e) Find any critical values for $f(x)$ —if any.
- (f) Classify any critical values you found in (e). If there were none, state so.
- (g) Find the x -value(s) for any point(s) of inflection for $f(x)$.

2. (16 points) Find the area of the largest rectangle that has its 'upper' vertices on the line $y = 20$ and its 'lower' vertices on the function $f(x) = 3x^2 + 2$.

3. (16 points) Showing all your work, complete the following parts:

(a) Find the linearization of $f(x) = \sqrt[3]{x}$ at $x = 8$.

(b) Use your answer in (a) to approximate $\sqrt[3]{11}$. Express your answer as a decimal.

4. (16 points) Showing all your work, compute the following limits:

(a) $\lim_{x \rightarrow \infty} \frac{7 \ln(3x + 1)}{2 \ln(5x - 4)}$

(b) $\lim_{x \rightarrow \infty} [\ln(6x + 5) - \ln(3x - 4)]$

(c) $\lim_{x \rightarrow 0} \frac{\tan(3x)}{x + \sin x}$

(d) $\lim_{x \rightarrow 0} (\cos x)^{6/x^2}$

5. (16 points) An extension ladder which is 20 feet long is leaning against the wall of a building. Suddenly, the latch of the ladder to the wall detaches and the ladder begins sliding down the wall at 4 ft per second. Simultaneously, the ladder begins to collapse shut at a rate of 3 ft per second. What is the rate at which the bottom of the ladder is moving away from the wall if the ladder is currently extended to a length of 15 ft and is leaning against a point 12 ft up the wall?

6. (16 points) An elliptic curve is a special type of curve widely used in cryptography. For its cryptographic applications, one needs to ‘add’ a point on the curve to itself. This process begins with finding the tangent line to the curve at the chosen point. The equation below is an elliptic curve. Find the tangent line to this curve at the point $(-1, 4)$.

$$y^2 + xy + y = x^3 - x^2 - 9x + 9$$