

MATH 115: Final Exam
Fall — 2024
12/11/2024
150 Minutes

Name: _____

Write your name on the appropriate line on the exam cover sheet. This exam contains 13 pages (including this cover page) and 13 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

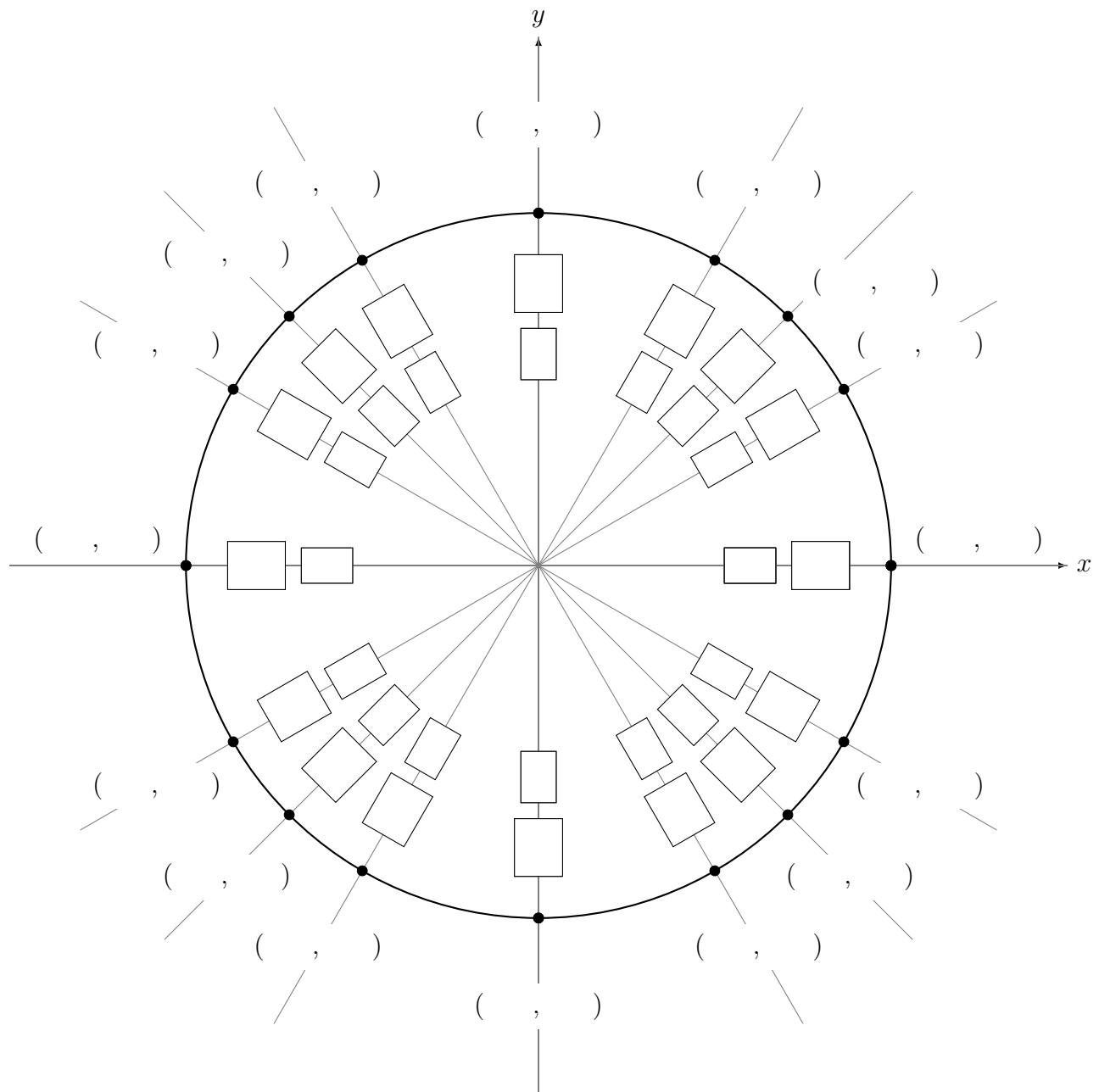
Question	Points	Score
1	10	
2	15	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	15	
12	10	
13	20	
Total:	150	

*'Twas the night before Christmas, up at the Pole,
Santa was frazzled—he'd lost all control!
The reindeer were restless, the sleight off its track,
And the toy distribution? A logistical smack!*

*Santa scratched his beard, eyes weary and red,
"The math here is tricky. I'm in over my head!
The children are waiting, and I need to take flight.
I need a Precalculus wizard to save Christmas tonight!"*

1. (10 points) *Santa's lost in the sky, the directions aren't right,*
"Help me compute these angles to save Christmas night!"

Below is a blank unit circle. Fill in each blank entry. The outside blank points should be the point on the unit circle. The innermost blank inside the unit circle should be the angle measure in degrees while the outer blank inside the unit circle is the angle measure in radians.



2. (15 points) *Santa's sleigh is in trouble, the tech's gone awry,
"Help me crunch these numbers, or we'll fall out of the sky!"*

Find the exact value for each of the following:

(a) $\log_3\left(\frac{1}{27}\right) =$

(i) $\sec(150^\circ) =$

(b) $\tan(135^\circ) =$

(j) $\arcsin(0.5) =$

(c) $\log_5\left(\sqrt[4]{5}\right) =$

(k) $\log_{\sqrt{2}}(4) =$

(d) $\cos\left(\frac{5\pi}{3}\right) =$

(l) $\ln(\sqrt{e^3}) =$

(e) $\tan^{-1}(' \infty ') =$

(m) $\arccos(-1) =$

(f) $\log_\pi(1) =$

(n) $\csc\left(-\frac{\pi}{6}\right) =$

(g) $\sin\left(-\frac{2\pi}{3}\right) =$

(o) $\log_7(7^{\sqrt{3}}) =$

(h) $\log_8(4) =$

(p) $5^{\log_5(0.71)} =$

3. (10 points) *Santa's sleigh is unsteady, the flight path's complex,
"Help me save Christmas—just find me the vertex!"*

Consider the function $f(x) = x^2 + 8x + 13$.

(a) Find the vertex form of $f(x)$.

(b) Use (a) to identify the vertex for $f(x)$.

(c) Use the previous parts to identify the range of $f(x)$.

4. (10 points) *Santa's puzzled by Rudolph, his math seems unclear,
"Help me check these divisions—Christmas' end is near!"*

Showing all your work. . .

- (a) Use synthetic division to find the quotient and remainder when $x^4 - 7x^2 - 6x$ is divided by $x + 2$.

- (b) Find the quotient and remainder when $3x^4 + 4x^3 - 2x^2 + 10x + 2$ is divided by $x^2 + 2x - 1$.

5. (10 points) *Santa's math seems off, his mind's in a bind,
"Check my rational computations—I fear I'm behind!"*

Consider the following function:

$$f(x) = \frac{5(x^2 - 1)}{x^2 + 4x - 5}$$

- (a) Find the domain for $f(x)$.
- (b) Find any vertical asymptotes for $f(x)$.
- (c) Find any horizontal asymptotes for $f(x)$.
- (d) Find any holes for $f(x)$.

*Santa's overwhelmed, his gift list is vast,
"Help me solve these equations—Christmas won't last!"*

6. (10 points) Showing all your work, solve the equation $\log_2(x + 3) + \log_2(x + 9) = 4$.

7. (10 points) Showing all your work, solve the equation $19 - e^{-2x} = 12$.

8. (10 points) Showing all your work, solve the equation $\sqrt{2x-1} + 3 = 8$.

9. (10 points) Showing all your work, find the solutions to $x(x+2) \geq 8$.

10. (10 points) Showing all your work, solve the equation $\frac{3-x}{x+3} = x+1$.

11. (15 points) *Santa's navigation's out, the sky's hard to read,
"Sketch me the path—help me finish this deed!"*

Consider the function $f(x) = 3 \sin(2x - \pi) + 1$.

- (a) Showing all your work, identify the period, amplitude, and phase shift for $f(x)$.
- (b) Use (a) to sketch a graph of $f(x)$. Your plot should contain any x - or y -intercepts and the location of any maxima or minima. You must graph $f(x)$ over at least one full period.

12. (10 points) *Santa's exhausted, his mind's in a blur;
"Help me verify this—I'm not sure what's a blur!"*

Showing all your work, verify the following trigonometric identity:

$$(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$$

13. (20 points) *Christmas almost over, you're nearly clear.*

"Help me find these identities, then we're done for this year."

Complete the following parts:

(a) Give the double angle identity for $\sin(2\theta)$.

(b) Give one of the double angle identities for $\cos(2\theta)$.

(c) Give the identity for $\cos(A \pm B)$.

(d) Give the half-angle identity for $\sin\left(\frac{\theta}{2}\right)$.

(e) Given $\csc^2 \theta$ in terms of $\cot^2 \theta$.

*Santa exclaimed, at the end of Christmas night,
“Thanks to you, the math’s worked out right!
Stay curious, dear student, keeping Mathematics in sight,
And merry Christmas to all, and to all a good night!*