

Check-In 01/15. (*True/False*)

Solution. Making the u -substitution $u = x^2 + 1$ in $\int_0^1 \frac{5x}{x^2 + 1} dx$, we obtain $\int_0^1 \frac{5}{2u} du$.

Solution. The statement is *false*. One must always remember to change the limits when making a substitution in a definite integral. If we choose $u = x^2 + 1$, then $du = 2x dx$. If $x = 0$, then $u = 0^2 + 1 = 1$. If $x = 1$, then $u = 1^2 + 1 = 2$. But then...

$$\int_0^1 \frac{5x}{x^2 + 1} dx = 5 \int_0^1 \frac{x}{x^2 + 1} dx = \frac{5}{2} \int_0^1 \frac{2x}{x^2 + 1} dx = \frac{5}{2} \int_1^2 \frac{du}{u}$$

We also have...

$$\frac{5}{2} \int_1^2 \frac{du}{u} = \frac{5}{2} \ln |u| \Big|_1^2 = \frac{5}{2} (\ln 2 - \ln 1) = \frac{5}{2} (\ln 2 - 0) = \frac{5}{2} \ln(2) = \ln(\sqrt{32}) \approx 1.73287$$