The Mathical Adventures of Robbie the Red
Rockets, Lamp Lights, and Quadratic Equations

To receive full or partial credit, you must show all work on your own paper.

Robbie the Red Robot has enjoyed one too many Red Bulls today and has decided to go shoot off rockets in his backyard. He grabs his launchpad attachment, heads outside, and gleefully fires the first rocket straight up into the air. He clocks its initially velocity at 304 feet per second, and knows the acceleration due to gravity on earth is -32 ft/sec^2. This allows him to concoct an equation that models the rocket's height h (in feet) as a function of time t (in seconds) since launch:

$$h(t) = -16t^2 + 304t.$$

Problem 1. *(10 points)* Using the equation Robbie developed for the rocket's height,

(a) Find the height of the rocket after 3 seconds.

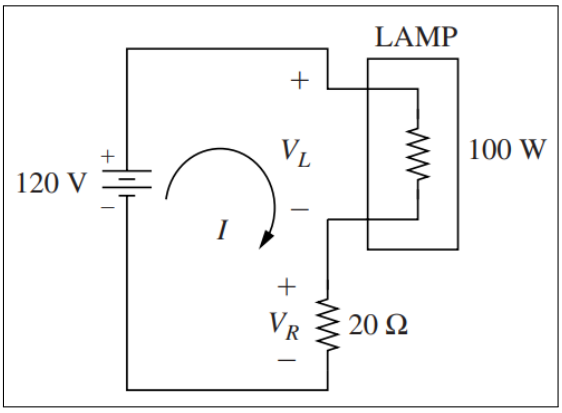
(b) Find the time it takes for the rocket to hit the ground.

(c) Find the times when the rocket has a height of 1,120 feet.
[HINT: 1,120 is divisible by 16.]

(d) Determine the time when the rocket reached its maximum height, and state this maximum height.

(e) Sketch a graph of $h(t)$ using the information you've collected.

As night falls, Robbie has gotten bored simply watching the rockets. He would like to fly! He has therefore attached himself to the next rocket! Robbie wants everyone to able to see his flight, so he sets up a lamp light to illuminate this daring feat. A picture of the lamp's circuitry is shown at the right. There's a 120 volt power supply (on the left side of the picture), the lamp has a power of 100 watts (top right of the picture), and there's a 20 ohm resistor (bottom right of the picture).



Recall Kirchhoff's voltage law: "sum of voltage rises equals sum of voltage drops". For this particular lamp, KVL says

$$120 = V_L + V_R$$

To express this equation entirely in terms of the current I, we'll first use the fact that "power = voltage · current" to rewrite V_L in terms of I:

$$\begin{aligned} P &= V_L I \\ 100 &= V_L I \\ V_L &= \frac{100}{I} \end{aligned}$$

Next, Ohm's law ("voltage = current · resistance") allows us to rewrite V_R in terms of I:

$$V_R = IR \quad \Rightarrow \quad V_R = 20I.$$

Now we can replace V_L and V_R in the equation $120 = V_L + V_R$ to obtain an equation entirely in terms of current:

$$120 = \frac{100}{I} + 20I.$$

Problem 2. *(6 points)* Referring to the equation just developed,

(a) Rewrite the equation in quadratic form (i.e. in the form $ax^2 + bx + c = 0$).
[HINT: multiply both sides by I to get started!]

(b) Solve the quadratic equation you constructed in part (a), and verify that your solutions are correct.

(c) Determine the voltage of the lamp light (i.e. find V_L) for each solution you obtained in part (b).