MATH 141: Exam 2 **Name:** _____
Fall — 2024
10/23/2024
75 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 11 pages (including this cover page) and 10 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total:	100	

1. (10 points) L'Hôpital's rule will not compute the limit below—the resulting limits essentially 'cycle.' Compute the limit below *without the use of l'Hôpital's*.

$$\lim_{x\to\infty} \frac{2x+5}{\sqrt{3x^2+1}}$$

2. (10 points) Let f be the function $f(x) = 2x^3 - 3x^2 - 12x + 7$. Show that $f(x)$ has a root on $[0, 1]$. Be sure to show all your work and fully justify your response.

3. (10 points) An open rectangular storage box is going to be designed to hold 36,000 cubic inches. The base of the box will be twice as long as it is wide. Find the dimensions of the box that will require the least amount of material, i.e. the box with minimal surface area. *For full credit, show all your work and justify that your dimensions actually give the minimal surface area.*

4. (10 points) Showing all your work, compute the following limits:

(a) $\lim_{x\to 0} \dfrac{\sin^2(x)}{\cos(x)-1} =$

(b) $\lim_{x\to \infty} 2x \tan\left(\dfrac{3}{x}\right) =$

(c) $\lim_{x\to 0^-} \dfrac{2x - e^x + 1}{x^2} =$

5. (10 points) A trisectrix of Maclaurin is a curve that can be given by the equation below:

$$x^3 + xy^2 = 3x^2 - y^2$$

Find the tangent line to this curve at the point $(1, -1)$.

6. (10 points) Consider the function $f(x) = \sqrt{x}$.

   (a) Find the linearization of $f(x)$ at $x = 64$. *Do not simplify your answer.*

   (b) Use (a) to approximate $\sqrt{60}$. Express your answer as a decimal.

7. (10 points) Showing all your work, compute the following limits:

(a) $\lim_{x\to\infty} \dfrac{\ln\left(3+e^{2x}\right)}{5x} =$

(b) $\lim_{x\to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{e^x - 1}\right) =$

(c) $\lim_{x\to\infty} \left(1 - \dfrac{2}{x}\right)^x =$

8. (10 points) A spotlight on the ground is illuminating a wall that is 40 ft away from the spotlight. A person that is 6 ft tall begins walking from the spotlight towards the wall at a speed of 6 ft/sec. How fast is the height of the resulting shadow along the wall changing when the person is 15 ft away from the spotlight?

9. (10 points) Suppose that $f(x)$ is a function such that $f(x)$ is differentiable on $[1, 5]$, $f(1) = 10$, and $-1 \leq f'(x) \leq 3$ on $[1, 5]$. Find the smallest and largest possible values for $f(5)$. Be sure to show all your work and fully justify your solutions.

10. (10 points) let $f(x) = 3x^4 - 4x^3 - 2$. Find the absolute minimum and absolute maximum *values* of $f(x)$ on the interval $[-1, 2]$. Be sure to show all your work and fully your answers. You may use any derivative test to justify your answers.