Check-In 01/15. *(True/False)* Making the u-substitution $u = x^2 + 1$ in $\displaystyle\int_0^1 \frac{5x}{x^2+1}\,dx$, we obtain $\displaystyle\int_0^1 \frac{5}{2u}\,du$.

Solution. The statement is *false*. One must always remember to change the limits when making a substitution in a definite integral. If we choose $u = x^2 + 1$, then $du = 2x\,dx$. If $x = 0$, then $u = 0^2 + 1 = 1$. If $x = 1$, then $u = 1^2 + 1 = 2$. But then…

$$\int_0^1 \frac{5x}{x^2+1}\,dx = 5\int_0^1 \frac{x}{x^2+1}\,dx = \frac{5}{2}\int_0^1 \frac{2x}{x^2+1}\,dx = \frac{5}{2}\int_1^2 \frac{du}{u}$$

We also have…

$$\frac{5}{2}\int_1^2 \frac{du}{u} = \frac{5}{2}\,\ln|u|\,\Big|_1^2 = \frac{5}{2}\,(\ln 2 - \ln 1) = \frac{5}{2}\,(\ln 2 - 0) = \frac{5}{2}\,\ln(2) = \ln(\sqrt{32}) \approx 1.73287$$

Check-In 01/20. *(True/False)* We should integrate $\displaystyle\int xe^{x^2}\,dx$ using integration-by-parts.

Solution. The statement is *false*. This certainly looks like an integration-by-parts integral—specifically, a tabular integral (because it is of the form polynomial times trig). The only possible choices for dv would be e^{x^2}, which cannot be integrated, or xe^{x^2}, which is the entire integral and we already 'don't know' how to integrate, or x. This last choices forces $u = e^{x^2}$. But then the new integral produced is $\displaystyle\int x^3 e^{x^2}\,dx$—which is even 'worse.' Instead, observe that the given integral can be found by u-substitution using $u = x^2$, so that $du = 2x\,dx$. But then…

$$\int xe^{x^2}\,dx = \frac{1}{2}\int 2xe^{x^2}\,dx = \frac{1}{2}\int e^u\,du = \frac{e^u}{2} + C = \frac{e^{x^2}}{2} + C$$

There is a heavy overlap between u-substitution and integration-by-parts in terms of what the integrals that require these methods 'look like.' One needs to be careful when determining when to use which method. Some integrals require both methods!

Check-In 01/22. *(True/False)* The integral $\displaystyle\int x^2 3^x\,dx$ can be integrated using tabular integration.

Solution. The statement is *true*. We anticipate that this might be an integration-by-parts tabular integral because it is of the form polynomial times exponential. Using LIATE, we choose $u = x^2$, which forces $dv = 3^x$. Note that $\displaystyle\int 3^x\,dx = \frac{3^x}{\ln 3} + C$. We then have…

u		dv
x^2	$+$	3^x
$2x$	$-$	$\frac{3^x}{\ln 3}$
2	$+$	$\frac{3^x}{(\ln 3)^2}$
0		$\frac{3^x}{(\ln 3)^3}$

Therefore, we have...

$$\int x^2 3^x \, dx = \boxed{\frac{x^2(3^x)}{\ln 3} - \frac{2x(3^x)}{(\ln 3)^2} + \frac{2(3^x)}{(\ln 3)^3} + C}$$

$$= \frac{3^x}{(\ln 3)^3}\left(x^2(\ln 3)^2 - 2x(\ln 3) + 2\right) + C$$

$$= \frac{3^x\left(x^2(\ln 3)^2 - x(2\ln 3) + 2\right)}{(\ln 3)^3} + C$$

$$= \frac{3^x\left(x^2(\ln 3)^2 - x\ln 9 + 2\right)}{(\ln 3)^3} + C$$