

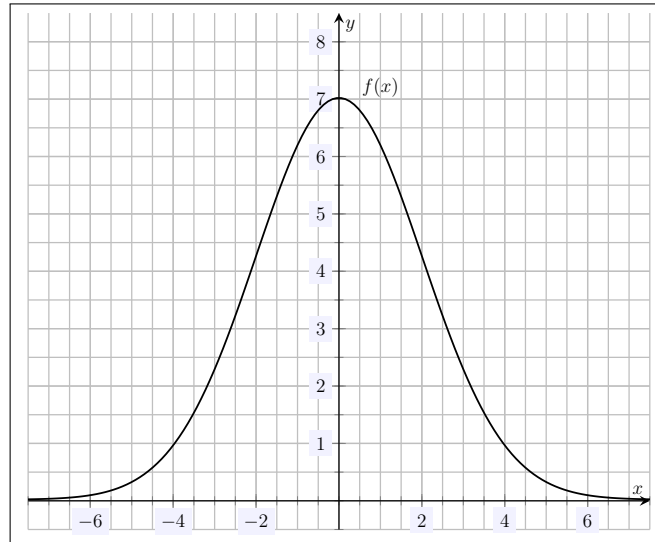
MATH 122: Exam 2
Fall — 2024
10/15/2024
75 Minutes

Name: _____

Write your name on the appropriate line on the exam cover sheet. This exam contains 8 pages (including this cover page) and 8 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	20	
2	15	
3	15	
4	15	
5	15	
6	5	
7	8	
8	7	
Total:	100	

1. (20 points) Consider the function $f(x)$ plotted below.



Based on the plot above, answer the following:

(a) On what interval(s)—if any—is $f'(x) > 0$?

(b) On what interval(s)—if any—is $f'(x) < 0$?

(c) Find any points of inflection—if any.

(d) On what interval(s)—if any—is $f''(x) > 0$?

(e) On what interval(s)—if any—is $f''(x) < 0$?

(f) Determine whether the following are positive (> 0), negative (< 0), or zero ($= 0$):

- | | |
|--------------------|--------------------|
| • $f(-2)$ _____ 0 | • $f'(-3)$ _____ 0 |
| • $f'(0)$ _____ 0 | • $f'(5)$ _____ 0 |
| • $f''(5)$ _____ 0 | • $f''(0)$ _____ 0 |

(g) Sketch the tangent line to $f(x)$ at $x = 4$ on the graph above.

2. (15 points) Showing all your work, compute the derivatives given below. *Do not simplify your answer.*

(a) $\frac{d}{dx} \left(x^6 - 3x + \frac{1}{\sqrt{x}} - 11 \right) =$

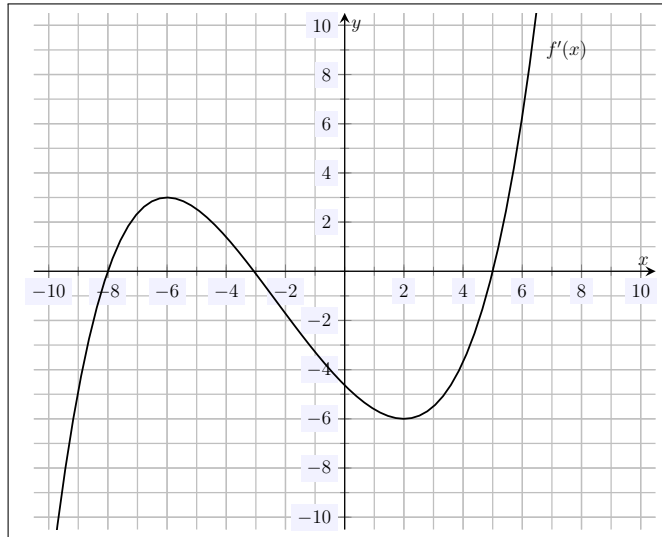
(b) $\frac{d}{dx} (x^{10} 2^x) =$

(c) $\frac{d}{dx} \ln(5x^2 - x) =$

(d) $\frac{d}{dx} (5\pi^3) =$

(e) $\frac{d}{dx} \left(\frac{6x - 1}{x^2 + 1} \right) =$

3. (15 points) Consider the *derivative* of a function, $f'(x)$, plotted below.



Based on the plot above, answer the following questions:

- (a) On what interval(s)—if any—is $f(x)$ increasing?
- (b) On what interval(s)—if any—is $f(x)$ decreasing?
- (c) On what interval(s)—if any—is $f(x)$ concave up?
- (d) On what interval(s)—if any—is $f(x)$ concave down?
- (e) Find any critical values for $f(x)$ —if any.
- (f) Classify any critical values you found in (e).
- (g) Find any points of inflection for $f(x)$.

4. (15 points) Showing all your work, compute the derivatives given below. *Do not simplify your answer.*

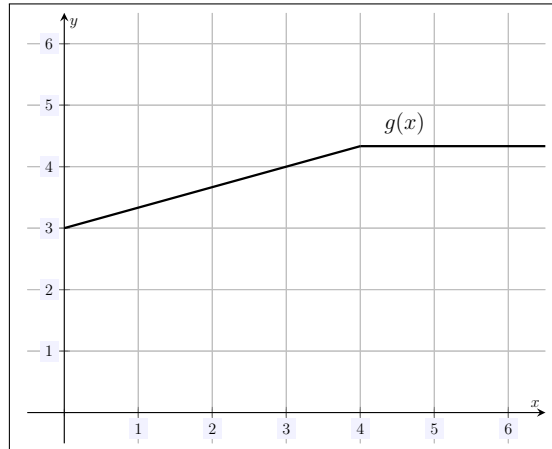
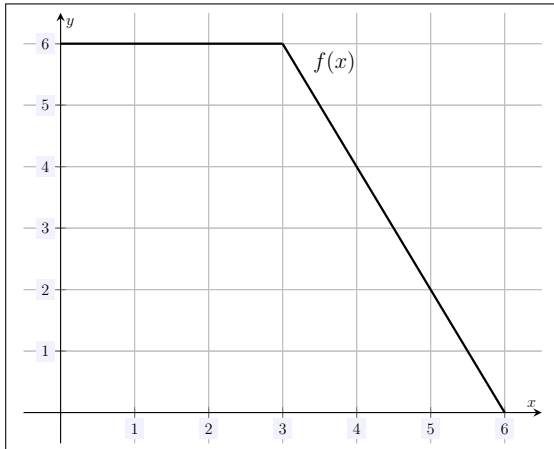
(a) $\frac{d}{dx}(3x^2 e^{-x} \log_5(x)) =$

(b) $\frac{d}{dx}(9^{x^3} - x)^7 =$

(c) $\frac{d}{dx} \left(\frac{(3x-1)e^{2x}}{\ln(1-x)} \right) =$

5. (15 points) A company produces widgets. They hire financial analysts to examine their production costs. The analysts determine that if q items are produced, then the total production cost for the widgets is given by $C(q) = 223,000 + 1,000q - q^2$.
- (a) Find the fixed costs for producing these widgets.
 - (b) Find the marginal costs at a production level of $q = 180$.
 - (c) What level of production maximizes the total cost of producing these widgets? Be sure to justify your answer with either the first or second derivative test.

6. (5 points) Consider the graphs of the functions $f(x)$ and $g(x)$ given below.



Based on these plot and showing all your work, compute $\left. \frac{d}{dx} f(g(x)) \right|_{x=3}$.

7. (8 points) Suppose $f(x)$ is a function with $f(10) = -2$, $f'(10) = 8$, and $f''(10) = 12$.

(a) Find the tangent line to $f(x)$ at $x = 10$.

(b) Use (a) to find an approximation for $f(10.3)$.

(c) Is your approximation (b) more likely an under-approximation or an over-approximation? Explain.

8. (7 points) Let $f(x) = 2x^2 + 8x - 5$. Using the definition of the derivative, approximate $f'(-1)$. *While you may check your answer using the derivative shortcuts, you will receive no credit for using derivative shortcuts to find this value.*