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MATH 111-I
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Quiz 5

Problem 1: Consider the quadratic function $f(x) = 3 - 6x - x^2$.

(a) Find a, b, c from the standard form for $f(x)$.

(b) Does the graph of $f(x)$ open upwards or downwards? Explain.

(c) Find the vertex and axis of symmetry for $f(x)$.

(a) The standard form for a quadratic function is $ax^2 + bx + c$. Writing $f(x)$ as $f(x) = -x^2 - 6x + 3$, we see that...

$$\boxed{a = 1, \quad b = -6, \quad c = 3}$$

(b) Because $a = -1 < 0$, we know that the graph of $f(x)$ opens downwards.

(c) We find the vertex form of $f(x)$.

Using completing the square, we first factor out -1 to make the x^2-coefficient 1. For the new term, we find half the 'middle' term ($\frac{6}{2} = 3$), square this term ($3^2 = 9$) and then add/subtract this value. This yields...

$$\begin{aligned} f(x) &= -x^2 - 6x + 3 \\ &= -(x^2 + 6x - 3) \\ &= -(x^2 + 6x + 9 - 9 - 3) \\ &= -\big((x^2 + 6x + 9) + (-9 - 3)\big) \\ &= -\big((x+3)^2 - 12\big) \\ &= -(x+3)^2 + 12 \end{aligned}$$

Therefore, the vertex form is $f(x) = 12 - (x+3)^2$. Alternatively, using the 'evaluation method', we know that the vertex is located at $x = -\frac{b}{2a} = -\frac{(-6)}{2(-1)} = \frac{6}{-2} = -3$. The y-value at this vertex location must be $f(-3) = -(-3)^2 - 6(-3) + 3 = -9 + 18 + 3 = 12$. We know that $a = -1$. The vertex form is $a(x - P)^2 + Q$, where the vertex is (P, Q). Therefore, we have $f(x) = -1\big(x - (-3)\big)^2 + 12 = -(x+3)^2 + 12 = 12 - (x+3)^2$.

In either case, the vertex must be $(-3, 12)$. This also implies that the axis of symmetry is $x = -3$.