The Mathical Adventures of Robbie the Red
Mixtures, Debris Removal, and Rational Functions

To receive full or partial credit, you must show all work on your own paper.

Robbie the Red Robot has received an a distress call from the nearby mixing lab! Much of the equipment there has inexplicably stopped working, and although Robbie has a small penchant for wanton destruction, he has all the right stuff to handle the lab's tasks today.

For starters, Robbie is given a pot containing 6 liters of brine (i.e. salt water) with salt concentration 120 grams per liter. The lab needs this concentration to be increased to 200 grams per liter, but there's no salt left. The only way out is for Robbie to boil off some water.

Problem 1. *(4 points)* Using the setup in the paragraph above,

(a) Determine how many grams of salt are in the pot of brine.
[HINT: use the units to help you!]

(b) Assuming the amount of salt in the brine doesn't change when boiling off the water, set up and solve a rational equation to determine how much water Robbie needs to boil off to reach the desired concentration of 200 grams per liter.

Robbie's success in boiling off the water was met with great enthusiasm! For the next task, Robbie is shuffled off to a room with a large cylindrical container. A careless employee has misplaced the records for this container, so Robbie will need to recover some of the information from scratch.

Explore. A right cylinder has volume $V = \pi r^2 h$ and surface area $S = 2\pi r^2 + 2\pi rh$.

(a) If the volume is given to be 32π cubic meters, express the surface area in terms of the radius.

(b) If the cylinder has a radius of 4 meters, what is its surface area? (Round your answer to two decimal places.)

A helpful robot's work is never done. Robbie is next taken to the boiler room, where the boiler has exploded and scattered debris everywhere...in fact, there are 100 kilograms of debris! Fortunately, Robbie has brought along some of his special drones which can pick up and dispose of this debris.

Problem 2. *(6 points)* The time (in minutes) it takes for the drones to collect x kilograms of debris is given by $f(x) = \dfrac{4x}{100 - x}$ (where $0 < x < 100$).

(a) Find the asymptotes of the rational function $f(x)$.

(b) Sketch the graph of $f(x)$ on the domain $0 < x < 100$.

(c) How much of the debris have the drones collected after 20 minutes?

The mixing lab is forever indebted to Robbie for all his help today...but that won't stop them from asking one more favor! One of their circuits has two resistors which are connected in parallel. They know the total resistance R_T is determined by the resistance R_1 and R_2 of each resistor according to the equation

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \text{where } R_1 \text{ and } R_2 \text{ are greater than 0.}$$

Explore. Robbie determines that $R_2 = 5$ Ohms. Using this,

(a) Express the total resistance R_T as a rational function of the resistance R_1.

(b) Will the total resistance ever exceed 5 Ohms? How do you know? (Remember that $R_1 > 0$.)