MATH 142: Exam 2 **Name:** ______________________________
Spring — 2025
03/20/2025
75 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 10 pages (including this cover page) and 8 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	25	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	15	
Total:	100	

1. (25 points) Determine whether the series below converges conditionally, converges absolutely, or diverges. Be sure to fully justify your answer using the appropriate series tests.

$$\sum_{n=1}^{\infty}(-1)^n \frac{n^2}{n^3+16}$$

2. (10 points) Determine whether the series below converges or diverges. Be sure to fully justify your response with the appropriate series tests.

$$\sum_{n=5}^{\infty} \frac{2n^3}{n^3+7}$$

3. (10 points) Determine whether the series below converges or diverges. Be sure to fully justify your response with the appropriate series tests.

$$\sum_{n=0}^{\infty} \frac{n^3\, 3^n}{n!}$$

4. (10 points) Determine whether the series below converges or diverges. Be sure to fully justify your response with the appropriate series tests.

$$\sum_{n=1}^{\infty} \frac{\ln n}{\sqrt[3]{n}}$$

5. (10 points) Determine whether the series below converges or diverges. Be sure to fully justify your response with the appropriate series tests.

$$\sum_{n=1}^{\infty} \left(\frac{5n+1}{4n+3} \right)^n$$

6. (10 points) Determine whether the series below converges or diverges. If the series converges, find the sum. Be sure to fully justify your response with the appropriate series tests and computations.

$$\sum_{n=1}^{\infty} \frac{(-4)^{n-1}}{3^{2n}}$$

7. (10 points) Determine whether the series below is convergent or divergent. Be sure to fully justify your response with the appropriate series tests.

$$\sum_{n=3}^{\infty} \frac{n^2 - 4}{n^6 + 5}$$

8. (15 points) Use the Integral Test to determine whether the series below converges or diverges. Be sure to fully justify your response and show all your work.

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^4}$$

Bonus I. The following parts are *each worth two bonus points* on the exam. You may answer one or more of them. Failing to answer any of these questions or answering any of them incorrectly ***will not*** count against your exam score.

(a) The name of the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is ____________________________.

(b) The sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is ____________________.

(c) The sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^4}$ is ____________________.

(d) The sum $\sum_{k=0}^{n} a\, r^n$ is __________________________.

(e) What is the most likely next term of the sequence $\{1, 11, 21, 1211, 111221, 312211, \ldots\}$?

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Bonus II. The following question is worth *ten bonus points*. Failing to answer this question or answering incorrectly ***will not*** count against your exam score.

Fully justifying your reasoning and using any appropriate series tests, determine whether the following series converges or diverges:

$$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^3}\right)$$