1 Online Updating of Gauss Distribution

This is implemented in goMultiGaussPDF.

$$p(x) = \frac{1}{\sqrt{\det \Sigma}(2\pi)^{\frac{n}{2}}} \exp\left(-\frac{1}{2}(x-\mu)^\top \Sigma^{-1}(x-\mu)\right)$$

We want to learn the covariance matrix Σ and mean μ.

$$\begin{aligned}
\Sigma &= \frac{1}{N}\sum_{i=1}^{N}(v_i-\mu)\cdot(v_i-\mu)^\top \\
&= \frac{1}{N}\sum_{i=1}^{N}(v_i-\mu)\cdot(v_i^\top-\mu^\top) \\
&= \frac{1}{N}\left[\sum_{i=1}^{N} v_i v_i^\top - 2\mu\sum_{i=1}^{N} v_i^\top\right] + \mu\mu^\top \\
&= \frac{1}{N}\left[\sum_{i=1}^{N} v_i v_i^\top\right] - \mu\mu^\top
\end{aligned}$$

Which means we can store $S := \frac{1}{N}\sum_{i=1}^{N} v_i v_i^\top$, μ, and N and then update with a new vector v

$$\begin{aligned}
\mu_{k+1} &= \frac{N_k}{N_k+1}\mu_k + \frac{v}{N_k+1} \\
S_{k+1} &= \frac{N_k}{N_k+1}S_k + \frac{1}{N_k+1}vv^\top \\
N_{k+1} &= N_k+1
\end{aligned}$$

with $\mu_0 := 0, \mu_0 \in \mathbb{R}^n$, $S_0 := 0, S_0 \in \mathbb{R}^{n\times n}$, $N_0 := 0, N_0 \in \mathbb{N}^+$.