Christopher Göbeler, Jingyi Lu, Wengchan Wang
7376835, XXXXXXX, 7430374

Applied Mathematical Optimization
- Project Report -

Project Report

Supervisor: Dr. Saber Talar
Second Examiner: Lennard Sund

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Team Composition

Team Members

1. Christopher Göbeler, 7376835

2. Jingyi Lu,

3. Wengchan Wang, 7430374

Abstract

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1 Introduction and Background

The transformation of the energy sector is driving the widespread adoption of renewable energy, but it also introduces new challenges related to grid congestion. In particular, Germany faces significant challenges due to the geographic mismatch between wind energy generation and electricity demand. Wind energy is predominantly concentrated in the northern regions, while demand centers are located in the south, resulting in increased demand for long-distance transmission and frequent grid bottlenecks(Appunn, 2018). This geographical imbalance is not unique to Germany; similar challenges are observed in other regions with high penetration of renewable energy, emphasizing the urgent need for innovative solutions.

Traditionally, grid operators have managed congestion through redispatching and curtailment. Redispatching involves adjusting generator outputs to balance the grid, whereas curtailment reduces renewable energy output to prevent overloading. However, these methods are costly and unsustainable as the penetration of renewable energy continues to rise. For instance, in Germany, the costs of redispatching exceeded 1 billion euros between 2015 and 2019, and curtailment can further lead to energy waste and lower returns on investment(Monforti-Ferrario & Blanco, 2021).

Advancements in home battery systems, such as the Tesla Powerwall, enhance residential solar photovoltaic (PV) efficiency by storing excess energy for use during low-generation periods, thereby reducing reliance on conventional grid electricity. These systems increase homeowner energy independence and minimize peak demand strain on the grid. On a larger scale, widespread adoption can support grid stability and accelerate the transition to a decentralized renewable energy system. As battery technologies improve and costs decline, these systems can play a critical role in mitigating the stochastic nature of a fully green grid. When combined with local PV production, they further reduce grid strain during peak demand periods.

Existing studies have largely focused on the technical feasibility and economic analyses of renewable energy integration. However, there has been limited assessment of the social benefits of distributed PV and local energy storage, particularly in grid systems similar to Germany's. This paper aims to address this gap by employing mathematical optimization techniques to evaluate the social benefits of these technologies. The insights from this study can guide regulators and policymakers regarding subsidies and other support mechanisms for renewable energy adoption.

2 Literature Review

2.1 Evaluation of Current Congestion Management Strategies

Grid congestion occurs when transmission lines approach their maximum capacity, potentially leading to reliability issues. In grids with high wind energy penetration, congestion is exacerbated by the variability and uncertainty of wind and solar energy. The case of Germany illustrates this challenge, as wind energy generated in the north must be transmitted to high-demand areas in the south, leading to frequent congestion on critical transmission lines(Appunn, 2018). Similar issues are observed in other regions, such as the United States and Australia(Schmidt, 2023).

Traditional strategies to address congestion include redispatching and curtailment. Redispatching involves adjusting generator outputs to manage power flows, a method extensively used in Germany between 2015 and 2019, with costs reaching 1 billion euros(Wohland, Reyers, Märker, & Witthaut, 2018). However, the sustainability of these strategies is increasingly questioned as renewable energy penetration grows(Hladik, Fraunholz, Kühnbach, Manz, & Kunze, 2020).

2.2 The Potential of Distributed PV and the Role of Local Energy Storage

Distributed PV has the potential to alleviate congestion by generating electricity near demand centers, thereby reducing the need for long-distance transmission. A study published in Utilities Policy (2017) demonstrated that distributed PV can decrease household reliance on the grid(Hanser, Lueken, Gorman, Mashal, & Group, 2017). While the system costs are relatively high, it provides system-wide benefits such as congestion mitigation. Research by NREL further indicates that distributed PV can reduce peak loads and delay the need for transmission upgrades(Cohn, 2024). In the case of Germany, increasing PV deployment in the southern regions could balance the north-south power flow and reduce congestion on critical transmission lines, though care must be taken to avoid creating new congestion within distribution networks(Chen & Heilscher, 2024).

Local energy storage systems can store excess energy during periods of low demand and release it during periods of high demand, helping to balance the grid. Vargas et al. (2014) analyzed the role of energy storage systems in managing transmission congestion, taking into account the ramping capabilities of power plants. Their findings suggest that storage can effectively reduce wind energy curtailment and congestion costs(Vargas, Bustos-Turu, & Larraín, 2014). Similarly, a study by IHS (2020) highlighted that energy storage can delay transmission upgrades and provide flexibility in managing power flows(Forsyth, 2020). In Germany, energy storage is regarded as a key solution for managing renewable energy variability and mitigating congestion(Stephan & Nitsch, 2024).

3 Modeling and Solution Approach

3.1 Related Work

In recent years, multi-period optimization techniques have been extensively studied for addressing renewable energy uncertainties, optimizing energy system scheduling, and mitigating grid congestion. Abunima et al. (2022) proposed a two-stage stochastic optimization model for managing renewable energy-based microgrids, minimizing operational costs through scenario analysis while addressing uncertainties in generation and load(Abunima, Park, Glick, & Kim, 2022). Similarly, Lazaroiu et al. (2016) optimized the sizing and operation of microgrids with renewable energy and energy storage systems using stochastic optimization, emphasizing the importance of scenario analysis in handling wind energy and load fluctuations(Lazaroiu, Dumbrava, Balaban, Longo, & Zaninelli, 2016).

Zhao et al. (2024) further expanded this field by exploring the application of stochastic optimization techniques in smart grids to enhance the integration of renewable energy and address the challenges of its intermittency(Zhao, Holland, & Nelson, 2024). In the context of network constraints, Mirzaei et al. (2019) developed a stochastic optimization framework for co-optimizing energy and reserve products in integrated electricity and gas networks, fully accounting for the impact of network constraints on system operations(Mirzaei et al., 2019). Teshome et al. (2015) proposed a stochastic optimization model for the scheduling of network-constrained power systems, incorporating stochastic inputs from wind power generation and load(Teshome, Correia, & Lian, 2015).

Additionally, for the integrated optimization of photovoltaic (PV) systems and energy storage, Garip and Ozdemir (2019) investigated the optimal sizing of PV and battery storage in grid-connected microgrids to minimize energy costs(Garip & Ozdemir, 2022), while Li et al. (2021) optimized the configuration of energy storage systems in distribution networks to regulate voltage through PV integration(Li, Zhou, Guo, Fan, & Huang, 2021).

These studies collectively demonstrate that stochastic optimization combined with scenario analysis is an effective tool for managing renewable energy uncertainties and network constraints, providing a theoretical foundation for this research. Building on this, our paper develops a multi-period stochastic optimization model that integrates wind energy, PV systems, and energy storage to evaluate their social benefits in alleviating congestion and balancing stochastic production. To show this, we approach from a macro perspective of a social planner aiming to minimize costs globally. A small model evaluating benefits on a micro level can be found in Appendix A.

4 Network and Scenario Setup

4.1 Network

The selected network setup consists of five interconnected buses (see Figure X). The central bus includes a conventional generator unit and a demand profile that varies over time periods. Upward in the network, a wind farm feeds energy into Bus 1 through stochastic power generation. Below the central bus, Bus 2 represents a demand point directly connected to the power source at Bus 1. Further downstream, there are two additional buses (Bus 3 and Bus 4) with demand that follows predefined scenarios. Notably, in the initial model, these buses are not directly connected to any power source. As a result, congestion issues arise when the aggregated demand at Buses 2, 3, and 4 exceeds the line capacity between Buses 1 and 2.

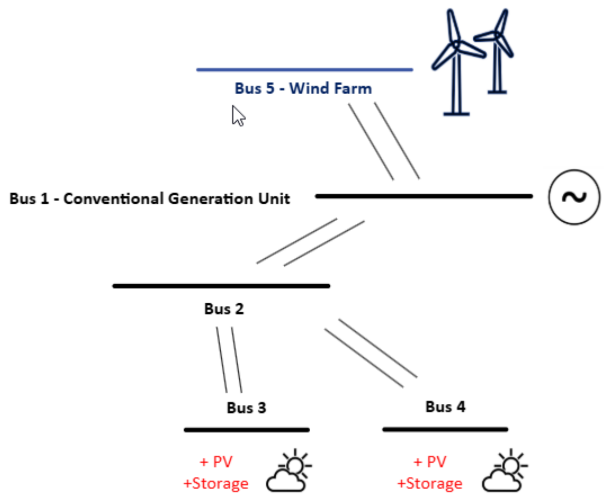

Figure 1: Figure 1: Network setup

4.2 Scenarios

The system operates over four time periods, with three scenarios representing different wind power availability and dynamic demand realizations. Each scenario occurs with equal probability of one-third. Conventional generation available at Bus 1 is subject to constrains regarding maximum generation as well as ramping limits between time periods. Wind generation at Bus 5 is scenario-dependent and can be subject to curtailment, although penalized in our objective function to incentivize the use of renewable energy. Demands follow scenarios in an aggregated manner (see Figure Y) while scenarios of stochastic renewables production can be taken from Figure Z and Z1 respectively.

4.2.1 Demand Profile

For our use case we decided to include stochastic demand in an agggregated manner. While independent scenarios for each Bus may represent real-world situations more accurately, we concluded that the added model complexity does not add particular value to the solution of our problem. However, we carefully selected the scenario set to illustrate different versions of congestion issues. Figure Y shows the scenario tree for the demand at each bus. While scenario 1 (low demand) does not lead to any congestion issues in the network, scenarios 2 (medium demand) and 3 (high demand) lead to partial and total load shedding at an outer bus.

4.2.2 Renewables Production

Production Scenarios were created using 2019 hourly data from *renewables.ninja* for the city of Cologne, Germany. Utilizing K-Means-Clustering, 3 different production scenarios for each wind and photovoltaic production, which is added into our model at later stages, were created. See the Scenarios and Production outputs for each scenario depicted in Figures Z and Z1.

Model	Cost	Conv. Power	Curtailment	Load Shed
Baseline	1248.33	72.94	28.43	5.19
Ext. A	158.33	15.83	70.2	0.0
Ext. B	37.22	3.72	16.26	0.0

Table 1: Comparison of overall Modelling Results

5 Mathematical Modelling Approach

The mathematical model represents a power system with wind generation, conventional generation, and extensions to incorporate photovoltaic (PV) generation and storage. The primary objective is to balance power supply and demand while considering network constraints, generation limits, curtailment, load shedding, and economic costs. The model evolves through three stages: (i) baseline with wind and conventional generation, (ii) addition of PV generation, and (iii) incorporation of storage.

For the Baseline model, we consider wind generation at Bus 5 and conventional generation at Bus 1. The transmission network connects these generation sources to demand locations, leading to congestion issues due to the geographical mismatch between supply and demand. The model accounts for network flow constraints and allows for load shedding at Buses 3 and 4 to ensure feasibility.

$$g_{\text{conv}}[s,t] \geq 0, \quad \text{bound by } G_{\max} \tag{1}$$

$$g_{\text{conv}}[s,t] - g_{\text{conv}}[s,t-1] \leq \text{rampup}\; g\text{conv}[s,t-1] - g_{\text{conv}}[s,t] \qquad \leq \text{ramp}_{\text{down}} \tag{2}$$

(1) ensures conventional generation to be within its maximum allowable output, while ramping constraints regulate the rate at which conventional generators can increase or decrease output between time periods.

$$g_{\text{wind}}[s,t] \geq 0 \tag{3}$$

$$c_w[s,t] \geq 0 \tag{4}$$

$$g_{\text{wind}}[s,t] + c_w[s,t] = W[s,t] \tag{5}$$

(2), (3) and (4) define wind power generation and curtailment, ensuring that total wind availability is either utilized or curtailed. Power flows between buses are modeled using voltage angles and the DC power flow equations. (6) also depicts the available transmissin lines in the network.

$$f_{51}[s,t], f_{12}[s,t], f_{23}[s,t], f_{24}[s,t] \tag{6}$$

$$\theta[bus,s,t] \text{for} bus \in 1,2,3,4,5 \tag{7}$$

$$\theta[1,s,t] = 0 \tag{8}$$

$$f_{ij}[s,t] = \frac{\theta[i,s,t] - \theta[j,s,t]}{X} \tag{9}$$

$$\tag{10}$$

$$-F_{\max} \leq f_{ij}[s,t] \leq F_{\max}, \quad \forall (i,j) \in (5,1),(1,2),(2,3),(2,4) \tag{11}$$

Line flow limits ensure that transmission lines operate within given technical limitations. To maintain feasibility, load shedding is permitted at Buses 3 and 4, ensuring demand balance in extreme congestion scenarios.

$$LS[s,t] \geq 0 \tag{12}$$

Power balance constraints (13) - (16) enforce energy conservation at each bus, ensuring that incoming power matches outgoing power plus demand.

$$g_{\text{wind}}(s,t) - f_{51}(s,t) = 0 \tag{13}$$

$$g_{\text{conv}}(s,t) + f_{51}(s,t) - f_{12}(s,t) - D_1(s,t) = 0 \tag{14}$$

$$f_{12}(s,t) - f_{23}(s,t) - f_{24}(s,t) - D_2(s,t) = 0 \tag{15}$$

$$f_{23}(s,t) + LS_3(s,t) - D_3(s,t) = 0 \tag{16}$$

$$f_{24}(s,t) + LS_4(s,t) - D_4(s,t) = 0, \forall s,t \tag{17}$$

$$\tag{18}$$

The objective function minimizes the cost associated with conventional generation, curtailment, and load shedding.

$$\min \sum_{s,t} \text{prob}[s]\left(c_g g_{\text{conv}}[s,t] + c_{\text{curt}} c_w[s,t] + c_{\text{ls}}(LS_3[s,t] + LS_4[s,t])\right) \tag{19}$$

5.1 Extension A: PV Generation at Bus 3 and 4

This extension introduces PV generation at Buses 3 and 4, expanding renewable energy sources while maintaining curtailment constraints. The objective function is updated as well as the power balance constraints of bus 3 and 4:

$$g_{\text{PV}}[s,t] \geq 0, \quad c_{\text{pv}}[s,t] \geq 0 \tag{20}$$

$$g_{\text{PV}}[s,t] + c_{\text{pv}}[s,t] = PV[s,t] \tag{21}$$

$$\tag{22}$$

$$f_{2i}(s,t) + g_{PV}[s,t] + LS_i(s,t) - D_i(s,t) = 0, \forall s,t,i = 3,4 \tag{23}$$

Updated Objective Function - Extension A

$$\min \sum_{s,t} \text{prob}[s]\left(c_g g_{\text{conv}}[s,t] + c_{\text{curt}}(c_w[s,t] + c_{\text{pv3}}[s,t] + c_{\text{pv4}}[s,t]) + c_{\text{ls}}(LS_3[s,t] + LS_4[s,t])\right) \tag{24}$$

5.2 Extension B: PV Generation and Storage Options at Bus 3 and 4

This extension incorporates storage systems at Buses 3 and 4, enhancing system flexibility by enabling intertemporal energy exchange. Storage is modeled through energy balance constraints and charge/discharge limits.

Battery Storage Variables

$$E[s,t] \geq 0, E[s,t] \leq E_{\max} \tag{25}$$

$$p_{\text{ch}}[s,t] \geq 0, p_{\text{dis}}[s,t] \geq 0 \; p_{\text{ch}}[s,t] \leq P_{\text{ch}\max}, p_{\text{dis}}[s,t] \leq P_{\text{dis}\max} \tag{26}$$

$$E[s,t] = E[s,t-1] + \eta_{\text{ch}} p_{\text{ch}}[s,t] - \frac{1}{\eta_{\text{dis}}} p_{\text{dis}}[s,t] \tag{27}$$

The power balance constraints are updated to integrate storage operations, improving system reliability and mitigating renewable energy variability. The objective function from Extension A is used with no adjustments.

$$f_{2i}(s,t) + g_{PV}[s,t] + LS_i(s,t) - D_i(s,t) + p_i^{disc} - p_i^{ch} = 0, \forall s,t,i = 3,4 \tag{28}$$

6 Results

Final Results can be taken from *Table 1.* The baseline setup shows a high reliance on conventional generation, particularly during peak demand periods. Wind curtailment occurs frequently, suggesting an inability to fully utilize available renewable energy due to system constraints. Although load shedding occurs only in exceptions, extreme demand periods exist where the full demand canot be met by generation. The overall costs are highest here, driven by expensive conventional power and load shedding penalties. These findings highlight the inefficiencies of a system without additional renewable capacity or storage which could reduce reliance on conventional generation, lower costs, and improve renewable energy integration.

Model	Cost	Conv. Power	Curtailment	Load Shed
Baseline	1248.33	72.94	28.43	5.19
Ext. A	158.33	15.83	70.2	0.0
Ext. B	37.22	3.72	16.26	0.0

Table 2: Comparison of overall Modelling Results

In Extension A, the introduction of PV generation at Buses 3 and 4 significantly reduces the need for conventional generation, leading to a substantial decrease in total system costs (from 1248.33 in the baseline to 158.33). Wind curtailment remains present but is reduced overall, as PV generation helps balance the load more efficiently. The system relies heavily on PV power, particularly in time periods with high solar availability, thereby minimizing conventional generation. However, curtailment persists when renewable production exceeds demand. This demonstrates that while PV integration lowers costs, further optimization can enhance system flexibility and reduce costs.

When considering PV and storage options, a dramatic reduction in overall system costs (from 158.33 in Extension A to 37.22) can be observed. Storage allows excess renewable energy to be stored and discharged later, minimizing wind curtailment and reducing reliance on conventional generation. During peak PV production periods, storage absorbs surplus energy, which is later discharged to balance supply and demand. This optimization significantly enhances renewable utilization, further lowering costs compared to Extension A. The system benefits from increased flexibility, demonstrating the economic and operational advantages of integrating battery storage in renewable-heavy power systems.

7 Discussion

This study examines different approaches to reducing reliance on conventional power, minimizing renewable energy curtailment, and enhancing grid flexibility through renewable energy and storage solutions. The results indicate that integrating PV generation significantly lowers costs and reduces curtailment compared to a baseline model reliant on conventional and wind generation setup similarly to the German grid. Further improvements arise when battery storage is added, as it enhances the intertemporal utilization of renewables, leading to the lowest overall system cost and minimal curtailment.

Our findings suggest that investment in battery storage and renewable infrastructure can yield economic and environmental benefits on a macro level. Expanding grid flexibility through storage can alleviate congestion, enhance energy security, and reduce dependence on fossil fuels. However, other complementary strategies, such as expanding transmission capacities or grid setup could also mitigate congestion and enable the efficient transport of renewable energy from rural generation sites to high-demand urban areas. Home battery systems, while relatively novel and not traditionally considered in grid planning, offer new opportunities for decentralizing energy storage. They allow households and small businesses to store excess renewable energy for later use, reducing stress on the transmission network and improving energy resilience. Incentivizing these systems alongside larger-scale storage solutions can further enhance grid stability and lower costs. Our work shows benefits beyond household independency and cost savings and highlights how such systems can help the whole grid mitigate congestion and fluctuating renewable energy production. In rural regions, where renewable energy potential is high but demand is lower, effective grid integration is crucial. The combination of localized battery storage and improved transmission infrastructure can maximize economic benefits of renewables while ensuring reliability. Future energy policies should thus consider a mix of strategies—including distributed generation, demand-side management, and storage incentives—to ensure a cost-effective and sustainable energy transition.

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