

# Problem Set 5

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## Linear Regression Error

1. [c] 100

$$\sigma = 0.1$$

$$d = 8$$

$$E_{in} > 0.008$$

$$\mathbb{E}_{\mathcal{D}}[E_{in}(\mathbf{w}_{lin})] > \sigma^2 \left(1 - \frac{d+1}{N}\right)$$

$$\frac{\mathbb{E}_{\mathcal{D}}[E_{in}(\mathbf{w}_{lin})]}{\sigma^2} > 1 - \frac{d+1}{N}$$

$$\frac{d+1}{N} > 1 - \frac{\mathbb{E}_{\mathcal{D}}[E_{in}(\mathbf{w}_{lin})]}{\sigma^2}$$

$$N > \frac{d+1}{1 - \frac{\mathbb{E}_{\mathcal{D}}[E_{in}(\mathbf{w}_{lin})]}{\sigma^2}}$$

$$N > \frac{8+1}{1 - \frac{0.008}{0.1^2}}$$

$$N > 45$$

## Nonlinear Transforms

2. [D]  $\tilde{w}_1 < 0, \tilde{w}_2 > 0$

$$\Phi(1, x_1, x_2) = (1, x_1^2, x_2^2)$$

$$\text{sign}(\tilde{\mathbf{w}}^T \cdot \tilde{x}) = \text{sign}(\tilde{w}_0 \cdot 1 + \tilde{w}_1 \cdot \tilde{x}_1 + \tilde{w}_2 \cdot \tilde{x}_2)$$

As  $\tilde{x}_1$  increases, the function should be more likely to return a negative number. Thus,  $\tilde{w}_1$  should be negative as well. As  $\tilde{x}_2$  increases, the function should be more likely to return a positive number. Thus,  $\tilde{w}_2$  should be positive as well.

3. [D] 20

The  $d_{vc} \leq d + 1$ . Here,  $d = 15$ , so  $d_{vc} \leq 16$ . Since none of the additional values are linear combinations of  $x_1$  and  $x_2$ , then they are each new dimensions.

## Gradient Descent

4. [D]  $\tilde{w}_1 < 0; \tilde{w}_2 > 0$

$$\begin{aligned} E(u, v) &= (ue^v - 2ve^{-u})^2 \\ \frac{\partial E}{\partial u} &= \frac{\partial (ue^v - 2ve^{-u})^2}{\partial u} \\ &= 2(ue^v - 2ve^{-u}) \cdot (e^v + 2ve^{-u}) \end{aligned}$$

5. [D] 10

See attached code.

6. [E] (0.045, 0.024)

7. [A]  $10^{-1}$

## Logistic Regression

8.

9.

## PLA as SGD

10.