MM2090 Assignment-4

mm20b008 me20b027 me20b055 me20b166
mm20b020

July 1, 2021

Contents

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1.1 Planck's Radiation Law

$$\rho(\nu, T) = \frac{8\pi h\nu^3}{c^3} \frac{1}{\exp_k^{\frac{h\nu}{k_B T}} - 1} \tag{1}$$

The Planck's Law describes the energy density of electromagnetic radiation emitted by a black body in thermal equilibrium at a given Temperature T, and there is no net flow of matter or energy between the body and its environment. The equation 1 depicts mathematical form of the Plank's Law.
Here
ρ - Energy Density
ν - Frequency of radiation emitted
k_B - Boltzmann Constant
k - Kaniadakis Parameter
T - Temperature
h - Plank Constant
The equation shows the fequency dependency of the the energy density [7].The figure 1 shows the graph of variation of energydensity vs frequency.

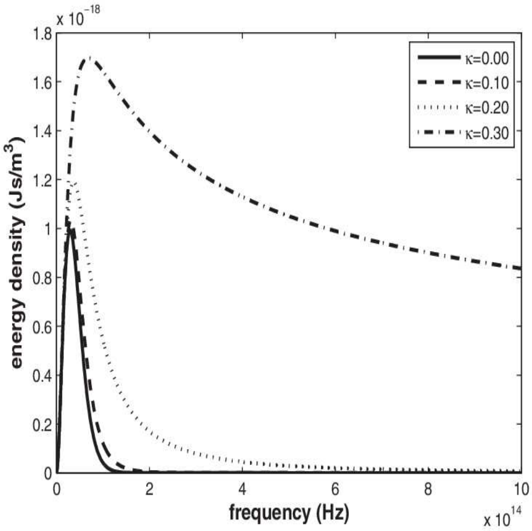

Figure 1: Variation of energy density with the frequency, for the different values of Kaniadakis Parameter

2 Anshid K,ME20B027

2.1 Reversible work done in Ideal gas systems

The main concern of the study of thermodynamics is the efficient extraction of useful work.Work can be extracted mainly in two ways,reversible and irreversible.A process,which can be retraced along the same path is called a reversible process.In this path,all the system properties are well defined and hence the work done ,W can be written as

$$W = \int_{V_1}^{V_2} PdV \tag{2}$$

Reversible work done [4]

where V_1 and V_2 are initial and final volumes and P,the instantaneous pressure of gas.

2.1.1 Isothermal expansion

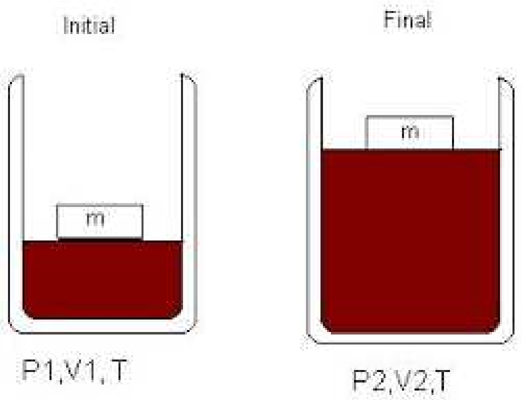

Figure 2: Reversible Isothermal Expansion [5]

The workdone in an isothermal process can be generalised from equation 2 as following

$$W = \int_{V_1}^{V_2} PdV$$

$$P = \frac{nRT}{V}$$

$$W = nRT \int_{V_1}^{V_2} \left(\frac{dV}{V}\right)$$
$$W = nRT \ln\left(\frac{V_2}{V_1}\right) \tag{3}$$

The figure 3 below,depicts the isothermal expansion of the ideal gas with area under the graph as the workdone

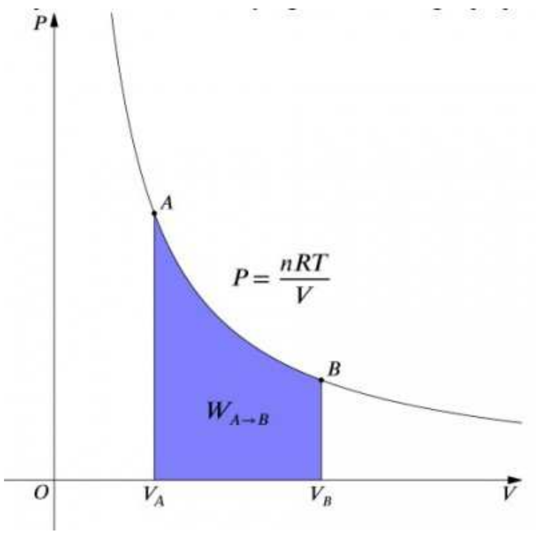

Figure 3: P-V diagram for isothermal expansion [3]

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3.1 Navier-Stokes Equations

Figure 4: Sir George Gabriel Stokes

3.2 Einstein summation convention

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} = 0 \tag{4}$$

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial[\rho u_i u_j]}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + \rho f_i \tag{5}$$

$$\frac{\partial(\rho e)}{\partial t} + (\rho e + p)\frac{\partial u_i}{\partial x_i} = \frac{\partial(\tau_{ij} u_j)}{\partial x_i} + \rho f_i u_i + \frac{\partial(\dot{q}_i)}{\partial x_i} + r \tag{6}$$

The Einstein summation convention dictates that: When a sub-indice (here i or j) is twice or more repeated in the same equation, one sums across the n-dimensions. This means, in the context of Navier-Stokes in 3 spacial dimensions, that one repeats the term 3 times, each time changing the indice for one representing the corresponding dimension (ie $1, 2, 3$ or x, y, z). Equation 1 is therefore a shorthand representation of: $\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_1)}{\partial x_1} + \frac{\partial(\rho u_2)}{\partial x_2} + \frac{\partial(\rho u_3)}{\partial x_3} = 0$. Equation 2 is actually a superposition of 3 separable equations which could be written in a 3-line form: one line equation for each i in each of which one sums the three terms for the j sub-indice.

3.3 Classic $\longrightarrow, \otimes, \nabla$ notation

$$\frac{\partial \rho}{\partial t} + \overrightarrow{\nabla} \cdot (\rho \overrightarrow{u}) = 0 \tag{7}$$

$$\frac{\partial(\rho \overrightarrow{u})}{\partial t} + \overrightarrow{\nabla} \cdot [\rho \overline{\overline{u \otimes u}}] = -\overrightarrow{\nabla p} + \overrightarrow{\nabla} \cdot \overline{\overline{\tau}} + \rho \overrightarrow{f} \tag{8}$$

$$\frac{\partial(\rho e)}{\partial t} + \overrightarrow{\nabla} \cdot ((\rho e + p) \overrightarrow{u}) = \overrightarrow{\nabla} \cdot (\overline{\overline{\tau}} \cdot \overrightarrow{u}) + \rho \overrightarrow{f} \overrightarrow{u} + \overrightarrow{\nabla} \cdot (\overrightarrow{q}) + r \tag{9}$$

Here $\otimes$ denotes the tensorial product, forming a tensor from the constituent vectors. A double bar denotes a tensor. The three equations $(4, 5, 6)$ are equivalent to $(1, 2, 3)$.

4 Shital

4.1 Special Relativity

4.1.1 introduction

Here we are going to introduce mass-energy equivalence relation.

In physics, mass–energy equivalence is the relationship between mass and energy as by equation 10 in a system's rest frame, where the two values differ only by a constant and the units of measurement.The principle is described by the physicist Albert Einstein's famous formula. [2]

$$E = mc^2 \tag{10}$$

The formula defines the energy E of a particle in its rest frame as the product of mass (m) with the speed of light squared (c^2). Because the speed of light is a large number in everyday units (approximately 3×10^8 meters per second), the formula implies that a small amount of rest mass corresponds to an enormous amount of energy, which is independent of the composition of the matter. Rest mass, also called invariant mass, is the mass that is measured when the system is at rest.

4.1.2 Spacetime

In physics, spacetime is any mathematical model which fuses the three dimensions of space and the one dimension of time into a single four-dimensional manifold. Spacetime diagrams can be used to visualize relativistic effects, such as why different observers perceive differently where and when events occur. In figure 5. [2]

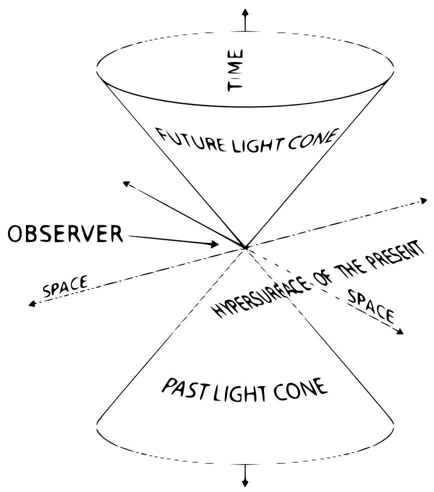

Figure 5: The world line:a diagrammatic representation of spacetime

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5.1 My favourite equation - Fourier series

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{2\pi nit} \tag{11}$$

$$c_n = \int e^{-2\pi nit} f(t)\ dt \tag{12}$$

5.1.1 Description

Fourier series (Equation-11) is a periodic function formed by weighted summation of harmincally related sinusoids. [6]

Any function can be written as fourier series by using appropriate weights. Theweights can be calculated by using Equation-12

Fourier series is an infinite series, but in practically it is impossible to compute a function of infinite length. Generally a finite number of terms(N) are considered, and the resolution of fourier series increases with increasing N, Figure-1 illustrates the above fact.

The computation of fourier series can be done by computers with the help sophisticated alogorithms. [1]

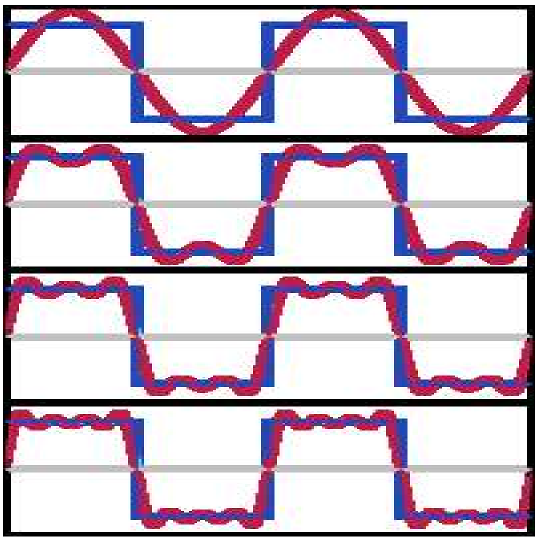

Figure 6: mm20b020 Resolution of Fourier series with N[8]

5.1.2 Importance

1. Signal Processing
2. JPEG compression
3. Solve partial differential equation
4. To seperate sounds of different frequency from microphone input

.... and the list goes on

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