1 Math

Bayes rule: $P(A|B) = \frac{P(A,B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$

$P(A,B|C) = \frac{P(A,B,C)}{P(C)}$

Cond indep: $P(A,B|C) = P(A|C)P(B|C)$

Chain rule: $P(A_n,\ldots,A_1) = \mathrm{P}(A_n|A_{n-1},\ldots,A_1)\cdot\mathrm{P}(A_{n-1},\ldots,A_1)$

Joint: $\mathrm{P}\left(\bigcap_{k=1}^n A_k\right) = \prod_{k=1}^n \mathrm{P}\left(A_k \,\middle|\, \bigcap_{j=1}^{k-1} A_j\right)$

Posterior: unobserved θ, observed x: $p(\theta|x) = \frac{p(x|\theta)p(\theta)}{p(x)}$

Prior: prior belief in dist. of θ: $p(\theta)$

Marginal: $\Pr(X=x) = \sum_y \Pr(X=x, Y=y) = \sum_y \Pr(X=x \mid Y=y)\Pr(Y=y)$

Likelihood: prob of observed given parameters: $p(x|\theta)$

MAP: $h_{MAP} = \arg\max_{h\in H} P(h|D) = \arg\max_{h\in H} P(D|h)P(h)$

Lagrange: $L(\mathbf{x},\lambda) = f(\mathbf{x}) + \lambda g(\mathbf{x})$ Set constraint $g(\mathbf{x})$ to zero and add multiplier.

$$\mathbf{xy^T} = \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} [x_1 \ldots x_n] = \begin{bmatrix} x_1y_1 & \ldots & x_1y_n \\ \vdots & \ddots & \vdots \\ x_my_1 & \ldots & x_my_n \end{bmatrix}$$

Matrix-vector product: $\mathbf{Ax} = \begin{bmatrix} a_1^T\mathbf{x} \\ \vdots \\ a_m^T\mathbf{x} \end{bmatrix}$

Matrix mult': $(\mathbf{AB})_{ij} = \sum_{k=1}^m A_{ik}B_{kj}$

$$\mathbf{AB} = \begin{bmatrix} a_1^Tb_1 & a_1^Tb_2 & \ldots & a_1^Tb_p \\ \vdots & \vdots & \ddots & \vdots \\ a_m^Tb_1 & a_m^Tb_2 & \ldots & a_m^Tb_n \end{bmatrix}$$

$(n\times p)(p\times m) = n\times p$

2 todo

- Eigendecomposition
- $\sin(x)' = \cos x$
- $\cos(x)' = -\sin x$
-
- $(AB)^T = B^TA$
- $\log_b(x^y) = y\log_b(x)$
- Uniform distribution: $P_{\theta_1,\theta_2}(x) = \frac{1}{\theta_2-\theta_1}$
- $AA^{-1} = A^{-1}A = I$ for square matrices.
- $F'(x) = f'(g(x))g'(x)$
- $(f\cdot g)' = f'\cdot g + f\cdot g'$
- Log properties
- Convexity
- $||\mathbf{x}||_d = \sum_i x_i^d$
- Polynomial multiplication
- Matrix transpose and inverse tproperties
- Gradient vs partial derivative
- $\sigma(x)' = \sigma(x)(1-\sigma(x))$
- $\mathrm{E}\left[\left(y-\hat{f}(x)\right)^2\right] = \mathrm{E}\left[\hat{f}(x) - f(x)\right]^2 + \mathrm{E}[\hat{f}(x)^2] - \mathrm{E}[\hat{f}(x)]^2 + \epsilon^2$

3 Regression

3.1 L2 regularization

Weights do not reach zero. Faster.

$$J_w = \frac{1}{2}(\Phi\mathbf{w}-\mathbf{y})^T(\Phi\mathbf{w}-\mathbf{y}) + \frac{\lambda}{2}\mathbf{w}^T\mathbf{w}$$

$$\mathbf{w} = (\Phi^T\Phi + \lambda\mathbf{I})^{-1}\Phi^Ty$$

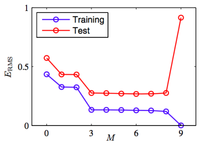
3.2 L1 regularization

Some weights set to zero. More expensive.

3.3 Gradient Descent

$\mathbf{w} \leftarrow \mathbf{w} - \alpha\nabla\log L(\mathbf{w})$ $\mathbf{w} \leftarrow \mathbf{w} - \alpha\nabla\log J(\mathbf{w})$

3.4 Bayesian regularization



4 Kernels

$k(\mathbf{x},\mathbf{z}) = \phi(\mathbf{x})\cdot\phi(\mathbf{z})$ **Mercer theorem:** $K(\mathbf{x},\mathbf{z})$ is kernel iff Gram matrix $\mathbf{K}$ symmetric and positive semidefinite: $\mathbf{K_{ij}} = \mathbf{K_{ji}}$ and $\mathbf{z}^T\mathbf{Kz} \geq 0$ $\mathbf{K} = \Phi\Phi^T$ where $\mathbf{K_{nm}} = \phi(\mathbf{x_n})^T\phi(\mathbf{x_m}) = k(\mathbf{x_n},\mathbf{x_m})$ Gram matrix size of input.

- Kernel properties
- Proving kernels

5 SVM

Minimize absolute error. More robust to outliers. Max margin is convex optimization.

$$h_{\mathbf{w}}(\mathbf{x}) = \mathrm{sign}(\sum_{i=1}^m \alpha_i y_i(\mathbf{x_i}\mathbf{x}) + \mathbf{w_0})$$

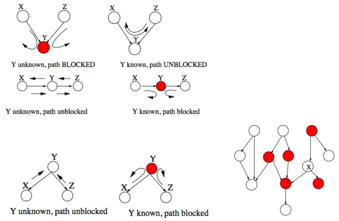
$\alpha_i > 0$ only for support vectors. Soft error SVM: $0 < \zeta \leq 1$ if inside margin. $\zeta > 1$ if misclassified. Total errors: $C\sum_i\zeta$. Large C higher variance.

6 EM/Active Learning/Missing Data

When missing data: many local maxima (normally likelihood has unique max), no closed form solutions. So we do gradient ascent or EM. $\log L(\theta) = \sum_{\text{complete data}} \log P(\mathbf{x_i}, y_i|\theta) + \sum_{\text{incomplete data}} \log\sum_y (\mathbf{x_i}|\theta)$. **E step:** compute expected assignment (hard or soft) of points to distributions (estimate $p(y_i = k|\theta)$). **M step:** recompute parameters to maximize likelihood of current assignments $p(\theta|y_i)$. Good for low dimensionality data.

7 Bayes Nets

$p(x_1,...,x_n) = \prod_{i-1}^n p(x_i|x_{\pi_i})$



Markov blanket: parents, children, spouses.
Moral graph: graph U: edge $(X,Y)\in U$ if X in Y's markov blanket.
Belief propagation: This is exact inference $m_{ji} = \sum_{x_j}\left(\psi^E(x_j)\psi(x_i,x_j)\prod_{k\in\mathrm{nghbr}(x_j)} m_{kj}(x_j)\right)$

$p(y|\hat{x}_E) \propto \psi^E(y)\prod_{k\in\mathrm{nghbr}(Y)} m_{ky}(y)$

Gibbs Sampling (approximate inference): 1. set evidence nodes $E = e$, all others random. 2. Sample x_i' from $P(X_i|x_1,..,x_n)$ i.e. markov blanket. 3. Obtain new $x_1'..x_n'$. Converges to true steady state distribution using makov chain properties.
Learning: likelihood of whole graph decomposes to likelihood over each node's parameter $L(\theta|D) = \prod_{i\in\text{nodes}}^n L(\theta_i|D)$. Use EM.

8 Markov Chains

Markov property: $P(s_{t+1}|s_t) = P(s_{t+1}|s_0,..s_t)$
$P(s_{t+1=s'}) = \sum_s P(s_0=s)P(s_1=s'|s_0=s)$
in matrix form: $\mathbf{p_t} = vekT^T\mathbf{p_{t-1}} = (\mathbf{T}^T)^t\mathbf{p_0}$

9 Hidden Markov Models

Parameters: states, observations: $\mathcal{S},\mathcal{O}$, $\mathbf{b_0}\in|\mathcal{S}|$, transition probs $\mathbf{T}\in|\mathcal{S}|\times|\mathcal{S}|$, emission probs $\mathbf{Q}\in|\mathcal{S}|\times|\mathcal{O}|$