Relative value tools for FX volatilities

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In this report, we summarise a few tools which our Cross-Asset Quant research team has already used when proposing tactical optional trade ideas. These tools are relevant for highlighting relative value opportunities in the FX volatility space.

We introduce the use of the SABR stochastic volatility model as a tool to interpolate the FX volatility smile: dislocations can be identified on the fitted parameters of the model or when the implied distribution deviates markedly from the historical one.

Measuring volatilities out of the spot market by using a heteroscedastic model such as the EWMA gives a benchmark for front-end gamma volatilities.

We apply the Principal Component Analysis (PCA) model as a relative value tool to investigate dislocations on the vega-segment of volatility surfaces, basket of ATM vols of fixed maturity and baskets of fixed maturity and delta risk reversals.

Screening the FX volatility space in detail allows us to identify the best opportunities for taking on directional risk, when looking for tactical optional trades or the cheapest ways for hedging a portfolio via FX options.

A collection of FX vol screeners, based on the models presented in this paper, is available on the FX vol section of the SG Quant Research website.

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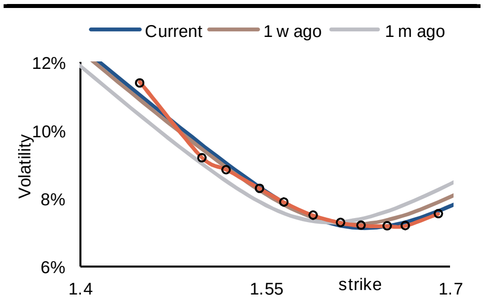
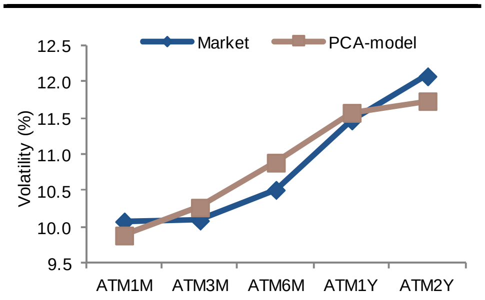
SABR-calibration of the GBP/USD 3M smile



Source: SG Cross Asset Research

PCA-based volatility curve for EUR/USD



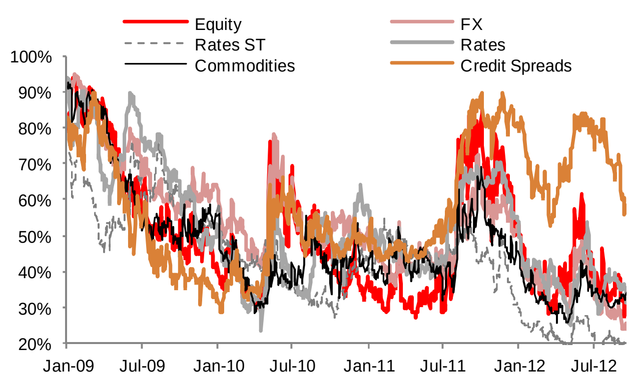
Source: SG Cross Asset Research

Executive summary

The relevance of the [FX market as an independent asset]() class has grown considerably in the past decade, while remaining an essential tool for trading goods and for investing in foreign currency-denominated capital markets. Within the FX market, the proportion of trades operated via derivatives products has also increased, when compared with the delta-one transactions. Furthermore, given that FX volatilities tend to be cheaper than those in equities and other asset classes, they are often used as proxies for other markets for implementing tactical or hedge trades. The number of macro hedge funds implementing long-term trades via cross-asset volatilities has grown in the past few years, which makes the FX vol asset class interesting for fund managers who manage not only FX but cross-asset portfolios.

All the considerations make the screening of FX vols an interesting topic, both within the FX market itself and at the wider cross-asset level. Our Cross-Asset Quant research team has developed a [cross-asset volatility screener]() to monitor the relative cheapness/expensiveness of volatilities across different asset classes (see charts below).

Normalised cross-asset volatilities…



Source: SG Cross Asset Research

…and cross-asset dislocations in the vol space

LONG-TERM FAIR VALUE

	Implied	Realised	Projection	Spread to: YTD	zscore
EuroStoxx 50	24.12	25.29	19.17	-14.8	1.33
S&P 500	16.26	15.50	14.92	-21.2	1.03
FTSE 100	16.58	13.92	14.70	-19.6	1.26
EURUSD	9.58	11.85	9.75	-13.7	-0.25
GBPUSD	7.22	7.89	8.30	-14	-1.31
USDJPY	7.28	6.79	9.16	-10.7	-1.34
AUDUSD	10.35	10.23	10.52	-16.2	-0.29
NZDUSD	10.73	10.19	11.99	-13.3	-1.34
EUR 2Y	43.20	42.38	54.55	-106	-1.11
EUR 10Y	85.37	82.87	71.20	-76.5	1.36
USD 2Y	26.97	24.80	54.46	-102	-1.21
USD 10Y	77.20	77.63	90.19	-117	-1.12
WTI	31.22	33.64	27.52	-61.2	1.16
Gold	17.00	18.54	15.60	-24.6	0.88
Wheat	30.67	25.47	33.19	-16.6	-0.77

In this report, we present a number of relative value tools we have developed specifically for the FX space, at the bottom-up level. In our previous articles on FX volatilities, we mostly looked at defining general rules for generating alpha, especially regarding the possibility of locking in the risk premium embedded in implied volatilities. In this paper, we face the more general task of screening the FX volatility space, looking for dislocations. Such signals are then to be interpreted via a discretionary, rather than a systematic, process: in other words, we suggest adding a human-brain dimension to discern whether a major dislocation also represents a trading opportunity or is it just indicative of a changing macro environment.

We do not provide a detailed description of the models in this report but rather describe how well-established techniques can be used for concrete applications for the detection of relative value signals in the volatility space: clients can contact the cross-asset quant team for more information on the implementation of the models.

This report, used in tandem with the cross-asset volatility screener mentioned above, could be informative to the portfolio or risk managers screening for opportunities to be implemented via FX volatilities.

Relative value tools for FX volatilities

In the following section, we present a number of tools that could be useful in highlighting opportunities for trading/hedging on a discretionary basis, rather than fully-automated trading systems. The first section introduces the use of a stochastic volatility model with the goal of interpolating the volatility smile and allowing a comparison of implied and historical probability distributions for different currencies. The second section details the use of heteroscedastic models for measuring volatilities out of the spot markets and providing benchmarks for the gamma segment of the implied volatility curves. The three following sections use the PCA model for highlighting dislocations on a basket of ATM vols and of (fixed maturity and deltas) risk reversals, and on volatility surfaces for different currencies: these latest tools are mostly useful on the vega segments of the respective volatility curves.

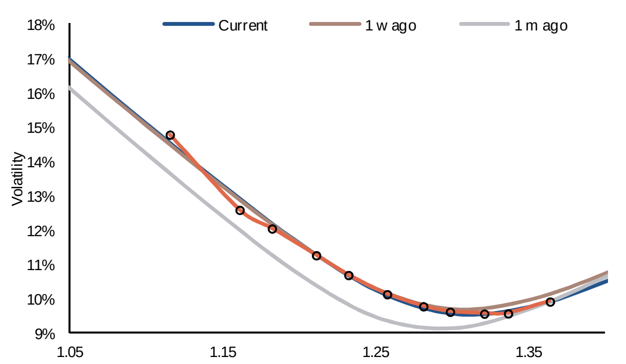
SABR calibration of the vol smile and probability distributions

The SABR model is a stochastic volatility model which allows for a simple calibration of FX volatility smiles. Different maturities of the surface are to be calibrated independently. These are the main equations describing the model:

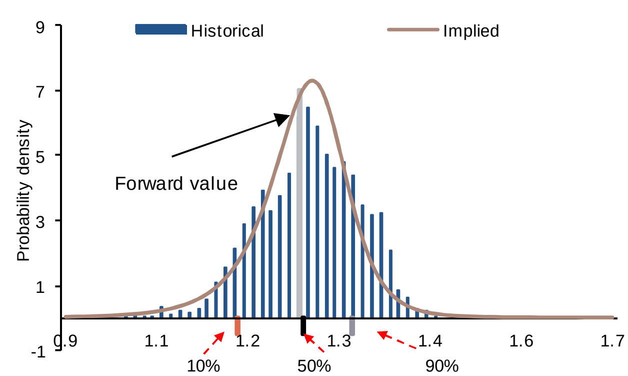
$$dS_t = \alpha\, S_t\, dW_t^1 \quad d\alpha_t = \nu\, \alpha_t\, dW_t^2 \quad dW_t^1\, dW_t^2 = \rho\, dt$$

Volatility itself is a random variable, affected by a random term correlated to that driving the spot market itself. We don't enter into technicalities in this paper, but present only the main applications of this model. The lower-left chart displays the interpolated EUR/USD 3M smile (we perform the calibration on 11 points of the smile for each maturity): by using only three parameters, the model can fit nicely a typical FX smile. The model can be used to compute the Greeks for an options strategy: given that it takes into account the sensitivity of volatility to moves in the spot, the delta of the model goes one step beyond the plain Black-Scholes delta.

Calibrated FX smile on EUR/USD 3M



Implied/historical distributions for EUR/USD 3M

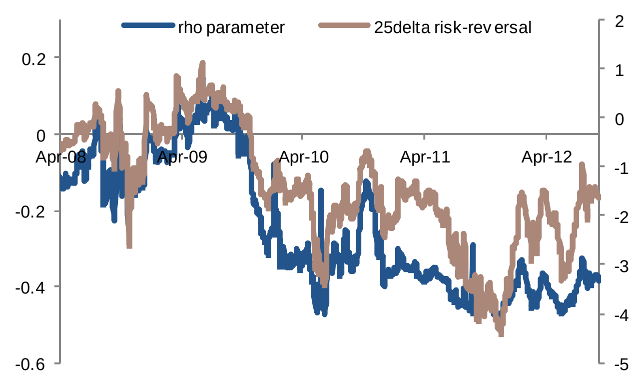


Source: SG Cross Asset Research

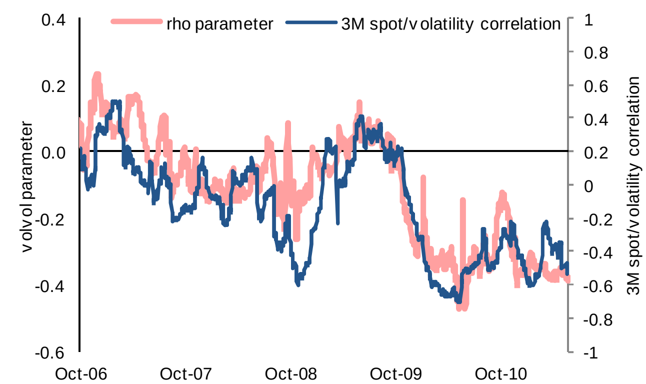
The implied probability distribution as obtained from the options space can be computed from the calibrated smile and compared with the historical distribution: this is shown in the right-hand chart above. Third (skewness) and fourth (kurtosis) moments of the implied and historical distributions can also be compared in order to see whether unlikely (far from the money) events are fairly priced or not. The parameters of the model bear close similarities with market variables. The ρ parameter, which in the model represents the correlation between the

Brownian motions for the spot and the volatility, follows closely the dynamics of market risk-reversals. Within the SABR framework, the parameter is also responsible for driving the skew of the distribution, which allows relating the value of riskies to the asymmetry of the probability distribution. Given that in the model the parameter represents the correlation between spot and volatility, we can see in the second graph that this relationship is well represented when computing the historical correlation between spot and volatility.

SABR ρ parameter and 25delta risk-reversal (EUR/USD 3M)



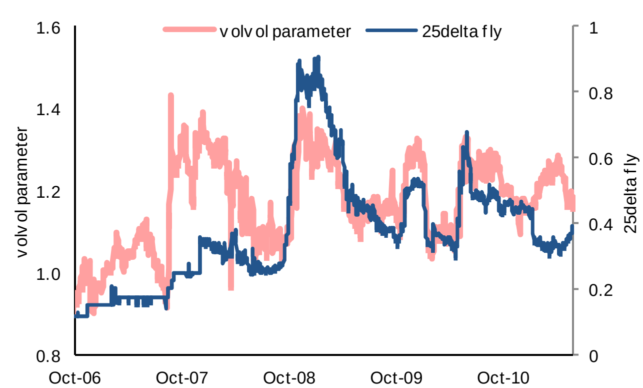
SABR ρ parameter and historical spot/vol correlation



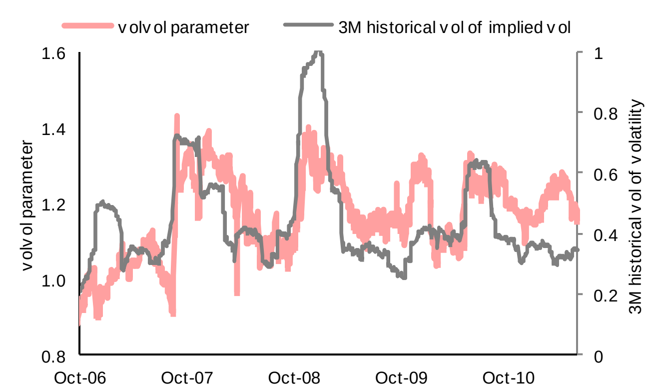
Source: SG Cross Asset Research

The ν parameter, which in the model plays the role of the volatility of the volatility (it is otherwise known as the volvol parameter), is closely related to the market dynamics of butterflies, as shown in the first chart below. In the model, the parameter is related to the kurtosis of the distribution, namely the measure of how fat the tails of the distribution are if compared to a Gaussian one. In the model, the parameter plays the role of the volatility of the volatility, which allows comparing directly (as we do in the second chart below) the parameter and the historical volatility of the implied volatility.

SABR ν parameter and butterflies (EUR/USD 3M)



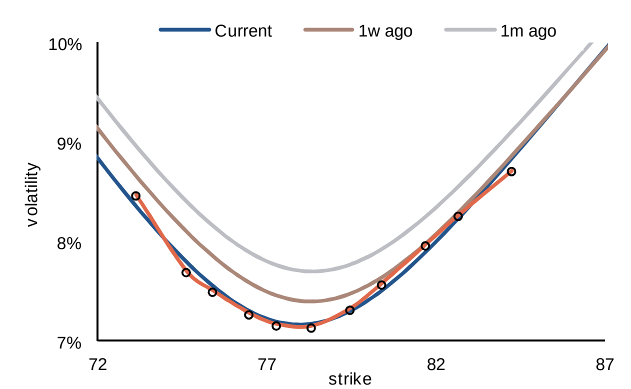
SABR ν parameter and historical volatility of volatility



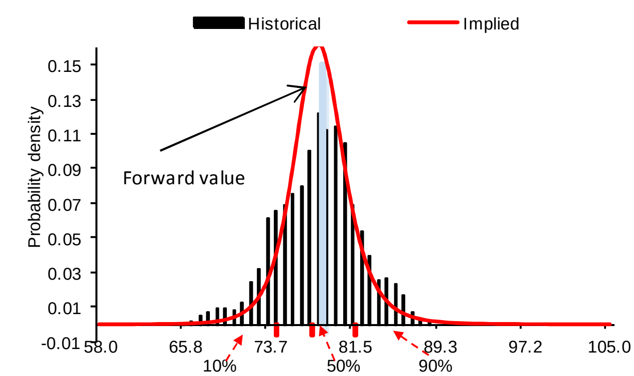
Source: SG Cross Asset Research

Therefore, drawing historical time series for the model parameters can be helpful for highlighting dislocations for the market variables which are closely linked to such parameters. The SABR calculation, so far described in the case of EUR/USD 3M only, can be applied to other FX smiles, as the two charts below show, and maturities. In the first chart, we show the interpolated smile and implied/historical 3M distributions for the USD/JPY.

Calibrated FX smile USD/JPY 3M



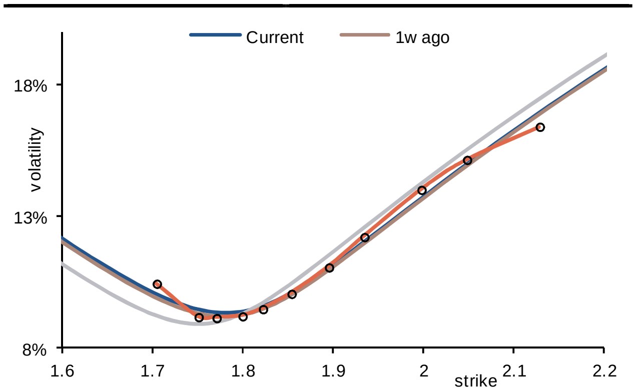
USD/JPY 3M probability distribution



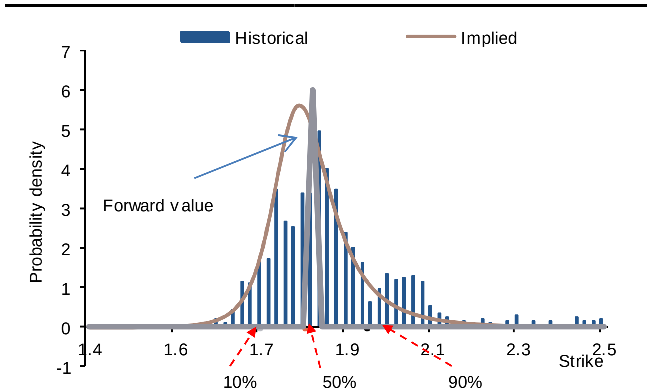
Source: SG Cross Asset Research

The next two charts display interpolated smile and implied/historical 3M distributions for the USD/TRY. As shown, the interpolation works well for both skewed and rather symmetric smiles, which gives the model a wide range of applicability (although the applicability of the model breaks down for very short maturities and deep OTM vols).

Calibrated FX smile USD/TRY 3M



USD/TRY 3M probability distribution



Source: SG Cross Asset Research

The table below displays the third and fourth moment of the implied distribution plus 10%/30% symmetric quantiles, for the EUR/USD, GBP/USD, USD/CHF and USD/JPY crosses.

Quantiles and third, fourth moments of the implied distributions

	Skewness	Kurtosis	Quantile(10%)	Quantile(30%)	Quantile(50%)	Quantile(70%)	Quantile(90%)
EUR/USD	-0.68	3.42	1.18	1.23	1.26	1.29	1.33
GBP/USD	-0.71	4.68	1.50	1.56	1.59	1.61	1.65
USD/JPY	0.41	3.68	74.90	77.03	78.36	79.73	82.09
USD/CHF	0.80	6.66	0.89	0.93	0.95	0.97	1.02

Source: SG Cross Asset Research

The model can be used as a tool for highlighting dislocations when the implied distribution deviates markedly from the historical one, or when a parameter of the model dislocates from its past historical time series.

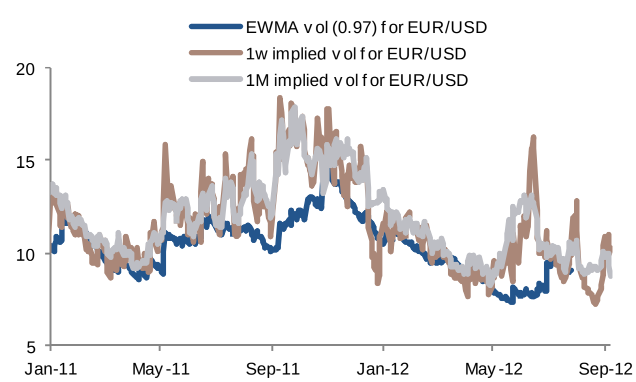
EWMA calculation for volatilities and correlations

In our latest article on FX vol, we described a systematic approach to benefit from the risk premia embedded in FX volatilities, by using a heteroscedastic model (EWMA) to provide a benchmark for gamma volatilities. Here, in the general spirit of this article, we only present how the EWMA can be used as a signal-generating tool. The following is the EWMA formula for calculating volatilities out of the spot market

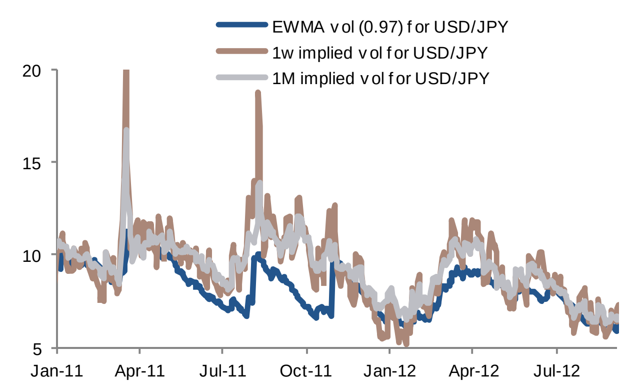
$$\sigma_t^2 = \alpha * (\sigma_{t-1}^2) + (1-\alpha) * (u_t^2)$$

where u_t^2 represents the squared log return of the underlying spot variable. At any step, the new update of the volatility can be obtained as an average of the previous update and of the latest market move. By working around the math, one can show that the impact of past shocks on the latest update of vol is exponentially suppressed, with a rate equal to $(1-\alpha)$. If compared with a plain moving average, the two calculations provide the same amount of smoothing if $(1-\alpha) = 2/(N+1)$, where N is the parameter to be used in the simple moving average calculation, but the EWMA gives rise to less lag for the same amount of smoothing. In the next two charts, where we compare gamma and EWMA volatilities for EUR/USD and USD/JPY, we use $\alpha = 0.97$, which is roughly equivalent to an average over 2-3 months.

1w/1M implied and EWMA(0.97) vols for EUR/USD



1w/1M implied and EWMA(0.97) vols for USD/JPY



Source: SG Cross Asset Research

The table below compares gamma volatilities with EWMA (0.97) vols for G10 crosses.

G10 EWMA (0.97) and gamma volatilities (1w to 3M)

FX	EWMA(0.97)	1w vol	1M vol	3M vol
EUR/USD	9.3	9.2	8.4	8.9
GBP/USD	6.4	6.0	6.1	6.8
USD/JPY	6.5	7.1	6.3	7.0
USD/CHF	9.1	9.5	8.8	9.3
AUD/USD	9.3	8.9	8.6	9.7
NZD/USD	9.9	9.1	8.8	10.0
USD/CAD	6.6	6.9	6.1	6.6
USD/NOK	9.3	9.0	8.7	9.6
USD/SEK	10.7	9.4	9.2	9.9

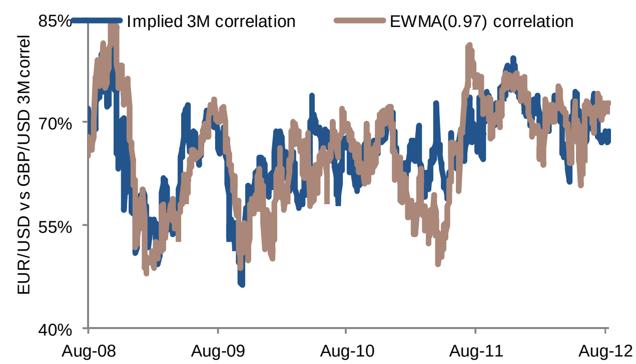
Source: SG Cross Asset Research

The table above can be used as a benchmark, rather than as a systematic tool, for screening opportunities in the gamma space. A similar approach can be applied for measuring covariance matrices. The off diagonal covariance can be calculated as

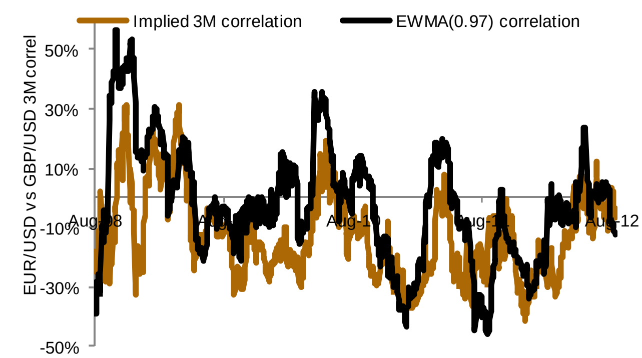
$$C_{t;1,2}^{2} = \alpha * \left(C_{t-1;1,2}^{2}\right) + (1-\alpha) * u_t * v_t$$

Where u_t , v_t represent the latest log-returns for the two assets. The correlation is obtained by rescaling the EWMA-covariance by the two corresponding EWMA-vols, as described above (we use the same α parameter when computing volatilities and covariance matrices). Implied correlations, on the other hand, can be calculated easily by using the three implied volatilities (for the two currencies whose correlation one wants to measure and for their cross). The two following graphs display 3M implied and EWMA-based correlations for EUR/USD vs GBP/USD and GBP/USD vs USD/JPY.

Implied vs EWMA(0.97) 3M correlation for EUR/USD vs GBP/USD



Implied vs EWMA(0.97) 3M correlation for GBP/USD vs USD/JPY



Source: SG Cross Asset Research

We introduced the use of EWMA correlations in our latest paper on Adaptive sentiment indicators as a way to measure the explanatory power of risk-related variables on different asset classes. In our 2009 paper "Volatility and correlation arbitrage strategies for the FX market" we had defined rules for replicating a covariance swap via plain vanillas. In the paper, we had highlighted how opportunities could be identified by comparing implied and historical correlation for entering long/short correlation positions: the same findings could be applied by replacing historical correlations with EWMA-correlations, while obtaining the advantages of the latter methodology (smoother estimate for an equal amount of lag).

PCA on baskets of ATM volatilities

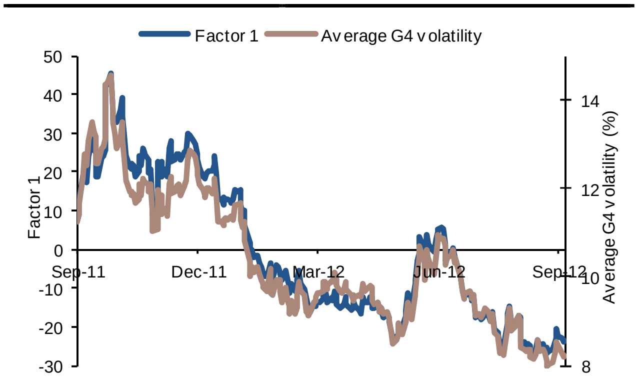
In the three final sections of this paper, we discuss how to use the PCA model for highlighting relative value opportunities in the volatility space. We start with a quick review of the approach applied to a basket of ATM volatilities of fixed maturities.

PCA is a statistical technique widely applied to financial markets: its main advantage is the applicability to a wide range of context, including spot and volatility variables on different asset classes. In broad terms, PCA performs a diagonalisation of the covariance (or correlation) matrix of the underlying system one wants to investigate, and returns as an output the main drivers of variance which can be interpreted as collective, and not idiosyncratic, phenomena. The risk-axes identified by PCA, obtained as linear combinations of the original variables, are by construction decorrelated from each other and can be sorted by the proportion of variance (or correlation) they explain. This allows for the introduction of a cut-off in the number of factors one retains: the high-variance factors, which are kept, are assumed to represent information and the real dynamics in the underlying covariance structure, whereas the low-variance ones, which are discarded, are assumed to be compatible with noise. In practical applications, there will not be such a clear-cut distinction between information and noise as assumed in the approach, but this can work as a first step to highlight dislocations and relative value opportunities.

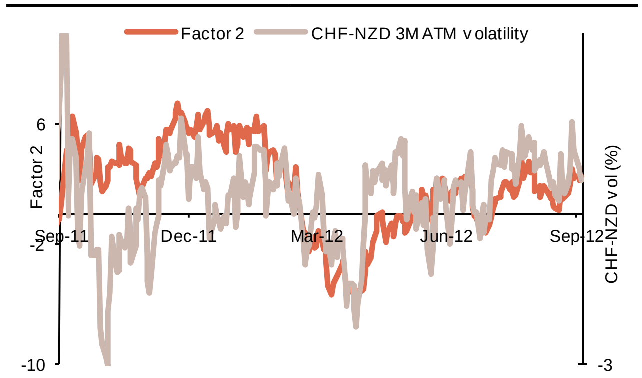
We use a concrete example for describing the model. We start by considering baskets of G10 ATM volatilities, all against the USD with the same maturity. We run the PCA algorithm on the correlation matrix of the daily changes of such variables, by using two years of data. With such settings, the model exhibits two main drivers of variance in the implied volatility space which, for the 3M maturity, are displayed in the two charts below. In parenthesis, the proportion of variance explained by each factor is displayed: the two factors together explain 81% of the total variance of the underlying basket. The time series below are plotted as the integrated time series of the difference PCA factors (given that we run the algorithm on the time series of the differences).

As shown in the first chart, the first factor can be easily understood as an average level of (G4/USD) volatilities, and is therefore responsible for joint movements of the volatility space up or down, on the back of shifts, for instance, in market sentiment: when the sentiment deteriorates, all volatilities tend to rise jointly, no matter what the specific driver has been behind sentiment's drop (and similarly when sentiment rises). So the factor acts as a long-only factor, to which all the underlying variables have roughly the same exposure to, modulo the different beta of each of the variables to this axis of risk.

Factor I (71.6%) and average value of G4 volatilities



Factor II (9.7%) and (USD/CHF-NZD/USD) volatility differential

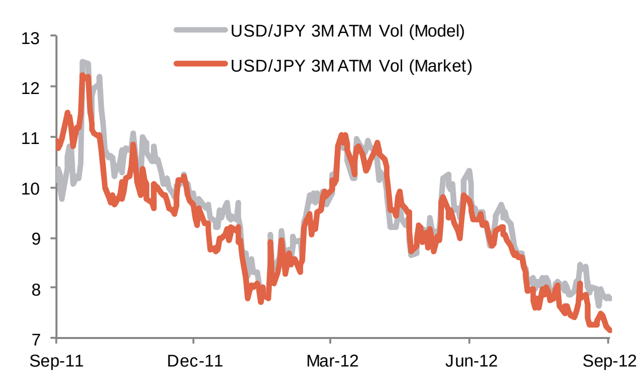


Source: SG Cross Asset Research

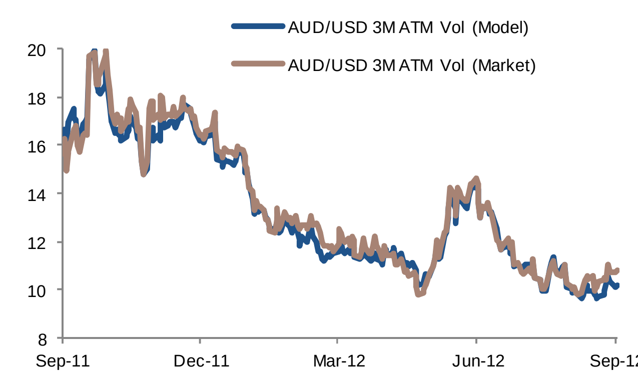
In most PCA models, higher-order factors play as relative value factors, where different segments of the whole market are compared to each other. In this specific case, what we find is that the second factor plays a relative rise of European volatilities against non-European volatilities, and as a proxy of this effect, we compare the factor with the USD/CHF, NZD/USD volatility differential, which shows a good match with the factor.

Once the factors are interpreted, the model allows for a simple calculation of fair values, for each of the variables composing the basket considered, by performing a multidimensional regression, using the factors as predictors. The result of the regression will highlight the sensitivities, or betas, of each variable against the interpreted factors, or drivers of variance. What we find is that, high-beta currencies in the spot market (like AUD, NZD) tend to have a high-beta in the volatility space against the benchmark level of volatility (first factor of the PCA). The two charts below display the time series of market and model values for the two most dislocated 3M volatilities, as of 5 September: USD/JPY and AUD/USD.

USD/JPY 3M ATM volatility (market and model values)



AUD/USD 3M ATM volatility (market and model values)



Source: SG Cross Asset Research

The table below summarizes the results collected for the PCA model applied on ATM volatilities with maturities ranging from 1M to 1Y (other maturities can be studied as well). Dislocations (i.e. deviations between model and market value by more than 1.5 standard deviations, either way) are highlighted in brown (market variable is rich) and blue (market variable is cheap). As we write, AUD vols tend to be expensive and JPY, SEK vols tend to be cheap: however, dislocations are not huge, as z-scores vary between -1.9 and 1.6.

PCA on ATM FX volatilities (vs the USD) for 1M to 1Y maturities

	1M			3M			6M			1Y		
Currency	Market	Model	z-score	Market	Model	z-score	Market	Model	z-score	Market	Model	z-score
EUR	10.1	10.1	-0.2	10.1	10.2	-0.4	10.5	10.6	0.0	11.5	11.5	0.0
JPY	6.5	7.1	-1.1	7.2	7.8	-1.4	8.0	8.8	-1.7	9.4	10.1	-1.5
GBP	7.1	7.3	-0.3	7.5	7.5	0.0	8.1	8.0	0.3	9.1	8.9	0.2
CHF	10.2	9.4	0.6	10.3	10.0	0.3	10.8	10.7	0.2	11.8	11.9	-0.1
AUD	10.3	9.5	1.3	10.8	10.2	1.6	11.5	11.0	1.5	12.7	12.3	1.4
NZD	10.5	9.9	0.9	11.0	10.6	1.0	11.8	11.5	1.0	13.0	12.7	0.8
CAD	6.8	6.8	0.0	7.1	6.9	0.4	7.6	7.3	0.8	8.5	8.2	0.7
NOK	10.3	10.9	-1.3	10.8	11.2	-1.3	11.5	11.9	-1.3	12.7	13.1	-1.3
SEK	10.9	11.1	-0.9	11.1	11.3	-1.1	11.6	12.0	-1.9	12.8	13.1	-1.6

Source: SG Cross Asset Research

Here, given that each maturity is investigated independently, we focus on relative-value opportunities for different currencies, at each given maturity, rather than by looking at the best opportunity along the vol curve for each currency. The interplay between volatility levels, skew and term-structure effects will be investigated in the final section of this paper, where the PCA model will be applied to the volatility surfaces for each currency pair.

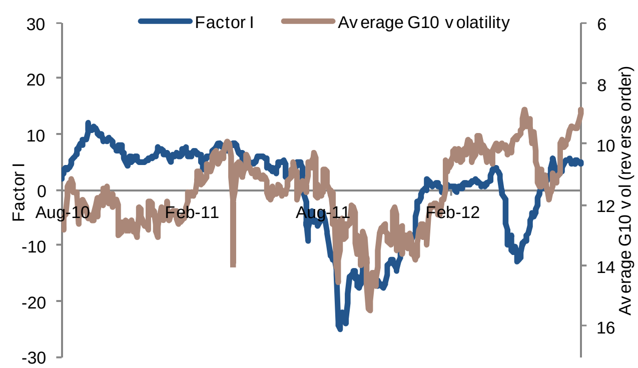
PCA on baskets of FX risk-reversals

As discussed on the earlier section about the SABR model, risk-reversals are intimately related to both the skewness of the distribution of a currency's returns and, alternatively, to the spot/volatility correlation of that currency. Fair value models for each currency's risk-reversal can be devised by comparing implied values with the historical time series of either spot/vol correlation or skewness. Here, we pursue a different approach. Rather than comparing implied risk-reversals with historical measures for each currency, we establish a relative-value approach in a basket of risk-reversals on different currencies.

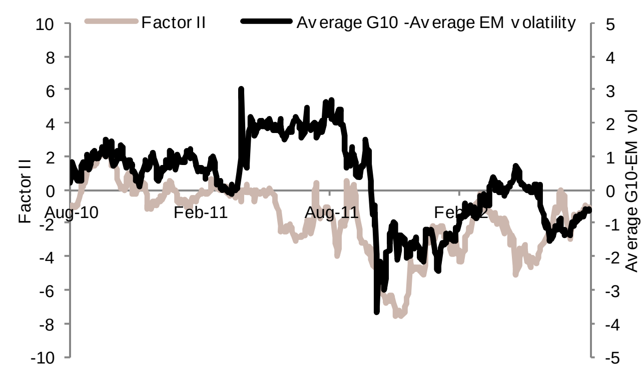
Compared to the previous section, here we apply the PCA technique to baskets of 25 delta risk-reversals, with maturities ranging from 1m to 1y: the baskets comprise G10 and the most liquid risk-reversals in the EM space. Given that EM vol variables tend to be less liquid than G10 ones, we apply the technique as described in the previous section on weekly, rather than daily, changes setting, and by using three years of data to increase the size of the sample.

Regarding the interpretation of the factors, the first one (which explains 43.1% of the total variance for 3M maturity) follows the dynamics of average G10 volatility levels, whereas the second (which explains 9.4% of the variance for the 3M basket), playing the relative increase of G10 against EM riskies, is sensitive to relative shifts in G10 vs EM average volatilities.

Factor I (43.1% of variance for the 3M basket) is linked to average vol levels



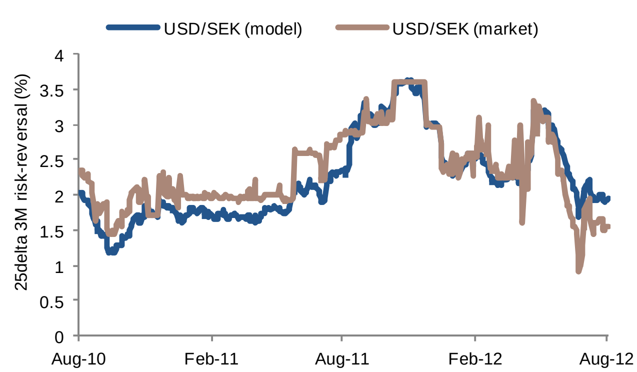
Factor II (9.4% of the variance for the 3M basket) is linked to G10-EM vol differentials



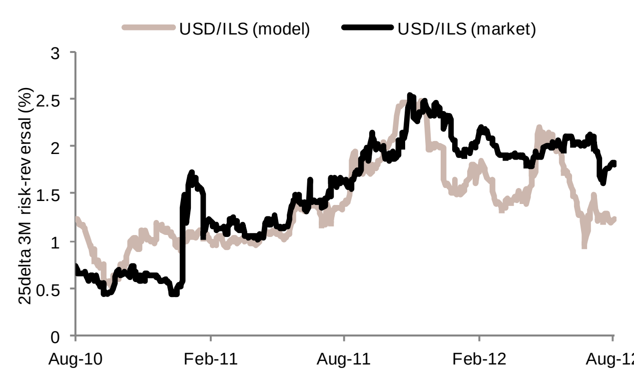
Source: SG Cross Asset Research

By following the same steps as detailed in the previous section, once the factors are selected, a multi-linear regression can lead to an estimate of fair values for the risk-reversals.

Risk-reversals on USD/SEK are cheap…



…whereas on USD/ILS are expensive



Source: SG Cross Asset Research

As we write, several skews, especially for USD/SEK, USD/NOK and NZD/USD, are cheap based on the calculation above, whereas some in the EM space, especially for USD/ZAR, USD/ILS and USD/TWD, are expensive. Given that there is no options market for some liquid EM crosses (like for instance BRL/MXN), optional trades on some EM crosses have to be expressed as long/short trades for the corresponding USD legs, which is another reason why the screener above can be useful if applied to the EM space.

The table below summarizes the main results for the 25delta risk-reversals of different maturities (other maturities and delta can be investigated as well). It can be used as a benchmark for highlighting, from a purely discretionary point of view, the best opportunities across different currencies for taking a position on OTM options. It can be used jointly with a screener developed by the team for highlighting OTM options opportunities across different asset classes, including FX, by making use of Extreme Value Theory.

PCA on 25delta risk-reversals (vs the USD) for 1M to 1Y maturities

Risk-reversal	1M			3M			6M			1Y		
(3M, 25delta)	Market	Model	z-score	Market	Model	z-score	Market	Model	z-score	Market	Model	z-score
EUR	-0.3	-0.4	0.1	-0.8	-1.1	0.8	-1.4	-1.7	0.9	-1.7	-2.1	0.9
JPY	0.7	-0.5	1.6	0.5	-1.2	2.1	0.3	-1.2	1.9	0.1	-1.8	2.1
GBP	-0.2	-0.5	0.7	-1.0	-1.3	0.7	-1.2	-1.7	1.1	-1.4	-2.0	1.4
CHF	0.6	-0.5	1.5	0.8	-0.3	1.7	0.9	0.1	1.3	1.0	0.1	1.3
AUD	-1.0	-0.8	-0.4	-1.9	-2.1	0.5	-2.5	-2.9	1.2	-2.8	-3.7	2.2
NZD	-0.8	-0.8	0.1	-1.8	-2.2	0.9	-2.3	-2.9	1.7	-2.6	-3.7	2.4
CAD	0.5	0.3	0.8	1.1	0.7	1.3	1.6	1.2	1.2	1.9	1.5	1.3
NOK	0.3	0.7	-1.1	0.8	1.3	-1.8	1.3	1.8	-1.4	1.7	2.1	-1.1
SEK	0.3	0.9	-1.7	0.8	1.5	-2.3	1.3	2.0	-2.4	1.7	2.2	-1.5
INR	0.1	0.8	-1.6	0.8	1.5	-1.4	1.6	2.3	-1.2	2.7	3.1	-0.8
BRL	1.5	2.4	-1.0	3.0	4.0	-1.3	4.1	4.9	-1.2	5.0	5.8	-1.1
CZK	1.1	1.1	-0.1	2.3	1.8	0.8	3.2	2.5	1.2	4.1	3.0	1.8
MXN	2.2	1.6	1.1	3.0	3.1	-0.4	4.0	3.9	0.3	4.7	4.7	0.1
ILS	1.1	0.7	1.1	1.8	0.9	2.4	2.0	1.2	2.2	2.1	1.3	2.3
ZAR	2.2	1.9	0.5	3.6	3.2	0.8	4.8	4.2	1.1	6.0	5.1	1.3
IDR	1.8	1.6	0.3	3.6	2.7	1.1	5.4	4.1	1.4	6.8	5.8	1.2
CLP	2.1	1.8	0.5	2.7	2.7	0.0	3.3	3.2	0.3	4.0	3.9	0.2
PLN	1.8	2.0	-0.7	3.2	3.1	0.1	4.1	4.0	0.4	5.0	4.7	0.7
SGD	0.6	0.4	0.7	1.1	0.8	0.9	1.5	1.3	0.6	2.0	1.9	0.3
HUF	1.8	1.9	-0.1	3.2	3.0	0.4	4.2	4.1	0.2	5.4	5.1	0.4
TWD	0.3	-0.4	1.6	0.6	-0.4	2.2	0.8	-0.2	2.3	1.2	0.1	2.4
MYR	0.7	0.7	-0.1	1.3	1.3	0.1	2.2	1.9	0.6	3.0	2.6	0.7
TRY	1.3	1.4	-0.2	2.6	2.7	-0.2	3.6	3.7	-0.2	4.6	4.5	0.2
KRW	0.6	1.8	-1.2	1.6	2.8	-1.5	2.6	3.6	-1.4	3.6	4.6	-1.5

Source: SG Cross Asset Research

PCA on FX Volatility surfaces

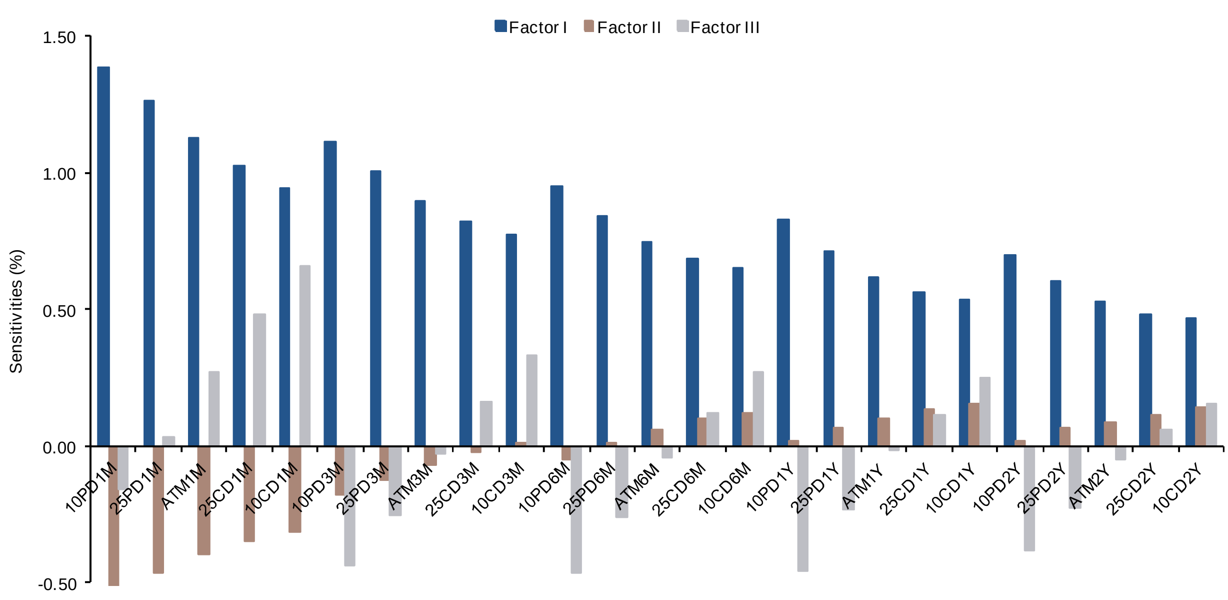
In this section, we apply the PCA model on each individual FX volatility surface. The use of PCA on curves/surfaces has been popular for several years, especially in the rates space: the empirical Nelson-Siegel model can in fact be reproduced by a more fundamental PCA. The reason why PCA models are applied to curves and surfaces so widely is because the latter by necessity exhibit a constrained dynamics, in the sense that different points on the curves/surfaces tend to move in line with each other, according to some predetermined patterns (which in some cases can be justified by means of no-arbitrage arguments).

We sample each surface by considering five strikes (symmetric deltas up to 0.1) for each maturity (five, ranging from 1m to 2y). The calculation is run by using 3y of data, on the correlation setting, by studying the 3day changes of the data (the use of 3day changes is motivated by the lack of liquidity of some OTM options, which makes the daily calculation too sensitive to the mediocre quality of the dataset).

The PCA factors reveal which are the main drivers behind the dynamics of each volatility surface. The table below highlights the sensitivity of each volatility point to the detected factors, which allows for an interpretation of the factors themselves. As shown, the first factor acts mostly as a long-only factor all volatilities have a similar exposure to: it tends to affect more short-term volatilities than longer-dated ones, which confirms the empirical finding that in a heated environment, where all vols move up, front-end volatilities are more impacted than longer-dated ones. The second factor plays mostly the term-structure of the volatility curve whereas the third factor impacts mostly the skew.

It has to be stressed that, by using three factors, the model explains more than 97% of the variance of the whole surface. This means that the model is very accurate in describing the shape of the volatility surface and that, even when dislocations are large if expressed as a z-score, they might correspond to volatility spreads falling within the bid-ask range. For this reason, one can still use the model by keeping only one or two principal factors, for widening the gap between market and model-based volatility points.

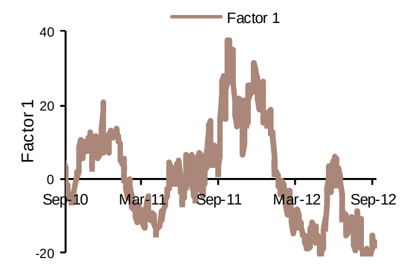
Sensitivities of the volatility points to the three main principal factors

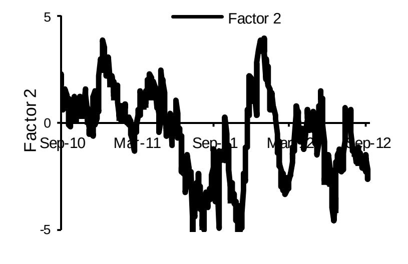


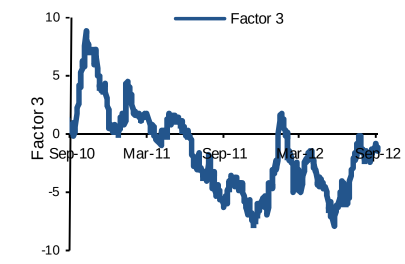
Source: SG Cross Asset Research

The three PCA principal factors, which explain around 97% of the total variance of the EUR/USD volatility surface, are shown in the charts below, in their integrated series: similar results hold for other currencies, although some currencies might exhibit peculiar features, especially when the skew of the currency is marked.

EUR/USD surface factors: Level (91.7%)	Term structure (3.4%)	Skew (2.7%)





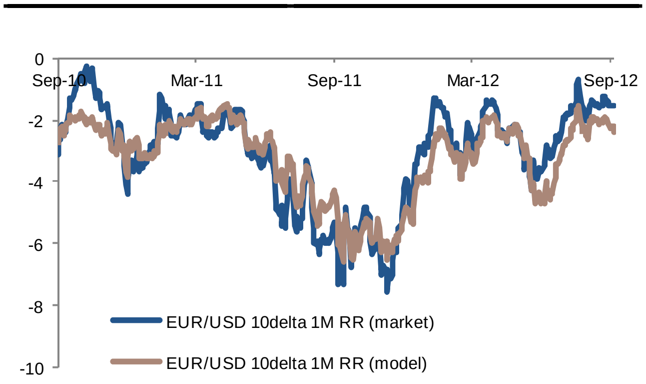


Source: SG Cross Asset Research

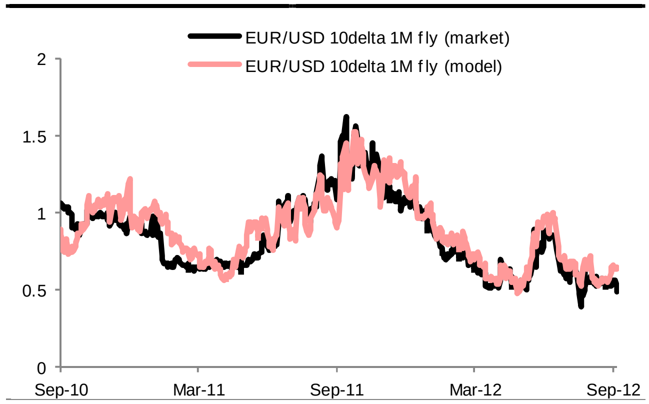
The dislocations as detected by the model can be expressed, rather than for the single volatility points, for risk-reversals and butterflies, which represent the market conventions for the shape of the smile: such variables can be expressed as linear functions of volatilities which allows for a simple calculation of the corresponding dislocations. Having a model for estimating the ideal shape of the smile can help in highlighting opportunities also for products like variance and volatility swaps which are highly sensitive to the shape of the smile, albeit via non-linear relationships.

The two charts below show PCA-fair values and market variables for 1M, 10delta EUR/USD risk-reversals and butterflies.

Market and PCA-fair value for the 10delta 1M EUR/USD risk-reversal



Market and PCA-fair value for the 10delta 1M EUR/USD Butterfly

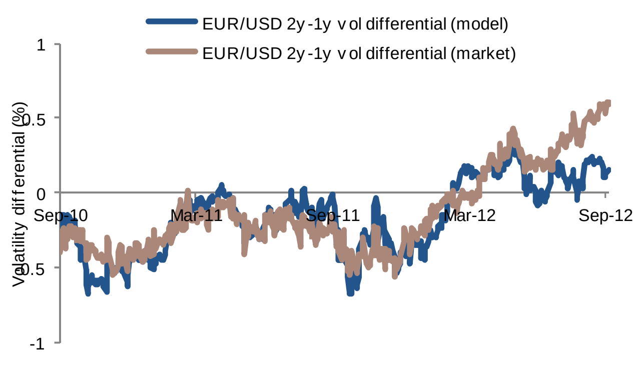


Source: SG Cross Asset Research

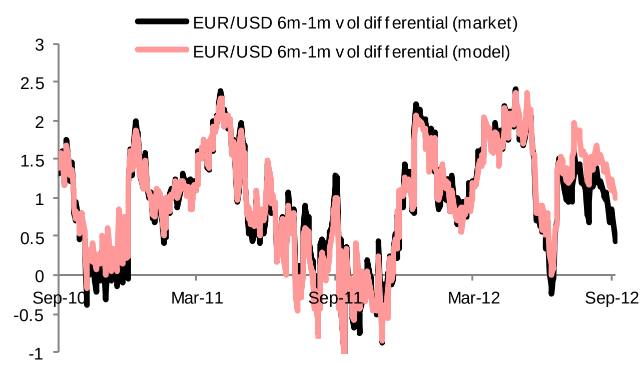
By using a similar linear transformation, the term-structure of the volatility curve can be investigated too: in other words, the model can investigate dislocations on the shape of the curve, in particular regarding differentials of volatilities corresponding to different maturity buckets. This comes in handy for proposing calendar spread ideas, to be implemented via plain vanillas or exotic products.

In the two charts below, we display PCA-values and market values for two selected ATM volatility differentials (2y-1y and 6m-1m) which are at the moment significantly dislocations.

Market and PCA-values for the 2y-1y EUR/USD vol differential



Market and PCA-values for the 6m-1m EUR/USD vol differential



Source: SG Cross Asset Research

The model values for the volatility curve can be summarized in the charts below, which plot market and model values for some volatility differentials and for the corresponding Forward Volatilities (assuming a linear time interpolation for the variance).

EUR/USD Term-structure dislocations…

Volatility differential	Market	Model	z-score
TS 3m-1m	0.25	-0.14	2.14
TS 6m-1m	0.50	0.38	0.70
TS 1y-1m	1.24	1.17	0.72
TS 2y-1m	1.84	1.55	2.95
TS 6m-3m	0.25	0.52	-2.64
TS 1y-3m	0.99	1.31	-2.37
TS 2y-3m	1.59	1.70	-0.51
TS 1y-6m	0.74	0.79	-0.42
TS 2y-6m	1.34	1.18	0.74
TS 2y-1y	0.60	0.39	1.52

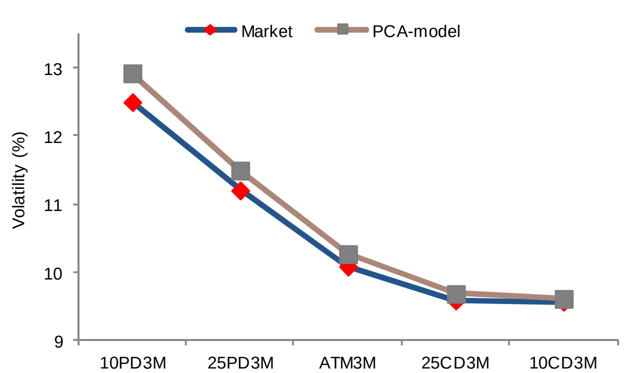
…and the corresponding signals on Forward Volatilities

XY Forward vol (Y maturity, X forward)	Market	Model
1m2m	8.92	8.53
1m5m	9.15	9.19
1m11m	9.90	10.01
1m23m	10.46	10.36
3m3m	9.29	9.61
3m9m	10.10	10.31
3m21m	10.60	10.52
6m6m	10.48	10.64
6m18m	10.80	10.66
1y1y	10.96	10.67

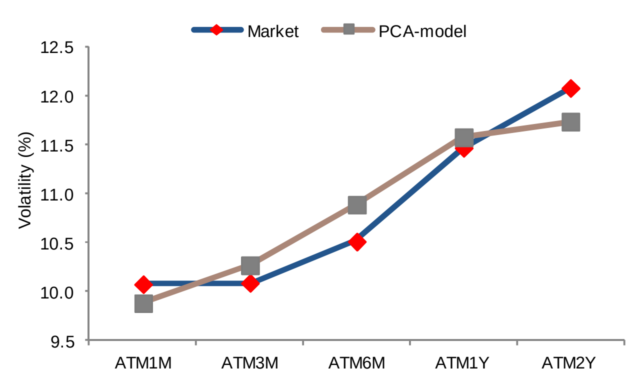
Source: SG Cross Asset Research

The two final charts display the market and PCA-based volatility smile (for 3M maturity) and ATM volatility curve for EUR/USD.

Market and PCA-model 3M volatility smile for EUR/USD



Market and PCA-model ATM volatility curve for EUR/USD



Source: SG Cross Asset Research

Conclusion

In this article we analyse several approaches that can be used to identify relative value opportunities in the FX volatility space. The SABR model is a stochastic volatility model which allows for a calibration of the volatility smile: aside from permitting a calculation of implied probability distributions, to be compared with historical ones, the model allows one to identify which points of the smile are deviating from each other and/or from long-term established patterns. If the heteroscedastic model is used mostly in establishing a benchmark for opportunities in the front-end (gamma) segment of the volatility curve, then the PCA model can bring value on the back-end (vega) segment of the vol curves, whereby volatilities are bought or sold depending on their relative value potential: more specifically, the model can be applied to detect dislocations within baskets of volatilities and risk-reversals, and on different points of a single volatility surface.

A collection of FX vol screeners, based on the models presented in this paper, is available on the FX vol section of the SG Quant Research website.

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APPENDIX

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