Extended Fibonacci Sequences

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1 The Classical Fibonacci Sequence [1]

The Fibonacci numbers are the sequence of numbers $\{F_n\}_{n=1}^{\infty}$ defined by the linear recurrence equation

$$F_n = F_{n-1} + F_{n-2}$$

with $F_1 = F_2 = 1$, and conventionally defining $F_0 = 0$.

We can thus calculate some of the first few Fibonacci numbers as 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1,597, 2,584, 4,181, 6,765, 10,946, 17,711, 28,657, 46,368, 75,025, 121,393, 196,418, 317,811, 514,229, 832,040, 1,346,269, and so forth.

2 Extending the Sequence

There are several extensions that obviously arise from this definition of the Fibonacci numbers. A commonly-known example are the Fibonacci polynomials, [2], which extend the Fibonacci sequence into the reals.

One in specific we are interested in is as such. We can view the Fibonacci numbers as merely the summation of the previous terms. In the traditional form we are summing the last two, but there is no reason we can't extend this concept for any arbitrary non-negative integer.

Thus, our extended Fibonacci sequence of numbers $\{F_{m,n}\}_{m,n=1}^{\infty}$ defined by the linear recurrence relation

$$F_{m,n} = \sum_{i=1}^{m} F_{m,n-i}$$

with $F_{m,j} = 1 \forall j \in [1, m]$, and conventionally defining $F_{m,0} = 0 \forall m$.

We can see that the traditional Fibonacci sequence F_n is $F_{2,n}$, since we sum only the previous two elements of the sequence. $F_{0,n}$ is quite uninteresting, since the only reasonable convention would be that

$$F_{0,n} = 0 \forall n$$

Similarly, $F_{1,n}$ is also quite uninteresting, as it would be

$$F_{1,n} = 1 \forall n$$

References

[1] CHANDRA, P., AND WEISSTEIN, E. W. Fibonacci number. [accessed 08/28/2011].

[2] WEISSTEIN, E. W. Fibonacci polynomial. [accessed 08/28/2011].