Photon Energy Calibration Systematics

Christophe Goudet

LABORATOIRE DE L'ACCÉLÉRATEUR LINÉAIRE

January 18, 2017

Model Comparison

Two correlation models of systematics are proposed :

- FULL model decorrelate all NP.
- ALL model sum in quadrature respectively all systematic sources into respectively a single nuisance parameter for scale and resolution.
- The fit with $m_{\gamma\gamma} \in [115, 135]$GeV is used as closest to the signal model tool default behaviour.
- Major (surprising?) difference for scale uncertainty

Total Inclusive Uncertainty :

%	Scale	Resolution
FULL	0.27	8.26
ALL	0.46	9.02

Exact uncertainty

(Thanks Guillaume for explanation)

- Consider N_{NP} NP with constant values N_B bins (in η, p_T, conversion status).
- The covariance matrix V is then defined as :

$$V_{ij} = \sum_{n}^{N_{NP}} \sigma_{n,i} \sigma_{n,j} \tag{1}$$

- The total inclusive uncertainty on measured mass is then,

$$\frac{\sigma_M}{M} = \frac{1}{N_\gamma} \sqrt{\sum_{ij}^{N_B} N_i N_j V_{ij}} \tag{2}$$

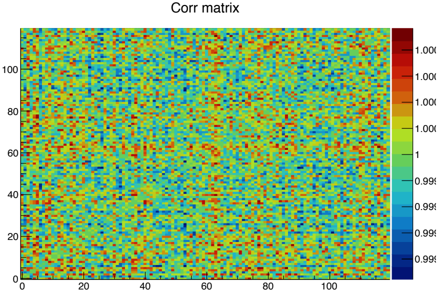
N_i = number of photons in bin i,

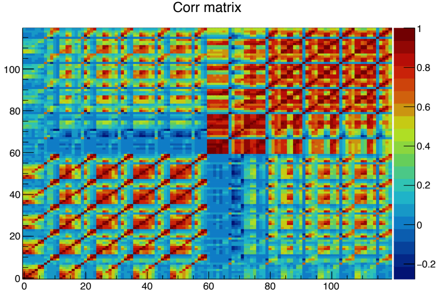
Models total mass uncertainty

(Thanks Guillaume for cross-check)

ALL | FULL

Correlation matrix





$$\frac{\sigma_M}{M} = 0.47\%$$

$$\frac{\sigma_M}{M} = 0.26\%$$

Cross-checks validate values obtained by mass fitting.

Category comparison

The previous inclusive results are compared to asimov with 13 categories.

Total Scale Uncertainty :

%	ALL	FULL	ratio
Mass fit	0.46	0.27	41
Formula	0.47	0.26	45
Asimov	0.53	0.34	36

Bad calibration categories have a higher contribution to the resolution.

Photon Energy Calibration Systematics

Christophe Goudet

LABORATOIRE
DE L'ACCÉLÉRATEUR
LINÉAIRE

January 18, 2017

Systematics in statistical framework

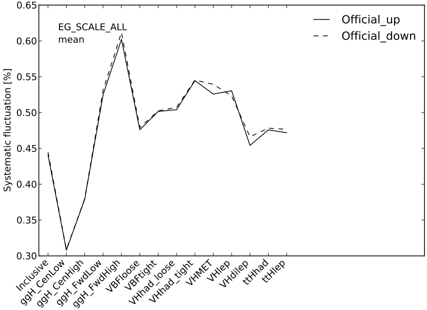
Changing an energy correction factor (by systematic fluctuation) affects several parameters. For current results, it is assumed that **only the parameter X targetted by the systematic (POI) is affected** (hence fixing the others).
Only the parametrization of X will contain the measured uncertainty δ_X :

$$X \to X(1 + \delta_X \theta)$$

with θ a gaussian constrained free parameter.

In the framework, the pulling of the nuisance parameters only affects the main parameter of the systematic

Typical plot

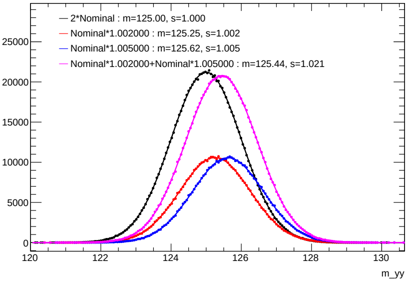


For a given fitted variable (here mean) :

- Full line : $\frac{\mu_{up} - \mu_{nominal}}{\mu_{nominal}}$
- Dashed line : $-\frac{\mu_{down} - \mu_{nominal}}{\mu_{nominal}}$

Both lines should be superimposed in case of symmetric systematics.

Scale impact on width

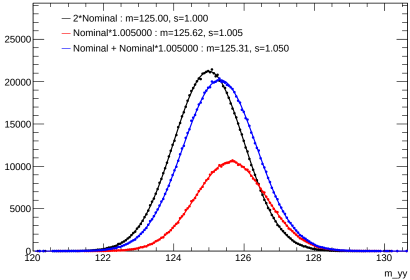


1M random numbers generated on a Gaussian($\mu = 125, \sigma = 1$).

- Initial numbers distribution.
- Half events multiplied by 1.002.
- Remaining events multiplied by 1.005.
- Combined distribution of red and blue.

Mean (m) and RMS (s) of a fitted gaussians are given in the legend. Interpretationof the curve in the next slides.

Uncertainty reduction



Lets assume a gaussian distributed energy distribution. Events are affected **either** by systematic A or B with same amplitude (0.3%).

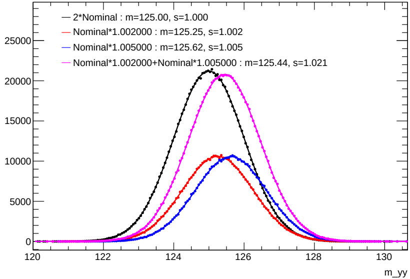
Summing quadratically the systematic gives $\delta E = 0.3\%$ for all event.
Hence $\Delta M = 0.3\%$.
Applying independently systematics to events gives :

$$\Delta M_A = \Delta M_B = \delta E/2$$

Because only half events are effectively modified. Finally :

$$\Delta M = \Delta M_A \oplus \Delta M_B = \frac{\delta E}{\sqrt{2}}$$

μ/σ scale correlation



Lets assume a gaussian distributed energy distribution. Applying energy scale correction gives :

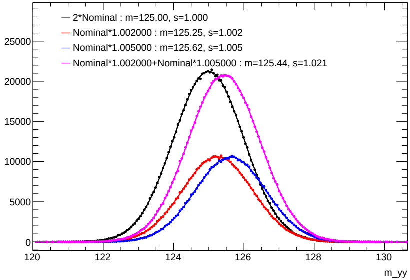
$$E \to E(1+a)$$

Hence the distribution will be changed to :

$$exp(-\frac{(E-\mu)^2}{2\sigma^2}) \ \to \ exp(-\frac{(\frac{E}{1+a}-\mu)^2}{2\sigma^2}) = exp(-\frac{(E-\mu(1+a))^2}{2\sigma^2(1+a)^2}) \quad (3)$$

The new distribution is a shifted gaussian with scaled RMS.
Given the medium shift of EG_SCALE_ALL, we expect $^{+0.4}_{-0.4}\%$ change in resolution.

Inhomegenous scale



The RMS of two points separated by d is $d/4$.
If d is the difference between two scale factors,

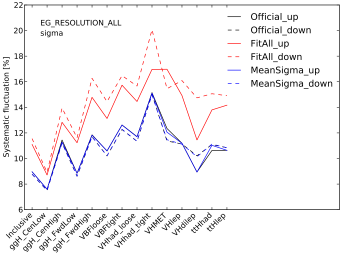
$$d \sim 3.10^{-3} \cdot E_\gamma = 0.18$$

$$\frac{\text{RMS}}{\text{Resolution}} = \frac{d/4}{1.5\text{GeV}} = 3\%$$

The inhomogeneity of the scale factors uncertainties changes the width of the distribution at the percent level. This effect will always increase the width.

Black and pink distribution show an illustration of this effect.

Fitting methods : σ fluctuation

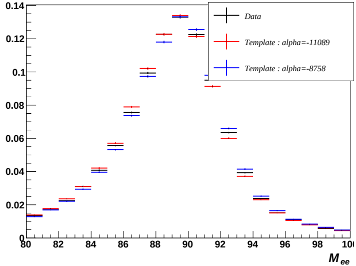


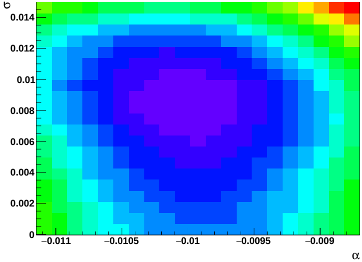
- σ is sensitive to the change of scale.
- Two understood explanations :
  - μ/σ correlation when scaling energies.
  - Resolution loss due to inhomogenous scaling.
- Effect on resolution is small but I propose to add a width effect linked to EG_SCALE_ALL nuisance parameter.
  - Add a second nuisance parameter to the width to limit overconstraints.
  - Good hopes to reduce Zee resolution systematics (dominant in EG_RESOLUTION_ALL) within medium timescale.

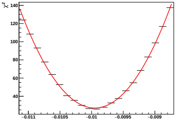
Cross-check : template method

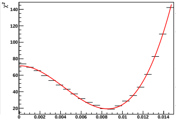
The template method is used to measure α and C simultaneously.

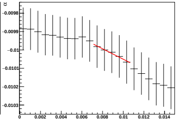
- Create distorded MC (templates) with test values of α and C.
- **Compute χ^2 between Z mass distribution of data and template**.
- **Fit the minimum of the χ^2 distribution** in the (α, C) plane.
- Fit performed in 2 steps of 1D fits :
  - fit $\chi^2 = f(\alpha)$ at constant C (lines) $\rightarrow (\alpha_{min}, \chi^2_{min})$.
  - fit $\chi^2_{min} = f(C) \rightarrow (C, \Delta C)$
  - project C in $\alpha_{min} = f(C)$, corresponding bin gives $(\alpha, \Delta\alpha)$.











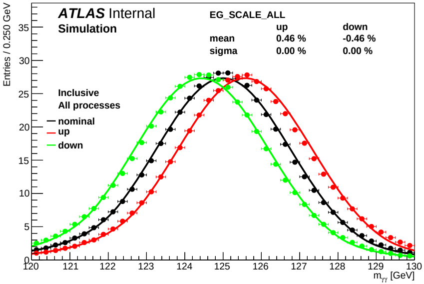
Scale factors interpretation

Assume the up fluctuation (red) as data and nominal distribution (black) as MC in the template method. One has

$$m_H^{up} = m_H^{nom}(1+\alpha)$$

Hence

$$\delta_{m_H} = \alpha$$



Furthermore :

$$\sigma_H^{up} = \sigma_H^{nom} \oplus cE$$

Hence

$$\delta_{\sigma_H} = \sqrt{1+\frac{c^2E^2}{\sigma_H^2}} - 1$$

One has to be carefull with resolution uncertainty as the template method is weak to measure small differences.