Answers to questions in Lab 1: Filtering operations

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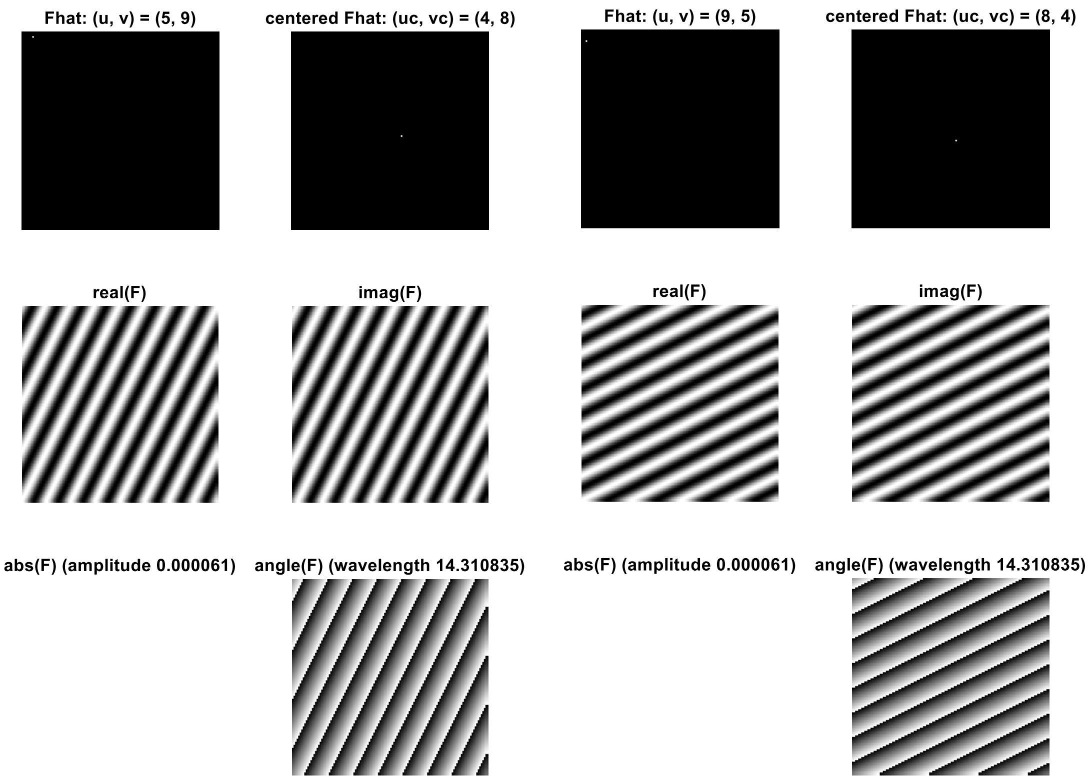
Instructions: Complete the lab according to the instructions in the notes and respond to the questions stated below. Keep the answers short and focus on what is essential. Illustrate with figures only when explicitly requested.

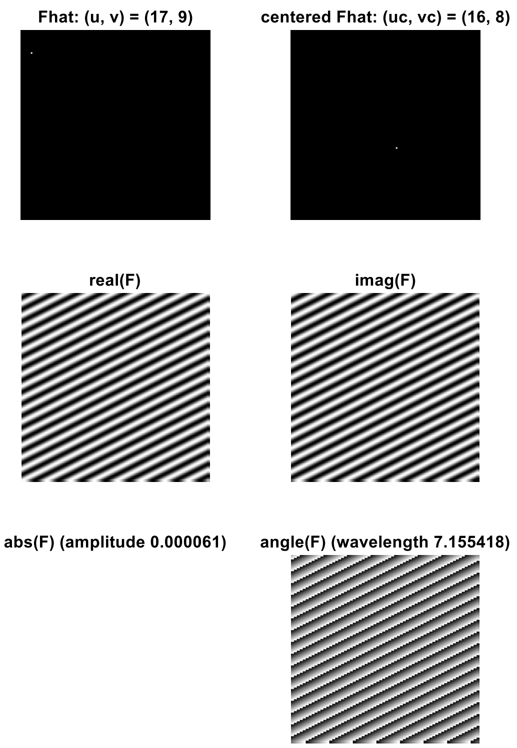
Good luck!

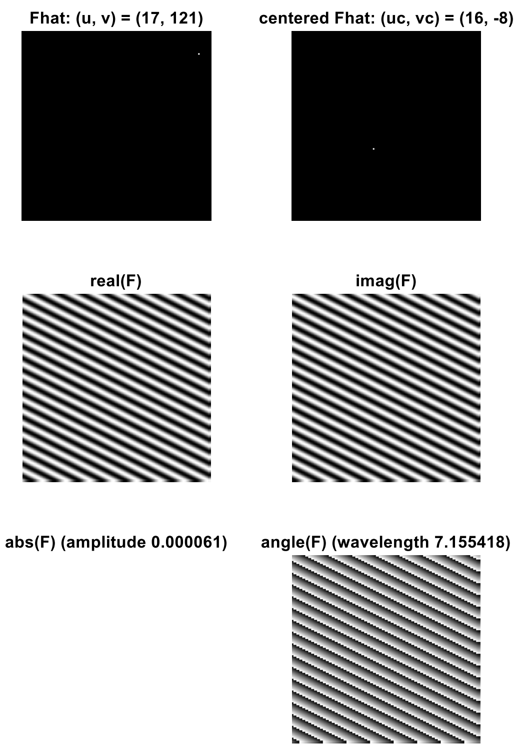
Question 1: Repeat this exercise with the coordinates p and q set to (5, 9), (9, 5), (17, 9), (17, 121), (5, 1) and (125, 1) respectively. What do you observe?

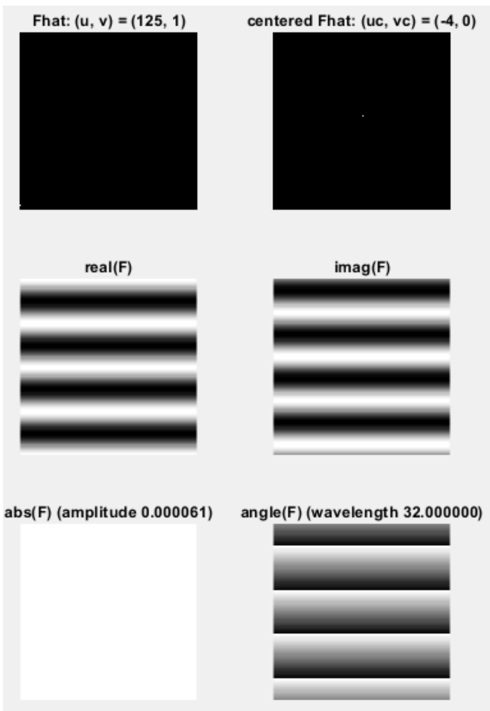
Answers:

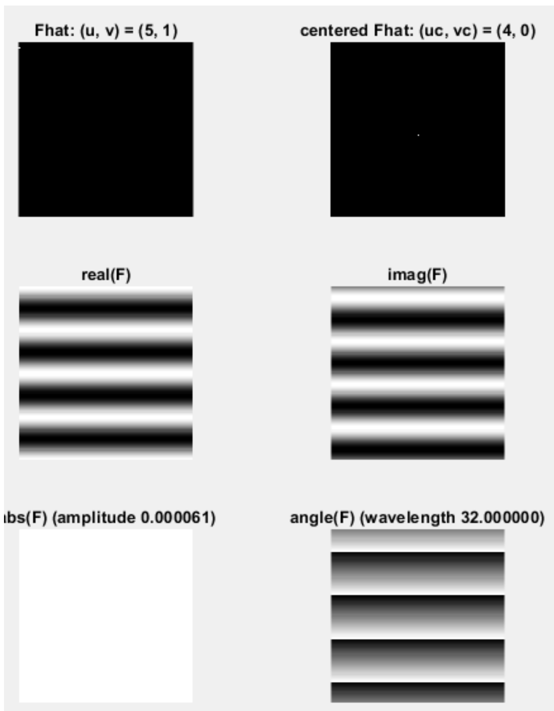
We can see sine waves. The vector (uc,vc) is orthogonal to the waves. The greater the norm of the vector (uc,vc) is, the greater the frequency of the waves is.




Fhat: (u, v) = (17, 9)
centered Fhat: (uc, vc) = (16, 8)
real(F)
imag(F)
abs(F) (amplitude 0.000061)
angle(F) (wavelength 7.155418)


Fhat: (u, v) = (17, 121)
centered Fhat: (uc, vc) = (16, -8)
real(F)
imag(F)
abs(F) (amplitude 0.000061)
angle(F) (wavelength 7.155418)

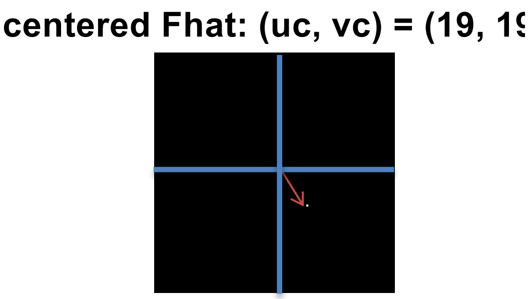

Fhat: (u, v) = (125, 1)
centered Fhat: (uc, vc) = (-4, 0)
real(F)
imag(F)
abs(F) (amplitude 0.000061)
angle(F) (wavelength 32.000000)

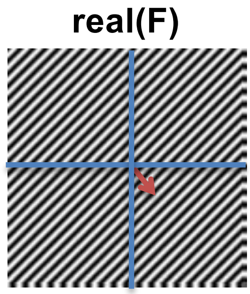

Fhat: (u, v) = (5, 1)
centered Fhat: (uc, vc) = (4, 0)
real(F)
imag(F)
abs(F) (amplitude 0.000061)
angle(F) (wavelength 32.000000)

Question 2: Explain how a position (p, q) in the Fourier domain will be projected as a sine wave in the spatial domain. Illustrate with a Matlab figure.

Answers:

A position (p, q) in the Fourier domain is a frequency (f1, f2) which is a vector. This vector gives the direction of the sine wave as shown on the figure below: the waves are orthogonal to the vector (f1, f2). The greater the norm of the vector (uc, vc) is, the greater the frequency of the waves is.





Question 3: How large is the amplitude? Write down the expression derived from Equation (4) in the notes. Complement the code (variable amplitude) accordingly.

Answers:

Since the Fourier transform is equal to 1 in f=(u,v) and 0 in other points, from equation 4:

$$F(x) = \frac{1}{N^2} e^{\frac{2\pi i f x}{N}}$$

So the amplitude is the factor $1/N^2 = 1/(128^2) = 0.000061$

Question 4: How does the direction and length of the sine wave depend on p and q? Write down the explicit expression that can be found in the lecture notes. Complement the code (variable wavelength) accordingly.

Answers:

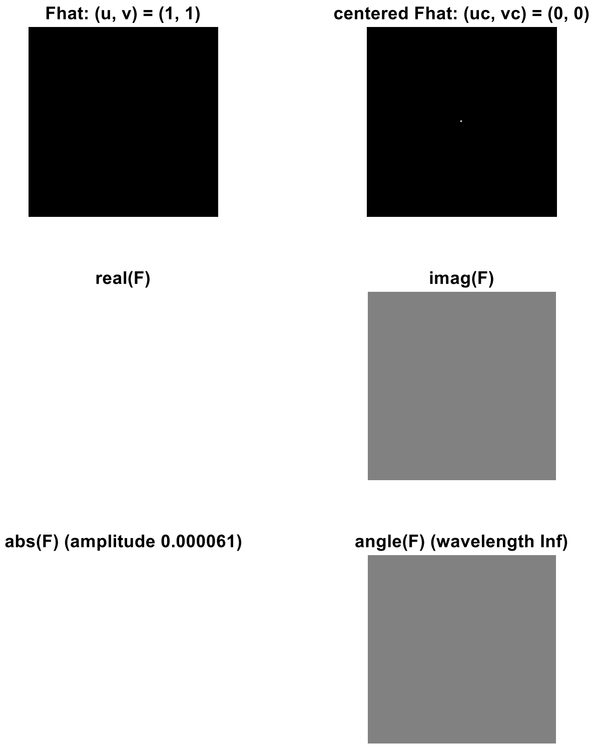
The direction of the sine wave is the vector f=(uc,vc)

With $w1 = \frac{2\pi u}{N}$ and $w2 = \frac{2\pi v}{N}$, the length of the sine wave is $\lambda = \frac{2\pi}{\sqrt{w1^2 + w2^2}}$

Question 5: What happens when we pass the point in the center and either p or q exceeds half the image size? Explain and illustrate graphically with Matlab!

Answers:

When we pass the point in the center, there is no sine wave (or we can say it is a sine way with frequency equal to 0). The inverse Fourier transform is a constant.



When either p or q exceeds half the image size, the size of the image is subtracted to the corresponding coordinates so that both p and q are in [-size/2, size/2] x [-size/2, size/2]. It does not change the inverse Fourier transform since the inverse Fourier transform is N-periodic.

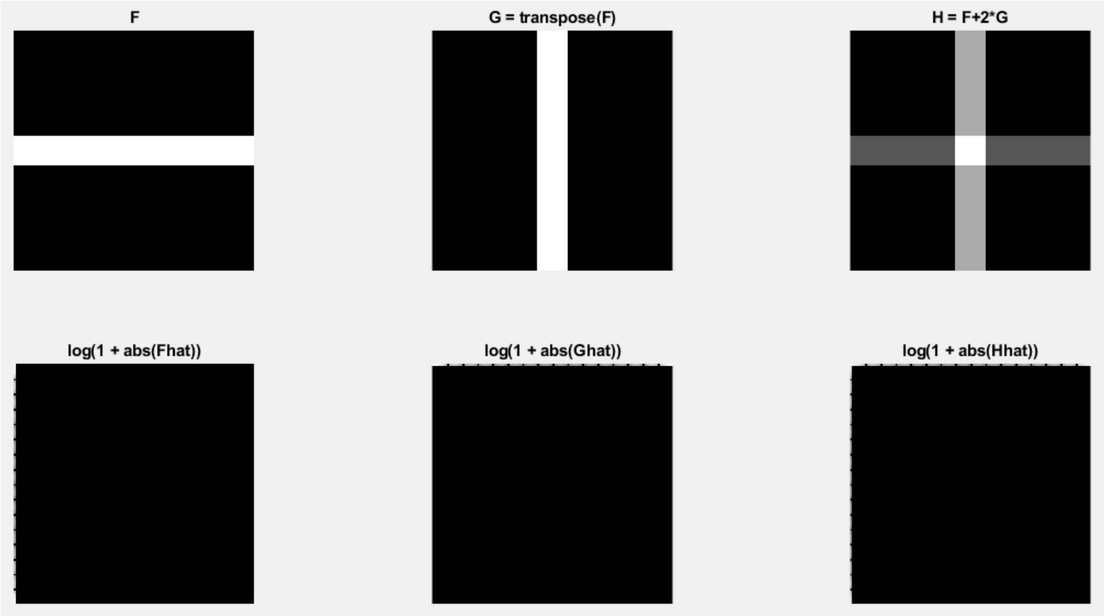
Question 6: What is the purpose of the instructions following the question *What is done by these instructions?* in the code?

Answers:

These instructions do what is explained in question 5, it makes a translation in the frequency domain to make sure the frequency is in the window [-size/2, size/2] x [-size/2, size/2].

Question 7: Why are these Fourier spectra concentrated to the borders of the images? Can you give a mathematical interpretation? Hint: think of the frequencies in the source image and consider the resulting image as a Fourier transform applied to a 2D function. It might be easier to analyze each dimension separately!

Answers:



If we consider F:
On the horizontal axis here is no periodicity; the frequency is 0; that is why the Fourier spectrum I concentrated to the left border of the image.
On the vertical axis there is a step function; that is why the Fourier spectrum is a dashed line, which corresponds to a sinc function.
We can observe the same results with G with a 90°-rotation.

Question 8: Why is the logarithm function applied?

Answers:

The logarithm is applied to observe minor frequencies (the absolute value of the sinc function decreases for very large or low values).

Question 9: What conclusions can be drawn regarding linearity? From your observations can you derive a mathematical expression in the general case?

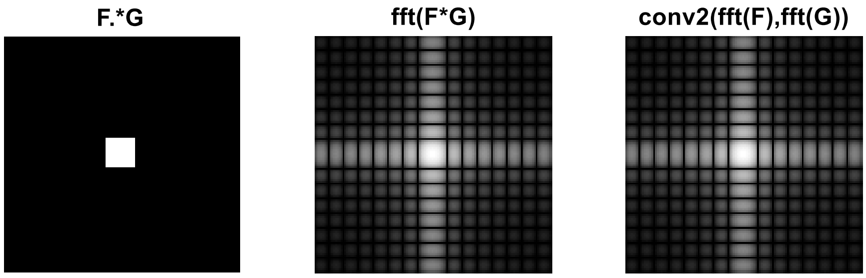
Answers:

From the figures we can see Fourier(F+2*G)=Fourier(F)+2*Fourier(G).
We can derive in the general case that Fourier(F+lambda*G)=Fourier(F)+lambda*Fourier(G) which is the definition of linearity.

Question 10: Are there any other ways to compute the last image? Remember what multiplication in Fourier domain equals to in the spatial domain! Perform these alternative computations in practice.

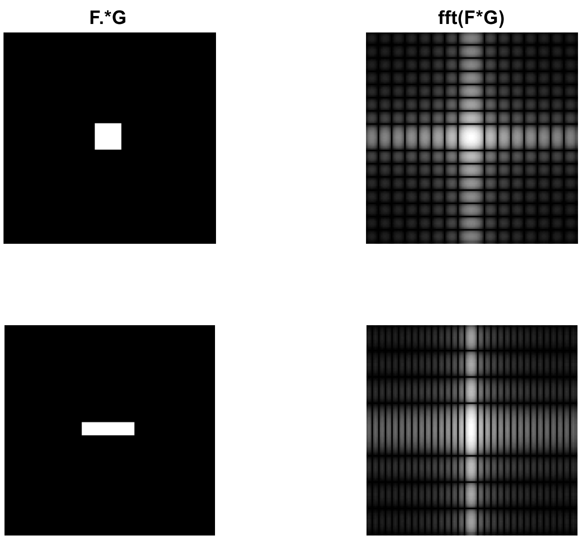
Answers:

A multiplication in Fourier domain equals to a convolution in the spatial domain so fourier(F*G)=convolution(fourier(F),fourier(G)) where * means ‘element-wise multiplication’



Question 11: What conclusions can be drawn from comparing the results with those in the previous exercise? See how the source images have changed and analyze the effects of scaling.

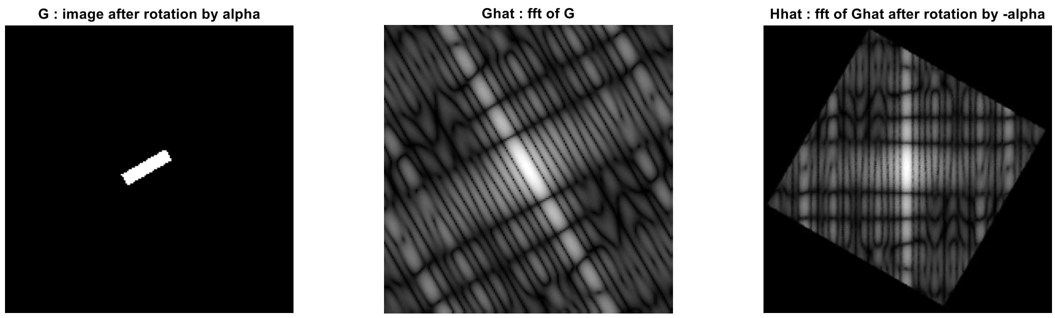
Answers:



The square in the bottom-image is wider and the shapes on the fft spectrum are narrower, the square in the bottom-image is shorter and the shapes on the fft spectrum are higher. Compression in spatial domain is same as expansion in Fourier domain (and vice versa)

Question 12: What can be said about possible similarities and differences? Hint: think of the frequencies and how they are affected by the rotation.

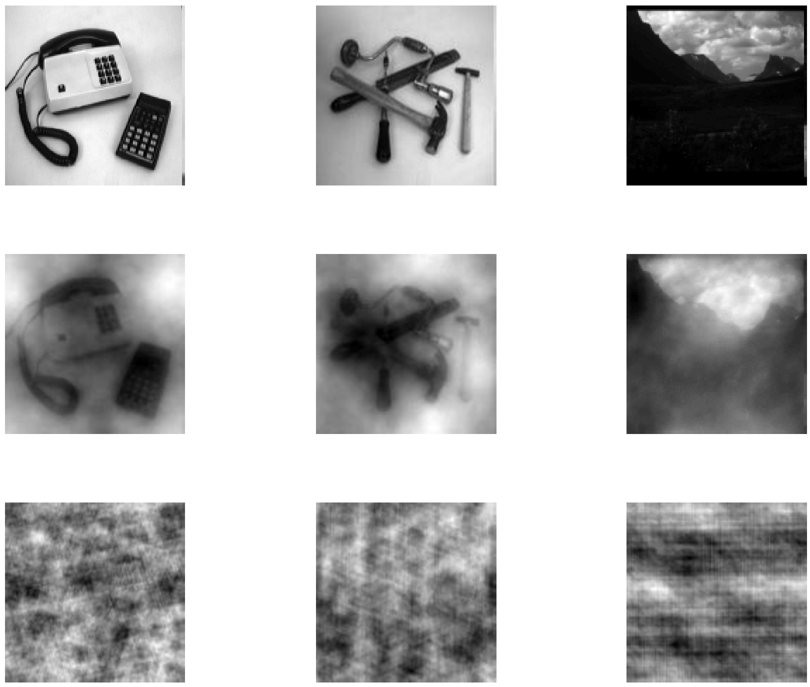
Answers:



From the figures above we can see that when we rotate an image by alpha, the Fourier transform also rotates by alpha. We can then rotate this Fourier transform by –alpha and we will get the Fourier transform of the original image. However this is not exactly the same image: when G is rotated the lines are not really straight because of the resolution and therefore it creates some noise.

Question 13: What information is contained in the phase and in the magnitude of the Fourier transform?

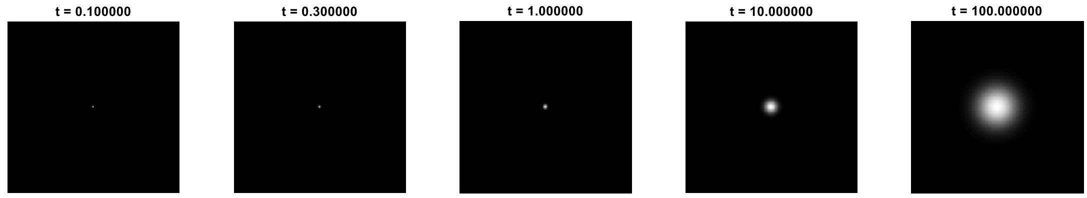
Answers:



The phase contains information about the position of the objects in the images but does not contain all the information about intensity. The magnitude contains information about the intensity of the different frequencies but no information about the position so we cannot extract a lot of meaning from the image reconstructed only from the magnitude.
Both phase and magnitude are needed to reconstruct the image properly.

Question 14: Show the impulse response and variance for the above-mentioned t-values.
What are the variances of your discretized Gaussian kernel for t = 0.1, 0.3, 1.0, 10.0 and 100.0?

Answers:




```
't = 0.100000'        't = 0.300000'        't = 1.000000'        't = 10.000000'        't = 100.000000'

var =                 var =                 var =                 var =                  var =

  0.0133   0.0000      0.2811   0.0000       1.0000   0.0000       10.0000    0.0000      100.0000     0.0000
  0.0000   0.0133      0.0000   0.2811       0.0000   1.0000        0.0000   10.0000        0.0000   100.0000
```

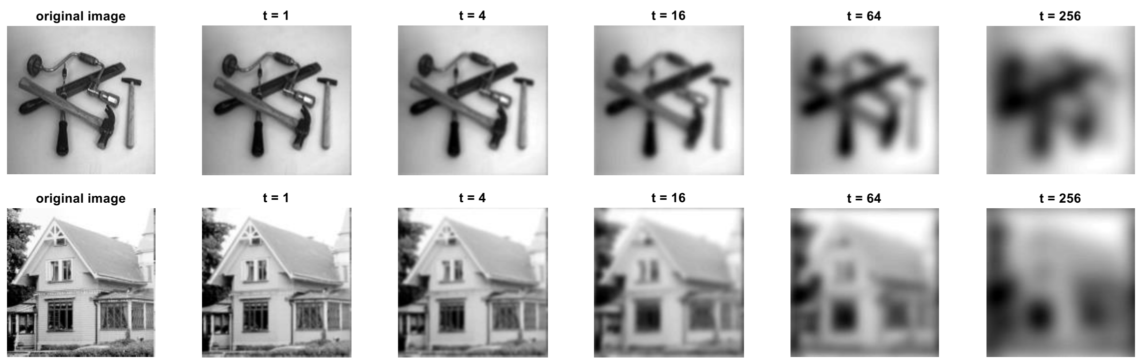
Question 15: Are the results different from or similar to the estimated variance? How does the result correspond to the ideal continuous case? Lead: think of the relation between spatial and Fourier domains for different values of t.

Answers:

For large t the variances of the discretized kernels are equal to t, this corresponds to the ideal continuous case. For smaller t there are non-negligible differences (0.133 instead of 0.1, 0.2811 instead of 0.3).

Question 16: Convolve a couple of images with Gaussian functions of different variances (like t = 1.0, 4.0, 16.0, 64.0 and 256.0) and present your results. What effects can you observe?

Answers:

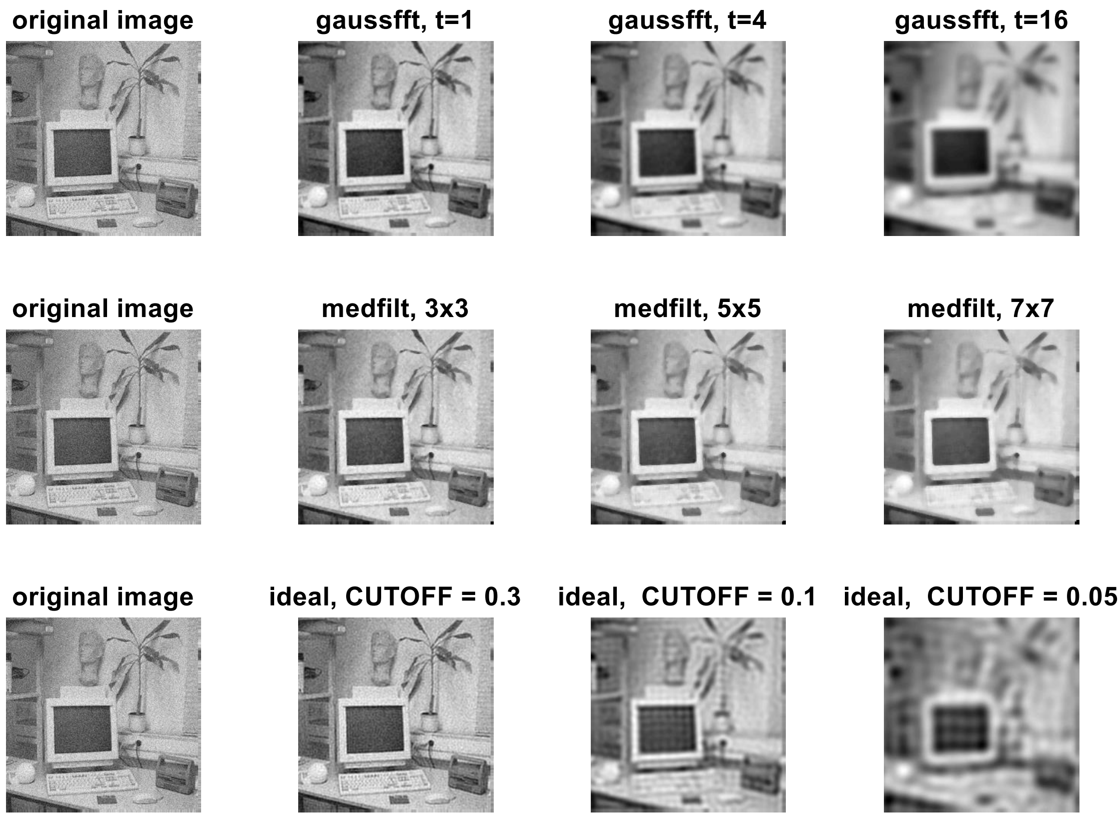


For small variances (t=1), the resulting image is very similar to the original image, it is difficult to see any difference. When t increases the image gets smoother to finally be completely blurred for very large t (t=256).

Question 17: What are the positive and negative effects for each type of filter? Describe what you observe and name the effects that you recognize. How do the results depend on the filter parameters? Illustrate with Matlab figure(s).

Answers:

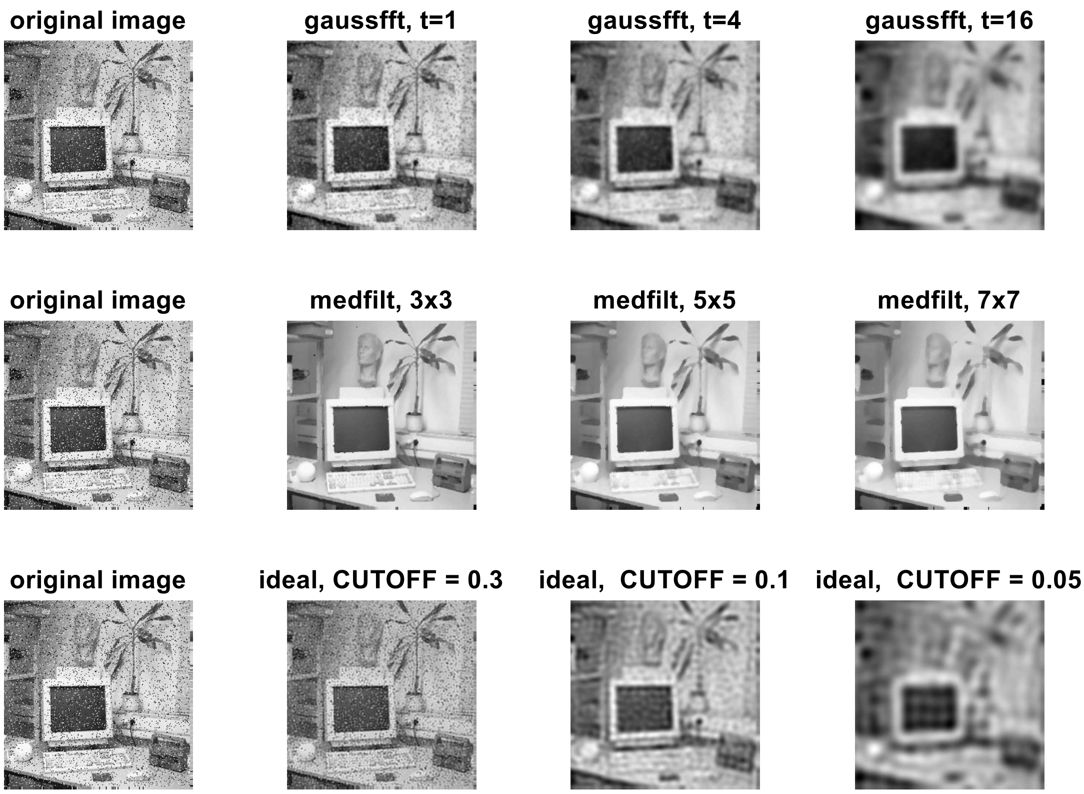
With Gaussian noise:



Gaussian filter: if t is too large, the image becomes very blurry; we cannot see small details and edges are destroyed.

Median filter: the main positive effect is that edges are not destroyed by the smoothing. However some thin lines may be removed if the kernel is too large. There is also a painting effect: some information is lost since it destroys the texture.

Ideal low pass filter: the main problem is that some unexpected periodicities appear when the frequency cutoff is too small.



Gaussian filter: The main problem is that noisy pixels have an influence on their neighbors. Like with Gaussian noise, if t is too large, the image becomes very blurry; we cannot see small details and edges are destroyed.

Median filter: Median filter performs very well on this kind of noise since almost all noisy pixels are removed. Another positive effect is that edges are not destroyed by the smoothing. However some thin lines may be removed if the kernel is too large. There is also a painting effect: some information is lost since it destroys the texture.

Ideal low pass filter: Like the Gaussian filter noisy pixels have an influence on their neighbors. Another problem is that some unexpected periodicities appear when the frequency cutoff is too small.

Question 18: What conclusions can you draw from comparing the results of the respective methods?

Answers:

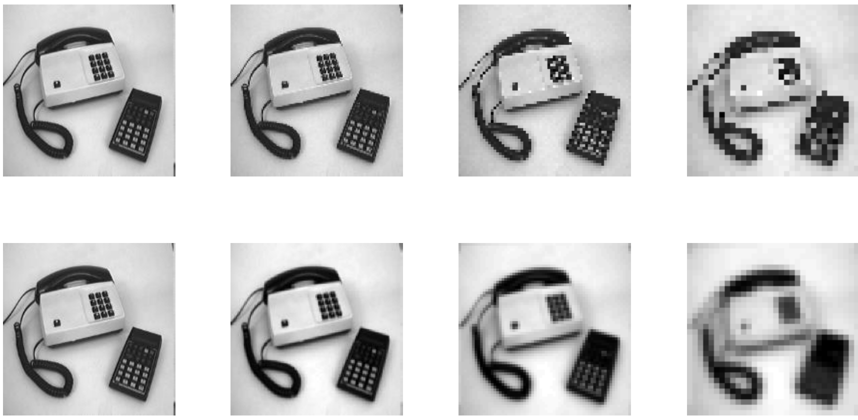
For SAP noise the median filtering is probably the filter to be used since it removes all the noisy pixels.

For Gaussian noise all the filters have both positive and negative effects: median filtering are good to preserve the shapes, especially the edges, whereas Gaussian and low-pass filtering are good to preserve the texture.

Question 19: What effects do you observe when subsampling the original image and the smoothed variants? Illustrate both filters with the best results found for iteration i = 4.

Answers:

With Gaussian filtering :



With ideal low-pass filtering:

For both low-pass filter and Gaussian filter the image is much smoother when smoothing is applied before the subsampling, especially after many iterations (i=4, right figure)

Question 20: What conclusions can you draw regarding the effects of smoothing when combined with subsampling? Hint: think in terms of frequencies and side effects.

Answers:

Since it reduces the number of pixel to represent an image, subsampling creates high frequencies. Gaussian filtering and low-pass filtering are low-pass filter so they remove these high frequencies and somehow preserve the quality of the image.
