HUDM6052 Psychometric II Homework_04

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Contents

Q1-a-Parameters-Estimation

Fit the 1PL, 2PL, and 3PL models. . . report the estimated item parameters in separated tables

My Solution:
To make the layout concise and good-looking, I intentionally omitted the codes for data cleaning and some instant display of running outcomes. I attached the estimated item parameters from all three models into one table to save space.

```
> # load the binary response dataset
> library(mirt)
> df <- read.csv("/Users/panpeter/Desktop/PhD_Learning/HUDM6052 Psychometric II/HUDM6052_Psychometic_II/
> df <- df[,-1]
>
> # --------------------------------------------------
> #                        Run 1PL
> # --------------------------------------------------
>
> # specify the model for 33 items loading on 1 dimension
> # and constrain all the item slope to be equal for 1PL estimation
> spec <- 'F = 1-33
+ CONSTRAIN = (1-33, a1)'
>
> # estimated the model
```

```
> # since I constrained all slopes to be equal, here the argument "2PL" is safe
> irt_1pl <- mirt(df, model = spec, itemtype = "2PL", SE=T)
Iteration: 1, Log-Lik: -20276.377, Max-Change: 0.18304Iteration: 2, Log-Lik: -20265.381, Max-Change: 0.

Calculating information matrix...
> irt_1pl

Call:
mirt(data = df, model = spec, itemtype = "2PL", SE = T)

Full-information item factor analysis with 1 factor(s).
Converged within 1e-04 tolerance after 25 EM iterations.
mirt version: 1.40
M-step optimizer: BFGS
EM acceleration: Ramsay
Number of rectangular quadrature: 61
Latent density type: Gaussian

Information matrix estimated with method: Oakes
Second-order test: model is a possible local maximum
Condition number of information matrix =  50.40657

Log-likelihood = -20260.22
Estimated parameters: 66
AIC = 40588.44
BIC = 40756.74; SABIC = 40648.75
```

```
> # --------------------------------------------------
> #                      Run 2PL
> # --------------------------------------------------
> irt_2pl <- mirt(df, model = 1, itemtype = "2PL", SE=T)
Iteration: 1, Log-Lik: -20276.377, Max-Change: 0.63072Iteration: 2, Log-Lik: -20095.717, Max-Change: 0.

Calculating information matrix...
> irt_2pl

Call:
mirt(data = df, model = 1, itemtype = "2PL", SE = T)

Full-information item factor analysis with 1 factor(s).
Converged within 1e-04 tolerance after 32 EM iterations.
mirt version: 1.40
M-step optimizer: BFGS
EM acceleration: Ramsay
Number of rectangular quadrature: 61
Latent density type: Gaussian

Information matrix estimated with method: Oakes
Second-order test: model is a possible local maximum
Condition number of information matrix =  11.16451

Log-likelihood = -20078.62
Estimated parameters: 66
```

```
AIC = 40289.23
BIC = 40615.92; SABIC = 40406.3
```

```
> # ---------------------------------------------------
> #                        Run 3PL
> # ---------------------------------------------------
> # specify the model
> spec <- 'F = 1-33
+ PRIOR = (1-33, g, norm, -1.1, 2)'
> irt_3pl <- mirt(df, model = spec, itemtype = "3PL", SE = T)
```

```
> print(irt_3pl)

Call:
mirt(data = df, model = spec, itemtype = "3PL", SE = T)

Full-information item factor analysis with 1 factor(s).
Converged within 1e-04 tolerance after 136 EM iterations.
mirt version: 1.40
M-step optimizer: BFGS
EM acceleration: Ramsay
Number of rectangular quadrature: 61
Latent density type: Gaussian

Information matrix estimated with method: Oakes
Second-order test: model is a possible local maximum
Condition number of information matrix =  783.5905

Log-posterior = -20064.48
Estimated parameters: 99
AIC = 40212.78
BIC = 40702.82; SABIC = 40388.38
```

Items	1PL				2PL				3PL			
	a	b	g	u	a	b	g	u	a	b	g	u
mathc1	0.960	-0.881	0	1	1.078	-0.816	0	1	1.385	-0.250	0.238	1
mathc2	0.960	-1.706	0	1	1.135	-1.516	0	1	1.198	-1.170	0.198	1
mathc3	0.960	-0.162	0	1	1.307	-0.145	0	1	1.529	0.080	0.095	1
mathc4	0.960	-0.412	0	1	1.308	-0.349	0	1	2.095	0.170	0.235	1
mathc5	0.960	1.104	0	1	0.627	1.556	0	1	2.282	1.443	0.187	1
mathc6	0.960	-0.696	0	1	1.352	-0.569	0	1	1.428	-0.432	0.060	1
mathc7	0.960	-0.457	0	1	1.044	-0.434	0	1	1.247	-0.081	0.141	1
mathc8	0.960	-0.562	0	1	0.897	-0.589	0	1	0.951	-0.395	0.071	1
mathc9	0.960	-2.089	0	1	1.415	-1.609	0	1	1.464	-1.443	0.129	1
mathc11	0.960	-0.668	0	1	1.243	-0.571	0	1	1.956	0.092	0.293	1
mathc12	0.960	0.470	0	1	0.856	0.514	0	1	1.130	0.802	0.114	1
mathc13	0.960	-1.212	0	1	1.535	-0.910	0	1	2.546	-0.207	0.360	1
mathc14	0.960	0.552	0	1	0.775	0.655	0	1	0.887	0.819	0.060	1
mathc15	0.960	0.498	0	1	0.726	0.624	0	1	0.858	0.838	0.075	1
mathc16	0.960	-0.216	0	1	0.829	-0.237	0	1	0.942	0.027	0.090	1
mathc17	0.960	0.715	0	1	0.837	0.795	0	1	1.254	1.050	0.125	1
mathc18	0.960	0.767	0	1	1.107	0.688	0	1	1.253	0.757	0.034	1
mathc19	0.960	0.290	0	1	0.551	0.475	0	1	1.432	1.302	0.297	1
mathc21	0.960	-0.004	0	1	0.607	0.011	0	1	3.411	1.149	0.407	1
mathc22	0.960	-0.253	0	1	0.990	-0.250	0	1	1.401	0.295	0.212	1
mathc23	0.960	-0.085	0	1	0.688	-0.099	0	1	1.428	0.863	0.316	1
mathc24	0.960	-0.052	0	1	0.986	-0.053	0	1	1.460	0.460	0.202	1
mathc25	0.960	-0.170	0	1	1.230	-0.155	0	1	1.517	0.132	0.122	1
mathc26	0.960	0.332	0	1	0.893	0.350	0	1	1.177	0.672	0.125	1
mathc27	0.960	-0.282	0	1	1.479	-0.232	0	1	1.719	-0.018	0.094	1
mathc28	0.960	-0.026	0	1	1.949	-0.044	0	1	2.260	0.100	0.058	1
mathc29	0.960	0.119	0	1	1.229	0.092	0	1	1.928	0.476	0.173	1
mathc31	0.960	1.250	0	1	0.723	1.564	0	1	1.699	1.576	0.150	1
mathc32	0.960	-0.476	0	1	0.837	-0.522	0	1	1.600	0.481	0.350	1
mathc33	0.960	0.176	0	1	1.284	0.134	0	1	2.790	0.582	0.217	1
mathc34	0.960	0.165	0	1	0.678	0.229	0	1	0.863	0.645	0.131	1
mathc35	0.960	0.667	0	1	0.360	1.577	0	1	1.480	1.899	0.288	1
mathc36	0.960	1.449	0	1	1.060	1.352	0	1	1.601	1.359	0.070	1

Due to the limitation of `mirt package`, I can't constrain all the α to be 1. Rather, I can only set them to be equal across all the items. Therefore, in the estimation for the 1PL, the estimated universal α is .96 here.

Q1-a-(1)

Does it appear reasonable to assume all the items having an equal slope...

My Solution:
No.

From a aspect of test development, since this is a test about math placement, we should expect that items can discriminate students with different traits well. In addition, comparing the estimated parameters from `2PL` model versus `1PL`, these items' levels of discrimination spread along a wide range. It is reasonable to have items with higher levels of discrimination than others.

From a mathematics perspective, since the 1PL model is nested in the 2PL model, I conducted the `Likelihood Ratio Test` to compare the two models as followed:

$$D = -2[ln(L_{1pl}) - ln(L_{2pl})].$$

Plug the log likelihood estimated from the above code chunck, then one can have $D = 363.2$ at the degree of freedom of $df = df_{2pl} - df_{1pl} = 66 - 34 = 32$. Based on the Chi-squared distribution, the p value is lower than .001. Therefore, 2PL is better than 1PL, which means the discrimination is preferred.

Q1-a-(2)

Does it appear useful to include a guessing parameter in the model...

My Solution:

Yes, it is useful.

Intuitively, it is reasonable to include a guessing parameter since this a test with multiple choice and guessing is very possible. In addition, by looking through all the guessing parameters, one can find that the `matchc13`, `matchc21`, and `matchc32` do have quite high guessing rate, i.e., all above .30.

However, in terms of model comparison, when using the LRT test again to compare the 2PL vs 3PL model, one can have $D = 28.28$ at 33 degree of freedom, $P = .701$. Based on the parsimony rule, one should endorse the simpler model, i.e., the 2PL.

Therefore, my overall conclusion is including a guessing parameter is useful in this scenario. A practitioner should choose the either model based on their purpose since these two models do not differ a lot.

Q1-a-(3)

Evaluate the goodness of fit of the items with the option of the chi-square test. . .

My Solution:

I conduct the item fit analysis on each model and summarize the results into one table to make the layout concise.

```
> # get the item fit indices for each model
> item_fit_1pl <- itemfit(irt_1pl, na.rm = T)
> item_fit_2pl <- itemfit(irt_2pl, na.rm = T)
> item_fit_3pl <- itemfit(irt_3pl, na.rm = T)
>
> # combine all the outputs into one table
> item_fit_all <- cbind(item_fit_1pl[,c("item")],
+                       round(item_fit_1pl[,c("S_X2", "p.S_X2")],4),
+                       round(item_fit_2pl[,c("S_X2", "p.S_X2")],4),
+                       round(item_fit_3pl[,c("S_X2", "p.S_X2")],4))
> names(item_fit_all)[1] <- "item"
>
> # get all the item fit indices for 1PL, 2PL, and 3PL model
> item_fit_all
      item    S_X2 p.S_X2    S_X2 p.S_X2    S_X2 p.S_X2
1   mathc1  24.8711 0.4128 23.0566 0.4575 23.8800 0.3536
2   mathc2  28.1234 0.2112 30.4743 0.1074 28.6024 0.1239
3   mathc3  27.2722 0.2919 22.4566 0.4929 21.7064 0.4775
4   mathc4  26.5449 0.3261 19.2254 0.6314 17.4558 0.6831
5   mathc5  97.3404 0.0000 52.8362 0.0014 39.3238 0.0183
6   mathc6  35.2844 0.0643 26.8461 0.2171 23.9949 0.2933
7   mathc7  30.6375 0.1645 30.4324 0.1708 30.8269 0.1271
8   mathc8  31.1674 0.1490 31.5235 0.1393 23.0747 0.4564
9   mathc9  31.5513 0.0854 16.4299 0.6896 12.8670 0.7994
10 mathc11  24.7300 0.4205 17.4787 0.7364 16.8747 0.7187
11 mathc12  23.5755 0.4861 22.8300 0.5298 20.6982 0.5995
12 mathc13  35.9083 0.0422 24.6484 0.2627 19.3900 0.3682
13 mathc14  27.5382 0.2800 23.1188 0.5128 20.5069 0.6112
14 mathc15  33.5261 0.0934 21.0409 0.6903 17.2351 0.7976
15 mathc16  26.6813 0.3195 25.3271 0.3882 22.7550 0.5343
16 mathc17  21.2673 0.6229 17.9679 0.8046 17.1584 0.8014
17 mathc18  21.5747 0.6046 22.3173 0.5012 19.1247 0.6376
```

```
18 mathc19  54.0475 0.0004 20.7629 0.7058 20.1995 0.6854
19 mathc21  61.7627 0.0000 33.2577 0.1247 23.3450 0.4408
20 mathc22  20.4045 0.6736 20.4046 0.6736 20.8114 0.5926
21 mathc23  20.8365 0.6483 14.4939 0.9524 13.5269 0.9566
22 mathc24  20.9306 0.6428 21.0813 0.6339 21.0628 0.5773
23 mathc25  24.9448 0.4088 21.3330 0.5608 21.4360 0.4939
24 mathc26  19.3543 0.7328 17.6730 0.8186 18.2240 0.7452
25 mathc27  45.0529 0.0057 29.1187 0.1112 27.4429 0.1567
26 mathc28  65.4501 0.0000 24.1670 0.2352 23.9470 0.1982
27 mathc29  20.6986 0.6564 16.3725 0.8389 14.2529 0.8923
28 mathc31  38.2013 0.0242 24.7382 0.4771 22.2939 0.5026
29 mathc32  26.7508 0.3162 22.5725 0.5451 21.6350 0.5424
30 mathc33  29.0666 0.2176 21.2045 0.5686  8.0644 0.9949
31 mathc34  37.3521 0.0403 26.8718 0.3623 24.8798 0.4123
32 mathc35 109.6358 0.0000 32.7703 0.2048 35.3966 0.1033
33 mathc36  28.9987 0.2202 25.1823 0.3410 23.7681 0.3595
```

Item `mathc5` shows bad item fit in all three models. In addition, `matchc19`, `matchc21`, `matchc27`, `matchc28`, `matchc31`, `matchc34`, and ‘`matchc35` show bad fit in 1PL model only.

Q1-a-(4)

Evaluate the overall fit of the model. Which model do you prefer for this data?...

My Solution:

```
> # get the fit indices for 1PL model
> M2(irt_1pl, na.rm = T)
            M2  df p      RMSEA    RMSEA_5   RMSEA_95      SRMSR       TLI
stats 992.2492 527 0 0.03104487 0.02805368 0.03398001 0.05781072 0.9632371
            CFI
stats 0.9633067
>
> # get the fit indices for 2PL model
> M2(irt_2pl, na.rm = T)
            M2  df            p      RMSEA    RMSEA_5   RMSEA_95      SRMSR
stats 678.0246 495 7.852727e-08 0.02009113 0.01617592 0.02371771 0.03274386
            TLI       CFI
stats 0.9846029 0.9855652
>
> # get the fit indices for 3PL model
> M2(irt_3pl, na.rm = T)
            M2  df            p      RMSEA    RMSEA_5   RMSEA_95      SRMSR
stats 604.7095 462 8.359281e-06 0.01836359 0.01400902 0.02228458 0.03106851
            TLI       CFI
stats 0.9871369 0.9887448
```

Past studies have recommended that a TLI of or above .95, a CFI of or above .95, an RMSEA of or below .05, and an SRMR of or below .05 could indicate a very good fit. Based on those criteria, all three model demonstrate very good model fit.

Next, by conducting the Likelihood Ratio Test (LRT) on 1PL vs. 2PL and 2PL vs. 3PL (finished in `Q1-a-(1)` and `Q1-a-(2)`), 2PL is preferred since it is significantly different from 1PL. In addition, LRT shows there is no significant difference in 2PL and 3PL. Based on the parsimony rule, I prefer 2PL model on this data.

Q1-a-(5)

Using the estimates obtained from the model you choose in part…

My Solution:

In the selected 2PL model, `matchc 28` shows the highest level of discrimination with $\alpha = 1.949$, and `matchc 35` has the lowest level of discrimination.

Next, to find the most informative item on a given trait level, I write a function that automatically search for the targeted item.

```
> # write a function to find the most informative item for a given trait
>
> most_info <- function(irt_model, trait){
+   # irt_model can be any estimated IRT model
+   # trait can be any given trait level
+   a <- 0
+   item_num <- 0
+   for (i in 1:33){
+     # extract the item information at a given trait
+     info_temp <- iteminfo(extract.item(irt_model, i), trait)
+     # dynamically updated the best target
+     if (info_temp > a){
+       a <- info_temp
+       item_num <- i
+     }
+   }
+   out <- list(item_num = item_num,
+               info_value = a)
+   return(out)
+ }
```

Then, I applied this function to find the most informative items on the given trait levels of $\theta = -1.5$, $\theta = 0$, and $\theta = 1.5$.

```
> # using a for loop to get all results in one sitting
> for (theta in c(-1.5, 0, 1.5)){
+   theta_out <- most_info(irt_2pl, theta)
+   print(paste0("The most informative item on given trait theta=", theta," is the item ", theta_out$it
+   print(paste0("     with the infomation value of ", round(theta_out$info_value,4)))
+ }
[1] "The most informative item on given trait theta=-1.5 is the item 9, i.e., the mathc9"
[1] "     with the infomation value of 0.4978"
[1] "The most informative item on given trait theta=0 is the item 26, i.e., the mathc28"
[1] "     with the infomation value of 0.9477"
[1] "The most informative item on given trait theta=1.5 is the item 33, i.e., the mathc36"
[1] "     with the infomation value of 0.2789"
```

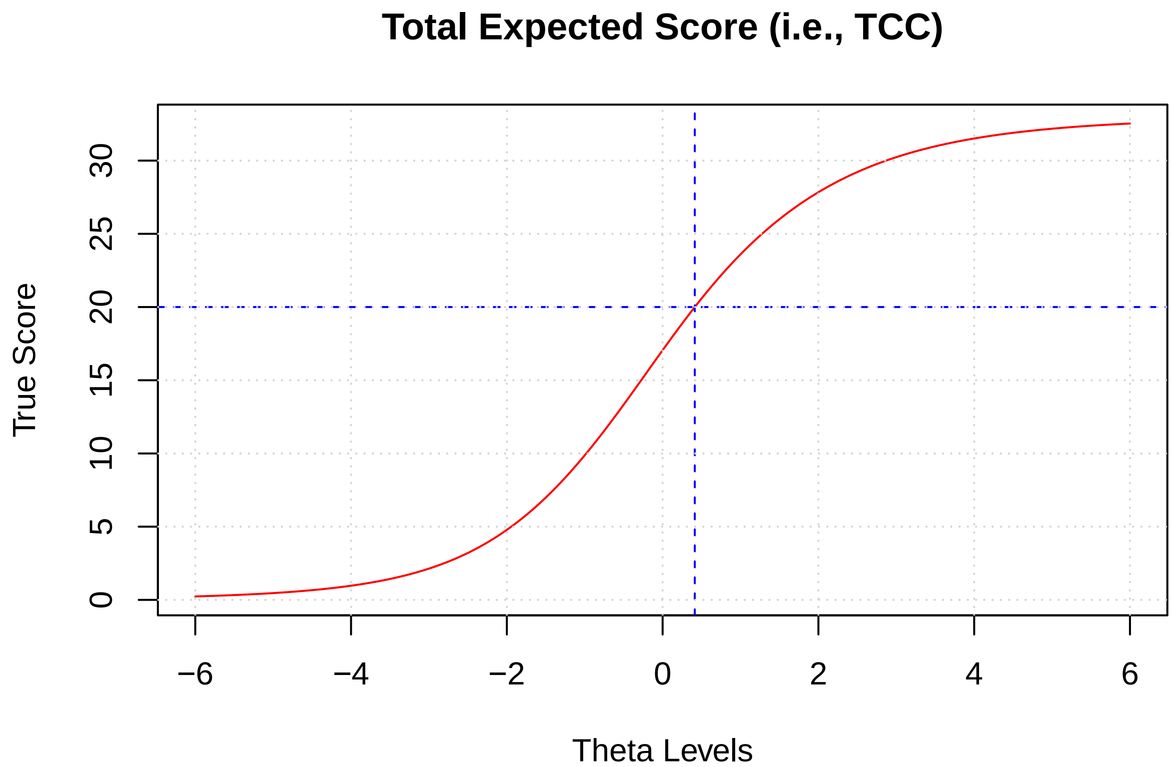
As shown above, the most informative item on $\theta = -1.5$ is the item `matchc9`. The most informative item on $\theta = 0$ is the item `matchc28`. The most informative item on $\theta = 1.5$ is the item `matchc36`.

Q1-b

Using the estimates obtained from the model you choose in part…

My Solution: As discussed above, I used the 2PL model to conduct the analysis. First, I got all the expected score (i.e., true score) for each trait levels using the `expected.test()` function. Second, I located the trait score with a corresponding true score 20.

```
> # get the thetas from this sample
> theta <- fscores(irt_2pl, method = 'EAP')
>
> # re-define a vector of trait levles
> theta_tempt <- as.matrix(seq(-6,6,0.01))
>
> # find the trait level who have the true score of 20
> tscore <- expected.test(irt_2pl, theta_tempt)
> theta_20 <- mean(theta_tempt[tscore >19.9 & tscore <20.1])
>
> # plot the TCC curve!
> plot(theta_tempt, tscore, type = "l", col="red",
+      xlim = c(-6,6),
+      main="Total Expected Score (i.e., TCC)",
+      xlab="Theta Levels", ylab="True Score")
> abline(h = 20,col="blue",lty=2)
> abline(v = 0.413,col="blue",lty=2)
> grid()
```



The TCC curve shows that the trait level of true score 20 is 0.41. Therefore, we need to select 10 items to have the maximum information on the 0.41.

Next, I try to get the information values of each item on 0.41 and sort these values in decreasing order. Finally, select the first ten items to make the shorter test.

```
> info_set <- c()
> # using a for loop to get all info value at this given trait
> for (i in 1:33) {
+   info_tempt <- iteminfo(extract.item(irt_2pl, i), theta_20)
+   info_set[i] <- info_tempt
+ }
> # make a new df
> info_matrix <- data.frame(
+   item = item_fit_2pl[,c("item")],
+   info_value = info_set
+ )
> # sort this df in decreasing order
> info_matrix <- info_matrix[order(-info_matrix$info_value),]
> # get the first 10 items
> info_matrix[c(1:10),]
      item info_value
26 mathc28  0.7855788
25 mathc27  0.4398556
30 mathc33  0.3991881
3   mathc3  0.3752177
27 mathc29  0.3633219
4   mathc4  0.3375869
23 mathc25  0.3359483
6   mathc6  0.3034735
17 mathc18  0.2993334
10 mathc11  0.2719404
```

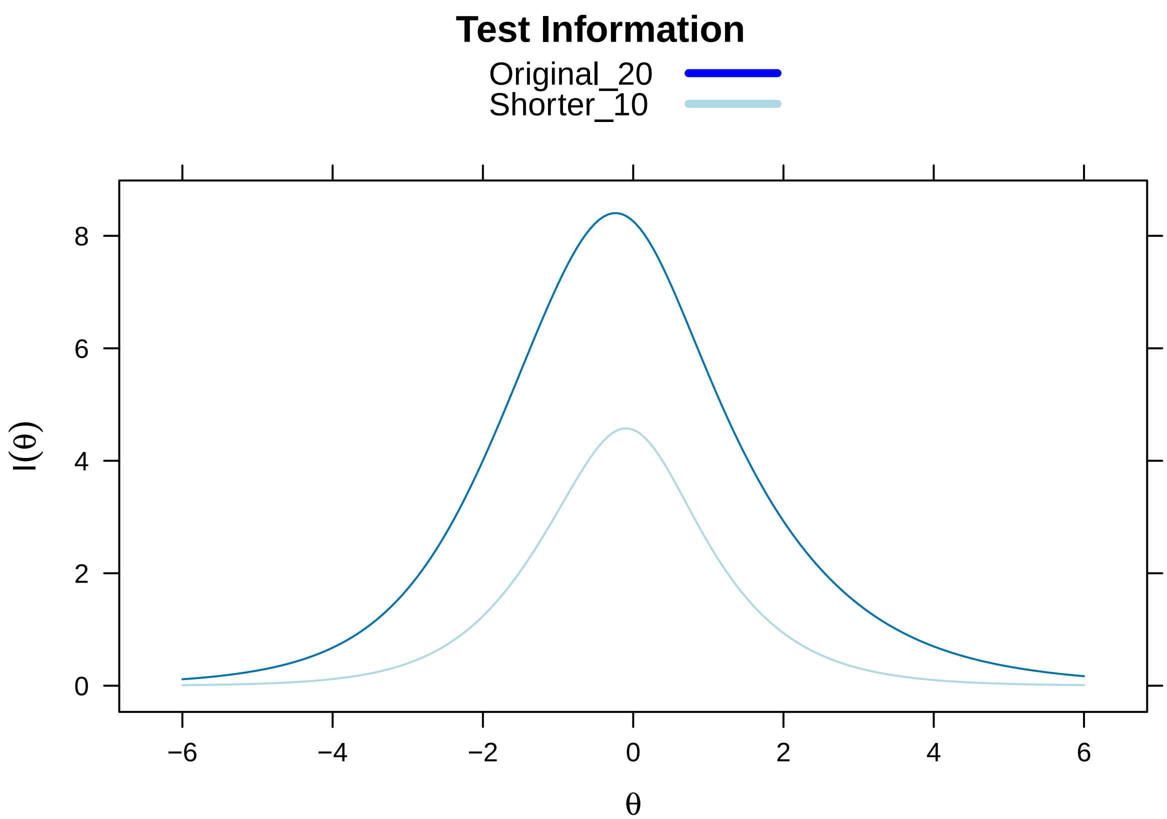
As shown above, I will select these ten items to create a shorter version of test.

```
> df_short <- df[,which(names(df) %in% info_matrix$item[1:10])]
> # fit 2PL on this short test
> irt_2pl_short <- mirt(df_short, model = 1, itemtype = "2PL", SE=T)
Iteration: 1, Log-Lik: -6358.356, Max-Change: 0.54171Iteration: 2, Log-Lik: -6291.212, Max-Change: 0.32

Calculating information matrix...
> irt_2pl_short

Call:
mirt(data = df_short, model = 1, itemtype = "2PL", SE = T)

Full-information item factor analysis with 1 factor(s).
Converged within 1e-04 tolerance after 19 EM iterations.
mirt version: 1.40
M-step optimizer: BFGS
EM acceleration: Ramsay
Number of rectangular quadrature: 61
Latent density type: Gaussian

Information matrix estimated with method: Oakes
Second-order test: model is a possible local maximum
Condition number of information matrix =  9.291311

Log-likelihood = -6266.591
```

```
Estimated parameters: 20
AIC = 12573.18
BIC = 12672.18; SABIC = 12608.66
```

Now to get the person trait estimation.

```
>
> library(latticeExtra)
> key = list(colums=1,
+            text=list(lab = c("Original_20","Shorter_10")),
+            lines=list(lwd=4, col=c("blue","lightblue")))
> p1 <- plot(irt_2pl, type= "info",key=key)
> p2 <- update(plot(irt_2pl_short, type="info"), col="lightblue")
> p1+p2
```



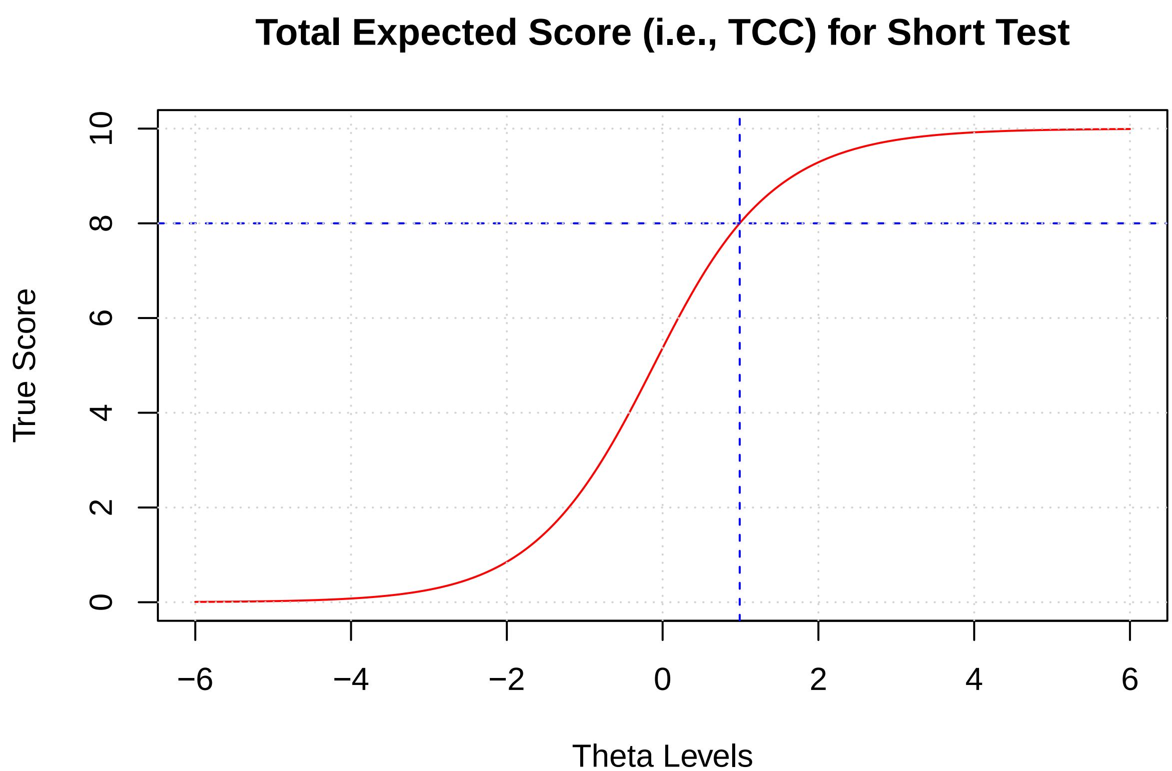
The plot shows that the peak of shorter test's information is closer to the target trait level 0.41. However, it provides less information comparing to the original 20 items test, which means shorter test is less accurate on estimating the given trait level comparing the to the original 20 items.

Q1-c

Assume a student with a true score of 8 on the ...

My Solution:

```
> # get the corresponding true score along the trait scores
> tscore <- expected.test(irt_2pl_short, theta_tempt)
> theta_8 <- mean(theta_tempt[tscore >7.9 & tscore <8.1])
> # plot the TCC
> # plot the TCC curve!
> plot(theta_tempt, tscore, type = "l", col="red",
+      xlim = c(-6,6),
+      main="Total Expected Score (i.e., TCC) for Short Test",
+      xlab="Theta Levels", ylab="True Score")
> abline(h = 8,col="blue",lty=2)
> abline(v = theta_8,col="blue",lty=2)
> grid()
```



The plot shows that the corresponding trait level is around 0.99. Next, I extract the information at this level and calculate the standard error.

```
> # get the test info at the theta = 2.255
> test_info <- testinfo(irt_2pl_short, theta_tempt)
> info_8 <- test_info[theta_tempt == round(theta_8,2)]
>
> # calculate the SE based on the relation between the Info and SE
> se_8 <- 1/ sqrt(info_8)
> se_8
numeric(0)
```

Based on the all the calculation above, for a student with true score of 8, his/ her corresponding trait level for this short test is 0.99 with standard error .

Q2

Suppose for three items ...

My Solution:
From the lecture note, we know that

$$P(\mu_{ij}|S_j) = \frac{\prod_{i=1}^{n} \epsilon^{\mu_{ij}}{}_i}{\sum_{(\mu_{ij})}^{r} \prod_{i=1}^{n} \epsilon^{\mu_{ij}}{}_i},$$

where (μ_{ij}) is all the possible response vectors given the total score is r. The possible response vectors are

$$X_1 = [1, 1, 0], X_2 = [1, 0, 1], X_3 = [0, 1, 1].$$

In Rasch model, since

$$\beta = log(\delta) = log(\frac{1}{\epsilon}) = -log(\epsilon),$$

one can have

$$\epsilon = e^{(-\beta)}.$$

Therefore, we can easily have $\epsilon_1 = 4.482$, $\epsilon_2 = .607$, and $\epsilon_3 = .135$.

Based on all the information above, one can have

$$P(X_1|\eta_j) = \frac{\epsilon_1\epsilon_2}{\epsilon_1\epsilon_2 + \epsilon_1\epsilon_3 + \epsilon_2\epsilon_3} = .798,$$

$$P(X_2|\eta_j) = \frac{\epsilon_1\epsilon_3}{\epsilon_1\epsilon_2 + \epsilon_1\epsilon_3 + \epsilon_2\epsilon_3} = .178,$$

$$P(X_3|\eta_j) = \frac{\epsilon_2\epsilon_3}{\epsilon_1\epsilon_2 + \epsilon_1\epsilon_3 + \epsilon_2\epsilon_3} = .024.$$