HUDM6052 Psychometric II Homework_03

Chenguang Pan (cp3280@tc.columbia.edu)

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Contents

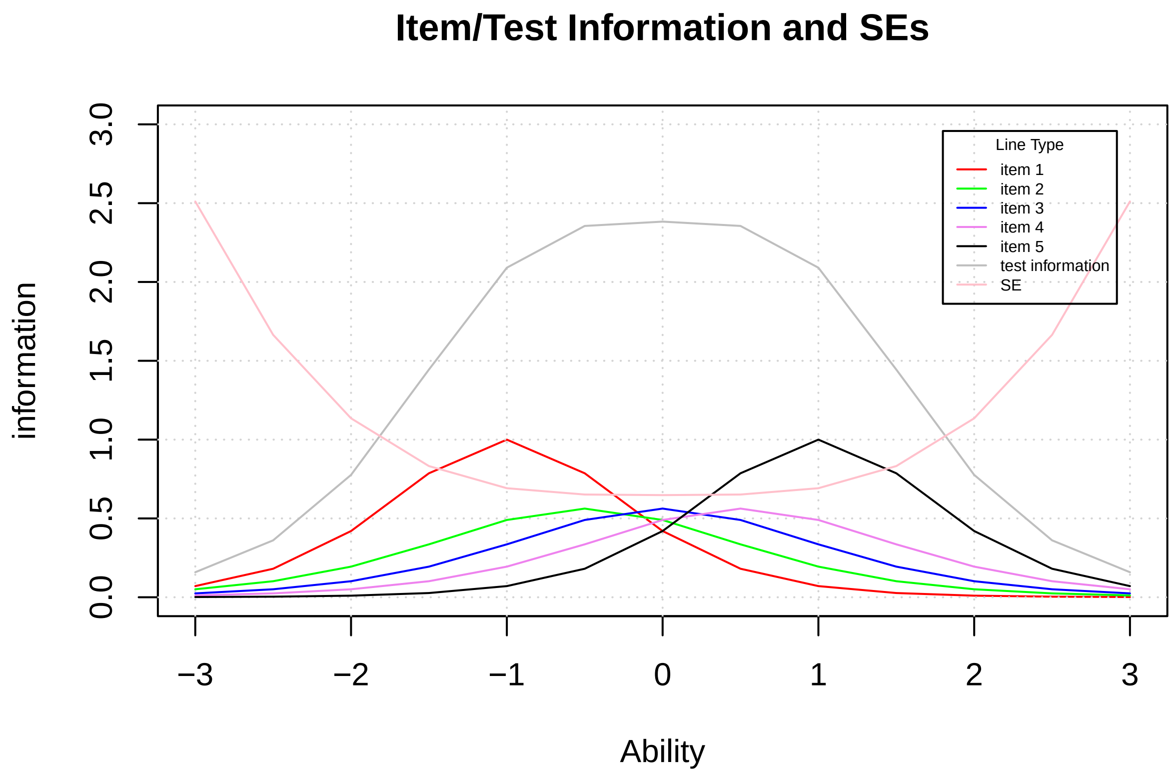
Q1

Using the item parameters given in the Table…

My Solution:

```
> # write the corresponding information function
> iif <- function(theta, a, b){
+   # get the logit
+   Z <- a*(theta-b)
+   # get the probability
+   out <- 1/(1 + exp(-Z))
+   info <- out*(1-out)*(a^2)
+   return(info)
+ }
>
> # set the ability range
> theta <- seq(-3,3, by=0.5)
> # set the item parameters
> a <- c(2, 1.5, 1.5, 1.5, 2)
> b <- c(-1, -0.5, 0, 0.5, 1)
> # define the color
> color_set <- c("red", "green", "blue","violet","black")
>
> # plot the item information function
> info_out <- iif(theta, a[1], b[1])
> # create a vector to sum up all the information function
> test_info <- info_out
>
> # initialize the plot by plotting the first item
> plot(theta, info_out,type = "l", col=color_set[1],
```

```
+      main = "Item/Test Information and SEs",
+      xlab = "Ability", ylab = "information",
+      ylim = c(0,3))
> grid()
>
> # plot the rest item using a for loop
> for (i in 2:5) {
+   info_out_i <- iif(theta,a[i],b[i])
+   lines(theta, info_out_i, type ="l", col=color_set[i])
+   test_info <- test_info +info_out_i
+ }
>
> # draw the test information function
> lines(theta, test_info, type = "l", col="gray")
> # plot the SE
> SE <- c()
> for (j in 1:length(theta)) {
+   se_j <- 1/sqrt(sum(iif(theta[j],a,b)))
+   SE[j] <- se_j
+ }
> lines(theta, SE, type = "l", col="pink")
>
> # add a legend
> legend('topright',inset=0.05,c("item 1","item 2","item 3","item 4","item 5",
+                                 "test information","SE"),
+        lty=1,col=c("red", "green","blue","violet","black","gray","pink"),
+        title="Line Type", cex = 0.5)
```



Q2.a

a. For each of the six items given in the Table below...

My Solution:
The maximum value of a 3PL's information function is at

$$\theta_{max} = \beta_i + \frac{1}{a_i} log[\frac{1 + \sqrt{1 + 8c_i}}{2}].$$

Therefore, I write a function to get the optimal θ_{max} first. And then send this value together with the parameters into the information function for 3PL model to get the results.

```
> # write a function to get the theta_max
> get_theta <- function(a,b,c){
+   out <- b + log(0.5+0.5*sqrt(1+8*c))/a
+   return(out)
+ }
>
> # write the 3pl information function
> iif_3pl <- function(theta, a, b, c){
+   z <- a*(theta - b)
+   p <- c + (1-c)/(1+exp(-z))
+   p_star <- 1/(1 + exp(-z))
+   I <- (a^2)*p*(1-p)*(p_star/p)^2
```

```
+   return(I)
+ }
```

Next, plug the given paramters into the functions to get the results.

```
> # load the given parameters vectors
> b <- c(1,1,1,-1.5,-0.5,0.5)
> a <- c(1.8,0.8,1.8,1.8,1.2,0.4)
> c <- c(0,0,0.25,0,0.1,0.15)
>
> # using a for loop to get all the required values
> theta_vec <- c()
> info_vec <- c()
> for (i in 1:length(b)) {
+   # get the optimal theta value
+   theta_max <- get_theta(a = a[i],b = b[i],c = c[i])
+   theta_vec[i] <- theta_max
+   # get the maximum value of information of item i
+   info_max <- iif_3pl(theta = theta_max,a = a[i],b = b[i],c = c[i])
+   info_vec[i] <- info_max
+ }
>
> # Merge all the values as a dataframe
> df_out <- data.frame(
+   item = seq(1,6),
+   theta_max = theta_vec,
+   info_max = info_vec
+ )
>
> df_out
  item  theta_max   info_max
1    1  1.0000000 0.81000000
2    2  1.0000000 0.16000000
3    3  1.1732808 0.50122974
4    4 -1.5000000 0.81000000
5    5 -0.3685794 0.29665631
6    6  1.0410421 0.02998276
```

The maximum values of information and corresponding θs are shown at end of the code chunk above.

Q2.b

b. Which item would you choose to make up a two-item...

My Solution:
I will choose the `item 1` and `item 2` to make a two-item test since they have the maximum information at the $\theta = 1.0$, which means this test can more accurately measure this given test-taker's ability. The test information at $\theta = 1.0$ is

$$0.81 + 0.16 = 0.97.$$

Q3.a

a. Determine the standard error of the estimate...

My Solution:

```
> # load the given paramters
> a <- c(1,1,2,2)
> b <- c(0,1,1,1.5)
> theta <- 1.5
>
> # using the information function created in the Q1 to get the
> # vector of information at the theta=1.5 for each item
> info_vec <- iif(theta=1.5, a, b)
>
> # get the SE
> SE_j <- 1/sqrt(sum(info_vec))
> SE_j
[1] 0.6787507
```

Therefore, the SE for the test-taker with estimated trait $\theta = 1.5$ is 0.679.

Q3.b

b. Construct a 95% confidence interval for...

My Solution:
The 95% confidence interval of the estimated $\theta = 1.5$ is

$$95\%CI = 1.5 \pm 1.96 \times 0.679.$$

Therefore, $95\%CI$ is $[.169, 2.831]$.