HUDM6052 Psychometric II Homework_04

Chenguang Pan

2023-11-14

Contents

Q1-a-Parameters-Estimation

Fit the 1PL, 2PL, and 3PL models…report the estimated item parameters in separated tables

My Solution:
To make the layout concise and good-looking, I intentionally omitted the codes for data cleaning and some instant display of running outcomes. I attached the estimated item parameters from all three models into one table to save space.

```
> # load the binary response dataset
> library(mirt)
> df <- read.csv("/Users/panpeter/Desktop/PhD_Learning/HUDM6052 Psychometric II/HUDM6052_Psychometic_II/
> df <- df[,-1]
>
> # ---------------------------------------------------
> #                       Run 1PL
> # ---------------------------------------------------
>
> # specify the model for 33 items loading on 1 dimension
> # and constrain all the item slope to be equal for 1PL estimation
> spec <- 'F = 1-33
+ CONSTRAIN = (1-33, a1)'
>
> # estimated the model
> # since I constrained all slopes to be equal, here the argument "2PL" is safe
> irt_1pl <- mirt(df, model = spec, itemtype = "2PL", SE=T)
Iteration: 1, Log-Lik: -20276.377, Max-Change: 0.18304Iteration: 2, Log-Lik: -20265.381, Max-Change: 0.0

Calculating information matrix...
> irt_1pl

Call:
mirt(data = df, model = spec, itemtype = "2PL", SE = T)
```

```
Full-information item factor analysis with 1 factor(s).
Converged within 1e-04 tolerance after 25 EM iterations.
mirt version: 1.40
M-step optimizer: BFGS
EM acceleration: Ramsay
Number of rectangular quadrature: 61
Latent density type: Gaussian

Information matrix estimated with method: Oakes
Second-order test: model is a possible local maximum
Condition number of information matrix =  50.40657

Log-likelihood = -20260.22
Estimated parameters: 66
AIC = 40588.44
BIC = 40756.74; SABIC = 40648.75
```

```
> # ---------------------------------------------------
> #                       Run 2PL
> # ---------------------------------------------------
> irt_2pl <- mirt(df, model = 1, itemtype = "2PL", SE=T)
Iteration: 1, Log-Lik: -20276.377, Max-Change: 0.63072Iteration: 2, Log-Lik: -20095.717, Max-Change: 0.2

Calculating information matrix...
> irt_2pl

Call:
mirt(data = df, model = 1, itemtype = "2PL", SE = T)

Full-information item factor analysis with 1 factor(s).
Converged within 1e-04 tolerance after 32 EM iterations.
mirt version: 1.40
M-step optimizer: BFGS
EM acceleration: Ramsay
Number of rectangular quadrature: 61
Latent density type: Gaussian

Information matrix estimated with method: Oakes
Second-order test: model is a possible local maximum
Condition number of information matrix =  11.16451

Log-likelihood = -20078.62
Estimated parameters: 66
AIC = 40289.23
BIC = 40615.92; SABIC = 40406.3
```

```
> # ---------------------------------------------------
> #                       Run 3PL
> # ---------------------------------------------------
> # specify the model
> spec <- 'F = 1-33
+ PRIOR = (1-33, g, norm, -1.1, 2)'
> irt_3pl <- mirt(df, model = spec, itemtype = "3PL", SE = T)
```

```
Iteration: 1, Log-Lik: -20990.083, Max-Change: 1.61517Iteration: 2, Log-Lik: -20300.703, Max-Change: 1.
```

Items	1PL				2PL				3PL			
	a	b	g	u	a	b	g	u	a	b	g	u
mathc1	0.960	-0.881	0	1	1.078	-0.816	0	1	1.385	-0.250	0.238	1
mathc2	0.960	-1.706	0	1	1.135	-1.516	0	1	1.198	-1.170	0.198	1
mathc3	0.960	-0.162	0	1	1.307	-0.145	0	1	1.529	0.080	0.095	1
mathc4	0.960	-0.412	0	1	1.308	-0.349	0	1	2.095	0.170	0.235	1
mathc5	0.960	1.104	0	1	0.627	1.556	0	1	2.282	1.443	0.187	1
mathc6	0.960	-0.696	0	1	1.352	-0.569	0	1	1.428	-0.432	0.060	1
mathc7	0.960	-0.457	0	1	1.044	-0.434	0	1	1.247	-0.081	0.141	1
mathc8	0.960	-0.562	0	1	0.897	-0.589	0	1	0.951	-0.395	0.071	1
mathc9	0.960	-2.089	0	1	1.415	-1.609	0	1	1.464	-1.443	0.129	1
mathc11	0.960	-0.668	0	1	1.243	-0.571	0	1	1.956	0.092	0.293	1
mathc12	0.960	0.470	0	1	0.856	0.514	0	1	1.130	0.802	0.114	1
mathc13	0.960	-1.212	0	1	1.535	-0.910	0	1	2.546	-0.207	0.360	1
mathc14	0.960	0.552	0	1	0.775	0.655	0	1	0.887	0.819	0.060	1
mathc15	0.960	0.498	0	1	0.726	0.624	0	1	0.858	0.838	0.075	1
mathc16	0.960	-0.216	0	1	0.829	-0.237	0	1	0.942	0.027	0.090	1
mathc17	0.960	0.715	0	1	0.837	0.795	0	1	1.254	1.050	0.125	1
mathc18	0.960	0.767	0	1	1.107	0.688	0	1	1.253	0.757	0.034	1
mathc19	0.960	0.290	0	1	0.551	0.475	0	1	1.432	1.302	0.297	1
mathc21	0.960	-0.004	0	1	0.607	0.011	0	1	3.411	1.149	0.407	1
mathc22	0.960	-0.253	0	1	0.990	-0.250	0	1	1.401	0.295	0.212	1
mathc23	0.960	-0.085	0	1	0.688	-0.099	0	1	1.428	0.863	0.316	1
mathc24	0.960	-0.052	0	1	0.986	-0.053	0	1	1.460	0.460	0.202	1
mathc25	0.960	-0.170	0	1	1.230	-0.155	0	1	1.517	0.132	0.122	1
mathc26	0.960	0.332	0	1	0.893	0.350	0	1	1.177	0.672	0.125	1
mathc27	0.960	-0.282	0	1	1.479	-0.232	0	1	1.719	-0.018	0.094	1
mathc28	0.960	-0.026	0	1	1.949	-0.044	0	1	2.260	0.100	0.058	1
mathc29	0.960	0.119	0	1	1.229	0.092	0	1	1.928	0.476	0.173	1
mathc31	0.960	1.250	0	1	0.723	1.564	0	1	1.699	1.576	0.150	1
mathc32	0.960	-0.476	0	1	0.837	-0.522	0	1	1.600	0.481	0.350	1
mathc33	0.960	0.176	0	1	1.284	0.134	0	1	2.790	0.582	0.217	1
mathc34	0.960	0.165	0	1	0.678	0.229	0	1	0.863	0.645	0.131	1
mathc35	0.960	0.667	0	1	0.360	1.577	0	1	1.480	1.899	0.288	1
mathc36	0.960	1.449	0	1	1.060	1.352	0	1	1.601	1.359	0.070	1

Due to the limitation of `mirt package`, I can't constrain all the α to be 1. Rather, I can only set them to be equal across all the items. Therefore, in the estimation for the 1PL, the estimated universal α is .96 here.

Q1-a-(1)

Does it appear reasonable to assume all the items having an equal slope…

My Solution:
No.

From a aspect of test development, since this is a test about math placement, we should expect that items can discriminate students with different traits well. In addition, comparing the estimated parameters from `2PL` model versus `1PL`, these items' levels of discrimination spread along a wide range. It is reasonable to have items with higher levels of discrimination than others.

From a mathematics perspective, since the 1PL model is nested in the 2PL model, I conducted the `Likelihood Ratio Test` to compare the two models as followed:

$$D = -2[ln(L_{1pl}) - ln(L_{2pl})].$$

Plug the log likelihood estimated from the above code chunck, then one can have $D = 363.2$ at the degree of freedom of $df = df_{2pl} - df_{1pl} = 66 - 34 = 32$. Based on the Chi-squared distribution, the p value is lower than .001. Therefore, 2PL is better than 1PL, which means the discrimination is preferred.

Q1-a-(2)

Does it appear useful to include a guessing parameter in the model…

My Solution: xw

Yes, it is useful.

Intuitively, it is reasonable to include a guessing parameter since this a test with multiple choice and guessing is very possible. In addition, by looking through all the guessing parameters, one can find that the `matchc13`, `matchc21`, and `matchc32` do have quite high guessing rate, i.e., all above .30.

However, in terms of model comparison, when using the LRT test again to compare the 2PL vs 3PL model, one can have $D = 28.28$ at 33 degree of freedom, $P = .701$. Based on the parsimony rule, one should endorse the simpler model, i.e., the 2PL.

Therefore, my overall conclusion is including a guessing parameter is useful in this scenario. A practitioner should choose the either model based on their purpose since these two models do not differ a lot.

```
> # get the fit indices for 1PL model
> M2(irt_1pl, na.rm = T)
            M2  df p      RMSEA    RMSEA_5   RMSEA_95      SRMSR       TLI
stats 992.2492 527 0 0.03104487 0.02805368 0.03398001 0.05781072 0.9632371
            CFI
stats 0.9633067
>
> # get the fit indices for 2PL model
> M2(irt_2pl, na.rm = T)
            M2  df            p      RMSEA    RMSEA_5   RMSEA_95      SRMSR
stats 678.0246 495 7.852727e-08 0.02009113 0.01617592 0.02371771 0.03274386
            TLI       CFI
stats 0.9846029 0.9855652
>
> # get the fit indices for 3PL model
> M2(irt_3pl, na.rm = T)
            M2  df            p      RMSEA    RMSEA_5   RMSEA_95      SRMSR
```

```
stats 604.7095 462 8.359281e-06 0.01836359 0.01400902 0.02228458 0.03106851
            TLI       CFI
stats 0.9871369 0.9887448
```