Introduction to Cognitive Diagnosis Modeling

Overview

- Background
- Conceptual example
- CDM specifics
- Common CDMs

Background

What are Diagnoses?

- The word and meaning of diagnosis is common in language.
- Meaning of diagnoses are deeply ingrained in our society –Seldom merits a second thought.

Definitions

- *American Heritage Dictionary definition of diagnosis:*

  –Generally
  - A critical analysis of the nature of something
  - The conclusion reached by such analysis

  –Medicine
  - The act or process of identifying of determining the nature and cause of a disease or injury through evaluation of a patient's history, examination, and review of laboratory data
  - The opinion derived from such an evaluation

  –Biology
  - A brief description of the distinguishing characteristics of an organism, as for taxonomic classification

Diagnosis

- The goal of diagnosis is to examine and analyze a condition for the purpose of identification and classification.

- In many contexts, diagnosis is carried out as a first step in determining appropriate interventions or remedial actions.
  - Example: Condition – "*My car does not start*"

  I need a mechanic to identify the reason (i.e., diagnose) why my car doesn't, and for him to take the necessary actions to address the problem

- A diagnosis is the *decision* that is being made based on information.

Diagnosis (Formalized)

- In diagnostic measurement, the procedures of diagnosis are formalized:
  - We make a set of observations
    - Usually through a set of test questions (items)

  - Based on these questions we make a decision as to the underlying state (or states) of a person
    - The competencies (skills) that a person has or has not mastered
    - The decision is the diagnosis
    - For this talk, a diagnosis is by its nature *discrete classification*

Background

- Assessments should aim to improve, and not merely ascertain the status of student learning (Wiggins, 1998; Stiggins, 2002).

- For test scores to facilitate learning, they need to be *interpretative*, *diagnostic*, *highly informative*, and *potentially prescriptive* (Pellegrino, Baxter, & Glaser, 1999).

- Most large-scale assessments are based on traditional (unidimensional) IRT models that provide single overall scores.
  - These scores are useful primarily for ranking students along a continuum.

Background (Cont'd)

- Within educational and psychological testing, providing a test score should give the information that is used for a diagnosis.

  - BUT, the score itself is not the diagnosis.
  - Need information about the competencies (skills) that a person has or has not mastered.
    - Helps classification.
    - Leads to possible tailored instruction and remediation.

Background (Cont'd)

- Alternative psychometric models that can provide inferences more relevant to classroom instruction and student learning currently exist.

- Such models are called *cognitive diagnosis models* (CDMs) or *diagnostic classification models* (DCMs).
  - They are developed specifically for diagnosing the mastery or non-mastery of multiple fine-grained ***attributes*** or ***skills***.
  - skills, cognitive processes, problem-solving strategies, knowledge states, etc.

Basic Terminology

- **Respondents:** The people on whom behavioral data are collected
- **Items:** Stimuli/prompts used to elicit (classify/diagnose) responses from respondents
  - Behavioral data considered primarily item responses; Not limited to only item responses → prompts or tasks
- **Diagnostic Assessment:** A method/instrument used to elicit behavioral data
  - Cognitive diagnostic assessment relies on applied cognitive psychology research

Basic Terminology

- **Attributes:** Unobserved dichotomous characteristics underlying the behavior (i.e., diagnostic status)
  - skills, cognitive processes, problem-solving strategies, knowledge states, etc
  - Latent variables linked to behaviors in diagnostic classification models

- **Diagnosis:** The classification of respondents into a particular attribute profile

- **Psychometric/Measurement Models:** Statistical models with latent variables used for analyzing item response data

- **Cognitive Diagnosis Models (CDMs)/ Diagnostic Classification Models (DCMs):** A class of measurement models with discrete latent variables used for developing statistically driven classifications/diagnoses of respondents

Diagnostic Classification Model Names

- Diagnostic classification models (DCMs) have been called many different things (Rupp & Templin, 2008)
  - Skills assessment models
  - Cognitive diagnosis models
  - Cognitive psychometric models
  - Latent response models
  - Restricted (constrained) latent class models
  - Multiple classification models
  - Structured located latent class models
  - Structured item response theory

Psychometric Soapbox

- CDMs are a small set of tools that must be adapted for common purpose
  - Part of a methodological toolbox that is used to classify respondents
  - Should also include content experts and end-users of the diagnoses

- CDMs link empirical observations and respondents characteristics
  - The models are only as good as underlying theories

The Role of CDMs (DCMs)

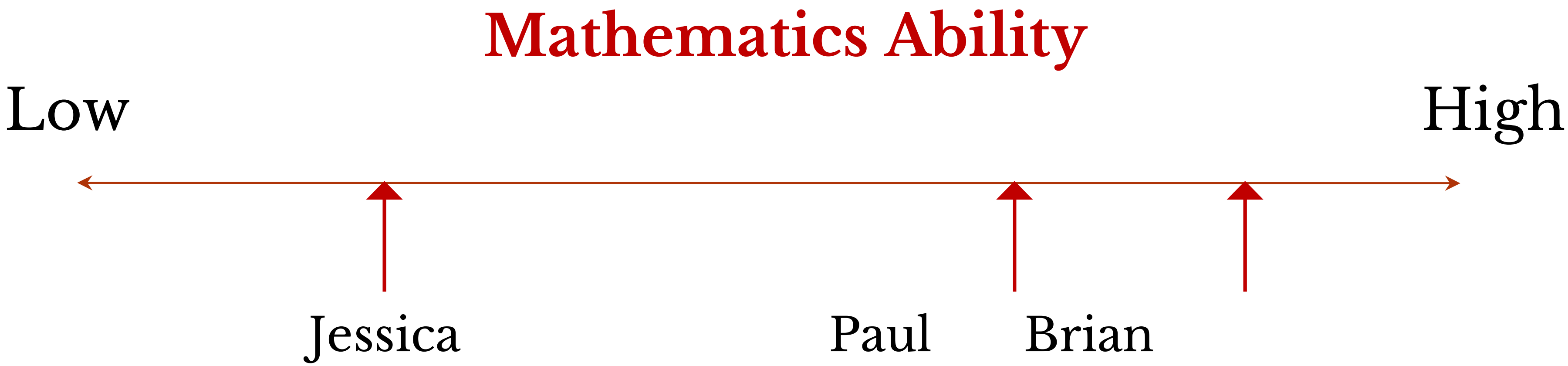
- Designing, implementing, and interpreting successful diagnostic assessments are challenging endeavors -- Involves subject-matter experts, measurement specialists, and potential stakeholders

  - CDMs play a small but crucial role in the comprehensive process
  - CDMs serve to support the validity of interpretations with empirical pieces of evidence

Conceptual Example

Traditional Psychometrics

- Imagine that an elementary teacher wants to test basic math ability.
- Using traditional psychometric approaches (CTT or IRT), the teacher could estimate an ability or test score for each student.
- By knowing each student's score, the students are ordered along a continuum as below.



Traditional Psychometrics (Cont'd)

- What results is an (weak) ordering of students.
  - Ordering is called weak because of error in estimates.
  - Brian > Paul> Jessica

- Questions that traditional psychometrics cannot answer:
  - Why is Jessica so low?
    - How can we get her some help?
  - How much ability is "enough" to pass?
    - How much is enough to be proficient?
  - What math skills have the students mastered?
    - If not mastered, what do we need to teach? What do they need to study?

Multiple Dimensions of Ability

- As an alternative, we could have expressed "mathematics ability" as a set of basic skills:
  - Addition
  - Subtraction
  - Multiplication
  - Division

- The set of skills represents the multiple dimensions of elementary mathematics ability.

Multiple Dimensions of Ability (Cont'd)

- Other psychometric approaches have been developed for multiple dimensions
  - Classical Test Theory – Scale subscores
  - Multidimensional Item Response Theory (MIRT)

- Yet, issues in application have remained:
  - Reliability of estimates is often poor for most practical test lengths
  - Dimensions are often very highly correlated
  - Large samples are needed to calibrate item parameters in MIRT

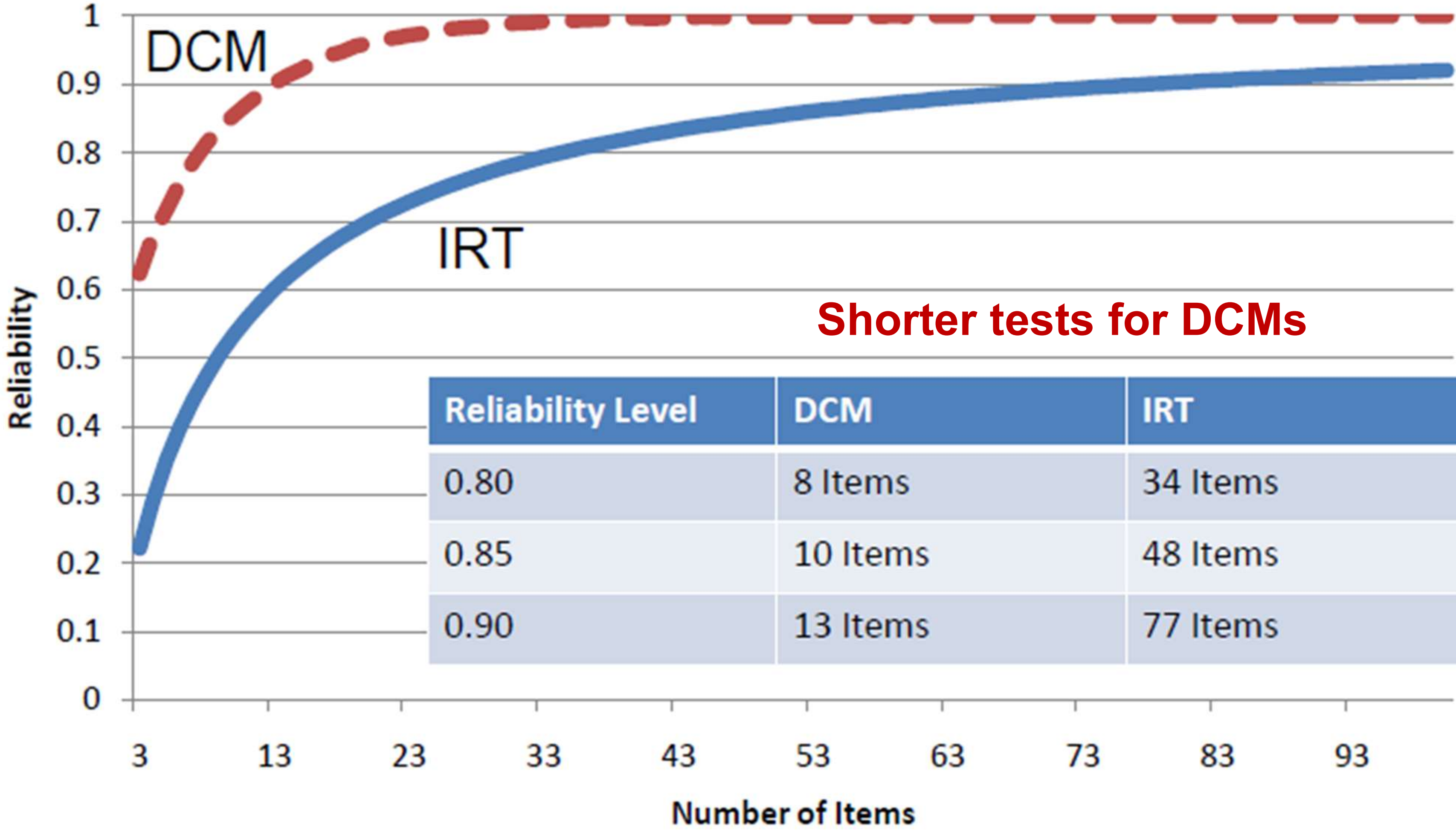
CDMs as an Alternative

- CDMs do not assign a single score. Instead, a ***profile*** of ***mastered attributes*** is given to respondents

	Mastered			Not Mastered		
Addition	Brian	Paul	Jessica			
Subtraction	Brian	Paul		Jessica		
Multiplication	Brian			Jessica	Paul	
Division	Brian			Jessica	Paul	

- CDMs provide respondents valuable information with fewer data demands
  - Higher reliability than comparable IRT/MIRT models
  - Complex item structures possible

Theoretical Reliability Comparison



Shorter tests for DCMs

Reliability Level	DCM	IRT
0.80	8 Items	34 Items
0.85	10 Items	48 Items
0.90	13 Items	77 Items

Psychometric Model Comparison

Using Traditional Models

- Has a score of 20
- Has a 75%, a grade of "C"
- Is in the 60th percentile of math
- Scored above the cut-off, passes math

Using CDMs

- *Is proficient* using Addition
- *Is proficient* using Subtraction
- *Should work* on Multiplication
- *Should work* on Division

CDM Specifics

CDM Specifics

- Let's expand on the idea of the basic math test
- Possible items may be
  2+3-1
  4/2
  (4×2)+3
- Not all items measure all attributes
- A Q-matrix is used to indicate the attributes measured by each item
  - This is the factor pattern matrix that assigns the loadings in confirmatory factor analysis

What is Q-matrix?

- Shows relationships between the items and the attributes (skills) that are used to classify respondents.

  - A Q-matrix traditionally contains the items in the rows and the attributes in the columns.
  - When each item is associated (or thought to measure) each attribute (skill)
    - 0 = an item does not measure the attribute
    - 1 = an item measures the attribute

The Q-Matrix

- An example of a Q-matrix using the math test

	Add	Sub	Mult	Div
2+3-1	1	1	0	0
4/2	0	0	0	1
(4 x 2)+3	1	0	1	0

Grain sizes of Attributes

- The degree of definitional specificity of an attribute is often referred to as the *definitional grain size* of the attributes.

- The grain size is driven by the level of specificity with which one would like to make statements about examinees.

- The grain size of an attribute is the resolution with which an investigator dissects a cognitive response process and describes its constituent components.

Grain Size of Attributes: $2\frac{4}{12}-\frac{7}{12}$

- Successful performance on the item requires a series of successful implementations of the attributes specified for the item.

- Required attributes:
  - (1) Borrowing from whole $=1\frac{16}{12}-\frac{7}{12}$
  - (2) Basic fraction subtraction $=1\frac{9}{12}$
  - (3) Reducing $=1\frac{3}{4}$
  - (4) Separating whole from fraction
  - (5) Converting whole to fraction

Different Levels of Grain Size

Construct: Number Subtraction Source: de la Torre & Douglas (2004)	
- convert a whole number to a fraction - separate a whole number from a fraction - simplify before subtracting - find a common denominator	- borrow from whole number part - column borrow to subtract - subtract numerators - simplify answer
Construct: Basic Artihmetic Skills Source: Kunina, Rupp, & Wilhelm (2008)	
- form a basic number representation - add and subtract (0 to 100) - add and subtract (100 to 1000) - multiply and divide (0 to 100)	- multiply and divide (100 to 1000) - perform inverse operations - solve problems with two operations - construct a mathematical model
Construct: Solving Linear Equations Source: Gierl, Leighton, & Hunka (2007)	
- understand meaning of symbols and conventions - comprehend textual description of problem - perform algebraic manipulations - solve linear equations	- solve quadratic equations - solve multiple equations simultaneously - construct a tabular representation - construct a graphical representation

Practical Issues with Grain sizes

- It is possible to further decompose individual attributes for more complex tasks.
  - That would increase the number of attributes.

- As the number of attributes increases, the number of latent variables in a CDM increases.
  - Attribute profiles and item parameters may become impossible to estimate statistically.

- It is important to fix the number of attributes to a statistically manageable number for a given diagnostic assessment length and respondent sample size.

Distinction for Attribute Specifications

- *Attribute label*
  - Is a word, phrase, or clause that reflects the key meaning of the attribute
- *Attribute definition*
  - Is a paragraph or short text that describes the different facets of the attribute in more detail
- *Coding instructions* for the attribute
  - Are specifications tied to a particular diagnostic assessment that help experts determine whether an attribute is measured by a particular item

Levels of Attribute Specifications

Attribute Label
Meaning of symbols and conventions
Attribute Definition
An item requires knowledge of the meaning of symbols and conventions, if the respondent needs to know…
(a) the arithmetic operations implied by +,-, ·, /, =, |term|, square root, exponent, >, <, and signed numbers
(b) implied operations in $2 \cdot n$, $2 \cdot (\text{term})$, $(\text{term})^2$, $n/2$
(c) that an inverse relationship exists between speed and distance travelled
Attribute Coding Instructions
When coding an item for presence (1) or absence (0) of this attribute, the following rules hold:
(i) Any combination of the operations (a) – (c) results in a code of (1).
(ii) No differential weight is assigned depending on how many operations listed in (a) – (c) are required to solve an item correctly. If at least one operation from (a), (b), or (c) is required, then a code of (1) is to be given.
(iii) If multiple solution paths to a problem exist and only selected solution paths require the use of this attribute, then a code of (1) is to be given.

Mapping to Attribute Specification

- It needs to be consistent with the cognitive theory that underlies the design of the diagnostic assessment in terms of the number of attributes, their level of definitional grain size, and their relationships to one another.
- It needs to be at a level of grain size that supports the desired interpretations and decisions about learners.
- It needs to be at a level of grain size that allows for the practical estimation of item parameters and attribute profiles with DCMs.

Respondent Profiles

- Examinees are characterized by profiles specifying which attributes have been mastered
  - Numeric values are arbitrary, but for our purposes
    - Mastery given a 1
    - Non-mastery given a 0
- For example,

	Add	Sub	Mult	Div
Examinee A	1	1	0	0

- Respondents profile estimates are in the form of probabilities of mastery (e.g., > .60 is "mastered")

Expected Responses to Items

Q-matrix

	Add	Sub	Mult	Div
2+3-1	1	1	0	0
4/2	0	0	0	1
(4 x 2)+3	1	0	1	0

By knowing which attributes are measured by each item and which attributes have been mastered by each respondent, we can determine the items that will likely be answered correctly by each respondent

Respondent Mastery

	Add	Sub	Mult	Div	
Respondent 1	1	1	0	0	Prob Ans #1
Respondent 2	0	1	0	1	Prob Ans #2
Respondent 3	1	0	1	0	Prob Ans #3
Respondent 4	1	1	1	0	Prob Ans #1 & #3

CDM Scoring and Score Reporting

Diagnostic Scoring Report

Student Name: Daphne

Review Your Answers

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Your Answer	✓	✓	✓	✓	a	c	✓	c	d	✓	✓	✓	c	✓	d	a	✓	b	a	a	d	c	b	a	c
Correct Answer	d	a	b	d	d	a	b	d	a	c	a	b	d	c	a	d	a	c	b	d	a	a	a	d	b
Difficulty	e	e	m	m	m	m	h	h	h	m	e	e	m	m	m	h	m	m	h	h	h	h	h	h	h

Score

You correctly answered 10 out of 25 questions.

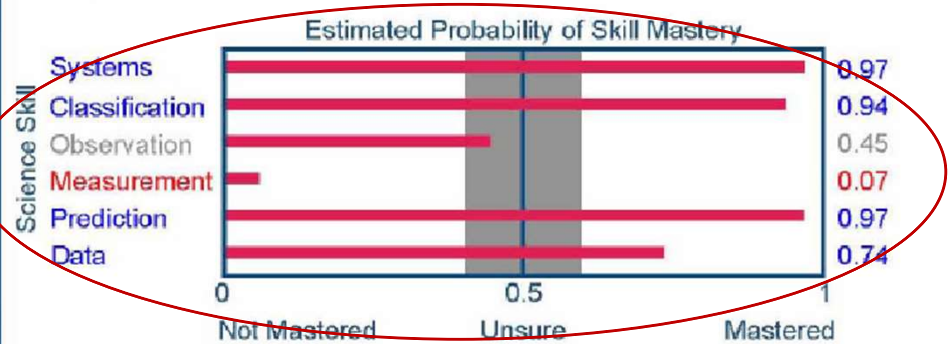
Easy: 4/4; Medium: 5/10; Hard: 1/11

Guide

✓ - Correct answer; o - Omitted answer

e - Easy; m - Medium; h - Hard

Improve Your Skills



Example Questions
3, 14, 2, 17, 19, 23, 9
3, 12, 13, 5, 2, 17, 18, 16, 24, 7
11, 15, 1, 8, 18
22, 20, 10, 11, 5, 6, 18, 25
4, 14, 20, 12, 5, 19, 9
22, 1, 19, 21

25-item benchmark test of basic 3rd grade science skills (Ackerman, et al., 2006)

Illustration

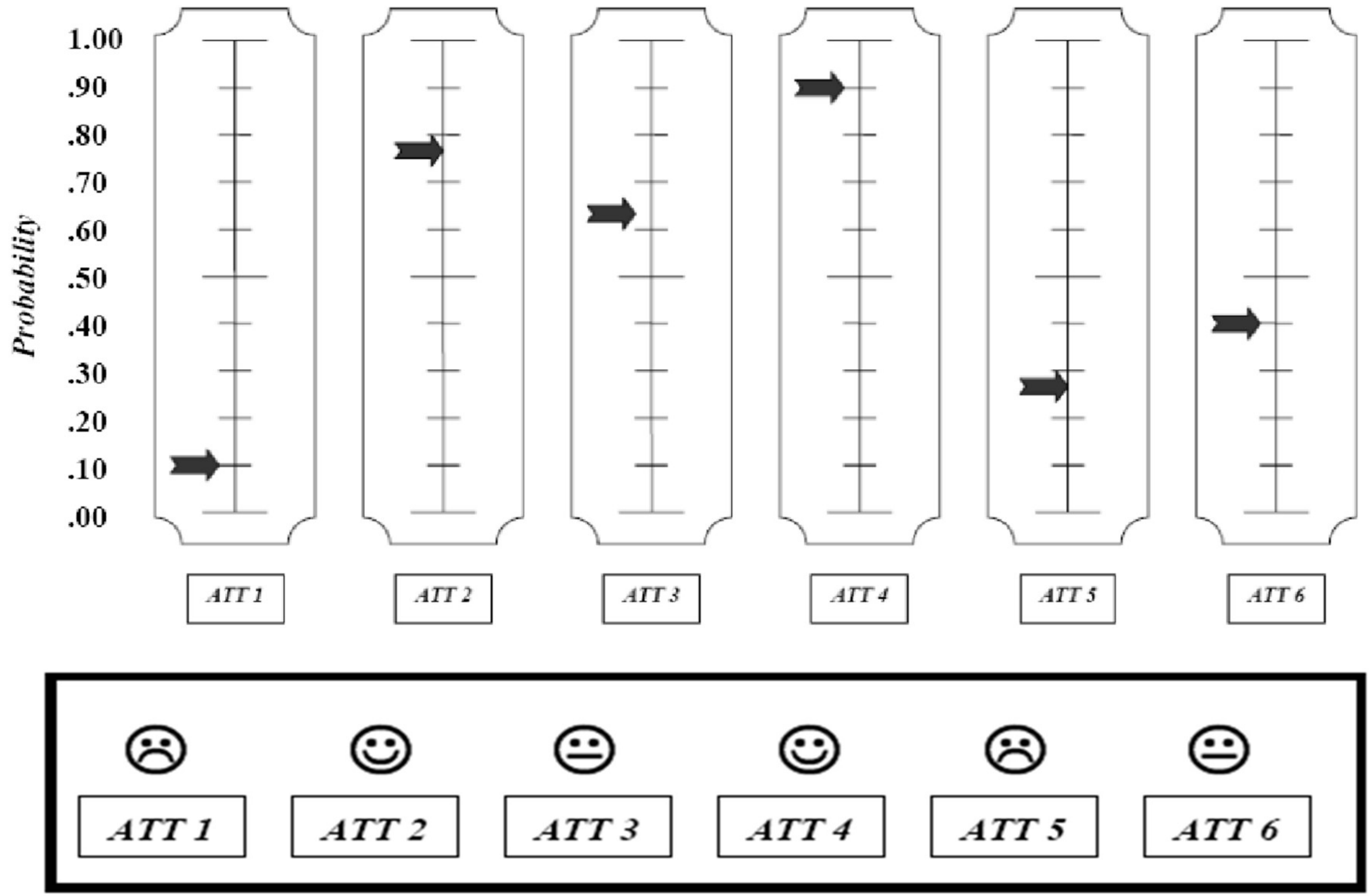
- Daphne has a high probability of mastering the skills associated with
  - Systems (.97); Classification (.94); Prediction (.97); Data (.74).

- Daphne is not a master of the Measurement skill (.07).

- For the Observation skill, Daphne has a probability of .45 of being a master, making her diagnosis on that skill *uncertain*.

- In CDMs, probabilities near .50 represent situations in which the classification is highly uncertain, meaning that the observable variables do not provide enough information to provide an unambiguous diagnosis.

Reporting Attribute Profiles



Explanatory Attribute Narratives

The questions that you have answered on this assessment show us a differentiated profile of what you can do at this point in time. The results show us with relative certainty that you are **able to apply basic definitions and theorems** (*ATT 2*) and that you can **solve a system of equations** once it is presented to you or once you have derived it (*ATT 4*). *This is excellent!*

On the other hand, they also show us that you still have difficulties in **translating a text problem into an appropriate set of mathematical equations** (*ATT 3*) and in **communicating this information meaningfully in writing** (*ATT 5*). *Here you need some additional practice!*

Your performance provides conflicting information about your ability to **interpret figures and graphs** (*ATT 3*) and your ability to **generate text problems that involve solving a system of multiple equations** (*ATT 6*). In some cases you were able to respond to these questions correctly while in other cases you made errors leaving us unsure about your actual ability. *We recommend that you try to review these items and find out in more detail what aspects you had problems with so you can practice them more intensively!*

Example Profile Report (Page 1)

DiagnOsis scoring report

Student Name: Margo — LanguEdge Reading Comprehension Test 1

Review Your Answers

Question	1	2	3	4	5	6	7	8	9	10	11	12	13
Your Answer	√	√	√	2	√	1	4	√	√	√	3	2	2
Correct Answer	2	3	2	3	3	3	1	1	1	4	4	3	2,4,6
Difficulty	e	m	e	h	m	m	h	h	m	m	h	m	m

Question	14	15	16	17	18	19	20	21	22	23	24	25
Your Answer	√	√	√	√	2	√	√	√	3	2	3	5
Correct Answer	2	3	2	1	3	2	3	4	2	4	1	1,5,6
Difficulty	e	m	m	m	e	e	e	e	h	m	m	m

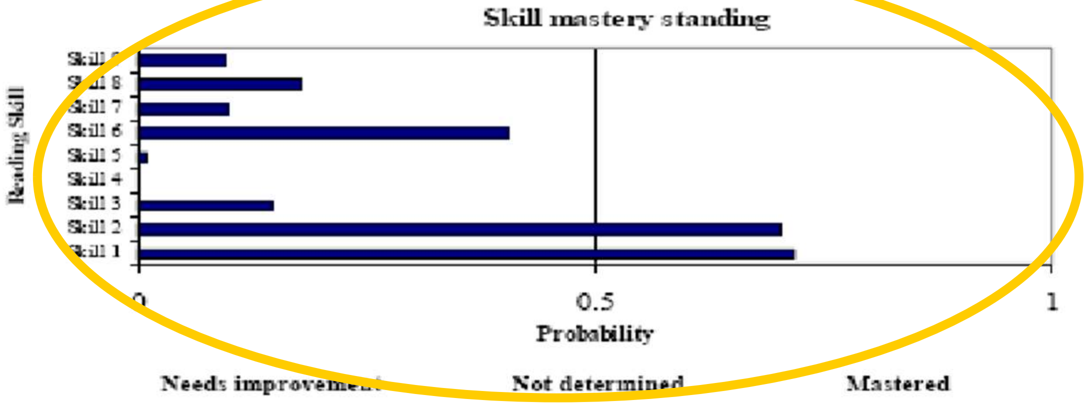
Question	26	27	28	29	30	31	32	33	34	35	36	37
Your Answer	1	√	1	4	4	√	√	√	o	√	3	1,4,6 2,3
Correct Answer	4	2	2	3	3	1	2	2	2	4	1	1,5,6 3,7
Difficulty	m	e	h	m	e	e	e	e	m	h	m	h

Scoring
Correct answer to questions with 4 choices = Plus 1 point
Wrong or omitted answer = No point
Q13 & 25: 3 correct = 2 points, 2 correct=1 point
Q37: 5 correct=3 points, 4 correct=2 points, 3 correct = 1 point

Key
√ Correct
o Omitted
+ Plus partial points
e = Easy, m = Medium, h = Hard
(Difficulty is based on 1372 students' performance on this test)

Score
You earned *20* out of maximum *41* points.
10 points from *12* easy questions
7 points from *17* medium questions
3 points from *8* hard questions
You omitted 1 question.

Improve Your Skills



How to Interpret Skill Mastery

- Nine primary reading skills are assessed in this reading comprehension test. Please review skill descriptions and example questions attached to this scoring report.
- The graph on the left side shows your probable mastery standing of each skill.
- The grey region indicates that your probable mastery standing cannot be determined.
- There may be some measurement error associated with the classification.
- This diagnostic information can be more useful when used in combination with your teacher's and your own evaluation of your reading skills.

Jang (2005)

Example Profile Report (Page 2)

DiagnOsis scoring report — **Primary Skill Descriptions and Example Questions** — **Margo**

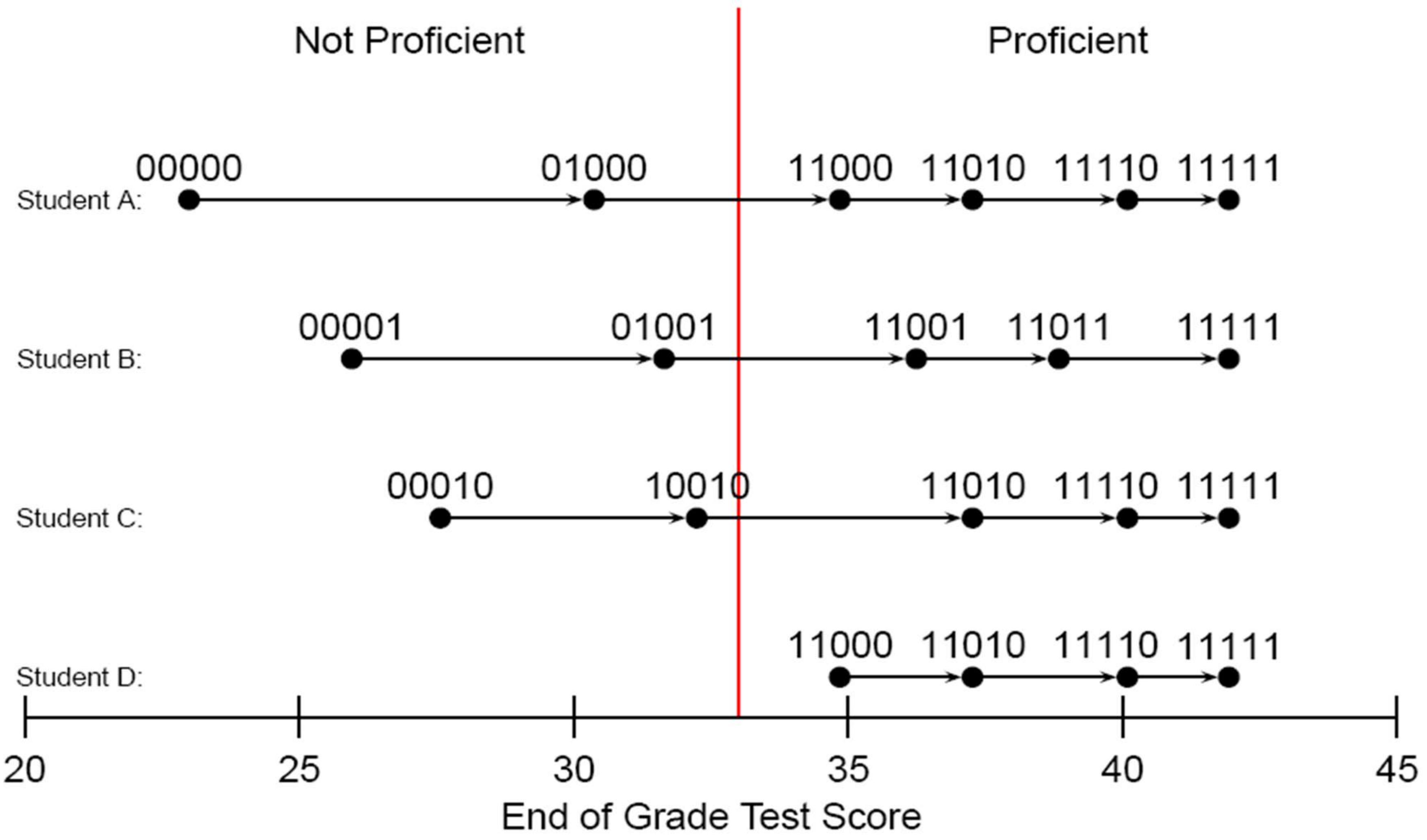
	Skill Descriptions	Example Questions
	Skill 1: Deduce word meaning from context Deducing the meaning of a word or a phrase by searching and analyzing text and by using contextual clues in the text.	33, 14, 32, 4, 3, 11
	Skill 2: Determine word meaning out of context Determine word meaning out of context with recourse to background knowledge	9, 27, 10, 29, 19, 21, 7
	Skill 3: Comprehend text through syntactic and semantic links Comprehend relations betweens parts of text through lexical and grammatical cohesion devices within and across successive sentences without logical problems	3, 26, 12, 36, 4, 2, 22, 33, 24
	Skill 4: Comprehension of text-explicit information Read quickly across sentences within a paragraph and comprehend literal meaning of explicitly stated information.	22, 18, 30, 17, 8, 24, 36, 20, 12, 25, 14
	Skill 5: Comprehend text-implicit information at global level Read selectively a paragraph or across paragraphs to recognize salient ideas paraphrased based on implicit information in text.	6, 34, 26, 4, 5, 35
?	**Skill 6: Infer major arguments or a writer's purpose** Skim through paragraphs and make propositional inferences about arguments or a writer's purpose with recourse to implicitly stated information or prior knowledge	31, 16, 23, 15, 28, 2, 11, 7, 32
	Skill 7: Comprehend negatively stated information Read carefully or expeditiously to locate relevant information in text and to determine which information is true or not true.	22, 7, 28, 5
	Skill 8: Summarize major ideas from minor details Analyze and evaluate relative importance of information in the text by distinguishing major ideas from supporting details.	13, 5, 17, 25, 20
	Skill 9: Determine contrasting ideas through diagrammatic display Recognize major contrasts and arguments in the text whose rhetorical structure contains the relationships such as compare/contrast, cause/effect or alternative arguments and map them into mental framework	37, 23, 35

- Not all example questions are equally informative in assessing related skills. Questions are listed in the order from most informative to least informative.
- indicates that these skills are weak areas you need to improve. '?' indicates that your mastery is not determined.

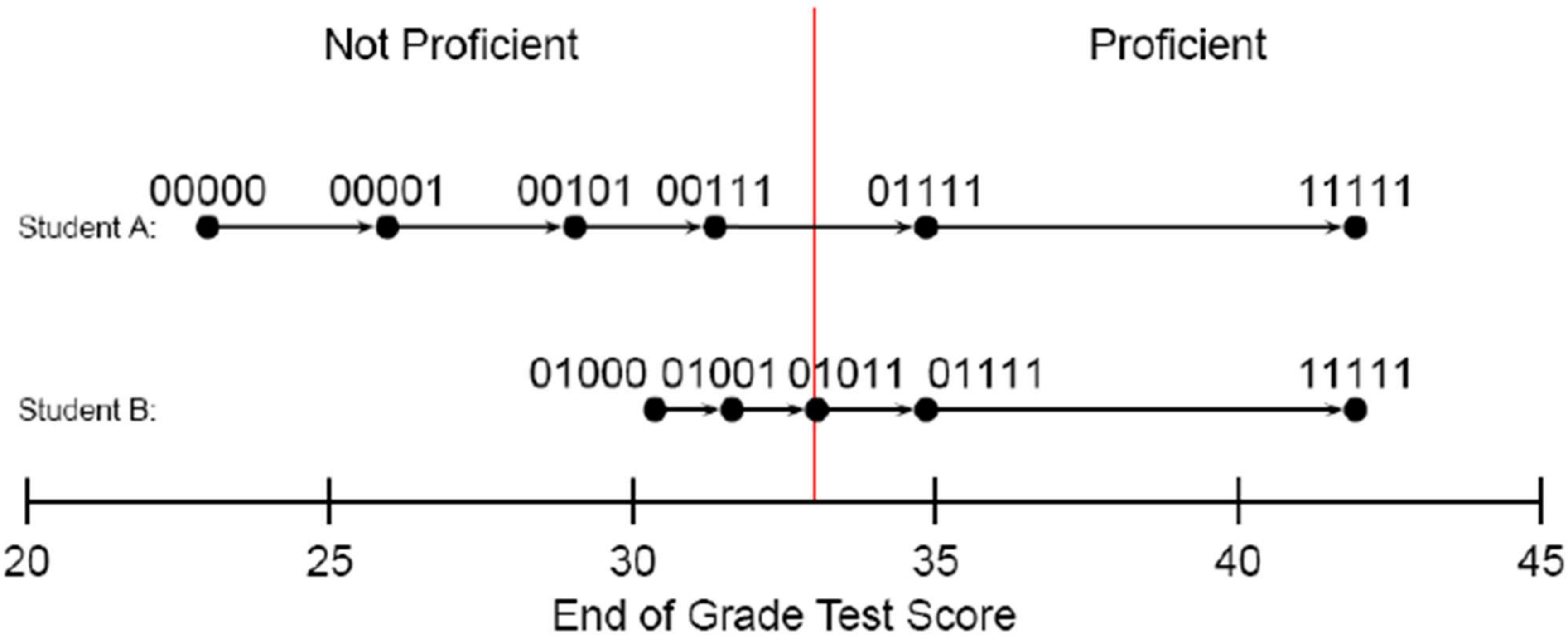
Pathways to Proficiency

- CDMs can be used to form of a "learning path" a respondent can follow that would most quickly lead to proficiency on the EOC test
- The pathway tells the respondent and the teacher the sequence of attributes to learn next that will provide the biggest increase in test score
- This mechanism may help teachers decide focus on when teaching a course
  - Balances time spent on instruction with impact on test score
- Provides a practical implementation of CDMs in today's classroom testing environment

Fast Path to Proficiency



Harder Path to Proficiency



- Some paths are less efficient at increasing EOC test scores

Additional Example

- In Templin and Henson (2006), the authors sought to develop a diagnostic assessment that could be used to screen respondents for a predisposition to be pathological gamblers.
- A set of 10 diagnostic criteria (i.e., attributes) are used in the CDM to classify a respondent as a pathological gambler.
- For a respondent to be diagnosed as a pathological gambler, he or she must meet *at least 5 of the 10 diagnostic criteria*.
  - By definition, such criteria are thus *categorical* in nature, because a respondent either meets or does not meet each one.

Example: DSM-IV-TR Criteria for Pathological Gambling

Diagnostic Classification Criteria for Pathological Gambling from the DSM-IV-TR

Pathological Gambling

A. Persistent and recurrent maladaptive gambling behavior as indicated by five (or more) of the following:

1. Is preoccupied with gambling (e.g., preoccupied with reliving past gambling experiences, handicapping or planning the next venture, or thinking of ways to get money with which to gamble).
2. Needs to gamble with increasing amounts of money in order to achieve the desired excitement.
3. Has repeated unsuccessful efforts to control, cut back, or stop gambling.
4. Is restless or irritable when attempting to cut down or stop gambling.
5. Gambles as a way of escaping from problems or of relieving a dysphoric mood (e.g., feelings of helplessness, guilt, anxiety, depression).
6. After losing money gambling, often returns another day to get even ("chasing" one's losses).
7. Lies to family members, therapist, or others to conceal the extent of involvement with gambling.
8. Has committed illegal acts such as forgery, fraud, theft, or embezzlement to finance gambling.
9. Has jeopardized or lost a significant relationship, job, or educational or career opportunity because of gambling.
10. Relies on others to provide money to relieve a desperate financial situation caused by gambling.

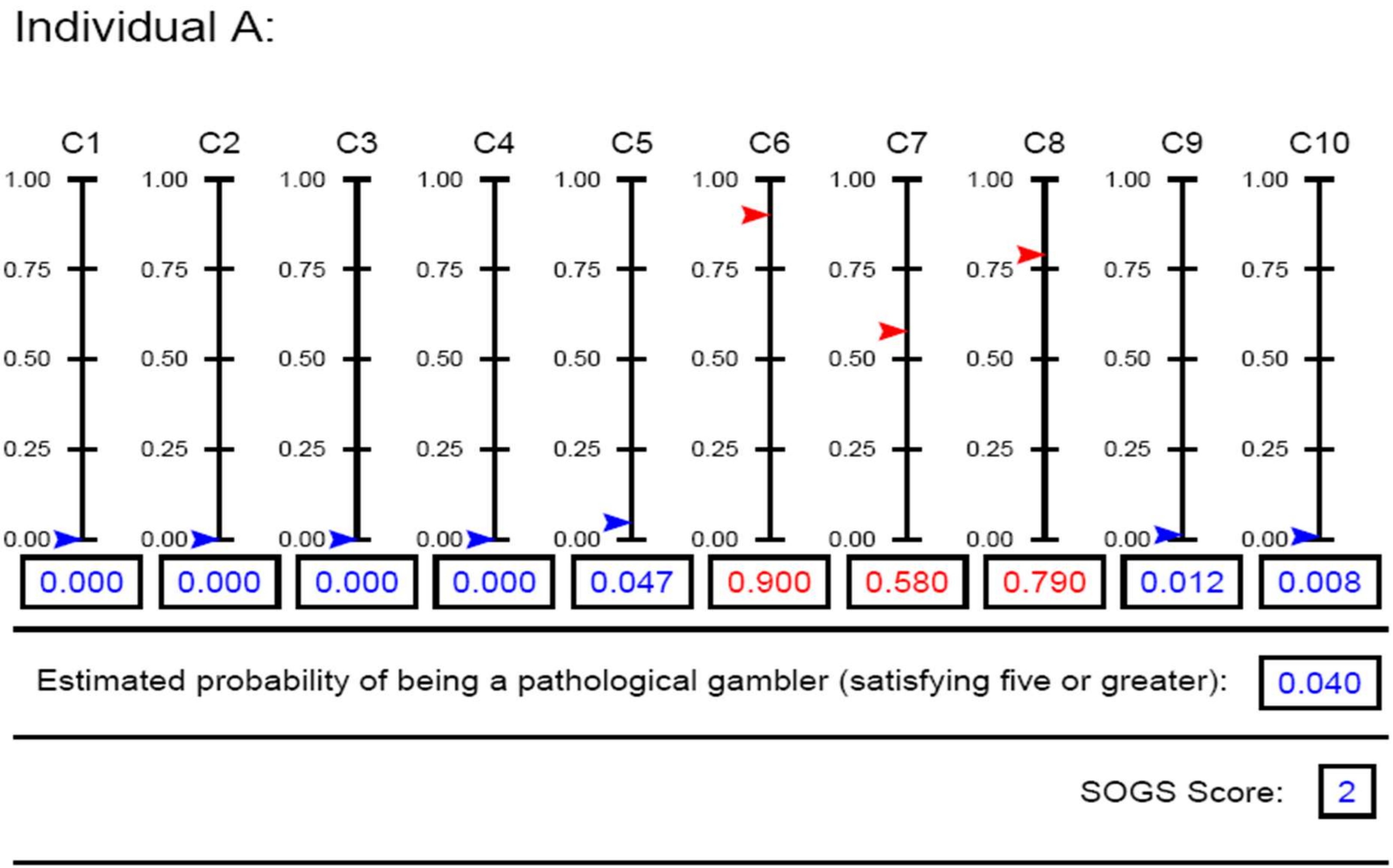
B. The gambling behavior is not better accounted for by a Manic Episode.

Example Item

- Item 22: “Gambling has hurt my financial situation.”
  - Related to :
    - Criterion 8: “Has committed illegal acts such as forgery, fraud, theft, or embezzlement to finance gambling.”
    - Criterion 10: “Relies on others to provide money to relieve a desperate financial situation caused by gambling.”

- Prior to their analysis, the authors did not know the proportion of respondents who had met either Criterion 8 or Criterion 10 and would respond positively to this item.

Example: DSM-IV-TR Criteria for Pathological Gambling Report



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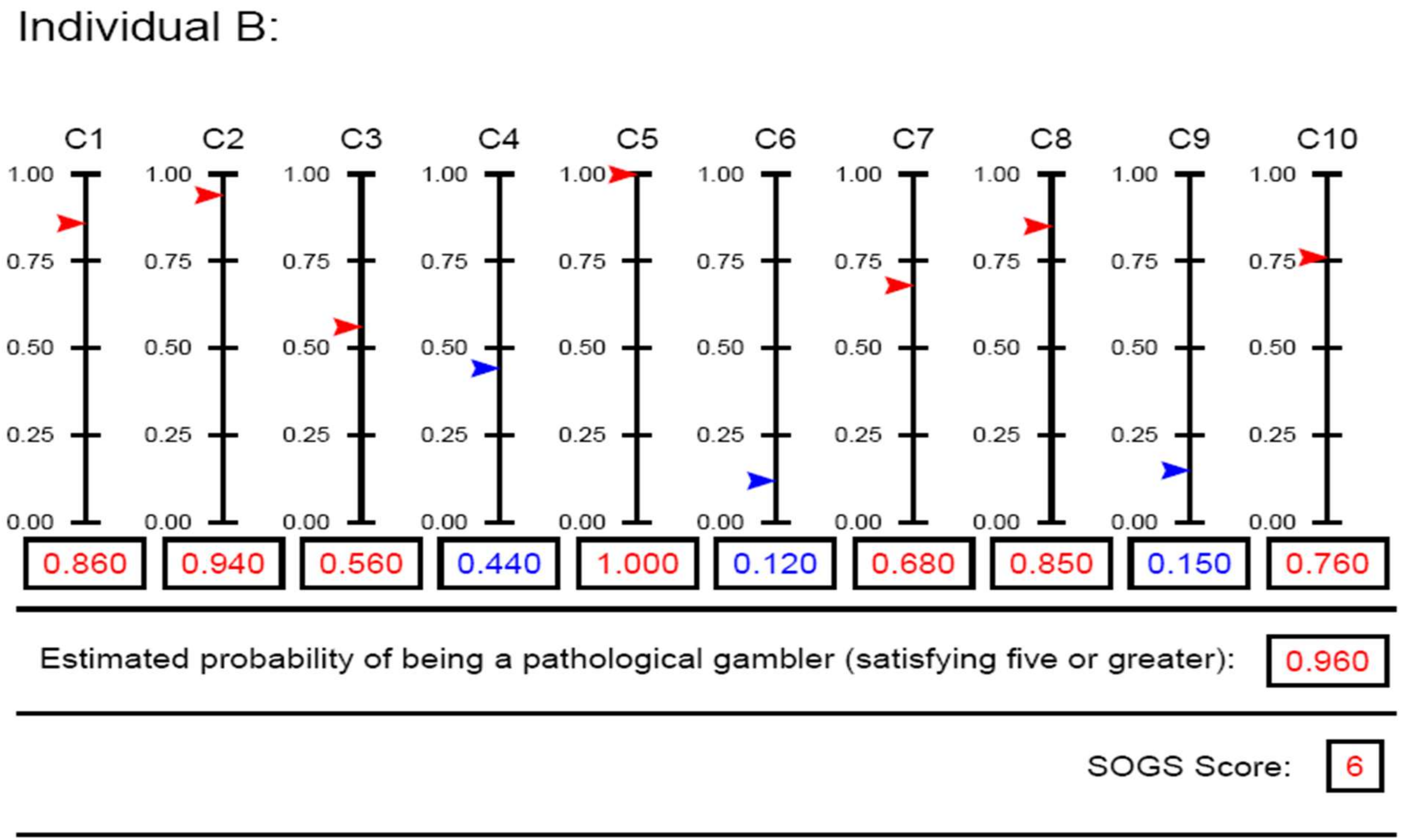


CDM Conceptual Summary

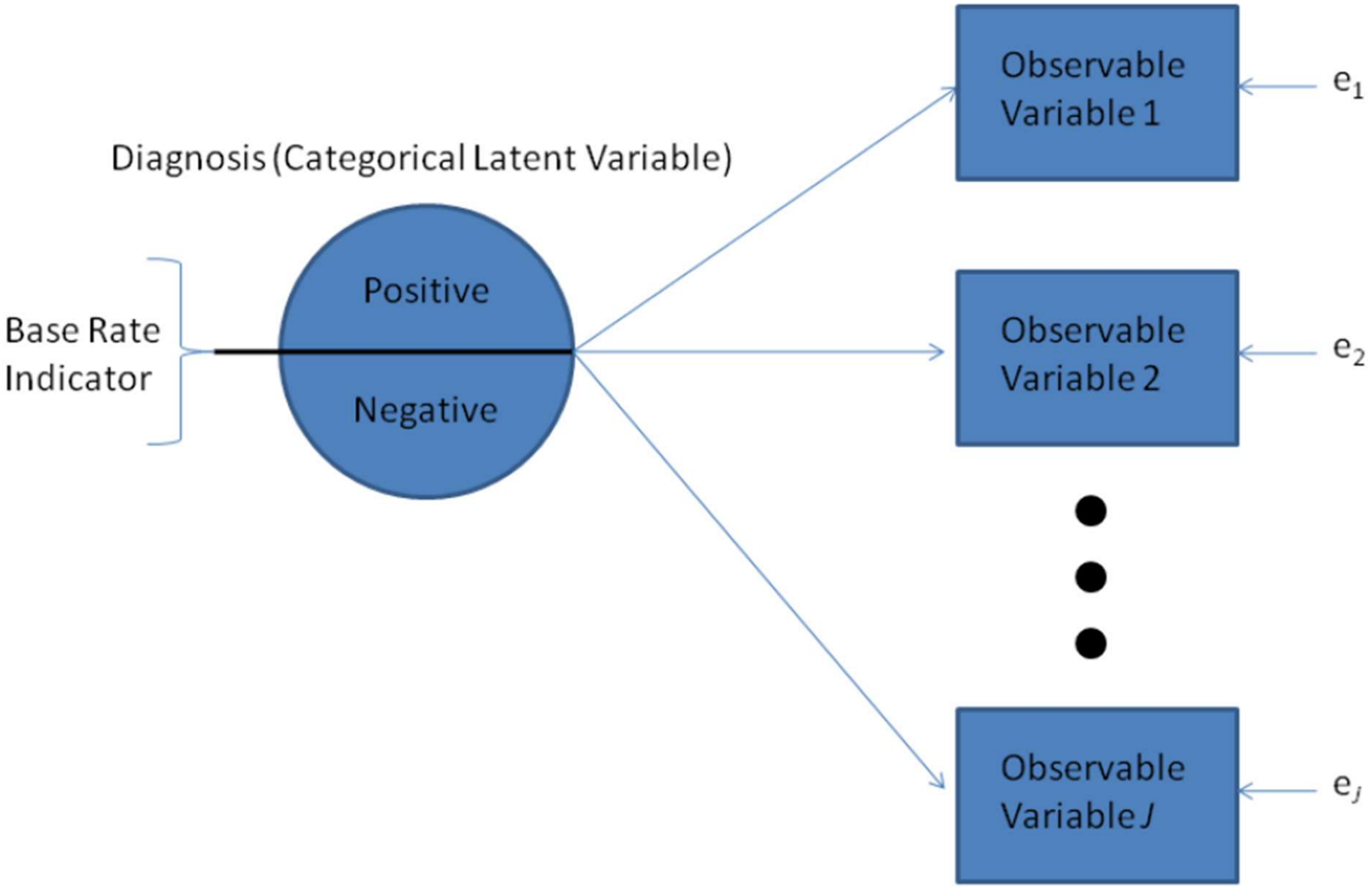
- CDMs focus on **WHY** a respondent is not performing well as compared to only focusing on **WHO**
- The models predict how respondents will answer each item based on respondent's attribute profile
  - Also allow for classification/diagnoses based on item responses
    - Optimally used when tests are used for classification – EOC Tests, Licensure/certification, Clinical screening, College entrance, Placement tests, etc.
- Many models have been created ranging in complexity
  - Later, we'll discuss DINA & DINO, which are commonly used in educational and psychological testing

CDM Conceptual Summary

- CDMs provide a direct link between the unobservable categorical latent variables and the observable response data collected on each respondent.
  - People are given a ***probabilistic (attribute) profile*** that indicates whether they meet the criteria for diagnosis (e.g., has "mastered" or "possesses") on one or more of the latent characteristics (i.e., attributes).

- CDMs are in their infancy
  - Time will tell their effectiveness
- Issues related to retrofitting

Conceptual Model Mapping in CDMs



Common CDMs

Compensatory vs. Non-compensatory

- In *compensatory latent variable models*, a low value on one latent variable can be compensated for by a high value on another latent variable.
  - Low levels of latent variables can be compensated for by higher levels of other latent variables.

- In *non-compensatory latent variable models*, a low value on one latent variable cannot be compensated by a high value on another latent variable.
  - Must be able to use all latent variables to answer an item.

Important Distinctions in CDM

- An important distinction in commonly used CDM is that of the model being either conjunctive or disjunctive.
- **Non-compensatory models** (such as DINA, NIDA, and RUM) model item responses by stating that the probability that an examinee correctly responds to an item that requires a skill an examinee has not mastered cannot be improved by some other required skill that an examinee has mastered.
  - This type of model works well with strictly defined skills, such as types of mathematics skills.
- **Compensatory models** (such as DINO, NIDO, and C-RUM) state the opposite, that the response probability of an item can be equally high if another skill has been mastered (can be thought of as having another skill be able to compensate for the lacked skill).
  - This type of model works well with more coarsely defined skills, such as reading or writing skills.

Condensation Rules

- A *condensation rule* describes how the attributes are 'condensed' to produce a latent response.
- *Conjunctive* condensation rule: all K latent attribute variables that are involved in the response process that is activated by a certain item are simply multiplied by one another.
  - All must be present.

- *Disjunctive* condensation rule: having at least one attribute suffices.

Non-compensatory=Conjunctive vs.
Compensatory=Disjunctive

The DINA Model

- The deterministic-input, noisy-and-gate (DINA) model, which is a non-compensatory model with a conjunctive rule, separates examinees into two classes per item:
  - Examinees who have mastered *all* necessary attributes
  - Examinees who are lacking mastery of *one or more* necessary attribute
  - The DINA model ensures all attributes missed are treated equally, resulting in the same chance of "guessing" correctly.
- Also, DINA specifies two parameters per item.
  - "***Slip***" – Possibility that students who have mastered all required attributes "slip" and incorrectly answer the item.
  - "***Guessing***" – Possibility that students who have not mastered at least one of the required attributes "guess" and correctly answer the item nevertheless.

The DINA model

$$\boxed{\xi_{ic} = \prod_{a=1}^{A} \alpha_{ca}^{q_{ia}}}$$

$$s_i = P(X_{ic} = 0 | \xi_{ic} = 1)$$

$$g_i = P(X_{ic} = 1 | \xi_{ic} = 0)$$

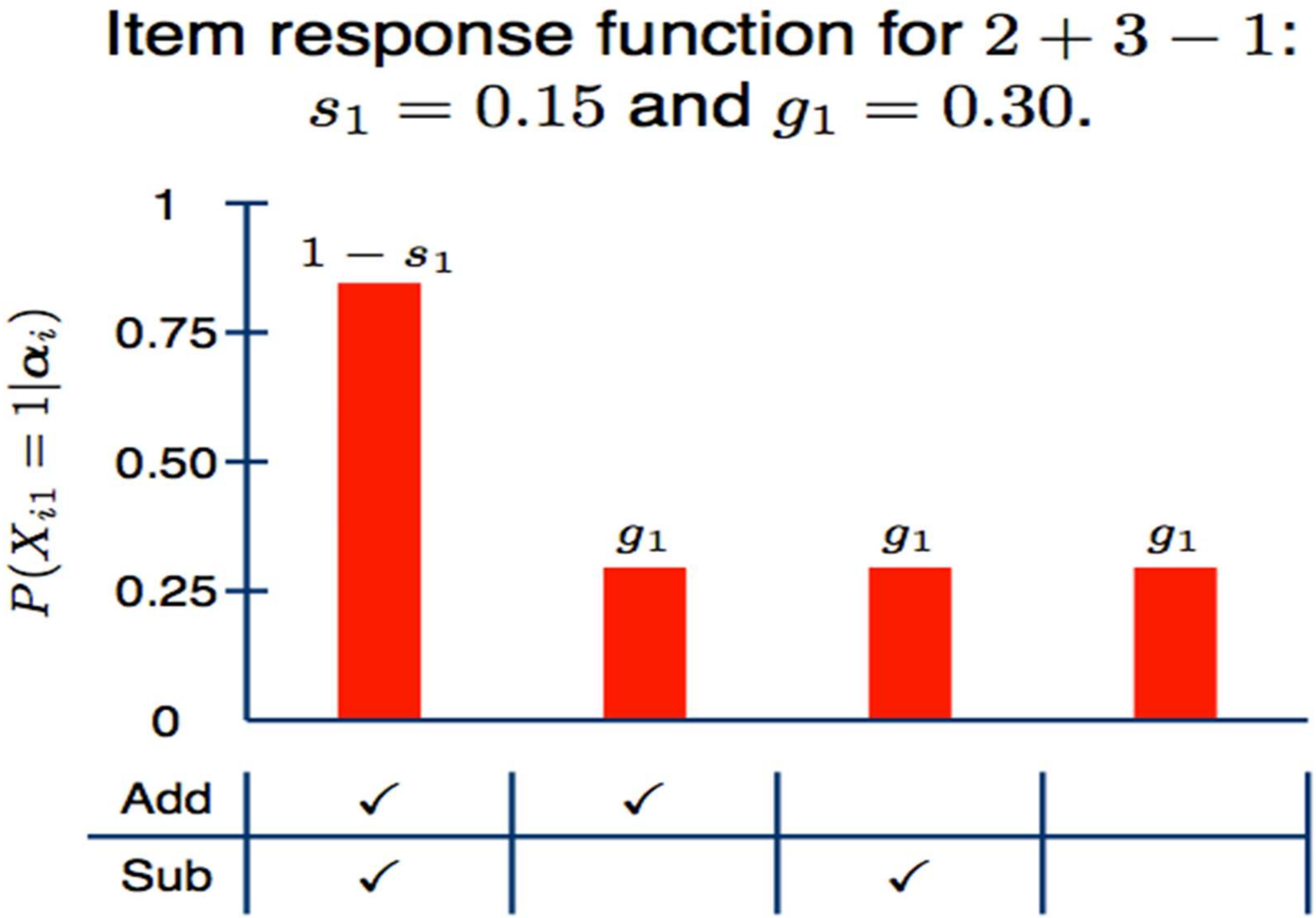
$$\pi_{ic} = P(X_{ic} = 1 | \zeta_{ic}) = (1 - s_i)^{\xi_{ic}} g_i^{(1-\xi_{ic})}$$

	$X_{ic} = 1$ (correct response)	$X_{ic} = 0$ (incorrect response)
$\xi_{ic} = 1$ (mastery of all required attributes)	$1 - s_i$	s_i
$\xi_{ic} = 0$ (non-mastery of at least one required attribute)	g_i	$1 - g_i$

DINA Illustration

- Consider the first item: 2+3-1=?
- Addition and subtraction are attributes needed to be correctly answer this problem.
- Imagine those examinees who have mastered both addition and subtraction ($\xi_{i1} = 1$).
  - If 85% of those examinees correctly responded to this item, then $s_1 = .15.$
- Now consider those examinees who have not mastered either addition or subtraction ($\xi_{i1} = 0$).
  - If 30% of those examinees correctly responded to this item, then $g_1 = .30.$

DINA Illustration

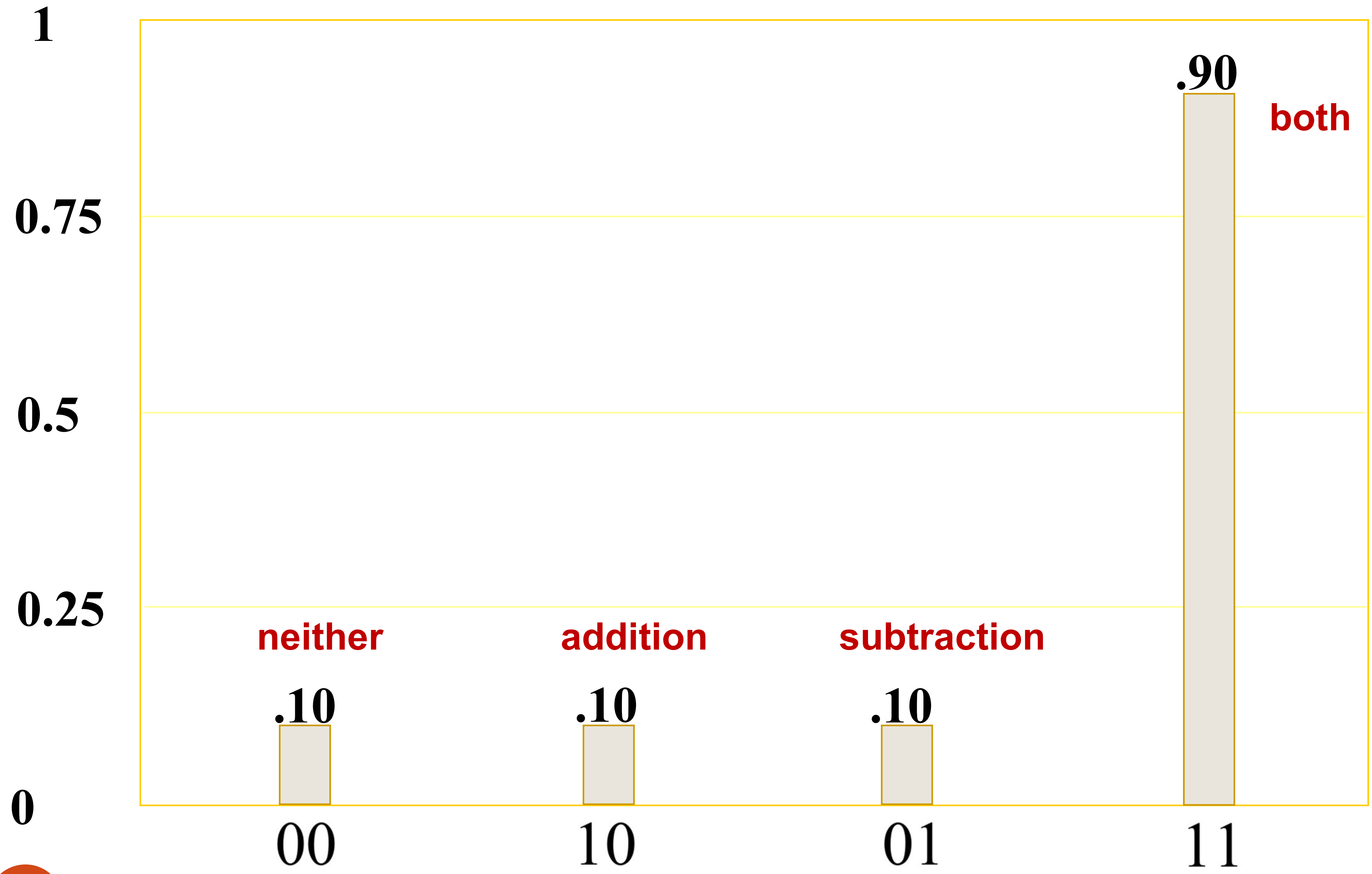


Item Parameter Estimates in DINA

Items	Addition	Subtraction	Multiplication	Division	Parameter	Estimate	Standard error
$2+3-1=?$	1	1	0	0	s_1	.08	.01
					g_1	.30	.02
$4/2=?$	0	0	0	1	s_2	.05	.01
					g_2	.12	.01
$5*3-4=?$	0	1	1	0	s_3	.15	.02
					g_3	.02	.01
$8+12=?$	1	0	0	0	s_4	.05	.01
					g_4	.02	.01

Most informative

DINA Illustration (with Two Attributes)



The DINO Model

- The deterministic-input, noisy-or-gate (DINO) model (e.g., Templin & Henson, 2006; Templin, 2006), which is a compensatory model with a disjunctive rule, classifies examinees into two classes per item:
  - Examinees who have mastered *at least one* of necessary attributes
  - Examinees who are lacking *all* necessary attributes
- Similarly, two parameters are estimated in the DINO model.
  - "***Slip***" – Probability of answering the item wrong when students have mastered at least one of measured attributes
  - "***Guessing***"– Probability of answering the item correctly when students have not mastered all measured attributes

The DINO Model

$$\omega_{ic} = 1 - \prod_{a=1}^{A} (1 - \alpha_{ca})^{q_{ia}}$$

$$g_i = P(X_{ic} = 1 | \omega_{ic} = 0)$$

$$s_i = P(X_{ic} = 0 | \omega_{ic} = 1)$$

$$\pi_{ic} = P(X_{ic} = 1 | \omega_{ic}) = (1 - s_i)^{\omega_{ic}} g_i^{1-\omega_{ic}}$$

	$X_{ic} = 1$ (positive response)	$X_{ic} = 0$ (negative response)
$\omega_{ic} = 1$ (presence of at least one required attribute)	$1 - s_i$	s_i
$\omega_{ic} = 0$ (absence of all required attributes)	g_i	$1 - g_i$

DINO Illustration (with Two Attributes)