HUDM6026 Homework_01

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Question 01 SCR 3.3

MY SOLUTION:
The inverse transformation of the `Pareto(a,b)`'s cdf function is as followed.

$$F^{-1}(u) = \frac{b}{(1-u)^{\frac{1}{a}}}$$

```
> # define the quantile function of Pareto(a,b) distribution
> quantile_Pareto <-function(prob, a, b){
+   x <- b * (1-prob)^(-1/a)
+   return(x)
+ }
> # define the simulated sample size
> n <- 100
> u <- runif(n)
> # based on the uniformly generated vector to get the random sample
> X <- quantile_Pareto(u, 2, 2)
> range(X)
[1]  2.018862 12.215290
```

This inverse function runs well. Before comparing the simulated density and the original density, I derivate the CDF to get the pdf function of Pareto(a,b), that is:

$$f(x) = \frac{ab^a}{x^{a+1}}$$

```
> # define the layout of graph's output
> #par(mfrow=c(1,2))
> # draw the density histogram of the simulated data
> hist(X, prob = T,
+      breaks = 50,
+      main = expression(f(x)==ab^a/x^(a+1)))
> # prepare the Pareto(2,2) distribution
> x <- seq(2,40,.38)
> y <- 2*(2^2)/(x^(2+1))
> # superimpose the lines on the simulated density
> lines(x, y, col="red")
> mtext("Figure 1. Comparing the simulated data with Pareto(a,b)",
+       side = 3,
+       line = -1,
+       outer = T)
```

Figure 1. Comparing the simulated data with Pareto(a,b)

$$f(x) = ab^a / x^{(a+1)}$$