HUDM6026 Homework_01

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Feb 03, 2023

Question 01 SCR 3.3

MY SOLUTION:
The inverse transformation of the `Pareto(a,b)`'s cdf function is as followed.

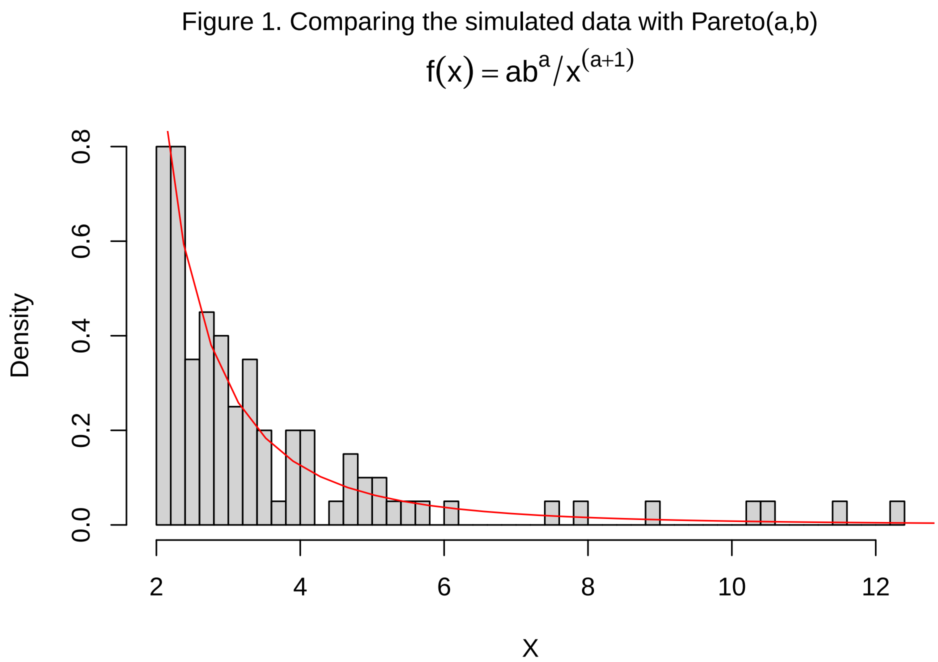
$$F^{-1}(u) = \frac{b}{(1-u)^{\frac{1}{a}}}$$

```
> # define the quantile function of Pareto(a,b) distribution
> quantile_Pareto <-function(prob, a, b){
+   x <- b * (1-prob)^(-1/a)
+   return(x)
+ }
> # define the simulated sample size
> n <- 100
> u <- runif(n)
> # based on the uniformly generated vector to get the random sample
> X <- quantile_Pareto(u, 2, 2)
> range(X)
[1]  2.018862 12.215290
```

This inverse function runs well. Before comparing the simulated density and the original density, I derivate the CDF to get the pdf function of Pareto(a,b), that is:

$$f(x) = \frac{ab^a}{x^{a+1}}$$

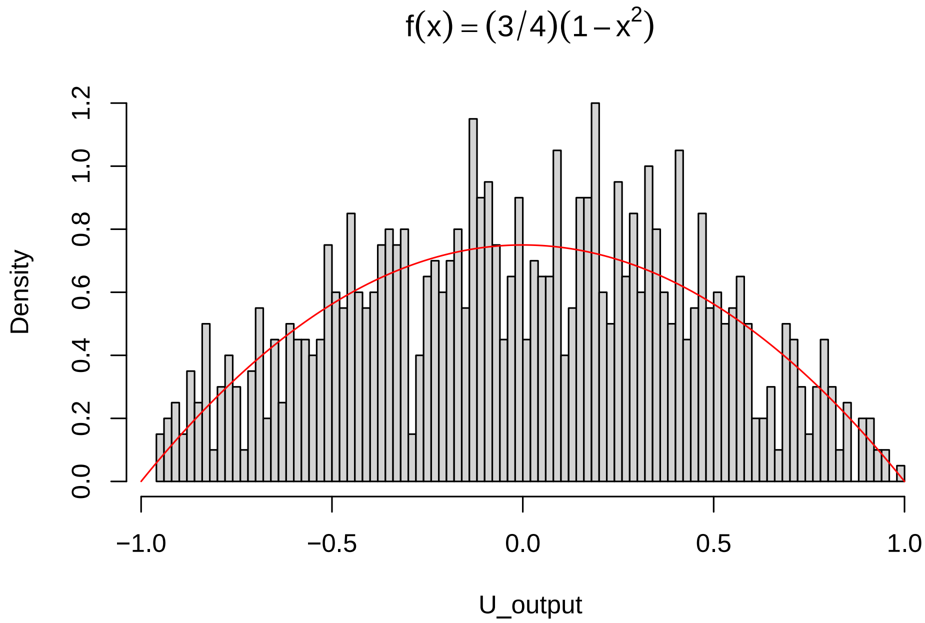
```
> # draw the density histogram of the simulated data
> hist(X, prob = T,
+      breaks = 50,
+      main = expression(f(x)==ab^a/x^(a+1)))
> # prepare the Pareto(2,2) distribution
> x <- seq(2,40,.38)
> y <- 2*(2^2)/(x^(2+1))
> # superimpose the lines on the simulated density
> lines(x, y, col="red")
> mtext("Figure 1. Comparing the simulated data with Pareto(a,b)",
+       side = 3,
+       line = -1,
+       outer = T)
```



Question 02 SCR 3.9

MY SOLUTION:
This question has already given the clues to generate random variable for the rescaled Epanechnikov kernel

```
> # write a function based on text's information
> gen_var <- function(n){ # n is the sample size
+   U_1 <- runif(n, -1, 1)
+   U_2 <- runif(n, -1, 1)
+   U_3 <- runif(n, -1, 1)
+   U_output <- c()
+   for (i in c(1:n)) {
+     if (abs(U_3[i]) > abs(U_2[i]) &
+         abs(U_3[i]) > abs(U_1[i]))
+       {U_output[i] <- U_2[i]}
+     else
+       {U_output[i] <- U_3[i]}
+   }
+   return(U_output)
+ }
>
> # generate 1000 data
> U_output <- gen_var(1000)
> hist(U_output, prob = T,
+      breaks = 100,
+      main = expression(f(x)==(3/4)*(1-x^2)))
> x_vec <- seq(-1,1,0.001)
> f_x <- 0.75*(1-x_vec^2)
> lines(x_vec, f_x, col="red")
```



f(x) = (3/4)(1 − x²)
Density
0.0
0.2
0.4
0.6
0.8
1.0
1.2
−1.0
−0.5
0.0
0.5
1.0
U_output