HUDM 6026 - Computational Statistics
HW 05 – Optimization

Instructions.

- You may use whatever software you like to write up your answers to the hw. For example, you might choose to use Rmarkdown for assignments that are primarily code-based; whereas, you might choose to use Word or LaTeX for more writing-heavy assignments. It's up to you.
- No matter which program you use, you should turn in a pdf or html version.
- **This is a group assignment that is meant to be completed by groups of size 2-4. Only one assignment will be turned in per group. You are responsible for finding one or more classmates to group up with. If you are having trouble doing so, please let me know and I'll try to find a group for you.**

Consider the univariate function

$$f(x) = -\ln(x^2+1) + x^{\frac{1}{3}}$$

on $[0, 4]$.

1. Determine the first derivative of f and encode it in a function called `f_prime`.
2. Create a plot of f and f' on $[0, 4]$ in different colors and line types and add a legend.
3. Finish the functions that I started in the R code notes for univariate optimization for the golden section search, the bisection method, and Newton's method.
4. Apply each of the three functions to this example to discover the minimum. Keep track of and report the number of iterations required for each method. Report the coordinates of the minimum discovered by each of the three functions as well as the number of iterations required.
5. Did the methods perform as you expected in terms of the number of iterations? Why or why not?

Consider the bivariate function

$$f(x_1, x_2) = x_1^4 + x_2^4 - 2x_1^2 + 2x_1x_2 - 3x_2^2 + 6x_1 - 4x_2 + 10$$

on $[-3, 3]^2$.

6. Use function `persp3d()` in package **rgl** to plot function f on $[-3, 3]^2$. Take a guess based on the plot about the coordinates of the absolute minimum.
7. Find the partial derivatives of f with respect to x_1 and x_2 and write a function called `grad_f()` that gives the gradient of f.

8. Write a function to implement gradient ascent with arguments for the start point, the number of iterations, the step size alpha, the x_1 values, and the x_2 values. The function should print the coordinates and iteration number at each iteration. Make the function plot for bonus points.

9. Discuss what changes would need to be made to the gradient ascent function to implement steepest ascent. You do not need to actually implement steepest ascent here but you can do so for bonus points.