HUDM6026 Homework_10

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0.1 ISLR_Chapter 6.10

We have seen that as the number of features used in a model increases, the training error will necessarily decrease, but the test error may not. We will now explore this in a simulated data set.

0.1.1 (a)

Generate a data set with p = 20 features, n = 1,000 observations, and an associated quantitative response vector generated according to the model

MY SOLUTION:

Thinking multiple variate regression in matrix form provides a more efficient way to estimate or simulate. Since generating the multivariate normal distribution needs the connivance matrix and mean vector, like shown in Dr.Keller's in-class R syntax, we need to define them first.

```
> # import the packages
> library(mvtnorm)
> library(clusterGeneration)
>
> # ---------------------
> # STAGE 1 PREPARE
> # ---------------------
> # set the random seed
> set.seed(666)
> # Generate a random covariance matrix with package clusterGeneration
> cov1 <- genPositiveDefMat(dim=20, # create 20 covariates
+                            covMethod = "eigen")
> # Generate a random vector of 20 means from a normal distribution with N(0, 20)
> mns1 <- rnorm(20,0,sd=20)
> # Generate coefficients vector for the output for Y from norm(0, 1)
> coef1 <- rnorm(21, # Beta0 + Beta1 +...+ Beta20
+                0,1) # drawn from a normal distribution N(0,1)
> ### Set ten of them equal to zero
> coef1[sample(2:21, 10, replace = FALSE)] <- 0
>
> # ---------------------------------------
> # STAGE 2 BUILD DATA GENERATING FUNCTION
> # ---------------------------------------
> dataGen <- function(N){
+   # Generate the X matrix
+   X <- rmvnorm(n=N, mean = mns1,sigma = cov1$Sigma)
```

```
+   # augmenting the X matrix
+   X_aug <- cbind(1,X)
+   # create the output Y with error term vector following the N(0,1)
+   Y <- X_aug %*% coef1 + rnorm(N,0,1)
+   # adjust the output
+   dfOut <-  data.frame(cbind(X,Y))
+   names(dfOut) <- c(paste0("X",1:20), "Y")
+   return(dfOut)
+ }
>
> # ----------------------
> # STAGE 3 GENERATE DATA
> # ----------------------
> df <- dataGen(1000)
> head(df)
         X1        X2       X3        X4        X5        X6        X7
1 -15.654375 -23.63880 23.30226 -9.037281  1.333409 -33.06151 -26.56916
2 -13.886764 -21.96735 27.40210 -8.405507 -1.067594 -30.14542 -21.91764
3 -16.544472 -24.79274 25.22519 -5.824548  2.412756 -32.54058 -24.17569
4 -12.779982 -20.92270 21.60266 -6.448326 -2.055508 -28.52098 -20.26761
5  -9.483628 -21.20340 27.07318 -8.032887  1.118195 -26.48337 -18.48465
6  -9.034796 -26.44232 24.74899 -6.218634  1.071316 -32.66566 -23.47574
        X8        X9       X10      X11       X12      X13       X14      X15
1 -38.65066 -22.00314 -26.93117 16.61306 -14.18094 26.11095 -14.44719 5.456897
2 -34.84637 -20.95672 -29.48340 12.95538 -13.58884 29.53441 -13.16991 3.997159
3 -36.27806 -22.09635 -24.52454 16.02539 -12.39565 28.99292 -11.51038 5.682973
4 -32.40778 -21.28272 -24.59206 17.65221 -13.14092 30.28706 -12.10753 7.614404
5 -34.09399 -20.28110 -24.05303 14.96859 -11.37569 33.35054 -11.67114 6.937878
6 -37.49877 -20.05385 -25.27227 18.48872 -14.45867 28.59119 -10.64753 3.655086
       X16      X17       X18      X19      X20         Y
1 9.945809 7.827704 -27.34912 4.186049 39.31443 -60.59024
2 6.217472 8.767564 -30.60701 4.419715 43.42606 -65.44828
3 7.294496 9.247503 -28.08444 7.811671 45.24822 -62.32994
4 7.385289 8.772892 -30.73892 2.614271 40.79408 -65.53384
5 3.612012 9.087225 -26.15211 5.705587 42.05414 -66.31407
6 4.453720 9.404593 -29.21027 5.087559 42.82547 -63.94680
> dim(df)
[1] 1000   21
```

The data looks good.

0.1.2 (b)

Split your dataset into a training set containing 100 observations and a test set containing 900 observations.

MY SOLUTION:

```
> # generate a random index from 1:1000
> set.seed(666)
> index_rdm <- sample(c(1:1000),100)
> # separate the dataset into train and test dataset
> df_train <- df[index_rdm,]
> df_test <- df[-index_rdm,]
```

```
> dim(df_train)
[1] 100  21
> dim(df_test)
[1] 900  21
```

The randomly-subseted train- and test-dataset look good.

0.1.3 (c)

Perform best subset selection on the training set, and plot the training set MSE associated with the best model of each size.

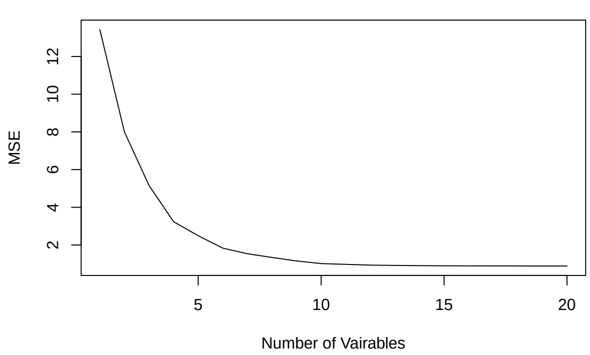
MY SOLUTION:

```
> library(bestglm)
> bss_out <- regsubsets(Y ~., data = df_train, nvmax=20)
> bss_out_summary <- summary(bss_out)
```

Note, by default, the `regsubsets()` only returns the first 8 models.
Based on the definition, we can get easily get the MSE since $MSE = \frac{RSS}{n}$.

```
> plot(bss_out_summary$rss/nrow(df_train),
+      xlab = "Number of Vairables",
+      ylab = "MSE",
+      type = "l")
> title(main = "Figure 1. Training set MSE associated with the best model of each size.",
+       cex.main = 1)
```

Figure 1. Training set MSE associated with the best model of each size.



0.1.4 (d)

Plot the test set MSE associated with the best model of each size.

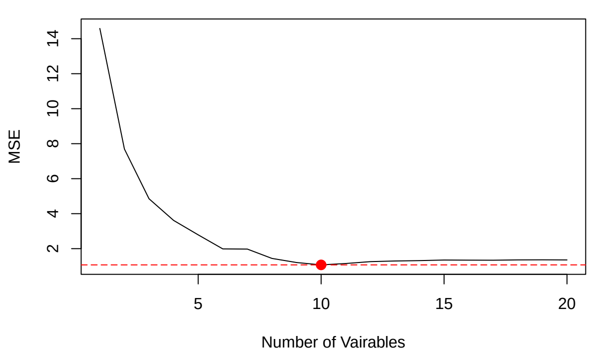
MY SOLUTION:
Since there is no `predict()` function built in the `regsubsets()`, I need to write by hand.

```
> # note the test dataset is in data.frame format need to be change to matrix
> df_test_matrix <- as.matrix(df_test)
> names(coef(bss_out, id=2))[-1]
[1] "X19" "X20"
>
> test_mse <- c()
> for (i in 1:20) {
+   # to extract the coefficient vector for each best model
+   coefi <- coef(bss_out, id=i)
+   # select the corresponding variables and times the coefficients
+   X_temp <- df_test_matrix[,names(coefi)[-1]]
+   X_temp_aug <- cbind(1, X_temp)
+   pred <- X_temp_aug %*% coefi
+   # use the estimated vector of outcome to get the MSE
+   mse <- mean((df_test[,"Y"]-pred)^2)
+   test_mse[i] <- mse
+ }
> test_mse
 [1] 14.587205  7.695202  4.852660  3.615263  2.790152  1.987071  1.975713
 [8]  1.442175  1.207953  1.073172  1.151072  1.257745  1.295256  1.316578
[15]  1.350052  1.343764  1.340008  1.357865  1.362500  1.356349
```

The result looks good. Next, I plot the MSE.

```
> plot(test_mse,
+      xlab = "Number of Vairables",
+      ylab = "MSE",
+      type = "l")
> title(main = "Figure 2. Test-set MSE associated with the best model of each size.",
+       cex.main = 1)
> min_index <- which.min(test_mse)
> points(min_index,test_mse[min_index],col="red",cex=2, pch=20)
> abline(h=min(test_mse), col="red",lty="longdash")
```

Figure 2. Test−set MSE associated with the best model of each size.



0.1.5 (e)

For which model size does the test set MSE take on its minimum value? Comment on your results.

MY SOLUTION:
Figure 2 shows that the model with 10 predictors has the lowest test MSE. From the results below, the ten covariates are $X1$,$X4$,$X7$,$X9$,$X10$,$X13$,$X14$,$X15$,$X19$, and $X20$.

```
> bss_out_summary$outmat[10,]
 X1   X2   X3   X4   X5   X6   X7   X8   X9  X10  X11  X12  X13  X14  X15  X16  X17  X18  X19  X20
"*"  " "  " "  "*"  " "  " "  "*"  " "  "*"  "*"  " "  " "  "*"  "*"  "*"  " "  " "  " "  "*"  "*"
```

0.1.6 (f)

How does the model at which the test set MSE is minimized compare to the true model used to generate the data?

MY SOLUTION:

```
> # extract the estimated 10-predictor model's coefficient
> round(coef(bss_out,id=10),3)
(Intercept)          X1          X4          X7          X9         X10
      9.692      -0.448       0.486       0.146       0.391       0.213
        X13         X14         X15         X19         X20
     -0.748      -0.353      -0.785       1.306      -1.017
> # extract the original 10-predictor model's coefficient
> original_coef <- as.data.frame(matrix(round(coef1[which(coef1 !=0)],3),
+                                        1,11,byrow=T))
> names(original_coef) <- c("Intercept","X1","X4","X7","X9","X10","X13","X14","X15","X19", "X20")
> original_coef
  Intercept     X1    X4    X7    X9   X10    X13    X14    X15  X19    X20
1     0.726 -0.489 0.515 0.091 0.375 0.198 -0.607 -0.506 -0.821 1.28 -0.994
```

Comparing the estimated coefficients and the original coefficients, we can see that:
- first, the number of the coefficient are the same;
- second, the value of each estimated coefficient is very close to the original one;

0.1.7 (g)

Create a plot displaying ... for a range of values of r, where . is the jth coeffcicient for the best model containing r coeeficients.Comment on what you observe. How does this compare to the test MSE plot from (d)

MY SOLUTION:

```
> coef1
 [1]  0.72560162 -0.48873390  0.00000000  0.00000000  0.51543421  0.00000000
 [7]  0.00000000  0.09143147  0.00000000  0.37483448  0.19752628  0.00000000
[13]  0.00000000 -0.60668500 -0.50569185 -0.82082351  0.00000000  0.00000000
[19]  0.00000000  1.27995052 -0.99367984
```

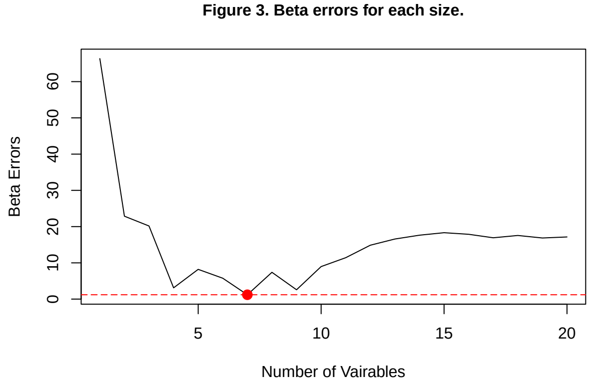
To solve this question, we need to manipulate the strings (a.k.a., the characters) to extract the selected variables and their index number. Here, I use the package `stringr`.

```
> # install.packages("stringr")
> library(stringr)
> beta_errors <- c()
```

```
> for (i in 1:20) {
+   # to extract the best coefficient and corresponding variables
+   coef_temp <- (coef(bss_out,id=i))
+   # to construct a dataframe for processing convenience
+   coef_temp_df <- as.data.frame(t(as.matrix(coef_temp)))
+   # make a null matrix(vector) with the size of 1*21
+   coef_temp_vec <- as.vector(rep(0,21))
+   # mapping all model-selected variables's names
+   for(name in names(coef_temp_df[-1])) {
+     # extract the location information
+     var_location <- as.numeric(str_sub(name, start = 2))
+     # write the coefficient to corresponding location
+     coef_temp_vec[var_location+1] <- coef_temp_df[1,name]
+   }
+   coef_temp_vec[1] <- coef_temp[1]
+
+   # to subtract the original coefficient vector and the estimated vector
+   beta_error <- as.vector(coef1)- coef_temp_vec
+   out_ <- sqrt(t(beta_error) %*% beta_error)
+
+   # write the outcome into the beta_errors set
+   beta_errors[i] <- out_
+ }
>
> round(beta_errors,3)
 [1] 66.353 22.873 20.167  3.107  8.206  5.759  1.201  7.390  2.575  8.969
[11] 11.422 14.863 16.576 17.628 18.318 17.880 16.917 17.554 16.858 17.143
```

The results looks good. Next, I plot the beta errors.

```
> plot(beta_errors,
+      xlab = "Number of Vairables",
+      ylab = "Beta Errors",
+      type = "l")
> title(main = "Figure 3. Beta errors for each size.",
+       cex.main = 1)
> min_index <- which.min(beta_errors)
> points(min_index,beta_errors[min_index],col="red",cex=2, pch=20)
> abline(h=min(beta_errors), col="red",lty="longdash")
```



From the plot we can see that this method selected the 7-predictors models as the best. However, we do not really know what is the performance on the testing dataset. We should not rely on this method to select the best model.