Supplementary materials: Orienting the causal relationship between imprecisely measured traits using GWAS summary data

## S1 Text. The influence of measurement error in the exposure on mediation-based estimated

We assume the following model

where is the exposure on the outcome , is an instrument that has a direct effect on , is the measured quantity of , where measurement error is incurred from linear transformation in and and imprecision from , and is the measured quantity of , where measurement error is incurred from linear transformation in and and imprecision from . Our objective is to estimate the expected magnitude of association between and after conditioning on . Under the CIT, this is expected to be when causes , where is the predicted value of using the measured value of .

We can split into two parts, and .

**Part 1**

**Part 2**

Simpifying further

which can be substituted back to give

Finally

thus if the measurement imprecision in is . However if there is any imprecision then the condition will not hold.