A summary of major probability distributions 1

| Distribution | Probability Density Function | Mean | Variance | $ \begin{array}{c c} \textbf{Moment} & \textbf{Generation} \\ \textbf{Function} & (M(t)) \end{array} $ |
|---|--|---|---|--|
| Uniform $U(a,b)$ | $U(x) = \frac{1}{b-a}, a \le x \le b$ | $\frac{b+a}{2}$ | $\frac{(b-a)^2}{12}$ | $\frac{e^{tb} - e^{ta}}{(b-a)t}, t \neq 0$ |
| NORMAL $N(\mu, \sigma)$ | $N(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\frac{(x-\mu)^2}{2\sigma^2}, -\infty < x < \infty$ | μ | σ^2 | $\exp^{\mu t + \frac{1}{2}\sigma^2 t^2}$ |
| EXPONENTIAL $f(\lambda)$ | $f(x) = \lambda e^{-\lambda x}, x \ge 0$ | $\frac{1}{\lambda}$ | $\frac{1}{\lambda^2}$ | $M_X(t) = \frac{\lambda}{\lambda - t}, t < \lambda$ |
| Gamma $f(\lambda, r)$ | $f(x) = \frac{\lambda^k}{\Gamma(k)} x^{k-1} e^{-x\lambda}, x > 0$ | $\frac{k}{\lambda}$ | $\frac{k}{\lambda^2}$ | $(\frac{\lambda}{\lambda - t})^k, t < \lambda$ |
| Chi-square $\chi^2(n)$ | $f(x) = \frac{\lambda^k}{\Gamma(k)} x^{k-1} e^{-x\lambda}, x > 0$ $\frac{1}{2^{\frac{k}{2}} \Gamma\left(\frac{k}{2}\right)} x^{\frac{k}{2}-1} e^{-\frac{x}{2}}, x > 0$ | k | 2k | $\frac{1}{(1-2t)^{-\frac{k}{2}}}$ |
| BINOMAL $f(n,p)$ | $\binom{n}{k} p^k (1-p)^{n-k}$ | np | np(1-p) | $(1 - p + pe^t)^n$ |
| Poisson $Pois(\lambda)$ | $\frac{\lambda^k}{k!} \cdot e^{-\lambda}$ | λ | λ | $\exp(\lambda(e^t - 1))$ |
| STUDENT'S T $f(x)$ | $\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$ | 0 | $\frac{\nu}{\nu-2}$, if $\nu>2$ | undefined |
| Bernoulli $f(k;p)$ | $\begin{cases} q = (1-p) & \text{for } k = 0 \\ p & \text{for } k = 1 \end{cases}$ | p | p(1-p) | $q + pe^t$ |
| DIRICHLET $f(x_1, \ldots, x_{K-1}; \alpha_1, \ldots, \alpha_K)$ | $\frac{1}{\mathrm{B}(\boldsymbol{\alpha})} \prod_{i=1}^{K} x_i^{\alpha_i - 1}, \text{ where } \mathrm{B}(\boldsymbol{\alpha}) = \frac{\prod_{i=1}^{K} \Gamma(\alpha_i)}{\Gamma(\sum_{i=1}^{K} \alpha_i)} \text{ and }$ $\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_K)$ | $E[X_i] = \frac{\alpha_i}{\sum_k \alpha_k}$ | $ \begin{aligned} \operatorname{Var}[X_i] &= \\ \alpha_i(\alpha_0 - \alpha_i) \end{aligned} $ | undefined |
| | $\alpha = (\alpha_1, \ldots, \alpha_K)$ | | $\overline{\alpha_0^2(\alpha_0+1)}$ | |

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