1 Introduction

Decisions:

1. When and how much of each item should be reordered?
2. How should the decision-maker balance costs and risks of unsatisfied demand?

Objective:

1. Minimize unsatisfied demand (weighted for item critically)

Constraints:

1. Supplier Capacity
2. Budget
3. Warehouse Capacity

2 Problem Formulation

The following notation and parameters are used to formulate the problem. A set of item~~s~~ types $K = \{1, \ldots, k, \ldots, K\}$ requested by demand sources are fulfilled through ~~the~~ centralized warehouse. Each item type k can be ordered from multiple suppliers $I = \{1, \ldots, i, \ldots, I\}$. The user will specify date ranges that ~~the~~ model will provide optimal ordering decisions. These ~~date ranges~~ will be ~~converted~~ to some ~~user specified~~ time-interval (days, weeks, bi-weekly, etc.), to generate a set of time intervals $T = \{1, \ldots, t, \ldots, T\}$, where T is the number of time-intervals within the specified ~~date range.~~

Each item type k is given a rank, r_k, by the decision-maker from 1 to 10, where 1 indicates an item is highly critical to operations and 10 indicates that an item is not critical to operations. The parameter p_k~~,~~ represents the relative penalty of unsatisfied demand of item $k \in K$, is calculated by taking the inverse the rank, so $p_k = 1/r_k$. So for example, if item type k is given a rank of 2, $r_k = 2$, then $p_k = 1/r_k = 1/2$.

~~We will~~ initialize inventory levels at $t = 1$, to ~~beginning inventory~~ $z_{k,i}^{init}$ for each item k from supplier i. Similarly, we will initialize unsatisfied demand for each item type k at $t = 1$ to α_k^{init}. We will use historical data based on expected delay time for item k from supplier i, to generate a discrete probability distribution $f_{k,i,t-\tilde{t}}^{DELAY}$

which represents the probability that item k order from supplier i ordered during time interval $\tilde{t}$ arrives on day t. Any units for item k from supplier i ordered previous to $t = 1$, that have not arrived at the warehouse prior to $t = 1$ arrive according $o_{k,i,t}$ where $o_{k,i,t}$ is calculated based on the expected delay distribution and number of units ordered previously.

The user will specify the measurement that they use to account for space capacities (e.g., square feet, pallets, etc.) and this will be used to generate size per units for each item type k denoted s_k and warehouse space capacities h. So for example, if a user wants to use pallets, s_k will represent the number of pallets, or percentage of a pallet, each unit takes up, and h will be represented by the number of pallets that fit in the warehouse. The model formulation also considers budget per time interval b. The unit capacity for each item k from supplier i is represented by the parameter $CAP_{k,i,t}$, where $CAP_{k,i,t}$ is a random variable with mean $\mu_{CAP_{k,i,t}}$ and standard deviation $\sigma_{CAP_{k,i,t}}$ if item type k is available for order from supplier i, 0 otherwise. Each item type k from supplier i is given a quality rating denoted $q_{k,i}$ if item type k is available for order from supplier i, 0 otherwise.

In this model, it is assumed that random variables for demand, denoted $D_{k,t}$ and supplier capacity denoted $CAP_{i,t}$ can be represented by closed-form probability distributions and are independent. The model uses mean and standard deviation for demand and supplier capacity.

The primary decision variables are $x_{k,i,t}$, which represents the number of units of item i to order during time interval t, $i \in I, t \in T$. We also introduce secondary decision variables that are impacted by the primary decision variables. Units of item type k received from supplier i during time interval t is represented by $y_{k,i,t}$~~, which includes expected arrivals of orders that are placed before and after $t = 1$~~. Units of item k available ~~that were given by~~ supplier i during interval t are represented by $z_{k,i,t}$. We keep track where the items came from to ensure that higher-quality items are prioritized over lower-quality items using $q_{k,i}$.

Unfulfilled demand for item type k at time interval t, denoted $\alpha_{k,t}$, is a function of demand requests and any demand requests left unfulfilled from the time period before. Fulfilled demand for item type k that came from supplier i during time interval t is denoted $beta_{k,i,t}$.

2.1 Notation

Sets	**Description**
I	Set of suppliers
K	Set of item types
T	Set of time intervals (e.g., days, weeks, etc.) in evaluation
Parameters	**Description**
r_k	Rank indicating critically of item type k to operations, $k \in K$
p_k	Relative penalty of unsatisfied demand of item type k, where $p_k = 1/r_k, k \in K$
$q_{k,i}$	Quality of item type k from supplier i, $k \in K, i \in I$
$o_{k,i,t}$	Expected units of item k from supplier i at time t from previously placed orders, $k \in K, i \in I, t \in T$
$z_{k,i}^{init}$	Beginning inventory of item type k at $t = 1$, $k \in K$
$DELAY_{k,i,t-\tilde{t}}$	Delay from the time an order placed for item type k from supplier i at time $\tilde{t}$ is received in time interval t, random variable based off a distribution with mean $\mu_{k,i,t-\tilde{t}}^{DELAY}$, standard deviation of $\sigma_{k,i,t-\tilde{t}}^{DELAY}$, cumulative distribution function of $F_{k,i,t-\tilde{t}}^{DELAY}$, and a probability distribution function of $f_{k,t,t-\tilde{t}}^{DELAY}$, $k \in K, i \in I, \tilde{t} \in T, t > \tilde{t}$
α_k^{init}	Unfulfilled demand at the beginning of the time horizon for item type k at $t = 1$, $k \in K$
$D_{k,t}$	Demand (in units) for item type k at time t, random variable with mean $\mu_{D_{k,t}}$, standard deviation of $\sigma_{D_{k,t}}$, and cumulative distribution function $F_{D_{k,t}}$
s_k	Size per unit (in pallets, sq ft, cubic ft, etc.) of item type k from supplier i, $k \in K, i \in I$
h	Warehouse space capacity (in pallets, sq ft, cubic ft, etc.)
$c_{k,i}$	Cost per unit for item type k from supplier i, $k \in K, i \in I$
b_t	Budget for each time interval, $t \in T$
$CAP_{i,t}$	The cumulative capacity of supplier(s) for item i at time t, random variable with mean $\mu_{CAP_{i,t}}$, standard deviation of $\sigma_{CAP_{i,t}}$, and cumulative distribution function $F_{CAP_{i,t}}$
Decision Variables	**Description**
$x_{k,i,t}$	Units of item k ordered from supplier i during time interval t, $k \in K, i \in I, t \in T$
$y_{k,i,t}$	Units of item k received from supplier i during time interval t $\forall k \in K, i \in I, t \in T$
$z_{k,i,t}$	Units of item k available from supplier i at the beginning of the time interval t, $k \in K, i \in I, t \in T$
$\alpha_{k,t}$	Unfulfilled demand for item type k at the beginning of time interval t, $k \in K, t \in T$
$\beta_{k,i,t}$	Fulfilled demand for item k from supplier i during time interval t, $k \in K, i \in I, t \in T$

2.2 Model Formulation

$$
\begin{aligned}
\max \quad & \sum_{k \in K} p_k \sum_{i \in I} \sum_{t \in T} \frac{1}{t} q_{k,i} \beta_{k,i,t} && && (1) \\
\text{s.t.} \quad & z_{k,i,1} = z_{k,i}^{init} && \forall k \in K, i \in I && (2) \\
& \alpha_{k,1} = \alpha_k^{init} && \forall k \in K && (3) \\
& y_{k,i,t} = \sum_{\tilde{t}=1}^{\tilde{t}=t} f_{k,i,t-\tilde{t}}^{DELAY} x_{k,i,\tilde{t}} + o_{k,i,t} && \forall k \in K, i \in I, t \in T && (4) \\
& z_{k,i,t} - z_{k,i,t-1} - y_{k,i,t-1} + \beta_{k,i,t-1} = 0 && \forall k \in K, i \in I, t \in \{2, \dots, T\} && (5) \\
& \beta_{k,i,t} - z_{k,i,t} \leq 0 && \forall k \in K, i \in I, t \in T && (6) \\
& \alpha_{k,t} - \alpha_{k,t-1} + \sum_{i \in I} \beta_{k,i,t} = \mu_{D_{k,t}} + \sigma_{D_{k,t}} && \forall k \in K, t \in \{2, \dots, T\} && (7) \\
& \sum_{k \in K} \sum_{i \in I} s_k z_{k,i,t} \leq h && \forall t \in T && (8) \\
& \sum_{k \in K} \sum_{i \in I} c_{k,i} x_{k,i,t} \leq b_t && \forall t \in T && (9) \\
& \sum_{k \in K} x_{k,i,t} \leq \mu_{CAP_{i,t}} + \sigma_{CAP_{i,t}} && \forall i \in I, t \in T && (10) \\
& x_{k,i,t}, y_{k,i,t}, z_{k,i,t}, \alpha_{k,t}, \beta_{k,i,t} \geq 0 && \forall k \in K, i \in I, t \in T && (11)
\end{aligned}
$$

The objective (1) maximized fulfilled demand with a higher priority of fulfilling highly critical items, and allocating higher-quality items, as early as possible. Constraint (2) initializes available supply at the beginning of the first time interval. Constraint (4) calculates the expected number of units received of type k from supplier i, during time t. Constraint (5) calculates the available supply by setting it equal to the amount available at the beginning of the day before, minus any items fulfilled the day before, plus the expected number of units received the day before. Constraint (3) initializes unfulfilled demand for each item type at time $t = 1$. Constraint (6) ensures that the amount of stock fulfilled is not more than the available supply. Constraint (7) calculates unsatisfied demand for each item type k at time t. Constraint (8) is the warehouse capacity constraint. Constraint (9) is the budget constraint. Constraint (10) is the supplier capacity constraint, which is formulated as a chance constraint, which states that the probability that supplier capacity is exceeded needs to below a user specified threshold. Constraint set (11) specifies the non-negative properties of the decision variables.

3 To Do

3.1 To Be Completed: Short Term - by September 15th

- Create generic file for input data
- Run against City of Seattle data
- Write report on how to use model
- Run/test over multiple items

3.2 To Be Completed: Medium Term

- Present method to use the model as a rolling forecast
- Separate cold and room-temperature capacities
- Minimum units per order constraints?

3.3 To Be Completed: Long Term

- Formulate as a two-stage stochastic model?
- Demand Sources Allocation Scheme with considerations for social vulnerabilities and a focus on outcomes vs outputs.
- Connect to separate model include supplier decisions (procurement) separate model
- Publish