# BONUS PROBLEMS

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. My primary resources were my notes from lectures , lab example problems and pauls notes. I combined these resources with AI so I could import these resources into chatGPT and ask specific questions about the parts I did not understand, or where I made an error in a step when I got stuck.

## 1. Problem 1

Let $\vec{F}(x,y,z) = \langle sin(x^2), xz, z^2\rangle$ Evaluate $\int_C \vec{F}\cdot d\vec{r}$ around the curve C of the intersection of the cylinder $x^2+y^2=4$ with the surface $z=x^2$ in the counter-clockwise direction as viewed from the top of the z-axis.

Use Stokes Theorem:
Which applies when:
-C is closed, simple and piecewise smooth
-Does not intersect
-Oriented counter clockwise when viewed from above.

Therefore for this problem we can say: $\int_C \vec{F}\cdot d\vec{r} = \iint_D \text{curl}\,\vec{F}\cdot d\vec{s}$

$$\text{Curl}(\tilde{\text{F}}) = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ F_x & F_y & F_z \\ sin(x^2) & xz & z^2 \end{bmatrix}$$

this yields: $\iint_D \langle -x, 0, z\rangle \cdot d\vec{s}$
$d\vec{s}$ can be defined as $r_u \times r_v$
Our surface is given by the intersection between this cylinder and parabolic sheet.

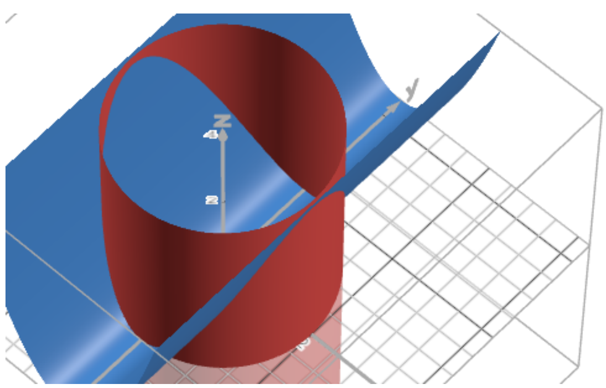


Figure 1. Intersection between cylinder and parabolic sheet

We can parameterize this surface to get our surface s: Let x = u , y = v and because $z = x^2 \quad z = u^2$
now we have $r(u,v) = \langle u, v, u^2\rangle$ for u and v $\in x^2+y^2 \le 4$

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$r_u = \langle 1, 0, 2u \rangle \quad r_v = \langle 0, 1, 0 \rangle$

$$r_u \times r_v = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 2u \\ 0 & 1 & 0 \end{bmatrix}$$

This gives our normal vector $\vec{n} = \langle -2u, 0, 1 \rangle$
to get the integrand: $\langle -u, 0, u^2 \rangle \cdot \langle -2u, 0, 1 \rangle$
$\int\int_D 2u^2 + 2u \cdot d\vec{s}$
$\int\int_D 3u^2 \cdot d\vec{s}$
let $\quad u = r\,cos\,(\theta), v = r\,sin\,(\theta)$
we now have:
$\int_0^{2\pi} \int_0^0 3r^2 cos^2(\theta)\, dr\, d\theta$
$\int_0^{2\pi} [\frac{3}{4} r^4 cos^2(\theta)]\Big|_0^2 d\theta$
$\int_0^{2\pi} 12cos^2(\theta)\, d\theta$
The double angle formula is given by: $cos^2(\theta) = \frac{1+cos(2\theta)}{2}$
$12 \int_0^{2\pi} \frac{1+cos(2\theta)}{2}$
$\int_0^{2\pi} 6 + 6cos(2\theta)$
$6 \int_0^{2\pi} 1 + cos(2\theta)$
$6[2\pi + \frac{1}{2} sin(2\theta)]$
$12\pi + 3sin(4\pi)$
Sin over any even number of pi over 1 period $= 0$
$12\pi + 0 = 12\pi$

## 2. Problem 2

Let E be the solid region between the plane $z = 4$ and the parabolid $z = x^2 + y^2$ Let $\vec{F}(x, y, z) = \langle \frac{-1}{3} x^3 + e^{z^2}, \frac{-1}{3} y^3 + xsin(z), 4z \rangle$ Let S be the surface that encloses E. Find $\int\int_S \vec{F} \cdot d\vec{s}$