# Project 1 Option 2

Auto Assembly Report with Recommendations

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Math 132A
06 February 2022

# 1 Problem Description

One plant close to Detroit, MI, assembles two models of midsized luxury cars. The first model, the Family Adventurer, is a four-door sedan with vinyl seats, plastic interior, standard features, and excellent gas mileage. It is marketed as a smart buy for middle-class families with tight budgets, and each Family Adventurer sold generates a modest profit of \$3,700 for the company. The second model, the Classic Transporter, is a two-door luxury sedan with leather seats, wooden interior, custom features, and navigational capabilities. It is marketed as a privilege of affluence for upper-middle -class families, and each Classic Transporter sold generates a profit of \$5,300 for the company.

We need to decide the production schedule for the next month. Specifically, we must decide how many Family Adventurers and how many Classic Transporters to assemble in the plant to maximize the profit for the company. We know that the plant possesses a capacity of 48,500 labor-hours during the month. We also know that it takes 6 labor-hours to assemble one Family Adventurer and 10.5 labor-hours to assemble one Classic Transporter.

Because the plant is simply an assembly plant, the parts required to assemble the two models are not produced at the plant. They are instead shipped from other plants around the Michigan area to the assembly plant. For the next month, we know that we will be able to obtain only 20,000 doors (10,000 left-hand doors and 10,000 right hand doors) from the door supplier. A recent labor strike forced the shutdown of that particular supplier plant for several days, and that plant will not be able to meet its production schedule for the next month. Both the Family Adventurer and the Classic Transporter use the same door part.

In addition, a recent company forecast of the monthly demands for different automobile models suggests that the demand for the Classic Transporter is limited to 3,500 cars. There is no limit on the demand for the Family Adventurer within the capacity limits of the assembly plant.

# 2 Linear Programming Model

Let $f, t$ represent the decision variables where $f$ represents the number of Family Adventurers we should produce and $t$ represents the number of Classic Transporters that should be produced in a month.
Our goal is to maximize profit. Thus, we represent this as $max\ 3700f + 5300t$ subject to the following constraints:

$$
\begin{aligned}
6f + 10.5t &\leq 48500 \\
4f + 2t &\leq 20000 \\
t &\leq 3500 \\
f, t &\geq 0
\end{aligned}
$$

## 2.1 Discussing the Use of a Linear Model

A linear model is the ideal fit for our situation, as the following four assumptions are maintained.

*Proportionality*

The constraints above all have constant coefficients. This indicates that regardless of which car is in production, we will consistently yield the same amount of labor hours, doors, etc. In other words, we assume producing a Family Adventurer will always take 6 labor hours to create and a Classic Transporter will always take 10.5 hours. We can extend the same logic to the other constraints. Thus these are proportional constants. Additionally, these coefficients are the only numbers multiplying variables (no variable is multiplying another variable), so this assumption is held.

*Additivity*

Clearly, the objective function and constraints in this problem only use addition between variables. Notice the only other operation being used is multiplication between a variable and its coefficient. Thus, our situation maintains the additivity assumption.

*Divisibility*

At first glance, we hesitate to accept that the divisibility assumption applies here. However, we can begin production on another Classic Transporter and Family Adventurer and allow it to rollover into next month's productions and sales. Thus, it is acceptable to make a fraction of a car, so it makes sense that divisibility applies to our model.

*Certainty*

There are no probabilistic elements in this problem. Notice that the Family Adventurer will always need four doors, and the other constraints also have certain coefficients. Also, we are assuming that we are subject to 48500 labor hours, 20000 doors, and a 3500 Class Transporters.

Because these four assumptions are held in this situation, a linear model is optimal for this problem.

## 3 Solution of Our LP Model

```
Global optimal solution found.
Objective value:                              0.2701000E+08
Infeasibilities:                               0.000000
Total solver iterations:                              2
Elapsed runtime seconds:                           0.15

Model Class:                                         LP

Total variables:                      2
Nonlinear variables:                  0
Integer variables:                    0

Total constraints:                    7
Nonlinear constraints:                0

Total nonzeros:                      11
Nonlinear nonzeros:                   0


                              Variable           Value        Reduced Cost
                                     F        3766.667            0.000000
                                     T        2466.667            0.000000

                                   Row    Slack or Surplus      Dual Price
                                     1       0.2701000E+08        1.000000
                                     2        0.000000            460.0000
                                     3        0.000000            0.000000
                                     4        0.000000            470.0000
                                     5        1033.333            0.000000
                                     6        2466.667            0.000000
                                     7        3766.667            0.000000
```

## 4 Interpreting Our Solutions

Based on our model, we should produce 3766.667 Family Adventurers and 2466.667 Classic Transporters in order to maximize profits given the constraints. Note that the hours and doors constraints were both fulfilled to it's maximum (i.e. 6f+10.5t=48500, 4f+2t=20000). However, even though the demand for Classic Transporters is higher than 2466.667, producing more would lower the optimal profit, so we are left with a surplus of 1033.333 for that constraint. Thus, this is a unique solution to our model. It yields an optimal value of $27,010,003 in profit.

# 5 Recommendations

## 5.1 Should we run a targeted \$500,000 advertising campaign?

| Variable | Value | Reduced Cost |
|---|---|---|
| F | 3766.667 | 0.000000 |
| T | 2466.667 | 0.000000 |

| Row | Slack or Surplus | Dual Price |
|---|---|---|
| 1 | 0.2701000E+08 | 1.000000 |
| 2 | 0.000000 | 460.0000 |
| 3 | 0.000000 | 235.0000 |
| 4 | 1733.333 | 0.000000 |
| 5 | 3766.667 | 0.000000 |
| 6 | 2466.667 | 0.000000 |

No. Despite the increase in demand from $t \leq 3500$ to $t \leq 4200$ (a 20% increase), the production amounts remain the same. Thus, we would just be losing \$500000 to run the campaign, so it should not be run.

## 5.2 Increasing Labor Capacity with Overtime

| Variable | Value | Reduced Cost |
|---|---|---|
| F | 3250.000 | 0.000000 |
| T | 3500.000 | 0.000000 |

| Row | Slack or Surplus | Dual Price |
|---|---|---|
| 1 | 0.3057500E+08 | 1.000000 |
| 2 | 4375.000 | 0.000000 |
| 3 | 0.000000 | 925.0000 |
| 4 | 0.000000 | 3450.000 |
| 5 | 3250.000 | 0.000000 |
| 6 | 3500.000 | 0.000000 |

Due to the 25% increase in labor capacity through overtime hours ($6f + 10.5f \leq 48500$ `to` $6f + 10.5f \leq 60625$), we should now produce 3250 Family Adventurer cars and 3500 Classic Transporters for a profit of \$30,575,000.

## 5.3 Overtime Labor Costs

In acknowledgement that overtime labor does not come without additional costs, the maximum amount we should be willing to pay is the difference of increased profit from the original model. In our case, that amount should be \$3,564,997.

### 5.4 Raising Demand and Labor-Hour Capacity

| Variable | Value | Reduced Cost |
|---|---|---|
| F | 2958.333 | 0.000000 |
| T | 4083.333 | 0.000000 |

| Row | Slack or Surplus | Dual Price |
|---|---|---|
| 1 | 0.3258750E+08 | 1.000000 |
| 2 | 0.000000 | 460.0000 |
| 3 | 0.000000 | 235.0000 |
| 4 | 116.6667 | 0.000000 |
| 5 | 2958.333 | 0.000000 |
| 6 | 4083.333 | 0.000000 |

When using both the targeted advertising campaign and the overtime labor-hours, the advertising campaign raises the demand for the Classic Transporter by 20 percent ( $t \leq 3500$ to $t \leq 4200$), and the overtime labor increases the plants labor-hour capacity by 25 percent ($6f + 10.5f \leq 48500$ `to` $6f + 10.5f \leq 60625$). This new model reveals that we should produce 2958.333 Family Adventurers and 4083.333 Classic Transporters for a profit of \$32,587,497 (before subtracting labor and advertising costs).

### 5.5 Which Scenario is More Optimal?

In the prior situation, the profit yielded is \$32,587,497, however, subtracting the costs of the advertising campaign and overtime labor-hours (\$500,000 and \$1,600,000, respectively) leaves us with \$30,487,497. This value is greater than the one in our original model (\$27,010,003). Thus we should proceed with these plans to increase our optimal profit.

### 5.6 Introducing a Discount

Due to dealers heavily discounting Family Adventurers and the profit-sharing agreement with its dealers, the company is now making a \$2,800 profit, as opposed to a \$3,700 one. We must now adjust our objective function to maximize profits from $3700f + 5300t$ to $2800f + 5300t$. We consider the following combinations of the plans mentioned above with the adjusted model.

*Not Using an Advertising Campaign or Overtime Labor*

| Variable | Value | Reduced Cost |
|---|---|---|
| F | 1958.333 | 0.000000 |
| T | 3500.000 | 0.000000 |

| Row | Slack or Surplus | Dual Price |
|---|---|---|
| 1 | 0.2403333E+08 | 1.000000 |
| 2 | 0.000000 | 466.6667 |
| 3 | 5166.667 | 0.000000 |
| 4 | 0.000000 | 400.0000 |
| 5 | 3500.000 | 0.000000 |
| 6 | 1958.333 | 0.000000 |

Using the new objective function and not implementing an advertising campaign or the overtime labor, we see that we should produce 19588.333 Family Adventurers and 3500.000 Classic Transporters. This yields a maximum profit of $24,003,332.40.

*Using an Advertising Campaign*

| Variable | Value | Reduced Cost |
|---|---|---|
| F | 733.3333 | 0.000000 |
| T | 4200.000 | 0.000000 |

| Row | Slack or Surplus | Dual Price |
|---|---|---|
| 1 | 0.2431333E+08 | 1.000000 |
| 2 | 0.000000 | 466.6667 |
| 3 | 8666.667 | 0.000000 |
| 4 | 0.000000 | 400.0000 |
| 5 | 4200.000 | 0.000000 |
| 6 | 733.3333 | 0.000000 |

Using an advertising campaign, but not overtime labor, we see that we should produce 733.333 Family Adventurers and 4200.00 Classic Transporters. This yields a maximum profit of $24,313,332.40, but subtracting the cost of the advertising campaign ($500,000) leaves us with $23,813,332.40.

*Using Overtime Labor*

| Variable | Value | Reduced Cost |
|---|---|---|
| F | 3250.000 | 0.000000 |
| T | 3500.000 | 0.000000 |

| Row | Slack or Surplus | Dual Price |
|---|---|---|
| 1 | 0.2765000E+08 | 1.000000 |
| 2 | 4375.000 | 0.000000 |
| 3 | 0.000000 | 700.0000 |
| 4 | 0.000000 | 3900.000 |
| 5 | 3500.000 | 0.000000 |
| 6 | 3250.000 | 0.000000 |

Using overtime labor, but not an advertising campaign, our solution reveals that we should produce 3250 Family Adventurers and 3500 Classic Transporters. This gives an optimal value of \$27,650,000, subtracting the cost of overtime labor (\$1,600,000) leaves us with \$26,050,000.

*Using an Advertising Campaign and Overtime Labor*

| Variable | Value | Reduced Cost |
|---|---|---|
| F | 2754.167 | 0.000000 |
| T | 4200.000 | 0.000000 |

| Row | Slack or Surplus | Dual Price |
|---|---|---|
| 1 | 0.2997167E+08 | 1.000000 |
| 2 | 0.000000 | 466.6667 |
| 3 | 583.3333 | 0.000000 |
| 4 | 0.000000 | 400.0000 |
| 5 | 4200.000 | 0.000000 |
| 6 | 2754.167 | 0.000000 |

Implementing both an advertising campaign and overtime labor, our model tells us that we should produce 2754.167 Family Transporters and 4200 Classic Transporters. This yields an optimal value of \$29,971,667.60, subtracting costs leaves us with \$27,871,667.60.

Out of the four models for this adjusted situation, it is clear that we should implement an advertising campaign and overtime labor in order to achieve an optimal profit of \$27,871,667.60.

## 5.7 Increasing Family Adventurer's Assembly Time

Due to quality problems associated with Family Adventurer's doors, we need to implement a quality control test for every Family Adventurer. As a result, the assembly time increases from 6 to 7.5 hours. Thus we need to adjust the hours constraint to $7.5f + 10.5t \leq 48500$.

| Variable | Value | Reduced Cost |
|---|---|---|
| F | 1566.667 | 0.000000 |
| T | 3500.000 | 0.000000 |

| Row | Slack or Surplus | Dual Price |
|---|---|---|
| 1 | 0.2434667E+08 | 1.000000 |
| 2 | 0.000000 | 493.3333 |
| 3 | 6733.333 | 0.000000 |
| 4 | 0.000000 | 120.0000 |
| 5 | 3500.000 | 0.000000 |
| 6 | 1566.667 | 0.000000 |

Assuming there is no longer a discount, our solution indicates that we should produce 1566.667 Family Adventurers and 3500 Classic Transporters with this adjusted assembly time. This yields a maximum value of \$23,563,334.40.

### 5.8 Should the Full Demand of the Classic Transporters be Met?

The board of directors of the automobile company wishes to capture a larger share of the luxury sedan market and therefore would like to meet the full demand for Classic Transporters. We are tasked to determine by how much the profit of his assembly plant would decrease as compared to the profit found in the original model. They then ask us to meet the full demand for Classic Transporters if the decrease in profit is not more than \$2,000,000.

| Variable | Value | Reduced Cost |
|---|---|---|
| F | 1958.333 | 0.000000 |
| T | 3500.000 | 0.000000 |

| Row | Slack or Surplus | Dual Price |
|---|---|---|
| 1 | 0.2579583E+08 | 1.000000 |
| 2 | 0.000000 | 616.6667 |
| 3 | 5166.667 | 0.000000 |
| 4 | 0.000000 | -1175.000 |
| 5 | 3500.000 | 0.000000 |
| 6 | 1958.333 | 0.000000 |

With this changed constraint ($t < 3500tot = 3500$), we should produce 1958.333 Family Adventurers and 3500 Classic Transporters. This yields a profit of \$24,816,665.60, which is a \$2,193,337.40 loss compared to the original model. Thus, we should not meet the full demand.

### 5.9 Making a Final Decision

Using our adjusted objective function $2800f + 5300t$ (i.e. the discount is still in place), we observe the following:

-Not implementing an advertising campaign or overtime labor, we see that we should produce 1958.333 Family Adventurers and 3500 Classic Transporters for a profit of \$24,033,332.40.
-Implementing an advertising campaign, but no overtime labor, we see that we should produce 733.333 Family Adventurers and 4200 Classic Transporters. This yields a final profit, after costs, of \$23,813,332.840.
-Implementing overtime labor, but not an advertising campaign, we should produce 3250 Family Adventurers and 3500 Classic Transporters. This yields a final profit, after costs, of \$26,050,000.
-Implementing overtime labor and an advertising campaign, we should produce 2754.167 Family Adventurers and 4200 Classic Transporters. This yields a final profit, after costs, of \$27,871,667.60.

Therefore, we can conclude that the best way to maximize profit in a discount scenario is to implement overtime labor and the advertising campaign as well as produce 2754.167 Family Adventurers and 4200 Classic Transporters. This leaves our company with an optimal profit of $27,871,667.60.