Math 132A Homework Three

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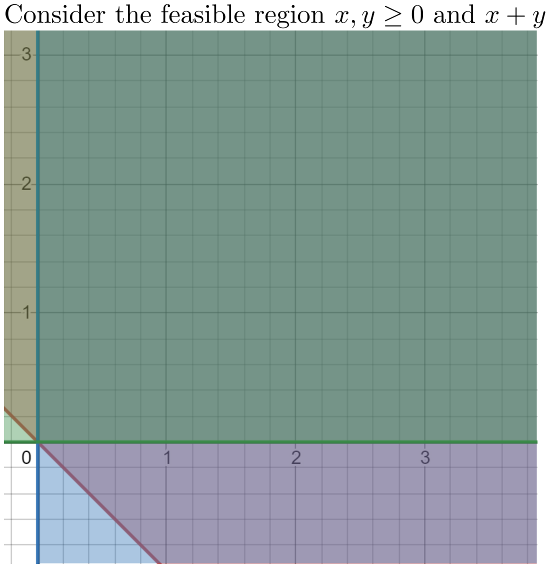
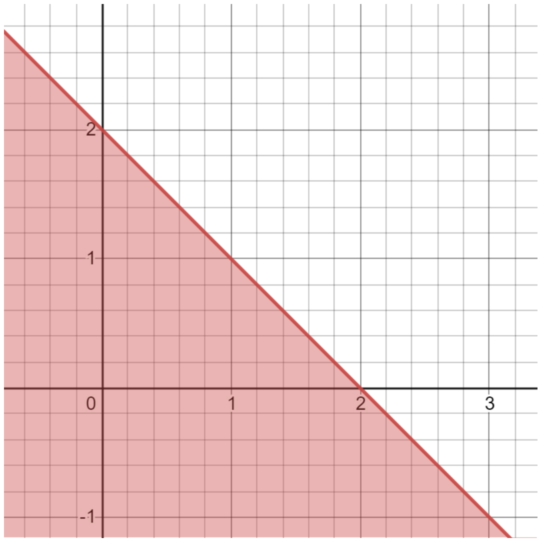
1 Part One

Exercise Two Consider the feasible region $x, y \geq 0$ and $x + y \geq 0$.



Notice it's unbounded, and the only point that cannot be written as a combination of $\lambda(x_1) + (1 - \lambda)(x_2)$, $x_1, x_2 \in$ the feasible region is $(0, 0)$, making it the only extreme point. $(0, 0)$ is also the intersection of 3 hyperplanes.

Exercise Three The converse, if x_0 is on the boundary of C, then x_0 is an extreme point of a convex set C is false. Consider the simple case where $x, y \geq 0$, $x + y \leq 2$, pictured below.



Let $x_0 = (0,1)$. Let $x_1 = (0,0)$ and $x_2 = (0,2)$. Clearly, $(0,1)$ is on the boundary of C, but is not an extreme point as it can be written as the combination between x_1, x_2 where $\lambda = 0.5$ so that $(0.5(0,0) + (1-0.5)(0,2) = (0,0) + (0.5)(0,2) = (0,0) + (0,1) = (0,1))$. Therefore x_0 is on the boundary, but is not an extreme point.

Exercise Four A feasible region of an LP problem cannot have infinitely many extreme points because there are a finite number of hyperplanes defining the region, so there are only a finite number intersections that can create an extreme point. Therefore, the number of extreme points must be finite.
A feasible region can have no extreme points if it is equal to the empty space (ϕ).