Math 132A Homework Six

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1 Exercise One

Consider the model

$$\text{MAX } x_1 + x_2 \text{ SUBJECT TO:}$$

$$x_1 - x_2 + s_1 = 0$$

$$-3x_1 + x_2 + s_2 = 1$$

$$x_1 + 3x_2 + s_3 = 4$$

$$x_1, x_2, s_1, s_2, s_3 \geq 0$$

Putting this in tableaux form, we observe:

	x_1	x_2	s_1	s_2	s_3	
$-z$	1	1	0	0	0	0
s_1	1	-1	1	0	0	0
s_2	-3	1	0	1	0	1
s_3	1	3	0	0	1	4

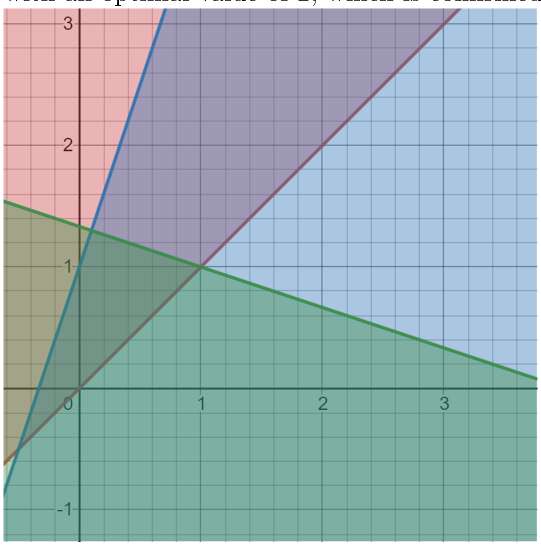
Thus or initial BFS is [0,0,0,1,4]. This point is not optimal, thus let x_1 enter the basis because of its smaller index. The MRT of 0 indicates s_1 should leave the basis. After some row calculations, our adjusted model is below.

	x_1	x_2	s_1	s_2	s_3	
$-z$	0	2	-1	0	0	0
x_1	1	-1	1	0	0	0
s_2	0	-2	3	1	0	1
s_3	0	4	-1	0	1	4

The MRT of 1 for x_2 indicates that it should enter the basis and s_3 should leave. This gives us:

	x_1	x_2	s_1	s_2	s_3	
$-z$	0	0	$\frac{-1}{2}$	0	$\frac{-1}{2}$	-2
x_1	1	0	$\frac{3}{4}$	0	1	1
s_2	0	0	$\frac{5}{2}$	1	2	3
x_2	0	1	$\frac{-1}{4}$	0	$\frac{1}{4}$	1

Now the RCC are all negative, we conclude there is one optimal solution [1,1,0,3,0] with an optimal value of 2, which is confirmed by the graph below.



2 Exercise Two

Consider the model, introduced with slack and artificial variables:

$$\text{-MAX} \quad -a \text{ SUBJECT TO:}$$

$$x_1 - x_2 + s_1 = 1$$

$$3x_1 + x_2 - s_2 + a = 3$$

$$x_1, x_2, s_1, s_2, a \geq 0$$

Putting this in tableaux form, we observe:

	x_1	x_2	s_1	s_2	a	
$-a$	0	0	0	0	-1	0
s_1	1	-1	1	0	0	1
a	3	1	0	-1	1	3

Converting the RCC of a to 0 yields:

	x_1	x_2	s_1	s_2	a	
$-a$	3	1	0	-1	0	3
s_1	1	-1	1	0	0	1
a	3	1	0	-1	1	3

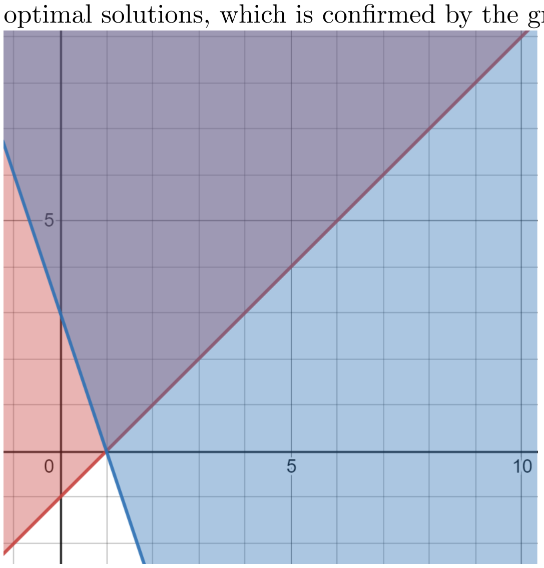
The MRT for x_1 is a tie, so we choose s_1 to leave the basis and x_1 to enter it. Then,

	x_1	x_2	s_1	s_2	a	
$-a$	0	4	-3	0	0	0
x_1	1	-1	1	0	0	1
a	0	4	-3	-1	1	0

The MRT for x_2 of 0 indicates a should leave the basis and x_2 to enter it.

	x_1	x_2	s_1	s_2	a	
$-a$	0	0	0	1	-1	0
x_1	1	0	$\frac{1}{4}$	$\frac{-1}{4}$	$\frac{1}{4}$	1
x_2	0	1	$\frac{-3}{4}$	$\frac{-1}{4}$	$\frac{1}{4}$	0

Note when conducting the MRT for s_2, we see that we can increase s_2 to infinity and not leave the feasible region. Thus, this problem is unbounded has no optimal solutions, which is confirmed by the graph below.



3 Exercise Five

Consider the model

$$\text{MAX } z(x_1, x_2) = x_1 + 2x_2 \text{ SUBJECT TO}$$

$$x_1 \leq 2$$

$$x_2 \leq 1$$

$$x_1 + 2x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

Given the last simplex tableaux,

	x_1	x_2	s_1	s_2	s_3	
$-z$	0	0	0	0	-1	-2
s_1	1	0	1	0	0	2
s_2	$\frac{-1}{2}$	0	0	1	$\frac{-1}{2}$	0
x_2	$\frac{1}{2}$	1	0	0	$\frac{1}{2}$	1

We see that $x_0 = [0, 1, 2, 0, 0]$ is a degenerate point (s_2 is a basic variable equal to 0). Since the RCC ≤ 0, this implies x_0 is the one optimal solution, which yields an optimal value of 2. This is confirmed below