Project 2

Crafting S.L. Market Expansion

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1 Introduction

Our company Crafting S.L located in California produces cloth, yarn, and crafting materials. So far we have only sold our products in the state but now we have decided to start selling our goods to other states. Since sales in California have not been very good lately, the goal of the company is maximizing our weekly profit from sales to other states and not sell in California anymore. The profit is computed as the difference between the revenue from sales and the cost of materials.

The marketing office has done some research and got to the conclusion that we could sell all that they produce at \$30 per meter of cloth, \$9 per skein of yarn, and \$25 per pound of crafting materials. In order to produce one meter of cloth, it is necessary to use 1 skein of yarn, 0.1 pounds of crafting materials and 200 yards of sewing thread (the company buys thread at \$0.5 per 100 yards). Other necessary materials and expenses, such as machinery, cost about \$4 per meter of cloth. In addition, it takes 30 min. of labor to produce 1 meter of cloth. The company can produce a maximum of 5,500 meters of cloth per week.

In order to produce 1 skein of yarn, the company uses 100 yards of sewing thread, 0.05 pounds of crafting materials, as well as, 5 min. of labor. Additionally, the cost of the necessary machinery is about \$2.5 per skein of yarn. The company can produce a maximum of 3,000 skeins of yarn per week.

To produce one pound of crafting materials, the company uses 50 min. of labor and the cost of the machinery used is \$1.5 per pound. Additionally, they use 0.75 skeins of yarn and 0.12 meters of cloth. The maximum amount of crafting materials that the company can produce is 500 pounds per week.

The company has a maximum of 2000 hrs of labor available each week. And since it does not have a warehouse, everything that is produced needs to be sold or used to produce other goods. The company has credit to buy thread and simple crafting materials every week. But the company cannot buy more than 1000 pounds of simple crafting materials per week and it loses \$2 per pound in this transaction (due to the payment of interests to the bank). Notice that our company does not make any profit in the sale of these units of crafting materials since it has to return the sales money to the bank.

2 Model

Let the decision variables be defined as follows:

M = Meters of cloth produced in a week
N = Meters of cloth sold in a week
S = Skeins of yarn produced in a week

T = Skeins of yarn sold in a week
P = Pounds of crafting materials produced in a week
Q = Pounds of crafting materials sold in a week
B = Pounds of simple crafting materials purchased in a week

Our objective function is to maximize profit, which is equivalent to revenue minus costs $(30N + 9T + 25Q - 2B - (M + 0.5S) - (4M + 2.5S + 1.5P))$. Thus, our objective function is

$$\max 30N + 9T + 25Q - 5M - 3S - 1.5P - 2B$$

Subject to the following constraints:

(Produced Cloth)

$$M = N + 0.12P$$

(Produced Yarn)

$$S = T + M + 0.75P$$

(Produced and Bought Materials)

$$P + B = Q + 0.1M + 0.05S$$

(Labor)

$$\frac{1}{2}M + \frac{1}{12}S + \frac{5}{6}P \leq 2000$$

$$M \leq 5500$$

$$S \leq 3000$$

$$P \leq 500$$

$$B \leq 1000$$

$$M, N, S, T, P, Q, B \geq 0$$

As we mentioned previously, the objective function was derived from the revenue (30N+9T+25Q) and cost functions (5M+3S+1.5P+2B). We know that every meter of cloth sold (N) generates \$30 in revenue, every skein of yarn sold (S) generates \$9, and every pound of crafting material generates \$25. In regard to cost, we added together the cost of machinery to produce one unit of each product (4M+2.5S+1.5P), the cost of sewing thread to produce one unit of each product (M+0.5S), and the cost of purchasing simple crafting materials (2B) to obtain the total cost.

To obtain the constraints, we started with labor. It takes 30 minutes, 5 minutes, and 50 minutes of labor to make one unit of each product (M, S, P) respectively, and we are limited to 2000 labor hours per week, which is how we obtain $\frac{1}{2}M + \frac{1}{12}S + \frac{5}{6}P \leq 2000$. Production limitations lead to the production of each product (produced and purchased) to be 5500, 3000, 500, 1000 with respect to produced cloth, yarn, and crafting materials, and bought/purchased crafting materials.

To reflect the necessity to use some produced products to make others, we define the amount of a product produced to be equivalent to the number of products sold plus the amount of product is needed to make other products. Then, the amount of cloth to produce (M) should equal the amount of cloth sold (N) plus the amount used to make crafting materials (0.12 per unit of produced crafting materials) to obtain $M = N + 0.12P$. We can apply similar logic to produced yarn to obtain S = T + M + 0.75P, since 1 skieve of yarn is needed to produce a unit of cloth and 0.75 skieves of yarn are used to make a pound of crafting materials. Notice we combine the amount of crafting materials produced and purchased since it does not matter which is used to produce cloth and yarn. Thus, this sum of materials is equivalent to how many units are sold and 0.1 pounds and 0.05 pounds are used to produce one unit of cloth and yarn. Thus, we get P + B = Q + 0.1M + 0.05S.

Discussing Our Use of a Linear Model

Proportionality

The constraints used all have consistent coefficients; they do not change depending on which unit of product is in production. Thus these are proportional constants. Additionally, these coefficients are the only numbers multiplying variables (no variable is multiplying another variable), so this assumption is held.

Additivity

Clearly, the objective function and constraints in this problem only use addition between variables. Notice the only other operation being used is multiplication between a variable and its coefficient. Thus, our situation maintains the additivity assumption.

Divisibility

At first glance, we hesitate to accept that the divisibility assumption applies here. However, we can begin production on another cloth, yarn, or crafting material and allow it to rollover into next week's productions. Furthermore, we do not need to sell/buy a standard unit of product, it is acceptable, for instance, to sell $\frac{2}{3}$ meters of cloth. Thus, it makes sense to use fractions, so it makes sense that divisibility applies to our model.

Certainty

There are no probabilistic elements in this problem. A meter of cloth will always be sold for \$30, buying a pound of crafting material will always end up costing the company \$2, and producing a skein of yarn will always cost \$3, for instance. Note this logic can be applied for every variable based on the data that was available at this time. Also, all the constraints were given determined limits (we knew production and purchasing was limited to 5500, 3000, 500, 1000).

3 Solution and Discussion of Different Scenarios

3.1 Solving Our Original Model

Now, solving our model (see Computer Generated Reports section (1)), we see we should sell 2565 meters of cloth, 0 skeins of yarn, 1087.5 pounds of crafting materials, produce 2625 meters of cloth, 3000 skeins of yarn, and 500 pounds of crafting materials, and buy 1000 pounds of simple crafting materials. This leads us to obtaining an optimal profit of $79,262.50.

3.2 Increasing Demand of Yarn

Due to an increase in demand for yarn, the company has increased the price of yarn by $1. Despite this revenue increase, the optimal production/sale mix and profit do not change. We can verify this by looking at computer report (2) since we can increase the revenue coefficient of yarn by $13.50 without changing profit.

3.3 Increasing Yarn Revenue to Ensure it is Profitable

In order to become profitable in this mix, the revenue of selling a skeins of yarn should be increased by, at least, $13.51. Thus the revenue we should obtain from yarn to become profitable is $22.51. Considering that the average sale price of a good quality skein of yarn is $10, this is an unrealistic sales price.

3.4 Selling 100 Skeins of Yarn

If a good client of our company asked us to sell them 100 skeins of yarn and we increased the sales price of yarn to $11.50/unit to minimize the loss, we would lose $1,100.00 of profit. Using computer report (1), we can see how we obtained this value from the Reduced Cost Coefficient of T (13.50). Since we sell a skeins of yarn at $11.50 instead of $9.00, so this reduced cost coefficient would be $13.50-$2.50=$11.00. Thus, every skeins of yarn we sell we lose $11.00. Selling 100 skeins to our client leaves us with our total profit loss, $11.00*1000=$1,100.00.

3.5 Identifying the High Costing Resource

The resource we should use as sparingly as possible in our new products should be cloth. We can see this in the CLOTH, YARN, CRAFTM, and LABOR constraints' shadow prices (-30, -22.5, -25, 0). These tell us that cloth will be the most expensive to produce more of for new products (our profit will go down -$30 with a one unit increase in cloth production.

3.6 Pricing a New Cloth Bag Product

We use the shadow prices to determine how much we should price a new cloth bag product in order to be profitable, considering they require 15 minutes of labor, 0.3 meters of cloth, and 0.01 pounds of crafting materials for each unit produced. Thus, the cost of this product would be $\frac{1}{4}*0+0.3*30+0.01*25 = 9.25$. To be profitable, the revenue should be at least $9.26.

3.7 Expanding Production Capacity

If the company had $5,000 extra to spend on expanding production capacity, and our choices were to spend it on increasing the production of cloth by 50 meters per week, or 30 skeins of yarn per week, or 40 pounds of crafting materials per week, we should spend it on yarn production. We can see this is the best choice by using the shadow prices, which tell us the increase in the optimal value if the production capacity increased by one unit. So we see, 50 meters increase in cloth production yields 0*50 = $0 increase in profit; 30 skein increase yields 18.25*30 = $547.50 increase in profit; 40 pound increase yields 3.025*40 = $121 increase in profit. Since $547.50 is the maximum optimal profit increase, we should invest in yarn production.

3.8 Effects of Increasing the Price of Thread to Produce Cloth

The price of thread used to produce cloth has increased by $0.40. The company should not reconsider the amount it produces for sale in order to maximize its profit because in our coefficient ranges, we see that the coefficient of M can decrease by 13.50 before the optimal mix changes and $0.4 < 13.50$. The profit change for the optimal mix in this scenario is $1,050 (2625*$0.4).

3.9 Small Warehouse Offer

If we were to rent the warehouse and stockpile yarn, the constraint YARN defined as $S = T + M + 0.75P$ must add 500 on the right side, since this is how many skeins of yarn are needed in order to fill the warehouse. The shadow price of this constraint is -22.5, so 22.5*500=$11,250+$300=$11,550 is the weekly profit loss.

3.10 Modifying Our Model to Consider Existing and Desired Inventories

It possible to modify our LP model to consider existing inventories at the start of each week and desired inventories at the end of the week. We do this by solving desired inventory at the end of the week minus the beginning inventory and adding that number to the right hand side of the equality constraints. This is a similar process to what we did above for yarn, so we can repeat this for the other products to derive our adjusted constraints.

3.11 Adjusting to Labor Strikes

Looking at the allowed decrease of labor hours in (2), we can see that even if a labor strike were to occur, we could lose 23.5 hours of labor a week without a change in our optimal mix. Thus in the first scenario, we can see that no change would occur as 500 minutes is equivalent to 8.33 hours which is much lower than 23.5. For the second scenario, there would also be no change since 1300 minutes is 21.67 hours which is also lower than 23.5. Thus for both cases our optimal profit will not be affected in the event of a labor strike.

3.12 Considering a New Scenario

Scenario 1

Suppose that an error was found in the production of cloth causing that week's maximum production to be reduced to 1000. What would the company have to change their selling price of cloth to in order to not lose any profit? After changing the cloth constraint to $M \leq 1000$ and keeping the initial \$30 cost of cloth we get a profit of \$57325.00 which is \$21937.50 lower than our initial profit. We keep having a decrease in profit until the price of cloth is changed to \$54 which gives us a optimal profit of \$79885.00 which is a \$622.50 increase in profit.

Scenario 2

Due to our company's long-standing loyalty and good credit, the bank has given us a lowered interest rate for purchasing simple crafting materials. Suppose now instead of costing us \$2 per pound, the company only loses \$0.50 per pound in each transaction. How much does the optimal profit change?

From our range report, we see that increasing the coefficient of simple crafting materials, B, is able to increase infinitely without changing the optimal mix. This, however, does not mean that the optimal profit does not change. We can expect to have a profit increase of \$1500 ($2 * 1000 - 0.5 * 1000 = 1500$).

4 Recommendations

To begin, in the event our company considers a new product, we should try to use products made using labor, then yarn, craft materials and finally cloth to save money. In the event we could expand our production capacity, we should invest in purchasing simple crafting materials, producing more yarn, then produced crafting materials, and finally cloth and labor since we can optimize our increased profit in this order according to their respective constraints' shadow prices (23, 18.25, 3.025, 0, 0). When pricing a new product, we should take into account the cost of producing one more unit of product of cloth, yarn, craft materials, and labor (-30, -22.5, -25, 0). Additionally, given our original model, we should produce yarn more then we sell it, since we will lose \$13.50 per unit if we do so.

5 Computer Generated Reports

Original Model Solution (1)

```
Global optimal solution found.
Objective value:                              79262.50
Infeasibilities:                              0.000000
Total solver iterations:                             2
Elapsed runtime seconds:                          0.04

Model Class:                                        LP

Total variables:                      7
Nonlinear variables:                  0
Integer variables:                    0

Total constraints:                    9
Nonlinear constraints:                0

Total nonzeros:                      26
Nonlinear nonzeros:                   0
```

Variable	Value	Reduced Cost
N	2565.000	0.000000
T	0.000000	13.50000
Q	1087.500	0.000000
M	2625.000	0.000000
S	3000.000	0.000000
P	500.0000	0.000000
B	1000.000	0.000000

Row	Slack or Surplus	Dual Price
1	79262.50	1.000000
CLOTH	0.000000	-30.00000
YARN	0.000000	-22.50000
CRAFTM	0.000000	-25.00000
LABOR	23.50000	0.000000
PCLOTH	2875.000	0.000000
SYARN	0.000000	18.25000
PCRAFT	0.000000	3.025000
BCRAFT	0.000000	23.00000

Original Model Range Solutions (2)

Ranges in which the basis is unchanged:

Objective Coefficient Ranges:

Variable	Current Coefficient	Allowable Increase	Allowable Decrease
N	30.00000	3.477011	13.50000
T	9.000000	13.50000	INFINITY
Q	25.00000	121.6667	2.813953
M	-5.000000	4.033333	13.50000
S	-3.000000	INFINITY	18.25000
P	-1.500000	INFINITY	3.025000
B	-2.000000	INFINITY	23.00000

Righthand Side Ranges:

Row	Current RHS	Allowable Increase	Allowable Decrease
CLOTH	0.000000	2565.000	INFINITY
YARN	0.000000	2565.000	47.00000
CRAFTM	0.000000	1087.500	INFINITY
LABOR	2000.000	INFINITY	23.50000
PCLOTH	5500.000	INFINITY	2875.000
SYARN	3000.000	40.30875	2565.000
PCRAFT	500.0000	51.64835	500.0000
BCRAFT	1000.000	INFINITY	1000.000

6 Honor Code

I, *Carly Greutert*, have completed this work without the help of the use of internet. I certify that the answers have been produced by my group. I am aware that breaking this honor code will result in a 0 in this assignment.

I, *Daniel Yu*, have completed this work without the help of the use of internet. I certify that the answers have been produced by my group. I am aware that breaking this honor code will result in a 0 in this assignment.