Math 132A HW 2

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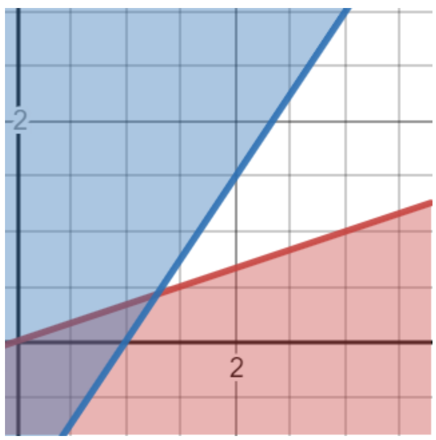
1 Part One

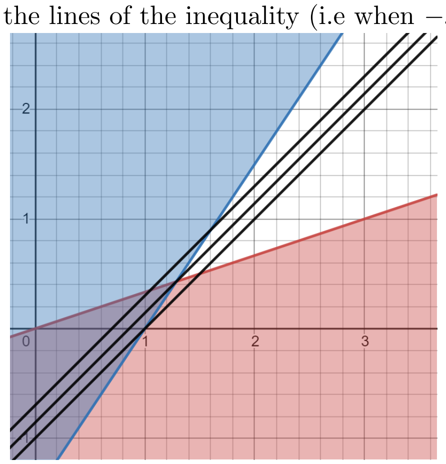
1.1 1.31a,b,c



1.31a)

Note the intersecting region in the first quadrant is the feasible region.

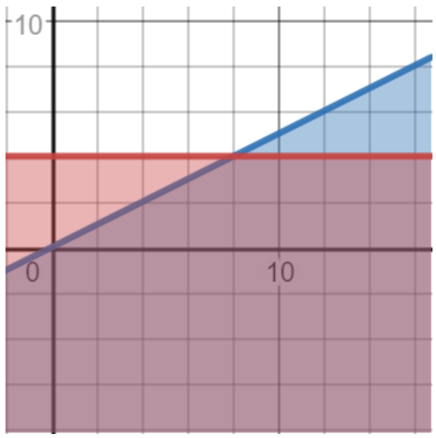
1.31b) The slack variables x_3, x_4 are equal to zero for all the points directly along the lines of the inequality (i.e when $-x_1 + 3x_2 = 0$, $-3x_1 + 2x_2 = -3$).



1.31c)

Note the intersection of the feasible region with the level curve $x - y = 0.8574$. Thus, there is only one optimal solution, (1.286, 0.4287) in order to achieve the optimal value, 0.8574.

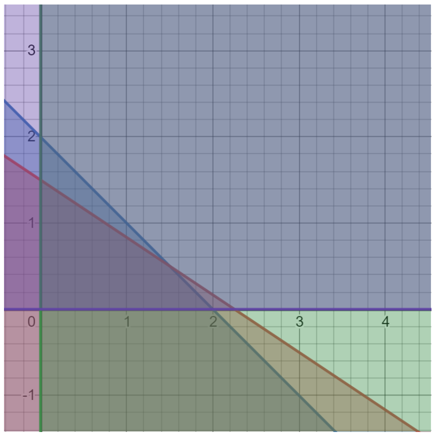
1.2 1.34

1.34a) 

The intersecting region is the feasible region.

1.34b) Notice the vertex of the feasible set (8,4) yields a value of 28. Consider, for example, the point (10, 4) in the feasible region. This point yields a value of $3(10) + 4 = 30 + 4 = 34$, which is a larger value than 28. Note, with a set value of $x_2 = 4$, you can increase x_1 infinitely, stay in the feasible region, and have a continually increasing optimal value. Thus, this problem has an unbounded optimal solution value.

1.3 1.35

1.35a) 

1.35b) Two alternative optimal extreme (corner) points are (1.5, 0.5) and (0, 1.5).

1.35c) Let S represent an infinite class of optimal solution values. We define S as the set

$$\{4x_1 + 6x_2 = 9 : 0 \leq x_1 \leq 1.5, 0.5 \leq x_2 \leq 1.5\}$$

2 Part Two

2.1 Exercise Two

Variables
Let x_1 = the number of ounces sold of Regular Brute
x_2 = the number of ounces sold of Regular Chanelle
x_3 = the number of ounces sold of Luxury Brute
x_4 = the number of ounces sold of Luxury Chanelle
x_5 = the number of pounds of raw material purchased annually

Objective Function
maximize $7x_1 + 6x_2 + 18x_3 + 14x_4 - 3x_5$ subject to the following

Constraints

$$\begin{aligned} x_5 &\leq 4000 \\ 3x_3 + 4x_4 + x_5 &\leq 6000 \end{aligned}$$

$$x_1 + x_3 - 3x_5 = 0$$
$$x_2 + x_4 - 4x_5 = 0$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Solution & Interpreting Results

The optimal solution is $x_1 = 11,333.33$, $x_2 = 16,000.00$, $x_3 = 666.6667$, $x_4 = 0.00$, and $x_5 = 4000.00$. This yields an optimal value (maximized profit) of \$175,333.31. The slack variables in the chart indicate that all the inequalities were satisfied exactly (the slack/surplus variable are all equivalent to 0). The divisibility assumption still holds because you can buy/sell a partial ounce of material. Thus, integer programming is not required.