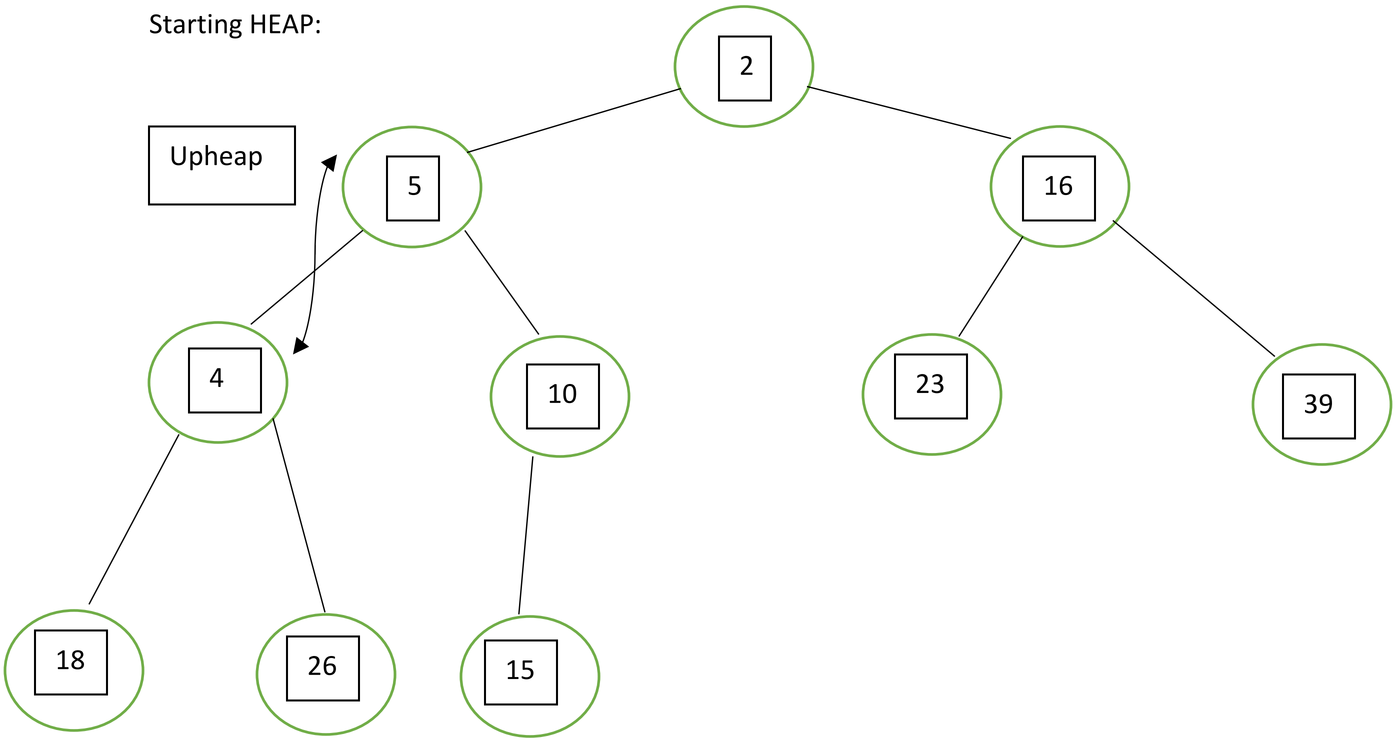
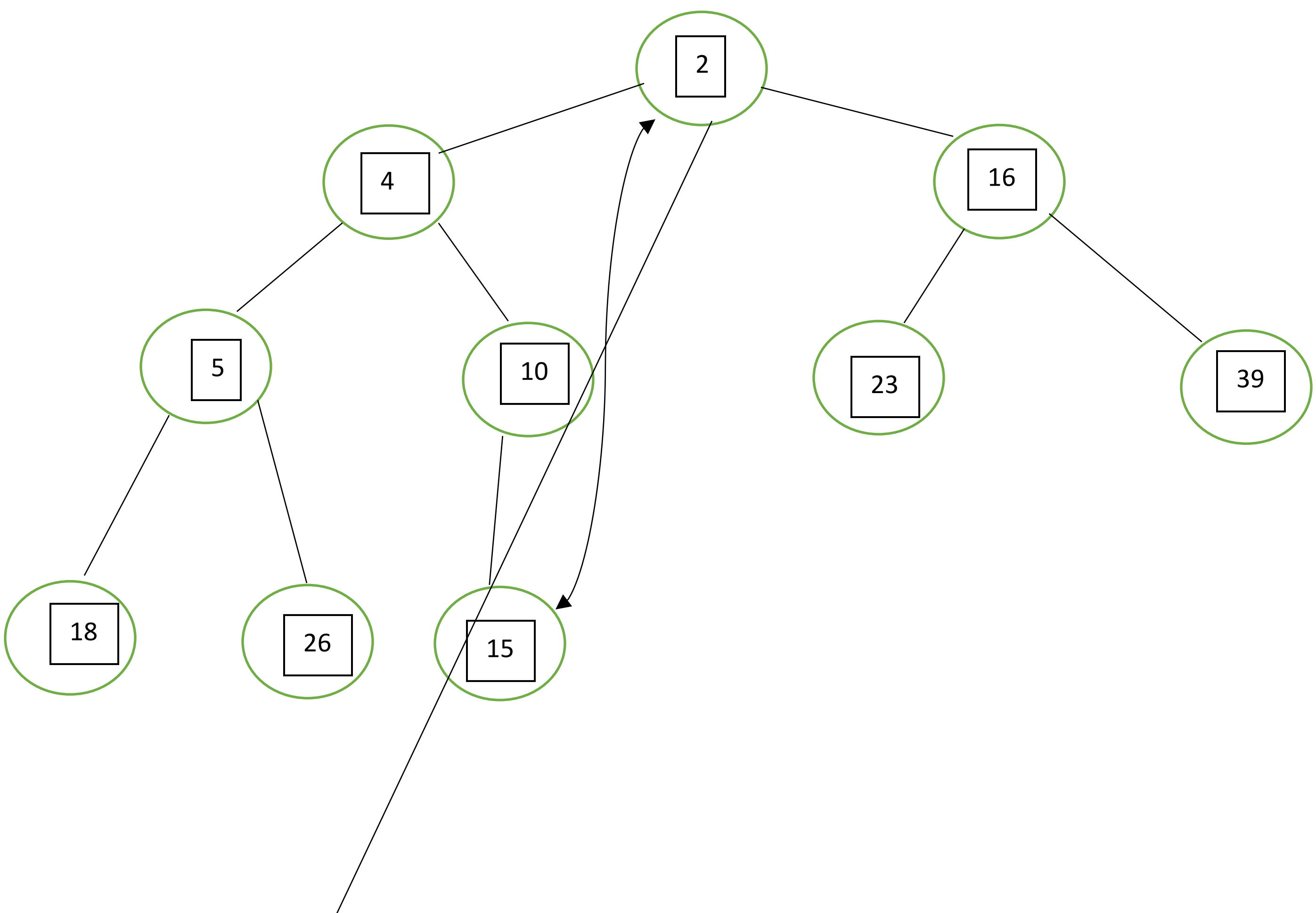
Chris Grimes
CS 46101
Homework #3

1. (a) (5 points) Illustrate the execution of the heap-sort algorithm on the following sequence: (2, 5, 16, 4, 10, 23, 39, 18, 26, 15). Show the contents of the (min) heap and the sequence at each step of the algorithm. Indicate upheap or downheap bubbling where appropriate.

Starting HEAP:



min HEAP:	0	0	0	0	0	0	0	0	0	0



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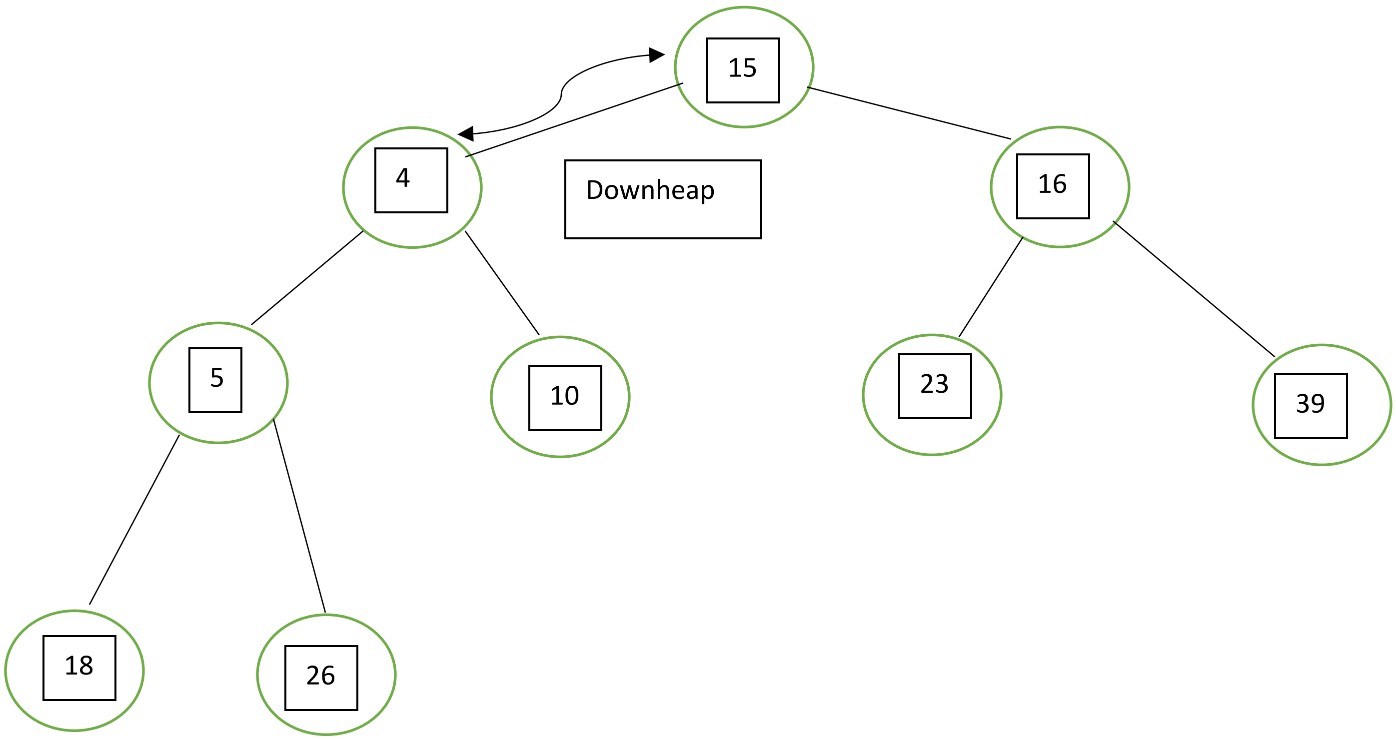
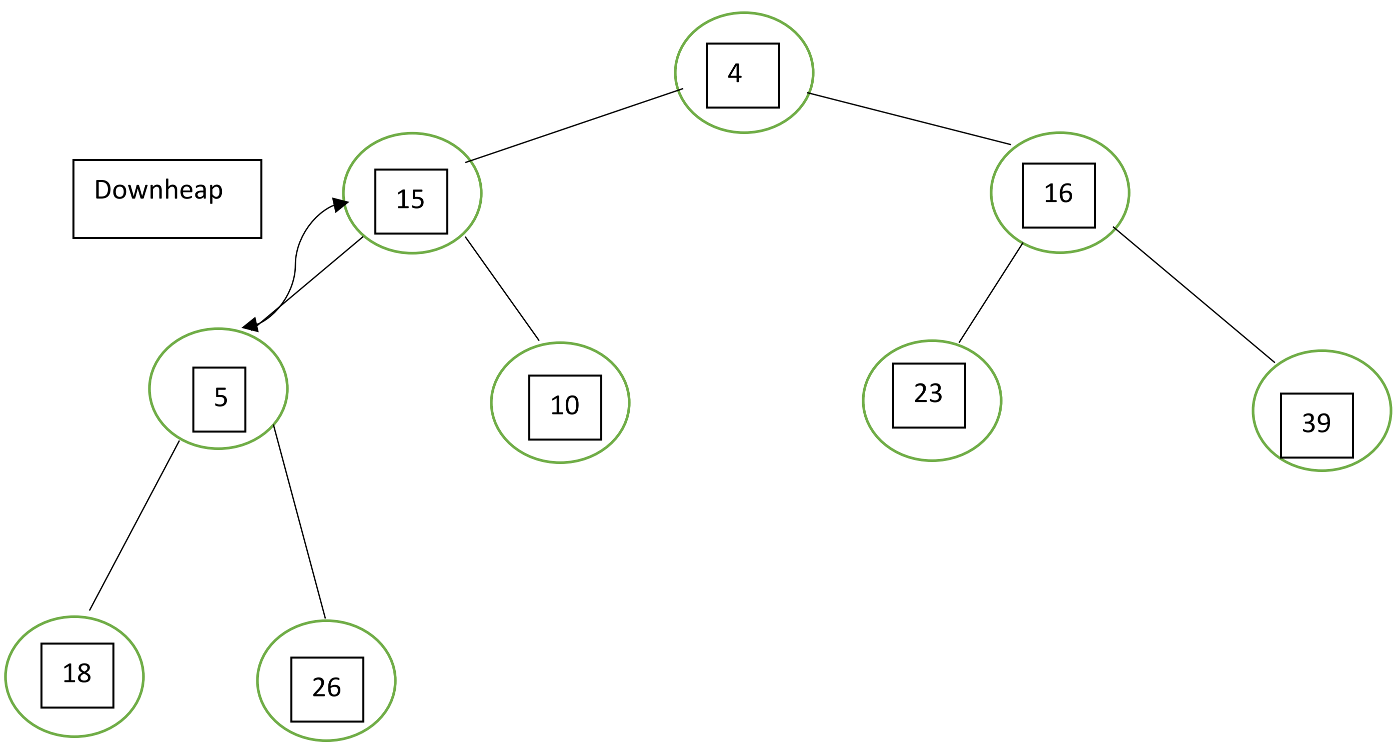
min HEAP:	2	0	0	0	0	0	0	0	0	0





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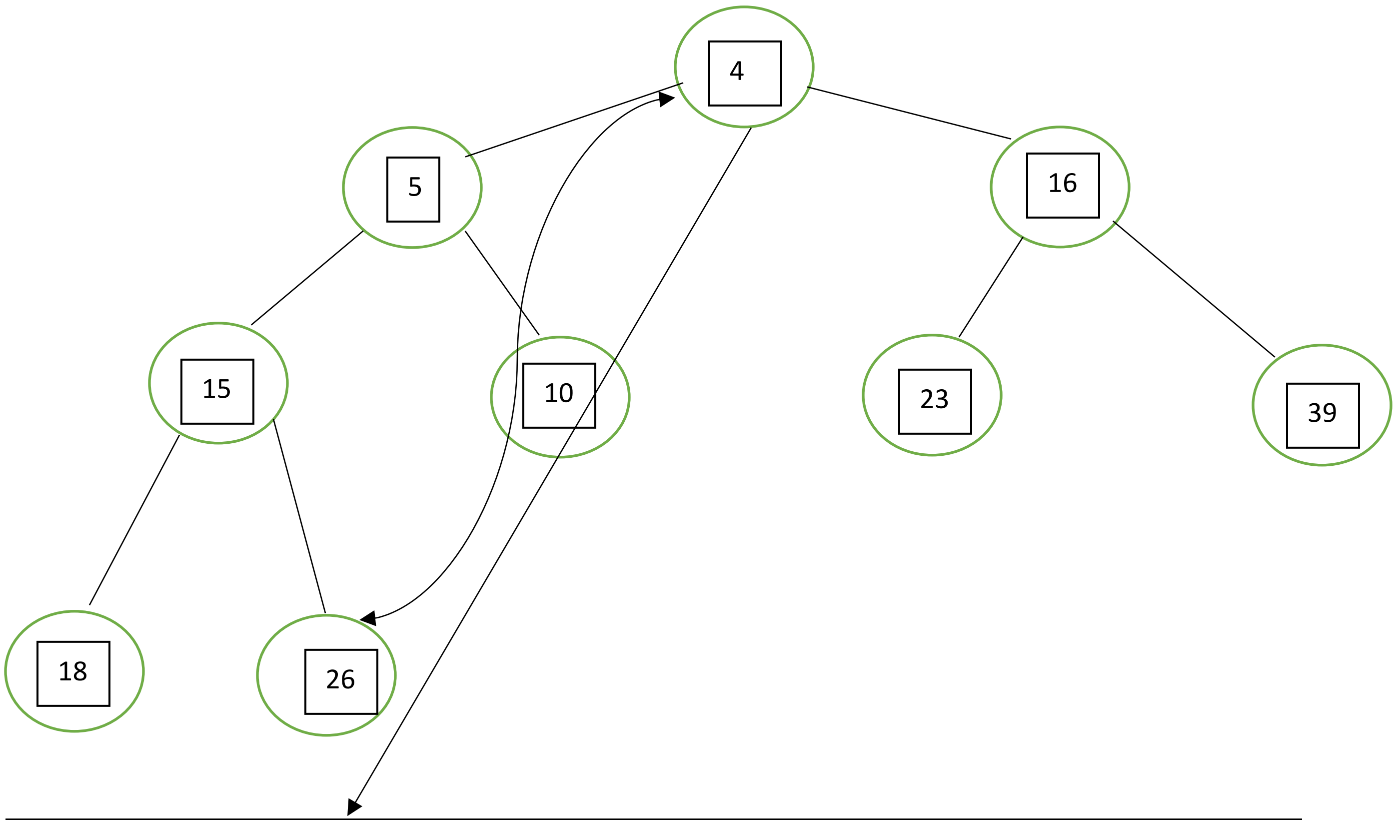

min HEAP:	2	4	0	0	0	0	0	0	0	0

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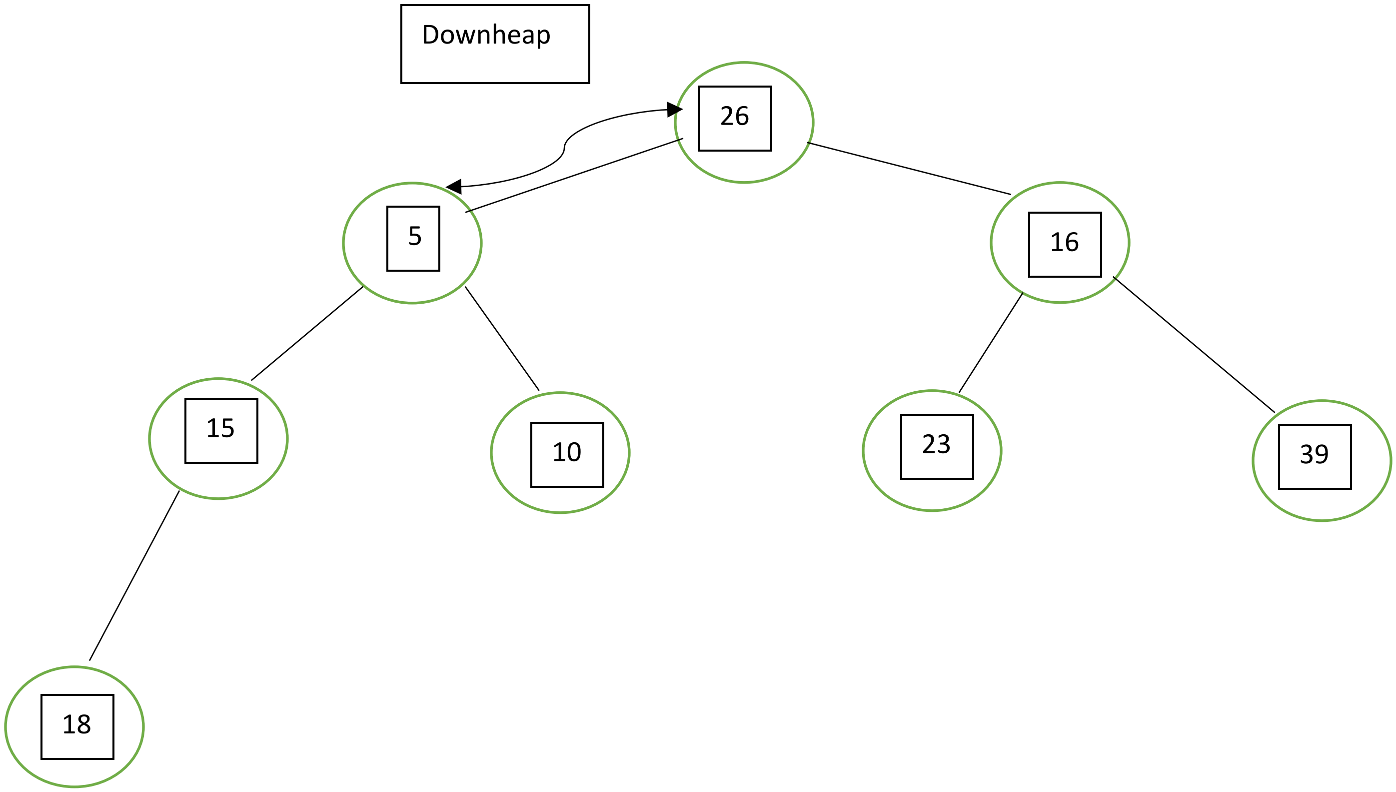

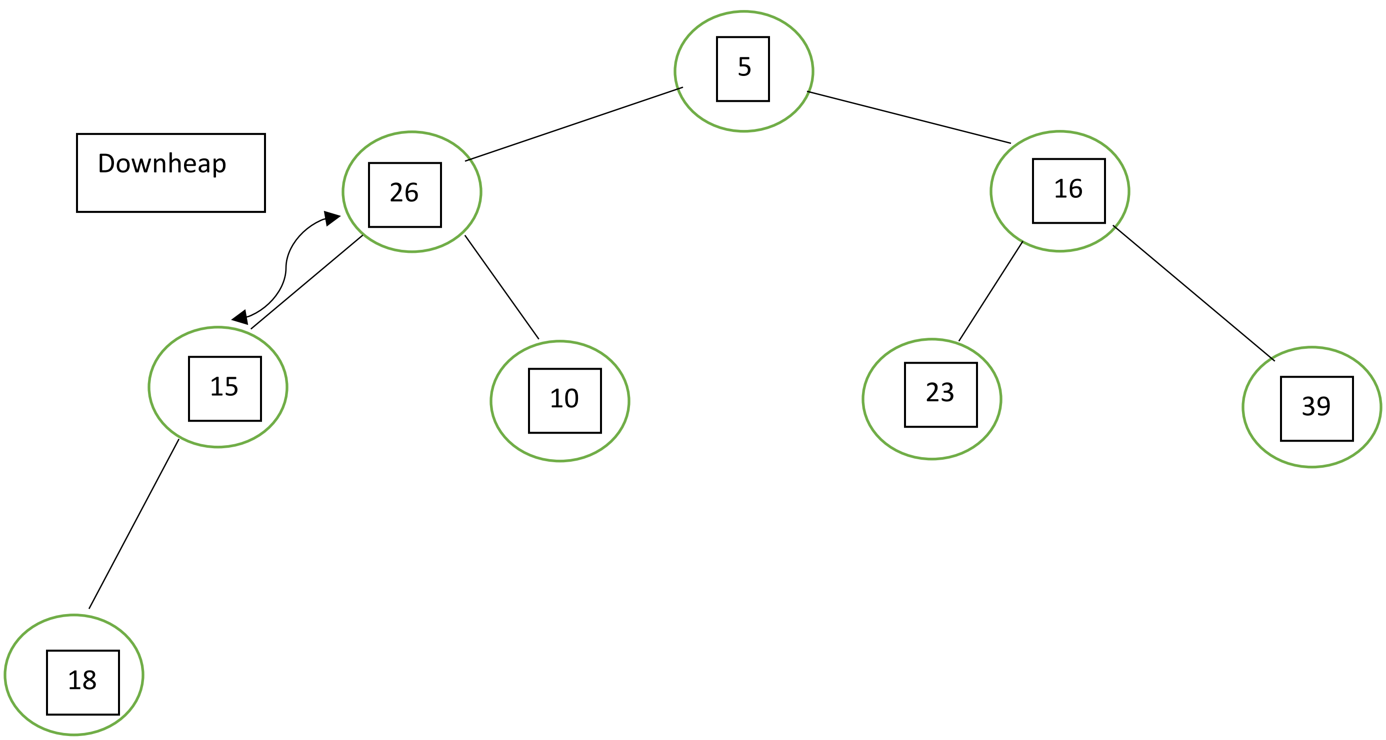

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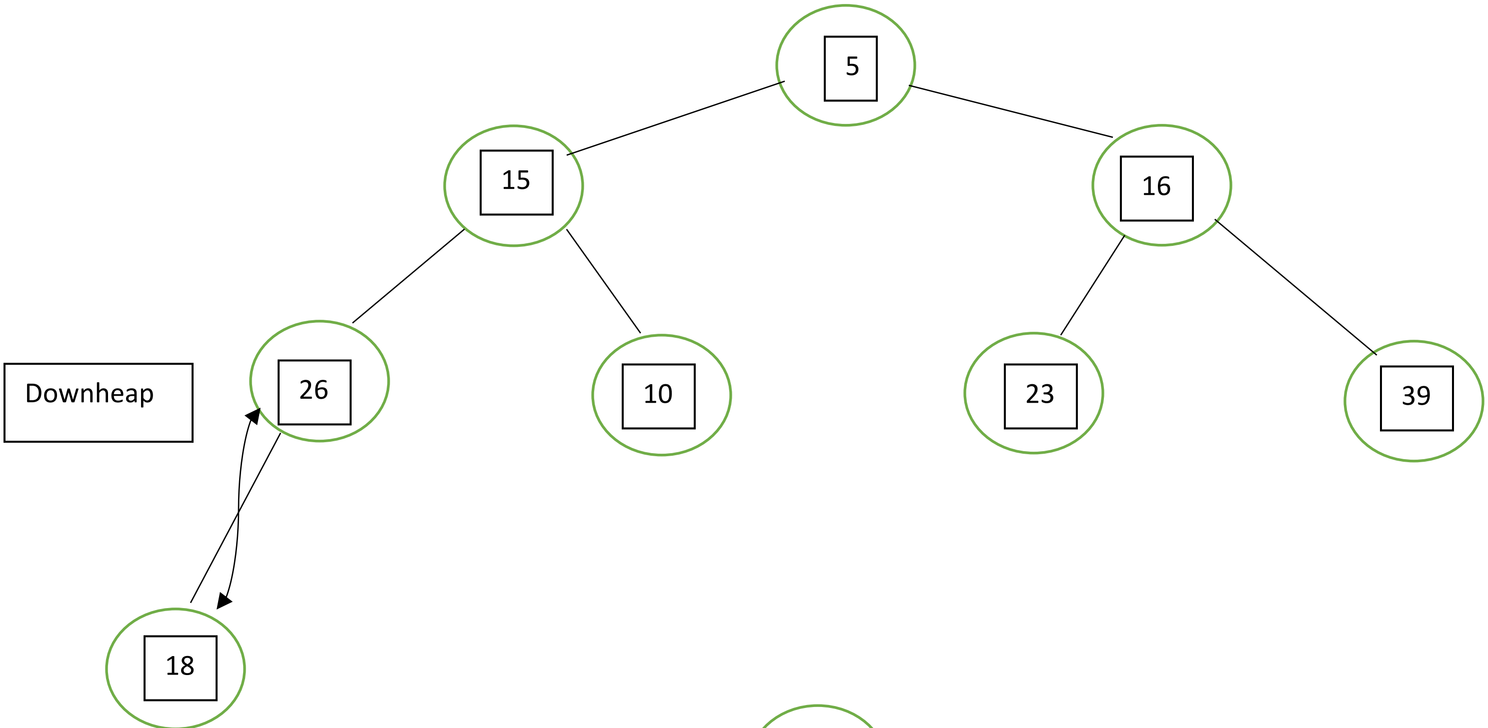

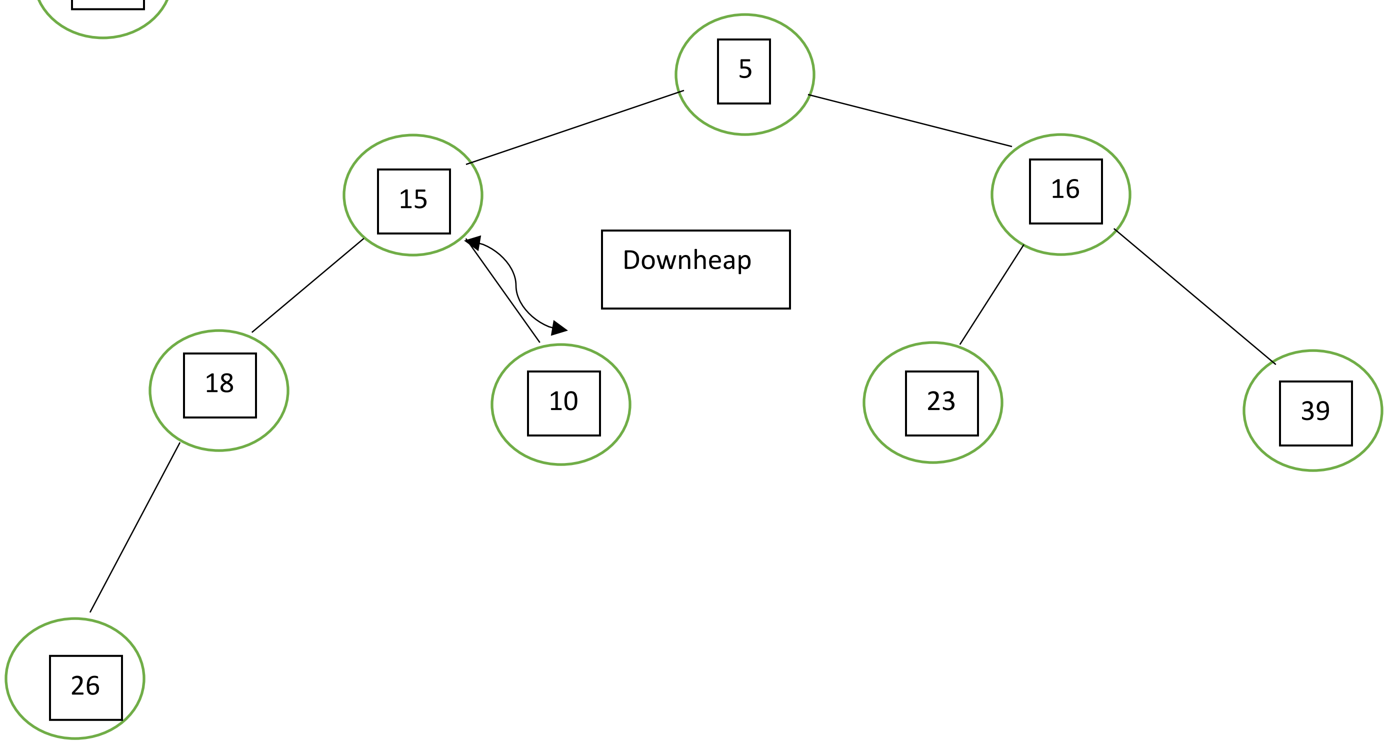

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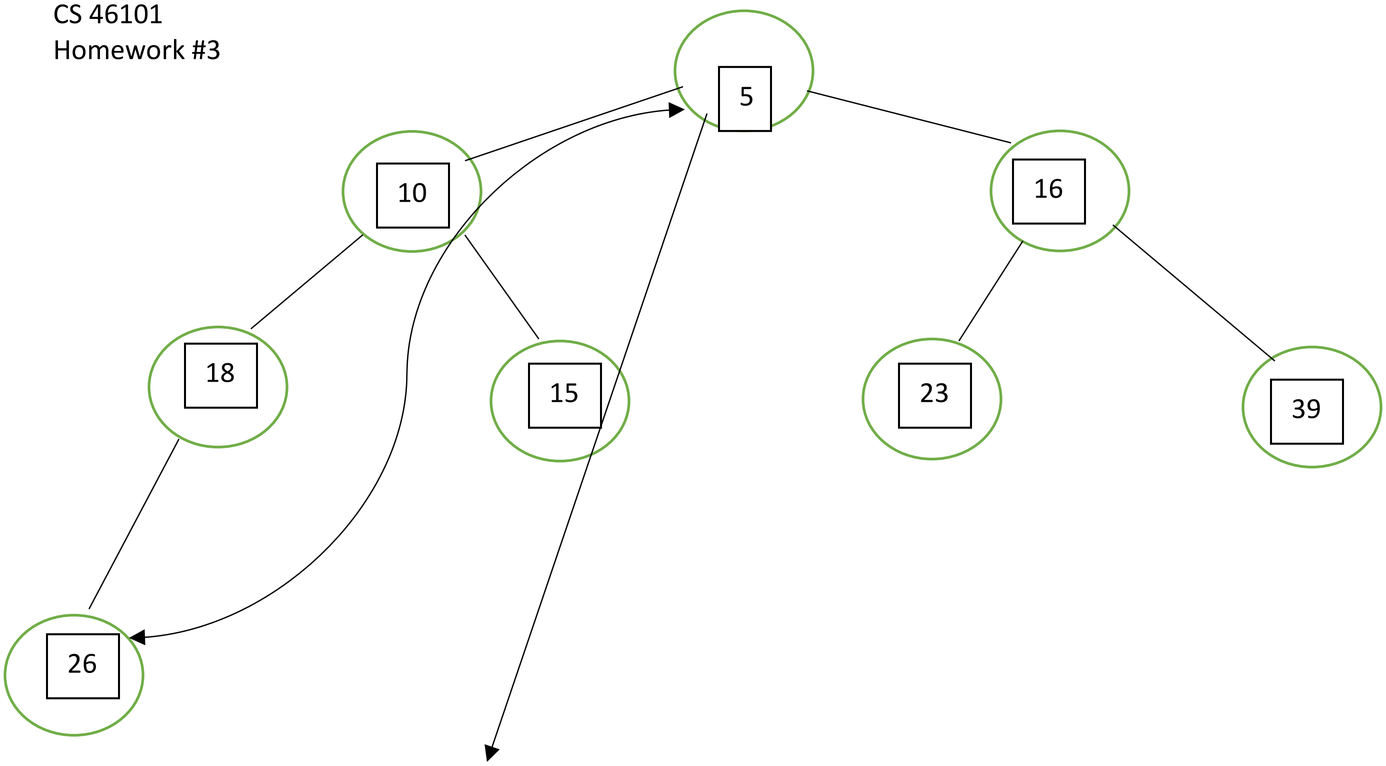
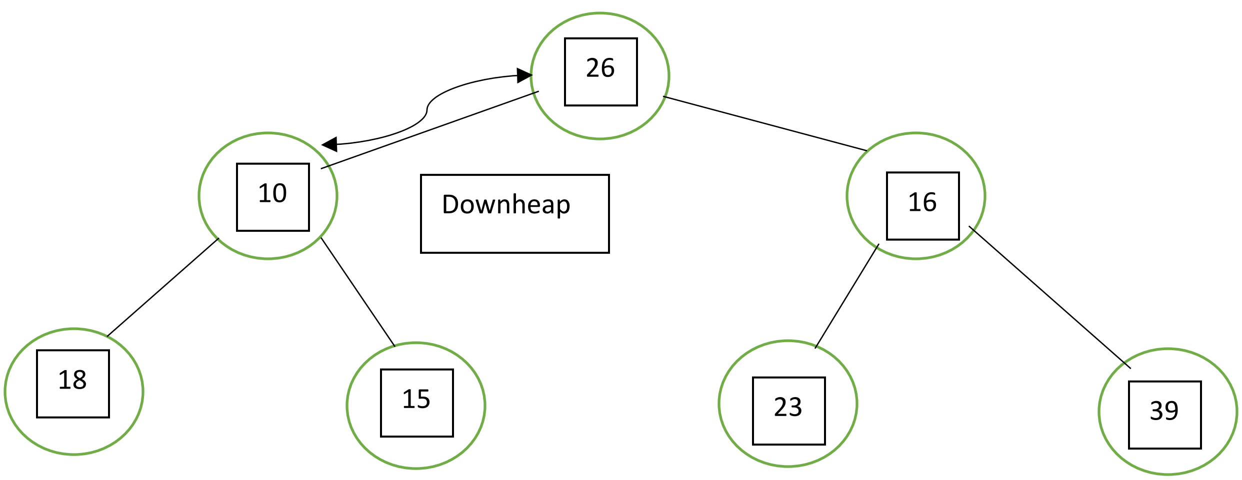


min HEAP:	2	4	5	0	0	0	0	0	0	0



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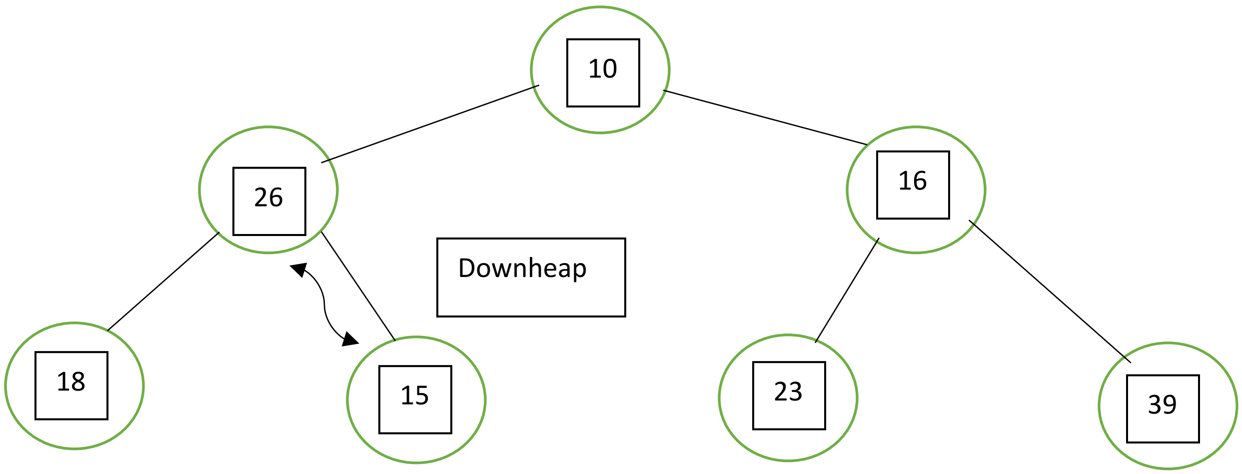

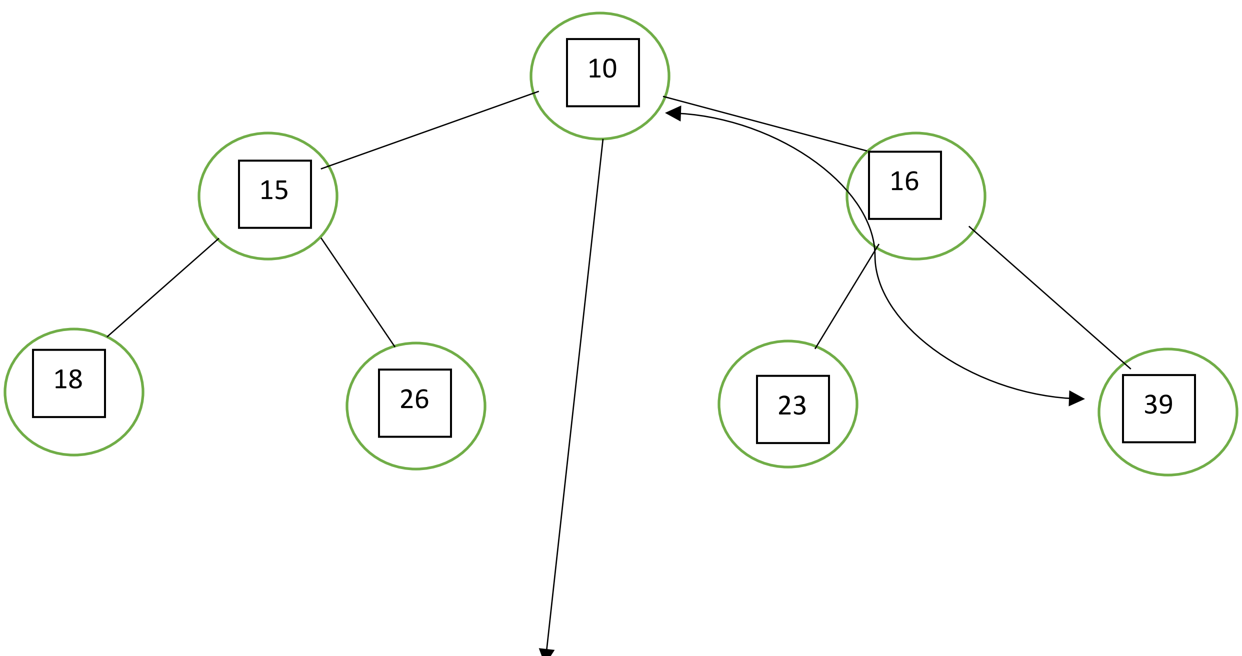

min HEAP:	2	4	5	10	0	0	0	0	0	0

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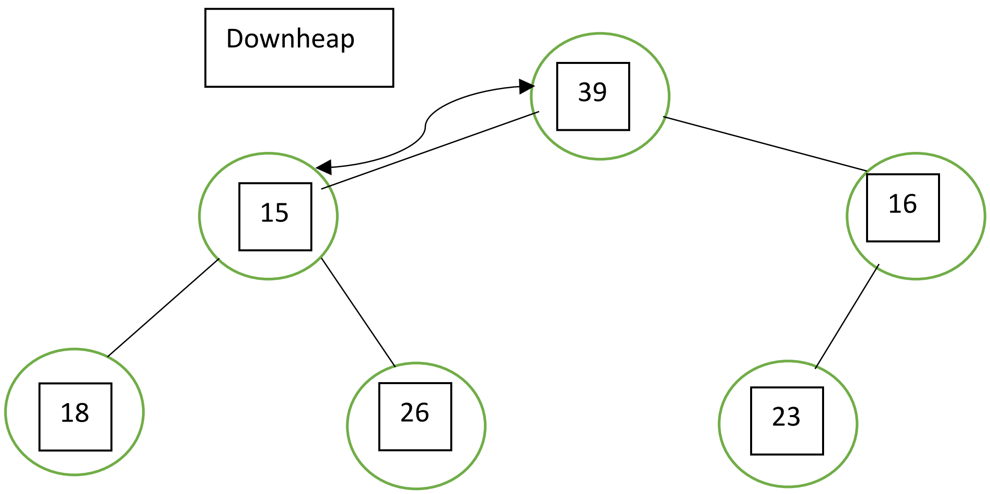

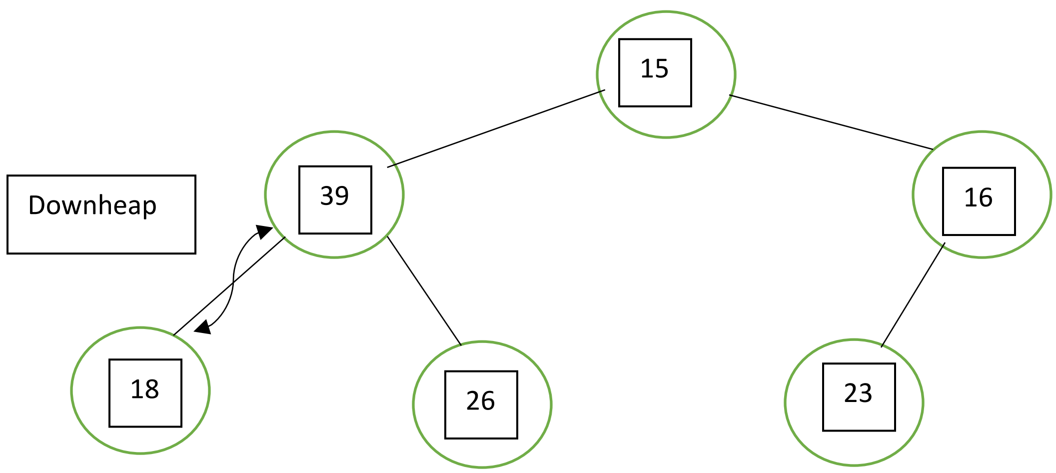

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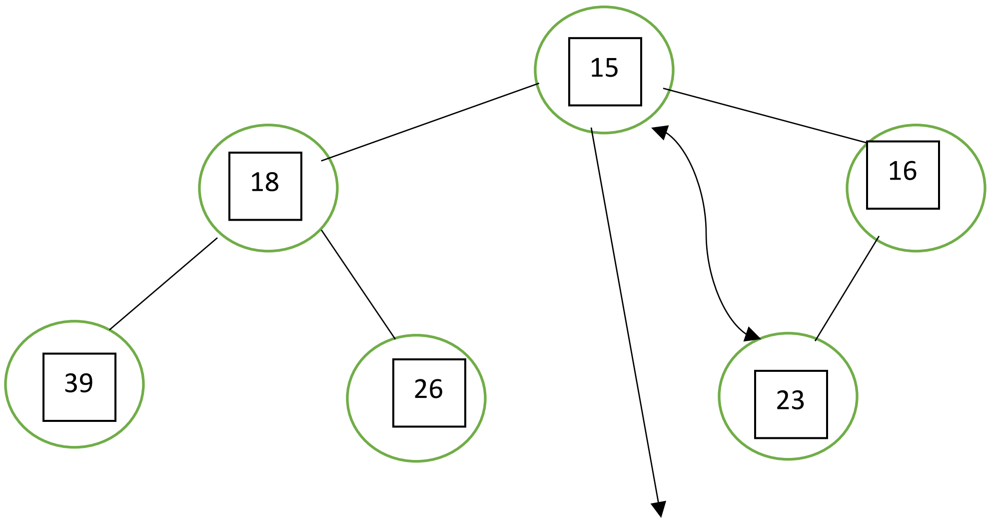
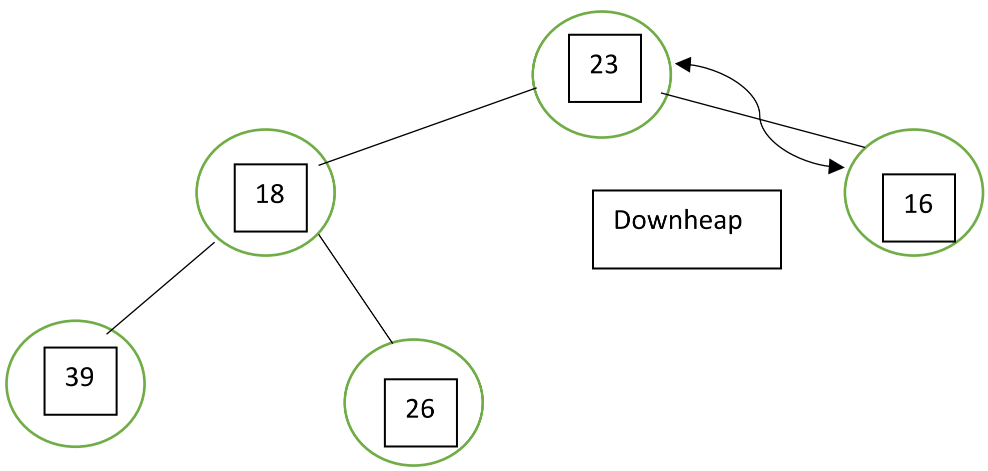


min HEAP:	2	4	5	10	15	0	0	0	0	0



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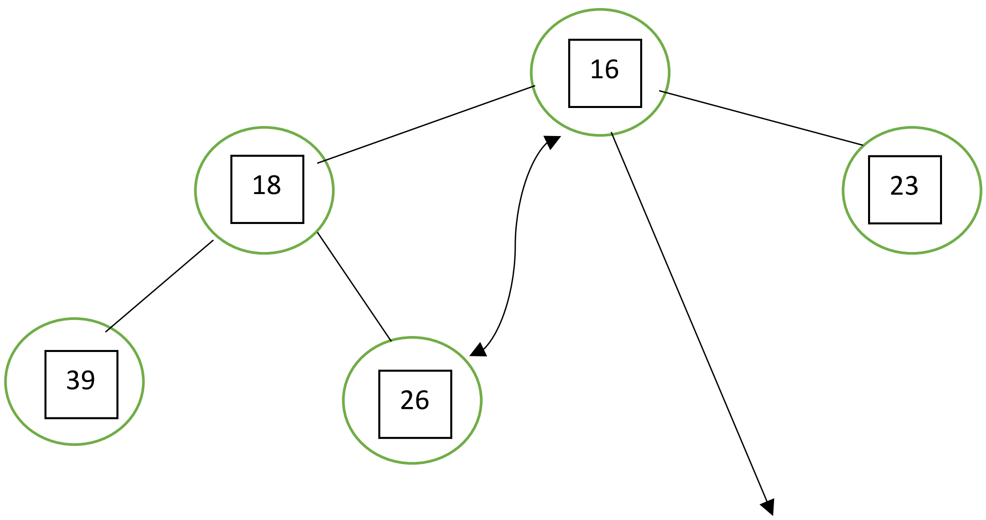
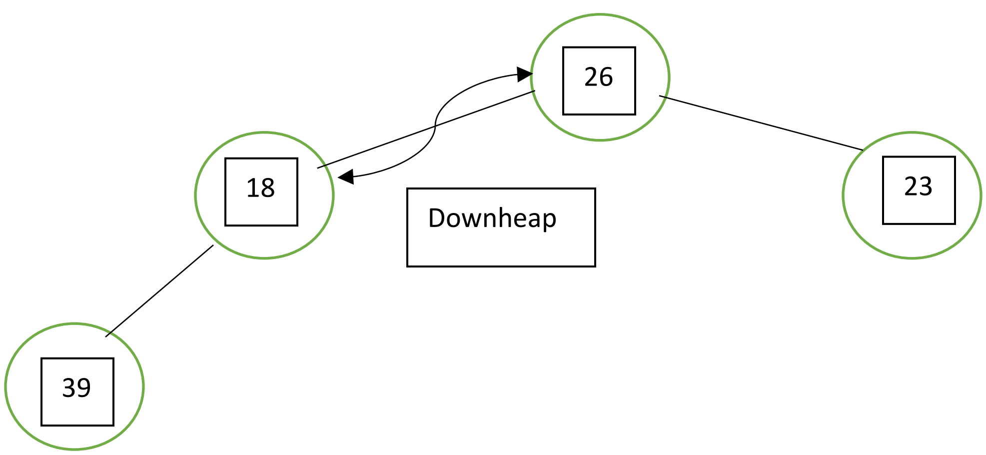


min HEAP:	2	4	5	10	15	16	0	0	0	0



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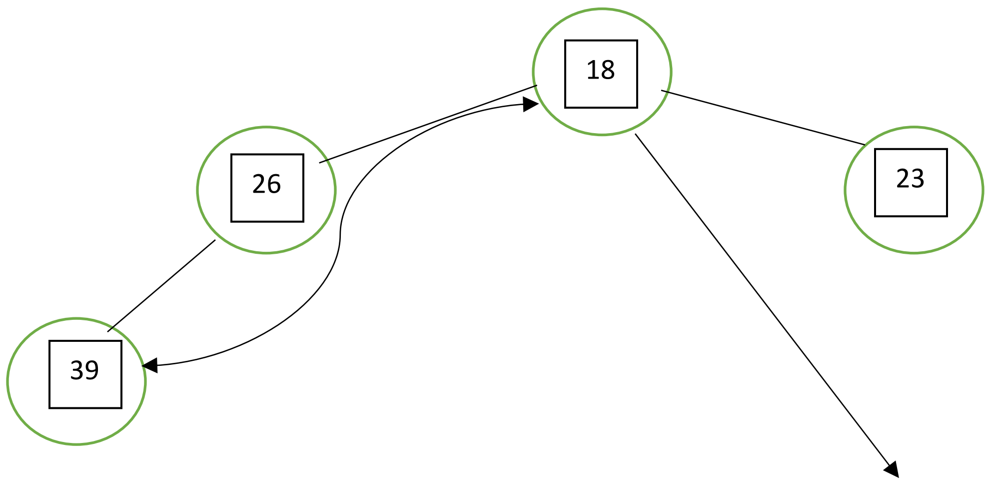
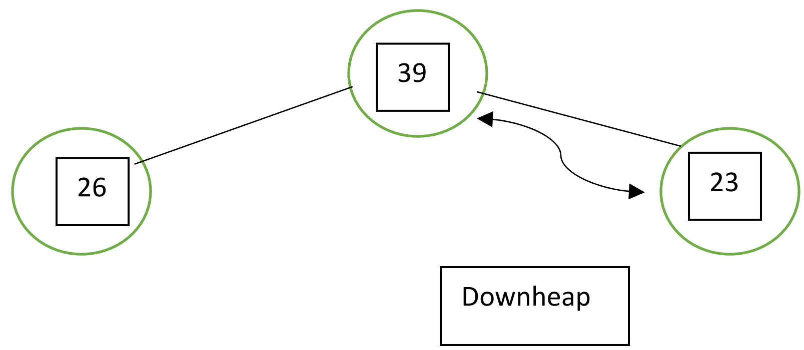
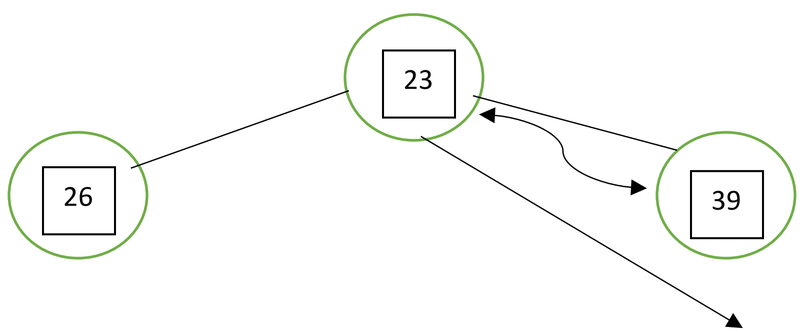


min HEAP:	2	4	5	10	15	16	18	0	0	0





min HEAP:	2	4	5	10	15	16	18	23	0	0

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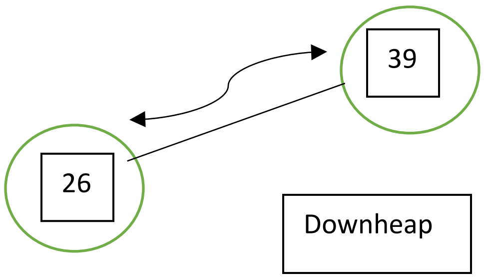
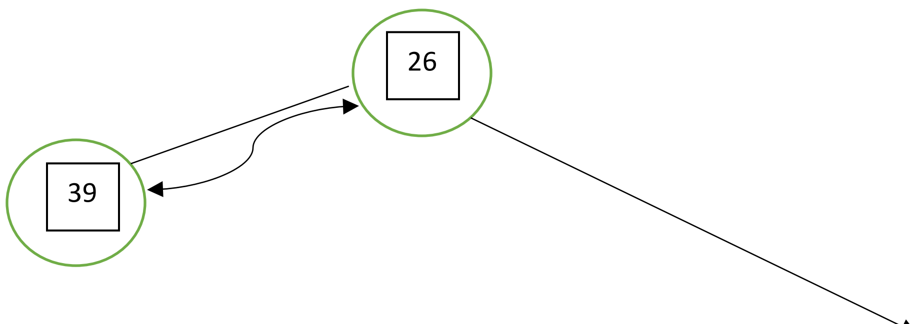
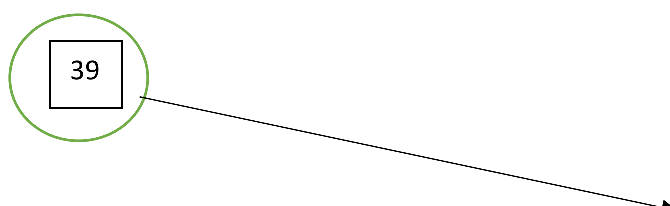




min HEAP:	2	4	5	10	15	16	18	23	26	0



min HEAP:	2	4	5	10	15	16	18	23	26	39

Final:

min HEAP:	2	4	5	10	15	16	18	23	26	39

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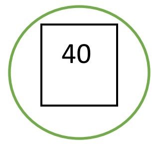
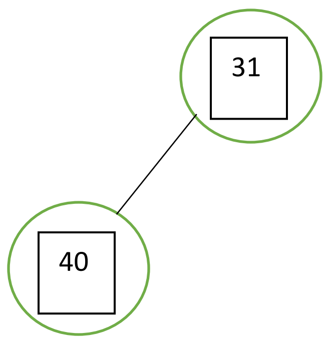
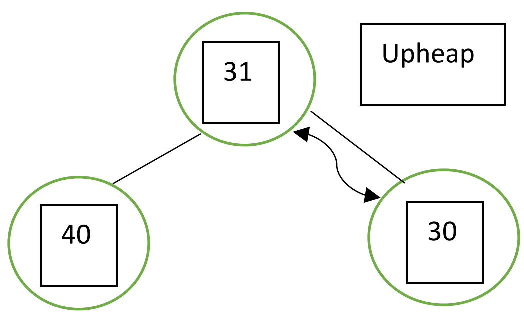
(b) (5 points) Illustrate the execution of the bottom-up construction of a (min) heap (like in Figure 2.49) on the following sequence: (2, 5, 16, 4, 10, 23, 39, 18, 26, 15, 7, 9, 30, 31, 40).

$n=2^h-1=15$







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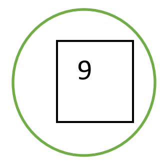

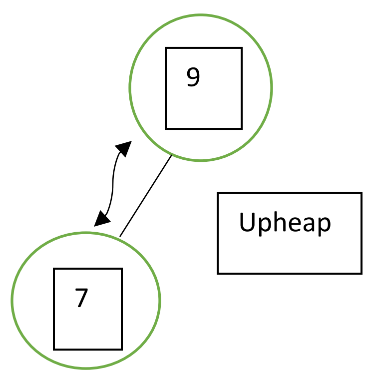

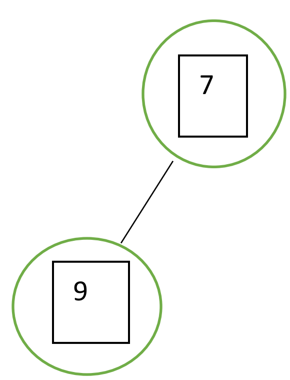

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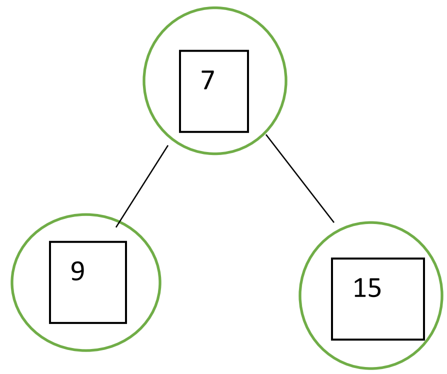

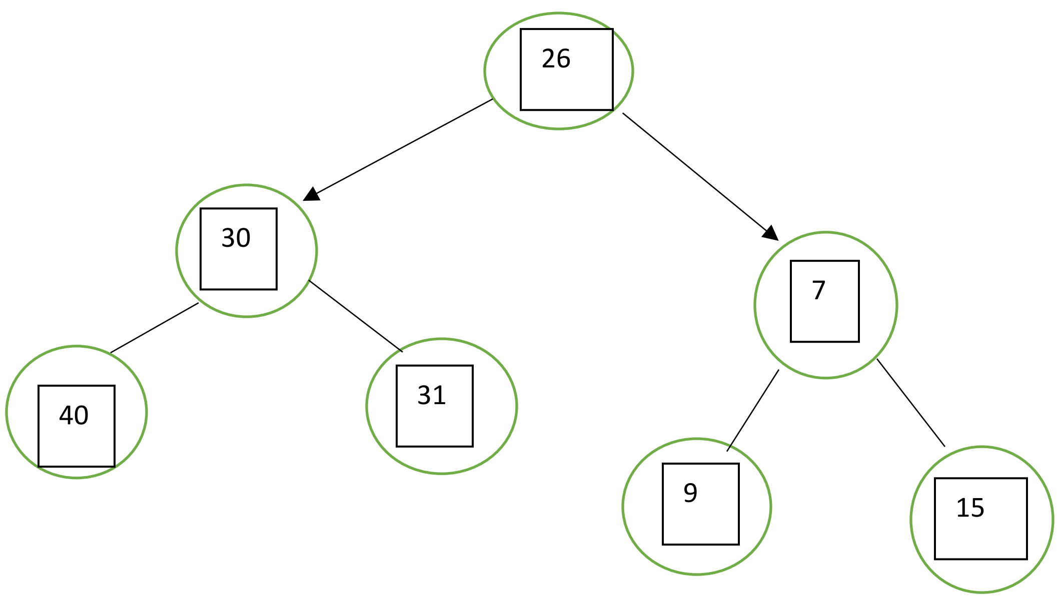

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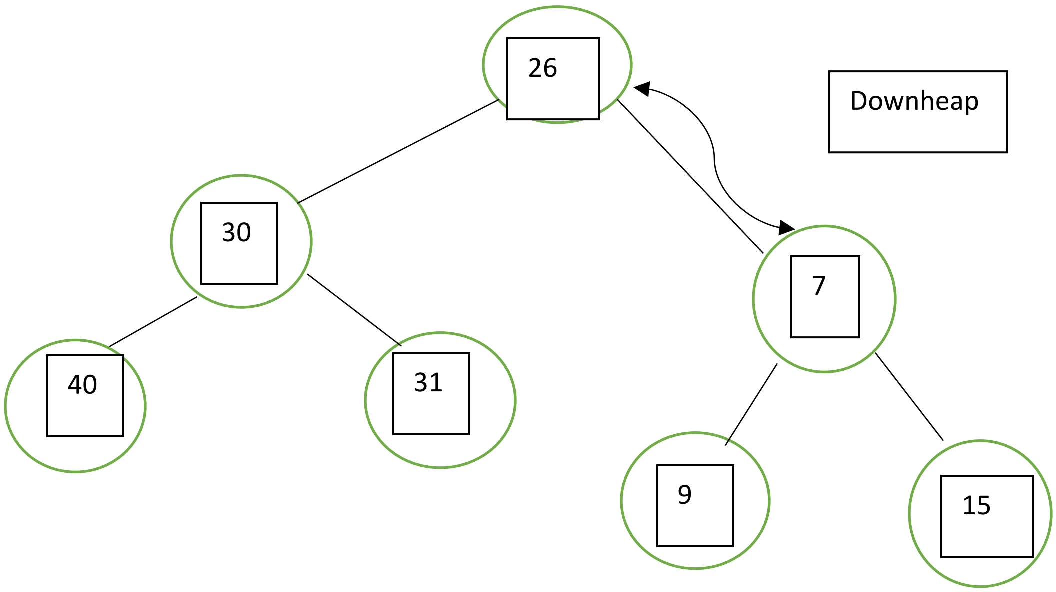

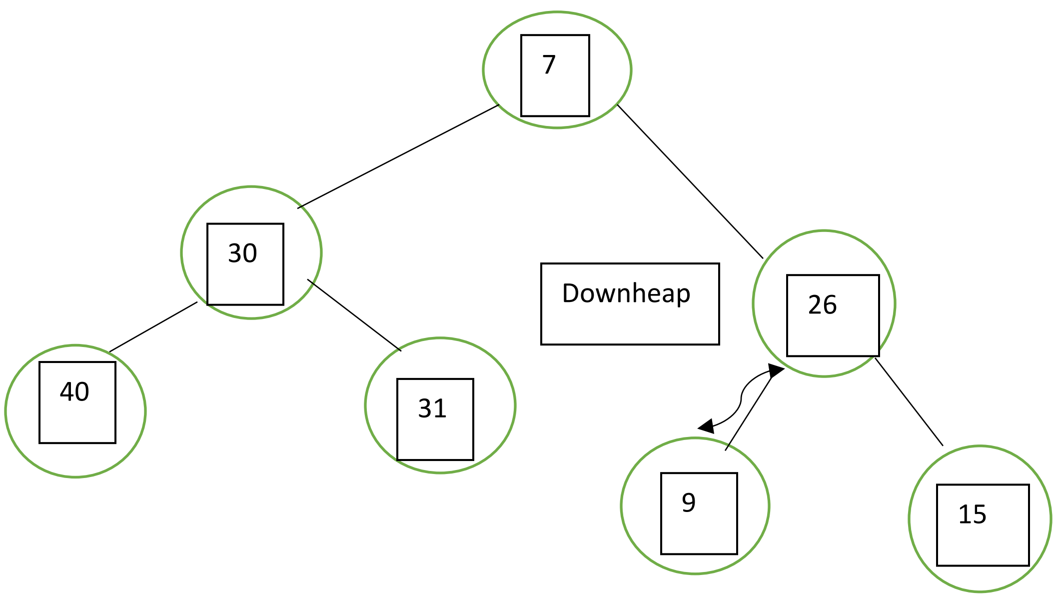

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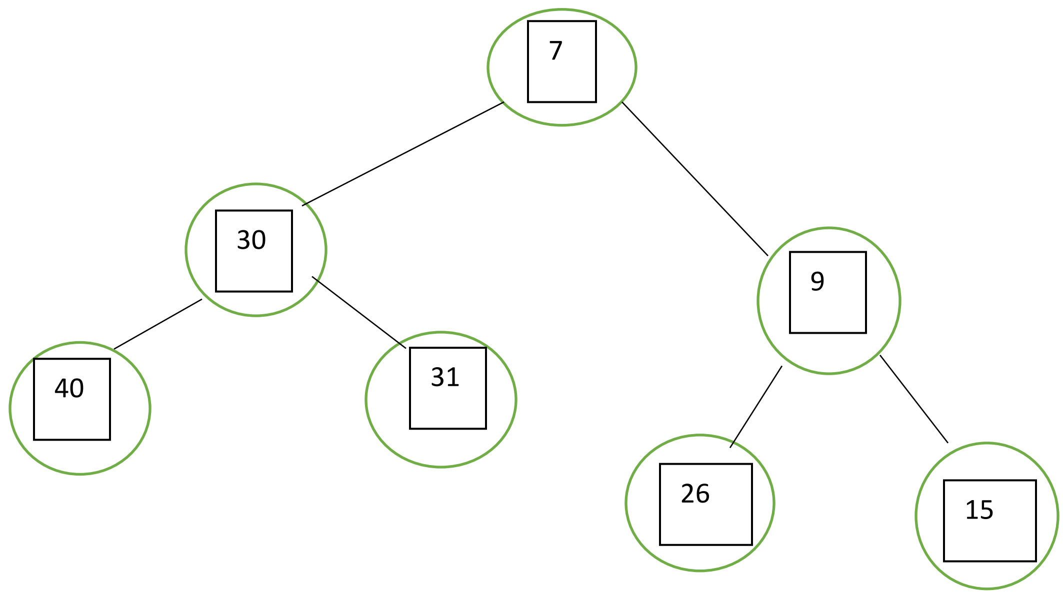

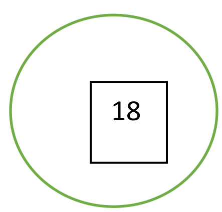

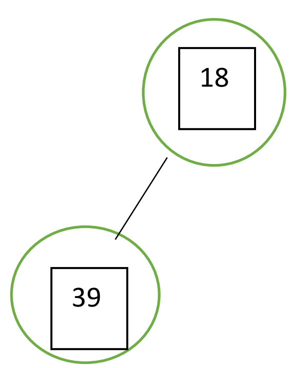

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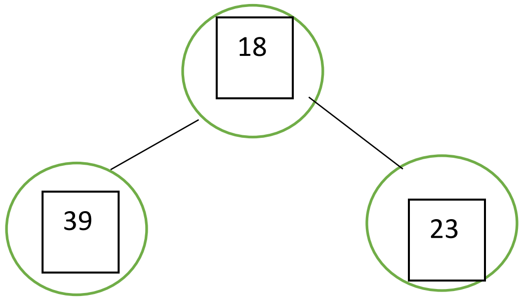

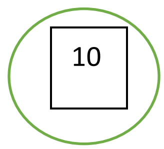

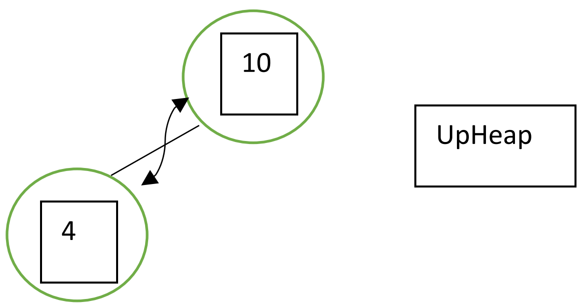

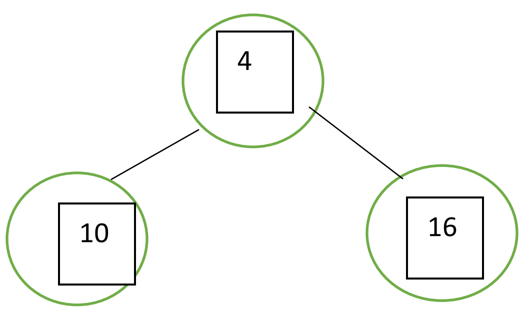

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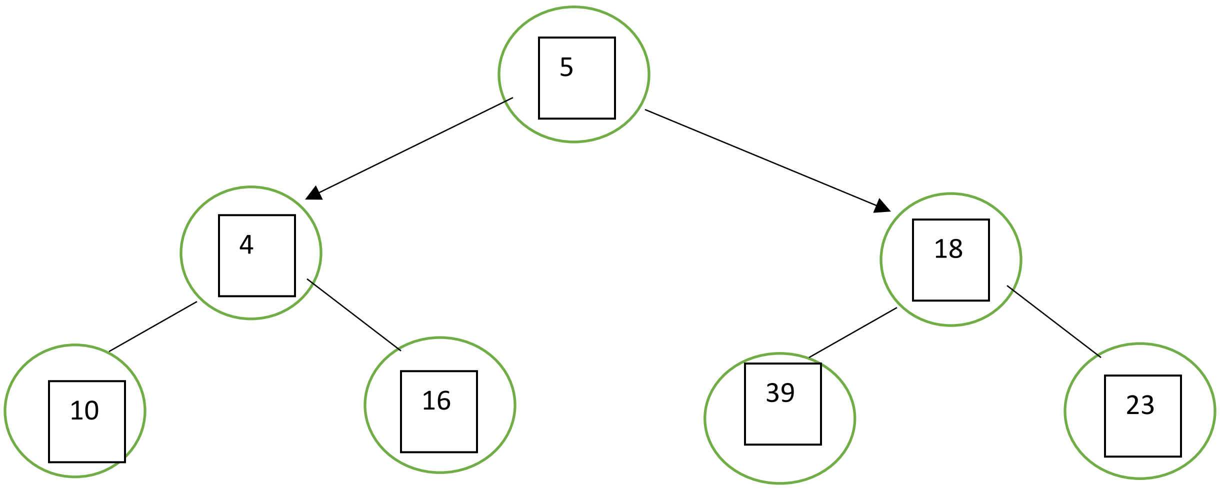
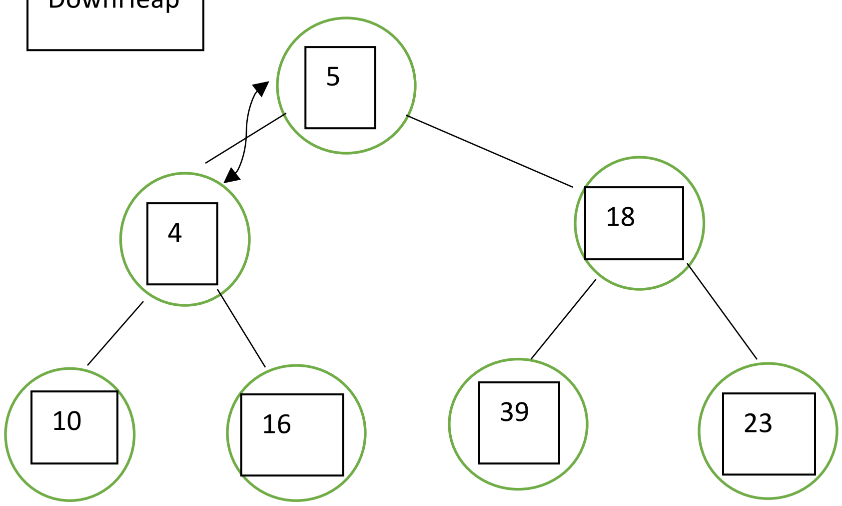


DownHeap



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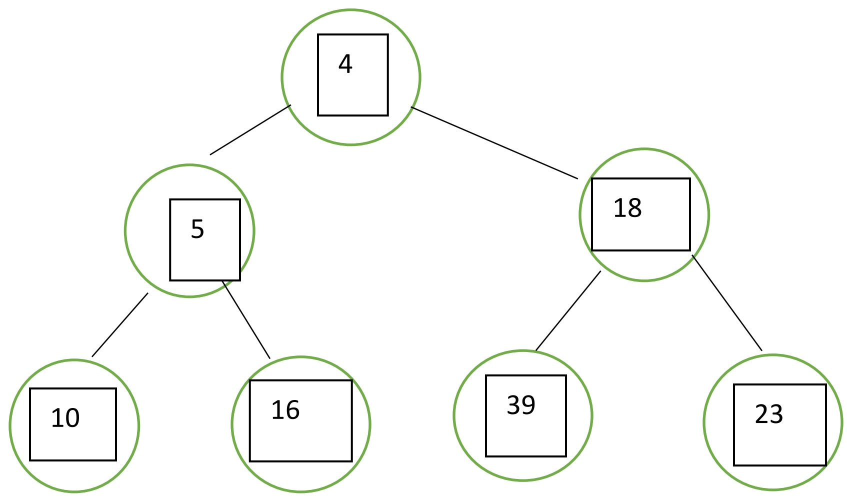

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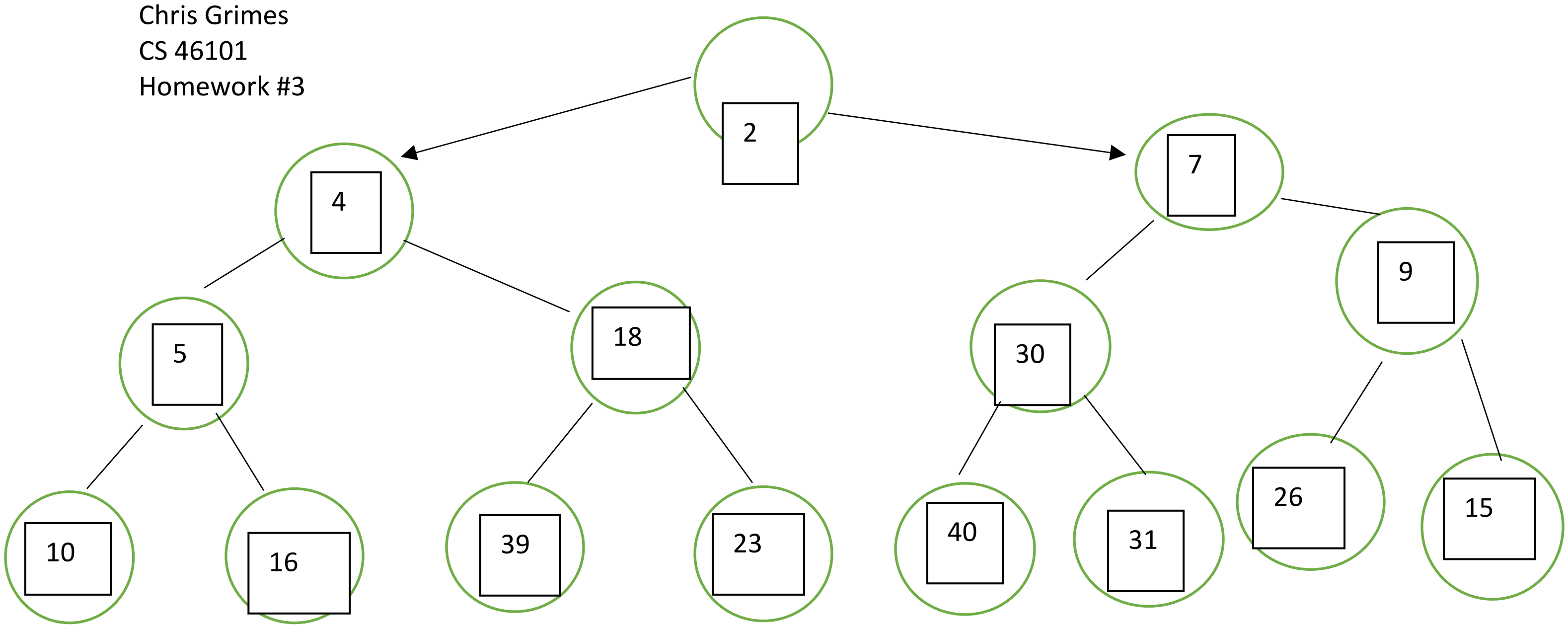

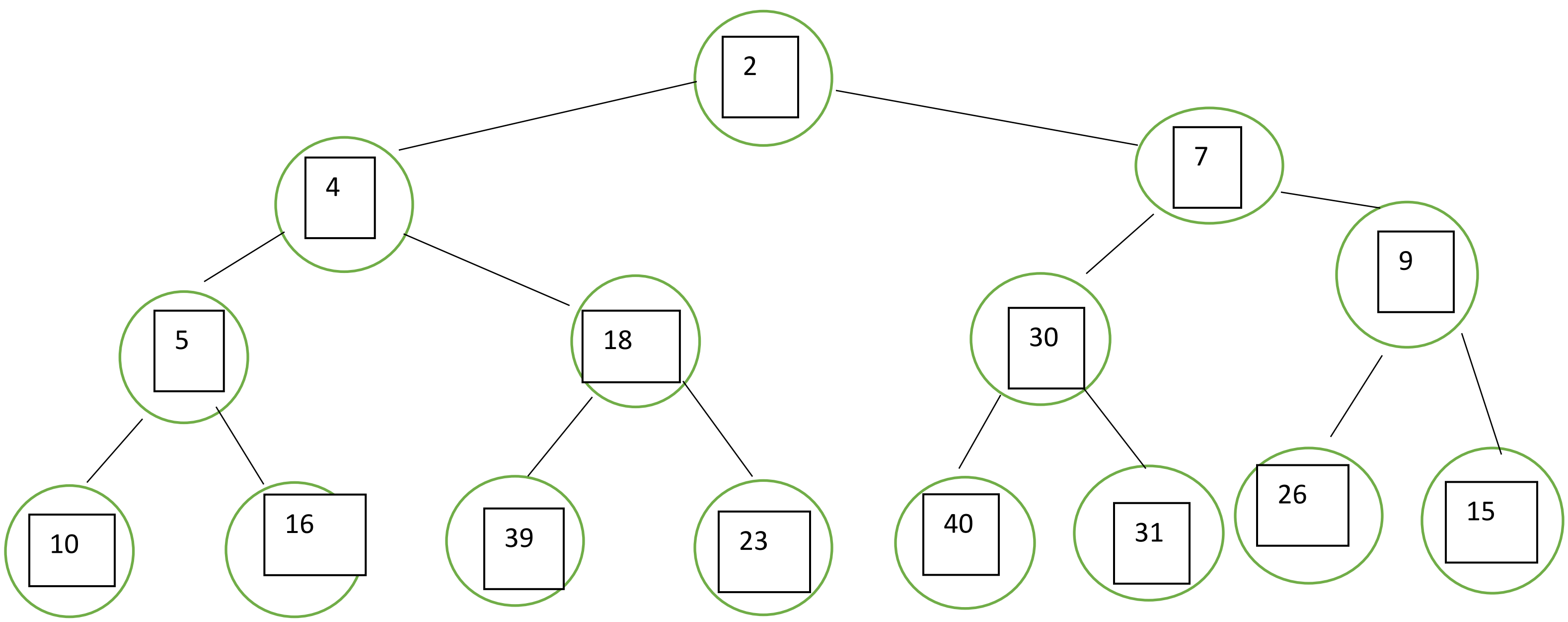

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2. (10 points) Let T be a (min) heap storing n keys. Give the pseudocode for an efficient algorithm for printing all the keys in T that are smaller than or equal to a given query key x (which is not necessarily in T). You can assume the existence of a O(1)-time print(key) function. For example, given the heap of Figure 2.41 and query key x = 7, the algorithm should report 4,5,6,7. Note that the keys do not need to be reported in sorted order. Your algorithm should run in O(k) time, where k is the number of keys reported.

```
Algorithm printless(T, x)

    Input: a binary tree T, a key x

    Output: print out all keys <= x

        if(root >x)

            return null

        else

            print(root)

            if(there is a left child)

                printless(the sub-tree whose root is left child, x)

            if(there is a right child)

                printless(the sub-tree whose root is right child, x)
```

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3. Use the table below to convert a character key to an integer for the following questions.

Letter	A	B	C	D	E	F	G	H	I	J	K	L	M
Key	0	1	2	3	4	5	6	7	8	9	10	11	12
Letter	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Key	13	14	15	16	17	18	19	20	21	22	23	24	25

(a) (5 points) Give the contents of the hash table that results when the following keys are inserted in that order into an initially empty 13-item hash table: (E_1, A, S_1, Y, Q, U, E_2, S_2, T, I, O, N). Use h(k) = k mod 13 for the hash function for the k-th letter of the alphabet (see above table for converting letter keys to integer values). Use linear probing.

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Value					E								
Key					4								

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Value	A				E								
Key	0				4								

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Value	A				E	S							
Key	0				4	18							

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Value	A				E	S						Y	
Key	0				4	18						24	

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Value	A			Q	E	S						Y	
Key	0			16	4	18						24	

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Value	A			Q	E	S		U				Y	
Key	0			16	4	18		20				24	

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Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Value	A			Q	E	S	E	U				Y	
Key	0			16	4	18	4	20				24	

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Value	A			Q	E	S	E	U	S			Y	
Key	0			16	4	18	4	20	18			24	

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Value	A			Q	E	S	E	U	S	T		Y	
Key	0			16	4	18	4	20	18	19		24	

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Value	A			Q	E	S	E	U	S	T	I	Y	
Key	0			16	4	18	4	20	18	19	8	24	

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Value	A	O		Q	E	S	E	U	S	T	I	Y	
Key	0	14		16	4	18	4	20	18	19	8	24	

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Value	A	O	N	Q	E	S	E	U	S	T	I	Y	
Key	0	14	13	16	4	18	4	20	18	19	8	24	

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Letter	A	B	C	D	E	F	G	H	I	J	K	L	M
Key	0	1	2	3	4	5	6	7	8	9	10	11	12
Letter	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Key	13	14	15	16	17	18	19	20	21	22	23	24	25

(b) (5 points) Give the contents of the hash table that results when the same keys are inserted in that order into an initially empty 13-item hash table. Use h(k) = k mod 13 for the hash function for the k-th letter of the alphabet (see above table for converting letter keys to integer values). Use double hashing and let h'(k) = 1 + (k mod 11) be the secondary hash function.

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Value					E								
Key					4								

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Value	A				E								
Key	0				4								

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Value	A				E	S							
Key	0				4	18							

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Value	A				E	S						Y	
Key	0				4	18						24	

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Value	A			Q	E	S						Y	
Key	0			16	4	18						24	

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Value	A			Q	E	S		U				Y	
Key	0			16	4	18		20				24	

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Value	A			Q	E	S		U		E		Y	
Key	0			16	4	18		20		4		24	

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Value	A			Q	E	S		U	S	E		Y	
Key	0			16	4	18		20	18	4		24	

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Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Value	A			Q	E	S	T	U	S	E		Y	
Key	0			16	4	18	19	20	18	4		24	

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Value	A	I		Q	E	S	T	U	S	E		Y	
Key	0	8		16	4	18	19	20	18	4		24	

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Value	A	I		Q	E	S	T	U	S	E		Y	O
Key	0	8		16	4	18	19	20	18	4		24	14

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Value	A	I	N	Q	E	S	T	U	S	E		Y	O
Key	0	8	13	16	4	18	19	20	18	4		24	14

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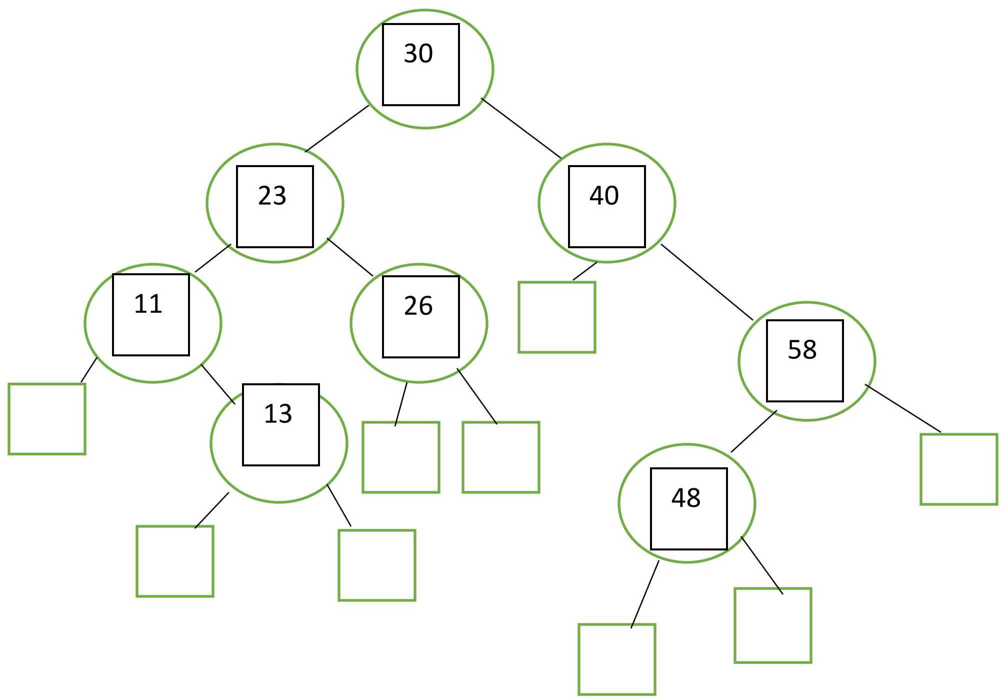
4. (a) (5 points) Insert into an initially empty binary search tree items with the following keys (in this order): 30, 40, 23, 58, 48, 26, 11, 13. Draw the resulting tree after all insertions.



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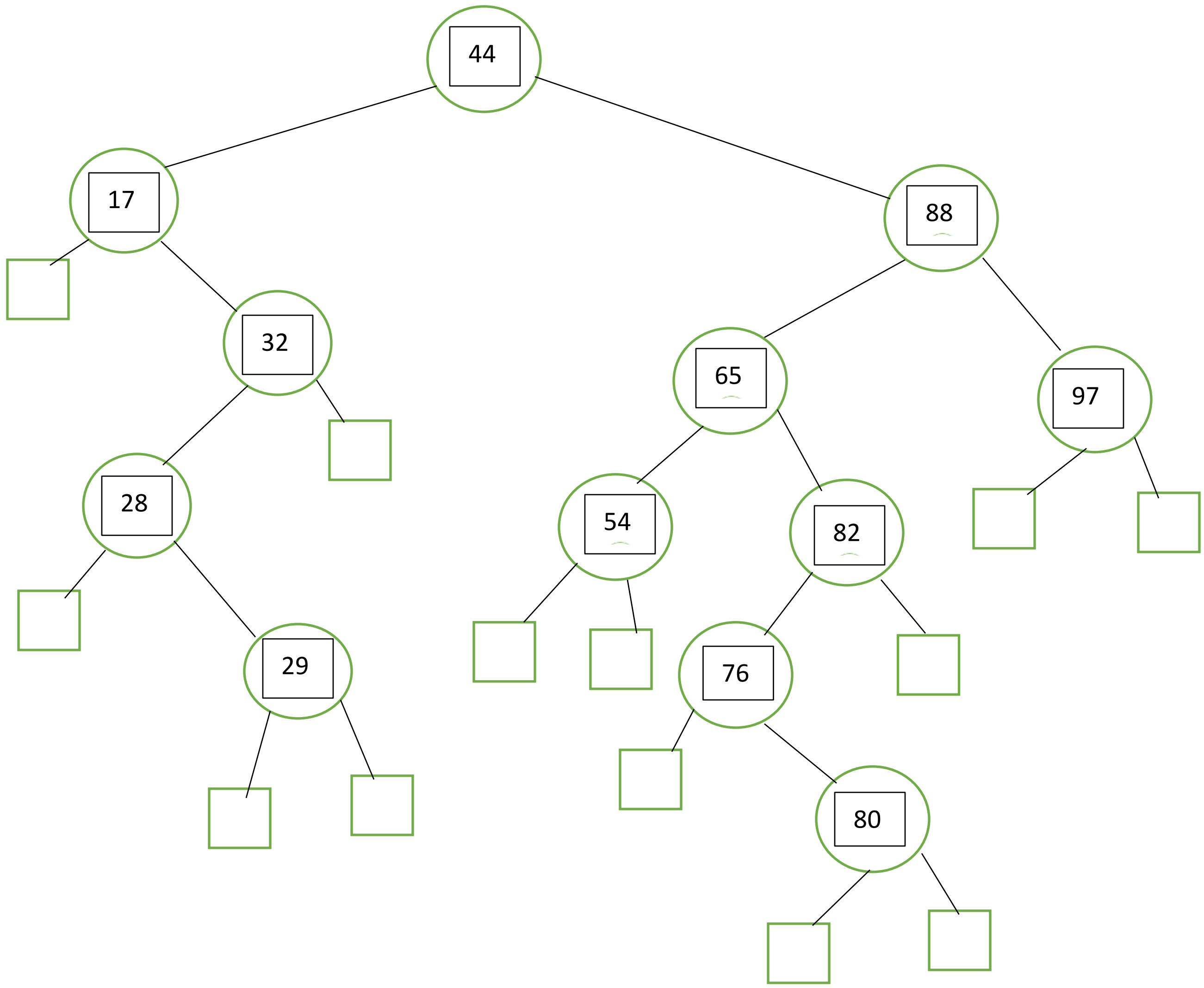
(b) (5 points) Remove from the binary search tree given below the following keys (in this order): 32, 65, 76, 88, 97. Draw the tree after each removal.



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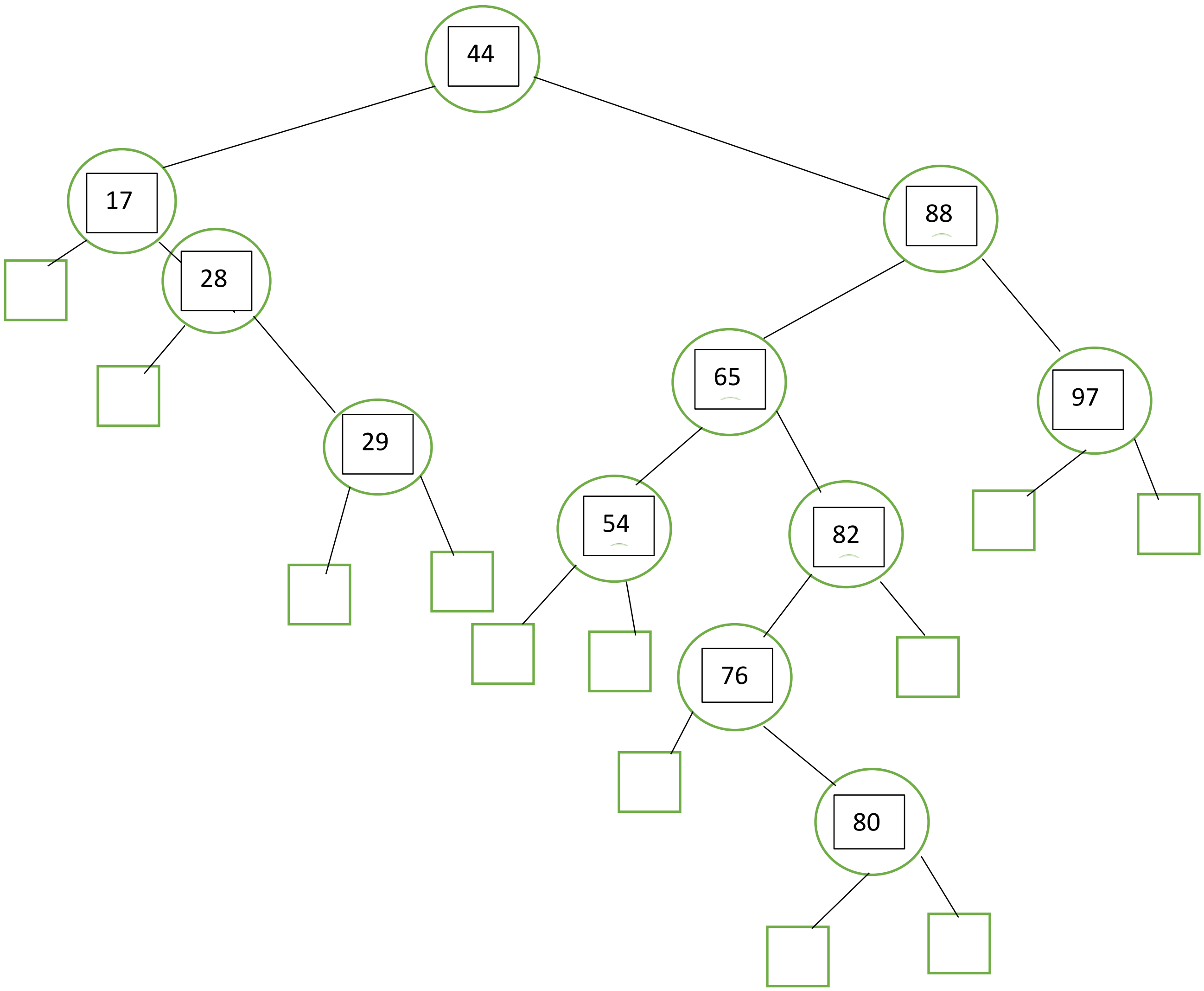

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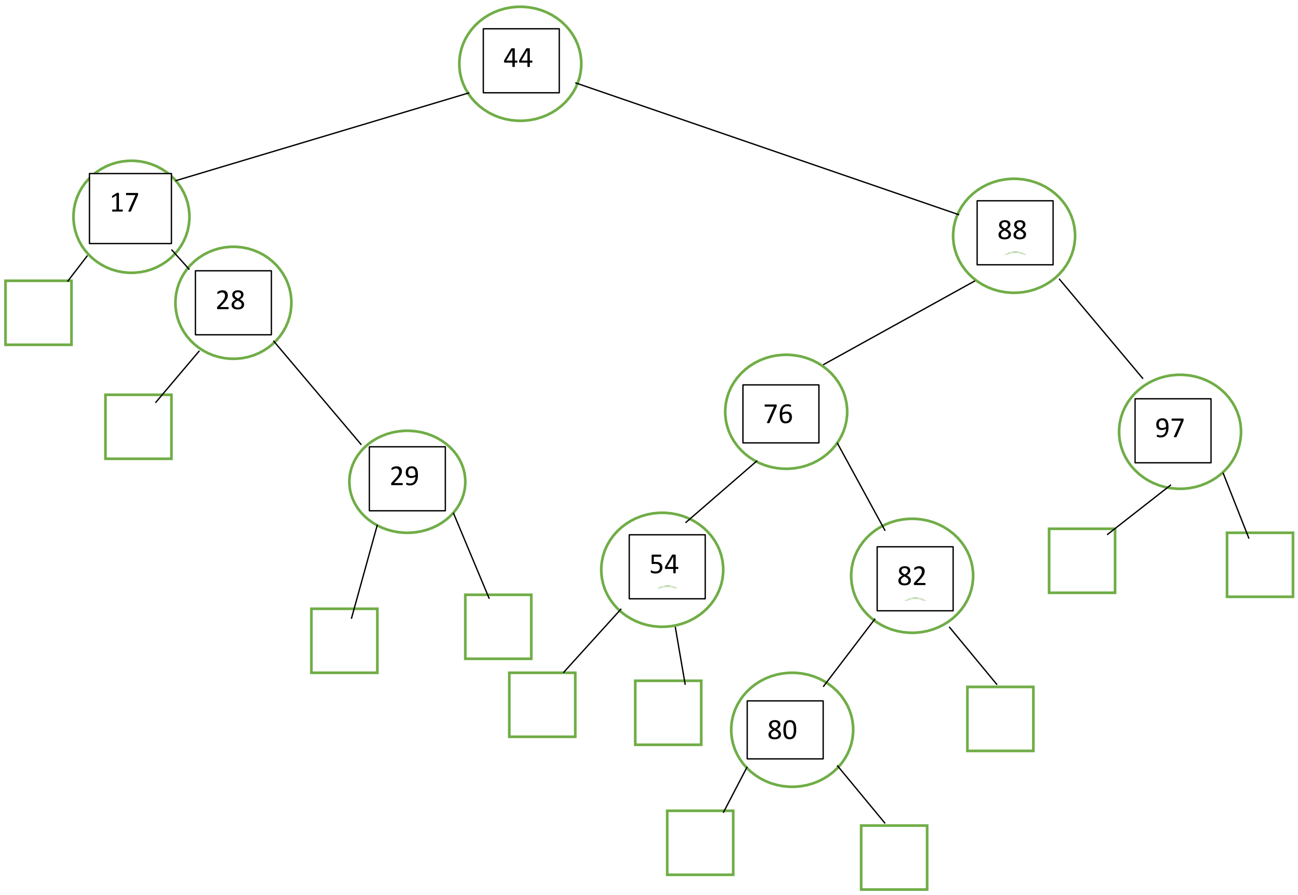

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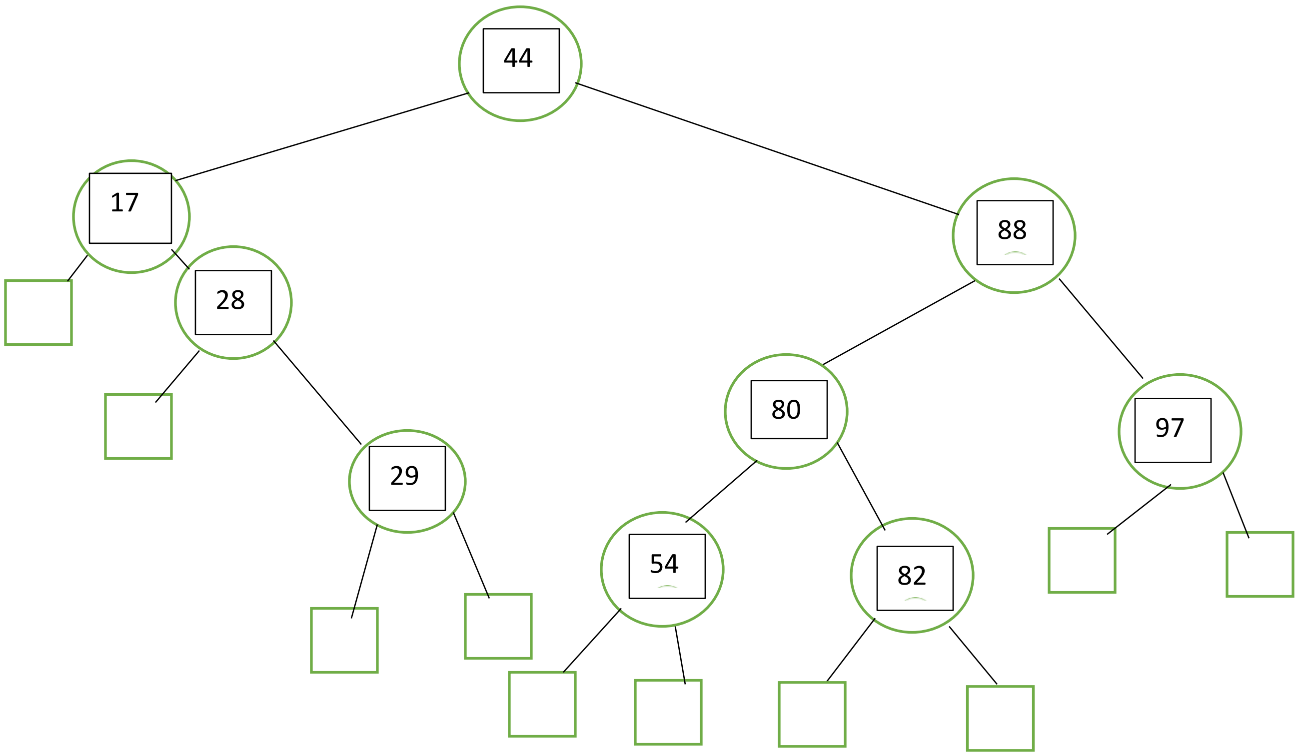

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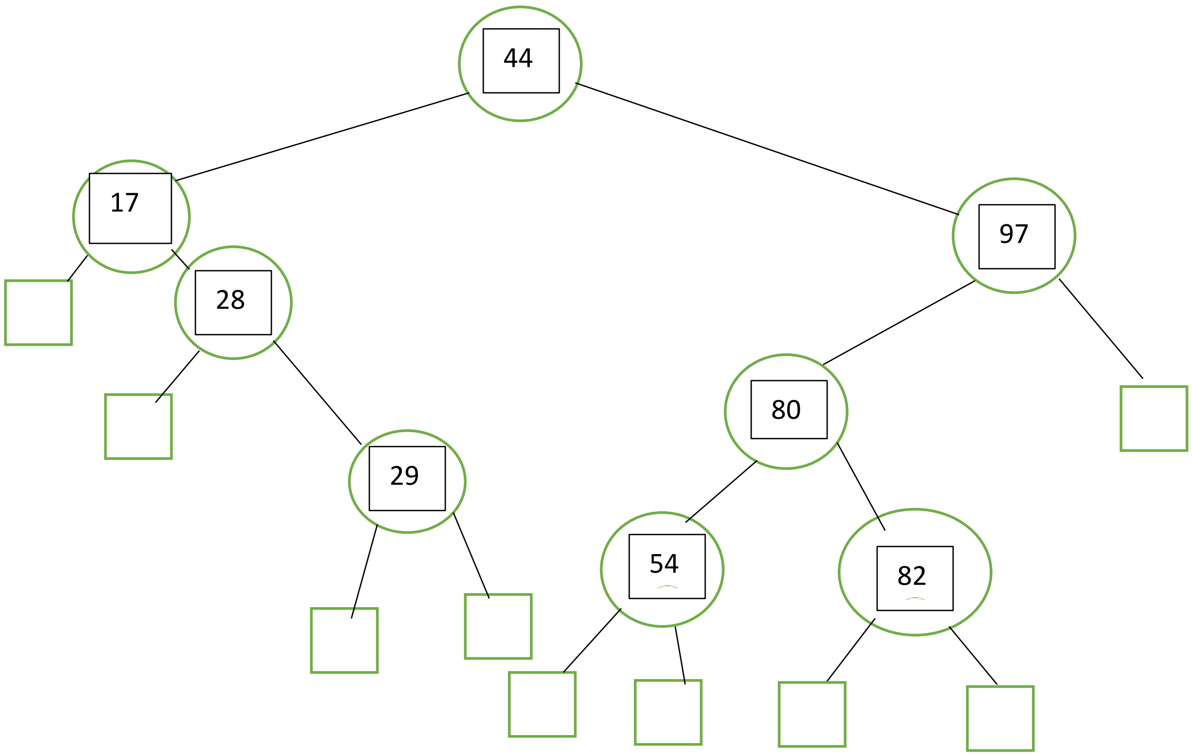

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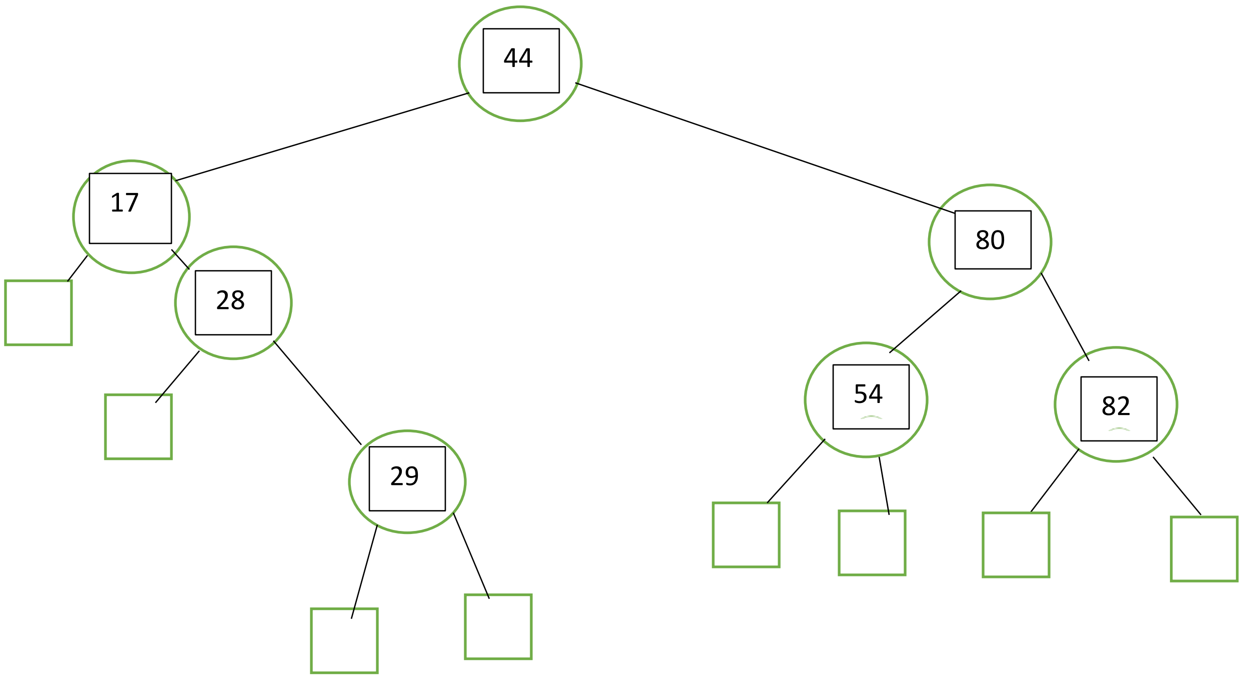

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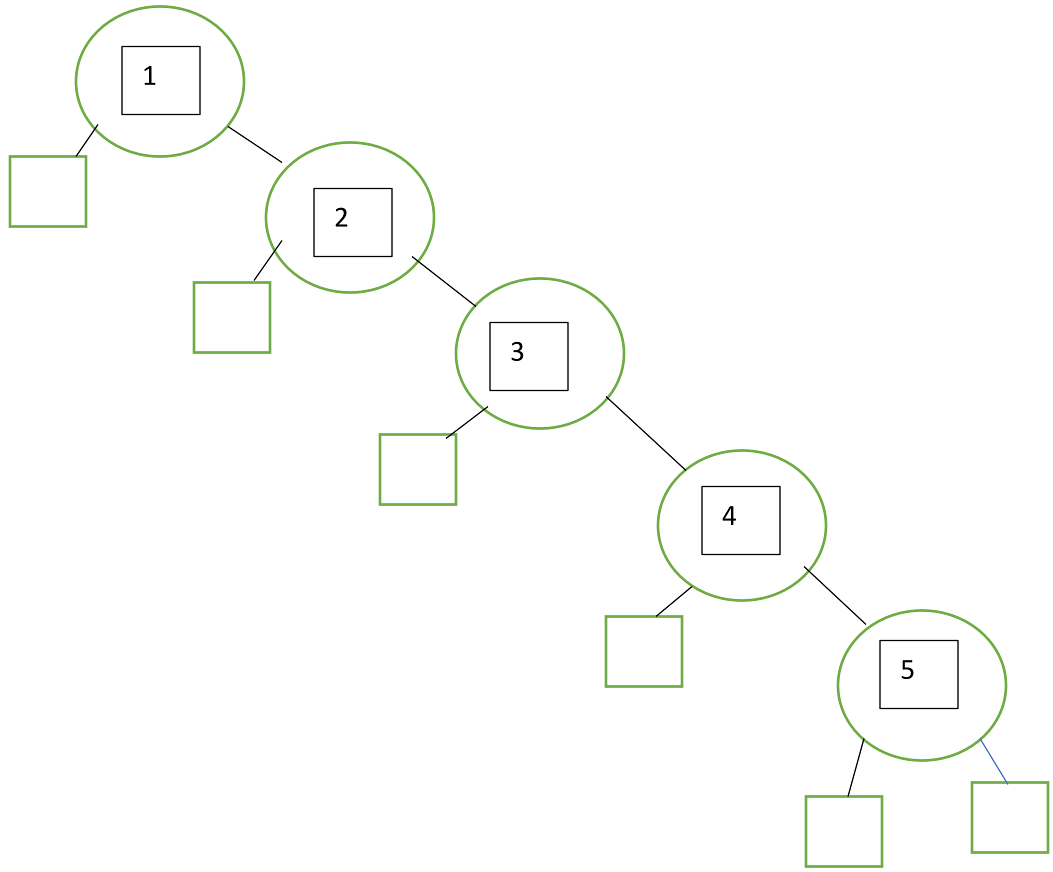
(c) (5 points) A different binary search tree results when we try to insert the same sequence into an empty BST in a different order. Give an example of this with at least 5 elements and show the two different binary search trees that result.

Using: {1,2,3,4,5}

Order: {1,2,3,4,5}



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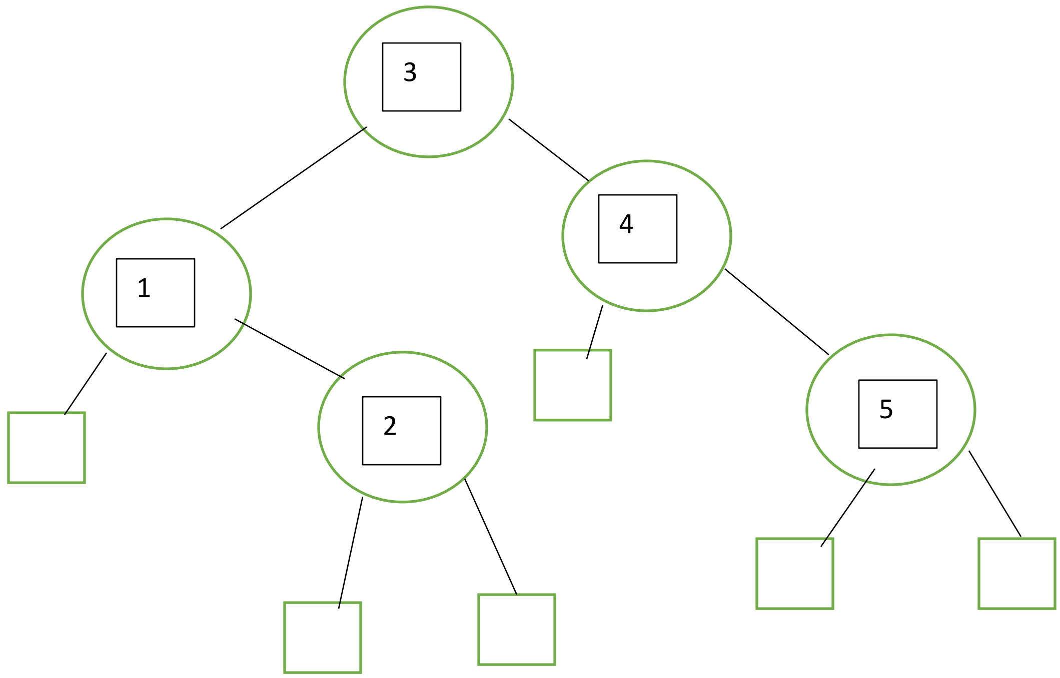
Order: {3,4,5,1,2}



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5. (10 points) Let T be a binary search tree, and let x be a key. Give an efficient algorithm for finding the smallest key y in T such that y > x. Note that x may or may not be in T. Explain why your algorithm has the running time it does.

```
Algorithm findMin>(T, x):

        Input: a tree T, a key x

        Output: the smallest key in the tree whose value is greater than x

                y<- 10^10

                if(root<y && root>x)

                        y<- root

                if(root has a left child && left child<y)
                                                findMin>(sub-tree whose root is left child, x)

                if(root has a right child && right child <y)
                                                findMin>(sub-tree whose root is right child, x)
```

The above algorithm should have a worst case running time of O(n) in which case all keys have values less than x, and the entire tree must be checked. This algorithm will be invoked on every node whose value is less than the current y value and if said node is less than the current value of y and greater than the passed value of x the key of said node will be the new value of y.