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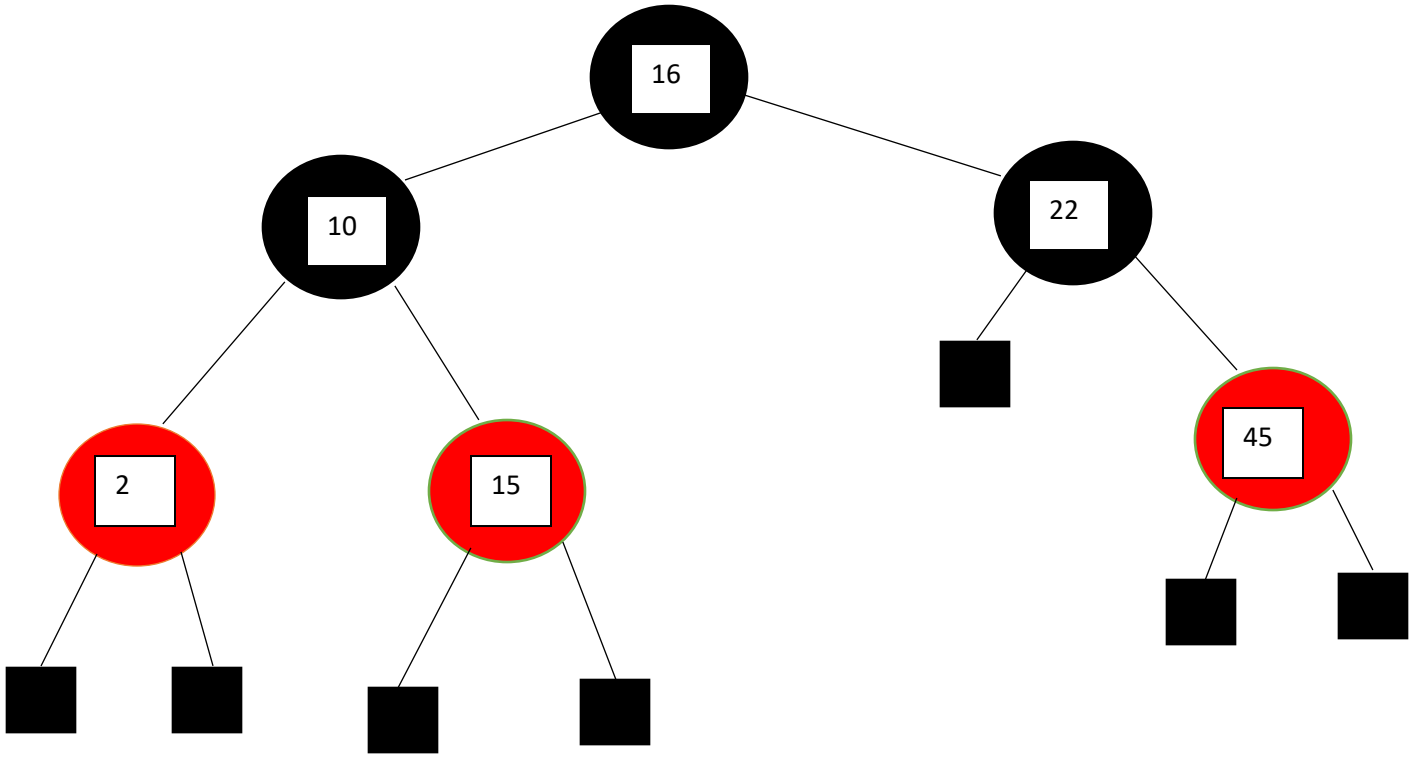
Homework #4

1. (5 points) Consider the following sequence of keys: (18, 30, 50, 12, 1). Insert the items with this set of keys in the order given into the red-black tree in the figure below. Draw the tree after each insertion.



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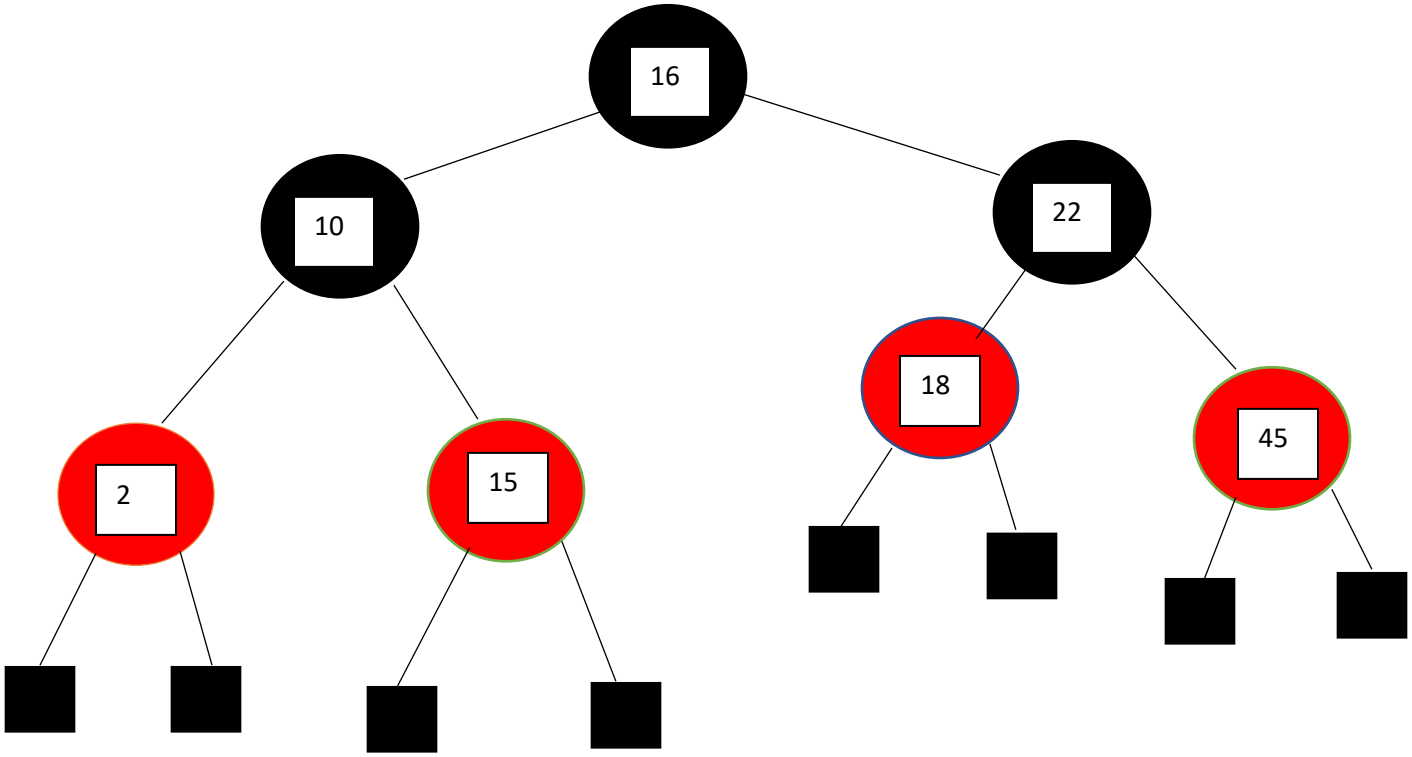

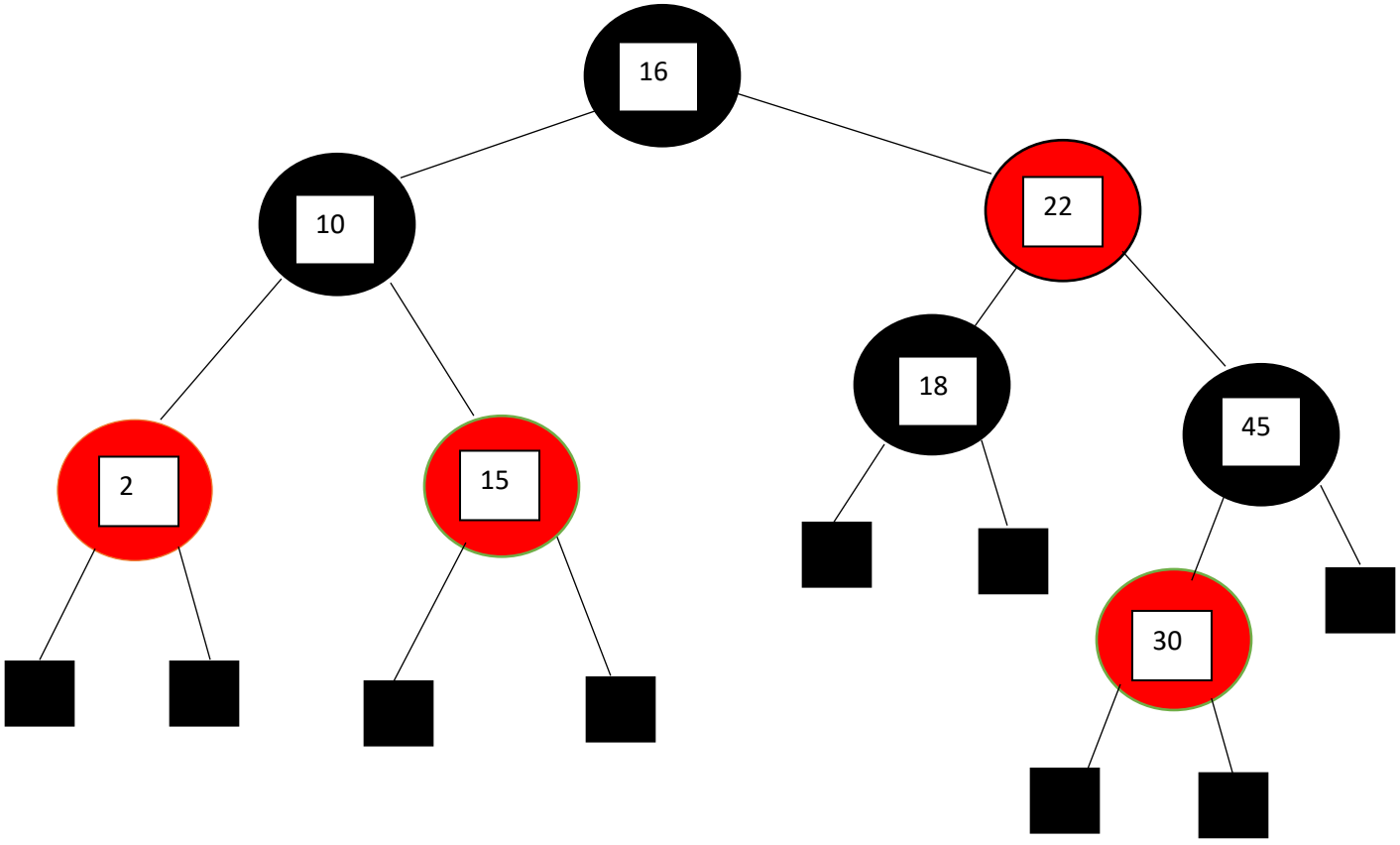

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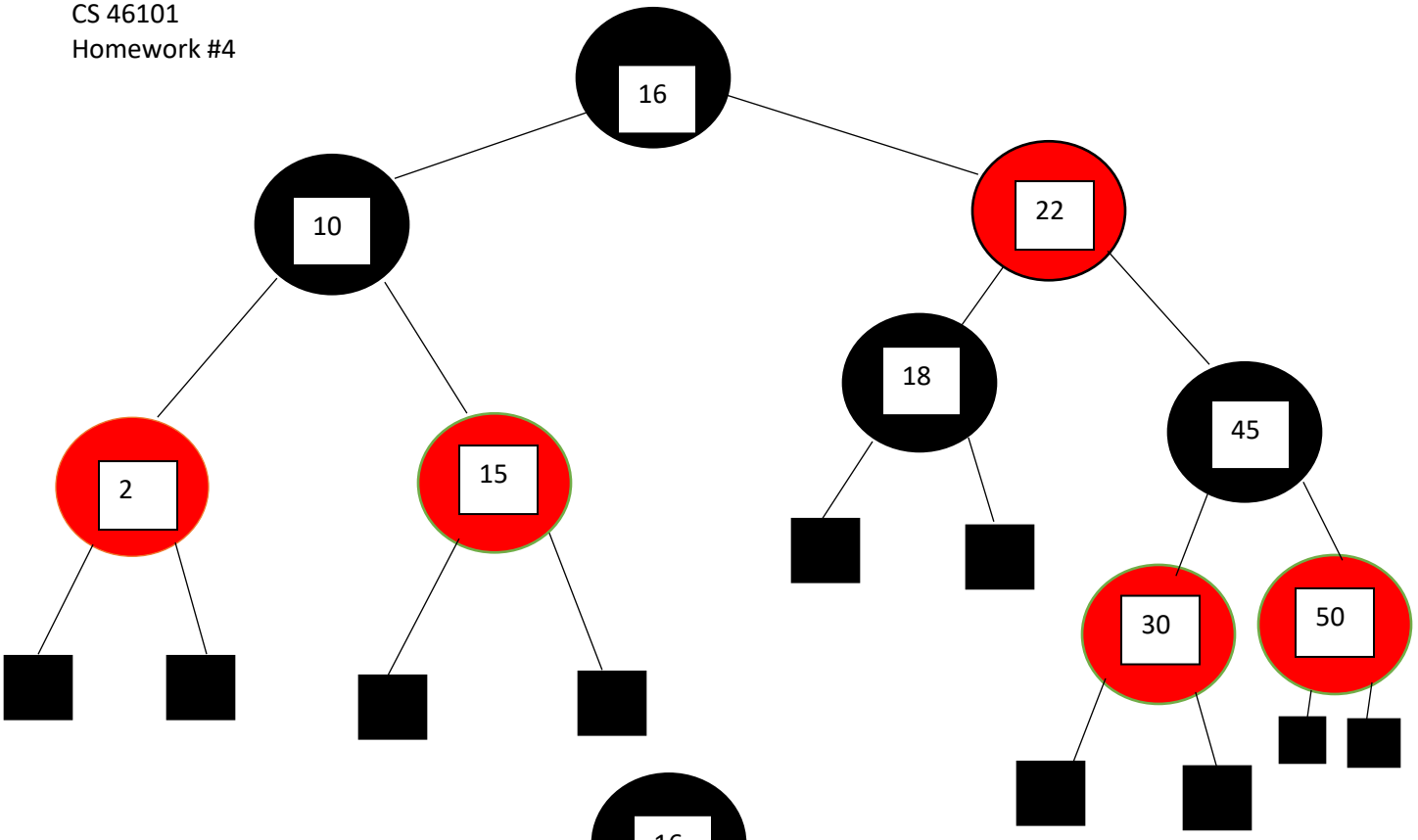

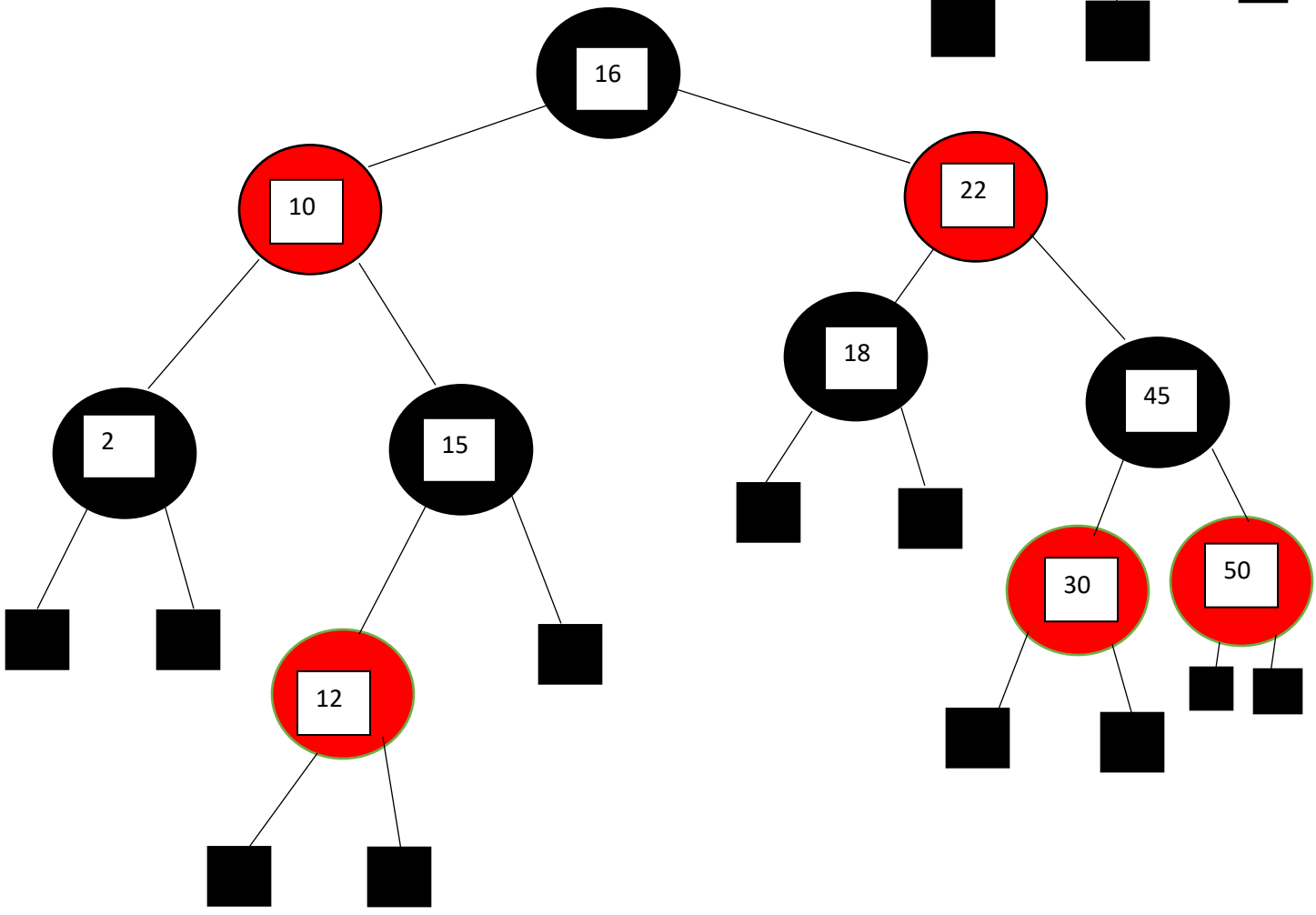

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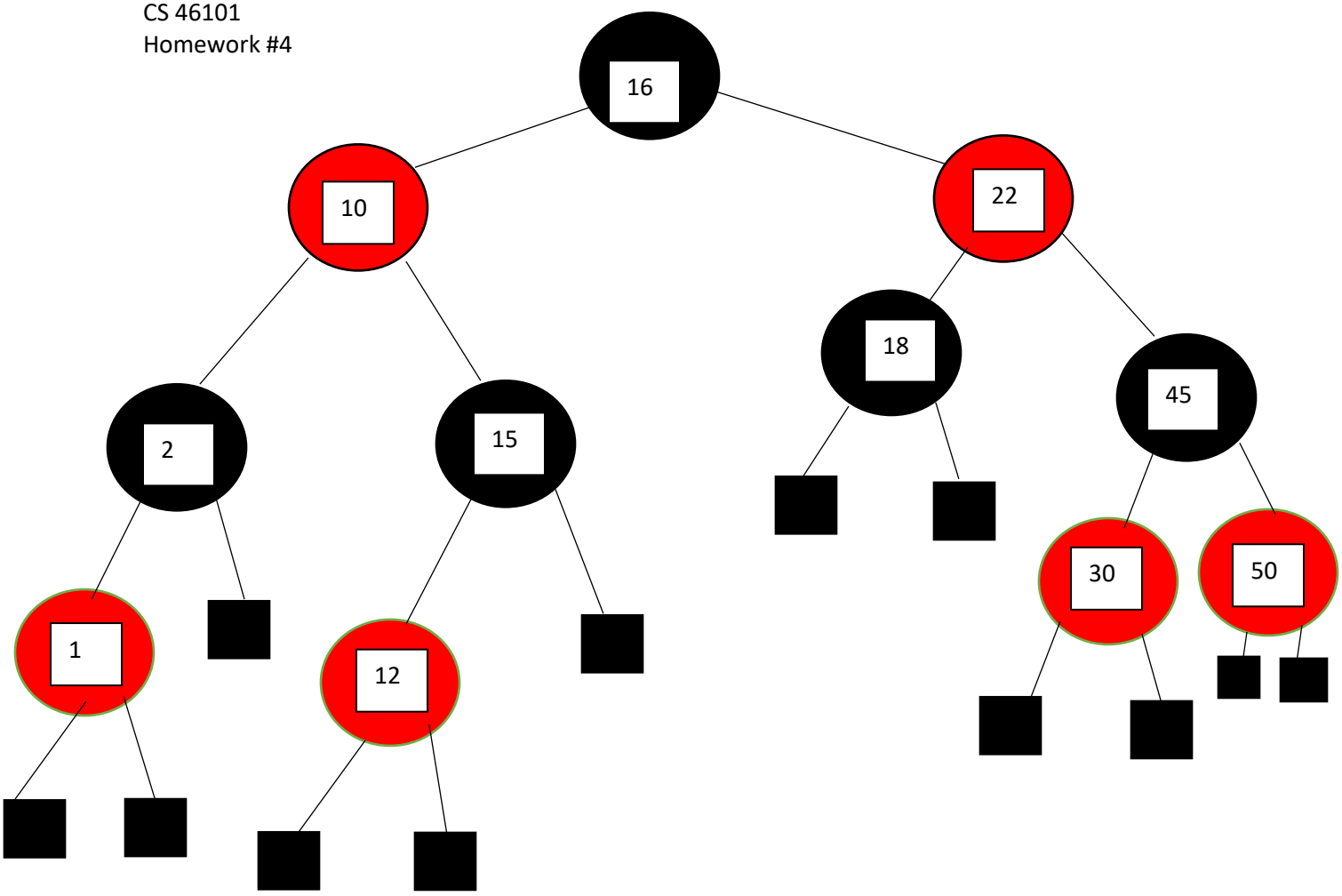

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2. (10 points) Design and give the pseudocode for an O(log n) algorithm that determines whether a red-black tree with n keys stores any keys within a certain (closed) interval. That is, the input to the algorithm is a red-black tree T and two keys, l and r, where $l \leq r$. If T has at least one key k such that $l \leq k \leq r$, then the algorithm returns true, otherwise it returns false. Hint: You can use the TreeSearch algorithm (page 146) as a subroutine.

```
Algorithm findRange(T, l, r)
        Input: a tree T, a lower value l, an upper value r
        Output: Boolean value true if said tree has at least one key between the values l and r
        if( root <= r && root>=l)
                return true
        else if (root has a left child)
                if(root>=r)
                        findRange(left child, l, r)
        else if (root has a right child)
                if(root <=l)
                        findRange(left child, l, r)
        else
                return false
```

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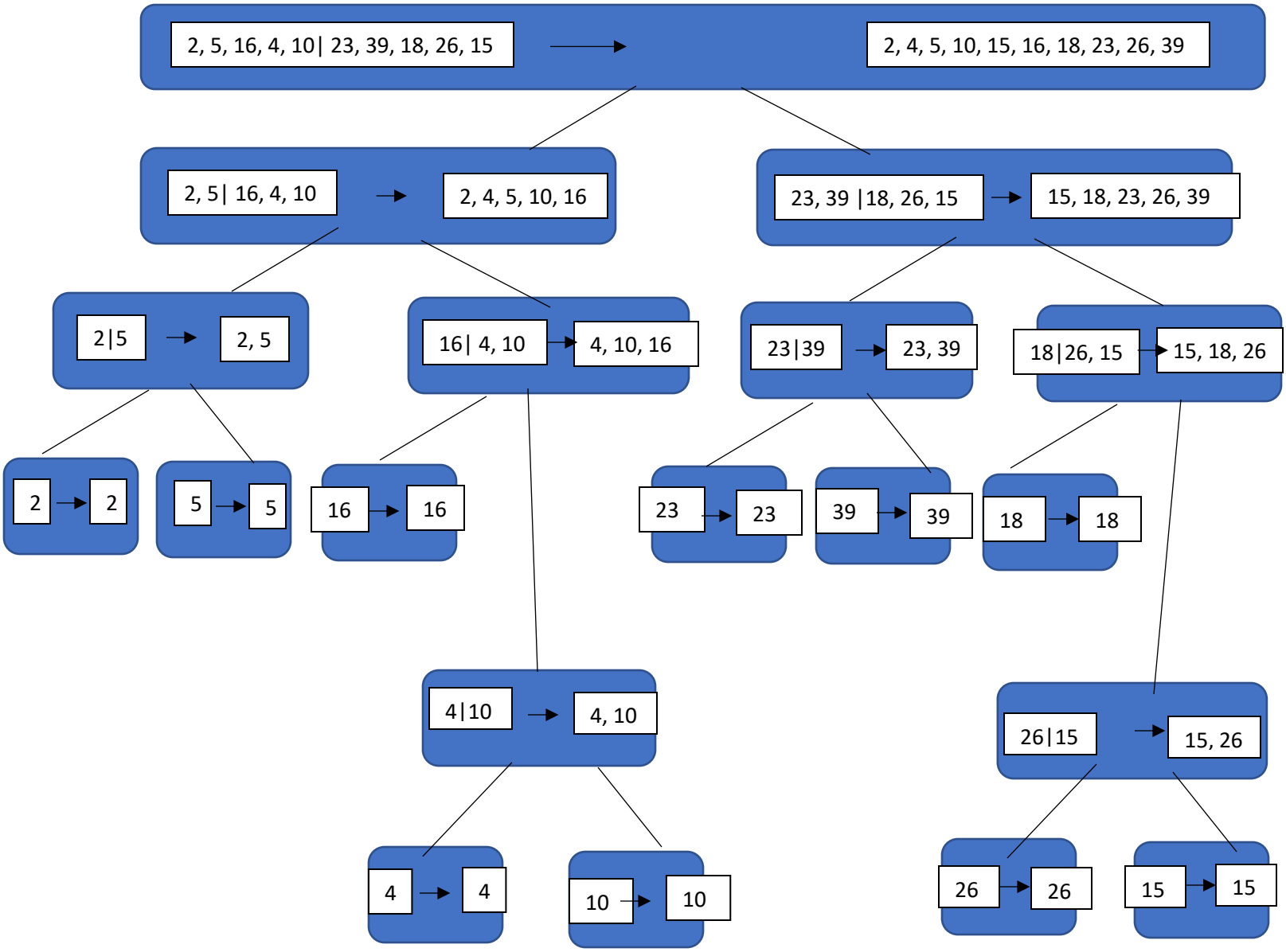
3. (a) (5 points) Draw the merge-sort tree for an execution of the merge-sort algorithm on the input sequence: (2, 5, 16, 4, 10, 23, 39, 18, 26, 15) (like in Figure. 4.2).



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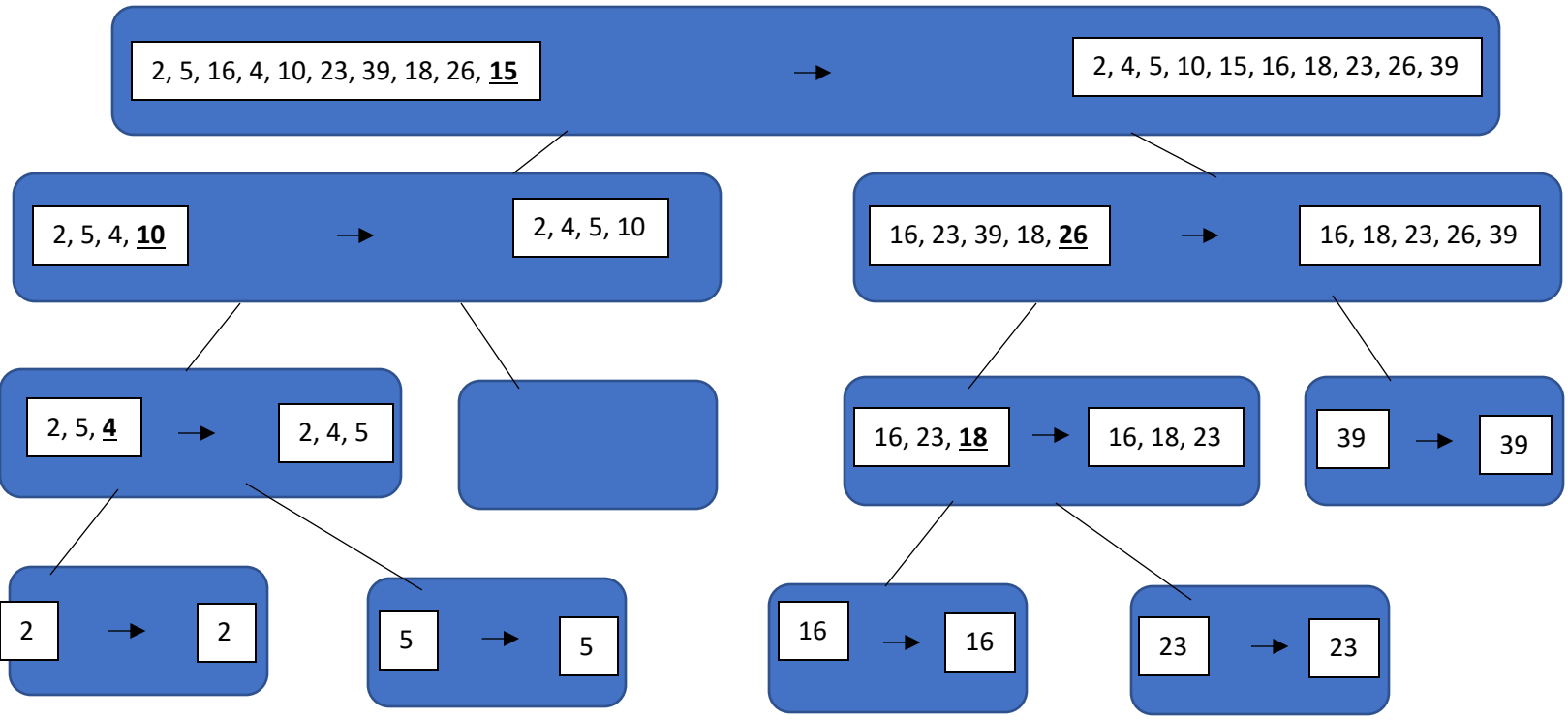
(b) (5 points) Draw the quick-sort tree for an execution of the quick-sort algorithm on the input sequence from part (a) (2, 5, 16, 4, 10, 23, 39, 18, 26, 15) (like in Figure 4.12). Use the last element as the pivot.



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(c) (3 points) What is the running time of the version of quick-sort that uses the element at rank floor of n/2 as the pivot, provided that the input sequence is already sorted? Explain.

If we are given a sorted sequence and we are to use the floor of n/2 as the pivot for quick-sort the expected run time should be O(n*log(n)) because our pivot will always give us two sequences of similar size and thus we will never have an empty, or nearly empty, sequence and a sequence with ¾ more elements.

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4. (10 points) Suppose we are given a sequence S of n elements, each of which is colored red or blue. Assuming S is represented by an array, give a linear-time in-place algorithm for ordering S so that all the blue elements are listed before all the red elements. What is the running time of your method?

```
Algorithm sortRvsB(S)
    Input: a sequence S
    Output: a sequence in which all the blue elements are listed before the red elements
    pointer low<- S[0]
    pointer high<- S[size of S -1}
    while(low <high && low< the last element of S)
        while(low != red element)
            ++low
        while(low=red element)
            if(low=red element && high=blue element && low <high)
                swap(low, high)
            else if(low< high)
                --high
    return S
```

The worst case running time of this algorithm is O(n) where n is the number of elements.

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5. (10 points) Let A and B be two sequences of n integers each. Give an integer m, describe an O(n log n) time algorithm for determining if there is an integer a in A and an integer b in B such that m = a + b.

```
Algorithm findSum(A, B, m)

        Input: an integer m and two sequences A and B

        Output: return 1 if there is one integer from each sequence that sums to m, if no pair is found return -1

        Sort(sequence A in ascending order)

        Sort(sequence B in desending order)

        pointer a<-A[0]

        pointer b<- B[0]

        while(a <= the last element in A && b<= the last element in B)

                if(*a+*b>m)

                        ++b

                else if(*a+*b<m)

                        ++a

                else if(*a+*b=m)

                        return 1

        return -1
```