

Assignment 3

Due 02/15/2019 by beginning of class

Instructions:

You may work with others on these problems but you must turn in your own work. You may type your solutions in any way you like (LaTeX, Markdown, Office, etc...) as long as you present your work clearly and in an organized way.

Unless otherwise specified, you must hand in a printed copy of your work at the beginning of class.

Problems:

1 Read Chapter 1 of *Linear Models R* to learn more qualitative background and history on regression analysis. Then complete the following exercises from the book:

1.1 Chapter 1, Exercise 1

1.2 Chapter 1, Exercise 2

1.3 Chapter 1, Exercise 3

1.4 Chapter 1, Exercise 4

1.5 Chapter 1, Exercise 5

Each part of Problem 1 is evaluated based on the following criteria:

- You make one plot of the data to summarize some relationship (x-axis versus y-axis) of the data set.
- You include a copy of a short R script corresponding to your plot.
- Your plot is clearly labeled in all axes, legends, and the plot includes a clear title.
- You include basic summary statistics of the data in a clear format.
- You provide exactly 1 short paragraph of explanation of your results (~ 3 sentences).

Note: You have written code similar to this in the last assignment. You can re-use scripts and this is encouraged. However, you will also want to modify your old scripts depending on your analysis.

2 We saw in class that the projection operator applied twice has the same result as applying the projection operator once, i.e., that the following relationship holds:

$$(\mathbf{I} - \mathbf{H})^2 = (\mathbf{I} - \mathbf{H}), \quad (1)$$

where we define

$$\mathbf{H} \triangleq \mathbf{X} (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \quad (2)$$

$$\mathbf{X} \in \mathbb{R}^{n \times p}, \quad (3)$$

and $p \neq n$.

For a matrix \mathbf{A} the property that $\mathbf{A}\mathbf{A} = \mathbf{A}$ is known as idempotence.

2.1 Prove that,

$$\mathbf{H}^2 = \mathbf{H}. \tag{4}$$

Note: \mathbf{X}^{-1} doesn't exist when $p \neq n$, and assuming its existence will in a proof will receive no credit.

2.2 Use the property in Eq. (4) to prove Eq. (1).

Note: If you are not familiar with typing mathematical equations, this problem may be handwritten.