Quiz 5

03/04/2019

Instructions:
Write your name at the top right. You are to work on this quiz alone without any help from any other resource *except for a single* 8.5×11 *inch page of handwritten notes.*

In each problem: suppose we have a data set which contains measurements of different cats': (i) age, (ii) weight, (iii) height and (iv) resting blood pressure. Answer the following questions:

1 Given a model for a cat's age in terms of weight, height and resting blood pressure, we wish to test the statistical significance of the parameter $\hat{\beta}_{blood\_pressure}$. State the null and alternative hypothesis. If ω represents the space of models under null hypothesis and Ω represents the space of models under the alternative, state a test statistic and an appropriate statistical test for this hypothesis (the name of the test is sufficient).

If we wish to test the statistical significance of the parameter $\hat{\beta}_{blood\_pressure}$ relative to the model in terms of weight, height and blood pressure, we obtain the following two hypotheses:

$$H_0 : \hat{\beta}_{blood\_pressure} = 0$$
$$H_1 : \hat{\beta}_i \neq 0 \text{ for each i=0,1,2,3}$$

This hypothesis test can be performed either with the t-statistic,

$$t \triangleq \frac{\hat{\beta}_{blood\_pressure}}{se\left(\hat{\beta}_{blood\_pressure}\right)}$$

or with the F-statistic,

$$F \triangleq \frac{(RSS_\omega - RSS_\Omega)/(p-q)}{RSS_\Omega/(n-p)}$$

corresponding to the t-test with $n-p$ degrees of freedom or the F-test with parameters $(p-q)$ and $(n-p)$ respectively.

2 Given two models for a cat's age in terms of

(a) weight, height and resting blood pressure; and

(b) weight,

suppose we wish to see if model (a) has an appreciably better performance than (b), or if we should simplify (a) to (b). State the null and alternative hypothesis. If ω represents the space of models under null hypothesis and Ω represents the space of models under the alternative, state a test statistic and an appropriate statistical test for this hypothesis (the name of the test is sufficient).

If we wish to determine if we can simplify the model (a) to the model (b), we obtain the two following hypotheses:

$$H_0 : \hat{\beta}_{height} = \hat{\beta}_{blood\_pressure} = 0$$
$$H_1 : \hat{\beta}_i \neq 0 \text{ for each i=0,1,2,3}$$

In this case, we can use the F-statistic,

$$F \triangleq \frac{(RSS_\omega - RSS_\Omega)/(p-q)}{RSS_\Omega/(n-p)}$$

and the F-test to determine the p-value.

3 Given two models for a cat's age in terms of

(a) weight and resting blood pressure; and

(b) weight and height

suppose we wish to see if model (a) has an appreciably better performance than (b), or if we should accept (b) over (a). Have we studied together a hypothesis testing procedure that is appropriate for this? Why or why not?

So far, we have only studied hypothesis testing that relates to *nested* or *embedded* sub-models. In this case, neither the F-test nor the t-test make sense.

4 Given a model for a cat's age in terms of weight, height and resting blood pressure, we find that the 95% confidence interval for $\boldsymbol{\beta}_{weight}$ is equal to

$$(4.35, 8.98). \tag{1}$$

Describe one hypothesis test that you can you conclude from the above interval at 5% significance. State the null and alternative hypothesis, and whether we accept the alternative or fail to reject the null.

There are many valid interpretations. A simple example we have seen is

$$H_0 : \hat{\beta}_{weight} = 0$$
$$H_1 : \hat{\beta}_i \neq 0 \text{ for each i=0,1,2,3}$$

In this case, the 95% confidence interval doesn't contain zero, so with 5% significance, we can reject the null hypothesis above.