Quiz 3

02/11/2019

Instructions:
Write your name at the top right. You are to work on this quiz alone without any help from any other resource *except for a single* 8.5×11 *inch page of handwritten notes.*

Problems:

1 We are studying cat data again, though with a different data set. The dataframe "*cats*" looks like the following:

	age	*weight*	*height*
1	1	2.1	7
2	4	5.0	15
3	6	3.2	9
4	7	4.0	10

We want to set up a linear model for "*age*" in terms of "*weight*" and "*height*". Answer the following:

1.1 How many "*explanatory variables*" are there in this relationship? How many "*observations*" are there of this relationship?

The explanatory variables are weight and height. There are two. The number of observations of the relationship are four, the number of rows in the dataframe.

1.2 What is the general equation for the linear model in terms of the vectors $\mathbf{Y}, \mathbf{X}, \boldsymbol{\beta}$ and $\boldsymbol{\epsilon}$?

This is simply written as

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$$

1.3 What values from *cats* go in $\mathbf{Y}$? Write out the vector $\mathbf{Y}$.

The vector $\mathbf{Y}$ is given as

$$\mathbf{Y} = \begin{pmatrix} 1 & 4 & 6 & 7 \end{pmatrix}^{\mathrm{T}}$$

1.4 What values from *cats* go in $\mathbf{X}$? Write out the matrix $\mathbf{X}$. **Remember the intercept term in X.**

The matrix $\mathbf{X}$ is given

$$\mathbf{X} = \begin{pmatrix} 1 & 2.1 & 7 \\ 1 & 5.0 & 15 \\ 1 & 3.2 & 9 \\ 1 & 4.0 & 10 \end{pmatrix}$$

1.5 Yes or no, is $\boldsymbol{\beta}$ known? What size is the vector $\boldsymbol{\beta}$? **Remember the intercept term**.

No, $\boldsymbol{\beta}$ is not known. $\boldsymbol{\beta}$ will be given

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_0 & \beta_1 & \beta_2 \end{pmatrix}^{\mathrm{T}}$$

where β_0 corresponds to the intercept. This is size 3×1 or simply length 3.

2 Suppose that $\mathbf{Y} \in \mathbb{R}^n$, $\mathbf{X} \in \mathbb{R}^{n \times p}$, and $\boldsymbol{\beta} \in \mathbb{R}^p$.

2.1 What dimension is the matrix product in equation (1)?

$$(\mathbf{Y} - \mathbf{X}\boldsymbol{\beta})^{\mathrm{T}}(\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}) \tag{1}$$

Notice that $\mathbf{Y}$ is a vector in $\mathbb{R}^n$, as is $\mathbf{X}\boldsymbol{\beta}$. Recall that for a vector $\mathbf{v} \in \mathbb{R}^n$, $\mathbf{v}^{\mathrm{T}}\mathbf{v}$ is simply the scalar product of the vector $\mathbf{v}$ with itself, which can be expressed as the product of a $1 \times n$ matrix times a $n \times 1$ matrix. This has one dimension, or is scalar.

2.2 Explain in general terms what does equation (1) have to do with solving a linear model.

Equation 1, when related to the answer of problem 1.2 can be seen equal to the sum of the square residual errors,

$$\begin{aligned}(\mathbf{Y} - \mathbf{X}\boldsymbol{\beta})^{\mathrm{T}}(\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}) &= \boldsymbol{\epsilon}^{\mathrm{T}}\boldsymbol{\epsilon} \\ &= \sum_{i=1}^{n} \epsilon_i^2\end{aligned}$$

This is the quantity we want to minimize when we solve for a "best" choice of β by least squares.