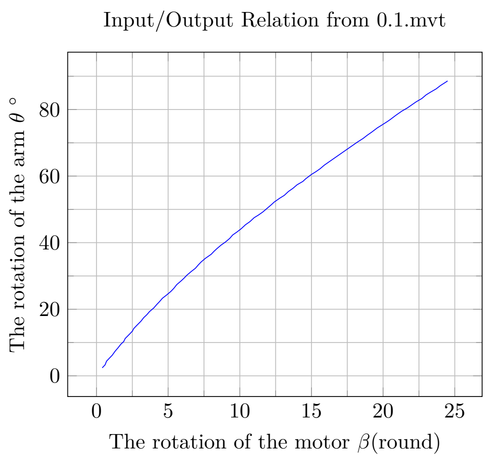
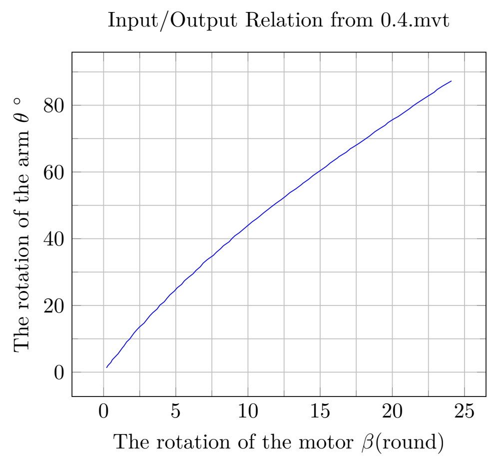
The Answer to the Experiment Maxpid Controlled Function Chain

July 4, 2018

Solution:
(a) The figure 1: Control board, arm, protection casting.

(b) The figure 2: Tachometric generator, motor, screw, nut, angular potentiometer.





Solution:

Solution: The approximative pitch of the screw is 0.400cm(4mm) and the screw has a right-hand thread. The relation is:

$$\Delta x = -p\Delta\beta$$

The hypotheses are as follow:

- Suppose the motor rotates clockwise
- and the positive part of x axis is along $\vec{x_1}$.

The known facts are as follow:

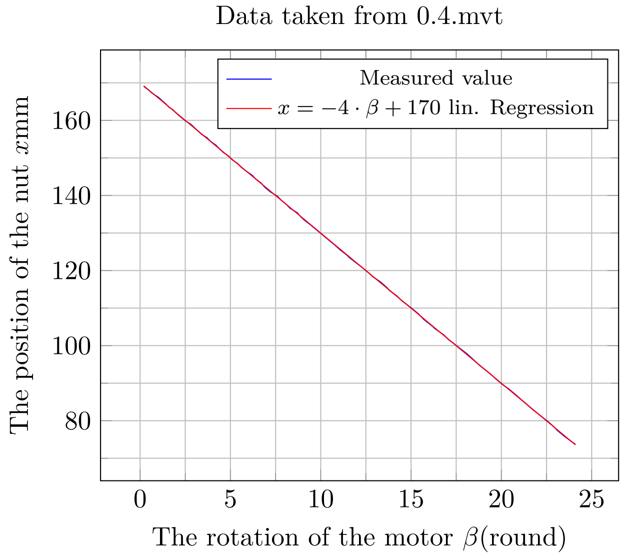
- By definition, a right-hand screw is the one that will be loosen when the nut revolves clockwise along it.
- In our case, the one that moves respecting to the protection casting is the *screw*, not the nut.

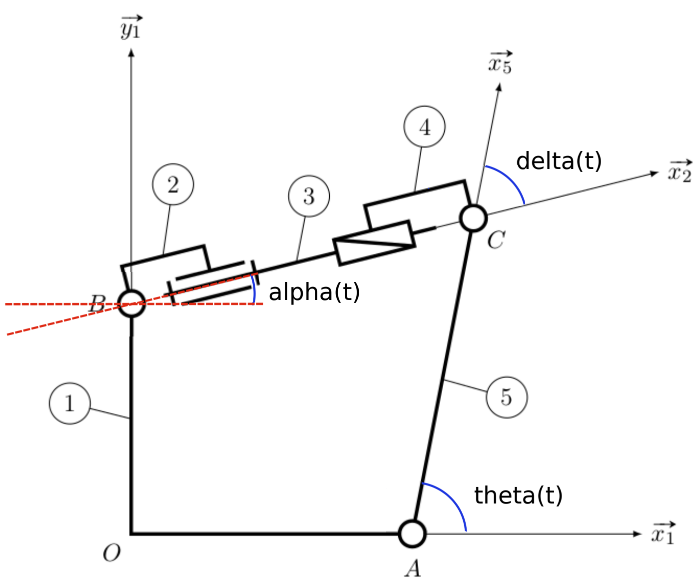
Thus, when the motor rotates clockwise, the nut moves along the negative part of the x. The sign of the relation is negative.

Solution: For the pitch we have:

$$p = \left|\frac{\Delta x}{\Delta \beta}\right|$$

From the linear regression we have $p = 4$mm, which matches the measurement.





Solution:

The angular parameters $\theta(t)$, $\alpha(t)$ and $\delta(t)$ are indicated in the image above.

Solution:

$$a = 70\text{mm}$$
$$b = 80\text{mm}$$
$$c = 80\text{mm}$$

Solution: The input parameter is $\beta(t)$ and the output parameter is $\theta(t)$.

Solution: The loop closure vector relation is:

$$\overrightarrow{OB} + \overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{AO} = \vec{0}$$
$$a\vec{x_1} + c\vec{x_5} - x(t)\vec{x_2} - b\vec{y_1} = \vec{0}$$

As $\vec{b_1}$ is represented by $\vec{x_1}$ and $\vec{y_1}$, we have two scalar equations:

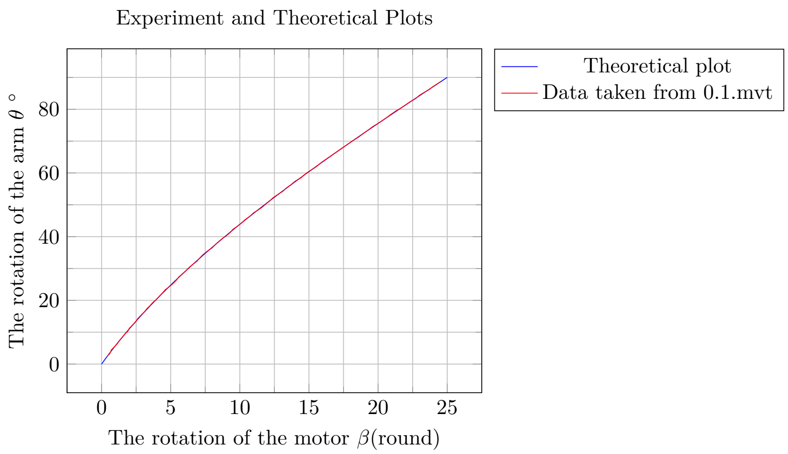
$$a\vec{x_1} \cdot \vec{x_1} + c\vec{x_5} \cdot \vec{x_1} - x(t)\vec{x_2} \cdot \vec{x_1} = 0$$
$$c\vec{x_5} \cdot \vec{y_1} - x(t)\vec{x_2} \cdot \vec{y_1} - b\vec{y_1} \cdot \vec{y_1} = 0$$

The final simplified equations are:

$$a + c\cos(\theta(t)) - x(t)\cos(\alpha(t)) = 0$$
$$-b + c\sin(\theta(t)) - x(t)\sin(\alpha(t)) = 0$$

Solution: Eliminating[1] α from the scalar equations using $\sin^2\alpha + \cos^2\alpha = 1$ yeilds:

$$\begin{aligned}
x^2 &= 2c(a\cos\theta - b\sin\theta) + a^2 + b^2 + c^2 \\
x^2 &= 2c\sqrt{a^2+b^2}\sin\left(\theta - \arctan\frac{a}{b}\right) + a^2 + b^2 + c^2 \\
\theta &= -\arcsin\left(\frac{x^2 - a^2 - b^2 - c^2}{2c\sqrt{a^2+b^2}}\right) + \arctan\frac{a}{b} \\
&= -\arcsin\left(\frac{(-4\beta+170)^2 - a^2 - b^2 - c^2}{2c\sqrt{a^2+b^2}}\right) + \arctan\frac{a}{b} \\
&= -\arcsin(\frac{16\beta^2 - 1360\beta + 11200}{17008}) + 41.19
\end{aligned}$$



Solution:

[1]Since t is not relevant to the relation between θ and β, we omit t in this solution.