

# GDC – Groups & Polarization

The similarity between two persons' dating hypotheses  $h$  respectively bodies of evidence  $e$  is assessed, that is  $SIM_{x,y}(h)$  respectively  $SIM_{x,y}(e)$ . For each time step, there is a scatter plot showing  $\{(SIM_{x,y}(e), SIM_{x,y}(h))\}$  for all persons participating at that time.

## Three Group Definitions in Terms of a Threshold

- $\forall x, y \in G_I : SIM_{x,y}(h) > s_0$
- $\forall x, y \in G_{II} : SIM_{x,y}(e) > s_0$
- $\forall x, y \in G_{III} : SIM_{x,y}(h) > s_0 \wedge SIM_{x,y}(e) > s_0$

If  $n$  persons form a group  $G$ , then there are  $\binom{n}{2} = \frac{n!}{2!(n-2)!}$  two-elements subgroups. For each of these subgroups, the corresponding condition applies.

Examples for  $G_I$ :

- Persons 1 and 4 form a  $G_I$ -group, iff
  - i.  $SIM_{1,4}(h) > s_0$
- Persons 1, 4 and 5 form a  $G_I$ -group, iff
  - i.  $SIM_{1,4}(h) > s_0$ ,
  - ii.  $SIM_{1,5}(h) > s_0$
  - iii.  $SIM_{4,5}(h) > s_0$
- Persons 1, 4, 5 and 6 form a  $G_I$ -group, iff
  - i.  $SIM_{1,4}(h) > s_0$ ,
  - ii.  $SIM_{1,5}(h) > s_0$ ,
  - iii.  $SIM_{1,6}(h) > s_0$ ,
  - iv.  $SIM_{4,5}(h) > s_0$ ,
  - v.  $SIM_{4,6}(h) > s_0$
  - vi.  $SIM_{5,6}(h) > s_0$
- Persons 1, 3, 4, 5 and 6 form a  $G_I$ -group, iff
  - i.  $SIM_{1,3}(h) > s_0$
  - ii.  $SIM_{1,4}(h) > s_0$ ,
  - iii.  $SIM_{1,5}(h) > s_0$ ,
  - iv.  $SIM_{1,6}(h) > s_0$ ,
  - v.  $SIM_{3,4}(h) > s_0$
  - vi.  $SIM_{3,5}(h) > s_0$
  - vii.  $SIM_{3,6}(h) > s_0$
  - viii.  $SIM_{4,5}(h) > s_0$ ,
  - ix.  $SIM_{4,6}(h) > s_0$
  - x.  $SIM_{5,6}(h) > s_0$
- Persons 1, 2, 3, 4, 5 and 6 form a  $G_I$ -group, iff
  - i.  $SIM_{1,2}(h) > s_0$
  - ii.  $SIM_{1,3}(h) > s_0$
  - iii.  $SIM_{1,4}(h) > s_0$ ,

- iv.  $SIM_{1,5}(h) > s_0$ ,
- v.  $SIM_{1,6}(h) > s_0$ ,
- vi.  $SIM_{2,3}(h) > s_0$
- vii.  $SIM_{2,4}(h) > s_0$
- viii.  $SIM_{2,5}(h) > s_0$
- ix.  $SIM_{2,6}(h) > s_0$
- x.  $SIM_{3,4}(h) > s_0$
- xi.  $SIM_{3,5}(h) > s_0$
- xii.  $SIM_{3,6}(h) > s_0$
- xiii.  $SIM_{4,5}(h) > s_0$ ,
- xiv.  $SIM_{4,6}(h) > s_0$
- xv.  $SIM_{5,6}(h) > s_0$

Groups have been identified for each time step after individual belief changes, that is  $Xb$ .

## Caveat

Using a threshold is somewhat arbitrary. Therefore, results are compared for three different thresholds.

## Results

### Groups

Table 1:  $s_0 = 0.9$

Time	Persons	$G_I$	$G_{II}$	$G_{III}$
1b	1,2,3	{3,2}	-	-
2b	+4	{3,2}, {4,1}	{4,1}	{4,1}
3b	+5	{2,3,5}, {4,1}	{1,5,4}	{4,1}
4b	-	{3,2},{5,4}	{4,1}, {5,4}	{5,4}
5b	-	{3,2}, {5,4}	{4,1}	-
6b	+6	{2,3,4,6}	{6,1},{6,4}	{6,4}
7b	-	{3,2},{6,4}	{3,2},{6,4},{6,1},{6,5}	{3,2},{6,4}
8b	-	{1,2,3,4,5,6}	{1,4,5,6},{2,3}	{1,4,5,6},{2,3}

Table 2:  $s_0 = 0.85$

Time	Persons	$G_I$	$G_{II}$	$G_{III}$
1b	1,2,3	{3,2}	-	-
2b	+4	{3,2}, {4,1}	{4,1}	{4,1}
3b	+5	{2,3,5}, {4,1}	{1,5,4}	{4,1}
4b	-	{2,3,4,5}	{1,4,5}	{4,5}

5b	-	{3,4,5}, {3,2}	{1,4,5}	{4,5}
6b	+6	{2,3,4,6}, {2,3,4,5}	{1,4,5,6}	{6,4}
7b	-	{2,3,6}, {6,4}, {6,5}	{3,2}, {4,5,6}, {6,1}	{3,2}, {6,4}, {5,4}
8b	-	{1,2,3,4,5,6}	{1,2,3,4,5,6}	{1,2,3,4,5,6}

Table 3:  $s_0 = 0.8$

Time	Persons	$G_I$	$G_{II}$	$G_{III}$
1b	1,2,3	{3,2}	-	-
2b	+4	{3,2}, {4,1}	{4,1}, {4,2}, {3,2}	{4,1}, {3,2}
3b	+5	{2,3,5}, {4,1}	{1,3,4,5}, {2,3,4}	{4,1}, {2,3}, {3,5}
4b	-	{2,3,4,5}	{1,4,5}, {2,5}, {2,3}, {1,3}	{4,5}, {2,3}, {2,5}
5b	-	{2,3,4,5}	{1,4,5}, {3,2}, {2,5}	{3,2}, {2,5}, {4,5}
6b	+6	{2,3,4,5,6}	{1,4,5,6}, {2,3,5}	{2,3,5}, {4,5,6}
7b	-	{2,3,4,6}, {4,5,6}	{2,3,6}, {1,4,5,6}	{2,3,6}, {4,5,6}
8b	-	{1,2,3,4,5,6}	{1,2,3,4,5,6}	{1,2,3,4,5,6}

## Polarization

Based on such a table, the following questions can be answered for each time step:

1. How many groups can be defined? How many subgroups can be maximally defined?  
(Community fragmentation)
2. How are participants distributed over groups? Are all groups more or less comparably sized?  
Or is there one dominant group? (Size parity)

Based on such a table and the specific similarity measure SIM, the following questions can be answered for each time step (Definition of SIM, see below):

1. To what extent do positions of members of the same group differ? (Group Consensus)

The difference can be assessed by  $1 - SIM_{x,y}(h)$  respectively  $1 - SIM_{x,y}(e)$ .

(This means that two persons differ (i) in respect to  $h$ , if there are contradictory atomic dating beliefs, (ii) and in respect to  $e$ , if there are contradictory evidential beliefs or abstentions.)

2. Are there shared beliefs between members of different groups? (Distinctness)

Two persons share no belief at all iff  $SIM_{x,y} = 0$ . It shows that this is never the case, even between members of different groups.

According to [1], community fragmentation, size parity, groups consensus and distinctness are four definitions of polarization (in terms of groups).

## Bibliography

[1] Aaron Bramson, Patrick Grim, Daniel J. Singer, William J. Berger, Graham Sack, Steven Fisher, Carissa Flocken, and Bennett Holman. Understanding polarization: Meanings, measures, and model evaluation. *Philosophy of Science*, 84(1):115–159, 1 2017.

## Reminder – Definition SIM

Similarity between two bodies of evidence respectively dating hypotheses

- $SIM_{x,y}(e) = 1 - \frac{-3 * |C| - |A|}{-3 * (|C| + |S|) - |A|}$
- $SIM_{x,y}(h) = \frac{|A|}{15}$

For two sets of statements, let  $|C|$  ,  $|A|$  and  $|S|$  , denote the number of contradictions, abstentions and agreements, respectively.

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