Generalized Parton Distribution Functions via Quantum Simulation of Quantum Field Theory in Light-front Coordinates

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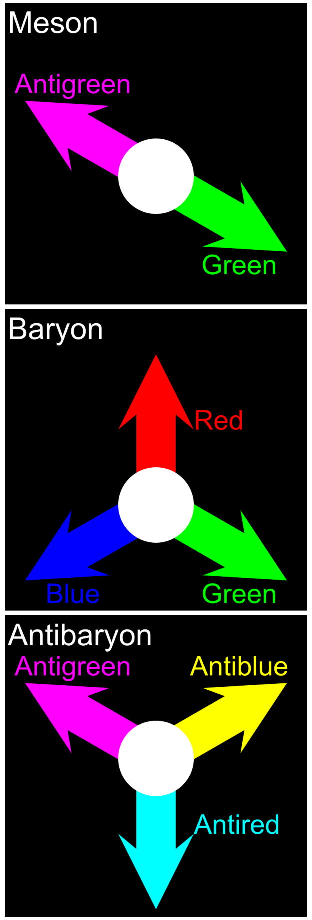
Overview

- The Partonic Model
- Description of the PDF & GPD
- LF coordinates
- Mapping
- Brief overview of VQE
- Results
- Future additions

The Partonic Model

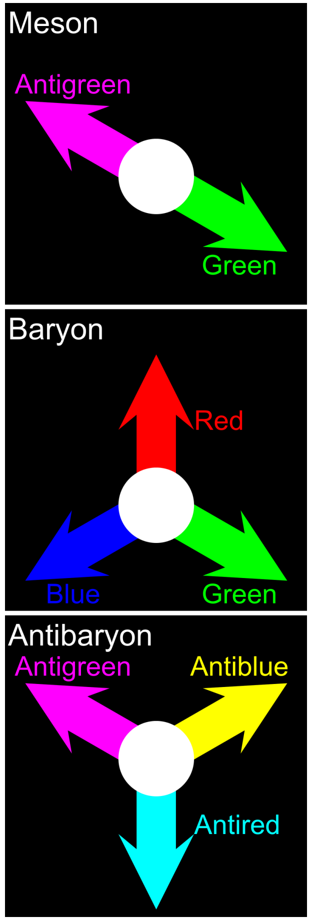
- The hadron is a composite bound state composed of quark/antiquarks (which we call partons)



1. https://en.wikipedia.org/wiki/Hadron

The Partonic Model

- The hadron is a composite bound state composed of quark/antiquarks (which we call partons)
- Hadrons come in two main types:
  - Meson ($|q\bar{q}\rangle$): $\pi, J/\psi, \rho$
  - Baryon($|qqq\rangle$): p, n



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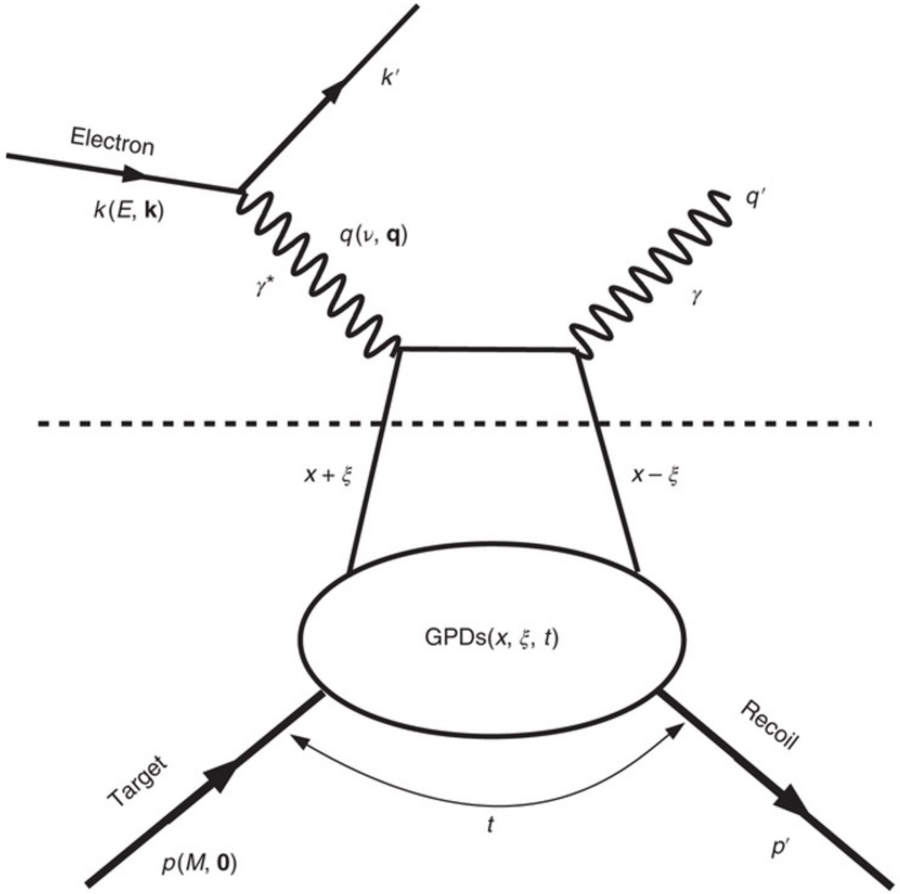
$$N_l(x) = l_n^\dagger l_n \qquad\qquad l = \begin{cases} a; bosons \\ b; fermions \\ d; antifermions \end{cases}$$

The Generalized Parton Distribution Function (GPD)



- The GPD represents the probability amplitude to find a parton with a fraction of the total light-front momentum after an interaction

$$F^q = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P'|\bar{q}(-\frac{z}{2})\gamma^+ q(\frac{z}{2}))|P\rangle\,|_{z^+=\boldsymbol{z}=0}$$

https://www.nature.com/articles/s41567-019-0774-3

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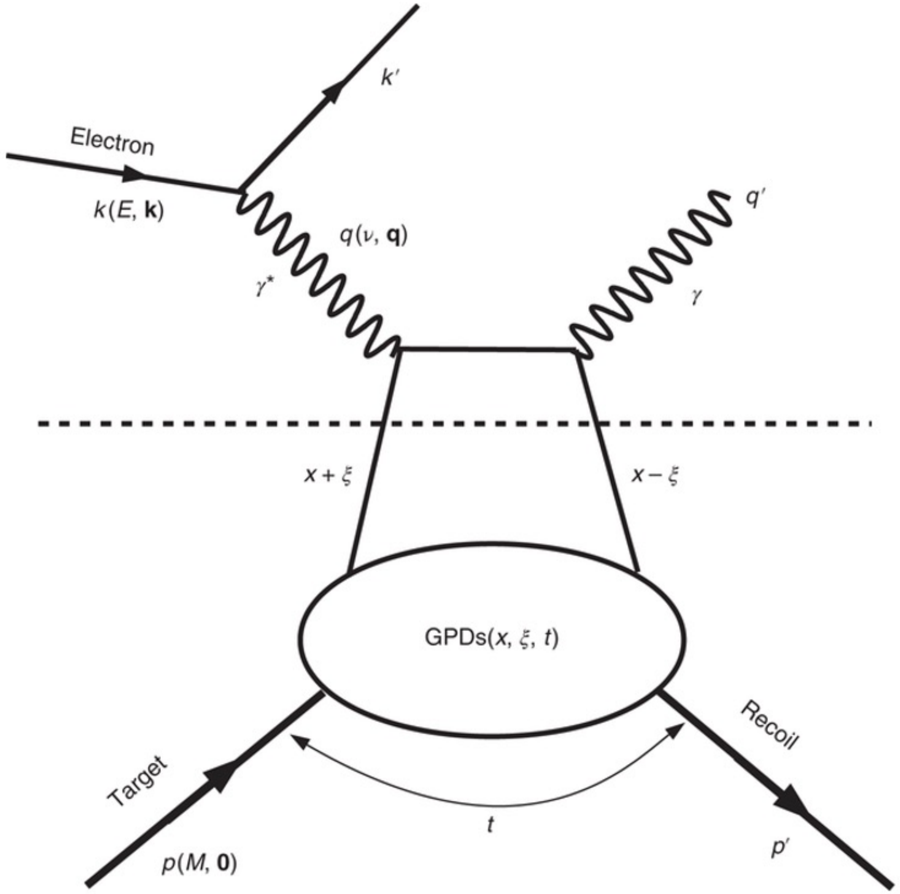

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$$x = \frac{n}{K} \qquad t = (P - P')^2$$

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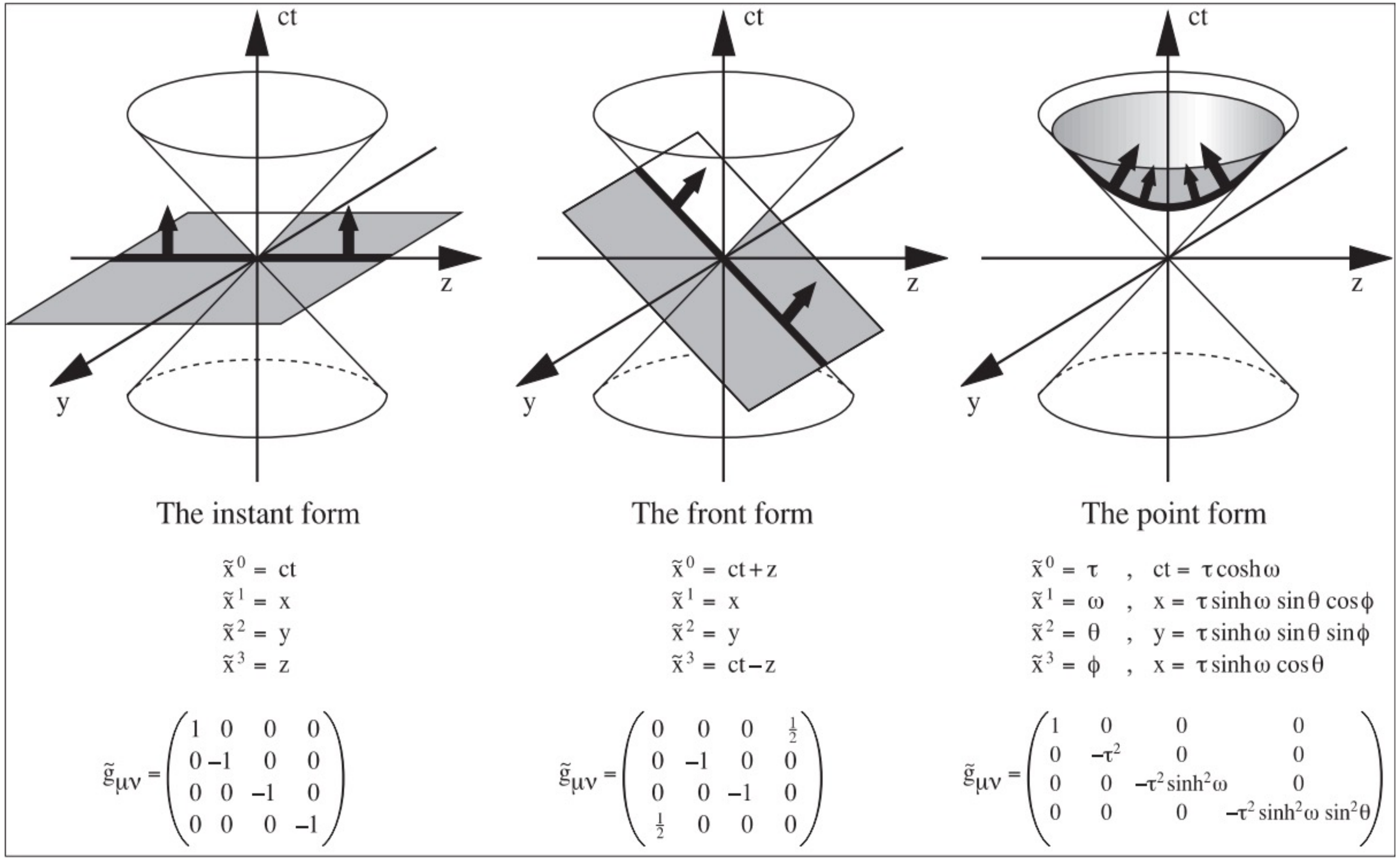
Light-front Coordinates



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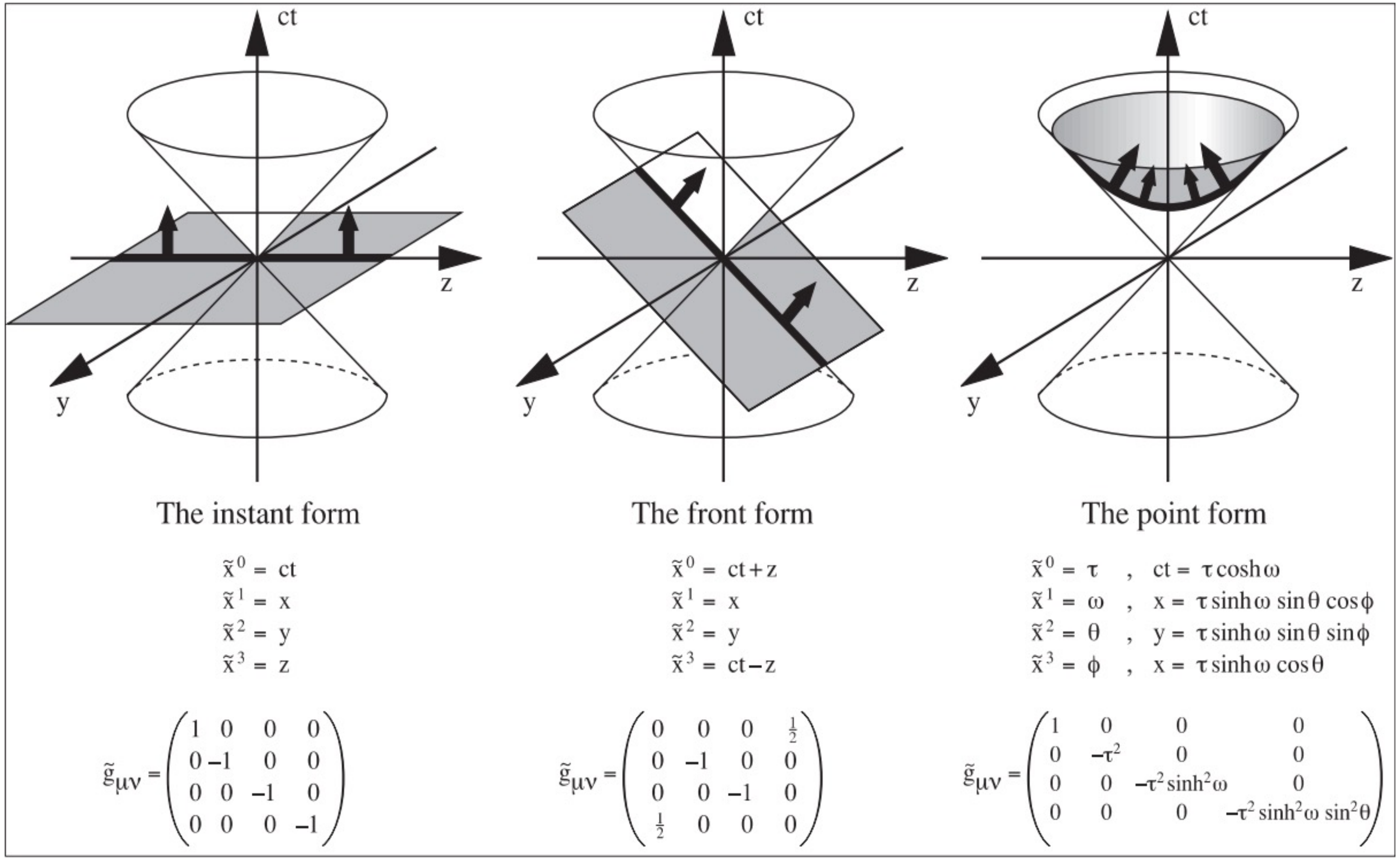

- $P^{\pm} = E \pm P$
- $P^{+} = \frac{2\pi}{L} K$
- $P^{-} = \frac{L}{2\pi} H$
- $M^2 = E^2 - P^2 = KH$

Fock state to qubit state

Wavefunction (First quantization)	Fock state (Second quantization)	Qubit State
$\Psi(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N) = \frac{1}{\sqrt{N!}} \det \begin{bmatrix} \phi_0(\boldsymbol{x}_0) & \cdots & \phi_M(\boldsymbol{x}_0) \\ \vdots & \ddots & \vdots \\ \phi_0(\boldsymbol{x}_N) & \cdots & \phi_M(\boldsymbol{x}_N) \end{bmatrix}$	$\lvert F\rangle = \left\lvert 1^{n_1}, \ldots, {N_F}^{n_{N_F}}; \tilde{1}^{\tilde{n}_1}, \ldots, {\tilde{N}_B}^{\tilde{n}_{N_B}} \right\rangle$ $n_i = \begin{cases} 0, 1; fermions \\ 0, 1, 2 \ldots; bosons \end{cases}$ $H_{F,F'} = \langle F \lvert H \rvert F' \rangle$	$\lvert \{n_i\}, \{\overline{n_j}\}, \{\widetilde{n_k}\} \rangle$ For fermions: $\lvert 1^{n_1}, \ldots, {N_F}^{n_{N_F}} \rangle \rightarrow$ $\left\lvert q_{N_F}, \ldots, q_0 \right\rangle$

- We reduce qubit count by looking at the $\lvert q\bar{q}\rangle$ sector of the Hamiltonian only (i.e. no bosonic modes)

Variational Quantum Eigensolver (VQE)

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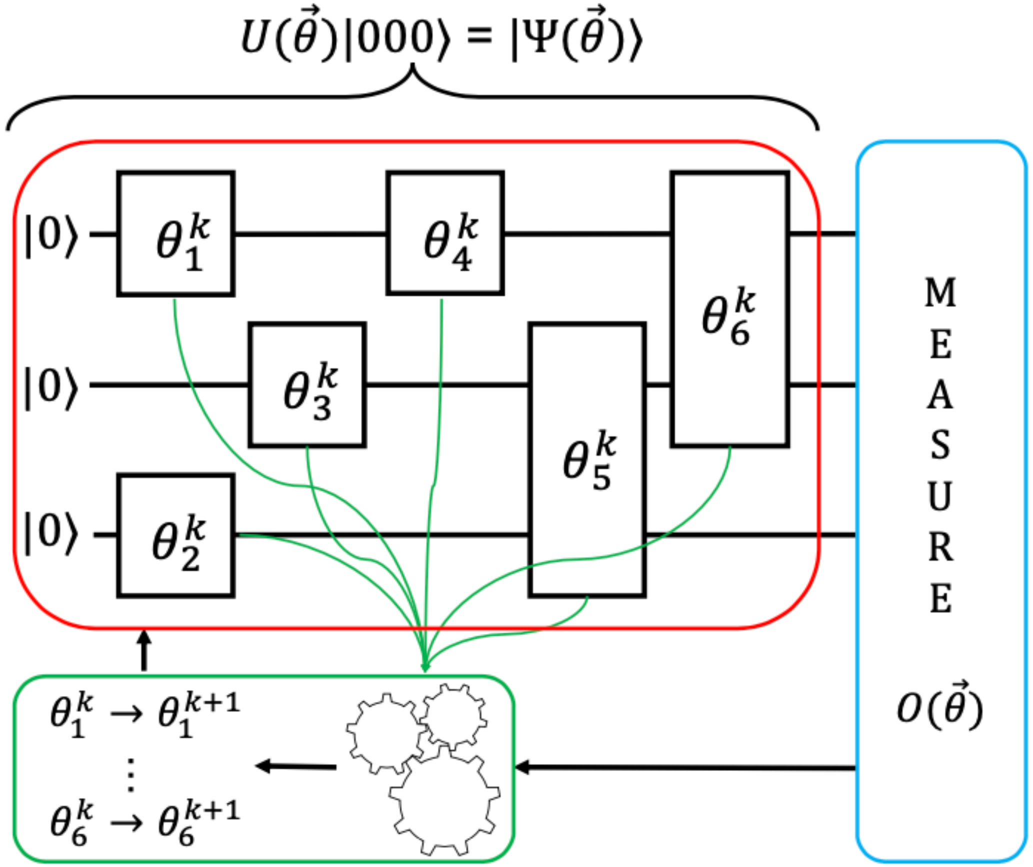
- In order to calculate the GPD, we need ground bound states of the Hamiltonian.
- Use VQE which is a hybrid-classical algorithm that finds the minimum eigenvalue of an operator (generally the Hamiltonian)
- VQE exploits the variational principle:

$$E_0 \leq \left\langle \Psi(\vec{\theta}) \middle| H \middle| \Psi(\vec{\theta}) \right\rangle$$

Variational Quantum Eigensolver (VQE)



Quantum Computational Chemistry, McArdle et. al.

Quantum Advantage

- The GPD calculates off-diagonal matrix elements:

$$F^q(n) = \frac{1}{4(2\pi)^{3/2}n}\sum_{k_T}\langle P'|G|P\rangle$$

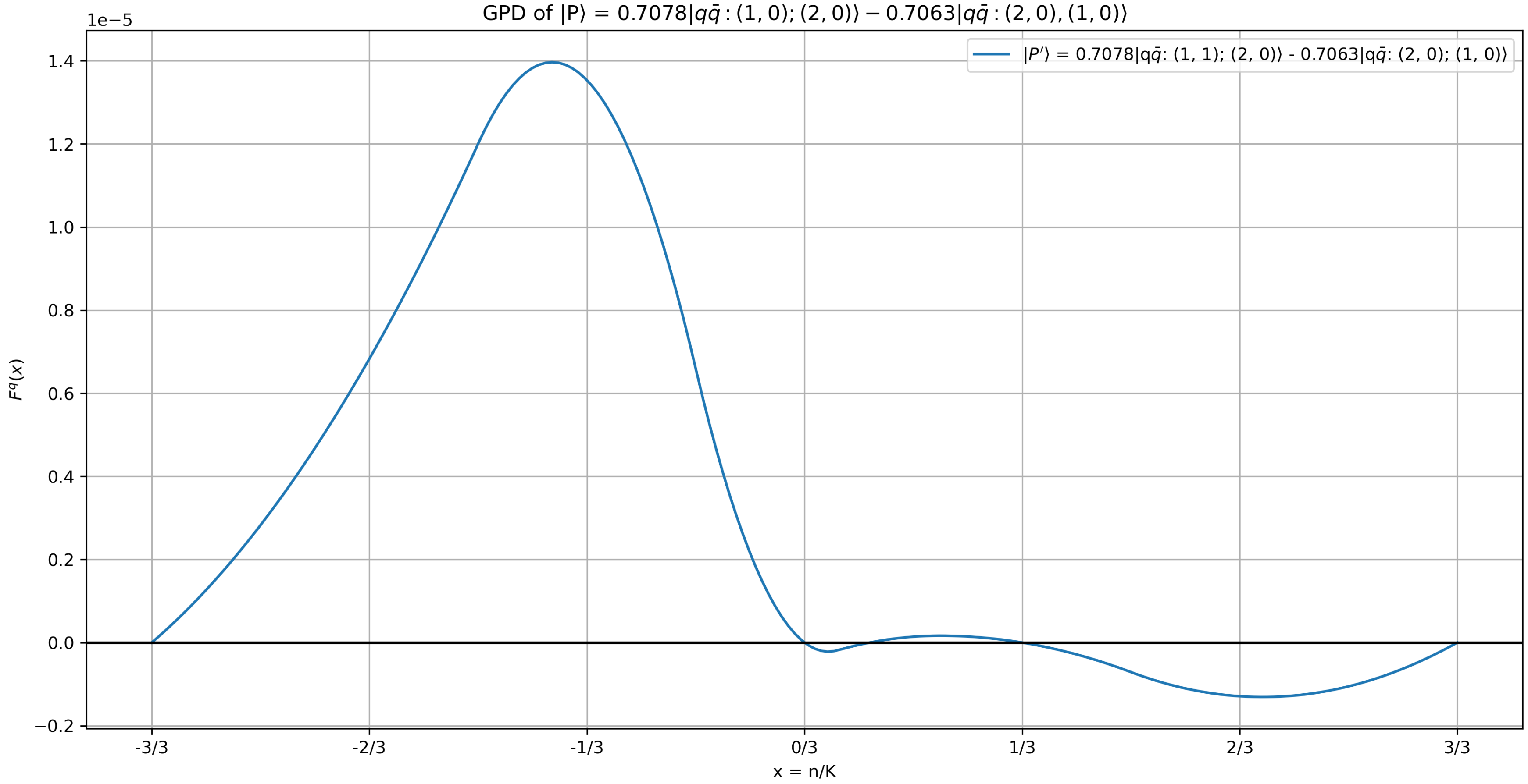
- A quantum computer can efficiently calculate expectation values: $\langle P|O|P\rangle$ (diagonal matrix elements)
- How can we efficiently calculate F^q on a quantum device?

Quantum Advantage

- Define an operator O: $|P'\rangle = O|P\rangle$
- We can now measure $F^q(n) = \frac{1}{4(2\pi)^{3/2}n}\sum_{k_T}\langle P|\mathcal{F}^q|P\rangle$ where

$$\mathcal{F}^q = O^\dagger G$$

Results



GPD of $|\mathrm{P}\rangle = 0.7078|q\bar{q}:(1,0);(2,0)\rangle - 0.7063|q\bar{q}:(2,0),(1,0)\rangle$
$|P'\rangle$ = 0.7078|q$\bar{q}$: (1, 1); (2, 0)⟩ - 0.7063|q$\bar{q}$: (2, 0); (1, 0)⟩
1e−5
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
$F^q(x)$
-3/3
-2/3
-1/3
0/3
1/3
2/3
3/3
x = n/K

Future Additions

- To encapsulate a more complete model for QCD, we must extend our Hamiltonian to include flavor, color, helicity, and a third space (3 + 1D) dimension.