Global Navigation and Satellite System

A basic introduction to concepts and applications

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Outline

1. Basic Concepts
   - Motivation
   - Mathematical concepts
2. Collecting Geospatial Data
3. Introducing GNSS
   - GPS Basics
   - GPS based Math Models
4. Applications and Software
   - Applications
   - Software and Hardware
5. Resources
6. Acknowledgements

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Where am I?

Locate and navigate

One of the most favourite questions of mankind

- Stone age
  - Using markers like stone, trees and mountains to find their way back
- Star age
  - Pole star was a constant reminder of the direction
  - Most world cultures refer to the Pole star in their literature
- Radio age
  - Radio signals for navigation purposes
- Satellite age:
  - Modern technology
  - Use of satellites

To understand GPS we need to revise some mathematical concepts.

Taylor's series expansion

If $f(x)$ is a given function, then its Taylor's series expansion is given as

$$f(x) = f(a) + f'(a)(x-a) + f''(a)\frac{(x-a)^2}{2!} + \ldots$$

Jacobian

If $f_1, f_2, \ldots, f_m$ be m different functions, and $x_1, x_2, \ldots, x_n$ be n independent variables, then the Jacobian matrix is given by:

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \ldots & \ldots & \vdots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

If $\mathbf{F}$ denotes the set of functions, and $\mathbf{X}$ denotes the set of variables, the Jacobian can be denoted by

$$\frac{\partial \mathbf{F}}{\partial \mathbf{X}}$$

Least squares adjustment

Fundamental example

Given Problem

Suppose $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ be n given points. It is required to fit a straight line through these points.

- Let the equation of "fitted" straight line be $Y = mX + c$.
- $y_1 + \delta_1 = m \cdot x_1 + c$, $y_2 + \delta_2 = m \cdot x_2 + c$, and so on ...
- Arrange into matrix form

$$\begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_n \end{bmatrix} = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \cdot \begin{bmatrix} m \\ c \end{bmatrix} - \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

Least squares adjustment

Fundamental example continued

To determine m and c we use the formula:

$$\begin{bmatrix} m \\ c \end{bmatrix} = \left(\begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix}^T \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix}^T \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

Least squares adjustment

For a general equation

- $\mathbf{L_b}$: set of observations,
- $\mathbf{X}$: set of parameters.
- $\mathbf{X}_0$: initial estimate of the parameters
- $\mathbf{L}_0$: initial estimate of the observations.
- $\mathbf{V}$: correction applied to $\mathbf{L_b}$
- $\mathbf{L}_a$: adjusted observations.

The set of equations is represented by

$$\mathbf{L_a} = \mathbf{F}(\mathbf{X_a})$$

Using Taylor's series expansion,

$$\mathbf{L}_b + \mathbf{V} = \mathbf{F}(\mathbf{X}_0)|_{\mathbf{X}_a=\mathbf{X}_0} + \left.\frac{\partial \mathbf{F}}{\partial \mathbf{X}_a}\right|_{\mathbf{X}_a=\mathbf{X}_0} (\mathbf{X}_a - \mathbf{X}_0) + \ldots$$

This can be represented as

$$\mathbf{V} = \mathbf{A}\Delta\mathbf{X} - \Delta\mathbf{L}$$

which can be solved by

$$\Delta\mathbf{X} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{L}$$

If case of weighted observations (P is the weight matrix),

$$\Delta\mathbf{X} = (\mathbf{A}^T P\mathbf{A})^{-1}\mathbf{A}^T P\mathbf{L}$$

Euclidean distance

Formula for finding distance between two points

If $\mathbf{p}_1(x_1, y_1, z_1)$ and $\mathbf{p}_2(x_2, y_2, z_2)$ be two different points in 3D space, then the distance between the two is given by:

$$d(\mathbf{p}_1, \mathbf{p}_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Outline

Collecting Geospatial Data

- Early: Chains, tapes and sextants
- Mid: Theodolites, auto levels
- Modern: EDM, GNSS

Older methods of collecting geospatial data

Pictures of some instruments

SEXTANT

GUNTER'S CHAIN

TAPE

THEODOLITE

AUTO LEVEL

TOTAL STATION

Coordinates of a point

How to find them?

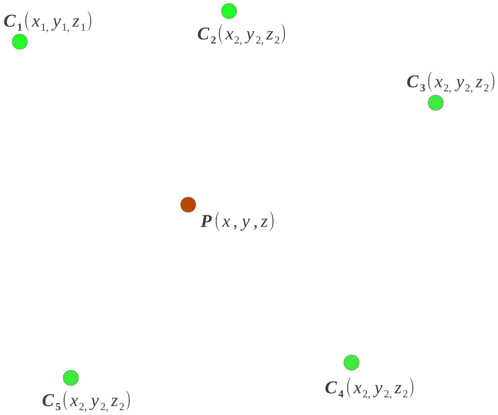

Figure: A typical resection problem in surveying. Given coordinates of the known points $\mathbf{C}_i$, find the coordinates of the unknown point $\mathbf{P}$

The resection problem

Issues of accuracy and geometry

- Resection problem requires the measurement of angles and distances
- All measurements made through theodolites, auto levels, EDMs etc contain errors
- Requirement for minimizing errors: The control points should be well distributed spatially.

Distance between objects

How to find it?

- What is the distance between the chalk and the blackboard?
- How to find the distance between a ball and a chair?
- How to find the distance between an aeroplane and the ground?

Distance between objects

How to find it?

- What is the distance between the chalk and the blackboard?
- How to find the distance between a ball and a chair?
- How to find the distance between an aeroplane and the ground?

- If there is a mathematical surface (e.g. plane, sphere etc.) then it is easier to find the distance.
- The Earth has a rough surface, and it has to be therefore approximated by a mathematical surface.

The mathematical surface

Datum and sphereoids

- Kalyanpur: Meaning of 127m above MSL?

The mathematical surface

Datum and sphereoids

- Kalyanpur: Meaning of 127m above MSL?
- There is no sea in sight? What is mean sea level?

The mathematical surface

Datum and sphereoids

- Kalyanpur: Meaning of 127m above MSL?
- There is no sea in sight? What is mean sea level?
- Geoid: The equipotential surface of gravity near the surface of the earth
- MSL - Mean sea level (an approximation to the Geoid
- Ellipsoid / spheroid: approximation to the MSL
- Mathematical surface: DATUM

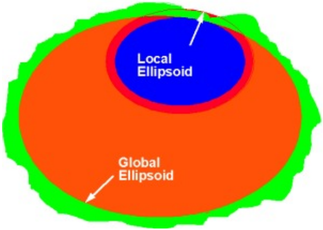

Figure: Src: http://www.dqts.net/wgs84.htm

World Geodetic System

One of the most prevalent datums

- It is an ellipsoid
- Semi-major Axis: $a = 63,78,137.0m$
- Semi-minor axis: $b = 63,56,752.314245m$
- Inverse flattening: $\frac{1}{f} = 298.257223563$
- Complete description given in the WGS84 Manual `http://www.dqts.net/files/wgsman24.pdf`

Parallels and Meridians

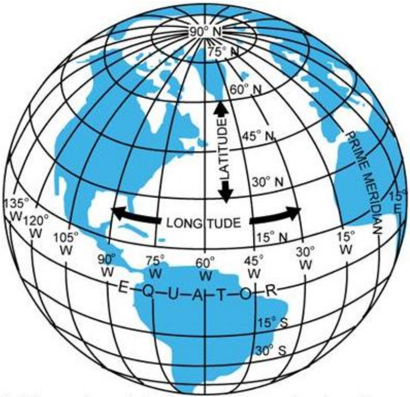

Figure: Latitudes and longitudes

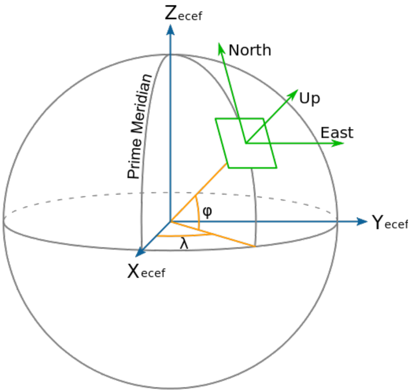

Figure: Measurement of angles

Map Projections

... and their needs

- It is a three dimensional world ...
- But the mapping paper is two-dimensional
- A mathematical function: Project 3D world to 2D Paper

Map Projections

... and their needs

- It is a three dimensional world ...
- But the mapping paper is two-dimensional
- A mathematical function: Project 3D world to 2D Paper

- Distance preserving
- Area preserving
- Angle preserving - conformal

- Conical
- Cylindrical
- Azimuthal

There are guidelines on how to chose a map projection for your map

Universal transverse mercator

A cylindrical and conformal projection

- The world divided into 60 zones - longitudewise
- Each zone of 6 degrees
- The developer surface is a cylinder placed tranverse
- The central meridian of the zone forms the central meridian of the projection
- UTM Zone for Kanpur: 44N
- Combination of projection and datum: UTM/WGS84 and Zone 44N

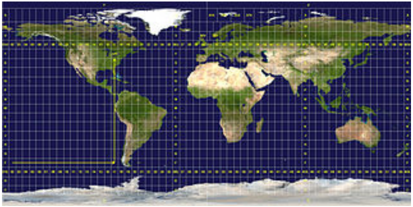

Figure: The UTM zone division (Src: Wikipedia)

Outline

NAVSTAR
The US-based initiative

- TRANSIT: The Naval Navigation System
- NAVSTAR: Navigation
- Principally designed for military action

Landmarks

- 1978: GPS satellite launched
- 1983: Ronald Raegan: GPS for public
- 1990-91: Gulf war: GPS used for first time
- 1993: Initial operational capability: 24 satellites
- 1995: Full operational capability
- 2000: Switching off selective availability

GLONASS

The Russian initiative

- Global Navigation Satellite System
- First satellite launched: 1982
- Orbital period: 11 hrs 15 minutes
- Number of satellites: 24
- Distance of satellite from earth: 19100 km
- Civilian availability: 2007

GALILEO

The European initiative

- 30 satellites
- 14 hours orbit time
- 23,222 km away from the earth
- At least 4 satellites visible from anywhere in the world
- First two satellites have been already launched
- 3 orbital planes at an angle of 56 degrees to the equator
  Src: `http://download.esa.int/docs/Galileo_IOV_Launch/Galileo_factsheet_20111003.pdf`

GAGAN
The Indian initiative

- **G**PS **A**nd **G**eo-**A**ugmented **N**avigation system
- Regional satellite based navigation
- **I**ndian **R**egional **N**avigational **S**atellite **S**ystem
- Improves the accuracy of the GNSS receivers by providing reference signals
- Work likely to be completed by 2014

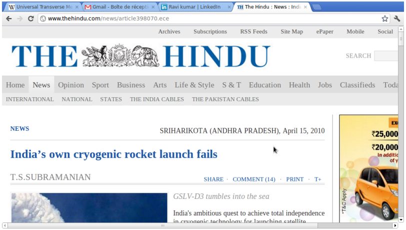

Figure: Some setbacks in the GAGAN programme

www.oosa.unvienna.org/pdf/icg/2008/expert/2-3.pdf

How does a GPS look like?

How does a GPS look like?

Three segments

- Control segment
- Space segment
- User segment

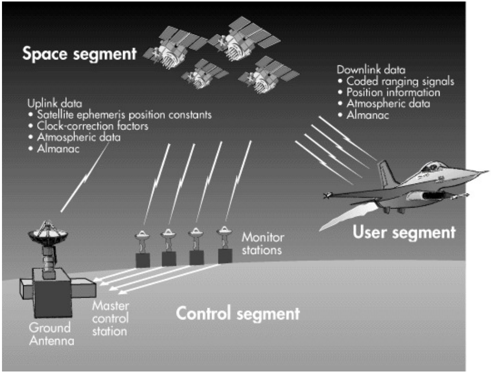

Figure: Src: http://infohost.nmt.edu/

The Space Segment

Concerns with satellites

Satellite constellation

- 24 satellites (30 on date)
- Downlinks
  - coded ranging signals
  - position information
  - atmospheric information
  - almanac
- Maintains accurate time using on-board Cesium and Rubidium clocks
- Runs small processes on the on-board computer

Figure: Src: www.kowoma.de

The Control Segment

Satellite control and information uplink

- GPS Master control station at Colorado Springs, Colorado
- Master control station is helped by different monitoring stations distributed across the world
- These stations track the satellites and keep on regularly updating the information on the satellites. Several updates per day.

Control Stations

- Colorado Springs (MCS)
- Ascension
- Diego Garcia
- Hawaii
- Kwajalein

What they do?

- Satellite orbit and clock performance parameters
- Heath status
- Requirement of repositioning

User segment

- Real time positioning with receiving units
- Hand-held receivers or antenna based receivers
- Calculation of position using “Resection”

GPS Signals
Choices and contents

- Atmosphere - refraction
- Ionosphere: biggest source of disturbance
- GPS Signals need minimum disturbance and minimum refraction

Signals

- **Basic frequency** (f_0): 10.23 MHz
- **L1 frequency** ($154 f_0$): 1575.42 MHz
- **L2 frequency** ($120 f_0$): 1227.60 MHz
- **C/A code** ($\frac{f_0}{10}$): 1.023 MHz
- **P(Y) code** (f_0): 10.23 MHz
- Another civilian code L5 is going to be there soon

L1 and L2 frequencies are known as carriers. The C/A and P(Y) carry the navigation message from the satellite to the reciever. The codes are modulated onto the carriers. In advanced receivers, L1 and L2 frequencies can be used to eliminate the iospheric errors from the data.

GPS Signals

Receivers and Policies

- Basic receiver: L1 carrier and C/A code (for general public)
- Dual frequency receiver: L1 and L2 carriers and C/A code (for advanced researchers)
- Military receivers: L1 + L2 carriers and C/A + P(Y) codes (for military purposes)

GPS and other GNSS policies are based on various scenarios at international levels

GPS Signals

Information available to the reciever

- Pseudoranges (?)
  - C/A code available to the civilian user
  - P1 and P2 codes available to the military user
- Carrier phases
  - L1, L2 mainly used for geodesy and surveying
- Range-rate: Doppler

Information is available in receiver dependent and receiver independent files (RINEX). Advanced receivers come with additional software to download information from them to the computer in proprietary and RINEX (Reciever INdependent EXchange) formats.

`ftp://ftp.unibe.ch/aiub/rinex/rinex300.pdf`

Range information
Code Pseudorange

- True Range (ρ): Geometric distance between the satellite and the receiver
- Time taken to travel = Receiver time at the time of reception - Satellite time at time of emission of the signal

$$\Delta t = t_r - t^s$$

$$t_r = t_{rGPS} + \delta_r, \qquad t^s = t^{sGPS} + \delta^s$$

- Pseudorange (P): ρ distorted by atmospheric disturbances, clock biases and delays
- Thus, true range is given by,

$$\rho = P - c \cdot (\delta_r - \delta^s) = P - c \cdot \delta_r^s$$

Mathematical model - code based

Receiver watching multiple satellites

Receiver coordinate: (X, Y, Z) and satellite coordinates for m satellites: (X^i, Y^i, Z^i)

Initial estimate of receiver position: (X_0, Y_0, Z_0)

True range

$$\rho^j(t) = \sqrt{(X^j(t) - X)^2 + (Y^j(t) - Y)^2 + (Z^j(t) - Z)^2} = f^j(X, Y, Z)$$

The “adjusted coordinates”: $X = X_0 + \Delta X$, $Y = Y_0 + \Delta Y$, $Z = Z_0 + \Delta Z$

Thus $f^j(X, Y, Z) =$

$$f^j(X_0, Y_0, Z_0) + \frac{\partial f^j(X_0, Y_0, Z_0)}{\partial X_0}\Delta X + \frac{\partial f^j(X_0, Y_0, Z_0)}{\partial Y_0}\Delta Y + \frac{\partial f^j(X_0, Y_0, Z_0)}{\partial Z_0}\Delta Z$$

The pseudo-range equations for the m satellites

$$P^j(t) = \rho^j(t) + c[\delta_r(t) - \delta^j(t)]$$

Therefore we have:

$$P^j(t) = f^j(X_0, Y_0, Z_0) + \frac{\partial f^j(X_0, Y_0, Z_0)}{\partial X_0}\Delta X + \frac{\partial f^j(X_0, Y_0, Z_0)}{\partial Y_0}\Delta Y + \frac{\partial f^j(X_0, Y_0, Z_0)}{\partial Z_0}\Delta Z + c[\delta_r(t) - \delta^j(t)]$$

Take all unknowns to one side

$$P^j(t) - f^j(X_0, Y_0, Z_0) + c\delta^j(t) = \frac{\partial f^j(X_0, Y_0, Z_0)}{\partial X_0}\Delta X + \frac{\partial f^j(X_0, Y_0, Z_0)}{\partial Y_0}\Delta Y + \frac{\partial f^j(X_0, Y_0, Z_0)}{\partial Z_0}\Delta Z + c \cdot \delta_r(t)$$

Put,

$$L = \begin{bmatrix} P^1(t) - f^1(X_0, Y_0, Z_0) + c\delta^1(t) \\ P^2(t) - f^2(X_0, Y_0, Z_0) + c\delta^2(t) \\ \dots \\ P^m(t) - f^m(X_0, Y_0, Z_0) + c\delta^m(t) \end{bmatrix}, \Delta X = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \\ c\delta_r(t) \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{\partial f^1(X_0,Y_0,Z_0)}{\partial X_0} & \frac{\partial f^1(X_0,Y_0,Z_0)}{\partial Y_0} & \frac{\partial f^1(X_0,Y_0,Z_0)}{\partial Z_0} & 1 \\ \frac{\partial f^2(X_0,Y_0,Z_0)}{\partial X_0} & \frac{\partial f^2(X_0,Y_0,Z_0)}{\partial Y_0} & \frac{\partial f^2(X_0,Y_0,Z_0)}{\partial Z_0} & 1 \\ \dots & \vdots & \vdots & \dots \\ \frac{\partial f^m(X_0,Y_0,Z_0)}{\partial X_0} & \frac{\partial f^m(X_0,Y_0,Z_0)}{\partial Y_0} & \frac{\partial f^m(X_0,Y_0,Z_0)}{\partial Z_0} & 1 \end{bmatrix}, V = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{bmatrix}$$

Use the arrangement:

$$V = A \cdot \Delta X - \Delta L$$

To obtain,

$$\Delta X = (A^T A)^{-1} A^T \Delta L$$

Mathematical model - code based

Generalized model

Adapted from Leick (2004),

$$P^j_{L1,CA}(t) = \rho^j(t) + c\Delta\delta^j_r(t) + I^j_{L1,CA} + r^j + T^j + d^j_{L1,CA} + \varepsilon_{L1,CA}$$

$$P^j_{L1}(t) = \rho^j(t) + c\Delta\delta^j_r(t) + I^j_{L1} + r^j + T^j + d^j_{L1} + \varepsilon_{L1}$$

$$P^j_{L2}(t) = \rho^j(t) + c\Delta\delta^j_r(t) + I^j_{L2} + r^j + T^j + d^j_{L2} + \varepsilon_{L2}$$

- I^j: Ionospheric error
- r^j: Relativistic error
- T^j: Tropospheric error
- d^j: Hardware delay - receiver, satellite, multipath (combination)
- ε: Receiver noise

Mathematical model - carrier phase based

Single receiver watching multiple satellites

The basic ranging equation is given as:

$$\lambda\phi^j(t) = \rho^j(t) + c(\delta_r(t) - \delta^j(t)) + \lambda N^j$$

Using Taylor's series, we have

$$\lambda\phi^j(t) = f^j(\mathbf{X}_0) + \left.\frac{\partial f^j}{\partial \mathbf{X}}\right|_{\mathbf{X}=\mathbf{X}_0} \Delta\mathbf{X} + c(\delta_r(t) - \delta^j(t)) + \lambda N^j$$

All unknowns to one side,

$$\lambda\phi^j(t) - f^j(\mathbf{X}_0) + c\delta^j(t) = \left.\frac{\partial f^j}{\partial \mathbf{X}}\right|_{\mathbf{X}=\mathbf{X}_0} \Delta\mathbf{X} + c\delta_r(t) + \lambda N^j$$

Number of unknowns: (a) **X**: 3, (b) $\delta_r(t)$: 1 and (c) N^j: Differs for each satellite. For a single instant and 4 satellites, the number of unknowns is: 8

Mathematical model - carrier phase based

Single receiver watching multiple satellites

Epoch-ID	ΔX	Bias	Int.Amb.	No.Var.	No.Eqns
1	3	+1	4	8	4
2	3	+1	4	9	8
3	3	+1	4	10	12
4	3	+1	4	11	16
5	3	+1	4	12	20

Table: Table showing the number of unknowns and equations as the epochs progress

Home Work: Compute the matrix notation for the phase-based model for at least 3-epochs and 4 satellites.

Mathematical model - Advanced carrier phase based

Single receiver watching multiple satellites

Suppose that f_1 and λ_1 denote the frequency and wavelength of L1 carrier and f_2 and λ_2 denote the frequency of L2 carrrier. The carrier phase model is given by

$$\phi^j_{L1}(t) = \frac{f_1}{c}\rho^j(t)+\frac{c}{\lambda_1}\Delta\delta^j_r(t)-I^j_{L1}(t)+\frac{f_1}{c}T^j(t)+R^j+d^j_{r,L1}(t)+N^j_{L1}+w_{L1}+\varepsilon_{r,L1}$$

$$\phi^j_{L2}(t) = \frac{f_2}{c}\rho^j(t)+\frac{c}{\lambda_2}\Delta\delta^j_r(t)-I^j_{L2}(t)+\frac{f_2}{c}T^j(t)+R^j+d^j_{L2}(t)+N^j_{L2}+w_{L2}+\varepsilon_{r,L2}$$

- T^j: Tropospheric error
- $d^j_{r,L1}$: Hardware error
- $I^j_{L1/L2}$: Ionospheric error
- R^j: Relativity error
- $N_{L1/L2}$: Integer ambiguity
- $w_{L1/L2}$: Phase winding error
- ε_r: Receiver noise

Advanced methods of positioning

- Differential and Relative Positioning
- Rapid static
- Stop and Go
- Real time kinematic (RTK)

RTK is difficult to implement in India for civilians owing to the defence restrictions.

Sources of errors

- Receiver clock bias
- Hardware error
- Receiver Noise
- Phase - wind up
- Multipath error
- Ionospheric error
- Troposheric error
- Antenna shift
- Satellite drift

Status of GPS Surveying

- 10-12 yrs earlier
  - For specialists
  - National and continental networks
  - Importance of accuracy
  - Absence of good GUI
- 5 yrs ago
  - Only post processing
  - Better and smaller receivers
  - Improvement in positioning methods
  - Better software

Today

- Demand for higher accuracies and better models
- Speed, ease-of-use, advanced features are key requirements
- Surveying to centimeter accuracy
- Automated and user friendly software
- Code measurements to cm level, phase measurements to mm level
- Better atmospheric models
- On its way to becoming a standard surveying tool

Outline

Applications of GPS

Civil Applications

- Aviation
- Agriculture
- Disaster Mitigation
- Location based services
- Mobile
- Rail
- Road Navigation
- Shipment tracking
- Surveying and Mapping
- Time based systems

Military Applications

- Soldier guidance for unfamiliar terrains
- Guided missiles

GPS
Partial list of GPS makes

- GARMIN
- Leica
- LOWRANCE
- TomTom
- Trimble
- Mio
- Navigon

Software for GPS

Commercial

- Leica Geooffice
- Leica GNSS Spider
- Trimble EZ Office
- Garmin Communicator Plugin

University Based

- GAMIT
- BERNESE

Outline

Resources

- `www.gps.gov`
- `www.kowoma.de`
- `www.gmat.unsw.edu.au/snap/gps/about_gps.htm`
- Book: Alfred Leick – GPS Satellite Surveying
- Book: Jan Van Sickle – GPS for Land Surveyors
- Book: Hofmann-Wellehof et al – Global Positioning System: Theory and Practice
- Book: Joel McNamara – GPS For Dummies

Outline

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Thank you

For the hearing and the bearing!