Course: CS 836

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1. Give the computational formulation of rough set approximations:

a). Give a definition of information table.

$T=(OB,AT,\{V_a|a\in AT\},\{I_a|a\in AT\})$

where OB is a finite non-empty set of objects which is the universe. AT is a finite non-empty set of attributes. V_a is the domain of attributes and I_a is an information function from OB to V_a which means each object is mapped to exactly one value from objects to attributes.

b). Define an equivalence relation in an information table.

In an information table, one can construct an equivalence relation by using a subset of attributes. For a subset of attributes $A\subseteq AT$, we define an equivalence relation E_A:

$xE_Ay \Longleftrightarrow \forall a \in A\ (I_a(x) = I_a(y))$.

c). Define the partition induced by the equivalence relation.

Consider the equivalence relation E_A induced by a subset of attributes $A \subseteq AT$. The equivalence relation E_A induces a partition OB/E_A of OB for instance a family of nonempty and pair-wise disjoint subsets whose union is the universe. For an object $x \in OB$, its equivalence class is given by: $[x]_A=\{y\in OB|xE_Ay\}$.

d). Define the atomic Boolean algebra by using the partition as the set of atoms.

One can construct an atomic sub-Boolean algebra $B(U/E_A)$ of 2^{OB} with OB/E_A as the set of atoms: $B(OB/E_A)=\{ \cup F|F\subseteq OB/E_A\}$. Each element in $B(OB/E_A)$ is the union of a family of equivalence classes. The Boolean algebra $B(OB/E_A)$ contains the empty set $\emptyset$, the whole set OB, and is closed under set complement, intersection, and union.

e). Define the rough set approximations.

For a subset $X\subseteq OB$, the lower and upper approximations of X are defined by:

Granule-based definition:

$\underline{aprA}(X)=\cup\{[x]_A \in OB/E_A \mid [x]_A \in X\}$, $\overline{aprA}(X)=\cup\{[x]_A \in OB/E_A \mid [x]_A \cap X \neq \emptyset\}$

Element-based definition:

$\underline{aprA}(X)=\{x \in OB \mid [x]_A \subseteq X\}$, $\overline{aprA}(X)=\{[x \in OB \mid [x]_A \cap X \neq \emptyset\}$

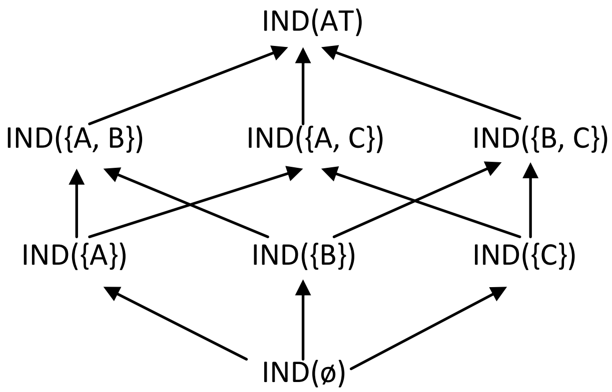
2. Consider the following information table:

Object	A	B	C
1	a_1	b_1	c_1
2	a_1	b_1	c_2
3	a_2	b_2	c_1
4	a_2	b_3	c_1
5	a_2	b_3	c_1
6	a_2	b_1	c_1
7	a_2	b_3	c_2
8	a_1	b_1	c_2

a). Construct the family of all equivalence relations defined by subsets of attributes.

IND(∅), IND({A}), IND({B}), IND({C}), IND({A,B}), IND({A,C}), IND({B,C}), IND(AT)

b). Draw a diagram to show the connections between those equivalence relations.



IND(AT) includes IND({A, B}), IND({A, C}), IND({B, C})

IND({A, B}) includes IND({A}) and IND({B})

IND({A, C}) includes IND({A}) and IND({C})

IND({B, C}) includes IND({B}) and IND({C})

IND({A}), IND({B}), IND({C}) include IND(∅)

c). Construct an equivalence relation on the power set of the set of attributes: two subsets of attributes are equivalent if and only if they produce the same equivalence relation on U. Give the partition of the power set of the set of attributes.

OB/ IND(ø)={{1,2,3,4,5,6,7,8}}

OB/ IND({A})={{1,2,8},{3,4,5,6,7}}

OB/ IND({B})={{1,2,6,8},{3},{4,5,7}}

OB/ IND({C})={{1,3,4,5,6},{2,7,8}}

OB/ IND({A, B})={{1,2,8},{3},{6},{4,5,7}}

OB/ IND({A, C})={{1},{2,8},{3,4,5,6},{7}}

OB/ IND({B, C})={{1,6},{2,8},{3},{4,5},{7}}

OB/ IND(AT)={{1},{2,8},{3},{4,5},{6},{7}}

From above, we can get none of them produce the same equivalence relation on U so there do not exist the equivalence relation on attributes. So the partition of the power set of the set of attributes is {{Ø},{A},{B},{C},{A, B},{A, C},{B, C},{A,B,C}}

3. Show that the conceptual formulation and the computational formulation are equivalent.

In an information table T, for a subset X⊆OB, the lower and upper approximations of X are defined by the following pair of definable sets, $\overline{aprA}$(X) is the least definable set in DEF_A(T) containing X and $\underline{aprA}$(X) is the greatest definable set in DEF_A(T) contained by X. We can easily get $\underline{aprA}$(X) ⊆ X ⊆ $\overline{aprA}$(X). From the fact that $[x]_A$ is a definable set and $\underline{aprA}$(X) is the greatest definable set contained in X, we can also conclude that $[x]_A \subseteq X \Longleftrightarrow [x]_A \subseteq \underline{aprA}(X)$, so then we combine these two and get $\underline{aprA}(X)=\cup\{[x]_A \in OB/E_A \mid [x]_A \in X\}$ and $\overline{aprA}(X)=\cup\{[x]_A \in OB/E_A \mid [x]_A \cap X \neq \emptyset\}$. For $\underline{aprA}(X)=\cup\{[x]_A \in OB/E_A \mid [x]_A \in X\}$ which has the same meaning with the greatest definable set in DEF_A(T) contained by X. And for $\overline{aprA}(X)=\cup\{[x]_A \in OB/E_A \mid [x]_A \cap X \neq \emptyset\}$, where $[x]_A \cap X \neq \emptyset$ is equal to $[x]_A \cap \overline{aprA}(X) \neq \emptyset$, it has the same meaning with the least definable set in DEF_A(T) containing X. Since the conceptual formulation and the computational formulation have the same meaning, they are equivalent.