1. A medical test for a certain disease has a 98% probability of giving a positive result if the patient actually has the disease, and 10% probability of giving a false positive. It is estimated that 0.5% of the population actually has this disease. If a patient tests positive, what is the probability they actually have the disease?

```python
# Given probabilities
P_A = 0.005
P_B_given_A = 0.98
P_B_given_not_A = 0.10

# Calculate complementary probability
P_not_A = 1 - P_A

# Calculate P(B)
P_B = (P_B_given_A * P_A) + (P_B_given_not_A * P_not_A)

# Calculate P(A|B)
P_A_given_B = (P_B_given_A * P_A) / P_B

print("The probability that a patient actually has the disease given they tested positive:", P_A_given_B)
```

```
The probability that a patient actually has the disease given they tested positive: 0.046934865900383135
```

2. An urn contains 3 red balls and 5 black balls. A ball is drawn at random, its color observed, and then replaced in the urn. A second ball is then drawn and observed. If the second ball drawn was red, what is the probability that the first ball was also red?

```python
[2] # Given number of red and black balls
red_balls = 3
black_balls = 5

# Total number of balls in the urn
total_balls = red_balls + black_balls

# Probability of drawing a red ball
P_red = red_balls / total_balls

# Probability of drawing a black ball
P_black = black_balls / total_balls

# Probability of drawing a red ball as the second draw
P_red_second = P_red

# Probability of drawing a red ball as the first and second draw
P_red_first_and_second = P_red * P_red_second

# Probability of the second ball being red
P_second_red = P_red_first_and_second + (P_black * P_red)

# Calculate P(A|B)
P_A_given_B = P_red_first_and_second / P_second_red

print("The probability that the first ball was red given the second ball was red:", P_A_given_B)
```

```
The probability that the first ball was red given the second ball was red: 0.375
```

3. A factory produces LCD monitors, of which 1% are defective. The factory has two quality control tests, test A and test B. Test A correctly identifies defective monitors 95% of the time, and never incorrectly flags a good monitor as defective. Test B correctly identifies defective monitors 80% of the time, and incorrectly flags 5% of good monitors as defective. If a monitor fails both tests, what is the probability it is actually defective?

```python
# Given probabilities
P_D = 0.01
P_not_A_given_D = 0.05
P_not_B_given_D = 0.20

# Calculate complementary probabilities
P_not_D = 1 - P_D

P_A_given_D = 1 - P_not_A_given_D
P_B_given_D = 1 - P_not_B_given_D

# Calculate the probability that both Test A and Test B do not indicate the monitor is defective
P_not_A_and_not_B_given_D = P_not_A_given_D * P_not_B_given_D

# Calculate the denominator of Bayes' theorem
P_A_and_B_given_D = P_A_given_D * P_B_given_D

# Calculate the numerator of Bayes' theorem
numerator = P_D * P_not_A_given_D * P_not_B_given_D

# Calculate P(D|Ā, B̄) using Bayes' theorem
P_D_given_not_A_and_not_B = numerator / P_A_and_B_given_D

print("The probability that the monitor is actually defective given it fails both Test A and Test B:",
      P_D_given_not_A_and_not_B)
```

```
The probability that the monitor is actually defective given it fails both Test A and Test B: 0.0001315789473684210
```

4. A weather forecaster predicts rain tomorrow with 80% probability. Historically, their rain predictions have been correct 70% of the time. What is the actual probability it will rain tomorrow?

```python
# Given probabilities
P_R = 0.8
P_P_given_R = 0.7
P_P_given_not_R = 0.3
P_not_R = 0.2

# Calculate P(P) using the law of total probability
P_P = P_P_given_R * P_R + P_P_given_not_R * P_not_R

# Calculate P(R|P) using Bayes' theorem
P_R_given_P = (P_P_given_R * P_R) / P_P

print("The actual probability that it will rain tomorrow given the forecaster's prediction:",
      P_R_given_P)
```

```
The actual probability that it will rain tomorrow given the forecaster's prediction: 0.903225806451613
```

5. A biased coin lands on heads 60% of the time. The coin is flipped twice, and the first flip lands on heads. What is the probability the second flip will also land on heads?

```
[5] # Given probability that the first flip lands on heads
    P_H1 = 0.6

    # Probability that the second flip lands on heads (assuming the same bias)
    P_H2 = 0.6

    # Calculate the probability that both flips land on heads
    P_H1_and_H2 = P_H1 * P_H2

    # Calculate P(H2|H1) using conditional probability formula
    P_H2_given_H1 = P_H1_and_H2 / P_H1

    print("The probability that the second flip will also land on heads given the first flip landed on heads:",
          P_H2_given_H1)
```

```
The probability that the second flip will also land on heads given the first flip landed on heads: 0.6
```