The Distance of a Storm Based on Thunder and Lightning

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Contents

1 Introduction

It is often asserted that the distance of a storm (in km) is the time between thunder and lightning divided by 3 [2]. I never did the maths on this and assumed it was correct but only now have I decided to actually figure that out.

2 Assumptions

1. The speed of light is 299 792 458 $\mathrm{m\,s^{-1}}$
2. The speed of sound is 343 $\mathrm{m\,s^{-1}}$
3. 'Storm clouds' (cumulonimbus clouds) exist between 335 m and 1981 m [1].

3 Solution

The problem can be represented by Figure 1, where:

h is the height of the cloud (m)

d is the distance of the cloud when the lightning strike occurs (m)

Figure 1: A diagram representation of the problem.

Therefore, the distance travelled by both the light and sound is:

$$\sqrt{h^2 + d^2} = \sqrt{[335, 1981]^2 + d^2} \tag{1}$$
$$= \sqrt{[112225, 3924361] + d^2} \tag{2}$$

Which would take the following times for light (t_l) and sound (t_s) to travel:

$$t_l = \frac{\sqrt{[112225, 3924361] + d^2}}{299792458} \tag{3}$$
$$t_s = \frac{\sqrt{[112225, 3924361] + d^2}}{343} \tag{4}$$

The difference of which is:

$$t_s - t_l = \frac{\sqrt{[112225, 3924361] + d^2}}{343} - \frac{\sqrt{[112225, 3924361] + d^2}}{299792458} \tag{5}$$
$$= \frac{(299792458 - 343)\sqrt{[112225, 3924361] + d^2}}{102828813094} \tag{6}$$
$$= \frac{299792115\sqrt{[112225, 3924361] + d^2}}{102828813094} \tag{7}$$

As the time difference is known (Δt), we can find d in terms of Δt:

$$\Delta t = \frac{299792115\sqrt{[112225, 3924361] + d^2}}{102828813094} \tag{8}$$

$$102828813094 \cdot \Delta t = 299792115\sqrt{[112225, 3924361] + d^2} \tag{9}$$

$$\frac{14689830442}{42827445} \cdot \Delta t = \sqrt{[112225, 3924361] + d^2} \tag{10}$$

$$\frac{215791118414709915364}{1834190045228025} \cdot (\Delta t)^2 = [112225, 3924361] + d^2 \tag{11}$$

Which leaves us with the following quadratic:

$$d^2 + [112225, 3924361] - \frac{215791118414709915364}{1834190045228025} \cdot (\Delta t)^2 = 0 \tag{12}$$

As $\frac{215791118414709915364}{1834190045228025} \approx 117649$, we can simplify to:

$$d^2 + [112225, 3924361] - 117649 \cdot (\Delta t)^2 = 0 \tag{13}$$

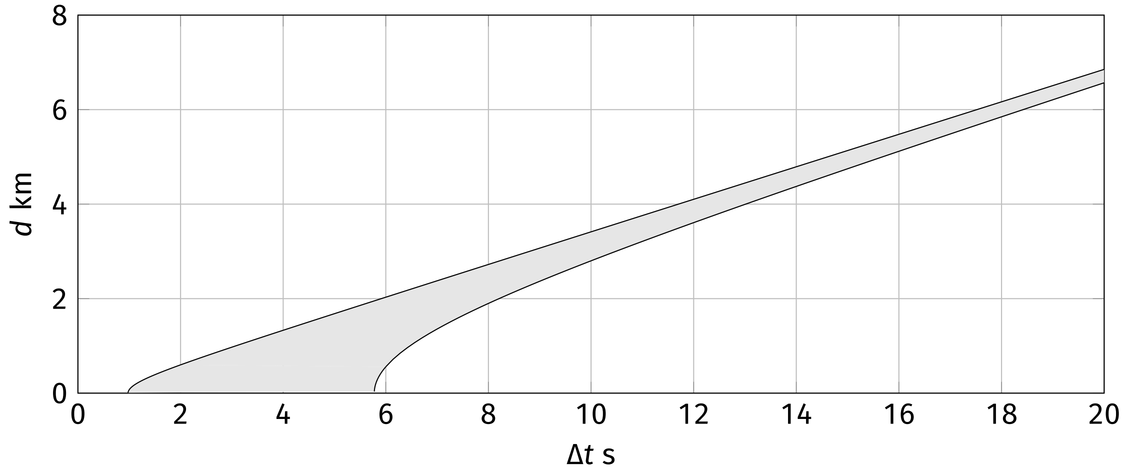
To understand this, we can rearrange the equation to be:

$$d^2 = 117649 \cdot (\Delta t)^2 - [112225, 3924361] \tag{14}$$

$$d = \sqrt{117649 \cdot (\Delta t)^2 - [112225, 3924361]}^1 \tag{15}$$

Which can then be plotted as:

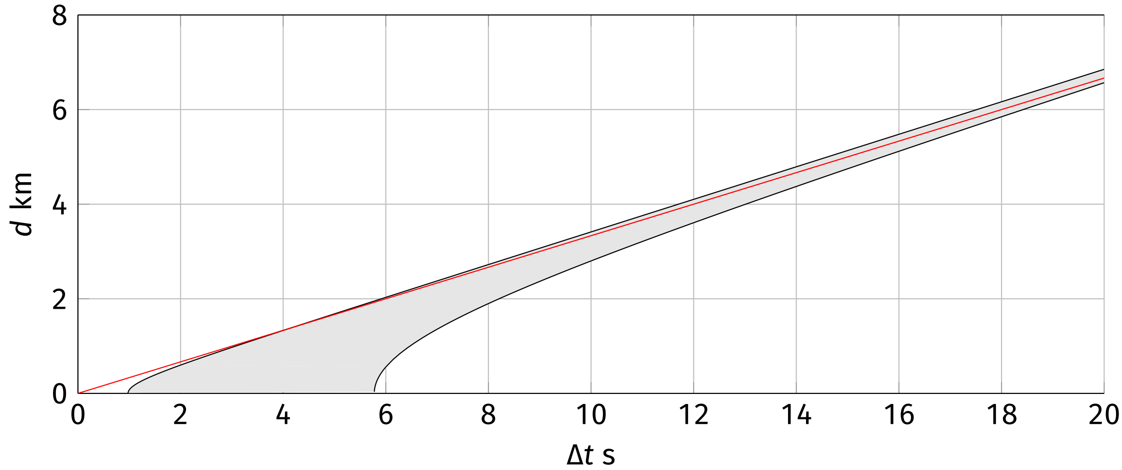
$$\sqrt{117649x^2 - 112225} \geq y \geq \sqrt{117649x^2 - 3924361}$$



[1](15): As distance cannot be negative.

4 The Solution vs. The Assertion

Now we have a ‘solution’ to this problem, we can test the assertion previously mentioned. To do this, we can overlay a graph of the assertion on the graph of our solution:



References

[1] Met Office. *Cumulonimbus clouds*. URL: `https://www.metoffice.gov.uk/weather/learn-about/weather/types-of-weather/clouds/low-level-clouds/cumulonimbus` (visited on 06/20/2023).

[2] WikiHow editors. *How to Calculate the Distance from Lightning*. 2023. URL: `https://www.wikihow.com/Calculate-the-Distance-from-Lightning` (visited on 06/20/2023).