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T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**A**	F4 ________________

2019
MCM/ICM

The Realistic Nature Role of Dragons

Summary

As sci-fi works gain popularity among the general audience, people are more concerned about the realistic ecological rationality of fictional creatures in science fiction. In this paper, we compare the dragon to the existing species Komodo dragon to study the characteristics, behaviors and diet of dragons. Besides, we use four specific islands located in Indonesia to simulate the living environment of dragons, so as to analyze the possible impact and requirements of dragons on ecological environment.

First of all, we establish the Dragon Invasion differential equations to analyze the ecological impact and requirements of the dragons. By comparing the results of the the predator-prey equations before and after dragon invasion, we obtain the impact of the invasion on the ecological environment. We find that after the invasion, the number of consumers in the environment decreases rapidly in a short period. Meanwhile, the cycle of population fluctuation is shortened from 7 years to 5 years. Therefore, we conclude that the survival of dragons relies on the ecological environment with a high species abundance.

Considering the energy expenditures and caloric intake requirements of the dragons, we build the Body Temperature Maintenance (BTM) model. We assume that the dragons are warm-blooded animal, and they burn calories through radiation, convection, breathing both air and fire, and mechanical movement. Based on the production and dissipation of heat balance, we determine that a daily intake of 27,486 $kcal$ is required for a dragon weighing 40 kg.

To determine the required area and community for the three dragons, we construct the Improved Ant Colony Optimization (IACO) model. By referring to the literature, we determine the distribution of Rusa deer on the four islands and simulate the hunting route of dragons with ACO algorithm. Then we figure out the minimum area of activity needed for the three dragons is 224.64 km^2, in which the number of preys could meet their needs.

Remarkably, we analyze the impact of climate change on the demand for resources during the dragon migration. Based on the BTM and IACO models, we reflect the change of environment between warmth, arid and arctic by varying the temperature, rainfall and species richness. We find that the caloric intake requirements of a dragon in arid, warm and arctic regions are 29,124 $kcal$, 31,184 $kcal$ and 39,726 $kcal$ per day respectively. The required living area are 165.46 km^2, 74.88 km^2, and 329.47 km^2 per year respectively.

Contents

1 Introduction

1.1 Problem Background

Whenever people surf the Internet, either the websites of news agencies or their home pages, it is not surprising to find the most frequently discussed topic regarding the fantastic three dragons in the fictional television series *Game of Throne*, which are based on the series of epic fantasy novel *A Song of Ice and Fire*. Nevertheless as science fiction works gain more and more attention among the general audience, people begin to pay more attention to the realistic ecological rationality of the fictional creatures in science fiction works and the existence forms of future creatures in science fiction works.

As sci-fi works gain popularity among the general audience, people are more concerned about the realistic ecological rationality of fictional creatures in science fiction.In addition to bringing people rich visual experience and ideological feast, the study of fictional creatures can be used in the study and learning of unknown species, which has great practical significance.Efforts to study dragons may help provide information and insight into the unknown.

1.2 Previous Research

So far, most of the common biological species on the earth have been studied in depth. At the same time, the curiosity of unknown alien species also leads human beings to make many meaningful explorations. In 1997, the discovery of a frozen corpse of a mammoth named Kirgilyakh proved the richness of tundra and forests in ancient Siberia. As a Quaternary paleomammal, mammoths have provided useful help in the study of climate change on the early Earth [1] Anthony P. Shillito and Neil S. Davies studied the dinosaur tracks in the Early Cretaceous orbits, revealing the development of microtopography and the new discovery of invertebrate cave dwelling activities [2]. Moreover, scientists have found that the study of paleontologists who can walk and fly provides new ideas for the study of vascular diseases [3].

1.3 Our Work

In the main report, we are required to analysis the following questions:

- The ecological impact and requirements of the dragons.
- Energy expenditures and caloric intake requirements of the dragons.
- Required area and community to support the dragons.
- The impact of climate change on dragons during migration.

2 Analysis of Overall and Key Points

2.1 Overall Analysis

As fictional creature, it is difficult to study the growth of dragons in the real natural environment. Therefore, in the process of analysis of dragons, we need to make many hypotheses that are in line with the reality of biological characteristics. According to the relevant novels and movies and TV dramas, we know that the dragons can continue to grow throughout their life. So how to determine the growth needs of dragons is one of the key points we shall consider. In addition to the commonalities of living creatures, some of the dragon's special skills, such as breathing fire, having hard armor that are deserve to be under our consideration.

2.2 Key Points Analysis

(1)Impact of Dragon on Environment

When analyzing the impact of dragons on the environment, we can mainly consider the strong predatory capacity and appetite of dragons.When analyzing the dragon's diet, we need to fully consider the dragon's eating habits, such as what prey to eat and how much prey to eat each time to meet the body's energy needs. In addition, dietary intake meets the dragon's consumption of various life activities.

(2)Living Area of Dragon

The dragon in the living environment will continue to expand their range of activities to increase their own prey. When prey range is large at the time of capture, the dragon's range changes accordingly. Not only do we need to determine the exact distribution of the prey that the dragon preys on, but we can also take into account the dragon's own mobility and more interaction with its environment.

(3)Impact of Climate Change on Dragon

In different climates, the food resources in dragons' living environment and their appetite may be different. On the other hand, climate may also affect the living area of dragons.

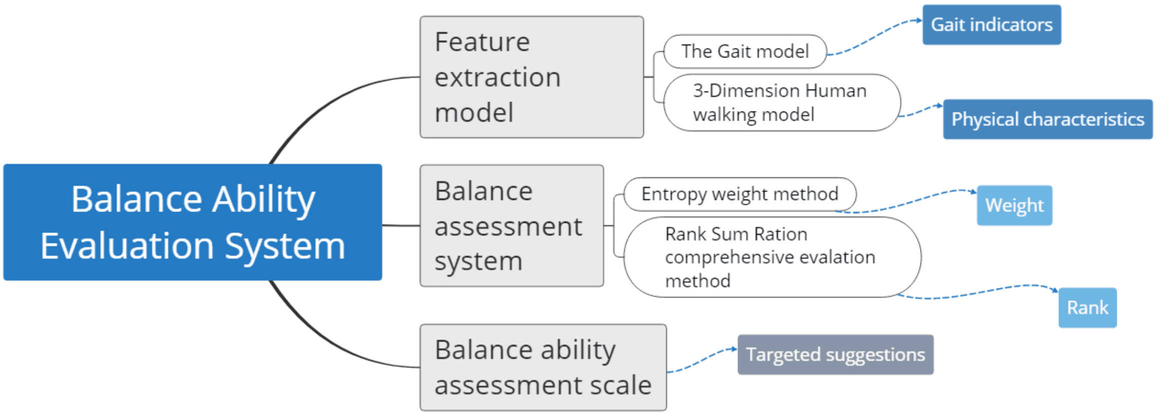


Figure 1: Modeling Ideas Mind Maps

3 Generall Assumptions

We make the following assumptions to complete our model through this paper. The supplement assumptions will be addressed later with detailed models.

- Assumption 1: We compare dragons to Komodo dragons, and use the individual characteristics of Komodo dragons to analyze dragons.Because Komodo dragons are the largest living lizards and are egg-laying animals. The Komodo dragon is also similar to the dragon in terms of activity ability, diet and temperament.
- Assumption 2: Dragons are assumed to be at the top of the food chain, regardless of their environmental conditions. Because the dragon has indestructible armor and breath fire. No known animal in nature can compete with dragons.
- Assumption 3: During the long life span of the dragons, they can only die of aging or lack of food and they can resist tremendous trauma.
- Assumption 4:In order to simpify our analysis, we do not take account of the influence of human activities on dragons in this paper.

4 Symbols and Definitions

In this paper,we use some symbols for constructing the model as follows, other symbol instructions will be given in the text.

Table 1: Symbols and Definitions

Symbols	Meanings
$X_i, i = 1, 2, 3$	Population of the i^{th} specials
Q	Calorie intake of the dragon
Q_1	Heat generated by metabolism
Q_2	Radiant heat dissipation
Q_3	Convective heat dissipation
Q_4	Heat dissipated by breathing air
Q_5	Heat dissipated by breathing fire
A_{Du}	Body surface area of the dragon
t_0	Body temperature of dragon
t_1	Temperature of the environment
h	Body length of dragon
m	Weight of dragon
M	Metabolic rate of dragon
S_{min}	Minimum living area of a dragon
S	Minimum living area of three dragons

5 Model I: Dragon Invasion Model

5.1 Modeling Ideas

When analyzing the impact and requirements of the dragon on the ecological environment, we can consider it as an invasive species. Therefore, we build the Dragon Invasion Model on the basis of the LotkaVolterra equations which is also known as the predator-prey equations[4].

In order to facilitate the analysis of the impact and demand of the dragon on the ecological environment, we consider introducing the dragon into a resource-limited environment. We only consider the two species that exist in the environment, Dracaena (a kind of grass) and Rusa deer, respectively, producers and consumers. After the invasion of dragons, the deer act as the prey for dragons (see **Figure 2**). Based on this, we can separately establish the predator-prey equations before and after the invasion of the dragon, and compare the results of the two equations to get the impact of the dragon's invasion on the environment.

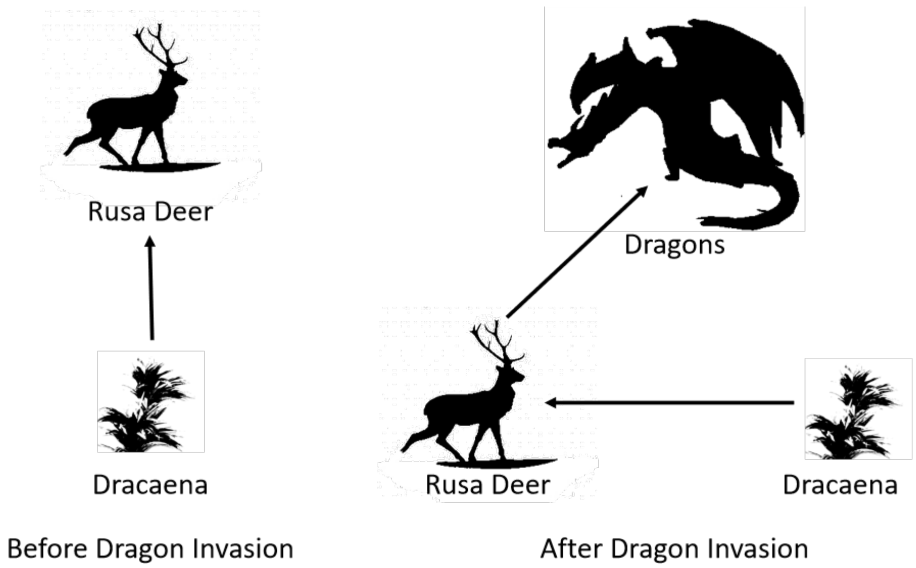


Figure 2: The food chain before and after the dragon invasion

5.2 Supplementary Assumptions

In the Dragon Invasion model, we make a number of supplementary assumptions, most of which are based on the assumptions of LotkaVolterra model.

- The food supply of the predator population depends entirely on the size of the prey population. That is, in addition to Dracaena, the deer does not eat any other plants. Similarly, the dragon has no other prey but deer.
- Dragons can normally progeny.
- There is reproductive isolation between the dragon and the deer, and no fertile offspring can be produced.
- The rate of change of population is proportional to its size.
- During the process, the environment does not change in favor of one species, and genetic adaptation is inconsequential.
- The dragon has a big enough appetite.

5.3 The Predator-Prey Equations

(1)Before dragon invasion

Before the dragon invasion, the original food chain consists of two species, the Rusa deer and Dracaena. The initial population of deer and Dracaena are x_1 and x_2 respectively. The population growth of Dracaena X_1' at time t is:

$$X_1' = r_1 x_1,$$

where r_1 is the relative growth rate when grass grows independently. Because of the consumption of deer, the growth rate of Dracaena is reduced. We use coefficient a to express the degree of reduction, which is proportional to the number of deer.

Because the predator cannot survive without the prey, the population of deer X_2' at time t is:

$$X_2{}' = -r_2 x_2,$$

where r_2 is the morality rate when the deer grow independently. Since they can graze, however, the mortality is decreased. We use coefficient b to express the degree of reduction, which is proportional to the population of grass.

Therefore, we get the system of ordinary differential equations (1):

$$\begin{cases} \frac{dX_1}{dt} = X_1 \left(r_1 - aX_2 \right) \\ \frac{dX_2}{dt} = X_2 \left(-r_2 + bX_1 \right) \end{cases} \tag{1}$$

where X_1 and X_2 are the population of Dracaena and deer, $\frac{dX_1}{dt}$ and $\frac{dX_2}{dt}$ represent the instantaneous growth rates of the two species.

(2)After dragon invasion

The depopulation of dragons after the invasion is:

$$X_3{}' = -r_3 x_3,$$

where r_3 is the mortality rate of dragon when growing independently. Since they can hunt the deer, the increase of population of dragon is represented by coefficient c, which is proportional to the amount of deer.

Combined with the equations (1), we get the equations after the dragon invasion:

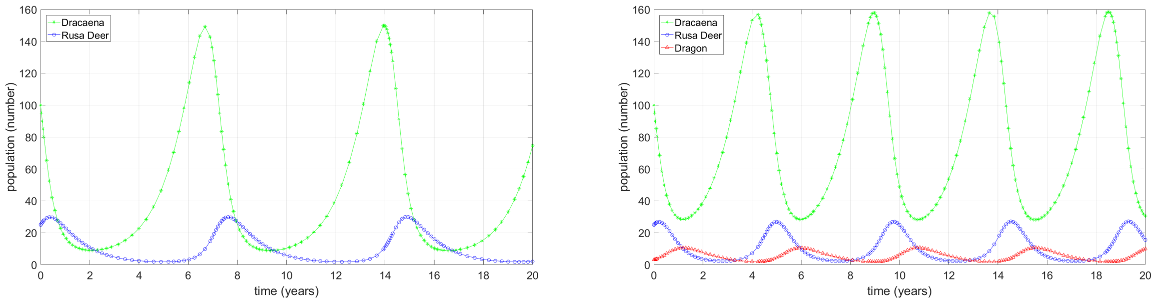
$$\begin{cases} \frac{dX_1}{dt} = X_1 \left(r_1 - aX_2 \right) \\ \frac{dX_2}{dt} = X_2 \left(-r_2 + bX_1 \right) \\ \frac{dX_3}{dt} = X_3 \left(-r_3 + cX_2 \right) \end{cases} \tag{2}$$

5.4 Model Calculation and Result Analysis

According to literature [5] , we get a growth rate of 1 for Dracaena in one year, a mortality rate of 1 for the deer when they grow independently, the predation capacity of the deer to Dracaena is 0.1, and the feeding ability of Dracaena to the deer is 0.02. Referring to the data of Komodo dragon , we conclude that the mortality rate when dragon grow independently is 1, the dragon's predation ability to the deer is 1, and the feeding ability to support the dragon is 0.05. Besides, we set the initial populations for Dracaena, Rusa deer and dragon population at 100, 25 and 3 respectively.

To solve the differential equations (1) and (2), we use the ODE45 numerical integrator in MATLAB to find the results in **Figure 3**.

Comparing **Figure 3(a)** and **Figure 3(b)**, we find that before the dragon invasion, the population fluctuation period of Dracaena and the Rusa deer is about 7 years, and the change cycle after the dragon invasion is reduced to about 5 years. Due to the



(a) Before dragon invasion (b) After dragon invasion

Figure 3: Changes in the number of species before and after the invasion of the dragon

introduction of dragons, the amplitude of the number of Dracaena decreased from 141 to 131 during a fluctuation period. At the same time, we notice that the peak of deer is smaller after dragon invasion.

It can be concluded that the addition of dragon is conductive to the renewal of the grass and the rusa deer, speeding up the evolution of both. Meanwhile, due to the dragons predation of the deer, the fluctuation of the population of Dracaena is stabilized. After the dragon invasion, the population of Dracaena remain above 30.

Ecological impact of dragons invasion: The invasion of dragons rapidly reduces the number of consumers (Rusa deer in this case) in the ecosystem, and the number of producers (Dracaena in this case) increases rapidly. In the long run, the dragon invasion accelerates the evolution of other consumers.

Ecological requirements of dragons: According to Elton (1958)[6], in order to maintain the stability of the dragon's living ecosystem, we shall appropriately extend the food chain and increase the biodiversity. At the same time, since island ecosystems are more likely to be destroyed by species invasion [7], we recommend that dragons be kept in species-rich continents rather than on islands.

6 Model II: Body Temperature Maintenance (BTM) Model

6.1 Modeling Ideas

Dragons maintain a constant body temperature through the generation and loss of heat in their daily activities. We consider that part of the energy the dragon consumes in food is converted into heat by the body. On the other hand, the dragon dissipates heat by radiation, convection, breathing both air and fire. Therefore, we establish a body heat balance equation to show the relationship between the calorie content consumed by the dragon and the increase of body weight, and then determined the weight of the dragon to obtain the corresponding calories consumed by the dragon.

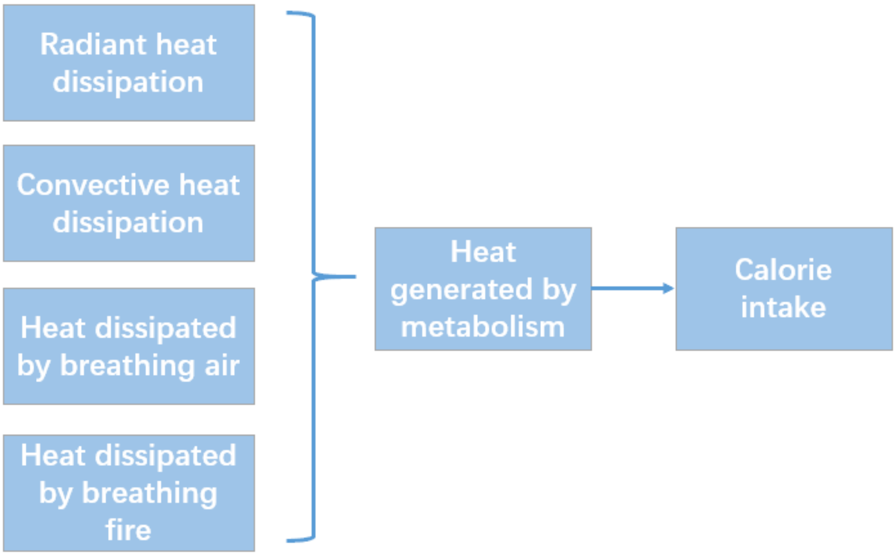


Figure 4: Modeling ideas of the BTM model

6.2 Supplementary Assumptions and Justification

- Dragons are warm-blooded animals and their body temperature remain constant at about 40 °C. Because they can fly swiftly even it is cold, however cold-blooded animals are unable to regulate their muscle when cold.
- The ratio of body weight to length of Komodo dragons and the dragons is approximately equal. Because they are similar in shape, and both have strong armor.
- The parameters in the human heat dissipation are also applicable to the dragon. Because dragons, like human beings, are warm-blooded and vertebrate, we do this to simplify the calculation.
- Dragons hunt or defend themselves in their daily activities by breathing fire.

6.3 Model Establishment

For dragons, the constant body temperature should be based on the balance of heat generated and lost [8]. The heat generated by metabolism is Q_1, radiant heat dissipation is Q_2, convective heat dissipation is Q_3, heat dissipated by breathing air and fire are Q_4 and Q_5 respectively. We take a day as the research cycle, and the dragon's body heat balance equation can be expressed as:

$$Q_1 = Q_2 + Q_3 + Q_4 + Q_5,$$

where the unit of $Q_i, i = 1, 2, ..., 5$ is $W \cdot h$.

1.Calculation of heat production Q_1

For warm-blooded animals, 50% of the energy taken from food is maintained in the form of heat, and most of the rest is re-transferred to ATP in the form of chemical energy [9]. We consider that the dragons are as efficient as humans at converting food into heat. Therefore, we have the heat generated by metabolism Q_1:

$$Q_1 = \eta Q,$$

where Q is the calories intake of the dragon, energy conversion efficiency $\eta = 0.5$.

2.Calculation of heat dissipation

The literature [10] gives the calculation formulas of radiant heat dissipation, convective heat dissipation and respiratory heat dissipation of human body, and we consider that these formulas are also applicable to dragons in the hypothesis.

(1)Radiant heat dissipation Q_2

When analyzing the radiation heat of the dragon, we consider that the dragon's skin is all exposed to the air. Since the temperature of the environment is constantly changing, we take the ambient temperature at 35°C.The radiant heat dissipation Q_2 can be denoted as:

$$Q_2 = T_1 A_{Du} \sigma[(t_0 + 273)^4 - (t_1 + 273)^4],$$

where the radiation time $T_1 = 12h$, A_{Du} is the body surface area of the dragon, Stephen Boltzmann constant σ=$5.67 \times 10^{-8} W/(m^2 \cdot K^4)$, t_0 is the body temperature. We apply the Stevenson human body surface area formula to obtain A_{Du} as follows:

$$A_{Du} = 0.0061h + 0.0128m - 0.1529,$$

where h and m are the body length and the weight of dragon. According to the ratio of body weight and length of Komodo dragons [11], we have $m = 0.29h$.

(2)Convective heat dissipation Q_3

The convective heat dissipation Q_3 can be expressed as:

$$Q_3 = T_2 A_{Du} h_c (t_0 - t_1),$$

where $T_2 = 12h$, convective heat transfer coefficient $h_c = 4.432W/(m^2 \cdot K)$ [12].

(3)Breathing air Q_4

Heat dissipated by breathing air is:

$$Q_4 = T_3[0.0173M(5.867 - P_a) + 0.0014M(t_0 - t_1)],$$

where $T_3 = 24h$, M is the metabolic rate of dragon, P_a is the partial pressure of water vapor in the air at temperature of t_1 and $P_a = 5.623KPa$ in this case.

Brody and Kleiber proposed that the metabolic rate of animals is positively correlated with weight to the power of 0.75. So we have the metabolic rate of the dragon:

$$M = 3.39m^{0.75},$$

where m is the weight of the dragon.

(4)Breathing fire Q_5

The electric eels hunt and protect themselves from threats during daily activities by discharging high voltage electricity. According to the literature [13], they can convert about 15 percent of the calories in their food into electric energy. Similarly, we consider the heat dissipated by breathing fire Q_5 can be written as:

$$Q_5{=}0.15Q.$$

6.4 Modeling Results and Analysis

Through the establishment of the body heat balance equation and the determination of the parameters, we obtain the relationship between the calories consumed by dragons and their weight:

$$Q = 65.41m + 2.61m^{0.75} - 294.14. \tag{3}$$

According to equation (3), we can figure out the calorie intake required by dragons at different body weights, as shown in **Table 2**.

Table 2: Changes in calorie intake within one year after birth

Weight of Dragon(kg)	Calorie Intake(kcal)
10	4356
15	8220
20	12078
25	15934
30	19786
35	23637
40	27486

As shown in **Table 2**, the change in calories intake as a function of body weight by the dragon from birth within one year old. As indicated in the table, the energy intake increases as their weight increases. A newborn dragon needs 4356kcal per day, when its weight gains four times, its calorie intake will increase by 5.3 times.

To further analyze the calorie intake levels of dragon, we compared the juvenile calorie intake of several other creatures at the top of the food chain with that of dragons, as shown in **Figure 5**.

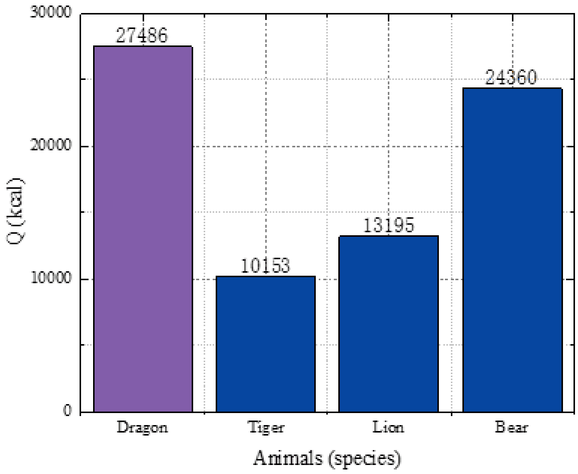


Figure 5: Comparison of calorie intake of dragon, tiger, lion and bear

As indicated in **Figure 5**, the calorie intake of dragon is 2.71 times that of the tiger and 2.08 times that of the lion. Combined with the heat dissipation mode of dragons and the above results, we can draw the following conclusions:

- Dragons consume energy through mechanical movement, radiation, convection, breathing both air and fire.
- As the weight of dragon increases, the caloric intake demanded by dragon is also increasing.
- Compared with other beasts at the top of the food chain, dragons consume more calories, which is consistent with their superior survivability in the real ecosystem.

7 Model III: Improved Ant Colony Optimization (IACO) Model

7.1 Modeling Ideas

In order to analyze the survival area required by the dragons, we consider calculating the minimum predation space of dragons.

According to the results of BTM model, we obtain the calorie intake required by the dragons in one year, so we can estimate the hunting ability of the dragons in one year. We select four islands in Indonesia namely Komodo, Rinca, Gili Motang and Nusa Kode as the feeding sites for dragons, Rusa deer as their solely prey, and then use the IACO algorithm to calculate the minimum living area of the dragons in one year.

Ant Colony Optimization (ACO) algorithm aims to search for an optimal path in a graph, based on the behavior of ants seeking a path between their colony and a source of food. [14] In the IACO model, we compare the process of the dragon hunting Rusa deer to the path selection process of the ant colony. To survive, the dragons won't stop hunting Rusa deer until their energy needs are met. Thus, we can find out the shortest route that dragon in the process of hunting and determine their minimum living area.

7.2 Supplementary Assumptions

- The dragons can fly across the straits and hunt on each island.
- Within a year, the deer population is reduced only by the predation of dragons, that is, we ignore deer breeding and natural death in a year.
- The dragon will constantly move to new places for hunting when there is no more deer for them to prey.
- The three dragons do not affect each other.

7.3 Model Calculation and Results Analysis

According to the BTM model, we suppose each dragon needs to hunt 100 Rusa deer per year for its survival. Literature [14] provides the Rusa deer density of 10 sites on the four islands.(**see Figure 6**).

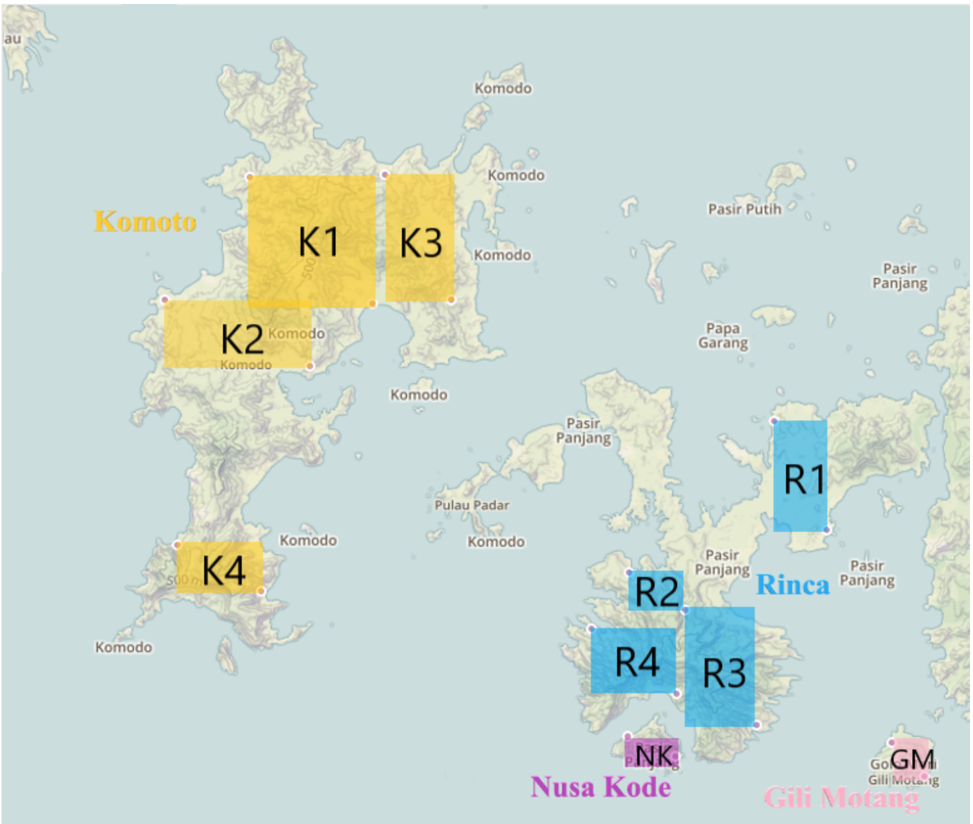


Figure 6: The sites studied in literature [14]

On the basis, we calculate the population of Rusa deer on the four islands, as shown in **Table 3**.

Table 3: The number of Javan rusa at various experimental sites

Island (km^2)	Site	Number
Komodo	Loh Liang (K1)	291
	Loh Lawi (K2)	82
	Loh Sebita (K3)	133
	Loh Wau (K4)	151
Rinca	Loh Buaya (R1)	42
	Loh Baru (R2)	23
	Loh Tongker (R3)	81
	Loh Dasami (R4)	53
Gili Motang	Gili Motang (GM)	1
Nusa Kode	Nusa Kode (NK)	1

(1) Algorithm steps

Step 1: Simulate the latitude and longitude positions of Rusa deer in each area by using MATLAB, the white dots in **Figure 6** represent the distribution of deer.

Step 2: Calculate the distance between each deer to form the distance matrix.

Step 3: Initialize the parameters,pheromone correlation factors and imortance factors of heuristic functions of the IACO algorithm.

Step 4: Iteratively finds the best path, constructs the solution space according to the ant's transition probability, and updates the pheromone concentration between each deer in real time according to the pheromone update formula after each iteration. We set the number of ant colonies to 75 and cycle iterations 200 times to record the final optimal path and length.

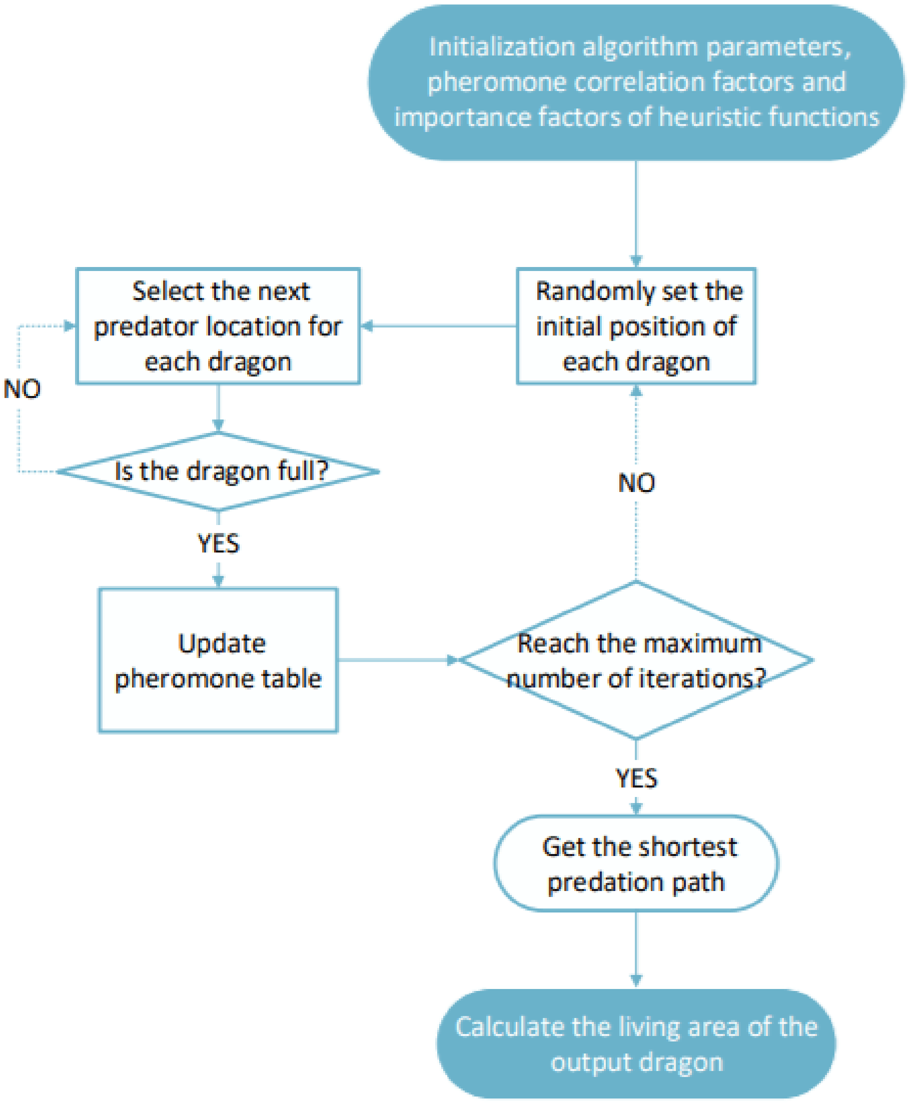


Figure 7: The flow chart of IACO algorithm

(2)Modeling results and analysis

We use MATLAB to calculate the shortest feeding route of the dragon, as shown in Figure 8.

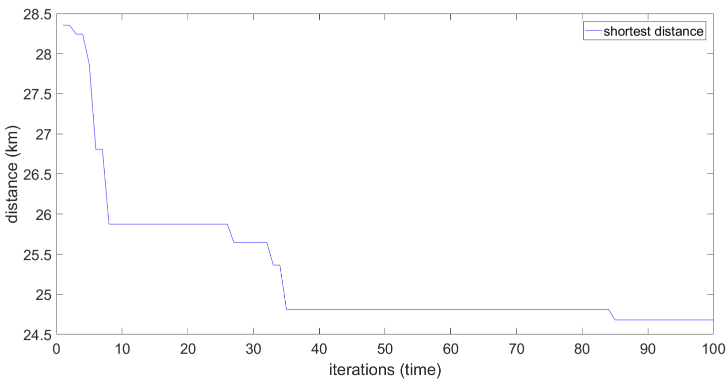


Figure 8: Culculation results of IACO algorithm using MATLAB

As indicated in **Figure 9**, the algorithm is convergent, and the length of the shortest predation route is 24.72 km.Then we presents the predation route of the dragon on map, as shown in **Figure 9**.

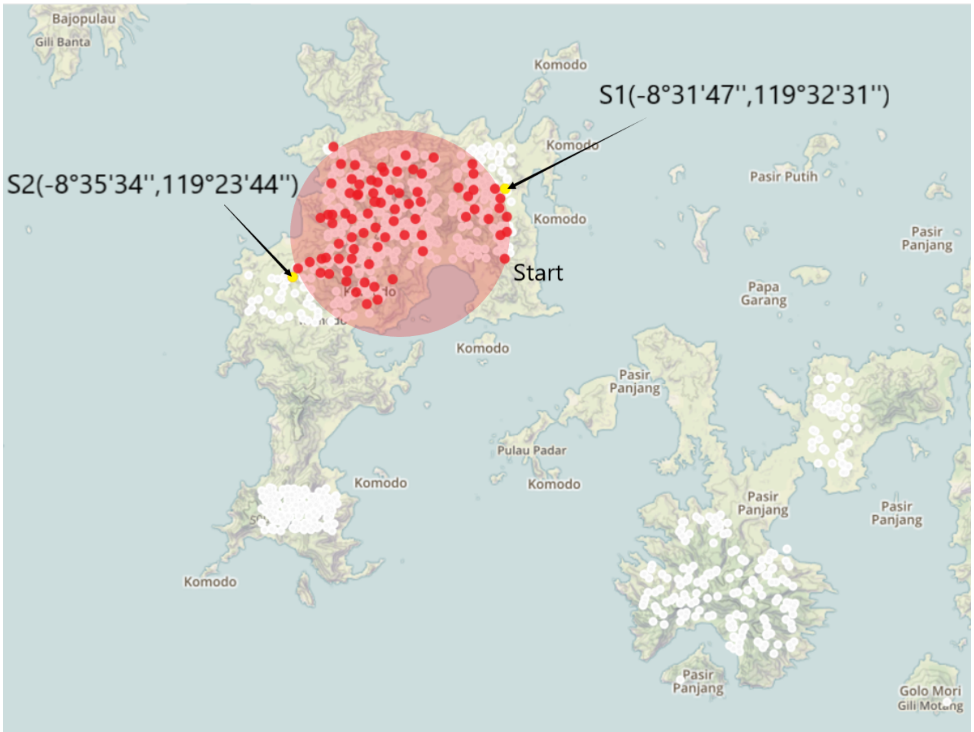


Figure 9: The predation route of the dragon(red dots represents the path of the dragon)

We use S_1 and S_2 to denote the two farthest points in the predation route, and $\overline{S_1S_2}$ is the distance from spot S_1 to S_2. Hence, we can obtain the minimum living area of a dragon

$$S_{\min} = \pi \left(\frac{\overline{S_1S_2}}{2} \right)^2 = 74.88km^2.$$

Since we assume the three dragons do not affect each other, their living area $S = 3S_{\min} = 224.64km^2$. Combined with the results in Model I, we consider a community larger than 314.88 square kilometers with a great species richness can support the survival of the three dragons within one year.

8 Impact of Climate on Models

8.1 The Analysis Process

Changes in climate cause changes in temperature, rainfall and species richness, which in turn reflect changes in the amount of caloric intake and the range of activity of the dragons. Considering that the temperature span of each region is relatively large, we determine the temperature range of each region according to the characteristics of climate type.The temperature ranges of each region are shown in **Table 4**.

Table 4: Temperature range of each region

region	arid	warm	arctic
temperature range	$15°C \sim 35°C$	$-5°C \sim 25°C$	$-45°C \sim -8°C$

Combined with BTM model, we can figure out the feeding requirements of dragons at different temperatures.

At the same time, the distribution density of organisms varies in different climates, so the distribution of dragon prey varies with the climate. In IACO model, we selected four islands in Indonesia to study the influence of the distribution of dragon's prey on the range of dragon's activities. By determining the distribution density of the dragon's prey in arid, warm and Arctic regions and combining the feeding conditions of the dragon in different regions, we can get the area of the dragon's activity. The distribution density of dragon's prey in different areas are shown in **Table 5**.

Table 5: Prey distribution density

region	arid	warm	arctic
Prey distribution density($\mathrm{deer} / \mathrm{km}^2$)	0.626	1.482	0.219

To simplify the analysis process, the group median of the temperature range in each region is used to replace the regional temperature level. With the help of BTM model, we can obtain the calorie intake of 29,124 $kcal$ in arid regions, 31,184 $kcal$ in warm regions and 39,726 $kcal$ in arctic regions.

Introducing the distribution density in $TableX$ into IACO model, we get the following results: The living area of dragon in arid, warmth and artic are 165.46 km^2 ,74.88 km^2 and 300.12 km^2.

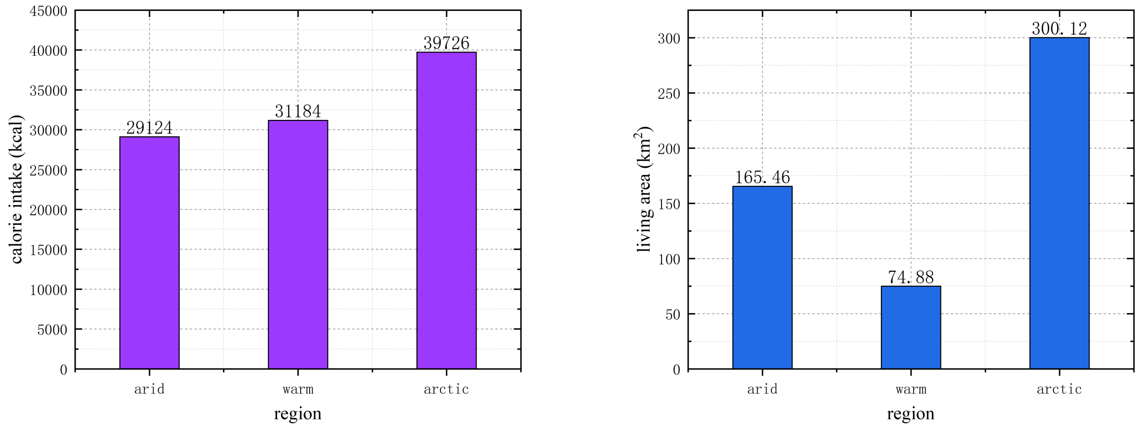
8.2 Results of the Analysis

(1)The impact of climate on caloric demand

The calorie intake of dragons in arctic area is much greater than that of dragons in warm and arid regions. We find that the lower the temperature, the higher the calorie requirement of the dragon. Although precipitation varies greatly between arid and warm regions, the calorie intake of the dragon is also close due to the approximation of temperature. Overall, the calorie requirement increases by 206 $kcal$ for every decrease in average temperature of 1 °C.(see **Figure 10(a)**)

(2)The impact of climate on living area

The different distribution densities of prey in different regions leads to different ranges of preying of dragons. In warm areas, the prey distribution is relatively dense. However, in arid and Arctic regions, the distribution density of prey is relatively low, which greatly increases the difficulty for the dragon to obtain food, so the dragon has to expand its hunting range to get enough food. The average living area required by the



(a) Effect on caloric intake (b) Effect on living area

Figure 10: Impact of Climate Change on Dragons

dragon increases 329.47 km^2 for each decrease in the density of the prey by 1 deer/km^2. (see **Figure 10(b)**)

9 Sensitivity Analysis

9.1 Sensitivity Analysis for Model II

Table 6: Sensitivity Analysis for t_1

	-5%	-3%	-1%	0%	1%	3%	5%
Q	0.802%	0.802%	0.266%	0%	0.266%	0.802%	0.802%

Table 7: Sensitivity Analysis for P_a

	-5%	-3%	-1%	0%	1%	3%	5%
Q	0.078%	0.048%	0.015%	0%	0.015%	0.048%	0.078%

As indicated in **Table 6** and **Table 7**, all the percentage presented on the tables represent the maximum fluctuation, so we can conclude that the change of parameters and has little effect on the results of Model II.

9.2 Sensitivity Analysis for Model III

(1)Parameter Sensitivity of Pheromone factor

Drawing on the concept of ant colony algorithm, the distance between the i^{th} deer and the j^{th} deer is d_{ij}. The pheromone concentration on the path is $\tau_{ij}(t)$.The calculation of $p_{ij}^{k}(t)$ is positively correlated with that of $[\tau_{ij}(t)]^{\alpha}$. Among them, α is the pheromone factor. The greater the value, the stronger the intensity of pheromone. Under the same number of iterations, the sensitivity of parameter pheromone concentration $\tau_{ij}(t)$ is analyzed.

By changing the concentration of pheromone by 1%, 3% and 5% respectively, the amount of change in the living area of the dragon can be calculated by running the program when the number of iterations is 100 generations. The results are shown in **Table 8**.

Table 8: Sensitivity analysis for pheromone factor

Variation of pheromone	-5%	-3%	-1%	0%	1%	3%	5%
Living area	4.140%	1.896%	0.548%	0%	2.230%	3.673%	5.623%

(2)Parameter sensitivity of heuristic function factor

Heuristic functions indicate the importance of dragons moving from one deer location to another. By changing 1%, 3% and 5% of the heuristic function factors respectively, the amount of change in the living area of the dragon can be calculated by running the program when the number of iterations is 100 generations. The results are shown in **Table 9**.

Table 9: Sensitivity analysis for heuristic function factor

Variation of heuristic	-5%	-3%	-1%	0%	1%	3%	5%
Living area	3.446%	1.656%	0.134%	0%	1.549%	4.874%	5.435%

Table A and Table B show that the pheromone concentration has a slightly greater impact on the living area of the Dragon than the heuristic function factor. Meanwhile, the heuristic function factor has slightly impact on the living area of the dragon.

10 Conclusions

According to the Dragon Invasion model, the dragon can reduce the number of consumers in the ecological environment. At the same time, it maintains the number of producers in the environment, and plays a positive role in the growth of vegetation. In addition, the invasion of dragons accelerated the evolution of other species. In terms of ecological demand, the survival of dragons relies on the ecological environment with a high species abundance.

Then we established BTM, a dynamic heat balance model for the study of the specific energy intake of the dragon. With the help of the heat balance equation, we analyze the energy consumption and energy production of the dragon. We find that the dragon need 27,486 $kcal$ per day when it weighed 40 kg a year after birth. Nearly half of the energy absorbed by dragons is converted into heat to maintain body temperature balance.

Using the conclusion in Model II, we can determine the amount of food required by dragons for one year. In the case of determining the specific distribution of the prey, we establish the Improved Ant Colony Optimization (IACO) model to obtain the required area to maintain the three dragons for the purpose of shortest-path prey.

Finally, we analyze the impact of climate change on the demand for resources during the dragon migration. Based on the BTM and IACO models, we reflect the change of environment between warmth, arid and arctic by varying the temperature, rainfall and species richness.

11 Strengths and Weaknesses

11.1 Strengths

1.We make use of the existing species Komodo dragon to study the characteristics, behaviors and diet of dragon. Since the Komodo dragon is the closet to dragon in the existing creatures, which makes our calculation process more reliable.

2. We use four islands located in Indonesia as well as the real data of them to simulate the living environment of dragons. Therefore, our IACO model is more in line with the real ecological environment.

3. When analyzing the living area of dragons, the ant colony optimization algorithm we use reduces the time complexity.

4.The sensitivity analysis shows that Model II and Model III are stable.

11.2 Weaknesses

1.When calculating the impact of dragons on the ecological environment, we only consider two original species, because the analysis of more species is too complicated.

2.We do not take into account the growth of body size when studying the impact of dragons on the environment.

12 Future Improvements

- When studying the environmental impact of dragons, we only consider the simplest predator-prey chain model. The quantitative changes among the three populations are not enough to reflect the impact of the dragon on the ecological environment. In addition, we should consider more complex food chains.

- When considering the food intake and energy consumption of dragons, we simplify the diet of dragons into a heat balance problem. The main energy consumption is heat dissipation. Dragon's energy consumption on flying and moving can be further considered.
- When calculating the living area of dragons, we only consider one kind of animal that dragons prey on. We can give a full consideration to the species of dragon predators, which is closely related to the living range of dragons.

A letter to George R.R. Martin

Dear George R.R. Martin,

We are your faithful readers, and we are also fond of mathematical modeling. The purpose of this letter is to provide advice about the ecological underpinning in your work, *A Song of Ice and Fire*. We are curious if dragons can live in the real ecological environment, so we ran a research on the characteristics, behaviors and diet of the three dragons in your work. Besides, we analyzed the determination of calorie intake and activity area of dragons, and the factor of climate has a significant impact on the physiological and behavioral characteristics of dragons.Here are some of the results of our research that may help you estimate the real ecological rationality of fiction creatures.

First of all, in order to maintain the stability of ecological balance and prevent the dragon from moving among various ecosystems to form species invasion. We establish the Dragon Invasion model to analyze the ecological impact and requirements of the dragons.The results of our dragon invasion on Komodo island show that the population fluctuation period of grass and the Rusa deer has been reduced from seven years to five years. In order to ensure the stable growth of the dragon in the new ecological environment, the number of baits provided to the dragon should be 8 times its number. With the addition of dragons, the difference between the maximum and minimum number of dragon blood trees decreased from 141 to 130 in one life cycle. At the same time the maximum Rusa deer population decreases. The addition of the dragon is beneficial to the regeneration of the dragon blood tree and Rusa deer and accelerates their evolution. At the same time the dragon's predation on the Rusa deer tends to stabilize the number of dragon blood trees. After the addition of the dragon, the number of Rusa deer is reduced by the dragon's predations, but it also keeps the dragon's blood tree population above 30 all year round.

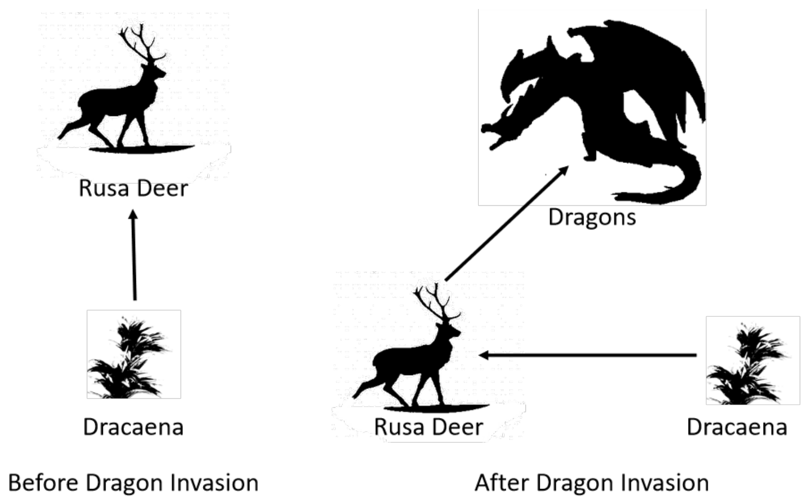
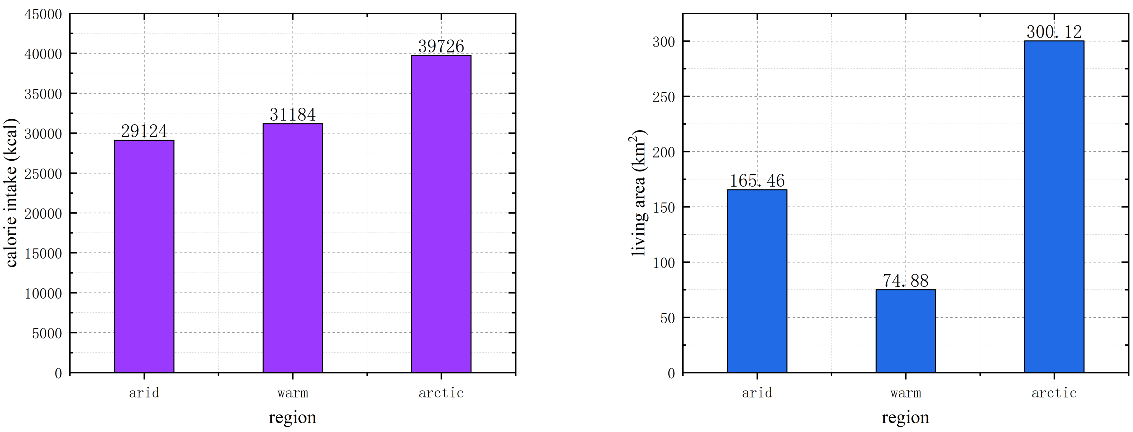


Figure 11: The food chain before and after the dragon invasion

Considering the energy expenditures and caloric intake requirements of the dragons, we build the Body Temperature Maintenance (BTM) model. Dragons need a lot of calories to sustain their growth. Since dragons are warm-blooded, most of the calories they consume are used to provide heat to maintain their body temperature balance. Since the dragon's food intake is only 50% efficient at turning into its own heat, and 15% of the intake is lost during the fire-breathing process, the dragon needs a lot of calories to meet its growth needs. Based on these facts, a 40kg dragon would need 27,486 $kcal$ per day. In order to get such a large number of calories per day, the dragon has to feed on a large number of preys in the ecosystem to meet its energy needs.

Third, accoding to the result of Improved Ant Colony Optimization (IACO) model,we need to provide 224.64 km^2 activity area for the three dragons. Based on our simulation of the dragon's life in the Indonesian islands, we found that, for a dragon, its activity area is not only dependent on its predation ability and movement ability, but also related to the specific distribution of the prey population. The daily calorie demand of the dragon is large, and the corresponding bait demand is large, so a wider area can meet the demand of the dragon. The consumption of sport also encourages the dragon to expand its hunting range.

In particular, as the dragons moved between arid regions, warm temperate regions and the arctic, their own conditions and environmental conditions changed. Due to less precipitation and higher temperature in arid regions, the daily calorie intake of dragons was 29124$kcal$ and the activity area was 165.46 km^2. The warmer climate is more comfortable and suitable for the growth of dragons. Daily dragon in warm areas. The daily calorie intake of the dragon in warm areas is 31184$kcal$, and the activity area is 74.88 km^2. So the demand for living area in warmth is relative small. The daily calorie intake of arctic dragons is 39,726$kcal$.The dragon's energy needs become more pronounced in extreme environments. And since there are fewer prey animals for the dragon in the arctic, the activity area of the dragon in the arctic is 300.12 km^2.



(a) Effect on caloric intake (b) Effect on living area

Figure 12: Impact of Climate Change on Dragons

In addition, we find that when the daily average temperature changes by 1 °C, the daily calorie intake changes 206$kcal$. Climate change affects the diet of dragons mainly from temperature. The colder the environment, the more food the dragon needs to metabolize to produce heat to maintain its body's temperature balance. Therefore, when migrating from a place with a favorable climate and a harsh climate, the dragon needs to search for enough food to ensure normal life activities. Many mammals in the harsh arid regions and the arctic are widely dispersed, and dragons need to move farther to hunt.

Hope that our suggestions can be helpful to you!

Sincerely

MCM Team 1923647

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