- Alphabet : finite, non-empty set of symbols. (Σ)
- Symbol : an entity which itself have no meaning
  eg: any keyboard character.
- Strings : finite sequence of symbols (W/S)
  - It can be empty, denoted by ε.
  - Length of a string : number of symbols, denoted as $|w|$.
  - Count : #
    eg: $w = 10101$, $\#_0 = 2$
    $\#_1 = 3$
  - Concatenation : $\cdot$ or $\circ$

Kleen closure / kleen star

$\boxed{\Sigma^* = \cup \Sigma^i}$, $i \geq 0$

- Σ^i : set of all strings which has length i

eg: $\Sigma^0 = \{\varepsilon\}$

Σ^1 : set of all possible strings using Σ which has length 1

$\Sigma^1 = \{0, 1\}$

Σ^2 : set of all possible strings using Σ which has length 2

$\Sigma^2 = \{10, 00, 01, 11\}$

$\Sigma^3 = \{000, 001 \dots 111\}$

- $\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \dots$

Q. Let $\Sigma = \{a, b, c\}$. Find Σ^*

$\Sigma^0 = \{\varepsilon\}$

$\Sigma^1 = \{a, b, c\}$

$\Sigma^2 = \{aa, ab, ac, bb, ba, bc, cc, ca, cb\}$

Hence, $\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \dots$

$\Sigma^* = \{\varepsilon, a, b, c, aa, ab, ac, bb, ba, bc, cc, ca, cb \ldots$

Positive Closure (Σ^+)

$\boxed{\Sigma^+ = \cup \Sigma^i}$, $i \geq 1$

Eg: If $W = \{0,1\}$, $\Sigma^+ = \{0, 1, 00, 01, 10, 11, 000 \ldots\}$
$W = \{a,b,c\}$, $\Sigma^+ = \{a, b, c, aa, ab, ac \ldots\}$

- If $W = \{a,b\}$, $W^3 = W \cdot W \cdot W$
$= \{a, b, a, b, a, b\}$

Language (L)

- It is a set of ~~al~~ strings which is defined over an alphabet Σ.
- $L \subseteq \Sigma^*$

Eg: • L_1 : set of strings that ends with zero
$W = \{0,1\}$
then, $L_1 = \{0, 10, 100, 110, 1110 \ldots\}$

• L_2 : set of all strings ~~th~~ with n no. of zeros followed by n no. of ones.
$L_1 = \{01, 0011, 000111 \ldots, \varepsilon\}$
ε means $n \geq 1$ not given so n can be 0.

- $L = \{w / \text{something about } w\}$

Q. If $L = \{cc, d\}$, find L^*
$L^0 = \{\varepsilon\}$
$L^1 = \{cc, d\}$
$L^2 = \{cc, dd\}$
$L^3 = \{ccd, dcc, ddd\}$

3/7

Finite Automata

1. DFA : Deterministic Finite automata

$M = (Q, \Sigma, \delta, q_0, F)$

Q : non-empty finite set of states

Σ : alphabet

q_0 : initial state $(q_0 \in Q)$

F : set of final/accepting states $(F \subseteq Q)$

δ : transition function

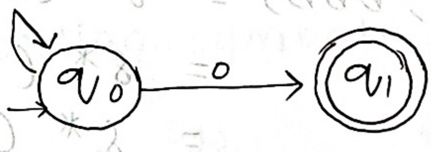
- δ(current state, current input symbol) = next state.

eg: $\delta(q_0, 0) = q_1$

$\delta(q_0, 1) = q_0$

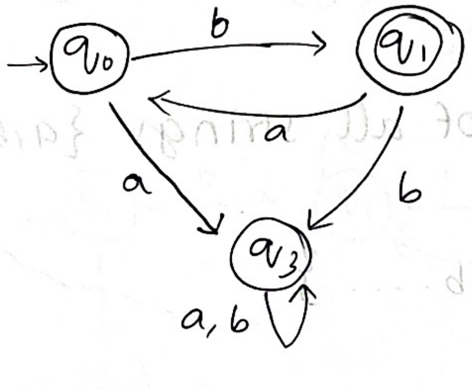
- $\boxed{Q \times \Sigma \to Q}$

Transition diagram :



- Initial state : represented by an arrow
- Final state : " " double circle.

Transition table

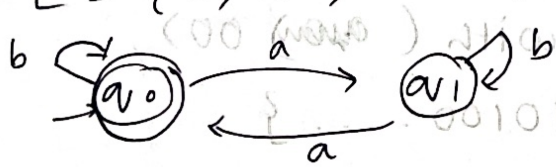


	a	b
q_0	q_3	q_1 } $\delta(q_0, b)$
q_1	q_0	q_3
q_3	q_3	q_3

8/7/25

Q. Design a DFA to accept set of all strings {a,b} as even number of a's.

→ $L = \{\varepsilon, aa, aaaa, \ldots. baa, aaaabbb \ldots\}$



	a	b
q_0	q_1	q_0
q_1	q_0	q_1

DFA for accept Σ

Tracing of a string

δ^* - Extended transition
- iteratively expand.

abab

$\delta^*(q_0, abab) = \delta^*(\delta(q_0,a), bab)$
$= \delta^*(q, bab)$
$= \delta^*(\delta(q,b), ab) = \delta^*(q, ab)$
$= \delta^*(\delta(q,0), b) = \delta^*(q_0, b)$
$= \delta^*(\delta(q_0,b))$
$= q_0$

Accepted

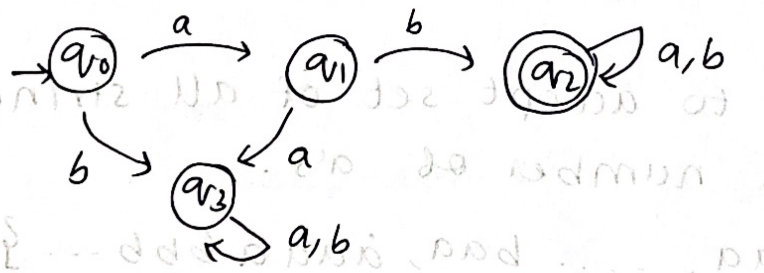
abbb

$\delta^*(\delta(q_0, a), bbb)$
$\delta^*(q, bbb) = \delta^*(\delta(q,b), bb)$
$= \delta^*(q, bb)$
$= \delta^*(\delta(q,b), b)$
$= \delta^*(q,b)$
$= \delta^*(\delta(q,b))$
$= q_1 \notin F$

Not accepted

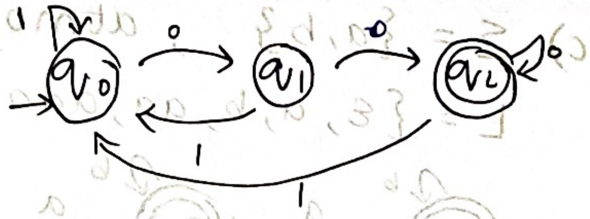
Q. Design a DFA to accept set of all strings $\{a,b\} = \Sigma$ starting with 'ab'.

→ L = { ab, abab, abaa, abbb.... }



Q. Design $\Sigma = \{0,1\}$ ending with (00)

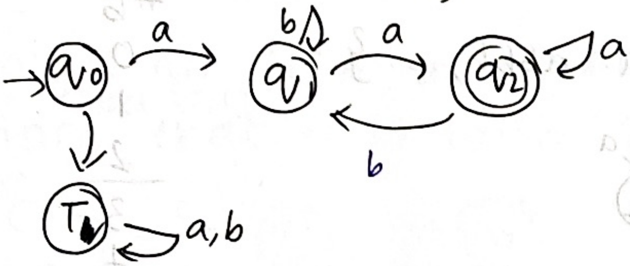
→ L = {00, 000..., 100, 1100, 10100 .... }



Q. Design a DFA for language $L = \{aWa, W \in (a,b)^*\}$

→ L = { any string starting and ending with a }

$L = \{aa, aa \ldots aa, abb \ldots ba, aabab \ldots a, ababa \ldots a\}$



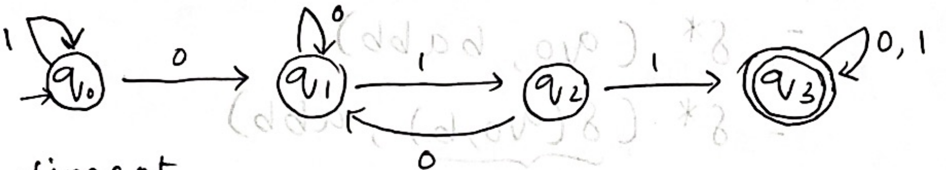
10/7/25

Q. Design DFA

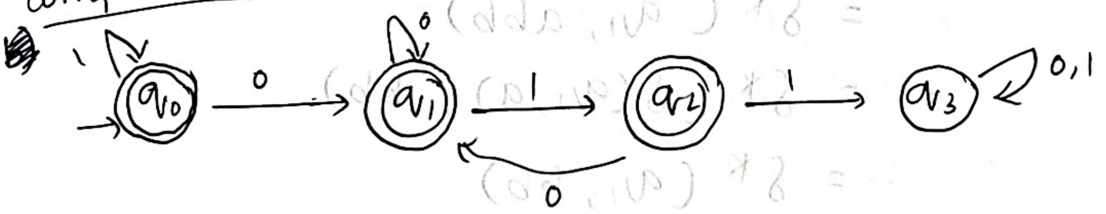
a) $\Sigma = \{0,1\}$ ~~ending~~ with substring '011'

b) $\Sigma = \{0,1\}$, starting and ending symbols donot match.

→ a)

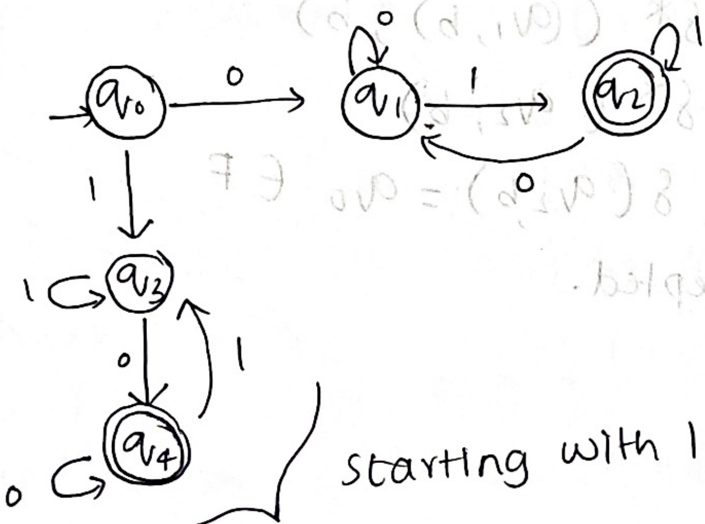


compliment



} with no substring '011'

b)



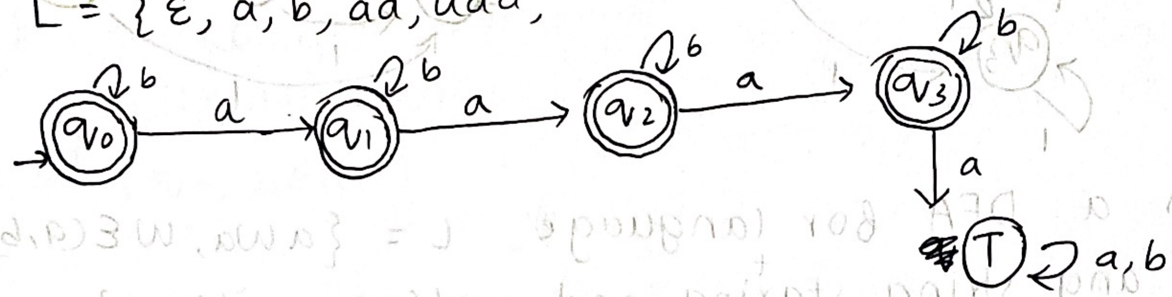
} starting with 0 and ending with 1

starting with 1 and ending with 0.

$L = \{01, 001, 0 \ldots 01, 011, 01 \ldots 1, 010101 \ldots\}$

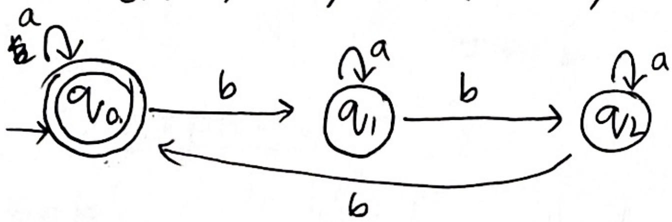
c) $\Sigma = \{a,b\}$, atmost 3 'a's

$L = \{\varepsilon, a, b, aa, aaa,$



d) $L = \{w \in (a,b)^*, \#_b(w) \bmod 3 = 0\}$

$= \{\varepsilon, a, a\ldots a, abbb, a\ldots abbb, \ldots\}$



$\#_b$
0
1
2
3
4
5
6
7
8
9
:

Trace for string of min. length 5:

$\delta(q_0, ababb)$

$\rightarrow \delta^*(q_0, ababb) = \delta^*(\underbrace{\delta(q_0,a)}, babb)$

$= \delta^*(q_0, babb)$

$= \delta^*(\underbrace{\delta(q_0,b)}, abb)$

$= \delta^*(q_1, abb)$

$= \delta^*(\delta(q_1,a), bb)$

$= \delta^*(q_1, bb)$

$= \delta^*((q_1,b), b)$

$= \delta^*(q_2, b)$

$= \delta(q_2,b) = q_0 \in F$

The string is accepted.

11/7 2. NFA (Non-deterministic FA)

$M = (Q, \Sigma, \delta, q_0, F)$

$\boxed{\delta : Q \times \Sigma \rightarrow 2^Q}$ / $P(Q)$

eg: $\delta(q_0, a) = \{q_0, q_1\}$

$\boxed{L_{NFA} = \{w, w \in \Sigma^*, \delta(q_0, w) \cap F \neq \phi\}}$

$\downarrow$

$\{q_0, q_1\}$

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Q.1) Design an NFA over $\{0,1\}$ to accept ~~all~~ the set of all strings that ends with 00.

Transition table :

	0	1
q_0	$\{q_0, q_1\}$	q_0
q_1	q_2	ϕ
q_2	ϕ	ϕ

Trace for 1100 :

$$\delta^*(q_0, 1100) = \delta^*(\delta(q_0, 1), 100)$$
$$= \delta^*(q_0, 100)$$
$$= \delta^*(\delta(q_0, 1), 00)$$
$$= \delta^*(q_0, 00)$$
$$= \delta^*(\delta(q_0, 0), 0)$$

here, $= \delta(\{q_0, q_1\}, 0)$

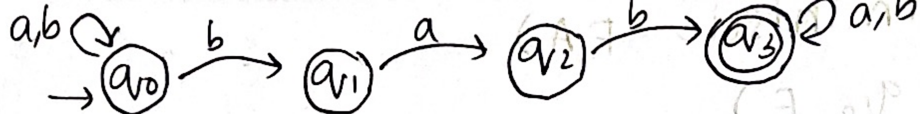
$$= \delta(q_0, 0) \cup \delta(q_1, 0)$$
$$= \{q_0, q_1\} \cup q_2$$
$$= \{q_0, q_1, q_2\}$$

here, $F = q_2$

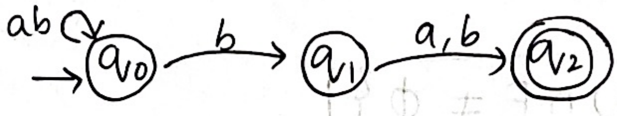
$\{q_0, q_1, q_2\} \cap F \neq \phi$

Hence string is accepted.

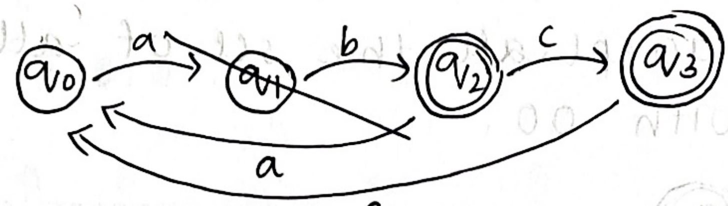
2) Design an NFA to accept set of all strings over $\{a, b\}$ with bab as a substring.



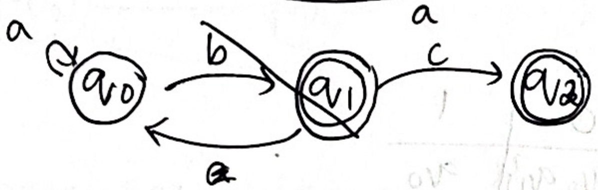
3) Design an NFA to accept set of all strings over $\{a,b\}$ where the second last symbol is b

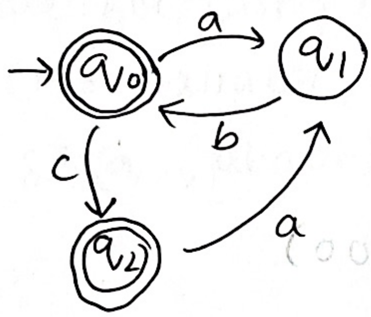


4) Design an NFA ~~to accept~~ with three states to accept a language $(ab, abc)^*$



ababab...
abcabc...
ababc





?

NFA-ε / Epsilon Transitions

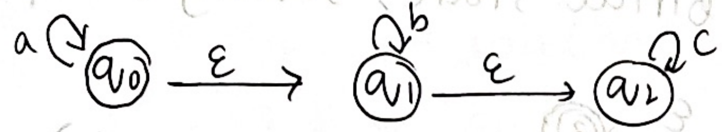
- Formally defined as 5 tuple :
  $(Q, \Sigma, \delta, q_0, F)$

Here, δ NFA-ε : $\boxed{Q \times \{\Sigma \cup \varepsilon\} = 2^Q}$

ε Closure (ε-close)

- Can be defined for all states.
- ε-closure of a state is the set of states that can be reached from that state without reading any input symbols or by only following ε-transitions.
- Never empty because a state itself is an ε transition

eg: String with any ~~of~~ no. of a's followed by any no. of b's and then followed by any no. of c's

ie, 

ε-close $(q_0) = \{q_0, q_1, q_2\}$

ε-close $(q_1) = \{q_1, q_2\}$

ε-close $(q_2) = \{q_2\}$

Transition table :

	a	b	c	ε
q_0	q_0	ϕ	ϕ	q_1
q_1	ϕ	q_1	ϕ	q_2
q_2	ϕ	ϕ	q_2	ϕ

String Tracing:

a) $\delta^*(q_0, abc)$

Here, $q_0 = \varepsilon\text{-close}(q_0) = \{q_0, q_1, q_2\}$

$\Rightarrow \delta^*(q_0, abc) = \delta^*(\delta(q_0, a), bc)$

$\delta(q_0, a) = \delta(\{q_0, q_1, q_2\}, a)$

$= \delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_2, a)$

$= q_0 \cup \cancel{q_1}\, \phi \cup \phi$

$= q_0$

$= \varepsilon\text{-close}(q_0) = \{q_0, q_1, q_2\}$

$\Rightarrow = \delta^*(\delta(q_0, b), c)$

~~$\delta(q_0, b)$~~ $= \delta^*(\delta(\{q_0, q_1, q_2\}, b), c)$

$= \delta^*(\delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_2, b), c)$

$= \delta^*((\phi \cup q_1 \cup \phi), c)$

$= \delta^*(q_1, c)$

$= \delta^*(\{q_1, q_2\}, c)$

$= \delta^*(\delta(q_1, c) \cup \delta(q_2, c))$

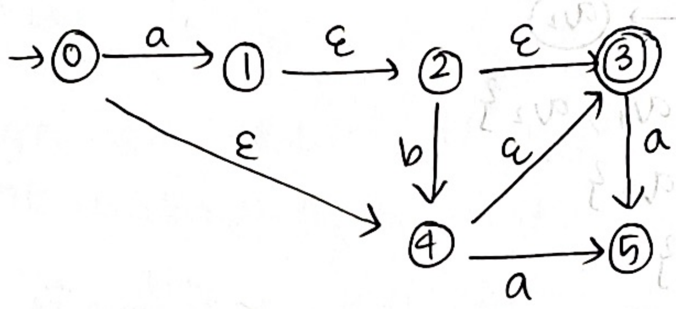
$= (\phi \cup q_2) = q_2$

$= \epsilon\text{-close}(q_2)$

$= \{q_2\}$

Hence q_2 is the final state, string accepted.

Q.



Trace $\delta(0, ab)$

$\epsilon\text{-close}(0) = \{0, 4, 3\}$

$\epsilon\text{-close}(1) = \{1, 2, 3\}$

$\epsilon\text{-close}(2) = \{2, 3\}$

$\epsilon\text{-close}(3) = \{3\}$

$\epsilon\text{-close}(4) = \{4, 3\}$

$\epsilon\text{-close}(5) = \{5\}$

$\delta(0, ab) = (\delta(0, a), b)$

- $\delta(0, a) = \delta(\epsilon\text{close}(0), a)$
  $= \delta(\{0, 4, 3\}, a)$
  $= \{1, 5\}$
  $= \epsilon\text{-close}(1) \cup \epsilon\text{-close}(5)$
  $= \{1, 2, 3\} \cup \{5\}$
  $= \{1, 2, 3, 5\}$

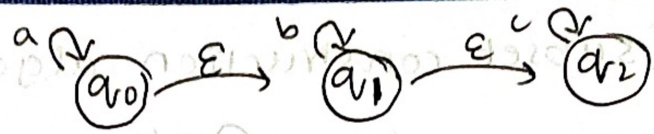
- $\delta(\{1, 2, 3, 5\}, b) = \delta(1, b) \cup \delta(2, b) \cup \delta(3, b) \cup \delta(5, b)$
  $= \phi \cup 4 \cup \phi \cup \phi$
  $= 4$
  $= \epsilon\text{-close}(4) = \{4, 3\}$

Since 3 is the final state, string accepted.

Eliminating ε transitions

- If ε-transitions are eliminated;

  NFA-ε $\longrightarrow$ NFA

NFA-ε : 

$\delta(q_0, a) = \delta(\{q_0, q_1, q_2\}, a) = q_0 \cup \phi \cup \phi$

$= q_0 = \varepsilon\text{-close}(q_0)$

$= \{q_0, q_1, q_2\}$

$\delta(q_0, b) = \delta(\{q_0, q_1, q_2\}, b) = \phi \cup q_1 \cup \phi$

$= q_1 = \varepsilon\text{-close}(q_1)$

$= \{q_1, q_2\}$

$\delta(q_0, c) = (\{q_0, q_1, q_2\}, c) = \phi \cup \phi \cup q_2$

$= q_2 = \varepsilon\text{-close}(q_2)$

$= \{q_2\}$

$\delta(q_1, a) = \phi$

$\delta(q_1, b) = \delta(\{q_1, q_2\}, b) = q_1 \cup \phi$

$= q_1 = \{q_1, q_2\}$

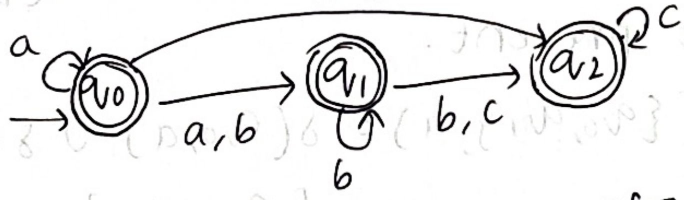
$\delta(q_1, c) = \delta(\{q_1, q_2\}, c) = \phi \cup q_2$

$= q_2 = \{q_2\}$

$\delta(q_2, a) = \phi$

$\delta(q_2, b) = \phi$

$\delta(q_2, c) = q_2$

$\therefore$ NFA : 

All states whose ε-close contains q_2 (final state of NFA-ε) will ~~have~~ be final states.

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Equivalence of NFA & DFA

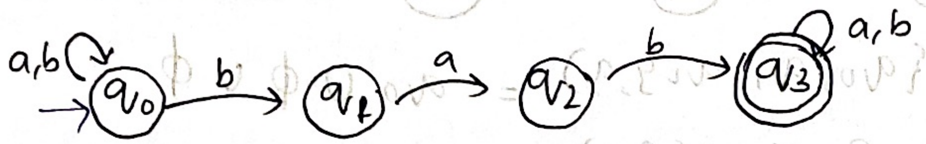
NFA $\equiv$ DFA

Part 1 : For DFA $\longrightarrow$ equivalent NFA (trivial)

Part 2 : For NFA $\longrightarrow$ " DFA

NFA $\longrightarrow$ DFA (subset construction algorithm)

Eg ÷



Step 1 : Initialise Q_D as ϕ

- NFA : $(Q_N, \Sigma, \delta_N, q_0, F_N)$ ← no change

  DFA : $(Q_D, \Sigma, \delta_D, q_0, F_D)$

Step 2 : Add q_0 to Q_D. Initial state of NFA is equivalent to initial state of DFA.

Step 3 : For every state q in Q_D compute transitions for every input symbol in Σ based on the transition ~~state~~ funcn of NFA

Eg ÷

	Q_D	
A	q_0	$\delta(q_0, a) = q_0$ $\delta(q_0, b) = \{q_0, q_1\}$

Step 4 : Add resulting state to Q_D if it is not present.

B $\{q_0, q_1\}$

$\delta(\{q_0, q_1\}, a) = \delta(q_0, a) \cup \delta(q_1, a)$
$= \{q_0, q_2\}$

$\delta(\{q_0, q_1\}, b) = \delta(q_0, b) \cup \delta(q_1, b)$
$= \{q_0 \cancel{\}} ; q_1\}$

C $\{q_0, q_2\}$

$\delta(\{q_0, q_2\}, a) = \delta(q_0, a) \cup \delta(q_2, a)$
$= \{q_0\}$

$\delta(\{q_0, q_2\}, b) = \delta(q_0, b) \cup \delta(q_2, b)$
$= \{q_0, q_3 \cancel{\}}, q_1\}$

~~$\delta(\{q_0, q_1, q_3\}, a)$~~

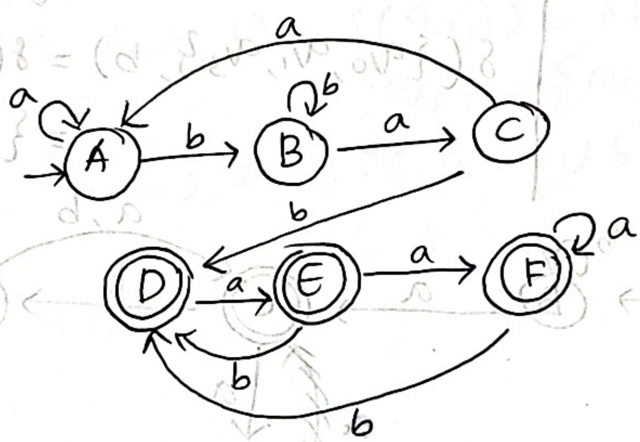
D $\{q_0, q_1, q_3\}$ | $\delta(\{q_0, q_1, q_3\}, a) = \{q_0, q_2, q_3\}$ ✓
$\delta(\{q_0, q_1, q_3\}, b) = \{q_0, q_1, q_3\}$ ×

E $\{q_0, q_2, q_3\}$ | $\delta(\{q_0, q_2, q_3\}, a) = \{q_0, q_3\}$ ✓
$\delta(\{q_0, q_2, q_3\}, b) = \{q_0, q_1, q_3\}$ ×

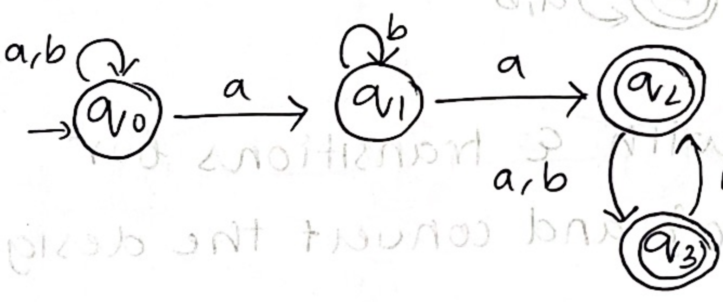
F $\{q_0, q_3\}$ | $\delta(\{q_0, q_3\}, a) = \{q_0, q_3\}$ ×
$\delta(\{q_0, q_3\}, b) = \{q_0, q_1, q_3\}$ ×

Step 5 : All states in Q_D which include atleast one member of F_N should be marked as final state of DFA.

q_0 : A
$\{q_0, q_1\}$: B
$\{q_0, q_2\}$: C
$\{q_0, q_1, q_3\}$: D
$\{q_0, q_2, q_3\}$: E
$\{q_0, q_3\}$: F



Q.



	Q_D	
(A)	q_0	$\delta(q_0, a) = \{q_0, q_1\}$ $\delta(q_0, b) = q_0$
B	$\{q_0, q_1\}$	$\delta(\{q_0, q_1\}, a) = \delta(q_0, a) \cup \delta(q_1, a) = \{q_0, q_1, q_2\}$ $\delta(\{q_0, q_1\}, b) = \delta(q_0, b) \cup \delta(q_1, b) = \{q_0, q_1\}$

C $\{q_0, q_1, q_2\}$

$\delta(\{q_0, q_1, q_2\}, a) = \delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_2$
$= \{q_0, q_1, q_2, q_3\}$

$\delta(\{q_0, q_1, q_2\}, b) = \delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_2 b)$
$= \{q_0, q_1, q_3\}$

D $\{q_0, q_1, q_2, q_3\}$

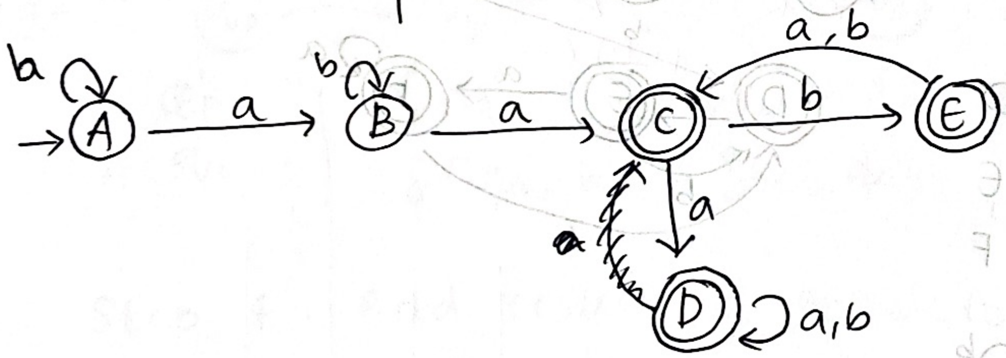
$\delta(\{q_0, q_1, q_2, q_3\}, a) = \delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_2, a) \cup \delta(q_3, a)$
$= \{q_0, q_1, q_2, q_3\}$

$\delta(\{q_0, q_1, q_2, q_3\}, b) = \delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_2, b) \cup \delta(q_3, b)$
$= \{q_0, q_1, q_3, q_2\}$

E $\{q_0, q_1, q_3\}$

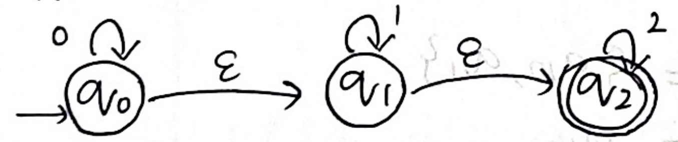
$\delta(\{q_0, q_1, q_3\}, a) = \delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_3, a)$
$= \{q_0, q_1, q_2\}$

$\delta(\{q_0, q_1, q_3\}, b) = \delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_3, b)$
$= \{q_0, q_1, q_2\}$



Q. Design an NFA ~~for~~ with ε transitions for $L = \{0^m, 1^n, 2^p \mid m, n, p \geq 0\}$ and convert the design to DFA.

→



Q\Σ	
q_0	

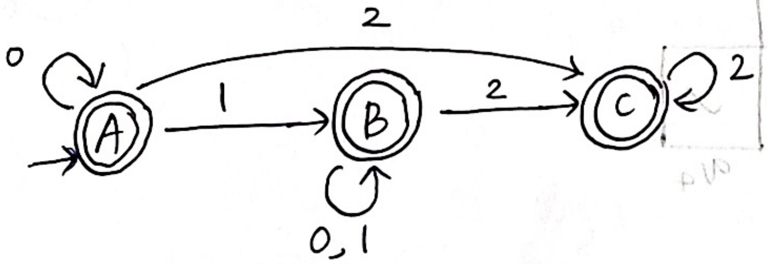
* Initial state of DFA is initial state of NFA.

$\epsilon\text{-close}(q_0) = \{q_0, q_1, q_2\}$

$\epsilon\text{-close}(q_1) = \{q_1, q_2\}$

$\epsilon\text{-close}(q_2) = \{q_2\}$

Q_D	
A $\{q_0, q_1, q_2\}$	$\delta(\{q_0,q_1,q_2\},0) = \delta(q_0,0) \cup \delta(q_1,0) \cup \delta(q_2,0) = \{q_0\} = \{q_1,q_2,q_3\}^*$ $\delta(\{q_0,q_1,q_2\},1) = \delta(q_0,1) \cup \delta(q_1,1) \cup \delta(q_2,1) = \{q_1\} = \{q_1,q_2\}$ $\delta(\{q_0,q_1,q_2\},2) = \delta(q_0,2) \cup \delta(q_1,2) \cup \delta(q_2,2) = \{q_2\} = \{q_2\}$
B $\{q_1, q_2\}$	$\delta(\{q_1,q_2\},0) = \delta(q_1,0) \cup \delta(q_2,0) = \{q_1\} = \{q_1,q_2,\cancel{q_3}\}$ $\delta(\{q_1,q_2\},1) = \delta(q_1,1) \cup \delta(q_2,1) = \{q_1\} = \{q_1,q_2\}$ $\delta(\{q_1,q_2\},2) = \delta(q_1,2) \cup \delta(q_2,2) = \{q_2\}$
C $\{q_2\}$	$\delta(q_2,0) = \phi$ $\delta(q_2,1) = \phi$ $\delta(q_2,2) = \{q_2\}$



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Minimization of DFA

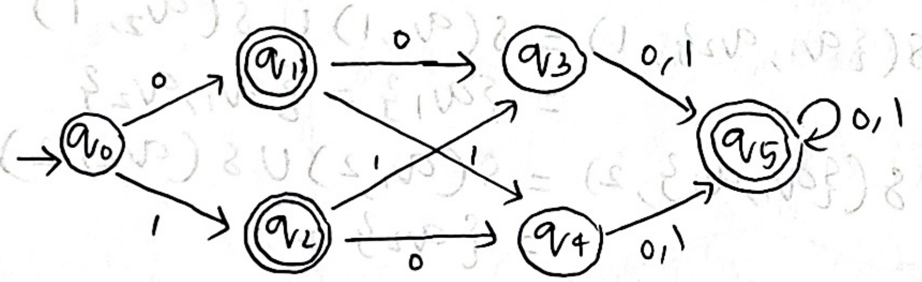
- Inaccessible states: states which can't be reached from q_0 (initial state)
- Indistinguishable states: can be defined for pair of (p,q) states. If for $q, w \in \Sigma^*$

$\delta^*(p,w) \in F \Rightarrow \delta^*(q,w) \in F$ or

$\delta^*(p,w) \notin F \Rightarrow \delta^*(q,w) \notin F$.

- These two types of states are removed to minimize DFA.
- To find ~~algorithm~~ indistinguishable pair there is table filling algorithm.

1. Remove all inaccessible states from DFA
2. Draw the table for identifying the pairs.
3. Mark all pairs (p, q) if $p \in F$, $q \notin F$ or viceversa.
4. Repeat the following until no new pairs get marked.
5. For every pair (p, q) and every symbol $a \in \Sigma$, compute $\delta(p, a) = p_a$ and $\delta(q, a) = q_a$.
   If (p_a, q_a) is already marked in the table then mark (p, q).
6. Combine all unmarked pairs to find minimised DFA.



no inaccessible state

	q_0	q_1	q_2	q_3	q_4
q_1	✓				
q_2	✓	×			
q_3	✓/	✓	✓		
q_4	✓/	✓	✓	×	
q_5	✓	✓/	✓/	✓	✓

final state : $\{q_1, q_2, q_5\}$
non-final state : $\{q_0, q_3, q_4\}$

1. (q_1, q_5)

   $\delta(q_1, 0) = q_3$

   $\delta(q_5, 0) = q_5$ $\Rightarrow (q_3, q_5)$ already marked then mark (q_1, q_5)

2. (q_2, q_5)

   $\delta(q_2, 0) = q_4$

   $\delta(q_5, 0) = q_5$ $\Rightarrow (q_4, q_5)$ already marked then mark (q_2, q_5)

3. (q_0, q_4)

$\delta(q_0, 0) = q_1$

$\delta(q_4, 0) = q_5 \Rightarrow (q_1, q_5)$ marked then mark (q_0, q_4)

4. (q_3, q_4)

$\left.\begin{array}{l}\delta(q_3, 0) = q_5 \\ \delta(q_4, 0) = q_5\end{array}\right\}$ not a pair

$\left.\begin{array}{l}\delta(q_3, 1) = q_5 \\ \delta(q_4, 1) = q_5\end{array}\right\}$ not a pair

5. (q_0, q_3)

$\delta(q_0, 0) = q_1 \quad \Rightarrow (q_1, q_5)$ is marked then mark

$\delta(q_3, 0) = q_5 \quad (q_0, q_3)$

6. (q_1, q_2)

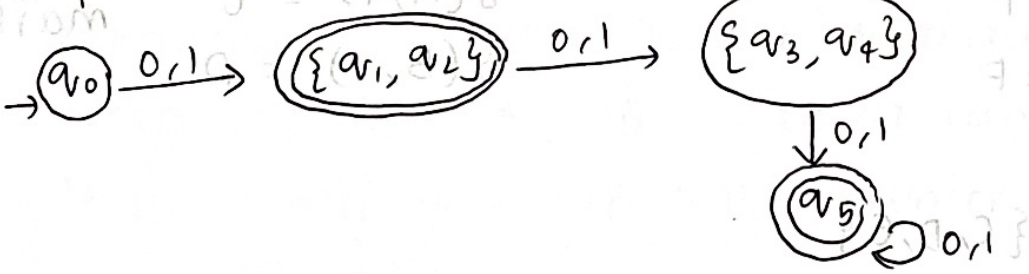
$\delta(q_1, 0) = q_3$

$\delta(q_2, 0) = q_4 \Rightarrow$ not marked

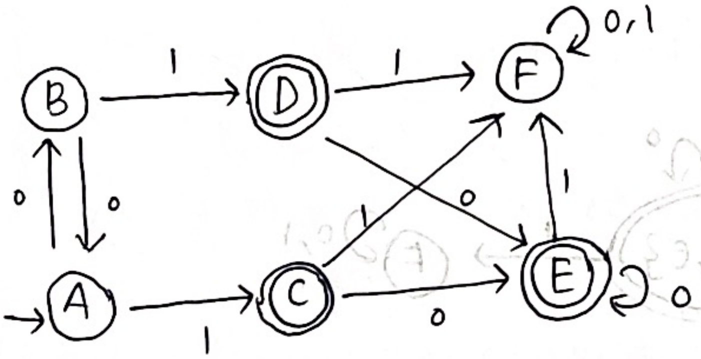
$\delta(q_1, 1) = q_4$

$\delta(q_2, 1) = q_3 \Rightarrow$ not marked

- In this step, we get 4 new markings.
- Two pair of indistinguishable state - merge these two pairs $\{q_1, q_2\}$, $\{q_3, q_4\}$



Q.



- no inaccessible state
- final : $\{C, D, E\}$, non final $\{A, B, F\}$

B	×				
C	✓	✓			
D	✓	✓	×		
E	✓	✓	×	×	
F	//	//	✓	✓	✓

1. (F, A)

$\delta(F,0) = F$
$\delta(A,0) = B$
$\delta(F,1) = F$
$\delta(A,1) = D$
$\Rightarrow$ marked (F, A)

2. (F, B)

$\delta(F,0) = F$
$\delta(B,0) = A$ $\Rightarrow$ marked (F, B)

3. (E, C)

$\delta(E,0) = E$
$\delta(C,0) = E$
$\delta(E,1) = F$
$\delta(C,1) = F$

4. (E, D)

$\delta(E,0) = E$
$\delta(D,0) = E$
$\delta(E,1) = F$
$\delta(D,1) = F$

5. (C, D)

$\delta(C,0) = E$
$\delta(D,0) = E$
$\delta(C,1) = F$
$\delta(D,1) = F$

6. (A, B)

$\delta(A,0) = B$
$\delta(B,0) = A$
$\delta(A,1) = C$
$\delta(B,1) = D$
$\Rightarrow$ not marked

(E, C)
(E, D)
(C, D) $\Rightarrow$ {C, D, E}
(A, B) {A, B}