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T1 ________________	**60083**	F1 ________________
T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**B**	F4 ________________

2017 Mathematical Contest in Modeling (MCM) Summary Sheet
(Attach a copy of this page to each copy of your solution paper.)

Abstract

The space of promotion of the merging area is little because of the traditional distribution of tollbooths. This paper uses a 2-stage model to gain a better design of merging area based on 2 models. The elementary model is newly constructed by our team to determine a newly designed tollbooths with an optimal merging pattern. The revised model gains the best size of merging area by traditional method, given our initial solution in the first stage.

Firstly, we regard the toll plaza as the $B \times B$ Grid and use BIP model to solve for the best location for tollbooth. The aim is minimizing the building cost and the constraints are about flow and safety. The flow constraint is built based on a model revised from queue model. And the safety constraint uses a model similar to what built in a therapy research paper. It can show how the distribution of tollbooths and the merging path affect the safety. More precisely, the danger presented to each discrete point on a N(B)ÃŮL coordinate system constructed on the merging area is considered severally.

Secondly, we build the improved model to determine the size of the minimal cost merging area. The merging path and the better shape of toll that we gain from the first stage are given now. In this stage, we use classic method to analyze the optimal problem and form a complete design of the tollbooths and merging area. Then we use Cellular Automata to mimic our solution and compare it with traditional ones.

Thirdly, we do a sensitive analysis by adjusting traffic flows and the tollbooth processing time, so that we can observe how different traffic condition and using of automated tollbooths affects the model.

A letter for a simple introduction of our idea about the design of toll and merging area to the New Jersey Turnpike Authority will be written at last.

Keywords: 2-stage model, newly designed toll, BIP model, revised queue model, danger model, Cellular Automata, sensitive analysis.

Merge After Toll

Contents

1 Introduction

1.1 Background

A toll plaza is consist of the fan-out area before the barrier toll, the toll barrier itself, and the fan-in area after the toll barrier. There are usually more tollbooths than there are incoming lanes of traffic. To design the shape, size, and merging pattern of the merging area, we should also consider the accident prevention, throughput and cost. Queue model and Cellular Automata are widely used in previous research to solve the problems related to traffic flow. Queue model offers a method to determine a continuous process of traffic flow and traffic condition as time goes by. Cellular Automata is an efficient program, which can mimic the process of traffic flow with some necessary parameters. The comprehensive use of these two models do work well on solving traffic flow problems.

1.2 Our Work

We take the accident prevention, throughput and cost into consideration when designing the shape, size and merging pattern of the area. We believe that the top priority for designing the merging area is reducing the cost. As the land and road construction are expensive, we need to minimize the scale of the merging area. However, if we reduce the length of roads in order to cut down the cost, the risk of traffic accidents will become higher. Safety is a very important factor that we should not ignore. So we wonder if there is a way we can prevent accidents as well as reducing the cost. Apart from optimizing the merging pattern, we can also change the distribution of tollbooths to achieve our goal. If we locate the tollbooths as an arc instead of a row, we can lower the risk of traffic accident when reducing the length of roads. What's more, we also need to think of how the new distribution of tollbooths affect the throughput. In order to solve all these problems, we use a two-stage model.
In the elementary model, we construct three models to determine the distribution of tollbooths. The first model measures the land and road cost of constructing the tollbooths, the second model gives a constraint to the traffic flow based on traditional queue model, and the last model offers a new way to measure the danger degree in merging area, which is related to both the driven route and the distribution of tollbooths. After the distribution of tollbooth zone and merging pattern is determined in the elementary model, an improved model can be used to solve the optimal size of the merging area. In this part, we use common models to solve for the problem. A complete design of both the tollbooth zone and merging area will be gained at this step and the design will be shown in a clear figure. Then we use Cellular Automata mimic the practical car flow condition in the tollbooth zone and merging area. This is can show that the new distribution of tollbooth zone do a better job in lower the risk of traffic accidents. Though the throughput is also lower, the difference is so little that can be ignored. After that, we do sensitive analysis to show the performance of our model in light and heavy traffic and how will the autonomous vehicles and automated tollbooths added to the system affect the solution by adjusting the parameters. Finally, we clarify the strengths and weakness of our model and write a letter to introduce our

new design to the New Jersey Turnpike Authority.

2 Assumptions and Parameters

2.1 Symbol Description

Abbreviaton	Description
B	the number of tollbooths
L	the number of lanes of a toll highway
b_{ij}	$\begin{cases} 1 & \text{if there is a tollbooth in row i, column j} \\ 0 & \text{otherwise} \end{cases}$
c_j	the marginal cost of builidng the ith tollbooth in the same row; we assum that $c_j = c$ in the elementary model
μ	average efficiency of service in the queue and flow model
L_s	the number of cars in the system of the queue and flow model
L_q	the number of cars waiting in a queue in the queue and flow model
W_s	the residence time in the system of a car in the queue and flow model
W_q	the waiting time in a queue of a car before the tollbooth the queue and flow model
C	the number of service stations in the queue and flow model, which is considered as the number of rows occupied by tollbooths in this model
$N(B)$	the scale of merging area, which equals the points on the horizontal axisin the discrete coridnates in the elementary model
P_n	posibility of n cars in one tollbooth; if n=0, that means the tollbooth is unoccupied adjust the unit length of the discrete cordinate of merging area to the ratio of the length of tollbooth to actual length
A	merging pattern,$(a_1, a_2 \cdots a_b) \quad b = 1, 2 \cdots L$ shows that cars passing by tollbooths in row b "fan out" into road a_b
$D(r)$	danger degree function, shows the degree of danger at a merging point, when the distance between a tollbooth and the point is r. Danger is caused by cars driving from the tollbooth to the point. According to the assumption that cars are speeding up and it's more dangerous if cars is driven faster, this is a increasing function
P	the coordiante of a merging point in the discrete coordinate system
$d(P, A, b_{ij})$	the danger coefficient of a merging point
Q	throughput boundary
d	danger degree boundary

2.2 Assumptions

- The constrution of toll plaza is symmetric in both directions. This means that we only need to optimize the design of a toll plaza in one single direction, and can get the optimal design for toll plaza in both directions.
- Cars "fan in" to the tollbooths follow a poison process. Merging pattern, which means the merging way for cars after passing by the tollbooths is prescribed. In the queue and flow model, whether the traffic is light or heavy will influence the average arrival rate λ, and whether the tollbooths are human-staffed or automated will affect the average efficiency of service μ.
- The area, which tollbooths locate in is a square. Cars must go straight through the tollbooths, then enter the fan area, where cars can merge.
- The optional location of tollbooths is finite. As long as we build one tollbooth along the straight way, we should construct the entire straight road. If we build more than one tollbooth along a straight way, it will take a marginal cost for relief roads.
- The space between two tollbooths is enough for cars to get to any tollbooths they want, even if two tollbooths locate in the same column. In this occasion, cars ca go through the relief road.
- Traffic accident won't happen in the tollbooth area, but will happen in the merging area.
- To impose restrictions on the transportation risk in merging area, we impose restriction on the risk of every point, which is dense and uniformly distributed in this area.
- After passing by a tollbooth, the car is in a accelerating process, the faster the car is driven, the higher possibility of accidents may happen.
- The transportation risk at one point in the merging area can be indicated as the sum of the risk causing by cars coms out of every tollbooth.
- The drivers won't violate the merging rules that have been set.
- In the elementary model, we think about the cost of construction of B tollbooths and mering area as sunk cost, and only consider the land and construciton cost of the roll plaza. We will take the construction cost of merging area into consideraton in the improved model.

3 Model

3.1 The Tollbooth Cost Model

We form a $B \times B$ grid to measure the different cost of the construction of tollbooths when the distribution of tollbooths is different. According to the assumption, when we choose to build a tollbooth, we have to construct the entire straight road, so we unitize this cost. It is obvious that building tollbooth on the road that has been already built can reduce the total cost. However, if we want to build more than one toolbooth on a stright road, we need to build a relief road to address the need of throughputs, the marginal cost of which is not zero. So we use a proper c to show the cost of the relief road. As B tollbooths at most occupy B rows or B conlumns, we can use a $B \times B$ frid to solve this problem.

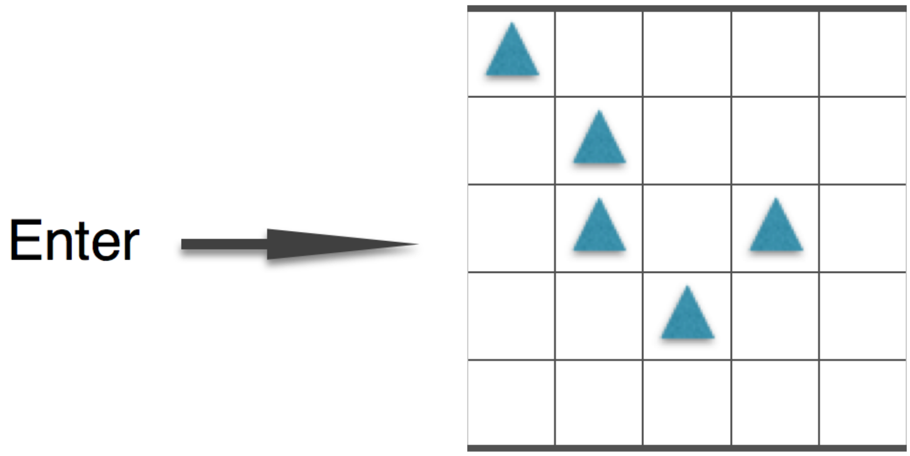

Figure 1

The triangles in the grid denote the locaton of tollbooths. In Figure 1 the total cost is 4+c. This is because the tollbooths ocuppy 4 rows, and we should build a relief road in the third row for cars to get to the second tollbooth. Cars enter the tollbooth through the relief road on the left side, and leave the tollbooth zone through the same relief road after passing by the tollbooth. A more general cost function can be expressed as

$$(1-c)\sum_{i=1}^{B}(1-\prod_{j=1}^{B}(1-b_{ij}))+Bc \tag{1}$$

subject to,

$$\sum_{i,j} b_{ij} = B, \qquad b_{ij} \in 0, 1$$

b_{ij} in the constraint is the binary variable that shows the location of (i,j) in the grid, which is decribed in the assumption. It is obvious that if we only consider how to minimize the cost of construction, the best way to located B tollbooths is as shown in Figure 2.

However when we take the risk factor and the throughput into consideration, this way is absurd.

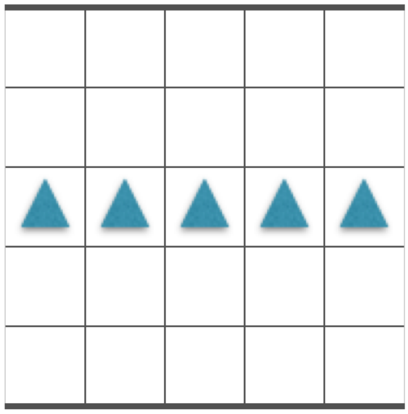

Figure 2

3.2 The Car Flow Model

Firstly, we think of a simple queue and flow model, and use the assumption that the traffic stream follows a Poisson Distribution. In a single queue service system, we assum that the average arrival rate $\lambda > 0$, and the average efficiency of service $\mu > 0$. P_n denotes the possibility of n cars heading for on tollbooth, then we have,

$$When \qquad \lambda < \mu : \quad P_0 = 1 - \frac{\lambda}{\mu}$$

$$P_n = \frac{\lambda}{\mu}^n (1 - \frac{\lambda}{\mu}) \qquad n = 1, 2, 3 \cdots$$

$$If \quad \rho = \frac{\lambda}{\mu}, we\ donote\ the\ above\ formula\ as \begin{cases} P_0 = 1 - \rho \\ P_n = \rho^n (1 - \rho) \qquad n = 1, 2, 3 \cdots \end{cases}$$

Only if $\lambda < \mu \quad \rho < 1$, the system has well reliability. Otherwise, the lines will become longer and longer. The evaluation index of the system,

$$queue\ length\ in\ the\ system \quad L_s = \frac{\lambda}{\mu - \lambda} \quad or \quad L_s = \frac{P}{1 - P}$$

$$queue\ length\ before\ the\ tollbooth \quad L_q = \frac{P^2}{1 - P} \quad or \quad L_q = \frac{P\lambda}{\mu - \lambda} = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

$$residence\ time\ of\ a\ car\ in\ the\ system \quad W_s = \frac{1}{\mu - \lambda}$$

$$waiting\ time\ of\ a\ queue\ before\ the\ tollbooth \quad W_q = \frac{\rho}{\mu - \lambda}$$

In this paper, the enter zone of traffic stream is the way from L lanes to B tollbooths. As a result of more than one tollbooths in a column, we assum that the extra tollbooths will increase efficiency by times. So we revisit our model as following.

$$In\ every\ line\ i \quad, \lambda' \to \lambda/C$$
$$\mu_i' = \mu \times \sum_{j=1}^{B} b_{ij}$$
$$C = \sum_j (1 - \prod_j (1 - b_{ij}))$$

We put this adjustment into queue and flow model,

$$W_{s_i} = \frac{1}{(\mu_i' - \lambda')}$$
$$W_{q_i} = \frac{\rho_i}{(\mu_i' - \lambda')}$$

We believe that in a $B \times B$ grid, passing through the point with tollbooth takes residence time, and passing through the point without a tollbooth but is heading for a tollbooth takes waiting time. We define the average time cost of every line i with at least one tollbooth as

$$W_i = q_i W_{s_i} + (1 - q_i) W_{q_i}$$
$$q_i = \sum_j b_{ij} / \max\{j : b_{ij} = 1\}$$

Define the average waiting time of line i, in which there isn't any tollbooth is positive infinity.

$$average\ traffic\ flow\ in\ unit\ time \quad \sum_{i=1}^{B} \frac{1}{W_i}$$

When we regard the traffic flow as constraint, we can rewrite the above formula as

$$\sum_{i=1}^{B} \frac{1}{W_i} > Q \tag{2}$$

$$Q\ is\ a\ constant\ , and\ denotes\ the\ constrait\ of\ the\ minimum\ of\ traffic\ flow$$

3.3 Danger Coefficient Model

In this model, we assume that traffic accidents won't happen in the tollbooth zone, but may hsppen in merging area. Using the assumption that B=5,L=3, we can get Figure 3.

In the assumption, we have already regarded the construction cost of merging area in the elementary model as sunk cost, so we mainly consider how do different merging pattern affect the risk of traffic accident in this area. We set the lower left corner in the grid as the origin of cordinates. Then, the merging area and tollbooth zone share the same unit of vertical axis. The unit of abscissa axis is set as a constant N in order to get N(B) discrete points.
A denotes merging pattern, we use $(a_1, a_2 \cdots a_b) \quad b = 1, 2 \cdots L$ to show that cars passing by tollbooths in row b "fan out" into road a_b, $b = 1, 2 \cdots L$. In the following part,

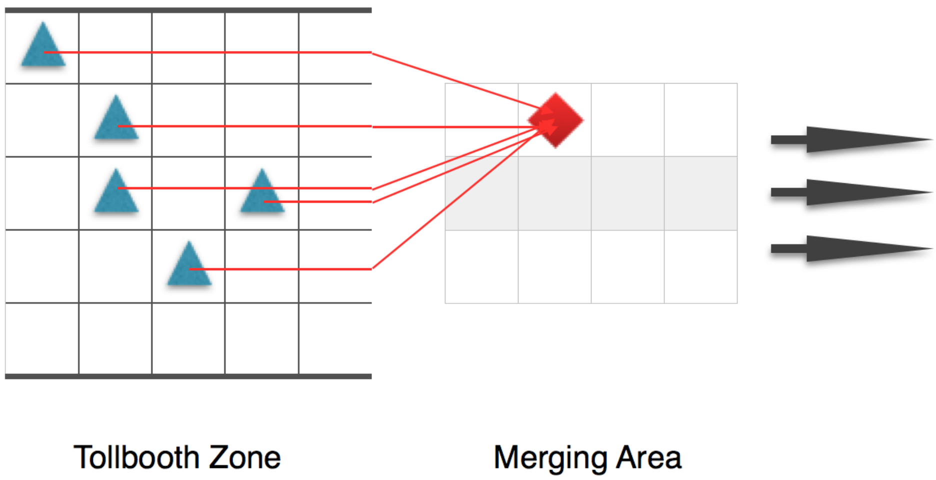

Figure 3

we set an array $A' = A + q$, q shows the adjustment of the cordination of A, which is because the merging area lies in the center of the road.
The car pass through a tollbooth(triangle in Figure 3) to point $P = (n, l)$ in the merging area(square in Figure 3), and cause a risk of accident $D(\|(i, B) - P\| + B - j)$, D is the danger degree funciton, which increases with the distance between the tollbooth and point P. As the assumption "the faster the car is driven, the higher possibility of accidents may happen", it is reasonable to set D as an increasing function. According to the assumption, if we do not take the merging pattern into consideration, and believe that each car passing through the tollbooths can choose whatever lanes they want to, the sum of risk at point P is $\sum_{i,j} b_{i,j} D(\|(i, B) - P\| + B - j)$.
When we specified a merging pattern, some tollbooths may not influence the location of some points in the merging area. For example, if we specified that cars passsing through the first line merge to the first lane, then cars "fan out" from the tollbooths located in the first row won't have an influence on the second and third lines in the merging area. So for a merging pattern A, an adjusted array A', the degree of danger at point P(n,l) is

$$
\begin{aligned}
& d(P, A, b_{i,j}) = \sum_{i,j} b_{i,j} \widetilde{D}(\check{A}, b_{i,j}) \\
& \widetilde{D}(A, b_{i,j}) = \begin{cases} D(\|(i, B) - P\| + B - j) & i \leq l \leq a_i \\ \qquad\qquad\qquad\qquad or & a_i \leq l \leq i \\ 0 \quad , \quad otherwise & \end{cases} \\
& d(P, A, b_{i,j}) < d, \ is\ obtained\ to\ any\ P \\
& d\ is\ a\ constraint\ of\ the\ degree\ of\ danger\ of\ every\ point\ at\ the\ merging\ area
\end{aligned}
\tag{3}
$$

3.4 Solve the Optimization Problem

The decision variable in this model isb_{ij}, which denotes the locaiton of the tollbooths, and $A = (a_1, a_2 \cdots a_B)$, which shows the merging pattern. We form the optimization problem as the following form.

$$Min \quad Z = (1-c)\sum_{i=1}^{B}(1-\prod_{j=1}^{B}(1-b_{ij})) + Bc \tag{4}$$

Subject to,

$$\sum_{i,j} b_{ij} = B$$
$$\sum_{i=1}^{B} \frac{1}{w_i} > Q$$
$$d(P, A, b_{i,j}) < d \; is \; obtained \; for \; any \; P \; in \; the \; merging \; area$$
$$a_1, a_2 \cdots a_B \in \{1, 2 \cdots L\}, \qquad b_{ij} \in (0, 1)$$

We use *MATLAB* and *LINGO* to solve for the optimization problem. Firstly, we assign a proper value to the parameters in the model. Then we can find that the number of tollbooths B and the number of lanes L can be selected according to the actual situation. As a result, we choose B=8, L=3, which is exactly the number in the example of the question.
In the flow model, λ denotes the traffic pressure and μ denotes the efficiency of the tollbooths. Under different traffic condition and human-sstaffed or autimated tollbooths, we set $\lambda = 1$, $\mu = 2$.
The lower bound of flow Q shows the average traffic stream in unit time that meets the minimun requirment. We set Q=20 in the first time to solve for the problem.
The danger coefficient function D(r) is a increasing funciton. However we should think about that the speeding up process of cars is finite. So the danger degree is increase from a stationary state and finally tend to be stable, and can be set as negative exponentioanl function $1 - e^{-x}$.
We find that if danger coefficient function is set in different function forms, the form of $d(P, A, b_{ij})$ will be different. So we should analyze data under different danger coefficient to get the upper bound of danger degree. Doing statistic analysis on New Jersey Department of Transportation Crash Rate On State and Interest Routes(2003-2015), we find that when there are eight tollbooths, the relationship between $d(P, A, b_{ij})$ of one point in the merging area and the crash rate is as follows.

d(P,A,b_{ij})	<7	<6.5	<6	<5.5	<5	<4.5	<4
Crashrates	<10.81	<9.41	<7.75	<4.29	<3.48	<2.68	<1.96

When cash rate is less than 2, we have ideal degree of safty. So we set the upper bound of danger degree as 4. According to the assumptions above, the scale of merging area N(B) is set as a constant in the elementary model N(B)=N=4.
The solution to the elementary model is as follows.
b_{ij} :

1	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0
0	0	0	0	1	0	0	0
0	0	0	0	0	1	0	0
0	0	0	0	0	1	0	0
0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0
1	0	0	0	0	0	0	0

$A = (1, 1, 1, 2, 3, 3, 3, 3)$
The location of tollbooths and the merging pattern is as follows:

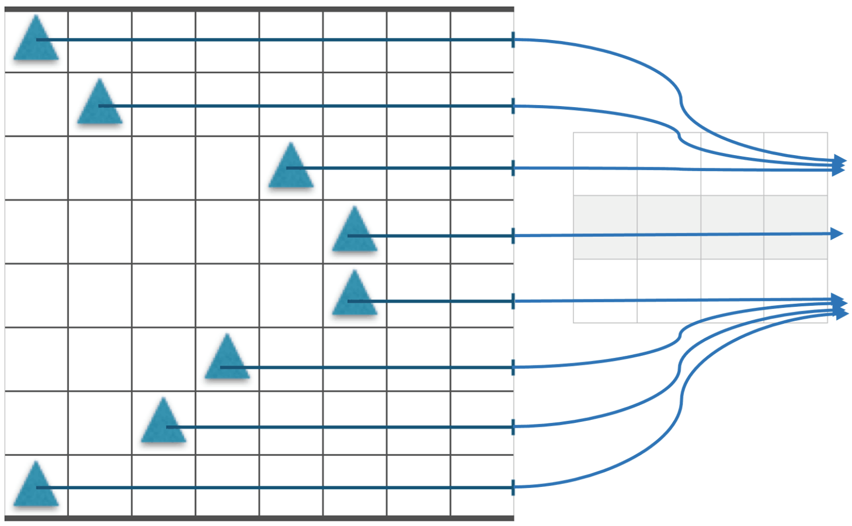

Figure 4

As it is shown in Figure 4, the occasion that two tollbooths locate in the same row, which can save the cost of land doesn't happen. This matches the reality, because if the tollbooths are human-staffed ones, it takes some time for a car to pay for the toll. Even if there are relief roads, we will have some trouble in road administration. But comparing with the traditional distribution of tollbooths, to locate the tollbooths as a arc may not only decrease the danger degree of the merging area but also address the need of thoughput.

3.5 The Improved Model

The improved model aims at determing the scale of the merging area. In the improved medel, we define that the ratio m between coordinate of merging area and tollbooth zone. According to the assumption above, if the scale of merging area is a constant $(N(B) \times m)$, the danger coefficient at every point is the sum of danger caused by cars that may get to the point. Whether the cars will reach the point is determined by the location of tollbooths and the merging pattern, while the danger degree is determined

by the speed of cars.
Now we are looking for the optimal scale of the merging area. To search for the possibility of a car reaching one point in the merging area, not only do we need to think of the location of the tollbooths and the merging pattern,but also consider the influence caused by the scale and size of the merging area. We suppose that the distribution of the possibiility of a car reaching one point in certain way is a uniform distribution. The danger degree is adjusted to: $D(A, b_{ij}, mN) = D(A, b_{ij})/s(A, b_{ij}, mN)$
And $S = (A, b_{ij}, mN) = mN \times |i - a_i' + 1|$ shows the area that is possible for a car driven from point (i,j) to reach. The merging pattern is given.
When taking the danger degree into consideration, we use double objective function to minimize the scale of the merging area

$$\begin{aligned} Min \qquad & \gamma mN(B) + (1-\gamma) d(A, b_{ij}, mN) \\ \gamma = & \frac{|((mN(B))^* - mN(B)|}{|((mN(B))^* - mN(B)| + |d^* - d|} \end{aligned} \tag{5}$$

Sloving this problem with the same parameters, we can get m=4.3. This means that in the merging area, the width of a coordinate frid is 4.3 times of the length of the tollbooth. So the total length of the merigng area is 17.2 times of the length of the tollbooth. The optimal scheme in this occasion is as follows.

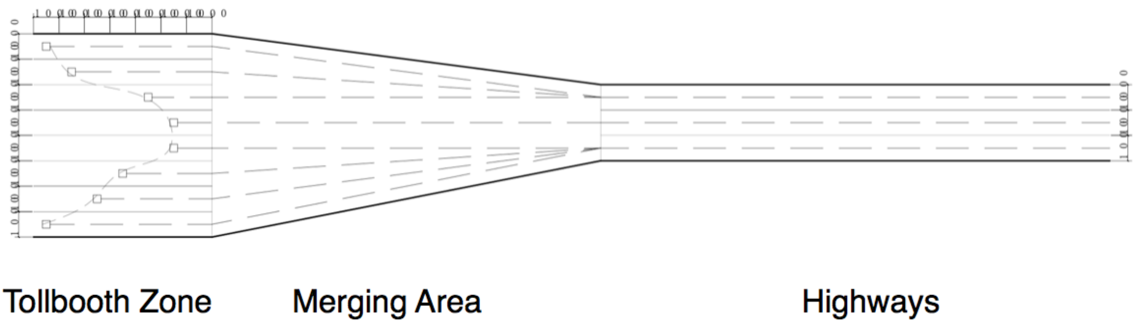

Figure 5

- Cellular Automaton is widely used in modeling diverse physical systems and natural phenomena, such as fluld flow, galaxy formation, snowslide, traffic flow calculation. Vehicles are allowed to change the way if they fulfill the follwing 4 rules of NaSch Model:
  acceleration:$x_i \rightarrow x_i + v_i$
  braking:$v_i \rightarrow min(v_i, d_i)$
  randomization:$v_i \rightarrow max(v_i - 1, 0)$
  foward:$x_i \rightarrow x_i + v_i$
  change the lanes: for cars that are near the tollbooths($\|x_n - X_i \leq 3$), it's not allowed to change the lanes;
  the way of changing lanes is$(x_i, y_i) > (x_i + 1, y_i + vi)$
  We use Cellular Automaton to mimic the traffic stream in the traditional locaiton of tollbooths and tollbooths locate as an arc.

(a) 1 (b) 2

4 Sensitivity Analysis

- According to different traffic condition, adjust the arrival rate λ to different value. When the traffic pressure is heavy, λ rises from 1 to 1.5. At this moment, the optimal location of tollbooths and the optimal merging pattern is as follows.

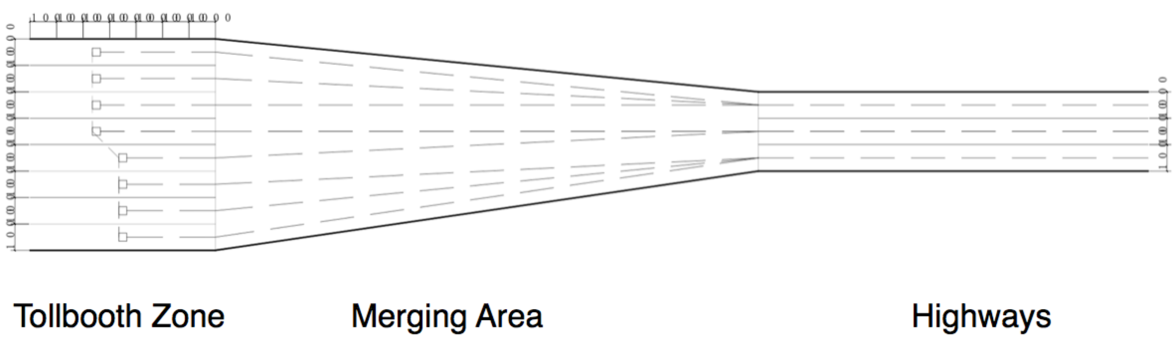

Figure 6

- If more autonomous vehicles are added and the proportion of automated tollbooths becomes higher, the service efficiency μ will become higher. Suppose μ is higher enough so that it takes little residnece time in teh tollbooth zone. At this moment, the solution is as follows.

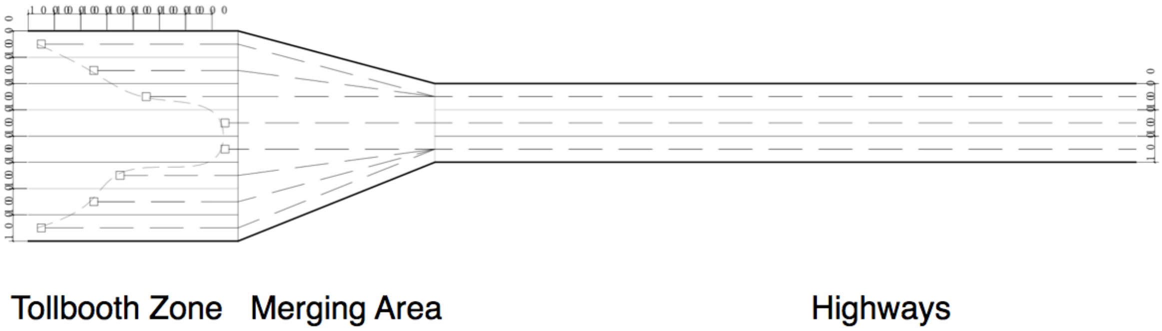

Figure 7

- Set λ/μ as 2, 5, 10, 20, and infinite, we can get the distribution of tollooths as follows.
  $\lambda/\mu = 2$

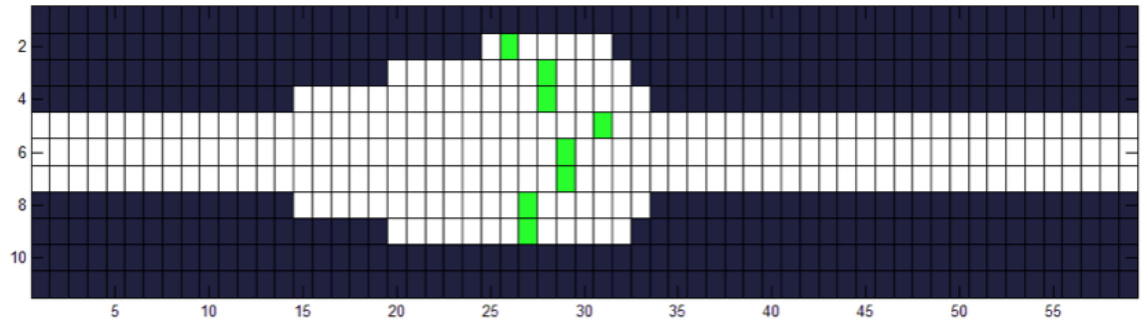

Figure 8

$\lambda/\mu = 5$

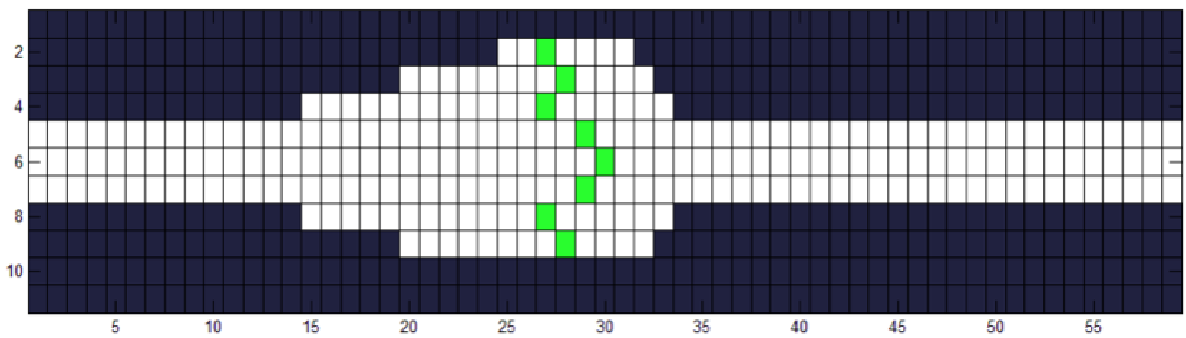

Figure 9

$\lambda/\mu = 10$

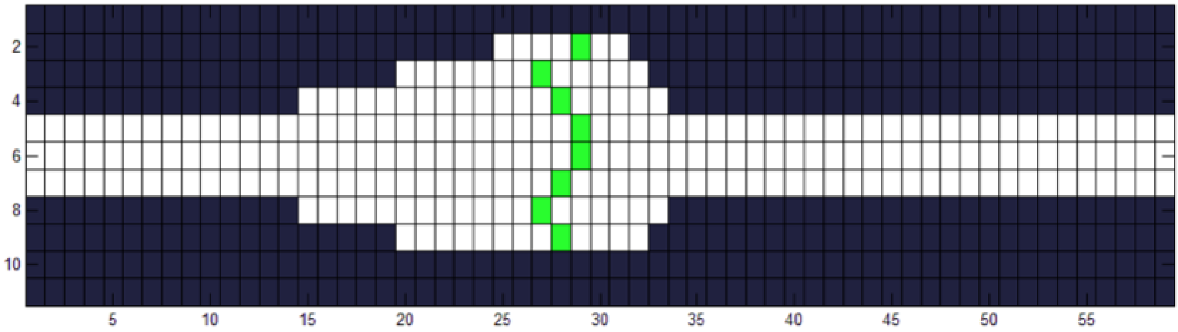

Figure 10

$\lambda/\mu = 20$

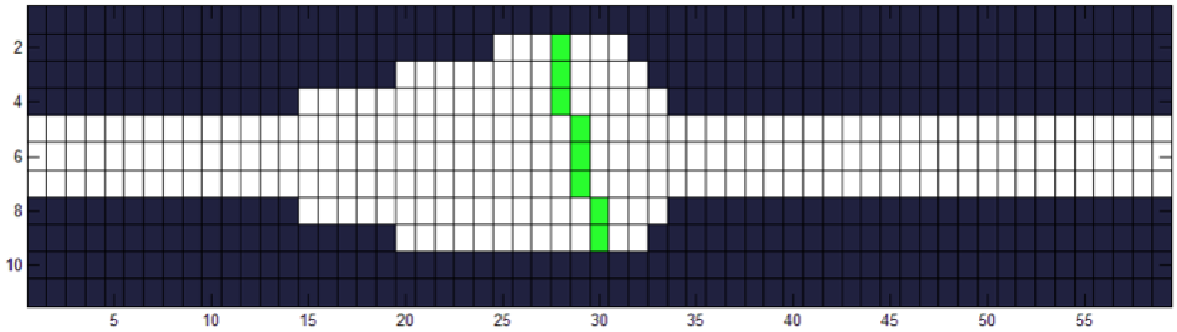

Figure 11

$\lambda/\mu = \infty$

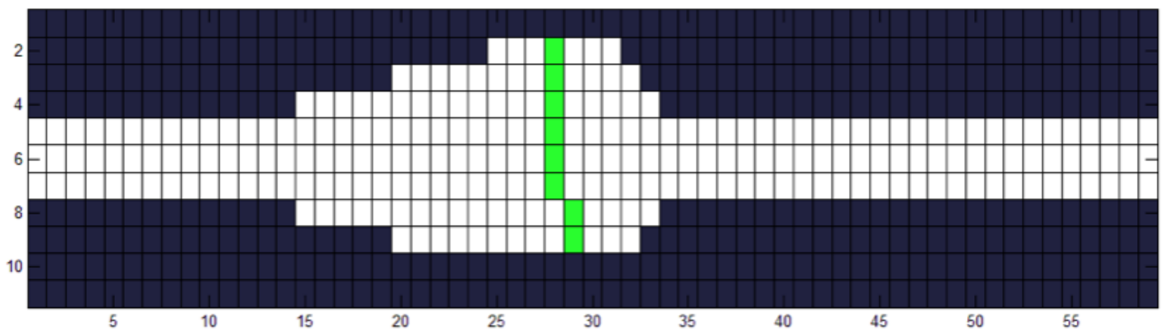

Figure 12

- We found that if $\rho = \lambda/\mu$ becomes larger, the relative pressure of traffic stream u becomes larger. When the location of tollbooths approaches a row perpendicular to the lanes, the relative presssure of traffic stream becomes less. The distribution of tollbooths approach an arc with a large radian to save the cost as well as satifying the requirement of safty.

5 Strengths ans Weaknesses

5.1 Strengths

- Our model breaks the traditional way of the location of tollbooths. We locate the tollbooths as a arc instead of a row, which performs better in reducing the total cost under the requirements of safety and throughput.
- We use two stage model to solve this problem. This model takes into account our innovation point and the main problem that need to be solved. In the elementary model, we take the new location way fo tollbooths, the cost of land, throughput and danger degree into consideration. We also use the traditional queue model as a constraint of the optimization problem. Regarding the cost of merging area as the sunk cost in the elementary model help us solve this problem more efficiently. The better location of tollbooths solved in elementary model is the premise of the improved model. Under this premise, we think about the merging pattern and the cost of merging area in the improved model. With this two stage model, we solved the problem better than only focus on the shape, size of merging area and merging pattern.
- Through seneitivity analysis, we shows different ways to construct the tollbooth zone and the merging area facing different traffic conditions, differnet types of cars and different types of tollbooths intuitively. This also providse us with a model that has universality.

5.2 Weaknesses

- There is an overwhelming emphasis on the distribution of tollbooths. As a result of this, we set too much assumptions in the improved model to solve for the minimun cost of the merging area. Although we get a better result, but the cost problem seems to simple with too much assumptions.
- The assumption that the tollbooth zone is a square limits us to search for a better plan, which may with a special shape tollbooth zone or merging area.
- The selection of paremeters is based on simple statistical analysis when solving problem in this paper. However, this may cause bias error. What's more, the selection of some parameters in the model must take the actual condition into consideration, which is lack of universality.

References

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Letter to the New Jersey Turnpike Authority

Dear official,

Hearing that New Jersey Turnpike Authority is looking for a better design for the area following the toll barrier, which we call it as merging area, my colleagues and I are very interested in it. During the past few weeks, we have been collecting data of the traffic stream, queueing time, traffic accidents, cost of road and land and throughput of the highways in New Jersey. Analyzing all the data, we come out with an optimal design of the merging area, which takes accident prevention, throughput and cost into consideration.We are going to make a brief description of the model we use to get the result and show you our design and its innovation point.

Firstly, we try our best to address the need of Turnpike Authority. We believe that reduce the cost of road and land is very important for you. However, it is also very significant to ensure safety and to meet the requirement of throughput. Considering all this factors, we will not only design the merging area, but also change the distribution of the tollbooths. This can help us maximize the total welfare.

Secondly, we use some models to solve the problem. One of the innovation point is that we use a two-stage model to solve the problem, which can make the plan better meet the various needs. In the first stage, we make a comprehensive use of the tollbooth cost model, the car flow model and the danger coefficient model to solve for the best distribution of the tollbooths. In the second stage, we use the improved model to solve for the optimal scale of the merging area under the new distribution of the tollbooths.

You may wonder why we spend so much time on changing the distribution of the tollbooths. This is because, we try to reduce the length of road in the merging area to reduce the cost of land and road. If we locate the tollbooths as an arc instead of a row, we can lower the risk of traffic accident as well as reducing the length of roads.

Then, we use the Cellular Automata to mimic the practical car flow condition in New Jersey. The result shows that the new distribution of tollbooth zone do a better job in lower the risk of traffic accidents. Though the throughput is also lower, the difference is so little that can be ignored. So please trust me that changing the distribution of tollbooths in this way has a lot of advantage.

In conclusion, our design of the distribution of tollbooths and the merging area will be helpful for you to reduce the total cost and increase the safety degree of the merging area. If you accept our design, New Jersey will be the first place in the world that has tollbooths located as arc.We sincerely hope that you can accept our suggestions, and we believe that our design can help you build a better toll plaza.

Best wishes,

♯60083

Appendices

Appendix A

New Jersey Department of Transportation
Crash Rates On State and Interstate Routes (2003 - 2015)

ROUTE	2003 RATE	2004 RATE	2005 RATE	2006 RATE	2007 RATE	2008 RATE	2009 RATE	2010 RATE	2011 RATE	2012 RATE	2013 RATE	2014 RATE	2015 RATE
1	3.66	4.35	4.04	3.86	3.67	3.60	3.53	3.66	3.84	4.00	3.97	3.81	3.97
1B	4.46	3.66	2.91	3.51	2.64	2.53	1.60	3.14	4.21	4.90	5.84	4.75	3.77
1T	4.91	4.81	3.64	0.68	0.38	1.12	1.14	2.18	3.66	3.39	3.19	4.49	3.99
3	2.38	2.66	2.49	2.24	2.70	2.62	2.68	2.52	3.17	3.19	2.67	2.51	2.33
4	3.41	3.74	3.34	3.22	3.19	3.18	3.15	3.39	3.24	3.64	3.46	3.62	3.61
5	6.69	7.93	6.34	4.95	5.00	4.82	6.11	5.06	4.95	5.18	5.97	8.56	10.14
7	5.96	9.17	8.11	7.27	7.29	7.99	7.66	7.54	9.53	9.14	7.75	8.46	9.08
9	3.68	3.85	3.66	3.66	3.64	3.64	3.99	3.85	3.71	3.87	3.62	3.63	3.74
9W	3.85	3.87	3.96	3.69	4.15	4.61	3.58	3.66	4.11	3.50	3.71	4.21	4.05
10	3.36	3.52	3.34	3.09	3.41	3.12	3.33	3.51	3.70	3.28	3.38	3.79	3.60
12	2.81	3.35	3.37	2.18	3.13	2.62	2.68	2.59	2.51	2.48	2.42	2.77	3.39
13	0.65	0.97	1.15	1.15	1.92	2.30	2.31	1.15	1.15	0.38	0.38	0.77	0.77
15	2.16	2.13	1.91	1.88	2.24	2.23	1.96	2.03	1.99	2.37	2.53	2.55	2.37
17	2.29	2.40	2.17	2.22	2.44	2.12	2.37	2.39	2.23	2.09	2.02	2.12	2.25
18	2.39	2.87	2.69	2.57	2.53	2.48	2.23	1.91	2.04	2.13	2.06	2.11	2.30
19	1.68	1.96	2.02	1.58	2.18	2.02	1.89	1.92	1.37	1.59	1.67	1.82	1.95
20	3.72	5.33	4.38	4.62	4.59	4.41	3.93	3.55	4.03	3.41	3.55	3.01	3.32
21	3.70	4.79	3.69	2.66	2.09	3.94	3.54	3.79	3.42	3.19	3.21	2.88	3.11
22	3.17	3.44	3.42	3.42	3.30	3.12	3.20	3.44	2.97	3.50	3.49	3.42	3.50
23	2.66	2.97	2.84	2.95	2.58	2.82	2.85	2.75	2.80	2.58	2.66	2.41	2.94
24	1.60	1.85	1.94	1.73	2.12	1.81	2.18	1.86	2.27	1.66	1.04	1.67	1.68
26	5.06	6.66	5.59	3.90	5.05	5.49	7.28	5.00	4.28	4.96	3.83	5.01	4.96
27	7.43	8.15	7.45	7.19	7.44	7.50	6.59	6.27	7.33	7.86	7.39	7.88	8.26
27Z	N/A	N/A	N/A	N/A	N/A	5.05	6.75	3.80	6.75	6.33	3.80	5.06	4.22
28	7.88	9.35	7.25	7.17	7.28	7.52	7.34	7.30	7.65	8.28	7.87	7.54	7.45
29	2.47	2.70	2.14	2.12	2.11	1.99	1.96	1.74	1.88	1.64	1.79	2.23	2.43
30	3.33	3.54	3.61	3.53	3.59	3.39	3.58	3.30	2.92	3.46	3.51	2.98	3.24
31	3.00	3.45	3.34	3.10	3.25	3.27	3.27	3.07	3.02	3.03	2.71	3.25	3.40

NOTE: Year 2015 does not include Private Property crashes with the exceptional of Fatal Crashes

Updated: 01/04/2017

Figure 13

ROUTE	2003 RATE	2004 RATE	2005 RATE	2006 RATE	2007 RATE	2008 RATE	2009 RATE	2010 RATE	2011 RATE	2012 RATE	2013 RATE	2014 RATE	2015 RATE
32	4.87	5.85	5.48	4.48	3.84	3.74	3.75	2.92	3.29	3.11	3.02	1.74	3.29
33	3.42	4.15	3.94	3.67	3.46	3.66	4.32	3.95	3.89	4.10	3.99	3.80	4.54
33B	2.94	2.24	3.95	2.44	2.65	2.51	2.28	2.49	2.83	3.32	3.77	3.01	3.70
34	2.10	2.40	2.45	2.34	2.39	2.31	2.62	2.58	2.52	2.89	3.05	2.79	3.15
35	5.06	5.52	5.52	5.23	5.54	5.25	5.38	5.37	5.22	4.94	4.64	4.69	4.92
36	3.22	4.19	4.45	3.98	3.95	4.14	3.94	4.18	3.83	3.53	3.04	2.99	3.33
37	3.74	3.64	4.05	4.00	3.85	3.61	3.86	3.89	4.53	4.59	4.14	4.24	4.56
38	2.65	3.09	3.05	3.06	3.11	3.18	3.46	3.23	3.37	3.64	3.62	3.70	3.89
40	2.93	3.32	3.03	3.05	3.16	2.71	3.07	2.72	2.45	2.63	2.14	2.20	2.48
41	4.13	3.83	3.20	3.04	3.20	3.42	3.71	4.17	3.68	4.20	3.72	2.94	6.49
42	2.04	2.35	2.53	2.42	2.55	2.41	2.80	2.29	2.18	2.30	2.07	2.18	2.46
44	2.03	2.39	2.92	2.79	3.01	3.01	3.28	2.52	3.01	3.64	3.83	2.54	2.88
45	4.21	4.93	4.16	4.26	4.45	4.47	4.26	4.89	4.87	5.10	4.12	3.46	2.81
46	3.62	4.03	3.91	3.77	3.66	3.49	3.48	3.46	3.42	3.59	3.51	3.46	3.63
47	3.41	4.01	3.70	3.68	3.46	3.66	3.95	3.50	3.40	3.46	3.30	3.81	4.33
48	2.72	2.98	1.46	2.65	1.06	2.12	3.05	2.79	2.79	2.67	1.60	2.14	2.94
49	3.09	3.59	3.10	2.84	2.87	2.78	2.88	2.59	2.53	1.86	2.15	2.52	2.75
50	2.84	2.78	2.93	2.75	3.23	2.76	2.81	2.47	2.59	2.15	2.03	1.73	2.12
52	0.60	2.11	1.87	1.64	1.60	2.35	1.29	1.29	1.60	0.92	0.58	0.79	0.67
53	4.17	6.02	5.66	4.59	4.72	5.05	4.25	4.55	4.47	4.77	4.60	3.93	3.60
54	3.49	4.01	3.06	3.04	2.35	1.96	2.31	2.58	2.47	2.16	2.51	2.89	2.55
55	0.92	1.00	0.96	0.87	0.88	0.90	0.95	0.96	0.97	0.94	0.76	1.08	1.12
56	1.50	1.74	2.17	2.99	2.31	2.92	2.14	3.10	2.93	3.14	4.21	3.96	3.57
57	2.00	2.73	2.36	2.74	2.58	2.85	2.93	2.97	2.92	2.72	2.81	2.76	2.92
59	1.69	0.00	0.00	3.19	3.19	0.00	3.19	3.19	7.97	2.39	4.78	7.97	19.14
63	9.89	11.23	9.26	9.34	8.75	6.73	8.80	8.92	9.76	10.81	9.49	10.06	10.63
64	1.68	5.43	6.71	5.45	5.87	7.52	3.77	7.13	2.51	1.26	3.77	3.35	4.19
66	1.58	2.90	5.45	5.28	5.21	4.55	5.34	4.74	4.49	6.11	5.30	4.89	3.54

NOTE: Year 2015 does not include Private Property crashes with the exceptional of Fatal Crashes **Updated: 01/04/2017**

Figure 14

ROUTE	2003 RATE	2004 RATE	2005 RATE	2006 RATE	2007 RATE	2008 RATE	2009 RATE	2010 RATE	2011 RATE	2012 RATE	2013 RATE	2014 RATE	2015 RATE
67	5.58	9.90	10.45	10.51	11.82	10.76	9.62	8.38	11.00	11.13	11.41	10.31	9.07
68	1.38	2.42	2.20	1.85	1.97	1.93	2.01	2.48	1.85	1.97	2.62	2.22	2.72
70	3.57	3.88	3.36	3.60	3.78	3.70	3.48	3.48	3.54	3.43	3.39	3.13	3.47
71	4.87	5.28	6.37	4.74	5.78	4.91	4.67	5.22	5.43	5.31	4.95	4.92	6.09
72	3.08	2.82	2.22	1.30	2.23	2.00	2.35	2.55	2.25	2.82	2.52	2.51	2.45
73	2.64	2.76	2.57	2.39	2.65	2.62	2.56	2.49	2.65	2.52	2.67	2.53	2.87
76	4.25	4.77	4.30	4.93	5.16	4.34	4.50	4.06	3.82	5.31	4.08	5.53	5.40
77	3.91	4.88	4.07	3.77	3.61	4.11	4.23	4.23	4.38	3.41	4.06	3.63	3.92
78	1.50	1.59	1.44	1.39	1.59	1.27	1.46	1.32	1.27	1.35	0.88	1.36	1.47
79	4.70	4.98	3.85	3.77	4.09	4.19	4.19	4.24	4.12	3.86	4.28	4.74	4.41
80	1.69	1.68	1.76	1.62	1.77	1.67	1.75	1.61	1.80	1.58	1.14	1.74	1.94
81	0.23	0.22	0.00	0.11	0.44	0.33	0.44	0.11	0.44	1.40	1.30	2.48	1.51
82	8.66	9.35	11.78	12.24	10.83	9.91	9.73	10.09	10.05	10.65	10.87	10.29	9.35
83	1.17	2.43	2.61	1.79	1.63	1.46	1.63	2.12	2.12	1.47	1.47	1.14	2.12
87	2.61	3.34	2.42	3.16	2.61	2.88	1.29	1.66	2.57	1.04	1.04	1.56	0.87
88	7.60	8.44	6.00	6.51	6.17	6.02	6.03	5.46	5.14	5.63	5.79	5.35	5.78
90	0.29	1.33	1.86	0.80	1.33	1.41	1.95	1.51	1.77	0.54	1.27	1.45	1.82
91	2.64	2.90	3.09	3.18	4.36	3.63	2.18	3.09	2.18	1.98	2.15	3.19	2.58
93	6.41	6.73	8.15	6.85	6.39	4.97	5.80	6.61	6.53	7.21	7.47	8.00	8.23
94	3.09	3.29	3.85	3.33	3.27	3.42	2.89	3.20	3.38	2.70	2.27	2.78	3.09
95M	N/A	N/A	N/A	1.07	1.32	1.22	1.45	1.41	1.37	1.51	0.94	1.59	1.57
109	3.35	2.78	3.04	9.41	6.95	9.99	6.54	7.36	5.52	3.39	3.99	3.19	4.09
120	2.46	6.41	2.84	3.15	3.60	3.00	2.93	2.12	1.90	1.72	1.62	1.47	1.72
122	2.53	11.80	9.40	3.14	4.93	3.73	4.04	4.04	4.34	3.40	4.77	2.89	4.60
124	9.59	4.28	5.41	9.61	9.84	8.88	9.04	8.33	9.54	9.64	10.28	10.27	9.67
129	4.45	3.36	3.15	4.98	6.13	4.68	3.94	4.15	5.92	3.24	4.56	4.95	6.43
130	3.14	1.38	1.75	2.99	2.98	2.71	2.87	2.91	3.08	3.46	3.39	3.41	3.42
133	1.01	3.08	2.48	1.66	1.57	1.75	1.38	1.20	1.94	1.14	1.30	0.83	1.33

NOTE: Year 2015 does not include Private Property crashes with the exceptional of Fatal Crashes **Updated: 01/04/2017**

Figure 15

ROUTE	2003 RATE	2004 RATE	2005 RATE	2006 RATE	2007 RATE	2008 RATE	2009 RATE	2010 RATE	2011 RATE	2012 RATE	2013 RATE	2014 RATE	2015 RATE
138	2.45	6.89	7.59	2.95	2.55	3.05	2.86	2.75	2.75	3.63	3.76	2.97	3.34
139	3.35	15.75	9.78	7.97	5.30	6.00	7.22	6.17	4.65	3.53	3.35	4.31	5.46
139U	17.30	9.50	10.81	0.30	6.62	4.20	5.11	5.57	14.89	14.59	17.60	8.88	3.01
140	12.86	2.02	4.03	10.81	11.24	8.19	8.22	9.51	6.92	5.25	7.44	11.82	6.13
143	1.01	1.23	0.73	2.04	2.01	4.02	2.01	7.05	4.03	6.04	5.03	3.02	2.01
147	0.97	0.86	0.75	1.10	0.78	1.05	0.50	0.82	0.60	1.16	1.33	1.16	1.94
152	0.32	0.86	0.75	0.89	0.75	0.75	0.96	0.75	1.02	1.04	1.17	1.10	1.38
154	5.74	6.02	5.54	5.64	5.54	4.34	4.45	5.93	5.74	5.02	6.47	5.98	7.24
156	28.22	17.42	19.36	19.73	15.97	12.18	12.21	13.15	17.85	9.39	8.45	14.09	9.39
157	8.53	3.55	4.29	3.86	2.79	4.06	3.00	4.08	3.65	5.15	3.43	4.93	3.00
159	0.33	0.55	1.00	1.77	2.99	2.32	1.88	2.99	2.21	1.66	3.32	2.55	4.98
161	2.24	2.39	4.69	3.65	4.35	3.81	5.21	4.52	6.78	6.95	5.04	3.13	3.65
162	N/A	N/A	0.74	0.00	0.00	0.00	2.31	3.08	2.31	3.08	2.31	0.00	0.00
163	N/A	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
165	9.46	18.88	5.65	4.85	10.50	6.44	4.04	9.59	10.50	6.46	12.12	7.27	11.31
166	8.36	7.71	7.89	7.41	5.98	7.14	6.81	7.13	6.50	6.13	7.64	7.01	6.29
167	N/A	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
168	7.39	8.17	8.16	8.02	8.45	8.65	7.79	7.03	7.21	5.59	5.76	5.51	4.28
169	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
171	11.37	11.57	6.33	3.77	7.39	9.02	8.90	7.99	7.24	6.79	6.94	7.39	8.14
172	1.39	1.16	1.86	1.63	9.06	6.95	3.95	2.73	2.18	1.64	1.09	1.91	1.64
173	5.49	4.81	3.19	3.16	3.19	3.61	3.38	1.77	3.25	3.55	2.66	2.64	2.64
175	8.92	2.74	2.90	1.74	3.48	5.78	4.74	4.21	1.58	2.17	1.63	4.17	3.57
179	1.89	4.03	1.19	1.64	1.94	1.49	1.50	2.43	2.38	1.04	0.82	2.38	1.79
181	3.81	3.82	3.07	2.87	3.03	3.33	3.34	2.83	3.18	3.89	3.79	3.74	3.84
182	6.69	9.68	9.87	9.87	9.40	9.26	9.52	7.08	9.75	9.29	8.01	7.43	7.89
183	3.68	4.23	3.70	3.88	3.98	3.04	1.85	2.50	2.87	1.40	2.45	1.93	3.33
184	9.36	11.79	9.60	8.00	8.58	7.18	7.42	6.04	4.58	5.79	6.68	8.38	7.34

NOTE: Year 2015 does not include Private Property crashes with the exceptional of Fatal Crashes **Updated: 01/04/2017**

Figure 16

ROUTE	2003 RATE	2004 RATE	2005 RATE	2006 RATE	2007 RATE	2008 RATE	2009 RATE	2010 RATE	2011 RATE	2012 RATE	2013 RATE	2014 RATE	2015 RATE
185	0.39	5.88	2.75	2.36	4.33	4.31	2.75	2.75	7.47	5.51	3.15	5.51	5.11
187	2.34	16.35	1.75	15.77	15.77	8.73	N/A	N/A	N/A	N/A	N/A	N/A	N/A
195	1.00	0.93	1.14	0.97	1.19	1.00	1.16	1.21	1.05	0.87	0.66	1.12	1.30
202	2.72	2.90	2.77	2.99	3.09	2.97	2.89	3.10	3.29	3.09	3.01	2.91	2.97
206	3.49	3.51	3.47	3.29	3.33	3.39	3.52	3.61	3.54	3.24	3.01	2.81	3.25
208	1.09	1.21	1.31	1.01	1.23	1.14	1.23	1.27	1.40	1.35	1.42	1.39	1.32
278	0.64	2.72	1.05	1.31	2.10	2.48	3.15	2.36	2.75	2.49	2.36	3.80	4.98
280	3.13	2.78	3.01	3.03	3.10	2.75	2.86	2.67	2.81	2.95	2.13	3.38	3.87
284	3.17	2.59	2.60	2.73	2.21	2.60	3.38	2.47	3.25	2.07	1.38	2.98	2.30
287	1.47	1.40	1.41	1.36	1.45	1.34	1.24	1.30	1.39	1.40	1.04	1.64	1.73
295	1.46	1.60	1.22	1.23	1.29	1.14	1.47	1.55	1.32	1.31	0.95	1.44	1.59
322	2.17	2.04	2.42	2.10	2.48	2.38	2.31	2.01	1.91	2.05	2.03	1.92	2.24
324	N/A	0.00	3.63	3.63	0.00	0.00	10.89	0.00	0.00	0.00	0.00	0.00	0.00
439	9.83	12.44	11.29	9.78	10.11	9.72	8.48	9.46	9.26	10.03	9.49	9.06	7.39
440	2.80	2.11	2.45	2.41	2.54	2.23	2.12	2.42	2.74	1.88	1.69	2.22	2.36
495	4.31	5.60	3.22	2.92	3.46	3.80	3.83	3.07	2.34	2.75	1.81	1.87	2.25
523	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	0.00	0.00
524	N/A	2.25	0.75	2.24	2.62	1.12	0.64	0.32	0.64	0.36	0.00	2.87	3.59
676	1.72	2.09	2.36	2.45	2.75	2.80	2.40	2.34	2.28	2.61	2.22	2.33	2.78

NOTE: Year 2015 does not include Private Property crashes with the exceptional of Fatal Crashes **Updated: 01/04/2017**

Figure 17

Appendix B