

- DFT: $F[m] = \sum_{n=0}^{N-1} f[n] e^{-j\frac{2\pi}{N}mn}$
 - When $f[n]$ is real, $F[m] = F^*[N - m]$
- We are applying DFT to two real sequences $f_1[n]$ and $f_2[n]$
 - Step 1: $f_3[n] = f_1[n] + jf_2[n]$
 - Step 2: $F_3[m] = \text{DFT}\{f_3[n]\}$
 - Step 3: $F_1[m] = \frac{F_3[m] + F_3^*[N-m]}{2}$, $F_2[m] = \frac{F_3[m] - F_3^*[N-m]}{2j}$

proof

Since DFT is a linear operation

$$F_3[m] = F_1[m] + jF_2[m]$$

$$\text{and } F_1[m] = F_1^*[N - m] \quad F_2[m] = F_2^*[N - m]$$

$$F_3[m] + F_3^*[N - m] = F_1[m] + jF_2[m] + F_1^*[N - m] - jF_2^*[N - m]$$

$$= 2F_1[m]$$

$$F_3[m] - F_3^*[N - m] = F_1[m] + jF_2[m] - F_1^*[N - m] + jF_2^*[N - m]$$

$$= 2jF_2[m]$$