- minimize $$f(x) = c^T x - \sum_{i=1}^{m} \log\left(-a_i^T x + b_i\right)$$

where $A \in \mathbf{R}^{m \times n}$ with a_i being the transpose of the ith row of A,

$b \in \mathbf{R}^m,\ c \in \mathbf{R}^n$.

- Here we set

$$A = \begin{bmatrix} 1 & 3 \\ 1 & -3 \\ -1 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}, \quad c = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Initial point $x^{(0)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

- Gradient $\nabla f(x) = \begin{bmatrix} \frac{\partial f(x)}{\partial x_1} \\ \frac{\partial f(x)}{\partial x_2} \end{bmatrix}$, and Hessian $\nabla^2 f(x) = \begin{bmatrix} \frac{\partial^2 f(x)}{\partial x_1^2} & \frac{\partial^2 f(x)}{\partial x_1 x_2} \\ \frac{\partial^2 f(x)}{\partial x_2 x_1} & \frac{\partial^2 f(x)}{\partial x_2^2} \end{bmatrix}$

- Newton step $\Delta x_{nt} = -(\nabla^2 f(x))^{-1} \nabla f(x)$ → $\Delta x_{nt}^{(0)}$

- Newton decrement $\lambda(x) = (-\nabla f(x)^T \Delta x_{nt})^{1/2}$ → $\lambda^{(0)}(x)$

- Backtracking line search along search direction using $\beta = 0.7$ starting from $t = 1$ until $x^{(0)} + t\Delta x_{nt}^{(0)} \in \mathbf{dom}\, f$ (Note: $t^+ := \beta t$)

- Continue the backtracking line search until $f(x + t\Delta x_{nt}^{(0)}) \leq \alpha t \lambda(x)^2$, where $\alpha = 0.1$

- Update $x^{(1)} = x^{(0)} + t\Delta x_{nt}^{(0)}$

- If $\lambda(x)^2 / 2 < \text{stopping criterion}$, then break the loop.