Materials Thickness

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$$R_0(GE180) = 7.040 \text{ cm} \tag{1}$$

$$\text{target length} = \frac{\text{target thickness}}{R_0} \tag{2}$$

For Polarized ^{3}He Target (Cell Dutch):

$$L_{window}(\text{upstream}) = 0.01435 \text{ cm} \tag{3}$$

$$L_{window}(\text{downstream}) = 0.01341 \text{ cm} \tag{4}$$

$$rl_{wall} = \frac{0.15}{R_0(GE180)} \approx 0.021307 \text{ cm} \tag{5}$$

$$rl_{target}(\text{upstream}) = \frac{L_{window}(\text{upstream})}{R_0(GE180)} \approx 0.002038 \text{ cm} \tag{6}$$

$$rl_{target}(\text{downstream}) = \frac{L_{window}(\text{downstream})}{R_0(GE180)} \approx 0.001905 \text{ cm} \tag{7}$$

$$\begin{aligned} rl_{pre\,target}(\text{upstream}) &= 10 \text{ mil(Be window)} + 10 \text{ cm(1 atm } N_2 \text{ gas)} \\ &= \frac{0.0254}{19.091} + \frac{10}{32742.54} \approx 0.001636 \text{ cm} \end{aligned} \tag{8}$$

$$\begin{aligned} rl_{pre\,target}(\text{downstream}) &= 10 \text{ mil(Be window)} + 10 \text{ cm(1 atm } N_2 \text{ gas)} + \text{upstream window} + {}^3\text{He in TC} \\ &= \frac{0.0254}{19.091} + \frac{10}{32742.54} + 1.014 * \frac{0.01435}{7.040} + \frac{40.0}{44947} \approx 0.004593 \text{ cm} \end{aligned}$$

$$\begin{aligned} rl_{post\,target}(\text{HMS}) &= 44 \text{ cm(air)} + 0.2 \text{ cm(nylon)} + 24.61 \text{ cm(air)} + 15 \text{ mil(Kevlar)} + 5 \text{ mil(mylar)} \\ &= \frac{44}{30420} + \frac{0.2}{35} + \frac{24.61}{30420} + \frac{0.015 * 2.54}{74.6} + \frac{0.005 * 2.54}{28.7} \approx 0.008923 \text{ cm} \end{aligned}$$

$$\begin{aligned} rl_{post\,target}(\text{SHMS}) &= 44 \text{ cm(air)} + 0.2 \text{ cm(nylon)} + 57.27 \text{ cm(air)} + 10 \text{ mil(aluminum)} \\ &= \frac{44}{30420} + \frac{0.2}{35} + \frac{57.27}{30420} + \frac{0.01 * 2.54}{8.89} \approx 0.011900 \text{ cm} \end{aligned}$$

$$
\begin{aligned}
rl_{before} &= rl_{pre\,target}(\text{upstream}) + L_{window}(\text{upstream}) + \frac{1}{2} *^3 \text{He in TC} \\
&= 0.001636 \text{ cm} + 0.01435 \text{ cm} + \frac{20.0 \text{ cm}}{44947} \\
&= 0.01643 \text{ cm}
\end{aligned}
$$

$$
\begin{aligned}
rl_{after} &= rl_{post\,target}(\text{SHMS/HMS}) + \frac{L_{window}(\text{downstream})}{sin(\theta_{scatt})} + \frac{\frac{1}{2} *^3 \text{He in TC}}{sin(\theta_{scatt})} \\
&= 0.011900(0.008923) \text{ cm} + \frac{0.01341 \text{ cm}}{sin(\theta_{scatt})} + \frac{\frac{20.0 \text{ cm}}{44947}}{sin(\theta_{scatt})} \\
&= 0.011900(0.008923) \text{ cm} + \frac{0.013855 \text{ cm}}{sin(\theta_{scatt})}
\end{aligned}
$$

For Polarized ^{3}He Target (Cell Bigbrother):

$$L_{window}(\text{upstream}) = 0.01009 \text{ cm} \tag{9}$$

$$L_{window}(\text{downstream}) = 0.01382 \text{ cm} \tag{10}$$

$$rl_{wall} = \frac{0.15}{R_0(GE180)} \approx 0.021307 \text{ cm} \tag{11}$$

$$rl_{target}(\text{upstream}) = \frac{L_{window}(\text{upstream})}{R_0(GE180)} \approx 0.001433 \text{ cm} \tag{12}$$

$$rl_{target}(\text{downstream}) = \frac{L_{window}(\text{downstream})}{R_0(GE180)} \approx 0.001963 \text{ cm} \tag{13}$$

$$
\begin{aligned}
rl_{pre\,target}(\text{upstream}) &= 10 \text{ mil(Be window)} + 10 \text{ cm(1 atm } N_2 \text{ gas)} \\
&= \frac{0.0254}{19.091} + \frac{10}{32742.54} \approx 0.001636 \text{ cm}
\end{aligned}
$$

$$
\begin{aligned}
rl_{pre\,target}(\text{downstream}) &= 10 \text{ mil(Be window)} + 10 \text{ cm(1 atm } N_2 \text{ gas)} + \text{upstream window} +^3 \text{He in TC} \\
&= \frac{0.0254}{19.091} + \frac{10}{32742.54} + 1.014 * \frac{0.01009}{7.040} + \frac{40.0}{44947} \approx 0.003979 \text{ cm}
\end{aligned}
$$

$$
\begin{aligned}
rl_{post\,target}(\text{HMS}) &= 44 \text{ cm(air)} + 0.2 \text{ cm(nylon)} + 24.61 \text{ cm(air)} + 15 \text{ mil(Kevlar)} + 5 \text{ mil(mylar)} \\
&= \frac{44}{30420} + \frac{0.2}{35} + \frac{24.61}{30420} + \frac{0.015 * 2.54}{74.6} + \frac{0.005 * 2.54}{28.7} \approx 0.008923 \text{ cm}
\end{aligned}
$$

$$
\begin{aligned}
rl_{post\,target}(\text{SHMS}) &= 44 \text{ cm(air)} + 0.2 \text{ cm(nylon)} + 57.27 \text{ cm(air)} + 10 \text{ mil(aluminum)} \\
&= \frac{44}{30420} + \frac{0.2}{35} + \frac{57.27}{30420} + \frac{0.01 * 2.54}{8.89} \approx 0.011900 \text{ cm}
\end{aligned}
$$

$$
\begin{aligned}
rl_{before} &= rl_{pre\,target}(\text{upstream}) + L_{window}(\text{upstream}) + \frac{1}{2} *^3 \text{He in TC} \\
&= 0.001636 \text{ cm} + 0.01009 \text{ cm} + \frac{20.0 \text{ cm}}{44947} \\
&= 0.01217 \text{ cm}
\end{aligned}
$$

$$
\begin{aligned}
rl_{after} &= rl_{post\,target}(\text{SHMS/HMS}) + \frac{L_{window}(\text{downstream})}{sin(\theta_{scatt})} + \frac{\frac{1}{2} *^3 \text{He in TC}}{sin(\theta_{scatt})} \\
&= 0.011900(0.008923) \text{ cm} + \frac{0.01009 \text{ cm}}{sin(\theta_{scatt})} + \frac{\frac{20.0 \text{ cm}}{44947}}{sin(\theta_{scatt})} \\
&= 0.011900(0.008923) \text{ cm} + \frac{0.010535 \text{ cm}}{sin(\theta_{scatt})}
\end{aligned}
$$

For Polarized ^{3}He Target (Cell Tommy):

$$L_{window}(\text{upstream}) = 0.0137 \text{ cm} \tag{14}$$

$$L_{window}(\text{downstream}) = 0.0145 \text{ cm} \tag{15}$$

$$rl_{wall} = \frac{0.15}{R_0(GE180)} \approx 0.021307 \text{ cm} \tag{16}$$

$$
\begin{aligned}
rl_{pre\,target}(\text{upstream}) &= 10 \text{ mil(Be window)} + 11 \text{ cm(1 atm } N_2 \text{ gas)} \\
&= \frac{0.0254}{19.091} + \frac{11}{32742.54} \approx 0.001666 \text{ cm}
\end{aligned}
$$

$$
\begin{aligned}
rl_{pre\,target}(\text{downstream}) &= 10 \text{ mil(Be window)} + 10 \text{ cm(1 atm } N_2 \text{ gas)} + \text{upstream window} +^3 \text{He in TC} \\
&= \frac{0.0254}{19.091} + \frac{10}{32742.54} + 1.014 * \frac{0.01009}{7.040} + \frac{40.0}{44947} \approx 0.003979 \text{ cm}
\end{aligned}
$$

$$
\begin{aligned}
rl_{post\,target}(\text{HMS}) &= 44 \text{ cm}(N_2) + 0.1588\text{cm(Lexan polycarbonate panels)} + 24.61 \text{ cm(air)} \\
&\quad + 15 \text{ mil(Kevlar)} + 5 \text{ mil(mylar)} \\
&= \frac{44}{32742.54} + \frac{0.1588}{34.6} + \frac{24.61}{30420} + \frac{0.015 * 2.54}{74.6} + \frac{0.005 * 2.54}{28.7} \\
&\approx 0.007696 \text{ cm}
\end{aligned}
$$

$$
\begin{aligned}
rl_{post\,target}(\text{SHMS}) &= 44 \text{ cm}(N_2) + 0.1588\text{cm(Lexan polycarbonate panels)} + 57.27 \text{ cm(air)} \\
&\quad + 10 \text{ mil(aluminum)} \\
&= \frac{44}{32742.54} + \frac{0.1588}{34.6} + \frac{57.27}{30420} + \frac{0.01 * 2.54}{8.89} \\
&\approx 0.01067 \text{ cm}
\end{aligned}
$$

$$
\begin{aligned}
rl_{before} &= rl_{pre\,target}(\text{upstream}) + L_{window}(\text{upstream}) + \frac{1}{2} *^3 \text{He in TC} \\
&= 0.001666 \text{ cm} + 0.0137 \text{ cm} + \frac{20.0 \text{ cm}}{44947} \\
&= 0.01581 \text{ cm}
\end{aligned}
$$

$$
\begin{aligned}
rl_{after} &= rl_{post\,target}(\text{SHMS/HMS}) + \frac{L_{window}(\text{downstream})}{sin(\theta_{scatt})} + \frac{\frac{1}{2} *^3 \text{He in TC}}{sin(\theta_{scatt})} \\
&= 0.01067(0.007696) \text{ cm} + \frac{0.0145 \text{ cm}}{sin(\theta_{scatt})} + \frac{\frac{20.0 \text{ cm}}{44947}}{sin(\theta_{scatt})} \\
&= 0.01067(0.007696) \text{ cm} + \frac{0.01494 \text{ cm}}{sin(\theta_{scatt})}
\end{aligned}
$$

1 Summary:

1.1 Dutch A_1^n:

$$rl_{wall} = 0.02133337455\text{(input to Chao's code)}$$
$$rl_{wall} = \frac{0.02133337455}{sin\theta_{scatt}}$$

$$rl_{before}\text{(upstream)} = 0.003455703613$$

$$rl_{after}\text{(HMS)} = 0.008660027879$$
$$rl_{after}\text{(SHMS)} = 0.01163692438$$

1.2 Bigbrother A_1^n:

$$rl_{wall} = 0.02133208241\text{(input to Chao's code)}$$
$$rl_{wall} = \frac{0.02133208241}{sin\theta_{scatt}}$$

$$rl_{before}\text{(upstream)} = 0.002823903788$$

$$rl_{after}\text{(HMS)} = 0.008660027879$$
$$rl_{after}\text{(SHMS)} = 0.01163692438$$

1.3 Tommy d_2^n:

$$rl_{wall} = 0.02133208241\text{(input to Chao's code)}$$
$$rl_{wall} = \frac{0.02133208241}{sin\theta_{scatt}}$$

$$rl_{before}\text{(upstream)} = 0.003359559998$$

$$rl_{after}\text{(HMS)} = 0.008660027879$$
$$rl_{after}\text{(SHMS)} = 0.01163692438$$

Materials	Thickness [cm]	Density [g/cm^3]	Density [amg]	X_0 [g/cm^2]	X_0 [cm]
Be Window	2.45E-02	1.848	–	65.19	35.276
N_2 gas	10	1.17E-03	–	37.99	32470.1*
GE-180	∼0.15	2.76	–	19.4246	7.038
^{3}He Dutch	∼20	1.472E-03	10.94±0.22	71.0736[1]	–
^{3}He Big Brother	∼20	1.378E-03	10.24±0.21	71.0736[1]	–
^{3}He Tommy	∼20	1.456E-03	10.82	71.0736[1]	–
Air	44/24.61/57.27	1.205E-03	–	36.62	30390*
Nylon	0.2	1.14	–	41.84	36.70
Kevlar	3.81E-02	1.43	–	106.7	74.6
Mylar	1.27E-02	1.39	–	39.9	28.7
Aluminum	2.54E-02	2.699	–	24.01	8.896

Table 2: Summary of material densities and radiation lengths
*For gases at STP only
[1] https://people.nscl.msu.edu/ singhj/docs/radthick.pdf