



## High-order Lagrange elements in FreeFem++

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## Finite element solution of time-harmonic wave propagation problems

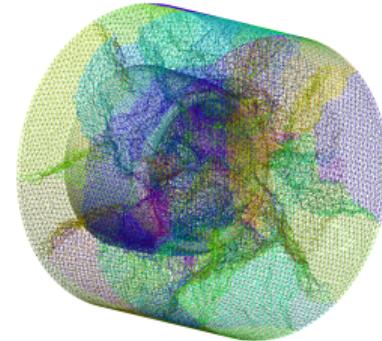
### Helmholtz problem:

$$\begin{cases} -\Delta \mathbf{p} - \kappa^2 \mathbf{p} = \mathbf{s} & \text{in } \Omega \\ \text{Boundary conditions} & \text{on } \partial\Omega \end{cases}$$

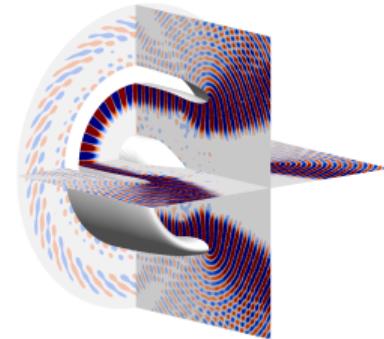
### Finite element error:

$$\|\mathbf{p} - \mathbf{p}_h\|_{H^1} \leq C_1(\kappa h)^p + C_2 \kappa^{(p+1)} h^p$$

Finite element mesh



Numerical field



High frequency (large  $\kappa$ )

Phenomena close to resonance

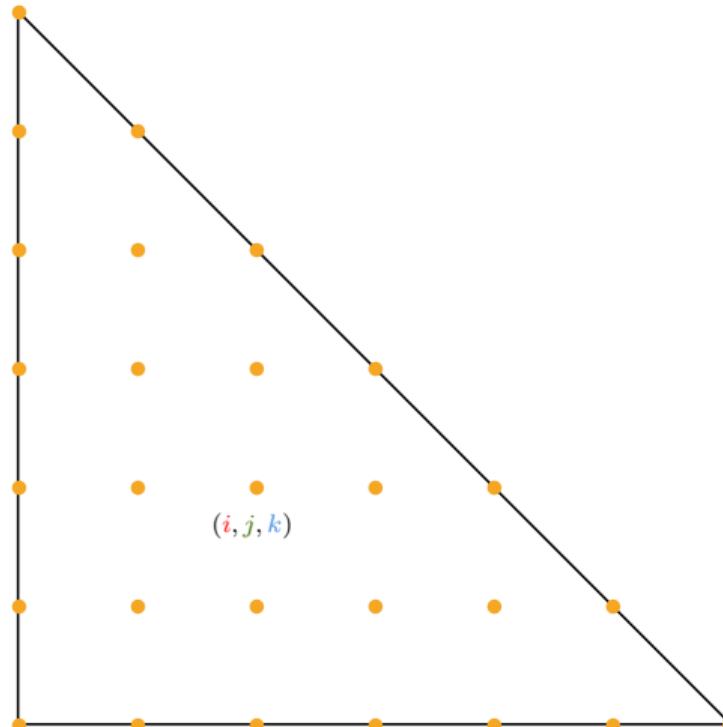
$\rightsquigarrow$

Low approximation quality

Fine mesh (small  $h$ )

High-order basis functions (large  $p$ )

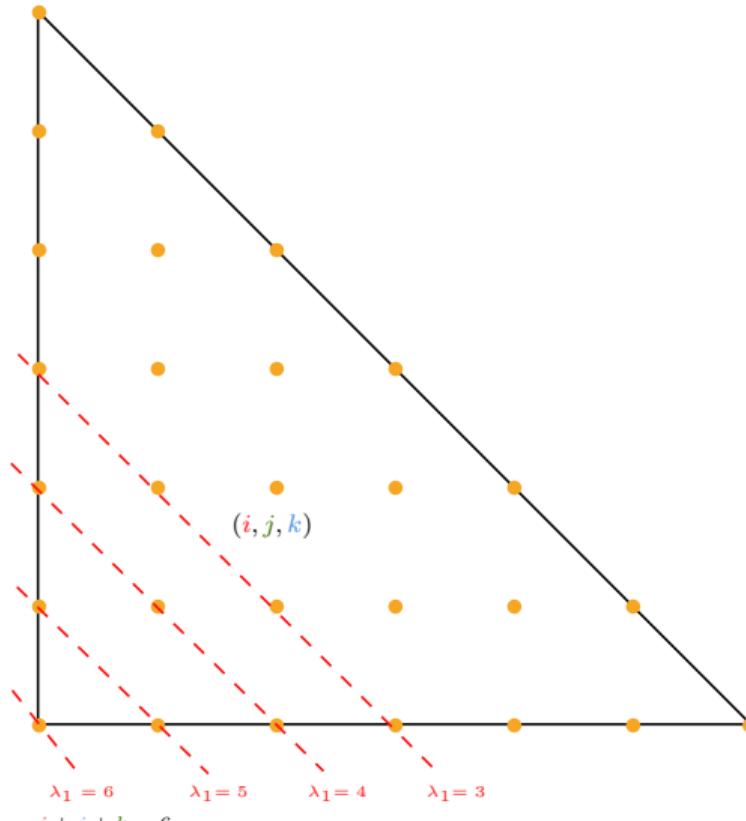
# Polynomial basis



$$i + j + k = 6$$

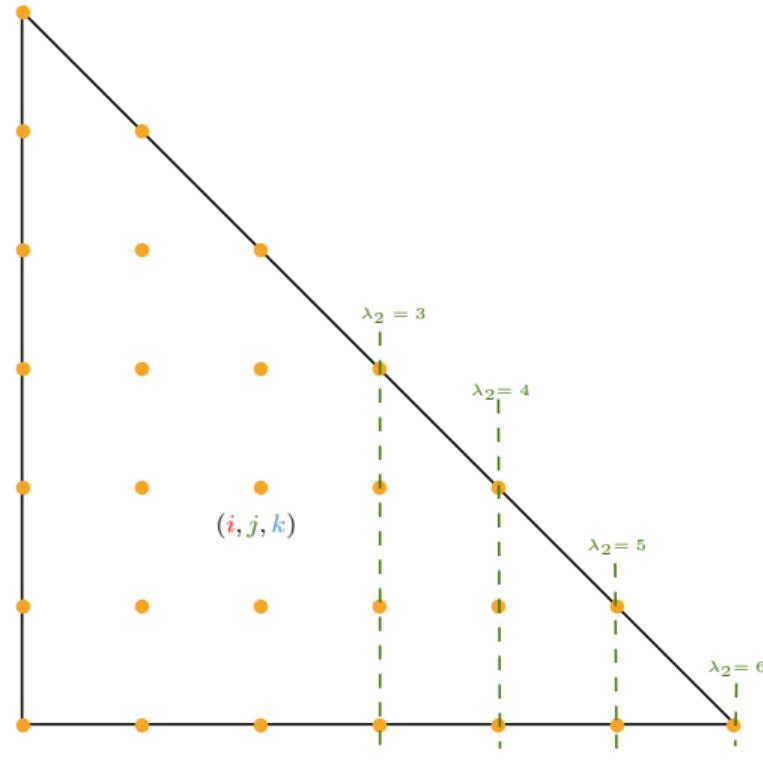
Finite element  $P_6$

# Polynomial basis



Finite element  $P_6$

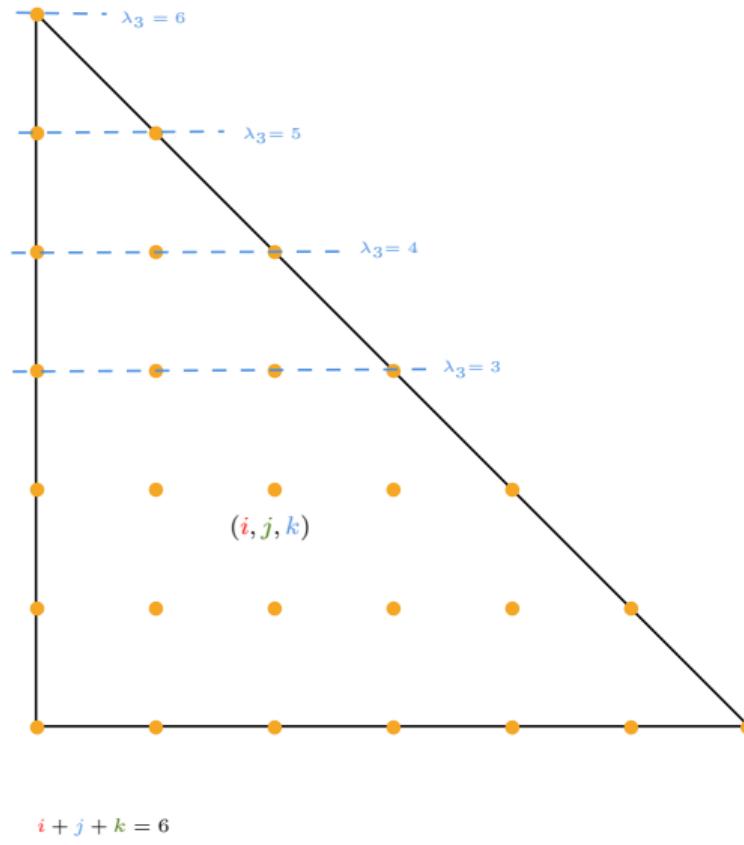
# Polynomial basis



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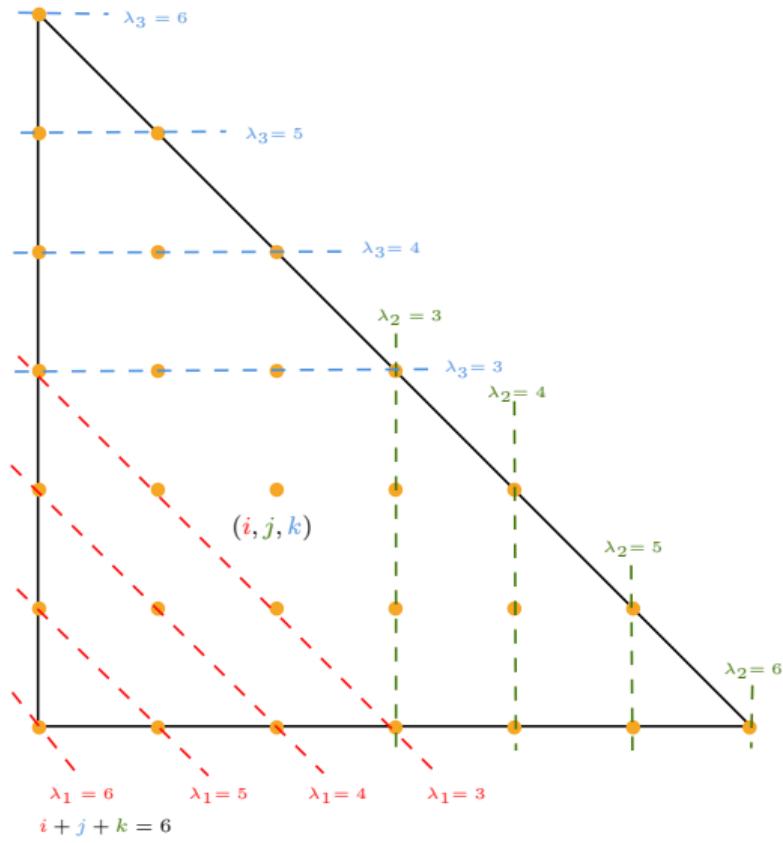
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# Polynomial basis



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Finite element  $P_6$

# Conclusion